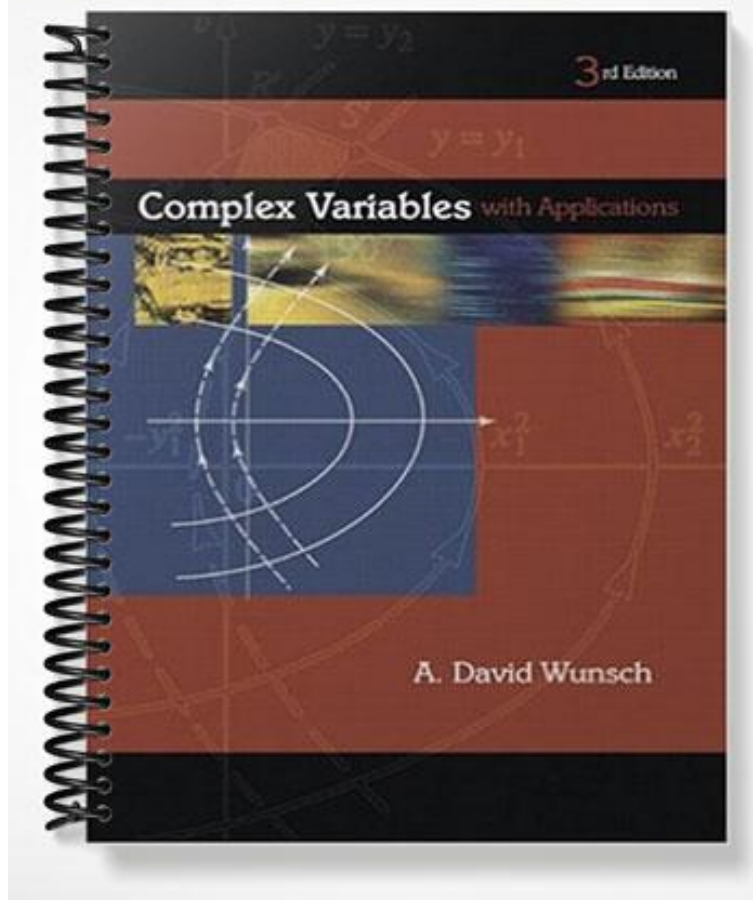


SOLUTIONS MANUAL



COMPLEX VARIABLES WITH APPLICATIONS

Third Edition

By A. David Wunsch
University of Massachusetts Lowell

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By

A. David Wunsch

&

Michael F. Brown

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The bulky volume you are holding represents the solutions to all the problems in the 3rd edition of my textbook *Complex Variables with Applications*. Both Michael Brown and I have separately worked through all the solutions, but I can say with overwhelming confidence that in spite of this redundancy there are some remaining errors. Please tell me of any that you find. My e mail address is David_Wunsch@UML.edu (note the underscore). Those preferring an older medium of communication may write to me at the Electrical and Computer Engineering Dept. University of Massachusetts Lowell, Lowell, MA 01854. I promise to acknowledge all e-mail and postal mail that I receive. I would also appreciate learning of any errors in the textbook itself.

I plan to post corrections to both the book and this manual at the web address http://faculty.uml.edu/awunsch/Wunsch_Complex_Variables/

This manual has been written primarily for college faculty who are teaching from my text. Whether it is to be made freely available to students- perhaps at the school library- is a matter I leave up to each individual instructor. Notice however, that there is little point in assigning the textbook problems involving computer programming if students already have the MATLAB code supplied in this manual. Regarding this code, I must assert that I am not a professional programmer and I'm certain that in many cases the reader will find more efficient ways of solving the same problem.

Finally, I must apologize for the idiosyncrasies of the handwriting. They are my own and not to be blamed on Mr. Brown.

A. David Wunsch
Belmont, Massachusetts
July 8, 2004

$$e^{i\pi} + 1 = 0$$

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1

Complex Numbers

Chapter 1
Section 1.1

- 1) $4x+3=0$, $x=-3/4$, rational number system
- 2) $x^2-x-1=0$, $x=\frac{1 \pm \sqrt{5}}{2}$ which is real but irrational. Need real number system
- 3) $x^2+x+1=0$ $x=\frac{-1 \pm \sqrt{-3}}{2}$ need complex number system
- 4) $\sin x=0$, $x=0$, integers 5) $\cos x=0$, $x=\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$ real number system odd ↑
- 6) $(x+2)(x+1)=0$ $x=-2, -1$, integers
- 7) $\sin(\log x)=0$, $x=1$, $\sin(\log 1)=0$, integers
- 8) $z^4=16$, $z=2$, integers 9) $z^4=-16$, $z=(-16)^{1/4}$ complex
- 10) a) $[1+10^{-2}+10^{-4}\dots] = \frac{1}{1-10^{-2}} = \frac{10^2}{100-1} = \frac{100}{99}$
- b) $23.2323\dots = 23 \frac{100}{99} = \frac{2300}{99}$
- c) $376.376376\dots = 376 [1+10^{-3}+10^{-6}+10^{-9}\dots]$
 $= 376 \frac{1}{1-10^{-3}} = 376 \frac{1000}{999} = \frac{376000}{999}$
- 11 (a) Consider $4.0404\dots = 4 [1+10^{-2}+10^{-4}\dots]$
 $= 4 \left[\frac{1}{1-10^{-2}} \right] = \frac{400}{99}$. Now $3.0404\dots$
 $= \frac{400}{99} - 1 = \frac{400-99}{99} = \frac{301}{99}$
- b) $.999\dots = .9 [1+10^{-1}+10^{-2}\dots]$
 $= .9 \frac{1}{1-10^{-1}} = \frac{.9}{1-.1} = \frac{.9}{.9} = 1$ q.e.d.
- 12(a) We begin by showing that the square of any odd number is odd. Let N_o be that number. Then $N_o = N_e + 1$ where N_e is even. Now $N_o^2 = N_e^2 + 2N_e + 1$
even
 $N_o^2 = \text{even} + 1 = \text{odd}$

Chapter 1
Sec. 1.1

12(a) continued. We showed that the square of an odd integer is odd. Thus the square root of a perfect square (that is even) must not be odd, \therefore is even.

12(b) $m^2 = 2n^2$, $2n^2$ is even, \therefore from part (a) the square root of $2n^2$, which is m , must be even.

12(c) $n^2 = \frac{m}{2} \cdot m$. Since m is even, $\frac{m}{2}$ is an integer. $\therefore n^2$ is even, and from (a) so is n .

12(d) We assumed that $\sqrt{2} = \frac{m}{n}$ can be expressed as the ratio of 2 integers having no common integer factor. Our assumption says that m and n both can't be even. This resulted in a contradiction, since in parts (b) and (c) we found that m and n were both even.

12(e). Assume $n + \sqrt{2}$ is rational = a

$a = n + \sqrt{2}$, $a - n = \sqrt{2}$. The left side is rational [the difference of rational numbers] but the right side is irrational. Have a contradiction.

Suppose $\sqrt{2}n^2$ is rational, then $\sqrt{2}n = a$ is rational $\frac{a}{n} = \sqrt{2}$. The left side is the quotient of rational numbers and is rational, the right side is irrational, have a contradiction.

Assume $a = \sqrt{\sqrt{2}}$ is rational $a^2 = \sqrt{2}$. Left side is rational, right side is irrational. Have contradiction.

(chap 1, page 2)

$$13(a) \quad x^2 + bx + c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4c}}{2} = 1 \pm \sqrt{2} \quad b = -2, \frac{\sqrt{4 - 4c}}{2} = \sqrt{2}$$

$$\therefore \boxed{x^2 - 2x - 1 = 0} \text{ will work } 13(b) \quad x = \sqrt{\sqrt{2}} \quad x^4 = 2 \quad \boxed{x^4 - 2 = 0}$$

$$14(a) \text{ Using Matlab: } \text{exp}(1) = 2.718281828 \underbrace{45905}_{\text{pattern breaks}}$$

Use long format

$$(b) \quad 201/26 = 7.7\overbrace{3076923076923}^{\text{repeating decimals}} \quad \boxed{\text{the digits } 307692 \text{ repeat}}$$

$$15 \quad \boxed{4 - 4i} \quad 16 \quad -5 + 2i + 15i + 7i = \boxed{16 + 22i}$$

$$17 \quad (3-2i)(4+3i)(3+2i) = (3-2i)(3+2i)(4+3i) =$$

$$(9+4)(4+3i) = \boxed{52 + i39}$$

chap 1
sec 1.1, continued

18) $(1+i)^3 = (1+i)^2 (1+i) = 2i(1+i) = -2+2i$

Imag part = $\boxed{2}$

19) $\text{Im}(1+i) = 1, [\text{Im}(1+i)]^3 = 1^3 = \boxed{1}$

20) $(x+iy)(u-iv)(x-iy)(u+iv) =$
 $(x+iy)(x-iy)(u-iv)(u+iv) = (x^2+y^2)(u^2+v^2)$
 $= \boxed{u^2x^2 + v^2x^2 + u^2y^2 + v^2y^2}$

21) (a) binomial theorem
 $(a+b)^n = \sum_{k=0}^n \frac{n!}{(n-k)! k!} a^{n-k} b^k$

let $a=1, b=iy$

$(1+iy)^n = \sum_{k=0}^n \frac{(iy)^k n!}{(n-k)! k!}$

b) $(1+2i)^5 = \sum_{k=0}^5 \frac{(2i)^k 5!}{(5-k)! k!} =$

$\frac{5!}{5!} + \frac{(2i)5!}{4!1!} + \frac{(-4)5!}{3!2!} + \frac{-i85!}{2!3!} + \frac{165!}{1!4!} + \frac{i325!}{5!}$

Real part is $1 + \frac{(-4)(120)}{12} + 0 = 41$

Imag part is $(2)5 - 8 \cdot 10 + 32 = -38$

c) Use $(1+2i)^5$ in Matlab code.
 Will get $41 - i38$

22) $z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)$

$\text{Re}(z_1 z_2) = x_1 x_2 - y_1 y_2 = \text{Re } z_1 \text{Re } z_2 - \text{Im } z_1 \text{Im } z_2$

23) From the above $\text{Im}(z_1 z_2) = x_1 y_2 + x_2 y_1 = \text{Re } z_1 \text{Im } z_2 + \text{Re } z_2 \text{Im } z_1$

24)

Chap 1, sec 1.1

$$i, i^2 = -1, i^3 = -i, i^4 = -i \cdot i = 1$$

$i^5 = i$, etc. \therefore the four possible

values are $i, -1, -i, 1$

$$i^{n+4} = i^4 i^n, \text{ but } i^4 = 1, \therefore i^{n+4} = i^n$$

as)

$$i^{1023} = i^{1020} i^3 = i^{1020} (-i)$$

$$= (i^4)^{255} (-i) = 1^{255} (-i) = \boxed{-i}$$

(25) Find: $(1-i)^{1025}$, note: $(1-i)^2 = -2i$

$$(1-i)^{1025} = (1-i)^{1024} (1-i) = [(1-i)^2]^{512} (1-i)$$

$$= (-2i)^{512} (1-i) = 2^{512} (-1)^{512} i^{512} (1-i)$$

$$= 2^{512} [(i^4)^{128}] (1-i) = 2^{512} 1^{128} (1-i) = \boxed{2^{512} (1-i)}$$

27)

$$x = 2x$$

$$\therefore x = 0$$

$$y + 1 = 2y$$

$$\therefore y = 1, \text{ ans } \boxed{x=0, y=1}$$

28)

$$x^2 - y^2 + i2xy = y + ix$$

$$x^2 - y^2 = y, \quad 2xy = x \Rightarrow \text{assume } x \neq 0$$

$$2y = 1, \quad y = 1/2$$

$$x^2 - \frac{1}{4} = \frac{1}{2} \quad x^2 = \frac{3}{4}, \quad x = \pm \sqrt{3}/2$$

Set of answers $\boxed{x = \pm \sqrt{3}/2, y = 1/2}$

Now assume $x = 0$, $2xy = x$ is satisfied

$$x^2 - y^2 = y \quad -y^2 = y \quad y + y^2 = 0$$

$$y(1+y) = 0, \quad y = 0 \text{ or } y = -1$$

Second set of answer $\boxed{x=0, y=0 \text{ or } -1}$

29)

$$e^{x^2+y^2} = e^{-2xy}$$

$$2y = 1, \therefore y = \frac{1}{2}$$

$$\text{Now } x^2 + y^2 = -2xy, \quad x^2 + y^2 + 2xy = 0$$

$$(x+y)^2 = 0 \quad \therefore x = -y, \quad x = -\frac{1}{2}$$

answer

$$x = -\frac{1}{2}, \quad y = \frac{1}{2}$$

30)

$$\text{Log}(x+y) = 1$$

$$y = xy$$

$$\text{If } \text{Log}(x+y) = 1, \quad x+y = e$$

$$\text{Now assume } y \neq 0 \quad y = xy \Rightarrow x = 1$$

$$\text{Since } x+y = e, \quad y = e-1 \quad \text{Answer: } x=1, y=e-1$$

Now assume $y = 0$, $y = xy$ is satisfied

$$x+y = e \Rightarrow x = e$$

Answer

$$x=e, y=0$$

31)

$$[\text{Log}(x)-1]^2 = 1, \quad [\text{Log}(y)-1]^2 = 0 \quad \therefore y=e$$

$$\text{Log}(x)-1 = \pm 1, \quad \text{Log } x - 1 = -1$$

$$\text{Log } x = 0, \quad x = 1$$

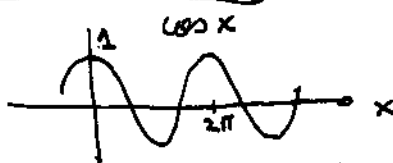
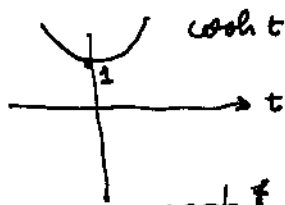
$$\text{or } \text{Log } x - 1 = 1$$

$$\text{Log } x = 2, \quad x = e^2$$

Answer

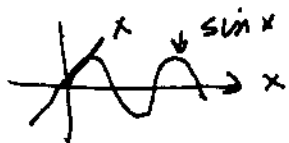
$$y=e, \quad x=1 \text{ or } e^2$$

32)



$\cosh(y-1) = \cos x$ can only be solved if $y=1$ and $x = 2n\pi, \quad n=0, \pm 1, \pm 2, \dots$

Now need $\sin x = xy \quad \sin x = x \quad (\sin y = 1)$



$\sin x = x$ if and only if $x=0$

so Answer

$$x=0, y=1$$

Sec 1.2

$$1) \quad z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2),$$

$$\overline{z_1 - z_2} = (x_1 - x_2) - i(y_1 - y_2) = x_1 - iy_1 - [x_2 - iy_2]$$

$$= \overline{z_1} - \overline{z_2}$$

$$2) \quad z_1 z_2 = (x_1 x_2 - y_1 y_2) + i[y_1 x_2 + y_2 x_1]$$

$$\overline{z_1 z_2} = (x_1 x_2 - y_1 y_2) - i[y_1 x_2 + y_2 x_1] =$$

$$(x_1 - iy_1)(x_2 - iy_2) = \overline{z_1} \overline{z_2}$$

$$3) \quad \frac{1}{z_1} = \frac{1}{x_1 + iy_1} = \frac{x_1 - iy_1}{x_1^2 + y_1^2}$$

$$\therefore \overline{\left(\frac{1}{z_1}\right)} = \frac{x_1 + iy_1}{x_1^2 + y_1^2} \quad \frac{1}{\overline{z_1}} = \frac{1}{x_1 - iy_1} = \frac{x_1 + iy_1}{x_1^2 + y_1^2}$$

$$\text{Thus } \overline{\left(\frac{1}{z_1}\right)} = \frac{1}{\overline{z_1}}$$

$$4) \quad \frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 + y_2^2} =$$

$$\frac{x_1 x_2 + y_1 y_2 + i[y_1 x_2 - y_2 x_1]}{x_2^2 + y_2^2} \quad \text{Now } z_1 \cdot \frac{1}{z_2}$$

$$= (x_1 + iy_1) \left[\frac{(x_2 - iy_2)}{x_2^2 + y_2^2} \right] = \frac{x_1 x_2 + y_1 y_2 + i[y_1 x_2 - y_2 x_1]}{x_2^2 + y_2^2}$$

$$\text{Thus } z_1 \cdot \frac{1}{z_2} = \left(\frac{z_1}{z_2}\right)$$

$$\Rightarrow \overline{\left(\frac{x_1 + iy_1}{x_2 + iy_2}\right)} = \overline{\left(\frac{z_1}{z_2}\right)} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 + y_2^2}$$

$$= \frac{x_1 x_2 + y_1 y_2 + i(y_1 x_2 - y_2 x_1)}{x_2^2 + y_2^2} = \frac{x_1 x_2 + y_1 y_2 + i[y_2 x_1 - y_1 x_2]}{x_2^2 + y_2^2}$$

$$\text{Now } \frac{\overline{z_1}}{\overline{z_2}} = \frac{x_1 - iy_1}{x_2 - iy_2} = \frac{(x_1 - iy_1)(x_2 + iy_2)}{x_2^2 + y_2^2} = \frac{x_1 x_2 + y_1 y_2 + i[y_2 x_1 - y_1 x_2]}{x_2^2 + y_2^2}$$

$$\text{Thus } \frac{\overline{z_1}}{\overline{z_2}} = \overline{\left(\frac{z_1}{z_2}\right)}$$

sec. 1.2 continued

$$6) \quad z_1 z_2 = x_1 x_2 - y_1 y_2 + i [y_1 x_2 + y_2 x_1]$$

$$\bar{z}_1 \bar{z}_2 = x_1 x_2 - y_1 y_2 - i [y_1 x_2 + y_2 x_1]$$

$$\operatorname{Re} [z_1 z_2] = \operatorname{Re} [\bar{z}_1 \bar{z}_2] \quad \text{Alternatively:}$$

$\operatorname{Re} [z_1 z_2] = \operatorname{Re} [\bar{z}_1 \bar{z}_2]$ since real part is unaffected by taking conj.

But $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ from prob 2

$$\therefore \operatorname{Re} [\overline{z_1 z_2}] = \operatorname{Re} [\bar{z}_1 \bar{z}_2]$$

$$7) \quad \text{From part 6) } z_1 z_2 = x_1 x_2 - y_1 y_2 + i [y_1 x_2 + y_2 x_1]$$

$$\text{thus: } \bar{z}_1 \bar{z}_2 = x_1 x_2 - y_1 y_2 - i [y_1 x_2 + y_2 x_1]$$

$$\text{Note } \operatorname{Im} [z_1 z_2] = -\operatorname{Im} \bar{z}_1 \bar{z}_2 = y_1 x_2 + y_2 x_1$$

$$8) \quad \frac{1}{1+2i} = \frac{1-2i}{1^2+2^2} = \boxed{\frac{1}{5} - i \frac{2}{5}}$$

$$9) \quad i + \frac{1}{1-2i} = i + \frac{1+2i}{5} = \frac{1}{5} + \frac{7i}{5}$$

$$\left(i + \frac{1}{1-2i}\right)^2 = \frac{1}{25} + \frac{14}{25}i - \frac{49}{25} =$$

$$= \boxed{-\frac{48}{25} + i \frac{14}{25}}$$

$$10) \quad \frac{3-4i}{1+2i} = \frac{(3-4i)(1-2i)}{5} = \frac{1}{5} [3-8+i[-4-6]]$$

$$= \frac{1}{5} [-5-10i] = \boxed{-1-2i}$$

$$11) \quad \frac{3-4i}{1+2i} + \frac{3+4i}{1-2i} \quad \text{This must equal}$$

$$2 \operatorname{Re} \left[\frac{3-4i}{1+2i} \right] = \boxed{-2} \quad 12) \text{ Use ans from 10}$$

$$12) \text{ cont'd } 2i + \frac{3-4i}{1+2i} = 2i + (-1-2i) = \boxed{-1}$$

Sec 1.2 cont'd

$$13) \left(\frac{4-4i}{2+2i} \right)^7 = \left(\frac{2-2i}{1+i} \right)^7 = \left[2^7 \frac{(1-i)^7}{1+i} \right]$$

$$= 2^7 \left[\frac{(1-i)^7}{1+i} \right] = \frac{2^7}{2^7} (1-i)^{14} =$$

$$\left[(1-i)^2 \right]^7 = (-2i)^7 = (-1)^7 2^7 i^7 = i \cdot 2^7 = \boxed{128i}$$

$$14) \left(\frac{4-4i}{2+2i} \right)^7 + \left(\frac{4+4i}{2-2i} \right)^7 \quad \text{Note the}$$

second term is the conjugate of the first.

∴ ans is $2 \operatorname{Re} \left[\frac{4-4i}{2+2i} \right]^7 = 2 \operatorname{Re} [128i]$ (see prob 13)

= $\boxed{0}$

PROBLEM 15.

» 8

» $1/(1+2i)$

ans =

$0.2000 - 0.4000i$

» 9

» $(i+1/(1-2i))^2$

ans =

$-1.9200 + 0.5600i$

» 10

» $(3-4i)/(1+2i)$

ans =

$-1.0000 - 2.0000i$

» 11

» $(3-4i)/(1+2i) + (3+4i)/(1-2i)$

ans =

-2

» 12

» $2i+(3-4i)/(1+2i)$

ans =

-1

» 13

» $((4-4i)/(2+2i))^7$

ans =

$0 + 1.2800e+002i$

» 214

» $((4-4i)/(2+2i))^7 + ((4+4i)/(2-2i))^7$

ans =

0

16) $\left(\frac{\bar{z}_1}{z_2 z_3} \right) = \frac{\bar{z}_1}{z_2 z_3} = \frac{\bar{z}_1}{\bar{z}_2 \bar{z}_3} = \bar{z}_1 \frac{1}{\bar{z}_2 \bar{z}_3}$

this is not equal to $\bar{z}_1 \frac{1}{z_2 z_3}$ ∴ not true

17) $\overline{z_1 \bar{z}_2 z_3} = \bar{z}_1 \bar{\bar{z}_2} \bar{z}_3 = \bar{z}_1 z_2 \bar{z}_3$ g.e.d. true

18) $\overline{i(z_1 + z_2 + z_3)} = \bar{i}(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) = -i(\bar{z}_1 + \bar{z}_2 + \bar{z}_3)$. This is not equal to $i(\bar{z}_1 + \bar{z}_2 + \bar{z}_3)$. ∴ not true

19) Consider z, \bar{z}_2, z_3 . Its conjugate is

$\overline{z \bar{z}_2 z_3} = \bar{z} \bar{\bar{z}_2} \bar{z}_3 = \bar{z} z_2 \bar{z}_3$
 ∴ $\text{Re}[z \bar{z}_2 z_3] = \text{Re}[\bar{z} z_2 \bar{z}_3]$ g.e.d. true

20) Consider z, \bar{z}_2, z_3 . Its conjugate is $\overline{z \bar{z}_2 z_3} = \bar{z} z_2 \bar{z}_3$. If 2 quantities are conjugates of each other, their imag. parts differ in sign. ∴ $\text{Im}(z \bar{z}_2 z_3) = -\text{Im}(\bar{z} z_2 \bar{z}_3)$
 Note the minus sign. ∴ the result given in the problem is not true.

sec 1.2 continued

21) Observe that $\bar{z}_1, z_2, \bar{z}_3$ is the conjugate of z_1, \bar{z}_2, z_3 . Suppose $z = a + ib$
 $i\bar{z} = i(a - ib) = b + ia$. Now $\text{Re}(i\bar{z}) = \text{Im}(z)$
 $\therefore \text{Re}(z_1, \bar{z}_2, z_3) = \text{Im}(i\bar{z}_1, z_2, \bar{z}_3)$. Is True

22)

a) If $p + iq = (k + il)(m + in)$

$$\overline{p + iq} = \overline{(k + il)(m + in)}$$

$$p - iq = (k - il)(m - in)$$

Now if $(p + iq) = (k + il)(m + in) = km - ln + i(lm + kn)$

Equating reals
 $p = km - ln$, require $p \geq 0$ \therefore take

$p = |km - ln|$ since only p^2 is of interest.

Now equate imaginaries in the above:

$$q = lm + kn \quad \text{which means}$$

that $(p + iq) = (k + il)(m + in)$ is satisfied and

so is $(p - iq) = (k - il)(m - in)$ and so is

$$(p^2 + q^2) = (k^2 + l^2)(m^2 + n^2)$$

b) $(p^2 + q^2) = (p + iq)(p - iq) = (k + il)(m + in)(k - il)(m - in)$

Take $(p + iq) = (k + il)(m + in)$, $(p - iq) = (k - il)(m - in)$

The second eqn. is the conjugate of the first.

So if the 1st is satisfied, so is the second.

$$p + iq = km + ln + i[lm - kn], \quad \text{Take } p = km + ln$$

$$q = |lm - kn| \quad [\text{sign is made pos.}]$$

Sec 1.2 continued.

prob 22] cont'd

$$c) \quad (3^2 + 5^2)(2^2 + 7^2) = p^2 + q^2$$

$$k=3, l=5, m=2, n=7$$

$$p = |km - nl| = 29, q = lm + kn = 31$$

$$29^2 + 31^2 = (3^2 + 5^2)(2^2 + 7^2) = 1802$$

$$\text{Try } p = km + nl = 41, q = |lm - kn|$$

$$= 11$$

$$p^2 + q^2 = (12^2 + 5^2) = (11^2 + 1^2)(7^2 + 2^2) = 6466$$

$$\text{Take } k=11, l=1, m=7, n=2$$

$$\therefore |km - nl| = 75, lm + kn = 29, \boxed{p=75, q=29}$$

$$\text{Note } 75^2 + 29^2 = 6466$$

$$km + nl = 79, |lm - kn| = 15 \quad \boxed{p=79, q=15}$$

$$23) \frac{(c,d)}{(a,b)} = (e,f) \quad (c,d) = (a,b)(e,f)$$

$$(c,d) = (ae - bf, be + af) \quad (b) \text{ Thus}$$

$$ae - bf = c$$

$$be + af = d$$

(c) Apply Cramer's rule to this pair of linear simult. equations with unknowns e, f . assume $a \neq 0, b \neq 0$

$$e = \frac{\begin{vmatrix} c & -b \\ d & a \end{vmatrix}}{(a^2 + b^2)} = \frac{ac + bd}{a^2 + b^2} = e \quad a^2 + b^2 \neq 0$$

$$f = \frac{\begin{vmatrix} a & c \\ b & d \end{vmatrix}}{(a^2 + b^2)} = \frac{ad - bc}{a^2 + b^2} = f$$

$$\frac{ct + id}{a + ib} = \frac{(ct + id)(a - ib)}{a^2 + b^2} = \frac{ac + bd + i(ad - bc)}{a^2 + b^2}$$

$$= e + if. \text{ Thus } e = \frac{ac + bd}{a^2 + b^2}, f = \frac{ad - bc}{a^2 + b^2}$$

(same result).

[assuming $a^2 + b^2 \neq 0$]

sec 1.2 cont'd

Prob 24 | For $\frac{p}{q} = \frac{r}{s}$ require $q \neq 0$ and $s \neq 0$

Mult both sides by qs

$$\frac{pqs}{q} = \frac{r}{s} qs \quad ps = qr$$

SUMMARY: $\frac{p}{q} = \frac{r}{s}$ Necessary and sufficient

conditions: $q \neq 0$ and $s \neq 0$ and $ps = qr$

$$1 \quad |3-i| = \sqrt{3^2+1^2} = \boxed{\sqrt{10}}$$

$$2 \quad |(2i)(3+i)| = |2i| |3+i| = 2\sqrt{3^2+1^2} = \boxed{2\sqrt{10}}$$

$$3 \quad |(2-3i)(3+i)| = |2-3i| |3+i| = \sqrt{2^2+3^2} \sqrt{3^2+1^2}$$

$$= \sqrt{13} \sqrt{10} = \boxed{\sqrt{130}}$$

$$4 \quad |(2-3i)^2(3+i)^3| = |(2-3i)^2| |(3+i)^3|$$

$$= (4+9) (\sqrt{9+9})^3 = \boxed{13 (\sqrt{18})^3} = \boxed{993}$$

$$5 \quad |2i + 2i(3+i)| = |2i + 6i - 2| = |-2 + 8i|$$

$$= \sqrt{4+64} = \sqrt{68} = \boxed{2\sqrt{17}}$$

$$6 \quad \left| 1+i + \frac{1}{1+i} \right| = \left| (1+i) + \frac{(1-i)}{2} \right| = \left| \frac{3}{2} + \frac{i}{2} \right| = \sqrt{\frac{9}{4} + \frac{1}{4}}$$

$$= \boxed{\frac{\sqrt{10}}{2}}$$

$$7 \quad \left| \frac{(1+i)^5}{(2+3i)^5} \right| = \left| \frac{1+i}{2+3i} \right|^5 = \left| \frac{\sqrt{2}}{\sqrt{13}} \right|^5 = \left[\frac{\sqrt{2}}{\sqrt{13}} \right]^5 \approx .0093$$

$$8 \quad \left| \frac{(1-i)^n}{(2+2i)^n} \right| = \left| \frac{(\sqrt{2})^n}{2^n |1+i|^n} \right| = \left| \frac{(\sqrt{2})^n}{2^n (\sqrt{2})^n} \right| = \boxed{\frac{1}{2^n}}$$

$$9 \quad \left| \frac{1}{(1-i)} + \frac{1}{(1+i)} + \frac{5}{(1+2i)} \right| = \left| \frac{1+i}{2} + \frac{1-i}{2} + \frac{5(1-2i)}{5} \right|$$

$$= |1 + 1 - 2i| = |2 - 2i| = \boxed{2\sqrt{2}}$$

Sec 1.3 continued

10) $|\alpha + i\beta + \alpha - i\beta| = 1$
 $|2\alpha| = 1$ take $\alpha = 1/2$

$$\sqrt{\alpha^2 + \beta^2} + \sqrt{\alpha^2 + \beta^2} = 2 \quad \sqrt{\alpha^2 + \beta^2} = 1$$

$$\sqrt{\frac{1}{4} + \beta^2} = 1 \quad \frac{1}{4} + \beta^2 = 1, \quad \beta^2 = 3/4$$

$$\beta = \pm \sqrt{3}/2. \quad \text{Take ans. as } \boxed{\frac{1+i\sqrt{3}}{2}}$$

and $\boxed{\frac{1-i\sqrt{3}}{2}}$

11) $z_1 = \alpha + i\beta, \quad z_2 = \alpha - i\beta$

$$|z_1 + z_2| = a \quad 2|\alpha| = a, \quad \text{Try } \alpha = a/2$$

$$|z_1| + |z_2| = \frac{1}{a} \quad \sqrt{\alpha^2 + \beta^2} + \sqrt{\alpha^2 + \beta^2} = \frac{1}{a}$$

$$2\sqrt{\alpha^2 + \beta^2} = \frac{1}{a} \quad \alpha^2 + \beta^2 = \frac{1}{4a^2}$$

$$\frac{a^2}{4} + \beta^2 = \frac{1}{4a^2} \quad \beta^2 = \frac{1}{4} \left[\frac{1}{a^2} - a^2 \right] > 0$$

$$\beta = \pm \frac{1}{2} \sqrt{\frac{1}{a^2} - a^2}$$

answer: $z_1 = \frac{a}{2} + \frac{i}{2} \sqrt{\frac{1}{a^2} - a^2}$

$$z_2 = \frac{a}{2} - \frac{i}{2} \sqrt{\frac{1}{a^2} - a^2}$$

check $|z_1 + z_2| = a$

$$\sqrt{\frac{a^2}{4} + \frac{1}{4a^2} - \frac{a^2}{4}} + \sqrt{\frac{a^2}{4} + \frac{1}{4} (\frac{1}{a^2} - a^2)}$$

$$= 2 \sqrt{\frac{1}{4a^2}} = \frac{1}{a}$$

sec 1.3 continued

12] Because their difference is purely imaginary, their real parts must be identical. Let the numbers be

$$z_1 = x + iy \quad \text{and} \quad z_2 = x + i\beta$$

$$\text{Now } z_1 - z_2 = i \quad \therefore y - \beta = 1$$

$$z_1 z_2 = x^2 - y\beta + ix[y + \beta] = 2$$

$$\text{Now } x[y + \beta] = 0$$

either $x = 0$ or $y = -\beta$

Suppose $x = 0$, then using $z_1 z_2 = 2$

$$\text{have } -y\beta = 2, \quad \beta = -2/y$$

$$\text{Since } y - \beta = 1 \quad \text{have } y + \frac{2}{y} = 1$$

Use quadratic formula, \uparrow has no real sol'n.

So Take $y = -\beta$, Now $y - \beta = 1$

$$y + y = 1, \quad y = 1/2, \quad \beta = -1/2$$

$$\text{Recall } x^2 - y\beta = 2 \quad x^2 + \frac{1}{4} = 2$$

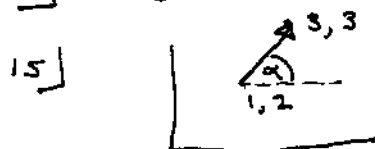
$$x = \pm \sqrt{7}/2$$

$$\text{answers: } \boxed{\sqrt{7}/2 + i/2, \quad \sqrt{7}/2 - i/2}$$

$$\text{also } \boxed{-\sqrt{7}/2 + i/2, \quad -\sqrt{7}/2 - i/2}$$

$$13] \quad 1 - (-1) + i[4 - (-3)] = \boxed{2 + 7i}$$

$$14] \quad 5 \cos 30^\circ + i 5 \sin 30^\circ = \boxed{\frac{5\sqrt{3}}{2} + i \frac{5}{2}}$$

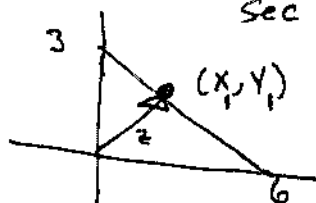


$$\cos \alpha = \frac{2}{\sqrt{5}}, \quad \sin \alpha = \frac{1}{\sqrt{5}}$$

$$a = 5 \cos \alpha = \boxed{2\sqrt{5} = a} \quad 5 \sin \alpha = \boxed{\sqrt{5} = b}$$

Sec 1.3 cont'd

16]



$$x + 2y = 6$$

$$z = a + ib \text{ or}$$

$$z = x_1 + iy_1, \quad x_1 + 2y_1 = 6, \quad y_1 = 3 - \frac{x_1}{2}$$

$$z = x_1 + i \left[3 - \frac{x_1}{2} \right]$$

slope of given line is $-1/2$

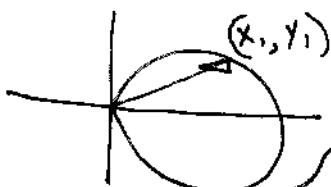
slope of vector is $\frac{3 - x_1/2}{x_1}$ must = 2

two slopes are neg. recip

$$3 - \frac{x_1}{2} = 2x_1; \quad x_1 = \frac{6}{5}, \quad y_1 = 3 - \frac{x_1}{2} = 3 - \frac{3}{5} = \frac{12}{5}$$

answer: $\boxed{\frac{6}{5} + i \frac{12}{5}}$ $\boxed{y_1 = \frac{12}{5}}$

17] Let $x_1 + iy_1$ be the vector = $a + ib$



$$(x_1 - 1)^2 + y_1^2 = 1$$

$$x_1^2 - 2x_1 + 1 + y_1^2 = 1$$

$$\text{Now } \sqrt{x_1^2 + y_1^2} = 3/2$$

use here $x_1^2 + y_1^2 = 9/4$

$$\therefore -2x_1 + 1 + 9/4 = 1$$

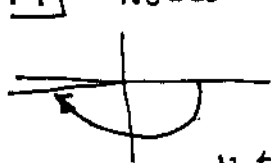
$$\boxed{9/8 = x_1}$$

$$\text{Now } \left(\frac{9}{8}\right)^2 + y_1^2 = 9/4$$

$$\sqrt{9/4 - (9/8)^2} = y_1, \quad \boxed{y_1 = \frac{3}{8}\sqrt{7}}$$

18] ans is $\boxed{3.14} = \theta$ since $-\pi < \theta \leq \pi$

19] Notice the angle 3.15 exceeds π



subtract off 2π
 $3.15 - 2\pi = \boxed{-3.133}$

Note $-\pi < \theta \leq \pi$

sec 1.3 cont'd

20] $-3 \operatorname{cis}(3.14) = 3 \operatorname{cis}(3.14 + \pi)$

The angle $3.14 + \pi$ does not satisfy $-\pi < \theta \leq \pi$, But we can subtract off 2π

Use $3.14 + \pi - 2\pi = 3.14 - \pi = \boxed{-0.001593}$

The preceding is the princ. value.

21] $-4 \operatorname{cis}(73.7\pi) = 4 \operatorname{cis}(74.7\pi)$

can subtract 74π from argument

set $\theta = \boxed{.7\pi}$ = princ. value.

22] $3 \operatorname{cis}(1.1\pi) + 4 \operatorname{cis}(1.2\pi) = 12 \operatorname{cis}(2.3\pi)$

subtract π from arg, set $\boxed{.3\pi}$

23] $\frac{3 \angle 1.5\pi}{3 \angle -1.5\pi} = 1 \angle 3.14$

$-\pi < 3.14 \leq \pi$, $\boxed{3.14}$ is a princ. value

24] $\frac{3 \angle 1.5\pi}{3 \angle -1.5\pi} = 1 \angle 3.15$ 3.15 not a

princ. value, but we can subtract 2π

$3.15 - 2\pi = \boxed{-3.133}$

25] $5 \operatorname{cis}(-98.5\pi)$ we can add 98π

to the argument set $-.5\pi = \boxed{-\pi/2}$

26] $5 \operatorname{cis}(\pi^2) = 5 \operatorname{cis} 29.61$ does not

lie between $-\pi$ and π . Subtract 10π

$\therefore 29.61 - 10\pi = \boxed{-1.81}$

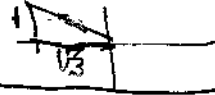
27] $3 [\cos 4 + i \sin(-4)] = \boxed{-1.96 + i 2.27}$

28] $4 [\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}] = \boxed{2\sqrt{2} + i 2\sqrt{2}}$

sec 1.3 cont'd

29] $r = \sqrt{3+1} = 2$

$\theta = \frac{5\pi}{6}$ princ.



$2 \angle \frac{5\pi}{6} + 2k\pi$

P.V. when $k=0$

30] $(1+i)(-\sqrt{3}+i) = \sqrt{2} \angle \frac{\pi}{4} \cdot 2 \angle \frac{5\pi}{6}$

$= 2\sqrt{2} \angle \frac{13\pi}{12} = 2\sqrt{2} \angle \frac{-11\pi}{12}$

$2\sqrt{2} \angle \frac{-11\pi}{12} + 2k\pi$

P.V. when $k=0$

31] $\sqrt{2} \angle \frac{-3\pi}{4} (2 \angle \frac{5\pi}{6})^3$

$= \sqrt{2} * 8 \angle \frac{-3\pi}{4} + \frac{15\pi}{6} =$

$8\sqrt{2} \angle \frac{-3\pi}{4} + 2\frac{1}{2}\pi = 8\sqrt{2} \angle \frac{-3\pi}{4} + \frac{\pi}{2}$

$= 8\sqrt{2} \angle \frac{-\pi}{4} = 8\sqrt{2} \angle \frac{-\pi}{4} + 2k\pi$ P.V. $k=0$

32] $(-4+3i)^2 = 7-24i = \sqrt{7^2+24^2} \angle -\tan^{-1} \frac{24}{7} + 2k\pi$

$= 25 \angle -1.287 + 2k\pi$

P.V. when $k=0$.

33] angle $(z_1 * z_2) = 2.879$

angle $(z_1) + \text{angle}(z_2) = 2.879$

angle $(z_1 * z_3) = -2.8797$, angle $z_1 + \text{angle } z_3 = 3.40$

angle $(z_1 * z_3) \neq \text{angle } z_1 + \text{angle } z_3$ because angle $z_1 + \text{angle } z_3 = 3.4$ not a princ value, but -2.87 is princ. value

34

$$\frac{-1-i}{\sqrt{3+i}} = \frac{\sqrt{2} \angle \frac{-3\pi}{4}}{2 \angle \frac{\pi}{6}} = \frac{1}{\sqrt{2}} \angle \frac{-22\pi}{24}$$

$$= \boxed{\frac{1}{\sqrt{2}} \angle \frac{-11\pi}{12}}$$

$$35 \quad \frac{\sqrt{2} \angle \frac{-3\pi}{4} \cdot \angle \frac{\pi}{4}}{(2 \angle 30^\circ)^2} = \frac{\sqrt{2} \angle \frac{-\pi}{2}}{4 \angle \frac{\pi}{3}} = \boxed{\frac{1}{2\sqrt{2}} \angle \frac{-5\pi}{6}}$$

$$36 \quad \frac{\text{cis}(2\pi)}{\text{cis}(-4\pi/3)} = \text{cis}\left[\frac{6\pi}{3} + \frac{4\pi}{3}\right] = \text{cis}\left[\frac{10\pi}{3}\right]$$

$$= \text{cis}\left[3\frac{1}{3}\pi\right] = \text{cis}\left[-\frac{2\pi}{3}\right]$$

37

Eqn (1.3-7) $|z_1 + z_2| \leq |z_1| + |z_2|$.

If ^{vectors} z_1 and z_2 are both pointing in same direction, then equality will hold. (They cannot point in opposite directions). Thus for equality $\arg z_1 = \arg z_2 + 2k\pi$ where k is any integer.

38

next page

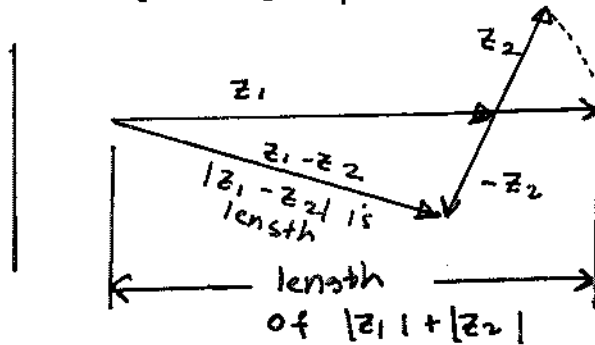
Sec 1.3 Cont'd

38] a) Given: $|z_1 + z_2| \leq |z_1| + |z_2|$

Use $-z_2$ in place of z_2

$|z_1 - z_2| \leq |z_1| + |-z_2|$ but $|-z_2| = |z_2|$

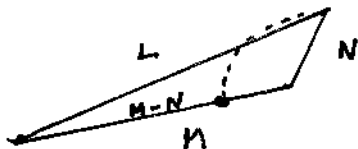
$\therefore |z_1 - z_2| \leq |z_1| + |z_2|$



The leg of the triangle with length $|z_1 - z_2|$ must be \leq sum of lengths of remaining two legs, i.e. $|z_1| + |z_2|$

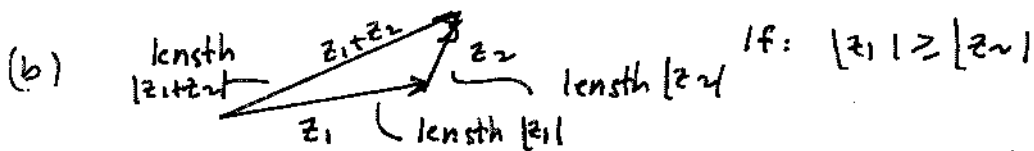
(b) If z_1 and $-z_2$ are in same direction equality will hold, i.e. z_1 and z_2 are in opposite directions, i.e. $\arg z_1 = -\arg z_2 + \pi + 2k\pi$, k is an integer.

39] a)



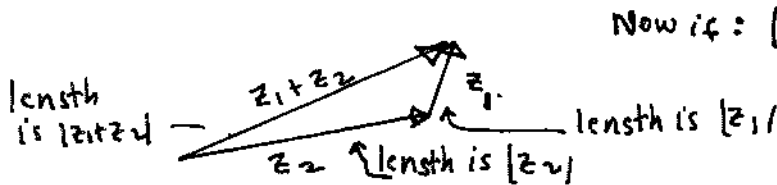
From figure,
 $L \geq M - N$

$M \geq N$



Referring to part (a) have

$|z_1 + z_2| \geq |z_1| - |z_2|$



$|z_1 + z_2| \geq |z_2| - |z_1|$

[continued next pg]

sec 1.3 cont'd prob 39, (b) cont'd.

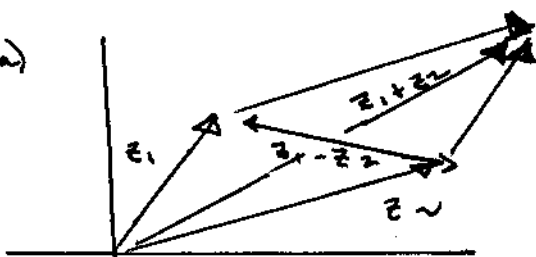
Explanation Eqn (1.3-20)

If $|z_2| \geq |z_1|$ then $||z_2| - |z_1|| = |z_2| - |z_1|$

If $|z_1| \geq |z_2|$ then $||z_2| - |z_1|| = |z_1| - |z_2|$

Thus by using $||z_2| - |z_1||$ we always get the required right hand side.

40) a)



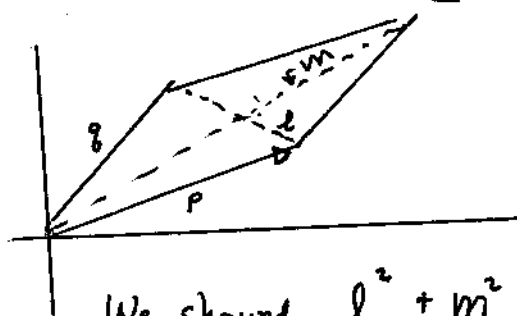
$$|z_1 - z_2|^2 + |z_1 + z_2|^2 =$$

$$(\bar{z}_1 - \bar{z}_2)(z_1 - z_2) + (\bar{z}_1 + \bar{z}_2)(z_1 + z_2) =$$

$$|z_1|^2 + |z_2|^2 - \bar{z}_1 z_2 - \bar{z}_2 z_1 + |z_1|^2 + |z_2|^2 + \bar{z}_1 z_2 + \bar{z}_2 z_1$$

$$= 2|z_1|^2 + 2|z_2|^2 \quad \text{g.e.d.}$$

b)



$$l = |z_1 - z_2|$$

$$m = |z_1 + z_2|$$

$$p = |z_2|, \quad q = |z_1|$$

We showed $l^2 + m^2 = 2(p^2 + q^2)$

9.11 (a) $(p - q)^2 \geq 0$ since $p - q$ is real.

$$p^2 - 2pq + q^2 \geq 0$$

$$p^2 + q^2 \geq 2pq \quad \text{reverse this}$$

$$2pq \leq p^2 + q^2, \quad \text{add } p^2 + q^2 \text{ to both sides}$$

$$p^2 + q^2 + 2pq \leq p^2 + q^2 + 2pq$$

$$(p + q)^2 \leq 2(p^2 + q^2) \quad \text{take square root both sides.}$$

$$p + q \leq \sqrt{2} \sqrt{p^2 + q^2} \quad \text{g.e.d.}$$