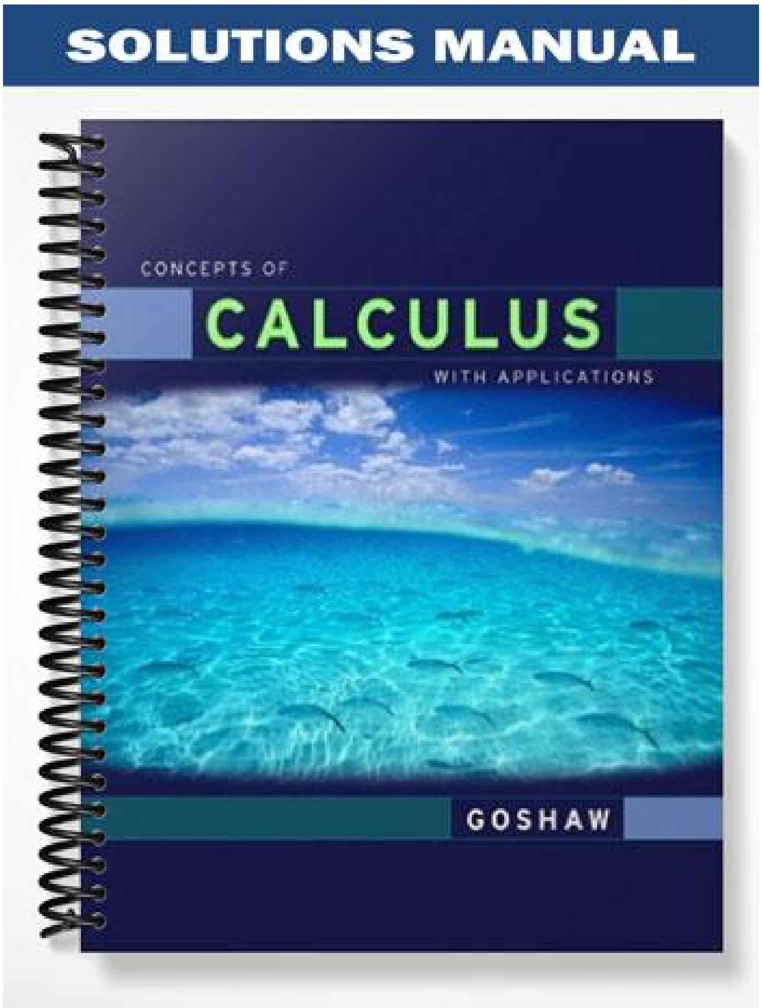

SOLUTIONS MANUAL
CONCEPTS OF
CALCULUS
WITH APPLICATIONS
GOSHAW

Unit 0
Function Review

Topic 1 Exercises

1. This example represents a function because each resident can only have one home address.

2. This example does not represent a function because each textbook could be paired with more than one author.

3. This set of ordered pairs does not describe a function because the domain element 5 is paired with both 7 and 3 in the range.

4. $x^2 + y^2 = 25$ does not describe a function. Choosing $x = 0$ yields $y^2 = 25,$ which means that $y = 5$ or 5.

5. $y = x^2 - 4x - 5$ describes a function. Every *x* will produce only one value of *y*.

6. This set of ordered pairs is a function because each domain element is paired with only one range element.

7. This set of ordered pairs is a function because each domain element is paired with only one range element.

8. This curve does not represent a function, because there is at least one vertical line that intersects the graph in more than one point.

9. $x + |y| = 4$ does not describe a function. Choosing $x = 0$ yields $y = 4$ or -4.

10. $y = x^3 - 4$ describes a function. Every *x* will produce only one value of *y*.

11. This curve does not represent a function, because there is at least one vertical line that intersects the graph in more than one point.

12. This set of ordered pairs does not describe a function because the domain element 1.1 is paired with both 2 and 9 in the range.

13. The curve describes a function because any vertical line would intersect the graph at only one point. In other words, no *x* value is paired with more than one *y* value.

14. The curve describes a function because any vertical line would intersect the graph at only one point. In other words, no *x* value is paired with more than one *y* value.

15. $$\begin{aligned} f(x) &= 3x - 5 \\ f(2) &= 3(2) - 5 \\ &= 1 \end{aligned}$$

16. $$\begin{aligned} f(x) &= 7 - 2x \\ f(-1) &= 7 - 2(-1) \\ &= 9 \end{aligned}$$

17. $$\begin{aligned} f(x) &= x^2 - 3x + 4 \\ f(1) &= 1^2 - 3(1) + 4 \\ &= 2 \end{aligned}$$

18. $$\begin{aligned} f(x) &= -2x^2 + x - 5 \\ f(-3) &= -2(-3)^2 + (-3) - 5 \\ &= -18 - 3 - 5 \\ &= -26 \end{aligned}$$

19. $$\begin{aligned} f(x) &= -2x^2 + x - 5 \\ f(a) &= -2a^2 + a - 5 \end{aligned}$$

20. $$\begin{aligned} f(x) &= x^3 + 2x \\ f(b) &= b^3 + 2b \end{aligned}$$

21. $$\begin{aligned} f(x) &= -2x^2 + x - 5 \\ f(a+h) &= -2(a+h)^2 + (a+h) - 5 \\ &= -2(a^2 + 2ah + h^2) + a + h - 5 \\ &= -2a^2 - 4ah - 2h^2 + a + h - 5 \end{aligned}$$

22. $$\begin{aligned} f(x) &= x^2 - 3x + 4 \\ f(a+h) &= (a+h)^2 - 3(a+h) + 4 \\ &= a^2 + 2ah + h^2 - 3a - 3h + 4 \end{aligned}$$

23. $f(x) = x^2 - 3x$

$f(a) = a^2 - 3a$

$$\begin{aligned} f(a+h) &= (a+h)^2 - 3(a+h) \\ &= a^2 + 2ah + h^2 - 3a - 3h \end{aligned}$$

$$\begin{aligned} f(a+h) - f(a) &= (a^2 + 2ah + h^2 - 3a - 3h) - (a^2 - 3a) \\ &= a^2 + 2ah + h^2 - 3a - 3h - a^2 + 3a \\ &= 2ah + h^2 - 3h \end{aligned}$$

24. $f(x) = 4 - x^2$

$f(a) = 4 - a^2$

$$\begin{aligned} f(a+h) &= 4 - (a+h)^2 \\ &= 4 - a^2 - 2ah - h^2 \end{aligned}$$

$$\begin{aligned} f(a+h) - f(a) &= (4 - a^2 - 2ah - h^2) - (4 - a^2) \\ &= 4 - a^2 - 2ah - h^2 - 4 + a^2 \\ &= -2ah - h^2 \end{aligned}$$

25. Using the four-step process for $f(x) = 3x - 5$, we know that

1. $f(a) = 3a - 5$
2. $f(a+h) = 3(a+h) - 5$

Then

3. $$\begin{aligned} f(a+h) - f(a) &= [3(a+h) - 5] - (3a - 5) \\ &= 3a + 3h - 5 - 3a + 5 \\ &= 3h \end{aligned}$$
4. Substituting $f(a+h) - f(a) = 3h$ into the difference quotient and simplifying yields
$$\frac{f(a+h) - f(a)}{h} = \frac{3h}{h} = 3$$

26. Using the four-step process for $f(x) = 10$, we know that

1. $f(a) = 10$
2. $f(a+h) = 10$

Then

3. $f(a+h) - f(a) = 10 - 10 = 0$
4. Substituting $f(a+h) - f(a) = 0$ into the difference quotient and simplifying yields
$$\frac{f(a+h) - f(a)}{h} = \frac{0}{h} = 0$$

27. Using the four-step process for $f(x) = x^2 + 4,$ we know that

1. $f(a) = a^2 + 4$
2. $f(a+h) = (a+h)^2 + 4$

Then

3. $f(a+h) - f(a) = [(a+h)^2 + 4] - (a^2 + 4)$
$$= a^2 + 2ah + h^2 + 4 - a^2 - 4$$
$$= 2ah + h^2$$
4. Substituting $f(a+h) - f(a) = 2ah + h^2$ into the difference quotient and simplifying yields
$$\frac{f(a+h) - f(a)}{h} = \frac{2ah + h^2}{h}$$
$$= \frac{h(2a+h)}{h}$$
$$= 2a + h$$

28. Using the four-step process for $f(x) = 3 - x^2,$ we know that

1. $f(a) = 3 - a^2$
2. $f(a+h) = 3 - (a+h)^2$

Then

3. $f(a+h) - f(a) = [3 - (a+h)^2] - (3 - a^2)$
$$= 3 - a^2 - 2ah - h^2 - 3 + a^2$$
$$= -2ah - h^2$$
4. Substituting $f(a+h) - f(a) = -2ah - h^2$ into the difference quotient and simplifying yields
$$\frac{f(a+h) - f(a)}{h} = \frac{-2ah - h^2}{h}$$
$$= \frac{h(-2a-h)}{h}$$
$$= -2a - h$$

29. Using the four-step process for $f(x) = 2x^2 - 3x + 5,$ we know that

1. $f(a) = 2a^2 - 3a + 5$
2. $f(a+h) = 2(a+h)^2 - 3(a+h) + 5$

Then

3. $f(a+h) - f(a) = [2(a+h)^2 - 3(a+h) + 5] - (2a^2 - 3a + 5)$
$$= 2a^2 + 4ah + 2h^2 - 3a - 3h + 5 - 2a^2 + 3a - 5$$
$$= 4ah + 2h^2 - 3h$$
4. Substituting $f(a+h) - f(a) = 4ah + 2h^2 - 3h$ into the difference quotient and simplifying yields
$$\frac{f(a+h) - f(a)}{h} = \frac{4ah + 2h^2 - 3h}{h}$$
$$= \frac{h(4a + 2h - 3)}{h}$$
$$= 4a + 2h - 3$$

30. Using the four-step process for $f(x) = 6 - 2x + x^2$, we know that

1. $f(a) = 6 - 2a + a^2$
2. $f(a+h) = 6 - 2(a+h) + (a+h)^2$

Then

3. $f(a+h) - f(a) = [6 - 2(a+h) + (a+h)^2] - (6 - 2a + a^2)$
$$= 6 - 2a - 2h + a^2 + 2ah + h^2 - 6 + 2a - a^2$$
$$= -2h + 2ah + h^2$$
4. Substituting $f(a+h) - f(a) = -2h + 2ah + h^2$ into the difference quotient and simplifying yields
$$\frac{f(a+h) - f(a)}{h} = \frac{-2h + 2ah + h^2}{h}$$
$$= \frac{h(-2 + 2a + h)}{h}$$
$$= -2 + 2a + h$$

31. Using the four-step process for $f(x) = x^3$, we know that

1. $f(a) = a^3$
2. $f(a+h) = (a+h)^3$

Then

3. $f(a+h) - f(a) = (a+h)^3 - a^3$
$$= a^3 + 3a^2h + 3ah^2 + h^3 - a^3$$
$$= 3a^2h + 3ah^2 + h^3$$
4. Substituting $f(a+h) - f(a) = 3a^2h + 3ah^2 + h^3$ into the difference quotient and simplifying yields
$$\frac{f(a+h) - f(a)}{h} = \frac{3a^2h + 3ah^2 + h^3}{h}$$
$$= \frac{h(3a^2 + 3ah + h^2)}{h}$$
$$= 3a^2 + 3ah + h^2$$

32. Using the four-step process for $f(x) = x^4$, we know that

1. $f(a) = a^4$
2. $f(a+h) = (a+h)^4$

Then

3. $f(a+h) - f(a) = (a+h)^4 - a^4$
$$= a^4 + 4a^3h + 6a^2h^2 + 4ah^3 + h^4 - a^4$$
$$= 4a^3h + 6a^2h^2 + 4ah^3 + h^4$$
4. Substituting $f(a+h) - f(a) = 4a^3h + 6a^2h^2 + 4ah^3 + h^4$ into the difference quotient and simplifying yields
$$\frac{f(a+h) - f(a)}{h} = \frac{4a^3h + 6a^2h^2 + 4ah^3 + h^4}{h}$$
$$= \frac{h(4a^3 + 6a^2h + 4ah^2 + h^3)}{h}$$
$$= 4a^3 + 6a^2h + 4ah^2 + h^3$$

33. $y = 3x - 2$ matches graph C; The slope is 3 and the *y*-intercept is $(0, -2)$.

34. $y = 3x + 2$ matches graph F; The slope is 3 and the *y*-intercept is $(0, 2)$.

35. $y = -3x + 2$ matches graph D; The slope is 3 and the *y*-intercept is $(0, 2)$.

36. $y = -3x - 2$ matches graph A; The slope is 3 and the *y*-intercept is $(0, -2)$.

37. $y = \frac{1}{3}x + 2$ matches graph E; The slope is $\frac{1}{3}$ and the *y*-intercept is $(0, 2)$.

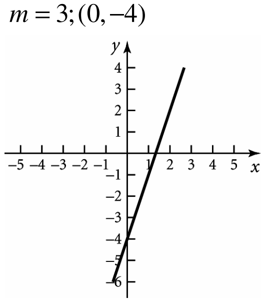
38. $y = -\frac{1}{2}x + 2$ matches graph G; The slope is $-\frac{1}{2}$ and the *y*-intercept is $(0, 2)$.

39. $y = -\frac{1}{2}x - 2$ matches graph B; The slope is $-\frac{1}{2}$ and the *y*-intercept is $(0, -2)$.

40. $y = \frac{1}{3}x - 2$ matches graph H; The slope is $\frac{1}{3}$ and the *y*-intercept is $(0, -2)$.

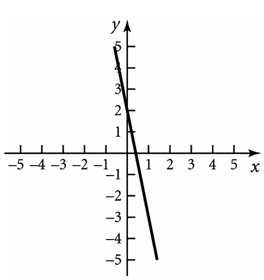
41. $y = 3x - 4$

$m = 3; (0, -4)$



42. $y = 2 - 5x$

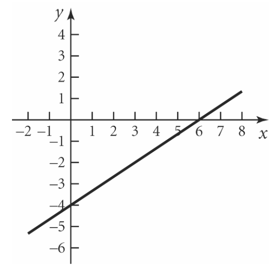
$m = -5; (0, 2)$



43. $2x - 3y = 12$

$$y = \frac{2}{3}x - 4$$

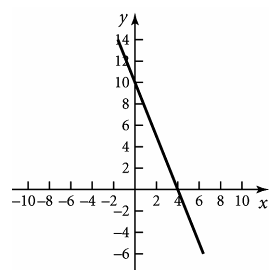
$m = \frac{2}{3}; (0, -4)$



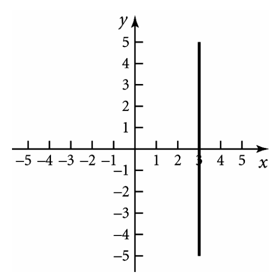
44. $5x + 2y = 20$

$$y = -\frac{5}{2}x + 10$$

$m = -\frac{5}{2}; (0, 10)$

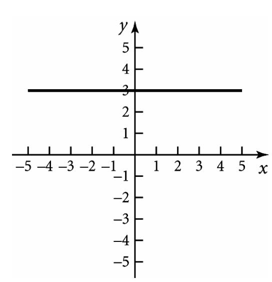


45. $x = 3$ has no slope and no *y*-intercept.



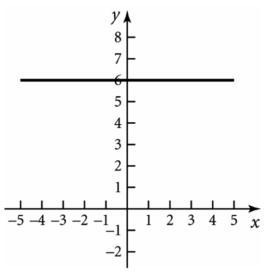
46. $y = 3$

$m = 0; (0, 3)$

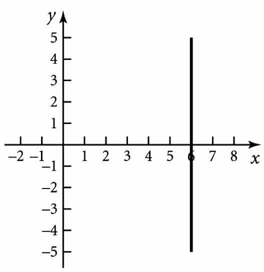


47. $y = 6$

$m = 0; (0, 6)$



48. $x = 6$ has no slope and no y-intercept.



49. The graph of $y = |x+3|$ is the graph of $y = |x|$ shifted to the left 3 units. Thus, it is graph C.

50. The graph of $y = 2 - |x|$ is the graph of $y = -|x|$ shifted up 2 units. Thus, it is graph E.

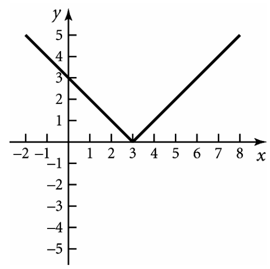
51. The graph of $y = |x| + 3$ is the graph of $y = |x|$ shifted up 3 units. Thus, it is graph A.

52. The graph of $y = |x-2|$ is the graph of $y = |x|$ shifted to the right 2 units. Thus, it is graph D.

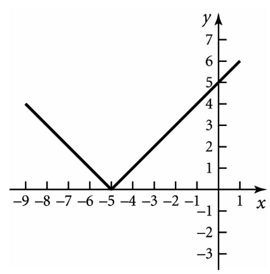
53. The graph of $y = 2 - |x+1|$ is the graph of $y = -|x|$ shifted up 2 units and 1 unit to the left. Thus, it is graph F.

54. The graph of $y = |x+3| - 2$ is the graph of $y = |x|$ shifted down 2 units and 3 units to the left. Thus, it is graph B.

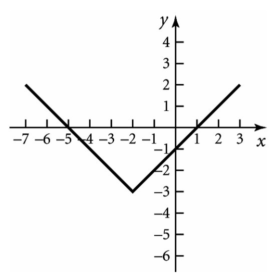
55. The graph of $f(x) = |x-3|$ is the graph of $f(x) = |x|$ shifted 3 units to the right. The corner point is $(3, 0)$. The domain is all real numbers; the range is $y \ge 0$.



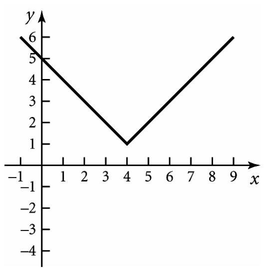
56. The graph of $f(x) = |x+5|$ is the graph of $f(x) = |x|$ shifted 5 units to the left. The corner point is $(-5, 0)$. The domain is all real numbers; the range is $y \ge 0$.



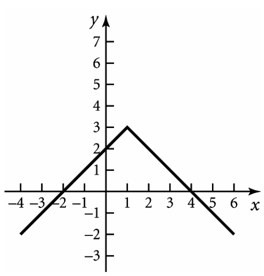
57. The graph of $f(x) = |x+2| - 3$ is the graph of $f(x) = |x|$ shifted 2 units to the left and down 3 units. The corner point is $(-2, -3)$. The domain is all real numbers; the range is $y \ge -3$.



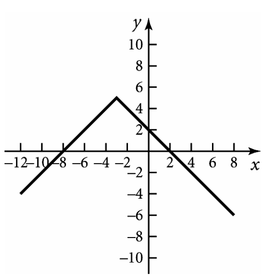
58. The graph of $f(x)=|x-4|+1$ is the graph of $f(x)=|x|$ shifted 4 units to the right and up 1 unit. The corner point is $(4,1)$. The domain is all real numbers; the range is $y \geq 1$.



59. The graph of $f(x)=3-|x-1|$ is the graph of $y=-|x|$ shifted up 3 units and 1 unit to the right. The corner point is $(1,3)$. The domain is all real numbers; the range is $y \leq 3$.



60. The graph of $f(x)=5-|x+3|$ is the graph of $y=-|x|$ shifted up 5 units and 3 units to the left. The corner point is $(-3,5)$. The domain is all real numbers; the range is $y \leq 5$.



61. a. Let $n =$ the number of copies made each month
Let $C =$ the monthly total rental charges
$C(n) = \$125 + \$0.02n$

b. To determine the charge to a business that makes 5600 copies per month, let $n = 5600$.
$C(5600) = \$125 + \$0.02(5600) = \$237$

62. a. Let $m =$ the number of miles
Let $C =$ the total rental charges
$C(m) = \$45 + \$0.23m$

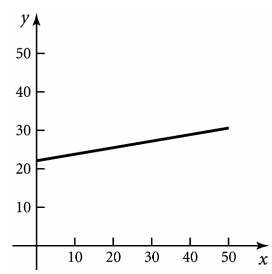
b. To determine the charge for a business that used the limousine for a 362-mile trip, let $m = 362$.
$C(362) = \$45 + \$0.23(362) = \$128.26$

63. a. To determine the median age of a first-time bride in 2002, let $x = 22$ and solve for y.

$$\begin{aligned} y &= 22.1 + 0.17x \\ &= 22.1 + 0.17(22) \\ &= 25.84 \end{aligned}$$

The median age of a first-time bride in 2002 is 25.84 years.

b.



c. To predict the year during which the median age for first-time brides will be 28, let $y = 28$ and solve for x.

$$\begin{aligned} y &= 22.1 + 0.17x \\ 28 &= 22.1 + 0.17x \\ 5.9 &= 0.17x \\ x &= 34.7 \end{aligned}$$

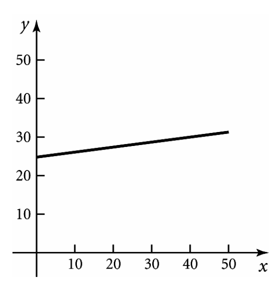
The median age for first-time brides will be 28, 34.7 years after 1980, so around 2015.

64. a. To determine the median age of a first-time groom in 2002, let $x = 22$ and solve for y.

$$\begin{aligned} y &= 24.8 + 0.13x \\ &= 22.1 + 0.17(22) \\ &= 27.66 \end{aligned}$$

The median age of a first-time bride in 2002 is 27.66 years.

b.



c. To predict the year during which the median age for first-time grooms will be 30, let $y = 30$ and solve for x.

$$y = 24.8 + 0.13x$$
$$30 = 24.8 + 0.13x$$
$$5.2 = 0.13x$$
$$x = 40$$

The median age for first-time grooms will be 30, 40 years after 1980, so around 2020.

65. a. To predict the life expectancy of a man born in 1990, let $x = 30$ and solve for y.

$$y = 0.203x + 65.92$$
$$= 0.203(30) + 65.92$$
$$= 72.01$$

The life expectancy of a man born in 1990 is 72.01 years.

b. The slope of the equation is 0.203. Life expectancy for men increases by 0.203 year per year for each birth year after 1960.

66. a. To predict the life expectancy of a woman born in 1990, let $x = 30$ and solve for y.

$$y = 0.171x + 73.32$$
$$= 0.171(30) + 73.32$$
$$= 78.45$$

The life expectancy of a woman born in 1990 is 78.45 years.

b. The slope of the equation is 0.171. Life expectancy for women increases by 0.171 year per year for each birth year after 1960.

67. Letting $C =$ the total charges, a function describing the total charges is $C(h) = \$50 + \$10h$.

If Malcolm uses the service for 6 hours, the total charges are $C(6) = \$50 + \$10(6) = \$110$.

68. Letting $F =$ the total fees, a function describing the total fees is $F(x) = \$125 + \$0.025x$.

If Barry's printed 25,300 copies this month, the total fees are $F(25,300) = \$125 + \$0.025(25,300) = \$757.50$.

69. With $P = \$950,000$, $L = 25$, and $C = \$75,000$, we have the function

$$V(t) = \$950,000 - \left(\frac{\$950,000 - \$75,000}{25}\right) \cdot t$$
$$V(t) = \$950,000 - \$35,000t$$

The lifetime of the press is 25 years, so the domain is $0 \le t \le 25$.

After 10 years, the press has a value of $V(10) = \$950,000 - \$35,000(10) = \$600,000$.

To determine when the press is worth \$125,000, let $V = \$125,000$ and solve for t.

$$V(t) = \$950,000 - \$35,000t$$
$$\$125,000 = \$950,000 - \$35,000t$$
$$-\$825,000 = -\$35,000t$$
$$t = 23.57$$

It will take nearly 24 years for the press to depreciate to a value of \$125,000.

70. With $P = \$85,000$, $L = 15$, and $C = \$10,000$, we have the function

$$V(t) = \$85,000 - \left(\frac{\$85,000 - \$10,000}{15}\right) \cdot t$$
$$V(t) = \$85,000 - \$5000t$$

The lifetime of the press is 15 years, so the domain is $0 \le t \le 15$.

After 5 years, the press has a value of $V(5) = \$85,000 - \$5000(5) = \$60,000$.

To determine when the press is worth \$50,000, let $V = \$50,000$ and solve for t.

$$V(t) = \$85,000 - \$5000t$$
$$\$50,000 = \$85,000 - \$5000t$$
$$-\$35,000 = -\$5000t$$
$$t = 7$$

It will take 7 years for the equipment to depreciate to a value of \$50,000.

71. Setting costs equal to revenue, $C(x) = R(x)$.

$$25000 + 540x = 1400x + 7800$$
$$17200 = 860x$$
$$x = 20$$

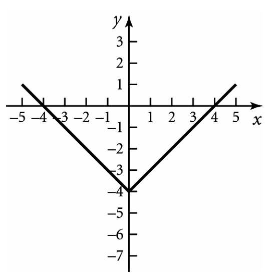
Digital Concepts will break even if 20 home theater systems are installed.

72. Setting costs equal to revenue, $C(x) = R(x)$.
$$800 + 12x = 30x + 440$$
$$360 = 18x$$
$$x = 20$$
Fifi's Dog Groomers will break even if 20 dogs are groomed weekly.

73. The equilibrium point is the price for which $S(x) = D(x)$.
$$-20 + x = 59.2 - 0.76x$$
$$1.76x = 79.2$$
$$x = 45$$
Equilibrium results if the monthly cost of the phone is \$45.

74. The equilibrium point is the price for which $S(x) = D(x)$.
$$4x - 1.5 = 10.5 - 2x$$
$$6x = 12$$
$$x = 2$$
Equilibrium results if the price of the bottled water is \$2.

75. a.

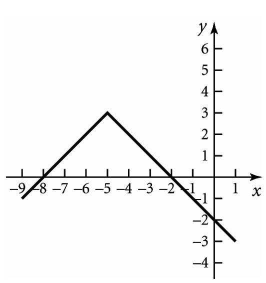


The x-intercepts are $(-4, 0)$ and $(4, 0)$.

b. $|x| - 4 = 0$
$$|x| = 4$$
$$x = 4, -4$$

c. The x-intercepts of the graph of $f(x)$ are the same as the solutions to $f(x) = 0$.

76. a.

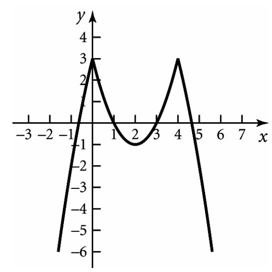


The x-intercepts are $(-8, 0)$ and $(-2, 0)$.

b. $3 - |x + 5| = 0$
$$3 = |x + 5|$$
$$x = -8, -2$$

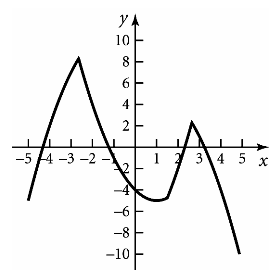
c. The x-intercepts of the graph of $f(x)$ are the same as the solutions to $f(x) = 0$.

77. a.



b. The four solutions are $x \approx -0.646$, 1, 3, 4.646.

78. a.



b. The four solutions are $x \approx -4.317$, -1.236, 2.317, 3.236.

Topic 2 Exercises

1. The domain is all real numbers; the vertex is $(0, -3)$. Because $a = 1 > 0$, the parabola opens upward and the range is $y \ge -3$. This describes graph D.

2. The domain is all real numbers; the vertex is $(0, 3)$. Because $a = -1 < 0$, the parabola opens downward and the range is $y \le 3$. This describes graph H.

3. The domain is all real numbers; the vertex is $(3, 0)$. Because $a = 1 > 0$, the parabola opens upward and the range is $y \ge 0$. This describes graph A.

4. The domain is all real numbers; the vertex is $(2, -1)$. Because $a = 1 > 0$, the parabola opens upward and the range is $y \geq -1$. This describes graph F.

5. The graph of $y = x^3 + 2$ is the graph of $y = x^3$ shifted up 2 units. The point of inflection is at $(0, 2)$. The graph is an increasing S-shape passing through $(0, 2)$. This describes graph E.

6. The graph of $y = (x+2)^3$ is the graph of $y = x^3$ shifted to the left 2 units. The point of inflection is at $(-2, 0)$. The graph is an increasing S-shape passing through $(-2, 0)$. This describes graph J.

7. The graph of $y = 4 - x^3$ is the graph of $y = -x^3$ shifted up 4 units. The point of inflection is at $(0, 4)$. The graph is a decreasing S-shape passing through $(0, 4)$. This describes graph I.

8. The graph of $y = 2 - (x-1)^3$ is the graph of $y = -x^3$ shifted up 2 units and 1 unit to the right. The point of inflection is at $(1, 2)$. The graph is a decreasing S-shape passing through $(1, 2)$. This describes graph B.

9. The domain is all $x \geq 0$. The graph is the graph of $y = \sqrt{x}$ reflected about the *x*-axis because of the negative sign in front of the radical. The range is all $y \leq 0$. The end point is $(0, 0)$. This describes graph C.

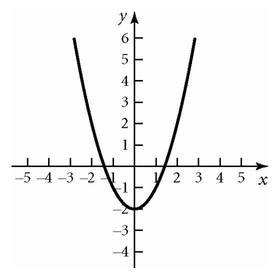
10. The domain is all values of *x* for which $x - 2 \geq 0$ or $x \geq 2$. The range is all nonnegative numbers. The end point of the graph is $(2, 0)$. The graph is the graph of $y = \sqrt{x}$ shifted 2 units to the right. This describes graph K.

11. The domain is all $x \geq 0$. The graph is the graph of $y = \sqrt{x}$ shifted 2 units downward, so the range is all $y \geq -2$ and the end point of the graph is $(0, -2)$ This describes graph L.

12. The domain is all values of *x* for which $-x \geq 0$ or $x \leq 0$. The range is all nonnegative numbers. The $-x$ under the radical sign reflects the graph about the *y*-axis. The graph of $y = \sqrt{-x}$ is the graph of $y = \sqrt{x}$ reflected about the *y*-axis. The end point of the graph is still at $(0, 0)$. This describes graph G.

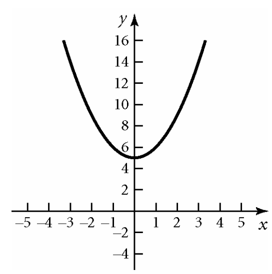
13. $y = x^2 - 2$

The domain is all real numbers; the vertex is $(0, -2)$. Because $a = 1 > 0$, the parabola opens upward and the range is $y \geq -2$.



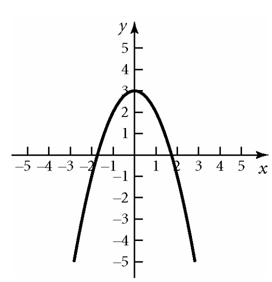
14. $y = x^2 + 5$

The domain is all real numbers; the vertex is $(0, 5)$. Because $a = 1 > 0$, the parabola opens upward and the range is $y \geq 5$.



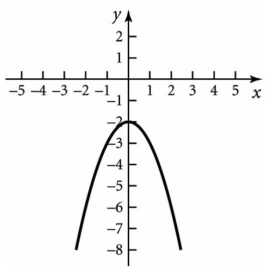
15. $y = 3 - x^2$

The domain is all real numbers; the vertex is $(0, 3)$. Because $a = -1 < 0$, the parabola opens downward and the range is $y \leq 3$.



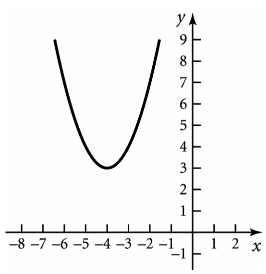
16. $y = -2 - x^2$

The domain is all real numbers; the vertex is $(0, -2)$. Because $a = -1 < 0$, the parabola opens downward and the range is $y \leq -2$.



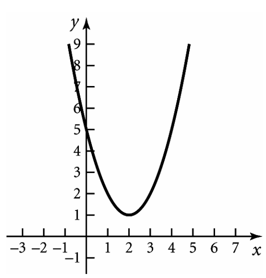
17. $y = (x + 4)^2 + 3$

The domain is all real numbers; the vertex is $(-4, 3)$. Because $a = 1 > 0$, the parabola opens upward and the range is $y \geq 3$.



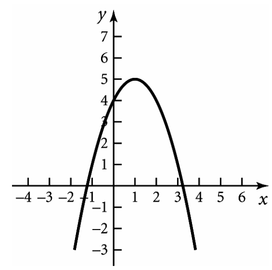
18. $y = (x - 2)^2 + 1$

The domain is all real numbers; the vertex is $(2, 1)$. Because $a = 1 > 0$, the parabola opens upward and the range is $y \geq 1$.



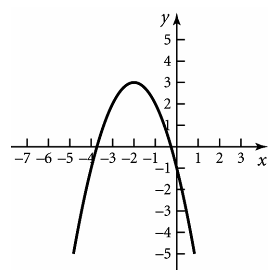
19. $y = 5 - (x - 1)^2$

The domain is all real numbers; the vertex is $(1, 5)$. Because $a = -1 < 0$, the parabola opens downward and the range is $y \leq 5$.

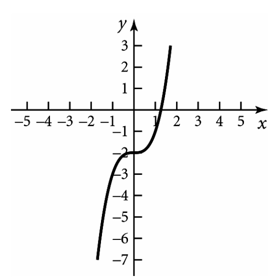


20. $y = 3 - (x + 2)^2$

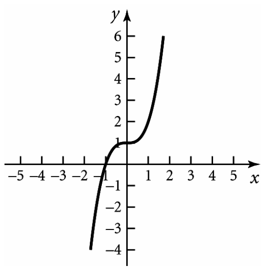
The domain is all real numbers; the vertex is $(-2, 3)$. Because $a = -1 < 0$, the parabola opens downward and the range is $y \leq 3$.



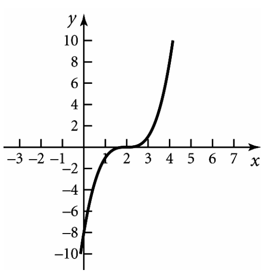
21. The graph of $y = x^3 - 2$ is the graph of $y = x^3$ shifted down 2 units. The point of inflection is at $(0, -2)$. The graph is an increasing S-shape passing through $(0, -2)$.



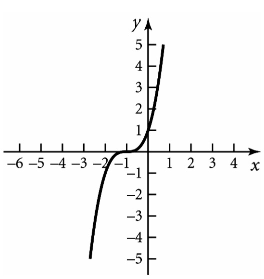
22. The graph of $y = x^3 + 1$ is the graph of $y = x^3$ shifted up 1 unit. The point of inflection is at $(0,1)$. The graph is an increasing S-shape passing through $(0,1)$.



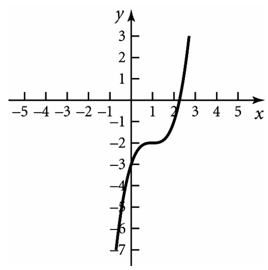
23. The graph of $y = (x-2)^3$ is the graph of $y = x^3$ shifted to the right 2 units. The point of inflection is at $(2,0)$. The graph is an increasing S-shape passing through $(2,0)$.



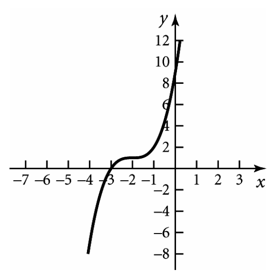
24. The graph of $y = (x+1)^3$ is the graph of $y = x^3$ shifted to the left 1 units. The point of inflection is at $(-1,0)$. The graph is an increasing S-shape passing through $(-1,0)$.



25. The graph of $y = (x-1)^3 - 2$ is the graph of $y = x^3$ shifted to the right 1 unit and down 2 units. The point of inflection is at $(1,-2)$. The graph is an increasing S-shape passing through $(1,-2)$.



26. The graph of $y = (x+2)^3 + 1$ is the graph of $y = x^3$ shifted to the left 2 units and up 1 unit. The point of inflection is at $(-2,1)$. The graph is an increasing S-shape passing through $(-2,1)$.



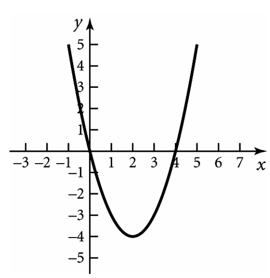
27. For $y = x^2 - 4x$, $a = 1$, $b = -4$, and $c = 0$. Substituting these values in the vertex formula yields

$$h = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

$$k = f(2) = 2^2 - 4(2) = -4$$

The vertex is at $(2,-4)$.

The domain is all real numbers and the range is $y \geq -4$.

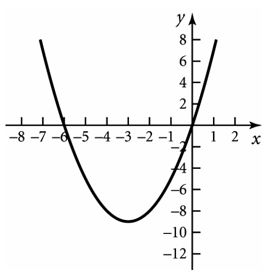


28. For $y = x^2 + 6x$, $a = 1$, $b = 6$, and $c = 0$.
Substituting these values in the vertex formula yields

$$h = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$$

$$k = f(-3) = (-3)^2 + 6(-3) = -9$$

The vertex is at $(-3, -9)$.
The domain is all real numbers and the range is $y \geq -9$.

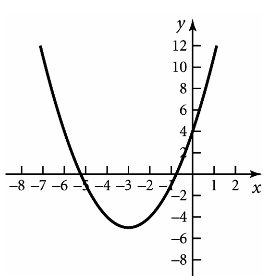


29. For $y = x^2 + 6x + 4$, $a = 1$, $b = 6$, and $c = 4$.
Substituting these values in the vertex formula yields

$$h = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$$

$$k = f(-3) = (-3)^2 + 6(-3) + 4 = -5$$

The vertex is at $(-3, -5)$.
The domain is all real numbers and the range is $y \geq -5$.

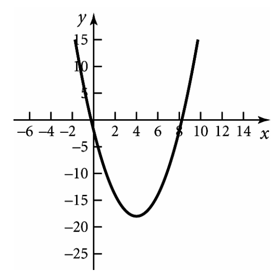


30. For $y = x^2 - 8x - 2$, $a = 1$, $b = -8$, and $c = -2$.
Substituting these values in the vertex formula yields

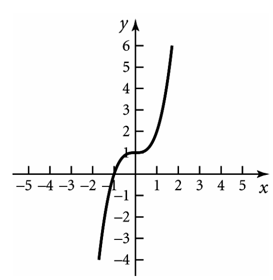
$$h = -\frac{b}{2a} = -\frac{-8}{2(1)} = 4$$

$$k = f(4) = (4)^2 - 8(4) - 2 = -18$$

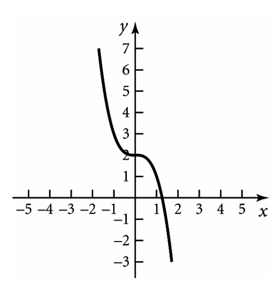
The vertex is at $(4, -18)$.
The domain is all real numbers and the range is $y \geq -18$.



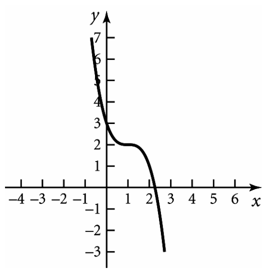
31. The graph of $y = x^3 + 1$ is the graph of $y = x^3$ shifted up 1 unit. The point of inflection is at $(0, 1)$. The graph is an increasing S-shape passing through $(0, 1)$. The domain and range are all real numbers.



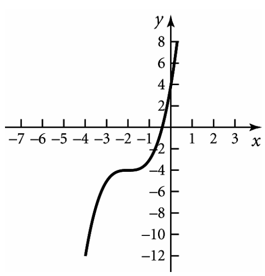
32. The graph of $y = 2 - x^3$ is the graph of $y = -x^3$ shifted up 2 units. The point of inflection is at $(0, 2)$. The graph is a decreasing S-shape passing through $(0, 2)$. The domain and range are all real numbers.



33. The graph of $y = 2-(x-1)^3$ is the graph of $y = -x^3$ shifted up 2 units and 1 unit to the right. The point of inflection is at $(1, 2)$. The graph is a decreasing S-shape passing through $(1, 2)$. The domain and range are all real numbers.

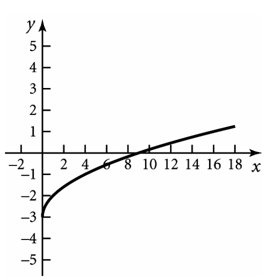


34. The graph of $y = (x+2)^3 - 4$ is the graph of $y = x^3$ shifted to the left 2 units and down 4 units. The point of inflection is at $(-2, -4)$. The graph is an increasing S-shape passing through $(-2, -4)$. The domain and range are all real numbers.



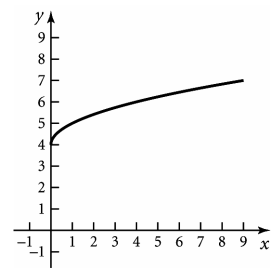
35. $y = \sqrt{x} - 3$

The domain is all $x \ge 0$. The graph is the graph of $y = \sqrt{x}$ shifted 3 units downward, so the range is all $y \ge -3$ and the end point of the graph is $(0, -3)$.



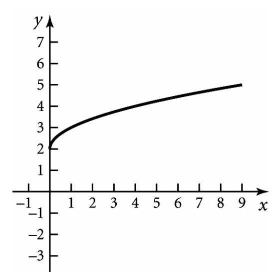
36. $y = \sqrt{x} + 4$

The domain is all $x \ge 0$. The graph is the graph of $y = \sqrt{x}$ shifted 4 units upward, so the range is all $y \ge 4$ and the end point of the graph is $(0, 4)$.



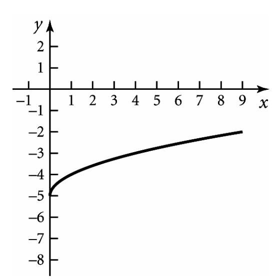
37. $y = \sqrt{x} + 2$

The domain is all $x \ge 0$. The graph is the graph of $y = \sqrt{x}$ shifted 2 units upward, so the range is all $y \ge 2$ and the end point of the graph is $(0, 2)$.



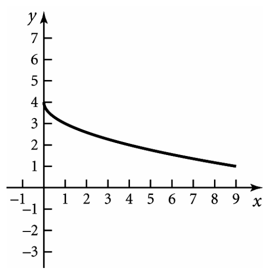
38. $y = \sqrt{x} - 5$

The domain is all $x \ge 0$. The graph is the graph of $y = \sqrt{x}$ shifted 5 units downward, so the range is all $y \ge -5$ and the end point of the graph is $(0, -5)$.



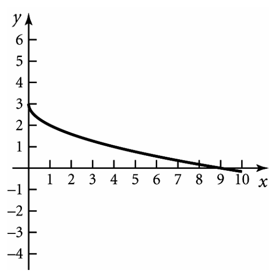
39. $y = 4 - \sqrt{x}$

The domain is all $x \geq 0$. The graph is the graph of $y = \sqrt{x}$ reflected about the x-axis because of the negative sign in front of the radical and shifted up 4 units. The range is all $y \leq 4$. The end point is $(0, 4)$.



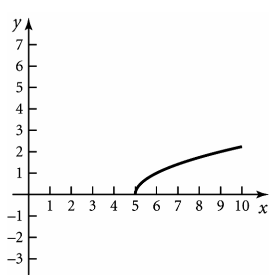
40. $y = 3 - \sqrt{x}$

The domain is all $x \geq 0$. The graph is the graph of $y = \sqrt{x}$ reflected about the x-axis because of the negative sign in front of the radical and shifted up 3 units. The range is all $y \leq 3$. The end point is $(0, 3)$.



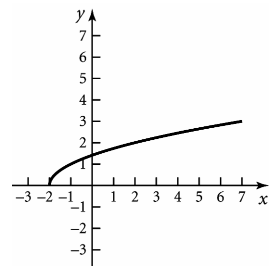
41. $y = \sqrt{x - 5}$

The domain is all values of x for which $x - 5 \geq 0$ or $x \geq 5$. The range is all nonnegative numbers. The end point of the graph is $(5, 0)$. The graph is the graph of $y = \sqrt{x}$ shifted 5 units to the right.



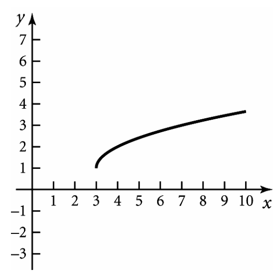
42. $y = \sqrt{x + 2}$

The domain is all values of x for which $x + 2 \geq 0$ or $x \geq -2$. The range is all nonnegative numbers. The end point of the graph is $(-2, 0)$. The graph is the graph of $y = \sqrt{x}$ shifted 2 units to the left.



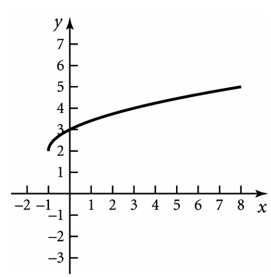
43. $y = 1 + \sqrt{x - 3}$

The domain is all values of x for which $x - 3 \geq 0$ or $x \geq 3$. The range is $y \geq 1$. The graph is the graph of $y = \sqrt{x}$ shifted 3 units to the right and 1 unit up. The end point of the graph is $(3,1)$.



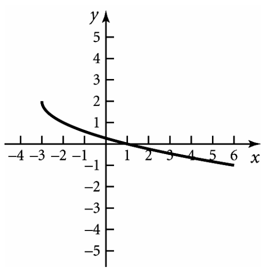
44. $y = 2 + \sqrt{x + 1}$

The domain is all values of x for which $x + 1 \geq 0$ or $x \geq -1$. The range is $y \geq 2$. The graph is the graph of $y = \sqrt{x}$ shifted 1 unit to the left and 2 units up. The end point of the graph is $(-1, 2)$.



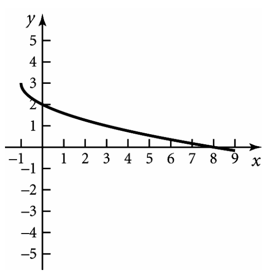
45. $y = 2 - \sqrt{x+3}$

The domain is all values of x for which $x+3 \geq 0$ or $x \geq -3$. The graph is the graph of $y = \sqrt{x}$ reflected about the x-axis because of the negative sign in front of the radical and shifted up 2 units and 3 units to the left. The range is all $y \leq 2$. The end point is $(-3, 2)$.



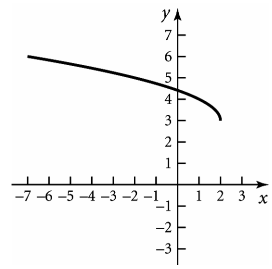
46. $y = 3 - \sqrt{x+1}$

The domain is all values of x for which $x+1 \geq 0$ or $x \geq -1$. The graph is the graph of $y = \sqrt{x}$ reflected about the x-axis because of the negative sign in front of the radical and shifted up 3 units and 1 unit to the left. The range is all $y \leq 3$. The end point is $(-1, 3)$.



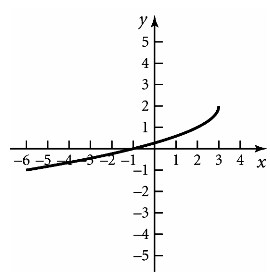
47. $y = 3 + \sqrt{2-x}$

The domain is all values of x for which $2-x \geq 0$ or $x \leq 2$. The graph is the graph of $y = \sqrt{x}$ shifted up 3 units and 2 units to the left and reflected about the y-axis because of the $-x$ under the radical. The range is all $y \geq 3$. The end point is $(2, 3)$.

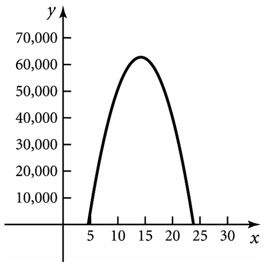


48. $y = 2 - \sqrt{3-x}$

The domain is all values of x for which $3-x \geq 0$ or $x \leq 3$. The graph is the graph of $y = \sqrt{x}$ reflected about the x-axis because of the negative sign in front of the radical, and shifted up 2 units and 3 units to the left. Lastly, reflected about the y-axis because of the $-x$ under the radical. The range is all $y \leq 2$. The end point is $(3, 2)$.



49. a. $A(x) = -680x^2 + 19,307x - 74,310$



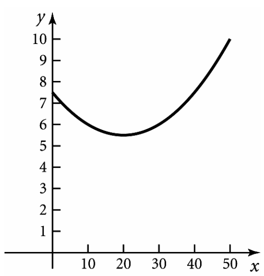
b. For $A(x) = -680x^2 + 19,307x - 74,310$, $a = -680$, $b = 19,370$, and $c = -74,310$. Substituting these values in the vertex formula yields

$$h = -\frac{b}{2a} = -\frac{19,307}{2(-680)} \approx 14.2$$

$$k = A(14.2) = -680(14.2)^2 + 19,307(14.2) - 74,310 = 62,734.2$$

The vertex is at $(14.2, 62734.2)$. This means the maximum number of new cases was 62,734.2, which occurred in 1994.

50. a. $E(x) = 0.005x^2 - 0.2x + 7.5$



b. For $E(x) = 0.005x^2 - 0.2x + 7.5$, $a = 0.005$, $b = -0.2$, and $c = 7.5$. Substituting these values in the vertex formula yields

$$h = -\frac{b}{2a} = -\frac{-0.2}{2(0.005)} = 20$$

$$k = E(20) = 0.005(20)^2 - 0.2(20) + 7.5 = 5.5$$

The vertex is at $(20, 5.5)$. This means the minimum unemployment rate was 5.5% in 2000.

51. a. To find the profit in 2007, let $x = 7$.

$$\begin{aligned} P &= 1.1x^2 - 5.97x + 22.1 \\ &= 1.1(7)^2 - 5.97(7) + 22.1 \\ &= 34.21 \end{aligned}$$

The estimated profit in 2007 is $34.21 billion.

b. For $P = 1.1x^2 - 5.97x + 22.1$, $a = 1.1$, $b = -5.97$, and $c = 22.1$. Substituting these values in the vertex formula yields

$$h = -\frac{b}{2a} = -\frac{-5.97}{2(1.1)} \approx 2.7$$

$$k = P(2.7) = 1.1(2.7)^2 - 5.97(2.7) + 22.1 = 14$$

This means the minimum profit was about \$14 billion in 2003.

52. a. To find the height of the ball after three seconds, let $t = 3$.

$$\begin{aligned} h &= -5.8t^2 + 46.5t - 2.7 \\ &= -5.8(3)^2 + 46.5(3) - 2.7 \\ &= 84.6 \end{aligned}$$

The height of the ball after three seconds is 84.6 feet.

b. For $h = -5.8t^2 + 46.5t - 2.7$, $a = -5.8$, $b = 46.5$, and $c = -2.7$. Substituting these values in the vertex formula yields

$$h = -\frac{b}{2a} = -\frac{46.5}{2(-5.8)} \approx 4$$

$$k = h(4) = -5.8(4)^2 + 46.5(4) - 2.7 = 90.5$$

This means the maximum height is approximately 90.5 feet after 4 seconds.

53. To estimate the CPI for 1950, let $x = 50$. For 1980, let $x = 80$. For 2005, let $x = 105$.

$$P(x) = 0.00047x^3 - 0.042x^2 + 1.25x + 2.77$$

$$P(50) = 0.00047(50)^3 - 0.042(50)^2 + 1.25(50) + 2.77 = 19.02$$

$$P(80) = 0.00047(80)^3 - 0.042(80)^2 + 1.25(80) + 2.77 = 74.61$$

$$P(105) = 0.00047(105)^3 - 0.042(105)^2 + 1.25(105) + 2.77 = 215.05$$

This means an item costing \$19.02 in 1950 cost \$74.61 in 1980 and \$215.05 in 2005.

54. To estimate the percent of young adult men (age 18-24) who smoked in 1975, let $x = 75$. For 1996, let $x = 96$. For 2005, let $x = 105$.

$$M(x) = 0.0013x^3 - 0.2713x^2 + 17.96x - 315$$

$$M(75) = 0.0013(75)^3 - 0.2713(75)^2 + 17.96(75) - 315 = 54.375$$

$$M(96) = 0.0013(96)^3 - 0.2713(96)^2 + 17.96(96) - 315 = 59.016$$

$$M(105) = 0.0013(105)^3 - 0.2713(105)^2 + 17.96(105) - 315 = 84.63$$

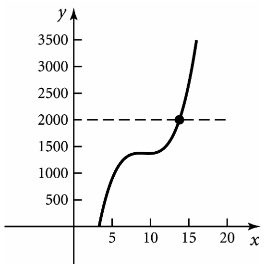
This means in 1975, 54.4% of young males smoked; in 1996, 59% smoked; and in 2005,84.63% smoked.

55. a. To find how many consumer bankruptcy filings there were in 2003, let $x = 8$.

$$\begin{aligned} B &= 6.9x^3 - 190.4x^2 + 1744x - 3931 \\ &= 6.9(8)^3 - 190.4(8)^2 + 1744(8) - 3931 \\ &= 1368.2 \end{aligned}$$

There were approximately 1,368,200 consumer bankruptcy filings in 2003.

b. $B = 6.9x^3 - 190.4x^2 + 1744x - 3931$



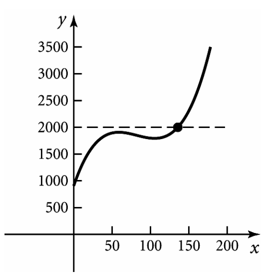
To estimate when there will be two million filings, find the point on the graph where $y = 2000$. The x-coordinate of that point, about 14, represents the number of years after 1995 when this will occur. So in 2009 there will be approximately two million filings.

56. a. To estimate the population of West Virginia in 2005, let $x = 105$.

$$\begin{aligned} P &= 0.00225x^3 - 0.554x^2 + 41.8x + 907 \\ &= 0.00225(105)^3 - 0.554(105)^2 + 41.8(105) + 907 \\ &= 1792.806 \end{aligned}$$

Thepopulation of West Virginia was approximately 1,792,806 in West Virginia in 2005.

b. $P = 0.00225x^3 - 0.554x^2 + 41.8x + 907$



To estimate when the population of West Virginia will be two million, find the point on the graph where $y = 2000$. The x-coordinate of that point, about 136, represents the number of years after 1900 when this will occur. So in 2036 there will be approximately two million people in West Virginia.

57. To find the demand if the price is \$10, let $p = 10$.

$$D = \sqrt{200 - p^2} = \sqrt{200 - 10^2} = \sqrt{100} = 10$$

The demand is 10,000 units if the price is \$10.

58. a. To find the dosage, let $h = 70$ and $w = 220$.

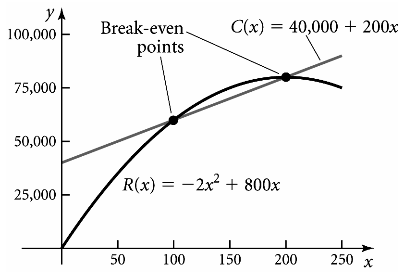
$$s = \frac{\sqrt{hw}}{15} = \frac{\sqrt{70(220)}}{15} \approx 8.3$$

The dosage for a man who is 70 inches tall and weighs 220 pounds is about 8.3 mg.

b. $$s = \frac{\sqrt{hw}}{15} = \frac{\sqrt{70(190)}}{15} \approx 7.7$$

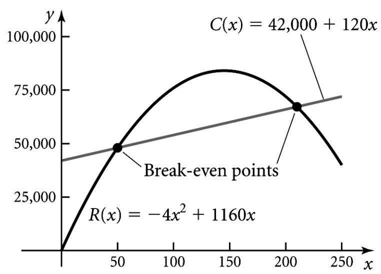
If the man's weight drops to 190 pounds the dosage is about 7.7 mg.

59. $C(x) = 40{,}000 + 200x$ and $R(x) = -2x^2 + 800x$



The break-even points are where the cost and revenue functions intersect. The manufacturer will break-even if either 100 or 200 motors are manufactured.

60. $C(x) = 42{,}000 + 120x$ and $R(x) = -4x^2 + 1160x$



The break-even points are where the cost and revenue functions intersect. A-to-Z Electronics will break-even if either 50 or 210 systems are installed.

61. The profit function is given by $P(x) = R(x) - C(x)$, where $R(x) = -2x^2 + 800x$ and $C(x) = 40{,}000 + 200x$.

$$P(x) = (-2x^2 + 800x) - (40{,}000 + 200x)$$
$$= -2x^2 + 600x - 40{,}000$$

The profit function is a quadratic function whose graph is a parabola opening downward. The maximum profit will occur at the vertex of the graph. Algebraically, the vertex is (h, k), where $h = -\dfrac{b}{2a}$ and $k = P(h)$.

$$h = -\frac{600}{2(-2)} = 150$$
$$k = P(150) = 5000$$

Thus, maximum profit, of \$5000, occurs if 150 motors are manufactured.

62. The profit function is given by $P(x) = R(x) - C(x)$, where $R(x) = -4x^2 + 1160x$ and $C(x) = 42{,}000 + 120x$.

$$P(x) = (-4x^2 + 1160x) - (42{,}000 + 120x)$$
$$= -4x^2 + 1040x - 42{,}000$$

The profit function is a quadratic function whose graph is a parabola opening downward. The maximum profit will occur at the vertex of the graph. Algebraically, the vertex is (h, k), where $h = -\dfrac{b}{2a}$ and $k = P(h)$.

$$h = -\frac{1040}{2(-4)} = 130$$
$$k = P(130) = 25{,}600$$

Thus, maximum profit, of \$25,600, occurs if 130 systems are installed.

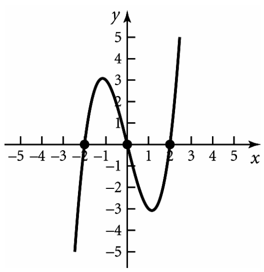
63. a. $x^3 - 4x = x(x^2 - 4) = x(x+2)(x-2)$

b. $x^3 - 4x = 0$

$x(x+2)(x-2) = 0$

$x = 0$ or $x = -2$ or $x = 2$

c. $y = x^3 - 4x$



The x-intercepts are $x = -2, 0, 2$.

d. Given $f(x)$, the solutions of $f(x) = 0$ are the x-intercepts of the graph of $y = f(x)$.

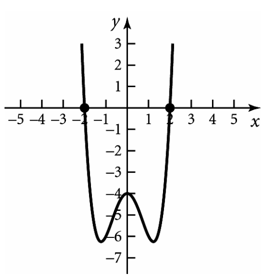
64. a. $x^4 - 3x^2 - 4 = (x^2 + 1)(x^2 - 4) = (x^2 + 1)(x+2)(x-2)$

b. $x^4 - 3x^2 - 4 = 0$

$(x^2 + 1)(x+2)(x-2) = 0$

The real solutions are $x = -2, 2$.

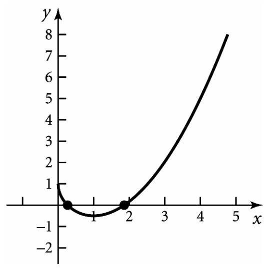
c. $y = x^4 - 3x^2 - 4$



The x-intercepts are $x = -2, 2$.

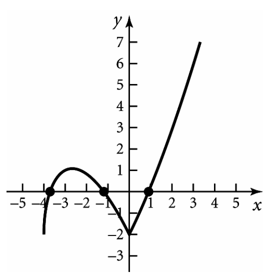
d. Given $f(x)$, the solutions of $f(x) = 0$ are the x-intercepts of the graph of $y = f(x)$.

65. **a.** $f(x)=0.5x^2-2\sqrt{x}+1$



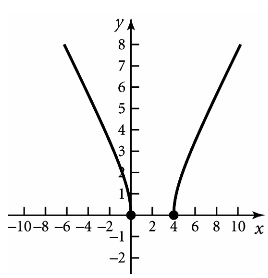
b. The solutions can be found at the x-intercepts. The solutions are $x\approx 0.268, 1.858$.

66. **a.** $f(x)=\sqrt{x^3+4x^2}-2$



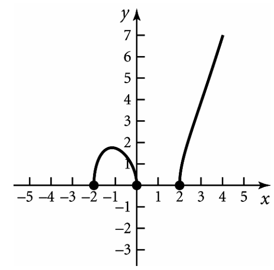
b. The solutions can be found at the x-intercepts. The solutions are $x\approx -3.709, -1.194, 0.903$.

67. $f(x)=\sqrt{x^2-4x}$



The domain is $x\le 0$ or $x\ge 4$. The range is $y\ge 0$.

68. $f(x)=\sqrt{x^3-4x}$



The domain is $-2\le x\le 0$ or $x\ge 2$. The range is $y\ge 0$.

69. $S(x)=200+15x$ and $D(x)=-1.5x^2+100x$, where x is the cost of a CD.
The equilibrium point is where supply equals demand.

$$S(x)=D(x)$$

$$200+15x=-1.5x^2+100x$$

$$1.5x^2-85x+200=0$$

By the Quadratic Formula,

$$x=\frac{-(-85)\pm\sqrt{(-85)^2-4(1.5)(200)}}{2(1.5)}$$

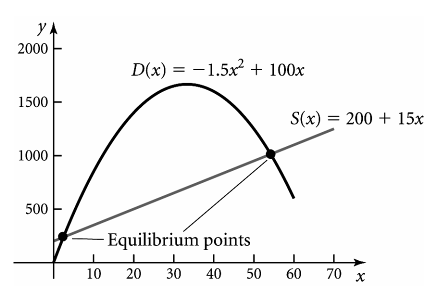
$$=\frac{85\pm\sqrt{6025}}{3}$$

$$\approx\frac{85\pm 77.62}{3}=\frac{162.62}{3}\text{ or }\frac{7.38}{3}$$

$$\approx 54.2\text{ or }2.46$$

Supply equals demand when the cost of a CD player is \$54.20. The other solution, \$2.46, is not a reasonable price for a CD player.

The graph of these functions confirms the points of equilibrium.



70. $S(x) = 50 + 5x$ and $D(x) = \sqrt{400 + 2500x}$, where x is the cost of the calculator.
The equilibrium point is where supply equals demand.

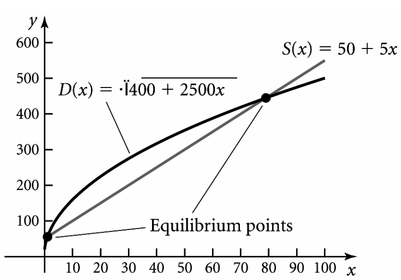
$$S(x) = D(x)$$
$$50 + 5x = \sqrt{400 + 2500x}$$
$$25x^2 - 2000x + 2100 = 0$$
$$x^2 - 80x + 84 = 0$$

By the Quadratic Formula,

$$x = \frac{-(-80) \pm \sqrt{(-80)^2 - 4(1)(84)}}{2(1)}$$
$$= \frac{80 \pm \sqrt{6064}}{2}$$
$$\approx \frac{80 \pm 77.872}{2} = \frac{157.872}{2} \text{ or } \frac{2.128}{2}$$
$$\approx 78.94 \text{ or } 1.06$$

Supply equals demand when the cost of the calculator is \$78.94. The other solution, \$1.06, is not a reasonable price for the calculator.

The graph of these functions confirms the points of equilibrium.



Topic 3 Exercises

1. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptote is $x = 2$.
The denominator has a higher degree than the numerator, so the horizontal asymptote is $y = 0$.

2. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptote is $x = -3$.
The denominator has a higher degree than the numerator, so the horizontal asymptote is $y = 0$.

3. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptote is $x = -\frac{7}{3}$.
The numerator and the denominator have the same degree, so the horizontal asymptote is $y = -\frac{2}{3}$.

4. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptote is $x = \frac{5}{2}$.
The numerator and the denominator have the same degree, so the horizontal asymptote is $y = \frac{3}{2}$.

5. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptotes are $x = 4$ and $x = -4$.
The numerator and the denominator have the same degree, so the horizontal asymptote is $y = 5$.

6. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptotes are $x = 3$ and $x = -3$.
The numerator and the denominator have the same degree, so the horizontal asymptote is $y = -2$.

7. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptotes are $x = 2$ and $x = -2$.
The denominator has a higher degree than the numerator, so the horizontal asymptote is $y = 0$.

8. There is no real value of x for which $x^2 + 4 = 0$. Thus, there is no vertical asymptote.
The denominator has a higher degree than the numerator, so the horizontal asymptote is $y = 0$.

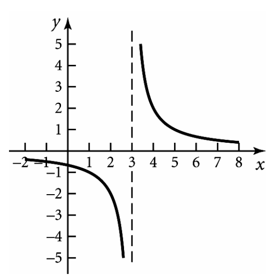
9. There is no real value of x for which $x^2 + 2 = 0$. Thus, there is no vertical asymptote.
The numerator and the denominator have the same degree, so the horizontal asymptote is $y = 1$.

10. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptotes are $x = 6$ and $x = -6$.
The numerator has the higher degree, so there is no horizontal asymptote.

11. There is no real value of x for which $x^2 + 1 = 0$. Thus, there is no vertical asymptote.
The denominator has a higher degree than the numerator, so the horizontal asymptote is $y = 0$.

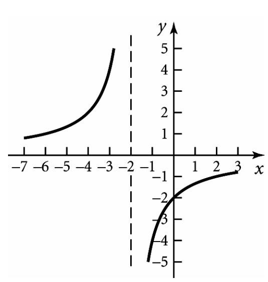
12. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptotes are $x = 2$ and $x = -2$.
The denominator has a higher degree than the numerator, so the horizontal asymptote is $y = 0$.

13. To graph a rational function, identify the asymptotes and intercepts, if any, and plot at least one point between and beyond each x-intercept and vertical asymptote before sketching the curve.



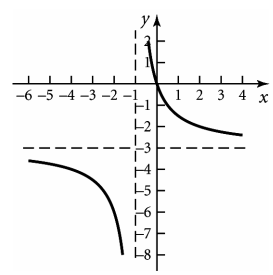
The domain of $y = \dfrac{2}{x-3}$ is all $x \neq 3$; the vertical asymptote is $x = 3$.
The range is $y \neq 0$; the horizontal asymptote is $y = 0$.

14. To graph a rational function, identify the asymptotes and intercepts, if any, and plot at least one point between and beyond each x-intercept and vertical asymptote before sketching the curve.



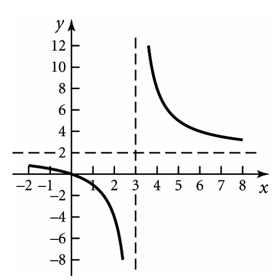
The domain of $y = \dfrac{-4}{x+2}$ is all $x \neq -2$; the vertical asymptote is $x = -2$.
The range is $y \neq 0$; the horizontal asymptote is $y = 0$.

15. To graph a rational function, identify the asymptotes and intercepts, if any, and plot at least one point between and beyond each x-intercept and vertical asymptote before sketching the curve.



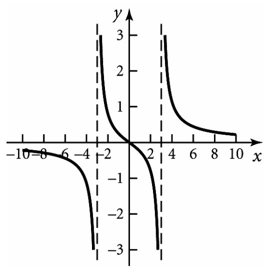
The domain of $y = \dfrac{-3x}{x+1}$ is all $x \neq -1$; the vertical asymptote is $x = -1$.
The range is $y \neq -3$; the horizontal asymptote is $y = -3$.

16. To graph a rational function, identify the asymptotes and intercepts, if any, and plot at least one point between and beyond each x-intercept and vertical asymptote before sketching the curve.



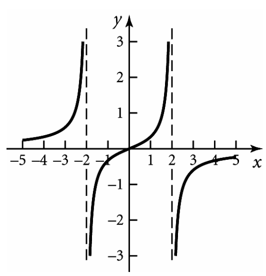
The domain of $y = \dfrac{2x}{x-3}$ is all $x \neq 3$; the vertical asymptote is $x = 3$.
The range is $y \neq 2$; the horizontal asymptote is $y = 2$.

17. $y = \dfrac{2x}{x^2 - 9}$



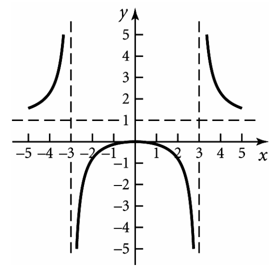
The domain of $y = \dfrac{2x}{x^2 - 9}$ is all $x \neq 3, -3$; the vertical asymptotes are $x = 3$ and $x = -3$. The range is all real numbers. The denominator has the higher degree, so the horizontal asymptote is $y = 0$.

18. $y = \dfrac{-x}{x^2 - 4}$



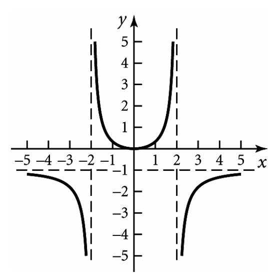
The domain of $y = \dfrac{-x}{x^2 - 4}$ is all $x \neq 2, -2$; the vertical asymptotes are $x = 2$ and $x = -2$. The range is all real numbers. The denominator has the higher degree, so the horizontal asymptote is $y = 0$.

19. $y = \dfrac{x^2}{x^2 - 9}$



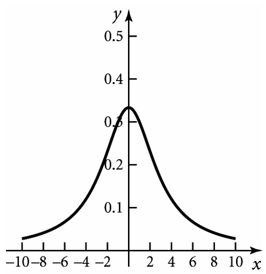
The domain of $y = \dfrac{x^2}{x^2 - 9}$ is all $x \neq 3, -3$; the vertical asymptotes are $x = 3$ and $x = -3$. The range is $y \leq 0$ or $y > 1$. The numerator and the denominator have the same degree, so the horizontal asymptote is $y = 1$.

20. $y = \dfrac{-x^2}{x^2 - 4}$



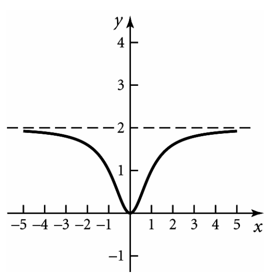
The domain of $y = \dfrac{-x^2}{x^2 - 4}$ is all $x \neq 2, -2$; the vertical asymptotes are $x = 2$ and $x = -2$. The range is $y \geq 0$ or $y < -1$. The numerator and the denominator have the same degree, so the horizontal asymptote is $y = -1$.

21. $y = \dfrac{3}{x^2 + 9}$



The domain of $y = \dfrac{3}{x^2 + 9}$ is all real numbers because there is no real value of x for which $x^2 + 9 = 0$. There are no vertical asymptotes. The range is $0 < y \le \dfrac{1}{3}$. The denominator has the higher degree, so the horizontal asymptote is $y = 0$.

22. $y = \dfrac{2x^2}{x^2 + 1}$



The domain of $y = \dfrac{2x^2}{x^2 + 1}$ is all real numbers because there is no value of x for which $x^2 + 1 = 0$. There are no vertical asymptotes. The range is $0 \le y < 2$. The numerator and the denominator have the same degree, so the horizontal asymptote is $y = 2$.

23. **a.** $x = -3$ belongs to the first domain interval, so $f(-3) = (-3) - 3 = -6$.

b. $x = 1$ belongs to the second domain interval, so $f(1) = 1^2 = 1$.

c. $x = 4$ belongs to the second domain interval, so $f(4) = 4^2 = 16$.

24. **a.** $x = -5$ belongs to the first domain interval, so $f(-5) = 5 - (-5) = 10$.

b. $x = 1$ belongs to the first domain interval, so $f(1) = 5 - 1 = 4$.

c. $x = 9$ belongs to the second domain interval, so $f(9) = \sqrt{9} = 3$.

25. **a.** $x = -3$ belongs to the first domain interval, so $f(-3) = 3$.

b. $x = 1$ belongs to the second domain interval, so $f(1) = 2(1) + 5 = 7$.

c. $x = 2$ belongs to the second domain interval, so $f(2) = 2(2) + 5 = 9$.

d. $x = 4$ belongs to the third domain interval, so $f(4) = 3 - 4 = -1$.

26. **a.** $x = -2$ belongs to the first domain interval, so $f(-2) = (-2)^3 = -8$.

b. $x = 0$ belongs to the second domain interval, so $f(0) = 1$.

c. $x = 2$ belongs to the third domain interval, so $f(2) = 2 - 1 = 1$.

d. $x = 4$ belongs to the third domain interval, so $f(4) = 4 - 1 = 3$.

27. **a.** $x = -3$ belongs to the first domain interval, so $f(-3) = (-3)^2 - 4 = 5$.

b. $x = 0$ belongs to the second domain interval, so $f(0) = 2$.

c. $x = 3$ belongs to the third domain interval, so $f(3) = 3^2 - 4 = 5$.

28. **a.** $x = 0$ belongs to the first domain interval, so $f(0) = 0 - 3 = -3$.

b. $x = 1$ belongs to the second domain interval, so $f(1) = 5$.

c. $x = 2$ belongs to the third domain interval, so $f(2) = 3 - 2 = 1$.

29. **a.** $x = 0$ belongs to the first domain interval, so $f(0) = 2(0) + 3 = 3$.

b. $x = 1$ belongs to the second domain interval, so $f(1) = 5$.

c. $x = 5$ belongs to the third domain interval, so $f(5) = \sqrt{5-1} = 2$.

30. **a.** $x = -3$ belongs to the first domain interval, so $f(-3) = 1 - (-3)^2 = -8$.

b. $x = 0$ belongs to the second domain interval, so $f(0) = 1$.

c. $x = 2$ belongs to the third domain interval, so $f(2) = 2^3 + 2 = 10$.

31. **a.** $x = -1$ belongs to the second domain interval, so $f(-1) = 4$.

b. $x = 1.99999$ belongs to the second domain interval, so $f(1.99999) = 4$.

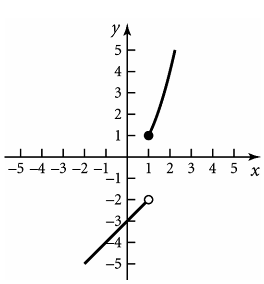
c. $x = 2.1$ belongs to the third domain interval, so $f(2.1) = 1$.

32. **a.** $x = -2$ belongs to the first domain interval, so $f(-2) = -2$.

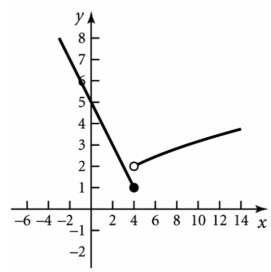
b. $x = 1.999999$ belongs to the second domain interval, so $f(1.99999) = 1$.

c. $x = 2$ belongs to the third domain interval, so $f(2) = 3$.

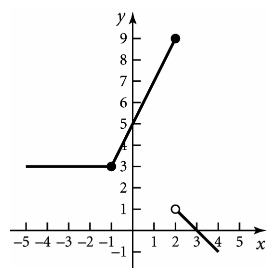
33. First graph $y_1 = x - 3$ on the domain $x < 1$, so only that part of the graph is drawn. Next, graph $y_2 = x^2$ on the domain $x \geq 1$, so that only that part of the graph is drawn.



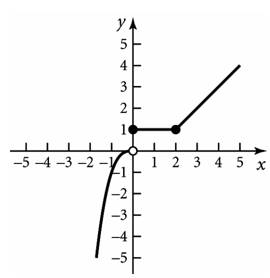
34. First graph $y_1 = 5 - x$ on the domain $x \leq 4$, so only that part of the graph is drawn. Next, graph $y_2 = \sqrt{x}$ on the domain $x > 4$, so that only that part of the graph is drawn.



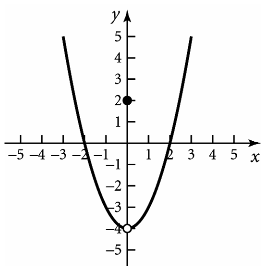
35. First graph $y_1 = 3$ on the domain $x \leq -1$, so only that part of the graph is drawn. Next, graph $y_2 = 2x + 5$ on the domain $-1 < x \leq 2$, so that only that part of the graph is drawn. Finally, graph $y_3 = 3 - x$ on the domain $x > 2$, so only that part of the graph is drawn.



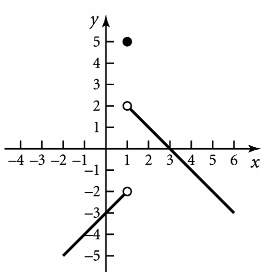
36. First graph $y_1 = x^3$ on the domain $x < 0$, so only that part of the graph is drawn. Next, graph $y_2 = 1$ on the domain $0 \leq x < 2$, so that only that part of the graph is drawn. Finally, graph $y_3 = x - 1$ on the domain $x \geq 2$, so only that part of the graph is drawn.



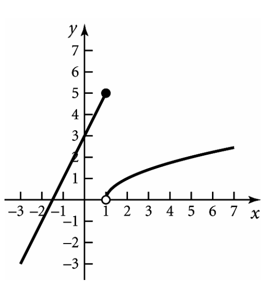
37. First graph $y_1 = x^2 - 4$ on the domain $x < 0$, so only that part of the graph is drawn. Next, graph $y_2 = 2$ on the domain $x = 0$, so that only that part of the graph is drawn. Finally, graph $y_3 = x^2 - 4$ on the domain $x > 0$, so only that part of the graph is drawn.



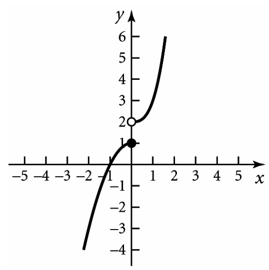
38. First graph $y_1 = x - 3$ on the domain $x < 1$, so only that part of the graph is drawn. Next, graph $y_2 = 5$ on the domain $x = 1$, so that only that part of the graph is drawn. Finally, graph $y_3 = 3 - x$ on the domain $x > 1$, so only that part of the graph is drawn.



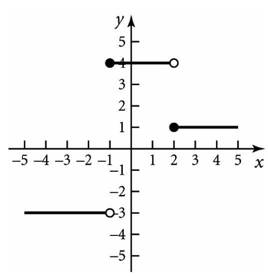
39. First graph $y_1 = 2x + 3$ on the domain $x < 1$, so only that part of the graph is drawn. Next, graph $y_2 = 5$ on the domain $x = 1$, so that only that part of the graph is drawn. Finally, graph $y_3 = \sqrt{x-1}$ on the domain $x > 1$, so only that part of the graph is drawn.



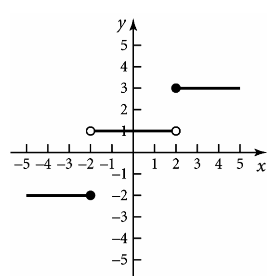
40. First graph $y_1 = 1 - x^2$ on the domain $x < 0$, so only that part of the graph is drawn. Next, graph $y_2 = 1$ on the domain $x = 0$, so that only that part of the graph is drawn. Finally, graph $y_3 = x^3 + 2$ on the domain $x > 0$, so only that part of the graph is drawn.



41. First graph $y_1 = -3$ on the domain $x < -1$, so only that part of the graph is drawn. Next, graph $y_2 = 4$ on the domain $-1 \le x < 2$, so that only that part of the graph is drawn. Finally, graph $y_3 = 1$ on the domain $x \ge 2$, so only that part of the graph is drawn.



42. First graph $y_1 = -2$ on the domain $x \le -2$, so only that part of the graph is drawn. Next, graph $y_2 = 1$ on the domain $-2 < x < 2$, so that only that part of the graph is drawn. Finally, graph $y_3 = 3$ on the domain $x \ge 2$, so only that part of the graph is drawn.



43. a. The fare for a 4-mile trip is \$2.20 for the first mile and \$1.50 for each of the three remaining miles or $f(4) = \$2.20 + \$1.50(3) = \$6.70$.

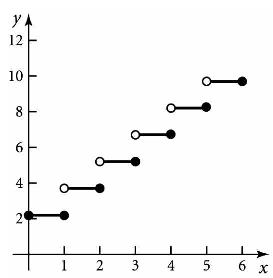
b. The fare for a 6.4-mile trip is \$2.20 for the first mile and \$1.50 for each of the six remaining miles or fraction of a mile or $f(6.4) = \$2.20 + \$1.50(6) = \$11.20$.

c. The piecewise defined function that describes cab fares in Las Vegas is

$$f(x) = \begin{cases} \$2.20 & \text{if } x \le 1 \\ \$3.70 & \text{if } 1 < x \le 2 \\ \$5.20 & \text{if } 2 < x \le 3 \\ \$6.70 & \text{if } 3 < x \le 4 \\ \$8.20 & \text{if } 4 < x \le 5 \\ \$9.70 & \text{if } 5 < x \le 6 \end{cases}$$

where x is the number of miles.

d.



44. a. The fare for a 4-mile trip is \$2.00 for the first mile and \$0.30 for each additional $\frac{1}{5}$ mile. The trip went 3 miles, or 15 fifths, past the first mile. So, $f(4) = \$2.00 + \$0.30(15) = \$6.50$.

b. The fare for a 2.4-mile trip is \$2.00 for the first mile and \$0.30 for each additional $\frac{1}{5}$ mile. The trip went 1.4 miles, or seven fifths, past the first mile. So, $f(2.4) = \$2.00 + \$0.30(7) = \$4.10$.

c. The fare for mileage is \$2.00 for the first mile and \$0.30 for each additional $\frac{1}{5}$ mile. The trip went 3.7 miles, or 19 fifths, past the first mile. So the fare for mileage is $f(4.7) = \$2.00 + \$0.30(19) = \$7.70$. The additional charge for idle time is \$0.20 per minute. If the cab was idle for 10.2 minutes, the additional charge would be $10.2(\$0.20) = \$2.04$. The total fare would be $\$7.70 + \$2.04 = \$9.74$.

45. a. $\$14,600 < \$15,800 \le \$59,400$, so

$$\begin{aligned} T(\$15,800) &= \$1460 + 0.15(\$15,800 - \$14,600) \\ &= \$1640 \end{aligned}$$

The income tax due is \$1640.

b. $\$59,400 < \$68,400 \le \$119,950$, so

$$\begin{aligned} T(\$68,400) &= \$8180 + 0.25(\$68,400 - \$59,400) \\ &= \$10,430 \end{aligned}$$

The income tax due is \$10,430.

c. $\$182,800 < \$202,600 \le \$326,450,$ so
$$T(\$202,600) = \$40,915.50 + 0.33(\$202,600 - \$182,800)$$
$$= \$47,449.50$$
The income tax due is $47,449.50.

46. **a.** $\$7300 < \$25,800 \le \$29,700,$ so
$$T(\$25,800) = \$730 + 0.15(\$25,800 - \$7,300)$$
$$= \$3505$$
The income tax due is $3505.

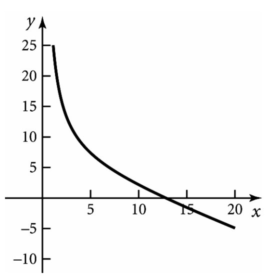
b. $\$29,700 < \$56,800 \le \$71,950,$ so
$$T(\$56,800) = \$4090 + 0.25(\$56,800 - \$29,700)$$
$$= \$10,865$$
The income tax due is $10,865.

c. $\$71,950 < \$92,600 \le \$150,150,$ so
$$T(\$92,600) = \$14,652.50 + 0.28(\$92,600 - \$71,950)$$
$$= \$20,434.50$$
The income tax due is $20,434.50.

47. **a.** The average annual profit function for *x* years after 2000 is
$$AveP(x) = \frac{-0.6x^2 + 5.97x + 22.1}{x} = -0.6x + 5.97 + \frac{22.1}{x}$$

b. To find the average profit for 2010, let $x = 10$.
$$AveP(10) = -0.6(10) + 5.97 + \frac{22.1}{10} = \$2.18 \text{ billion}$$

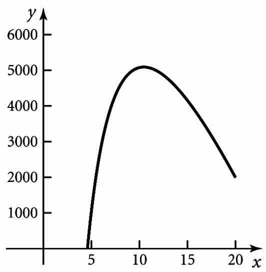
c.



48. **a.** The average number of cases per year for *x* years after 1990 is
$$AveA(x) = \frac{-680x^2 + 19,307x - 74,310}{x} = -680x + 19,307 - \frac{74,310}{x}$$

b. To find the average number of new cases for 2008, let $x = 18$.
$$AveA(18) = -680(18) + 19,307 - \frac{74,310}{18} = 2938.7 \approx 2939 \text{ cases}$$

c.



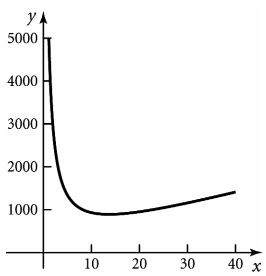
49. a. The average cost of the mower per year for *x* years is

$$AveC(x) = \frac{5600 + 70x + 30x^2}{x} = \frac{5600}{x} + 70 + 30x$$

b. $AveC(10) = \frac{5600}{10} + 70 + 30(10) = \$930$ cost per year after 10 years

$AveC(30) = \frac{5600}{30} + 70 + 30(30) = \$1156.67 \approx \$1157$ cost per year after 30 years

c.



d. Bernie should replace the mower when the average cost per year starts to increase. The graph shows that to be around 14 years.

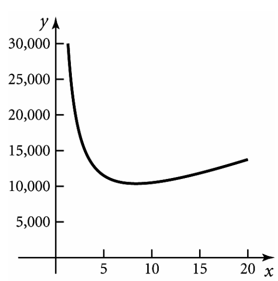
50. a. The average cost of the copier per year for *x* years is

$$AveC(x) = \frac{35,000 + 2000x + 500x^2}{x} = \frac{35,000}{x} + 2000 + 500x$$

b. $AveC(5) = \frac{35,000}{5} + 2000 + 500(5) = \$11,500$ cost per year after 5 years

$AveC(15) = \frac{35,000}{15} + 2000 + 500(15) = \$11,833.33 \approx \$11,833$ cost per year after 15 years

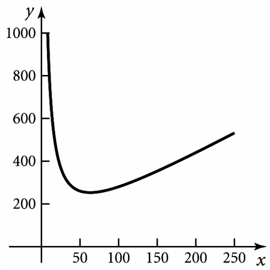
c.



d. The copier should be replaced when the average cost per year starts to increase. The graph shows that to be around 8 or 9 years.

51. **a.** The domain of the perimeter function is $x > 0$.

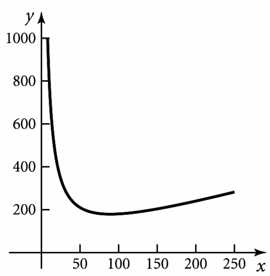
b.



c. Using the graph the length of side *x* that gives the smallest perimeter is $x \approx 63.25$ feet.

52. **a.** The domain of the fence function is $x > 0$.

b.



c. Using the graph the length of side *x* that gives the smallest amount of fencing is $x \approx 89.44$ feet.

53. **a.** The postage for a parcel weighing 1.5 pounds is \$0.45 because $1.5 \le 2$.

b. The postage for a parcel weighing 4 pounds is \$1.25 because $3 < 4 \le 4$.

c. The postage for a parcel weighing 20 pounds is \$1.70 because $4 < 20 \le 30$.

d. A function that describes postage rates for parcel airlift based on weight is

$$P(x) = \begin{cases} \$0.45 & \text{if } x \le 2 \\ \$0.85 & \text{if } 2 < x \le 3 \\ \$1.25 & \text{if } 3 < x \le 4 \\ \$1.70 & \text{if } 4 < x \le 30 \end{cases}$$

where *P* is the postage rate and *x* is the parcel weight in pounds.

54. **a.** The postage for an express mail package weighing 5 ounces is \$13.65 because $5 \le 8$.

b. The postage for an express mail package weighing 2 pounds is \$17.85 because $0.5 < 2 \le 2$.

c. The postage for an express mail package weighing 3.8 pounds is \$24.20 because $3 < 3.8 \le 4$.

d. A function that describes express mail rate based on weight is

$$P(x) = \begin{cases} \$13.65 & \text{if } x \le 0.5 \\ \$17.85 & \text{if } 0.5 < x \le 2 \\ \$21.05 & \text{if } 2 < x \le 3 \\ \$24.20 & \text{if } 3 < x \le 4 \\ \$27.30 & \text{if } 4 < x \le 5 \\ \$30.40 & \text{if } 5 < x \le 6 \end{cases}$$

where P is the postage rate and x is the express mail package weight in pounds.

55. a. A piecewise defined function that describes the tax due is

$$T(d) = \begin{matrix} 0.10d & \text{if } d \le \$10{,}450 \\ \$1045 + 0.15(d - \$10{,}450) & \text{if } \$10{,}450 < d \le \$39{,}800 \\ \$5447.50 + 0.25(d - \$39{,}800) & \text{if } \$39{,}800 < d \le \$102{,}800 \\ \$21{,}197.50 + 0.28(d - \$102{,}800) & \text{if } \$102{,}800 < d \le \$166{,}450 \\ \$39{,}019.50 + 0.33(d - \$166{,}450) & \text{if } \$166{,}450 < d \le \$326{,}450 \\ \$91{,}819.50 + 0.35(d - \$326{,}450) & \text{if } d < \$326{,}450 \end{matrix}$$

where T is income tax due and d is the taxable income.

b. $\$10{,}450 < \$20{,}000 \le \$39{,}800$, so

$$\begin{aligned} T(\$20{,}000) &= \$1045 + 0.15(\$20{,}000 - \$10{,}450) \\ &= \$2477.50 \end{aligned}$$

The income tax due is $2477.50.

c. $\$39{,}800 < \$50{,}200 \le \$102{,}800$, so

$$\begin{aligned} T(\$50{,}200) &= \$5447.50 + 0.25(\$50{,}200 - \$39{,}800) \\ &= \$8047.50 \end{aligned}$$

The income tax due is $8047.50.

56. a. A piecewise defined function that describes the tax due is

$$T(d) = \begin{cases} 0.10d & \text{if } d \le \$7300 \\ \$730 + 0.15(d - \$7300) & \text{if } \$7300 < d \le \$29{,}700 \\ \$4090 + 0.25(d - \$29{,}700) & \text{if } \$29{,}700 < d \le \$59{,}975 \\ \$11{,}658.75 + 0.28(d - \$59{,}975) & \text{if } \$59{,}975 < d \le \$91{,}400 \\ \$20{,}457.75 + 0.33(d - \$91{,}400) & \text{if } \$91{,}400 < d \le \$163{,}225 \\ \$44{,}160 + 0.35(d - \$163{,}225) & \text{if } d > \$163{,}225 \end{cases}$$

where T is income tax due and d is the taxable income.

b. $\$7300 < \$20{,}000 \le \$29{,}700$, so

$$\begin{aligned} T(\$20{,}000) &= \$730 + 0.15(\$20{,}000 - \$7300) \\ &= \$2635 \end{aligned}$$

The income tax due is $2635.

c. $\$29{,}700 < \$50{,}200 \le \$59{,}975$, so

$$\begin{aligned} T(\$50{,}200) &= \$4090 + 0.25(\$50{,}200 - \$29{,}700) \\ &= \$9215 \end{aligned}$$

The income tax due is $9215.

Topic 4 Exercises

1. The graph shows that as x increases in value, y also increases at a fairly constant rate. Thus, a linear model would be the best model of this data.

2. The graph shows that y decreases until $x = 0$ and then increases in value. Thus, a quadratic model would be the best model of this data.

3. The graph shows that y increases until $x = 0$ and then decreases in value. Thus, a quadratic model would be the best model of this data.

4. The graph shows a cubic model would be the best model of this data.

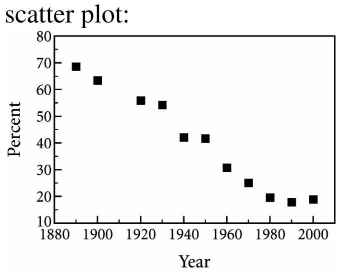
5. The graph shows an upward trend in the data, but the increase is not always at a constant rate. The data might be modeled by a linear function or by a power function. It appears to be more likely a power function.

6. The graph shows that y generally decreases until $x = 0$ and then increases in value. Thus, a quadratic model would be the best model of this data.

7. The graph shows a cubic model would be the best model of this data.

8. The graph shows a cubic model would be the best model of this data.

9. **a.** scatter plot:

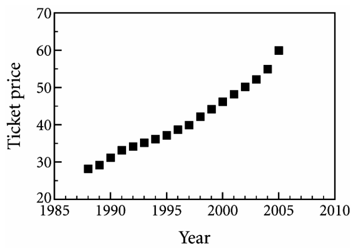


b. On a TI-83/84 use STAT→EDIT→and enter the x data in list 1 and the y data in list 2. To find the linear regression use STAT→CALC→LinReg→2^{nd} 1, 2^{nd} 2→ENTER.
The linear regression function is $y = -0.5102x + 69.23,$ where y is the percent of men aged 65 or older in the labor force and x is the number of years after 1890.

c. Let $x = 70$ to use the model in part b to estimate the percent of men aged 65 or older in the work force in 1960.
$y = -0.5102(70) + 69.23 = 33.516$
The estimate of 33.516% is fairly close to the actual 30.5%.

d. Let $x = 118$ to use the model in part b to estimate the percent of men aged 65 or older in the work force in 2008.
$y = -0.5102(118) + 69.23 = 9.0264$
Approximately 9.0264% of men aged 65 or older are predicted to be in the work force in 2008.

10. a. scatter plot:

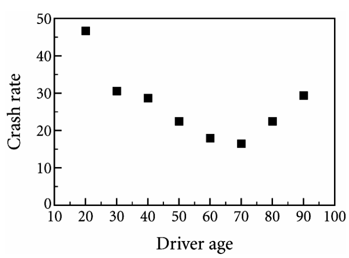


b. On a TI-83/84 use STAT→EDIT→and enter the x data in list 1 and the y data in list 2. To find the linear regression use STAT→CALC→LinReg→2nd 1, 2nd 2→ENTER.
The linear regression function is $y = 1.6979x + 21.46,$ where y is the ticket price and x is the number of years after 1985.

c. Let $x = 17$ to use the model in part b to estimate the ticket price in 2002.
$y = 1.6979(17) + 21.46 = 50.32$
The estimate of \$50.32 is very close to the actual ticket price of \$50.00.

d. Let $x = 23$ to use the model in part b to estimate the ticket price in 2008.
$y = 1.6979(23) + 21.46 = 60.51$
The ticket price is predicted to be \$60.51 in 2008.

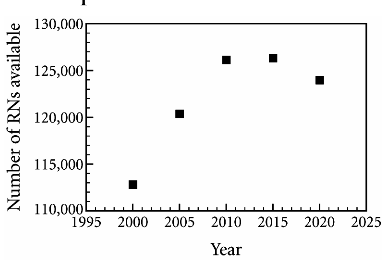
11. a. scatter plot:



b. On a TI-83/84 use STAT→EDIT→and enter the x data in list 1 and the y data in list 2. To find the quadratic regression use STAT→CALC→QuadReg→2nd 1, 2nd 2→ENTER.
The quadratic regression function is $y = 0.0147x^2 - 1.8632x + 76.795,$ where y is the crash rate per 1000 drivers and x is the midpoint of each age group.

c. Let $x = 40$ to use the model in part b to estimate the crash rate for 40-year-old drivers.
$y = 0.0147(40)^2 - 1.8632(40) + 76.795 = 25.787$ The estimate of 25.787 per 1000 drivers is fairly close to the actual rate of 28.5 per 1000 drivers.

12. a. scatter plot:



b. On a TI-83/84 use STAT→EDIT→and enter the x data in list 1 and the y data in list 2. To find the quadratic regression use STAT→CALC→QuadReg→2nd 1, 2nd 2→ENTER.

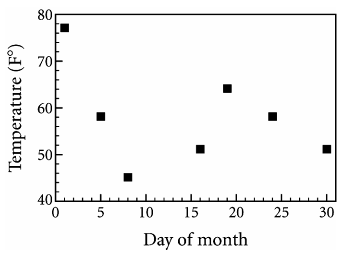
The quadratic regression function is $y = -72.611x^2 + 2018.4x + 112{,}559,$ where y is the number of registered nurses available and x is the amount of years after 2000.

c. Let $x = 7$ to use the model in part b to estimate the number of registered nurses available in 2007.

$y = -72.611(7)^2 + 2018.4(7) + 112{,}559 \approx 123{,}130$

The model predicts there will be 123,130 registered nurses available in 2007.

13. a. scatter plot:



b. On a TI-83/84 use STAT→EDIT→and enter the x data in list 1 and the y data in list 2. To find the cubic regression use STAT→CALC→CubicReg→2nd 1, 2nd 2→ENTER.

The cubic regression function is $y = -0.0121x^3 + 0.6192x^2 - 8.8497x + 85.619,$ where y is the temperature $(^\circ\text{F})$ and x is the day of the month.

c. Let $x = 16$ to use the model in part b to estimate the temperature on November 16.

$y = -0.0121(16)^3 + 0.6192(16)^2 - 8.8497(16) + 85.619 \approx 53$

The estimate of 53° F is very close to the actual temperature of 51° F.

d. Let $x = 12$ to use the model in part b to estimate the temperature on November 12.

$y = -0.0121(12)^3 + 0.6192(12)^2 - 8.8497(12) + 85.619 \approx 47.7$

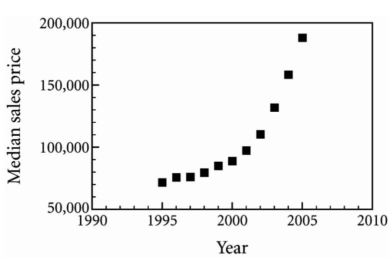
The model predicts the temperature will be 47.7°F on November 12.

e. Let $x = 33$ to use the model in part b to estimate the temperature on December 3.

$y = -0.0121(33)^3 + 0.6192(33)^2 - 8.8497(33) + 85.619 \approx 33.1$

The model predicts the temperature will be 33.1°F on December 3.

14. a. scatter plot:



b. On a TI-83/84 use STAT→EDIT→and enter the x data in list 1 and the y data in list 2. To find the cubic regression use STAT→CALC→CubicReg→2nd 1, 2nd 2→ENTER.

The cubic regression function is $y = 173.05x^3 - 3531.9x^2 + 26133x + 7381.7,$ where y is the median sale price and x is number of years after 1990.

c. Let $x = 11$ to use the model in part b to estimate the median sales price in 2001.

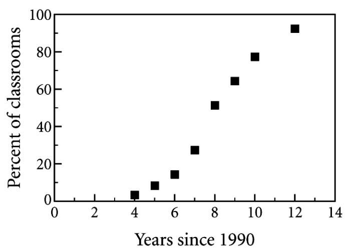
$y = 173.05(11)^3 - 3531.9(11)^2 + 26133(11) + 7381.7 = 97,814.35$

The estimate of \$97,814.35 is close to the actual median sales price of \$96,770.

d. Let $x = 18$ to use the model in part b to estimate the median sales price in 2008.

$y = 173.05(18)^3 - 3531.9(18)^2 + 26133(18) + 7381.7 = \$342,667.70$

15. a. scatter plot:



b. On a TI-83/84 use STAT→EDIT→and enter the *x* data in list 1 and the *y* data in list 2. To find the power regression use STAT→CALC→PwrReg→2^{nd} 1, 2^{nd} 2→ENTER.

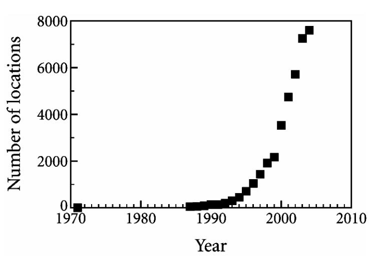
The power regression function is $y = 0.0405x^{3.2742}$, where *y* is the percent of classrooms with Internet access and *x* is number of years after 1990.

c. Let $x = 7$ to use the model in part b to estimate the percent of classrooms with Internet access in 1997.

$y = 0.0405(7)^{3.2742} \approx 23.7$

The estimate of 23.7% is fairly close to the actual percent of 27%.

16. a. scatter plot:



b. On a TI-83/84 use STAT→EDIT→and enter the *x* data in list 1 and the *y* data in list 2. To find the power regression use STAT→CALC→PwrReg→2^{nd} 1, 2^{nd} 2→ENTER.

The power regression function is $y = 0.2013x^{2.5164}$, where *y* is the number of Starbucks locations and *x* is number of years after 1970.

c. Let $x = 29$ to use the model in part b to estimate the number of Starbucks locations in 1999.

$y = 0.2013(29)^{2.5164} \approx 963$

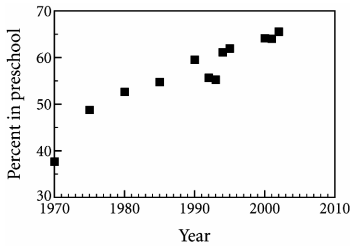
The estimate of 963 locations is not very close actual amount of locations in 1999 of 2135.

d. Let $x = 40$ to use the model in part b to estimate the number of Starbucks locations in 2010.

$y = 0.2013(29)^{2.5164} \approx 2164$

The estimate of 2164 locations does not seem very reasonable.

17. a. scatter plot:



b. Letting x represent the number of years after 1970, we obtain the following regression models.

Linear Model	$y = 0.7274x + 42.242$	$r^2 = 0.8923$
Quadratic Model	$y = -0.0107x^2 + 1.0836x + 40.433$	$r^2 = 0.9089$
Cubic Model	$y = 0.0019x^3 - 0.1092x^2 + 2.3665x + 38.037$	$r^2 = 0.9486$

c. For 1975, let $x = 5$. Using the linear model, $y \approx 45.9$. Using the quadratic model, $y \approx 45.6$. Using the cubic model, $y \approx 47.4$, which is the closest to the actual value of 48.6%.

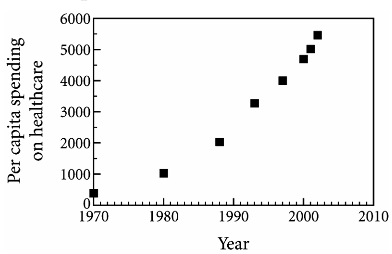
For 1993, let $x = 23$. Using the linear model, $y \approx 59$. Using the quadratic model, $y \approx 59.7$. Using the cubic model, $y \approx 57.8$, which is the closest to the actual value of 55.1%.

d. Let $x = 39$ to use the cubic model in part b to estimate the percent of 3- to 5-year-olds that will be in preschool in 2009.

$y = 0.0019x^3 - 0.1092x^2 + 2.3665x + 38.037 \approx 76.9$

The cubic model in part b predicts that 76.9% of 3- to 5-year-olds will be in preschool in 2009.

18. a. scatter plot:



b. Letting x represent the number of years after 1960, we obtain the following regression models.

Linear Model	$y = 162.16x - 1873.4$	$r^2 = 0.948$
Quadratic Model	$y = 4.1426x^2 - 59.8019x + 540.555$	$r^2 = 0.9974$
Cubic Model	$y = -0.0116x^3 + 5.0587x^2 - 81.419x + 683.13$	$r^2 = 0.9974$
Power Model	$y = 3.8592x^{1.9179}$	$r^2 = 0.9878$

c. For 1988, let $x = 28$. Using the linear model, $y \approx \$2667$. Using the quadratic model, $y \approx \$2114$. Using the cubic model, $y \approx \$2115$. Using the power model, $y \approx \$2301$. The actual value is \$2012, so the quadratic model is the closest.

For 2001, let $x = 41$. Using the linear model, $y \approx \$4775$. Using the quadratic model, $y \approx \$5052$. Using the cubic model, $y \approx \$5049$. Using the power model, $y \approx \$4783$. The actual value is \$4995, so the cubic model is the closest.

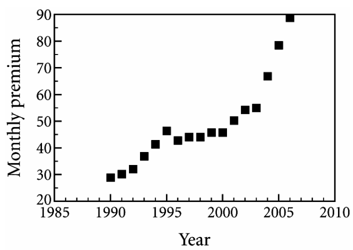
This means the quadratic and the cubic models are fairly equal for this situation.

d. Let $x = 48$ to use both the quadratic and cubic model in part b to estimate the per capita spending on health care in 2008.

Quadratic Model $y = 4.1426(48)^2 - 59.8019(48) + 540.555 \approx \$7215$

Cubic Model $y = -0.0116(48)^3 + 5.0587(48)^2 - 81.419(48) + 683.13 \approx \$7147$

19. a. scatter plot:



b. Letting *x* represent the number of years after 1980, we obtain the following regression models.

Linear Model $y = 2.9108x - 3.7353$ $r^2 = 0.8228$

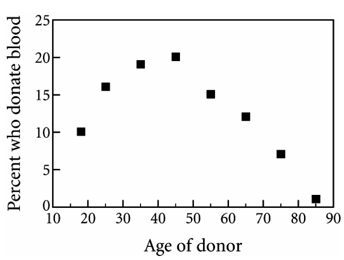
Cubic Model $y = 0.0495x^3 - 2.4669x^2 + 41.492x - 192.45$ $r^2 = 0.9819$

Power Model $y = 2.9101x^{0.9717}$ $r^2 = 0.8684$

The domain is $1990 \le x \le 2006$. Based on r^2 values, the cubic model is most likely the best predictor.

c. For 2009, let $x = 29$. Using the cubic model, $y \approx \$143.41$. The model predicts the cost of premiums in 2009 will be \$143.41.

20. a. scatter plot:



b. The scatter plot showed a quadratic model. Letting *x* represent the age of the donor, we obtain the following quadratic model: $y = -0.0105x^2 + 0.9015x - 1.069$, with $18 \le x \le 85$.

c. To estimate the percentage of 20-year-olds who donate blood, let $x = 20$. Using the quadratic model, $y \approx 12.76\%$. The model seems to predict the percentage very well as compared to the nearest data points.

d. To estimate the percentage of 50-year-olds who donate blood, let $x = 50$. Using the quadratic model, $y \approx 17.76\%$. The model seems to predict the percentage very well as compared to the nearest data points.

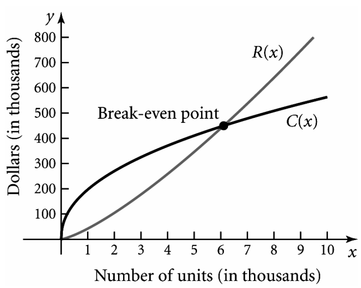
21. a. The linear regression function that models the demand for registered nurses is $D(x) = 3437.5x + 112,915,$ where x represents the number of years after 2000.

b. The quadratic regression function that models the supply of registered nurses is $S(x) = -72.611x^2 + 2018.4x + 112,559,$ where x is the amount of years after 2000.

c. To predict the shortage of registered nurses in 2008 and 2015, let $x = 8$ and $x = 15,$ in both the supply and demand models. Then subtract the supply from the demand to find the shortage.
$D(8) - S(8) = 140,415 - 124,059 = 16,356$ registered nurses short in 2008
$D(15) - S(15) = 164,478 - 126,498 = 37,980$ registered nurses short in 2015

22. a. The power regression model for cost is $C(x) = 196.83x^{0.4565},$ where x is the number of units in thousands and C is the cost in thousands of dollars.
The power regression model for revenue is $R(x) = 42.163x^{1.308},$ where x is the number of units in thousands and R is the revenue in thousands of dollars.

b. Graph of the cost and revenue functions:



The point of intersection of the graphs show the break-even point is about 6108 books.

c. To estimate the profit if 8000 textbooks are sold, let $x = 8,$ in both models. Then subtract the costs from the revenue to find the profit in thousands of dollars.
$P(8) = R(8) - S(8) \approx 639.99 - 508.571 = 131.419$
Thus, the profit is $131.419(1000) = \$131,419$.

Unit 0 Test

1. This set of ordered pairs does not describe a function because the domain element 3 is paired with both 5 and 7 in the range.

2. This equation describes a function. Each x will produce only one value of y. The domain and range is all real numbers.

3. The curve describes a function because any vertical line would intersect the graph at only one point. In other words, no x value is paired with more than one y value. The domain is $x \geq 2$ and the range is $y \geq 0$.

4. This curve does not represent a function, because there is at least one vertical line that intersects the graph in more than one point.

5. This equation does not describe a function. Choosing $x = 2$ yields $y^2 = 1,$ which means that $y = 1$ or -1.

6. This set of ordered pairs is a function because each domain element is paired with only one range element. The domain is $\{-1, 3, 6\}$ and the range is $\{4, 5\}$.

7. a. $f(x) = -2x^2 + 3x - 4$
$f(3) = -2(3)^2 + 3(3) - 4 = -13$

b. $f(-1) = -2(-1)^2 + 3(-1) - 4 = -9$

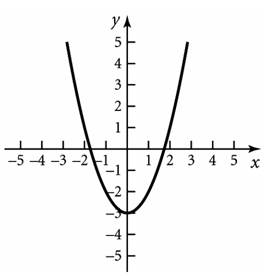
c. $f(a) = -2a^2 + 3a - 4$

d. $f(a+h) = -2(a+h)^2 + 3(a+h) - 4$
$= -2(a^2 + 2ah + h^2) + 3a + 3h - 4$
$= -2a^2 - 4ah - 2h^2 + 3a + 3h - 4$

8. a. $m = -3; (0, 5)$

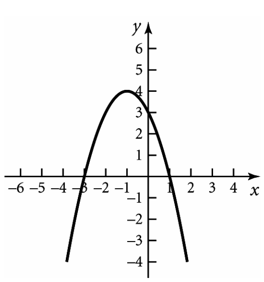
b. $5x - 2y = 10$
$y = \frac{5}{2}x - 5$
So, $m = \frac{5}{2}; (0, -5)$

9. $f(x) = x^2 - 3$



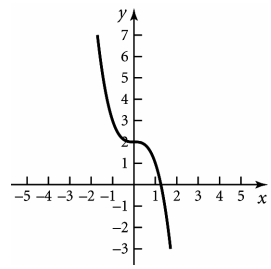
The domain is all real numbers; the vertex is $(0, -3)$. Because $a = 1 > 0$, the parabola opens upward and the range is $y \geq -3$.

10. $f(x) = 4 - (x+1)^2$



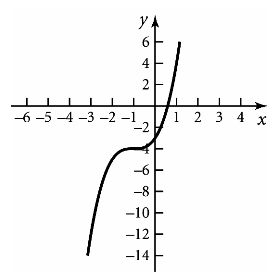
The domain is all real numbers; the vertex is $(-1, 4)$. Because $a = -1 < 0$, the parabola opens downward and the range is $y \leq 4$.

11. $f(x) = 2 - x^3$



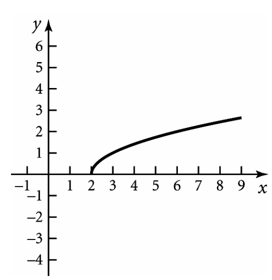
The graph of $y = 2 - x^3$ is the graph of $y = -x^3$ shifted up 2 units. The point of inflection is at $(0, 2)$. The graph is a decreasing S-shape passing through $(0, 2)$. The domain and range are all real numbers.

12. $f(x) = (x+1)^3 - 4$



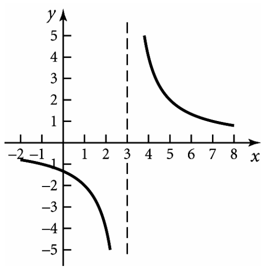
The graph of $y = (x+1)^3 - 4$ is the graph of $y = x^3$ shifted to the left 1 unit and down 4 units. The point of inflection is at $(-1, -4)$. The graph is an increasing S-shape passing through $(-1, -4)$. The domain and range are all real numbers.

13. $f(x) = \sqrt{x - 2}$



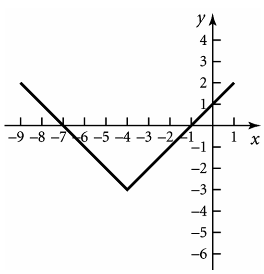
The domain is all values of x for which $x - 2 \geq 0$ or $x \geq 2$. The range is all nonnegative numbers. The end point of the graph is $(2, 0)$. The graph is the graph of $y = \sqrt{x}$ shifted 2 units to the right.

14. $f(x)=\dfrac{4}{x-3}$



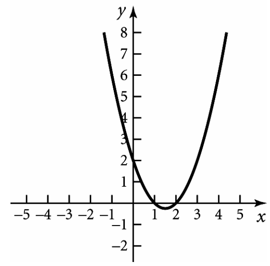
The domain of $y=\dfrac{4}{x-3}$ is all $x\neq 3;$ the vertical asymptote is $x=3.$
The range is $y\neq 0;$ the horizontal asymptote is $y=0.$

15. $f(x)=|x+4|-3$



The graph of $f(x)=|x+4|-3$ is the graph of $f(x)=|x|$ shifted 4 units to the left and down 3 units. The corner point is $(-4,-3).$ The domain is all real numbers; the range is $y\geq -3.$

16. $y=2-3x+x^2$



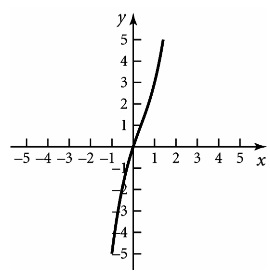
For $y=2-3x+x^2,$ $a=1,$ $b=-3,$ and $c=2.$ Substituting these values in the vertex formula yields

$$h=-\frac{b}{2a}=-\frac{-3}{2(1)}=1.5$$

$$k=f(1.5)=2-3(1.5)+(1.5)^2=-0.25$$

The vertex is at $(1.5,-0.25).$
The domain is all real numbers and the range is $y\geq -0.25.$

17. $y=x^3-x^2+3x$



The domain and the range are all real numbers.

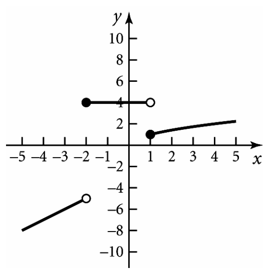
18. **a.** $x = 0$ belongs to the second domain interval, so $f(0) = 4$.

b. $x = 1$ belongs to the third domain interval, so $f(1) = \sqrt{1} = 1$.

c. $x = -2$ belongs to the second domain interval, so $f(-2) = 4$.

d. $x = -4$ belongs to the first domain interval, so $f(-4) = -4 - 3 = -7$.

e.



19. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptote is $x = 3$.
The denominator has a higher degree than the numerator, so the horizontal asymptote is $y = 0$.

20. Vertical asymptotes occur where the denominator is equal to zero. Thus, the vertical asymptote is $x = 4$.
The numerator and the denominator have the same degree, so the horizontal asymptote is $y = 6$.

21. The domain of $y = \dfrac{3x^2}{x^2 + 1}$ is all real numbers because there is no real value of x for which $x^2 + 1 = 0$. There are no vertical asymptotes.
The numerator and the denominator have the same degree, so the horizontal asymptote is $y = 3$.

22. The vertical asymptotes are $x = 1$ and $x = -1$.
The numerator has the higher degree, so there is no horizontal asymptote.

23. $x = 5$ belongs to the second domain interval, so $f(5) = \$35,900$.
Steve's starting salary will be $35,900.

24. **a.** A function that describes the total charge is $C = \$65 + \$30h$, where h is the number of hours worked. If a fractional part of an hour is worked, the entire hour is charged.

b. A service call lasting 1.4 hours will cost the same as a service call lasting 2 hours.
$C(2) = \$65 + \$30(2) = \$125$

25. **a.** With $P = \$325,000$, $L = 20$, and $C = \$50,000$, we have the function

$$V(t) = \$325,000 - \left(\frac{\$325,000 - \$50,000}{20}\right) \cdot t$$

$$V(t) = \$325,000 - \$13,750t$$

The lifetime of the backhoe is 20 years, so the domain is $0 \le t \le 20$.

b. After 7 years, the backhoe has a value of $V(7) = \$325,000 - \$13,750(7) = \$228,750$.

c. To determine how long until the backhoe is worth $100,000, let $V = \$100,000$ and solve for t.

$$V(t) = \$325,000 - \$13,750t$$
$$\$100,000 = \$325,000 - \$13,750t$$
$$-\$225,000 = -\$13,750t$$
$$t \approx 16.36 \text{ years}$$

It will take approximately 16 to 17 years for the backhoe to depreciate to a value of $100,000.

26. Setting costs equal to revenue, $C(x) = R(x)$.

$$200 + 7x = -x^2 + 40x$$
$$x^2 - 33x + 200 = 0$$
$$(x - 25)(x - 8) = 0$$
$$x = 8 \text{ or } x = 25$$

Moodz Hair Salon will break even with either 8 or 25 customers.

27. The profit function is given by $P(x) = R(x) - C(x)$, so

$P(x) = (-x^2 + 40x) - (200 + 7x)$

$= -x^2 + 33x - 200$

The profit function is a quadratic function whose graph is a parabola opening downward. The maximum profit will occur at the vertex of the graph. Algebraically, the vertex is (h, k), where

$h = -\frac{b}{2a}$ and $k = P(h)$.

$h = -\frac{33}{2(-1)} = 16.5$

Thus, the maximum profit occurs with 16 or 17 customers.

$k = P(16) = P(17) = 72$

Thus, the maximum profit, of \$72, occurs with 16 or 17 customers.

28. The equilibrium point is the price for which $S(p) = D(p)$.

$0.1p - 10 = -0.2p + 130$

$0.3p = 140$

$p \approx 466.67$

Equilibrium results if the cost of the camera is \$466.67.

29. a. The linear regression function is $y = 0.3943x + 5.0182$, where y is the percent of U.S. schoolchildren aged 5 to 17 who speak another language at home and x is the number of years after 1970.

b. Let $x = 24$ to use the model in part a to estimate the percent of U.S. schoolchildren aged 5 to 17 who spoke another language at home in 1994.

$y = 0.3943(24) + 5.0182 \approx 14.48\%$

c. Let $x = 40$ to use the model in part a to predict the percent of U.S. schoolchildren aged 5 to 17 who will speak another language at home in 2010.

$y = 0.3943(40) + 5.0182 \approx 20.8\%$

30. a. The quadratic regression function is $y = 22.033x^2 - 1028.9x + 24,254$, where y is the number of bachelor's degrees in mathematics conferred and x is the number of years after 1970.

b. Let $x = 15$ to use the model in part a to estimate the number of bachelor's degrees in mathematics conferred in 1970.

$y = 22.033(15)^2 - 1028.9(15) + 24,254 \approx 13,778$ degrees

c. Let $x = 39$ to use the model in part a to predict the number of bachelor's degrees in mathematics conferred in 2009.

$y = 22.033(39)^2 - 1028.9(39) + 24,254 \approx 17,639$ degrees