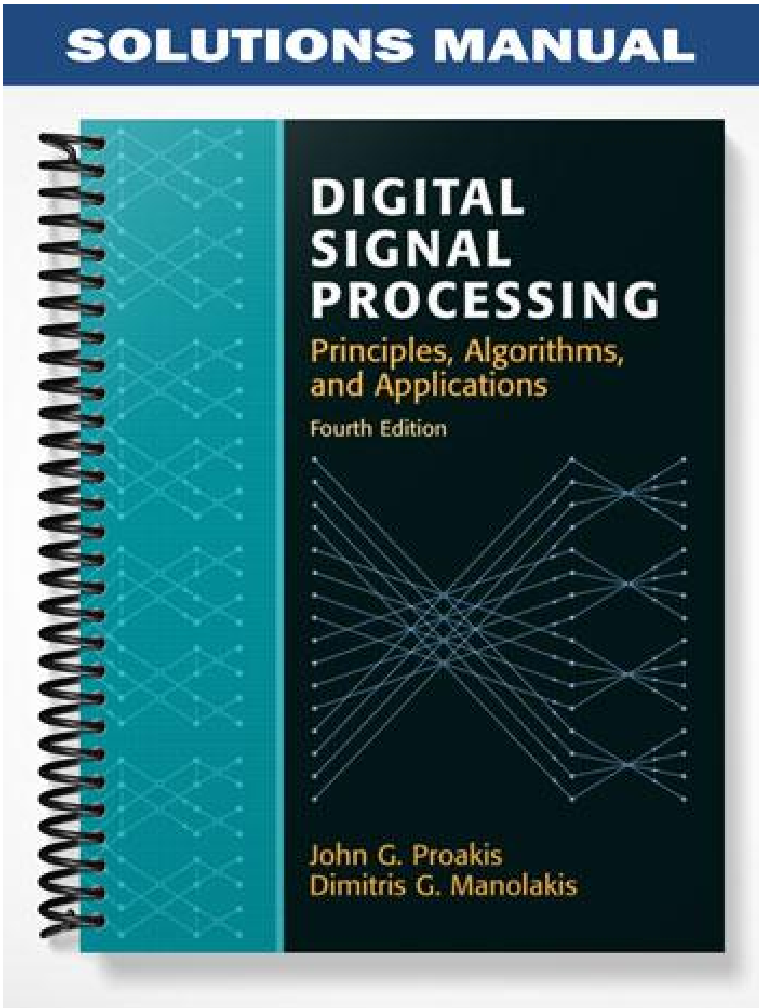
SOLUTIONS MANUAL
DIGITAL
SIGNAL
PROCESSING
Principles, Algorithms,
and Applications
Fourth Edition
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Chapter 2

2.1

(a)

$$x(n) = \left\{ \ldots 0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots \right\}$$

. Refer to fig 2.1-1.
(b) After folding s(n) we have

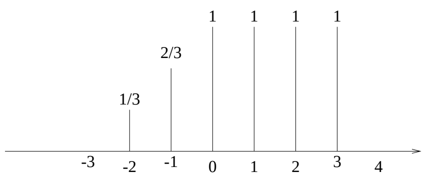

Figure 2.1-1:

$$x(-n) = \left\{ \ldots 0, 1, 1, 1, \underset{\uparrow}{1}, \frac{2}{3}, \frac{1}{3}, 0, \ldots \right\}.$$

After delaying the folded signal by 4 samples, we have

$$x(-n+4) = \left\{ \ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots \right\}.$$

On the other hand, if we delay x(n) by 4 samples we have

$$x(n-4) = \left\{ \ldots \underset{\uparrow}{0}, 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0, \ldots \right\}.$$

Now, if we fold $x(n-4)$ we have

$$x(-n-4) = \left\{ \ldots 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \underset{\uparrow}{0}, \ldots \right\}$$

(c)

$$x(-n+4) = \left\{\ldots \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots\right\}$$

(d) To obtain $x(-n+k)$, first we fold $x(n)$. This yields $x(-n)$. Then, we shift $x(-n)$ by k samples to the right if $k > 0$, or k samples to the left if $k < 0$.
(e) Yes.

$$x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$$

2.2

$$x(n) = \left\{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(a)

$$x(n-2) = \left\{\ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(b)

$$x(4-n) = \left\{\ldots 0, \underset{\uparrow}{\frac{1}{2}}, \frac{1}{2}, 1, 1, 1, 1, 0, \ldots\right\}$$

(see 2.1(d))
(c)

$$x(n+2) = \left\{\ldots 0, 1, 1, 1, \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(d)

$$x(n)u(2-n) = \left\{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, 0, 0, \ldots\right\}$$

(e)

$$x(n-1)\delta(n-3) = \left\{\ldots \underset{\uparrow}{0}, 0, 1, 0, \ldots\right\}$$

(f)

$$\begin{aligned} x(n^2) &= \{\ldots 0, x(4), x(1), x(0), x(1), x(4), 0, \ldots\} \\ &= \left\{\ldots 0, \frac{1}{2}, 1, \underset{\uparrow}{1}, 1, \frac{1}{2}, 0, \ldots\right\} \end{aligned}$$

(g)

$$\begin{aligned} x_e(n) &= \frac{x(n) + x(-n)}{2}, \\ x(-n) &= \left\{\ldots 0, \frac{1}{2}, \frac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0, \ldots\right\} \\ &= \left\{\ldots 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \ldots\right\} \end{aligned}$$

(h)

$$\begin{aligned} x_o(n) &= \frac{x(n)-x(-n)}{2} \\ &= \left\{\ldots 0, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \ldots\right\} \end{aligned}$$

2.3

(a)

$$u(n) - u(n-1) = \delta(n) = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 0, & n > 0 \end{cases}$$

(b)

$$\sum_{k=-\infty}^{n} \delta(k) = u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

2.4

Let

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)],$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)].$$

Since

$$x_e(-n) = x_e(n)$$

and

$$x_o(-n) = -x_o(n),$$

it follows that

$$x(n) = x_e(n) + x_o(n).$$

The decomposition is unique. For

$$x(n) = \left\{2, 3, \underset{\uparrow}{4}, 5, 6\right\},$$

we have

$$x_e(n) = \left\{4, 4, \underset{\uparrow}{4}, 4, 4\right\}$$

and

$$x_o(n) = \left\{-2, -1, \underset{\uparrow}{0}, 1, 2\right\}.$$

2.5

First, we prove that

$$\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) = 0$$

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) &= \sum_{m=-\infty}^{\infty} x_e(-m)x_o(-m) \\
&= -\sum_{m=-\infty}^{\infty} x_e(m)x_o(m) \\
&= -\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) \\
&= \sum_{n=-\infty}^{\infty} x_e(n)x_o(n) \\
&= 0
\end{aligned}$$

Then,

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x^2(n) &= \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2 \\
&= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n)x_o(n) \\
&= E_e + E_o
\end{aligned}$$

2.6

(a) No, the system is time variant. Proof: If

$$\begin{aligned}
x(n) \rightarrow y(n) &= x(n^2) \\
x(n-k) \rightarrow y_1(n) &= x[(n-k)^2] \\
&= x(n^2 + k^2 - 2nk) \\
&\neq y(n-k)
\end{aligned}$$

(b) (1)

$$x(n) = \left\{0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\right\}$$

(2)

$$y(n) = x(n^2) = \left\{\ldots, 0, 1, \underset{\uparrow}{1}, 1, 0, \ldots\right\}$$

(3)

$$y(n-2) = \left\{\ldots, 0, \underset{\uparrow}{0}, 1, 1, 1, 0, \ldots\right\}$$

(4)

$$x(n-2) = \left\{\ldots, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, \ldots\right\}$$

(5)

$$y_2(n) = \mathcal{T}[x(n-2)] = \left\{\ldots, 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0, \ldots\right\}$$

(6)

$$y_2(n) \neq y(n-2) \Rightarrow \text{system is time variant.}$$

(c) (1)

$$x(n) = \left\{\underset{\uparrow}{1}, 1, 1, 1\right\}$$

(2)

$$y(n) = \left\{\underset{\uparrow}{1}, 0, 0, 0, 0, -1\right\}$$

(3)

$$y(n-2) = \left\{\underset{\uparrow}{0}, 0, 1, 0, 0, 0, 0, -1\right\}$$

(4)

$$x(n-2) = \left\{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1\right\}$$

(5)

$$y_2(n) = \left\{\underset{\uparrow}{0}, 0, 1, 0, 0, 0, 0, -1\right\}$$

(6)

$$y_2(n) = y(n-2).$$

The system is time invariant, but this example alone does not constitute a proof.
(d) (1)

$$y(n) = nx(n),$$

$$x(n) = \left\{\ldots, 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\right\}$$

(2)

$$y(n) = \left\{\ldots, \underset{\uparrow}{0}, 1, 2, 3, \ldots\right\}$$

(3)

$$y(n-2) = \left\{\ldots, \underset{\uparrow}{0}, 0, 0, 1, 2, 3, \ldots\right\}$$

(4)

$$x(n-2) = \left\{\ldots, 0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, \ldots\right\}$$

(5)

$$y_2(n) = \mathcal{T}[x(n-2)] = \{\ldots, 0, 0, 2, 3, 4, 5, \ldots\}$$

(6)

$$y_2(n) \neq y(n-2) \Rightarrow \text{the system is time variant.}$$

2.7

(a) Static, nonlinear, time invariant, causal, stable.
(b) Dynamic, linear, time invariant, noncausal and unstable. The latter is easily proved. For the bounded input $x(k) = u(k)$, the output becomes

$$y(n) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0, & n < -1 \\ n+2, & n \geq -1 \end{cases}$$

since $y(n) \to \infty$ as $n \to \infty$, the system is unstable.
(c) Static, linear, timevariant, causal, stable.
(d) Dynamic, linear, time invariant, noncausal, stable.
(e) Static, nonlinear, time invariant, causal, stable.
(f) Static, nonlinear, time invariant, causal, stable.
(g) Static, nonlinear, time invariant, causal, stable.
(h) Static, linear, time invariant, causal, stable.
(i) Dynamic, linear, time variant, noncausal, unstable. Note that the bounded input $x(n) = u(n)$ produces an unbounded output.
(j) Dynamic, linear, time variant, noncausal, stable.
(k) Static, nonlinear, time invariant, causal, stable.
(l) Dynamic, linear, time invariant, noncausal, stable.
(m) Static, nonlinear, time invariant, causal, stable.
(n) Static, linear, time invariant, causal, stable.

2.8

(a) True. If

$$v_1(n) = \mathcal{T}_1[x_1(n)] \text{ and}$$
$$v_2(n) = \mathcal{T}_1[x_2(n)],$$

then

$$\alpha_1 x_1(n) + \alpha_2 x_2(n)$$

yields

$$\alpha_1 v_1(n) + \alpha_2 v_2(n)$$

by the linearity property of $\mathcal{T}_1$. Similarly, if

$$y_1(n) = \mathcal{T}_2[v_1(n)] \text{ and}$$

$$y_2(n) = \mathcal{T}_2[v_2(n)],$$

then

$$\beta_1 v_1(n) + \beta_2 v_2(n) \to y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$$

by the linearity property of $\mathcal{T}_2$. Since

$$v_1(n) = \mathcal{T}_1[x_1(n)] \text{ and}$$

$$v_2(n) = \mathcal{T}_2[x_2(n)],$$

it follows that

$$A_1x_1(n) + A_2x_2(n)$$

yields the output

$$A_1\mathcal{T}[x_1(n)] + A_2\mathcal{T}[x_2(n)],$$

where $\mathcal{T} = \mathcal{T}_1\mathcal{T}_2$. Hence $\mathcal{T}$ is linear.
(b) True. For $\mathcal{T}_1$, if

$$x(n) \rightarrow v(n) \text{ and}$$

$$x(n-k) \rightarrow v(n-k),$$

For $\mathcal{T}_2$, if

$$v(n) \rightarrow y(n)$$

$$\text{and} v(n-k) \rightarrow y(n-k).$$

Hence, For $\mathcal{T}_1\mathcal{T}_2$, if

$$x(n) \rightarrow y(n) \text{ and}$$

$$x(n-k) \rightarrow y(n-k)$$

Therefore, $\mathcal{T} = \mathcal{T}_1\mathcal{T}_2$ is time invariant.
(c) True. $\mathcal{T}_1$ is causal $\Rightarrow v(n)$ depends only on x(k) for $k \leq n$. $\mathcal{T}_2$ is causal $\Rightarrow y(n)$ depends only on $v(k)$ for $k \leq n$. Therefore, $y(n)$ depends only on $x(k)$ for $k \leq n$. Hence, $\mathcal{T}$ is causal.
(d) True. Combine (a) and (b).
(e) True. This follows from $h_1(n) * h_2(n) = h_2(n) * h_1(n)$
(f) False. For example, consider

$$\mathcal{T}_1 : y(n) = nx(n) \text{ and}$$

$$\mathcal{T}_2 : y(n) = nx(n+1).$$

Then,

$$\begin{aligned}
\mathcal{T}_2[\mathcal{T}_1[\delta(n)]] &= \mathcal{T}_2(0) = 0. \\
\mathcal{T}_1[\mathcal{T}_2[\delta(n)]] &= \mathcal{T}_1[\delta(n+1)] \\
&= -\delta(n+1) \\
&\neq 0.
\end{aligned}$$

(g) False. For example, consider

$$\mathcal{T}_1 : y(n) = x(n) + b \text{ and}$$

$$\mathcal{T}_2 : y(n) = x(n) - b, \text{ where } b \neq 0.$$

Then,

$$\mathcal{T}[x(n)] = \mathcal{T}_2[\mathcal{T}_1[x(n)]] = \mathcal{T}_2[x(n) + b] = x(n).$$

Hence $\mathcal{T}$ is linear.
(h) True.

$$\mathcal{T}_1 \text{ is stable } \Rightarrow v(n) \text{ is bounded if } x(n) \text{ is bounded.}$$

$$\mathcal{T}_2 \text{ is stable } \Rightarrow y(n) \text{ is bounded if } v(n) \text{ is bounded .}$$

Hence, y(n) is bounded if x(n) is bounded $\Rightarrow \mathcal{T} = \mathcal{T}_1\mathcal{T}_2$ is stable.
(i) Inverse of (c). $\mathcal{T}_1$ and for $\mathcal{T}_2$ are noncausal $\Rightarrow \mathcal{T}$ is noncausal. Example:

$$\begin{aligned} \mathcal{T}_1 : y(n) &= x(n+1) \text{ and} \\ \mathcal{T}_2 : y(n) &= x(n-2) \\ \Rightarrow \mathcal{T} : y(n) &= x(n-1), \end{aligned}$$

which is causal. Hence, the inverse of (c) is false.
Inverse of (h): $\mathcal{T}_1$ and/or $\mathcal{T}_2$ is unstable, implies $\mathcal{T}$ is unstable. Example:

$$\mathcal{T}_1 : y(n) = e^{x(n)}, \text{ stable and } \mathcal{T}_2 : y(n) = ln[x(n)], \text{ which is unstable.}$$

But $\mathcal{T} : y(n) = x(n),$ which is stable. Hence, the inverse of (h) is false.

2.9

(a)

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{n} h(k)x(n-k), x(n) = 0, n < 0 \\ y(n+N) &= \sum_{k=-\infty}^{n+N} h(k)x(n+N-k) = \sum_{k=-\infty}^{n+N} h(k)x(n-k) \\ &= \sum_{k=-\infty}^{n} h(k)x(n-k) + \sum_{k=n+1}^{n+N} h(k)x(n-k) \\ &= y(n) + \sum_{k=n+1}^{n+N} h(k)x(n-k) \end{aligned}$$

For a BIBO system, $\lim_{n\to\infty}|h(n)| = 0$. Therefore,

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)x(n-k) = 0 \text{ and}$$

$$\lim_{n\to\infty} y(n+N) = y(N).$$

(b) Let $x(n) = x_o(n) + au(n),$ where a is a constant and

$$x_o(n) \text{ is a bounded signal with } \lim_{n\to\infty} x_o(n) = 0.$$

Then,

$$\begin{aligned} y(n) &= a\sum_{k=0}^{\infty} h(k)u(n-k) + \sum_{k=0}^{\infty} h(k)x_o(n-k) \\ &= a\sum_{k=0}^{n} h(k) + y_o(n) \end{aligned}$$

clearly, $\sum_n x_o^2(n) < \infty \Rightarrow \sum_n y_o^2(n) < \infty$ (from (c) below) Hence,

$$\lim_{n\to\infty}|y_o(n)| = 0.$$

and, thus, $\lim_{n\to\infty} y(n) = a\sum_{k=0}^{n} h(k) = \text{constant}$.
(c)

$$\begin{aligned}
y(n) &= \sum_k h(k)x(n-k) \\
\sum_{-\infty}^{\infty} y^2(n) &= \sum_{-\infty}^{\infty}\left[\sum_k h(k)x(n-k)\right]^2 \\
&= \sum_k \sum_l h(k)h(l)\sum_n x(n-k)x(n-l)
\end{aligned}$$

But

$$\sum_n x(n-k)x(n-l) \le \sum_n x^2(n) = E_x.$$

Therefore,

$$\sum_n y^2(n) \le E_x \sum_k |h(k)| \sum_l |h(l)|.$$

For a BIBO stable system,

$$\sum_k |h(k)| < M.$$

Hence,

$$E_y \le M^2 E_x, \text{ so that}$$
$$E_y < 0 \text{ if } E_x < 0.$$

2.10

The system is nonlinear. This is evident from observation of the pairs

$$x_3(n) \leftrightarrow y_3(n) \text{ and } x_2(n) \leftrightarrow y_2(n).$$

If the system were linear, $y_2(n)$ would be of the form

$$y_2(n) = \{3, 6, 3\}$$

because the system is time-invariant. However, this is not the case.

2.11

since

$$x_1(n) + x_2(n) = \delta(n)$$

and the system is linear, the impulse response of the system is

$$y_1(n) + y_2(n) = \left\{0, \underset{\uparrow}{3}, -1, 2, 1\right\}.$$

If the system were time invariant, the response to $x_3(n)$ would be

$$\left\{\underset{\uparrow}{3}, 2, 1, 3, 1\right\}.$$

But this is not the case.

2.12

(a) Any weighted linear combination of the signals $x_i(n), i = 1, 2, \ldots, N$.
(b) Any $x_i(n-k)$, where k is any integer and $i = 1, 2, \ldots, N$.

2.13

A system is BIBO stable if and only if a bounded input produces a bounded output.

$$\begin{aligned} y(n) &= \sum_k h(k)x(n-k) \\ |y(n)| &\leq \sum_k |h(k)||x(n-k)| \\ &\leq M_x \sum_k |h(k)| \end{aligned}$$

where $|x(n-k)| \leq M_x$. Therefore, $|y(n)| < \infty$ for all n, if and only if

$$\sum_k |h(k)| < \infty.$$

2.14

(a) A system is causal $\Leftrightarrow$ the output becomes nonzero after the input becomes non-zero. Hence,

$$x(n) = 0 \text{ for } n < n_o \Rightarrow y(n) = 0 \text{ for } n < n_o.$$

(b)

$$y(n) = \sum_{-\infty}^{n} h(k)x(n-k), \text{ where } x(n) = 0 \text{ for } n < 0.$$

If $h(k) = 0$ for $k < 0$, then

$$y(n) = \sum_{0}^{n} h(k)x(n-k), \text{ and hence, } y(n) = 0 \text{ for } n < 0.$$

On the other hand, if $y(n) = 0$ for $n < 0$, then

$$\sum_{-\infty}^{n} h(k)x(n-k) \Rightarrow h(k) = 0, k < 0.$$

2.15

(a)

$$\begin{aligned} \text{For } a = 1, \sum_{n=M}^{N} a^n &= N - M + 1 \\ \text{for } a \neq 1, \sum_{n=M}^{N} a^n &= a^M + a^{M+1} + \ldots + a^N \\ (1-a)\sum_{n=M}^{N} a^n &= a^M + a^{M+1} - a^{M+1} + \ldots + a^N - a^N - a^{N+1} \\ &= a^M - a^{N+1} \end{aligned}$$

(b) For $M = 0, |a| < 1$, and $N \to \infty$,

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, |a| < 1.$$

2.16

(a)

$$\begin{aligned} y(n) &= \sum_k h(k)x(n-k) \\ \sum_n y(n) &= \sum_n \sum_k h(k)x(n-k) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k) \\ &= \left(\sum_k h(k)\right)\left(\sum_n x(n)\right) \end{aligned}$$

(b) (1)

$$y(n) = h(n) * x(n) = \{1, 3, 7, 7, 7, 6, 4\}$$

$$\sum_n y(n) = 35, \qquad \sum_k h(k) = 5, \qquad \sum_k x(k) = 7$$

(2)

$$y(n) = \{1, 4, 2, -4, 1\}$$

$$\sum_n y(n) = 4, \qquad \sum_k h(k) = 2, \qquad \sum_k x(k) = 2$$

(3)

$$y(n) = \left\{0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, -2\right\}$$

$$\sum_n y(n) = -5, \qquad \sum_n h(n) = 2.5, \qquad \sum_n x(n) = -2$$

(4)

$$y(n) = \{1, 2, 3, 4, 5\}$$

$$\sum_n y(n) = 15, \qquad \sum_n h(n) = 1, \qquad \sum_n x(n) = 15$$

(5)

$$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$$

$$\sum_n y(n) = 8, \qquad \sum_n h(n) = 4, \qquad \sum_n x(n) = 2$$

(6)

$$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$$

$$\sum_n y(n) = 8, \qquad \sum_n h(n) = 2, \qquad \sum_n x(n) = 4$$

(7)

$$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$$

$$\sum_n y(n) = -2, \qquad \sum_n h(n) = -1, \qquad \sum_n x(n) = 2$$

(8)

$$y(n) = u(n) + u(n-1) + 2u(n-2)$$

$$\sum_n y(n) = \infty, \qquad \sum_n h(n) = \infty, \qquad \sum_n x(n) = 4$$

(9)

$$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$$

$$\sum_n y(n) = 0, \qquad \sum_n h(n) = 0, \qquad \sum_n x(n) = 4$$

(10)

$$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$$

$$\sum_n y(n) = 36, \qquad \sum_n h(n) = 6, \qquad \sum_n x(n) = 6$$

(11)

$$y(n) = [2(\frac{1}{2})^n - (\frac{1}{4})^n]u(n)$$

$$\sum_n y(n) = \frac{8}{3}, \qquad \sum_n h(n) = \frac{4}{3}, \qquad \sum_n x(n) = 2$$

2.17

(a)

$$\begin{aligned}
x(n) &= \left\{\underset{\uparrow}{1}, 1, 1, 1\right\} \\
h(n) &= \left\{\underset{\uparrow}{6}, 5, 4, 3, 2, 1\right\} \\
y(n) &= \sum_{k=0}^{n} x(k)h(n-k) \\
y(0) &= x(0)h(0) = 6, \\
y(1) &= x(0)h(1) + x(1)h(0) = 11 \\
y(2) &= x(0)h(2) + x(1)h(1) + x(2)h(0) = 15 \\
y(3) &= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0) = 18 \\
y(4) &= x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0) = 14 \\
y(5) &= x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + x(4)h(1) + x(5)h(0) = 10 \\
y(6) &= x(1)h(5) + x(2)h(4) + x(3)h(2) = 6 \\
y(7) &= x(2)h(5) + x(3)h(4) = 3 \\
y(8) &= x(3)h(5) = 1 \\
y(n) &= 0, n \geq 9 \\
y(n) &= \left\{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\right\}
\end{aligned}$$

(b) By following the same procedure as in (a), we obtain

$$y(n) = \left\{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\right\}$$

(c) By following the same procedure as in (a), we obtain

$$y(n) = \left\{1, \underset{\uparrow}{2}, 2, 2, 1\right\}$$

(d) By following the same procedure as in (a), we obtain

$$y(n) = \left\{\underset{\uparrow}{1}, 2, 2, 2, 1\right\}$$

2.18

(a)

$$\begin{aligned}
x(n) &= \left\{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\right\} \\
h(n) &= \left\{1, 1, \underset{\uparrow}{1}, 1, 1\right\} \\
y(n) &= x(n) * h(n) \\
&= \left\{\frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\right\}
\end{aligned}$$

(b)

$$\begin{aligned}
x(n) &= \frac{1}{3}n[u(n) - u(n-7)], \\
h(n) &= u(n+2) - u(n-3) \\
y(n) &= x(n) * h(n) \\
&= \frac{1}{3}n[u(n) - u(n-7)] * [u(n+2) - u(n-3)] \\
&= \frac{1}{3}n[u(n) * u(n+2) - u(n) * u(n-3) - u(n-7) * u(n+2) + u(n-7) * u(n-3)] \\
y(n) &= \frac{1}{3}\delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3) + \frac{20}{3}\delta(n-4) + 6\delta(n-5) \\
&\quad +5\delta(n-6) + 5\delta(n-6) + \frac{11}{3}\delta(n-7) + \delta(n-8)
\end{aligned}$$

2.19

$$\begin{aligned}
y(n) &= \sum_{k=0}^{4} h(k)x(n-k), \\
x(n) &= \left\{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \ldots, \alpha^{5}\right\} \\
h(n) &= \left\{\underset{\uparrow}{1}, 1, 1, 1, 1\right\}
\end{aligned}$$

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{4} x(n-k), -3 \leq n \leq 9 \\
&= 0, \text{ otherwise.}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
y(-3) &= \alpha^{-3}, \\
y(-2) &= x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}, \\
y(-1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1}, \\
y(0) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 \\
y(1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha, \\
y(2) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2} \\
y(3) &= \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}, \\
y(4) &= \alpha^{4} + \alpha^{3} + \alpha^{2} + \alpha + 1 \\
y(5) &= \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(6) &= \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5} \\
y(7) &= \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(8) &= \alpha^{4} + \alpha^{5}, \\
y(9) &= \alpha^{5}
\end{aligned}
$$

2.20

(a) 131 x 122 = 15982
(b) $\{1_{\uparrow}, 3, 1\} * \{1_{\uparrow}, 2, 2\} = \{1, 5, 9, 8, 2\}$
(c) $(1 + 3z + z^2)(1 + 2z + 2z^2) = 1 + 5z + 9z^2 + 8z^3 + 2z^4$
(d) 1.31 x 12.2 = 15.982.
(e) These are different ways to perform convolution.

2.21

(a)

$$
y(n) = \sum_{k=0}^{n} a^k u(k) b^{n-k} u(n-k) = b^n \sum_{k=0}^{n} (ab^{-1})^k
$$

$$
y(n) = \begin{cases} \frac{b^{n+1} - a^{n+1}}{b-a} u(n), & a \neq b \\ b^n (n+1) u(n), & a = b \end{cases}
$$

(b)

$$
\begin{aligned}
x(n) &= \left\{ 1, 2, \underset{\uparrow}{1}, 1 \right\} \\
h(n) &= \left\{ \underset{\uparrow}{1}, -1, 0, 0, 1, 1 \right\} \\
y(n) &= \left\{ 1, 1, -\underset{\uparrow}{1}, 0, 0, 3, 3, 2, 1 \right\}
\end{aligned}
$$

(c)

$$x(n) = \left\{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\right\},$$
$$h(n) = \left\{1, 2, \underset{\uparrow}{3}, 2, 1\right\}$$
$$y(n) = \left\{1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -2, -1\right\}$$

(d)

$$x(n) = \left\{\underset{\uparrow}{1}, 1, 1, 1, 1\right\},$$
$$h'(n) = \left\{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1\right\}$$
$$h(n) = h'(n) + h'(n-9),$$
$$y(n) = y'(n) + y'(n-9), \text{ where}$$
$$y'(n) = \left\{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\right\}$$

2.22

(a)

$$\begin{aligned}
y_i(n) &= x(n) * h_i(n) \\
y_1(n) &= x(n) + x(n-1) \\
&= \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}, \text{ similarly} \\
y_2(n) &= \{1, 6, 11, 11, 13, 16, 14, 13, 15, 21, 25, 28, 24, 9\} \\
y_3(n) &= \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\} \\
y_4(n) &= \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75, 5.25, 6.25, 7, 6, 2.25\} \\
y_5(n) &= \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}
\end{aligned}$$

(b)

$$\begin{aligned}
y_3(n) &= \frac{1}{2}y_1(n), \text{ because} \\
h_3(n) &= \frac{1}{2}h_1(n) \\
y_4(n) &= \frac{1}{4}y_2(n), \text{ because} \\
h_4(n) &= \frac{1}{4}h_2(n)
\end{aligned}$$

(c) $y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$, but $y_4(n)$ will appear even smoother because of the smaller scale factor.

(d) System 4 results in a smoother output. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$

(e)

$$y_6(n) = \left\{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2}\right\}$$

$y_2(n)$ is smoother than $y_6(n)$.

2.23

We can express the unit sample in terms of the unit step function as $\delta(n) = u(n) - u(n-1)$. Then,

$$\begin{aligned} h(n) &= h(n) * \delta(n) \\ &= h(n) * (u(n) - u(n-1) \\ &= h(n) * u(n) - h(n) * u(n-1) \\ &= s(n) - s(n-1) \end{aligned}$$

Using this definition of $h(n)$

$$\begin{aligned} y(n) &= h(n) * x(n) \\ &= (s(n) - s(n-1)) * x(n) \\ &= s(n) * x(n) - s(n-1) * x(n) \end{aligned}$$

2.24

If

$$\begin{aligned} y_1(n) &= ny_1(n-1) + x_1(n) \text{ and} \\ y_2(n) &= ny_2(n-1) + x_2(n) \text{ then} \\ x(n) &= ax_1(n) + bx_2(n) \end{aligned}$$

produces the output

$$y(n) = ny(n-1) + x(n), \text{ where}$$

$$y(n) = ay_1(n) + by_2(n).$$

Hence, the system is linear. If the input is $x(n-1)$, we have

$$\begin{aligned} y(n-1) &= (n-1)y(n-2) + x(n-1). \text{ But} \\ y(n-1) &= ny(n-2) + x(n-1). \end{aligned}$$

Hence, the system is time variant. If $x(n) = u(n)$, then $|x(n)| \leq 1$. But for this bounded input, the output is

$$y(0) = 1, \quad y(1) = 1 + 1 = 2, \quad y(2) = 2\text{x}2 + 1 = 5, \ldots$$

which is unbounded. Hence, the system is unstable.

2.25

(a)

$$\begin{aligned} \delta(n) &= \gamma(n) - a\gamma(n-1) \text{ and,} \\ \delta(n-k) &= \gamma(n-k) - a\gamma(n-k-1). \text{ Then,} \\ x(n) &= \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \\ &= \sum_{k=-\infty}^{\infty} x(k)[\gamma(n-k) - a\gamma(n-k-1)] \end{aligned}$$

$$
\begin{aligned}
x(n) &= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\gamma(n-k-1) \\
x(n) &= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k) \\
&= \sum_{k=-\infty}^{\infty} [x(k) - ax(k-1)]\gamma(n-k) \\
\text{Thus, } c_k &= x(k) - ax(k-1)
\end{aligned}
$$

(b)

$$
\begin{aligned}
y(n) &= \mathcal{T}[x(n)] \\
&= \mathcal{T}[\sum_{k=-\infty}^{\infty} c_k\gamma(n-k)] \\
&= \sum_{k=-\infty}^{\infty} c_k\mathcal{T}[\gamma(n-k)] \\
&= \sum_{k=-\infty}^{\infty} c_k g(n-k)
\end{aligned}
$$

(c)

$$
\begin{aligned}
h(n) &= \mathcal{T}[\delta(n)] \\
&= T[\gamma(n) - a\gamma(n-1)] \\
&= g(n) - ag(n-1)
\end{aligned}
$$

2.26

With x(n) = 0, we have

$$
\begin{aligned}
y(n-1) + \frac{4}{3}y(n-1) &= 0 \\
y(-1) &= -\frac{4}{3}y(-2) \\
y(0) &= (-\frac{4}{3})^2 y(-2) \\
y(1) &= (-\frac{4}{3})^3 y(-2) \\
&\vdots \\
y(k) &= (-\frac{4}{3})^{k+2} y(-2) \leftarrow \text{ zero-input response.}
\end{aligned}
$$

2.27

Consider the homogeneous equation:

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = 0.$$

The characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0.\lambda = \frac{1}{2}, \frac{1}{3}.$$

Hence,

$$y_h(n) = c_1(\frac{1}{2})^n + c_2(\frac{1}{3})^n$$

The particular solution to

$$x(n) = 2^n u(n) \text{ is}$$

$$y_p(n) = k(2^n)u(n).$$

Substitute this solution into the difference equation. Then, we obtain

$$k(2^n)u(n) - k(\frac{5}{6})(2^{n-1})u(n-1) + k(\frac{1}{6})(2^{n-2})u(n-2) = 2^n u(n)$$

For n = 2,

$$4k - \frac{5k}{3} + \frac{k}{6} = 4 \Rightarrow k = \frac{8}{5}.$$

Therefore, the total solution is

$$y(n) = y_p(n) + y_h(n) = \frac{8}{5}(2^n)u(n) + c_1(\frac{1}{2})^n u(n) + c_2(\frac{1}{3})^n u(n).$$

To determine c_1 and c_2, assume that $y(-2) = y(-1) = 0$. Then,

$$y(0) = 1 \text{ and}$$

$$y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$$

Thus,

$$\begin{aligned} \frac{8}{5} + c_1 + c_2 &= 1 \Rightarrow c_1 + c_2 = -\frac{3}{5} \\ \frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 &= \frac{17}{6} \Rightarrow 3c_1 + 2c_2 = -\frac{11}{5} \end{aligned}$$

and, therefore,

$$c_1 = -1, c_2 = \frac{2}{5}.$$

The total solution is

$$y(n) = \left[\frac{8}{5}(2)^n - (\frac{1}{2})^n + \frac{2}{5}(\frac{1}{3})^n\right] u(n)$$

2.28

Fig. 2.28-1 shows the transient response, $y_{zi}(n)$, for $y(-1) = 1$ and the steady state response, $y_{zs}(n)$.

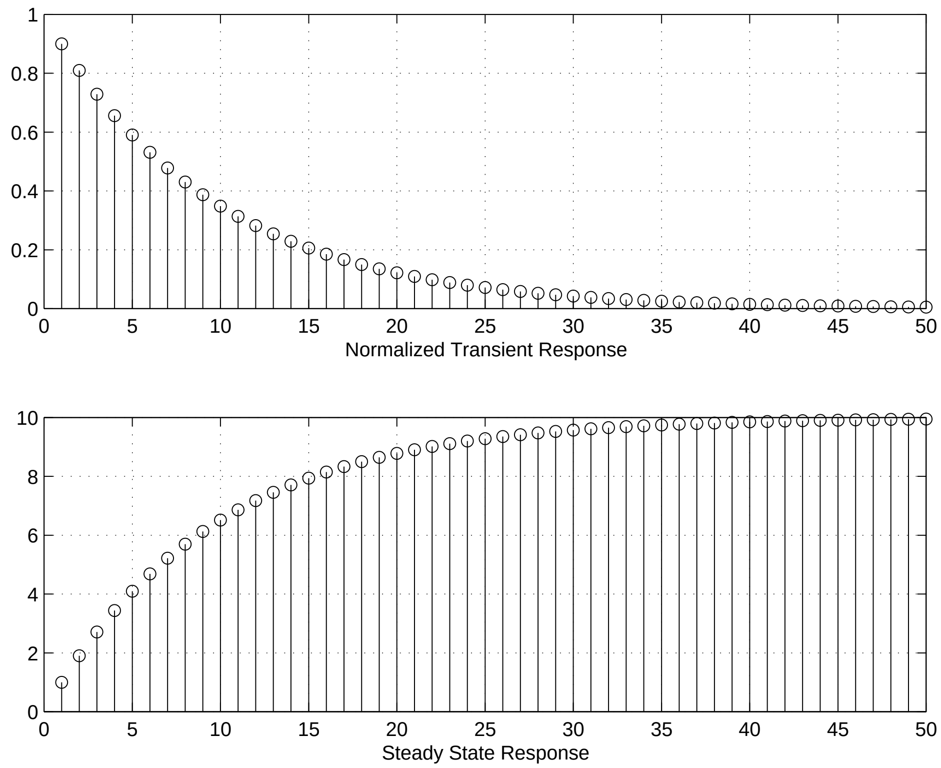

Figure 2.28-1:

2.29

$$
\begin{aligned}
h(n) &= h_1(n) * h_2(n) \\
&= \sum_{k=-\infty}^{\infty} a^k [u(k) - u(k-N)][u(n-k) - u(n-k-M)] \\
&= \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k-M) \\
&\quad - \sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k-M) \\
&= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k \right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-M} a^k \right) \\
&= 0
\end{aligned}
$$

2.30

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

The characteristic equation is

$$\lambda^2 - 3\lambda - 4 = 0.$$

Hence, $\lambda = 4, -1$ and

$$y_h(n) = c_1(n)4^n + c_2(-1)^n.$$

Since 4 is a characteristic root and the excitation is

$$x(n) = 4^n u(n),$$

we assume a particular solution of the form

$$y_p(n) = kn4^n u(n).$$

Then

$$kn4^n u(n) - 3k(n-1)4^{n-1}u(n-1) - 4k(n-2)4^{n-2}u(n-2)$$
$$= 4^n u(n) + 2(4)^{n-1}u(n-1)$$

. For $n = 2$,

$$k(32 - 12) = 4^2 + 8 = 24 \rightarrow k = \frac{6}{5}.$$

The total solution is

$$\begin{aligned} y(n) &= y_p(n) + y_h(n) \\ &= \left[\frac{6}{5}n4^n + c_1 4^n + c_2(-1)^n\right]u(n) \end{aligned}$$

To solve for c_1 and c_2, we assume that $y(-1) = y(-2) = 0$. Then,

$$y(0) = 1 \text{ and}$$

$$y(1) = 3y(0) + 4 + 2 = 9$$

Hence,

$$c_1 + c_2 = 1 \text{ and}$$

$$\frac{24}{5} + 4c_1 - c_2 = 9$$

$$4c_1 - c_2 = \frac{21}{5}$$

Therefore,

$$c_1 = \frac{26}{25} \text{ and } c_2 = -\frac{1}{25}$$

The total solution is

$$y(n) = \left[\frac{6}{5}n4^n + \frac{26}{25}4^n - \frac{1}{25}(-1)^n\right]u(n)$$

2.31

From 2.30, the characteristic values are $\lambda = 4, -1$. Hence

$$y_h(n) = c_1 4^n + c_2(-1)^n$$

When $x(n) = \delta(N)$, we find that

$$y(0) = 1 \text{ and}$$

$$y(1) - 3y(0) = 2 \text{ or}$$

$$y(1) = 5.$$

Hence,

$$c_1 + c_2 = 1 \text{ and } 4c_1 - c_2 = 5$$

This yields, $c_1 = \frac{6}{5}$ and $c_2 = -\frac{1}{5}$. Therefore,

$$h(n) = \left[\frac{6}{5}4^n - \frac{1}{5}(-1)^n\right]u(n)$$

2.32

(a) $L_1 = N_1 + M_1$ and $L_2 = N_2 + M_2$
(b) Partial overlap from left:

$$\text{low } N_1 + M_1 \qquad \text{high } N_1 + M_2 - 1$$

$$\text{Full overlap: low } N_1 + M_2 \qquad \text{high } N_2 + M_1$$

Partial overlap from right:

$$\text{low } N_2 + M_1 + 1 \qquad \text{high } N_2 + M_2$$

(c)

$$\begin{aligned} x(n) &= \left\{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\right\} \\ h(n) &= \left\{2, \underset{\uparrow}{2}, 2, 2\right\} \\ N_1 &= -2, \\ N_2 &= 4, \\ M_1 &= -1, \\ M_2 &= 2, \end{aligned}$$

$$\text{Partial overlap from left: } n = -3 \qquad n = -1 \qquad L_1 = -3$$

$$\text{Full overlap: } n = 0 \qquad n = 3$$

$$\text{Partial overlap from right:} n = 4 \qquad n = 6 \qquad L_2 = 6$$

2.33

(a)

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n).$$

The characteristic equation is

$$\lambda^2 - 0.6\lambda + 0.08 = 0.$$

$\lambda = 0.2, 0.4$ Hence,

$$y_h(n) = c_1 \frac{1}{5}^n + c_2 \frac{2}{5}^n.$$

With $x(n) = \delta(n)$, the initial conditions are

$$\begin{aligned} y(0) &= 1, \\ y(1) - 0.6y(0) &= 0 \Rightarrow y(1) = 0.6. \\ \text{Hence,} c_1 + c_2 &= 1 \text{ and} \\ \frac{1}{5}c_1 + \frac{2}{5} &= 0.6 \Rightarrow c_1 = -1, c_2 = 3. \\ \text{Therefore } h(n) &= \left[-(\frac{1}{5})^n + 2(\frac{2}{5})^n\right] u(n) \end{aligned}$$

The step response is

$$\begin{aligned} s(n) &= \sum_{k=0}^{n} h(n-k), n \geq 0 \\ &= \sum_{k=0}^{n} \left[2(\frac{2}{5})^{n-k} - (\frac{1}{5})^{n-k}\right] \\ &= \left\{\frac{1}{0.12}\left[(\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[(\frac{1}{5}^{n+1} - 1\right]\right\} u(n) \end{aligned}$$

(b)

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2).$$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0.$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ Hence,

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n.$$

With $x(n) = \delta(n)$, we have

$$\begin{aligned} y(0) &= 2, \\ y(1) - 0.7y(0) &= 0 \Rightarrow y(1) = 1.4. \\ \text{Hence,} c_1 + c_2 &= 2 \text{ and} \\ \frac{1}{2}c_1 + \frac{1}{5} &= 1.4 = \frac{7}{5} \\ \Rightarrow c_1 + \frac{2}{5}c_2 &= \frac{14}{5}. \\ \text{These equations yield} & \\ c_1 = \frac{10}{3}, c_2 = -\frac{4}{3}. & \\ h(n) &= \left[\frac{10}{3}(\frac{1}{2})^n - \frac{4}{3}(\frac{1}{5})^n\right] u(n) \end{aligned}$$

The step response is

$$
\begin{aligned}
s(n) &= \sum_{k=0}^{n} h(n-k), \\
&= \frac{10}{3}\sum_{k=0}^{n}(\frac{1}{2})^{n-k} - \frac{4}{3}\sum_{k=0}^{n}(\frac{1}{5})^{n-k} \\
&= \frac{10}{3}(\frac{1}{2})^{n}\sum_{k=0}^{n}2^{k} - \frac{4}{3}(\frac{1}{5})^{n}\sum_{k=0}^{n}5^{k} \\
&= \frac{10}{3}(\frac{1}{2}^{n}(2^{n+1}-1)u(n) - \frac{1}{3}(\frac{1}{5}^{n}(5^{n+1}-1)u(n)
\end{aligned}
$$

2.34

$$
\begin{aligned}
h(n) &= \left\{\underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right\} \\
y(n) &= \left\{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0\right\} \\
x(0)h(0) &= y(0) \Rightarrow x(0) = 1 \\
\frac{1}{2}x(0) + x(1) &= y(1) \Rightarrow x(1) = \frac{3}{2}
\end{aligned}
$$

By continuing this process, we obtain

$$
x(n) = \left\{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \ldots\right\}
$$

2.35

(a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$
(b)

$$
\begin{aligned}
h_3(n) * h_4(n) &= (n-1)u(n-2) \\
h_2(n) - h_3(n) * h_4(n) &= 2u(n) - \delta(n) \\
h_1(n) &= \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) \\
\text{Hence } h(n) &= \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * [2u(n) - \delta(n)] \\
&= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)
\end{aligned}
$$

(c)

$$
\begin{aligned}
x(n) &= \left\{1, 0, \underset{\uparrow}{0}, 3, 0, -4\right\} \\
y(n) &= \left\{\frac{1}{2}, \frac{5}{4}, \underset{\uparrow}{2}, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \ldots\right\}
\end{aligned}
$$

2.36

First, we determine

$$\begin{aligned} s(n) &= u(n) * h(n) \\ s(n) &= \sum_{k=0}^{\infty} u(k)h(n-k) \\ &= \sum_{k=0}^{n} h(n-k) \\ &= \sum_{k=0}^{\infty} a^{n-k} \\ &= \frac{a^{n+1}-1}{a-1}, n \geq 0 \end{aligned}$$

For $x(n) = u(n+5) - u(n-10)$, we have the response

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1}u(n+5) - \frac{a^{n-9}-1}{a-1}u(n-10)$$

From figure P2.33,

$$\begin{aligned} y(n) &= x(n) * h(n) - x(n) * h(n-2) \\ \text{Hence, } y(n) &= \frac{a^{n+6}-1}{a-1}u(n+5) - \frac{a^{n-9}-1}{a-1}u(n-10) \\ &\quad -\frac{a^{n+4}-1}{a-1}u(n+3) + \frac{a^{n-11}-1}{a-1}u(n-12) \end{aligned}$$

2.37

$$\begin{aligned} h(n) &= [u(n) - u(n-M)]/M \\ s(n) &= \sum_{k=-\infty}^{\infty} u(k)h(n-k) \\ &= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases} \end{aligned}$$

2.38

$$\begin{aligned} \sum_{n=-\infty}^{\infty} |h(n)| &= \sum_{n=0, neven}^{\infty} |a|^n \\ &= \sum_{n=0}^{\infty} |a|^{2n} \\ &= \frac{1}{1-|a|^2} \end{aligned}$$

Stable if $|a| < 1$

2.39

$h(n) = a^n u(n)$. The response to $u(n)$ is

$$\begin{aligned} y_1(n) &= \sum_{k=0}^{\infty} u(k)h(n-k) \\ &= \sum_{k=0}^{n} a^{n-k} \\ &= a^n \sum_{k=0}^{n} a^{-k} \\ &= \frac{1-a^{n+1}}{1-a} u(n) \\ \text{Then, } y(n) &= y_1(n) - y_1(n-10) \\ &= \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right] \end{aligned}$$

2.40

We may use the result in problem 2.36 with $a = \frac{1}{2}$. Thus,

$$y(n) = 2\left[1 - (\frac{1}{2})^{n+1}\right] u(n) - 2\left[1 - (\frac{1}{2})^{n-9}\right] u(n-10)$$

2.41

(a)

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\ &= \sum_{k=0}^{n} (\frac{1}{2})^k 2^{n-k} \\ &= 2^n \sum_{k=0}^{n} (\frac{1}{4})^k \\ &= 2^n \left[1 - (\frac{1}{4})^{n+1}\right] (\frac{4}{3}) \\ &= \frac{2}{3}\left[2^{n+1} - (\frac{1}{2})^{n+1}\right] u(n) \end{aligned}$$

(b)

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\ &= \sum_{k=0}^{\infty} h(k) \\ &= \sum_{k=0}^{\infty} (\frac{1}{2})^k = 2, n < 0 \end{aligned}$$

$$
\begin{aligned}
y(n) &= \sum_{k=n}^{\infty} h(k) \\
&= \sum_{k=n}^{\infty} (\frac{1}{2})^k \\
&= \sum_{k=0}^{\infty} (\frac{1}{2})^k - \sum_{k=0}^{n-1} (\frac{1}{2})^k \\
&= 2 - (\frac{1-(\frac{1}{2})^n}{\frac{1}{2}}) \\
&= 2(\frac{1}{2})^n, n \geq 0.
\end{aligned}
$$

2.42

(a)

$$
\begin{aligned}
h_e(n) &= h_1(n) * h_2(n) * h_3(n) \\
&= [\delta(n) - \delta(n-1)] * u(n) * h(n) \\
&= [u(n) - u(n-1)] * h(n) \\
&= \delta(n) * h(n) \\
&= h(n)
\end{aligned}
$$

(b) No.

2.43

(a) $x(n)\delta(n-n_0) = x(n_0)$. Thus, only the value of $x(n)$ at $n = n_0$ is of interest.
$x(n) * \delta(n-n_0) = x(n-n_0)$. Thus, we obtain the shifted version of the sequence $x(n)$.
(b)

$$
\begin{aligned}
y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\
&= h(n) * x(n) \\
\text{Linearity:} x_1(n) \rightarrow y_1(n) &= h(n) * x_1(n) \\
x_2(n) \rightarrow y_2(n) &= h(n) * x_2(n) \\
\text{Then } x(n) &= \alpha x_1(n) + \beta x_2(n) \rightarrow y(n) = h(n) * x(n) \\
y(n) &= h(n) * [\alpha x_1(n) + \beta x_2(n)] \\
&= \alpha h(n) * x_1(n) + \beta h(n) * x_2(n) \\
&= \alpha y_1(n) + \beta y_2(n) \\
\text{Time Invariance:} & \\
x(n) \rightarrow y(n) &= h(n) * x(n) \\
x(n-n_0) \rightarrow y_1(n) &= h(n) * x(n-n_0) \\
&= \sum_k h(k)x(n-n_0-k) \\
&= y(n-n_0)
\end{aligned}
$$

(c) $h(n) = \delta(n-n_0)$.

2.44

(a) $s(n) = -a_1 s(n-1) - a_2 s(n-2) - \ldots - a_N s(n-N) + b_0 v(n)$. Refer to fig 2.44-1.
(b) $v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \ldots + a_N s(n-N)\right]$. Refer to fig 2.44-2

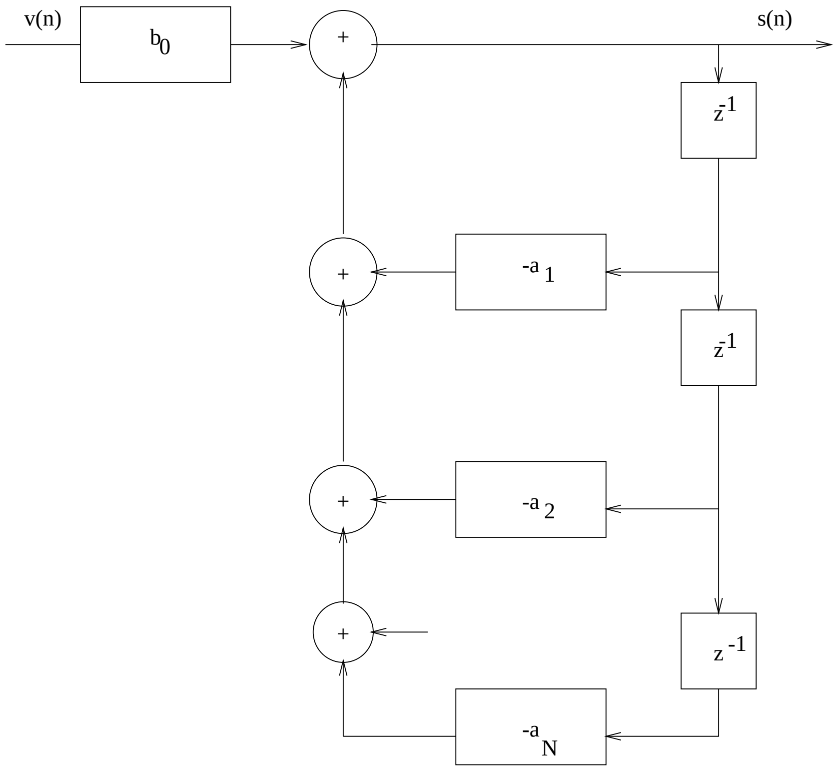

Figure 2.44-1:

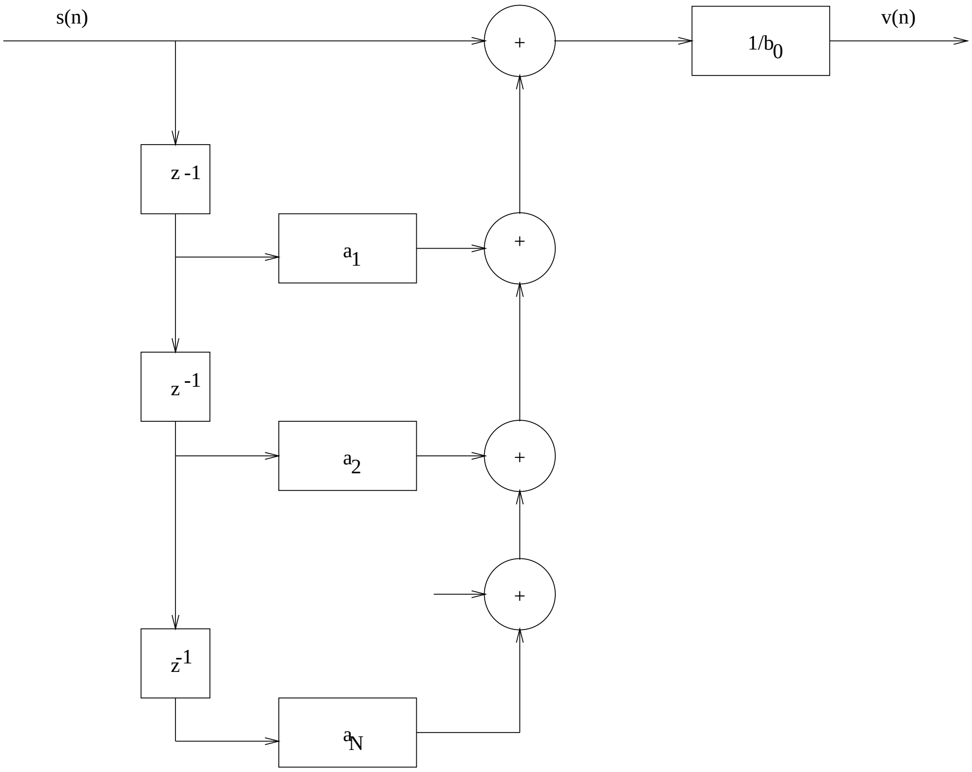

Figure 2.44-2:

2.45

$$
\begin{aligned}
y(n) &= -\frac{1}{2}y(n-1) + x(n) + 2x(n-2) \\
y(-2) &= -\frac{1}{2}y(-3) + x(-2) + 2x(-4) = 1 \\
y(-1) &= -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = \frac{3}{2} \\
y(0) &= -\frac{1}{2}y(-1) + 2x(-2) + x(0) = \frac{17}{4} \\
y(1) &= -\frac{1}{2}y(0) + x(1) + 2x(-1) = \frac{47}{8}, \text{ etc}
\end{aligned}
$$

2.46

(a) Refer to fig 2.46-1
(b) Refer to fig 2.46-2

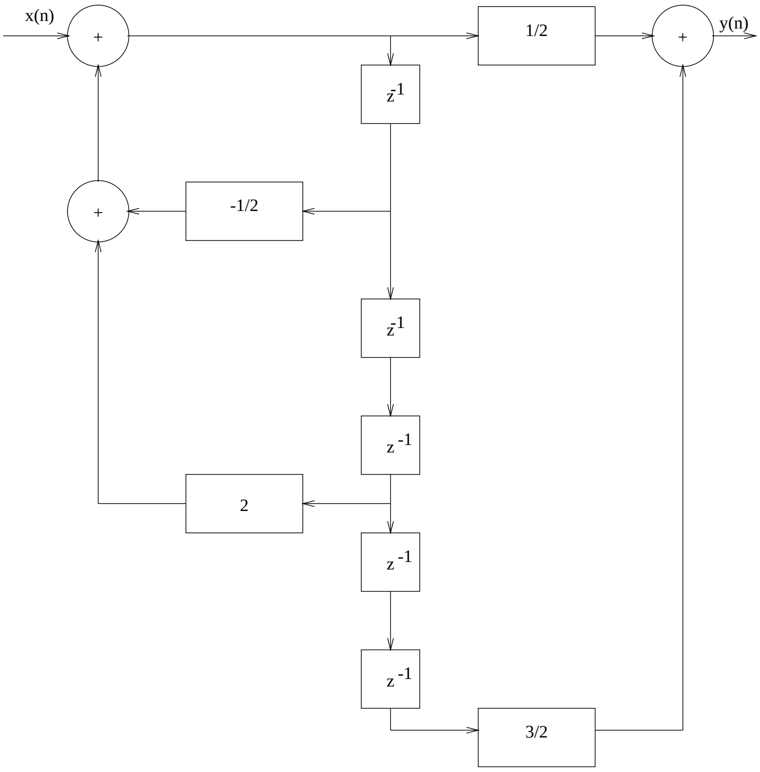

Figure 2.46-1:

2.47

(a)

$$
\begin{aligned}
x(n) &= \left\{ \underset{\uparrow}{1}, 0, 0, \ldots \right\} \\
y(n) &= \frac{1}{2}y(n-1) + x(n) + x(n-1) \\
y(0) &= x(0) = 1, \\
y(1) &= \frac{1}{2}y(0) + x(1) + x(0) = \frac{3}{2} \\
y(2) &= \frac{1}{2}y(1) + x(2) + x(1) = \frac{3}{4}. \text{ Thus, we obtain} \\
y(n) &= \left\{ 1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \ldots \right\}
\end{aligned}
$$

(b) $y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$
(c) As in part(a), we obtain

$$y(n) = \left\{ 1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, \ldots \right\}$$

(d)

$$
\begin{aligned}
y(n) &= u(n) * h(n) \\
&= \sum_k u(k)h(n-k)
\end{aligned}
$$

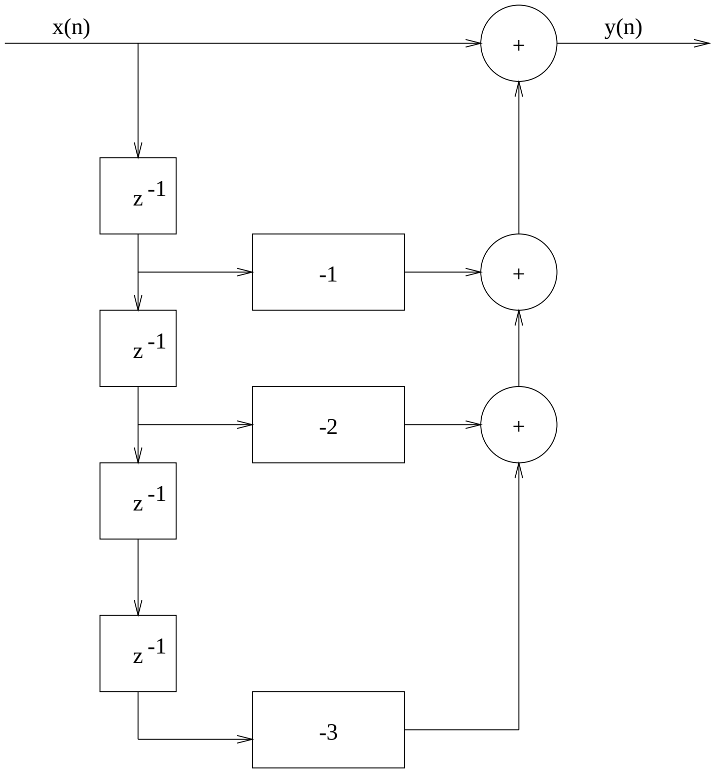

Figure 2.46-2:

$$\begin{aligned} &= \sum_{k=0}^{n} h(n-k) \\ y(0) &= h(0) = 1 \\ y(1) &= h(0) + h(1) = \frac{5}{2} \\ y(2) &= h(0) + h(1) + h(2) = \frac{13}{4}, \text{ etc} \end{aligned}$$

(e) from part(a), $h(n) = 0$ for $n < 0 \Rightarrow$ the system is causal.

$$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}(1 + \frac{1}{2} + \frac{1}{4} + \ldots) = 4 \Rightarrow \text{ system is stable}$$

2.48

(a)

$$\begin{aligned} y(n) &= ay(n-1) + bx(n) \\ \Rightarrow h(n) &= ba^n u(n) \\ \sum_{n=0}^{\infty} h(n) &= \frac{b}{1-a} = 1 \\ \Rightarrow b &= 1 - a. \end{aligned}$$

(a)

$$
\begin{aligned}
s(n) &= \sum_{k=0}^{n} h(n-k) \\
&= b\left[\frac{1-a^{n+1}}{1-a}\right]u(n) \\
s(\infty) &= \frac{b}{1-a} = 1 \\
\Rightarrow b &= 1-a.
\end{aligned}
$$

(c) $b = 1 - a$ in both cases.

2.49

(a)

$$
\begin{aligned}
y(n) &= 0.8y(n-1) + 2x(n) + 3x(n-1) \\
y(n) - 0.8y(n-1) &= 2x(n) + 3x(n-1) \\
\text{The characteristic equation is} & \\
\lambda - 0.8 &= 0 \\
\lambda &= 0.8. \\
y_h(n) &= c(0.8)^n
\end{aligned}
$$

Let us first consider the response of the sytem

$$y(n) - 0.8y(n-1) = x(n)$$

to $x(n) = \delta(n)$. Since $y(0) = 1$, it folows that $c = 1$. Then, the impulse response of the original system is

$$
\begin{aligned}
h(n) &= 2(0.8)^n u(n) + 3(0.8)^{n-1}u(n-1) \\
&= 2\delta(n) + 4.6(0.8)^{n-1}u(n-1)
\end{aligned}
$$

(b) The inverse system is characterized by the difference equation

$$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1)$$

Refer to fig 2.49-1

2.50

$y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$
(a)For $x(n) = \delta(n)$, we have

$$
\begin{aligned}
y(0) &= 1, \\
y(1) &= 2.9, \\
y(2) &= 5.61, \\
y(3) &= 5.049, \\
y(4) &= 4.544, \\
y(5) &= 4.090, \ldots
\end{aligned}
$$

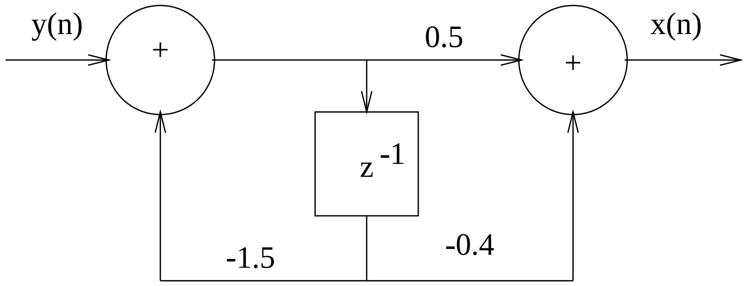

Figure 2.49-1:

(b)

$$\begin{aligned} s(0) &= y(0) = 1, \\ s(1) &= y(0) + y(1) = 3.91 \\ s(2) &= y(0) + y(1) + y(2) = 9.51 \\ s(3) &= y(0) + y(1) + y(2) + y(3) = 14.56 \\ s(4) &= \sum_0^4 y(n) = 19.10 \\ s(5) &= \sum_0^5 y(n) = 23.19 \end{aligned}$$

(c)

$$\begin{aligned} h(n) &= (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2) \\ &= \delta(n) + 2.9\delta(n-1) + 5.61(0.9)^{n-2} u(n-2) \end{aligned}$$

2.51

(a)

$$\begin{aligned} y(n) &= \frac{1}{3}x(n) + \frac{1}{3}x(n-3) + y(n-1) \\ \text{for } x(n) &= \delta(n), \text{ we have} \\ h(n) &= \left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \ldots\right\} \end{aligned}$$

(b)

$$\begin{aligned} y(n) &= \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + \frac{1}{2}x(n-2) \\ \text{with } x(n) &= \delta(n), \text{ and} \\ y(-1) &= y(-2) = 0, \text{ we obtain} \\ h(n) &= \left\{0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \frac{15}{256}, \frac{41}{1024}, \ldots\right\} \end{aligned}$$

(c)

$$\begin{aligned}
y(n) &= 1.4y(n-1) - 0.48y(n-2) + x(n) \\
\text{with } x(n) &= \delta(n), \text{ and} \\
y(-1) &= y(-2) = 0, \text{ we obtain} \\
h(n) &= \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086, \ldots\}
\end{aligned}$$

(d) All three systems are IIR.
(e)

$$y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$$

The characteristic equation is

$$\begin{aligned}
\lambda^2 - 1.4\lambda + 0.48 &= 0 \text{ Hence} \\
\lambda &= 0.8, 0.6. \text{ and} \\
y_h(n) &= c_1(0.8)^n + c_2(0.6)^n \text{For } x(n) = \delta(n). \text{ We have,} \\
c_1 + c_2 &= 1 \text{ and} \\
0.8c_1 + 0.6c_2 &= 1.4 \\
\Rightarrow c_1 &= 4, \\
c_2 &= -3. \text{ Therefore} \\
h(n) &= [4(0.8)^n - 3(0.6)^n]\, u(n)
\end{aligned}$$

2.52

(a)

$$\begin{aligned}
h_1(n) &= c_0\delta(n) + c_1\delta(n-1) + c_2\delta(n-2) \\
h_2(n) &= b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2) \\
h_3(n) &= a_0\delta(n) + (a_1 + a_0a_2)\delta(n-1) + a_1a_2\delta(n-2)
\end{aligned}$$

(b) The only question is whether

$$\begin{aligned}
h_3(n) &\stackrel{?}{=} h_2(n) = h_1(n) \\
\text{Let } a_0 &= c_0, \\
a_1 + a_2c_0 &= c_1, \\
a_2a_1 &= c_2. \text{ Hence} \\
\frac{c_2}{a_2} + a_2c_0 - c_1 &= 0 \\
\Rightarrow c_0a_2^2 - c_1a_2 + c_2 &= 0
\end{aligned}$$

For $c_0 \neq 0$, the quadratic has a real solution if and only if

$$c_1^2 - 4c_0c_2 \geq 0$$

2.53

(a)

$$\begin{aligned}
y(n) &= \frac{1}{2}y(n-1) + x(n) + x(n-1) \\
\text{For } y(n) - \frac{1}{2}y(n-1) &= \delta(n), \text{ the solution is} \\
h(n) &= (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1}u(n-1)
\end{aligned}$$

(b) $h_1(n) * [\delta(n) + \delta(n-1)] = (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1} u(n-1)$.

2.54

(a)

$$\begin{aligned} \text{convolution: } y_1(n) &= \left\{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\right\} \\ \text{correlation: } \gamma_1(n) &= \left\{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\right\} \end{aligned}$$

(b)

$$\begin{aligned} \text{convolution: } y_2(n) &= \left\{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\right\} \\ \text{correlation: } \gamma_1(n) &= \left\{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\right\} \end{aligned}$$

Note that $y_2(n) = \gamma_2(n)$, because $h_2(-n) = h_2(n)$ (c)

$$\begin{aligned} \text{convolution: } y_3(n) &= \left\{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\right\} \\ \text{correlation: } \gamma_1(n) &= \left\{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\right\} \end{aligned}$$

(c)

$$\begin{aligned} \text{convolution: } y_4(n) &= \left\{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\right\} \\ \text{correlation: } \gamma_4(n) &= \left\{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\right\} \\ \text{Note that } h_3(-n) &= h_4(n+3), \\ \text{hence, } \gamma_3(n) &= y_4(n+3) \\ \text{and } h_4(-n) &= h_3(n+3), \\ \Rightarrow \gamma_4(n) &= y_3(n+3) \end{aligned}$$

2.55

Obviously, the length of $h(n)$ is 2, i.e.

$$\begin{aligned} h(n) &= \{h_0, h_1\} \\ h_0 &= 1 \\ 3h_0 + h_1 &= 4 \\ \Rightarrow h_0 &= 1, h_1 = 1 \end{aligned}$$

2.56

$$(2.5.6) \qquad y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$\begin{aligned}(2.5.9) \quad w(n) &= -\sum_{k=1}^{N} a_k w(n-k) + x(n) \\ (2.5.10) \quad y(n) &= \sum_{k=0}^{M} b_k w(n-k)\end{aligned}$$

From (2.5.9) we obtain

$$x(n) = w(n) + \sum_{k=1}^{N} a_k w(n-k) \qquad \text{(A)}$$

By substituting (2.5.10) for $y(n)$ and (A) into (2.5.6), we obtain L.H.S = R.H.S.

2.57

$$\begin{aligned}y(n) - 4y(n-1) + 4y(n-2) &= x(n) - x(n-1) \\ \text{The characteristic equation is} & \\ \lambda^2 - 4\lambda + 4 &= 0 \\ \lambda &= 2, 2. \text{ Hence,} \\ y_h(n) &= c_1 2^n + c_2 n 2^n\end{aligned}$$

The particular solution is

$$y_p(n) = k(-1)^n u(n).$$

Substituting this solution into the difference equation, we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2, k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$. The total solution is

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

From the initial condtions, we obtain $y(0) = 1, y(1) = 2$. Then,

$$\begin{aligned}c_1 + \frac{2}{9} &= 1 \\ \Rightarrow c_1 &= \frac{7}{9}, \\ 2c_1 + 2c_2 - \frac{2}{9} &= 2 \\ \Rightarrow c_2 &= \frac{1}{3},\end{aligned}$$

2.58

From problem 2.57,

$$h(n) = \left[c_1 2^n + c_2 n 2^n\right] u(n)$$

With $y(0) = 1, y(1) = 3$, we have

$$\begin{aligned}c_1 &= 1 \\ 2c_1 + 2c_2 &= 3\end{aligned}$$

$$\begin{aligned} \Rightarrow c_2 &= \frac{1}{2} \\ \text{Thus } h(n) &= \left[2^n + \frac{1}{2} n 2^n\right] u(n) \end{aligned}$$

2.59

$$\begin{aligned} x(n) &= x(n) * \delta(n) \\ &= x(n) * [u(n) - u(n-1)] \\ &= [x(n) - x(n-1)] * u(n) \\ &= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k) \end{aligned}$$

2.60

Let $h(n)$ be the impulse response of the system

$$\begin{aligned} s(k) &= \sum_{m=-\infty}^{k} h(m) \\ \Rightarrow h(k) &= s(k) - s(k-1) \\ y(n) &= \sum_{k=-\infty}^{\infty} h(k) x(n-k) \\ &= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k) \end{aligned}$$

2.61

$$x(n) = \begin{cases} 1, & n_0 - N \leq n \leq n_0 + N \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) x(n-l)$$

The range of non-zero values of $\gamma_{xx}(l)$ is determined by

$$n_0 - N \leq n \leq n_0 + N$$

$$n_0 - N \leq n - l \leq n_0 + N$$

which implies

$$-2N \leq l \leq 2N$$

For a given shift l, the number of terms in the summation for which both $x(n)$ and $x(n-l)$ are non-zero is $2N + 1 - |l|$, and the value of each term is 1. Hence,

$$\gamma_{xx}(l) = \begin{cases} 2N + 1 - |l|, & -2N \leq l \leq 2N \\ 0, & \text{otherwise} \end{cases}$$

For $\gamma_{xy}(l)$ we have

$$\gamma_{xy}(l) = \begin{cases} 2N+1-|l-n_0|, & n_0-2N \le l \le n_0+2N \\ 0, & \text{otherwise} \end{cases}$$

2.62

(a)

$$\begin{aligned}
\gamma_{xx}(l) &= \sum_{n=-\infty}^{\infty} x(n)x(n-l) \\
\gamma_{xx}(-3) &= x(0)x(3) = 1 \\
\gamma_{xx}(-2) &= x(0)x(2) + x(1)x(3) = 3 \\
\gamma_{xx}(-1) &= x(0)x(1) + x(1)x(2) + x(2)x(3) = 5 \\
\gamma_{xx}(0) &= \sum_{n=0}^{3} x^2(n) = 7 \\
\text{Also } \gamma_{xx}(-l) &= \gamma_{xx}(l) \\
\text{Therefore } \gamma_{xx}(l) &= \left\{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\right\}
\end{aligned}$$

(b)

$$\gamma_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)y(n-l)$$

We obtain

$$\gamma_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$$

we observe that $y(n) = x(-n+3)$, which is equivalent to reversing the sequence $x(n)$. This has not changed the autocorrelation sequence.

2.63

$$\begin{aligned}
\gamma_{xx}(l) &= \sum_{n=-\infty}^{\infty} x(n)x(n-l) \\
&= \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases} \\
\gamma_{xx}(0) &= 2N+1
\end{aligned}$$

Therefore, the normalized autocorrelation is

$$\begin{aligned}
\rho_{xx}(l) &= \frac{1}{2N+1}(2N+1-|l|), -2N \le l \le 2N \\
&= 0, \text{ otherwise}
\end{aligned}$$

2.64

(a)

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} [s(n) + \gamma_1 s(n-k_1) + \gamma_2 s(n-k_2)] *$$
$$[s(n-l) + \gamma_1 s(n-l-k_1) + \gamma_2 s(n-l-k_2)]$$
$$= (1+\gamma_1^2+\gamma_2^2)\gamma_{ss}(l) + \gamma_1 [\gamma_{ss}(l+k_1) + \gamma_{ss}(l-k_1)]$$
$$+\gamma_2 [\gamma_{ss}(l+k_2) + \gamma_{ss}(l-k_2)]$$
$$+\gamma_1\gamma_2 [\gamma_{ss}(l+k_1-k_2) + \gamma_{ss}(l+k_2-k_1)]$$

(b) $\gamma_{xx}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$. Suppose that $k_1 < k_2$. Then, we can determine γ_1 and k_1. The problem is to determine γ_2 and k_2 from the other peaks.
(c) If $\gamma_2 = 0$, the peaks occur at $l = 0$ and $l = \pm k_1$. Then, it is easy to obtain γ_1 and k_1.

2.65

(a) The shift at which the crosscorrelation is maximum is the amount of delay D.
(b) variance = 0.01. Refer to fig 2.65-1.
(b) Delay D = 20. Refer to fig 2.65-1.

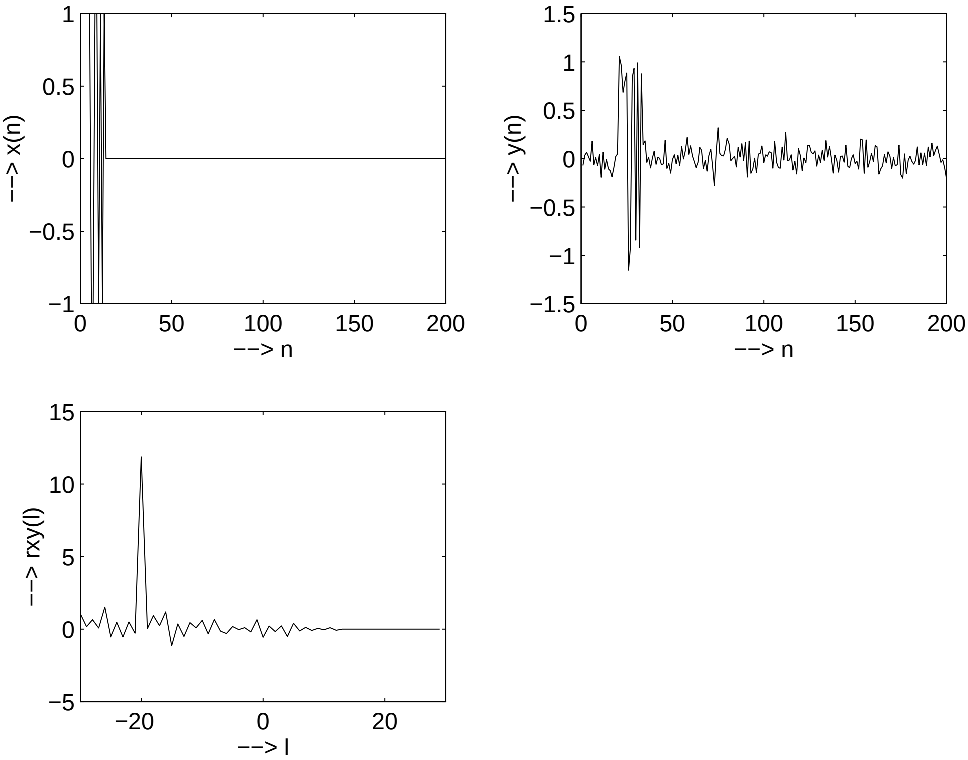

Figure 2.65-1: variance = 0.01

(c) variance = 0.1. Delay D = 20. Refer to fig 2.65-2.
(d) Variance = 1. delay D = 20. Refer to fig 2.65-3.
(e) $x(n) = \{-1,-1,-1,+1,+1,+1,+1,-1,+1,-1,+1,+1,-1,-1,+1\}$. Refer to fig 2.65-4.
(f) Refer to fig 2.65-5.

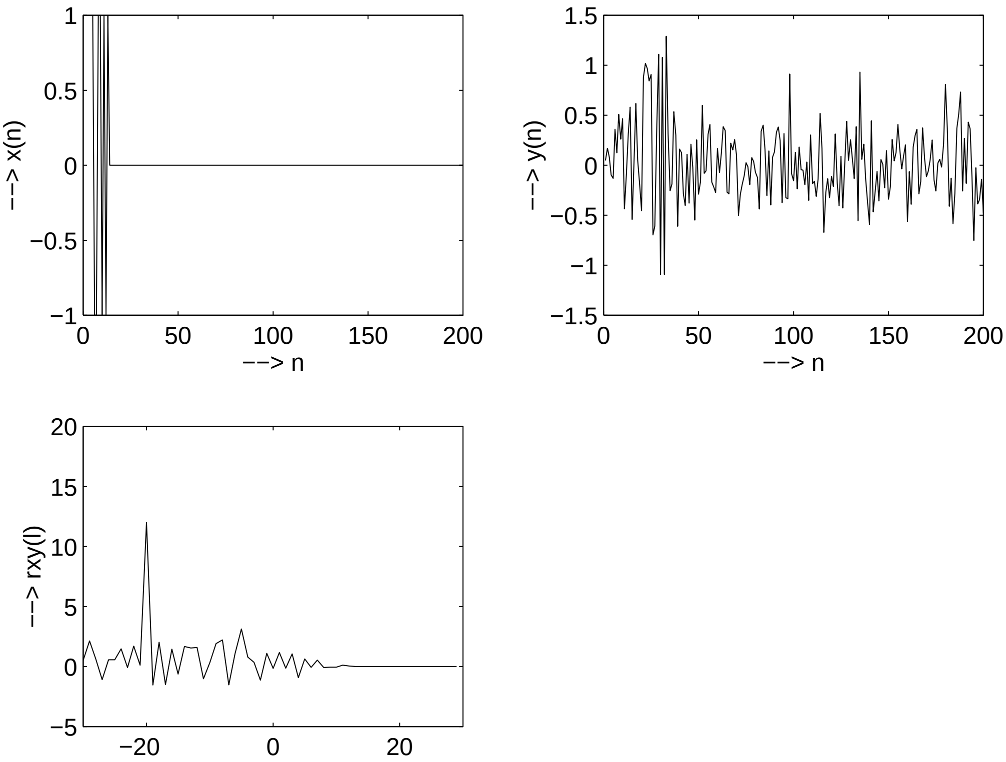

Figure 2.65-2: variance = 0.1

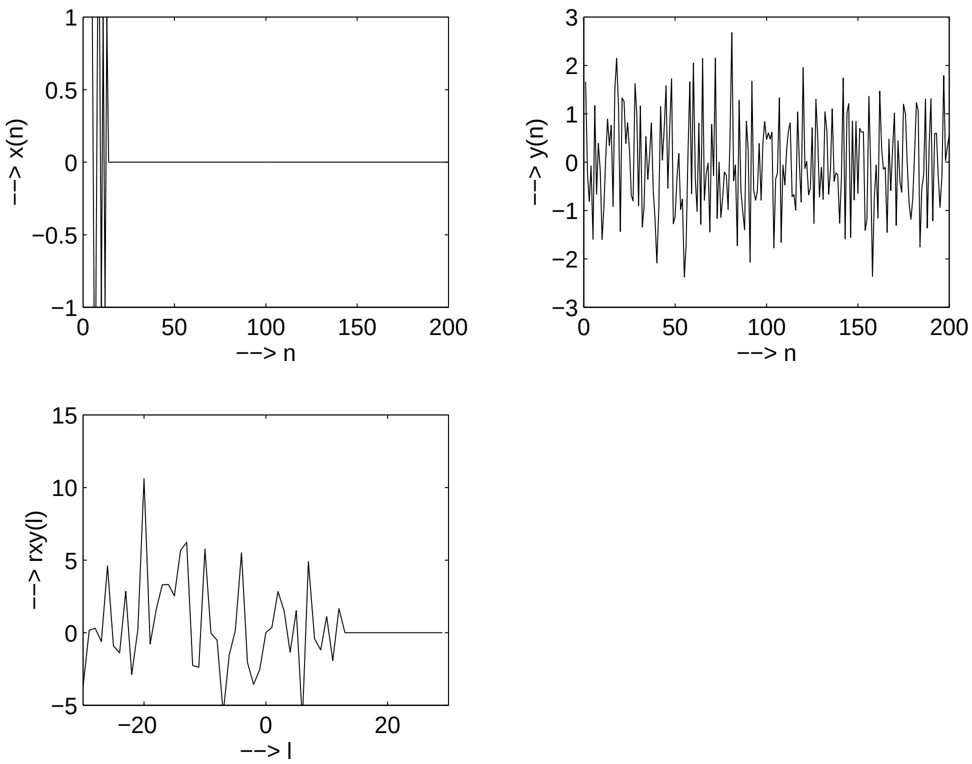

Figure 2.65-3: variance = 1

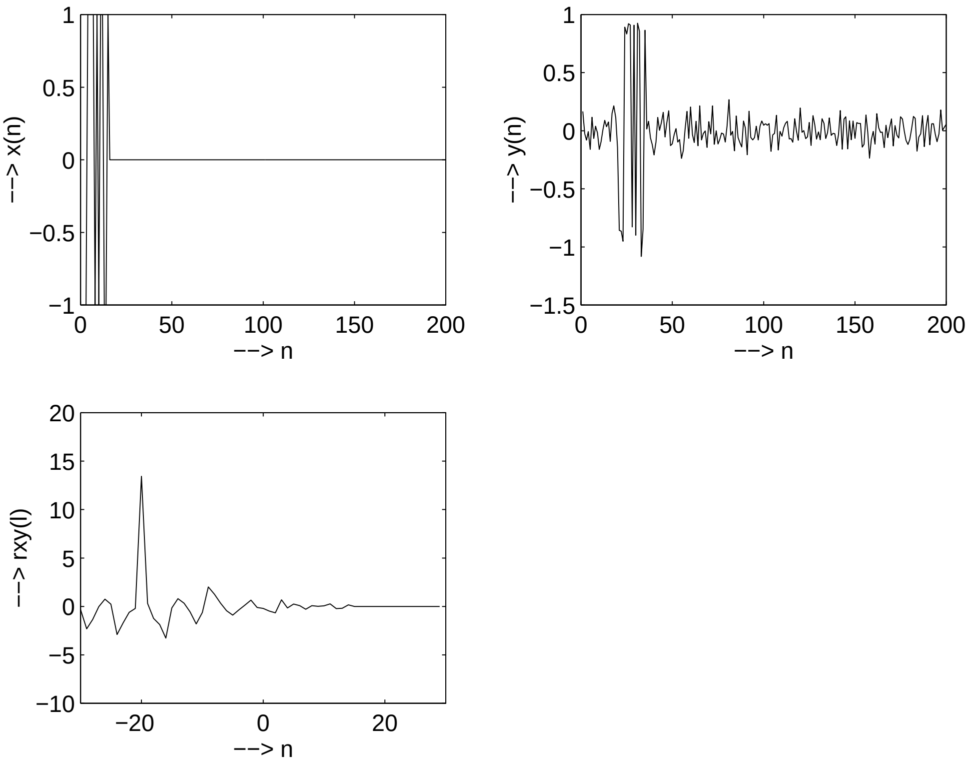

Figure 2.65-4:

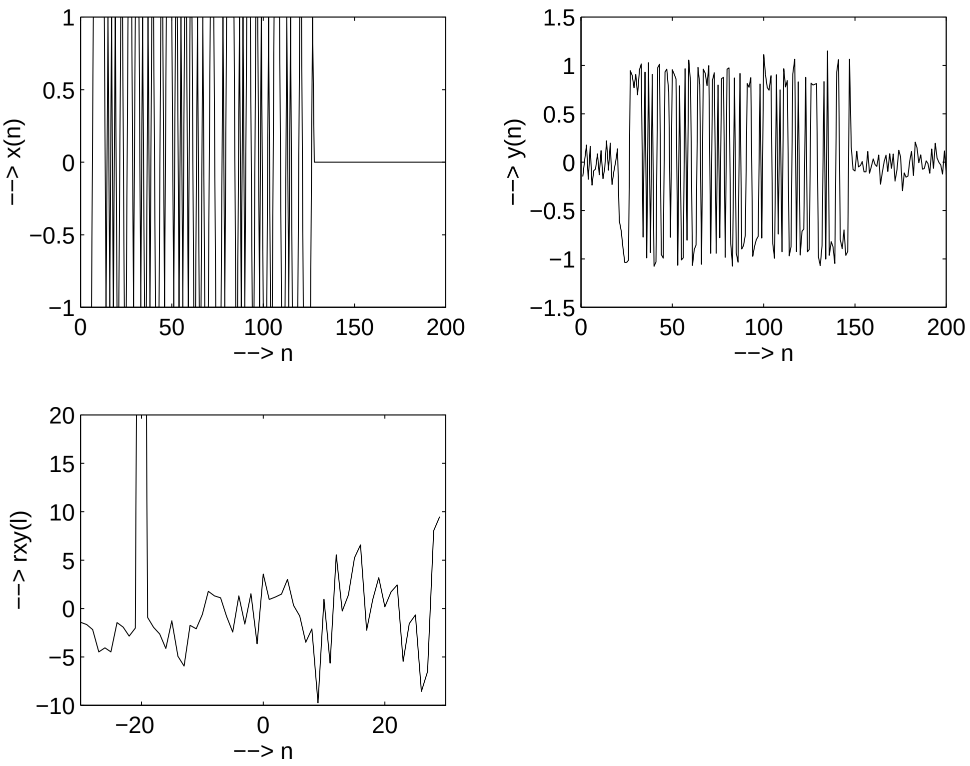

Figure 2.65-5:

2.66

(a) Refer to fig 2.66-1.
(b) Refer to fig 2.66-2.

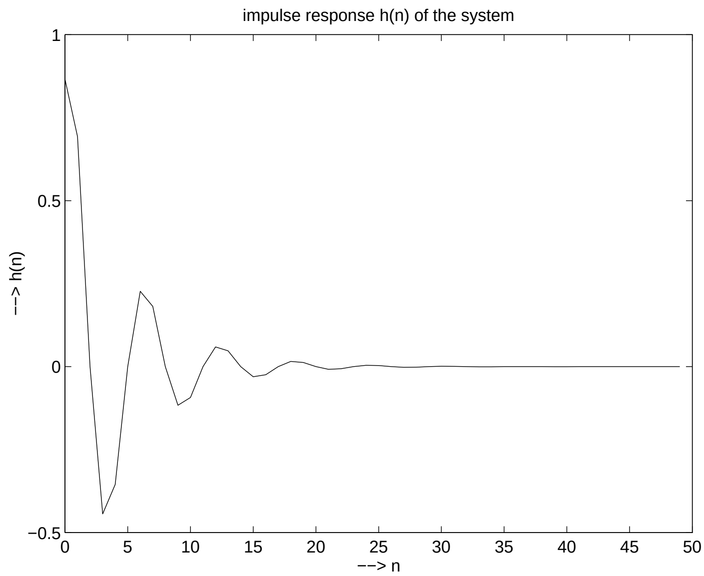

Figure 2.66-1:

(c) Refer to fig 2.66-3.
(d) The step responses in fig 2.66-2 and fig 2.66-3 are similar except for the steady state value after n=20.

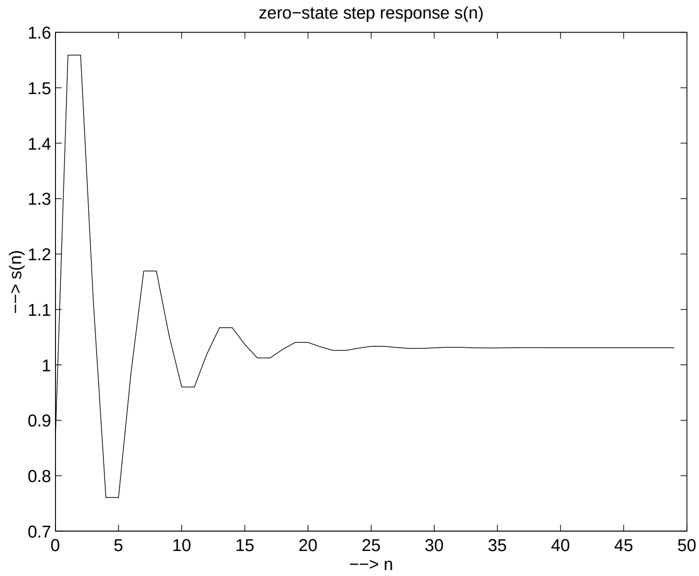

Figure 2.66-2:

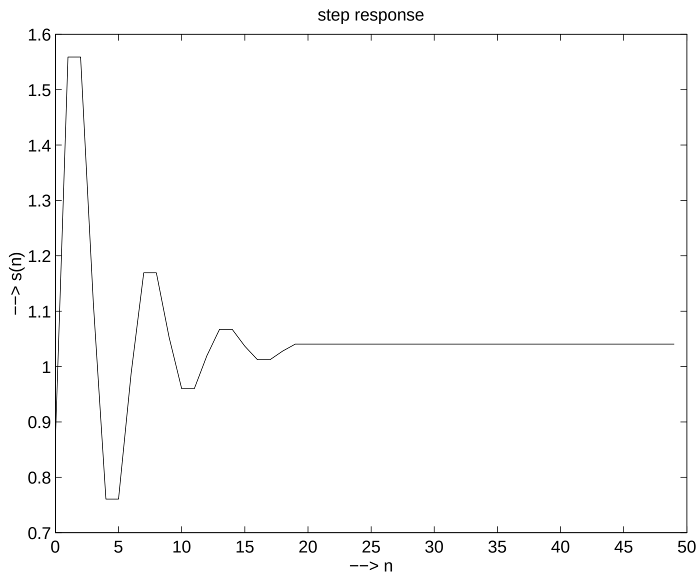

Figure 2.66-3:

2.67

Refer to fig 2.67-1.

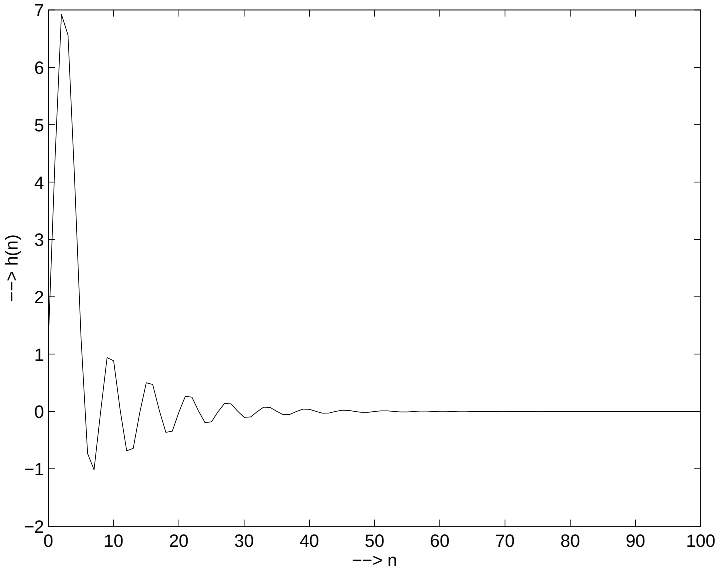

Figure 2.67-1: