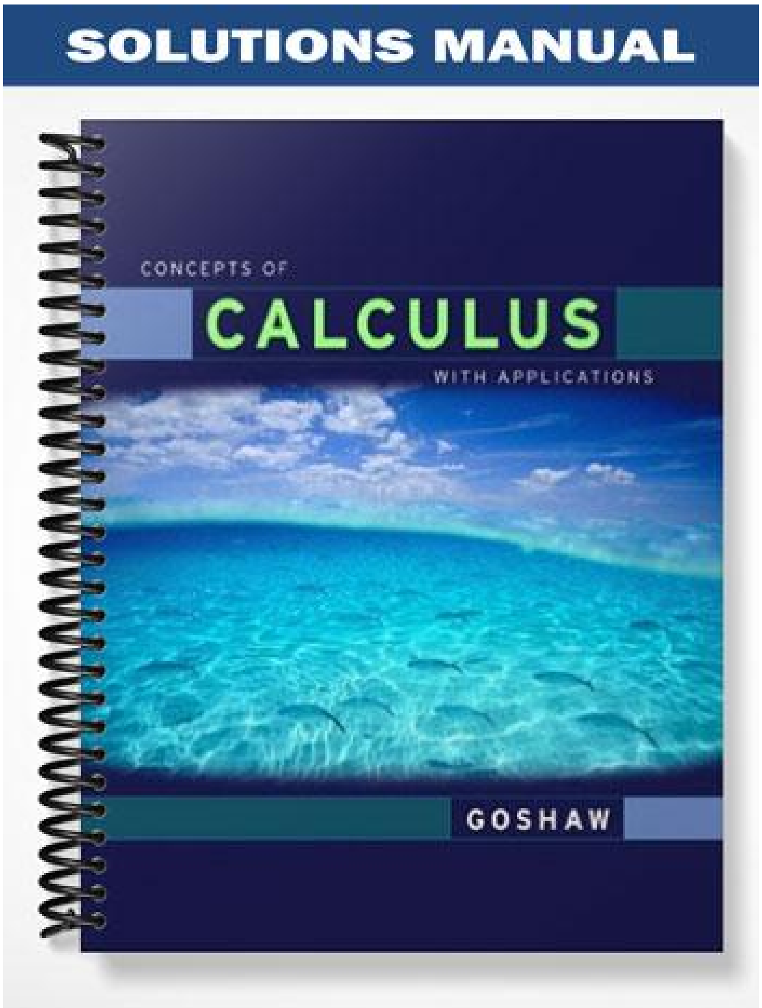

SOLUTIONS MANUAL
CONCEPTS OF
CALCULUS
WITH APPLICATIONS
GOSHAW

Unit 1
Limits and Derivatives

Topic 5 Exercises

1. As x approaches 3 from the right, $f(x)$ is approaching 1. So, $\lim_{x \to 3^+} f(x) = 1.$

2. As x approaches 2 from the right, $f(x)$ is approaching 2. So, $\lim_{x \to 2^+} f(x) = 2.$

3. As x approaches 4 from the left, $f(x)$ is approaching -4. So, $\lim_{x \to 4^-} f(x) = -4.$

4. As x approaches 4 from the left, $f(x)$ is approaching -7. So, $\lim_{x \to 4^-} f(x) = -7.$

5. As x approaches 2 from the left, $f(x)$ is approaching $-\infty$. As x approaches 2 from the right, $f(x)$ is approaching ∞. So, $\lim_{x \to 2} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

6. As x approaches 3 from the left, $f(x)$ is approaching $-\infty$. As x approaches 3 from the right, $f(x)$ is approaching ∞. So, $\lim_{x \to 3} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

7. As x approaches 1 from the left, $f(x)$ is approaching 1. As x approaches 1 from the right, $f(x)$ is approaching 4. So, $\lim_{x \to 1} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

8. As x approaches 2 from the left, $f(x)$ is approaching $-\infty$. As x approaches 2 from the right, $f(x)$ is approaching ∞. So, $\lim_{x \to 2} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

9. Choose a small interval around $x = 3$ and allow that interval to "close in" on $x = 3.$ Notice that the corresponding y values are closing in on the point $(3,2)$. So, $\lim_{x \to 3} f(x) = 2.$

10. Choose a small interval around $x = 2$ and allow that interval to "close in" on $x = 2.$ Notice that the corresponding y values are closing in on the point $(2,1)$. So, $\lim_{x \to 2} f(x) = 1.$

11. Choose a small interval around $x = 3$ and allow that interval to "close in" on $x = 3.$ Notice that the corresponding y values are closing in on the point $(3,2)$. So, $\lim_{x \to 3} f(x) = 2.$ Remember that $f(x)$ does not have to include the limit, it just has to get extremely close to that limit.

12. Choose a small interval around $x = 2$ and allow that interval to "close in" on $x = 2.$ Notice that the corresponding y values are closing in on the point $(2,1)$. So, $\lim_{x \to 2} f(x) = 1.$ Remember that $f(x)$ does not have to include the limit, it just has to get extremely close to that limit.

13. Choose a small interval around $x = 3$ and allow that interval to "close in" on $x = 3.$ Notice that the corresponding y values are closing in on the point $(3,2)$. So, $\lim_{x \to 3} f(x) = 2.$ Defining $f(3)$ by adding a point to the graph does not change the limit.

14. Choose a small interval around $x = 2$ and allow that interval to "close in" on $x = 2.$ Notice that the corresponding y values are closing in on the point $(2,1)$. So, $\lim_{x \to 2} f(x) = 1.$ Defining $f(2)$ by adding a point to the graph does not change the limit.

15. As x approaches 3 from the left, $f(x)$ is approaching ∞. As x approaches 3 from the right, $f(x)$ is approaching $-\infty$. So, $\lim_{x \to 3} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

16. As x approaches 2 from the left, $f(x)$ is approaching $-\infty$. As x approaches 2 from the right, $f(x)$ is approaching ∞. So, $\lim_{x \to 2} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

17. As x approaches 3 from the left and right, $f(x)$ is approaching ∞. So, $\lim_{x\to 3} f(x)=\infty$.

18. As x approaches 2 from the left and right, $f(x)$ is approaching $-\infty$. So, $\lim_{x\to 3} f(x)=-\infty$.

19. **a.** $\lim_{x\to 3^-} f(x)=3$

b. $\lim_{x\to 3^+} f(x)=-1$

c. $\lim_{x\to 3} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

20. **a.** $\lim_{x\to 2^-} f(x)=1$

b. $\lim_{x\to 2^+} f(x)=3$

c. $\lim_{x\to 2} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

21. **a.** $\lim_{x\to 1^-} f(x)=2$

b. $\lim_{x\to 1^+} f(x)=-2$

c. $\lim_{x\to 1} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

22. **a.** $\lim_{x\to 1^-} f(x)=3$

b. $\lim_{x\to 1^+} f(x)=-3$

c. $\lim_{x\to 1} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

23. **a.** $\lim_{x\to -4} f(x)=2$

b. $\lim_{x\to -2} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

c. $\lim_{x\to -1} f(x)=-1$

d. $\lim_{x\to 2} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

e. $\lim_{x\to 3} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

f. $\lim_{x\to 5} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

24. **a.** $\lim_{x\to -3} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

b. $\lim_{x\to -1} f(x)$ does not exist, because the left-hand limit and the right-hand limit are not the same.

c. $\lim_{x\to 0} f(x)=-4$

d. $\lim_{x\to 1} f(x)=-\infty$

e. $\lim_{x\to 3} f(x)=-3$

f. $\lim_{x\to 4} f(x)=-2$

25. Part a is false because a hole at $x=a$ does not mean the limit does not exist. Part b is true because we are given that $f(a)$ is not defined. So, the correct answer is choice b.

26. Part a is true because a jump at $x=a$ will cause the left-hand and right-hand limits to be different. Part b is false because the function may be undefined at both endpoints at which the jump occurs (identified as open circles). So, the correct answer is choice a.

27. **a.** true

b. true

c. false; because the left-hand limit and the right-hand limit are not the same.

d. true

e. false; because as x is approaching –3, $f(x)$ is approaching 1.

f. true

28. **a.** false; because the left-hand limit and the right-hand limit are not the same.

b. true

c. true

d. true

e. true

f. false; because as x is approaching 3 from the left, $f(x)$ is approaching 1.

29. $\lim_{x\to 3}(3x-2)=7$ because $f(3)=3(3)-2=7.$

30. $\lim_{x\to 2}(x^2-5)=-1$ because $f(2)=2^2-5=-1.$

31. $\lim_{x\to 7} 42=42$ because $f(x)=42$ is a constant function. So, $f(7)=42.$

32. $\lim_{x\to 11} 54=54$ because $f(x)=54$ is a constant function. So, $f(11)=54.$

33. $\lim_{x\to -3}\frac{2x}{x+4}=-6$ because $f(-3)=\frac{2(-3)}{-3+4}=\frac{-6}{1}=-6.$

34. $\lim_{y\to -2}\frac{3y}{y+5}=-2$ because $f(-2)=\frac{3(-2)}{-2+5}=\frac{-6}{3}-2.$

35. $\lim_{d\to 2}\sqrt{2d+5}=3$ because $f(2)=\sqrt{2(2)+5}=\sqrt{9}=3.$ Only the principal (nonnegative) square root is considered because functions can only have one y value for any given d value.

36. $\lim_{a\to 5}\sqrt[3]{5a+2}=3$ because $f(5)=\sqrt[3]{5(5)+2}=\sqrt[3]{27}=3.$

37. $\lim_{t\to 1}(2t^3-5t+4)=1$ because $f(1)=2(1)^3-5(1)+4=2-5+4=1.$

38. $\lim_{x\to 2}\frac{-2x}{x-3}=4$ because $f(2)=\frac{-2(2)}{2-3}=\frac{-4}{-1}=4.$

39. $\lim_{x\to 1}\frac{3x}{x^2-1}=\lim_{x\to 1}\frac{3x}{(x+1)(x-1)}=\frac{3}{0}=$ undefined

Examining one-sided limits shows that $\lim_{x\to 1^-}\frac{3x}{x^2-1}=-\infty$ and $\lim_{x\to 1^+}\frac{3x}{x^2-1}=\infty.$

So, $\lim_{x\to 1}\frac{3x}{x^2-1}$ does not exist.

40. $\lim_{x\to 3}\frac{4x+12}{x^2-9}=\lim_{x\to 3}\frac{4(x+3)}{(x+3)(x-3)}=\lim_{x\to 3}\frac{4}{(x-3)}=\frac{4}{0}=\text{undefined}$

Examining one-sided limits shows that $\lim_{x\to 3^-}\frac{4x+12}{x^2-9}=-\infty$ and $\lim_{x\to 3^+}\frac{4x+12}{x^2-9}=\infty$.

So, $\lim_{x\to 3}\frac{4x+12}{x^2-9}$ does not exist.

41. Direct substitution of $x=1$ gives the indeterminate form $\frac{0}{0}$.

Simplify the rational expression by factoring and reducing, then evaluate the limit.

$\lim_{x\to 1}\frac{3x-3}{x^2-1}=\lim_{x\to 1}\frac{3(x-1)}{(x+1)(x-1)}=\lim_{x\to 1}\frac{3}{x+1}=\frac{3}{2}$

42. Direct substitution of $x=3$ gives the indeterminate form $\frac{0}{0}$.

Simplify the rational expression by factoring and reducing, then evaluate the limit.

$\lim_{x\to 3}\frac{4x-12}{x^2-9}=\lim_{x\to 3}\frac{4(x-3)}{(x+3)(x-3)}=\lim_{x\to 3}\frac{4}{x+3}=\frac{4}{6}=\frac{2}{3}$

43. Direct substitution of $x=-3$ gives the indeterminate form $\frac{0}{0}$.

Simplify the rational expression by factoring and reducing, then evaluate the limit.

$\lim_{x\to -3}\frac{x^2+3x}{x^2-9}=\lim_{x\to -3}\frac{x(x+3)}{(x+3)(x-3)}=\lim_{x\to -3}\frac{x}{x-3}=\frac{-3}{-6}=\frac{1}{2}$

44. Direct substitution of $x=2$ gives the indeterminate form $\frac{0}{0}$.

Simplify the rational expression by factoring and reducing, then evaluate the limit.

$\lim_{x\to 2}\frac{3x-6}{x^2-4}=\lim_{x\to 2}\frac{3(x-2)}{(x+2)(x-2)}=\lim_{x\to 2}\frac{3}{x+2}=\frac{3}{4}$

45. $\lim_{x\to 4}\frac{3x+12}{x^2-16}=\lim_{x\to 4}\frac{3(x+4)}{(x+4)(x-4)}=\lim_{x\to 4}\frac{3}{(x-4)}=\frac{3}{0}=\text{undefined}$

Examining one-sided limits shows that $\lim_{x\to 4^-}\frac{3x+12}{x^2-16}=-\infty$ and $\lim_{x\to 4^+}\frac{3x+12}{x^2-16}=\infty$.

So, $\lim_{x\to 4}\frac{3x+12}{x^2-16}$ does not exist.

46. $\lim_{x\to 2}\frac{x^2+4}{x^2-2x}=\lim_{x\to 2}\frac{x^2+4}{x(x-2)}=\frac{8}{0}=\text{undefined}$

Examining one-sided limits shows that $\lim_{x\to 2^-}\frac{x^2+4}{x^2-2x}=-\infty$ and $\lim_{x\to 2^+}\frac{x^2+4}{x^2-2x}=\infty$.

So, $\lim_{x\to 2}\frac{x^2+4}{x^2-2x}$ does not exist.

47. $\lim_{x\to 4}\frac{x^3}{x^2-16}=\lim_{x\to 4}\frac{x^3}{(x+4)(x-4)}=\frac{64}{0}=\text{undefined}$

Examining one-sided limits shows that $\lim_{x\to 4^-}\frac{x^3}{x^2-16}=-\infty$ and $\lim_{x\to 4^+}\frac{x^3}{x^2-16}=\infty.$

So, $\lim_{x\to 4}\frac{x^3}{x^2-16}$ does not exist.

48. $\lim_{x\to 5}\frac{-x^3}{x^2-25}=\lim_{x\to 5}\frac{-x^3}{(x+5)(x-5)}=\frac{-125}{0}=\text{undefined}$

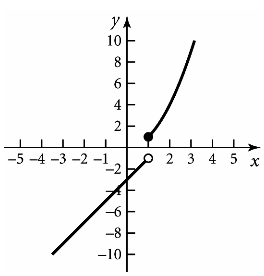
Examining one-sided limits shows that $\lim_{x\to 5^-}\frac{-x^3}{x^2-25}=\infty$ and $\lim_{x\to 5^+}\frac{-x^3}{x^2-25}=-\infty.$

So, $\lim_{x\to 5}\frac{-x^3}{x^2-25}$ does not exist.

49. $\lim_{h\to 0}\frac{4a^2h-3ah+h^2}{h}=\lim_{h\to 0}\frac{h(4a^2-3a+h)}{h}=\lim_{h\to 0}(4a^2-3a+h)=4a^2-3a$

50. $\lim_{h\to 0}\frac{-6a^2h-12ah-h^2}{h}=\lim_{h\to 0}\frac{h(-6a^2-12a-h)}{h}=\lim_{h\to 0}(-6a^2-12a-h)=-6a^2-12a$

51. $f(x)=\begin{cases}2x-3 & \text{if } x<1\\ x^2 & \text{if } x\ge 1\end{cases}$



a. x is approaching 0, which belongs to the domain of the first part of the function. So, $\lim_{x\to 0} f(x)=-3.$

b. x is approaching 3, which belongs to the domain of the second part of the function. So, $\lim_{x\to 3} f(x)=9.$

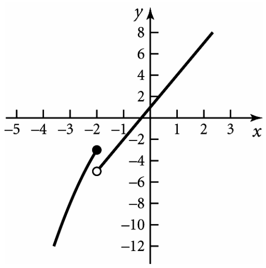
c. x is approaching 1, which is the break-point of the domain of the two parts of the function. Thus, the left-hand and the right-hand limits need to be evaluated.

$\lim_{x\to 1^-} f(x)=-1$

$\lim_{x\to 1^+} f(x)=1$

Because the left-hand and the right-hand limit are not equal, $\lim_{x\to 1} f(x)$ does not exist.

52. $f(x)=\begin{cases}1-x^2 & \text{if } x\le -2\\ 3x+1 & \text{if } x>-2\end{cases}$



a. x is approaching -5, which belongs to the domain of the first part of the function. So, $\lim_{x\to -5} f(x)=-24$.

b. x is approaching 0, which belongs to the domain of the second part of the function. So, $\lim_{x\to 0} f(x)=1$.

c. x is approaching -2, which is the break-point of the domain of the two parts of the function. Thus, the left-hand and the right-hand limits need to be evaluated.

$\lim_{x\to -2^-} f(x)=-3$

$\lim_{x\to -2^+} f(x)=-5$

Because the left-hand and the right-hand limit are not equal, $\lim_{x\to 2} f(x)$ does not exist.

53. $f(x)=\begin{cases}x+5 & \text{if } x<-1\\ -x^2-3 & \text{if } -1\le x<2\\ \sqrt{x-1} & \text{if } x\ge 2\end{cases}$

a. x is approaching 0, which belongs to the domain of the second part of the function. So,

$\lim_{x\to 0} f(x)=f(0)=-0^2-3=-3.$

b. x is approaching -1, which is the break-point of the domain of the first and second parts of the function. Thus, the left-hand and the right-hand limits need to be evaluated.

$\lim_{x\to -1^-} f(x)=f(-1)=-1+5=4$

$\lim_{x\to -1^+} f(x)=f(-1)=-(-1)^2-3=-4$

Because the left-hand and the right-hand limit are not equal, $\lim_{x\to -1} f(x)$ does not exist.

c. x is approaching 2, which is the break-point of the domain of the second and third parts of the function. Thus, the left-hand and the right-hand limits need to be evaluated.

$\lim_{x\to 2^-} f(x)=f(2)=-2^2-3=-7$

$\lim_{x\to 2^+} f(x)=f(2)=\sqrt{2-1}=1$

Because the left-hand and the right-hand limit are not equal, $\lim_{x\to 2} f(x)$ does not exist.

54. $f(x)=\begin{cases}\dfrac{x^2}{2} & \text{if } x\le -3\\ -x^2-3 & \text{if } -3<x\le 2\\ \sqrt{x+2} & \text{if } x>2\end{cases}$

a. x is approaching 0, which belongs to the domain of the second part of the function. So,
$\lim_{x\to 0} f(x)=f(0)=-0^2-3=-3.$

b. x is approaching -3, which is the break-point of the domain of the first and second parts of the function. Thus, the left-hand and the right-hand limits need to be evaluated.
$\lim_{x\to -3^-} f(x)=f(-3)=\dfrac{(-3)^2}{2}=4.5$
$\lim_{x\to -3^+} f(x)=f(-3)=-(-3)^2-3=-12$
Because the left-hand and the right-hand limit are not equal, $\lim_{x\to -3} f(x)$ does not exist.

c. x is approaching 2, which is the break-point of the domain of the second and third parts of the function. Thus, the left-hand and the right-hand limits need to be evaluated.
$\lim_{x\to 2^-} f(x)=f(2)=-2^2-3=-7$
$\lim_{x\to 2^+} f(x)=f(2)=\sqrt{2+2}=2$
Because the left-hand and the right-hand limit are not equal, $\lim_{x\to 2} f(x)$ does not exist.

55. **a**. $f(x)=5x-7$
Step 1: $f(a)=5a-7$
Step 2: $f(a+h)=5(a+h)-7$
$=5a+5h-7$
Step3: $f(a+h)-f(a)=(5a+5h-7)-(5a-7)$
$=5a+5h-7-5a+7$
$=5h$
Step 4: $\dfrac{f(a+h)-f(a)}{h}=\dfrac{5h}{h}=5$

b. $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}=\lim_{h\to 0}5=5$

56. **a**. $f(x)=-46$
Step 1: $f(a)=-46$
Step 2: $f(a+h)=-46$
Step3: $f(a+h)-f(a)=(-46)-(-46)=0$
Step 4: $\dfrac{f(a+h)-f(a)}{h}=\dfrac{0}{h}=0$

b. $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}=\lim_{h\to 0}0=0$

57. **a**. $f(x) = x^2 + 2$

Step 1: $f(a) = a^2 + 2$

Step 2: $f(a+h) = (a+h)^2 + 2$
$= a^2 + 2ah + h^2 + 2$

Step3: $f(a+h) - f(a) = (a^2 + 2ah + h^2 + 2) - (a^2 + 2)$
$= a^2 + 2ah + h^2 + 2 - a^2 - 2$
$= 2ah + h^2$

Step 4: $\dfrac{f(a+h) - f(a)}{h} = \dfrac{2ah + h^2}{h}$
$= \dfrac{h(2a+h)}{h}$
$= 2a + h$

b. $\lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h\to 0} (2a + h) = 2a$

58. **a**. $f(x) = 3 - 2x$

Step 1: $f(a) = 3 - 2a$

Step 2: $f(a+h) = 3 - 2(a+h)$
$= 3 - 2a - 2h$

Step3: $f(a+h) - f(a) = (3 - 2a - 2h) - (3 - 2a)$
$= 3 - 2a - 2h - 3 + 2a$
$= -2h$

Step 4: $\dfrac{f(a+h) - f(a)}{h} = \dfrac{-2h}{h} = -2$

b. $\lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h\to 0} -2 = -2$

59. **a**. $f(x) = 12$

Step 1: $f(a) = 12$

Step 2: $f(a+h) = 12$

Step3: $f(a+h) - f(a) = 12 - 12 = 0$

Step 4: $\dfrac{f(a+h) - f(a)}{h} = \dfrac{0}{h} = 0$

b. $\lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h\to 0} 0 = 0$

60. **a**. $f(x)=3-x^2$

Step 1: $f(a)=3-a^2$

Step 2: $f(a+h)=3-(a+h)^2$

$=3-a^2-2ah-h^2$

Step3: $f(a+h)-f(a)=(3-a^2-2ah-h^2)-(3-a^2)$

$=3-a^2-2ah-h^2-3+a^2$

$=-2ah-h^2$

Step 4: $\dfrac{f(a+h)-f(a)}{h}=\dfrac{-2ah-h^2}{h}$

$=\dfrac{h(-2a-h)}{h}$

$=-2a-h$

b. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}=\lim\limits_{h\to 0}(-2a-h)=-2a$

61. **a**. $f(x)=-2x^2+7x-1$

Step 1: $f(a)=-2a^2+7a-1$

Step 2: $f(a+h)=-2(a+h)^2+7(a+h)-1$

$=-2a^2-4ah-2h^2+7a+7h-1$

Step3: $f(a+h)-f(a)=(-2a^2-4ah-2h^2+7a+7h-1)-(-2a^2+7a-1)$

$=-2a^2-4ah-2h^2+7a+7h-1+2a^2-7a+1$

$=-4ah-2h^2+7h$

Step 4: $\dfrac{f(a+h)-f(a)}{h}=\dfrac{-4ah-2h^2+7h}{h}$

$=\dfrac{h(-4a-2h+7)}{h}$

$=-4a-2h+7$

b. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}=\lim\limits_{h\to 0}(-4a-2h+7)=-4a+7$

62. a. $f(x) = 3x^2 - 2x + 4$

Step 1: $f(a) = 3a^2 - 2a + 4$

Step 2: $f(a+h) = 3(a+h)^2 - 2(a+h) + 4$

$= 3a^2 + 6ah + 3h^2 - 2a - 2h + 4$

Step3: $f(a+h) - f(a) = (3a^2 + 6ah + 3h^2 - 2a - 2h + 4) - (3a^2 - 2a + 4)$

$= 3a^2 + 6ah + 3h^2 - 2a - 2h + 4 - 3a^2 + 2a - 4$

$= 6ah + 3h^2 - 2h$

Step 4: $\dfrac{f(a+h) - f(a)}{h} = \dfrac{6ah + 3h^2 - 2h}{h}$

$= \dfrac{h(6a + 3h - 2)}{h}$

$= 6a + 3h - 2$

b. $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h \to 0} (6a + 3h - 2) = 6a - 2$

63. a. $f(x) = \dfrac{2}{x+1}$

Step 1: $f(a) = \dfrac{2}{a+1}$

Step 2: $f(a+h) = \dfrac{2}{a+h+1}$

Step 3: $f(a+h) - f(a) = \dfrac{2}{a+h+1} - \dfrac{2}{a+1}$

$= \dfrac{2(a+1)}{(a+h+1)(a+1)} - \dfrac{2(a+h+1)}{(a+1)(a+h+1)}$

$= \dfrac{2a+2}{(a+h+1)(a+1)} - \dfrac{2a+2h+2}{(a+1)(a+h+1)}$

$= \dfrac{2a + 2 - (2a + 2h + 2)}{(a+h+1)(a+1)}$

$= \dfrac{2a + 2 - 2a - 2h - 2}{(a+h+1)(a+1)}$

$= \dfrac{-2h}{(a+h+1)(a+1)}$

Step 4: $\dfrac{f(a+h) - f(a)}{h} = \dfrac{-2h}{h(a+h+1)(a+1)}$

$= \dfrac{-2}{(a+h+1)(a+1)}$

b. $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h \to 0} \dfrac{-2}{(a+h+1)(a+1)} = \dfrac{-2}{(a+1)^2}$