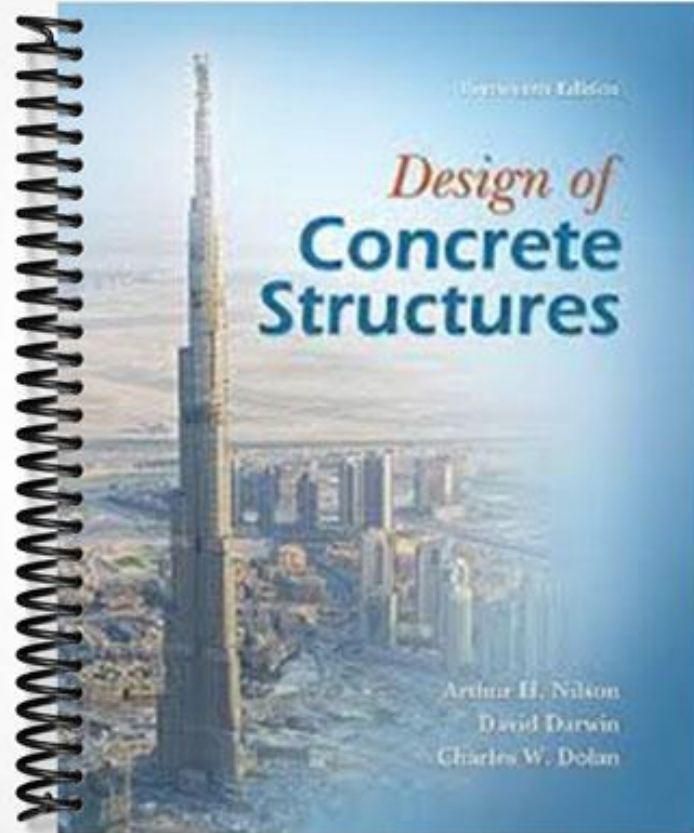


SOLUTIONS MANUAL



1.1

$$A_s = 6.0 \text{ in}^2$$

$$f_y = 40,000 \text{ psi}$$

$$E_s = 29,000,000 \text{ psi}$$

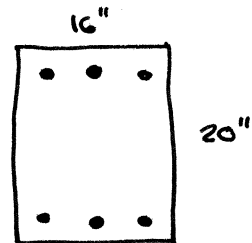
$$n = \frac{29,000,000}{3,600,000} = 8$$

$$A_g = 16 \times 20 = 320 \text{ in}^2$$

$$A_c = 320 - 6 = 314 \text{ in}^2$$

$$f_c' = 4,000 \text{ psi}$$

$$E_c = 3,600,000$$



a) $f_c = 1200 \text{ psi}$, $f_y = 40,000 \text{ psi}$

$$P = 1200(314 + 8 \times 6)$$

$$= 434,000 \text{ lbs}$$

$$P_s = 1200(8 \times 6) = 57,600 \text{ lbs}$$

$$P_s = 13.3\% \text{ of } P$$

b) $\epsilon_y = \frac{40,000}{29,000,000} = 0.00140$

for slow loading $f_c = 3000 \text{ psi}$

$$P = 3000(314) + 40,000(6) =$$

$$= 1,182,000 \text{ lbs}$$

$$P_s = 40,000(6) = 240,000 \text{ lb}$$

$$= 20.3\% P$$

c) $f_c = 3400 \text{ psi}$

$$P_u = 3400(314) + 40,000(6)$$

$$= 1,308,000 \text{ lb}$$

$$P_s = 240,000 \text{ lb} (18.3\% P_u)$$

a) $f_y = 60,000 \text{ psi}$
Same

$$\epsilon_y = \frac{60,000}{29,000,000} = 0.00207$$

$$f_c = 3300 \text{ psi}$$

$$P = 3300(314) + 60,000(6)$$

$$= 1,396,000 \text{ lbs}$$

$$P_s = 60,000(6) = 360,000 \text{ lb}$$

$$= 25.6\% P$$

$$P_u = 3400(314) + 60,000(6)$$

$$= 1,428,000 \text{ lb}$$

$$P_s = 360,000 \text{ lb} (25.2\% P_u)$$

Comments:

1. There is no difference in performance at $f_c = 1200 \text{ psi}$
2. As the strain increases, the steel with $f_y = 60,000 \text{ psi}$ contributes more to the total load and the column has a higher total load.
3. For the same cost, $f_y = 60,000 \text{ psi}$ provides a 9% increase in capacity.

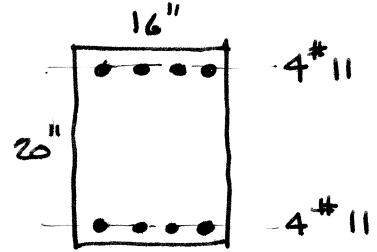
1.2

$$A_s = 8(1.56) = 12.48 \text{ in}^2$$

$$A_g = 320 \text{ in}^2 \quad A_c = 307.5 \text{ in}^2$$

$$n = 8 \quad E_s = 29,000,000 \text{ psi}$$

$$E_c = 3,600,000 \text{ psi}$$



$$f_y = 60,000 \text{ psi}$$

a)

$$P = 1200(307.5 + 8(12.48)) =$$

$$= 489,000 \text{ lb}$$

$$P_s = 1200(8)(12.48) = 120,000 \text{ lb}$$

(24.5% P)

b)

$$e_y = 0.00140, \quad f_c = 3000 \text{ psi}$$

$$P = 3000(307.5) + 40,000(12.48)$$

$$= 1,424,000 \text{ lb}$$

$$P_s = 40,000(12.48) = 500,000 \text{ lb}$$

(35.1% of P)

c)

$$f_c = 3400 \text{ psi} \text{ both cases}$$

$$P_0 = 3400(307.5) + 40,000(12.48)$$

$$= 1,547,000 \text{ lb}$$

$$P_s = 500,000 \text{ lb}$$

(32.3% of P_0)

• Same as $f_y = 40,000$

$$e_y = 0.00207 \quad f_c = 3300 \text{ psi}$$

$$P = 3300(307.5) + 60,000(12.48)$$

$$= 1,766,000 \text{ lb}$$

$$P_s = 60,000(12.48) = 750,000$$

(42.5% of P)

$$P_0 = 3400(307.5) + 60,000(12.48)$$

$$= 1,797,000 \text{ lb}$$

$$P_s = 750,000 \text{ lb}$$

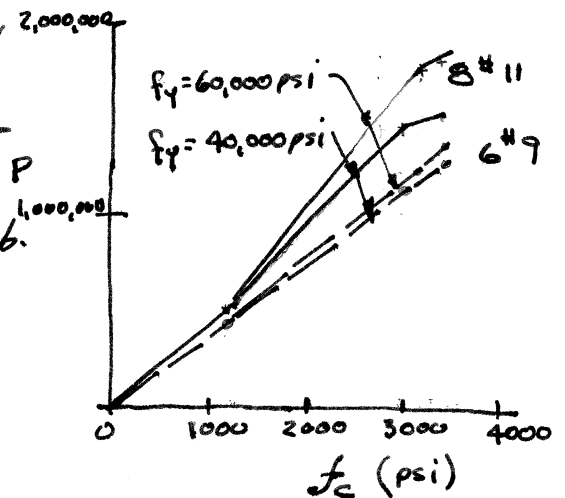
(41.7% of P_0)

Comments

1. There is no strength difference at $f_c = 1200 \text{ psi}$

2. There is a 16% strength increase at ultimate using $f_y = 60,000 \text{ psi}$. This occurs at virtually no cost increase

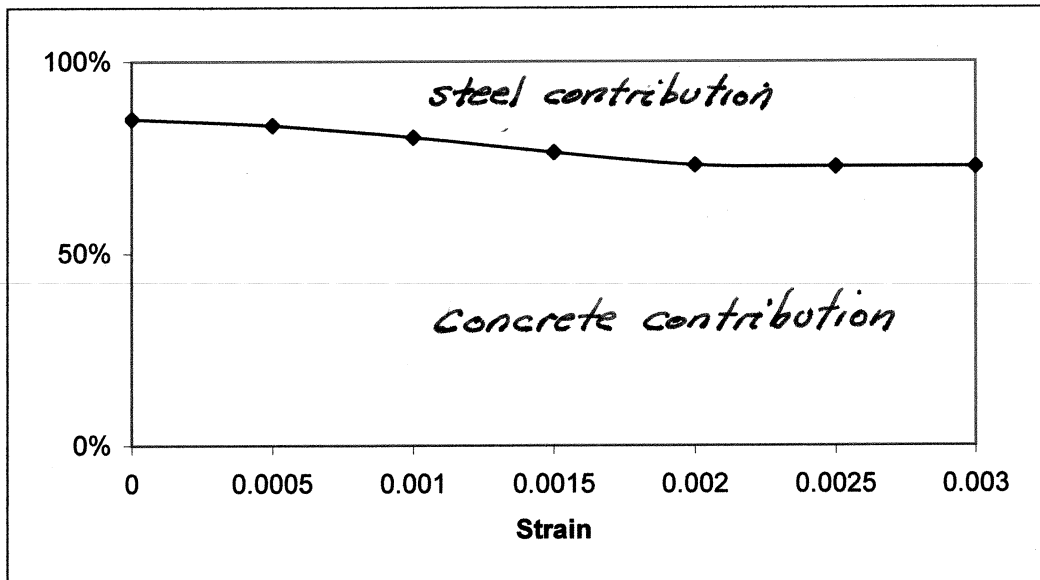
3. The higher steel ratio produces a stronger column - compare to prob. 1.1



1.3

$A_s = 10.12 \text{ in}^2$
 $A_c = 474 \text{ in}^2$
 $f_y = 60000 \text{ psi}$
 $f'_c = 4000 \text{ psi}$

$\epsilon_c = \epsilon_s$	f_c (psi)	P_c (kips)	f_s (psi)	P_s (kips)	P_{total} (kips)	P_c/P_{total}	P_s/P_{total}
0	0	0	0	0	0	85.0%	15.0%
0.0005	1600	758.4	15000	151.8	910.2	83.3%	16.7%
0.001	2600	1232.4	30000	303.6	1536	80.2%	19.8%
0.0015	3100	1469.4	45000	455.4	1924.8	76.3%	23.7%
0.002	3300	1564.2	57000	576.84	2141.04	73.1%	26.9%
0.0025	3400	1611.6	60000	607.2	2218.8	72.6%	27.4%
0.003	3400	1611.6	60000	607.2	2218.8	72.6%	27.4%



1.4 A 20 x 24 in. column is made of the same concrete as Examples 1.1 and 1.2 but reinforced with six No. 11 (No. 36) bars with $f_y = 60$ ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1400 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly and (d) the maximum load if the section is loaded rapidly. The area of one No. 11 (No. 36) bar is 1.56 in^2 . Determine the percent of the load carried by the steel and the concrete for each combination.

▣ Reinforcement Areas

Given Properties

$$f_c := 4000 \text{ psi} \quad f_y := 60000 \text{ psi} \quad f_c := 1400 \text{ psi} \quad n := 8 \quad E_s := 29000000 \text{ psi}$$

Column Properties

$$b := 20 \text{ in} \quad h := 24 \text{ in} \quad A_{st} := 6 \cdot A_{s11} = 9.36 \text{ in}^2 \quad \text{The total area of steel } A_{st} \text{ is six no. 11 bars}$$

Part (a) Compute the axial capacity of the section loaded below the elastic limit.

Solution: The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is $(n-1)A_{st}$.

$$A_g := b \cdot h \quad A_g = 480 \text{ in}^2 \quad A_{st} = 9.36 \text{ in}^2 \quad \text{Reinforcement ratio} = \frac{A_{st}}{A_g} = 0.0195$$

$$P := f_c \cdot [A_g + (n - 1) \cdot A_{st}] \quad P = 764 \text{ kip}$$

$$P_c := f_c \cdot (A_g - A_{st}) \quad P_c = 659 \text{ kip}$$

$$P_s := f_c \cdot n \cdot A_{st} \quad P_s = 105 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 86.3 \quad 100 \cdot \frac{P_s}{P} = 13.7$$

Part (b): Compute the capacity of the column when the steel begins to yield $\epsilon := 0.002069$ or 2/10 of one percent

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve, but we are at the elastic limit of the steel.

$$f_s := \epsilon \cdot E_s \quad f_s = 60001 \text{ psi}$$

$$\text{From Figure 1.16} \quad f_c := 3100 \text{ psi} \quad \text{for slow loading}$$

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$A_c := A_g - A_{st}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st} \quad P = 2021 \text{ kip}$$

$$P_c := f_c \cdot A_c \quad P_c = 1459 \text{ kip}$$

$$P_s := f_s \cdot A_{st} \quad P_s = 562 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 72.2 \quad 100 \cdot \frac{P_s}{P} = 27.8$$

Part (c): Compute the maximum load capacity of the section

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve, but we are in the plastic range of the steel.

$$f_s := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_c := 3400 \text{ psi} \quad \text{for slow loading}$$

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$A_c := A_g - A_{st}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st} \quad P = 2162 \text{ kip}$$

$$P_c := f_c \cdot A_c \quad P_c = 1600 \text{ kip}$$

$$P_s := f_s \cdot A_{st} \quad P_s = 562 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 74.0 \quad 100 \cdot \frac{P_s}{P} = 26.0$$

If we reexamine the problem with a fast loading as would occur in a building, then the concrete stress would be

$$f_c := 4000 \text{ psi}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st} \quad P = 2444 \text{ kip}$$

$$P_c := f_c \cdot A_c \quad P_c = 1883 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 77.0$$

$$P_s := f_s \cdot A_{st} \quad P_s = 562 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 23.0$$

Comments

1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the concrete.
2. The slow loading is approximately 88% of the fast load scenario

1.5 A 24 in. diameter column is made of the same concrete as Examples 1.1 and 1.2. The area of reinforcement equals 2.1 percent of the gross cross section (i.e., $A_s = 0.021A_g$) and $f_y = 60$ ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1200 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly, (d) the maximum load if the section is loaded rapidly and (e) the maximum load the reinforcement in the column is raised to 6.5 percent and the column is loaded slowly. Comment on your answer, especially the percent of the load carried by the steel and the concrete for each combination.

Given Properties

$$f_c := 4000\text{psi} \quad f_y := 60000\text{psi} \quad f_c := 1200\text{psi} \quad n := 8 \quad E_g := 29000000\text{psi}$$

Column Properties

$$d := 24\text{in} \quad A_g := \pi \frac{d^2}{4} \quad \rho := 0.021 \quad \rho \text{ is the reinforcement ratio or the fraction of the section that is steel}$$

$$A_{st} := \rho A_g \quad \text{The total area of steel } A_{st} \text{ is } A_{st} = 9.5 \text{ in}^2$$

Part (a) Compute the axial capacity of the section loaded below the elastic limit.

Solution: The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is $(n-1)A_{st}$.

$$A_c := A_g - A_{st} \quad A_g = 452 \text{ in}^2 \quad A_{st} = 9.50 \text{ in}^2 \quad A_c = 443 \text{ in}^2$$

$$P := f_c [A_g + (n-1)A_{st}] \quad P = 623 \text{ kip} \quad \text{Concrete and steel contribution}$$

$$P_c := f_c (A_g - A_{st}) \quad P_c = 531 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 85.4$$

$$P_s := f_c \cdot n \cdot A_{st} \quad P_s = 91 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 14.6$$

$$\epsilon_y := \frac{f_y}{E_s}$$

Part (b): Compute the capacity of the column when the steel begins to yield

$$\epsilon_y = 0.00207 \quad \text{or } 2/10 \text{ of one percent}$$

Examining Figure 1.16, we are **beyond the elastic portion of the concrete stress strain curve**, but we are at the elastic limit of the steel.

$$f_s := \epsilon_y \cdot E_s \quad f_s = 60000 \text{ psi}$$

From Figure 1.16 $f_c := 3100\text{psi}$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$P := f_c A_c + f_s A_{st} \quad P = 1943 \text{ kip}$$

$$P_c := f_c A_c \quad P_c = 1373 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 27.4$$

$$P_s := f_s A_{st} \quad P_s = 570 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 29.3$$

Part (c): Compute the maximum load capacity of the section if loaded slowly

Examining Figure 1.16, we are **beyond the elastic portion of the concrete stress strain curve**

and we are in the plastic range of the steel.

$$f_s := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_c := 3400 \text{ psi for slow loading}$$

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1:

$$P := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 2076 \text{ kip}$$

$$P_c := f_c \cdot A_c$$

$$P_c = 1506 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 72.5$$

$$P_s := f_s \cdot A_{st}$$

$$P_s = 570 \text{ kip}$$

$$100 \cdot \frac{P_s}{P} = 27.5$$

Part (d): If we reexamine the problem with a fast loading as would occur in a building, then the concrete stress would be

$$f_c := 4000 \text{ psi}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 2342 \text{ kip}$$

$$P_c := f_c \cdot A_c$$

$$P_c = 1772 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 75.7 \quad 100 \cdot \frac{P_s}{P} = 24.3$$

$$P_s := f_s \cdot A_{st}$$

$$P_s = 570 \text{ kip}$$

Part (e): Determine the capacity for a slow loaded column with the steel changed to 6.5%

$$A_{st} := 0.065 \cdot A_g$$

$$A_{st} = 29.4 \text{ in}^2$$

$$f_s := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_c := 3400 \text{ psi for slow loading}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 3270 \text{ kip}$$

$$P_c := f_c \cdot A_c$$

$$P_c = 1506 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 46.0$$

$$P_s := f_s \cdot A_{st}$$

$$P_s = 1764 \text{ kip}$$

$$100 \cdot \frac{P_s}{P} = 54.0$$

Comments

1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the concrete.
2. The slow loading is approximately 88% of the fast load scenario - This is slightly higher than the 0.85 given in eq. 1.8.

1.5 A 24 in. diameter column is made of the same concrete as Examples 1.1 and 1.2. The area of reinforcement equals 2.1 percent of the gross cross section (i.e., $A_s = 0.021A_g$) and $f_y = 60$ ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1200 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly, (d) the maximum load if the section is loaded rapidly and (e) the maximum load the reinforcement in the column is raised to 6.5 percent and the column is loaded slowly. Comment on your answer, especially the percent of the load carried by the steel and the concrete for each combination.

Given Properties

$$f_c := 4000\text{psi} \quad f_y := 60000\text{psi} \quad f_c := 1200\text{psi} \quad n := 8 \quad E_s := 29000000\text{psi}$$

Column Properties

$$d := 24\text{in} \quad A_g := \pi \frac{d^2}{4} \quad \rho := 0.021 \quad \rho \text{ is the reinforcement ratio or the fraction of the section that is steel}$$

$$A_{st} := \rho A_g \quad \text{The total area of steel } A_{st} \text{ is } A_{st} = 9.5 \text{ in}^2$$

Part (a) Compute the axial capacity of the section loaded below the elastic limit.

Solution: The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is $(n-1)A_{st}$.

$$A_c := A_g - A_{st} \quad A_g = 452 \text{ in}^2 \quad A_{st} = 9.50 \text{ in}^2 \quad A_c = 443 \text{ in}^2$$

$$P := f_c [A_g + (n-1)A_{st}] \quad P = 623 \text{ kip} \quad \text{Concrete and steel contribution}$$

$$P_c := f_c (A_g - A_{st}) \quad P_c = 531 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 85.4$$

$$P_s := f_y n A_{st} \quad P_s = 91 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 14.6$$

Part (b): Compute the capacity of the column when the steel begins to yield $\epsilon_y := \frac{f_y}{E_s}$

$$\epsilon_y = 0.00207 \quad \text{or } 2/10 \text{ of one percent}$$

Examining Figure 1.16, we are **beyond the elastic portion of the concrete stress strain curve**, but we are at the elastic limit of the steel.

$$f_s := \epsilon_y E_s \quad f_s = 60000 \text{ psi}$$

From Figure 1.16 $f_{s,max} := 3100\text{psi}$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$P_{max} := f_c A_c + f_s A_{st} \quad P = 1943 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 27.4$$

$$P_{c,max} := f_c A_c \quad P_c = 1373 \text{ kip}$$

$$P_{s,max} := f_s A_{st} \quad P_s = 570 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 29.3$$

Part (c): Compute the maximum load capacity of the section if loaded slowly

Examining Figure 1.16, we are **beyond the elastic portion of the concrete stress strain curve**

and we are in the plastic range of the steel.

$$f_{s,max} := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_{s,max} := 3400 \text{ psi for slow loading}$$

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$P_{s,max} := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 2076 \text{ kip}$$

$$P_{c,max} := f_c \cdot A_c$$

$$P_c = 1506 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 72.5$$

$$P_{s,max} := f_s \cdot A_{st}$$

$$P_s = 570 \text{ kip}$$

$$100 \cdot \frac{P_s}{P} = 27.5$$

Part (d): If we reexamine the problem with a fast loading as would occur in a building, then the concrete stress would be

$$f_{c,max} := 4000 \text{ psi}$$

$$P_{s,max} := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 2342 \text{ kip}$$

$$P_{c,max} := f_c \cdot A_c$$

$$P_c = 1772 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 75.7 \quad 100 \cdot \frac{P_s}{P} = 24.3$$

$$P_{s,max} := f_s \cdot A_{st}$$

$$P_s = 570 \text{ kip}$$

Part (e): Determine the capacity for a slow loaded column with the steel changed to 6.5%

$$A_{s,max} := 0.065 \cdot A_g$$

$$A_{st} = 29.4 \text{ in}^2$$

$$f_{s,max} := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_{s,max} := 3400 \text{ psi for slow loading}$$

$$P_{s,max} := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 3270 \text{ kip}$$

$$P_{c,max} := f_c \cdot A_c$$

$$P_c = 1506 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 46.0$$

$$P_{s,max} := f_s \cdot A_{st}$$

$$P_s = 1764 \text{ kip}$$

$$100 \cdot \frac{P_s}{P} = 54.0$$

Comments

1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the concrete.
2. The slow loading is approximately 88% of the fast load scenario - This is slightly higher than the 0.85 given in eq. 1.8.

2.1

$$f'_c = 6000 \text{ psi}$$

(a) No prior results

$$f'_{cr} = f'_c + 0.1 f'_c + 700 \text{ psi} = 6000 + 0.1 \times 6000 + 700 = 7300 \text{ psi}$$

(b) 20 prior tests for concrete with f'_c within 1000 psi of f'_c for project. $S_s = 580 \text{ psi}$ -

$$\text{From Table 1.1, } 1.08 \times 580 = 626 \text{ psi}$$

Because $f'_c > 5000 \text{ psi}$, use Eqs. (2.1) + (2.2b)

$$f'_{cr} = f'_c + 1.34 S_s = 6000 + 1.34 \times 626 = 6840 \text{ psi}$$

$$f'_{cr} = 0.9 f'_c + 2.33 S_s = 0.9 \times 6000 + 2.33 \times 626 = 6860 \text{ psi}$$

$$\text{Use } f'_{cr} = 6860 \text{ psi}$$

(c) 30 prior tests for concrete with f'_c within 1000 psi of f'_c for project. $S_s = 590 \text{ psi}$ -

$$f'_{cr} = f'_c + 1.34 S_s = 6000 + 1.34 \times 590 = 6790 \text{ psi}$$

$$f'_{cr} = 0.9 f'_c + 2.33 S_s = 0.9 \times 6000 + 2.33 \times 590 = 6770 \text{ psi}$$

$$\text{Use } f'_{cr} = 6790 \text{ psi}$$

2.2

(a) For $f'_c = 4000 \text{ psi}$, the strength results indicate satisfactory concrete quality because (1) no individual test is below $f'_c - 500 \text{ psi} = 3500 \text{ psi}$, and (2) every arithmetic average of any three consecutive tests equals or exceeds f'_c .

(b) For $S_s = 510 \text{ psi}$ for 30 consecutive tests,

calculate f'_{cr} using Eqs (2.1) and (2.2a)

$$f'_{cr} = f'_c + 1.34S_s = 4000 + 1.34 \times 510 = 4680 \text{ psi}$$

$$f'_{cr} = f'_c + 2.33S_s - 500 \text{ psi} = 4000 + 2.33 \times 510 - 500 \\ = 4690 \text{ psi}$$

Use 4690 psi

$$(4590 + 4750 + 5280 + 4210 + 4460 + 4170 + 3750 + 5110 + 4640 + 4170) / 10 \\ = 4510 < f'_{cr}$$

Because the average compressive strength is less than f'_{cr} , the water-cement ratio must be decreased, either by adding cement or reducing water, to increase strength. If the water is reduced, a water reducer must be added or the quantity of water reducer must be increased to maintain concrete workability.

Problem 3.1 A rectangular beam made using concrete with $f'_c = 6000$ psi and steel with $f_y = 60,000$ psi had a width $b = 20$ in., and an effective depth of $d = 17.5$ in and an $h = 20$ in. The Concrete modulus of rupture $f_r = 530$ psi. The elastic modulus of the steel and concrete are, respectively $E_c = 4,030,000$ psi and $E_s = 29,000,000$ psi. The area of steel is four No. 11 (No. 36) bars.

- Find the maximum service load that can be resisted without stressing the concrete above $0.45 f'_c$ or the steel above $0.40 f_y$.
- Determine if the beam will show cracking before reaching the service load
- Compute the nominal moment capacity of the beam
- Compute the ratio of the nominal capacity of the beam to the maximum service level capacity and compare your findings to the ACI load factors and strength reduction factor.

▣ Reinforcement sizes

Given data

Note: for all MathCAD based solutions, the area and diameter of reinforcement bars is in a common database. Hence the notation A_{s11} indicates the area of a single No. 11 (No. 36) bar.

$$A_s := 4 \cdot A_{s11} \quad A_s = 6.24 \text{ in}^2 \quad E_s := 29000000 \text{ psi}$$

$$b := 20 \text{ in} \quad d := 17.5 \text{ in} \quad h := 20 \text{ in}$$

$$f'_c := 6000 \text{ psi} \quad f_y := 60000 \text{ psi} \quad E_c := 57000 \sqrt{f'_c} \text{ psi} \quad E_c = 4415 \text{ ksi}$$

$$f_r := 7.5 \sqrt{f'_c} \text{ psi} \quad f_r = 581 \text{ psi} \quad n := \frac{E_s}{E_c} \quad n = 6.6$$

- Find the maximum service load that can be resisted without stressing the concrete above $0.45 f'_c$ or the steel above $0.40 f_y$.

$$f_c := 0.45 f'_c \quad f_c = 2700 \text{ psi}$$

$$f_s := 0.40 f_y \quad f_s = 36000 \text{ psi}$$

$$\rho := \frac{A_s}{b \cdot d} \quad \rho = 0.018$$

$$k := \sqrt{(\rho \cdot n)^2 + 2 \rho \cdot n} - \rho \cdot n \quad k = 0.381$$

$$j := 1 - \frac{k}{3} \quad j = 0.873$$

Moment due to concrete limits

$$M_{sc} := \frac{1}{2} \cdot f_c \cdot b \cdot k \cdot d \cdot \left(d - \frac{k \cdot d}{3} \right) \quad M_{sc} = 229 \text{ ft} \cdot \text{kip}$$

Moment due to steel limit

$$M_{ss} := A_s \cdot f_s \cdot j \cdot d \quad M_{ss} = 286 \text{ ft} \cdot \text{kip}$$

The maximum service moment is the minimum of the two values.

$$M_s := \min(M_{ss}, M_{sc}) \quad M_s = 229 \text{ ft} \cdot \text{kip}$$