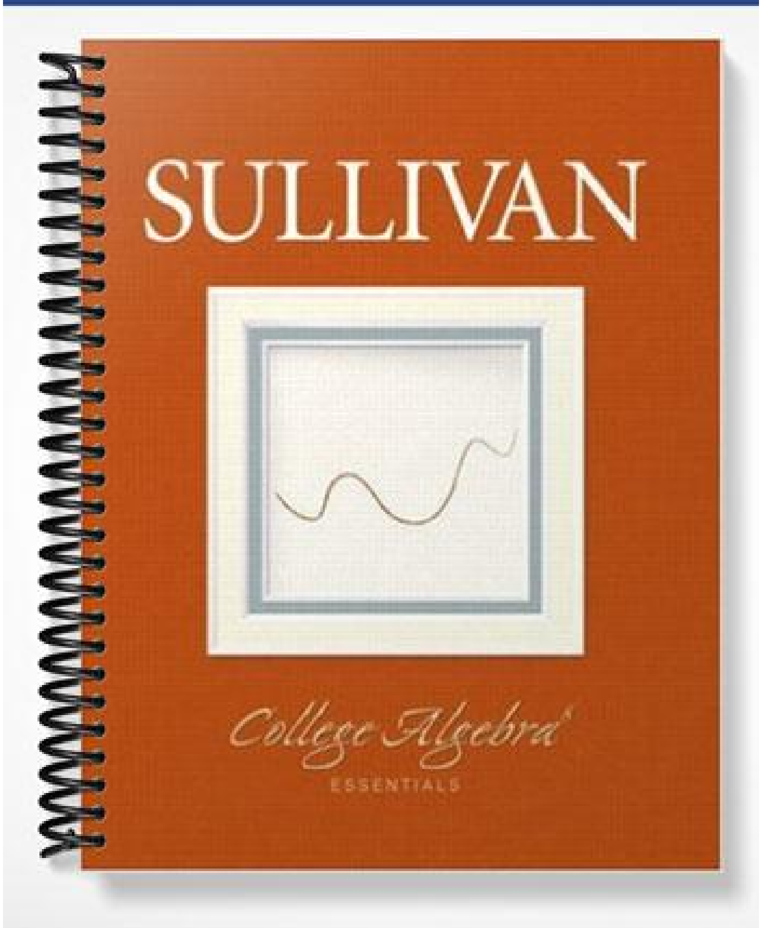
SOLUTIONS MANUAL



Chapter 1
Equations and Inequalities

Section 1.1

1. Distributive

2. Zero-Product

3. $\{x | x \neq 4\}$

4. False. Multiplying both sides of an equation by zero will not result in an equivalent equation.

5. identity

6. linear; first-degree

7. False. The solution is $\frac{8}{3}$.

8. True

9.
$$7x = 21$$
$$\frac{7x}{7} = \frac{21}{7}$$
$$x = 3$$
The solution set is $\{3\}$.

10.
$$6x = -24$$
$$\frac{6x}{6} = \frac{-24}{6}$$
$$x = -4$$
The solution set is $\{-4\}$.

11.
$$3x + 15 = 0$$
$$3x + 15 - 15 = 0 - 15$$
$$3x = -15$$
$$\frac{3x}{3} = \frac{-15}{3}$$
$$x = -5$$
The solution set is $\{-5\}$.

12.
$$6x + 18 = 0$$
$$6x + 18 - 18 = 0 - 18$$
$$6x = -18$$
$$\frac{6x}{6} = \frac{-18}{6}$$
$$x = -3$$
The solution set is $\{-3\}$.

13.
$$2x - 3 = 0$$
$$2x - 3 + 3 = 0 + 3$$
$$2x = 3$$
$$\frac{2x}{2} = \frac{3}{2}$$
$$x = \frac{3}{2}$$
The solution set is $\left\{\frac{3}{2}\right\}$.

14.
$$3x + 4 = 0$$
$$3x + 4 - 4 = 0 - 4$$
$$3x = -4$$
$$\frac{3x}{3} = \frac{-4}{3}$$
$$x = -\frac{4}{3}$$
The solution set is $\left\{-\frac{4}{3}\right\}$.

15.
$$\frac{1}{3}x = \frac{5}{12}$$
$$3\left(\frac{1}{3}x\right) = 3\left(\frac{5}{12}\right)$$
$$x = \frac{5}{4}$$
The solution set is $\left\{\frac{5}{4}\right\}$.

16. $\frac{2}{3}x = \frac{9}{2}$

$6\left(\frac{2}{3}x\right) = 6\left(\frac{9}{2}\right)$

$4x = 27$

$\frac{4x}{4} = \frac{27}{4}$

$x = \frac{27}{4}$

The solution set is $\left\{\frac{27}{4}\right\}$.

17. $3x + 4 = x$

$3x + 4 - 4 = x - 4$

$3x = x - 4$

$3x - x = x - 4 - x$

$2x = -4$

$\frac{2x}{2} = \frac{-4}{2}$

$x = -2$

The solution set is $\{-2\}$.

18. $2x + 9 = 5x$

$2x + 9 - 9 = 5x - 9$

$2x = 5x - 9$

$2x - 5x = 5x - 9 - 5x$

$-3x = -9$

$\frac{-3x}{-3} = \frac{-9}{-3}$

$x = 3$

The solution set is $\{3\}$.

19. $2t - 6 = 3 - t$

$2t - 6 + 6 = 3 - t + 6$

$2t = 9 - t$

$2t + t = 9 - t + t$

$3t = 9$

$\frac{3t}{3} = \frac{9}{3}$

$t = 3$

The solution set is $\{3\}$.

20. $5y + 6 = -18 - y$

$5y + 6 - 6 = -18 - y - 6$

$5y = -y - 24$

$5y + y = -y - 24 + y$

$6y = -24$

$\frac{6y}{6} = \frac{-24}{6}$

$y = -4$

The solution set is $\{-4\}$.

21. $6 - x = 2x + 9$

$6 - x - 6 = 2x + 9 - 6$

$-x = 2x + 3$

$-x - 2x = 2x + 3 - 2x$

$-3x = 3$

$\frac{-3x}{-3} = \frac{3}{-3}$

$x = -1$

The solution set is $\{-1\}$.

22. $3 - 2x = 2 - x$

$3 - 2x - 3 = 2 - x - 3$

$-2x = -x - 1$

$-2x + x = -x - 1 + x$

$-x = -1$

$\frac{-x}{-1} = \frac{-1}{-1}$

$x = 1$

The solution set is $\{1\}$.

23. $3 + 2n = 4n + 7$

$3 + 2n - 3 = 4n + 7 - 3$

$2n = 4n + 4$

$2n - 4n = 4n + 4 - 4n$

$-2n = 4$

$\frac{-2n}{-2} = \frac{4}{-2}$

$n = -2$

The solution set is $\{-2\}$.

24.
$$\begin{aligned}6-2m&=3m+1\\6-2m-6&=3m+1-6\\-2m&=3m-5\\-2m-3m&=3m-5-3m\\-5m&=-5\\\frac{-5m}{-5}&=\frac{-5}{-5}\\m&=1\end{aligned}$$
The solution set is $\{1\}$.

25.
$$\begin{aligned}2(3+2x)&=3(x-4)\\6+4x&=3x-12\\6+4x-6&=3x-12-6\\4x&=3x-18\\4x-3x&=3x-18-3x\\x&=-18\end{aligned}$$
The solution set is $\{-18\}$.

26.
$$\begin{aligned}3(2-x)&=2x-1\\6-3x&=2x-1\\6-3x-6&=2x-1-6\\-3x&=2x-7\\-3x-2x&=2x-7-2x\\-5x&=-7\\\frac{-5x}{-5}&=\frac{-7}{-5}\\x&=\frac{7}{5}\end{aligned}$$
The solution set is $\left\{\frac{7}{5}\right\}$.

27.
$$\begin{aligned}8x-(3x+2)&=3x-10\\8x-3x-2&=3x-10\\5x-2&=3x-10\\5x-2+2&=3x-10+2\\5x&=3x-8\\5x-3x&=3x-8-3x\\2x&=-8\\\frac{2x}{2}&=\frac{-8}{2}\\x&=-4\end{aligned}$$
The solution set is $\{-4\}$.

28.
$$\begin{aligned}7-(2x-1)&=10\\7-2x+1&=10\\8-2x&=10\\8-2x-8&=10-8\\-2x&=2\\\frac{-2x}{-2}&=\frac{2}{-2}\\x&=-1\end{aligned}$$
The solution set is $\{-1\}$.

29.
$$\begin{aligned}\frac{3}{2}x+2&=\frac{1}{2}-\frac{1}{2}x\\2\left(\frac{3}{2}x+2\right)&=2\left(\frac{1}{2}-\frac{1}{2}x\right)\\3x+4&=1-x\\3x+4-4&=1-x-4\\3x&=-3-x\\3x+x&=-3-x+x\\4x&=-3\\\frac{4x}{4}&=\frac{-3}{4}\\x&=-\frac{3}{4}\end{aligned}$$
The solution set is $\left\{-\frac{3}{4}\right\}$.

30.
$$\begin{aligned}\frac{1}{3}x&=2-\frac{2}{3}x\\3\left(\frac{1}{3}x\right)&=3\left(2-\frac{2}{3}x\right)\\x&=6-2x\\x+2x&=6-2x+2x\\3x&=6\\\frac{3x}{3}&=\frac{6}{3}\\x&=2\end{aligned}$$
The solution set is $\{2\}$.

31.
$$\frac{1}{2}x-5=\frac{3}{4}x$$
$$4\left(\frac{1}{2}x-5\right)=4\left(\frac{3}{4}x\right)$$
$$2x-20=3x$$
$$2x-20-2x=3x-2x$$
$$-20=x$$
$$x=-20$$
The solution set is $\{-20\}$.

32.
$$1-\frac{1}{2}x=6$$
$$2\left(1-\frac{1}{2}x\right)=2(6)$$
$$2-x=12$$
$$2-x-2=12-2$$
$$-x=10$$
$$\frac{-x}{-1}=\frac{10}{-1}$$
$$x=-10$$
The solution set is $\{-10\}$.

33.
$$\frac{2}{3}p=\frac{1}{2}p+\frac{1}{3}$$
$$6\left(\frac{2}{3}p\right)=6\left(\frac{1}{2}p+\frac{1}{3}\right)$$
$$4p=3p+2$$
$$4p-3p=3p+2-3p$$
$$p=2$$
The solution set is $\{2\}$.

34.
$$\frac{1}{2}-\frac{1}{3}p=\frac{4}{3}$$
$$6\left(\frac{1}{2}-\frac{1}{3}p\right)=6\left(\frac{4}{3}\right)$$
$$3-2p=8$$
$$3-2p-3=8-3$$
$$-2p=5$$
$$\frac{-2p}{-2}=\frac{5}{-2}$$
$$p=-\frac{5}{2}$$
The solution set is $\left\{-\frac{5}{2}\right\}$.

35.
$$0.9t=0.4+0.1t$$
$$0.9t-0.1t=0.4+0.1t-0.1t$$
$$0.8t=0.4$$
$$\frac{0.8t}{0.8}=\frac{0.4}{0.8}$$
$$t=0.5$$
The solution set is $\{0.5\}$.

36.
$$0.9t=1+t$$
$$0.9t-t=1+t-t$$
$$-0.1t=1$$
$$\frac{-0.1t}{-0.1}=\frac{1}{-0.1}$$
$$t=-10$$
The solution set is $\{-10\}$.

37.
$$\frac{x+1}{3}+\frac{x+2}{7}=2$$
$$21\left(\frac{x+1}{3}+\frac{x+2}{7}\right)=21(2)$$
$$7(x+1)+(3)(x+2)=42$$
$$7x+7+3x+6=42$$
$$10x+13=42$$
$$10x+13-13=42-13$$
$$10x=29$$
$$\frac{10x}{10}=\frac{29}{10}$$
$$x=\frac{29}{10}$$
The solution set is $\left\{\frac{29}{10}\right\}$.

38.
$$\frac{2x+1}{3}+16=3x$$
$$3\left(\frac{2x+1}{3}+16\right)=3(3x)$$
$$2x+1+48=9x$$
$$2x+49=9x$$
$$2x+49-2x=9x-2x$$
$$49=7x$$
$$\frac{49}{7}=\frac{7x}{7}$$
$$x=7$$
The solution set is $\{7\}$.

39.
$$\frac{2}{y}+\frac{4}{y}=3$$
$$y\left(\frac{2}{y}+\frac{4}{y}\right)=y(3)$$
$$2+4=3y$$
$$6=3y$$
$$\frac{6}{3}=\frac{3y}{3}$$
$$2=y$$
Since $y = 2$ does not cause a denominator to equal zero, the solution set is $\{2\}$.

40.
$$\frac{4}{y}-5=\frac{5}{2y}$$
$$2y\left(\frac{4}{y}-5\right)=2y\left(\frac{5}{2y}\right)$$
$$8-10y=5$$
$$8-10y-8=5-8$$
$$-10y=-3$$
$$\frac{-10y}{-10}=\frac{-3}{-10}$$
$$y=\frac{3}{10}$$
Since $y=\frac{3}{10}$ does not cause a denominator to equal zero, the solution set is $\left\{\frac{3}{10}\right\}$.

41.
$$\frac{1}{2}+\frac{2}{x}=\frac{3}{4}$$
$$4x\left(\frac{1}{2}+\frac{2}{x}\right)=4x\left(\frac{3}{4}\right)$$
$$2x+8=3x$$
$$2x+8-2x=3x-2x$$
$$8=x$$
Since $x = 8$ does not cause any denominator to equal zero, the solution set is $\{8\}$.

42.
$$\frac{3}{x}-\frac{1}{3}=\frac{1}{6}$$
$$6x\left(\frac{3}{x}-\frac{1}{3}\right)=6x\left(\frac{1}{6}\right)$$
$$18-2x=x$$
$$18-2x+2x=x+2x$$
$$18=3x$$
$$\frac{18}{3}=\frac{3x}{3}$$
$$6=x$$
Since $x=6$ does not cause a denominator to equal zero, the solution set is $\{6\}$.

43.
$$(x+7)(x-1)=(x+1)^2$$
$$x^2+6x-7=x^2+2x+1$$
$$x^2+6x-7-x^2=x^2+2x+1-x^2$$
$$6x-7=2x+1$$
$$6x-7+7=2x+1+7$$
$$6x=2x+8$$
$$6x-2x=2x+8-2x$$
$$4x=8$$
$$\frac{4x}{4}=\frac{8}{4}$$
$$x=2$$
The solution set is $\{2\}$.

44.
$$(x+2)(x-3)=(x+3)^2$$
$$x^2-x-6=x^2+6x+9$$
$$x^2-x-6-x^2=x^2+6x+9-x^2$$
$$-x-6=6x+9$$
$$-x-6+6=6x+9+6$$
$$-x=6x+15$$
$$-x-6x=6x+15-6x$$
$$-7x=15$$
$$\frac{-7x}{-7}=\frac{15}{-7}$$
$$x=-\frac{15}{7}$$
The solution set is $\left\{-\frac{15}{7}\right\}$.

45.
$$\begin{aligned} x(2x-3) &= (2x+1)(x-4) \\ 2x^2-3x &= 2x^2-7x-4 \\ 2x^2-3x-2x^2 &= 2x^2-7x-4-2x^2 \\ -3x &= -7x-4 \\ -3x+7x &= -7x-4+7x \\ 4x &= -4 \\ \frac{4x}{4} &= \frac{-4}{4} \\ x &= -1 \end{aligned}$$
The solution set is $\{-1\}$.

46.
$$\begin{aligned} x(1+2x) &= (2x-1)(x-2) \\ x+2x^2 &= 2x^2-5x+2 \\ x+2x^2-2x^2 &= 2x^2-5x+2-2x^2 \\ x &= -5x+2 \\ x+5x &= -5x+2+5x \\ 6x &= 2 \\ \frac{6x}{6} &= \frac{2}{6} \\ x &= \frac{1}{3} \end{aligned}$$
The solution set is $\left\{\frac{1}{3}\right\}$.

47.
$$\begin{aligned} z(z^2+1) &= 3+z^3 \\ z^3+z &= 3+z^3 \\ z^3+z-z^3 &= 3+z^3-z^3 \\ z &= 3 \end{aligned}$$
The solution set is $\{3\}$.

48.
$$\begin{aligned} w(4-w^2) &= 8-w^3 \\ 4w-w^3 &= 8-w^3 \\ 4w-w^3+w^3 &= 8-w^3+w^3 \\ 4w &= 8 \\ \frac{4w}{4} &= \frac{8}{4} \\ w &= 2 \end{aligned}$$
The solution set is $\{2\}$.

49.
$$\begin{aligned} \frac{x}{x-2}+3 &= \frac{2}{x-2} \\ \left(\frac{x}{x-2}+3\right)(x-2) &= \left(\frac{2}{x-2}\right)(x-2) \\ x+3(x-2) &= 2 \\ x+3x-6 &= 2 \\ 4x-6 &= 2 \\ 4x-6+6 &= 2+6 \\ 4x &= 8 \\ \frac{4x}{4} &= \frac{8}{4} \\ x &= 2 \end{aligned}$$
Since $x = 2$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

50.
$$\begin{aligned} \frac{2x}{x+3} &= \frac{-6}{x+3}-2 \\ \left(\frac{2x}{x+3}\right)(x+3) &= \left(\frac{-6}{x+3}-2\right)(x+3) \\ 2x &= -6-(2)(x+3) \\ 2x &= -6-2x-6 \\ 2x &= -12-2x \\ 2x+2x &= -12-2x+2x \\ 4x &= -12 \\ \frac{4x}{4} &= \frac{-12}{4} \\ x &= -3 \end{aligned}$$
Since $x = -3$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

51.
$$\frac{2x}{x^2-4}=\frac{4}{x^2-4}-\frac{3}{x+2}$$
$$\frac{2x}{(x+2)(x-2)}=\frac{4}{(x+2)(x-2)}-\frac{3}{x+2}$$
$$\left(\frac{2x}{(x+2)(x-2)}\right)(x+2)(x-2)=\left(\frac{4}{(x+2)(x-2)}-\frac{3}{x+2}\right)(x+2)(x-2)$$
$$2x=4-3(x-2)$$
$$2x=4-3x+6$$
$$2x=10-3x$$
$$2x+3x=10-3x+3x$$
$$5x=10$$
$$\frac{5x}{5}=\frac{10}{5}$$
$$x=2$$

Since $x = 2$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

52.
$$\frac{x}{x^2-9}+\frac{4}{x+3}=\frac{3}{x^2-9}$$
$$\frac{x}{(x+3)(x-3)}+\frac{4}{x+3}=\frac{3}{(x+3)(x-3)}$$
$$\left(\frac{x}{(x+3)(x-3)}+\frac{4}{x+3}\right)(x+3)(x-3)=\left(\frac{3}{(x+3)(x-3)}\right)(x+3)(x-3)$$
$$x+4(x-3)=3$$
$$x+4x-12=3$$
$$5x-12=3$$
$$5x-12+12=3+12$$
$$5x=15$$
$$\frac{5x}{5}=\frac{15}{5}$$
$$x=3$$

Since $x = 3$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

53.
$$\frac{x}{x+2}=\frac{3}{2}$$
$$2(x+2)\left(\frac{x}{x+2}\right)=2(x+2)\left(\frac{3}{2}\right)$$
$$2x=3(x+2)$$
$$2x=3x+6$$
$$2x-3x=3x+6-3x$$
$$-x=6$$
$$\frac{-x}{-1}=\frac{6}{-1}$$
$$x=-6$$

Since $x = -6$ does not cause any denominator to equal zero, the solution set is $\{-6\}$.

54.
$$\frac{3x}{x-1}=2$$
$$\left(\frac{3x}{x-1}\right)(x-1)=2(x-1)$$
$$3x=2x-2$$
$$3x-2x=2x-2-2x$$
$$x=-2$$

Since $x = -2$ does not cause any denominator to equal zero, the solution set is $\{-2\}$.

55.
$$\frac{5}{2x-3}=\frac{3}{x+5}$$
$$\left(\frac{5}{2x-3}\right)(2x-3)(x+5)=\left(\frac{3}{x+5}\right)(2x-3)(x+5)$$
$$5(x+5)=3(2x-3)$$
$$5x+25=6x-9$$
$$5x+25-6x=6x-9-6x$$
$$25-x=-9$$
$$25-x-25=-9-25$$
$$-x=-34$$
$$\frac{-x}{-1}=\frac{-34}{-1}$$
$$x=34$$
Since x = 34 does not cause any denominator to equal zero, the solution is {34}.

56.
$$\frac{-4}{x+4}=\frac{-3}{x+6}$$
$$\left(\frac{-4}{x+4}\right)(x+6)(x+4)=\left(\frac{-3}{x+6}\right)(x+6)(x+4)$$
$$-4(x+6)=-3(x+4)$$
$$-4x-24=-3x-12$$
$$-4x-24+4x=-3x-12+4x$$
$$-24=-12+x$$
$$-24+12=-12+x+12$$
$$-12=x$$
Since x = –12 does not cause any denominator to equal zero, the solution set is {–12}.

57.
$$\frac{6t+7}{4t-1}=\frac{3t+8}{2t-4}$$
$$\left(\frac{6t+7}{4t-1}\right)(4t-1)(2t-4)=\left(\frac{3t+8}{2t-4}\right)(4t-1)(2t-4)$$
$$(6t+7)(2t-4)=(3t+8)(4t-1)$$
$$12t^2-24t+14t-28=12t^2-3t+32t-8$$
$$12t^2-10t-28=12t^2+29t-8$$
$$12t^2-10t-28-12t^2=12t^2+29t-8-12t^2$$
$$-10t-28=29t-8$$
$$-10t-28-29t=29t-8-29t$$
$$-28-39t=-8$$
$$-28-39t+28=-8+28$$
$$-39t=20$$
$$\frac{-39t}{-39}=\frac{20}{-39}$$
$$t=-\frac{20}{39}$$
Since $t=-\frac{20}{39}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{20}{39}\right\}$.

58.
$$\frac{8w+5}{10w-7}=\frac{4w-3}{5w+7}$$
$$\left(\frac{8w+5}{10w-7}\right)(10w-7)(5w+7)=\left(\frac{4w-3}{5w+7}\right)(10w-7)(5w+7)$$
$$(8w+5)(5w+7)=(4w-3)(10w-7)$$
$$40w^2+56w+25w+35=40w^2-28w-30w+21$$
$$40w^2+81w+35=40w^2-58w+21$$
$$40w^2+81w+35-40w^2=40w^2-58w+21-40w^2$$
$$81w+35=-58w+21$$
$$81w+35+58w=-58w+21+58w$$
$$139w+35=21$$
$$139w+35-35=21-35$$
$$139w=-14$$
$$\frac{139w}{139}=\frac{-14}{139}$$
$$w=-\frac{14}{139}$$
Since $w=-\frac{14}{139}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{14}{139}\right\}$.

59.
$$\frac{4}{x-2}=\frac{-3}{x+5}+\frac{7}{(x+5)(x-2)}$$
$$\left(\frac{4}{x-2}\right)(x+5)(x-2)=\left(\frac{-3}{x+5}+\frac{7}{(x+5)(x-2)}\right)(x+5)(x-2)$$
$$4(x+5)=-3(x-2)+7$$
$$4x+20=-3x+6+7$$
$$4x+20=-3x+13$$
$$4x+20+3x=-3x+13+3x$$
$$7x+20=13$$
$$7x+20-20=13-20$$
$$7x=-7$$
$$\frac{7x}{7}=\frac{-7}{7}$$
$$x=-1$$
Since $x=-1$ does not cause any denominator to equal zero, the solution set is $\{-1\}$.

60.

$$\frac{-4}{2x+3}+\frac{1}{x-1}=\frac{1}{(2x+3)(x-1)}$$

$$\left(\frac{-4}{2x+3}+\frac{1}{x-1}\right)(2x+3)(x-1)=\left(\frac{1}{(2x+3)(x-1)}\right)(2x+3)(x-1)$$

$$-4(x-1)+1(2x+3)=1$$

$$-4x+4+2x+3=1$$

$$-2x+7=1$$

$$-2x+7-7=1-7$$

$$-2x=-6$$

$$\frac{-2x}{-2}=\frac{-6}{-2}$$

$$x=3$$

Since $x=3$ does not cause any denominator to equal zero, the solution set is $\{3\}$.

61.

$$\frac{2}{y+3}+\frac{3}{y-4}=\frac{5}{y+6}$$

$$\left(\frac{2}{y+3}+\frac{3}{y-4}\right)(y+3)(y-4)(y+6)=\left(\frac{5}{y+6}\right)(y+3)(y-4)(y+6)$$

$$2(y-4)(y+6)+3(y+3)(y+6)=5(y+3)(y-4)$$

$$2\left(y^2+6y-4y-24\right)+3\left(y^2+6y+3y+18\right)=5\left(y^2-4y+3y-12\right)$$

$$2\left(y^2+2y-24\right)+3\left(y^2+9y+18\right)=5\left(y^2-y-12\right)$$

$$2y^2+4y-48+3y^2+27y+54=5y^2-5y-60$$

$$5y^2+31y+6=5y^2-5y-60$$

$$5y^2+31y+6-5y^2=5y^2-5y-60-5y^2$$

$$31y+6=-5y-60$$

$$31y+6+5y=-5y-60+5y$$

$$36y+6=-60$$

$$36y+6-6=-60-6$$

$$36y=-66$$

$$\frac{36y}{36}=\frac{-66}{36}$$

$$y=-\frac{11}{6}$$

Since $y=-\frac{11}{6}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{11}{6}\right\}$.

62.

$$\frac{5}{5z-11}+\frac{4}{2z-3}=\frac{-3}{5-z}$$
$$\left(\frac{5}{5z-11}+\frac{4}{2z-3}\right)(5z-11)(2z-3)(5-z)=\left(\frac{-3}{5-z}\right)(5z-11)(2z-3)(5-z)$$
$$5(2z-3)(5-z)+4(5z-11)(5-z)=-3(5z-11)(2z-3)$$
$$5\left(10z-2z^2-15+3z\right)+4\left(25z-5z^2-55+11z\right)=-3\left(10z^2-15z-22z+33\right)$$
$$5\left(-2z^2+13z-15\right)+4\left(-5z^2+36z-55\right)=-3\left(10z^2-37z+33\right)$$
$$-10z^2+65z-75-20z^2+144z-220=-30z^2+111z-99$$
$$-30z^2+209z-295=-30z^2+111z-99$$
$$-30z^2+209z-295+30z^2=-30z^2+111z-99+30z^2$$
$$209z-295=111z-99$$
$$209z-295-209z=111z-99-209z$$
$$-295=-98z-99$$
$$-295+99=-98z-99+99$$
$$-196=-98z$$
$$\frac{-196}{-98}=\frac{-118z}{-98}$$
$$2=z$$

Since $z=2$ does not cause any denominator to equal zero, the solution set is $\{2\}$.

63.

$$\frac{x}{x^2-1}-\frac{x+3}{x^2-x}=\frac{-3}{x^2+x}$$
$$\frac{x}{(x+1)(x-1)}-\frac{x+3}{x(x-1)}=\frac{-3}{x(x+1)}$$
$$\left(\frac{x}{(x+1)(x-1)}-\frac{x+3}{x(x-1)}\right)x(x+1)(x-1)=\left(\frac{-3}{x(x+1)}\right)x(x+1)(x-1)$$
$$(x)(x)-(x+3)(x+1)=-3(x-1)$$
$$x^2-\left(x^2+x+3x+3\right)=-3x+3$$
$$x^2-\left(x^2+4x+3\right)=-3x+3$$
$$x^2-x^2-4x-3=-3x+3$$
$$-4x-3=-3x+3$$
$$-4x-3+4x=-3x+3+4x$$
$$-3=3+x$$
$$-3-3=3+x-3$$
$$-6=x$$

Since $x=-6$ does not cause any denominator to equal zero, the solution set is $\{-6\}$.

64.

$$\frac{x+1}{x^2+2x}-\frac{x+4}{x^2+x}=\frac{-3}{x^2+3x+2}$$
$$\frac{x+1}{x(x+2)}-\frac{x+4}{x(x+1)}=\frac{-3}{(x+2)(x+1)}$$
$$\left(\frac{x+1}{x(x+2)}-\frac{x+4}{x(x+1)}\right)x(x+2)(x+1)=\left(\frac{-3}{(x+2)(x+1)}\right)x(x+2)(x+1)$$
$$(x+1)(x+1)-(x+4)(x+2)=-3x$$
$$\left(x^2+x+x+1\right)-\left(x^2+2x+4x+8\right)=-3x$$
$$x^2+2x+1-\left(x^2+6x+8\right)=-3x$$
$$x^2+2x+1-x^2-6x-8=-3x$$
$$2x+1-6x-8=-3x$$
$$-4x-7=-3x$$
$$-4x-7+4x=-3x+4x$$
$$-7=x$$

Since $x=-7$ does not cause any denominator to equal zero, the solution set is $\{-7\}$.

65.

$$3.2x+\frac{21.3}{65.871}=19.23$$
$$3.2x+\frac{21.3}{65.871}-\frac{21.3}{65.871}=19.23-\frac{21.3}{65.871}$$
$$3.2x=19.23-\frac{21.3}{65.871}$$
$$\left(\frac{1}{3.2}\right)(3.2x)=\left(19.23-\frac{21.3}{65.871}\right)\left(\frac{1}{3.2}\right)$$
$$x=\left(19.23-\frac{21.3}{65.871}\right)\left(\frac{1}{3.2}\right)\approx 5.91$$

The solution set is approximately $\{5.91\}$.

66.

$$6.2x-\frac{19.1}{83.72}=0.195$$
$$6.2x-\frac{19.1}{83.72}+\frac{19.1}{83.72}=0.195+\frac{19.1}{83.72}$$
$$6.2x=0.195+\frac{19.1}{83.72}$$
$$\left(\frac{1}{6.2}\right)(6.2x)=\left(0.195+\frac{19.1}{83.72}\right)\left(\frac{1}{6.2}\right)$$
$$x=\left(0.195+\frac{19.1}{83.72}\right)\left(\frac{1}{6.2}\right)\approx 0.07$$

The solution set is approximately $\{0.07\}$.

67.

$$14.72-21.58x=\frac{18}{2.11}x+2.4$$

$$14.72-21.58x-\frac{18}{2.11}x=\frac{18}{2.11}x+2.4-\frac{18}{2.11}x$$

$$14.72-21.58x-\frac{18}{2.11}x=2.4$$

$$14.72-21.58x-\frac{18}{2.11}x-14.72=2.4-14.72$$

$$-21.58x-\frac{18}{2.11}x=-12.32$$

$$\left(-21.58-\frac{18}{2.11}\right)x=-12.32$$

$$\left(\frac{1}{-21.58-\frac{18}{2.11}}\right)\left(-21.58-\frac{18}{2.11}\right)x=-12.32\left(\frac{1}{-21.58-\frac{18}{2.11}}\right)$$

$$x=-12.32\left(\frac{1}{-21.58-\frac{18}{2.11}}\right)\approx 0.41$$

The solution set is approximately $\{0.41\}$.

68.

$$18.63x-\frac{21.2}{2.6}=\frac{14}{2.32}x-20$$

$$18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x=\frac{14}{2.32}x-20-\frac{14}{2.32}x$$

$$18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x=-20$$

$$18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x+\frac{21.2}{2.6}=-20+\frac{21.2}{2.6}$$

$$18.63x-\frac{14}{2.32}x=-20+\frac{21.2}{2.6}$$

$$\left(18.63-\frac{14}{2.32}\right)x=-20+\frac{21.2}{2.6}$$

$$\left(\frac{1}{18.63-\frac{14}{2.32}}\right)\left(18.63-\frac{14}{2.32}\right)x=\left(-20+\frac{21.2}{2.6}\right)\left(\frac{1}{18.63-\frac{14}{2.32}}\right)$$

$$x=\left(-20+\frac{21.2}{2.6}\right)\left(\frac{1}{18.63-\frac{14}{2.32}}\right)\approx -0.94$$

The solution set is approximately $\{-0.94\}$.

69.
$$\begin{aligned} ax-b&=c, \quad a\neq 0\\ ax-b+b&=c+b\\ ax&=b+c\\ \frac{ax}{a}&=\frac{b+c}{a}\\ x&=\frac{b+c}{a} \end{aligned}$$

70.
$$\begin{aligned} 1-ax&=b, \;\; a\neq 0\\ 1-ax-1&=b-1\\ -ax&=b-1\\ \frac{-ax}{-a}&=\frac{b-1}{-a}\\ x&=\frac{b-1}{-a}=\frac{1-b}{a} \end{aligned}$$

71.
$$\begin{aligned} \frac{x}{a}+\frac{x}{b}&=c, \quad a\neq 0, b\neq 0, a\neq -b\\ ab\left(\frac{x}{a}+\frac{x}{b}\right)&=ab\cdot c\\ bx+ax&=abc\\ (a+b)x&=abc\\ \frac{(a+b)x}{a+b}&=\frac{abc}{a+b}\\ x&=\frac{abc}{a+b} \end{aligned}$$

72.
$$\begin{aligned} \frac{a}{x}+\frac{b}{x}&=c, \quad c\neq 0\\ x\left(\frac{a}{x}+\frac{b}{x}\right)&=x\cdot c\\ a+b&=cx\\ \frac{a+b}{c}&=\frac{cx}{c}\\ x&=\frac{a+b}{c} \end{aligned}$$

73.
$$\begin{aligned} \frac{1}{x-a}+\frac{1}{x+a}&=\frac{2}{x-1}\\ \left(\frac{1}{x-a}+\frac{1}{x+a}\right)(x-a)(x+a)(x-1)&=\left(\frac{2}{x-1}\right)(x-a)(x+a)(x-1)\\ (x+a)(x-1)+(x-a)(x-1)&=2(x-a)(x+a)\\ x^2-x+ax-a+x^2-x-ax+a&=2\left(x^2+ax-ax-a^2\right)\\ 2x^2-2x&=2\left(x^2-a^2\right)\\ 2x^2-2x&=2x^2-2a^2\\ -2x&=-2a^2\\ \frac{-2x}{-2}&=\frac{-2a^2}{-2}\\ x&=a^2 \end{aligned}$$

such that $x\neq \pm a, x\neq 1$

74.
$$\frac{b+c}{x+a}=\frac{b-c}{x-a}, c\neq 0, a\neq 0$$
$$\left(\frac{b+c}{x+a}\right)(x+a)(x-a)=\left(\frac{b-c}{x-a}\right)(x+a)(x-a)$$
$$(b+c)(x-a)=(b-c)(x+a)$$
$$bx-ba+cx-ca=bx+ba-cx-ca$$
$$-ba+cx-ca=ba-cx-ca$$
$$-ba+cx=ba-cx$$
$$-ba+cx+ba=ba-cx+ba$$
$$2cx=2ba$$
$$\frac{2cx}{2c}=\frac{2ba}{2c}$$
$$x=\frac{ba}{c}$$
such that $x\neq \pm a$

75. $x+2a=16+ax-6a$, if $x=4$
$$4+2a=16+a(4)-6a$$
$$4+2a=16+4a-6a$$
$$4+2a=16-2a$$
$$4a=12$$
$$\frac{4a}{4}=\frac{12}{4}$$
$$a=3$$

76. $x+2b=x-4+2bx$, for $x=2$
$$2+2b=2-4+2b(2)$$
$$2+2b=2-4+4b$$
$$2+2b=-2+4b$$
$$4=2b$$
$$\frac{4}{2}=b$$
$$b=2$$

77.
$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$
$$RR_1R_2\left(\frac{1}{R}\right)=RR_1R_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$R_1R_2=RR_2+RR_1$$
$$R_1R_2=R(R_2+R_1)$$
$$\frac{R_1R_2}{R_2+R_1}=\frac{R(R_2+R_1)}{R_2+R_1}$$
$$\frac{R_1R_2}{R_2+R_1}=R$$

78.
$$A=P(1+rt)$$
$$A=P+Prt$$
$$A-P=Prt$$
$$\frac{A-P}{Pt}=\frac{Prt}{Pt}$$
$$\frac{A-P}{Pt}=r$$

79.
$$F=\frac{mv^2}{R}$$
$$RF=R\left(\frac{mv^2}{R}\right)$$
$$RF=mv^2$$
$$\frac{RF}{F}=\frac{mv^2}{F}$$
$$R=\frac{mv^2}{F}$$

80.
$$PV=nRT$$
$$\frac{PV}{nR}=\frac{nRT}{nR}$$
$$\frac{PV}{nR}=T$$

81.
$$S=\frac{a}{1-r}$$
$$S(1-r)=\left(\frac{a}{1-r}\right)(1-r)$$
$$S-Sr=a$$
$$S-Sr-S=a-S$$
$$-Sr=a-S$$
$$\frac{-Sr}{-S}=\frac{a-S}{-S}$$
$$r=\frac{S-a}{S}$$

82.
$$v=-gt+v_0$$
$$v-v_0=-gt$$
$$\frac{v-v_0}{-g}=\frac{-gt}{-g}$$
$$t=\frac{v-v_0}{-g}=\frac{v_0-v}{g}$$

83.

Amount in bonds	Amount in CDs	Total
x	$x-3000$	$20,000$

$$x+(x-3000)=20,000$$
$$2x-3000=20,000$$
$$2x=23,000$$
$$x=11,500$$

\$11,500 will be invested in bonds and \$8500 will be invested in CD's.

84.

Sean's Amount	George's Amount	Total
x	$x-3000$	$10,000$

$$x+(x-3000)=10,000$$
$$2x-3000=10,000$$
$$2x=13,000$$
$$x=6500$$

Sean will receive \$6500 and George will receive \$3500.

85.

Yahoo! searches	Google searches	Total
x	$x+0.53$	3.57

$$x+(x+0.53)=3.57$$
$$2x+0.53=3.57$$
$$2x=3.04$$
$$x=1.52$$

Yahoo! was used for 1.52 billion searches and Google was used for 2.05 billion searches.

86.

Judy's Amount	Tom's Amount	Total
x	$\frac{2}{3}x$	18

$$x+\frac{2}{3}x=18$$
$$\frac{5}{3}x=18$$
$$x=\frac{3}{5}(18)$$
$$x=10.80$$

Judy pays \$10.80 and Tom pays \$7.20.

87.

	Dollars per hour	Hours worked	Money earned
Regular wage	x	40	$40x$
Overtime wage	$1.5x$	8	$8(1.5x)$

$$40x+8(1.5x)=442$$
$$40x+12x=442$$
$$52x=442$$
$$x=\frac{442}{52}=8.50$$

Sandra's regular hourly wage is \$8.50.

88.

	Dollars per hour	Hours worked	Money earned
Regular wage	x	40	$40x$
Overtime wage	$1.5x$	6	$6(1.5x)$
Sunday wage	$2x$	4	$4(2x)$

$$40x+6(1.5x)+4(2x)=342$$
$$40x+9x+8x=342$$
$$57x=342$$
$$x=\frac{342}{57}=6$$

Leigh's regular hourly wage is \$6.00.

89. Let x represent the score on the final exam.

$$\frac{80+83+71+61+95+x+x}{7}=80$$
$$\frac{390+2x}{7}=80$$
$$390+2x=560$$
$$2x=170$$
$$x=85$$

Brooke needs a score of 85 on the final exam.

90. Let x represent the score on the final exam. Note: since the final exam counts for two-thirds of the overall grade, the average of the four test scores count for one-third of the overall grade. For a B, the average score must be 80.

$$\frac{1}{3}\left(\frac{86+80+84+90}{4}\right)+\frac{2}{3}x=80$$
$$\frac{1}{3}\left(\frac{340}{4}\right)+\frac{2}{3}x=80$$
$$\frac{85}{3}+\frac{2}{3}x=80$$
$$3\left(\frac{85}{3}+\frac{2}{3}x\right)=3(80)$$
$$85+2x=240$$
$$2x=155$$
$$x=77.5$$

Mike needs a score of 78 to earn a B.

For an A, the average score must be 90.

$$\frac{1}{3}\left(\frac{86+80+84+90}{4}\right)+\frac{2}{3}x=90$$
$$\frac{1}{3}\left(\frac{340}{4}\right)+\frac{2}{3}x=90$$
$$\frac{85}{3}+\frac{2}{3}x=90$$
$$3\left(\frac{85}{3}+\frac{2}{3}x\right)=3(90)$$
$$85+2x=270$$
$$2x=185$$
$$x=92.5$$

Mike needs a score of 93 to earn an A.

91. Let x represent the original price of the house. Then $0.15x$ represents the reduction in the price of the house. The new price of the home is $425,000.

original price − reduction = new price

$$x-0.15x=425{,}000$$
$$0.85x=425{,}000$$
$$x=500{,}000$$

The original price of the house was $500,000. The amount of the reduction (i.e., the savings) is 0.15($500,000) = $75,000.

92. Let x represent the original price of the car. Then $0.15x$ represents the reduction in the price of the car. The new price of the car is $8000.

list price − reduction = new price

$$x-0.15x=8000$$
$$0.85x=8000$$
$$x\approx 9411.76$$

The list price of the car was $9411.76. The amount of the reduction (i.e., the savings) is 0.15($9411.76) ≈ $1411.76 .

93. Let x represent the price the bookstore pays for the book. Then $0.35x$ represents the markup on the book. The selling price of the book is $92.00.

publisher price + markup = selling price

$$x+0.35x=92.00$$
$$1.35x=92.00$$
$$x\approx 68.15$$

The bookstore paid $68.15 for the book.

94. Let x represent selling price for the new car. The dealer's cost is 0.85($18,000) = $15,300. The markup is $100.

selling price = dealer's cost + markup

$x=15{,}300+100=\$15{,}400$

At $100 over the dealer's cost, the price of the care is $15,400.

95.

	Tickets sold	Price per ticket	Money earned
Adults	x	7.50	$7.50x$
Children	$5200-x$	4.50	$4.50(5200-x)$

$$7.50x+4.50(5200-x)=29{,}961$$
$$7.50x+23{,}400-4.50x=29{,}961$$
$$3.00x+23{,}400=29{,}961$$
$$3.00x=6561$$
$$x=2187$$

There were 2187 adult patrons.

96. Let p represent the original price for the suit. Then, $0.30p$ represents the discounted amount.

original price − discount = clearance price

$$p-0.30p=399$$
$$0.70p=399$$
$$p=570$$

The suit originally cost $570.

97. Let w represent the width of the rectangle.
Then $w+8$ is the length.
Perimeter is given by the formula $P = 2l + 2w$.

$$2(w+8)+2w=60$$
$$2w+16+2w=60$$
$$4w+16=60$$
$$4w=44$$
$$w=11$$

Now, 11 + 8 = 19.
The width of the rectangle is 11 feet and the length is 19 feet.

98. Let w represent the width of the rectangle.
Then $2w$ is the length.
Perimeter is given by the formula $P = 2l + 2w$.

$$2(2w)+2w=42$$
$$4w+2w=42$$
$$6w=42$$
$$w=7$$

Now, 2(7) = 14.
The width of the rectangle is 7 meters and the length is 14 meters.

99. Let x represent the number of worldwide Internet users in March 2006.
Then $0.219x$ represents the number U.S. Internet users, which equals 152 million

$$0.219x=152$$
$$x \approx 694.06$$

In March 2006, there were about 694.06 million Internet users worldwide.

100. To move from step (6) to step (7), we divided both sides of the equation by the expression $x-2$. From step (1), however, we know $x = 2$, so this means we divided both sides of the equation by zero.

101 – 102. Answers will vary.

Section 1.2

1. $x^2-5x-6=(x-6)(x+1)$

2. $2x^2-x-3=(2x-3)(x+1)$

3. $\left\{-\frac{5}{3}, 3\right\}$

4. True

5. add; $\left(\frac{5}{2}\right)^2=\frac{25}{4}$

6. discriminant; negative

7. False; a quadratic equation may have no real solutions.

8. False; if the discriminant is positive, the equation has two distinct real solutions.

9.
$$x^2-9x=0$$
$$x(x-9)=0$$
$$x=0 \text{ or } x-9=0$$
$$x=0 \text{ or } x=9$$
The solution set is {0, 9}.

10.
$$x^2+4x=0$$
$$x(x+4)=0$$
$$x=0 \text{ or } x+4=0$$
$$x=0 \text{ or } x=-4$$
The solution set is {–4, 0}.

11.
$$x^2-25=0$$
$$(x+5)(x-5)=0$$
$$x+5=0 \text{ or } x-5=0$$
$$x=-5 \text{ or } x=5$$
The solution set is {–5, 5}.

12.
$$x^2-9=0$$
$$(x+3)(x-3)=0$$
$$x+3=0 \text{ or } x-3=0$$
$$x=-3 \text{ or } x=3$$
The solution set is {–3, 3}.

13.
$$z^2+z-6=0$$
$$(z+3)(z-2)=0$$
$$z+3=0 \text{ or } z-2=0$$
$$z=-3 \text{ or } z=2$$
The solution set is {–3, 2}.

14.
$$v^2+7v+6=0$$
$$(v+6)(v+1)=0$$
$$v+6=0 \text{ or } v+1=0$$
$$v=-6 \text{ or } v=-1$$
The solution set is {–6, –1}

15.
$$2x^2-5x-3=0$$
$$(2x+1)(x-3)=0$$
$$2x+1=0 \quad \text{or} \quad x-3=0$$
$$x=-\frac{1}{2} \quad \text{or} \quad x=3$$
The solution set is $\left\{-\frac{1}{2}, 3\right\}$

16.
$$3x^2+5x+2=0$$
$$(3x+2)(x+1)=0$$
$$3x+2=0 \quad \text{or} \quad x+1=0$$
$$x=-\frac{2}{3} \quad \text{or} \quad x=-1$$
The solution set is $\left\{-1,-\frac{2}{3}\right\}$.

17.
$$3t^2-48=0$$
$$3(t^2-16)=0$$
$$3(t+4)(t-4)=0$$
$$t+4=0 \quad \text{or} \quad t-4=0$$
$$t=-4 \quad \text{or} \quad t=4$$
The solution set is {–4, 4}.

18.
$$2y^2-50=0$$
$$2(y^2-25)=0$$
$$2(y+5)(y-5)=0$$
$$y+5=0 \quad \text{or} \quad y-5=0$$
$$y=-5 \quad \text{or} \quad y=5$$
The solution set is {–5, 5}.

19.
$$x(x-8)+12=0$$
$$x^2-8x+12=0$$
$$(x-6)(x-2)=0$$
$$x-6=0 \quad \text{or} \quad x-2=0$$
$$x=6 \quad \text{or} \quad x=2$$
The solution set is $\{2, 6\}$.

20.
$$x(x+4)=12$$
$$x^2+4x-12=0$$
$$(x+6)(x-2)=0$$
$$x+6=0 \quad \text{or} \quad x-2=0$$
$$x=-6 \quad \text{or} \quad x=2$$
The solution set is $\{-6,\ 2\}$.

21.
$$4x^2+9=12x$$
$$4x^2-12x+9=0$$
$$(2x-3)^2=0$$
$$2x-3=0$$
$$x=\frac{3}{2}$$
The solution set is $\left\{\frac{3}{2}\right\}$.

22.
$$25x^2+16=40x$$
$$25x^2-40x+16=0$$
$$(5x-4)^2=0$$
$$5x-4=0$$
$$x=\frac{4}{5}$$
The solution set is $\left\{\frac{4}{5}\right\}$.

23.
$$6(p^2-1)=5p$$
$$6p^2-6=5p$$
$$6p^2-5p-6=0$$
$$(3p+2)(2p-3)=0$$
$$3p+2=0 \quad \text{or} \quad 2p-3=0$$
$$p=-\frac{2}{3} \quad \text{or} \quad p=\frac{3}{2}$$
The solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

24.
$$2(2u^2-4u)+3=0$$
$$4u^2-8u+3=0$$
$$(2u-1)(2u-3)=0$$
$$2u-1=0 \quad \text{or} \quad 2u-3=0$$
$$u=\frac{1}{2} \quad \text{or} \quad u=\frac{3}{2}$$
The solution set is $\left\{\frac{1}{2}, \frac{3}{2}\right\}$.

25.
$$6x-5=\frac{6}{x}$$
$$(6x-5)x=\left(\frac{6}{x}\right)x$$
$$6x^2-5x=6$$
$$6x^2-5x-6=0$$
$$(3x+2)(2x-3)=0$$
$$3x+2=0 \quad \text{or} \quad 2x-3=0$$
$$x=-\frac{2}{3} \quad \text{or} \quad x=\frac{3}{2}$$
Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

26.
$$x+\frac{12}{x}=7$$
$$\left(x+\frac{12}{x}\right)x=7x$$
$$x^2+12=7x$$
$$x^2-7x+12=0$$
$$(x-3)(x-4)=0$$
$$x-3=0 \quad \text{or} \quad x-4=0$$
$$x=3 \quad \text{or} \quad x=4$$
Neither of these values causes a denominator to equal zero, so the solution set is {3, 4}.

27.
$$\frac{4(x-2)}{x-3}+\frac{3}{x}=\frac{-3}{x(x-3)}$$
$$\left(\frac{4(x-2)}{x-3}+\frac{3}{x}\right)x(x-3)=\left(\frac{-3}{x(x-3)}\right)x(x-3)$$
$$4x(x-2)+3(x-3)=-3$$
$$4x^2-8x+3x-9=-3$$
$$4x^2-5x-6=0$$
$$(4x+3)(x-2)=0$$
$$4x+3=0 \quad \text{or} \quad x-2=0$$
$$x=-\frac{3}{4} \quad \text{or} \quad x=2$$
Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{3}{4}, 2\right\}$.

28.
$$\frac{5}{x+4}=4+\frac{3}{x-2}$$
$$\left(\frac{5}{x+4}\right)(x+4)(x-2)=\left(4+\frac{3}{x-2}\right)(x+4)(x-2)$$
$$5(x-2)=4(x+4)(x-2)+3(x+4)$$
$$5x-10=4\left(x^2+2x-8\right)+3x+12$$
$$5x-10=4x^2+8x-32+3x+12$$
$$0=4x^2+6x-10$$
$$0=2\left(2x^2+3x-5\right)$$
$$0=2(2x+5)(x-1)$$
$$2x+5=0 \quad \text{or} \quad x-1=0$$
$$x=-\frac{5}{2} \quad \text{or} \quad x=1$$
Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{5}{2}, 1\right\}$.

29.
$$x^2=25$$
$$x=\pm\sqrt{25}$$
$$x=\pm 5$$
The solution set is $\{-5, 5\}$.

30.
$$x^2=36$$
$$x=\pm\sqrt{36}$$
$$x=\pm 6$$
The solution set is $\{-6, 6\}$.

31.
$$(x-1)^2=4$$
$$x-1=\pm\sqrt{4}$$
$$x-1=\pm 2$$
$$x-1=2 \quad \text{or} \quad x-1=-2$$
$$x=3 \quad \text{or} \quad x=-1$$
The solution set is $\{-1, 3\}$.

32.
$$(x+2)^2=1$$
$$x+2=\pm\sqrt{1}$$
$$x+2=\pm 1$$
$$x+2=1 \quad \text{or} \quad x+2=-1$$
$$x=-1 \quad \text{or} \quad x=-3$$
The solution set is $\{-3, -1\}$.

33. $(2x+3)^2 = 9$

$2x+3 = \pm\sqrt{9}$

$2x+3 = \pm 3$

$2x+3 = 3$ or $2x+3 = -3$

$2x = 0$ or $2x = -6$

$x = 0$ or $x = -3$

The solution set is $\{-3, 0\}$.

34. $(3x-2)^2 = 4$

$3x-2 = \pm\sqrt{4}$

$3x-2 = \pm 2$

$3x-2 = 2$ or $3x-2 = -2$

$3x = 4$ or $3x = 0$

$x = \frac{4}{3}$ or $x = 0$

The solution set is $\left\{0, \frac{4}{3}\right\}$.

35. $\left(\frac{1}{2}\cdot(-8)\right)^2 = (-4)^2 = 16$

36. $\left(\frac{1}{2}\cdot(-4)\right)^2 = (-2)^2 = 4$

37. $\left(\frac{1}{2}\cdot\frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

38. $\left(\frac{1}{2}\cdot\left(-\frac{1}{3}\right)\right)^2 = \left(-\frac{1}{6}\right)^2 = \frac{1}{36}$

39. $\left(\frac{1}{2}\cdot\left(-\frac{2}{3}\right)\right)^2 = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}$

40. $\left(\frac{1}{2}\cdot\left(-\frac{2}{5}\right)\right)^2 = \left(-\frac{1}{5}\right)^2 = \frac{1}{25}$

41. $x^2+4x = 21$

$x^2+4x+4 = 21+4$

$(x+2)^2 = 25$

$x+2 = \pm\sqrt{25}$

$x+2 = \pm 5$

$x = -2\pm 5$

$x = 3$ or $x = -7$

The solution set is $\{-7, 3\}$.

42. $x^2-6x = 13$

$x^2-6x+9 = 13+9$

$(x-3)^2 = 22$

$x-3 = \pm\sqrt{22}$

$x = 3\pm\sqrt{22}$

The solution set is $\{3-\sqrt{22}, 3+\sqrt{22}\}$.

43. $x^2-\frac{1}{2}x-\frac{3}{16} = 0$

$x^2-\frac{1}{2}x = \frac{3}{16}$

$x^2-\frac{1}{2}x+\frac{1}{16} = \frac{3}{16}+\frac{1}{16}$

$\left(x-\frac{1}{4}\right)^2 = \frac{1}{4}$

$x-\frac{1}{4} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$x = \frac{1}{4}\pm\frac{1}{2}$

$x = \frac{3}{4}$ or $x = -\frac{1}{4}$

The solution set is $\left\{-\frac{1}{4}, \frac{3}{4}\right\}$.

44. $x^2+\frac{2}{3}x-\frac{1}{3} = 0$

$x^2+\frac{2}{3}x = \frac{1}{3}$

$x^2+\frac{2}{3}x+\frac{1}{9} = \frac{1}{3}+\frac{1}{9}$

$\left(x+\frac{1}{3}\right)^2 = \frac{4}{9}$

$x+\frac{1}{3} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$x = -\frac{1}{3}\pm\frac{2}{3}$

$x = \frac{1}{3}$ or $x = -1$

The solution set is $\left\{-1, \frac{1}{3}\right\}$.

45. $3x^2 + x - \frac{1}{2} = 0$

$$x^2 + \frac{1}{3}x - \frac{1}{6} = 0$$

$$x^2 + \frac{1}{3}x = \frac{1}{6}$$

$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{1}{6} + \frac{1}{36}$$

$$\left(x + \frac{1}{6}\right)^2 = \frac{7}{36}$$

$$x + \frac{1}{6} = \pm\sqrt{\frac{7}{36}}$$

$$x + \frac{1}{6} = \pm\frac{\sqrt{7}}{6}$$

$$x = \frac{-1 \pm \sqrt{7}}{6}$$

The solution set is $\left\{\frac{-1-\sqrt{7}}{6}, \frac{-1+\sqrt{7}}{6}\right\}$.

46. $2x^2 - 3x - 1 = 0$

$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$

$$x^2 - \frac{3}{2}x = \frac{1}{2}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm\sqrt{\frac{17}{16}}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{3-\sqrt{17}}{4}, \frac{3+\sqrt{17}}{4}\right\}$.

47. $x^2 - 4x + 2 = 0$

$a = 1, \quad b = -4, \quad c = 2$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{16-8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

The solution set is $\left\{2 - \sqrt{2}, 2 + \sqrt{2}\right\}$.

48. $x^2 + 4x + 2 = 0$

$a = 1, \quad b = 4, \quad c = 2$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{16-8}}{2}$$

$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

The solution set is $\left\{-2 - \sqrt{2}, -2 + \sqrt{2}\right\}$.

49. $x^2 - 4x - 1 = 0$

$a = 1, \quad b = -4, \quad c = -1$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{16+4}}{2}$$

$$= \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

The solution set is $\left\{2 - \sqrt{5}, 2 + \sqrt{5}\right\}$.

50. $x^2 + 6x + 1 = 0$

$a = 1, \quad b = 6, \quad c = 1$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)} = \frac{-6 \pm \sqrt{36-4}}{2}$$

$$= \frac{-6 \pm \sqrt{32}}{2} = \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$$

The solution set is $\left\{-3 - 2\sqrt{2}, -3 + 2\sqrt{2}\right\}$.

51. $2x^2 - 5x + 3 = 0$

$a = 2, \quad b = -5, \quad c = 3$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25-24}}{4} = \frac{5 \pm \sqrt{1}}{4} = \frac{5 \pm 1}{4}$$

$$x = \frac{5+1}{4} \quad \text{or} \quad x = \frac{5-1}{4}$$

$$x = \frac{6}{4} \quad \text{or} \quad x = \frac{4}{4}$$

$$x = \frac{3}{2} \quad \text{or} \quad x = 1$$

The solution set is $\left\{1, \frac{3}{2}\right\}$.

52. $2x^2+5x+3=0$

$a=2,\quad b=5,\quad c=3$

$$x=\frac{-5\pm\sqrt{5^2-4(2)(3)}}{2(2)}=\frac{-5\pm\sqrt{25-24}}{4}=\frac{-5\pm\sqrt{1}}{4}=\frac{-5\pm1}{4}$$

$x=\frac{-5+1}{4}$ or $x=\frac{-5-1}{4}$

$x=\frac{-4}{4}$ or $x=\frac{-6}{4}$

$x=-1$ or $x=-\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2},-1\right\}$.

53. $4y^2-y+2=0$

$a=4,\quad b=-1,\quad c=2$

$$y=\frac{-(-1)\pm\sqrt{(-1)^2-4(4)(2)}}{2(4)}=\frac{1\pm\sqrt{1-32}}{8}=\frac{1\pm\sqrt{-31}}{8}$$

No real solution.

54. $4t^2+t+1=0$

$a=4,\quad b=1,\quad c=1$

$$t=\frac{-1\pm\sqrt{1^2-4(4)(1)}}{2(4)}=\frac{-1\pm\sqrt{1-16}}{8}=\frac{-1\pm\sqrt{-15}}{8}$$

No real solution.

55. $4x^2=1-2x$

$4x^2+2x-1=0$

$a=4,\quad b=2,\quad c=-1$

$$x=\frac{-2\pm\sqrt{2^2-4(4)(-1)}}{2(4)}=\frac{-2\pm\sqrt{4+16}}{8}=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm2\sqrt{5}}{8}=\frac{-1\pm\sqrt{5}}{4}$$

The solution set is $\left\{\frac{-1-\sqrt{5}}{4},\frac{-1+\sqrt{5}}{4}\right\}$.

56. $2x^2=1-2x$

$2x^2+2x-1=0$

$a=2,\quad b=2,\quad c=-1$

$$x=\frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}=\frac{-2\pm\sqrt{4+8}}{4}=\frac{-2\pm\sqrt{12}}{4}=\frac{-2\pm2\sqrt{3}}{4}=\frac{-1\pm\sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{3}}{2},\frac{-1+\sqrt{3}}{2}\right\}$.

57. $4x^2=9x$

$4x^2-9x=0$

$x(4x-9)=0$

$x=0$ or $4x-9=0$

$x=0$ or $x=\frac{9}{4}$

The solution set is $\left\{0,\frac{9}{4}\right\}$.

58. $5x=4x^2$

$0=4x^2-5x$

$0=x(4x-5)$

$x=0$ or $4x-5=0$

$x=0$ or $x=\frac{5}{4}$

The solution set is $\left\{0,\frac{5}{4}\right\}$.

59. $9t^2-6t+1=0$

$a=9,\quad b=-6,\quad c=1$

$$t=\frac{-(-6)\pm\sqrt{(-6)^2-4(9)(1)}}{2(9)}=\frac{6\pm\sqrt{36-36}}{18}=\frac{6\pm0}{18}=\frac{1}{3}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

60. $4u^2-6u+9=0$

$a=4,\quad b=-6,\quad c=9$

$$u=\frac{-(-6)\pm\sqrt{(-6)^2-4(4)(9)}}{2(4)}=\frac{6\pm\sqrt{36-144}}{8}=\frac{6\pm\sqrt{-108}}{8}$$

No real solution.

61. $\frac{3}{4}x^2-\frac{1}{4}x-\frac{1}{2}=0$

$4\left(\frac{3}{4}x^2-\frac{1}{4}x-\frac{1}{2}\right)=4(0)$

$3x^2-x-2=0$

$a=3,\quad b=-1,\quad c=-2$

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-2)}}{2(3)}$

$=\frac{1\pm\sqrt{1+24}}{6}=\frac{1\pm\sqrt{25}}{6}=\frac{1\pm5}{6}$

$x=\frac{1+5}{6}$ or $x=\frac{1-5}{6}$

$x=\frac{6}{6}$ or $x=\frac{-4}{6}$

$x=1$ or $x=-\frac{2}{3}$

The solution set is $\left\{-\frac{2}{3},1\right\}$.

62. $\frac{2}{3}x^2-x-3=0$

$3\left(\frac{2}{3}x^2-x-3\right)=3(0)$

$2x^2-3x-9=0$

$a=2,\quad b=-3,\quad c=-9$

$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-9)}}{2(2)}$

$=\frac{3\pm\sqrt{9+72}}{4}=\frac{3\pm\sqrt{81}}{4}=\frac{3\pm9}{4}$

$x=\frac{3+9}{4}$ or $x=\frac{3-9}{4}$

$x=\frac{12}{4}$ or $x=\frac{-6}{4}$

$x=3$ or $x=-\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2},3\right\}$.

63. $\frac{5}{3}x^2-x=\frac{1}{3}$

$3\left(\frac{5}{3}x^2-x\right)=3\left(\frac{1}{3}\right)$

$5x^2-3x=1$

$5x^2-3x-1=0$

$a=5,\quad b=-3,\quad c=-1$

$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(5)(-1)}}{2(5)}$

$=\frac{3\pm\sqrt{9+20}}{10}=\frac{3\pm\sqrt{29}}{10}$

The solution set is $\left\{\frac{3-\sqrt{29}}{10},\frac{3+\sqrt{29}}{10}\right\}$.

64. $\frac{3}{5}x^2-x=\frac{1}{5}$

$5\left(\frac{3}{5}x^2-x\right)=5\left(\frac{1}{5}\right)$

$3x^2-5x=1$

$3x^2-5x-1=0$

$a=3,\quad b=-5,\quad c=-1$

$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-1)}}{2(3)}$

$=\frac{5\pm\sqrt{25+12}}{6}=\frac{5\pm\sqrt{37}}{6}$

The solution set is $\left\{\frac{5-\sqrt{37}}{6},\frac{5+\sqrt{37}}{6}\right\}$.

65. $2x(x+2)=3$

$2x^2+4x-3=0$

$a=2,\quad b=4,\quad c=-3$

$x=\frac{-4\pm\sqrt{4^2-4(2)(-3)}}{2(2)}=\frac{-4\pm\sqrt{16+24}}{4}$

$=\frac{-4\pm\sqrt{40}}{4}=\frac{-4\pm2\sqrt{10}}{4}=\frac{-2\pm\sqrt{10}}{2}$

The solution set is $\left\{\frac{-2-\sqrt{10}}{2},\frac{-2+\sqrt{10}}{2}\right\}$.

66. $3x(x+2)=1$

$3x^2+6x-1=0$

$a=3,\quad b=6,\quad c=-1$

$$x=\frac{-6\pm\sqrt{6^2-4(3)(-1)}}{2(3)}=\frac{-6\pm\sqrt{36+12}}{6}$$
$$=\frac{-6\pm\sqrt{48}}{6}=\frac{-6\pm4\sqrt{3}}{6}=\frac{-3\pm2\sqrt{3}}{3}$$

The solution set is $\left\{\frac{-3-2\sqrt{3}}{3},\frac{-3+2\sqrt{3}}{3}\right\}$.

67. $4-\frac{1}{x}-\frac{2}{x^2}=0$

$$x^2\left(4-\frac{1}{x}-\frac{2}{x^2}\right)=x^2(0)$$
$$4x^2-x-2=0$$

$a=4,\quad b=-1,\quad c=-2$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(4)(-2)}}{2(4)}$$
$$=\frac{1\pm\sqrt{1+32}}{8}=\frac{1\pm\sqrt{33}}{8}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{\frac{1-\sqrt{33}}{8},\frac{1+\sqrt{33}}{8}\right\}$.

68. $4+\frac{1}{x}-\frac{1}{x^2}=0$

$$x^2\left(4+\frac{1}{x}-\frac{1}{x^2}\right)=x^2(0)$$
$$4x^2+x-1=0$$

$a=4,\quad b=1,\quad c=-1$

$$x=\frac{-1\pm\sqrt{1^2-4(4)(-1)}}{2(4)}$$
$$=\frac{-1\pm\sqrt{1+16}}{8}=\frac{-1\pm\sqrt{17}}{8}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{\frac{-1-\sqrt{17}}{8},\frac{-1+\sqrt{17}}{8}\right\}$.

69. $\frac{3x}{x-2}+\frac{1}{x}=4$

$$\left(\frac{3x}{x-2}+\frac{1}{x}\right)x(x-2)=4x(x-2)$$
$$3x(x)+(x-2)=4x^2-8x$$
$$3x^2+x-2=4x^2-8x$$
$$0=x^2-9x+2$$

$a=1,\quad b=-9,\quad c=2$

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(1)(2)}}{2(1)}$$
$$=\frac{9\pm\sqrt{81-8}}{2}=\frac{9\pm\sqrt{73}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{\frac{9-\sqrt{73}}{2},\frac{9+\sqrt{73}}{2}\right\}$.

70. $\frac{2x}{x-3}+\frac{1}{x}=4$

$$\left(\frac{2x}{x-3}+\frac{1}{x}\right)x(x-3)=4x(x-3)$$
$$2x(x)+(x-3)=4x^2-12x$$
$$2x^2+x-3=4x^2-12x$$
$$0=2x^2-13x+3$$

$a=2,\quad b=-13,\quad c=3$

$$x=\frac{-(-13)\pm\sqrt{(-13)^2-4(2)(3)}}{2(2)}$$
$$=\frac{13\pm\sqrt{169-24}}{4}=\frac{13\pm\sqrt{145}}{4}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{\frac{13-\sqrt{145}}{4},\frac{13+\sqrt{145}}{4}\right\}$.

71. $x^2-4.1x+2.2=0$

$a=1,\quad b=-4.1,\quad c=2.2$

$$x=\frac{-(-4.1)\pm\sqrt{(-4.1)^2-4(1)(2.2)}}{2(1)}$$
$$=\frac{4.1\pm\sqrt{16.81-8.8}}{2}=\frac{4.1\pm\sqrt{8.01}}{2}$$

$x\approx3.47$ or $x\approx0.63$

The solution set is $\{0.63,\ 3.47\}$.

72. $x^2 + 3.9x + 1.8 = 0$

$a = 1, \quad b = 3.9, \quad c = 1.8$

$$x = \frac{-3.9 \pm \sqrt{(3.9)^2 - 4(1)(1.8)}}{2(1)}$$

$$= \frac{-3.9 \pm \sqrt{15.21 - 7.2}}{2} = \frac{-3.9 \pm \sqrt{8.01}}{2}$$

$x \approx -0.53$ or $x \approx -3.37$

The solution set is $\{-3.37, -0.53\}$.

73. $x^2 + \sqrt{3}x - 3 = 0$

$a = 1, \quad b = \sqrt{3}, \quad c = -3$

$$x = \frac{-\sqrt{3} \pm \sqrt{\left(\sqrt{3}\right)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-\sqrt{3} \pm \sqrt{3 + 12}}{2} = \frac{-\sqrt{3} \pm \sqrt{15}}{2}$$

$x \approx 1.07$ or $x \approx -2.80$

The solution set is $\{-2.80, 1.07\}$.

74. $x^2 + \sqrt{2}x - 2 = 0$

$a = 1, \quad b = \sqrt{2}, \quad c = -2$

$$x = \frac{-\sqrt{2} \pm \sqrt{\left(\sqrt{2}\right)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2 + 8}}{2} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}$$

$x \approx 0.87$ or $x \approx -2.29$

The solution set is $\{-2.29, 0.87\}$.

75. $\pi x^2 - x - \pi = 0$

$a = \pi, \quad b = -1, \quad c = -\pi$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(\pi)(-\pi)}}{2(\pi)}$$

$$= \frac{1 \pm \sqrt{1 + 4\pi^2}}{2\pi}$$

$x \approx 1.17$ or $x \approx -0.85$

The solution set is $\{-0.85, 1.17\}$.

76. $\pi x^2 + \pi x - 2 = 0$

$a = \pi, \quad b = \pi, \quad c = -2$

$$x = \frac{-\pi \pm \sqrt{(\pi)^2 - 4(\pi)(-2)}}{2(\pi)}$$

$$= \frac{-\pi \pm \sqrt{\pi^2 + 8\pi}}{2\pi}$$

$x \approx 0.44$ or $x \approx -1.44$

The solution set is $\{-1.44, 0.44\}$.

77. $3x^2 + 8\pi x + \sqrt{29} = 0$

$a = 3, \quad b = 8\pi, \quad c = \sqrt{29}$

$$x = \frac{-8\pi \pm \sqrt{(8\pi)^2 - 4(3)\left(\sqrt{29}\right)}}{2(3)}$$

$$= \frac{-8\pi \pm \sqrt{64\pi^2 - 12\sqrt{29}}}{6}$$

$x \approx -0.22$ or $x \approx -8.16$

The solution set is $\{-8.16, -0.22\}$.

78. $\pi x^2 - 15\sqrt{2}x + 20 = 0$

$a = \pi, \quad b = -15\sqrt{2}, \quad c = 20$

$$x = \frac{-\left(-15\sqrt{2}\right) \pm \sqrt{\left(-15\sqrt{2}\right)^2 - 4(\pi)(20)}}{2(\pi)}$$

$$= \frac{15\sqrt{2} \pm \sqrt{450 - 80\pi}}{2\pi}$$

$x \approx 5.62$ or $x \approx 1.13$

The solution set is $\{1.13, 5.62\}$.

79. $x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

The solution set is $\left\{-\sqrt{5}, \sqrt{5}\right\}$.

80. $x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

The solution set is $\left\{-\sqrt{6}, \sqrt{6}\right\}$.

81. $16x^2 - 8x + 1 = 0$

$(4x-1)(4x-1) = 0$

$4x - 1 = 0$

$x = \frac{1}{4}$

The solution set is $\left\{\frac{1}{4}\right\}$.

82. $9x^2 - 12x + 4 = 0$

$(3x-2)(3x-2) = 0$

$3x - 2 = 0$

$x = \frac{2}{3}$

The solution set is $\left\{\frac{2}{3}\right\}$.

83. $10x^2 - 19x - 15 = 0$

$(5x+3)(2x-5) = 0$

$5x + 3 = 0$ or $2x - 5 = 0$

$x = -\frac{3}{5}$ or $x = \frac{5}{2}$

The solution set is $\left\{-\frac{3}{5}, \frac{5}{2}\right\}$.

84. $6x^2 + 7x - 20 = 0$

$(3x-4)(2x+5) = 0$

$3x - 4 = 0$ or $2x + 5 = 0$

$x = \frac{4}{3}$ or $x = -\frac{5}{2}$

The solution set is $\left\{-\frac{5}{2}, \frac{4}{3}\right\}$.

85. $2 + z = 6z^2$

$0 = 6z^2 - z - 2$

$0 = (3z-2)(2z+1)$

$3z - 2 = 0$ or $2z + 1 = 0$

$z = \frac{2}{3}$ or $z = -\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$.

86. $2 = y + 6y^2$

$0 = 6y^2 + y - 2$

$0 = (3y+2)(2y-1)$

$3y + 2 = 0$ or $2y - 1 = 0$

$y = -\frac{2}{3}$ or $y = \frac{1}{2}$

The solution set is $\left\{-\frac{2}{3}, \frac{1}{2}\right\}$.

87. $x^2 + \sqrt{2}x = \frac{1}{2}$

$x^2 + \sqrt{2}x - \frac{1}{2} = 0$

$2\left(x^2 + \sqrt{2}x - \frac{1}{2}\right) = 2(0)$

$2x^2 + 2\sqrt{2}x - 1 = 0$

$a = 2, \quad b = 2\sqrt{2}, \quad c = -1$

$$x = \frac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{8+8}}{4} = \frac{-2\sqrt{2} \pm \sqrt{16}}{4}$$

$$= \frac{-2\sqrt{2} \pm 4}{4} = \frac{-\sqrt{2} \pm 2}{2}$$

The solution set is $\left\{\frac{-\sqrt{2}-2}{2}, \frac{-\sqrt{2}+2}{2}\right\}$

88. $\frac{1}{2}x^2 = \sqrt{2}x + 1$

$\frac{1}{2}x^2 - \sqrt{2}x - 1 = 0$

$2\left(\frac{1}{2}x^2 - \sqrt{2}x - 1\right) = 2(0)$

$x^2 - 2\sqrt{2}x - 2 = 0$

$a = 1, \quad b = -2\sqrt{2}, \quad c = -2$

$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{2\sqrt{2} \pm \sqrt{8+8}}{2} = \frac{2\sqrt{2} \pm \sqrt{16}}{2}$$

$$= \frac{2\sqrt{2} \pm 4}{2} = \frac{\sqrt{2} \pm 2}{1}$$

The solution set is $\left\{\sqrt{2}-2, \sqrt{2}+2\right\}$.

89.

$$x^2 + x = 4$$
$$x^2 + x - 4 = 0$$
$$a = 1, \quad b = 1, \quad c = -4$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{17}}{2}, \frac{-1+\sqrt{17}}{2}\right\}$.

90.

$$x^2 + x = 1$$
$$x^2 + x - 1 = 0$$
$$a = 1, \quad b = 1, \quad c = -1$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right\}$.

91.

$$\frac{x}{x-2} + \frac{2}{x+1} = \frac{7x+1}{x^2 - x - 2}$$
$$\frac{x}{x-2} + \frac{2}{x+1} = \frac{7x+1}{(x-2)(x+1)}$$
$$\left(\frac{x}{x-2} + \frac{2}{x+1}\right)(x-2)(x+1) = \left(\frac{7x+1}{(x-2)(x+1)}\right)(x-2)(x+1)$$
$$x(x+1) + 2(x-2) = 7x+1$$
$$x^2 + x + 2x - 4 = 7x + 1$$
$$x^2 + 3x - 4 = 7x + 1$$
$$x^2 - 4x - 5 = 0$$
$$(x+1)(x-5) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = -1 \quad \text{or} \quad x = 5$$

The value $x = -1$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\{5\}$.

92.

$$\frac{3x}{x+2} + \frac{1}{x-1} = \frac{4-7x}{x^2 + x - 2}$$
$$\frac{3x}{x+2} + \frac{1}{x-1} = \frac{4-7x}{(x+2)(x-1)}$$
$$\left(\frac{3x}{x+2} + \frac{1}{x-1}\right)(x+2)(x-1) = \left(\frac{4-7x}{(x+2)(x-1)}\right)(x+2)(x-1)$$
$$3x(x-1) + (x+2) = 4 - 7x$$
$$3x^2 - 3x + x + 2 = 4 - 7x$$
$$3x^2 - 2x + 2 = 4 - 7x$$
$$3x^2 + 5x - 2 = 0$$
$$(3x-1)(x+2) = 0$$
$$3x - 1 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = \frac{1}{3} \quad \text{or} \quad x = -2$$

The value $x = -2$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\left\{\frac{1}{3}\right\}$.

93. $2x^2 - 6x + 7 = 0$
$a = 2, \quad b = -6, \quad c = 7$
$b^2 - 4ac = (-6)^2 - 4(2)(7) = 36 - 56 = -20$
Since the $b^2 - 4ac < 0$, the equation has no real solution.

94. $x^2 + 4x + 7 = 0$
$a = 1, \quad b = 4, \quad c = 7$
$b^2 - 4ac = (4)^2 - 4(1)(7) = 16 - 28 = -12$
Since the $b^2 - 4ac < 0$, the equation has no real solution.

95. $9x^2 - 30x + 25 = 0$
$a = 9, \quad b = -30, \quad c = 25$
$b^2 - 4ac = (-30)^2 - 4(9)(25) = 900 - 900 = 0$
Since $b^2 - 4ac = 0$, the equation has one repeated real solution.

96. $25x^2 - 20x + 4 = 0$
$a = 25, \quad b = -20, \quad c = 4$
$b^2 - 4ac = (-20)^2 - 4(25)(4) = 400 - 400 = 0$
Since $b^2 - 4ac = 0$, the equation has one repeated real solution.

97. $3x^2 + 5x - 8 = 0$
$a = 3, \quad b = 5, \quad c = -8$
$b^2 - 4ac = (5)^2 - 4(3)(-8) = 25 + 96 = 121$
Since $b^2 - 4ac > 0$, the equation has two unequal real solutions.

98. $2x^2 - 3x - 7 = 0$
$a = 2, \quad b = -3, \quad c = -7$
$b^2 - 4ac = (-3)^2 - 4(2)(-7) = 9 + 56 = 65$
Since $b^2 - 4ac > 0$, the equation has two unequal real solutions.

99. $20.2x^2 + 314.5x + 3467.6 = 8000$
$20.2x^2 + 314.5x - 4532.4 = 0$
$a = 20.2, \quad b = 314.5, \quad c = -4532.4$

$$x = \frac{-(314.5) \pm \sqrt{(314.5)^2 - 4(20.2)(-4532.4)}}{2(20.2)}$$
$$= \frac{-314.5 \pm \sqrt{465{,}128.17}}{40.4}$$

~~$x \approx -24.7$~~ or $x \approx 9.1$

Disregard the negative solution since we are looking beyond the 2000-2001 academic year. Thus, according to the equation, the average annual tuition-and-fee charges will be \$8000 approximately 9.1 years after 2000-2001, which is roughly the academic year 2009-2010.

100. $0.14x^2 + 7.8x + 540 = 632$
$0.14x^2 + 7.8x - 92 = 0$
$a = 0.14, \quad b = 7.8, \quad c = -92$

$$x = \frac{-(7.8) \pm \sqrt{(7.8)^2 - 4(0.14)(-92)}}{2(0.14)}$$
$$= \frac{-7.8 \pm \sqrt{112.36}}{0.28} = \frac{-7.8 \pm 10.6}{0.28}$$

~~$x \approx -65.7$~~ or $x = 10$

Disregard the negative solution since we are looking beyond the year 2000. Thus, 10 years after 2000, the median weekly earnings for women 16 years and older will be \$632. 10 years after 2000. This would be the year 2010.

101. Let w represent the width of window.
Then $l = w + 2$ represents the length of the window.
Since the area is 143 square feet, we have:
$w(w + 2) = 143$
$w^2 + 2w - 143 = 0$
$(w + 13)(w - 11) = 0$
~~$w = -13$~~ or $w = 11$
Discard the negative solution since width cannot be negative. The width of the rectangular window is 11 feet and the length is 13 feet.

102. Let w represent the width of window.
Then $l = w + 1$ represents the length of the window.
Since the area is 306 square centimeters, we have: $w(w + 1) = 306$
$w^2 + w - 306 = 0$
$(w + 18)(w - 17) = 0$
~~$w = -18$~~ or $w = 17$
Discard the negative solution since width cannot be negative. The width of the rectangular window is 17 centimeters and the length is 18 centimeters.

103. Let l represent the length of the rectangle.
Let w represent the width of the rectangle.
The perimeter is 26 meters and the area is 40 square meters.

$$2l + 2w = 26$$
$$l + w = 13 \quad \text{so} \quad w = 13 - l$$
$$l\,w = 40$$
$$l(13 - l) = 40$$
$$13l - l^2 = 40$$
$$l^2 - 13l + 40 = 0$$
$$(l - 8)(l - 5) = 0$$
$$l = 8 \text{ or } l = 5$$
$$w = 5 \qquad w = 8$$

The dimensions are 5 meters by 8 meters.

104. Let r represent the radius of the circle.
Since the field is a square with area 1250 square feet, the length of a side of the square is $\sqrt{1250} = 25\sqrt{2}$ feet. The length of the diagonal is $2r$.
Use the Pythagorean Theorem to solve for r:

$$(2r)^2 = \left(25\sqrt{2}\right)^2 + \left(25\sqrt{2}\right)^2$$
$$4r^2 = 1250 + 1250$$
$$4r^2 = 2500$$
$$r^2 = 625$$
$$r = 25$$

The shortest radius setting for the sprinkler is 25 feet.

105. Let x = length of side of original sheet in feet.
Length of box: $x - 2$ feet
Width of box: $x - 2$ feet
Height of box: 1 foot

$$V = l \cdot w \cdot h$$
$$4 = (x-2)(x-2)(1)$$
$$4 = x^2 - 4x + 4$$
$$0 = x^2 - 4x$$
$$0 = x(x-4)$$
$$x = 0 \quad \text{or} \quad x = 4$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

106. Let x = width of original sheet in feet.
Length of sheet: $2x$
Length of box: $2x - 2$ feet
Width of box: $x - 2$ feet
Height of box: 1 foot

$$V = l \cdot w \cdot h$$
$$4 = (2x-2)(x-2)(1)$$
$$4 = 2x^2 - 6x + 4$$
$$0 = 2x^2 - 6x$$
$$0 = x^2 - 3x$$
$$0 = x(x-3)$$
$$x = 0 \quad \text{or} \quad x = 3$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet is 3 feet wide and 6 feet long.

107. **a.** When the ball strikes the ground, the distance from the ground will be 0. Therefore, we solve

$$96 + 80t - 16t^2 = 0$$
$$-16t^2 + 80t + 96 = 0$$
$$t^2 - 5t - 6 = 0$$
$$(t-6)(t+1) = 0$$
$$t = 6 \quad \text{or} \quad t = -1$$

Discard the negative solution since the time of flight must be positive. The ball will strike the ground after 6 seconds.

b. When the ball passes the top of the building, it will be 96 feet from the ground. Therefore, we solve

$$96 + 80t - 16t^2 = 96$$
$$-16t^2 + 80t = 0$$
$$t^2 - 5t = 0$$
$$t(t-5) = 0$$
$$t = 0 \quad \text{or} \quad t = 5$$

The ball is at the top of the building at time $t = 0$ when it is thrown. It will pass the top of the building on the way down after 5 seconds.

108. a. To find when the object will be 15 meters above the ground, we solve

$-4.9t^2 + 20t = 15$

$-4.9t^2 + 20t - 15 = 0$

$a = -4.9,\ b = 20,\ c = -15$

$t = \dfrac{-20 \pm \sqrt{20^2 - 4(-4.9)(-15)}}{2(-4.9)}$

$= \dfrac{-20 \pm \sqrt{106}}{-9.8} = \dfrac{20 \pm \sqrt{106}}{9.8}$

$t \approx 0.99$ or $t \approx 3.09$

The object will be 15 meters above the ground after about 0.99 seconds (on the way up) and about 3.09 seconds (on the way down).

b. The object will strike the ground when the distance from the ground is 0. Therefore, we solve

$-4.9t^2 + 20t = 0$

$t(-4.9t + 20) = 0$

$t = 0$ or $-4.9t + 20 = 0$

$-4.9t = -20$

$t \approx 4.08$

The object will strike the ground after about 4.08 seconds.

c. $-4.9t^2 + 20t = 100$

$-4.9t^2 + 20t - 100 = 0$

$a = -4.9,\ b = 20,\ c = -100$

$t = \dfrac{-20 \pm \sqrt{20^2 - 4(-4.9)(-100)}}{2(-4.9)}$

$= \dfrac{-20 \pm \sqrt{-1560}}{-9.8}$

There is no real solution. The object never reaches a height of 100 meters.

109. Let x represent the number of centimeters the length and width should be reduced.
$12 - x$ = the new length, $7 - x$ = the new width.
The new volume is 90% of the old volume.

$(12 - x)(7 - x)(3) = 0.9(12)(7)(3)$

$3x^2 - 57x + 252 = 226.8$

$3x^2 - 57x + 25.2 = 0$

$x^2 - 19x + 8.4 = 0$

$x = \dfrac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(8.4)}}{2(1)} = \dfrac{19 \pm \sqrt{327.4}}{2}$

$x \approx 0.45$ or $x \approx 18.55$

Since 18.55 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.55 cm by 6.55 cm by 3 cm.

110. Let x represent the number of centimeters the length and width should be reduced.
$12 - x$ = the new length, $7 - x$ = the new width.
The new volume is 80% of the old volume.

$(12 - x)(7 - x)(3) = 0.8(12)(7)(3)$

$3x^2 - 57x + 252 = 201.6$

$3x^2 - 57x + 50.4 = 0$

$x^2 - 19x + 16.8 = 0$

$x = \dfrac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(16.8)}}{2(1)} = \dfrac{19 \pm \sqrt{293.8}}{2}$

$x \approx 0.93$ or $x \approx 18.07$

Since 18.07 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.07 cm by 6.07 cm by 3 cm.

111. Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x + 5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is

$A_T = \pi(x+5)^2$.

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is

$A_B = A_T - A_P = \pi(x+5)^2 - 25\pi$.

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$0.25A_B = 0.25\left(\pi(x+5)^2 - 25\pi\right)$

Solving the volume equation:

$0.25\left(\pi(x+5)^2 - 25\pi\right) = 27$

$\pi\left(x^2 + 10x + 25 - 25\right) = 108$

$\pi x^2 + 10\pi x - 108 = 0$

$x = \dfrac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-108)}}{2(\pi)}$

$= \dfrac{-31.42 \pm \sqrt{100\pi^2 + 432\pi}}{6.28}$

$x \approx 2.71$ or $x \approx -12.71$

Discard the negative solution. The width of the border is roughly 2.71 feet.

112. Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x+5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is
$A_T = \pi(x+5)^2$.

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is
$A_B = A_T - A_P = \pi(x+5)^2 - 25\pi$.

Since the concrete is 4 inches = $\frac{1}{3}$ foot thick, the volume of the concrete in the border is
$\frac{1}{3}A_B = \frac{1}{3}\left(\pi(x+5)^2 - 25\pi\right)$

Solving the volume equation:

$$\frac{1}{3}\left(\pi(x+5)^2 - 25\pi\right) = 27$$
$$\pi\left(x^2+10x+25-25\right) = 81$$
$$\pi x^2 + 10\pi x - 81 = 0$$
$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-81)}}{2(\pi)}$$
$$= \frac{-31.42 \pm \sqrt{100\pi^2 + 324\pi}}{6.28}$$
$$x \approx 2.13 \text{ or } x \approx -12.13$$

Discard the negative solution. The width of the border is approximately 2.13 feet.

113. Let x represent the width of the border measured in feet.
The total area is $A_T = (6+2x)(10+2x)$.

The area of the garden is $A_G = 6\cdot 10 = 60$.

The area of the border is
$A_B = A_T - A_G = (6+2x)(10+2x) - 60$.

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is
$0.25A_B = 0.25\left((6+2x)(10+2x) - 60\right)$

Solving the volume equation:

$$0.25\left((6+2x)(10+2x) - 60\right) = 27$$
$$60 + 32x + 4x^2 - 60 = 108$$
$$4x^2 + 32x - 108 = 0$$
$$x^2 + 8x - 27 = 0$$
$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-27)}}{2(1)} = \frac{-8 \pm \sqrt{172}}{2}$$
$$x \approx 2.56 \text{ or } x \approx -10.56$$

Discard the negative solution. The width of the border is approximately 2.56 feet.

114. Let x = the width and $2x$ = the length of the patio. The height is $\frac{1}{3}$ foot and the concrete available is $8(27) = 216$ cubic feet..

$$V = lwh = x(2x)\cdot\frac{1}{3} = 216$$
$$\frac{2}{3}x^2 = 216$$
$$x^2 = 324$$
$$x = \pm 18$$

The dimensions of the patio are 18 feet by 36 feet.

115. Let x = the length of a traditional 4:3 format TV. Then $\frac{3}{4}x$ = the width of the traditional TV.
The diagonal of the 37-inch traditional TV is 37 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{3}{4}x\right)^2 = 37^2$$
$$x^2 + \frac{9}{16}x^2 = 1369$$
$$16\left(x^2 + \frac{9}{16}x^2\right) = 16(1369)$$
$$16x^2 + 9x^2 = 21{,}904$$
$$25x^2 = 21{,}904$$
$$x^2 = 876.16$$
$$x = \pm\sqrt{876.16} = \pm 29.6$$

Since the length cannot be negative, the length of the traditional 37-inch TV is 29.6 inches and the width is $\frac{3}{4}(29.6) = 22.2$ inches. Thus, the area of the traditional 37-inch TV is
$(29.6)(22.2) = 657.12$ square inches.

Let y = the length of a 37-inch 16:9 LCD TV. Then $\frac{9}{16}y$ = the width of the LCD TV.
The diagonal of a 37-inch LCD TV is 37 inches, so by the Pythagorean theorem we have:

$$y^2 + \left(\frac{9}{16}y\right)^2 = 37^2$$
$$y^2 + \frac{81}{256}y^2 = 1369$$
$$256\left(y^2 + \frac{81}{256}y^2\right) = 256(1369)$$

$$256y^2 + 81y^2 = 350,464$$
$$337y^2 = 350,464$$
$$y^2 = \frac{350,464}{337}$$
$$y = \pm\sqrt{\frac{350,464}{337}} \approx \pm 32.248$$

Since the length cannot be negative, the length of the LCD TV is $\sqrt{\frac{350,464}{337}} \approx 32.248$ inches and the width is $\frac{9}{16}\sqrt{\frac{350,464}{337}} \approx 18.140$ inches. Thus, the area of the 37-inch 16:9 format LCD TV is

$$\left(\sqrt{\frac{350,464}{337}}\right)\left(\frac{9}{16}\sqrt{\frac{350,464}{337}}\right)$$
$$= \frac{197,136}{337} \approx 584.97 \text{ square inches.}$$

The traditional 4:3 format TV has the larger screen since its area is larger.

116. Let x = the length of a traditional 4:3 format TV. Then $\frac{3}{4}x$ = the width of the traditional TV. The diagonal of the 50-inch traditional TV is 50 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{3}{4}x\right)^2 = 50^2$$
$$x^2 + \frac{9}{16}x^2 = 2500$$
$$16\left(x^2 + \frac{9}{16}x^2\right) = 16(2500)$$
$$16x^2 + 9x^2 = 40,000$$
$$25x^2 = 40,000$$
$$x^2 = 1600$$
$$x = \pm\sqrt{1600} = \pm 40$$

Since the length cannot be negative, the length of the traditional TV is 40 inches and the width is $\frac{3}{4}(40) = 30$ inches. Thus, the area of the 50-inch traditional TV is $(40)(30) = 1200$ square inches.

Let y = the length of a 50-inch 16:9 Plasma TV. Then $\frac{9}{16}y$ = the width of the Plasma TV. The diagonal of the 50-inch Plasma TV is 50 inches, so by the Pythagorean theorem we have:

$$y^2 + \left(\frac{9}{16}y\right)^2 = 50^2$$
$$y^2 + \frac{81}{256}y^2 = 2500$$
$$256\left(y^2 + \frac{81}{256}y^2\right) = 256(2500)$$
$$256y^2 + 81y^2 = 640,000$$
$$337y^2 = 640,000$$
$$y^2 = \frac{640,000}{337}$$
$$y = \pm\sqrt{\frac{640,000}{337}} \approx \pm 43.578$$

Since the length cannot be negative, the length of the Plasma TV is $\sqrt{\frac{640,000}{337}} \approx 43.578$ inches and the width is $\frac{9}{16}\sqrt{\frac{640,000}{337}} \approx 24.513$ inches. Thus, the area of the 50-inch Plasma TV is

$$\left(\sqrt{\frac{640,000}{337}}\right)\left(\frac{9}{16}\sqrt{\frac{640,000}{337}}\right)$$
$$= \frac{360,000}{337} \approx 1068.25 \text{ square inches.}$$

The traditional 4:3 format TV has the larger screen since its area is larger.

117.
$$\frac{1}{2}n(n+1) = 666$$
$$n(n+1) = 1332$$
$$n^2 + n - 1332 = 0$$
$$(n-36)(n+37) = 0$$
$$n = 36 \quad \text{or} \quad n = -37$$

Since the number of consecutive integers cannot be negative, we discard the negative value. We must add 36 consecutive integers, beginning at 1, in order to get a sum of 666.

118. $\frac{1}{2}n(n-3)=65$

$n(n-3)=130$

$n^2-3n-130=0$

$(n-13)(n+10)=0$

$n=13$ or $n=-10$

Since the number of sides cannot be negative, we discard the negative value. A polygon with 65 diagonals will have 13 sides.

$\frac{1}{2}n(n-3)=80$

$n(n-3)=160$

$n^2-3n-160=0$

$a=1, b=-3, c=-160$

$$n=\frac{3\pm\sqrt{(-3)^2-4(1)(-160)}}{2(1)}=\frac{3\pm\sqrt{646}}{2}$$

Neither solution is an integer, so there is no polygon that has 80 diagonals.

119. The roots of a quadratic equation are

$$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x_1+x_2=\frac{-b-\sqrt{b^2-4ac}}{2a}+\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-b-\sqrt{b^2-4ac}-b+\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2b}{2a}$$

$$=-\frac{b}{a}$$

120. The roots of a quadratic equation are

$$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x_1\cdot x_2=\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)$$

$$=\frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{(2a)^2}=\frac{b^2-b^2+4ac}{4a^2}$$

$$=\frac{4ac}{4a^2}$$

$$=\frac{c}{a}$$

121. In order to have one repeated solution, we need the discriminant to be 0.

$b^2-4ac=0$

$1^2-4(k)(k)=0$

$1-4k^2=0$

$4k^2=1$

$k^2=\frac{1}{4}$

$k=\pm\sqrt{\frac{1}{4}}$

$k=\frac{1}{2}$ or $k=-\frac{1}{2}$

122. In order to have one repeated solution, we need the discriminant to be 0.

$b^2-4ac=0$

$(-k)^2-4(1)(4)=0$

$k^2-16=0$

$(k-4)(k+4)=0$

$k=4$ or $k=-4$

123. For $ax^2+bx+c=0$:

$$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

For $ax^2-bx+c=0$:

$$x_1^*=\frac{-(-b)-\sqrt{(-b)^2-4ac}}{2a}$$

$$=\frac{b-\sqrt{b^2-4ac}}{2a}$$

$$=-\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)$$

$$=-x_2$$

and

$$x_2^*=\frac{-(-b)+\sqrt{(-b)^2-4ac}}{2a}$$

$$=\frac{b+\sqrt{b^2-4ac}}{2a}$$

$$=-\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$=-x_1$$

124. For $ax^2+bx+c=0$:

$$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

For $cx^2+bx+a=0$:

$$\begin{aligned}x_1^*&=\frac{-b-\sqrt{b^2-4(c)(a)}}{2c}=\frac{-b-\sqrt{b^2-4ac}}{2c}\\&=\frac{-b-\sqrt{b^2-4ac}}{2c}\cdot\frac{-b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\\&=\frac{b^2-\left(b^2-4ac\right)}{2c\left(-b+\sqrt{b^2-4ac}\right)}=\frac{4ac}{2c\left(-b+\sqrt{b^2-4ac}\right)}\\&=\frac{2a}{-b+\sqrt{b^2-4ac}}\\&=\frac{1}{x_2}\end{aligned}$$

and

$$\begin{aligned}x_2^*&=\frac{-b+\sqrt{b^2-4(c)(a)}}{2c}=\frac{-b+\sqrt{b^2-4ac}}{2c}\\&=\frac{-b+\sqrt{b^2-4ac}}{2c}\cdot\frac{-b-\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}}\\&=\frac{b^2-\left(b^2-4ac\right)}{2c\left(-b-\sqrt{b^2-4ac}\right)}=\frac{4ac}{2c\left(-b-\sqrt{b^2-4ac}\right)}\\&=\frac{2a}{-b-\sqrt{b^2-4ac}}\\&=\frac{1}{x_1}\end{aligned}$$

125. **a.** $x^2=9$ and $x=3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x=-3$.

b. $x=\sqrt{9}$ and $x=3$ are equivalent because $\sqrt{9}=3$.

c. $(x-1)(x-2)=(x-1)^2$ and $x-2=x-1$ are not equivalent because they do not have the same solution set.
The first equation has the solution set $\{1\}$ while the second equation has no solutions.

126. Answers will vary. Methods may include the quadratic formula, completing the square, graphing, etc.

127. Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.

128. Answers will vary. One possibility:
Two distinct: $x^2-3x-18=0$
One repeated: $x^2-14x+49=0$
No real: $x^2+x+4=0$

129. Answers will vary.

Section 1.3

1. Integers: $\{-3,0\}$
Rationals: $\left\{-3,0,\frac{6}{5}\right\}$

2. True; the set of real numbers consists of all rational and irrational numbers.

3. $$\begin{aligned}\frac{3}{2+\sqrt{3}}&=\frac{3}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}\\&=\frac{3\left(2-\sqrt{3}\right)}{2^2-\left(\sqrt{3}\right)^2}\\&=\frac{3\left(2-\sqrt{3}\right)}{4-3}\\&=3\left(2-\sqrt{3}\right)\end{aligned}$$

4. real; imaginary; imaginary unit

5. $\{-2i,2i\}$

6. False; the conjugate of $2+5i$ is $2-5i$.

7. True; the set of real numbers is a subset of the set of complex numbers.

8. False; if $2-3i$ is a solution of a quadratic equation with real coefficients, then its conjugate, $2+3i$, is also a solution.

9. $(2-3i)+(6+8i)=(2+6)+(-3+8)i=8+5i$

10. $$\begin{aligned}(4+5i)+(-8+2i)&=(4+(-8))+(5+2)i\\&=-4+7i\end{aligned}$$

11. $(-3+2i)-(4-4i) = (-3-4)+(2-(-4))i$
$= -7+6i$

12. $(3-4i)-(-3-4i) = (3-(-3))+(-4-(-4))i$
$= 6+0i = 6$

13. $(2-5i)-(8+6i) = (2-8)+(-5-6)i$
$= -6-11i$

14. $(-8+4i)-(2-2i) = (-8-2)+(4-(-2))i$
$= -10+6i$

15. $3(2-6i) = 6-18i$

16. $-4(2+8i) = -8-32i$

17. $2i(2-3i) = 4i-6i^2 = 4i-6(-1) = 6+4i$

18. $3i(-3+4i) = -9i+12i^2 = -9i+12(-1) = -12-9i$

19. $(3-4i)(2+i) = 6+3i-8i-4i^2$
$= 6-5i-4(-1)$
$= 10-5i$

20. $(5+3i)(2-i) = 10-5i+6i-3i^2$
$= 10+i-3(-1)$
$= 13+i$

21. $(-6+i)(-6-i) = 36+6i-6i-i^2$
$= 36-(-1)$
$= 37$

22. $(-3+i)(3+i) = -9-3i+3i+i^2$
$= -9+(-1)$
$= -10$

23. $\dfrac{10}{3-4i} = \dfrac{10}{3-4i}\cdot\dfrac{3+4i}{3+4i} = \dfrac{30+40i}{9+12i-12i-16i^2}$
$= \dfrac{30+40i}{9-16(-1)} = \dfrac{30+40i}{25}$
$= \dfrac{30}{25}+\dfrac{40}{25}i$
$= \dfrac{6}{5}+\dfrac{8}{5}i$

24. $\dfrac{13}{5-12i} = \dfrac{13}{5-12i}\cdot\dfrac{5+12i}{5+12i}$
$= \dfrac{65+156i}{25+60i-60i-144i^2}$
$= \dfrac{65+156i}{25-144(-1)} = \dfrac{65+156i}{169}$
$= \dfrac{65}{169}+\dfrac{156}{169}i$
$= \dfrac{5}{13}+\dfrac{12}{13}i$

25. $\dfrac{2+i}{i} = \dfrac{2+i}{i}\cdot\dfrac{-i}{-i} = \dfrac{-2i-i^2}{-i^2}$
$= \dfrac{-2i-(-1)}{-(-1)} = \dfrac{1-2i}{1} = 1-2i$

26. $\dfrac{2-i}{-2i} = \dfrac{2-i}{-2i}\cdot\dfrac{i}{i} = \dfrac{2i-i^2}{-2i^2}$
$= \dfrac{2i-(-1)}{-2(-1)} = \dfrac{1+2i}{2} = \dfrac{1}{2}+i$

27. $\dfrac{6-i}{1+i} = \dfrac{6-i}{1+i}\cdot\dfrac{1-i}{1-i} = \dfrac{6-6i-i+i^2}{1-i+i-i^2}$
$= \dfrac{6-7i+(-1)}{1-(-1)} = \dfrac{5-7i}{2} = \dfrac{5}{2}-\dfrac{7}{2}i$

28. $\dfrac{2+3i}{1-i} = \dfrac{2+3i}{1-i}\cdot\dfrac{1+i}{1+i} = \dfrac{2+2i+3i+3i^2}{1+i-i-i^2}$
$= \dfrac{2+5i+3(-1)}{1-(-1)} = \dfrac{-1+5i}{2} = -\dfrac{1}{2}+\dfrac{5}{2}i$

29. $\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^2 = \dfrac{1}{4}+2\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}i\right)+\dfrac{3}{4}i^2$
$= \dfrac{1}{4}+\dfrac{\sqrt{3}}{2}i+\dfrac{3}{4}(-1) = -\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

30. $\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i\right)^2 = \dfrac{3}{4}-2\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}i\right)+\dfrac{1}{4}i^2$
$= \dfrac{3}{4}-\dfrac{\sqrt{3}}{2}i+\dfrac{1}{4}(-1) = \dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

31. $(1+i)^2 = 1+2i+i^2 = 1+2i+(-1) = 2i$

32. $(1-i)^2 = 1-2i+i^2 = 1-2i+(-1) = -2i$

33. $i^{23} = i^{22+1} = i^{22} \cdot i = \left(i^2\right)^{11} \cdot i = (-1)^{11} i = -i$

34. $i^{14} = \left(i^2\right)^7 = (-1)^7 = -1$

35. $i^{-15} = \dfrac{1}{i^{15}} = \dfrac{1}{i^{14+1}} = \dfrac{1}{i^{14} \cdot i} = \dfrac{1}{(i^2)^7 \cdot i}$

$= \dfrac{1}{(-1)^7 i} = \dfrac{1}{-i} = \dfrac{1}{-i} \cdot \dfrac{i}{i} = \dfrac{i}{-i^2} = \dfrac{i}{-(-1)} = i$

36. $i^{-23} = \dfrac{1}{i^{23}} = \dfrac{1}{i^{22+1}} = \dfrac{1}{i^{22} \cdot i} = \dfrac{1}{(i^2)^{11} \cdot i}$

$= \dfrac{1}{(-1)^{11} i} = \dfrac{1}{-i} = \dfrac{1}{-i} \cdot \dfrac{i}{i} = \dfrac{i}{-i^2} = \dfrac{i}{-(-1)} = i$

37. $i^6 - 5 = \left(i^2\right)^3 - 5 = (-1)^3 - 5 = -1 - 5 = -6$

38. $4 + i^3 = 4 + i^2 \cdot i = 4 + (-1)\, i = 4 - i$

39. $6i^3 - 4i^5 = i^3(6 - 4i^2)$

$= i^2 \cdot i(6 - 4(-1)) = -1 \cdot i(10) = -10\, i$

40. $4i^3 - 2i^2 + 1 = 4i^2 \cdot i - 2i^2 + 1$

$= 4(-1)\, i - 2(-1) + 1$

$= -4i + 2 + 1$

$= 3 - 4i$

41. $(1+i)^3 = (1+i)(1+i)(1+i) = (1 + 2i + i^2)(1+i)$

$= (1 + 2i - 1)(1+i) = 2i(1+i)$

$= 2i + 2i^2 = 2i + 2(-1)$

$= -2 + 2i$

42. $(3i)^4 + 1 = 81i^4 + 1 = 81(1) + 1 = 82$

43. $i^7(1 + i^2) = i^7(1 + (-1)) = i^7(0) = 0$

44. $2i^4(1 + i^2) = 2(1)(1 + (-1)) = 2(0) = 0$

45. $i^6 + i^4 + i^2 + 1 = \left(i^2\right)^3 + \left(i^2\right)^2 + i^2 + 1$

$= (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0$

46. $i^7 + i^5 + i^3 + i = \left(i^2\right)^3 \cdot i + \left(i^2\right)^2 \cdot i + i^2 \cdot i + i$

$= (-1)^3 \cdot i + (-1)^2 \cdot i + (-1) \cdot i + i$

$= -i + i - i + i$

$= 0$

47. $\sqrt{-4} = 2i$

48. $\sqrt{-9} = 3i$

49. $\sqrt{-25} = 5\, i$

50. $\sqrt{-64} = 8\, i$

51. $\sqrt{(3+4i)(4i-3)} = \sqrt{12i - 9 + 16i^2 - 12i}$

$= \sqrt{-9 + 16(-1)}$

$= \sqrt{-25}$

$= 5\, i$

52. $\sqrt{(4+3i)(3i-4)} = \sqrt{12i - 16 + 9i^2 - 12i}$

$= \sqrt{-16 + 9(-1)}$

$= \sqrt{-25}$

$= 5\, i$

53. $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$x = \pm 2i$

The solution set is $\{-2i, 2i\}$.

54. $x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2$ or $x = 2$

The solution set is $\{-2, 2\}$.

55. $x^2 - 16 = 0$

$(x+4)(x-4) = 0$

$x = -4$ or $x = 4$

The solution set is $\{-4, 4\}$.

56. $x^2+25=0$

$x^2=-25$

$x=\pm\sqrt{-25}=\pm 5i$

The solution set is $\{-5i, 5i\}$.

57. $x^2-6x+13=0$

$a=1, b=-6, c=13,$

$b^2-4ac=(-6)^2-4(1)(13)=36-52=-16$

$x=\dfrac{-(-6)\pm\sqrt{-16}}{2(1)}=\dfrac{6\pm 4i}{2}=3\pm 2i$

The solution set is $\{3-2i, 3+2i\}$.

58. $x^2+4x+8=0$

$a=1, b=4, c=8$

$b^2-4ac=4^2-4(1)(8)=16-32=-16$

$x=\dfrac{-4\pm\sqrt{-16}}{2(1)}=\dfrac{-4\pm 4i}{2}=-2\pm 2i$

The solution set is $\{-2-2i, -2+2i\}$.

59. $x^2-6x+10=0$

$a=1, b=-6, c=10$

$b^2-4ac=(-6)^2-4(1)(10)=36-40=-4$

$x=\dfrac{-(-6)\pm\sqrt{-4}}{2(1)}=\dfrac{6\pm 2i}{2}=3\pm i$

The solution set is $\{3-i, 3+i\}$.

60. $x^2-2x+5=0$

$a=1, b=-2, c=5$

$b^2-4ac=(-2)^2-4(1)(5)=4-20=-16$

$x=\dfrac{-(-2)\pm\sqrt{-16}}{2(1)}=\dfrac{2\pm 4i}{2}=1\pm 2i$

The solution set is $\{1-2i, 1+2i\}$.

61. $8x^2-4x+1=0$

$a=8, b=-4, c=1$

$b^2-4ac=(-4)^2-4(8)(1)=16-32=-16$

$x=\dfrac{-(-4)\pm\sqrt{-16}}{2(8)}=\dfrac{4\pm 4i}{16}=\dfrac{1}{4}\pm\dfrac{1}{4}i$

The solution set is $\left\{\dfrac{1}{4}-\dfrac{1}{4}i, \dfrac{1}{4}+\dfrac{1}{4}i\right\}$.

62. $10x^2+6x+1=0$

$a=10, b=6, c=1$

$b^2-4ac=6^2-4(10)(1)=36-40=-4$

$x=\dfrac{-6\pm\sqrt{-4}}{2(10)}=\dfrac{-6\pm 2i}{20}=-\dfrac{3}{10}\pm\dfrac{1}{10}i$

The solution set is $\left\{-\dfrac{3}{10}-\dfrac{1}{10}i, -\dfrac{3}{10}+\dfrac{1}{10}i\right\}$.

63. $5x^2+1=2x$

$5x^2-2x+1=0$

$a=5, b=-2, c=1$

$b^2-4ac=(-2)^2-4(5)(1)=4-20=-16$

$x=\dfrac{-(-2)\pm\sqrt{-16}}{2(5)}=\dfrac{2\pm 4i}{10}=\dfrac{1}{5}\pm\dfrac{2}{5}i$

The solution set is $\left\{\dfrac{1}{5}-\dfrac{2}{5}i, \dfrac{1}{5}+\dfrac{2}{5}i\right\}$.

64. $13x^2+1=6x$

$13x^2-6x+1=0$

$a=13, b=-6, c=1$

$b^2-4ac=(-6)^2-4(13)(1)=36-52=-16$

$x=\dfrac{-(-6)\pm\sqrt{-16}}{2(13)}=\dfrac{6\pm 4i}{26}=\dfrac{3}{13}\pm\dfrac{2}{13}i$

The solution set is $\left\{\dfrac{3}{13}-\dfrac{2}{13}i, \dfrac{3}{13}+\dfrac{2}{13}i\right\}$.

65. $x^2+x+1=0$

$a=1, b=1, c=1,$

$b^2-4ac=1^2-4(1)(1)=1-4=-3$

$x=\dfrac{-1\pm\sqrt{-3}}{2(1)}=\dfrac{-1\pm\sqrt{3}\,i}{2}=-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$

The solution set is $\left\{-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i, -\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right\}$.

66. $x^2-x+1=0$

$a=1, b=-1, c=1$

$b^2-4ac=(-1)^2-4(1)(1)=1-4=-3$

$x=\dfrac{-(-1)\pm\sqrt{-3}}{2(1)}=\dfrac{1\pm\sqrt{3}\,i}{2}=\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$

The solution set is $\left\{\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i, \dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right\}$.

67. $x^3 - 8 = 0$

$(x-2)\left(x^2+2x+4\right)=0$

$x-2=0 \Rightarrow x=2$

or $x^2+2x+4=0$

$a=1, b=2, c=4$

$b^2-4ac=2^2-4(1)(4)=4-16=-12$

$$x=\frac{-2\pm\sqrt{-12}}{2(1)}=\frac{-2\pm 2\sqrt{3}\,i}{2}=-1\pm\sqrt{3}i$$

The solution set is $\left\{2, -1-\sqrt{3}i, -1+\sqrt{3}i\right\}$.

68. $x^3+27=0$

$(x+3)\left(x^2-3x+9\right)=0$

$x+3=0 \Rightarrow x=-3$

or $x^2-3x+9=0$

$a=1, b=-3, c=9$

$b^2-4ac=(-3)^2-4(1)(9)=9-36=-27$

$$x=\frac{-(-3)\pm\sqrt{-27}}{2(1)}=\frac{3\pm 3\sqrt{3}\,i}{2}=\frac{3}{2}\pm\frac{3\sqrt{3}}{2}i$$

The solution set is $\left\{-3, \frac{3}{2}-\frac{3\sqrt{3}}{2}i, \frac{3}{2}+\frac{3\sqrt{3}}{2}i\right\}$.

69. $x^4=16$

$x^4-16=0$

$\left(x^2-4\right)\left(x^2+4\right)=0$

$(x-2)(x+2)\left(x^2+4\right)=0$

$x-2=0$ or $x+2=0$ or $x^2+4=0$

$x=2$ or $x=-2$ or $x^2=-4$

$x=2$ or $x=-2$ or $x=\pm\sqrt{-4}=\pm 2i$

The solution set is $\{-2, 2, -2i, 2i\}$.

70. $x^4=1$

$x^4-1=0$

$\left(x^2-1\right)\left(x^2+1\right)=0$

$(x-1)(x+1)\left(x^2+1\right)=0$

$x-1=0$ or $x+1=0$ or $x^2+1=0$

$x=1$ or $x=-1$ or $x^2=-1$

$x=1$ or $x=-1$ or $x=\pm\sqrt{-1}=\pm i$

The solution set is $\{-1, 1, -i, i\}$.

71. $x^4+13x^2+36=0$

$\left(x^2+9\right)\left(x^2+4\right)=0$

$x^2+9=0$ or $x^2+4=0$

$x^2=-9$ or $x^2=-4$

$x=\pm\sqrt{-9}$ or $x=\pm\sqrt{-4}$

$x=\pm 3i$ or $x=\pm 2i$

The solution set is $\{-3i, 3i, -2i, 2i\}$.

72. $x^4+3x^2-4=0$

$\left(x^2-1\right)\left(x^2+4\right)=0$

$(x-1)(x+1)\left(x^2+4\right)=0$

$x-1=0$ or $x+1=0$ or $x^2+4=0$

$x=1$ or $x=-1$ or $x^2=-4$

$x=1$ or $x=-1$ or $x=\pm\sqrt{-4}=\pm 2i$

The solution set is $\{-1, 1, -2i, 2i\}$.

73. $3x^2-3x+4=0$

$a=3, b=-3, c=4$

$b^2-4ac=(-3)^2-4(3)(4)=9-48=-39$

The equation has two complex solutions that are conjugates of each other.

74. $2x^2-4x+1=0$

$a=2, b=-4, c=1$

$b^2-4ac=(-4)^2-4(2)(1)=16-8=8$

The equation has two unequal real number solutions.

75. $2x^2+3x=4$

$2x^2+3x-4=0$

$a=2, b=3, c=-4$

$b^2-4ac=3^2-4(2)(-4)=9+32=41$

The equation has two unequal real solutions.

76. $x^2+6=2x$

$x^2-2x+6=0$

$a=1, b=-2, c=6$

$b^2-4ac=(-2)^2-4(1)(6)=4-24=-20$

The equation has two complex solutions that are conjugates of each other.

77. $9x^2 - 12x + 4 = 0$
$a = 9, b = -12, c = 4$
$b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$
The equation has a repeated real solution.

78. $4x^2 + 12x + 9 = 0$
$a = 4, b = 12, c = 9$
$b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0$
The equation has a repeated real solution.

79. The other solution is $\overline{2+3i} = 2 - 3i$.

80. The other solution is $\overline{4-i} = 4 + i$.

81. $z + \overline{z} = 3 - 4i + \overline{3-4i} = 3 - 4i + 3 + 4i = 6$

82.
$$\begin{aligned} w - \overline{w} &= 8 + 3i - \left(\overline{8+3i}\right) \\ &= 8 + 3i - (8 - 3i) \\ &= 8 + 3i - 8 + 3i \\ &= 0 + 6i \\ &= 6i \end{aligned}$$

83.
$$\begin{aligned} z \cdot \overline{z} &= (3-4i)(\overline{3-4i}) \\ &= (3-4i)(3+4i) \\ &= 9 + 12i - 12i - 16i^2 \\ &= 9 - 16(-1) \\ &= 25 \end{aligned}$$

84.
$$\begin{aligned} \overline{z - w} &= \overline{3 - 4i - (8+3i)} \\ &= \overline{3 - 4i - 8 - 3i} \\ &= \overline{-5 - 7i} \\ &= -5 + 7i \end{aligned}$$

85.
$$\begin{aligned} Z &= \frac{V}{I} = \frac{18+i}{3-4i} = \frac{18+i}{3-4i} \cdot \frac{3+4i}{3+4i} \\ &= \frac{54 + 72i + 3i + 4i^2}{9 + 12i - 12i - 16i^2} = \frac{54 + 75i - 4}{9 + 16} \\ &= \frac{50 + 75i}{25} = 2 + 3i \end{aligned}$$
The impedance is $2 + 3i$ ohms.

86.
$$\begin{aligned} \frac{1}{Z} &= \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{2+i} + \frac{1}{4-3i} = \frac{(4-3i) + (2+i)}{(2+i)(4-3i)} \\ &= \frac{6-2i}{8 - 6i + 4i - 3i^2} = \frac{6-2i}{8 - 2i + 3} = \frac{6-2i}{11-2i} \end{aligned}$$
So,
$$\begin{aligned} Z &= \frac{11-2i}{6-2i} = \frac{11-2i}{6-2i} \cdot \frac{6+2i}{6+2i} \\ &= \frac{66 + 22i - 12i - 4i^2}{36 + 12i - 12i - 4i^2} = \frac{66 + 10i + 4}{36 + 4} \\ &= \frac{70 + 10i}{40} = \frac{7}{4} + \frac{1}{4}i \end{aligned}$$
The total impedance is $\frac{7}{4} + \frac{1}{4}i$ ohms.

87.
$$\begin{aligned} z + \overline{z} &= (a + bi) + (\overline{a + bi}) \\ &= a + bi + a - bi \\ &= 2a \end{aligned}$$
$$\begin{aligned} z - \overline{z} &= a + bi - (\overline{a + bi}) \\ &= a + bi - (a - bi) \\ &= a + bi - a + bi \\ &= 2bi \end{aligned}$$

88. $\overline{\overline{z}} = \overline{\overline{a + bi}} = \overline{a - bi} = a + bi = z$

89.
$$\begin{aligned} \overline{z + w} &= \overline{(a + bi) + (c + di)} \\ &= \overline{(a + c) + (b + d)i} \\ &= (a + c) - (b + d)i \\ &= (a - bi) + (c - di) \\ &= \overline{a + bi} + \overline{c + di} \\ &= \overline{z} + \overline{w} \end{aligned}$$

90.
$$\begin{aligned} \overline{z \cdot w} &= \overline{(a + bi) \cdot (c + di)} \\ &= \overline{ac + adi + bci + bdi^2} \\ &= \overline{(ac - bd) + (ad + bc)i} \\ &= (ac - bd) - (ad + bc)i \end{aligned}$$
$$\begin{aligned} \overline{z} \cdot \overline{w} &= \overline{a + bi} \cdot \overline{c + di} \\ &= (a - bi)(c - di) \\ &= ac - adi - bci + bdi^2 \\ &= (ac - bd) - (ad + bc)i \end{aligned}$$

91 – 92. Answers will vary.

Section 1.4

1. True

2. $\sqrt[3]{-8}=-2$

3. $6x^3-2x^2=2x^2(3x-1)$

4. extraneous

5. quadratic in form

6. True

7. $$\begin{aligned}\sqrt{2t-1}&=1\\ \left(\sqrt{2t-1}\right)^2&=1^2\\ 2t-1&=1\\ 2t&=2\\ t&=1\end{aligned}$$
Check: $\sqrt{2(1)-1}=\sqrt{1}=1$
The solution set is $\{1\}$.

8. $$\begin{aligned}\sqrt{3t+4}&=2\\ \left(\sqrt{3t+4}\right)^2&=2^2\\ 3t+4&=4\\ 3t&=0\\ t&=0\end{aligned}$$
Check: $\sqrt{3(0)+4}=\sqrt{4}=2$
The solution set is $\{0\}$.

9. $\sqrt{3t+4}=-6$
Since the principal square root is never negative, the equation has no real solution.

10. $\sqrt{5t+3}=-2$
Since the principal square root is never negative, the equation has no real solution.

11. $$\begin{aligned}\sqrt[3]{1-2x}-3&=0\\ \sqrt[3]{1-2x}&=3\\ \left(\sqrt[3]{1-2x}\right)^3&=3^3\\ 1-2x&=27\\ -2x&=26\\ x&=-13\end{aligned}$$
Check: $\sqrt[3]{1-2(-13)}-3=\sqrt[3]{27}-3=0$
The solution set is $\{-13\}$.

12. $$\begin{aligned}\sqrt[3]{1-2x}-1&=0\\ \sqrt[3]{1-2x}&=1\\ \left(\sqrt[3]{1-2x}\right)^3&=1^3\\ 1-2x&=1\\ -2x&=0\\ x&=0\end{aligned}$$
Check: $\sqrt[3]{1-2(0)}-1=\sqrt[3]{1}-1=0$
The solution set is $\{0\}$.

13. $$\begin{aligned}\sqrt[4]{5x-4}&=2\\ \left(\sqrt[4]{5x-4}\right)^4&=2^4\\ 5x-4&=16\\ 5x&=20\\ x&=4\end{aligned}$$
Check: $\sqrt[4]{5(4)-4}=\sqrt[4]{16}=2$
The solution set is $\{4\}$.

14. $$\begin{aligned}\sqrt[5]{2x-3}&=-1\\ \left(\sqrt[5]{2x-3}\right)^5&=(-1)^5\\ 2x-3&=-1\\ 2x&=2\\ x&=1\end{aligned}$$
Check: $\sqrt[5]{2(1)-3}=\sqrt[5]{-1}=-1$
The solution set is $\{1\}$.

15. $$\begin{aligned}\sqrt[5]{x^2+2x}&=-1\\ \left(\sqrt[5]{x^2+2x}\right)^5&=(-1)^5\\ x^2+2x&=-1\\ x^2+2x+1&=0\\ (x+1)^2&=0\\ x+1&=0\\ x&=-1\end{aligned}$$
Check: $\sqrt[5]{(-1)^2+2(-1)}=\sqrt[5]{1-2}=\sqrt[5]{-1}=-1$
The solution set is $\{-1\}$.

16. $\sqrt[4]{x^2+16}=\sqrt{5}$

$\left(\sqrt[4]{x^2+16}\right)^4=\left(\sqrt{5}\right)^4$

$x^2+16=25$

$x^2=9$

$x=\pm 3$

Check -3: $\sqrt[4]{(-3)^2+16}=\sqrt[4]{9+16}=\sqrt[4]{25}=\sqrt{5}$

Check 3: $\sqrt[4]{(3)^2+16}=\sqrt[4]{9+16}=\sqrt[4]{25}=\sqrt{5}$

The solution set is $\{-3,3\}$.

17. $x=8\sqrt{x}$

$(x)^2=\left(8\sqrt{x}\right)^2$

$x^2=64x$

$x^2-64x=0$

$x(x-64)=0$

$x=0$ or $x=64$

Check 0: $0=8\sqrt{0}$, $0=0$ Check 64: $64=8\sqrt{64}$, $64=64$

The solution set is $\{0,64\}$.

18. $x=3\sqrt{x}$

$(x)^2=\left(3\sqrt{x}\right)^2$

$x^2=9x$

$x^2-9x=0$

$x(x-9)=0$

$x=0$ or $x=9$

Check 0: $0=3\sqrt{0}$, $0=0$ Check 9: $9=3\sqrt{9}$, $9=9$

The solution set is $\{0,9\}$.

19. $\sqrt{15-2x}=x$

$\left(\sqrt{15-2x}\right)^2=x^2$

$15-2x=x^2$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5$ or $x=3$

Check –5: $\sqrt{15-2(-5)}=\sqrt{25}=5\neq -5$

Check 3: $\sqrt{15-2(3)}=\sqrt{9}=3=3$

Disregard $x=-5$ as extraneous.

The solution set is {3}.

20. $\sqrt{12-x}=x$

$\left(\sqrt{12-x}\right)^2=x^2$

$12-x=x^2$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ or $x=3$

Check –4: $\sqrt{12-(-4)}=\sqrt{16}=4\neq -4$

Check 3: $\sqrt{12-3}=\sqrt{9}=3=3$

Disregard $x=-4$ as extraneous.

The solution set is {3}.

21. $x=2\sqrt{x-1}$

$x^2=\left(2\sqrt{x-1}\right)^2$

$x^2=4(x-1)$

$x^2=4x-4$

$x^2-4x+4=0$

$(x-2)^2=0$

$x=2$

Check: $2=2\sqrt{2-1}$

$2=2$

The solution set is {2}.

22. $x=2\sqrt{-x-1}$

$x^2=\left(2\sqrt{-x-1}\right)^2$

$x^2=4(-x-1)$

$x^2=-4x-4$

$x^2+4x+4=0$

$(x+2)^2=0$

$x=-2$

Check: $-2=2\sqrt{-(-2)-1}$

$-2\neq 2$

The equation has no real solution.

23. $\sqrt{x^2-x-4}=x+2$

$$\left(\sqrt{x^2-x-4}\right)^2=(x+2)^2$$
$$x^2-x-4=x^2+4x+4$$
$$-8=5x$$
$$-\frac{8}{5}=x$$

Check: $\sqrt{\left(-\frac{8}{5}\right)^2-\left(-\frac{8}{5}\right)-4}=\left(-\frac{8}{5}\right)+2$

$$\sqrt{\frac{64}{25}+\frac{8}{5}-4}=\frac{2}{5}$$
$$\sqrt{\frac{4}{25}}=\frac{2}{5}$$
$$\frac{2}{5}=\frac{2}{5}$$

The solution set is $\left\{-\frac{8}{5}\right\}$.

24. $\sqrt{3-x+x^2}=x-2$

$$\left(\sqrt{3-x+x^2}\right)^2=(x-2)^2$$
$$3-x+x^2=x^2-4x+4$$
$$3x=1$$
$$x=\frac{1}{3}$$

Check: $\sqrt{3-\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^2}=\left(\frac{1}{3}\right)-2$

$$\sqrt{3-\frac{1}{3}+\frac{1}{9}}=-\frac{5}{3}$$

Since the principal square root is always a non-negative number; $x=\frac{1}{3}$ does not check.
Therefore this equation has no real solution.

25. $3+\sqrt{3x+1}=x$

$$\sqrt{3x+1}=x-3$$
$$\left(\sqrt{3x+1}\right)^2=(x-3)^2$$
$$3x+1=x^2-6x+9$$
$$0=x^2-9x+8$$
$$0=(x-1)(x-8)$$
$$x=1 \text{ or } x=8$$

Check 1: $3+\sqrt{3(1)+1}=3+\sqrt{4}=5\neq 1$

Check 8: $3+\sqrt{3(8)+1}=3+\sqrt{25}=8=8$

Discard $x=1$ as extraneous.
The solution set is $\{8\}$.

26. $2+\sqrt{12-2x}=x$

$$\sqrt{12-2x}=x-2$$
$$\left(\sqrt{12-2x}\right)^2=(x-2)^2$$
$$12-2x=x^2-4x+4$$
$$0=x^2-2x-8$$
$$(x+2)(x-4)=0$$
$$x=-2 \text{ or } x=4$$

Check -2: $2+\sqrt{12-2(-2)}=2+\sqrt{16}=6\neq -2$

Check 4: $2+\sqrt{12-2(4)}=2+\sqrt{4}=4=4$

Discard $x=-2$ as extraneous.
The solution set is $\{4\}$.

27. $\sqrt{2x+3}-\sqrt{x+1}=1$

$$\sqrt{2x+3}=1+\sqrt{x+1}$$
$$\left(\sqrt{2x+3}\right)^2=\left(1+\sqrt{x+1}\right)^2$$
$$2x+3=1+2\sqrt{x+1}+x+1$$
$$x+1=2\sqrt{x+1}$$
$$(x+1)^2=\left(2\sqrt{x+1}\right)^2$$
$$x^2+2x+1=4(x+1)$$
$$x^2+2x+1=4x+4$$
$$x^2-2x-3=0$$
$$(x+1)(x-3)=0$$
$$x=-1 \text{ or } x=3$$

Check -1: $\sqrt{2(-1)+3}-\sqrt{-1+1}$
$=\sqrt{1}-\sqrt{0}=1-0=1=1$

Check 3: $\sqrt{2(3)+3}-\sqrt{3+1}$
$=\sqrt{9}-\sqrt{4}=3-2=1=1$

The solution set is $\{-1,3\}$.

28. $\sqrt{3x+7}+\sqrt{x+2}=1$

$$\sqrt{3x+7}=1-\sqrt{x+2}$$
$$\left(\sqrt{3x+7}\right)^2=\left(1-\sqrt{x+2}\right)^2$$
$$3x+7=1-2\sqrt{x+2}+x+2$$
$$2x+4=-2\sqrt{x+2}$$
$$-x-2=\sqrt{x+2}$$
$$(-x-2)^2=\left(\sqrt{x+2}\right)^2$$
$$x^2+4x+4=x+2$$
$$x^2+3x+2=0$$
$$(x+1)(x+2)=0$$
$$x=-1 \text{ or } x=-2$$

Check –1: $\sqrt{3(-1)+7}+\sqrt{-1+2}$
$=\sqrt{4}+\sqrt{1}=2+1=3\neq 1$

Check -2: $\sqrt{3(-2)+7}+\sqrt{-2+2}$
$=\sqrt{1}+\sqrt{0}=1+0=1=1$

Discard $x=-1$ as extraneous.
The solution set is {–2}.

29. $\sqrt{3x+1}-\sqrt{x-1}=2$

$$\sqrt{3x+1}=2+\sqrt{x-1}$$
$$\left(\sqrt{3x+1}\right)^2=\left(2+\sqrt{x-1}\right)^2$$
$$3x+1=4+4\sqrt{x-1}+x-1$$
$$2x-2=4\sqrt{x-1}$$
$$(2x-2)^2=\left(4\sqrt{x-1}\right)^2$$
$$4x^2-8x+4=16(x-1)$$
$$x^2-2x+1=4x-4$$
$$x^2-6x+5=0$$
$$(x-1)(x-5)=0$$
$$x=1 \text{ or } x=5$$

Check 1: $\sqrt{3(1)+1}-\sqrt{1-1}$
$=\sqrt{4}-\sqrt{0}=2-0=2=2$

Check 5: $\sqrt{3(5)+1}-\sqrt{5-1}$
$=\sqrt{16}-\sqrt{4}=4-2=2=2$

The solution set is $\{1,5\}$.

30. $\sqrt{3x-5}-\sqrt{x+7}=2$

$$\sqrt{3x-5}=2+\sqrt{x+7}$$
$$\left(\sqrt{3x-5}\right)^2=\left(2+\sqrt{x+7}\right)^2$$
$$3x-5=4+4\sqrt{x+7}+x+7$$
$$2x-16=4\sqrt{x+7}$$
$$(2x-16)^2=\left(4\sqrt{x+7}\right)^2$$
$$4x^2-64x+256=16(x+7)$$
$$4x^2-64x+256=16x+112$$
$$4x^2-80x+144=0$$
$$x^2-20x+36=0$$
$$(x-2)(x-18)=0$$
$$x=2 \text{ or } x=18$$

Check 2: $\sqrt{3(2)-5}-\sqrt{2+7}$
$=\sqrt{1}-\sqrt{9}=1-3=-2\neq 2$

Check 18: $\sqrt{3(18)-5}-\sqrt{18+7}$
$=\sqrt{49}-\sqrt{25}=7-5=2=2$

Discard $x=2$ as extraneous.
The solution set is {18}.

31. $\sqrt{3-2\sqrt{x}}=\sqrt{x}$

$$\left(\sqrt{3-2\sqrt{x}}\right)^2=\left(\sqrt{x}\right)^2$$
$$3-2\sqrt{x}=x$$
$$-2\sqrt{x}=x-3$$
$$\left(-2\sqrt{x}\right)^2=\left(x-3\right)^2$$
$$4x=x^2-6x+9$$
$$0=x^2-10x+9$$
$$0=\left(x-1\right)\left(x-9\right)$$
$$x=1 \quad \text{or} \quad x=9$$

Check 1:	Check 9:
$\sqrt{3-2\sqrt{1}}=\sqrt{1}$	$\sqrt{3-2\sqrt{9}}=\sqrt{9}$
$\sqrt{3-2}=1$	$\sqrt{3-2\cdot 3}=3$
$\sqrt{1}=1$	$\sqrt{-3}\neq 3$
$1=1$	

Discard $x=9$ as extraneous.
The solution set is {1}.

32. $\sqrt{10+3\sqrt{x}}=\sqrt{x}$

$$\left(\sqrt{10+3\sqrt{x}}\right)^2=\left(\sqrt{x}\right)^2$$
$$10+3\sqrt{x}=x$$
$$3\sqrt{x}=x-10$$
$$\left(3\sqrt{x}\right)^2=\left(x-10\right)^2$$
$$9x=x^2-20x+100$$
$$0=x^2-29x+100$$
$$0=(x-4)(x-25)$$
$$x=4 \quad \text{or} \quad x=25$$

Check 4:
$$\sqrt{10+3\sqrt{4}}=\sqrt{4}$$
$$\sqrt{10+3\cdot 2}=2$$
$$\sqrt{16}=2$$
$$4\neq 2$$

Check 25:
$$\sqrt{10+3\sqrt{25}}=\sqrt{25}$$
$$\sqrt{10+3\cdot 5}=5$$
$$\sqrt{25}=5$$
$$5=5$$

Discard $x=4$ as extraneous.
The solution set is {25}.

33. $(3x+1)^{1/2}=4$

$$\left((3x+1)^{1/2}\right)^2=(4)^2$$
$$3x+1=16$$
$$3x=15$$
$$x=5$$

Check: $(3(5)+1)^{1/2}=16^{1/2}=4$
The solution set is {5}.

34. $(3x-5)^{1/2}=2$

$$\left((3x-5)^{1/2}\right)^2=(2)^2$$
$$3x-5=4$$
$$3x=9$$
$$x=3$$

Check: $(3(3)-5)^{1/2}=4^{1/2}=2$
The solution set is {3}.

35. $(5x-2)^{1/3}=2$

$$\left((5x-2)^{1/3}\right)^3=(2)^3$$
$$5x-2=8$$
$$5x=10$$
$$x=2$$

Check: $(5(2)-2)^{1/3}=8^{1/3}=2$
The solution set is {2}.

36. $(2x+1)^{1/3}=-1$

$$\left((2x+1)^{1/3}\right)^3=(-1)^3$$
$$2x+1=-1$$
$$2x=-2$$
$$x=-1$$

Check: $(2(-1)+1)^{1/3}=(-1)^{1/3}=-1$
The solution set is $\{-1\}$.

37. $\left(x^2+9\right)^{1/2}=5$

$$\left(\left(x^2+9\right)^{1/2}\right)^2=(5)^2$$
$$x^2+9=25$$
$$x^2=16$$
$$x=\pm\sqrt{16}=\pm 4$$

Check -4: $\left((-4)^2+9\right)^{1/2}=25^{1/2}=5$

Check 4: $\left((4)^2+9\right)^{1/2}=25^{1/2}=5$

The solution set is $\{-4,4\}$.

38. $\left(x^2-16\right)^{1/2}=9$

$$\left(\left(x^2-16\right)^{1/2}\right)^2=(9)^2$$
$$x^2-16=81$$
$$x^2=97$$
$$x=\pm\sqrt{97}$$

Check $-\sqrt{97}$: $\left(\left(-\sqrt{97}\right)^2-16\right)^{1/2}=81^{1/2}=9$

Check $\sqrt{97}$: $\left(\left(\sqrt{97}\right)^2-16\right)^{1/2}=81^{1/2}=9$

The solution set is $\left\{-\sqrt{97},\sqrt{97}\right\}$.

39. $x^{3/2} - 3x^{1/2} = 0$

$x^{1/2}(x-3) = 0$

$x^{1/2} = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$

Check 0: $0^{3/2} - 3 \cdot 0^{1/2} = 0 - 0 = 0$

Check 3: $3^{3/2} - 3 \cdot 3^{1/2} = 3\sqrt{3} - 3\sqrt{3} = 0$

The solution set is $\{0, 3\}$.

40. $x^{3/4} - 9x^{1/4} = 0$

$x^{1/4}\left(x^{1/2} - 9\right) = 0$

$x^{1/4} = 0$ or $x^{1/2} = 9$

$x = 0$ $\quad x = 81$

Check 0: $0^{3/4} - 9 \cdot 0^{1/4} = 0 - 0 = 0$

Check 81: $81^{3/4} - 9 \cdot 81^{1/4} = 27 - 27 = 0$

The solution set is $\{0, 81\}$.

41. $x^4 - 5x^2 + 4 = 0$

$\left(x^2 - 4\right)\left(x^2 - 1\right) = 0$

$x^2 - 4 = 0$ or $x^2 - 1 = 0$

$x = \pm 2$ or $x = \pm 1$

The solution set is $\{-2, -1, 1, 2\}$.

42. $x^4 - 10x^2 + 25 = 0$

$\left(x^2 - 5\right)\left(x^2 - 5\right) = 0$

$x^2 - 5 = 0$

$x = \pm\sqrt{5}$

The solution set is $\left\{-\sqrt{5}, \sqrt{5}\right\}$.

43. $3x^4 - 2x^2 - 1 = 0$

$\left(3x^2 + 1\right)\left(x^2 - 1\right) = 0$

$3x^2 + 1 = 0$ or $x^2 - 1 = 0$

$3x^2 = -1$ or $x^2 = 1$

Not real or $x = \pm 1$

The solution set is $\{-1, 1\}$.

44. $2x^4 - 5x^2 - 12 = 0$

$\left(2x^2 + 3\right)\left(x^2 - 4\right) = 0$

$2x^2 + 3 = 0$ or $x^2 - 4 = 0$

$2x^2 = -3$ or $x^2 = 4$

Not real or $x = \pm 2$

The solution set is $\{-2, 2\}$.

45. $x^6 + 7x^3 - 8 = 0$

$\left(x^3 + 8\right)\left(x^3 - 1\right) = 0$

$x^3 + 8 = 0$ or $x^3 - 1 = 0$

$x^3 = -8$ or $x^3 = 1$

$x = -2$ or $x = 1$

The solution set is $\{-2, 1\}$.

46. $x^6 - 7x^3 - 8 = 0$

$\left(x^3 - 8\right)\left(x^3 + 1\right) = 0$

$x^3 - 8 = 0$ or $x^3 + 1 = 0$

$x^3 = 8$ or $x^3 = -1$

$x = 2$ or $x = -1$

The solution set is $\{-1, 2\}$.

47. $(x+2)^2 + 7(x+2) + 12 = 0$

Let $u = x + 2$, so that $u^2 = (x+2)^2$.

$u^2 + 7u + 12 = 0$

$(u+3)(u+4) = 0$

$u + 3 = 0$ or $u + 4 = 0$

$u = -3$ or $u = -4$

$x + 2 = -3$ or $x + 2 = -4$

$x = -5$ or $x = -6$

The solution set is $\{-6, -5\}$.

48. $(2x+5)^2 - (2x+5) - 6 = 0$

Let $u = 2x + 5$ so that $u^2 = (2x+5)^2$.

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u - 3 = 0$ or $u + 2 = 0$

$u = 3$ or $u = -2$

$2x + 5 = 3$ or $2x + 5 = -2$

$x = -1$ or $x = -\frac{7}{2}$

The solution set is $\left\{-\frac{7}{2}, -1\right\}$.

49. $(3x+4)^2-6(3x+4)+9=0$

Let $u=3x+4$ so that $u^2=(3x+4)^2$.

$u^2-6u+9=0$

$(u-3)^2=0$

$u-3=0$

$u=3$

$3x+4=3$

$x=-\frac{1}{3}$

The solution set is $\left\{-\frac{1}{3}\right\}$.

50. $(2-x)^2+(2-x)-20=0$

Let $u=2-x$ so that $u^2=(2-x)^2$.

$u^2+u-20=0$

$(u+5)(u-4)=0$

$u+5=0$ or $u-4=0$

$u=-5$ or $u=4$

$2-x=-5$ or $2-x=4$

$x=7$ or $x=-2$

The solution set is $\{-2,7\}$.

51. $2(s+1)^2-5(s+1)=3$

Let $u=s+1$ so that $u^2=(s+1)^2$.

$2u^2-5u=3$

$2u^2-5u-3=0$

$(2u+1)(u-3)=0$

$2u+1=0$ or $u-3=0$

$u=-\frac{1}{2}$ or $u=3$

$s+1=-\frac{1}{2}$ or $s+1=3$

$s=-\frac{3}{2}$ or $s=2$

The solution set is $\left\{-\frac{3}{2},2\right\}$.

52. $3(1-y)^2+5(1-y)+2=0$

Let $u=1-y$ so that $u^2=(1-y)^2$.

$3u^2+5u+2=0$

$(3u+2)(u+1)=0$

$3u+2=0$ or $u+1=0$

$u=-\frac{2}{3}$ or $u=-1$

$1-y=-\frac{2}{3}$ or $1-y=-1$

$y=\frac{5}{3}$ or $y=2$

The solution set is $\left\{\frac{5}{3},2\right\}$.

53. $x-4x\sqrt{x}=0$

$x\left(1-4\sqrt{x}\right)=0$

$x=0$ or $1-4\sqrt{x}=0$

$1=4\sqrt{x}$

$\frac{1}{4}=\sqrt{x}$

$\left(\frac{1}{4}\right)^2=\left(\sqrt{x}\right)^2$

$\frac{1}{16}=x$

Check:

$x=0$: $0-4(0)\sqrt{0}=0$

$0=0$

$x=\frac{1}{16}$: $\left(\frac{1}{16}\right)-4\left(\frac{1}{16}\right)\sqrt{\frac{1}{16}}=0$

$\frac{1}{16}-4\left(\frac{1}{16}\right)\left(\frac{1}{4}\right)=0$

$\frac{1}{16}-\frac{1}{16}=0$

$0=0$

The solution set is $\left\{0,\frac{1}{16}\right\}$.

54. $x+8\sqrt{x}=0$

$8\sqrt{x}=-x$

$\left(8\sqrt{x}\right)^2=(-x)^2$

$64x=x^2$

$0=x^2-64x$

$0=x(x-64)$

$x=0$ or $x=64$

Check: $x=0$: $0+8\sqrt{0}=0$

$0=0$

$x=64$: $64+8\sqrt{64}=0$

$64+64\neq 0$

The solution set is $\{0\}$.

55. $x+\sqrt{x}=20$

Let $u=\sqrt{x}$ so that $u^2=x$.

$$u^2+u=20$$
$$u^2+u-20=0$$
$$(u+5)(u-4)=0$$
$$u+5=0 \quad \text{or} \quad u-4=0$$
$$u=-5 \quad \text{or} \quad u=4$$
$$\sqrt{x}=-5 \quad \text{or} \quad \sqrt{x}=4$$
$$\text{not possible} \quad \text{or} \quad x=16$$

Check: $16+\sqrt{16}=20$
$$16+4=20$$

The solution set is $\{16\}$.

56. $x+\sqrt{x}=6$

Let $u=\sqrt{x}$ so that $u^2=x$.

$$u^2+u=6$$
$$u^2+u-6=0$$
$$(u+3)(u-2)=0$$
$$u+3=0 \quad \text{or} \quad u-2=0$$
$$u=-3 \quad \text{or} \quad u=2$$
$$\sqrt{x}=-3 \quad \text{or} \quad \sqrt{x}=2$$
$$\text{not possible} \quad \text{or} \quad x=4$$

Check: $4+\sqrt{4}=6$
$$4+2=6$$

The solution set is $\{4\}$.

57. $t^{1/2}-2t^{1/4}+1=0$

Let $u=t^{1/4}$ so that $u^2=t^{1/2}$.

$$u^2-2u+1=0$$
$$(u-1)^2=0$$
$$u-1=0$$
$$u=1$$
$$t^{1/4}=1$$
$$t=1$$

Check: $1^{1/2}-2(1)^{1/4}+1=0$
$$1-2+1=0$$
$$0=0$$

The solution set is $\{1\}$.

58. $z^{1/2}-4t^{1/4}+4=0$

Let $u=z^{1/4}$ so that $u^2=z^{1/2}$.

$$u^2-4u+4=0$$
$$(u-2)^2=0$$
$$u-2=0$$
$$u=2$$
$$z^{1/4}=2$$
$$z=16$$

Check: $16^{1/2}-4(16)^{1/4}+4=0$
$$4-8+4=0$$
$$0=0$$

The solution set is $\{16\}$.

59. $4x^{1/2}-9x^{1/4}+4=0$

Let $u=x^{1/4}$ so that $u^2=x^{1/2}$.

$$4u^2-9u+4=0$$
$$u=\frac{-(-9)\pm\sqrt{(-9)^2-4(4)(4)}}{2(4)}=\frac{9\pm\sqrt{17}}{8}$$
$$x^{1/4}=\frac{9\pm\sqrt{17}}{8}$$
$$x=\left(\frac{9\pm\sqrt{17}}{8}\right)^4$$

Check $x=\left(\frac{9+\sqrt{17}}{8}\right)^4$:

$$4\left(\left(\frac{9+\sqrt{17}}{8}\right)^4\right)^{1/2}-9\left(\left(\frac{9+\sqrt{17}}{8}\right)^4\right)^{1/4}+4=0$$
$$4\left(\frac{9+\sqrt{17}}{8}\right)^2-9\left(\frac{9+\sqrt{17}}{8}\right)+4=0$$
$$4\frac{\left(9+\sqrt{17}\right)^2}{64}-9\left(\frac{9+\sqrt{17}}{8}\right)+4=0$$
$$64\left(4\frac{\left(9+\sqrt{17}\right)^2}{64}-9\left(\frac{9+\sqrt{17}}{8}\right)+4\right)=(0)(64)$$
$$4\left(9+\sqrt{17}\right)^2-72\left(9+\sqrt{17}\right)+256=0$$
$$4\left(81+18\sqrt{17}+17\right)-72\left(9+\sqrt{17}\right)+256=0$$
$$324+72\sqrt{17}+68-648-72\sqrt{17}+256=0$$
$$0=0$$

Check $x=\left(\frac{9-\sqrt{17}}{8}\right)^4$:

$$4\left(\left(\frac{9-\sqrt{17}}{8}\right)^4\right)^{1/2}-9\left(\left(\frac{9-\sqrt{17}}{8}\right)^4\right)^{1/4}+4=0$$

$$4\left(\frac{9-\sqrt{17}}{8}\right)^2-9\left(\frac{9-\sqrt{17}}{8}\right)+4=0$$

$$4\left(81-18\sqrt{17}+17\right)-72\left(9-\sqrt{17}\right)+256=0$$

$$324-72\sqrt{17}+68-648+72\sqrt{17}+256=0$$

$$0=0$$

The solution set is $\left\{\left(\frac{9-\sqrt{17}}{8}\right)^4,\left(\frac{9+\sqrt{17}}{8}\right)^4\right\}$.

60. $x^{1/2}-3x^{1/4}+2=0$

Let $u=x^{1/4}$ so that $u^2=x^{1/2}$.

$u^2-3u+2=0$

$(u-2)(u-1)=0$

$u=2$ or $u=1$

$x^{1/4}=2$ or $x^{1/4}=1$

$x=16$ or $x=1$

Check:

$x=16$: $16^{1/2}-3(16)^{1/4}+2=0$

$4-6+2=0$

$0=0$

$x=1$: $1^{1/2}-3(1)^{1/4}+2=0$

$1-3+2=0$

$0=0$

The solution set is $\{1,16\}$.

61. $\sqrt[4]{5x^2-6}=x$

$\left(\sqrt[4]{5x^2-6}\right)^4=x^4$

$5x^2-6=x^4$

$0=x^4-5x^2+6$

Let $u=x^2$ so that $u^2=x^4$.

$0=u^2-5u+6$

$0=(u-3)(u-2)$

$u=3$ or $u=2$

$x^2=3$ or $x^2=2$

$x=\pm\sqrt{3}$ or $x=\pm\sqrt{2}$

Check:

$x=-\sqrt{3}$: $\sqrt[4]{5\left(-\sqrt{3}\right)^2-6}=-\sqrt{3}$

$\sqrt[4]{15-6}=-\sqrt{3}$

$\sqrt[4]{9}\neq-\sqrt{3}$

$x=\sqrt{3}$: $\sqrt[4]{5\left(\sqrt{3}\right)^2-6}=\sqrt{3}$

$\sqrt[4]{15-6}=\sqrt{3}$

$\sqrt[4]{9}=\sqrt{3}$

$\sqrt{3}=\sqrt{3}$

$x=-\sqrt{2}$: $\sqrt[4]{5\left(-\sqrt{2}\right)^2-6}=-\sqrt{2}$

$\sqrt[4]{10-6}=-\sqrt{2}$

$\sqrt[4]{4}\neq-\sqrt{2}$

$x=\sqrt{2}$: $\sqrt[4]{5\left(\sqrt{2}\right)^2-6}=\sqrt{2}$

$\sqrt[4]{10-6}=\sqrt{2}$

$\sqrt[4]{4}=\sqrt{2}$

$\sqrt{2}=\sqrt{2}$

The solution set is $\left\{\sqrt{2},\sqrt{3}\right\}$.

62. $\sqrt[4]{4-5x^2}=x$

$\left(\sqrt[4]{4-5x^2}\right)^4=x^4$

$4-5x^2=x^4$

$0=x^4+5x^2-4$

Let $u=x^2$ so that $u^2=x^4$.

$0=u^2+5u-4$

$$u=\frac{-5\pm\sqrt{5^2-4(1)(-4)}}{2}=\frac{-5\pm\sqrt{41}}{2}$$

$$x^2=\frac{-5\pm\sqrt{41}}{2}$$

$$x=\pm\sqrt{\frac{-5\pm\sqrt{41}}{2}}$$

Since $-5-\sqrt{41}<0$, $x=\pm\sqrt{\frac{-5-\sqrt{41}}{2}}$ is not real.

Since x is a fourth root, $x=-\sqrt{\frac{-5+\sqrt{41}}{2}}$ is also not real. Therefore, we have only one possible solution to check: $x=\sqrt{\frac{-5+\sqrt{41}}{2}}$:

Check $x=\sqrt{\frac{-5+\sqrt{41}}{2}}$:

$$\sqrt[4]{4-5\left(\pm\sqrt{\frac{-5+\sqrt{41}}{2}}\right)^2}=\sqrt{\frac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{4-5\left(\frac{-5+\sqrt{41}}{2}\right)}=\sqrt{\frac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\frac{8-5\left(-5+\sqrt{41}\right)}{2}}=\sqrt{\frac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\frac{33-5\sqrt{41}}{2}}=\sqrt{\frac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\frac{66-10\sqrt{41}}{4}}=\sqrt{\frac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\frac{25-10\sqrt{41}+41}{4}}=\sqrt{\frac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\frac{\left(-5+\sqrt{41}\right)^2}{4}}=\sqrt{\frac{-5+\sqrt{41}}{2}}$$
$$\sqrt{\frac{-5+\sqrt{41}}{2}}=\sqrt{\frac{-5+\sqrt{41}}{2}}$$

The solution set is $\left\{\sqrt{\frac{-5+\sqrt{41}}{2}}\right\}$.

63. $x^2+3x+\sqrt{x^2+3x}=6$

Let $u=\sqrt{x^2+3x}$ so that $u^2=x^2+3x$.

$$u^2+u=6$$
$$u^2+u-6=0$$
$$(u+3)(u-2)=0$$
$$u=-3 \quad\text{or}\quad u=2$$
$$\sqrt{x^2+3x}=-3 \quad\text{or}\quad \sqrt{x^2+3x}=2$$
$$\text{Not possible} \quad\text{or}\quad x^2+3x=4$$
$$x^2+3x-4=0$$
$$(x+4)(x-1)=0$$
$$x=-4 \text{ or } x=1$$

Check $x=-4$:

$$(-4)^2+3(-4)+\sqrt{(-4)^2+3(-4)}=6$$
$$16-12+\sqrt{16-12}=6$$
$$16-12+\sqrt{4}=6$$
$$6=6$$

Check $x=1$:

$$(1)^2+3(1)+\sqrt{(1)^2+3(1)}=6$$
$$1+3+\sqrt{1+3}=6$$
$$4+\sqrt{4}=6$$
$$6=6$$

The solution set is $\{-4,\ 1\}$.

64. $x^2-3x-\sqrt{x^2-3x}=2$

Let $u=\sqrt{x^2-3x}$ so that $u^2=x^2-3x$.

$$u^2-u=2$$
$$u^2-u-2=0$$
$$(u+1)(u-2)=0$$
$$u=-1 \quad\text{or}\quad u=2$$
$$\sqrt{x^2-3x}=-1 \quad\text{or}\quad \sqrt{x^2-3x}=2$$
$$\text{Not possible} \quad\text{or}\quad x^2-3x=4$$
$$x^2-3x-4=0$$
$$(x-4)(x+1)=0$$
$$x=4 \text{ or } x=-1$$

Check $x=4$:

$$(4)^2-3(4)-\sqrt{(4)^2-3(4)}=16-12-\sqrt{4}$$
$$=4-2=2$$

Check $x=-1$:

$$(-1)^2-3(-1)-\sqrt{(-1)^2-3(-1)}=1+3-\sqrt{4}$$
$$=4-2=2$$

The solution set is $\{-1,\ 4\}$.

65. $\frac{1}{(x+1)^2}=\frac{1}{x+1}+2$

Let $u=\frac{1}{x+1}$ so that $u^2=\left(\frac{1}{x+1}\right)^2$.

$$u^2=u+2$$
$$u^2-u-2=0$$
$$(u+1)(u-2)=0$$
$$u=-1 \quad\text{or}\quad u=2$$
$$\frac{1}{x+1}=-1 \quad\text{or}\quad \frac{1}{x+1}=2$$
$$1=-x-1 \quad\text{or}\quad 1=2x+2$$
$$x=-2 \quad\text{or}\quad -2x=1$$
$$x=-\frac{1}{2}$$

Check:

$x=-2$: $\dfrac{1}{(-2+1)^2}=\dfrac{1}{-2+1}+2$

$1=-1+2$

$1=1$

$x=-\frac{1}{2}$: $\dfrac{1}{\left(-\frac{1}{2}+1\right)^2}=\dfrac{1}{\left(-\frac{1}{2}+1\right)}+2$

$4=2+2$

$4=4$

The solution set is $\left\{-2,-\frac{1}{2}\right\}$.

66. $\dfrac{1}{(x-1)^2}+\dfrac{1}{x-1}=12$

Let $u=\dfrac{1}{x-1}$ so that $u^2=\left(\dfrac{1}{x-1}\right)^2$.

$u^2+u=12$

$u^2+u-12=0$

$(u+4)(u-3)=0$

$u=-4$ or $u=3$

$\dfrac{1}{x-1}=-4$ or $\dfrac{1}{x-1}=3$

$1=-4x+4$ or $1=3x-3$

$4x=3$ or $4=3x$

$x=\dfrac{3}{4}$ or $x=\dfrac{4}{3}$

Check:

$x=\frac{3}{4}$: $\dfrac{1}{\left(\frac{3}{4}-1\right)^2}+\dfrac{1}{\left(\frac{3}{4}-1\right)}=12$

$\dfrac{1}{\left(\frac{1}{16}\right)}+\dfrac{1}{\left(-\frac{1}{4}\right)}=12$

$16-4=12$

$12=12$

$x=\frac{4}{3}$: $\dfrac{1}{\left(\frac{4}{3}-1\right)^2}+\dfrac{1}{\left(\frac{4}{3}-1\right)}=12$

$\dfrac{1}{\left(\frac{1}{9}\right)}+\dfrac{1}{\left(\frac{1}{3}\right)}=12$

$9+3=12$

$12=12$

The solution set is $\left\{\frac{3}{4},\frac{4}{3}\right\}$.

67. $3x^{-2}-7x^{-1}-6=0$

Let $u=x^{-1}$ so that $u^2=x^{-2}$.

$3u^2-7u-6=0$

$(3u+2)(u-3)=0$

$u=-\dfrac{2}{3}$ or $u=3$

$x^{-1}=-\dfrac{2}{3}$ or $x^{-1}=3$

$\left(x^{-1}\right)^{-1}=\left(-\dfrac{2}{3}\right)^{-1}$ or $\left(x^{-1}\right)^{-1}=(3)^{-1}$

$x=-\dfrac{3}{2}$ or $x=\dfrac{1}{3}$

Check:

$x=-\frac{3}{2}$: $3\left(-\frac{3}{2}\right)^{-2}-7\left(-\frac{3}{2}\right)^{-1}-6=0$

$3\left(\frac{4}{9}\right)-7\left(-\frac{2}{3}\right)-6=0$

$\dfrac{4}{3}+\dfrac{14}{3}-6=0$

$0=0$

$x=\frac{1}{3}$: $3\left(\dfrac{1}{3}\right)^{-2}-7\left(\dfrac{1}{3}\right)^{-1}-6=0$

$3(9)-7(3)-6=0$

$27-21-6=0$

$0=0$

The solution set is $\left\{-\dfrac{3}{2},\dfrac{1}{3}\right\}$.

68. $2x^{-2}-3x^{-1}-4=0$

Let $u=x^{-1}$ so that $u^2=x^{-2}$.

$2u^2-3u-4=0$

$u=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(2)(-4)}}{2(2)}=\dfrac{3\pm\sqrt{41}}{4}$

$u=\dfrac{3+\sqrt{41}}{4}$ or $u=\dfrac{3-\sqrt{41}}{4}$

$x^{-1}=\dfrac{3+\sqrt{41}}{4}$ or $x^{-1}=\dfrac{3-\sqrt{41}}{4}$

$\left(x^{-1}\right)^{-1}=\left(\dfrac{3+\sqrt{41}}{4}\right)^{-1}$ or $\left(x^{-1}\right)^{-1}=\left(\dfrac{3-\sqrt{41}}{4}\right)^{-1}$

$x=\dfrac{4}{3+\sqrt{41}}\left(\dfrac{3-\sqrt{41}}{3-\sqrt{41}}\right)$ or $x=\dfrac{4}{3-\sqrt{41}}\left(\dfrac{3+\sqrt{41}}{3+\sqrt{41}}\right)$

$=\dfrac{12-4\sqrt{41}}{-32}$ $\qquad$ $=\dfrac{12+4\sqrt{41}}{-32}$

$=\dfrac{-3+\sqrt{41}}{8}$ $\qquad$ $=\dfrac{-3-\sqrt{41}}{8}$

Check $x=\frac{-3+\sqrt{41}}{8}$:

$$2\left(\frac{-3+\sqrt{41}}{8}\right)^{-2}-3\left(\frac{-3+\sqrt{41}}{8}\right)^{-1}-4=0$$

$$2\left(\frac{64}{\left(-3+\sqrt{41}\right)^2}\right)-3\left(\frac{8}{-3+\sqrt{41}}\right)-4=0$$

$$2(64)-3(8)\left(-3+\sqrt{41}\right)-4\left(-3+\sqrt{41}\right)^2=0$$

$$128+72-24\sqrt{41}-4\left(9-6\sqrt{41}+41\right)=0$$

$$128+72-24\sqrt{41}-36+24\sqrt{41}-164=0$$

$$0=0$$

Check $x=\frac{-3-\sqrt{41}}{8}$:

$$2\left(\frac{-3-\sqrt{41}}{8}\right)^{-2}-3\left(\frac{-3-\sqrt{41}}{8}\right)^{-1}-4=0$$

$$2\left(\frac{64}{\left(-3-\sqrt{41}\right)^2}\right)-3\left(\frac{8}{-3-\sqrt{41}}\right)-4=0$$

$$2(64)-3(8)\left(-3-\sqrt{41}\right)-4\left(-3-\sqrt{41}\right)^2=0$$

$$128+72+24\sqrt{41}-4\left(9+6\sqrt{41}+41\right)=0$$

$$128+72+24\sqrt{41}-36-24\sqrt{41}-164=0$$

$$0=0$$

The solution set is $\left\{\frac{-3-\sqrt{41}}{8},\frac{-3+\sqrt{41}}{8}\right\}$.

69. $2x^{2/3}-5x^{1/3}-3=0$

Let $u=x^{1/3}$ so that $u^2=x^{2/3}$.

$$2u^2-5u-3=0$$

$$(2u+1)(u-3)=0$$

$$u=-\frac{1}{2} \quad \text{or} \quad u=3$$

$$x^{1/3}=-\frac{1}{2} \quad \text{or} \quad x^{1/3}=3$$

$$\left(x^{1/3}\right)^3=\left(-\frac{1}{2}\right)^3 \quad \text{or} \quad \left(x^{1/3}\right)^3=(3)^3$$

$$x=-\frac{1}{8} \quad \text{or} \quad x=27$$

Check $x=-\frac{1}{8}$: $2\left(-\frac{1}{8}\right)^{2/3}-5\left(-\frac{1}{8}\right)^{1/3}-3=0$

$$2\left(\frac{1}{4}\right)-5\left(-\frac{1}{2}\right)-3=0$$

$$\frac{1}{2}+\frac{5}{2}-3=0$$

$$3-3=0$$

$$0=0$$

Check $x=27$: $2(27)^{2/3}-5(27)^{1/3}-3=0$

$$2(9)-5(3)-3=0$$

$$18-15-3=0$$

$$3-3=0$$

$$0=0$$

The solution set is $\left\{-\frac{1}{8},27\right\}$.

70. $3x^{4/3}+5x^{2/3}-2=0$

Let $u=x^{2/3}$ so that $u^2=x^{4/3}$.

$$3u^2+5u-2=0$$

$$(3u-1)(u+2)=0$$

$$u=\frac{1}{3} \quad \text{or} \quad u=-2$$

$$x^{2/3}=\frac{1}{3} \quad \text{or} \quad x^{2/3}=-2$$

$$\left(x^{2/3}\right)^3=\left(\frac{1}{3}\right)^3 \quad \text{or} \quad \left(x^{2/3}\right)^3=(-2)^3$$

$$x^2=\frac{1}{27} \quad \text{or} \quad x^2=-8$$

$$x=\pm\sqrt{\frac{1}{27}} \qquad \text{not real}$$

Check: $3\left(\pm\sqrt{\frac{1}{27}}\right)^{4/3}+5\left(\pm\sqrt{\frac{1}{27}}\right)^{2/3}-2=0$

$$3\left(\frac{1}{27}\right)^{2/3}+5\left(\pm\frac{1}{27}\right)^{1/3}-2=0$$

$$3\left(\frac{1}{3}\right)^2+5\left(\frac{1}{3}\right)-2=0$$

$$3\left(\frac{1}{9}\right)+\frac{5}{3}-2=0$$

$$\frac{1}{3}+\frac{5}{3}-2=0$$

$$2-2=0$$

$$0=0$$

Note: $\pm\sqrt{\frac{1}{27}}=\pm\sqrt{\frac{3}{81}}=\pm\frac{\sqrt{3}}{9}$

The solution set is $\left\{-\frac{\sqrt{3}}{9},\frac{\sqrt{3}}{9}\right\}$.

71. $\left(\frac{v}{v+1}\right)^2+\frac{2v}{v+1}=8$

$\left(\frac{v}{v+1}\right)^2+2\left(\frac{v}{v+1}\right)=8$

Let $u=\frac{v}{v+1}$ so that $u^2=\left(\frac{v}{v+1}\right)^2$.

$u^2+2u=8$

$u^2+2u-8=0$

$(u+4)(u-2)=0$

$u=-4$ or $u=2$

$\frac{v}{v+1}=-4$ or $\frac{v}{v+1}=2$

$v=-4v-4$ or $v=2v+2$

$v=-\frac{4}{5}$ or $v=-2$

Check $v=-\frac{4}{5}$: $\left(\frac{-\frac{4}{5}}{-\frac{4}{5}+1}\right)^2+\frac{2\left(-\frac{4}{5}\right)}{\left(-\frac{4}{5}\right)+1}=8$

$\frac{\left(\frac{16}{25}\right)}{\left(\frac{1}{25}\right)}+\frac{\left(-\frac{8}{5}\right)}{\left(\frac{1}{5}\right)}=8$

$16-8=8$

$8=8$

Check $v=-2$: $\left(\frac{-2}{-2+1}\right)^2+\frac{2(-2)}{(-2)+1}=8$

$4+4=8$

$8=8$

The solution set is $\left\{-2,-\frac{4}{5}\right\}$.

72. $\left(\frac{y}{y-1}\right)^2=6\left(\frac{y}{y-1}\right)+7$

Let $u=\frac{y}{y-1}$ so that $u^2=\left(\frac{y}{y-1}\right)^2$.

$u^2=6u+7$

$u^2-6u-7=0$

$(u-7)(u+1)=0$

$u=-1$ or $u=7$

$\frac{y}{y-1}=-1$ or $\frac{y}{y-1}=7$

$y=-y+1$ or $y=7y-7$

$2y=1$ or $-6y=-7$

$y=\frac{1}{2}$ or $y=\frac{7}{6}$

Check $y=\frac{1}{2}$: $\left(\frac{\frac{1}{2}}{\frac{1}{2}-1}\right)^2=6\left(\frac{\frac{1}{2}}{\frac{1}{2}-1}\right)+7$

$\frac{\frac{1}{4}}{\frac{1}{4}}=6\cdot\frac{\frac{1}{2}}{\left(-\frac{1}{2}\right)}+7$

$1=6(-1)+7$

$1=1$

Check $y=\frac{7}{6}$: $\left(\frac{\frac{7}{6}}{\frac{7}{6}-1}\right)^2=6\left(\frac{\frac{7}{6}}{\frac{7}{6}-1}\right)+7$

$\frac{\left(\frac{49}{36}\right)}{\left(\frac{1}{36}\right)}=6\left(\frac{\left(\frac{7}{6}\right)}{\left(\frac{1}{6}\right)}\right)+7$

$49=42+7$

$49=49$

The solution set is $\left\{\frac{1}{2},\frac{7}{6}\right\}$.

73. $x^3-9x=0$

$x\left(x^2-9\right)=0$

$x(x-3)(x+3)=0$

$x=0$ or $x-3=0$ $x+3=0$

$x=3$ $x=-3$

The solution set is $\{-3,0,3\}$.

74. $x^4-x^2=0$

$x^2\left(x^2-1\right)=0$

$x^2(x-1)(x+1)=0$

$x^2=0$ or $x-1=0$ or $x+1=0$

$x=0$ $x=1$ $x=-1$

The solution set is $\{-1,0,1\}$.

75.
$$4x^3 = 3x^2$$
$$4x^3 - 3x^2 = 0$$
$$x^2(4x-3) = 0$$
$$x^2 = 0 \quad \text{or} \quad 4x - 3 = 0$$
$$x = 0 \qquad 4x = 3$$
$$x = \frac{3}{4}$$
The solution set is $\left\{0, \frac{3}{4}\right\}$.

76.
$$x^5 = 4x^3$$
$$x^5 - 4x^3 = 0$$
$$x^3\left(x^2 - 4\right) = 0$$
$$x^3(x-2)(x+2) = 0$$
$$x^3 = 0 \quad \text{or} \quad x - 2 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 0 \qquad x = 2 \qquad x = -2$$
The solution set is $\{-2, 0, 2\}$.

77.
$$x^3 + x^2 - 20x = 0$$
$$x\left(x^2 + x - 20\right) = 0$$
$$x(x+5)(x-4) = 0$$
$$x = 0 \quad \text{or} \quad x + 5 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -5 \qquad x = 4$$
The solution set is $\{-5, 0, 4\}$.

78.
$$x^3 + 6x^2 - 7x = 0$$
$$x\left(x^2 + 6x - 7\right) = 0$$
$$x(x+7)(x-1) = 0$$
$$x = 0 \quad \text{or} \quad x + 7 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -7 \qquad x = 1$$
The solution set is $\{-7, 0, 1\}$.

79.
$$x^3 + x^2 - x - 1 = 0$$
$$x^2(x+1) - 1(x+1) = 0$$
$$(x+1)\left(x^2 - 1\right) = 0$$
$$(x+1)(x-1)(x+1) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -1 \qquad x = 1$$
The solution set is $\{-1, 1\}$.

80.
$$x^3 + 4x^2 - x - 4 = 0$$
$$x^2(x+4) - 1(x+4) = 0$$
$$(x+4)\left(x^2 - 1\right) = 0$$
$$(x+4)(x-1)(x+1) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 1 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -4 \qquad x = 1 \qquad x = -1$$
The solution set is $\{-4, -1, 1\}$.

81.
$$x^3 - 3x^2 - 4x + 12 = 0$$
$$x^2(x-3) - 4(x-3) = 0$$
$$(x-3)\left(x^2 - 4\right) = 0$$
$$(x-3)(x-2)(x+2) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x - 2 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 3 \qquad x = 2 \qquad x = -2$$
The solution set is $\{-2, 2, 3\}$.

82.
$$x^3 - 3x^2 - x + 3 = 0$$
$$x^2(x-3) - 1(x-3) = 0$$
$$(x-3)\left(x^2 - 1\right) = 0$$
$$(x-3)(x-1)(x+1) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x - 1 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 3 \qquad x = 1 \qquad x = -1$$
The solution set is $\{-1, 1, 3\}$.

83.
$$2x^3 + 4 = x^2 + 8x$$
$$2x^3 - x^2 - 8x + 4 = 0$$
$$x^2(2x-1) - 4(2x-1) = 0$$
$$(2x-1)\left(x^2 - 4\right) = 0$$
$$(2x-1)(x-2)(x+2) = 0$$
$$2x - 1 = 0 \quad \text{or} \quad x - 2 = 0 \quad \text{or} \quad x + 2 = 0$$
$$2x = 1 \qquad x = 2 \qquad x = -2$$
$$x = \frac{1}{2}$$
The solution set is $\left\{-2, \frac{1}{2}, 2\right\}$.

84. $$3x^3+4x^2=27x+36$$
$$3x^3+4x^2-27x-36=0$$
$$x^2(3x+4)-9(3x+4)=0$$
$$(3x+4)(x^2-9)=0$$
$$(3x+4)(x-3)(x+3)=0$$
$$3x+4=0 \quad \text{or} \quad x-3=0 \quad \text{or} \quad x+3=0$$
$$3x=-4 \qquad x=3 \qquad x=-3$$
$$x=-\frac{4}{3}$$

The solution set is $\left\{-3,-\frac{4}{3},3\right\}$.

85. $$5x^3+45x=2x^2+18$$
$$5x^3-2x^2+45x-18=0$$
$$x^2(5x-2)+9(5x-2)=0$$
$$(5x-2)(x^2+9)=0$$
$$5x-2=0 \quad \text{or} \quad x^2+9=0$$
$$5x=2 \qquad x^2=-9$$
$$x=\frac{2}{5} \qquad \text{no real solutions}$$

The solution set is $\left\{\frac{2}{5}\right\}$.

86. $$3x^3+12x=5x^2+20$$
$$3x^3-5x^2+12x-20=0$$
$$x^2(3x-5)+4(3x-5)=0$$
$$(3x-5)(x^2+4)=0$$
$$3x-5=0 \quad \text{or} \quad x^2+4=0$$
$$3x=5 \qquad x^2=-4$$
$$x=\frac{5}{3} \qquad \text{no real solutions}$$

The solution set is $\left\{\frac{5}{3}\right\}$.

87. $$x\left(x^2-3x\right)^{1/3}+2\left(x^2-3x\right)^{4/3}=0$$
$$\left(x^2-3x\right)^{1/3}\left[x+2\left(x^2-3x\right)\right]=0$$
$$\left(x^2-3x\right)^{1/3}\left(x+2x^2-6x\right)=0$$
$$\left(x^2-3x\right)^{1/3}\left(2x^2-5x\right)=0$$
$$\left(x^2-3x\right)^{1/3}=0 \quad \text{or} \quad 2x^2-5x=0$$
$$x^2-3x=0 \quad \text{or} \quad 2x^2-5x=0$$
$$x(x-3)=0 \quad \text{or} \quad x(2x-5)=0$$
$$x=0 \text{ or } x=3 \quad \text{or} \quad x=0 \text{ or } x=\frac{5}{2}$$

The solution set is $\left\{0,\frac{5}{2},3\right\}$.

88. $$3x\left(x^2+2x\right)^{1/2}-2\left(x^2+2x\right)^{3/2}=0$$
$$\left(x^2+2x\right)^{1/2}\left[3x-2\left(x^2+2x\right)\right]=0$$
$$\left(x^2+2x\right)^{1/2}\left(3x-2x^2-4x\right)=0$$
$$\left(x^2+2x\right)^{1/2}\left(-2x^2-x\right)=0$$
$$\left(x^2+2x\right)^{1/2}=0 \quad \text{or} \quad -2x^2-x=0$$
$$x^2+2x=0 \quad \text{or} \quad 2x^2+x=0$$
$$x(x+2)=0 \quad \text{or} \quad x(2x+1)=0$$
$$x=0 \text{ or } x=-2 \quad \text{or} \quad x=0 \text{ or } x=-\frac{1}{2}$$

Check $x=0$:
$$3\cdot 0\left(0^2+2\cdot 0\right)^{1/2}-2\left(0^2+2\cdot 0\right)^{3/2}=0$$
$$3\cdot 0(0)^{1/2}-2(0)^{3/2}=0$$
$$0=0$$

Check $x=-2$:
$$3(-2)\left((-2)^2+2(-2)\right)^{1/2}-2\left((-2)^2+2(-2)\right)^{3/2}=0$$
$$3(-2)(4-4)^{1/2}-2(4-4)^{3/2}=0$$
$$3(-2)(0)^{1/2}-2(0)^{3/2}=0$$
$$3(-2)(0)-2(0)=0$$
$$0=0$$

Check $x=-\frac{1}{2}$:
$$3\left(-\tfrac{1}{2}\right)\left(\left(-\tfrac{1}{2}\right)^2+2\left(-\tfrac{1}{2}\right)\right)^{1/2}-2\left(\left(-\tfrac{1}{2}\right)^2+2\left(-\tfrac{1}{2}\right)\right)^{3/2}=0$$
$$3\left(-\tfrac{1}{2}\right)\left(\tfrac{1}{4}-1\right)^{1/2}-2\left(\tfrac{1}{4}-1\right)^{3/2}=0$$
$$3\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{4}\right)^{1/2}-2\left(-\tfrac{3}{4}\right)^{3/2}=0$$
Not real

The solution set is $\{-2,0\}$.

89. $x - 4x^{1/2} + 2 = 0$

Let $u = x^{1/2}$ so that $u^2 = x^2$.

$u^2 - 4u + 2 = 0$

$$u = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$$u = 2 + \sqrt{2} \quad \text{or} \quad u = 2 - \sqrt{2}$$
$$x^{1/2} = 2 + \sqrt{2} \quad \text{or} \quad x^{1/2} = 2 - \sqrt{2}$$
$$\left(x^{1/2}\right)^2 = \left(2 + \sqrt{2}\right)^2 \quad \text{or} \quad \left(x^{1/2}\right)^2 = \left(2 - \sqrt{2}\right)^2$$
$$x = \left(2 + \sqrt{2}\right)^2 \quad \text{or} \quad x = \left(2 - \sqrt{2}\right)^2$$

Check $x = \left(2 + \sqrt{2}\right)^2$:

$$\left(2 + \sqrt{2}\right)^2 - 4\left(2 + \sqrt{2}\right) + 2 = 0$$
$$4 + 4\sqrt{2} + 2 - 8 - 4\sqrt{2} + 2 = 0$$
$$0 = 0$$

Check $x = \left(2 - \sqrt{2}\right)^2$:

$$\left(2 - \sqrt{2}\right)^2 - 4\left(2 - \sqrt{2}\right) + 2 = 0$$
$$4 - 4\sqrt{2} + 2 - 8 + 4\sqrt{2} + 2 = 0$$
$$0 = 0$$

The solution set is $\left\{\left(2 - \sqrt{2}\right)^2,\ \left(2 + \sqrt{2}\right)^2\right\} \approx \{0.34,\ 11.66\}$.

90. $x^{2/3} + 4x^{1/3} + 2 = 0$

Let $u = x^{1/3}$ so that $u^2 = x^{2/3}$.

$u^2 + 4u + 2 = 0$

$$u = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

$$u = -2 + \sqrt{2} \quad \text{or} \quad u = -2 - \sqrt{2}$$
$$x^{1/3} = -2 + \sqrt{2} \quad \text{or} \quad x^{1/3} = -2 - \sqrt{2}$$
$$x = \left(-2 + \sqrt{2}\right)^3 \quad \text{or} \quad x = \left(-2 - \sqrt{2}\right)^3$$

Check $x = \left(-2 + \sqrt{2}\right)^3$:

$$\left(\left(-2 + \sqrt{2}\right)^3\right)^{2/3} + 4\left(\left(-2 + \sqrt{2}\right)^3\right)^{1/3} + 2 = 0$$
$$\left(-2 + \sqrt{2}\right)^2 + 4\left(-2 + \sqrt{2}\right) + 2 = 0$$
$$4 - 4\sqrt{2} + 2 - 8 + 4\sqrt{2} + 2 = 0$$
$$0 = 0$$

Check $x = \left(-2 - \sqrt{2}\right)^3$:

$$\left(\left(-2 - \sqrt{2}\right)^3\right)^{2/3} + 4\left(\left(-2 - \sqrt{2}\right)^3\right)^{1/3} + 2 = 0$$
$$\left(-2 - \sqrt{2}\right)^2 + 4\left(-2 - \sqrt{2}\right) + 2 = 0$$
$$4 + 4\sqrt{2} + 2 - 8 - 4\sqrt{2} + 2 = 0$$
$$0 = 0$$

The solution set is $\left\{\left(-2 - \sqrt{2}\right)^3,\ \left(-2 + \sqrt{2}\right)^3\right\} \approx \{-39.80,\ -0.20\}$.

91. $x^4 + \sqrt{3}x^2 - 3 = 0$

Let $u = x^2$ so that $u^2 = x^4$.

$u^2 + \sqrt{3}u - 3 = 0$

$$u = \frac{-\sqrt{3} \pm \sqrt{\left(\sqrt{3}\right)^2 - 4(1)(-3)}}{2(1)} = \frac{-\sqrt{3} \pm \sqrt{15}}{2}$$

$$u = \frac{-\sqrt{3} + \sqrt{15}}{2} \quad \text{or} \quad u = \frac{-\sqrt{3} - \sqrt{15}}{2}$$
$$x^2 = \frac{-\sqrt{3} + \sqrt{15}}{2} \quad \text{or} \quad x^2 = \frac{-\sqrt{3} - \sqrt{15}}{2}$$
$$x = \pm\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}} \quad \text{or} \quad x = \pm\sqrt{\frac{-\sqrt{3} - \sqrt{15}}{2}}$$

Not real

Check $x = \sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}}$:

$$\left(\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}}\right)^4 + \sqrt{3}\left(\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}}\right)^2 - 3 = 0$$
$$\left(\frac{-\sqrt{3} + \sqrt{15}}{2}\right)^2 + \sqrt{3}\left(\frac{-\sqrt{3} + \sqrt{15}}{2}\right) - 3 = 0$$
$$\frac{3 - 2\sqrt{3}\sqrt{15} + 15}{4} + \frac{\sqrt{3}\left(-\sqrt{3}\right) + \sqrt{3}\sqrt{15}}{2} - 3 = 0$$
$$\frac{18 - 2\sqrt{45}}{4} + \frac{-3 + \sqrt{45}}{2} - 3 = 0$$
$$\frac{9 - \sqrt{45}}{2} + \frac{-3 + \sqrt{45}}{2} - 3 = 0$$
$$\frac{9 - \sqrt{45} - 3 + \sqrt{45}}{2} - 3 = 0$$
$$3 - 3 = 0$$
$$0 = 0$$

Check $x = -\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}}$:

$$\left(-\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}\right)^4+\sqrt{3}\left(-\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}\right)^2-3=0$$
$$\left(\frac{-\sqrt{3}+\sqrt{15}}{2}\right)^2+\sqrt{3}\left(\frac{-\sqrt{3}+\sqrt{15}}{2}\right)-3=0$$
$$\frac{3-2\sqrt{3}\sqrt{15}+15}{4}+\frac{\sqrt{3}\left(-\sqrt{3}\right)+\sqrt{3}\sqrt{15}}{2}-3=0$$
$$\frac{18-2\sqrt{45}}{4}+\frac{-3+\sqrt{45}}{2}-3=0$$
$$\frac{9-\sqrt{45}}{2}+\frac{-3+\sqrt{45}}{2}-3=0$$
$$\frac{9-\sqrt{45}-3+\sqrt{45}}{2}-3=0$$
$$3-3=0$$
$$0=0$$

The solution set is
$\left\{-\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}},\ \sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}\right\}\approx\{-1.03,\ 1.03\}$.

92. $x^4+\sqrt{2}x^2-2=0$

Let $u=x^2$ so that $u^2=x^4$.

$u^2+\sqrt{2}u-2=0$

$$u=\frac{-\sqrt{2}\pm\sqrt{\left(\sqrt{2}\right)^2-4(1)(-2)}}{2(1)}=\frac{-\sqrt{2}\pm\sqrt{10}}{2}$$

$u=\frac{-\sqrt{2}+\sqrt{10}}{2}$ or $u=\frac{-\sqrt{2}-\sqrt{10}}{2}$

$x^2=\frac{-\sqrt{2}+\sqrt{10}}{2}$ or $x^2=\frac{-\sqrt{2}-\sqrt{10}}{2}$

$x=\pm\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}$ or $x=\pm\sqrt{\frac{-\sqrt{2}-\sqrt{10}}{2}}$
Not real

Check $x=\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}$:

$$\left(\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right)^4+\sqrt{2}\left(\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right)^2-2=0$$
$$\left(\frac{-\sqrt{2}+\sqrt{10}}{2}\right)^2+\sqrt{2}\left(\frac{-\sqrt{2}+\sqrt{10}}{2}\right)-2=0$$
$$\frac{12-2\sqrt{20}}{4}+\frac{-2+\sqrt{20}}{2}-2=0$$
$$\frac{6-\sqrt{20}-2+\sqrt{20}}{2}-2=0$$
$$2-2=0$$
$$0=0$$

Check $x=-\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}$:

$$\left(-\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right)^4+\sqrt{2}\left(-\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right)^2-2=0$$
$$\left(\frac{-\sqrt{2}+\sqrt{10}}{2}\right)^2+\sqrt{2}\left(\frac{-\sqrt{2}+\sqrt{10}}{2}\right)-2=0$$
$$\frac{12-2\sqrt{20}}{4}+\frac{-2+\sqrt{20}}{2}-2=0$$
$$\frac{6-\sqrt{20}-2+\sqrt{20}}{2}-2=0$$
$$2-2=0$$
$$0=0$$

The solution set is
$\left\{-\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}},\ \sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right\}\approx\{-0.93,\ 0.93\}$.

93. $\pi(1+t)^2=\pi+1+t$

Let $u=1+t$ so that $u^2=(1+t)^2$.

$\pi u^2=\pi+u$

$\pi u^2-u-\pi=0$

$$u=\frac{-(-1)\pm\sqrt{(-1)^2-4(\pi)(-\pi)}}{2(\pi)}=\frac{1\pm\sqrt{1+4\pi^2}}{2\pi}$$

$$1+t=\frac{1\pm\sqrt{1+4\pi^2}}{2\pi}$$
$$t=-1+\frac{1\pm\sqrt{1+4\pi^2}}{2\pi}$$

Check $t=-1+\frac{1+\sqrt{1+4\pi^2}}{2\pi}$:

$$\pi\left(\frac{1+\sqrt{1+4\pi^2}}{2\pi}\right)^2=\pi+\frac{1+\sqrt{1+4\pi^2}}{2\pi}$$
$$\pi\left(\frac{1+2\sqrt{1+4\pi^2}+1+4\pi^2}{4\pi^2}\right)=\pi+\frac{1+\sqrt{1+4\pi^2}}{2\pi}$$
$$\frac{2+2\sqrt{1+4\pi^2}+4\pi^2}{4\pi}=\frac{2\pi^2+1+\sqrt{1+4\pi^2}}{2\pi}$$
$$\frac{1+\sqrt{1+4\pi^2}+2\pi^2}{2\pi}=\frac{2\pi^2+1+\sqrt{1+4\pi^2}}{2\pi}$$

Check $t=-1+\frac{1-\sqrt{1+4\pi^2}}{2\pi}$:

$$\pi\left(\frac{1-\sqrt{1+4\pi^2}}{2\pi}\right)^2 = \pi + \frac{1-\sqrt{1+4\pi^2}}{2\pi}$$

$$\pi\left(\frac{1-2\sqrt{1+4\pi^2}+1+4\pi^2}{4\pi^2}\right) = \pi + \frac{1-\sqrt{1+4\pi^2}}{2\pi}$$

$$\frac{2-2\sqrt{1+4\pi^2}+4\pi^2}{4\pi} = \frac{2\pi^2+1-\sqrt{1+4\pi^2}}{2\pi}$$

$$\frac{1-\sqrt{1+4\pi^2}+2\pi^2}{2\pi} = \frac{2\pi^2+1-\sqrt{1+4\pi^2}}{2\pi}$$

The solution set is

$$\left\{-1+\frac{1-\sqrt{1+4\pi^2}}{2\pi}, -1+\frac{1+\sqrt{1+4\pi^2}}{2\pi}\right\}$$

$\approx\{-1.85,\ 0.17\}$.

94. $\pi(1+r)^2 = 2+\pi(1+r)$

Let $u = 1+r$ so that $u^2 = (1+r)^2$.

$$\pi u^2 = 2+\pi u$$

$$\pi u^2 - \pi u - 2 = 0$$

$$u = \frac{-(-\pi)\pm\sqrt{(-\pi)^2+4(\pi)(-2)}}{2(\pi)}$$

$$= \frac{\pi\pm\sqrt{\pi^2-8\pi}}{2\pi}$$

$$1+r = \frac{\pi\pm\sqrt{\pi^2+8\pi}}{2\pi}$$

$$r = -1+\frac{\pi\pm\sqrt{\pi^2+8\pi}}{2\pi}$$

Check $r = -1+\frac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}$:

$$\pi\left(\frac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}\right)^2 = 2+\pi\left(\frac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}\right)$$

$$\pi\left(\frac{\pi^2+2\pi\sqrt{\pi^2+8\pi}+\pi^2+8\pi}{4\pi^2}\right) = 2+\pi\left(\frac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}\right)$$

$$\frac{2\pi^2+2\pi\sqrt{\pi^2+8\pi}+8\pi}{4\pi} = 2+\frac{\pi+\sqrt{\pi^2+8\pi}}{2}$$

$$\frac{\pi+\sqrt{\pi^2+8\pi}+4}{2} = \frac{4+\pi+\sqrt{\pi^2+8\pi}}{2}$$

Check $r = -1+\frac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}$:

$$\pi\left(\frac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}\right)^2 = 2+\pi\left(\frac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}\right)$$

$$\pi\left(\frac{\pi^2-2\pi\sqrt{\pi^2+8\pi}+\pi^2+8\pi}{4\pi^2}\right) = 2+\pi\left(\frac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}\right)$$

$$\frac{2\pi^2-2\pi\sqrt{\pi^2+8\pi}+8\pi}{4\pi} = 2+\frac{\pi-\sqrt{\pi^2+8\pi}}{2}$$

$$\frac{\pi-\sqrt{\pi^2+8\pi}+4}{2} = \frac{4+\pi-\sqrt{\pi^2+8\pi}}{2}$$

The solution set is

$$\left\{-1+\frac{\pi-\sqrt{\pi^2+8\pi}}{2\pi},\ -1+\frac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}\right\}$$

$\approx\{-1.44,\ 0.44\}$.

95.

$$k^2-k=12$$

$$k^2-k-12=0$$

$$(k-4)(k+3)=0$$

$k=4$ or $k=-3$

$\frac{x+3}{x-3}=4$ or $\frac{x+3}{x-3}=-3$

$x+3=4x-12$ or $x+3=-3x+9$

$3x=15$ or $4x=6$

$x=5$ or $x=\frac{6}{4}=1.5$

Neither of these values causes a denominator to equal zero, so the solution set is $\{1.5, 5\}$.

96.

$$k^2-3k=28$$

$$k^2-3k-28=0$$

$$(k+4)(k-7)=0$$

$k=-4$ or $k=7$

$\frac{x+3}{x-4}=-4$ or $\frac{x+3}{x-4}=7$

$x+3=-4x+16$ or $x+3=7x-28$

$5x=13$ or $-6x=-31$

$x=\frac{13}{5}=2.6$ or $x=\frac{31}{6}\approx 5.17$

Neither of these values causes a denominator to equal zero, so the solution set is

$\left\{\frac{13}{5},\frac{31}{6}\right\}\approx\{2.6, 5.17\}$.

97. Solve the equation $\frac{\sqrt{s}}{4}+\frac{s}{1100}=4$.

$$\frac{s}{1100}+\frac{\sqrt{s}}{4}-4=0$$
$$(1100)\left(\frac{s}{1100}+\frac{\sqrt{s}}{4}-4\right)=(0)(1100)$$
$$s+275\sqrt{s}-4400=0$$

Let $u=\sqrt{s}$, so that $u^2=s$.

$$u^2+275u-4400=0$$
$$u=\frac{-275\pm\sqrt{275^2-4(1)(-4400)}}{2}=\frac{-275\pm\sqrt{93,225}}{2}$$
$$u\approx 15.1638 \text{ or } u\approx -290.1638$$

Since $u=\sqrt{s}$, it must be positive, so

$$s=u^2\approx(15.1638)^2\approx 229.94$$

The distance to the water's surface is approximately 229.94 feet.

98. $T=\sqrt[4]{\frac{LH^2}{25}}$

Let $T=4$ and $H=10$, and solve for L.

$$4=\sqrt[4]{\frac{L(10)^2}{25}}$$
$$4=\sqrt[4]{4L}$$
$$(4)^4=\left(\sqrt[4]{4L}\right)^4$$
$$256=4L$$
$$64=L$$

The crushing load is 64 tons.

99. $T=2\pi\sqrt{\frac{l}{32}}$

Let $T=16.5$ and solve for l.

$$16.5=2\pi\sqrt{\frac{l}{32}}$$
$$\frac{16.5}{2\pi}=\sqrt{\frac{l}{32}}$$
$$\left(\frac{16.5}{2\pi}\right)^2=\left(\sqrt{\frac{l}{32}}\right)^2$$
$$\left(\frac{16.5}{2\pi}\right)^2=\frac{l}{32}$$
$$l=32\left(\frac{16.5}{2\pi}\right)^2\approx 220.7$$

The length was approximately 220.7 feet.

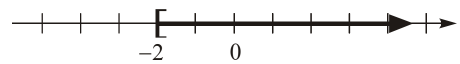
100. Answers will vary. One example: $\sqrt{x+1}=-1$.

101. Answers will vary. One example: $x-\sqrt{x}-2=0$.

102. Answers will vary.

Section 1.5

1. $x\ge -2$



2. False. -5 is to the left of -2 on the number line, so $-5>-2$.
3. negative
4. closed interval
5. multiplication properties (for inequalities)
6. True. This follows from the addition property for inequalities.
7. True. This follows from the addition property for inequalities.
8. True;. This follows from the multiplication property for inequalities.
9. False. Since both sides of the inequality are being divided by a negative number, the sense, or direction, of the inequality must be reversed. That is, $\frac{a}{c}>\frac{b}{c}$.
10. True
11. Interval: $[0,2]$
    Inequality: $0\le x\le 2$
12. Interval: $(-1,2)$
    Inequality: $-1<x<2$
13. Interval: $[2,\infty)$
    Inequality: $x\ge 2$
14. Interval: $(-\infty,0]$
    Inequality: $x\le 0$
15. Interval: $[0,3)$
    Inequality: $0\le x<3$

16. Interval: $(-1,1]$
Inequality: $-1 < x \le 1$

17. **a.**
$$3 < 5$$
$$3+3 < 5+3$$
$$6 < 8$$

b.
$$3 < 5$$
$$3-5 < 5-5$$
$$-2 < 0$$

c.
$$3 < 5$$
$$3(3) < 3(5)$$
$$9 < 15$$

d.
$$3 < 5$$
$$-2(3) > -2(5)$$
$$-6 > -10$$

18. **a.**
$$2 > 1$$
$$2+3 > 1+3$$
$$5 > 4$$

b.
$$2 > 1$$
$$2-5 > 1-5$$
$$-3 > -4$$

c.
$$2 > 1$$
$$3(2) > 3(1)$$
$$6 > 3$$

d.
$$2 > 1$$
$$-2(2) < -2(1)$$
$$-4 < -2$$

19. **a.**
$$4 > -3$$
$$4+3 > -3+3$$
$$7 > 0$$

b.
$$4 > -3$$
$$4-5 > -3-5$$
$$-1 > -8$$

c.
$$4 > -3$$
$$3(4) > 3(-3)$$
$$12 > -9$$

d.
$$4 > -3$$
$$-2(4) < -2(-3)$$
$$-8 < 6$$

20. **a.**
$$-3 > -5$$
$$-3+3 > -5+3$$
$$0 > -2$$

b.
$$-3 > -5$$
$$-3-5 > -5-5$$
$$-8 > -10$$

c.
$$-3 > -5$$
$$3(-3) > 3(-5)$$
$$-9 > -15$$

d.
$$-3 > -5$$
$$-2(-3) < -2(-5)$$
$$6 < 10$$

21. **a.**
$$2x+1 < 2$$
$$2x+1+3 < 2+3$$
$$2x+4 < 5$$

b.
$$2x+1 < 2$$
$$2x+1-5 < 2-5$$
$$2x-4 < -3$$

c.
$$2x+1 < 2$$
$$3(2x+1) < 3(2)$$
$$6x+3 < 6$$

d.
$$2x+1 < 2$$
$$-2(2x+1) > -2(2)$$
$$-4x-2 > -4$$

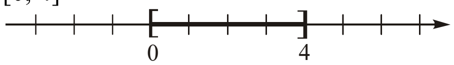
22. **a.**
$$1-2x > 5$$
$$1-2x+3 > 5+3$$
$$4-2x > 8$$

b.
$$1-2x > 5$$
$$1-2x-5 > 5-5$$
$$-4-2x > 0$$

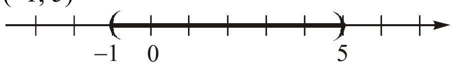
c.
$$1-2x > 5$$
$$3(1-2x) > 3(5)$$
$$3-6x > 15$$

d.
$$1-2x > 5$$
$$-2(1-2x) < -2(5)$$
$$-2+4x < -10$$

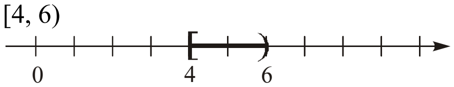
23. $[0, 4]$



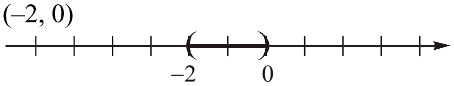
24. $(-1, 5)$



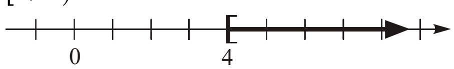
25. $[4, 6)$



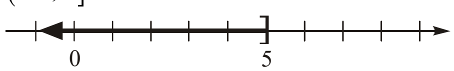
26. $(-2, 0)$



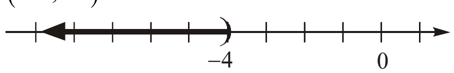
27. $[4, \infty)$



28. $(-\infty, 5]$

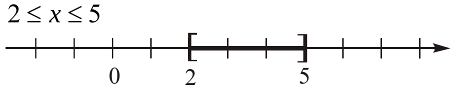


29. $(-\infty, -4)$

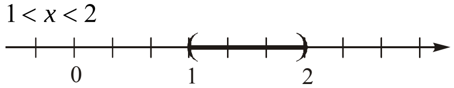


30. $(1, \infty)$

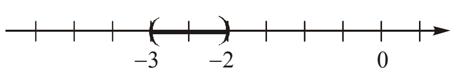
31. $2 \le x \le 5$



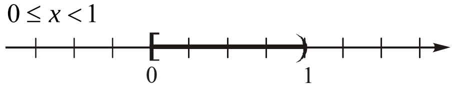
32. $1 < x < 2$



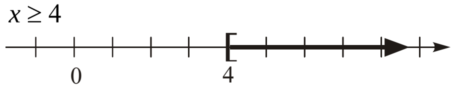
33. $-3 < x < -2$



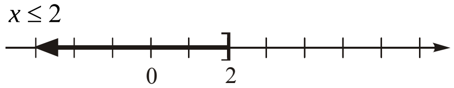
34. $0 \le x < 1$



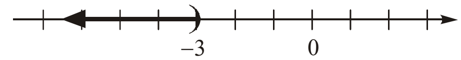
35. $x \ge 4$



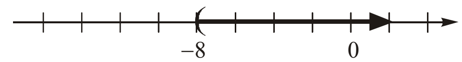
36. $x \le 2$



37. $x < -3$



38. $x > -8$



39. If $x < 5$, then $x - 5 < 0$.

40. If $x < -4$, then $x + 4 < 0$.

41. If $x > -4$, then $x + 4 > 0$.

42. If $x > 6$, then $x - 6 > 0$.

43. If $x \ge -4$, then $3x \ge -12$.

44. If $x \le 3$, then $2x \le 6$.

45. If $x > 6$, then $-2x < -12$.

46. If $x > -2$, then $-4x < 8$.

47. If $x \ge 5$, then $-4x \le -20$.

48. If $x \le -4$, then $-3x \ge 12$.

49. If $2x > 6$, then $x > 3$.

50. If $3x \le 12$, then $x \le 4$.

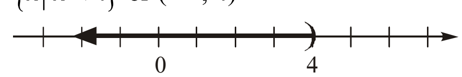
51. If $-\frac{1}{2}x \le 3$, then $x \ge -6$.

52. If $-\frac{1}{4}x > 1$, then $x < -4$.

53.

$$x + 1 < 5$$
$$x + 1 - 1 < 5 - 1$$
$$x < 4$$

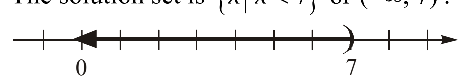
$\{x \mid x < 4\}$ or $(-\infty, 4)$



54.

$$x - 6 < 1$$
$$x - 6 + 6 < 1 + 6$$
$$x < 7$$

The solution set is $\{x \mid x < 7\}$ or $(-\infty, 7)$.

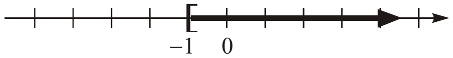


55. $1-2x \le 3$

$-2x \le 2$

$x \ge -1$

The solution set is $\{x \mid x \ge -1\}$ or $[-1, \infty)$.

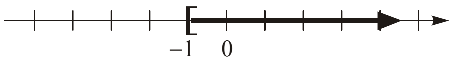


56. $2-3x \le 5$

$-3x \le 3$

$x \ge -1$

The solution set is $\{x \mid x \ge -1\}$ or $[-1, \infty)$.

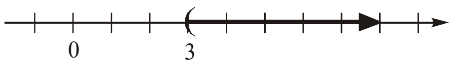


57. $3x-7 > 2$

$3x > 9$

$x > 3$

The solution set is $\{x \mid x > 3\}$ or $(3, \infty)$.

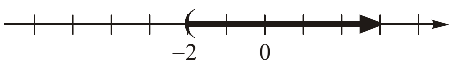


58. $2x+5 > 1$

$2x > -4$

$x > -2$

The solution set is $\{x \mid x > -2\}$ or $(-2, \infty)$.

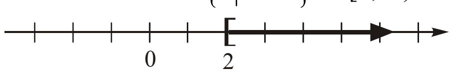


59. $3x-1 \ge 3+x$

$2x \ge 4$

$x \ge 2$

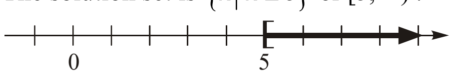
The solution set is $\{x \mid x \ge 2\}$ or $[2, \infty)$.



60. $2x-2 \ge 3+x$

$x \ge 5$

The solution set is $\{x \mid x \ge 5\}$ or $[5, \infty)$.



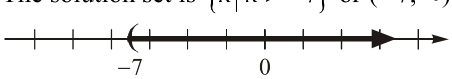
61. $-2(x+3) < 8$

$-2x-6 < 8$

$-2x < 14$

$x > -7$

The solution set is $\{x \mid x > -7\}$ or $(-7, \infty)$.



62. $-3(1-x) < 12$

$-3+3x < 12$

$3x < 15$

$x < 5$

The solution set is $\{x \mid x < 5\}$ or $(-\infty, 5)$.

63. $4-3(1-x) \le 3$

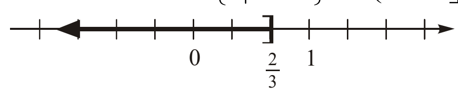
$4-3+3x \le 3$

$3x+1 \le 3$

$3x \le 2$

$x \le \frac{2}{3}$

The solution set is $\left\{x \,\middle|\, x \le \frac{2}{3}\right\}$ or $\left(-\infty, \frac{2}{3}\right]$.



64. $8-4(2-x) \le -2x$

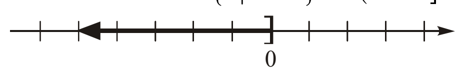
$8-8+4x \le -2x$

$4x \le -2x$

$6x \le 0$

$x \le 0$

The solution set is $\{x \mid x \le 0\}$ or $(-\infty, 0]$.



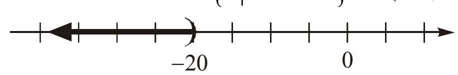
65. $\frac{1}{2}(x-4) > x+8$

$\frac{1}{2}x-2 > x+8$

$-\frac{1}{2}x > 10$

$x < -20$

The solution set is $\{x \mid x < -20\}$ or $(-\infty, -20)$.



66. $3x+4>\frac{1}{3}(x-2)$

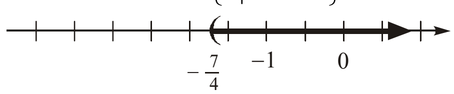
$3x+4>\frac{1}{3}x-\frac{2}{3}$

$9x+12>x-2$

$8x>-14$

$x>-\frac{7}{4}$

The solution set is $\left\{x \middle| x>-\frac{7}{4}\right\}$ or $(-\frac{7}{4},\infty)$.



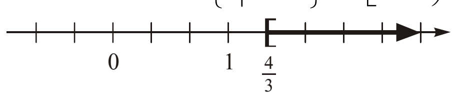
67. $\frac{x}{2}\ge 1-\frac{x}{4}$

$2x\ge 4-x$

$3x\ge 4$

$x\ge \frac{4}{3}$

The solution set is $\left\{x \middle| x\ge\frac{4}{3}\right\}$ or $\left[\frac{4}{3},\infty\right)$.

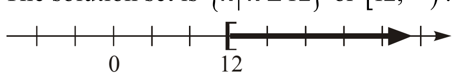


68. $\frac{x}{3}\ge 2+\frac{x}{6}$

$2x\ge 12+x$

$x\ge 12$

The solution set is $\{x \mid x\ge 12\}$ or $[12,\infty)$.

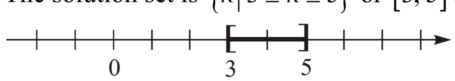


69. $0\le 2x-6\le 4$

$6\le 2x\le 10$

$3\le x\le 5$

The solution set is $\{x \mid 3\le x\le 5\}$ or $[3,5]$.

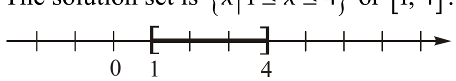


70. $4\le 2x+2\le 10$

$2\le 2x\le 8$

$1\le x\le 4$

The solution set is $\{x \mid 1\le x\le 4\}$ or $[1,4]$.

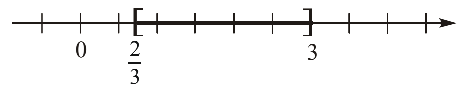


71. $-5\le 4-3x\le 2$

$-9\le -3x\le -2$

$3\ge x\ge \frac{2}{3}$

The solution set is $\left\{x \middle| \frac{2}{3}\le x\le 3\right\}$ or $\left[\frac{2}{3},3\right]$.

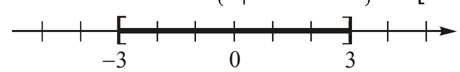


72. $-3\le 3-2x\le 9$

$-6\le -2x\le 6$

$3\ge x\ge -3$

The solution set is $\{x \mid -3\le x\le 3\}$ or $[-3,3]$.



73. $-3<\frac{2x-1}{4}<0$

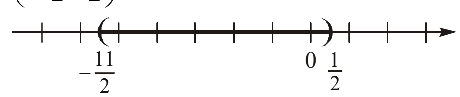
$-12<2x-1<0$

$-11<2x<1$

$-\frac{11}{2}<x<\frac{1}{2}$

The solution set is $\left\{x \middle| -\frac{11}{2}<x<\frac{1}{2}\right\}$ or

$\left(-\frac{11}{2},\frac{1}{2}\right)$.



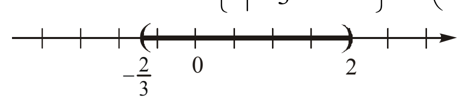
74. $0<\frac{3x+2}{2}<4$

$0<3x+2<8$

$-2<3x<6$

$-\frac{2}{3}<x<2$

The solution set is $\left\{x \middle| -\frac{2}{3}<x<2\right\}$ or $\left(-\frac{2}{3},2\right)$.



75. $1 < 1 - \frac{1}{2}x < 4$

$0 < -\frac{1}{2}x < 3$

$0 > x > -6$ or $-6 < x < 0$

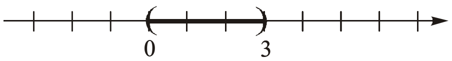
The solution set is $\{x \mid -6 < x < 0\}$ or $(-6, 0)$.

76. $0 < 1 - \frac{1}{3}x < 1$

$-1 < -\frac{1}{3}x < 0$

$3 > x > 0$ or $0 < x < 3$

The solution set is $\{x \mid 0 < x < 3\}$ or $(0, 3)$.



77. $(x+2)(x-3) > (x-1)(x+1)$

$x^2 - x - 6 > x^2 - 1$

$-x - 6 > -1$

$-x > 5$

$x < -5$

The solution set is $\{x \mid x < -5\}$ or $(-\infty, -5)$.

78. $(x-1)(x+1) > (x-3)(x+4)$

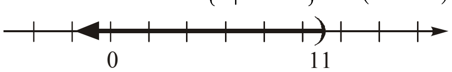
$x^2 - 1 > x^2 + x - 12$

$-1 > x - 12$

$-x > -11$

$x < 11$

The solution set is $\{x \mid x < 11\}$ or $(-\infty, 11)$.



79. $x(4x+3) \le (2x+1)^2$

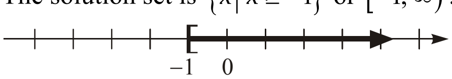
$4x^2 + 3x \le 4x^2 + 4x + 1$

$3x \le 4x + 1$

$-x \le 1$

$x \ge -1$

The solution set is $\{x \mid x \ge -1\}$ or $[-1, \infty)$.



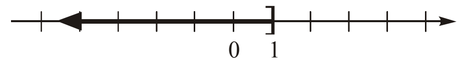
80. $x(9x-5) \le (3x-1)^2$

$9x^2 - 5x \le 9x^2 - 6x + 1$

$-5x \le -6x + 1$

$x \le 1$

The solution set is $\{x \mid x \le 1\}$ or $(-\infty, 1]$.



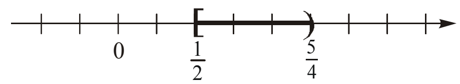
81. $\frac{1}{2} \le \frac{x+1}{3} < \frac{3}{4}$

$6 \le 4x + 4 < 9$

$2 \le 4x < 5$

$\frac{1}{2} \le x < \frac{5}{4}$

The solution set is $\left\{x \middle| \frac{1}{2} \le x < \frac{5}{4}\right\}$ or $\left[\frac{1}{2}, \frac{5}{4}\right)$.



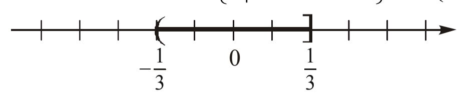
82. $\frac{1}{3} < \frac{x+1}{2} \le \frac{2}{3}$

$2 < 3x + 3 \le 4$

$-1 < 3x \le 1$

$-\frac{1}{3} < x \le \frac{1}{3}$

The solution set is $\left\{x \middle| -\frac{1}{3} < x \le \frac{1}{3}\right\}$ or $\left(-\frac{1}{3}, \frac{1}{3}\right]$.



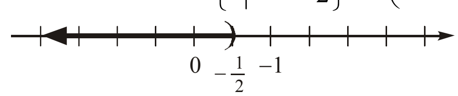
83. $(4x+2)^{-1} < 0$

$\frac{1}{4x+2} < 0$

$4x + 2 < 0$

$x < -\frac{1}{2}$

The solution set is $\left\{x \middle| x < -\frac{1}{2}\right\}$ or $\left(-\infty, -\frac{1}{2}\right)$.



84. $(2x-1)^{-1} > 0$

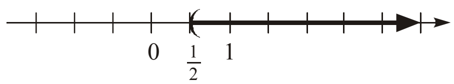
$$\frac{1}{2x-1} > 0$$

Since $\frac{1}{2x-1} > 0$, this means $2x-1>0$.

Therefore,

$$2x-1>0$$
$$x > \frac{1}{2}$$

The solution set is $\left\{x \middle| x > \frac{1}{2}\right\}$ or $\left(\frac{1}{2}, \infty\right)$.



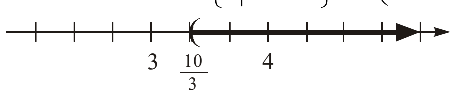
85. $0 < \frac{2}{x} < \frac{3}{5}$

$0 < \frac{2}{x}$ and $\frac{2}{x} < \frac{3}{5}$

Since $\frac{2}{x} > 0$, this means that $x > 0$. Therefore,

$$\frac{2}{x} < \frac{3}{5}$$
$$5x\left(\frac{2}{x}\right) < 5x\left(\frac{3}{5}\right)$$
$$10 < 3x$$
$$\frac{10}{3} < x$$

The solution set is $\left\{x \middle| x > \frac{10}{3}\right\}$ or $\left(\frac{10}{3}, \infty\right)$.



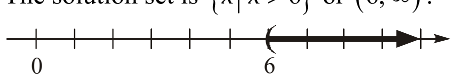
86. $0 < \frac{4}{x} < \frac{2}{3}$

$0 < \frac{4}{x}$ and $\frac{4}{x} < \frac{2}{3}$

Since $\frac{4}{x} > 0$, this means that $x > 0$. Therefore,

$$\frac{4}{x} < \frac{2}{3}$$
$$3x\left(\frac{4}{x}\right) < 3x\left(\frac{2}{3}\right)$$
$$12 < 2x$$
$$6 < x$$

The solution set is $\{x \mid x > 6\}$ or $(6, \infty)$.



87. $0 < (2x-4)^{-1} < \frac{1}{2}$

$$0 < \frac{1}{2x-4} < \frac{1}{2}$$

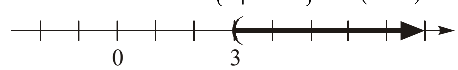
$0 < \frac{1}{2x-4}$ and $\frac{1}{2x-4} < \frac{1}{2}$

Since $\frac{1}{2x-4} > 0$, this means that $2x-4>0$.

Therefore,

$$\frac{1}{2x-4} < \frac{1}{2}$$
$$\frac{1}{2(x-2)} < \frac{1}{2}$$
$$2(x-2)\left(\frac{1}{2(x-2)}\right) < 2(x-2)\left(\frac{1}{2}\right)$$
$$1 < x-2$$
$$3 < x$$

The solution set is $\{x \mid x > 3\}$ or $(3, \infty)$.



88. $0 < (3x+6)^{-1} < \frac{1}{3}$

$$0 < \frac{1}{3x+6} < \frac{1}{3}$$

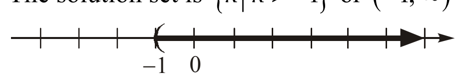
$0 < \frac{1}{3x+6}$ and $\frac{1}{3x+6} < \frac{1}{3}$

Since $\frac{1}{3x+6} > 0$, this means that $3x+6>0$.

Therefore,

$$\frac{1}{3x+6} < \frac{1}{3}$$
$$\frac{1}{3(x+2)} < \frac{1}{3}$$
$$3(x+2)\left(\frac{1}{3(x+2)}\right) < 3(x+2)\left(\frac{1}{3}\right)$$
$$1 < x+2$$
$$-1 < x$$

The solution set is $\{x \mid x > -1\}$ or $(-1, \infty)$.



89. If $-1 < x < 1$, then

$$-1+4 < x+4 < 1+4$$
$$3 < x+4 < 5$$

So, $a = 3$ and $b = 5$.

90. If $-3 < x < 2$, then
$-3-6 < x-6 < 2-6$
$-9 < x-6 < -4$
So, $a = -9$ and $b = -4$.

91. If $2 < x < 3$, then
$-4(2) < -4(x) < -4(3)$
$-12 < -4x < -8$
So, $a = -12$ and $b = -8$.

92. If $-4 < x < 0$, then
$\frac{1}{2}(-4) < \frac{1}{2}(x) < \frac{1}{2}(0)$
$-2 < \frac{1}{2}x < 0$
So, $a = -2$ and $b = 0$.

93. If $0 < x < 4$, then
$2(0) < 2(x) < 2(4)$
$0 < 2x < 8$
$0+3 < 2x+3 < 8+3$
$3 < 2x+3 < 11$
So, $a = 3$ and $b = 11$.

94. If $-3 < x < 3$, then
$-2(-3) > -2(x) > -2(3)$
$6 > -2x > -6$
$6+1 > -2x+1 > -6+1$
$7 > 1-2x > -5$
$-5 < 1-2x < 7$
So, $a = -5$ and $b = 7$.

95. If $-3 < x < 0$, then
$-3+4 < x+4 < 0+4$
$1 < x+4 < 4$
$1 > \frac{1}{x+4} > \frac{1}{4}$
$\frac{1}{4} < \frac{1}{x+4} < 1$
So, $a = \frac{1}{4}$ and $b = 1$.

96. If $2 < x < 4$, then
$2-6 < x-6 < 4-6$
$-4 < x-6 < -2$
$-\frac{1}{4} > \frac{1}{x-6} > -\frac{1}{2}$
$-\frac{1}{2} < \frac{1}{x-6} < -\frac{1}{4}$
So, $a = -\frac{1}{2}$ and $b = -\frac{1}{4}$.

97. If $6 < 3x < 12$, then
$\frac{6}{3} < \frac{3x}{3} < \frac{12}{3}$
$2 < x < 4$
$2^2 < x^2 < 4^2$
$4 < x^2 < 16$
So, $a = 4$ and $b = 16$.

98. If $0 < 2x < 6$, then
$\frac{0}{2} < \frac{2x}{2} < \frac{6}{2}$
$0 < x < 3$
$0^2 < x^2 < 3^2$
$0 < x^2 < 9$
So, $a = 0$ and $b = 9$.

99. $\sqrt{3x+6}$
We need $3x+6 \ge 0$
$3x \ge -6$
$x \ge -2$
To the domain is $\{x \mid x \ge -2\}$ or $[-2, \infty)$.

100. $\sqrt{8+2x}$
We need $8+2x \ge 0$
$2x \ge -8$
$x \ge -4$
To the domain is $\{x \mid x \ge -4\}$ or $[-4, \infty)$.

101. 21 < young adult's age < 30

102. 40 ≤ middle-aged < 60

103. **a.** Let x = age at death.
$x - 30 \ge 49.66$
$x \ge 79.66$
Therefore, the average life expectancy for a 30-year-old male in 2005 will be greater than or equal to 79.66 years.

b. Let x = age at death.
$x - 30 \ge 53.58$
$x \ge 83.58$
Therefore, the average life expectancy for a 30-year-old female in 2005 will be greater than or equal to 83.58 years.

c. By the given information, a female can expect to live $83.58 - 79.66 = 3.92$ years longer.

104. $V = 20T$

$80^\circ \le T \le 120^\circ$

$80^\circ \le \frac{V}{20} \le 120^\circ$

$1600 \le V \le 2400$

The volume ranges from 1600 to 2400 cubic centimeters, inclusive.

105. Let P represent the selling price and C represent the commission.

Calculating the commission:

$C = 45,000 + 0.25(P - 900,000)$

$= 45,000 + 0.25P - 225,000$

$= 0.25P - 180,000$

Calculate the commission range, given the price range:

$900,000 \le P \le 1,100,000$

$0.25(900,000) \le 0.25P \le 0.25(1,100,000)$

$225,000 \le 0.25P \le 275,000$

$225,000 - 180,000 \le 0.25P - 180,000 \le 275,000 - 180,000$

$45,000 \le C \le 95,000$

The agent's commission ranges from \$45,000 to \$95,000, inclusive.

$\frac{45,000}{900,000} = 0.05 = 5\%$ to $\frac{95,000}{1,100,000} = 0.086 = 8.6\%$, inclusive.

As a percent of selling price, the commission ranges from 5% to 8.6%, inclusive.

106. Let C represent the commission.

Calculate the commission range:

$25 + 0.4(200) \le C \le 25 + 0.4(3000)$

$105 \le C \le 1225$

The commissions are at least \$105 and at most \$1225.

107. Let W = weekly wages and T = tax withheld.

Calculating the withholding tax range, given the range of weekly wages:

$700 \le W \le 900$

$700 - 620 \le W - 620 \le 900 - 620$

$80 \le W - 620 \le 280$

$0.25(80) \le 0.25(W - 620) \le 0.25(280)$

$20 \le 0.25(W - 620) \le 70$

$20 + 78.30 \le 0.25(W - 620) + 78.30 \le 70 + 78.30$

$98.30 \le T \le 148.30$

The amount withheld varies from \$98.30 to \$148.30, inclusive.

108. Let x represent the length of time Sue should exercise on the seventh day.

$200 \le 40 + 45 + 0 + 50 + 25 + 35 + x \le 300$

$200 \le 195 + x \le 300$

$5 \le x \le 105$

Sue will stay within the ACSM guidelines by exercising from 5 to 105 minutes.

109. Let K represent the monthly usage in kilowatt-hours and let C represent the monthly customer bill.

Calculating the bill: $C = 0.08275K + 7.58$

Calculating the range of kilowatt-hours, given the range of bills:

$63.47 \le C \le 214.53$

$63.47 \le 0.08275K + 7.58 \le 214.53$

$55.89 \le 0.08275K \le 206.95$

$675.41 \le K \le 2500.91$

The usage varies from 675.41 kilowatt-hours to 2500.91 kilowatt-hours, inclusive.

110. Let W represent the amount of water used (in thousands of gallons). Let C represent the customer charge (in dollars).

Calculating the charge:

$C = 28.84 + 2.28(W - 12)$

$= 28.84 + 2.28W - 27.36$

$= 2.28W + 1.48$

Calculating the range of water usage, given the range of charges:

$42.52 \le C \le 74.44$

$42.52 \le 2.28W + 1.48 \le 74.44$

$41.04 \le 2.28W \le 72.96$

$18 \le W \le 32$

The range of water usage ranged from 18,000 to 32,000 gallons.

111. Let C represent the dealer's cost and M represent the markup over dealer's cost.

If the price is \$18,000, then

$18,000 = C + MC = C(1 + M)$

Solving for C yields: $C = \frac{18,000}{1 + M}$

Calculating the range of dealer costs, given the range of markups:

$0.12 \le M \le 0.18$

$1.12 \le 1 + M \le 1.18$

$\frac{1}{1.12} \ge \frac{1}{1 + M} \ge \frac{1}{1.18}$

$$\frac{18,000}{1.12} \geq \frac{18,000}{1+M} \geq \frac{18,000}{1.18}$$
$$16,071.43 \geq C \geq 15,254.24$$

The dealer's cost varies from \$15,254.24 to \$16,071.43, inclusive.

112. Let T represent the test scores of the people in the top 2.5%.

$$T > 1.96(12)+100 = 123.52$$

People in the top 2.5% will have test scores greater than 123.52. That is, $T > 123.52$ or $(123.52, \infty)$.

113. a. Let T represent the score on the last test and G represent the course grade.
Calculating the course grade and solving for the last test:

$$G = \frac{68+82+87+89+T}{5}$$
$$G = \frac{326+T}{5}$$
$$5G = 326+T$$
$$T = 5G-326$$

Calculating the range of scores on the last test, given the grade range:

$$80 \leq G < 90$$
$$400 \leq 5G < 450$$
$$74 \leq 5G-326 < 124$$
$$74 \leq T < 124$$

To get a grade of B, you need at least a 74 on the fifth test.

b. Let T represent the score on the last test and G represent the course grade.
Calculating the course grade and solving for the last test:

$$G = \frac{68+82+87+89+2T}{6}$$
$$G = \frac{326+2T}{6}$$
$$G = \frac{163+T}{3}$$
$$T = 3G-163$$

Calculating the range of scores on the last test, given the grade range:

$$80 \leq G < 90$$
$$240 \leq 3G < 270$$
$$77 \leq 3G-163 < 107$$
$$77 \leq T < 107$$

To get a grade of B, you need at least a 77 on the fifth test.

114. Let C represent the number of calories in a serving of regular Miracle Whip®, and let F represent the grams of fat in a serving of regular Miracle Whip®.

One possibility for a "light" classification is that the 20 calories in a serving of Miracle Whip® Light is less than or equal to one-third the calories in regular Miracle Whip®. That is,

$$20 \leq \frac{1}{3}C.$$

The second possibility for a "light" classification is that the 1.5 grams of fat in a serving of Miracle Whip® Light is less than or equal to one-half the grams of fat in regular Miracle Whip®.

That is, $1.5 \leq \frac{1}{2}F$.

We have:

$$20 \leq \frac{1}{3}C \quad \text{or} \quad 1.5 \leq \frac{1}{2}F$$
$$60 \leq C \quad \text{or} \quad 3 \leq F$$

A serving of regular Miracle Whip® either contains at least 60 calories or at least 3 grams of fat, or both.

115. Since $a < b$,

$$\frac{a}{2} < \frac{b}{2} \quad \text{and} \quad \frac{a}{2} < \frac{b}{2}$$
$$\frac{a}{2}+\frac{a}{2} < \frac{a}{2}+\frac{b}{2} \quad \text{and} \quad \frac{a}{2}+\frac{b}{2} < \frac{b}{2}+\frac{b}{2}$$
$$a < \frac{a+b}{2} \quad \text{and} \quad \frac{a+b}{2} < b$$

Thus, $a < \frac{a+b}{2} < b$.

116. From problem 115, $a < \frac{a+b}{2} < b$, so

$$d\left(a, \frac{a+b}{2}\right) = \frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{b-a}{2} \quad \text{and}$$
$$d\left(b, \frac{a+b}{2}\right) = b - \frac{a+b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2}.$$

Therefore, $\frac{a+b}{2}$ is equidistant from a and b.

117. If $0 < a < b$, then

$$ab > a^2 > 0 \quad \text{and} \quad b^2 > ab > 0$$
$$\left(\sqrt{ab}\right)^2 > a^2 \quad \text{and} \quad b^2 > \left(\sqrt{ab}\right)^2$$
$$\sqrt{ab} > a \quad \text{and} \quad b > \sqrt{ab}$$

Thus, $a < \sqrt{ab} < b$.

118. Show that $\sqrt{ab} < \frac{a+b}{2}$.

$$\frac{a+b}{2} - \sqrt{ab} = \frac{1}{2}\left(a - 2\sqrt{ab} + b\right)$$
$$= \frac{1}{2}\left(\sqrt{a} - \sqrt{b}\right)^2 > 0, \text{ since } a \neq b.$$

Therefore, $\sqrt{ab} < \frac{a+b}{2}$.

119. For $0 < a < b$, $\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$

$$h \cdot \frac{1}{h} = \frac{1}{2}\left(\frac{b+a}{ab}\right) \cdot h$$
$$1 = \frac{1}{2}\left(\frac{b+a}{ab}\right) \cdot h$$
$$h = \frac{2ab}{a+b}$$

$$h - a = \frac{2ab}{a+b} - a = \frac{2ab - a(a+b)}{a+b}$$
$$= \frac{2ab - a^2 - ab}{a+b} = \frac{ab - a^2}{a+b}$$
$$= \frac{a(b-a)}{a+b} > 0$$

Therefore, $h > a$.

$$b - h = b - \frac{2ab}{a+b} = \frac{b(a+b) - 2ab}{a+b}$$
$$= \frac{ab + b^2 - 2ab}{a+b} = \frac{b^2 - ab}{a+b}$$
$$= \frac{b(b-a)}{a+b} > 0$$

Therefore, $h < b$, and we have $a < h < b$.

120. Show that $h = \frac{(\text{geometric mean})^2}{\text{arithmetic mean}} = \frac{\left(\sqrt{ab}\right)^2}{\left(\frac{1}{2}(a+b)\right)}$

From Problem 119, we know:

$$\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$
$$\frac{2}{h} = \frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$$
$$\frac{h}{2} = \frac{ab}{a+b}$$
$$h = 2 \cdot \frac{ab}{a+b} = \frac{\left(\sqrt{ab}\right)^2}{\left(\frac{1}{2}(a+b)\right)}$$

121. Since $0 < a < b$, then $a - b < 0$ and $ab > 0$.

Therefore, $\frac{a-b}{ab} < 0$. So,

$$\frac{a}{ab} - \frac{b}{ab} < 0$$
$$\frac{1}{b} - \frac{1}{a} < 0$$
$$\frac{1}{b} < \frac{1}{a}$$

Now, since $b > 0$, then $\frac{1}{b} > 0$, so we have

$0 < \frac{1}{b} < \frac{1}{a}$.

122. Answers will vary. One possibility:
No solution: $4x + 6 \le 2(x-5) + 2x$
One solution: $3x + 5 \le 2(x+3) + 1 \le 3(x+2) - 1$

123. Since $x^2 \ge 0$, we have

$x^2 + 1 \ge 0 + 1$

$x^2 + 1 \ge 1$

Therefore, the expression $x^2 + 1$ can never be less than -5.

124 – 125. Answers will vary.

Section 1.6

1. $|-2| = 2$

2. True

3. $\{-5, 5\}$

4. $\{x \mid -5 < x < 5\}$

5. True

6. True

7. $|2x| = 6$

$2x = 6$ or $2x = -6$

$x = 3$ or $x = -3$

The solution set is $\{-3, 3\}$.

8. $|3x| = 12$

$3x = 12$ or $3x = -12$

$x = 4$ or $x = -4$

The solution set is $\{-4, 4\}$.

9. $|2x+3| = 5$

$2x+3 = 5$ or $2x+3 = -5$

$2x = 2$ or $2x = -8$

$x = 1$ or $x = -4$

The solution set is $\{-4, 1\}$.

10. $|3x-1| = 2$

$3x-1 = 2$ or $3x-1 = -2$

$3x = 3$ or $3x = -1$

$x = 1$ or $x = -\frac{1}{3}$

The solution set is $\left\{-\frac{1}{3}, 1\right\}$.

11. $|1-4t|+8 = 13$

$|1-4t| = 5$

$1-4t = 5$ or $1-4t = -5$

$-4t = 4$ or $-4t = -6$

$t = -1$ or $t = \frac{3}{2}$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

12. $|1-2z|+6 = 9$

$|1-2z| = 3$

$1-2z = 3$ or $1-2z = -3$

$-2z = 2$ or $-2z = -4$

$z = -1$ or $z = 2$

The solution set is $\{-1, 2\}$.

13. $|-2x| = |8|$

$|-2x| = 8$

$-2x = 8$ or $-2x = -8$

$x = -4$ or $x = 4$

The solution set is $\{-4, 4\}$.

14. $|-x| = |1|$

$|-x| = 1$

$-x = 1$ or $-x = -1$

The solution set is $\{-1, 1\}$.

15. $|-2|x = 4$

$2x = 4$

$x = 2$

The solution set is $\{2\}$.

16. $|3|x = 9$

$3x = 9$

$x = 3$

The solution set is $\{3\}$.

17. $\frac{2}{3}|x| = 9$

$|x| = \frac{27}{2}$

$x = \frac{27}{2}$ or $x = -\frac{27}{2}$

The solution set is $\left\{-\frac{27}{2}, \frac{27}{2}\right\}$.

18. $\frac{3}{4}|x| = 9$

$|x| = 12$

$x = 12$ or $x = -12$

The solution set is $\{-12, 12\}$.

19. $\left|\frac{x}{3}+\frac{2}{5}\right| = 2$

$\frac{x}{3}+\frac{2}{5} = 2$ or $\frac{x}{3}+\frac{2}{5} = -2$

$5x+6 = 30$ or $5x+6 = -30$

$5x = 24$ or $5x = -36$

$x = \frac{24}{5}$ or $x = -\frac{36}{5}$

The solution set is $\left\{-\frac{36}{5}, \frac{24}{5}\right\}$.

20. $\left|\frac{x}{2}-\frac{1}{3}\right| = 1$

$\frac{x}{2}-\frac{1}{3} = 1$ or $\frac{x}{2}-\frac{1}{3} = -1$

$3x-2 = 6$ or $3x-2 = -6$

$3x = 8$ or $3x = -4$

$x = \frac{8}{3}$ or $x = -\frac{4}{3}$

The solution set is $\left\{-\frac{4}{3}, \frac{8}{3}\right\}$.

21. $|u-2| = -\frac{1}{2}$

No solution, since absolute value always yields a non-negative number.

22. $|2-v| = -1$

No solution, since absolute value always yields a non-negative number.

23. $4-|2x| = 3$

$-|2x| = -1$

$|2x| = 1$

$2x = 1$ or $2x = -1$

$x = \frac{1}{2}$ or $x = -\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

24. $5-\left|\frac{1}{2}x\right| = 3$

$-\left|\frac{1}{2}x\right| = -2$

$\left|\frac{1}{2}x\right| = 2$

$\frac{1}{2}x = 2$ or $\frac{1}{2}x = -2$

$x = 4$ or $x = -4$

The solution set is $\{-4, 4\}$.

25. $|x^2-9| = 0$

$x^2-9 = 0$

$x^2 = 9$

$x = \pm 3$

The solution set is $\{-3, 3\}$.

26. $|x^2-16| = 0$

$x^2-16 = 0$

$x^2 = 16$

$x = \pm 4$

The solution set is $\{-4, 4\}$.

27. $|x^2-2x| = 3$

$x^2-2x = 3$ or $x^2-2x = -3$

$x^2-2x-3 = 0$ or $x^2-2x+3 = 0$

$(x-3)(x+1) = 0$ or $x = \frac{2 \pm \sqrt{4-12}}{2}$

$x = 3$ or $x = -1$ or $x = \frac{2 \pm \sqrt{-8}}{2}$ no real sol.

The solution set is $\{-1, 3\}$.

28. $|x^2+x| = 12$

$x^2+x = 12$ or $x^2+x = -12$

$x^2+x-12 = 0$ or $x^2+x+12 = 0$

$(x-3)(x+4) = 0$ or $x = \frac{-1 \pm \sqrt{1-48}}{2}$

$x = 3$ or $x = -4$ or $x = \frac{1 \pm \sqrt{-47}}{2}$ no real sol.

The solution set is $\{-4, 3\}$.

29. $|x^2+x-1| = 1$

$x^2+x-1 = 1$ or $x^2+x-1 = -1$

$x^2+x-2 = 0$ or $x^2+x = 0$

$(x-1)(x+2) = 0$ or $x(x+1) = 0$

$x = 1, x = -2$ or $x = 0, x = -1$

The solution set is $\{-2, -1, 0, 1\}$.

30. $|x^2+3x-2| = 2$

$x^2+3x-2 = 2$ or $x^2+3x-2 = -2$

$x^2+3x = 4$ or $x^2+3x = 0$

$x^2+3x-4 = 0$ or $x(x+3) = 0$

$(x+4)(x-1) = 0$ or $x = 0, x = -3$

$x = -4, x = 1$

The solution set is $\{-4, -3, 0, 1\}$.

31. $\left|\dfrac{3x-2}{2x-3}\right| = 2$

$\dfrac{3x-2}{2x-3} = 2$ or $\dfrac{3x-2}{2x-3} = -2$

$3x-2 = 2(2x-3)$ or $3x-2 = -2(2x-3)$

$3x-2 = 4x-6$ or $3x-2 = -4x+6$

$-x = -4$ or $7x = 8$

$x = 4$ or $x = \dfrac{8}{7}$

Neither of these values cause the denominator to equal zero, so the solution set is $\left\{\dfrac{8}{7},\ 4\right\}$.

32. $\left|\dfrac{2x+1}{3x+4}\right| = 1$

$\dfrac{2x+1}{3x+4} = 1$ or $\dfrac{2x+1}{3x+4} = -1$

$2x+1 = 1(3x+4)$ or $2x+1 = -1(3x+4)$

$2x+1 = 3x+4$ or $2x+1 = -3x-4$

$-x = 3$ or $5x = -5$

$x = -3$ or $x = -1$

Neither of these values cause the denominator to equal zero, so the solution set is $\{-3,-1\}$.

33. $|x^2+3x| = |x^2-2x|$

$x^2+3x = x^2-2x$ or $x^2+3x = -(x^2-2x)$

$3x = -2x$ or $x^2+3x = -x^2+2x$

$5x = 0$ or $2x^2+x = 0$

$x = 0$ or $x(2x+1) = 0$

$x = 0$ or $x = 0$ or $x = -\dfrac{1}{2}$

The solution set is $\left\{-\dfrac{1}{2},\ 0\right\}$.

34. $|x^2-2x| = |x^2+6x|$

$x^2-2x = x^2+6x$ or $x^2-2x = -(x^2+6x)$

$-2x = 6x$ or $x^2-2x = -x^2-6x$

$-8x = 0$ or $2x^2+4x = 0$

$x = 0$ or $2x(x+2) = 0$

$x = 0$ or $x = 0$ or $x = -2$

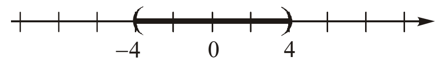
The solution set is $\{-2,\ 0\}$.

35. $|2x| < 8$

$-8 < 2x < 8$

$-4 < x < 4$

$\{x \mid -4 < x < 4\}$ or $(-4,4)$

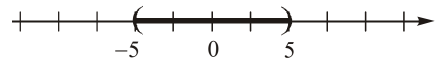


36. $|3x| < 15$

$-15 < 3x < 15$

$-5 < x < 5$

$\{x \mid -5 < x < 5\}$ or $(-5,5)$

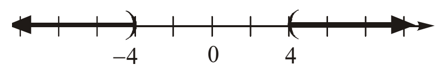


37. $|3x| > 12$

$3x < -12$ or $3x > 12$

$x < -4$ or $x > 4$

$\{x \mid x < -4 \text{ or } x > 4\}$ or $(-\infty,-4)\cup(4,\infty)$

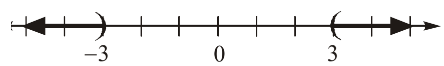


38. $|2x| > 6$

$2x < -6$ or $2x > 6$

$x < -3$ or $x > 3$

$\{x \mid x < -3 \text{ or } x > 3\}$ or $(-\infty,-3)\cup(3,\infty)$



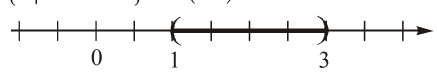
39. $|x-2|+2 < 3$

$|x-2| < 1$

$-1 < x-2 < 1$

$1 < x < 3$

$\{x \mid 1 < x < 3\}$ or $(1,3)$



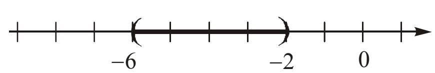
40. $|x+4|+3 < 5$

$|x+4| < 2$

$-2 < x+4 < 2$

$-6 < x < -2$

$\{x \mid -6 < x < -2\}$ or $(-6,-2)$



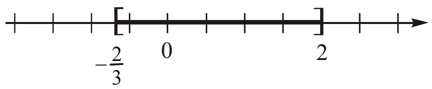
41. $|3t-2| \le 4$

$-4 \le 3t-2 \le 4$

$-2 \le 3t \le 6$

$-\frac{2}{3} \le t \le 2$

$\left\{t \middle| -\frac{2}{3} \le t \le 2\right\}$ or $\left[-\frac{2}{3}, 2\right]$



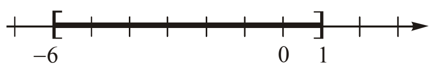
42. $|2u+5| \le 7$

$-7 \le 2u+5 \le 7$

$-12 \le 2u \le 2$

$-6 \le u \le 1$

$\{u \mid -6 \le u \le 1\}$ or $[-6, 1]$



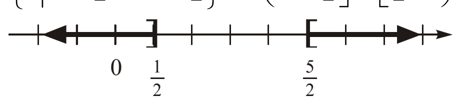
43. $|2x-3| \ge 2$

$2x-3 \le -2$ or $2x-3 \ge 2$

$2x \le 1$ or $2x \ge 5$

$x \le \frac{1}{2}$ or $x \ge \frac{5}{2}$

$\left\{x \middle| x \le \frac{1}{2} \text{ or } x \ge \frac{5}{2}\right\}$ or $\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{5}{2}, \infty\right)$



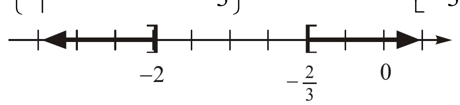
44. $|3x+4| \ge 2$

$3x+4 \le -2$ or $3x+4 \ge 2$

$3x \le -6$ or $3x \ge -2$

$x \le -2$ or $x \ge -\frac{2}{3}$

$\left\{x \middle| x \le -2 \text{ or } x \ge -\frac{2}{3}\right\}$ or $(-\infty, -2] \cup \left[-\frac{2}{3}, \infty\right)$



45. $|1-4x| - 7 < -2$

$|1-4x| < 5$

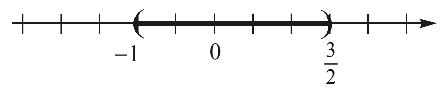
$-5 < 1-4x < 5$

$-6 < -4x < 4$

$\frac{-6}{-4} > x > \frac{4}{-4}$

$\frac{3}{2} > x > -1$ or $-1 < x < \frac{3}{2}$

$\left\{x \middle| -1 < x < \frac{3}{2}\right\}$ or $\left(-1, \frac{3}{2}\right)$



46. $|1-2x| - 4 < -1$

$|1-2x| < 3$

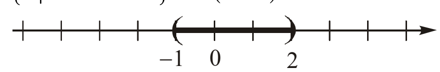
$-3 < 1-2x < 3$

$-4 < -2x < 2$

$\frac{-4}{-2} > x > \frac{2}{-2}$

$2 > x > -1$ or $-1 < x < 2$

$\{x \mid -1 < x < 2\}$ or $(-1, 2)$



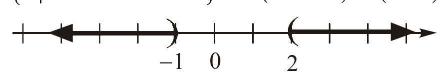
47. $|1-2x| > 3$

$1-2x < -3$ or $1-2x > 3$

$-2x < -4$ or $-2x > 2$

$x > 2$ or $x < -1$

$\{x \mid x < -1 \text{ or } x > 2\}$ or $(-\infty, -1) \cup (2, \infty)$



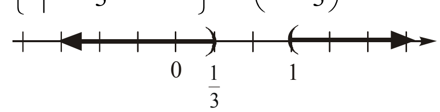
48. $|2-3x| > 1$

$2-3x < -1$ or $2-3x > 1$

$-3x < -3$ or $-3x > -1$

$x > 1$ or $x < \frac{1}{3}$

$\left\{x \middle| x < \frac{1}{3} \text{ or } x > 1\right\}$ or $\left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$

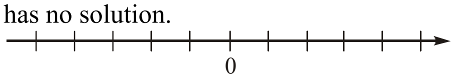


49. $|-4x|+|-5|\le 1$

$|-4x|+5\le 1$

$|-4x|\le -4$

This is impossible since absolute value always yields a non-negative number. The inequality has no solution.



50. $|-x|-|4|\le 2$

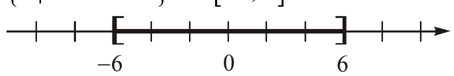
$|-x|-4\le 2$

$|-x|\le 6$

$-6\le -x\le 6$

$6\ge x\ge -6$

$\{x\,|-6\le x\le 6\}$ or $[-6, 6]$



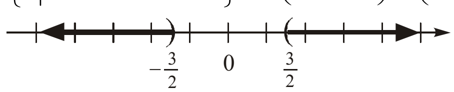
51. $|-2x|>|-3|$

$|2x|>3$

$2x<-3$ or $2x>3$

$x<-\frac{3}{2}$ or $x>\frac{3}{2}$

$\left\{x\,\middle|\, x<-\frac{3}{2} \text{ or } x>\frac{3}{2}\right\}$ or $\left(-\infty,-\frac{3}{2}\right)\cup\left(\frac{3}{2},\infty\right)$



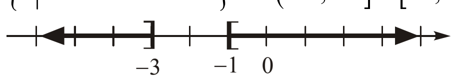
52. $|-x-2|\ge 1$

$-x-2\le -1$ or $-x-2\ge 1$

$-x\le 1$ or $-x\ge 3$

$x\ge -1$ or $x\le -3$

$\{x\,|\,x\le -3 \text{ or } x\ge -1\}$ or $(-\infty,-3]\cup[-1,\infty)$



53. $-|2x-1|\ge -3$

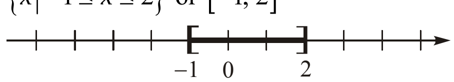
$|2x-1|\le 3$

$-3\le 2x-1\le 3$

$-2\le 2x\le 4$

$-1\le x\le 2$

$\{x\,|-1\le x\le 2\}$ or $[-1, 2]$



54. $-|1-2x|\ge -3$

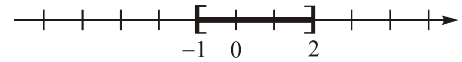
$|1-2x|\le 3$

$-3\le 1-2x\le 3$

$-4\le -2x\le 2$

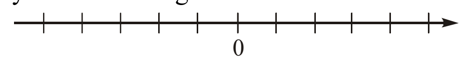
$2\ge x\ge -1$

$\{x\,|-1\le x\le 2\}$ or $[-1, 2]$



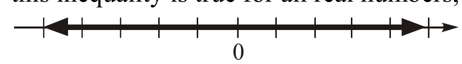
55. $|2x|<-1$

This is impossible since absolute value always yields a non-negative number. No solution.



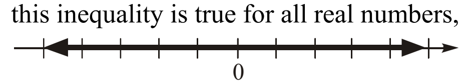
56. $|3x|\ge 0$

Absolute value yields a non-negative number, so this inequality is true for all real numbers, $(-\infty,\infty)$.



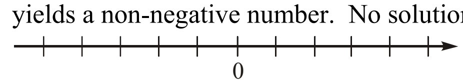
57. $|5x|\ge -1$

Absolute value yields a non-negative number, so this inequality is true for all real numbers, $(-\infty,\infty)$.



58. $|6x|<-2$

This is impossible since absolute value always yields a non-negative number. No solution.



59. $\left|\frac{2x+3}{3}-\frac{1}{2}\right|<1$

$-1<\frac{2x+3}{3}-\frac{1}{2}<1$

$6(-1)<6\left(\frac{2x+3}{3}-\frac{1}{2}\right)<6(1)$

$-6<2(2x+3)-3<6$

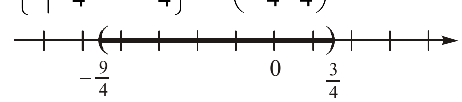
$-6<4x+6-3<6$

$-6<4x+3<6$

$-9<4x<3$

$-\frac{9}{4}<x<\frac{3}{4}$

$\left\{x\,\middle|-\frac{9}{4}<x<\frac{3}{4}\right\}$ or $\left(-\frac{9}{4},\frac{3}{4}\right)$



60. $3-|x+1|<\frac{1}{2}$

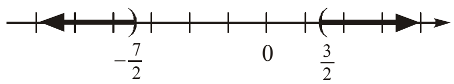
$$-|x+1|<-\frac{5}{2}$$

$$|x+1|>\frac{5}{2}$$

$$x+1<-\frac{5}{2} \text{ or } x+1>\frac{5}{2}$$

$$x<-\frac{7}{2} \text{ or } x>\frac{3}{2}$$

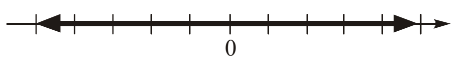
$$\left\{x \middle| x<-\frac{7}{2} \text{ or } x>\frac{3}{2}\right\} \text{ or } \left(-\infty,-\frac{7}{2}\right)\cup\left(\frac{3}{2},\infty\right)$$



61. $5+|x-1|>\frac{1}{2}$

$$|x-1|>-\frac{9}{2}$$

Absolute value yields a non-negative number, so this inequality is true for all real numbers, $(-\infty,\infty)$.



62. $\left|\frac{2x-3}{2}+\frac{1}{3}\right|>1$

$$\frac{2x-3}{2}+\frac{1}{3}<-1 \quad \text{or} \quad \frac{2x-3}{2}+\frac{1}{3}>1$$

$$6\left(\frac{2x-3}{2}+\frac{1}{3}\right)<6(-1) \quad \text{or} \quad 6\left(\frac{2x-3}{2}+\frac{1}{3}\right)>6(1)$$

$$3(2x-3)+2<-6 \quad \text{or} \quad 3(2x-3)+2>6$$

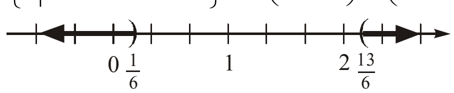
$$6x-9+2<-6 \quad \text{or} \quad 6x-9+2>6$$

$$6x-7<-6 \quad \text{or} \quad 6x-7>6$$

$$6x<1 \quad \text{or} \quad 6x>13$$

$$x<\frac{1}{6} \quad \text{or} \quad x>\frac{13}{6}$$

$$\left\{x \middle| x<\frac{1}{6} \text{ or } x>\frac{13}{6}\right\} \text{ or } \left(-\infty,\frac{1}{6}\right)\cup\left(\frac{13}{6},\infty\right)$$



63. A temperature x that differs from $98.6°$ F by at least $1.5°$F.

$$|x-98.6°|\ge 1.5°$$

$$x-98.6°\le -1.5° \text{ or } x-98.6°\ge 1.5°$$

$$x\le 97.1° \quad \text{or} \quad x\ge 100.1°$$

The temperatures that are considered unhealthy are those that are less than 97.1˚F or greater than 100.1˚F, inclusive.

64. A voltage x that differs from 110 volts by at most 5 volts.

$$|x-110|\le 5$$

$$-5\le x-110\le 5$$

$$105\le x\le 115$$

The actual voltage is between 105 and 115 volts, inclusive.

65. The true average number of books read x should differ from 13.4 by less than 1.35 books.

$$|x-13.4|<1.35$$

$$-1.35<x-13.4<1.35$$

$$12.05<x<14.75$$

Gallup is 99% confident that the actual average number of books read per year is between 12.05 and 14.75 books.

66. The speed x varies from 707 mph by up to 55 mph.

a. $|x-707|\le 55$

b. $-55\le x-707\le 55$

$-55\le x-707\le 55$

$652\le x\le 762$

The speed of sound is between 652 and 762 miles per hour, depending on conditions.

67. x differs from 3 by less than $\frac{1}{2}$.

$$|x-3|<\frac{1}{2}$$

$$-\frac{1}{2}<x-3<\frac{1}{2}$$

$$\frac{5}{2}<x<\frac{7}{2}$$

$$\left\{x \middle| \frac{5}{2}<x<\frac{7}{2}\right\}$$

68. x differs from -4 by less than 1

$$|x-(-4)|<1$$

$$|x+4|<1$$

$$-1<x+4<1$$

$$-5<x<-3$$

$$\{x|-5<x<-3\}$$

69. x differs from -3 by more than 2.

$$|x-(-3)|>2$$
$$|x+3|>2$$
$$x+3<-2 \text{ or } x+3>2$$
$$x<-5 \text{ or } x>-1$$
$$\{x \mid x<-5 \text{ or } x>-1\}$$

70. x differs from 2 by more than 3.

$$|x-2|>3$$
$$x-2<-3 \text{ or } x-2>3$$
$$x<-1 \text{ or } x>5$$
$$\{x \mid x<-1 \text{ or } x>5\}$$

71. $|x-1|<3$

$$-3<x-1<3$$
$$-3+5<(x-1)+5<3+5$$
$$2<x+4<8$$
$$a=2,\ b=8$$

72. $|x+2|<5$

$$-5<x+2<5$$
$$-5-4<(x+2)-4<5-4$$
$$-9<x-2<1$$
$$a=-9,\ b=1$$

73. $|x+4|\le 2$

$$-2\le x+4\le 2$$
$$-6\le x\le -2$$
$$-12\le 2x\le -4$$
$$-15\le 2x-3\le -7$$
$$a=-15,\ b=-7$$

74. $|x-3|\le 1$

$$-1\le x-3\le 1$$
$$2\le x\le 4$$
$$6\le 3x\le 12$$
$$7\le 3x+1\le 13$$
$$a=7,\ b=13$$

75. $|x-2|\le 7$

$$-7\le x-2\le 7$$
$$-5\le x\le 9$$
$$-15\le x-10\le -1$$
$$-\frac{1}{15}\ge \frac{1}{x-10}\ge -1$$
$$-1\le \frac{1}{x-10}\le -\frac{1}{15}$$
$$a=-1,\ b=-\frac{1}{15}$$

76. $|x+1|\le 3$

$$-3\le x+1\le 3$$
$$-4\le x\le 2$$
$$1\le x+5\le 7$$
$$1\ge \frac{1}{x+5}\ge \frac{1}{7}$$
$$\frac{1}{7}\le \frac{1}{x+5}\le 1$$
$$a=\frac{1}{7},\ b=1$$

77. Given that $a>0,\ b>0,$ and $\sqrt{a}<\sqrt{b}$, show that $a<b$.

Note that $b-a=\left(\sqrt{b}+\sqrt{a}\right)\left(\sqrt{b}-\sqrt{a}\right)$.

Since $\sqrt{a}<\sqrt{b}$ means $\sqrt{b}-\sqrt{a}>0$, we have $b-a=\left(\sqrt{b}+\sqrt{a}\right)\left(\sqrt{b}-\sqrt{a}\right)>0$.

Therefore, $b-a>0$ which means $a<b$.

78. Show that $a\le |a|$.

We know $0\le |a|$. So if $a<0$, then we have $a<0\le |a|$ which means $a\le |a|$.. Now, if $a\ge 0$, then $|a|=a$. So $a\le |a|$.

79. Prove $|a+b|\le |a|+|b|$.

Note that $|a+b|^2=|a+b|\cdot|a+b|$.

<u>Case 1</u>: If $a+b\ge 0$, then $|a+b|=a+b$, so

$$\begin{aligned}|a+b|\cdot|a+b| &= (a+b)(a+b)\\ &= a^2+2ab+b^2\\ &\le |a|^2+2|a|\cdot|b|+|b|^2\\ &= \left(|a|+|b|\right)^2 \quad \text{by problem 78}\end{aligned}$$

Thus, $\left(|a+b|\right)^2\le\left(|a|+|b|\right)^2$

$$|a+b|\le |a|+|b|.$$

<u>Case 2</u>: If $a+b<0$, then $|a+b|=-(a+b)$, so

$$\begin{aligned}|a+b|\cdot|a+b| &= \left(-(a+b)\right)\left(-(a+b)\right)\\ &= (a+b)(a+b)\\ &= a^2+2ab+b^2\\ &\le |a|^2+2|a|\cdot|b|+|b|^2\\ &= \left(|a|+|b|\right)^2 \quad \text{by problem 78}\end{aligned}$$

Thus, $\left(|a+b|\right)^2\le\left(|a|+|b|\right)^2$

$$|a+b|\le |a|+|b|$$

80. Prove $|a-b| \ge |a|-|b|$.

$|a| = |(a-b)+b| \le |a-b|+|b|$ by the Triangle Inequality, so $|a| \le |a-b|+|b|$ which means $|a|-|b| \le |a-b|$. Therefore, $|a-b| \ge |a|-|b|$.

81. Given that $a > 0$,

$$x^2 < a$$
$$x^2 - a < 0$$
$$\left(x+\sqrt{a}\right)\left(x-\sqrt{a}\right) < 0$$

If $x < -\sqrt{a}$, then $x+\sqrt{a} < 0$ and $x-\sqrt{a} < -2\sqrt{a} < 0$. Therefore, $\left(x+\sqrt{a}\right)\left(x-\sqrt{a}\right) > 0$, which is a contradiction.

If $-\sqrt{a} < x < \sqrt{a}$, then $0 < x+\sqrt{a} < 2\sqrt{a}$ and $-2\sqrt{a} < x-\sqrt{a} < 0$.
Therefore, $\left(x+\sqrt{a}\right)\left(x-\sqrt{a}\right) < 0$.

If $x > \sqrt{a}$, then $x+\sqrt{a} > 2\sqrt{a} > 0$ and $x-\sqrt{a} > 0$. Therefore, $\left(x+\sqrt{a}\right)\left(x-\sqrt{a}\right) > 0$, which is a contradiction. So the solution set for $x^2 < a$ is $\left\{x \middle| -\sqrt{a} < x < \sqrt{a}\right\}$.

82. Given that $a > 0$,

$$x^2 > a.$$
$$x^2 - a > 0$$
$$\left(x+\sqrt{a}\right)\left(x-\sqrt{a}\right) > 0$$

If $x < -\sqrt{a}$, then $x+\sqrt{a} < 0$ and $x-\sqrt{a} < -2\sqrt{a} < 0$.
Therefore, $\left(x+\sqrt{a}\right)\left(x-\sqrt{a}\right) > 0$.

If $-\sqrt{a} < x < \sqrt{a}$, then $0 < x+\sqrt{a} < 2\sqrt{a}$ and $-2\sqrt{a} < x-\sqrt{a} < 0$ ..Therefore, $\left(x+\sqrt{a}\right)\left(x-\sqrt{a}\right) < 0$, which is a contradiction.

If $x > \sqrt{a}$, then $x+\sqrt{a} > 2\sqrt{a} > 0$ and $x-\sqrt{a} > 0$. Therefore, $\left(x+\sqrt{a}\right)\left(x-\sqrt{a}\right) > 0$.

So the solution set for $x^2 > a$ is $\left\{x \middle| x < -\sqrt{a} \text{ or } x > \sqrt{a}\right\}$.

83. $x^2 < 1$

$-\sqrt{1} < x < \sqrt{1}$

$-1 < x < 1$

The solution set is $\{x \mid -1 < x < 1\}$.

84. $x^2 < 4$

$-\sqrt{4} < x < \sqrt{4}$

$-2 < x < 2$

The solution set is $\{x \mid -2 < x < 2\}$.

85. $x^2 \ge 9$

$x \le -\sqrt{9}$ or $x \ge \sqrt{9}$

$x \le -3$ or $x \ge 3$

The solution set is $\{x \mid x \le -3 \text{ or } x \ge 3\}$.

86. $x^2 \ge 1$

$x \le -\sqrt{1}$ or $x \ge \sqrt{1}$

$x \le -1$ or $x \ge 1$

The solution set is $\{x \mid x \le -1 \text{ or } x \ge 1\}$.

87. $x^2 \le 16$

$-\sqrt{16} \le x \le \sqrt{16}$

$-4 \le x \le 4$

The solution set is $\{x \mid -4 \le x \le 4\}$.

88. $x^2 \le 9$

$-\sqrt{9} \le x \le \sqrt{9}$

$-3 \le x \le 3$

The solution set is $\{x \mid -3 \le x \le 3\}$.

89. $x^2 > 4$

$x < -\sqrt{4}$ or $x > \sqrt{4}$

$x < -2$ or $x > 2$

The solution set is $\{x \mid x < -2 \text{ or } x > 2\}$.

90. $x^2 \ge 16$

$x \le -\sqrt{16}$ or $x \ge \sqrt{16}$

$x \le -4$ or $x \ge 4$

The solution set is $\{x \mid x < -4 \text{ or } x > 4\}$.

91. $\left|3x-|2x+1|\right|=4$

$3x-|2x+1|=4$ or $3x-|2x+1|=-4$

$3x-|2x+1|=4$

$3x-4=|2x+1|$

$2x+1=3x-4$ or $2x+1=-(3x-4)$

$-x=-5$ or $2x+1=-3x+4$

$x=5$ or $5x=3$

$x=5$ or $x=\frac{3}{5}$

or

$3x-|2x+1|=-4$

$3x+4=|2x+1|$

$2x+1=3x+4$ or $2x+1=-(3x+4)$

$-x=3$ or $2x+1=-3x-4$

$x=-3$ or $5x=-5$

$x=-3$ or $x=-1$

The only values that check in the original equation are $x=5$ and $x=-1$.

The solution set is $\{-1, 5\}$.

92. $\left|x+|3x-2|\right|=2$

$x+|3x-2|=2$ or $x+|3x-2|=-2$

$x+|3x-2|=2$

$|3x-2|=2-x$

$3x-2=2-x$ or $3x-2=-(2-x)$

$4x=4$ or $3x-2=-2+x$

$x=1$ or $2x=0$

$x=1$ or $x=0$

or

$x+|3x-2|=-2$

$|3x-2|=-2-x$

$3x-2=-2-x$ or $3x-2=-(-2-x)$

$4x=0$ or $3x-2=2+x$

$x=0$ or $2x=4$

$x=0$ or $x=2$

The only values that check in the original equation are $x = 0$ and $x = 1$.

The solution set is $\{0, 1\}$.

93 – 95. Answers will vary.

Section 1.7

1. mathematical modeling

2. interest

3. uniform motion

4. False; the amount charged for the use of principal is the interest.

5. True; this is the uniform motion formula.

6. If there are x pounds of coffee A, then there are $100-x$ pounds of coffee B.

7. Let A represent the area of the circle and r the radius. The area of a circle is the product of π times the square of the radius: $A=\pi r^2$

8. Let C represent the circumference of a circle and r the radius. The circumference of a circle is the product of π times twice the radius: $C=2\pi r$

9. Let A represent the area of the square and s the length of a side. The area of the square is the square of the length of a side: $A=s^2$

10. Let P represent the perimeter of a square and s the length of a side. The perimeter of a square is four times the length of a side: $P=4s$

11. Let F represent the force, m the mass, and a the acceleration. Force equals the product of the mass times the acceleration: $F=ma$

12. Let P represent the pressure, F the force, and A the area. Pressure is the force per unit area: $P=\frac{F}{A}$

13. Let W represent the work, F the force, and d the distance. Work equals force times distance: $W=Fd$

14. Let K represent the kinetic energy, m the mass, and v the velocity. Kinetic energy is one-half the product of the mass and the square of the velocity: $K=\frac{1}{2}mv^2$

15. $C =$ total variable cost in dollars, $x =$ number of dishwashers manufactured: $C = 150x$

16. $R =$ total revenue in dollars, $x =$ number of dishwashers sold: $R = 250x$

17. Let x represent the amount of money invested in bonds. Then $50,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$6,000, we have:

$$\begin{aligned} 0.15x + 0.07(50,000 - x) &= 6,000 \\ (100)(0.15x + 0.07(50,000 - x)) &= (6,000)(100) \\ 15x + 7(50,000 - x) &= 600,000 \\ 15x + 350,000 - 7x &= 600,000 \\ 8x + 350,000 &= 600,000 \\ 8x &= 250,000 \\ x &= 31,250 \end{aligned}$$

\$31,250 should be invested in bonds at 15% and \$18,750 should be invested in CD's at 7%.

18. Let x represent the amount of money invested in bonds. Then $50,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$7,000, we have:

$$\begin{aligned} 0.15x + 0.07(50,000 - x) &= 7,000 \\ (100)(0.15x + 0.07(50,000 - x)) &= (7,000)(100) \\ 15x + 7(50,000 - x) &= 700,000 \\ 15x + 350,000 - 7x &= 700,000 \\ 8x + 350,000 &= 700,000 \\ 8x &= 350,000 \\ x &= 43,750 \end{aligned}$$

\$43,750 should be invested in bonds at 15% and \$6,250 should be invested in CD's at 7%.

19. Let x represent the amount of money loaned at 8%. Then $12,000 - x$ represents the amount of money loaned at 18%. Since the total interest is to be \$1,000, we have:

$$\begin{aligned} 0.08x + 0.18(12,000 - x) &= 1,000 \\ (100)(0.08x + 0.18(12,000 - x)) &= (1,000)(100) \\ 8x + 18(12,000 - x) &= 100,000 \\ 8x + 216,000 - 18x &= 100,000 \\ -10x + 216,000 &= 100,000 \\ -10x &= -116,000 \\ x &= 11,600 \end{aligned}$$

\$11,600 is loaned at 8% and \$400 is at 18%.

20. Let x represent the amount of money loaned at 16%. Then $1,000,000 - x$ represents the amount of money loaned at 19%. Since the total interest is to be \$1,000,000(0.18), we have:

$$\begin{aligned} 0.16x + 0.19(1,000,000 - x) &= 1,000,000(0.18) \\ 0.16x + 190,000 - 0.19x &= 180,000 \\ -0.03x + 190,000 &= 180,000 \\ -0.03x &= -10,000 \\ x &= \frac{-10,000}{-0.03} \\ x &= \$333,333.33 \end{aligned}$$

Wendy can lend \$333,333.33 at 16%.

21. Let x represent the number of pounds of Earl Gray tea. Then $100 - x$ represents the number of pounds of Orange Pekoe tea.

$$\begin{aligned} 5x + 3(100 - x) &= 4.50(100) \\ 5x + 300 - 3x &= 450 \\ 2x + 300 &= 450 \\ 2x &= 150 \\ x &= 75 \end{aligned}$$

75 pounds of Earl Gray tea must be blended with 25 pounds of Orange Pekoe.

22. Let x represent the number of pounds of the first kind of coffee. Then $100 - x$ represents the number of pounds of the second kind of coffee.

$$\begin{aligned} 2.75x + 5(100 - x) &= 3.90(100) \\ 2.75x + 500 - 5x &= 390 \\ -2.25x + 500 &= 390 \\ -2.25x &= -110 \\ x &\approx 48.9 \end{aligned}$$

Approximately 49 pounds of the first kind of coffee must be blended with approximately 51 pounds of the second kind of coffee.

23. Let x represent the number of pounds of cashews. Then $x + 60$ represents the number of pounds in the mixture.

$$\begin{aligned} 9x + 3.50(60) &= 7.50(x + 60) \\ 9x + 210 &= 7.50x + 450 \\ 1.5x &= 240 \\ x &= 160 \end{aligned}$$

160 pounds of cashews must be added to the 60 pounds of almonds.

24. Let x represent the number of caramels in the box. Then $30-x$ represents the number of cremes in the box.

$$\text{Revenue} - \text{Cost} = \text{Profit}$$
$$12.50-\left(0.25x+0.45(30-x)\right)=3.00$$
$$12.50-\left(0.25x+13.5-0.45x\right)=3.00$$
$$12.50-\left(13.5-0.20x\right)=3.00$$
$$12.50-13.50+0.20x=3.00$$
$$-1.00+0.20x=3.00$$
$$0.20x=4.00$$
$$x=20$$

The box should contain 20 caramels and 10 cremes.

25. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$16-r$	$\frac{20}{60}=\frac{1}{3}$	$\frac{16-r}{3}$
Downstream	$16+r$	$\frac{15}{60}=\frac{1}{4}$	$\frac{16+r}{4}$

Since the distance is the same in each direction:

$$\frac{16-r}{3}=\frac{16+r}{4}$$
$$4(16-r)=3(16+r)$$
$$64-4r=48+3r$$
$$16=7r$$
$$r=\frac{16}{7}\approx 2.286$$

The speed of the current is approximately 2.286 miles per hour.

26. Let r represent the speed of the motorboat.

	Rate	Time	Distance
Upstream	$r-3$	5	$5(r-3)$
Downstream	$r+3$	2.5	$2.5(r+3)$

The distance is the same in each direction:

$$5(r-3)=2.5(r+3)$$
$$5r-15=2.5r+7.5$$
$$2.5r=22.5$$
$$r=9$$

The speed of the motorboat is 9 miles per hour.

27. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$15-r$	$\frac{10}{15-r}$	10
Downstream	$15+r$	$\frac{10}{15+r}$	10

Since the total time is 1.5 hours, we have:

$$\frac{10}{15-r}+\frac{10}{15+r}=1.5$$
$$10(15+r)+10(15-r)=1.5(15-r)(15+r)$$
$$150+10r+150-10r=1.5(225-r^2)$$
$$300=1.5(225-r^2)$$
$$200=225-r^2$$
$$r^2-25=0$$
$$(r-5)(r+5)=0$$
$$r=5 \text{ or } r=-5$$

Speed must be positive, so disregard $r=-5$. The speed of the current is 5 miles per hour.

28. Let r represent the rate of the slower car. Then $r+10$ represents the rate of the faster car.

	Rate	Time	Distance
Slower car	r	3.5	$3.5r$
Faster car	$r+10$	3	$3(r+10)$

$$3.5r=3(r+10)$$
$$3.5r=3r+30$$
$$0.5r=30$$
$$r=60$$

The slower car travels at a rate of 60 miles per hour. The faster car travels at a rate of 70 miles per hour. The distance is (70)(3) = 210 miles.

29. Let r represent Karen's normal walking speed.

	Rate	Time	Distance
With walkway	$r+2.5$	$\frac{50}{r+2.5}$	50
Against walkway	$r-2.5$	$\frac{50}{r-2.5}$	50

Since the total time is 40 seconds:

$$\frac{50}{r+2.5}+\frac{50}{r-2.5}=40$$
$$50(r-2.5)+50(r+2.5)=40(r-2.5)(r+2.5)$$
$$50r-125+50r+125=40(r^2-6.25)$$
$$100r=40r^2-250$$
$$0=40r^2-100r-250$$
$$0=4r^2-10r-25$$
$$r=\frac{-(-10)\pm\sqrt{(-10)^2-4(4)(-25)}}{2(4)}$$
$$=\frac{10\pm\sqrt{500}}{8}=\frac{10\pm10\sqrt{5}}{8}=\frac{5\pm5\sqrt{5}}{4}$$
$$r\approx 4.05 \text{ or } r\approx -1.55$$

Speed must be positive, so disregard $r\approx -1.55$. Karen' normal walking speed is approximately 4.05 feet per second.

30. Let r represent the speed of the Montparnasse walkway.

	Rate	Time	Distance
Walking with	$1.5+r$	$\frac{200}{1.5+r}$	200
Standing still	r	$\frac{200}{r}$	200

Walking with the walkway takes 30 seconds less time than standing still on the walkway:

$$\frac{200}{1.5+r}=\frac{200}{r}-30$$
$$200r=200(1.5+r)-30r(r+1.5)$$
$$200r=300+200r-30r^2-45r$$
$$30r^2+45r-300=0$$
$$2r^2+3r-20=0$$
$$(2r+5)(r+4)=0$$
$$2r-5=0 \quad \text{or} \quad r+4=0$$
$$r=\frac{5}{2}=2.5 \quad \text{or} \quad r=-4$$

Speed must be positive, so disregard $r=-4$. The speed of the Montparnasse walkways is 2.5 meters per second.

31. Let w represent the width of a regulation doubles tennis court. Then $2w+6$ represents the length. The area is 2808 square feet:

$$w(2w+6)=2808$$
$$2w^2+6w=2808$$
$$2w^2+6w-2808=0$$
$$w^2+3w-1404=0$$
$$(w+39)(w-36)=0$$
$$w+39=0 \quad \text{or} \quad w-36=0$$
$$w=-39 \quad \text{or} \quad w=36$$

The width must be positive, so disregard $w=-39$. The width of a regulation doubles tennis court is 36 feet and the length is 2(36) + 6 = 78 feet.

32. Let t represent the time it takes the HP LaserJet 2420 to complete the print job alone. Then $t+10$ represents the time it takes the HP LaserJet 1300 to complete the print job alone.

	Time to do job	Part of job done in one minute
HP LJ 2420	t	$\frac{1}{t}$
HP LJ 1300	$t+10$	$\frac{1}{t+10}$
Together	12	$\frac{1}{12}$

$$\frac{1}{t}+\frac{1}{t+10}=\frac{1}{12}$$
$$12(t+10)+12t=t(t+10)$$
$$12t+120+12t=t^2+10t$$
$$0=t^2-14t-120$$
$$0=(t-20)(t+6)$$
$$t-20=0 \quad \text{or} \quad t+6=0$$
$$t=20 \quad \text{or} \quad t=-6$$

Time must be positive, so disregard $t=-6$. The HP LaserJet 2420 takes 20 minutes to complete the job alone, printing $\frac{600}{20}=30$ pages per minute. The HP LaserJet 1300 takes 20 + 10 = 30 minutes to complete the job alone, printing $\frac{600}{30}=20$ pages per minute.

33. Let t represent the time it takes to do the job together.

	Time to do job	Part of job done in one minute
Trent	30	$\frac{1}{30}$
Lois	20	$\frac{1}{20}$
Together	t	$\frac{1}{t}$

$$\frac{1}{30}+\frac{1}{20}=\frac{1}{t}$$
$$2t+3t=60$$
$$5t=60$$
$$t=12$$

Working together, the job can be done in 12 minutes.

34. Let t represent the time it takes April to do the job working alone.

	Time to do job	Part of job done in one hour
Patrice	10	$\frac{1}{10}$
April	t	$\frac{1}{t}$
Together	6	$\frac{1}{6}$

$$\frac{1}{10}+\frac{1}{t}=\frac{1}{6}$$
$$3t+30=5t$$
$$2t=30$$
$$t=15$$

April would take 15 hours to paint the rooms.

35. $l =$ length of the garden
$w =$ width of the garden

a. The length of the garden is to be twice its width. Thus, $l = 2w$.
The dimensions of the fence are $l + 4$ and $w + 4$.
The perimeter is 46 feet, so:
$$2(l+4)+2(w+4)=46$$
$$2(2w+4)+2(w+4)=46$$
$$4w+8+2w+8=46$$
$$6w+16=46$$
$$6w=30$$
$$w=5$$
The dimensions of the garden are 5 feet by 10 feet.

b. Area $= l \cdot w = 5 \cdot 10 = 50$ square feet

c. If the dimensions of the garden are the same, then the length and width of the fence are also the same $(l+4)$. The perimeter is 46 feet, so:
$$2(l+4)+2(l+4)=46$$
$$2l+8+2l+8=46$$
$$4l+16=46$$
$$4l=30$$
$$l=7.5$$
The dimensions of the garden are 7.5 feet by 7.5 feet.

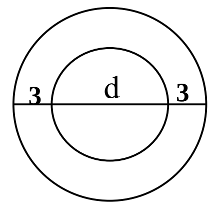
d. Area $= l \cdot w = 7.5(7.5) = 56.25$ square feet.

36. $l =$ length of the pond
$w =$ width of the pond

a. The pond is to be a square. Thus, $l = w$.
The dimensions of the fenced area are $w + 6$ on each side. The perimeter is 100 feet, so:
$$4(w+6)=100$$
$$4w+24=100$$
$$4w=76$$
$$w=19$$
The dimensions of the pond are 19 feet by 19 feet.

b. The length of the pond is to be three times the width. Thus, $l = 3w$. The dimensions of the fenced area are $w+6$ and $l+6$. The perimeter is 100 feet, so:
$$2(w+6)+2(l+6)=100$$
$$2(w+6)+2(3w+6)=100$$
$$2w+12+6w+12=100$$
$$8w+24=100$$
$$8w=76$$
$$w=9.5$$
$$l=3(9.5)=28.5$$
The dimensions of the pond are 9.5 feet by 28.5 feet.

c. If the pond is circular, the diameter is d and the diameter of the circle with the pond and the deck is $d+6$.



The perimeter is 100 feet, so:
$$\pi(d+6)=100$$
$$\pi d+6\pi=100$$
$$\pi d=100-6\pi$$
$$d=\frac{100}{\pi}-6\approx 25.83$$
The diameter of the pond is 25.83 feet.

d. $\text{Area}_{\text{square}} = l \cdot w = 19(19) = 361 \text{ ft}^2$.
$\text{Area}_{\text{rectangle}} = l \cdot w = 28.5(9.5) = 270.75 \text{ ft}^2$.
$\text{Area}_{\text{circle}} = \pi r^2 = \pi\left(\frac{25.83}{2}\right)^2 \approx 524 \text{ ft}^2$.
The circular pond has the largest area.

37. Let t represent the time it takes for the defensive back to catch the tight end.

	Time to run 100 yards	Time	Rate	Distance
Tight End	12 sec	t	$\frac{100}{12}=\frac{25}{3}$	$\frac{25}{3}t$
Def. Back	10 sec	t	$\frac{100}{10}=10$	$10t$

Since the defensive back has to run 5 yards farther, we have:
$$\frac{25}{3}t+5=10t$$
$$25t+15=30t$$
$$15=5t$$
$$t=3 \quad \rightarrow \quad 10t=30$$
The defensive back will catch the tight end at the 45 yard line (15 + 30 = 45).

38. Let x represent the number of highway miles traveled. Then $30,000-x$ represents the number of city miles traveled.

$$\frac{x}{40}+\frac{30,000-x}{25}=900$$
$$200\left(\frac{x}{40}+\frac{30,000-x}{25}\right)=200(900)$$
$$5x+240,000-8x=180,000$$
$$-3x+240,000=180,000$$
$$-3x=-60,000$$
$$x=20,000$$

Therese is allowed to claim 20,000 miles as a business expense.

39. Let x represent the number of gallons of pure water. Then $x+1$ represents the number of gallons in the 60% solution.

$$(\%)(\text{gallons})+(\%)(\text{gallons})=(\%)(\text{gallons})$$
$$0(x)+1(1)=0.60(x+1)$$
$$1=0.6x+0.6$$
$$0.4=0.6x$$
$$x=\frac{4}{6}=\frac{2}{3}$$

$\frac{2}{3}$ gallon of pure water should be added.

40. Let x represent the number of liters to be drained and replaced with pure antifreeze.

$$(\%)(\text{liters})+(\%)(\text{liters})=(\%)(\text{liters})$$
$$1(x)+0.40(15-x)=0.60(15)$$
$$x+6-0.40x=9$$
$$0.60x=3$$
$$x=5$$

5 liters should be drained and replaced with pure antifreeze.

41. Let x represent the number of ounces of water to be evaporated; the amount of salt remains the same. Therefore, we get

$$0.04(32)=0.06(32-x)$$
$$1.28=1.92-0.06x$$
$$0.06x=0.64$$
$$x=\frac{0.64}{0.06}=\frac{64}{6}=\frac{32}{3}=10\tfrac{2}{3}$$

$10\frac{2}{3}\approx 10.67$ ounces of water need to be evaporated.

42. Let x represent the number of gallons of water to be evaporated; the amount of salt remains the same.

$$0.03(240)=0.05(240-x)$$
$$7.2=12-0.05x$$
$$0.05x=4.8$$
$$x=\frac{4.8}{0.05}=96$$

96 gallons of water need to be evaporated.

43. Let x represent the number of grams of pure gold. Then $60-x$ represents the number of grams of 12 karat gold to be used.

$$x+\frac{1}{2}(60-x)=\frac{2}{3}(60)$$
$$x+30-0.5x=40$$
$$0.5x=10$$
$$x=20$$

20 grams of pure gold should be mixed with 40 grams of 12 karat gold.

44. Let x represent the number of atoms of oxygen. $2x$ represents the number of atoms of hydrogen. $x+1$ represents the number of atoms of carbon.

$$x+2x+x+1=45$$
$$4x=44$$
$$x=11$$

There are 11 atoms of oxygen and 22 atoms of hydrogen in the sugar molecule.

45. Let t represent the time it takes for Mike to catch up with Dan. Since the distances are the same, we have:

$$\frac{1}{6}t=\frac{1}{9}(t+1)$$
$$3t=2t+2$$
$$t=2$$

Mike will pass Dan after 2 minutes, which is a distance of $\frac{1}{3}$ mile.

46. Let t represent the time of flight with the wind. The distance is the same in each direction:

$$330t=270(5-t)$$
$$330t=1350-270t$$
$$600t=1350$$
$$t=2.25$$

The distance the plane can fly and still return safely is 330(2.25) = 742.5 miles.

47. Let t represent the time the auxiliary pump needs to run. Since the two pumps are emptying one tanker, we have:

$$\frac{3}{4}+\frac{t}{9}=1$$
$$27+4t=36$$
$$4t=9$$
$$t=\frac{9}{4}=2.25$$

The auxiliary pump must run for 2.25 hours. It must be started at 9:45 a.m.

48. Let x represent the number of pounds of pure cement. Then $x+20$ represents the number of pounds in the 40% mixture.

$$x+0.25(20)=0.40(x+20)$$
$$x+5=0.4x+8$$
$$0.6x=3$$
$$x=\frac{30}{6}=5$$

5 pounds of pure cement should be added.

49. Let t represent the time for the tub to fill with the faucets on and the stopper removed. Since one tub is being filled, we have:

$$\frac{t}{15}+\left(-\frac{t}{20}\right)=1$$
$$4t-3t=60$$
$$t=60$$

60 minutes is required to fill the tub.

50. Let t be the time the 5 horsepower pump needs to run to finish emptying the pool. Since the two pumps are emptying one pool, we have:

$$\frac{t+2}{5}+\frac{2}{8}=1$$
$$4(2+t)+5=20$$
$$8+4t+5=20$$
$$4t=7$$
$$t=1.75$$

The 5 horsepower pump must run for an additional 1.75 hours or 1 hour and 45 minutes to empty the pool.

51. Let t represent the time spent running. Then $5-t$ represents the time spent biking.

	Rate	Time	Distance
Run	6	t	$6t$
Bike	25	$5-t$	$25(5-t)$

The total distance is 87 miles:

$$6t+25(5-t)=87$$
$$6t+125-25t=87$$
$$-19t+125=87$$
$$-19t=-38$$
$$t=2$$

The time spent running is 2 hours, so the distance of the run is $6(2)=12$ miles. The distance of the bicycle race is $25(5-2)=75$ miles.

52. Let r represent the speed of the eastbound cyclist. Then $r+5$ represents the speed of the westbound cyclist.

	Rate	Time	Distance
Eastbound	r	6	$6r$
Westbound	$r+5$	6	$6(r+5)$

The total distance is 246 miles:

$$6r+6(r+5)=246$$
$$6r+6r+30=246$$
$$12r+30=246$$
$$12r=216$$
$$r=18$$

The speed of the eastbound cyclist is 18 miles per hour, and the speed of the westbound cyclist is $18+5=23$ miles per hour.

53. Burke's rate is $\frac{100}{12}$ meters/sec. In 9.99 seconds, Burke will run $\frac{100}{12}(9.99)=83.25$ meters. Lewis would win by 16.75 meters.

54. $A=2\pi r^2+2\pi r h$. Since $A=188.5$ square inches and $h=7$ inches,

$$2\pi r^2+2\pi r(7)=188.5$$
$$2\pi r^2+14\pi r-188.5=0$$
$$r=\frac{-14\pi\pm\sqrt{(14\pi)^2-4(2\pi)(-188.5)}}{2(2\pi)}.$$
$$=\frac{-14\pi\pm\sqrt{6671.9642}}{4\pi}$$

$r\approx 3$ or ~~$r\approx -10$~~

The radius of the coffee can is approximately 3 inches.

55. Let x be the original selling price of the shirt.

$\text{Profit} = \text{Revenue} - \text{Cost}$

$4 = x - 0.40x - 20 \rightarrow 24 = 0.60x \rightarrow x = 40$

The original price should be \$40 to ensure a profit of \$4 after the sale.

If the sale is 50% off, the profit is:

$40 - 0.50(40) - 20 = 40 - 20 - 20 = 0$

At 50% off there will be no profit.

56. Answers will vary.

57. It is impossible to mix two solutions with a lower concentration and end up with a new solution with a higher concentration.

Algebraic Solution:

Let x = the number of liters of 25% solution.

$$(\%)(\text{liters}) + (\%)(\text{liters}) = (\%)(\text{liters})$$

$$\begin{aligned} 0.25x + 0.48(20) &= 0.58(20 + x) \\ 0.25x + 9.6 &= 10.6 + 0.58x \\ -0.33x &= 1 \\ x &\approx -3.03 \text{ liters} \\ &\text{(not possible)} \end{aligned}$$

58. Let t_1 and t_2 represent the times for the two segments of the trip. Since Atlanta is halfway between Chicago and Miami, the distances are equal.

$$\begin{aligned} 45t_1 &= 55t_2 \\ t_1 &= \frac{55}{45}t_2 \\ t_1 &= \frac{11}{9}t_2 \end{aligned}$$

Computing the average speed:

$$\begin{aligned} \text{Avg Speed} &= \frac{\text{Distance}}{\text{Time}} = \frac{45t_1 + 55t_2}{t_1 + t_2} \\ &= \frac{45\left(\frac{11}{9}t_2\right) + 55t_2}{\frac{11}{9}t_2 + t_2} = \frac{55t_2 + 55t_2}{\left(\frac{11t_2 + 9t_2}{9}\right)} \\ &= \frac{110t_2}{\left(\frac{20t_2}{9}\right)} = \frac{990t_2}{20t_2} \\ &= \frac{99}{2} = 49.5 \text{ miles per hour} \end{aligned}$$

The average speed for the trip from Chicago to Miami is 49.5 miles per hour.

59. The time traveled with the tail wind was:

$t = \dfrac{919}{550} \approx 1.67091$ hours .

Since they were 20 minutes $\left(\frac{1}{3}\text{ hour}\right)$ early, the time in still air would have been:

$$\begin{aligned} 1.67091 \text{ hrs } + 20 \text{ min} &= (1.67091 + 0.33333) \text{ hrs} \\ &\approx 2.00424 \text{ hrs} \end{aligned}$$

Thus, with no wind, the ground speed is $\dfrac{919}{2.00424} \approx 458.53$. Therefore, the tail wind is $550 - 458.53 = 91.47$ knots .

Chapter 1 Review

1.

$$\begin{aligned} 2 - \frac{x}{3} &= 8 \\ 6 - x &= 24 \\ x &= -18 \end{aligned}$$

The solution set is $\{-18\}$.

2.

$$\begin{aligned} \frac{x}{4} - 2 &= 4 \\ x - 8 &= 16 \\ x &= 24 \end{aligned}$$

The solution set is $\{24\}$.

3.

$$\begin{aligned} -2(5 - 3x) + 8 &= 4 + 5x \\ -10 + 6x + 8 &= 4 + 5x \\ 6x - 2 &= 4 + 5x \\ x &= 6 \end{aligned}$$

The solution set is $\{6\}$.

4.

$$\begin{aligned} (6 - 3x) - 2(1 + x) &= 6x \\ 6 - 3x - 2 - 2x &= 6x \\ -5x + 4 &= 6x \\ -11x &= -4 \\ x &= \frac{4}{11} \end{aligned}$$

The solution set is $\left\{\frac{4}{11}\right\}$.

5. $\frac{3x}{4}-\frac{x}{3}=\frac{1}{12}$

$9x-4x=1$

$5x=1$

$x=\frac{1}{5}$

The solution set is $\left\{\frac{1}{5}\right\}$.

6. $\frac{4-2x}{3}+\frac{1}{6}=2x$

$2(4-2x)+1=12x$

$8-4x+1=12x$

$9=16x$

$x=\frac{9}{16}$

The solution set is $\left\{\frac{9}{16}\right\}$.

7. $\frac{x}{x-1}=\frac{6}{5}$

$5x=6x-6$

$6=x$

Since $x = 6$ does not cause a denominator to equal zero, the solution set is $\{6\}$.

8. $\frac{4x-5}{3-7x}=2$

$4x-5=6-14x$

$18x=11$

$x=\frac{11}{18}$

Since $x=\frac{11}{18}$ does not cause a denominator to equal zero, the solution set is $\left\{\frac{11}{18}\right\}$.

9. $x(1-x)=6$

$x-x^2=6$

$0=x^2-x+6$

$b^2-4ac=(-1)^2-4(1)(6)$

$=1-24=-23$

Therefore, there are no real solutions.

10. $x(1+x)=6$

$x+x^2=6$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$ or $x=2$

The solution set is $\{-3,2\}$.

11. $\frac{1}{2}\left(x-\frac{1}{3}\right)=\frac{3}{4}-\frac{x}{6}$

$(12)\left(\frac{1}{2}\right)\left(x-\frac{1}{3}\right)=\left(\frac{3}{4}-\frac{x}{6}\right)(12)$

$6x-2=9-2x$

$8x=11$

$x=\frac{11}{8}$

The solution set is $\left\{\frac{11}{8}\right\}$.

12. $\frac{1-3x}{4}=\frac{x+6}{3}+\frac{1}{2}$

$(12)\left(\frac{1-3x}{4}\right)=\left(\frac{x+6}{3}+\frac{1}{2}\right)(12)$

$3(1-3x)=4(x+6)+6$

$3-9x=4x+24+6$

$-13x=27$

$x=-\frac{27}{13}$

The solution set is $\left\{-\frac{27}{13}\right\}$.

13. $(x-1)(2x+3)=3$

$2x^2+x-3=3$

$2x^2+x-6=0$

$(2x-3)(x+2)=0$

$x=\frac{3}{2}$ or $x=-2$

The solution set is $\left\{-2,\frac{3}{2}\right\}$.

14. $x(2-x)=3(x-4)$

$2x-x^2=3x-12$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ or $x=3$

The solution set is $\{-4,3\}$.

15. $2x+3=4x^2$

$0=4x^2-2x-3$

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(4)(-3)}}{2(4)}$

$=\frac{2\pm\sqrt{52}}{8}=\frac{2\pm2\sqrt{13}}{8}=\frac{1\pm\sqrt{13}}{4}$

The solution set is $\left\{\frac{1-\sqrt{13}}{4},\frac{1+\sqrt{13}}{4}\right\}$.

16. $1+6x=4x^2$

$0=4x^2-6x-1$

$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(4)(-1)}}{2(4)}$

$=\frac{6\pm\sqrt{52}}{8}=\frac{6\pm2\sqrt{13}}{8}=\frac{3\pm\sqrt{13}}{4}$

The solution set is $\left\{\frac{3-\sqrt{13}}{4},\frac{3+\sqrt{13}}{4}\right\}$.

17. $\sqrt[3]{x^2-1}=2$

$\left(\sqrt[3]{x^2-1}\right)^3=(2)^3$

$x^2-1=8$

$x^2=9$

$x=\pm3$

Check $x=-3$:

$\sqrt[3]{(-3)^2-1}=2$

$\sqrt[3]{9-1}=2$

$\sqrt[3]{8}=2$

$2=2$

Check $x=3$:

$\sqrt[3]{(3)^2-1}=2$

$\sqrt[3]{9-1}=2$

$\sqrt[3]{8}=2$

$2=2$

The solution set is $\{-3,3\}$.

18. $\sqrt{1+x^3}=3$

$\left(\sqrt{1+x^3}\right)^2=(3)^2$

$1+x^3=9$

$x^3=8$

$x=\sqrt[3]{8}=2$

Check $x=2$: $\sqrt{1+(2)^3}=3$

$\sqrt{9}=3$

$3=3$

The solution set is $\{2\}$.

19. $x(x+1)+2=0$

$x^2+x+2=0$

$x=\frac{-1\pm\sqrt{(1)^2-4(1)(2)}}{2(1)}=\frac{-1\pm\sqrt{-7}}{2}$

No real solutions.

20. $3x^2-x+1=0$

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(1)}}{2(3)}=\frac{1\pm\sqrt{-11}}{6}$

No real solutions.

21. $x^4-5x^2+4=0$

$(x^2-4)(x^2-1)=0$

$x^2-4=0$ or $x^2-1=0$

$x=\pm2$ or $x=\pm1$

The solution set is $\{-2,-1,1,2\}$.

22. $3x^4+4x^2+1=0$

$(3x^2+1)(x^2+1)=0$

$3x^2+1=0$ or $x^2+1=0$

$3x^2=-1$ or $x^2=-1$

No real solutions.

23. $\sqrt{2x-3}+x=3$

$\sqrt{2x-3}=3-x$

$2x-3=9-6x+x^2$

$x^2-8x+12=0$

$(x-2)(x-6)=0$

$x=2$ or $x=6$

Check $x=2$: $\sqrt{2(2)-3}+2=\sqrt{1}+2=3$

Check $x=6$: $\sqrt{2(6)-3}+6=\sqrt{9}+6=9\neq3$

The solution set is $\{2\}$.

24. $\sqrt{2x-1}=x-2$

$2x-1=x^2-4x+4$

$x^2-6x+5=0$

$(x-1)(x-5)=0$

$x=1$ or $x=5$

Check $x=1$:

$\sqrt{2(1)-1}=1-2$

$1\neq-1$

Check $x=5$:

$\sqrt{2(5)-1}=5-2$

$3=3$

The solution set is $\{5\}$.

25. $\sqrt[4]{2x+3}=2$

$\left(\sqrt[4]{2x+3}\right)^4=2^4$

$2x+3=16$

$2x=13$

$x=\dfrac{13}{2}$

Check $x=\dfrac{13}{2}$:

$\sqrt[4]{2\left(\dfrac{13}{2}\right)+3}=\sqrt[4]{13+3}=\sqrt[4]{16}=2$

The solution set is $\left\{\dfrac{13}{2}\right\}$.

26. $\sqrt[5]{3x+1}=-1$

$\left(\sqrt[5]{3x+1}\right)^5=(-1)^5$

$3x+1=-1$

$3x=-2$

$x=-\dfrac{2}{3}$

Check $x=-\dfrac{2}{3}$:

$\sqrt[5]{3\left(-\dfrac{2}{3}\right)+1}=\sqrt[5]{-2+1}=\sqrt[5]{-1}=-1$

The solution set is $\left\{-\dfrac{2}{3}\right\}$.

27. $\sqrt{x+1}+\sqrt{x-1}=\sqrt{2x+1}$

$\left(\sqrt{x+1}+\sqrt{x-1}\right)^2=\left(\sqrt{2x+1}\right)^2$

$x+1+2\sqrt{x+1}\sqrt{x-1}+x-1=2x+1$

$2x+2\sqrt{x+1}\sqrt{x-1}=2x+1$

$2\sqrt{x+1}\sqrt{x-1}=1$

$\left(2\sqrt{x+1}\sqrt{x-1}\right)^2=(1)^2$

$4(x+1)(x-1)=1$

$4x^2-4=1$

$4x^2=5$

$x^2=\dfrac{5}{4}$

$x=\pm\dfrac{\sqrt{5}}{2}$

Check $x=\dfrac{\sqrt{5}}{2}$:

$\sqrt{\dfrac{\sqrt{5}}{2}+1}+\sqrt{\dfrac{\sqrt{5}}{2}-1}=\sqrt{2\left(\dfrac{\sqrt{5}}{2}\right)+1}$

$1.79890743995=1.79890743995$

Check $x=-\dfrac{\sqrt{5}}{2}$:

$\sqrt{-\dfrac{\sqrt{5}}{2}+1}+\sqrt{-\dfrac{\sqrt{5}}{2}-1}=\sqrt{2\left(-\dfrac{\sqrt{5}}{2}\right)+1},$

The second solution is not possible because it makes the radicand negative.

The solution set is $\left\{\dfrac{\sqrt{5}}{2}\right\}$.

28. $\sqrt{2x-1}-\sqrt{x-5}=3$

$\sqrt{2x-1}=3+\sqrt{x-5}$

$\left(\sqrt{2x-1}\right)^2=\left(3+\sqrt{x-5}\right)^2$

$2x-1=9+6\sqrt{x-5}+x-5$

$x-5=6\sqrt{x-5}$

$(x-5)^2=\left(6\sqrt{x-5}\right)^2$

$x^2-10x+25=36(x-5)$

$x^2-10x+25=36x-180$

$x^2-46x+205=0$

$(x-41)(x-5)=0$

$x=41$ or $x=5$

Check $x=41$:

$\sqrt{2(41)-1}-\sqrt{41-5}=\sqrt{81}-\sqrt{36}=9-6=3$

Check $x=5$:

$\sqrt{2(5)-1}-\sqrt{5-5}=\sqrt{9}-\sqrt{0}=3-0=3$

The solution set is $\{5,41\}$.

29. $2x^{1/2}-3=0$

$$2x^{1/2}=3$$
$$\left(2x^{1/2}\right)^2=3^2$$
$$4x=9$$
$$x=\frac{9}{4}$$

Check $x=\frac{9}{4}$:

$$2\left(\frac{9}{4}\right)^{1/2}-3=2\left(\frac{3}{2}\right)-3=3-3=0$$

The solution set is $\left\{\frac{9}{4}\right\}$.

30. $3x^{1/4}-2=0$

$$3x^{1/4}=2$$
$$\left(3x^{1/4}\right)^4=2^4$$
$$81x=16$$
$$x=\frac{16}{81}$$

Check $x=\frac{16}{81}$:

$$3\left(\frac{16}{81}\right)^{1/4}-2=3\left(\frac{2}{3}\right)-2=2-2=0$$

The solution set is $\left\{\frac{16}{81}\right\}$.

31. $x^{-6}-7x^{-3}-8=0$

Let $u=x^{-3}$ so that $u^2=x^{-6}$.

$$u^2-7u-8=0$$
$$(u-8)(u+1)=0$$
$$u=8 \quad \text{or} \quad u=-1$$
$$x^{-3}=8 \quad \text{or} \quad x^{-3}=-1$$
$$\left(x^{-3}\right)^{-1/3}=(8)^{-1/3} \quad \text{or} \quad \left(x^{-3}\right)^{-1/3}=(-1)^{-1/3}$$
$$x=\frac{1}{2} \quad \text{or} \quad x=-1$$

Check $\frac{1}{2}$: $\left(\frac{1}{2}\right)^{-6}-7\left(\frac{1}{2}\right)^{-3}-8=64-56-8=0$

Check -1: $(-1)^{-6}-7(-1)^{-3}-8=1+7-8=0$

The solution set is $\left\{-1,\frac{1}{2}\right\}$.

32. $6x^{-1}-5x^{-1/2}+1=0$

Let $u=x^{-1/2}$ so that $u^2=x^{-1}$.

$$6u^2-5u+1=0$$
$$(3u-1)(2u-1)=0$$
$$u=\frac{1}{3} \quad \text{or} \quad u=\frac{1}{2}$$
$$x^{-1/2}=\frac{1}{3} \quad \text{or} \quad x^{-1/2}=\frac{1}{2}$$
$$\left(x^{-1/2}\right)^{-2}=\left(\frac{1}{3}\right)^{-2} \quad \text{or} \quad \left(x^{-1/2}\right)^{-2}=\left(\frac{1}{2}\right)^{-2}$$
$$x=9 \quad \text{or} \quad x=4$$

Check $x=9$:

$$6(9)^{-1}-5(9)^{-1/2}+1=6\left(\frac{1}{9}\right)-5\left(\frac{1}{3}\right)+1$$
$$=\frac{2}{3}-\frac{5}{3}+1=-1+1=0$$

Check $x=4$:

$$6(4)^{-1}-5(4)^{-1/2}+1=6\left(\frac{1}{4}\right)-5\left(\frac{1}{2}\right)+1$$
$$=\frac{3}{2}-\frac{5}{2}+1=-1+1=0$$

The solution set is $\{4,9\}$.

33.

$$x^2+m^2=2mx+(nx)^2$$
$$x^2+m^2=2mx+n^2x^2$$
$$x^2-n^2x^2-2mx+m^2=0$$
$$\left(1-n^2\right)x^2-2mx+m^2=0$$
$$x=\frac{-(-2m)\pm\sqrt{(-2m)^2-4\left(1-n^2\right)m^2}}{2\left(1-n^2\right)}$$
$$=\frac{2m\pm\sqrt{4m^2-4m^2+4m^2n^2}}{2\left(1-n^2\right)}$$
$$=\frac{2m\pm\sqrt{4m^2n^2}}{2\left(1-n^2\right)}=\frac{2m\pm 2mn}{2\left(1-n^2\right)}$$
$$=\frac{2m(1\pm n)}{2\left(1-n^2\right)}=\frac{m(1\pm n)}{1-n^2}$$
$$x=\frac{m(1+n)}{1-n^2}=\frac{m(1+n)}{(1+n)(1-n)}=\frac{m}{1-n}$$

or

$$x=\frac{m(1-n)}{1-n^2}=\frac{m(1-n)}{(1+n)(1-n)}=\frac{m}{1+n}$$

The solution set is $\left\{\frac{m}{1-n},\frac{m}{1+n}\right\}$, $n\neq 1$, $n\neq -1$.

34. $b^2x^2 + 2ax = x^2 + a^2$

$b^2x^2 + 2ax - x^2 - a^2 = 0$

$b^2x^2 - x^2 + 2ax - a^2 = 0$

$(b^2 - 1)x^2 + 2ax - a^2 = 0$

$$x = \frac{-(2a) \pm \sqrt{(2a)^2 - 4(b^2-1)(-a^2)}}{2(b^2-1)}$$

$$= \frac{-2a \pm \sqrt{4a^2 + 4a^2b^2 - 4a^2}}{2(b^2-1)} = \frac{-2a \pm \sqrt{4a^2b^2}}{2(b^2-1)}$$

$$= \frac{-2a \pm 2ab}{2(b^2-1)} = \frac{2a(-1 \pm b)}{2(b^2-1)} = \frac{a(-1 \pm b)}{b^2-1}$$

$$x = \frac{a(-1-b)}{b^2-1} = \frac{-a(b+1)}{(b+1)(b-1)} = \frac{-a}{b-1} = \frac{a}{1-b}$$

or

$$x = \frac{a(-1+b)}{b^2-1} = \frac{a(b-1)}{(b+1)(b-1)} = \frac{a}{b+1} = \frac{a}{1+b}$$

The solution set is $\left\{\frac{a}{1-b}, \frac{a}{1+b}\right\}$, $b \neq 1$, $b \neq -1$.

35. $10a^2x^2 - 2abx - 36b^2 = 0$

$5a^2x^2 - abx - 18b^2 = 0$

$(5ax + 9b)(ax - 2b) = 0$

$5ax + 9b = 0$ or $ax - 2b = 0$

$5ax = -9b$ $\quad ax = 2b$

$x = -\frac{9b}{5a}$ $\quad x = \frac{2b}{a}$

The solution set is $\left\{-\frac{9b}{5a}, \frac{2b}{a}\right\}$, $a \neq 0$.

36. $$\frac{1}{x-m} + \frac{1}{x-n} = \frac{2}{x}$$

$$\frac{(x-n)+(x-m)}{(x-m)(x-n)} = \frac{2}{x}$$

$$\frac{2x-m-n}{(x-m)(x-n)} = \frac{2}{x}$$

$x(2x - m - n) = 2(x-m)(x-n)$

$2x^2 - xm - xn = 2x^2 - 2xn - 2xm + 2mn$

$xm + xn - 2mn = 0$

$xn + xm = 2mn$

$x(n+m) = 2mn$

$$x = \frac{2mn}{n+m}$$

The solution set is $\left\{\frac{2mn}{n+m}\right\}$ where

$n \neq -m, x \neq m, x \neq n, x \neq 0$.

37. $\sqrt{x^2+3x+7} - \sqrt{x^2-3x+9} + 2 = 0$

$\sqrt{x^2+3x+7} = \sqrt{x^2-3x+9} - 2$

$\left(\sqrt{x^2+3x+7}\right)^2 = \left(\sqrt{x^2-3x+9} - 2\right)^2$

$x^2 + 3x + 7 = x^2 - 3x + 9 - 4\sqrt{x^2-3x+9} + 4$

$6x - 6 = -4\sqrt{x^2-3x+9}$

$(6(x-1))^2 = \left(-4\sqrt{x^2-3x+9}\right)^2$

$36(x^2 - 2x + 1) = 16(x^2 - 3x + 9)$

$36x^2 - 72x + 36 = 16x^2 - 48x + 144$

$20x^2 - 24x - 108 = 0$

$5x^2 - 6x - 27 = 0$

$(5x+9)(x-3) = 0$

$x = -\frac{9}{5}$ or $x = 3$

Check $x = -\frac{9}{5}$:

$$\sqrt{\left(-\frac{9}{5}\right)^2 + 3\left(-\frac{9}{5}\right) + 7} - \sqrt{\left(-\frac{9}{5}\right)^2 - 3\left(-\frac{9}{5}\right) + 9} + 2$$

$$= \sqrt{\frac{81}{25} - \frac{27}{5} + 7} - \sqrt{\frac{81}{25} + \frac{27}{5} + 9} + 2$$

$$= \sqrt{\frac{81 - 135 + 175}{25}} - \sqrt{\frac{81 + 135 + 225}{25}} + 2$$

$$= \sqrt{\frac{121}{25}} - \sqrt{\frac{441}{25}} + 2 = \frac{11}{5} - \frac{21}{5} + 2 = 0$$

Check $x = 3$:

$$\sqrt{(3)^2 + 3(3) + 7} - \sqrt{(3)^2 - 3(3) + 9} + 2$$

$= \sqrt{9+9+7} - \sqrt{9-9+9} + 2$

$= \sqrt{25} - \sqrt{9} + 2 = 2 + 2$

$= 4 \neq 0$

The solution set is $\left\{-\frac{9}{5}\right\}$.

38. $\sqrt{x^2+3x+7} - \sqrt{x^2+3x+9} = 2$

$\sqrt{x^2+3x+7} = \sqrt{x^2+3x+9} + 2$

$\left(\sqrt{x^2+3x+7}\right)^2 = \left(\sqrt{x^2+3x+9} + 2\right)^2$

$x^2 + 3x + 7 = x^2 + 3x + 9 + 4\sqrt{x^2+3x+9} + 4$

$-6 = 4\sqrt{x^2+3x+9}$

This is impossible since the principal square root always yields a non-negative number. Therefore, there is no real solution.

39. $|2x+3|=7$

$2x+3=7$ or $2x+3=-7$

$2x=4$ or $2x=-10$

$x=2$ or $x=-5$

The solution set is $\{-5, 2\}$.

40. $|3x-1|=5$

$3x-1=5$ or $3x-1=-5$

$3x=6$ or $3x=-4$

$x=2$ or $x=-\frac{4}{3}$

The solution set is $\left\{-\frac{4}{3}, 2\right\}$.

41. $|2-3x|+2=9$

$|2-3x|=7$

$2-3x=7$ or $2-3x=-7$

$-3x=5$ or $-3x=-9$

$x=-\frac{5}{3}$ or $x=3$

The solution set is $\left\{-\frac{5}{3},3\right\}$

42. $|1-2x|+1=4$

$|1-2x|=3$

$1-2x=3$ or $1-2x=-3$

$2x=-2$ or $2x=4$

$x=-1$ or $x=2$

The solution set is $\{-1, 2\}$.

43. $2x^3=3x^2$

$2x^3-3x^2=0$

$x^2(2x-3)=0$

$x^2=0$ or $2x-3=0$

$x=0$ or $x=\frac{3}{2}$

The solution set is $\left\{0,\frac{3}{2}\right\}$.

44. $5x^4=9x^3$

$5x^4-9x^3=0$

$x^3(5x-9)=0$

$x^3=0$ or $5x-9=0$

$x=0$ or $x=\frac{9}{5}$

The solution set is $\left\{0,\frac{9}{5}\right\}$.

45. $2x^3+5x^2-8x-20=0$

$x^2(2x+5)-4(2x+5)=0$

$(2x+5)(x^2-4)=0$

$2x+5=0$ or $x^2-4=0$

$2x=-5$ or $x^2=4$

$x=-\frac{5}{2}$ or $x=\pm 2$

The solution set is $\left\{-\frac{5}{2},-2,2\right\}$.

46. $3x^3+5x^2-3x-5=0$

$x^2(3x+5)-1(3x+5)=0$

$(3x+5)(x^2-1)=0$

$3x+5=0$ or $x^2-1=0$

$3x=-5$ or $x^2=1$

$x=-\frac{5}{3}$ or $x=\pm 1$

The solution set is $\left\{-\frac{5}{3},-1,1\right\}$.

47. $\frac{2x-3}{5}+2\le\frac{x}{2}$

$2(2x-3)+10(2)\le 5x$

$4x-6+20\le 5x$

$14\le x$

$x\ge 14$

$\{x \mid x\ge 14\}$ or $[14,\infty)$

48. $\dfrac{5-x}{3} \le 6x-4$

$5-x \le 3(6x-4)$

$5-x \le 18x-12$

$-19x \le -17$

$x \ge \dfrac{17}{19}$

$\left\{x \,\middle|\, x \ge \dfrac{17}{19}\right\}$ or $\left[\dfrac{17}{19}, \infty\right)$

49. $-9 \le \dfrac{2x+3}{-4} \le 7$

$36 \ge 2x+3 \ge -28$

$33 \ge \quad 2x \quad \ge -31$

$\dfrac{33}{2} \ge \quad x \quad \ge -\dfrac{31}{2}$

$-\dfrac{31}{2} \le \quad x \quad \le \dfrac{33}{2}$

$\left\{x \,\middle|\, -\dfrac{31}{2} \le x \le \dfrac{33}{2}\right\}$ or $\left[-\dfrac{31}{2}, \dfrac{33}{2}\right]$

50. $-4 < \dfrac{2x-2}{3} < 6$

$-12 < 2x-2 < 18$

$-10 < \quad 2x \quad < 20$

$-5 < \quad x \quad < 10$

$\{x \mid -5 < x < 10\}$ or $(-5, 10)$

51. $2 < \dfrac{3-3x}{12} < 6$

$24 < 3-3x < 72$

$21 < \; -3x \; < 69$

$-7 > \quad x \quad > -23$

$\{x \mid -23 < x < -7\}$ or $(-23, -7)$

52. $-3 \le \dfrac{5-3x}{2} \le 6$

$-6 \le 5-3x \le 12$

$-11 \le \; -3x \; \le 7$

$\dfrac{11}{3} \ge \quad x \quad \ge -\dfrac{7}{3}$

$\left\{x \,\middle|\, -\dfrac{7}{3} \le x \le \dfrac{11}{3}\right\}$ or $\left[-\dfrac{7}{3}, \dfrac{11}{3}\right]$

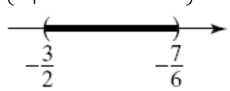
53. $|3x+4| < \dfrac{1}{2}$

$-\dfrac{1}{2} < 3x+4 < \dfrac{1}{2}$

$-\dfrac{9}{2} < \quad 3x \quad < -\dfrac{7}{2}$

$-\dfrac{3}{2} < \quad x \quad < -\dfrac{7}{6}$

$\left\{x \,\middle|\, -\dfrac{3}{2} < x < -\dfrac{7}{6}\right\}$ or $\left(-\dfrac{3}{2}, -\dfrac{7}{6}\right)$



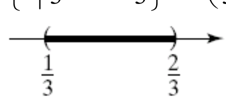
54. $|1-2x| < \dfrac{1}{3}$

$-\dfrac{1}{3} < 1-2x < \dfrac{1}{3}$

$-\dfrac{4}{3} < \; -2x \; < -\dfrac{2}{3}$

$\dfrac{2}{3} > \quad x \quad > \dfrac{1}{3}$

$\left\{x \,\middle|\, \dfrac{1}{3} < x < \dfrac{2}{3}\right\}$ or $\left(\dfrac{1}{3}, \dfrac{2}{3}\right)$



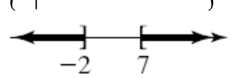
55. $|2x-5| \ge 9$

$2x-5 \le -9$ or $2x-5 \ge 9$

$2x \le -4$ or $2x \ge 14$

$x \le -2$ or $x \ge 7$

$\{x \mid x \le -2 \text{ or } x \ge 7\}$ or $(-\infty, -2] \cup [7, \infty)$



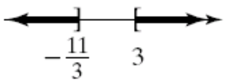
56. $|3x+1| \ge 10$

$3x+1 \le -10 \text{ or } 3x+1 \ge 10$

$3x \le -11 \text{ or } 3x \ge 9$

$x \le -\frac{11}{3} \text{ or } x \ge 3$

$\left\{x \middle| x \le -\frac{11}{3} \text{ or } x \ge 3\right\}$ or $\left(-\infty, -\frac{11}{3}\right] \cup [3, \infty)$



57. $2+|2-3x| \le 4$

$|2-3x| \le 2$

$-2 \le 2-3x \le 2$

$-4 \le -3x \le 0$

$\frac{4}{3} \ge x \ge 0$

$\left\{x \middle| 0 \le x \le \frac{4}{3}\right\}$ or $\left[0, \frac{4}{3}\right]$

58. $\frac{1}{2}+\left|\frac{2x-1}{3}\right| \le 1$

$\left|\frac{2x-1}{3}\right| \le \frac{1}{2}$

$-\frac{1}{2} \le \frac{2x-1}{3} \le \frac{1}{2}$

$-\frac{3}{2} \le 2x-1 \le \frac{3}{2}$

$-\frac{1}{2} \le 2x \le \frac{5}{2}$

$-\frac{1}{4} \le x \le \frac{5}{4}$

$\left\{x \middle| -\frac{1}{4} \le x \le \frac{5}{4}\right\}$ or $\left[-\frac{1}{4}, \frac{5}{4}\right]$

59. $1-|2-3x| < -4$

$-|2-3x| < -5$

$|2-3x| > 5$

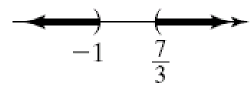
$2-3x < -5 \text{ or } 2-3x > 5$

$7 < 3x \text{ or } -3 > 3x$

$\frac{7}{3} < x \text{ or } -1 > x$

$x < -1 \text{ or } x > \frac{7}{3}$

$\left\{x \middle| x < -1 \text{ or } x > \frac{7}{3}\right\}$ or $(-\infty, -1) \cup \left(\frac{7}{3}, \infty\right)$



60. $1-\left|\frac{2x-1}{3}\right| < -2$

$-\left|\frac{2x-1}{3}\right| < -3$

$\left|\frac{2x-1}{3}\right| > 3$

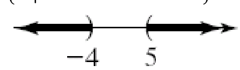
$\frac{2x-1}{3} < -3 \text{ or } \frac{2x-1}{3} > 3$

$2x-1 < -9 \text{ or } 2x-1 > 9$

$2x < -8 \text{ or } 2x > 10$

$x < -4 \text{ or } x > 5$

$\{x | x < -4 \text{ or } x > 5\}$ or $(-\infty, -4) \cup (5, \infty)$



61. $\left(\frac{1}{2} \cdot 6\right)^2 = 9$

62. $\left(\frac{1}{2} \cdot (-10)\right)^2 = 25$

63. $\left(\frac{1}{2} \cdot \left(-\frac{4}{3}\right)\right)^2 = \frac{4}{9}$

64. $\left(\frac{1}{2} \cdot \frac{4}{5}\right)^2 = \frac{4}{25}$

65. $(6+3i)-(2-4i)=(6-2)+(3-(-4))i=4+7i$

66. $(8-3i)+(-6+2i)=(8-6)+(-3+2)i=2-i$

67. $4(3-i)+3(-5+2i)=12-4i-15+6i=-3+2i$

68. $2(1+i)-3(2-3i)=2+2i-6+9i=-4+11i$

69. $\frac{3}{3+i}=\frac{3}{3+i}\cdot\frac{3-i}{3-i}=\frac{9-3i}{9-3i+3i-i^2}$
$=\frac{9-3i}{10}=\frac{9}{10}-\frac{3}{10}i$

70. $\frac{4}{2-i}=\frac{4}{2-i}\cdot\frac{2+i}{2+i}=\frac{8+4i}{4+2i-2i-i^2}$
$=\frac{8+4i}{5}=\frac{8}{5}+\frac{4}{5}i$

71. $i^{50}=i^{48}\cdot i^2=\left(i^4\right)^{12}\cdot i^2=1^{12}(-1)=-1$

72. $i^{29}=i^{28}\cdot i=\left(i^4\right)^7\cdot i=1^7\cdot i=i$

73. $(2+3i)^3=(2+3i)^2(2+3i)$
$=\left(4+12i+9i^2\right)(2+3i)$
$=(-5+12i)(2+3i)$
$=-10-15i+24i+36i^2$
$=-46+9i$

74. $(3-2i)^3=(3-2i)^2(3-2i)$
$=\left(9-12i+4i^2\right)(3-2i)$
$=(5-12i)(3-2i)$
$=15-10i-36i+24i^2$
$=-9-46i$

75. $x^2+x+1=0$
$a=1, b=1, c=1,$
$b^2-4ac=1^2-4(1)(1)=1-4=-3$
$x=\frac{-1\pm\sqrt{-3}}{2(1)}=\frac{-1\pm\sqrt{3}\,i}{2}=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$
The solution set is $\left\{-\frac{1}{2}-\frac{\sqrt{3}}{2}i,\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i\right\}$.

76. $x^2-x+1=0$
$a=1, b=-1, c=1,$
$b^2-4ac=(-1)^2-4(1)(1)=1-4=-3$
$x=\frac{-(-1)\pm\sqrt{-3}}{2(1)}=\frac{1\pm\sqrt{3}\,i}{2}=\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$
The solution set is $\left\{\frac{1}{2}-\frac{\sqrt{3}}{2}i,\ \frac{1}{2}+\frac{\sqrt{3}}{2}i\right\}$.

77. $2x^2+x-2=0$
$a=2, b=1, c=-2,$
$b^2-4ac=1^2-4(2)(-2)=1+16=17$
$x=\frac{-1\pm\sqrt{17}}{2(2)}=\frac{-1\pm\sqrt{17}}{4}$
The solution set is $\left\{\frac{-1-\sqrt{17}}{4},\ \frac{-1+\sqrt{17}}{4}\right\}$.

78. $3x^2-2x-1=0$
$(3x+1)(x-1)=0$
$x=-\frac{1}{3}$ or $x=1$
The solution set is $\left\{-\frac{1}{3},\ 1\right\}$.

79. $x^2+3=x$
$x^2-x+3=0$
$a=1, b=-1, c=3,$
$b^2-4ac=(-1)^2-4(1)(3)=1-12=-11$
$x=\frac{-(-1)\pm\sqrt{-11}}{2(1)}=\frac{1\pm\sqrt{11}\,i}{2}=\frac{1}{2}\pm\frac{\sqrt{11}}{2}i$
The solution set is $\left\{\frac{1}{2}-\frac{\sqrt{11}}{2}i,\ \frac{1}{2}+\frac{\sqrt{11}}{2}i\right\}$.

80. $2x^2+1=2x$
$2x^2-2x+1=0$
$a=2, b=-2, c=1,$
$b^2-4ac=(-2)^2-4(2)(1)=4-8=-4$
$x=\frac{-(-2)\pm\sqrt{-4}}{2(2)}=\frac{2\pm2\,i}{4}=\frac{1}{2}\pm\frac{1}{2}i$
The solution set is $\left\{\frac{1}{2}-\frac{1}{2}i,\ \frac{1}{2}+\frac{1}{2}i\right\}$.

81. $x(1-x)=6$

$-x^2+x-6=0$

$a=-1, b=1, c=-6,$

$b^2-4ac=1^2-4(-1)(-6)=1-24=-23$

$x=\frac{-1\pm\sqrt{-23}}{2(-1)}=\frac{-1\pm\sqrt{23}\,i}{-2}=\frac{1}{2}\pm\frac{\sqrt{23}}{2}i$

The solution set is $\left\{\frac{1}{2}-\frac{\sqrt{23}}{2}i,\ \frac{1}{2}+\frac{\sqrt{23}}{2}i\right\}$.

82. $x(1+x)=2$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2$ or $x=1$

The solution set is $\{-2,1\}$.

83. $p=2l+2w$

84. $c=50{,}000+95x$

85. $I=P\cdot r\cdot t$

$I=(9000)(0.07)(1)=\$630$

86. Let x represent the amount of money invested in bonds. Then $70{,}000-x$ represents the amount of money invested in CD's.
Since the total interest is to be \$5000, we have:

$0.08x+0.05(70{,}000-x)=5000$

$(100)(0.08x+0.05(70{,}000-x))=(5000)(100)$

$8x+350{,}000-5x=500{,}000$

$3x+350{,}000=500{,}000$

$3x=150{,}000$

$x=50{,}000$

\$50,000 should be invested in bonds at 8% and \$20,000 should be invested in CD's at 5%.

87. Using $s=vt$, we have $t=3$ and $v=1100$.
Finding the distance s in feet:

$s=1100(3)=3300$

The storm is 3300 feet away.

88. $1600\le I\le 3600$

$1600\le\frac{900}{x^2}\le 3600$

$\frac{1}{1600}\ge\frac{x^2}{900}\ge\frac{1}{3600}$

$\frac{9}{16}\ge x^2\ge\frac{1}{4}$

$\frac{3}{4}\ge x\ge\frac{1}{2}$

The range of distances is from 0.5 meters to 0.75 meters, inclusive.

89. Let s represent the distance the plane can travel.

	With wind	Against wind
Rate	$250+30=280$	$250-30=220$
Time	$\frac{(s/2)}{280}$	$\frac{(s/2)}{220}$
Dist.	$\frac{s}{2}$	$\frac{s}{2}$

Since the total time is at most 5 hours, we have:

$\frac{(s/2)}{280}+\frac{(s/2)}{220}\le 5$

$\frac{s}{560}+\frac{s}{440}\le 5$

$11s+14s\le 5(6160)$

$25s\le 30{,}800$

$s\le 1232$

The plane can travel at most 1232 miles or 616 miles one way and return 616 miles.

90. Let s represent the distance the plane can travel.

	With wind	Against wind
Rate	$250+30=280$	$250-30=220$
Time	$\frac{(s/2)}{280}$	$\frac{(s/2)}{220}$
Dist.	$\frac{s}{2}$	$\frac{s}{2}$

Since the total time is at most 7 hours, we have:

$\frac{(s/2)}{280}+\frac{(s/2)}{220}\le 7$

$\frac{s}{560}+\frac{s}{440}\le 7$

$11s+14s\le 7(6160)$

$25s\le 43{,}120$

$s\le 1724.8$

The plane can travel at most 1724.8 miles or 862.4 miles one way. This is 246.4 miles farther than in Problem 89.

91. Let t represent the time it takes the helicopter to reach the raft.

	Raft	Helicopter
Rate	5	90
Time	t	t
Dist.	$5t$	$90t$

Since the total distance is 150 miles, we have:

$$5t + 90t = 150$$
$$95t = 150$$
$$t \approx 1.58 \text{ hours } \approx \text{ 1 hour and 35 minutes}$$

The helicopter will reach the raft in about 1 hour and 35 minutes.

92. Let d represent the distance flown by the bee traveling at 3 meters per second.

$$\frac{d}{3} = \frac{150-d}{5} \quad \text{(Times needed to meet are equal.)}$$
$$5d = 450 - 3d$$
$$8d = 450$$
$$d = 56.25 \text{ meters}$$
$$t = \frac{56.25}{3} = 18.75 \text{ seconds}$$

The bees meet for the first time after 18.75 seconds.

The bees will meet a second time on the second lap. The first bee will have traveled 150 + x meters and the second bee will have traveled 150 + (150 – x) meters.
Solving for time, we have:

$$\frac{150+x}{3} = \frac{150+(150-x)}{5}$$
$$\frac{150+x}{3} = \frac{300-x}{5}$$
$$750 + 5x = 900 - 3x$$
$$8x = 150$$
$$x = 18.75 \text{ meters into the second lap}$$
$$t = \frac{168.75}{3} = 56.25 \text{ seconds}$$

The bees meet the second time after 56.25 seconds, or 37.5 seconds after their first meeting.

93. Let r represent the rate of the Metra train in miles per hour.

	Metra Train	Amtrak Train
Rate	r	$r+50$
Time	3	1
Dist.	$3r$	$r+50$

The Amtrak Train has traveled 10 fewer miles than the Metra Train.

$$r + 50 = 3r - 10$$
$$60 = 2r$$
$$r = 30$$

The Metra Train is traveling at 30 mph, and the Amtrak Train is traveling at $30 + 50 = 80$ mph.

94. Given that $s = 1280 - 32t - 16t^2$,

a. The object hits the ground when $s = 0$.

$$0 = 1280 - 32t - 16t^2$$
$$t^2 + 2t - 80 = 0$$
$$(t+10)(t-8) = 0$$
$$t = -10, t = 8$$

The object hits the ground after 8 seconds.

b. After 4 seconds, the object's height is $s = 1280 - 32(4) - 16(4)^2 = 896$ feet.

95. Let t represent the time it takes Clarissa to complete the job by herself.

	Clarissa	Shawna
Time to do job alone	t	$t+5$
Part of job done in 1 day	$\frac{1}{t}$	$\frac{1}{t+5}$
Time on job (days)	6	6
Part of job done by each person	$\frac{6}{t}$	$\frac{6}{t+5}$

Since the two people paint one house, we have:

$$\frac{6}{t} + \frac{6}{t+5} = 1$$
$$6(t+5) + 6t = t(t+5)$$
$$6t + 30 + 6t = t^2 + 5t$$
$$t^2 - 7t - 30 = 0$$
$$(t-10)(t+3) = 0$$
$$t = 10 \text{ or } t = -3$$

It takes Clarissa 10 days to paint the house when working by herself.

96. Let t represent the time it takes the smaller pump to empty the tank.

	Small Pump	Large Pump
Time to do job alone	t	$t-4$
Part of job done in 1 hr	$\frac{1}{t}$	$\frac{1}{t-4}$
Time on job (hrs)	5	5
Part of job done by each pump	$\frac{5}{t}$	$\frac{5}{t-4}$

Since the two pumps empty one tank, we have:

$$\frac{5}{t}+\frac{5}{t-4}=1$$

$$5(t-4)+5t=t(t-4)$$

$$5t-20+5t=t^2-4t$$

$$t^2-14t+20=0$$

We can solve this equation for t by using the quadratic formula:

$$t=\frac{-(-14)\pm\sqrt{(-14)^2-4(1)(20)}}{2(1)}$$

$$=\frac{14\pm\sqrt{116}}{2}=\frac{14\pm 2\sqrt{29}}{2}$$

$$=7\pm\sqrt{29}\approx 7+5.385$$

$t=12.385$ or $t=1.615$ (not feasible)

It takes the small pump approximately 12.385 hours (12 hr 23 min) to empty the tank.

97. Let x represent the amount of water added.

% salt	Tot. amt.	amt. of salt
10%	64	$(0.10)(64)$
0%	x	$(0.00)(x)$
2%	$64+x$	$(0.02)(64+x)$

$$(0.10)(64)+(0.00)(x)=(0.02)(64+x)$$

$$6.4=1.28+0.02x$$

$$5.12=0.02x$$

$$x=256$$

256 ounces of water must be added.

98. Let x represent the amount of water evaporated.

% salt	Tot. amt.	amt. of salt
2%	64	$(0.02)(64)$
0%	x	$(0.00)(x)$
10%	$64-x$	$(0.10)(64-x)$

$$(0.02)(64)-(0.00)(x)=(0.10)(64-x)$$

$$1.28=6.4-0.10x$$

$$0.10x=5.12$$

$$x=51.2$$

51.2 ounces of water must be evaporated.

99. Let the length of leg 1 = x.
Then the length of leg 2 = $17-x$.
By the Pythagorean Theorem we have

$$x^2+(17-x)^2=(13)^2$$

$$x^2+x^2-34x+289=169$$

$$2x^2-34x+120=0$$

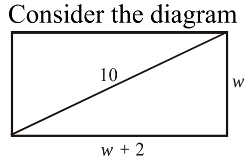
$$x^2-17x+60=0$$

$$(x-12)(x-5)=0$$

$$x=12 \quad\text{or}\quad x=5$$

The legs are 5 centimeters and 12 centimeters long.

100. Consider the diagram



By the Pythagorean Theorem we have

$$w^2+(w+2)^2=(10)^2$$

$$w^2+w^2+4w+4=100$$

$$2w^2+4w-96=0$$

$$w^2+2w-48=0$$

$$(w+8)(w-6)=0$$

$$w=-8 \quad\text{or}\quad w=6$$

The width is 6 inches and the length is 6 + 2 = 8 inches.

101. Let x represent the amount of the 15% solution added.

% acid	tot. amt.	amt. of acid
40%	60	$(0.40)(60)$
15%	x	$(0.15)(x)$
25%	$60+x$	$(0.25)(60+x)$

$$(0.40)(60)+(0.15)(x)=(0.25)(60+x)$$

$$24+0.15x=15+0.25x$$

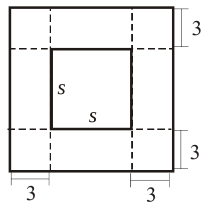
$$9=0.1x$$

$$x=90$$

90 cubic centimeters of the 15% solution must be added, producing 150 cubic centimeters of the 25% solution.

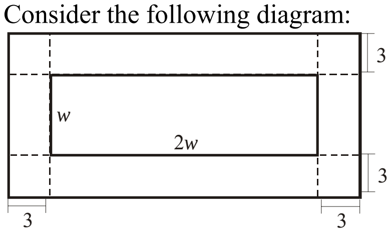
102. a. Consider the following diagram:

$$4(s+6)=50$$
$$4s+24=50$$
$$4s=26$$
$$s=6.5$$



The painting is 6.5 inches by 6.5 inches. $s+6=12.5$, so the frame is 12.5 inches by 12.5 inches.

b. Consider the following diagram:



$$2(2w+6)+2(w+6)=50$$
$$4w+12+2w+12=50$$
$$6w=26$$
$$w=\frac{26}{6}=4\tfrac{1}{3}$$
$$l=2w=8\tfrac{2}{3}$$

The painting is $8\frac{2}{3}$ inches by $4\frac{1}{3}$ inches.

The frame is $14\frac{2}{3}$ inches by $10\frac{1}{3}$ inches.

103. Let t represent the time it takes the smaller pump to finish filling the tank.

	3hp Pump	8hp Pump
Time to do job alone	12	8
Part of job done in 1 hr	$\frac{1}{12}$	$\frac{1}{8}$
Time on job (hrs)	$t+4$	4
Part of job done by each pump	$\frac{t+4}{12}$	$\frac{4}{8}$

Since the two pumps fill one tank, we have:

$$\frac{t+4}{12}+\frac{4}{8}=1$$
$$\frac{t+4}{12}=\frac{1}{2}$$
$$t+4=6$$
$$t=2$$

It takes the small pump a total of 2 more hours to fill the tank.

104. Let $w=4$. Solve for the length:

$$l^2=4(l+4)$$
$$l^2=4l+16$$
$$l^2-4l-16=0$$
$$l=\frac{-(-4)+\sqrt{(-4)^2-4(1)(-16)}}{2(1)}=\frac{4+\sqrt{80}}{2}$$
$$=2+2\sqrt{5}\approx 6.47$$

The length of the plasterboard should be cut to a length of approximately 6.47 feet.

105. Let x represent the amount Scott receives. Then $\frac{3}{4}x$ represents the amount Alice receives and $\frac{1}{2}x$ represents the amount Tricia receives. The total amount is \$900,000, so we have:

$$x+\frac{3}{4}x+\frac{1}{2}x=900,000$$
$$4\left(x+\frac{3}{4}x+\frac{1}{2}x\right)=4(900,000)$$
$$4x+3x+2x=3,600,000$$
$$9x=3,600,000$$
$$x=400,000$$

So, $\frac{3}{4}x=\frac{3}{4}(400,000)=300,000$ and

$\frac{1}{2}x=\frac{1}{2}(400,000)=200,000$.

Scott receives \$400,000, Alice receives \$300,000, and Tricia receives \$200,000.

106. Let x represent the number of passengers over 20. Then $20+x$ represents the total number of passengers, and $15-0.1x$ represents the fare for each passenger. Solving the equation for total cost, \$482.40, we have:

$$(20+x)(15-0.1x)=482.40$$
$$300+13x-0.1x^2=482.40$$
$$-0.1x^2+13x-182.40=0$$
$$x^2-130x+1824=0$$
$$(x-114)(x-16)=0$$
$$x=114 \text{ or } x=16$$

Since the capacity of the bus is 44, we discard the 114. Therefore, $20+16=36$ people went on the trip; each person paid $15-0.1(16)=\$13.40$.

107. Let t represent the time it takes the older machine to complete the job by itself.

	Old copier	New copier
Time to do job alone	t	$t-1$
Part of job done in 1 hr	$\frac{1}{t}$	$\frac{1}{t-1}$
Time on job (hrs)	1.2	1.2
Part of job done by each copier	$\frac{1.2}{t}$	$\frac{1.2}{t-1}$

Since the two copiers complete one job, we have:

$$\frac{1.2}{t}+\frac{1.2}{t-1}=1$$
$$1.2(t-1)+1.2t=t(t-1)$$
$$1.2t-1.2+1.2t=t^2-t$$
$$t^2-3.4t+1.2=0$$
$$5t^2-17t+6=0$$
$$(5t-2)(t-3)=0$$
$$t=0.4 \text{ or } t=3$$

It takes the old copier 3 hours to do the job by itself. (0.4 hour is impossible since together it takes 1.2 hours.)

108. Let r_S represent Scott's rate and let r_T represent Todd's rate. The time for Scott to run 95 meters is the same as for Todd to run 100 meters.

$$\frac{95}{r_S}=\frac{100}{r_T}$$
$$r_S=0.95r_T$$
$$d_S=t\cdot r_s=t\left(0.95r_T\right)=0.95d_T$$

If Todd starts from 5 meters behind the start:

$$d_T=105$$
$$d_S=0.95d_T=0.95(105)=99.75$$

a. The race does not end in a tie.

b. Todd wins the race.

c. Todd wins by 0.25 meters.

d. To end in a tie:
$$100=0.95(100+x)$$
$$100=95+0.95x$$
$$5=0.95x$$
$$x\approx 5.26 \text{ meters}$$

e. 95 = 0.95(100) Therefore, the race ends in a tie.

109. The effective speed of the train (i.e., relative to the man) is 30 – 4 = 26 miles per hour. The time is $5 \text{ sec}=\frac{5}{60}\text{ min }=\frac{5}{3600}\text{ hr }=\frac{1}{720}\text{ hr.}$

$$s=vt$$
$$=26\left(\frac{1}{720}\right)$$
$$=\frac{26}{720}\text{ miles}$$
$$=\frac{26}{720}\cdot 5280\approx 190.67\text{ feet}$$

The freight train is about 190.67 feet long.

Chapter 1 Test

1.
$$\frac{2x}{3}-\frac{x}{2}=\frac{5}{12}$$
$$12\left(\frac{2x}{3}-\frac{x}{2}\right)=12\left(\frac{5}{12}\right)$$
$$8x-6x=5$$
$$2x=5$$
$$x=\frac{5}{2}$$

The solution set is $\left\{\frac{5}{2}\right\}$.

2.
$$x(x-1)=6$$
$$x^2-x=6$$
$$x^2-x-6=0$$
$$(x-3)(x+2)=0$$
$$x-3=0 \quad\text{or}\quad x+2=0$$
$$x=3 \quad\text{or}\quad x=-2$$

The solution set is $\{-2, 3\}$.

3.
$$x^4-3x^2-4=0$$
$$\left(x^2-4\right)\left(x^2+1\right)=0$$
$$x^2-4=0 \quad\text{or}\quad x^2+1=0$$
$$x^2=4 \quad\text{or}\quad x^2=-1$$
$$x=\pm 2 \quad\text{or}\quad \text{Not real}$$

The solution set is $\{-2, 2\}$.

4. $\sqrt{2x-5}+2=4$

$$\sqrt{2x-5}=2$$
$$\left(\sqrt{2x-5}\right)^2=(2)^2$$
$$2x-5=4$$
$$2x=9$$
$$x=\frac{9}{2}$$

Check: $\sqrt{2\left(\frac{9}{2}\right)-5}+2=4$

$$\sqrt{9-5}+2=4$$
$$\sqrt{4}+2=4$$
$$2+2=4$$
$$4=4$$

The solution set is $\left\{\frac{9}{2}\right\}$.

5. $|2x-3|+7=10$

$$|2x-3|=3$$
$$2x-3=3 \quad \text{or} \quad 2x-3=-3$$
$$2x=6 \quad \text{or} \quad 2x=0$$
$$x=3 \quad \text{or} \quad x=0$$

The solutions set is $\{0, 3\}$.

6. $3x^3+2x^2-12x-8=0$

$$x^2(3x+2)-4(3x+2)=0$$
$$\left(x^2-4\right)(3x+2)=0$$
$$(x+2)(x-2)(3x+2)=0$$
$$x+2=0 \quad \text{or} \quad x-2=0 \quad \text{or} \quad 3x+2=0$$
$$x=-2 \quad \text{or} \quad x=2 \quad \text{or} \quad x=-\frac{2}{3}$$

The solution set is $\left\{-2,-\frac{2}{3}, 2\right\}$.

7. $3x^2-x+1=0$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(1)}}{2(3)}$$
$$=\frac{1\pm\sqrt{-11}}{6} \quad \text{(Not real)}$$

This equation has no real solutions.

8. $-3\le\frac{3x-4}{2}\le 6$

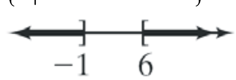
$$2(-3)\le 2\left(\frac{3x-4}{2}\right)\le 2(6)$$
$$-6\le 3x-4\le 12$$
$$-2\le 3x\le 16$$
$$-\frac{2}{3}\le x\le\frac{16}{3}$$
$$\left\{x\middle|-\frac{2}{3}\le x\le\frac{16}{3}\right\} \text{ or } \left[-\frac{2}{3},\frac{16}{3}\right]$$

9. $|3x+4|<8$

$$-8<3x+4<8$$
$$-12<3x<4$$
$$-4<x<\frac{4}{3}$$
$$\left\{x\middle|-4<x<\frac{4}{3}\right\} \text{ or } \left(-4,\frac{4}{3}\right)$$

10. $2+|2x-5|\ge 9$

$$|2x-5|\ge 7$$
$$2x-5\le -7 \quad \text{or} \quad 2x-5\ge 7$$
$$2x\le -2 \quad \text{or} \quad 2x\ge 12$$
$$x\le -1 \quad \text{or} \quad x\ge 6$$
$$\{x|x\le -1 \text{ or } x\ge 6\} \text{ or } (-\infty,-1]\cup[6,\infty).$$



11. $\frac{-2}{3-i}=\frac{-2}{3-i}\cdot\frac{3+i}{3+i}=\frac{-6-2i}{9+3i-3i-i^2}=\frac{-6-2i}{9-(-1)}$

$$=\frac{-6-2i}{10}=\frac{-3-i}{5}=-\frac{3}{5}-\frac{1}{5}i$$

12. $4x^2-4x+5=0$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(4)(5)}}{2(4)}$$
$$=\frac{4\pm\sqrt{-64}}{8}=\frac{4\pm 8i}{8}=\frac{1}{2}\pm i$$

This solution set is $\left\{\frac{1}{2}-i,\frac{1}{2}+i\right\}$.

13. Let x represent the amount of the \$8-per-pound coffee.

Amt. of coffee (pounds)	Price (\$)	Total \$
20	4	$(20)(4)$
x	8	$(8)(x)$
$20+x$	5	$(5)(20+x)$

$$80+8x=(5)(20+x)$$
$$80+8x=100+5x$$
$$3x=20$$
$$x=\frac{20}{3}=6\tfrac{2}{3}$$

Add $6\frac{2}{3}$ pounds of \$8/lb coffee to get $26\frac{2}{3}$ pounds of \$5/lb coffee.

Chapter 1 Projects

Project I

$$P=L\left[\frac{\frac{r}{12}}{1-\left(1+\frac{r}{12}\right)^{-t}}\right]$$

P = monthly payment, L = loan amount, r = annual rate of interest, expressed as a decimal, t = length of loan, in months

1. a. $P=200000\left[\frac{\frac{0.0658}{12}}{1-\left(1+\frac{0.0658}{12}\right)^{-360}}\right]\approx \$1274.68$

b. $P=200000\left[\frac{\frac{0.0617}{12}}{1-\left(1+\frac{0.0617}{12}\right)^{-180}}\right]\approx \$1706.14$

2. Total paid = (Life of loan)(Monthly payment)

a. Total amount paid $=(360)(1274.68)$
$=\$458{,}884.80$

b. Total amount paid $=(180)(1706.14)$
$=\$307{,}105.20$

3. Interest = Total paid − original loan amount

a. Interest paid $=458{,}884.80-200{,}000$
$=\$258{,}884.80$

b. Interest paid $=307{,}105.20-200{,}000$
$=\$107{,}105.20$

4. a. $P=200000\left[\frac{\frac{0.0659}{12}}{1-\left(1+\frac{0.0659}{12}\right)^{-360}}\right]\approx \$1276.00$

b. $P=200000\left[\frac{\frac{0.0622}{12}}{1-\left(1+\frac{0.0622}{12}\right)^{-180}}\right]\approx \$1711.58$

5. Total paid = (Life of loan)(Monthly payment)

a. Total paid $=(360)(1276.00)$
$=\$459{,}360.00$

b. Total paid $=(180)(1711.58)$
$=\$308{,}084.40$

6. Interest = Total paid − original loan

a. Interest $=459{,}360.00-200{,}000$
$=\$259{,}360.00$

b. Interest $=308{,}084.40-200{,}000$
$=\$108{,}084.40$

7. $P=L\left[\frac{\frac{r}{12}}{1-\left(1+\frac{r}{12}\right)^{-t}}\right]$

$$L=\frac{P}{\left[\frac{\frac{r}{12}}{1-\left(1+\frac{r}{12}\right)^{-t}}\right]}=P\left[\frac{1-\left(1+\frac{r}{12}\right)^{-t}}{\frac{r}{12}}\right]$$

8. $L = P\left[\dfrac{1-\left(1+\frac{r}{12}\right)^{-t}}{\frac{r}{12}}\right]$

a. $L = 1000\left[\dfrac{1-\left(1+\frac{0.0659}{12}\right)^{-360}}{\frac{0.0659}{12}}\right] \approx \$156{,}740.19$

b. $L = 1000\left[\dfrac{1-\left(1+\frac{0.0622}{12}\right)^{-180}}{\frac{0.0622}{12}}\right] \approx \$116{,}851.28$

9. $L = P\left[\dfrac{1-\left(1+\frac{r}{12}\right)^{-t}}{\frac{r}{12}}\right]$

a. $L = 1000\left[\dfrac{1-\left(1+\frac{0.0658}{12}\right)^{-360}}{\frac{0.0658}{12}}\right] \approx \$156{,}902.52$

b. $L = 1000\left[\dfrac{1-\left(1+\frac{0.0617}{12}\right)^{-180}}{\frac{0.0617}{12}}\right] \approx \$117{,}223.84$

10. Answers will vary.

11. Answers will vary. (Use P = \$1300 and the interest rates you obtained for problem 10.)

12. Answers will vary.

13. Answers will vary.

Project II

1. $T = \dfrac{n}{Cnp + L + M}, \; n = 3, \; L = 5, \; M = 1, \; C = 0.2$

$$T = \frac{3}{0.2(3)p + 5 + 1} = \frac{3}{0.6p + 6} = \frac{1}{0.2p + 2}$$

2. All of the times given in problem 1 were in seconds, so $T = 0.1$ board per second needs to used as the value for T in the equation found in problem 1.

$$0.1 = \frac{1}{0.2p + 2}$$
$$(0.2p + 2)(0.1) = 1$$
$$0.02p + 0.2 = 1$$
$$0.02p = 0.8$$
$$p = 40 \text{ parts per board}$$

3. $T = 0.15$ board per second

$$0.15 = \frac{1}{0.2p + 2}$$
$$(0.2p + 2)(0.15) = 1$$
$$0.03p + 0.3 = 1$$
$$0.03p = 0.7$$
$$p \approx 23.3 \text{ parts per board}$$

Thus, only 23 parts per board will work.

For problems 4 – 6, C is requested, so solve for C first:

$$T = \frac{n}{Cnp + L + M}$$
$$(Cnp + L + M)T = n$$
$$CnpT + LT + MT = n$$
$$CnpT = n - LT - MT$$
$$C = \frac{n - LT - MT}{npT}$$

4. $T = 0.06$, $n = 3$, $p = 100$, $M = 1$, $L = 5$

$$C = \frac{3 - 5(0.06) - 1(0.06)}{3(100)(0.06)} \approx 0.147 \text{ sec}$$

5. $T = 0.06$, $n = 3$, $p = 150$, $M = 1$, $L = 5$

$$C = \frac{3 - 5(0.06) - 1(0.06)}{3(150)(0.06)} \approx 0.098 \text{ sec}$$

6. $T = 0.06$, $n = 3$, $p = 200$, $M = 1$, $L = 5$

$$C = \frac{3 - 5(0.06) - 1(0.06)}{3(200)(0.06)} \approx 0.073 \text{ sec}$$

7. As the number of parts per board increases, the tact time decreases, if all the other factors remain constant.