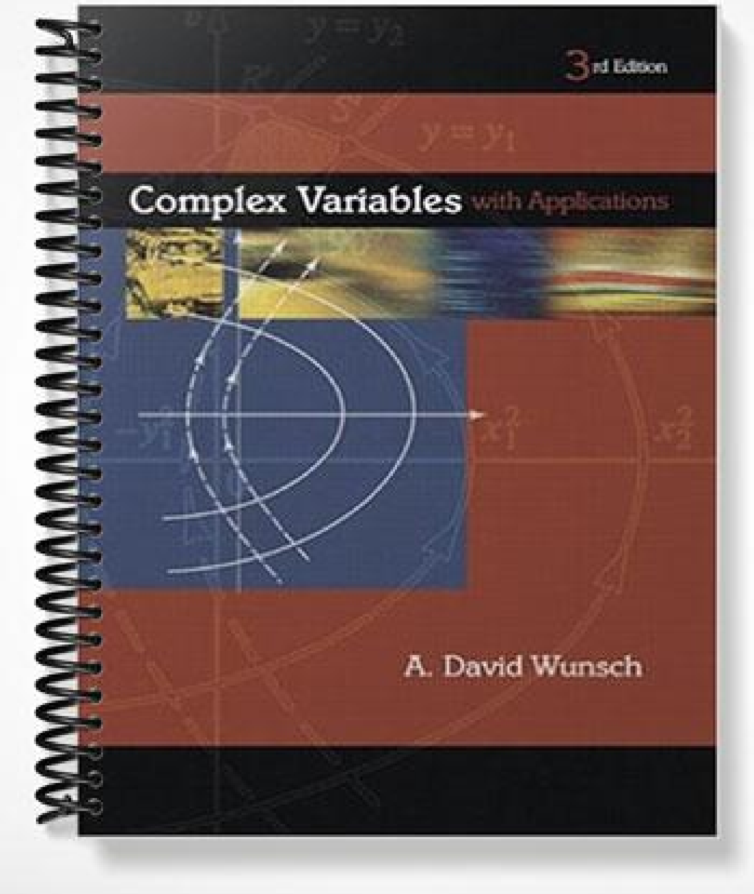
SOLUTIONS MANUAL



COMPLEX VARIABLES WITH APPLICATIONS

Third Edition
By A. David Wunsch
University of Massachusetts Lowell

SOLUTIONS MANUAL

By

A. David Wunsch

&

Michael F. Brown

Publishing as Pearson Addison-Wesley, 75 Arlington Street, Boston, MA 02116

COMPLEX VARIABLES WITH APPLICATIONS

Third Edition

By A. David Wunsch

University of Massachusetts Lowell

SOLUTIONS MANUAL

By

A. David Wunsch

&

Michael F. Brown

Publishing as Pearson Addison-Wesley, 75 Arlington Street, Boston, MA 02116

The bulky volume you are holding represents the solutions to all the problems in the 3rd edition of my textbook *Complex Variables with Applications*. Both Michael Brown and I have separately worked through all the solutions, but I can say with overwhelming confidence that in spite of this redundancy there are some remaining errors. Please tell me of any that you find. My e mail address is David_Wunsch@UML.edu (note the underscore). Those preferring an older medium of communication may write to me at the Electrical and Computer Engineering Dept. University of Massachusetts Lowell, Lowell, MA 01854. I promise to acknowledge all e-mail and postal mail that I receive. I would also appreciate learning of any errors in the textbook itself.

I plan to post corrections to both the book and this manual at the web address http://faculty.uml.edu/awunsch/Wunsch_Complex_Variables/

This manual has been written primarily for college faculty who are teaching from my text. Whether it is to be made freely available to students- perhaps at the school library- is a matter I leave up to each individual instructor. Notice however, that there is little point in assigning the textbook problems involving computer programming if students already have the MATLAB code supplied in this manual. Regarding this code, I must assert that I am not a professional programmer and I'm certain that in many cases the reader will find more efficient ways of solving the same problem.

Finally, I must apologize for the idiosyncrasies of the handwriting. They are my own and not to blamed on Mr. Brown.

A. David Wunsch
Belmont, Massachusetts
July 8, 2004

$$e^{i\pi}+1=0$$

TABLE OF CONTENTS

1

Complex Numbers

Chapter 1
Section 1.1

1) $4x+3=0$, $x=-3/4$, rational number system

2) $x^2-x-1=0$, $x=\frac{1\pm\sqrt{5}}{2}$ which is real but irrational. Need real number system

3) $x^2+x+1=0$ $x=\frac{-1\pm\sqrt{-3}}{2}$ need complex number system

4) $\sin x=0$, $x=0$, integers

5) $\cos x=0$, $x=\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, -\frac{\pm n\pi}{2}$ real number system n odd ↑

6) $(x+2)(x+1)=0$ $x=-2,-1$, integers

7) $\sin(\log x)=0$, $x=1$, $\sin(\log 1)=0$, integers

8) $z^4=16$, $z=2$, integers

9) $z^4=-16$, $z=(-16)^{1/4}$ complex

10 a) $[1+10^{-2}+10^{-4}\ldots] = \frac{1}{1-10^{-2}} = \frac{10^2}{100-1} = \boxed{\frac{100}{99}}$

b) $23.2323\ldots = 23\,\frac{(100)}{99} = \frac{2300}{99}$

c) $376.376376 = 376\left[1+10^{-3}+10^{-6}+10^{-9}\ldots\right]$

$= 376\,\frac{1}{1-10^{-3}} = 376\,\frac{1000}{999} = \frac{376000}{999}$

11 (a) Consider $4.0404\ldots = 4\left[1+10^{-2}+10^{-4}\ldots\right]$

$= 4\left[\frac{1}{1-10^{-2}}\right] = \frac{400}{99}$. Now $3.0404\ldots$

$= \frac{400}{99} - 1 = \frac{400-99}{99} = \boxed{\frac{301}{99}}$

b) $.999\ldots = .9\left[1+10^{-1}+10^{-2}\ldots\right]$

$= .9\,\frac{1}{1-10^{-1}} = \frac{.9}{1-.1} = \frac{.9}{.9} = 1$ q.e.d.

12(a) We begin by showing that the square of any odd number is odd. Let N_o be that number. Then $N_o = N_e + 1$ where N_e is even. Now $N_o^2 = \underbrace{N_e^2 + 2N_e}_{\text{even}} + 1$

N_o^2 = even + 1 = odd

Chapter 1

Sec. 1.1

12 (a) continued We showed that the square of an odd integer is odd. Thus the square root of a perfect square (that is even) must not be odd, ∴ is even.

12 (b) $m^2 = 2n^2$, $2n^2$ is even, ∴ from part (a) the square root of $2n^2$, which is m, must be even.

12 (c) $n^2 = \frac{m}{2} * m$. Since m is even, $\frac{m}{2}$ is an integer. ∴ n^2 is even, and from (a) so is n.

12 (d) We assumed that $\sqrt{2} = \frac{m}{n}$ can be expressed as the ratio of 2 integers having no common integer factor. Our assumption says that m and n both can't be even. This resulted in a contradiction, since in parts (b) and (c) we found that m and n were both even

12 (e). Assume $n + \sqrt{2}$ is rational $= a$

$a = n + \sqrt{2}$, $a - n = \sqrt{2}$. The left side is rational [the difference of rational numbers] but the right side is irrational. Have a contradiction.

Suppose $\sqrt{2n^2}$ is rational. Then $\sqrt{2}\, n = a$ is rational $\frac{a}{n} = \sqrt{2}$. The left side is the quotient of rational numbers and is rational, the right side is irrational, have a contradiction.

Assume $a = \sqrt{\sqrt{2}}$ is rational $a^2 = \sqrt{2}$. Left side is rational, right side is irrational. Have contradiction.

13(a) $x^2+bx+c=0$ $x=\frac{-b\pm\sqrt{b^2-4c}}{2}=1+\sqrt{2}$ $b=-2$, $c=-1$, $\frac{\sqrt{4-4c}}{2}=\sqrt{2}$

$\therefore$ $\boxed{x^2-2x-1=0}$ will work 13(b) $x=\sqrt{\sqrt{2}}$ $x^4=2$ $\boxed{x^4-2=0}$

14 a) Using Matlab: exp(1) = $2.718281828\underbrace{45905}_{\text{pattern}}$ breaks.

Use long format

(b) $201/26 = 7.7\underbrace{307692}_{\text{repeating decimals}}\overbrace{307692}3$ $\boxed{\text{the digits 307692 repeat}}$

15 $\boxed{4-4i}$ 16 $-5+21+15i+7i=\boxed{16+22i}$

17 $(3-2i)(4+3i)(3+2i) = (3-2i)(3+2i)(4+3i) =$

$(9+4)(4+3i)=\boxed{52+i\,39}$

chap I

sec 1.1, continued

18| $(1+i)^3 = (1+i)^2(1+i) = 2i\,(1+i) = -2+2i$

Imag part $= \boxed{2}$

19| $\mathrm{Im}(1+i) = 1,\ \left[\mathrm{Im}(1+i)\right]^3 = 1^3 = \boxed{1}$

20| $(x+iy)(u-iv)(x-iy)(u+iv) =$

$(x+iy)(x-iy)(u-iv)(u+iv) = (x^2+y^2)(u^2+v^2)$

$= \boxed{u^2x^2+v^2x^2+u^2y^2+v^2y^2}$

21| (a) binomial theorem

$$(a+b)^n = \sum_{k=0}^{n} \frac{a^{n-k}b^k\, n!}{(n-k)!\,k!}$$

let $a=1,\ b=iy$

$$(1+iy)^n = \sum_{k=0}^{n} \frac{(iy)^k\, n!}{(n-k)!\,k!}$$

b) $$(1+2i)^5 = \sum_{k=0}^{5} \frac{(i2)^k\, 5!}{(5-k)!\,k!} =$$

$$\frac{5!}{5!} + \frac{(i2)\,5!}{4!\,1!} + \frac{(-4)\,5!}{3!\,2!} + \frac{-i8\,5!}{2!\,3!} + \frac{16\,5!}{1!\,4!} + \frac{i32\cdot 5!}{5!}$$

Real part is $1 + \frac{(-4)(120)}{12} + 80 = 41$

Imag part is $(i2)5 - 8\cdot 10 + 32 = -38$

c) Use $(1+2i)^\wedge 5$ in Matlab code.
Will get $41 - i38$

22| $z_1z_2 = (x_1+iy_1)(x_2+iy_2) = x_1x_2 - y_1y_2 + ix_1y_2 + ix_2y_1$

$\mathrm{Re}(z_1z_2) = x_1x_2 - y_1y_2 = \mathrm{Re}\,z_1\,\mathrm{Re}\,z_2 - \mathrm{Im}\,z_1\,\mathrm{Im}\,z_2$

23| From the above $\mathrm{Im}(z_1z_2) = x_1y_2 + x_2y_1 = \mathrm{Re}\,z_1\,\mathrm{Im}\,z_2 + \mathrm{Re}\,z_2\,\mathrm{Im}\,z_1$

24| $i,\ i^2=-1,\ i^3=-i,\ i^4=-i\cdot i=1$

$i^5=i$, etc. ∴ the four possible

Values are $i, -1, -i, 1$

$i^{n+4}=i^4\, i^n$, but $i^4=1$, ∴ $i^{n+4}=i^n$

25| $i^{1023}=i^{1020}\, i^3 = i^{1020}(-i)$

$= (i^4)^{255}(-i) = 1^{255}(-i) = \boxed{-i}$

26| Find: $(1-i)^{1025}$, Note: $(1-i)^2=-2i$

$(1-i)^{1025} = (1-i)^{1024}(1-i) = \left[(1-i)^2\right]^{512}(1-i)$

$= (-2i)^{512}(1-i) = 2^{512}(-1)^{512}\, i^{512}(1-i)$

$= 2^{512}\left[(i)^4\right]^{128}(1-i) = 2^{512}\, 1^{128}(1-i) = \boxed{2^{512}(1-i)}$

27|

$x=2x$

∴ $x=0$

$y+1=2y$

∴ $y=1$, ans $\boxed{x=0,\ y=1}$

28|

$x^2-y^2+i2xy = y+ix$

$x^2-y^2=y,\quad 2xy=x \Rightarrow$ assume $x\neq 0$

$2y=1,\ y=1/2$

$x^2-\frac{1}{4}=1/2 \quad x^2=3/4,\quad x=\pm\sqrt{3}/2$

Set of answers $\boxed{x=\pm\sqrt{3}/2,\ y=1/2}$

Now assume $x=0$, $2xy=x$ is satisfied

$x^2-y^2=y \qquad -y^2=y \qquad y+y^2=0$

$y(1+y)=0,\quad y=0$ or $y=-1$

Second set of answer $\boxed{x=0,\ y=0 \text{ or } -1}$

29)

$$e^{x^2+y^2} = e^{-2xy} \qquad 2y = 1, \quad \therefore \quad y = \tfrac{1}{2}$$

Now $x^2+y^2 = -2xy$, $\quad x^2+y^2+2xy = 0$

$(x+y)^2 = 0 \quad \therefore \quad x = -y, \quad x = -1/2$

answer $\boxed{x = \frac{-1}{2}, \quad y = 1/2}$

30)

$$\operatorname{Log}(x+y) = 1 \qquad y = xy$$

If $\operatorname{Log}(x+y) = 1$, $\quad x+y = e$

Now assume $y \neq 0 \qquad y = xy \quad \Rightarrow \quad x = 1$

Since $x+y = e$, $\quad y = e-1 \quad$ Answer: $\boxed{x=1, \ y = e-1}$

Now assume $y = 0$, $\quad y = xy \quad$ is satisfied

$x+y = e \quad \Rightarrow x = e \qquad$ Answer $\boxed{x = e, \ y = 0}$

31)

$$[\operatorname{Log}(x) - 1]^2 = 1, \qquad [\operatorname{Log}(y) - 1]^2 = 0 \quad \therefore \boxed{y = e}$$

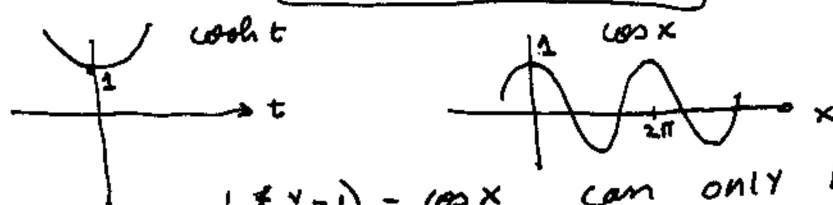
$\operatorname{Log}(x) - 1 = \pm 1$, $\quad \operatorname{Log} x - 1 = -1$

$\operatorname{Log} x = 0, \quad x = 1 \qquad$ or $\quad \operatorname{Log} x - 1 = 1$

$\operatorname{Log} x = 2, \quad x = e^2$

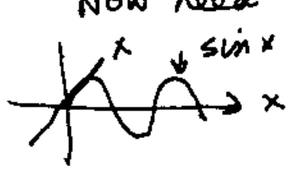
Answer $\boxed{y = e, \quad x = 1 \text{ or } e^2}$

32)



$\cosh(y-1) = \cos x \quad$ can only be solved if $\quad y = 1 \quad$ and $\quad x = 2n\pi, \quad n = 0, \pm 1, \pm 2 \ldots$

Now need $\sin x = xy \qquad \sin x = x \quad$ (since $y=1$)



$\sin x = x$ if and only if $x = 0$

so Answer $\boxed{x = 0, \ y = 1}$

Sec 1.2

1) $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$,

$\overline{z_1 - z_2} = (x_1 - x_2) - i(y_1 - y_2) = x_1 - iy_1 - [x_2 - iy_2]$

$= \bar{z}_1 - \bar{z}_2$

2) $z_1 z_2 = (x_1 x_2 - y_1 y_2) + i[y_1 x_2 + y_2 x_1]$

$\overline{z_1 z_2} = (x_1 x_2 - y_1 y_2) - i[y_1 x_2 + y_2 x_1] =$

$(x_1 - iy_1)(x_2 - iy_2) = \bar{z}_1 \bar{z}_2$

3) $\frac{1}{z_1} = \frac{1}{x_1 + iy_1} = \frac{x_1 - iy_1}{x_1^2 + y_1^2}$

$\therefore \overline{\left(\frac{1}{z_1}\right)} = \frac{x_1 + iy_1}{x_1^2 + y_1^2}$ $\qquad \frac{1}{\bar{z}_1} = \frac{1}{x_1 - iy_1} = \frac{x_1 + iy_1}{x_1^2 + y_1^2}$

Thus $\overline{\left(\frac{1}{z_1}\right)} = \frac{1}{\bar{z}_1}$

4) $\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 + y_2^2} =$

$\frac{x_1 x_2 + y_1 y_2 + i[y_1 x_2 - y_2 x_1]}{x_2^2 + y_2^2}$. Now $z_1 \frac{1}{z_2}$

$= (x_1 + iy_1)\left[\frac{(x_2 - iy_2)}{x_2^2 + y_2^2}\right] = \frac{x_1 x_2 + y_1 y_2 + i[y_1 x_2 - x_1 y_2]}{x_2^2 + y_2^2}$

Thus $z_1 \frac{1}{z_2} = \left(\frac{z_1}{z_2}\right)$

5) $\overline{\left(\frac{x_1 + iy_1}{x_2 + iy_2}\right)} = \overline{\left(\frac{z_1}{z_2}\right)} = \overline{\frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 + y_2^2}}$

$= \overline{\frac{x_1 x_2 + y_1 y_2 + i(y_1 x_2 - y_2 x_1)}{x_2^2 + y_2^2}} = \frac{x_1 x_2 + y_1 y_2 + i[y_2 x_1 - y_1 x_2]}{x_2^2 + y_2^2}$

Now $\frac{\bar{z}_1}{\bar{z}_2} = \frac{x_1 - iy_1}{x_2 - iy_2} = \frac{(x_1 - iy_1)(x_2 + iy_2)}{x_2^2 + y_2^2} = \frac{x_1 x_2 + y_1 y_2 + i[y_2 x_1 - y_1 x_2]}{x_2^2 + y_2^2}$

Thus $\frac{\bar{z}_1}{\bar{z}_2} = \overline{\left(\frac{z_1}{z_2}\right)}$

sec. 1.2 continued

6] $z_1 z_2 = x_1 x_2 - y_1 y_2 + i\left[y_1 x_2 + y_2 x_1\right]$

$\bar{z}_1 \bar{z}_2 = x_1 x_2 - y_1 y_2 - i\left[y_1 x_2 + y_2 x_1\right]$

$\operatorname{Re}\left[z_1 z_2\right] = \operatorname{Re}\left[\bar{z}_1 \bar{z}_2\right]$. Alternatively:

$\operatorname{Re}\left[z_1 z_2\right] = \operatorname{Re}\left[\overline{z_1 z_2}\right]$ since real part is unaffected by taking conj.

But $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ from prob 2

$\therefore \operatorname{Re}\left[z_1 z_2\right] = \operatorname{Re}\left[\bar{z}_1 \bar{z}_2\right]$

7] From part 6] $z_1 z_2 = x_1 x_2 - y_1 y_2 + i\left[y_1 x_2 + y_2 x_1\right]$

thus: $\bar{z}_1 \bar{z}_2 = x_1 x_2 - y_1 y_2 - i\left[y_1 x_2 + y_2 x_1\right]$

Note $\operatorname{Im}(z_1 z_2) = -\operatorname{Im} \bar{z}_1 \bar{z}_2 = y_1 x_2 + y_2 x_1$

8] $\dfrac{1}{1+2i} = \dfrac{1-2i}{1^2+2^2} = \boxed{\dfrac{1}{5} - i\dfrac{2}{5}}$

9] $i + \dfrac{1}{1-2i} = i + \dfrac{1+2i}{5} = \dfrac{1}{5} + \dfrac{7i}{5}$

$\left(i + \dfrac{1}{1-2i}\right)^2 = \dfrac{1}{25} + \dfrac{14}{25} i - \dfrac{49}{25} =$

$= \boxed{-\dfrac{48}{25} + i\dfrac{14}{25}}$

10] $\dfrac{3-4i}{1+2i} = \dfrac{(3-4i)(1-2i)}{5} = \dfrac{1}{5}\left[3 - 8 + i\left[-4-6\right]\right]$

$= \dfrac{1}{5}\left[-5 - i10\right] = \boxed{-1 - 2i}$

11] $\dfrac{3-4i}{1+2i} + \dfrac{3+4i}{1-2i}$. This must equal

$2\operatorname{Re}\left[\dfrac{3-4i}{1+2i}\right] = \boxed{-2}$ 12] Use ans from 10

12] cont'd $2i + \dfrac{3-4i}{1+2i} = 2i + (-1-2i) = \boxed{-1}$

sec 1.2 cont'd

13] $\left(\frac{4-4i}{2+2i}\right)^7 = \left(\frac{2-2i}{1+i}\right)^7 = \left[2^7 \left[\frac{(1-i)}{1+i}\right]^7\right]$

$= 2^7 \left[\frac{(1-i)^2}{2}\right]^7 = \frac{2^7}{2^7} (1-i)^{14} =$

$\left[(1-i)^2\right]^7 = (-2i)^7 = (-1)^7 2^7 i^7 = i \cdot 2^7 = \boxed{128i}$

14] $\left(\frac{4-4i}{2+2i}\right)^7 + \left(\frac{4+4i}{2-2i}\right)^7$. Note the second term is the conjugate of the first.

$\therefore$ ans is $2 \,\mathrm{Re}\left[\frac{4-4i}{2+2i}\right]^7 = 2 \,\mathrm{Re}\left[128i\right]$ (see prob 13)

$= \boxed{0}$

PROBLEM 15

```
» %8
» 1/(1+2i)

ans =

   0.2000 - 0.4000i
```

```
» %9
» (i+1/(1-2i))^2

ans =

  -1.9200 + 0.5600i
```

```
» %10
» (3-4i)/(1+2i)

ans =

  -1.0000 - 2.0000i
```

```
» %11
» (3-4i)/(1+2i)+(3+4i)/(1-2i)

ans =

    -2
```

```
» %12
» 2i+(3-4i)/(1+2i)

ans =

    -1
```

```
» %13
» ((4-4i)/(2+2i))^7

ans =

                        0 +1.2800e+002i
```

```
» %14
» ((4-4i)/(2+2i))^7+((4+4i)/(2-2i))^7

ans =

     0
```

16| $\overline{\left(\frac{z_1}{z_2 z_3}\right)} = \frac{\bar{z}_1}{\overline{z_2 z_3}} = \frac{\bar{z}_1}{\bar{z}_2\,\bar{z}_3} = \bar{z}_1 \frac{1}{\bar{z}_2\bar{z}_3}$

this is <u>not</u> equal to $\bar{z}_1 \frac{1}{\bar{z}_2 z_3}$ ∴ $\boxed{\text{not true}}$

17| $\overline{z_1 \bar{z}_2 z_3} = \bar{z}_1 \bar{\bar{z}}_2 \bar{z}_3 = \bar{z}_1\, z_2\, \bar{z}_3$ q.e.d $\boxed{\text{true}}$

18| $\overline{i\,(z_1+z_2+z_3)} = \bar{i}\,(\bar{z}_1+\bar{z}_2+\bar{z}_3) =$

$-i\,(\bar{z}_1+\bar{z}_2+\bar{z}_3)$. This is <u>not</u> equal to $i\,(\bar{z}_1+\bar{z}_2+\bar{z}_3)$. ∴ $\boxed{\text{not true}}$

19| Consider $z_1\bar{z}_2 z_3$. Its conjugate is

$\overline{z_1 \bar{z}_2 z_3} = \bar{z}_1 \bar{\bar{z}}_2 \bar{z}_3 = \bar{z}_1\, z_2\, \bar{z}_3$

∴ $\mathrm{Re}\left[z_1\bar{z}_2\, z_3\right] = \mathrm{Re}\left[\bar{z}_1\, z_2\, \bar{z}_3\right]$ q.e.d. $\boxed{\text{true}}$

20| Consider $z_1\bar{z}_2 z_3$. Its conjugate is $\overline{z_1 \bar{z}_2 z_3} = \bar{z}_1 z_2 \bar{z}_3$. If 2 quantities are conjugates of each other, their imag. parts differ in sign. ∴ $\mathrm{Im}\,(z_1\bar{z}_2 z_3) = -\,\mathrm{Im}\,(\bar{z}_1 z_2 \bar{z}_3)$
Note the minus sign. ∴ the result given in the problem is $\boxed{\text{not true}}$.

21| Observe that $\bar{z}_1 z_2 \bar{z}_3$ is the conjugate of $z_1 \bar{z}_2 z_3$. Suppose $z = a + ib$

$i\bar{z} = i(a - ib) = b + ia$. Note $\mathrm{Re}(z) = \mathrm{Im}(i\bar{z})$

$\therefore \mathrm{Re}(z_1\bar{z}_2 z_3) = \mathrm{Im}(i\bar{z}_1 z_2 \bar{z}_3)$. Is $\boxed{\text{true}}$

22|

a) If $p + iq = (k + il)(m + in)$

$\overline{p+iq} = \overline{(k+il)}\ \overline{(m+in)}$

$p - iq = (k - il)(m - in)$

Now if $(p+iq) = (k+il)(m+in) = km - ln + i(lm + kn)$

Equate reals

$p = km - ln$, require $p \geq 0$ $\therefore$ take

$p = |km - ln|$ since only p^2 is of interest.

Now equate imaginaries in the above:

$q = lm + kn$ which means

that $(p+iq) = (k+il)(m+in)$ is satisfied and so is $(p - iq) = (k - il)(m - in)$ and so is

$(p^2 + q^2) = (k^2 + l^2)(m^2 + n^2)$

b) $(p^2 + q^2) = (p+iq)(p-iq) = (k+il)(m-in)(k-il)(m+in)$

Take $(p+iq) = (k+il)(m-in)$. $(p-iq) = (k-il)(m+in)$

The second eqn. is the conjugate of the first.

So if the 1st is satisfied, so is the second.

$p + iq = km + ln + i[lm - kn]$. Take $p = km + ln$

$q = |lm - kn|$ [sign is made pos.]

prob 22] cont'd

c] $(3^2+5^2)(2^2+7^2) = p^2+q^2$

$k=3,\ l=5,\ m=2,\ n=7$

$p = |km-nl| = 29,\ q = lm+kn = 31$

$29^2+31^2 = (3^2+5^2)(2^2+7^2) = 1802$

Try $p = km+nl = 41,\ q = |lm-kn|$
$= 11$

$p^2+q^2 = (122)(53) = (11^2+1^2)(7^2+2^2) = 6466$

Take $k=11,\ l=1,\ m=7,\ n=2$

$\therefore\ |km-nl| = 75,\ lm+kn = 29,\ \boxed{p=75,\ q=29}$

Note $75^2+29^2 = 6466$

$km+nl = 79\ ;\ |lm-kn| = 15\ \boxed{p=79,\ q=15}$

23] a) $\dfrac{(c,d)}{(a,b)} = (e,f) \qquad (c,d) = (a,b)(e,f)$

$(c,d) = (ae-bf,\ be+af)$. (b) Thus

$ae-bf=c$
$be+af=d$

(c) Apply Cramer's rule to this pair of linear simult. equations with unknowns e, f. assume $a\neq0,\ b\neq0$

$e = \begin{bmatrix} c & -b \\ d & a \end{bmatrix} \Big/ (a^2+b^2) = \boxed{\dfrac{ac+bd}{a^2+b^2}} = e \qquad a^2+b^2\neq0$

$f = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \Big/ (a^2+b^2) = \boxed{\dfrac{ad-bc}{a^2+b^2}} = f$

$\dfrac{c+id}{a+ib} = \dfrac{(c+id)(a-ib)}{a^2+b^2} = \dfrac{ac+bd+i(ad-bc)}{a^2+b^2}$

$= e+if$. Thus $e = \dfrac{ac+bd}{a^2+b^2},\quad f = \dfrac{ad-bc}{a^2+b^2}$

(same result). [assuming $a^2+b^2\neq0$]

sec 1.2 cont'd

Prob 24 For $\frac{p}{q} = \frac{r}{s}$ require $q \neq 0$ and $s \neq 0$

mult both sides by qs

$$\frac{p\,qs}{q} = \frac{r}{s}\,qs \qquad ps = qr$$

Summary: $\frac{p}{q} = \frac{r}{s}$ Necessary and sufficient conditions : $\boxed{q \neq 0 \text{ and } s \neq 0 \quad \text{and } ps = qr}$

chap 1, sec 1.3

1] $|3-i| = \sqrt{3^2+1^2} = \boxed{\sqrt{10}}$

2] $|(2i)(3+i)| = |2i|\,|3+i| = 2\sqrt{3^2+1} = \boxed{2\sqrt{10}}$

3] $|(2-3i)(3+i)| = |2-3i|\,|3+i| = \sqrt{2^2+3^2}\sqrt{3^2+1^2}$

$= \sqrt{13}\sqrt{10}$

$= \boxed{\sqrt{130}}$

4] $|(2-3i)^2(3+3i)^3| = |(2-3i)^2|\,|(3+3i)^3|$

$= (4+9)\left(\sqrt{9+9}\right)^3 = \boxed{13\cdot\left(\sqrt{18}\right)^3} \doteq \boxed{993}$

5] $|2i+2i(3+i)| = |2i+6i-2| = |-2+8i|$

$= \sqrt{4+64} = \sqrt{68} = \boxed{2\sqrt{17}}$

6] $\left|1+i+\frac{1}{1+i}\right| = \left|(1+i)+\frac{(1-i)}{2}\right| = \left|\frac{3}{2}+\frac{i}{2}\right| = \sqrt{\frac{9}{4}+\frac{1}{4}}$

$= \boxed{\frac{\sqrt{10}}{2}}$

7] $\left|\frac{(1+i)^5}{(2+3i)^5}\right| = \left|\frac{1+i}{2+3i}\right|^5 = \left|\frac{\sqrt{2}}{\sqrt{13}}\right|^5 = \boxed{\left[\sqrt{\frac{2}{13}}\right]^5 \doteq .0093}$

8] $\left|\frac{(1-i)^n}{(2+2i)^n}\right| = \left|\frac{(\sqrt{2})^n}{2^n\,|1+i|^n}\right| = \left|\frac{(\sqrt{2})^n}{2^n(\sqrt{2})^n}\right| = \boxed{\frac{1}{2^n}}$

9] $\left|\frac{1}{(1-i)}+\frac{1}{(1+i)}+\frac{5}{(1+2i)}\right| = \left|\frac{1+i}{2}+\frac{1-i}{2}+\frac{5(1-2i)}{5}\right|$

$= |1+1-2i| = |2-2i| = \boxed{2\sqrt{2}}$

10| $|\alpha + i\beta + \alpha - i\beta| = 1$

$|2\alpha| = 1$ take $\alpha = 1/2$

$\sqrt{\alpha^2+\beta^2} + \sqrt{\alpha^2+\beta^2} = 2$ $\sqrt{\alpha^2+\beta^2} = 1$

$\sqrt{\frac{1}{4}+\beta^2} = 1$ $1/4 + \beta^2 = 1$, $\beta^2 = 3/4$

$\beta = \pm\sqrt{3}/2$. Take ans. as $\boxed{\frac{1}{2}+i\frac{\sqrt{3}}{2}}$

and $\boxed{\frac{1}{2} - i\frac{\sqrt{3}}{2}}$

11| $z_1 = \alpha + i\beta$, $z_2 = \alpha - i\beta$

$|z_1 + z_2| = a$ $2|\alpha| = a$. Try $\alpha = a/2$

$|z_1| + |z_2| = \frac{1}{a}$ $\sqrt{\alpha^2+\beta^2} + \sqrt{\alpha^2+\beta^2} = \frac{1}{a}$

$2\sqrt{\alpha^2+\beta^2} = \frac{1}{a}$ $\alpha^2+\beta^2 = \frac{1}{4a^2}$

$\frac{a^2}{4} + \beta^2 = \frac{1}{4a^2}$ $\beta^2 = \frac{1}{4}\left[\frac{1}{a^2} - a^2\right]$ > 0

$\beta = \pm\frac{1}{2}\sqrt{\frac{1}{a^2} - a^2}$

answer: $z_1 = \frac{a}{2} + \frac{i}{2}\sqrt{\frac{1}{a^2} - a^2}$

$z_2 = \frac{a}{2} - \frac{i}{2}\sqrt{\frac{1}{a^2} - a^2}$

Check $|z_1 + z_2| = a$

$\sqrt{\frac{a^2}{4} + \frac{1}{4a^2} - \frac{a^2}{4}} + \sqrt{\frac{a^2}{4} + \frac{1}{4}\left(\frac{1}{a^2} - a^2\right)}$

$= 2\sqrt{\frac{1}{4a^2}} = \frac{1}{a}$

12] Because their difference is purely imaginary, their real parts must be identical. Let the numbers be

$z_1 = x + iy$ and $z_2 = x + i\beta$

Now $z_1 - z_2 = i$ $\therefore$ $y - \beta = 1$

$z_1 z_2 = x^2 - y\beta + ix[y+\beta] = 2$

Now $x[y+\beta] = 0$

either $x = 0$ or $y = -\beta$

suppose $x = 0$, then using $z_1 z_2 = 2$

have $-y\beta = 2$, $\beta = -2/y$

Since $y - \beta = 1$ have $y + \frac{2}{y} = 1$

Use quadratic formula, has no real sol'n.

So Take $y = -\beta$, Now $y - \beta = 1$

$y + y = 1$, $y = 1/2$, $\beta = -1/2$

Recall $x^2 - y\beta = 2$ $\quad x^2 + \frac{1}{4} = 2$

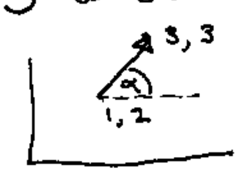
$x = \pm\sqrt{7}/2$

answers: $\boxed{\sqrt{7}/2 + i/2, \quad \sqrt{7}/2 - i/2}$

also $\boxed{-\sqrt{7}/2 + i/2, \quad -\sqrt{7}/2 - i/2}$

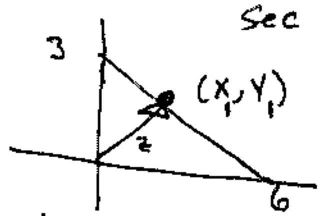
13] $1 - (-1) + i[4 - (-3)] = \boxed{2 + i7}$

14] $5\cos 30° + i\, 5 \sin 30° = \boxed{\frac{5\sqrt{3}}{2} + i\frac{5}{2}}$

15] 

$\cos\alpha = \frac{2}{\sqrt{5}}$, $\sin\alpha = \frac{1}{\sqrt{5}}$

$a = 5\cos\alpha = \boxed{2\sqrt{5} = a}$ $\quad 5\sin\alpha = \boxed{\sqrt{5} = b}$

16] 

$x + 2y = 6$

$z = a + ib$ or

$z = x_1 + i y_1 \qquad x_1 + 2y_1 = 6, \quad y_1 = 3 - \frac{x_1}{2}$

$z = x_1 + i \left[3 - \left(\frac{x_1}{2}\right)\right]$

slope of given line is $-1/2$

slope of vector is $\frac{3 - x_1/2}{x_1}$ must $= 2$

two slopes are neg. recips

$3 - x_1/2 = 2x_1$; $\boxed{x_1 = \frac{6}{5}}$ $\quad y_1 = 3 - \frac{x_1}{2} = 3 - \frac{3}{5} = 2\,2/5$

answer: $\boxed{\frac{6}{5} + i\,\frac{12}{5}}$ $\qquad \boxed{y_1 = \frac{12}{5}}$

17] Let $x_1 + i y_1$ be the vector $= a + ib$

$(x_1 - 1)^2 + y_1^2 = 1$

$x_1^2 - 2x_1 + 1 + y_1^2 = 1$

Now $\sqrt{x_1^2 + y_1^2} = 3/2$

use here $\quad x_1^2 + y_1^2 = 9/4$

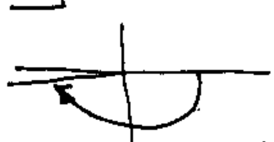
$\therefore \quad -2x_1 + 1 + 9/4 = 1 \qquad \boxed{9/8 = x_1}$

Now $\left(\frac{9}{8}\right)^2 + y_1^2 = 9/4$

$\sqrt{9/4 - (9/8)^2} = y_1 \qquad \boxed{y_1 = \frac{3}{8}\sqrt{7}}$

18] ans is $\boxed{3.14} = \theta$ since $-\pi < \theta \leq \pi$

19] Notice the angle 3.15 exceeds π



subtract off 2π

$3.15 - 2\pi = \boxed{-3.133}$

Note $-\pi < \theta \leq \pi$

20] $-3 \text{ cis}(3.14) = 3 \text{ cis}(3.14+\pi)$
The angle $3.14+\pi$ does not satisfy $-\pi < \theta \le \pi$, But we can subtract off 2π
Use $3.14+\pi-2\pi = 3.14-\pi = \boxed{-.001593}$
The preceding is the princ. value.

21] $-4 \text{ cis}(73.7\pi) = 4 \text{ cis}(74.7\pi)$
can subtract 74π from argument
get $\boxed{\theta = .7\pi}$ = princ. value.

22] $3 \text{ cis}(1.1\pi) * 4 \text{ cis}(1.2\pi) = 12 \text{ cis}(2.3\pi)$
subtract 2π from arg, get $\boxed{.3\pi}$

23] $\dfrac{3 \angle 1.5\pi}{3 \angle -1.57} = 1 \angle 3.14$
$-\pi < 3.14 \le \pi$, $\boxed{3.14 \text{ is a princ value}}$

24] $\dfrac{3 \angle 1.57}{3 \angle -1.58} = 1 \angle 3.15$ 3.15 not a princ. value, but we can subtract 2π
$3.15 - 2\pi = \boxed{-3.133}$

25] $5 \text{ cis}(-98.5\pi)$ we can add 98π to the argument get $-.5\pi = \boxed{-\pi/2}$

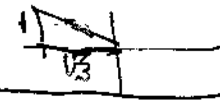
26] $5 \text{ cis}(3\pi^2) = 5 \text{ cis } 29.61$ does not lie between $-\pi$ and π. Subtract 10π
$\therefore$ $29.61 - 10\pi = \boxed{-1.81}$

27] $3[\cos 4 + i \sin(-4)] = \boxed{-1.96 + i\, 2.27}$

28] $4\left[\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right] = \boxed{2\sqrt{2} + i\, 2\sqrt{2}}$

sec 1.3 cont'd

29] $r = \sqrt{3+1} = 2$



$\theta = \frac{5\pi}{6}$ princ.

$\boxed{2 \angle \frac{5\pi}{6} + 2k\pi}$

$\boxed{\text{p.v. when } k=0}$

30] $(1+i)(-\sqrt{3}+i) = \sqrt{2} \angle \frac{\pi}{4} \; 2 \angle \frac{5\pi}{6}$

$= 2\sqrt{2} \angle \frac{13\pi}{12} = 2\sqrt{2} \angle \frac{-11\pi}{12}$

$\boxed{2\sqrt{2} \angle \frac{-11\pi}{12} + 2k\pi}$ p.v. when $k=0$

31] $\sqrt{2} \angle \frac{-3\pi}{4} \left(2 \angle + \frac{5}{6}\pi\right)^3$

$= \sqrt{2} * 8 \angle \frac{-3\pi}{4} + \frac{15\pi}{6} =$

$8\sqrt{2} \angle \frac{-3\pi}{4} + 2\frac{1}{2}\pi = 8\sqrt{2} \angle \frac{-3\pi}{4} + \frac{\pi}{2}$

$= 8\sqrt{2} \angle \frac{-\pi}{4} = \boxed{8\sqrt{2} \angle \frac{-\pi}{4} + 2k\pi}$ $\boxed{\text{p.v. } k=0}$

32] $(-4+3i)^2 = 7-24i = \sqrt{7^2+24^2} \angle -\tan^{-1}\frac{24}{7} + 2k\pi$

$= \boxed{25 \angle -1.287 + 2k\pi}$ p.v. when $k=0$.

33] angle $(z1 * z2) = \boxed{2.879}$

angle (z_1) + angle $(z_2) = \boxed{2.879}$

angle $(z1 * z3) = -2.8797$, angle z_1 + angle $z_3 = \boxed{3.40}$

angle $(z1 * z3) \neq$ angle z_1 + angle z_3 because

angle z_1 + angle z_3 $= 3.4$ not a princ value, but $\boxed{-2.87}$ is princ. value

34)

$$\frac{-1-i}{\sqrt{3}+i} = \frac{\sqrt{2}\angle\frac{-3\pi}{4}}{2\angle\pi/6} = \frac{1}{\sqrt{2}}\angle -\frac{22\pi}{24}$$

$$= \boxed{\frac{1}{\sqrt{2}}\angle\frac{-11\pi}{12}}$$

35) $$\frac{\sqrt{2}\angle\frac{-3\pi}{4} \quad 1\angle\pi/4}{(2\angle 30^\circ)^2} = \frac{\sqrt{2}\angle\frac{-\pi}{2}}{4\angle\pi/3} = \boxed{\frac{1}{2\sqrt{2}}\angle\frac{-5\pi}{6}}$$

36) $$\frac{cis\ (2\pi)}{cis\ (-4\pi/3)} = cis\left[\frac{6\pi}{3}+\frac{4\pi}{3}\right] = cis\left[\frac{10\pi}{3}\right]$$

$$= cis\left[3\tfrac{1}{3}\pi\right] = cis\left[-\frac{2}{3}\pi\right]$$

37)

Eqn (1.3-7) $\quad |z_1+z_2| \leq |z_1| + |z_2|$.

If vectors z_1 and z_2 are both pointing in same direction, then equality will hold. (They cannot point in opposite directions). Thus for equality $\arg z_1 = \arg z_2 + 2k\pi$ where k is any integer.

38)

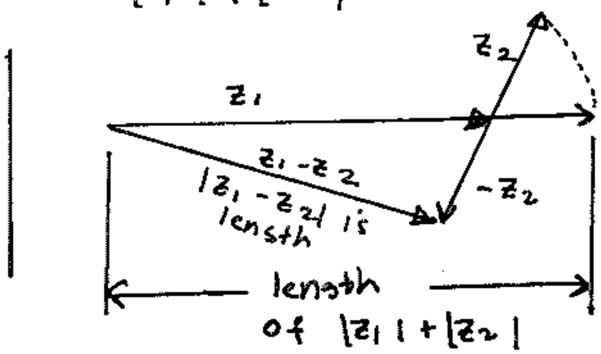
next page

38] a) Given: $|z_1 + z_2| \leq |z_1| + |z_2|$

Use $-z_2$ in place of z_2

$|z_1 - z_2| \leq |z_1| + |-z_2|$ but $|-z_2| = |z_2|$

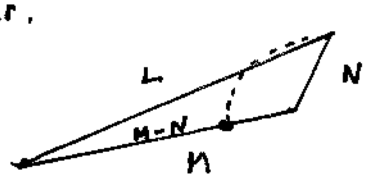
$\therefore$ $|z_1 - z_2| \leq |z_1| + |z_2|$



The leg of the triangle with length $|z_1 - z_2|$ must be $\leq$ sum of lengths of remaining two legs, i.e. $|z_1| + |z_2|$

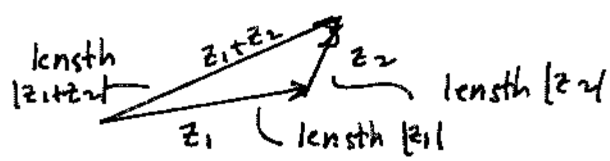
(b) If z_1 and $-z_2$ are in same direction equality will hold, i.e. z_1 and z_2 are in opposite directions, i.e. $\arg z_1 = \arg z_2 + \pi + 2k\pi$, k is an integer.

39] a)



From figure, $L \geq M - N$ $\boxed{M \geq N}$

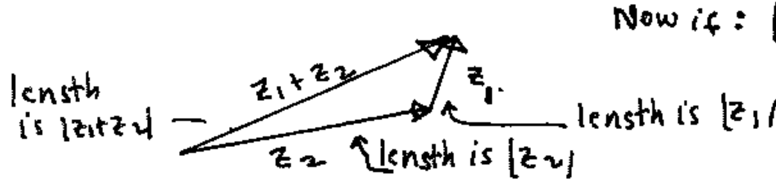
(b)



If: $|z_1| \geq |z_2|$

Referring to part (a) have $|z_1 + z_2| \geq |z_1| - |z_2|$

Now if: $|z_2| \geq |z_1|$



$|z_1 + z_2| \geq |z_2| - |z_1|$,

[continued next pg]

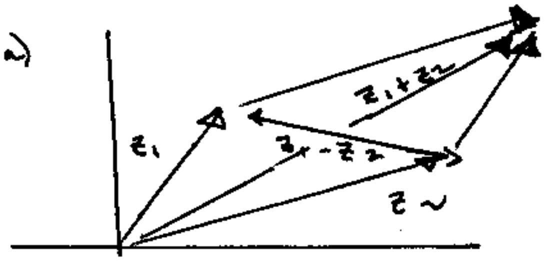
Explanation Eqn (1.3-20)

If $|z_2| \geq |z_1|$ then $||z_2|-|z_1|| = |z_2|-|z_1|$

If $|z_1| \geq |z_2|$ then $||z_2|-|z_1|| = |z_1|-|z_2|$

Thus by using $||z_2|-|z_1||$ we always get the required right hand side.

40) a)



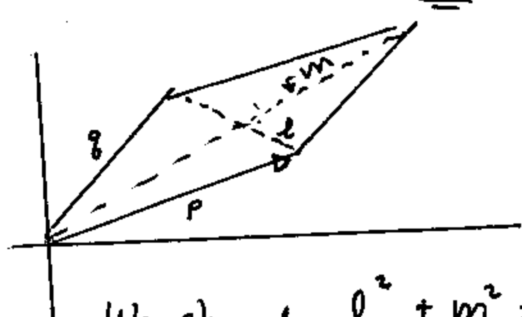
$$|z_1-z_2|^2 + |z_1+z_2|^2 =$$

$$(\bar{z}_1-\bar{z}_2)(z_1-z_2) + (\bar{z}_1+\bar{z}_2)(z_1+z_2) =$$

$$|z_1|^2+|z_2|^2 - \bar{z}_1 z_2 - \bar{z}_2 z_1 + |z_1|^2+|z_2|^2 + \bar{z}_1 z_2 + z_1\bar{z}_2$$

$$= 2|z_1|^2 + 2|z_2|^2 \quad \text{q.e.d}$$

b)



$l = |z_1-z_2|$

$m = |z_1+z_2|$

$p = |z_2|$, $q = |z_1|$

We showed $l^2 + m^2 = 2(p^2+q^2)$

41) (a) $(p-q)^2 \geq 0$ since $p-q$ is real.

$p^2 - 2pq + q^2 \geq 0$

$p^2+q^2 \geq 2pq$ reverse this

$2pq \leq p^2+q^2$, add p^2+q^2 to both sides

$p^2+q^2+2pq \leq 2p^2+2q^2$

$(p+q)^2 \leq 2(p^2+q^2)$ take square root both sides.

$p+q \leq \sqrt{2}\sqrt{p^2+q^2}$ q.e.d.