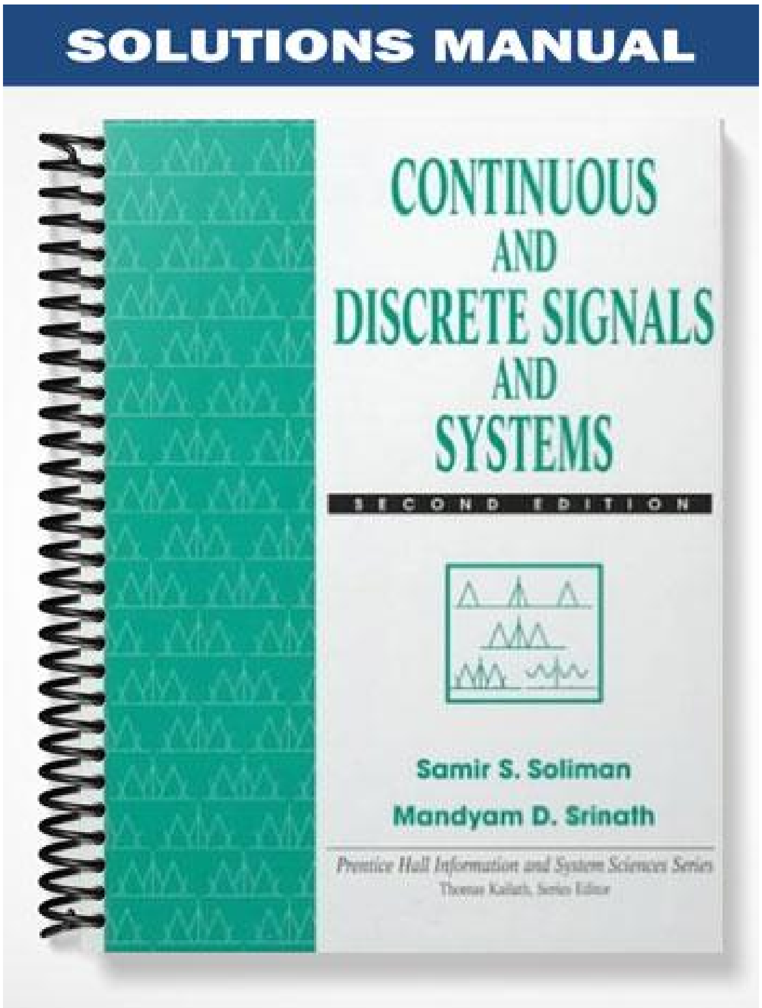
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AND
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SECOND EDITION
Samir S. Soliman
Mandyam D. Srinath
Prentice Hall Information and System Sciences Series
Thomas Kailath, Series Editor

SOLUTIONS MANUAL

CONTINUOUS AND DISCRETE SIGNALS AND SYSTEMS

SECOND EDITION

Samir S. Soliman
Mandyam D. Srinath

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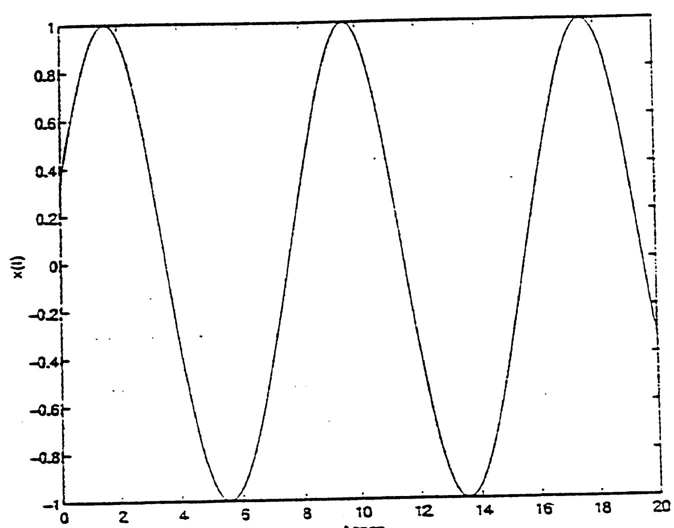
CONTENTS

Chapter 1

1.1

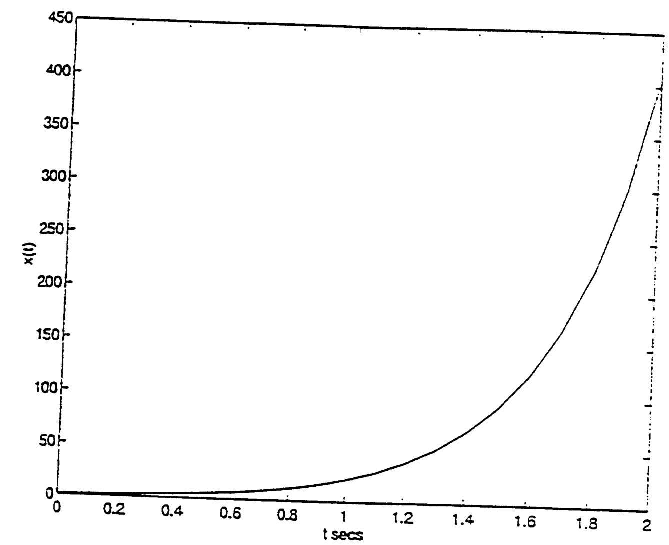
Signal	Period
$\cos(\pi t)$	2
$\sin(2\pi t)$	1
$\cos(3\pi t)$	$\frac{1}{3}$
$\sin(4\pi t)$	$\frac{1}{2}$
$\cos(\frac{\pi}{2} t)$	4
$\sin(\frac{\pi}{3} t)$	6
$\cos(\frac{5\pi}{2} t)$	$\frac{4}{5}$
$\sin(\frac{4\pi}{3} t)$	$\frac{3}{2}$
$\cos(\frac{\pi}{4} t)$	8
$\sin(\frac{2\pi}{3} t)$	$\frac{1}{3}$
$\cos(\frac{3\pi}{5} t)$	$\frac{10}{3}$

1.2

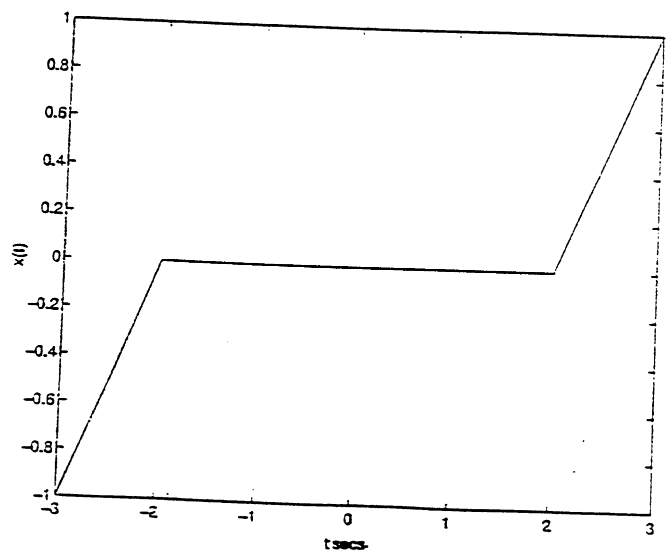


(a) $x(t) = \sin(\frac{\pi}{4} t + 20°)$

1.2

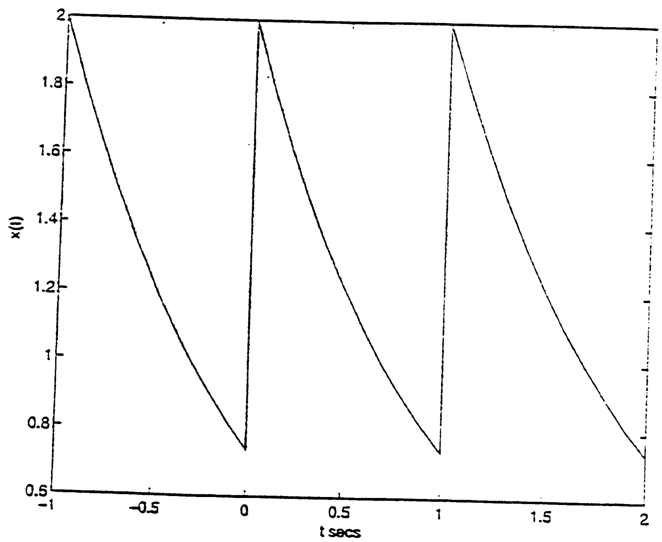


(b) $x(t) = t + e^{3t}$



(c) $x(t) = \begin{cases} t+2 & t \le -2 \\ 0 & -2 \le t \le 2 \\ t-2 & 2 \le t \end{cases}$

1.2



(d) $x(t) = 2\exp[-t]$, $0 \le t < 1$, and $x(t+1) = x(t)$

1.3 $x(t+T) = x(t)$ for every t.

if we substitute $t = t+T$

$x(t+2T) = x(t+T) = x(t)$

i.e $x(t)$ is periodic with period $2T$

if we substitute $t = t+2T$

$x(t+3T) = x(t+2T) = x(t)$

i.e $x(t)$ is periodic with period $3T$

In general $x(t)$ is periodic with period nT

1.4 $x_3(t) = a x_1(t) + b x_2(t)$

$x_3(t+T) = a x_1(t+T) + b x_2(t+T)$

$= a x_1(t) + b(x_2(t)$

$= x_3(t)$

1.5 (a) $\sin \frac{2\pi}{3}t$ is periodic with period 3 while $\sin \frac{16\pi t}{3}$ is peridic with period $\frac{3}{8}$. Then $x(t)$ is periodic with period 3.

1.5 (b) $\exp[j\frac{2\pi}{6}t]$ is periodic with period 6 and

$\exp[-j\pi t]$ is periodic with period 2.

Then $x(t)$ is periodic with period 6.

(c) $\exp[j\frac{2\pi}{5}t]$ is periodic 5 while

$\exp[j3t]$ is periodic with period $\frac{2\pi}{3}$

Since the ratio of these two periods, $\frac{5}{\frac{2\pi}{3}} = \frac{15}{2\pi}$

is not a rational number, $x(t)$ is not periodic.

1.6 (a) $T_1 = 6$, $T_2 = \frac{3}{4}$, $\frac{T_1}{T_2} = 8$. Periodic, with period $T = 6$.

(b) $T_1 = \frac{7}{6}$, $T_2 = \frac{5}{6}$, $\frac{T_1}{T_2} = \frac{7}{5}$. Periodic, with period $T = 35$.

(d) $\exp[\frac{5}{6}t]$ is not periodic, so that $x(t)$ is not periodic.

(e) $T_1 = \frac{16}{3}$, $T_2 = \frac{8\pi}{3}$, $\frac{T_1}{T_2} = \frac{8\pi}{3}$ is not a rational number. $x(t)$ is not periodic.

1.7 $\exp[j\omega t] = \cos\omega t + j\sin\omega t$ (Euler's formula)

Since $\cos\omega t$ and $\sin\omega t$ are periodic with period $2\pi/\omega$, then their linear combination is also periodic with period $2\pi/\omega$.

1.8 Since $x(t)$ is periodic with period T, then

$$x(at) = x(at+T)$$
$$= x(a(t+T/a))$$

For $x(at)$ to be periodic with period T_1, one needs $T_1 = T/a$

Similarly,

$$x(t/b) = x(t/b+T)$$
$$= x\left(\frac{1}{b}(t+bT)\right)$$

For $x(t/b)$ to be periodic with period T_2, one needs

$$T_2 = bT$$

For $\sin t$, $T = 2\pi$

For $\sin 2t$, $T_1 = \pi = T/2$

For $\sin t/2$ $T_2 = 4\pi = 2T$

1.9 (a)

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} t^2\sin^2\left(\frac{\pi}{3}t\right)dt$$

$$= \lim_{T\to\infty} \frac{1}{4T}\int_{-T}^{T} t^2\left(1-\cos\left(\frac{2\pi}{3}t\right)\right)dt$$

$$= \lim_{T\to\infty} \frac{1}{4T}\left[\frac{2T^3}{3} - \int_{-T}^{T}\cos\left(\frac{2\pi}{3}t\right)dt\right] \longrightarrow \infty$$

so that $x(t)$ is neither an energy nor a power signal.

(b)

$$E = \lim_{T\to\infty}\int_{-T}^{T}\exp[-4|t|]\sin^2(\pi t)dt$$

$$= \lim_{T\to\infty}\int_{0}^{T}\exp[-4t][1-\cos(2\pi t)]dt = \frac{\pi^2}{\pi^2+4}$$

so that $x(t)$ is an energy signal with $P = 0$.

(c) $$P = \lim \frac{1}{2T}\int_{-T}^{T} \exp(8|t|)dt = \lim \frac{e^{8T}-1}{T} \longrightarrow \infty$$

so that $x(t)$ is neither an energy nor a power signal.

(d) $x(t)$ is periodic with period $T = 12/5$s, so that it is a power signal with

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{T}\int_0^T |\exp[j\frac{5\pi}{6}t|^2 dt = 1$$

(e) $x(t) = \frac{1}{2}[\sin(\frac{25\pi}{24}t) - \sin(\frac{7\pi}{24}t)]$ and is periodic with $T = 48$ secs. Thus

$$P = \frac{1}{4T}\int_0^T [\sin(\frac{25\pi}{24}t) - \sin(\frac{7\pi}{24}t)]^2 dt$$

$$= \frac{1}{4T}\int_0^T \sin^2(\frac{25\pi}{24}t)dt + \frac{1}{4T}\int_0^T \sin^2(\frac{7\pi}{24}t)]dt - \frac{1}{2T}\int_0^T \sin(\frac{25\pi}{24}t)\sin(\frac{7\pi}{24}t)dt$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

(f) $$P = \lim_{T\to\infty} \frac{1}{2T}[\int_{-T}^{0} 1dt + \int_0^T e^{-6t}dt]$$

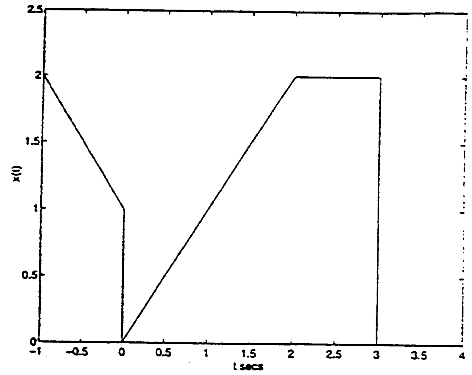
$$= \lim_{T\to\infty} \frac{1}{2T}[T + \frac{1-e^{-6t}}{6}] = \frac{1}{2}$$

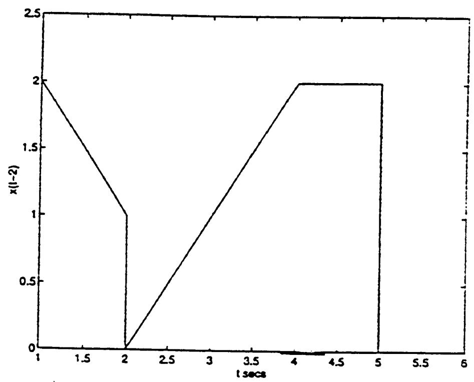
so that $x(t)$ is a power signal.

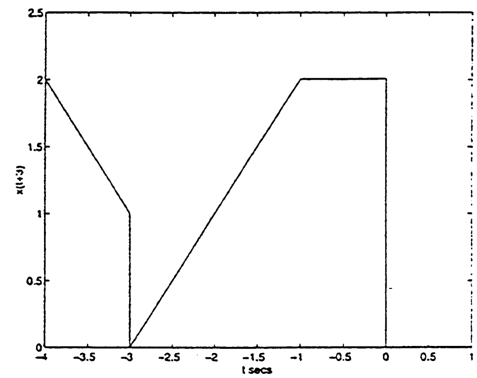
1.10 $$\left|\int_0^T x(t)\,dt\right|^2 \le \int_0^T |x(t)|^2 dt = PT$$

$$\left|\int_0^T x(t)\,dt\right| \le \sqrt{PT}$$

1.11

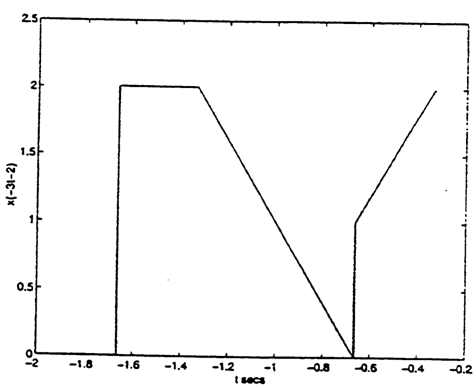


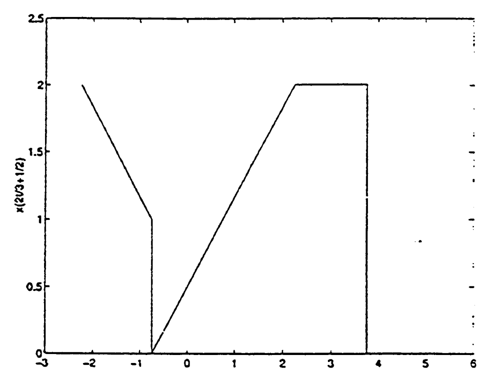




$$x(t-2)=\begin{cases} -t+3 & 1 \le t < 2 \\ t-2 & 2 \le t < 2 \\ 2 & 4 \le t < 5 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t+3)=\begin{cases} -t-3 & -4 \le t < -3 \\ t+3 & -3 \le t < -1 \\ 2 & -1 \le t < 0 \\ 0 & \text{otherwise} \end{cases}$$

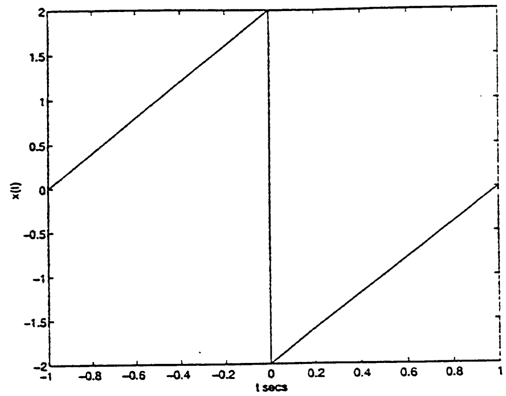


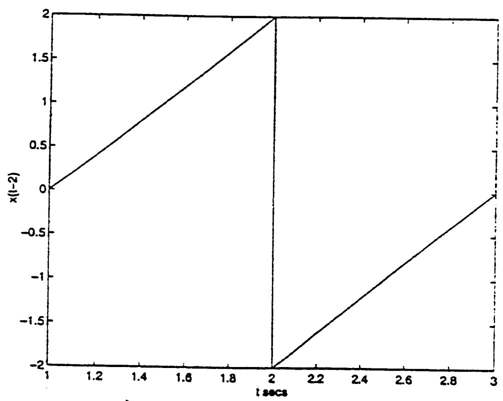


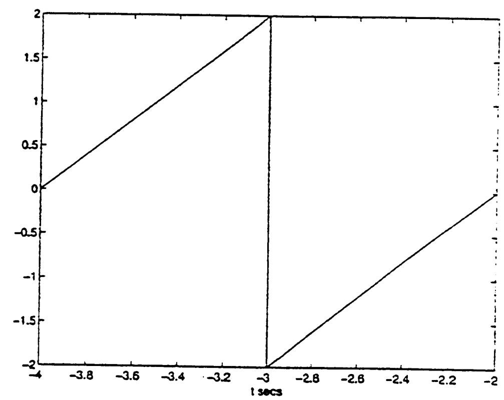
$$x(-3t-2)=\begin{cases} 3t+5 & -\frac{2}{3} < t \le -\frac{1}{3} \\ -3t-2 & -\frac{4}{3} < t \le -\frac{2}{3} \\ 2 & -\frac{5}{3} < t \le -\frac{4}{3} \\ 0 & \text{otherwise} \end{cases}$$

$$x(\frac{2}{3}t+\frac{1}{2})=\begin{cases} \frac{2}{3}t+\frac{1}{2} & -\frac{9}{4} \le t \leftarrow \frac{3}{4} \\ \frac{2}{3}t+\frac{1}{2} & -\frac{3}{4} \le t < \frac{9}{4} \\ 2 & \frac{9}{4} \le t < \frac{15}{4} \\ 0 & \text{otherwise} \end{cases}$$

1.12

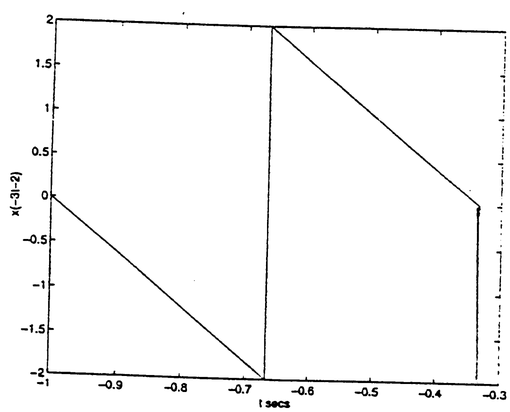


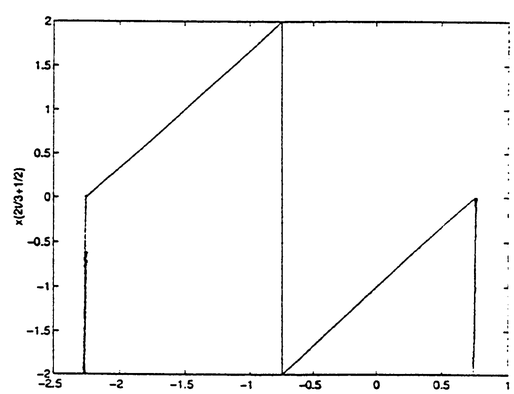




$$x(t-2) = \begin{cases} 2t-2 & 1 \le t < 2 \\ 2t-6 & 2 \le t < 3 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t+3) = \begin{cases} 2t+8 & -4 \le t < -3 \\ 2t+4 & -3 \le t < -2 \\ 0 & \text{otherwise} \end{cases}$$

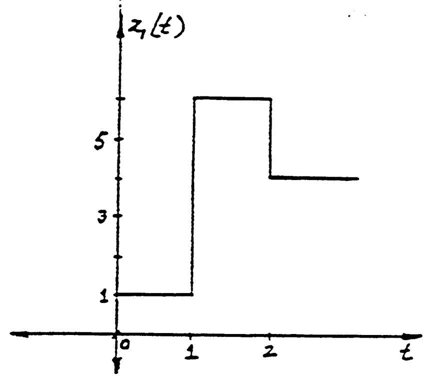




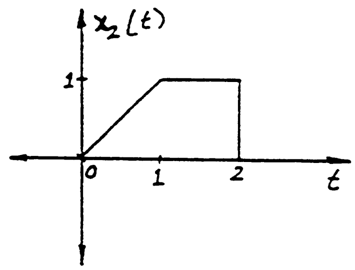
$$x(-3t-2) = \begin{cases} -6t-6 & -1 < t \le -\frac{2}{3} \\ -6t-2 & -\frac{2}{3} < t \le -\frac{1}{3} \\ 0 & \\ & \text{otherwise} \end{cases}$$

$$x(\frac{2}{3}t+\frac{1}{2}) = \begin{cases} \frac{4}{3}t+3 & -\frac{9}{4} \le t \leftarrow \frac{3}{4} \\ \frac{4}{3}t-1 & -\frac{3}{4} \le t < \frac{3}{4} \\ 2 & \\ & \text{otherwise} \\ 0 & \end{cases}$$

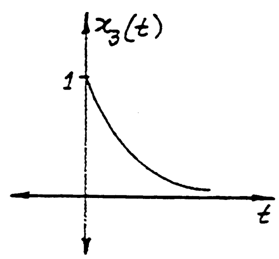
1.13 (a) $x_1(t) = u(t) + 5u(t-1) - 2u(t-2)$



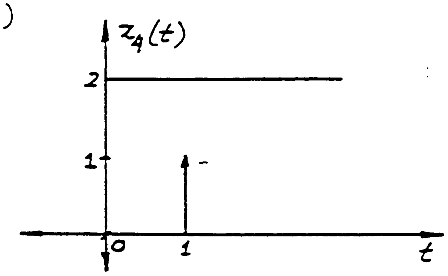
1.13 (b) $x_2(t) = r(t) - r(t-1) - u(t-2)$



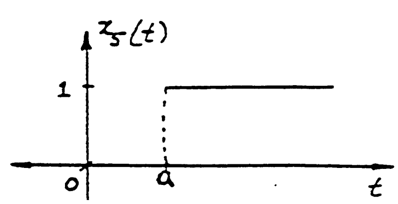
(c) $x_3(t) = e^{-t}u(t)$



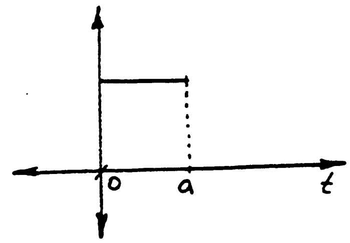
(d) $x_4(t) = 2u(t) + \delta(t-1)$



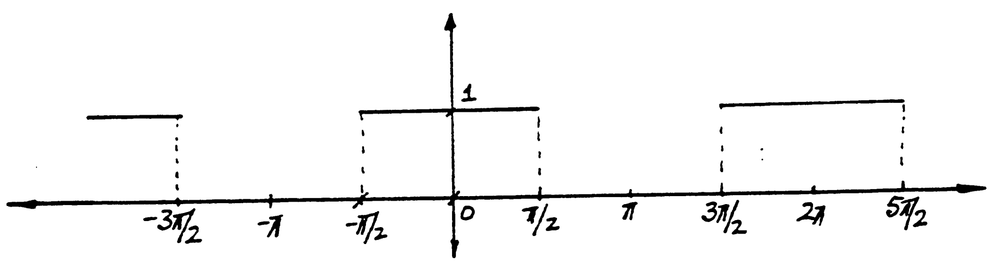
(e) $x_5(t) = u(t)\,u(t-a)$, $a > 0$



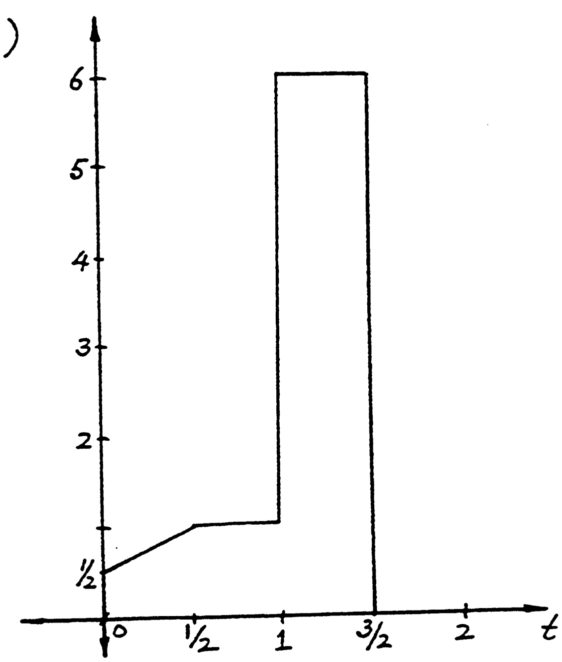
(f) $x_6(t) = u(t)\,u(a-t)$, $a>0$



(g) $x_7(t) = u(\cos t) = \begin{cases} 1 & \cos t \geq 0 \\ 0 & o.\omega \end{cases}$

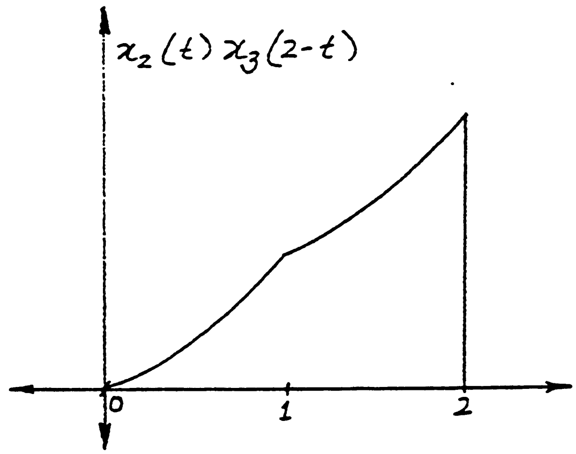


(h) $x_1(t)\ x_2(t+\frac{1}{2})$



(i) $x_1\left(-\frac{t}{3}+\frac{1}{2}\right) x_3(t-2) = 0$ for all t

(j)



1.14 (a) $x_e(t) = \frac{1}{2}\left[x(-t) + x(-t)\right]$

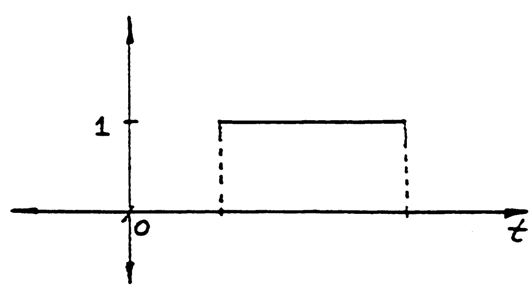
$= x_e(t)$

(b) $x_o(-t) = \frac{1}{2}\left[x(-t) - x(t)\right]$

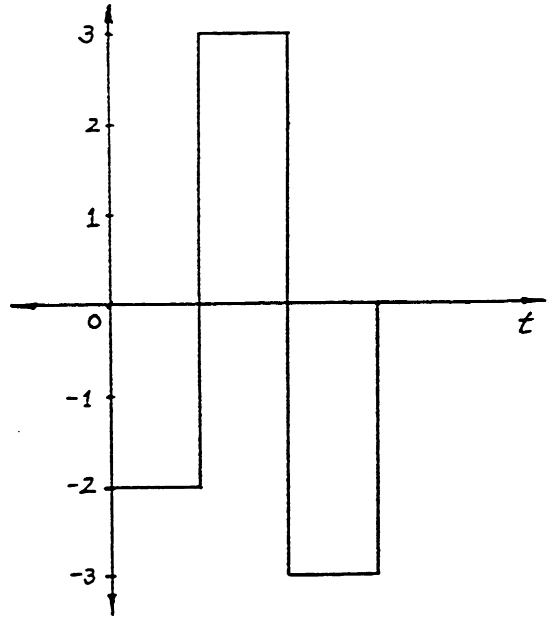
$= -\frac{1}{2}\left[x(t) - x(-t)\right]$

$= -x_o(t)$

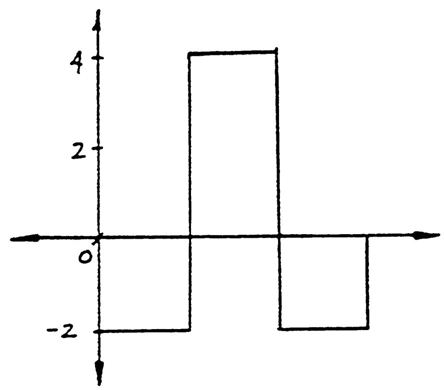
1.15 (a) (i) L + R



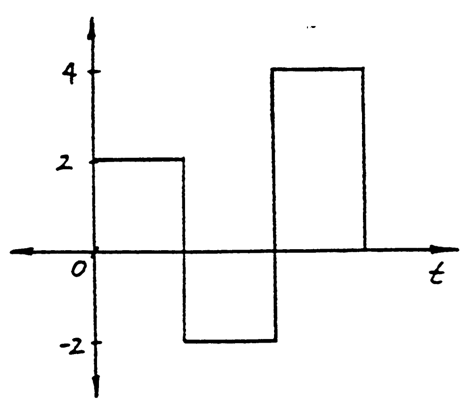
(ii) L−R



(b) (L+R) + (L−R) = 2L



(c) (L+R) + (−(L − R)) = (L+R) + (R− L) = 2R



1.16 $x_1(t) = u(t) - u(t-a)$

$x_2(t) = r(t) - r(t-a) - a\,u(t-b)$

$x_3(t) = \frac{a}{b-a}\left[r(t+b) - r(t+a) - r(t-a) + r(t-b)\right]$

$x_4(t) = (r(t) - r(-t))\,(u(t+a) - u(t-a))$

$x_5(t) = \frac{1}{c}r(t) - \frac{a}{c(a-c)}r(t-c) + \frac{1}{c}r(t-2a+c)$

$x_6(t) = \frac{1}{b-a}\left[r(t+b) - r(t+a)\right] + u(t) - \frac{2}{b-a}\left[r(t-a) - r(t-b)\right]$

1.17 (a) $x_1(t) = A e^{-t/T} u(t)$

$x_1(0) = A \qquad x_1(t') = \frac{A}{e} = A e^{-t'/T}$

$\therefore\ t' = T$

The duration of $x_1(t) = T$

(b) $x_2(t) = x_1(3t) = A e^{-3t/T} u(3t) = A e^{-3t/T} u(t)$

$x_2(t') = \frac{A}{e} = A e^{-3t'/T} \Rightarrow t' = \frac{T}{3}$

The duration of $x_2(t) = \frac{T}{3}$

(c) $x_3(t) = x_1(t/2) = A e^{-t/2T} u(t/2) = A e^{-t/2T} u(t)$

$x_3(0) = A$

$x_3(t') = \frac{A}{e} = A e^{-t'/2T} \Rightarrow t' = 2T$

The duration of $x_3(t) = 2T$

(d) $x_4(t) = 2x_1(t) = 2Ae^{-t/T}\, u(t)$

$x_4(0) = 2A$

$x_4(t') = \frac{2A}{e} = 2Ae^{-t'/T} \Rightarrow t' = T$

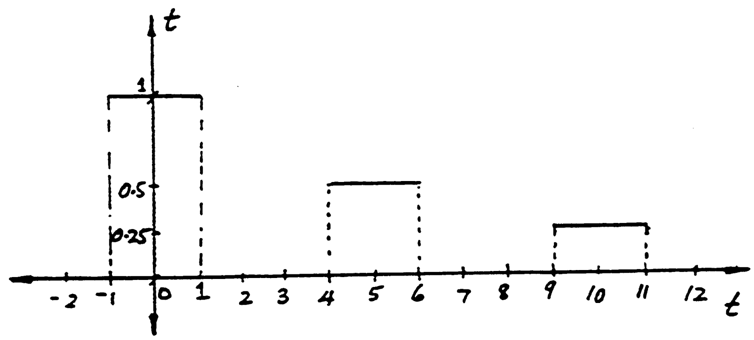
The duration of $x_4(t) = T$

1.18 $x(t) = \text{rect}(t/2)$

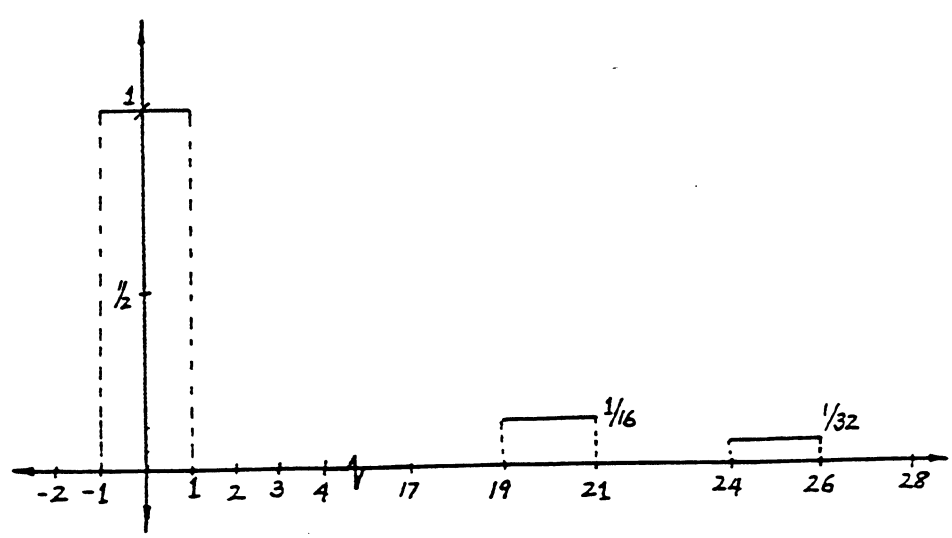
$y(t) = x(t) + 0.5\,x(t - T/2) + 0.25\,x(t - T) \qquad T >> 2$

(a) $T = 10$

$y(t) = x(t) + 0.5\,x(t-5) + 0.25\,x(t-10) \qquad T >> 2$



(b) $T = 10$, $z(t) = y(t) - \frac{1}{2}y(t-5) + \frac{1}{8}y(t-15)$



1.19 (a) $P_1(t) = \lim_{\epsilon \to 0} \dfrac{1}{\epsilon \pi \cosh t/\epsilon}$

note that $P_1(t)$ is an even symmetric function with

$$P_1(0) = \lim_{\epsilon \to 0} \frac{1}{\epsilon \pi} \to \infty$$

for $t \neq 0$

$$P_1(t) = \lim_{\epsilon \to 0} \frac{1}{\epsilon \pi \left(\dfrac{e^{t/\epsilon} + e^{-t/\epsilon}}{2} \right)}$$

$$= \lim_{\epsilon \to 0} \frac{2}{\epsilon \pi (e^{t/\epsilon} + e^{-t/\epsilon})}$$

$$= \lim_{\epsilon \to 0} \frac{2/\epsilon}{\pi (e^{t/\epsilon} + e^{-t/\epsilon})}$$

$$= \lim_{\epsilon \to 0} \frac{-\frac{2}{\epsilon^2}}{\pi (e^{t/\epsilon} + e^{-t/\epsilon})(-\frac{1}{\epsilon^2})}$$

$$= \lim_{\epsilon \to 0} \frac{1}{\pi (e^{t/\epsilon} + e^{-t/\epsilon})} = 0$$

$$\int_{-\infty}^{\infty} P_1(t)\,dt = \lim_{\epsilon \to 0} \frac{1}{\epsilon \pi} \int_{-\infty}^{\infty} \frac{1}{\cosh (t/\epsilon)}\,dt$$

$$= \lim_{\epsilon \to 0} \frac{1}{\epsilon \pi} \int_{-\infty}^{\infty} \operatorname{sech}(t/\epsilon)\,dt \qquad t' = \frac{t}{\epsilon}$$

$$= \lim_{\epsilon \to 0} \frac{1}{\pi} \int_{-\infty}^{\infty} \operatorname{sech}(t')\,dt'$$

$$= \lim_{\epsilon \to 0} \frac{1}{\pi} \tan^{-1} (\sinh (t')) \Big|_{-\infty}^{\infty}$$

$$= \lim_{\epsilon \to 0} \frac{1}{\pi} \left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) \right) = 1$$

$\therefore$ $P_1(t)$ can be used as a mathematical model for the delta function

$$\int_{-\infty}^{\infty} P_3(t)\, dt$$

$$= \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \frac{2\epsilon}{\epsilon^2 + 4\pi^2 t^2}\, dt = \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \frac{1}{\pi} \frac{1}{1 + \left(\frac{2\pi t}{\epsilon}\right)^2}\, d\left(\frac{2\pi t}{\epsilon}\right)$$

$$= \lim_{\epsilon \to 0} \frac{1}{\pi} \tan^{-1}\left(\frac{2\pi t}{\epsilon}\right) \Big|_{-\infty}^{\infty}$$

$$= \frac{1}{\pi}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 1$$

$\therefore$ $P_3(t)$ can be modelled as a delta function

(d) $P_4(t) = \lim_{\epsilon \to 0} \frac{1}{\pi} \frac{\epsilon}{t^2 + \epsilon^2}$

note that $P_4(t)$ is an even symmetric function with

$$P_4(0) = \lim_{\epsilon \to 0} \frac{\epsilon}{\pi \epsilon^2} = 0$$

for $t \neq 0$

$$P_4(t) = \lim_{\epsilon \to 0} \frac{\epsilon}{\pi (t^2 + \epsilon^2)} = 0$$

$$\int_{-\infty}^{\infty} P_4(t)\, dt = \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \frac{\epsilon}{\pi (t^2 + \epsilon^2)}\, dt$$

$$= \frac{1}{\pi} \tan^{-1} \frac{t}{\epsilon} \Big|_{-\infty}^{\infty} = \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = 1$$

$\therefore$ $P_4(t)$ can be used as a model for the delta function.

(e) $P_5(t) = \lim_{\epsilon \to 0} \epsilon \exp[-\epsilon / |t|]$

note that $P_5(t)$ is an even symmetric function with

$P_5(0) = \lim_{\epsilon \to 0} \epsilon = 0$

$P_5(t)$ cannot be used as a model for the delta function.

(f) $P_6(t) = \lim_{\epsilon \to 0} \frac{1}{\pi} \frac{\sin \epsilon t}{t}$

note that $P_6(t)$ is an even symmetric function with

$P_6(0) = \lim_{\epsilon \to 0} \frac{\epsilon}{\pi} \frac{\sin \epsilon t}{\epsilon t} = 0$

$P_6(t)$ cannot be used as a model for the delta function.

1.20 (a) $\int_{-\infty}^{\infty} (\frac{2}{3}t - \frac{3}{2})\delta(t-1)dt = \frac{2}{3} - \frac{3}{2} = -\frac{5}{6}$

(b) $\int_{-\infty}^{\infty} (t-1)\delta(\frac{2}{3}t - \frac{3}{2})dt = \int_{-\infty}^{\infty} (t-1)\delta(\frac{2}{3}t - \frac{3}{2})dt = \int_{-\infty}^{\infty} (t-1)\frac{3}{2}\delta(t-1)dt = 0$

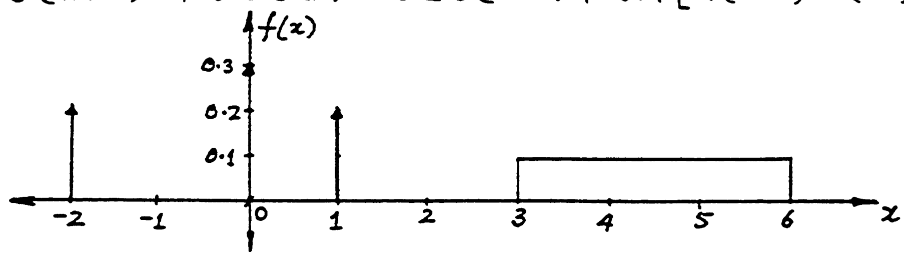
(c) $\int_{-3}^{-2} [\exp(-t+1) + \sin(\frac{2\pi}{3}t)]\delta(t - \frac{3}{2})dt = 0$

(d) $\int_{-3}^{2} [\exp(-t+1) + \sin(\frac{2\pi}{3}t)]\delta(t - \frac{3}{2})dt = e^{-0.5} + \sin(\pi) = e^{-0.5}$

(e) $\int_{-\infty}^{\infty} \exp[-5t+1]\delta'(t-5)dt = -5e^{-5(5)} = -5e^{-25}$

1.21 $P[X \le \alpha] = \int_{-\infty}^{\alpha^+} f(x)\,dx$

$$f(x) = 0.2\,\delta(x+2) + 0.3\,\delta(x) + 0.2\,\delta(x-1) + 0.1\,[u(x-3) - u(x-6)]$$



(a) $P[X \le -3] = \int_{-\infty}^{-3} f(x)\,dx = 0$

(b) $P[X \le 1.5] = \int_{-\infty}^{1.5} f(x)\,dx = \int_{-\infty}^{1.5} [0.2\,\delta(x+2) + 0.3\,\delta(x) + 0.2\,\delta(x-1)]\,dx$

$= 0.2 + 0.3 + 0.2 = 0.7$

(c) $P[X \le 4] = \int_{-\infty}^{4} f(x)\,dx$

$= 0.2 + 0.3 + 0.2 + 0.1 \int_{3}^{4} dx$

$= 0.2 + 0.3 + 0.2 + 0.1 = 0.8$

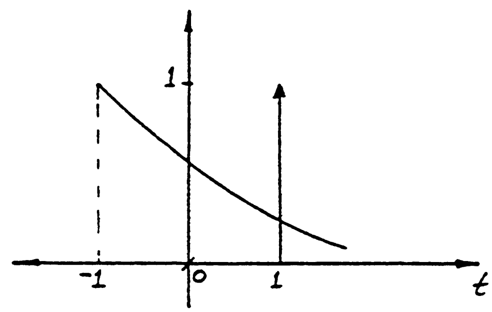
(d) $P[X \le 6] = \int_{-\infty}^{6} f(x)\,dx$

$= 0.2 + 0.3 + 0.2 + 0.1 \int_{3}^{6} dx$

$= 0.2 + 0.3 + 0.2 + 0.3 = 1$

1.22 $V(t) = e^{-(t+1)} u(t+1) + \delta(t-1)$

(a)



(b) $f(t) = m \frac{d}{dt} [v(t)]$

$= \frac{d}{dt} [\exp[-(t+1)] u(t+1) + \delta(t-1)] \times 10^{-3}$

$= (\exp[-(t+1)] \delta(t+1) - \exp[-(t+1)] u(t+1)$

$+ \delta'(t-1)) \times 10^{-3}$

$= (\delta(t+1) - e^{-(t+1)} u(t+1) + \delta'(t-1) \times 10^{-3}$ N

(c) $f_k(t) = k \int_{-\infty}^{t} u(\tau) d\tau$

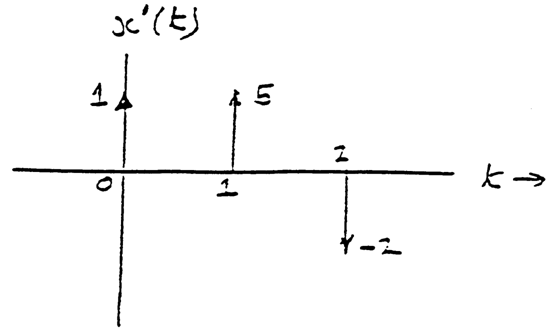
$= k \int_{-\infty}^{t} \exp[-(\tau+1)] u(\tau+1) + \delta(\tau-1) d\tau$

(1) $t < -1$ $f_k(t) = 0$

(2) $-1 < t < 1$ $f_k(t) = 1 - \exp[-(t+1)]$ N

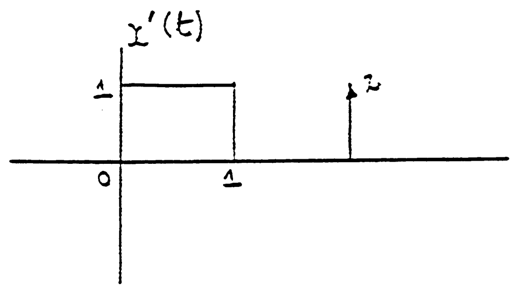
(3) $t > 1$ $f_k(t) = 1 - \exp[-(t+1)] + u(t-1)$ N

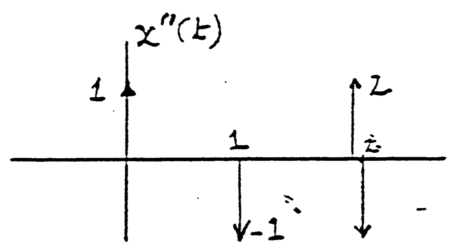
1.23 (a) $x'(t) = \delta(t) + 5\delta(t-1) - 2\delta(t-2)$



(b) $x'(t) = u(t) - u(t-1) + 2\delta(t-2)$

$x''(t) = \delta(t) - \delta(t-1) + 2\delta'(t-2)$





(c) From Equation (1.12), we see that

$$x'(t) = 2u(t+1) - 4\delta(t) - 2u(t-1)$$

$$x''(t) = 2\delta(t+1) - 4\delta'(t) - 2\delta(t-1)$$