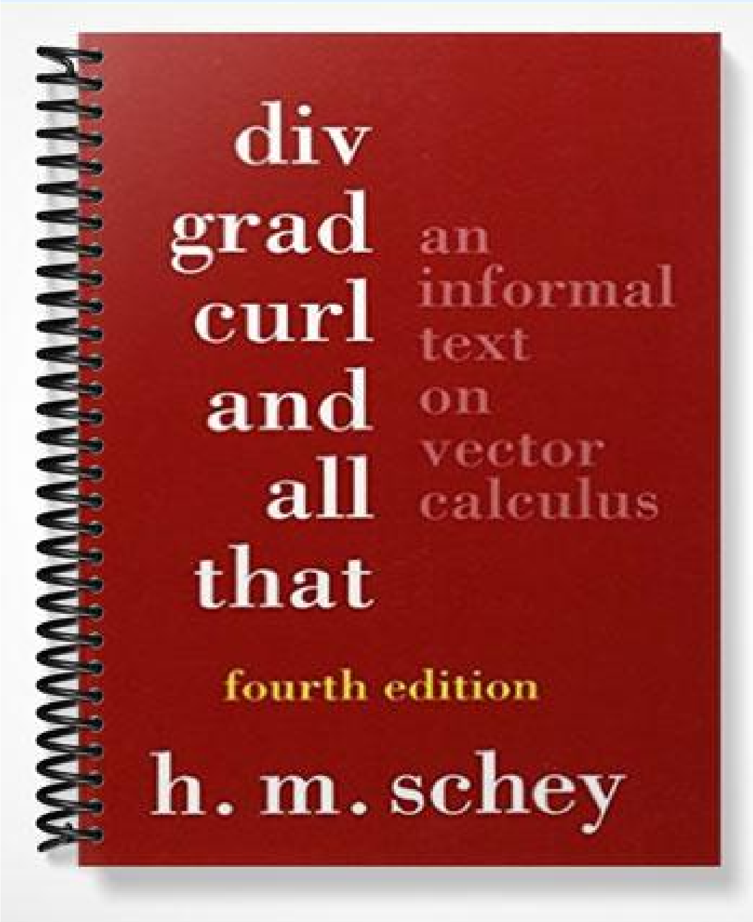
SOLUTIONS MANUAL



DIV, GRAD, CURL, AND ALL THAT

AN INFORMAL TEXT ON VECTOR ANALYSIS

FOURTH EDITION

SOLUTION MANUAL

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ROCHESTER INSTITUTE OF TECHNOLOGY
ROCHESTER, NEW YORK

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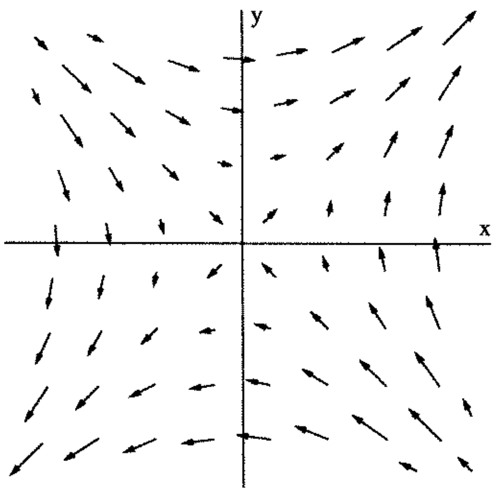
The diagrams in this manual were made by Nathan D. Clinch using Mathwriter. We are grateful to Professor Rebecca E. Hill for her generous assistance in the use of Mathwriter.

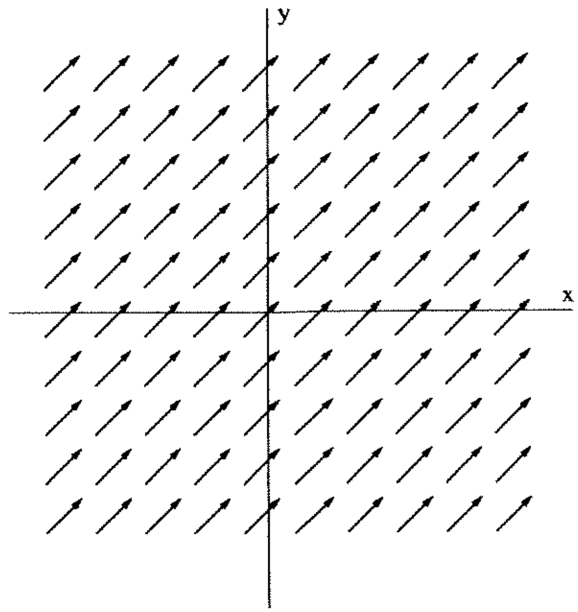
There is no harm in being sometimes wrong--especially if one is promptly found out.

--Maynard Keynes

CHAPTER I



(a)



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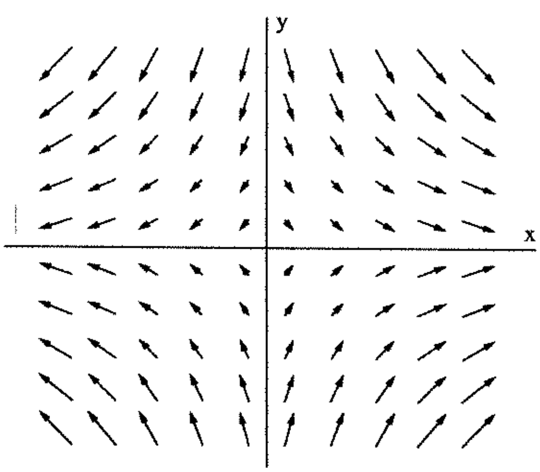

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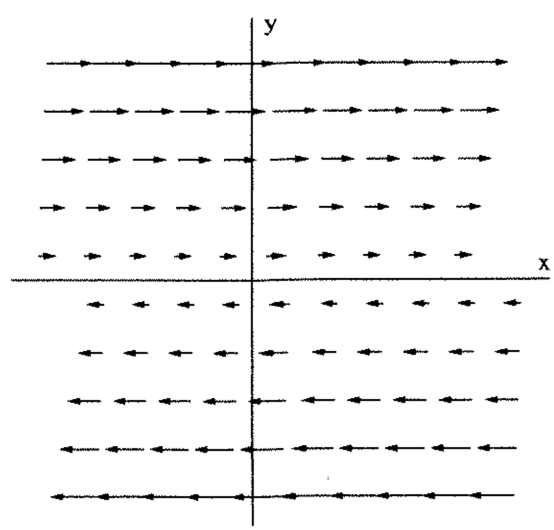

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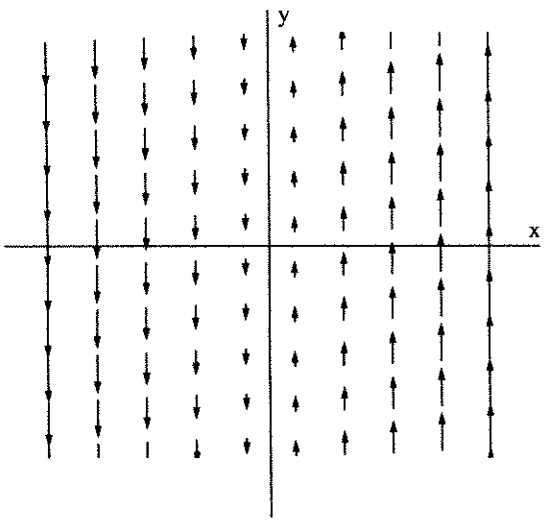

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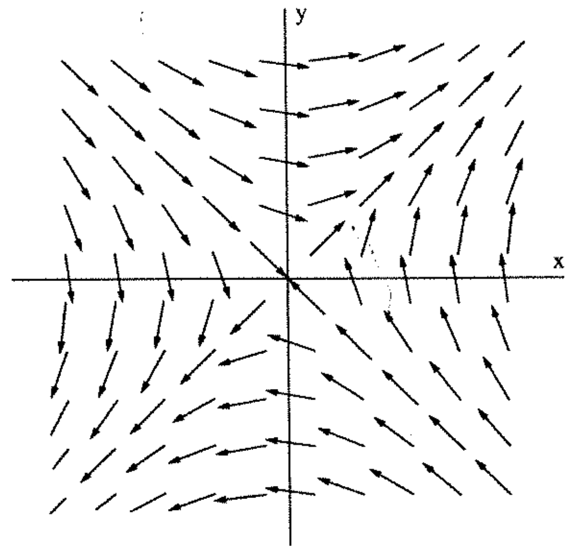

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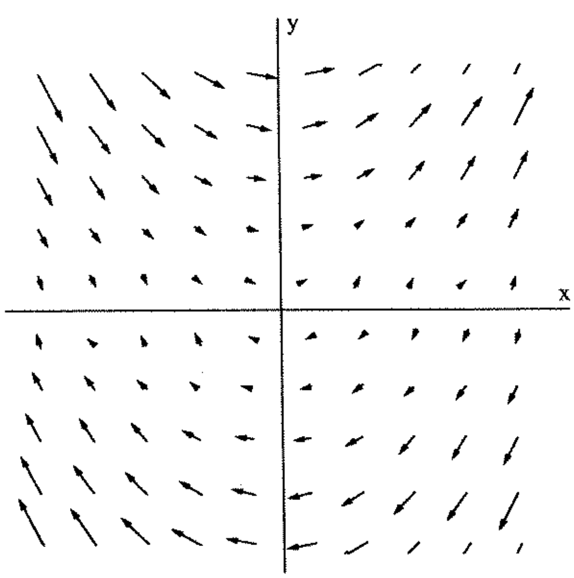
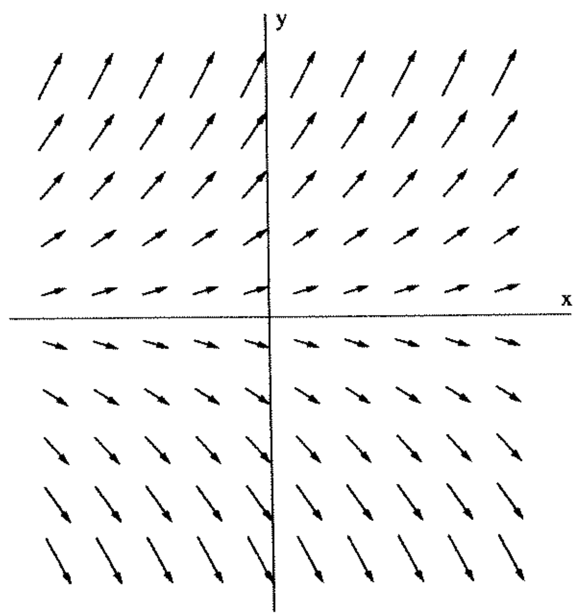
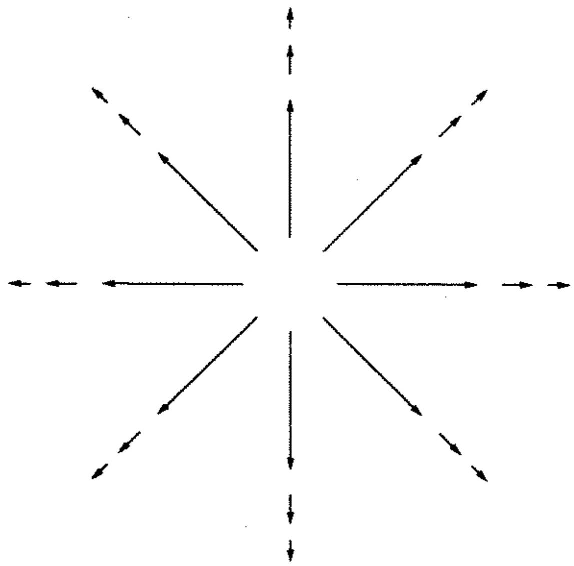


(g)



(h)

2.



3. a. $\dfrac{\mathbf{i}x + \mathbf{j}y}{\sqrt{x^2 + y^2}}$ c. $-\mathbf{i}y + \mathbf{j}x$

b. $(x^2 + y^2)\dfrac{\mathbf{i} + \mathbf{j}}{\sqrt{2}}$ d. $\dfrac{\mathbf{i}x + \mathbf{j}y + \mathbf{k}z}{\sqrt{x^2 + y^2 + z^2}}$

4. a. $|\mathbf{r}| = \sqrt{a^2\cos^2\omega t + b^2\sin^2\omega t}$

b. $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = -\mathbf{i}a\omega\sin\omega t + \mathbf{j}b\omega\cos\omega t$

c. $\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = -\mathbf{i}a\omega^2\cos\omega t - \mathbf{j}b\omega^2\sin\omega t = -\omega^2\mathbf{r}$

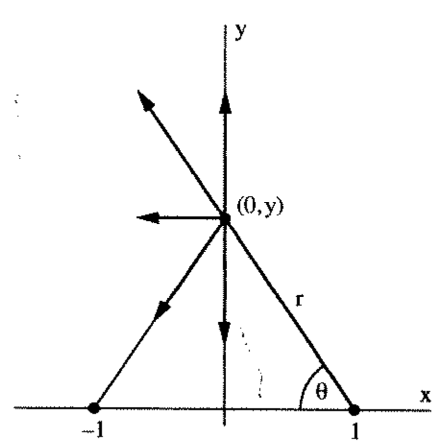
d. The x and y coordinates of the object at time t are given by $x = a\cos\omega t$ and $y = b\sin\omega t$. Hence $(x/a)^2 + (y/b)^2 = \cos^2\omega t + \sin^2\omega t = 1$.

5. It is clear from the figure that the y component of the field is 0. The x component of the field due to the charge at (1,0,0) is

$$E_x^{(1)} = -\frac{1}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}.$$

But $r = \sqrt{1 + y^2}$ and $\cos\theta = 1/r$. Hence

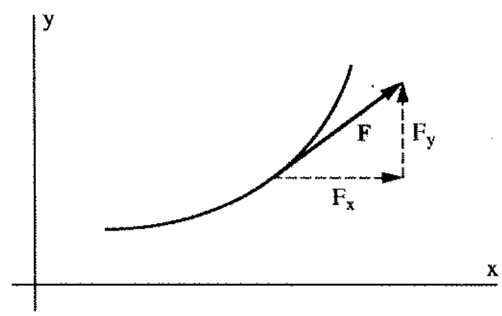
$$E_x^{(1)} = -\frac{1}{4\pi\varepsilon_0}\frac{1}{r^3} = -\frac{1}{4\pi\varepsilon_0}\frac{1}{(1 + y^2)^{3/2}}.$$



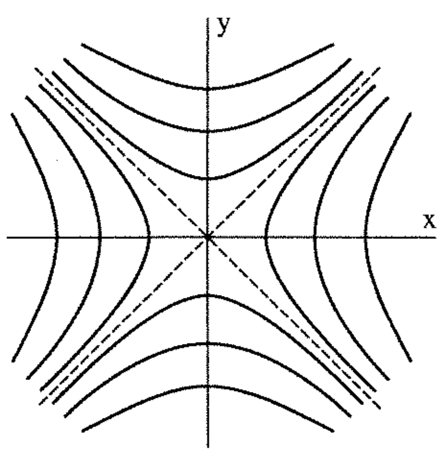
The x component of the field due to the charge at (−1,0,0), $E_x^{(-1)}$, is the same as this. Hence

$$\mathbf{E} = -\frac{1}{2\pi\varepsilon_0}\frac{\mathbf{i}}{(1 + y^2)^{3/2}}.$$

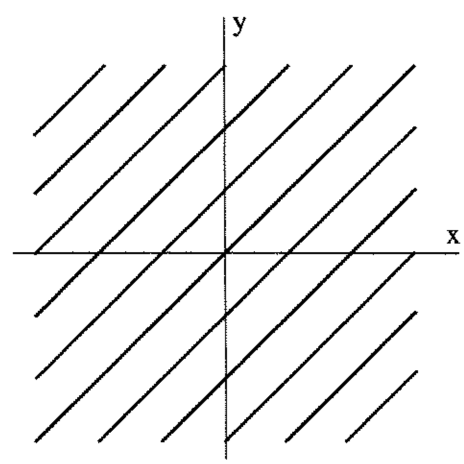
6\. a. Since the function **F** is tangent to the field line, the slope of the field line at any point is F_y/F_x (see figure). But the slope is also given by dy/dx. Hence $dy/dx = F_y/F_x$.



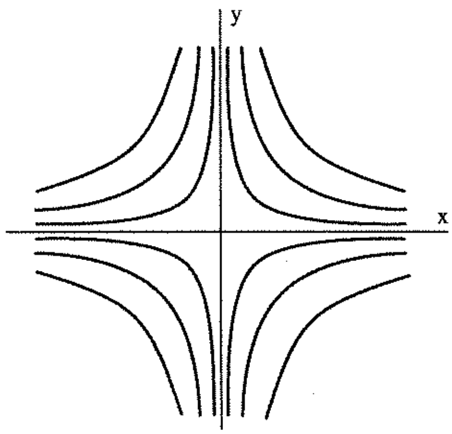
b. In the following, c represents an arbitrary constant.



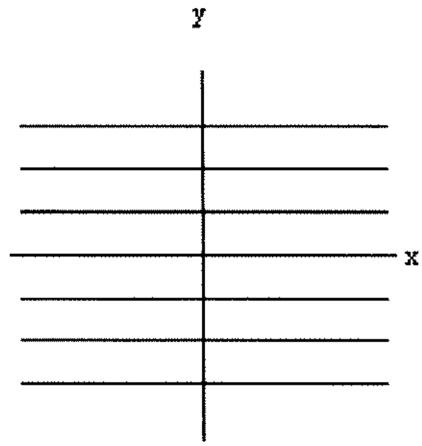
(i). $dy/dx = x/y$, $x^2 - y^2 = c$. A family of hyperbolas with asymptotes $y = \neq x$.



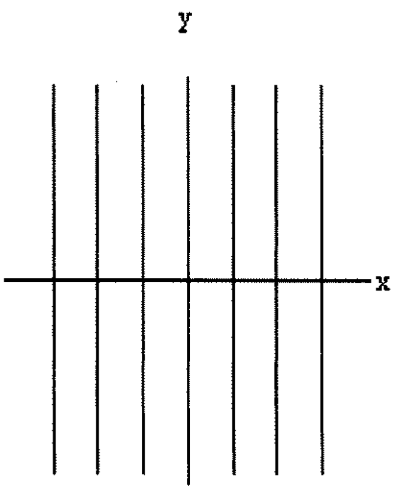
(ii). $dy/dx = 1$, $y = x + c$. A family of lines with slope 1.



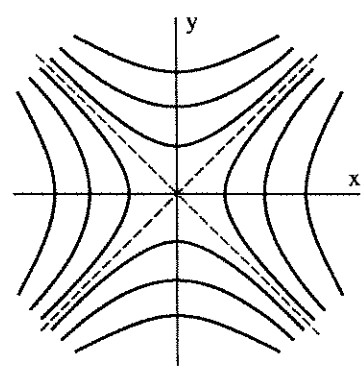
(iii). $dy/dx = -y/x$, $y = c/x$
A family of hyperbolas with asymptotes on the axes.



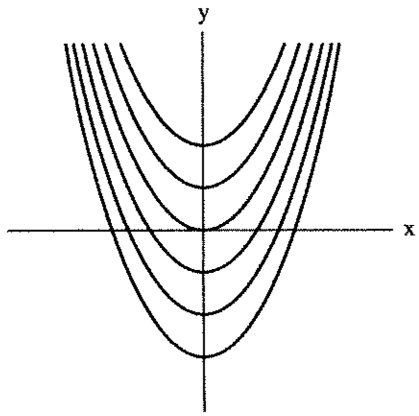
(iv). $dy/dx = 0$, $y = c$. A family of lines parallel to the x-axis.



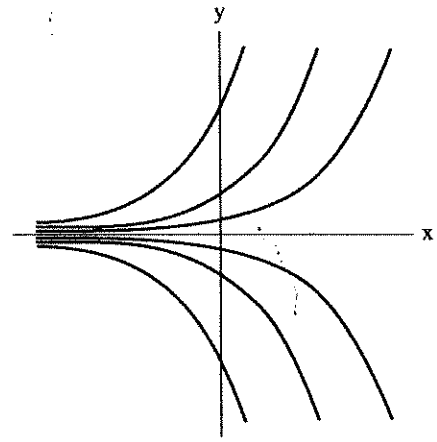
(v). dy/dx is not defined, but $dx/dy = 0$, $x = c$. A family of lines parallel to the y-axis.



(vi). $dy/dx = x/y$, $x^2 - y^2 = c$.
A family of hyperbolas with asymptotes $y = \pm x$.



(vii). $dy/dx = x$, $y = x^2/2 + c$. A family of parabolas.



(viii). $dy/dx = y$, $y = ce^x$. A family of exponentials.