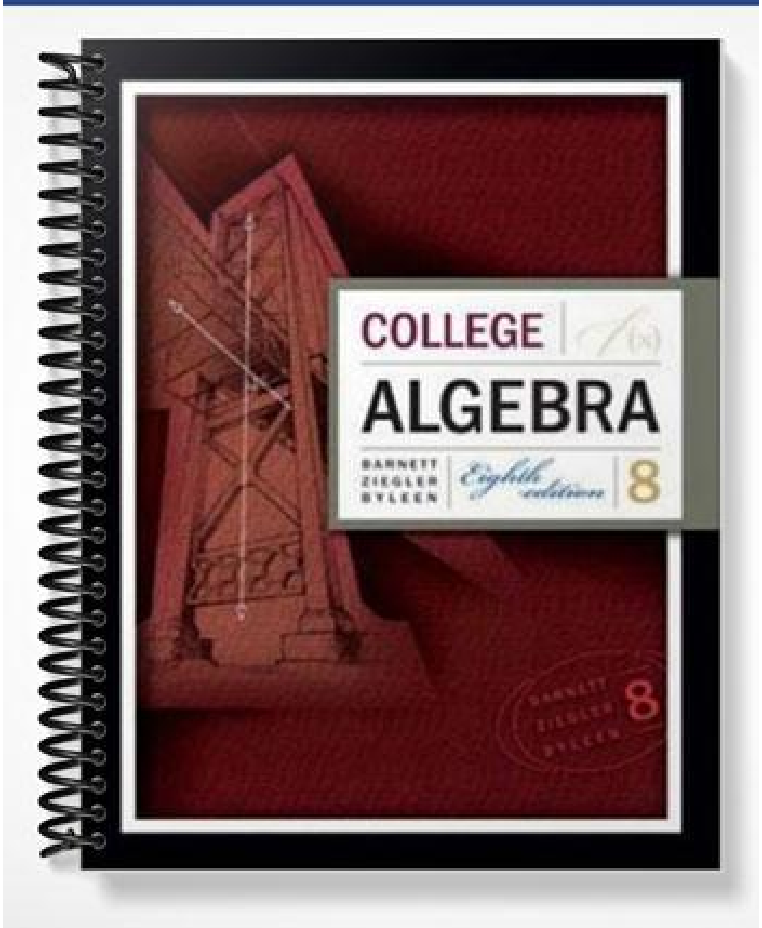
SOLUTIONS MANUAL



CHAPTER 2

Section 2-1

2. The set of all points for which the x and y coordinates are both positive is quadrant I.

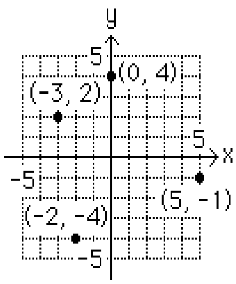
4. The set of all points for which the y coordinate is 0 is the x axis.

6. The set of all points for which y is negative, excluding those points for which $x = 0$ (negative y axis), includes quadrants III and IV.

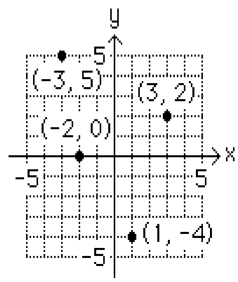
8. The set of all points for which x is negative and y is positive is Quadrant II.

10. The set of all points for which $xy > 0$ includes those points for which both coordinates are positive (Quadrant I) and also those points for which both coordinates are negative (Quadrant III).

12.



14.



16. Point A has coordinates $(0, 3)$. Its reflection through the x axis is $A'(0, -3)$.
Point B has coordinates $(-4, -5)$. Its reflection through the x axis is $B'(-4, 5)$.
Point C has coordinates $(4, 1)$. Its reflection through the x axis is $C'(4, -1)$.
Point D has coordinates $(1, -3)$. Its reflection through the x axis is $D'(1, 3)$.

18. Point A has coordinates $(4, 2)$. Reflection through the x axis gives $(4, -2)$; reflection of this through the y axis gives $A'(-4, -2)$.
Point B has coordinates $(-2, -4)$. Reflection through the x axis gives $(-2, 4)$; reflection of this through the y axis gives $B'(2, 4)$.
Point C has coordinates $(-4, 3)$. Reflection through the x axis gives $(-4, -3)$; reflection of this through the y axis is $C'(4, -3)$.
Point D has coordinates $(5, 0)$. This is unchanged by reflection through the x axis; reflection through the y axis gives $D'(-5, 0)$.

20. $y = \frac{1}{2}x + 1$

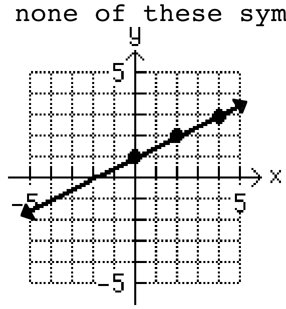
Test y axis
Replace x with $-x$:
$y = \frac{1}{2}(-x) + 1$
$y = -\frac{1}{2}x + 1$

Test x axis
Replace y with $-y$:
$-y = \frac{1}{2}x + 1$
$y = -\frac{1}{2}x - 1$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = \frac{1}{2}(-x) + 1$
$y = \frac{1}{2}x - 1$

The graph has none of these symmetries.

x	y
0	1
2	2
4	3



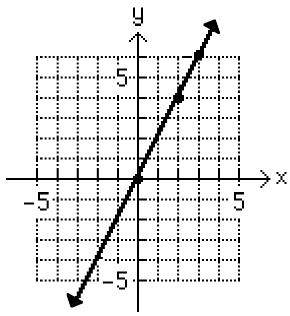
22. $y = 2x$

Test y axis
Replace x with $-x$:
$y = 2(-x)$
$y = -2x$

Test x axis
Replace y with $-y$:
$-y = 2x$
$y = -2x$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = 2(-x)$
$y = 2x$

x	y
0	0
2	4
3	6



The graph has symmetry with respect to the origin.

We reflect the portion of the graph in quadrant I through the origin, using the origin symmetry.

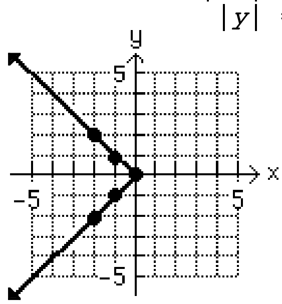
24. $|y| = -x$

Test y axis
Replace x with $-x$:
$|y| = -(-x)$
$|y| = x$

Test x axis
Replace y with $-y$:
$|-y| = -x$
$|y| = -x$

Test origin
Replace x with $-x$ and y with $-y$:
$|-y| = -(-x)$
$|y| = x$

x	y
0	0
−1	±1
−2	±2



The graph has symmetry with respect to the x axis.

We reflect the portion of the graph where $y \geq 0$ through the x axis, using the x axis symmetry.

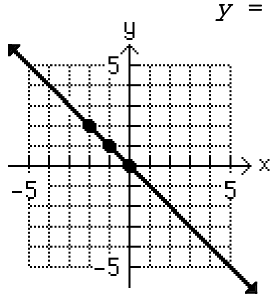
26. $y = -x$

Test y axis
Replace x with $-x$:
$y = -(-x)$
$y = x$

Test x axis
Replace y with $-y$:
$-y = -x$
$y = x$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = -(-x)$
$y = -x$

x	y
0	0
−1	1
−2	2



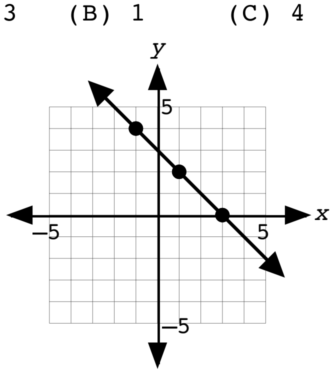
The graph has symmetry with respect to the origin.

We reflect the portion of the graph in quadrant II through the origin, using the origin symmetry.

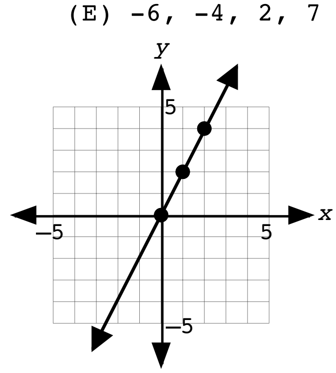
28. (A) 5 (B) -8 (C) 6 (D) -2, 4 (E) -4, 6 (F) -3, 5

30. (A) -3 (B) 1 (C) 4 (D) 3, 6 (E) -6, -4, 2, 7 (F) -5, 2, 7

32. (A)

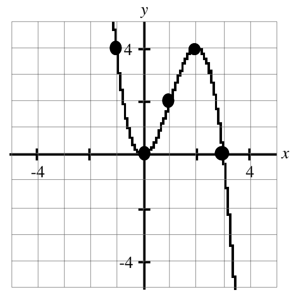


(B)

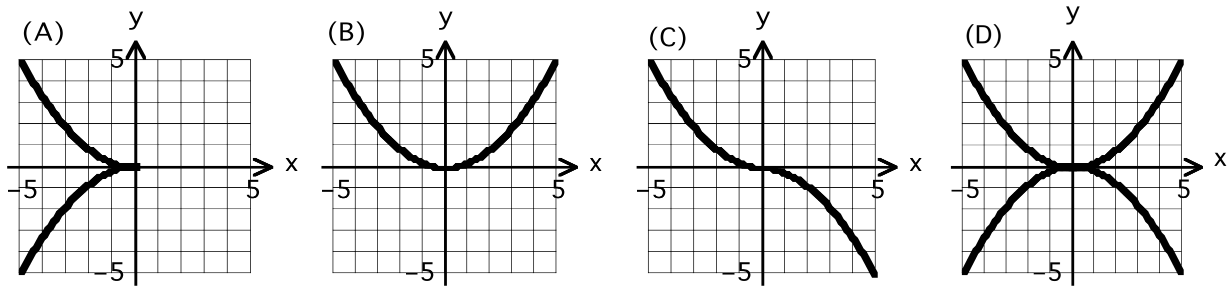


(C)

x	-1	0	1	2	3
y	4	0	2	4	0



34.



36. $x^3y^3 = 8$

Test y axis	Test x axis	Test origin
Replace x with $-x$:	Replace y with $-y$:	Replace x with $-x$ and y with $-y$:
$(-x)^3y^3 = 8$	$x^3(-y)^3 = 8$	$(-x)^3(-y)^3 = 8$
$-x^3y^3 = 8$	$-x^3y^3 = 8$	$x^3y^3 = 8$

The graph has origin symmetry.

38. $x + 4xy + 2y^2 = 7$

Test y axis	Test x axis	Test origin
Replace x with $-x$:	Replace y with $-y$:	Replace x with $-x$ and y with $-y$:
$-x + 4(-x)y + 2y^2 = 7$	$x + 4x(-y) + 2(-y)^2 = 7$	$-x + 4(-x)(-y) + 2(-y)^2 = 7$
$-x - 4xy + 2y^2 = 7$	$x - 4xy + 2y^2 = 7$	$-x + 4xy + 2y^2 = 7$

The graph has none of these symmetries.

40. $x^2 - 2xy + 3y^2 = 4$

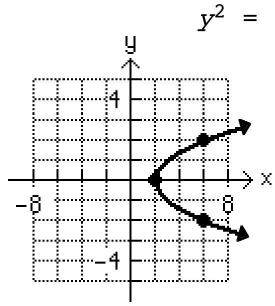
Test y axis	Test x axis	Test origin
Replace x with $-x$:	Replace y with $-y$:	Replace x with $-x$ and y with $-y$:
$(-x)^2 - 2(-x)y + 3y^2 = 4$	$x^2 - 2x(-y) + 3(-y)^2 = 4$	$(-x)^2 - 2(-x)(-y) + 3(-y)^2 = 4$
$x^2 + 2xy + 3y^2 = 4$	$x^2 + 2xy + 3y^2 = 4$	$x^2 - 2xy + 3y^2 = 4$

The graph has symmetry with respect to the origin.

42. $y^2 = x - 2$

Test y axis	Test x axis	Test origin
Replace x with $-x$:	Replace y with $-y$:	Replace x with $-x$ and y with $-y$:
$y^2 = -x - 2$	$(-y)^2 = x - 2$	$(-y)^2 = (-x) - 2$
	$y^2 = x - 2$	$y^2 = -x - 2$

x	y
2	0
6	± 2



The graph has symmetry with respect to the x axis.

To obtain the portion of the graph for $y \geq 0$, we sketch $y = \sqrt{x - 2}$, $x \geq 2$. We reflect the portion of the graph for $y \geq 0$ across the x axis, using the x axis symmetry.

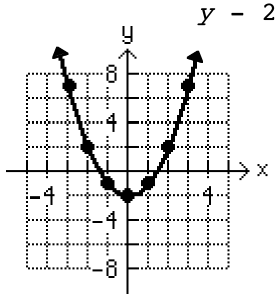
44. $y + 2 = x^2$

Test y axis
Replace x with $-x$:
$y + 2 = (-x)^2$
$y + 2 = x^2$

Test x axis
Replace y with $-y$:
$(-y) + 2 = x^2$
$y - 2 = -x^2$

Test origin
Replace x with $-x$ and y with $-y$:
$(-y) + 2 = (-x)^2$
$y - 2 = -x^2$

x	y
0	−2
±1	−1
±2	2
±3	7



The graph has symmetry with respect to the y axis. We reflect the portion of the graph for $x \ge 0$ across the y axis, using the y axis symmetry.

46. $4x^2 - y^2 = 1$

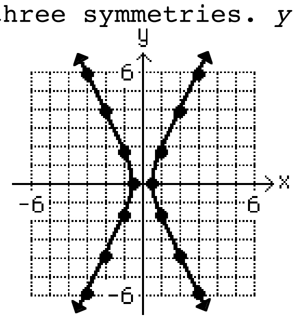
Test y axis
Replace x with $-x$:
$4(-x)^2 - y^2 = 1$
$4x^2 - y^2 = 1$

Test x axis
Replace y with $-y$:
$4x^2 - (-y)^2 = 1$
$4x^2 - y^2 = 1$

Origin symmetry follows automatically.

The graph has all three symmetries. $y = \pm\sqrt{4x^2 - 1}$.

x	y
$\pm\frac{1}{2}$	0
±1	$\pm\sqrt{3}$
±2	$\pm\sqrt{15}$
±3	$\pm\sqrt{35}$



To obtain the quadrant I portion of the graph, we sketch $y = \sqrt{4x^2 - 1}$, $x \ge 0$. We reflect this graph across the y axis, then reflect everything across the x axis.

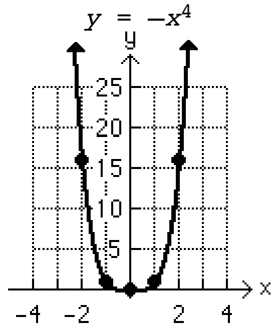
48. $y = x^4$

Test y axis
Replace x with $-x$:
$y = (-x)^4$
$y = x^4$

Test x axis
Replace y with $-y$:
$-y = x^4$
$y = -x^4$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = (-x)^4$
$y = -x^4$

x	y
0	0
±1	1
±2	16



The graph has symmetry with respect to the y axis. We reflect the portion of the graph in quadrant I through the y axis, using the y axis symmetry.

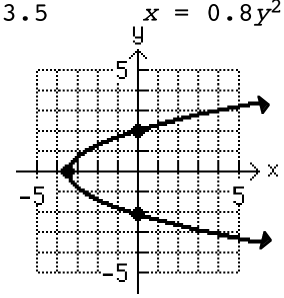
50. $x = 0.8y^2 - 3.5$

Test y axis
Replace x with $-x$:
$-x = 0.8y^2 - 3.5$
$x = -0.8y^2 + 3.5$

Test x axis
Replace y with $-y$:
$x = 0.8(-y)^2 - 3.5$
$x = 0.8y^2 - 3.5$

Test origin
Replace x with $-x$ and y with $-y$:
$-x = 0.8(-y)^2 - 3.5$
$x = -0.8y^2 + 3.5$

x	y
−3.5	0
0	$\pm\sqrt{\frac{35}{8}}$



The graph has symmetry with respect to the x axis. We reflect the portion of the graph for $y \ge 0$ across the x axis, using the x axis symmetry.

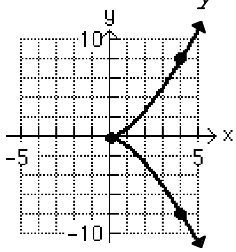
52. $y^{2/3} = x$

Test y axis
Replace x with $-x$:
$y^{2/3} = -x$

Test x axis
Replace y with $-y$:
$(-y)^{2/3} = x$
$y^{2/3} = x$

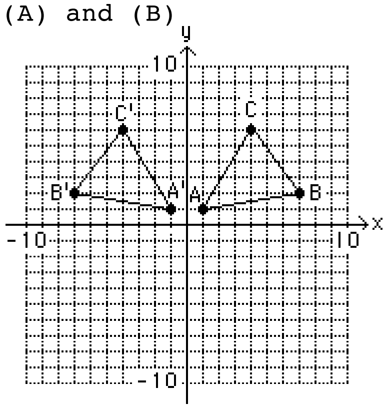
Test origin
Replace x with $-x$ and y with $-y$:
$(-y)^{2/3} = -x$
$y^{2/3} = -x$

x	y
0	0
4	±8



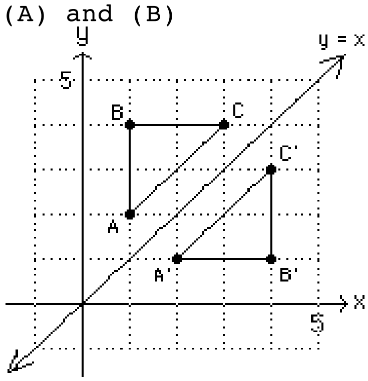
The graph has symmetry with respect to the x axis. We reflect the portion of the graph for $y \geq 0$ across the x axis, using the x axis symmetry.

54. (A) and (B)



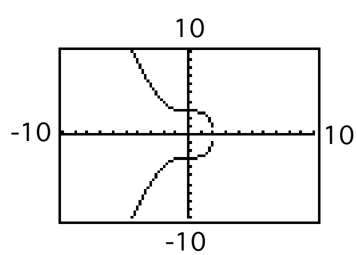
(C) The triangles are mirror images of each other, reflected across the y axis. Changing the sign of the x coordinate reflects the graph across the y axis.

56. (A) and (B)

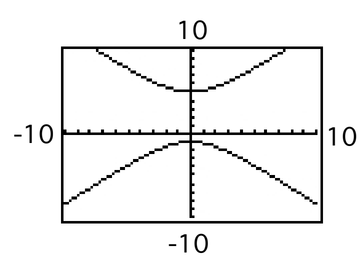


(C) The triangles are mirror images of each other, reflected across the line $y = x$. Reversing the coordinates reflects the graph across the line $y = x$.

58. $x^3 + y^2 = 8$
$y^2 = 8 - x^3$
$y = \pm\sqrt{8 - x^3}$



60. $(y - 2)^2 - x^2 = 9$
$(y - 2)^2 = 9 + x^2$
$y - 2 = \pm\sqrt{9 + x^2}$
$y = 2 \pm \sqrt{9 + x^2}$



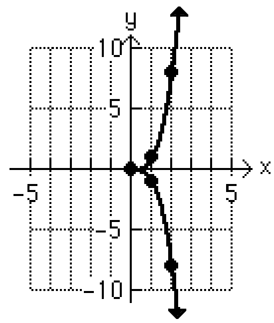
62. $|y| = x^3$

Test y axis
Replace x with $-x$:
$|y| = (-x)^3$
$|y| = -x^3$

Test x axis
Replace y with $-y$:
$|-y| = x^3$
$|y| = x^3$

Test origin
Replace x with $-x$ and y with $-y$:
$|-y| = (-x)^3$
$|y| = -x^3$

x	y
0	0
1	±1
2	±8



The graph has symmetry with respect to the x axis. We reflect the portion of the graph for $y \geq 0$ across the x axis, using the x axis symmetry.

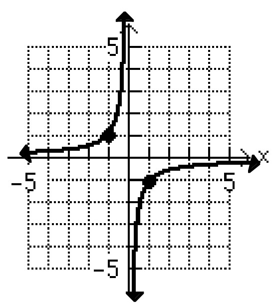
64. $xy = -1$

Test y axis
Replace x with $-x$:
$(-x)y = -1$
$xy = 1$

Test x axis
Replace y with $-y$:
$x(-y) = -1$
$xy = 1$

Test origin
Replace x with $-x$ and y with $-y$:
$(-x)(-y) = -1$
$xy = -1$

x	y
1	−1
−1	1



The graph has symmetry with respect to the origin.
We reflect the portion of the graph in quadrant II through the origin, using the origin symmetry.

66. $y = x^2 - 6x$

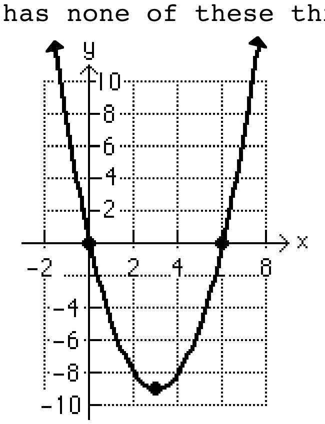
Test y axis
Replace x with $-x$:
$y = (-x)^2 - 6(-x)$
$y = x^2 + 6x$

Test x axis
Replace y with $-y$:
$-y = x^2 - 6x$
$y = -x^2 + 6x$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = (-x)^2 - 6(-x)$
$y = -x^2 - 6x$

The graph has none of these three symmetries.

x	y
0	0
3	−9
6	0



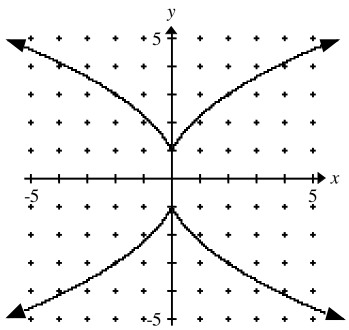
68. $y^2 = 4|x| + 1$

Test y axis
Replace x with $-x$:
$y^2 = 4|-x| + 1$
$y^2 = 4|x| + 1$

Test x axis
Replace y with $-y$:
$(-y)^2 = 4|x| + 1$
$y^2 = 4|x| + 1$

Origin symmetry follows automatically.

x	y
0	1
±1	$\sqrt{5} \approx 2.2$
±2	3



The graph has symmetry with respect to the x axis, the y axis, and the origin. $y = \pm\sqrt{4|x| + 1}$
To obtain the quadrant I portion of this graph, we sketch $y = \sqrt{4|x| + 1}$, $x \geq 0$. We reflect this graph across the y axis, then reflect everything across the x axis.

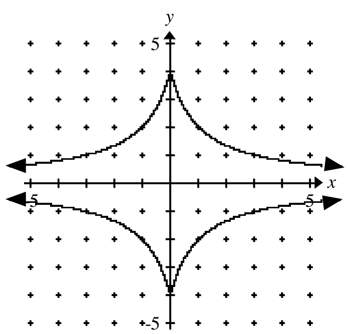
70. $|xy| + |y| = 4$

Test y axis
Replace x with $-x$:
$|(-x)y| + |y| = 4$
$|xy| + |y| = 4$

Test x axis
Replace y with $-y$:
$|x(-y)| + |-y| = 4$
$|xy| + |y| = 4$

Origin symmetry follows automatically.

x	y
0	±4
±1	±2
±3	±1

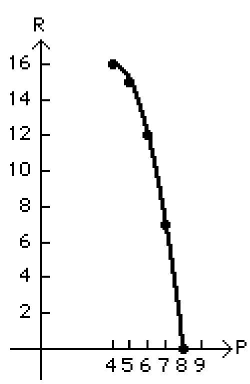


The graph has symmetry with respect to the x axis, the y axis, and the origin.
We reflect the portion of the graph in quadrant I across the y axis, then reflect everything across the x axis.

72. Reflecting a point (x, y) across the x axis yields the point $(-x, y)$. Reflecting this point through the origin yields the point $(x, -y)$. This point is the same point that would result from reflecting the original point across the x axis. Therefore, if the graph is unchanged by reflecting across the y axis and through the origin, it will be unchanged by reflecting across the x axis and will necessarily have symmetry with respect to the x axis.

74.

P	$R = (8 - p)p$
4	16
5	15
6	12
7	7
8	0



76. (A) The supply is 3000 cases when the price is \$5.60.

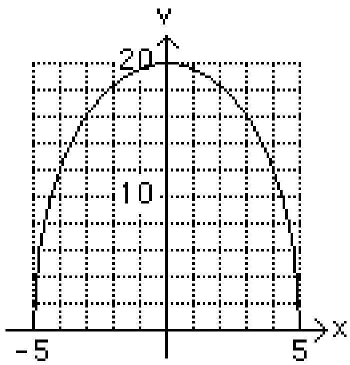
(B) As the price increases from \$5.60 to \$5.80 the supply increases by about 300 cases.

(C) As the price decreases from \$5.60 to \$5.40 the supply decreases by about 400 cases.

(D) As price increases so does supply. As price decreases so does supply.

78. (A) The temperature at 7 p.m. is about 60°. (B) The lowest temperature is 44° at 5 a.m. (C) The temperature is 52° at about 9 a.m. and 10 p.m.

80. (A) $V = 4\sqrt{25 - x^2}$



(B) The speed of the ball is zero at the top and bottom of the oscillation and has a maximum speed of 4 at the rest position.

Section 2-2

2. $d = \sqrt{(3-0)^2 + (5-1)^2} = \sqrt{25} = 5$

Midpoint $= \left(\frac{0+3}{2}, \frac{1+5}{2}\right) = \left(\frac{3}{2}, 3\right)$

4. $d = \sqrt{(-2-3)^2 + (3-0)^2} = \sqrt{34}$

Midpoint $= \left(\frac{3+-2}{2}, \frac{0+(-3)}{2}\right) = \left(\frac{1}{2}, -\frac{3}{2}\right)$

6. $d = \sqrt{(6-(-5))^2 + (-1-4)^2} = \sqrt{146}$

Midpoint $= \left(\frac{-5+6}{2}, \frac{4+(-1)}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

8. $d = \sqrt{(-5-(-1))^2 + (-2-2)^2} = \sqrt{32} = 4\sqrt{2}$

Midpoint $= \left(\frac{(-5)+(-1)}{2}, \frac{-2+2}{2}\right) = (-3, 0)$

10. $C(0, 0)$, $r = 5$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = 5^2$

$x^2 + y^2 = 25$

12. $C(5, 6)$, $r = 2$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-5)^2 + (y-6)^2 = 4$

14. $C(-5, 6)$, $r = \sqrt{11}$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-(-5))^2 + (y-6)^2 = (\sqrt{11})^2$

$(x+5)^2 + (y-6)^2 = 11$

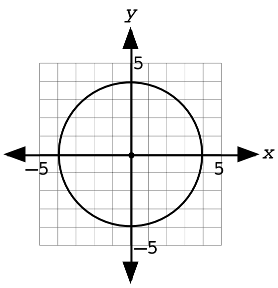
16. $C(4, -1)$, $r = \sqrt{5}$

$(x-h)^2 + (y-k)^2 = r^2$

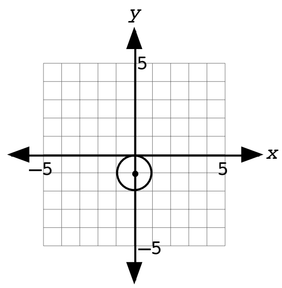
$(x-4)^2 + (y-(-1))^2 = (\sqrt{5})^2$

$(x-4)^2 + (y+1)^2 = 5$

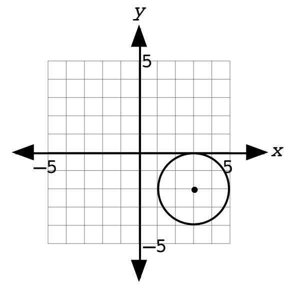
18. This is a circle with center (0, 0) and radius 4.
$x^2 + y^2 = 16$



20. This is a circle with center (0, -1) and radius 1.
$x^2 + (y + 1)^2 = 1$



22. This is a circle with center (3, -2) and radius 2.
$(x - 3)^2 + (y + 2)^2 = 4$



24. (A) $\frac{-3 + b_1}{2} = 4 \Rightarrow -3 + b_1 = 8 \Rightarrow b_1 = 11$

(B) $\frac{5 + b_2}{2} = -2 \Rightarrow 5 + b_2 = -4 \Rightarrow b_2 = -9$

(C) $d(A, M) = \sqrt{(4 - (-3))^2 + (-2 - 5)^2} = \sqrt{98}$

$d(M, b) = \sqrt{(11 - 4)^2 + (-9 - (-2))^2} = \sqrt{98}$

26. $(x, 2)$ is 4 units from $(3, -3)$:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$4 = \sqrt{(x - 3)^2 + (2 - (-3))^2}$$
$$16 = (x - 3)^2 + 25$$
$$-9 = (x - 3)^2$$

There is no solution.

28. $(3, y)$ is 13 units from $(-9, 2)$:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$13 = \sqrt{(3 - (-9))^2 + (y - 2)^2}$$
$$169 = 144 + (y - 2)^2$$
$$25 = (y - 2)^2$$
$$\pm 5 = y - 2$$
$$y = 2 + 5 = 7$$
$$y = 2 - 5 = -3$$

30. This is a circle with center (-1, 0) and radius 1. That is, the set of all points that are one unit away from (-1, 0).

$$(x + 1)^2 + y^2 = 1$$

32. This is a circle with center (2, -1) and radius 3. That is, the set of all points that are three units away from (2, -1).

$$(x - 2)^2 + (y + 1)^2 = 9$$

34. $M = \left(\frac{2.8 - 4.1}{2}, \frac{-3.5 + 7.6}{2}\right) = (-0.65, 2.05)$

$d(A, M) = \sqrt{(-0.65 - 2.8)^2 + (2.05 - (-3.5))^2} = 6.53$

$d(M, B) = \sqrt{(-4.1 - (-0.65))^2 + (7.6 - 2.05)^2} = 6.53$

$\frac{1}{2}d(A, B) = \frac{1}{2}\sqrt{(-4.1 - 2.8)^2 + (7.6 - (-3.5))^2} = 6.53$

36. Let $A = (a_1, a_2)$

$\frac{a_1 + 12}{2} = 2.5 \Rightarrow a_1 = -7$ $\qquad \frac{a_2 + 10}{2} = 3.5 \Rightarrow a_2 = -3$

$d(A, M) = \sqrt{(2.5 - (-7))^2 + (3.5 - (-3))^2} = 11.5$

$d(M, B) = \sqrt{(12 - 2.5)^2 + (10 - 3.5)^2} = 11.5$

$\frac{1}{2}d(A, B) = \frac{1}{2}\sqrt{(12 - (-7))^2 + (10 - (-3))^2} = 11.5$

38. Let $B = (b_1, b_2)$

$$\frac{-4 + b_1}{2} = -1.5 \Rightarrow b_1 = 1 \qquad \frac{-2 + b_2}{2} = -4.5 \Rightarrow b_2 = -7$$

$$d(A, M) = \sqrt{(-1.5 - 1)^2 + (-4.5 - (-7))^2} = 3.54$$

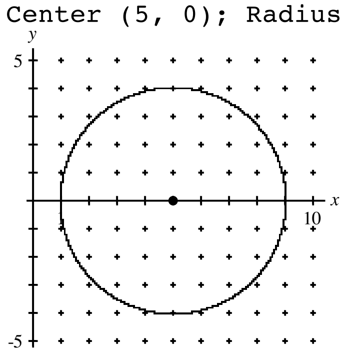
$$d(M, B) = \sqrt{(1 - (-1.5))^2 + (-7 - (-4.5))^2} = 3.54$$

$$\frac{1}{2}d(A, B) = \frac{1}{2}\sqrt{(1 - (-4))^2 + (-7 - (-2))^2} = 3.54$$

40. $(x - 5)^2 + y^2 = 16$

$(x - 5)^2 + (y - 0)^2 = 4^2$

Center (5, 0); Radius = 4

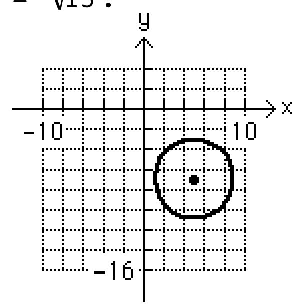


42. $(x - 5)^2 + (y + 7)^2 = 15$

$(x - 5)^2 + (y - (-7))^2 = (\sqrt{15})^2$

from which $(h, k) = (5, -7)$

and $r = \sqrt{15}$.



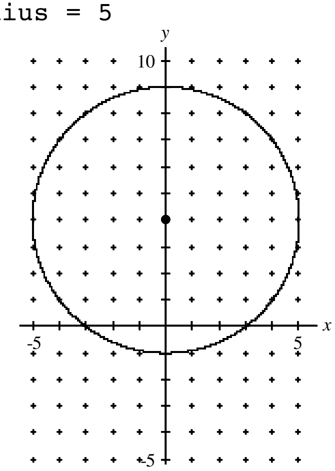
44. $x^2 + y^2 - 8y = 9$

$x^2 + y^2 - 8y + 16 = 9 + 16$

$x^2 + (y - 4)^2 = 25 = 5^2$

from which center = (0, 4);

radius = 5



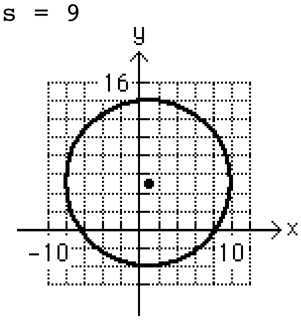
46. $x^2 + y^2 - 2x - 10y = 55$

$x^2 - 2x + 1 + y^2 - 10y + 25 = 55 + 26$

$(x - 1)^2 + (y - 5)^2 = 81 = 9^2$

from which center = (1, 5);

radius = 9



48. $2x^2 + 2y^2 + 8x + 20y + 30 = 0$

$x^2 + y^2 + 4x + 10y + 15 = 0$

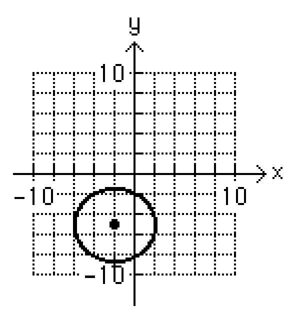
$x^2 + 4x + 4 + y^2 + 10y + 25 = -15 + 4 + 25$

$(x + 2)^2 + (y + 5)^2 = 14$

$(x - (-2))^2 + (y - (-5))^2 = (\sqrt{14})^2$

from which center = (-2, -5);

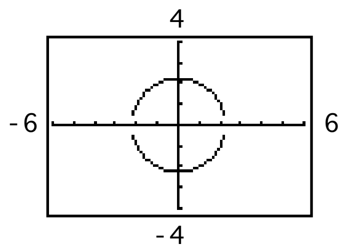
radius = $\sqrt{14}$



50. $x^2 + y^2 = 5$

$y^2 = 5 - x^2$

$y = \pm\sqrt{5 - x^2}$

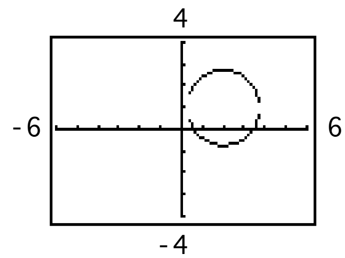


52. $(x - 2)^2 + (y - 1)^2 = 3$

$(y - 1)^2 = 3 - (x - 2)^2$

$y - 1 = \pm\sqrt{3 - (x - 2)^2}$

$y = 1 \pm\sqrt{3 - (x - 2)^2}$



54. Let $A = (-1, 3)$, $B = (3,5)$, $C = (5, 1)$

$d(A, B) = \sqrt{(3 - (-1))^2 + (5 - 3)^2} = \sqrt{20}$

$d(B, C) = \sqrt{(5 - 3)^2 + (1 - 5)^2} = \sqrt{20}$

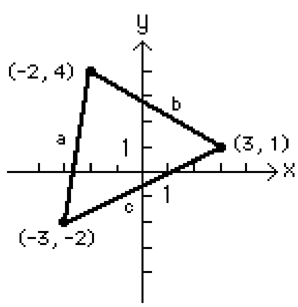
$d(A, C) = \sqrt{(5 - (-1))^2 + (1 - 3)^2} = \sqrt{40}$

$d(A, B)^2 + d(B, C)^2 = d(A, C)^2$, so the points are vertices of a right triangle.

Midpoint of $AC = \left(\frac{-1 + 5}{2}, \frac{3 + 1}{2}\right) = (2, 2)$

$d(M, B) = \sqrt{(3 - 2)^2 + (5 - 2)^2} = \sqrt{10}$

56.



$a = \sqrt{(-2 - (-3))^2 + (4 - (-2))^2} = \sqrt{37}$

$b = \sqrt{(-2 - 3)^2 + (4 - 1)^2} = \sqrt{34}$

$c = \sqrt{(3 - (-3))^2 + (1 - (-2))^2} = \sqrt{45}$

$p = a + b + c$

$= \sqrt{37} + \sqrt{34} + \sqrt{45}$

≈ 18.62

58. (A) Midpoint of $AC = \left(\frac{0 + a + c}{2}, \frac{0 + b}{2}\right) = \left(\frac{a + c}{2}, \frac{b}{2}\right)$.

(B) Midpoint of $BD = \left(\frac{a + c}{2}, \frac{b + 0}{2}\right) = \left(\frac{a + c}{2}, \frac{b}{2}\right)$.

(C) They intersect at their midpoints.

60. $(2, 0)$, $(-8, 0)$

Center: $\left(\frac{2 + (-8)}{2}, \frac{0 + 0}{2}\right) = (-3, 0)$

Diameter: $d = \sqrt{[2 - (-8)]^2 + (0 - 0)^2}$

$= \sqrt{100}$

$= 10$

Radius $= \frac{d}{2} = \frac{10}{2} = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 3)^2 + y^2 = 25$

62. $(-3, 2)$, $(7, -4)$

Center: $\left(\frac{-3 + 7}{2}, \frac{(2 + (-4))}{2}\right) = (2, -1)$

Diameter: $d = \sqrt{(-3 - 7)^2 + (2 + 4)^2}$

$= \sqrt{100 + 36}$

$= \sqrt{136}$

$= 2\sqrt{34}$

Radius $= \frac{d}{2} = \sqrt{34}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y + 1)^2 = 34$

64. The radius of a circle is the distance from the center to any point on the circle. Since the center of this circle is (0, 0) and (-2, 6) is a point on the circle, the radius is given by

$$r = \sqrt{(-2-0)^2 + (6-0)^2} = \sqrt{4+36} = \sqrt{40}$$

Hence the equation of the circle is given by

$$x^2 + y^2 = (\sqrt{40})^2$$
$$x^2 + y^2 = 40$$

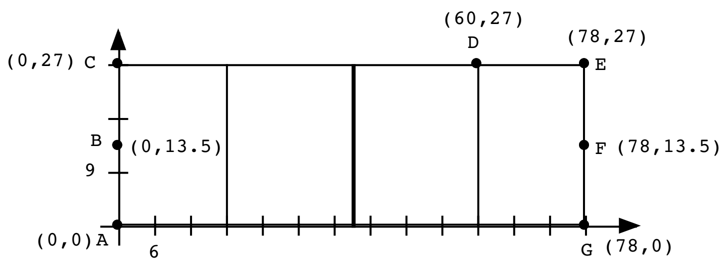
66. The radius of a circle is the distance from the center to any point on the circle. Since the center of this circle is (-5, 4) and (2, -3) is a point on the circle, the radius is given by

$$r = \sqrt{(-5-2)^2 + [4-(-3)]^2} = \sqrt{49+49} = \sqrt{98}$$

Hence the equation of the circle is given by

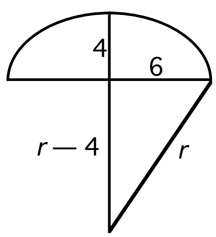
$$(x - (-5))^2 + (y - 4)^2 = (\sqrt{98})^2$$
$$(x + 5)^2 + (y - 4)^2 = 98$$

68.



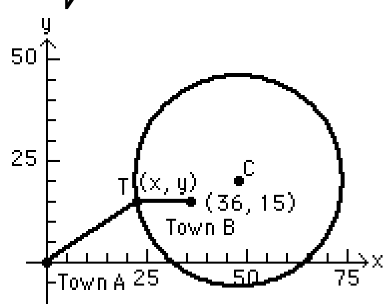
$$d(A, D) = \sqrt{60^2 + 27^2} = 66 \text{ feet}$$
$$d(C, G) = \sqrt{78^2 + 27^2} = 83 \text{ feet}$$

70.



$$r^2 = (r-4)^2 + 6^2$$
$$r^2 = r^2 - 8r + 16 + 36$$
$$8r = 52$$
$$r = 6.5 \text{ mm}$$

72. (A)



$$AT = 2TB$$
$$\sqrt{(x-0)^2 + (y-0)^2} = 2\sqrt{(x-36)^2 + (y-15)^2}$$
$$x^2 + y^2 = 4(x^2 - 72x + 36^2) + 4(y^2 - 30y + 15^2)$$
$$x^2 + y^2 = 4x^2 - 288x + 5184 + 4y^2 - 120y + 900$$
$$3x^2 - 288x + 3y^2 - 120y = -6084$$
$$x^2 - 96x + 1728 + y^2 - 40y + 300 = -2028$$
$$x^2 - 96x + 2304 + y^2 - 40y + 400 = -2028 + 2304 + 400$$
$$(x - 48)^2 + (y - 20)^2 = 676 = 26^2\text{: circle}$$
$$\text{center} = (48, 20);\ \text{radius} = 26$$

(B) On the circle, find y when $x = 0$:

$$(x - 48)^2 + (0 - 20)^2 = 676$$
$$(x - 48)^2 = 276$$
$$x - 48 = \pm 16.613$$
$$x = 64.6 \text{ miles or}$$
$$x = 31.4 \text{ miles}$$

Section 2-3

2. Using the points (-3, -3) and (1, 3),

Rise = 3 - (-3) = 6; Run = 1 - (-3) = 4; Slope = $\frac{6}{4} = \frac{3}{2}$

$$y - 3 = \frac{3}{2}(x - 1)$$
$$y - 3 = \frac{3}{2}x - \frac{3}{2}$$
$$y = \frac{3}{2}x + \frac{3}{2}$$
$$3x - 2y = -3$$

4. Using the points (0, -3) and (5, 3),

Rise = 3 - (-3) = 6; Run = 5 - 0 = 5; Slope = $\frac{6}{5}$

$$y - 3 = \frac{6}{5}(x - 5)$$
$$y - 3 = \frac{6}{5}x - 6$$
$$y = \frac{6}{5}x - 3$$
$$6x - 5y = 15$$

6. Using the points (-5, 3) and (-1, -2),

Rise = -2 - 3 = -5; Run = -1 - (-5) = 4; Slope = $-\frac{5}{4}$

$$y - (-2) = -\frac{5}{4}(x - (-1))$$
$$y + 2 = -\frac{5}{4}x - \frac{5}{4}$$
$$y = -\frac{5}{4}x - \frac{13}{4}$$
$$5x + 4y = -13$$

8. The x intercept is 1. The y intercept is 1. From the point (0, 1) to the point (1, 0) the value of y decreases by 1 unit as the value of x increases by 1 unit. Thus slope = $\frac{\text{rise}}{\text{run}} = \frac{-1}{1} = -1$.

Equation: $y = mx + b$
$y = -1x + 1$ or $y = -x + 1$

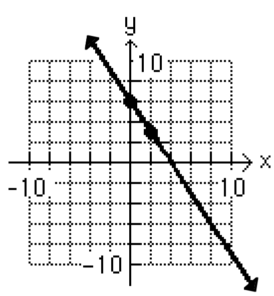
10. There is no x intercept. The y intercept is 3. The slope of this horizontal line is 0. The equation of this horizontal line is $y = 3$.

12. The x intercept is -2. There is no y intercept. The slope of this vertical line is undefined. The equation of this vertical line is $x = -2$.

14. $y = -\frac{3}{2}x + 6$

x	y
0	6
2	3

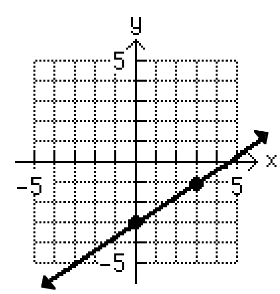
slope = $-\frac{3}{2}$



16. $y = \frac{2}{3}x - 3$

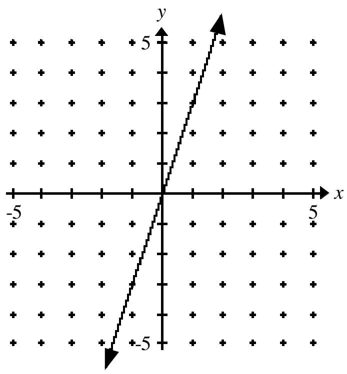
x	y
0	−3
3	−1

slope = $\frac{2}{3}$



18. $6x - 2y = 0$
$-2y = -6x$
$y = 3x$
slope 3

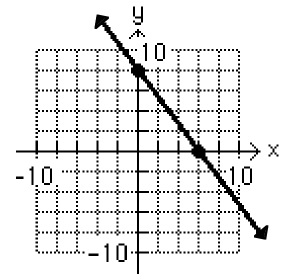
x	y
0	0
1	3
2	6



20. $4x + 3y = 24$
$3y = -4x + 24$
$y = -\frac{4}{3}x + 8$

x	y
0	8
6	0

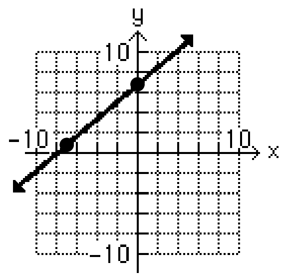
slope $= -\frac{4}{3}$



22. $6x - 7y = -49$
$7y = 6x + 49$
$y = \frac{6}{7}x + 7$

x	y
0	7
–7	1

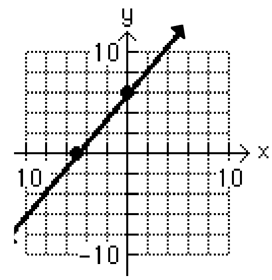
slope $= \frac{6}{7}$



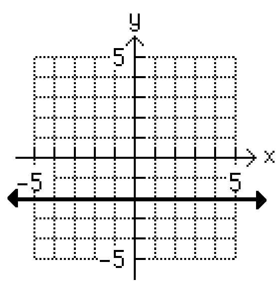
24. $\frac{y}{6} - \frac{x}{5} = 1$
$\frac{y}{6} = \frac{x}{5} + 1$
$y = \frac{6}{5}x + 6$

x	y
0	6
–5	0

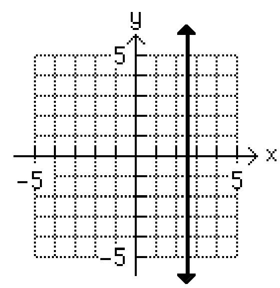
slope $= \frac{6}{5}$



26. $y = -2$,
horizontal line
slope = 0



28. $x = 2.5$,
vertical line
undefined slope

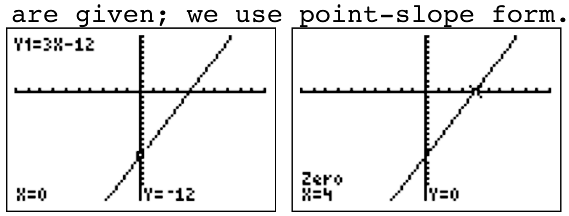


30. $m = -1$, $b = 7$:
$y = mx + b$
$y = -x + 7$
$x + y = 7$

32. $m = \frac{5}{3}$, $b = 6$:
$y = mx + b$
$y = \frac{5}{3}x + 6$
$3y = 5x + 18$
$-5x + 3y = 18$
$5x - 3y = -18$

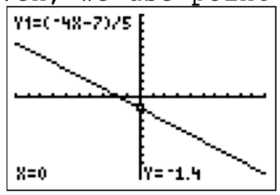
34. The equation of this vertical line is $x = 3$.

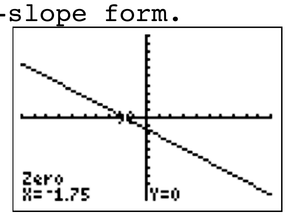
36. A point and the slope are given; we use point-slope form.
$y - 0 = 3(x - 4)$
$y = 3x - 12$



38. A point and the slope are given; we use point-slope form.

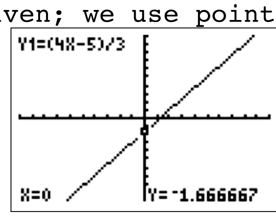
$$y - (-3) = -\frac{4}{5}(x - 2)$$
$$y + 3 = -\frac{4}{5}x + \frac{8}{5}$$
$$y = -\frac{4}{5}x - \frac{7}{5}$$

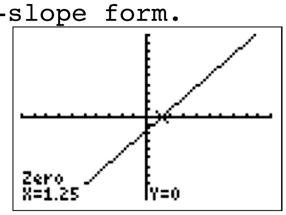




40. A point and the slope are given; we use point-slope form.

$$y - 1 = \frac{4}{3}(x - 2)$$
$$y - 1 = \frac{4}{3}x - \frac{8}{3}$$
$$y = \frac{4}{3}x - \frac{5}{3}$$





42. (-1, 4); (3, 2):

$$m = \frac{4 - 2}{-1 - 3} = \frac{2}{-4} = -\frac{1}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{1}{2}(x - (-1))$$
$$y - 4 = -\frac{1}{2}x - \frac{1}{2}$$
$$y = -\frac{1}{2}x - \frac{1}{2} + \frac{8}{2}$$
$$y = -\frac{1}{2}x + \frac{7}{2}$$

44. (0, 5); (2, 5):

$$m = \frac{5 - 5}{0 - 2} = 0$$

Equation of the form $y = b$

$$y = 5$$

46. (5, -4); (5, 6):

$m = \frac{-4 - 6}{5 - 5}$; undefined slope

Equation of the form $x = a$

$$x = 5$$

48. Sketch a graph with the given information.

(2, 0); $m = 2$:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 2)$$
$$y = 2x - 4$$

50. Sketch a graph with the given information.

(-4, -2); $m = \frac{1}{2}$:

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \frac{1}{2}(x - (-4))$$
$$y + 2 = \frac{1}{2}x + 2$$
$$y = \frac{1}{2}x$$

52. Sketch a graph with the given information.

(-3, 4), (6, 1):

$m = \frac{y_2 - y_1}{x_2 - x_1}$;

$$m = \frac{1 - 4}{6 - (-3)}$$
$$m = -\frac{1}{3}$$
$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{1}{3}(x - (-3))$$
$$y - 4 = -\frac{1}{3}x - 1$$
$$y = -\frac{1}{3}x + 3$$

54. Sketch a graph with the given information.

(2, -1), (10, 5):

$m = \frac{y_2 - y_1}{x_2 - x_1}$;

$$m = \frac{5 - (-1)}{10 - 2}$$
$$m = \frac{3}{4}$$
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{3}{4}(x - 2)$$
$$y + 1 = \frac{3}{4}x - \frac{3}{2}$$
$$y = \frac{3}{4}x - \frac{5}{2}$$

56. Sketch a graph with the given information.

(0, -2), (4, -2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-2 - (-2)}{4 - 0}$$

$m = 0$, horizontal line

$y = -2$

58. Sketch a graph with the given information.
(-3, 1), (-3, -4):
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$m = \frac{-4 - 1}{-3 - (-3)}$
$m = \frac{-5}{0}$, vertical line
$x = -3$

60. Sketch a graph with the given information.
(-4, 0), (0, -5):
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 0}{0 - (-4)} = -\frac{5}{4}$
$y - y_1 = m(x - x_1)$
$y - 0 = -\frac{5}{4}(x - (-4))$
$y = -\frac{5}{4}x - 5$

62. (-4, 0); || to $y = -2x + 1$:
$y = -2x + 1$
$m = -2$, $|| m = -2$
$y - y_1 = m(x - x_1)$
$y - 0 = -2(x - (-4))$
$y = -2x - 8$
$2x + y = -8$

64. (-2, -4); $\perp$ to $y = \frac{2}{3}x - 5$
$y = \frac{2}{3}x - 5$; $m = \frac{2}{3}$
$\perp m = -\frac{3}{2}$
$y - y_1 = m(x - x_1)$
$y - (-4) = -\frac{3}{2}(x - (-2))$
$y + 4 = -\frac{3}{2}(x + 2)$
$2y + 8 = -3x - 6$
$3x + 2y = -14$

66. parallel to x axis $\Rightarrow m = 0$
(7, 3) is contained; $y = 3$

68. horizontal $\Rightarrow m = 0$;
(-2, -3) is contained; $y = -3$

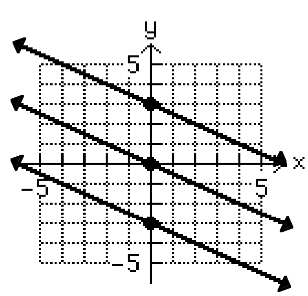
70. $3x + 4y = 8$
$4y = -3x + 8$
$y = -\frac{3}{4}x + 2$
$m = -\frac{3}{4}$
$|| m = -\frac{3}{4}$

$y - y_1 = m(x - x_1)$
$y - 5 = -\frac{3}{4}(x - 3)$
$4y - 20 = -3x + 9$
$3x + 4y = 29$

72. $4x + 5y = 0$
$y = -\frac{4}{5}x$
$m = -\frac{4}{5}$
$\perp m = \frac{5}{4}$

$m = \frac{5}{4}$; (-2, 4)
$y - y_1 = m(x - x_1)$
$y - 4 = \frac{5}{4}(x - (-2))$
$4y - 16 = 5x + 10$
$5x - 4y = -26$

74. $y = -\frac{1}{2}x + b$
$m = -\frac{1}{2}$
y int $= b$
Lines will be parallel.



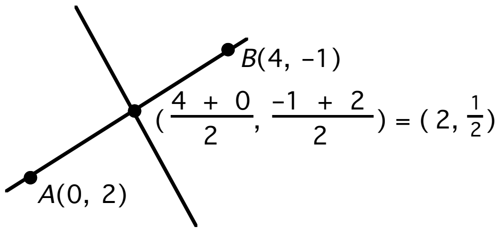
76. $m_{DA} = \dfrac{2 - (-2)}{0 - (-3)} = \dfrac{4}{3}$; $m_{CB} = \dfrac{-5 - (-1)}{1 - 4} = \dfrac{4}{3}$

Since $m_{DA} = m_{CB} = \dfrac{4}{3}$, $DA \parallel CB$.

78. $m_{AD} = \dfrac{4}{3}$ from problem 76

$m_{DC} = \dfrac{-5 - (-2)}{1 - (-3)} = -\dfrac{3}{4}$ from which $m_{AD} \cdot m_{DC} = -1$ which shows $AD \perp DC$.

80.



$m_{AB} = \dfrac{-1 - 2}{4 - 0} = -\dfrac{3}{4}$

$\perp m = \dfrac{4}{3}$

$y - y_1 = m(x - x_1)$

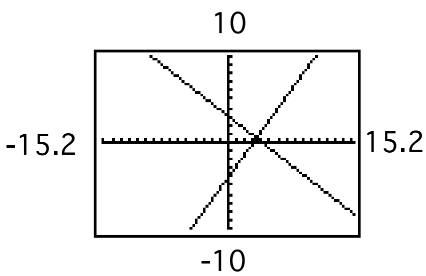
$y - \dfrac{1}{2} = \dfrac{4}{3}(x - 2)$

$6y - 3 = 8(x - 2)$

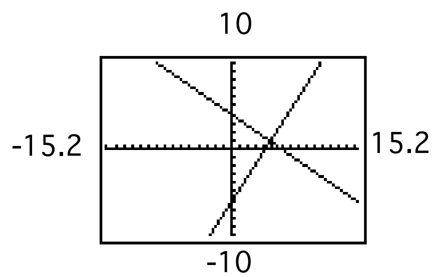
$6y - 3 = 8x - 16$

$8x - 6y = 13$

82. (A) The graphs of $3x + 4y = 12$ and $4x - 3y = 12$ are



(B) The graphs of $2x + 3y = 12$ and $3x - 2y = 12$ are



(C) The graphs of $Ax + By = C$ and $Bx - Ay = C$ are perpendicular lines.

(D) $Ax + By = C$ may be written $y = -\dfrac{A}{B}x + \dfrac{C}{B}$ and $Bx - Ay = C$ may be written $y = \dfrac{B}{A}x - \dfrac{C}{A}$. Multiplying the slopes gives $-\dfrac{A}{B} \cdot \dfrac{B}{A} = -1$ which shows the lines are perpendicular.

84. Two points are given; we first find the slope, then use the point-slope form.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$y - y_1 = m(x - x_1)$

$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Multiply both sides by

$x_2 - x_1$ $(x_2 \neq x_1)$,

then

$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$

86. Using the result of problem 83, we have

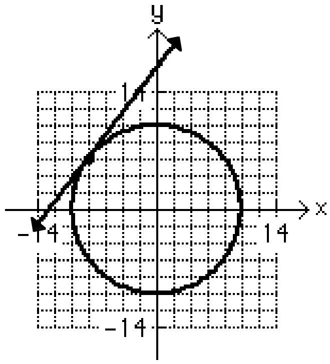
$a = -2$, $b = 7$

The equation is

$\dfrac{x}{-2} + \dfrac{y}{7} = 1$

$7x - 2y = -14$

88.



$x^2 + y^2 = 100$; $(-8, 6)$

Find m from the center, $(0, 0)$, to $(-8, 6)$:

$$m = \frac{6 - 0}{-8 - 0} = -\frac{3}{4}$$

$$\perp m = \frac{4}{3}$$

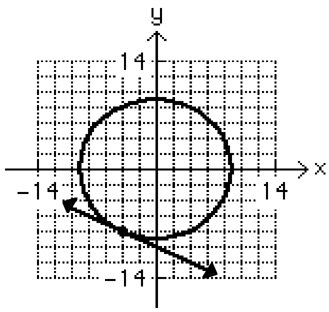
$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{4}{3}(x - (-8))$$

$$3y - 18 = 4x + 32$$

$$4x - 3y = -50$$

90.



$x^2 + y^2 = 80$; $(-4, -8)$

Find m from the center, $(0, 0)$, to $(-4, -8)$:

$$m = \frac{-8 - 0}{-4 - 0} = 2$$

$$\perp m = -\frac{1}{2}$$

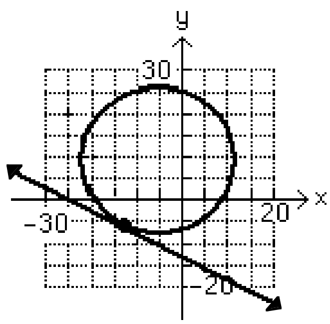
$$y - y_1 = m(x - x_1)$$

$$y - (-8) = -\frac{1}{2}(x - (-4))$$

$$2y + 16 = -x - 4$$

$$x + 2y = -20$$

92.



$(x + 5)^2 + (y - 9)^2 = 289$; $(-13, -6)$

center = $(-5, 9)$

radius = 17

Find m from the center $(-5, 9)$ to $(-13, -6)$:

$$m = \frac{9 - (-6)}{-5 - (-13)} = \frac{15}{8}$$

$$\perp m = \frac{8}{15}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-6) = -\frac{8}{15}(x - (-13))$$

$$15y + 90 = -8x - 104$$

$$8x + 15y = -194$$

94. (A)

x	0	1	2	3	4	5
A	25	16	7	−2	−11	−20

$A = 25 - 9x$

(B) For every kilometer increase in altitude the air temperature decreases 9°C.

96. The charge is \$15 for travel to site plus 70 cents for each minute it takes to do the installation.

98.

$$C(x) = 1{,}200 + 45x$$

$$4{,}800 = 1{,}200 + 45x$$

$$3{,}600 = 45x$$

$$80 = x$$

80 tables can be produced.

100. (A) We write $d = mw + b$. Since $d_1 = 18$ when $w_1 = 3$ and $d_2 = 10$ when $w_2 = 5$, the slope is given by

$$m = \frac{d_2 - d_1}{w_2 - w_1} = \frac{10 - 18}{5 - 3} = -4$$

Then $d = -4w + b$.
Substituting $d_1 = 18$ when $w_1 = 3$, we obtain

$$18 = -4(3) + b$$
$$b = 30$$

Hence $d = -4w + 30$

(B) If $w = 0$, $d = 30$ inches.
(C) If $d = 0$, solve $0 = -4w + 30$ to obtain $w = 7.5$ pounds.

102. We write $R = mK + b$. Since $R_1 = 492$ when $K_1 = 273$ and $R_2 = 672$ when $K_2 = 373$, the slope is given by

$$m = \frac{R_2 - R_1}{K_2 - K_1} = \frac{672 - 492}{373 - 273} = 1.8$$

Then $R = 1.8K + b$.
Substituting $R_1 = 492$ when $K_1 = 273$, we obtain

$$492 = 1.8(273) + b$$
$$b = 0.6$$

Hence $R = 1.8K + 0.6$

104. (A) We write $h = mt + b$. Since $h_1 = 7$ when $t_1 = 9$ and $h_2 = 11$ when $t_2 = 25$, the slope is given by

$$m = \frac{h_2 - h_1}{t_2 - t_1} = \frac{11 - 7}{25 - 9} = \frac{1}{4} = 0.25$$

Then $h = 0.25t + b$.
Substituting $h_1 = 7$ when $t_1 = 9$, we obtain

$$7 = 0.25(9) + b$$
$$b = 4.75$$

Hence $h = 0.25t + 4.75$.

(B) Solve $20 = 0.25t + 4.75$

$$15.25 = 0.25t$$
$$t = 61 \text{ hours}$$

106. (A) We write $N = mt + b$. Since $N_1 = 4.76$ when $t_1 = 0$ and $N_2 = 2.50$ when $t_2 = 90$, the slope is given by

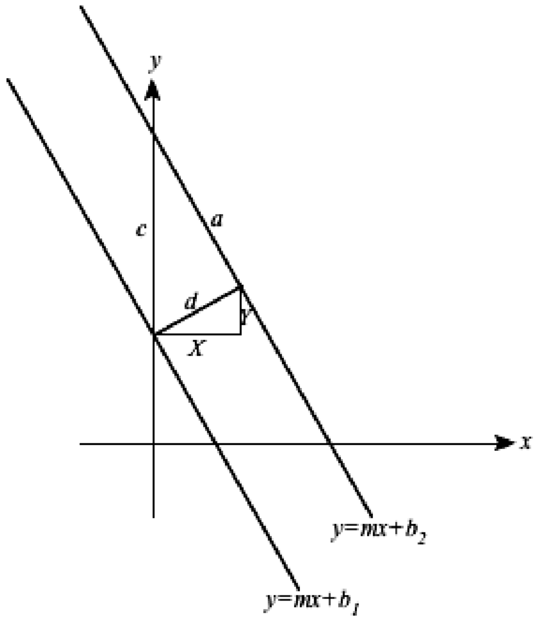
$$m = \frac{N_2 - N_1}{t_2 - t_1} = \frac{2.50 - 4.76}{90 - 0} \approx -0.0251$$

Then $N = -0.0251t + b$.
Substituting $N_1 = 4.76$ when $t_1 = 0$, we obtain $4.76 = b$.
Hence $N = -0.0251t + 4.76$.

(B) We are asked for N when $t = 100$.

$$N = -0.0251(100) + 4.76$$
$$N = 2.25 \text{ persons per household}$$

108.



In general, we can show that $d = \dfrac{c}{\sqrt{1 + m^2}}$ as follows:

The Pythagorean Theorem gives:

$$d^2 + a^2 = c^2$$

The two triangles shown are similar, hence corresponding sides are proportional. Thus

$$\frac{a}{d} = \frac{X}{Y}$$

The slope of the line segment labeled d is the negative reciprocal of m.

Thus $\dfrac{Y}{X} = -\dfrac{1}{m}$

$$\frac{X}{Y} = -m$$

It follows that $a = \dfrac{X}{Y}d = -md$.

Hence $d^2 + (-md)^2 = c^2$

$$d^2(1 + m^2) = c^2$$

$$d^2 = \frac{c^2}{1 + m^2}$$

$$d = \frac{c}{\sqrt{1 + m^2}}$$

In particular, avenue A is shown to have a rise of -5000 and a run of 4000, hence $m = -\dfrac{5000}{4000} = -1.25$. The equation of avenue A is then (using the slope-intercept form $y = mx + b$) $y = -1.25x + 4000$. Avenue B has the same slope, and y intercept 4000. Avenue C has the same slope, and y intercept 2000. Substituting in the above formula, with $c = 4000 - 2000 = 2000$, yields

$$d_2 = \frac{2000}{\sqrt{1 + (-1.25)^2}} = 1{,}249 \text{ ft.}$$

Section 2-4

2. (A) If cost y is linearly related to the number of tennis rackets x_1 then we are looking for an equation whose graph passes through $(x_1, y_1) = (50, 4{,}174)$ and $(x_2, y_2) = (60, 4{,}634)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4{,}634 - 4{,}174}{60 - 50} = 46$$

$$y - y_1 = m(x - x_1)$$
$$y - 4{,}174 = 46(x - 50)$$
$$y - 4{,}174 = 46x - 2300$$
$$y = 46x + 1{,}874$$

(B) The slope of 46 is the rate of change of cost with respect to production, \$46 per tennis racket.

(C) Increasing production by 1 unit increases cost by \$46.

4. (A) The rate of change of height with respect to DBH is 2.27 feet per inch.

(B) Increasing DBH by 1 inch increases height by 2.27 feet.

(C) Substitute $d = 12$ into $h = 2.27d + 33.1$ to obtain

$$h = 2.27(12) + 33.1$$
$$h = 60 \text{ feet}$$

(D) Substitute $h = 100$ into $h = 2.27d + 33.1$ and solve.

$$100 = 2.27d + 33.1$$
$$66.9 = 2.27d$$
$$d = 29 \text{ inches}$$

6. (A) Robinson: The rate of change of weight with respect to height is 4.2 pounds per inch.
Miller: The rate of change of weight with respect to height is 3.1 pounds per inch.

(B) 5′10″ = 10 inches over 5 feet
Substitute $h = 10$ into each model.
Robinson: $w = 115 + 4.2(10) = 157$ pounds
Miller: $w = 124 + 3.1(10) = 155$ pounds

(C) Substitute $w = 160$ into each model and solve.
Robinson: $160 = 115 + 4.2h$
$45 = 4.2h$
$h = 11$ inches, predicting 5′11″.
Miller: $160 = 124 + 3.1h$
$36 = 3.1h$
$h = 12$ inches, predicting 6′.

8. If speed s is linearly related to temperature t, then we are looking for an equation whose graph passes through $(t_1, s_1) = (10, 337)$ and $(t_2, s_2) = (20, 343)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{s_2 - s_1}{t_2 - t_1} = \frac{343 - 337}{20 - 10} = 0.6$$
$$s - s_1 = m(t - t_1)$$
$$s - 337 = 0.6(t - 10)$$
$$s - 337 = 0.6t - 6$$
$$s = 0.6t + 331$$

The speed of sound at sea level increases by 0.6 mph for each 1°C change in temperature.

10. If percentage f is linearly related to time t, then we are looking for an equation whose graph passes through $(t_1, f_1) = (0, 22.9)$ and $(t_2, f_2) = (10, 21.1)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{f_2 - f_1}{t_2 - t_1} = \frac{21.1 - 22.9}{10 - 0} = -0.18$$
$$f - f_1 = m(t - t_1)$$
$$f - 22.9 = -0.18(t - 0)$$
$$f = -0.18t + 22.9$$

To find t when $f = 18$, substitute $f = 18$ and solve.

$$18 = -0.18t + 22.9$$
$$-4.9 = -0.18t$$
$$t = 27$$

27 years after 1990 will be 2017.

12. (A) If value V is linearly related to time t, then we are looking for an equation whose graph passes through $(t_1, V_1) = (0, 154{,}900)$ and $(t_2, V_2) = (16, 46{,}100)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{V_2 - V_1}{t_2 - t_1} = \frac{46{,}100 - 154{,}900}{16 - 0} = -6{,}800$$

$$V - V_1 = m(t - t_1)$$

$$V - 154{,}900 = -6{,}800(t - 0)$$

$$V = -6{,}800t + 154{,}900$$

(B) The boat's value decreases at the rate of \$6,800 per year.

(C) To find t when $V = 100{,}000$ substitute $V = 100{,}000$ and solve.

$$100{,}000 = -6{,}800t + 154{,}900$$

$$-54{,}900 = -6{,}800t$$

$$t = 8.07\text{, that is, during the ninth year}$$

14. (A) If price R is linearly related to cost C, then we are looking for an equation whose graph passes through $(C_1, R_1) = (20, 33)$ and $(C_2, R_2) = (60, 93)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{R_2 - R_1}{C_2 - C_1} = \frac{93 - 33}{60 - 20} = 1.5$$

$$R - R_1 = m(C - C_1)$$

$$R - 33 = 1.5(C - 20)$$

$$R - 33 = 1.5C - 30$$

$$R = 1.5C + 3$$

(B) The slope is 1.5. This is the rate of change of retail price with respect to cost.

(C) To find C when $R = 240$, substitute $R = 240$ and solve.

$$240 = 1.5C + 3$$

$$237 = 1.5C$$

$$C = \$158$$

16. (A) Since the true airspeed is 2% more than the indicated airspeed for each 1000 feet of altitude, an indicated airspeed of 200 mph must be adjusted by 2% (200) = 4 mph for each 1000 feet of altitude. Thus T is linearly related to A with a slope of $4 = m$. Then $T = 4A + b$. Since $T = 200$ (true airspeed = indicated airspeed) when $A = 0$, the y intercept $b = 200$. Thus $T = 4A + 200$.

(B) Substitute $A = 6.5$ to obtain $T = 4(6.5) + 200 = 226$ mph.

18. (A) If altitude a is linearly related to time t, then we are looking for an equation whose graph passes through $(t_1, a_1) = (0, 2{,}880)$ and $(t_2, a_2) = (180, 0)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{a_2 - a_1}{t_2 - t_1} = \frac{0 - 2{,}880}{180 - 0} = -16$$

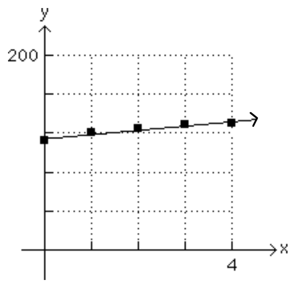
$$a - a_1 = m(t - t_1)$$

$$a - 2{,}880 = -16(t - 0)$$

$$a = -16t + 2{,}880$$

(B) Since altitude is decreasing at the rate of 16 feet per second, this is the rate of descent.

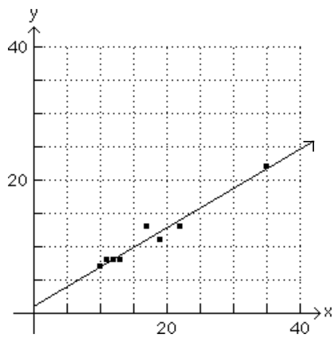
20. (A)



(B) Substitute $x = 10$ into $y = 4.4x + 114$ to obtain $y = 4.4(10) + 114 = 158$. This represents $158,000.

(C) The median price is increasing at the rate of $4,400 per year.

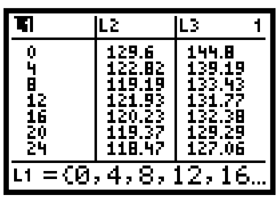
22. (A)

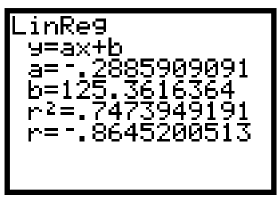


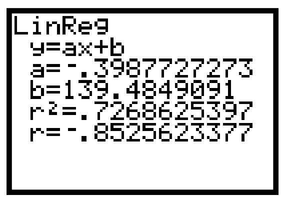
(B) Substitute $x = 35.5$ into $y = 0.59x + 1.06$ to obtain $y = 0.59(35.5) + 1.06 \approx 22.0$ million.

(C) Substitute $x = 19.2$ into $y = 0.59x + 1.06$ to obtain $y = 0.59(19.2) + 1.06 \approx 12.4$ million.

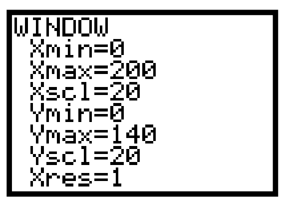
24. A portion of the entered data is shown here along with the results of the linear regression calculations.

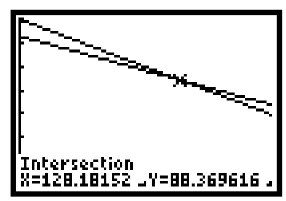






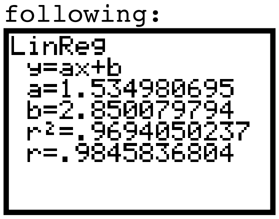
The linear regression model for men's 200-meter backstroke data is seen to be $y = -0.28859x + 125.36$. The linear regression model for women's 200-meter backstroke data is seen to be $y = -0.39877x + 139.48$. A plausible window is shown here, along with the results of the intersection calculation.

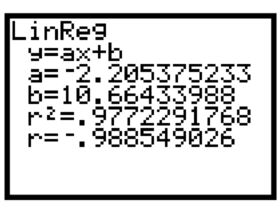




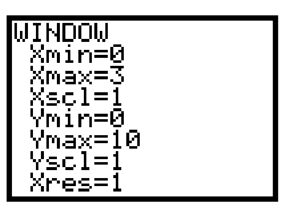
The intersection for $x \approx 128$ implies that the women will catch up with the men in $1968 + 128 = 2096$.

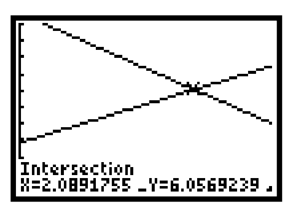
26. Entering the data and applying the linear regression routine yields the following:





The linear regression model for the price-supply data is seen to be $y = 1.53x + 2.85$. The linear regression model for the price-demand data is seen to be $y = -2.21x + 10.7$. A plausible window is shown here, along with the results of the intersection calculation.





The intersection for $y = 6.06$ implies an equilibrium price of $6.06.

Chapter 2 Group Activity

1. (A) average rate $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{D + D}{\frac{D}{r} + \frac{D}{s}} = \dfrac{2D}{\frac{D(s+r)}{rs}} = \dfrac{2rs}{s + r}$

(B) r = rate upstream = 10 – 5 = 5 mph; time upstream $= \dfrac{60}{5} = 12$ hr

s = rate downstream = 10 + 5 = 15 mph; time downstream $= \dfrac{60}{15} = 4$ hr

average rate $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{60 + 60}{12 + 4} = \dfrac{120}{16} = 7.5$ mph

The formula from part (A) gives

average rate $= \dfrac{2rs}{s + r} = \dfrac{2 \cdot 5 \cdot 15}{5 + 15} = \dfrac{150}{20} = 7.5$ mph

(C) If we try to solve $6 = \dfrac{2rs}{s + r}$ for s with $r = 3$, we find

$$6 = \frac{6s}{3 + s}$$
$$18 + 6s = 6s$$
$$18 = 0$$

There is no solution. It can't be done.

2. (A) average rate in 1st second $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{16}{1} = 16$ fps

average rate in 2nd second $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{64 - 16}{1} = 48$ fps

average rate in 3rd second $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{144 - 64}{1} = 80$ fps

(B)

Time Interval	[1.9, 2]	[1.99, 2]	[1.999, 2]	[1.9999, 2]
Distance fallen	$16 \cdot 2^2 - 16 \cdot 1.9^2$ = 6.24 ft	$16 \cdot 2^2 - 16 \cdot 1.99^2$ = .6384 ft	$16 \cdot 2^2 - 16 \cdot 1.999^2$ = .063984 ft	$16 \cdot 2^2 - 16 \cdot 1.9999^2$ = .00639984 ft
Average rate	$\frac{6.24}{0.1}$ = 62.4 fps	$\frac{.6384}{.01}$ = 63.84 fps	$\frac{.063984}{.001}$ = 63.984 fps	$\frac{.00639984}{.0001}$ = 63.9984 fps

These numbers appear to approach 64 fps.

(C) average rate $= \dfrac{16 \cdot 2^2 - 16 \cdot t^2}{2 - t} = \dfrac{64 - 16t^2}{2 - t}$

$= \dfrac{16(2 - t)(2 + t)}{(2 - t)} = 16(2 + t)$. Near 2 this appears to be very close to 64.

(D) 64 fps

CHAPTER 2

Section 2-1

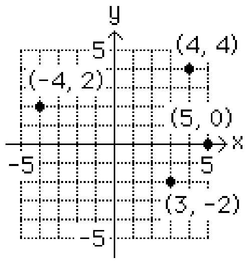
1. The *y* axis **2.** Quadrant I **3.** Quadrant III **4.** The *x* axis

5. Quadrant IV **6.** Quadrant III and quadrant IV

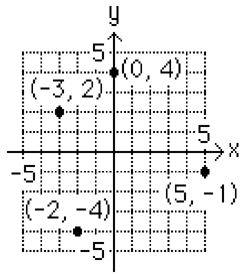
7. Quadrant II and quadrant IV **8.** Quadrant II

9. Quadrant I and quadrant IV **10.** Quadrant I and quadrant III

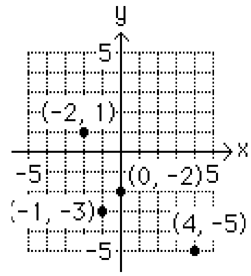
11.



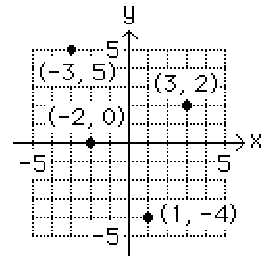
12.



13.



14.



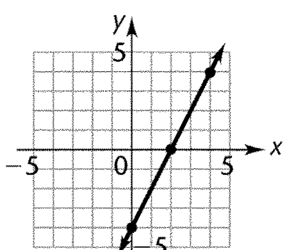
15. Points: $A = (2, 4)$, $B = (3, -1)$, $C = (-4, 0)$, $D = (-5, 2)$
Reflections: $A' = (-2, 4)$, $B' = (-3, -1)$, $C' = (4, 0)$, $D' = (5, 2)$

16. Points: $A = (0, 3)$, $B = (-4, -5)$, $C = (4, 1)$, $D = (1, -3)$
Reflections: $A' = (0, -3)$, $B' = (-4, 5)$, $C' = (4, -1)$, $D' = (1, 3)$

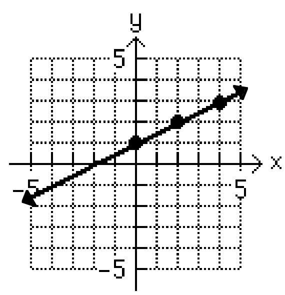
17. Points: $A = (-3, -3)$, $B = (0, 4)$, $C = (-3, 2)$, $D = (5, -1)$
Reflections: $A' = (3, 3)$, $B' = (0, -4)$, $C' = (3, -2)$, $D' = (-5, 1)$

18. Points: $A = (4, 2)$, $B = (-2, -4)$, $C = (-4, 3)$, $D = (5, 0)$
Reflections: $A' = (-4, -2)$, $B' = (2, 4)$, $C' = (4, -3)$, $D' = (-5, 0)$

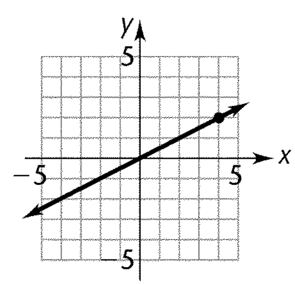
19. No symmetry with respect to *x* axis, *y* axis, or origin



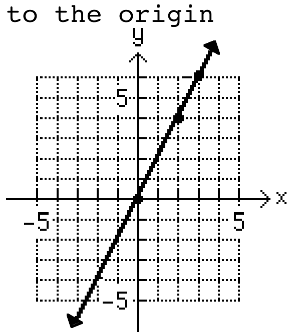
20.



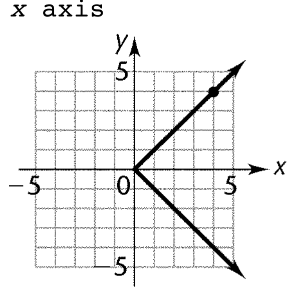
21. Symmetric with respect to the origin



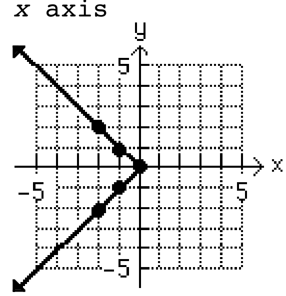
22. Symmetric with respect
to the origin



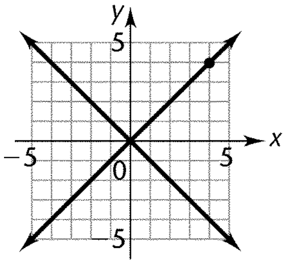
23. Symmetric with respect to the *x* axis



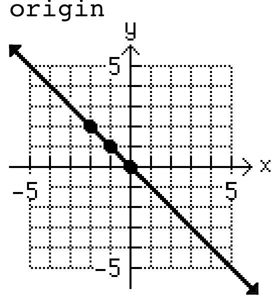
24. Symmetric with respect to the *x* axis



25. Symmetric with respect to the *x* axis, *y* axis, and origin



26. Symmetric with respect to the origin



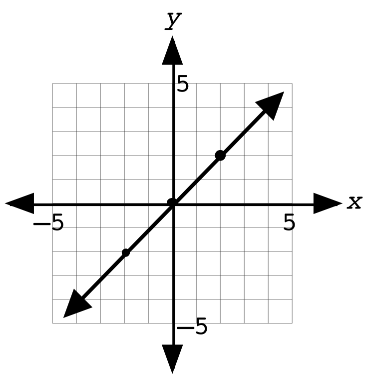
27. (A) 6 (B) −5 (C) −1 (D) 8 (E) −5 (F) 5

28. (A) 5 (B) −8 (C) 6 (D) −2, 4 (E) −4, 6 (F) −3, 5

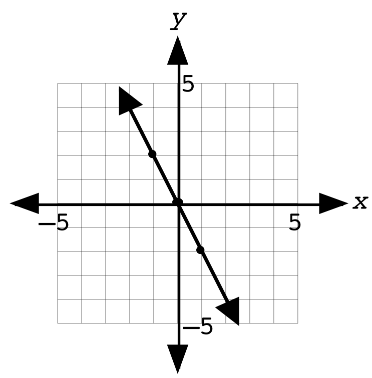
29. (A) 6 (B) 4 (C) 4 (D) 8 (E) −8, 0, 6 (F) −7, −2, 7

30. (A) −3 (B) 1 (C) 4 (D) 3, 6 (E) −6, −4, 2, 7 (F) −5, 2, 7

31. (A)

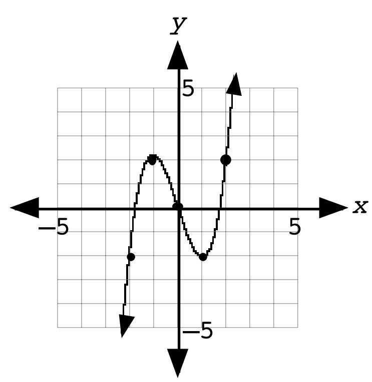


(B)

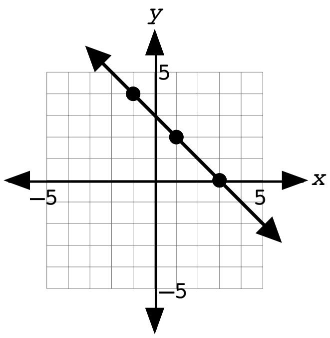


(C)

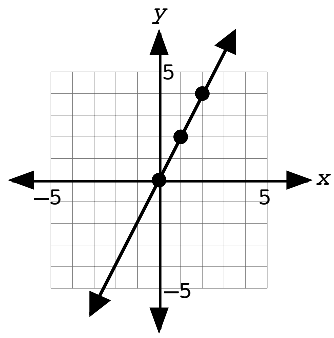
x	−2	−1	0	1	2
y	−2	2	0	−2	2



32. (A)

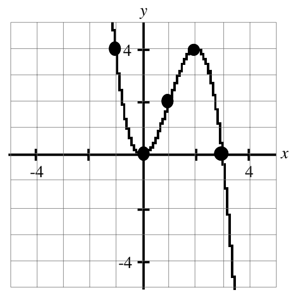


(B)

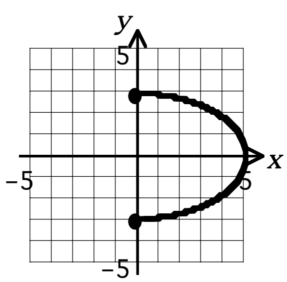


(C)

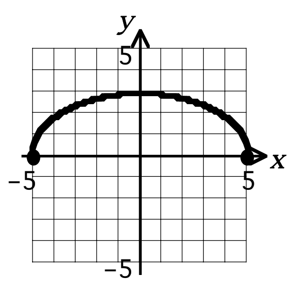
x	−1	0	1	2	3
y	4	0	2	4	0



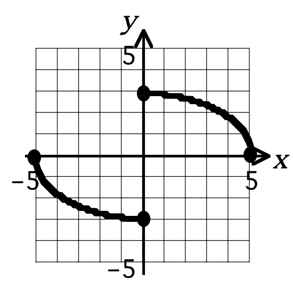
33. (A)



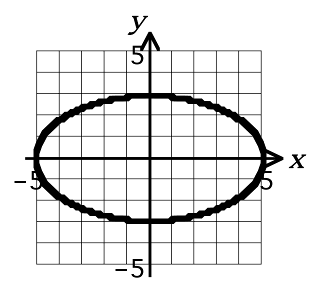
(B)



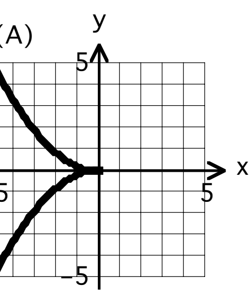
(C)



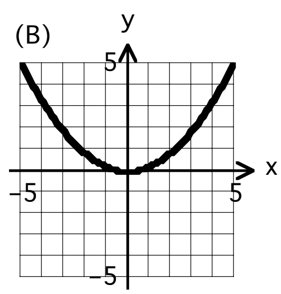
(D)



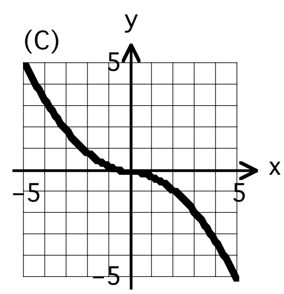
34. (A)



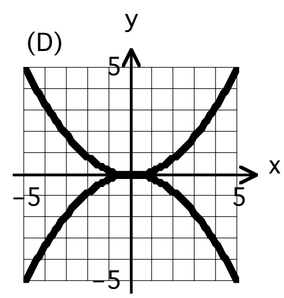
(B)



(C)



(D)



35. Symmetric with respect to the *x* axis, *y* axis, and origin

36. Symmetric with respect to the origin

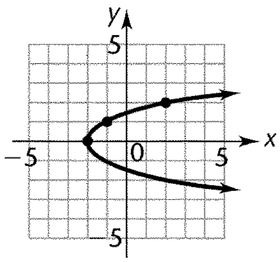
37. Symmetric with respect to the *y* axis

38. No symmetry with respect to the *x* axis, *y* axis, and origin

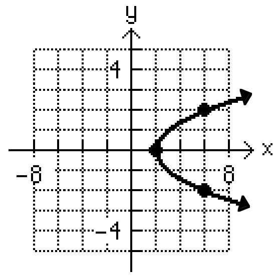
39. Symmetric with respect to the *x* axis

40. Symmetric with respect to the origin

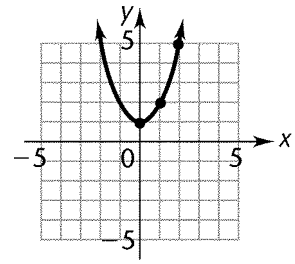
41. Symmetric with respect to the x axis



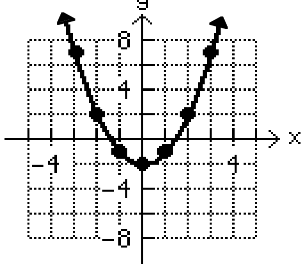
42. Symmetric with respect to the x axis



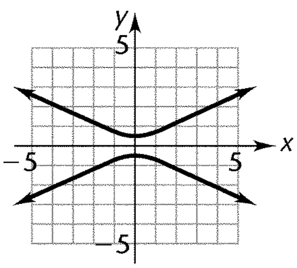
43. Symmetric with respect to the y axis



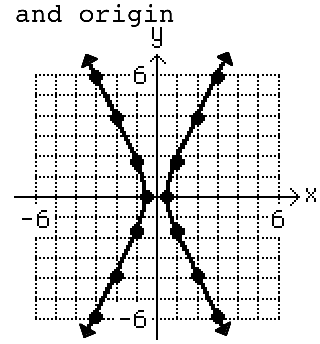
44. Symmetric with respect to the y axis



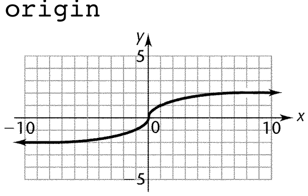
45. Symmetric with respect to the x axis, y axis and origin



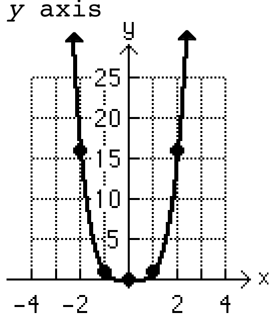
46. Symmetric with respect to the x axis, y axis and origin



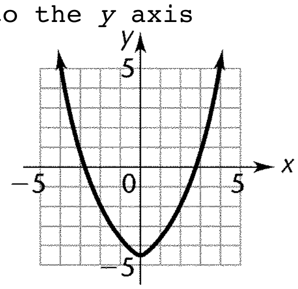
47. Symmetric with respect to the origin



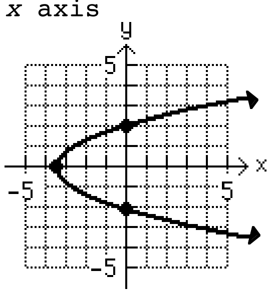
48. Symmetric with respect to the y axis



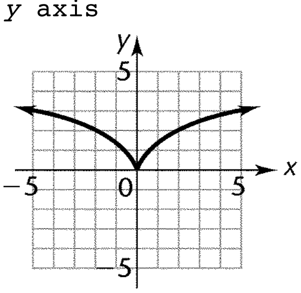
49. Symmetric with respect to the y axis



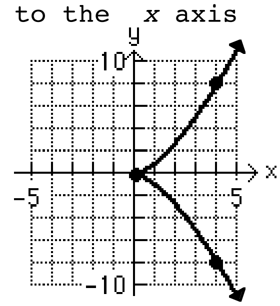
50. Symmetric with respect to the x axis



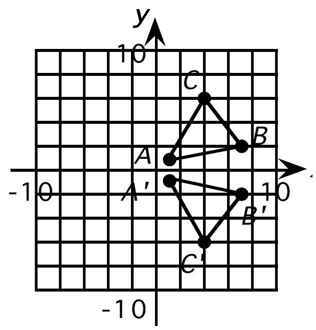
51. Symmetric with respect to the y axis



52. Symmetric with respect to the x axis

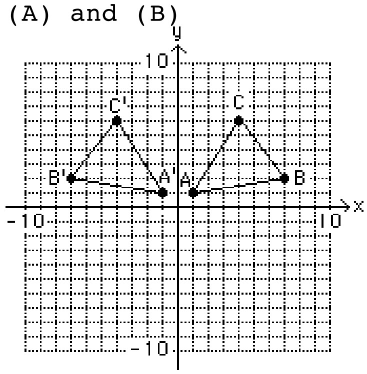


53. (A) and (B)



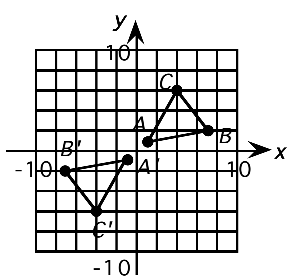
(C) The triangles are mirror images of each other, reflected across the x axis. Changing the sign of the y coordinate reflects the graph across the x axis.

54. (A) and (B)



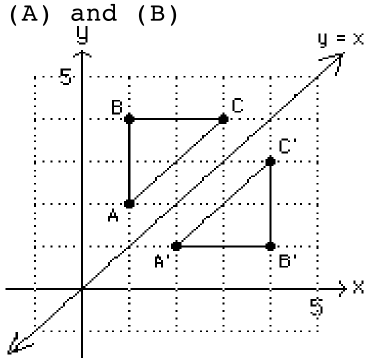
(C) The triangles are mirror images of each other, reflected across the y axis. Changing the sign of the x coordinate reflects the graph across the y axis.

55. (A) and (B)



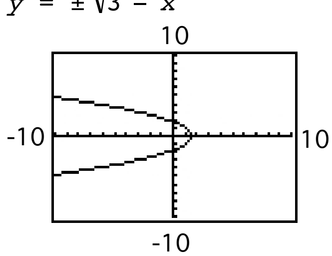
(C) The triangles are mirror images of each other, reflected across the origin. Changing the signs of both coordinates reflects the graph through the origin.

56. (A) and (B)

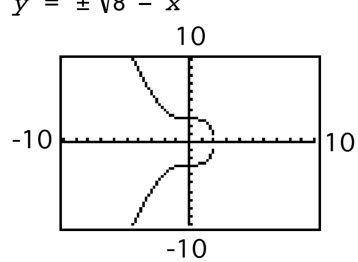


(C) The triangles are mirror images of each other, reflected across the line $y = x$. Reversing the coordinates reflects the graph across the line $y = x$.

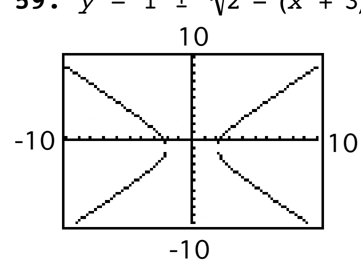
57. $y = \pm\sqrt{3 - x^2}$



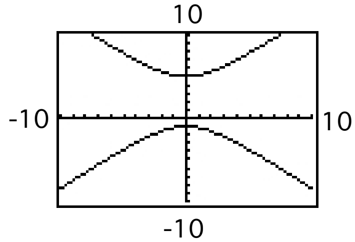
58. $y = \pm\sqrt{8 - x^3}$



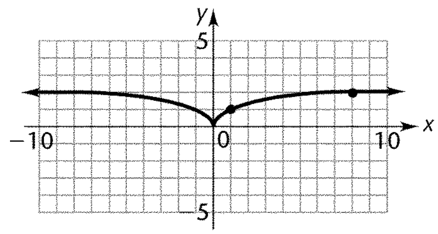
59. $y = 1 \pm \sqrt{2 - (x + 3)^2}$



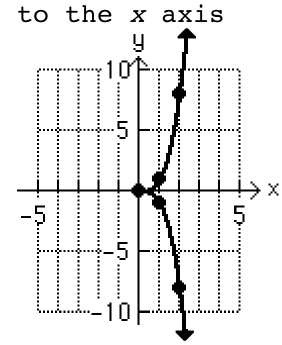
60. $y = 2 \pm \sqrt{9 + x^2}$



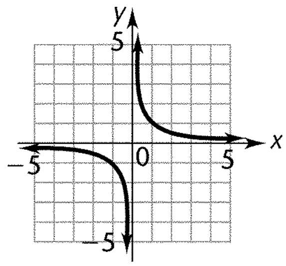
61. Symmetric with respect to the y axis



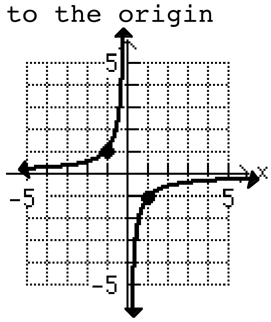
62. Symmetric with respect to the x axis



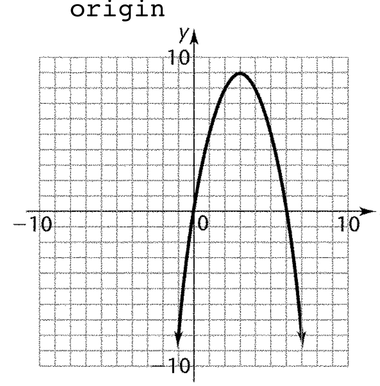
63. Symmetric with respect to the origin



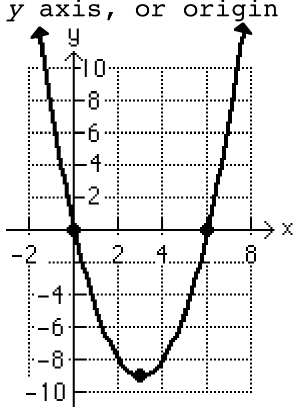
64. Symmetric with respect to the origin



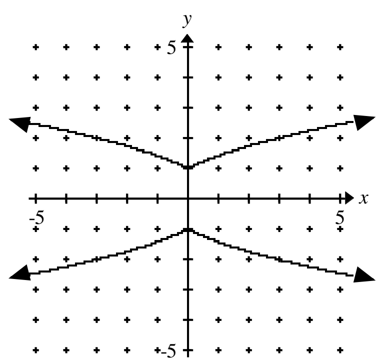
65. No symmetry with respect to the x axis, y axis, or origin



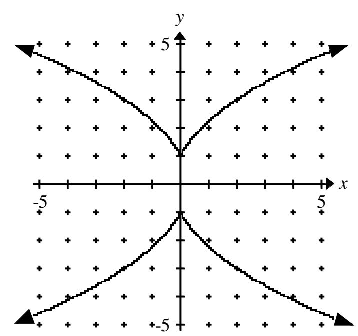
66. No symmetry with respect to the x axis, y axis, or origin



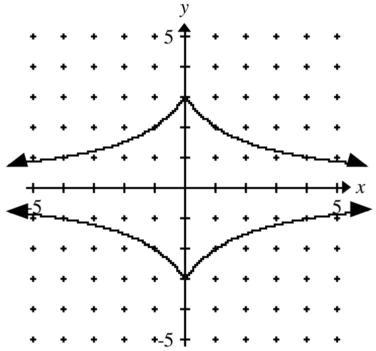
67. Symmetric with respect to the x axis, y axis, and origin



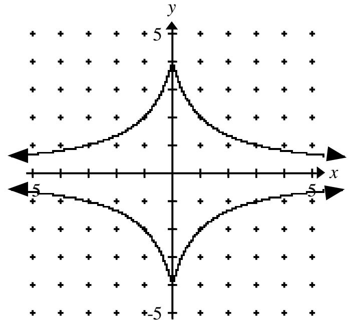
68. Symmetric with respect to the x axis, y axis, and origin



69. Symmetric with respect to the x axis, y axis, and origin



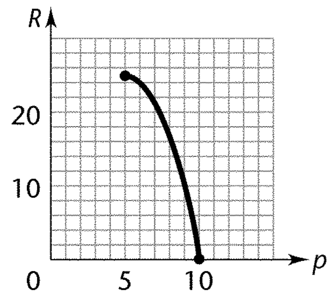
70. Symmetric with respect to the x axis, y axis, and origin



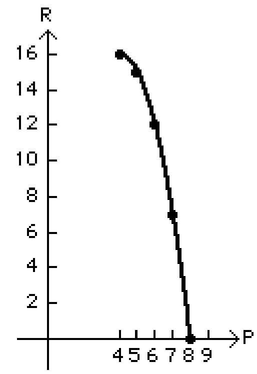
71. Yes

72. Yes

73.



74.



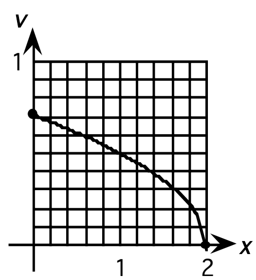
75. (A) 3,000 cases (B) Demand decreases by 400 cases
(C) Demand increases by 600 cases

76. (A) 3,000 cases (B) Supply increases by 300 cases
(C) Supply decreases by 400 cases

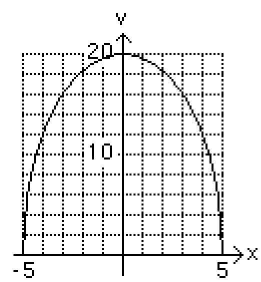
77. (A) 53° (B) 68° at 3 PM (C) 1 AM, 7 AM, 11 PM

78. (A) 60° (B) 44° at 5 AM (C) 9 AM, 10 PM

79.



80.



Section 2-2

1. $d = 5$, $M = (2.5, 2)$

2. $d = 5$, $M = (1.5, 3)$

3. $d = 13$, $M = (2.5, 4)$

4. $d = \sqrt{34}$, $M = (0.5, -1.5)$

5. $d = \sqrt{145}$, $M = (-1.5, 0)$

6. $d = \sqrt{146}$, $M = (0.5, 1.5)$

7. $d = 2\sqrt{5}$, $M = (-4, -2)$

8. $d = 4\sqrt{2}$, $M = (-3, 0)$

9. $x^2 + y^2 = 49$

10. $x^2 + y^2 = 25$

11. $(x - 2)^2 + (y - 3)^2 = 36$

12. $(x - 5)^2 + (y - 6)^2 = 4$

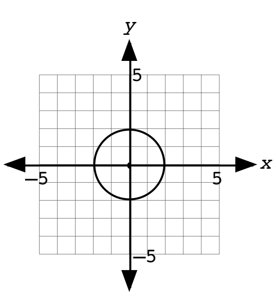
13. $(x + 4)^2 + (y - 1)^2 = 7$

14. $(x + 5)^2 + (y - 6)^2 = 11$

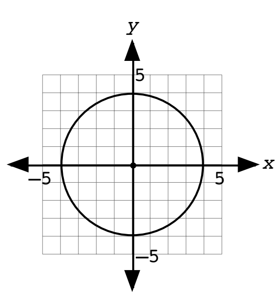
15. $(x + 3)^2 + (y + 4)^2 = 2$

16. $(x - 4)^2 + (y + 1)^2 = 5$

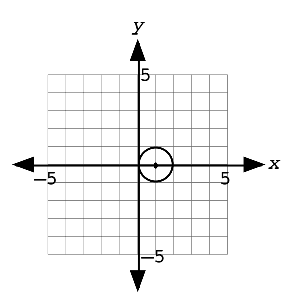
17. $x^2 + y^2 = 4$



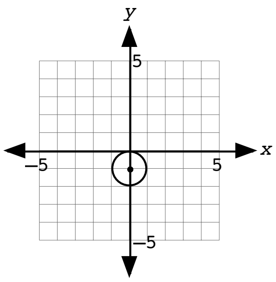
18. $x^2 + y^2 = 16$



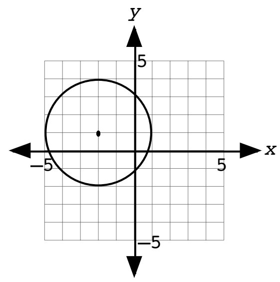
19. $(x - 1)^2 + y^2 = 1$



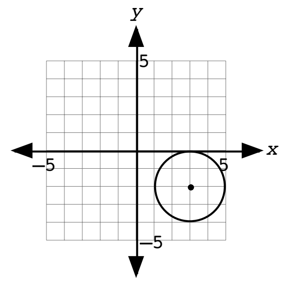
20. $x^2 + (y + 1)^2 = 1$



21. $(x + 2)^2 + (y - 1)^2 = 9$



22. $(x - 3)^2 + (y + 2)^2 = 4$



23. (A) -5 (B) 9 (C) $\sqrt{18}$ **24.** (A) 11 (B) -9 (C) $\sqrt{98}$ **25.** $x = -12, 4$

26. No solution **27.** $y = 4$ **28.** $y = -3, 7$

29. The set of all points that are two units from the point (0, 2).
$x^2 + (y - 2)^2 = 4$

30. The set of all points that are 1 unit from the point (-1, 0). $(x + 1)^2 + y^2 = 1$

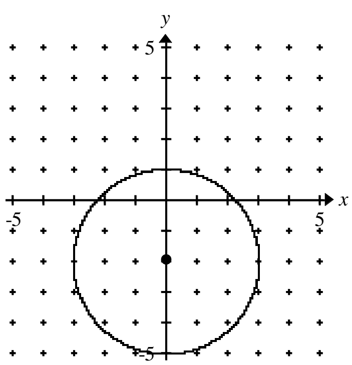
31. The set of all points that are 4 units from the point (1, 1).
$(x - 1)^2 + (y - 1)^2 = 16$

32. The set of all points that are 3 units from the point (2, -1).
$(x - 2)^2 + (y + 1)^2 = 9$

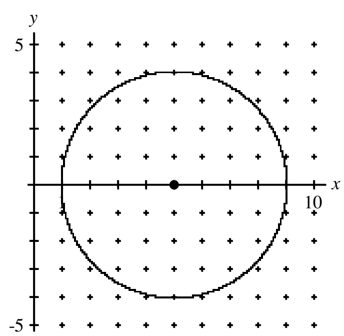
33. (2.65, 1.75) **34.** (-0.65, 2.05) **35.** (-35, -14) **36.** (-7, -3)

37. (-18, -16) **38.** (1, -7)

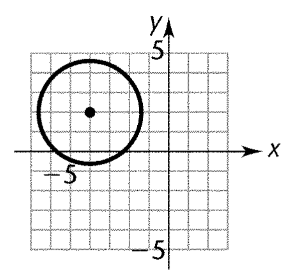
39. Center: (0, -2);
radius: 3



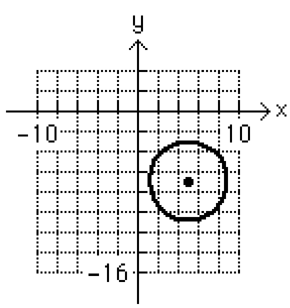
40. Center: (5, 0);
radius: 4



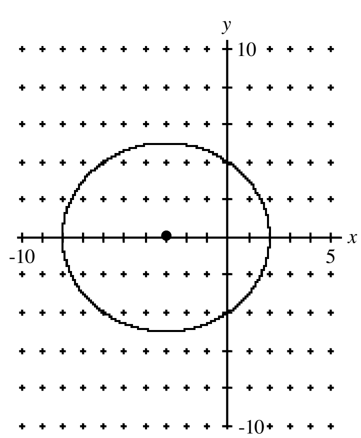
41. Center: (-4, 2);
radius: $\sqrt{7}$



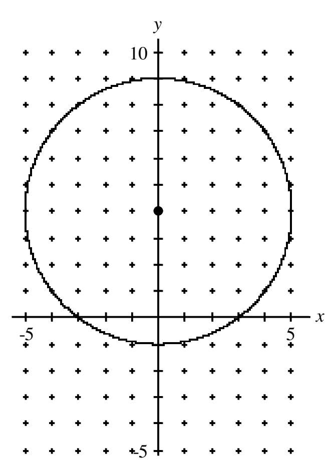
42. $(h, k) = (5,-7)$;
$r = \sqrt{15}$



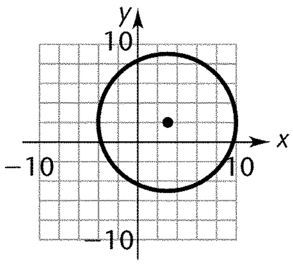
43. Center: (-3, 0);
radius: 5



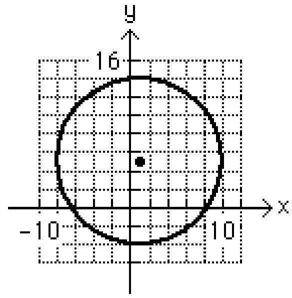
44. Center: (0, 4);
radius: 5



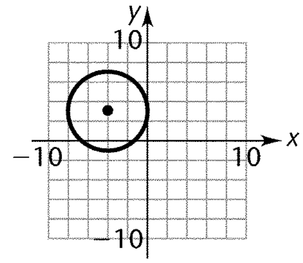
45. Center: (3, 2); radius: 7



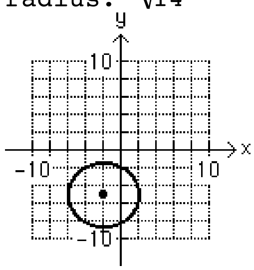
46. Center: (1, 5); radius: 9



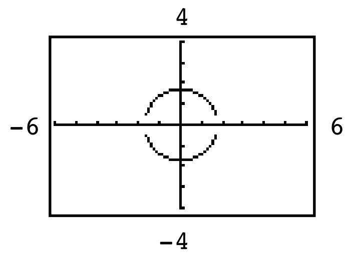
47. Center: (−4, 3); radius: $\sqrt{17}$



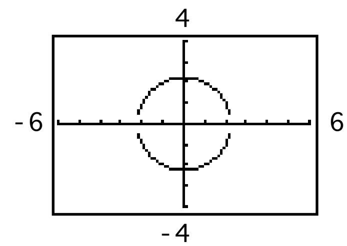
48. Center: (−2, −5); radius: $\sqrt{14}$



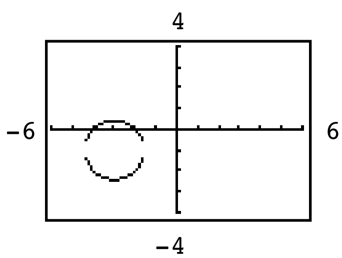
49. $y = \pm\sqrt{3 - x^2}$



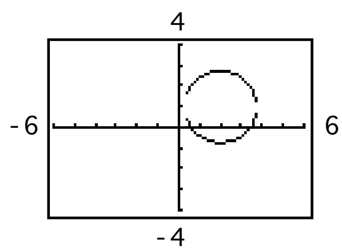
50. $y = \pm\sqrt{5 - x^2}$



51. $y = -1 \pm \sqrt{2 - (x + 3)^2}$



52. $y = 1 \pm \sqrt{3 - (x - 2)^2}$



53. $\sqrt{32.5}$

54. $\sqrt{10}$

55. 18.11

56. 18.62

58. (A) $\left(\frac{a + c}{2}, \frac{b}{2}\right)$ (B) $\left(\frac{a + c}{2}, \frac{b}{2}\right)$

59. $x^2 + (y - 1)^2 = 16$

60. $(x + 3)^2 + y^2 = 25$

61. $(x - 4)^2 + (y - 2)^2 = 34$

62. $(x - 2)^2 + (y + 1)^2 = 34$

63. $x^2 + y^2 = 90$

64. $x^2 + y^2 = 40$

65. $(x - 2)^2 + (y - 2)^2 = 50$

66. $(x + 5)^2 + (y - 4)^2 = 98$

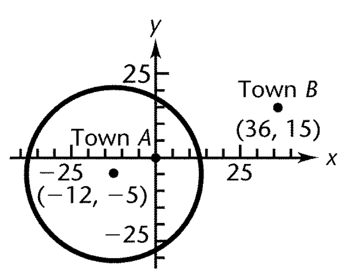
67. (A) $A = (0, 0)$, $B = (0, 13.5)$, $C = (0, 27)$, $D = (60, 27)$, $E = (78, 27)$, $F = (78, 13.5)$, $G = (78, 0)$
(B) 62 feet, 79 feet

68. 66 ft, 83 ft

69. 2.5 feet

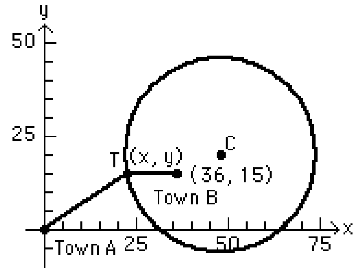
70. $r = 6.5$ mm

71. (A) $(x + 12)^2 + (y + 5)^2 = 26^2$; center: (−12, −5); radius: 26



(B) 13.5 miles

72. (A) Center = (48, 20); radius = 26



(B) x = 64.6 mi or x = 31.4 mi

Section 2-3

1. Rise = 3, run = 5, slope = $\frac{3}{5}$ = 0.6, $3x - 5y = -4$

2. Rise = 6, run = 4, slope = $\frac{6}{4}$ = 1.5, $3x - 2y = -3$

3. Rise = 2, run = 8, slope = $\frac{2}{8}$ = 0.25, $x - 4y = -8$

4. Rise = 6, run = 5, slope = $\frac{6}{5}$ = 1.2, $6x - 5y = 15$

5. Rise = -3, run = 5, slope = $-\frac{3}{5}$ = -0.6, $3x + 5y = -2$

6. Rise = -5, run = 4, slope = $-\frac{5}{4}$ = -1.25, $5x + 4y = -13$

7. x intercept = -2; y intercept = 2; slope = 1; equation: $y = x + 2$

8. x intercept = 1; y intercept = 1; slope = -1; equation: $y = -x + 1$

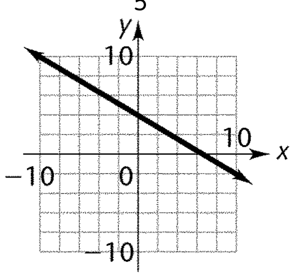
9. x intercept = -2; y intercept = -4; slope = -2; equation: $y = -2x - 4$

10. x intercept: none; y intercept: 3; slope: 0; equation: $y = 3$

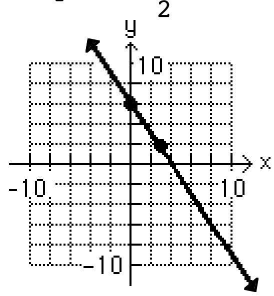
11. x intercept = 3; y intercept = -1; slope = $\frac{1}{3}$; equation: $y = \frac{1}{3}x - 1$

12. x intercept: -2; y intercept: none; slope: undefined; equation: $x = -2$

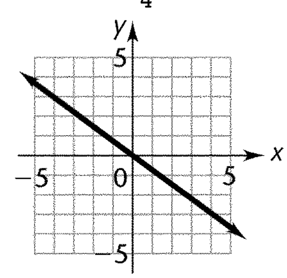
13. slope = $-\frac{3}{5}$



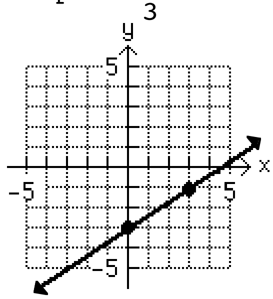
14. slope = $-\frac{3}{2}$



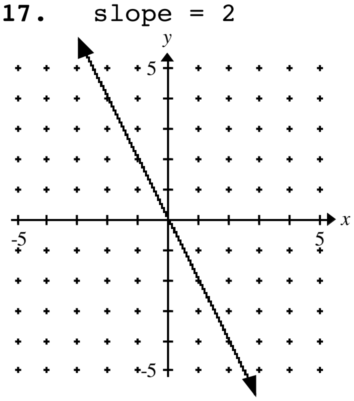
15. slope = $-\frac{1}{4}$



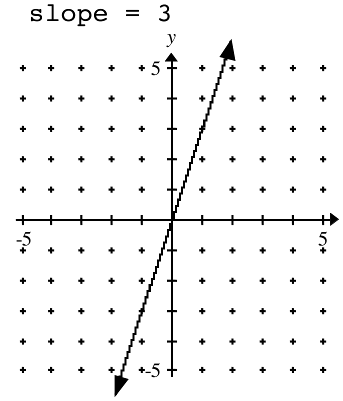
16. slope = $\frac{2}{3}$



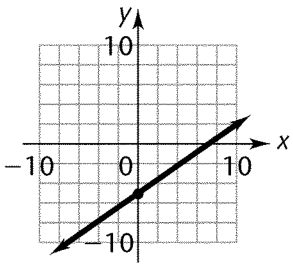
17. slope = 2



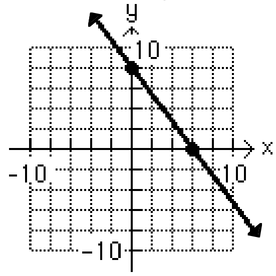
18. slope = 3



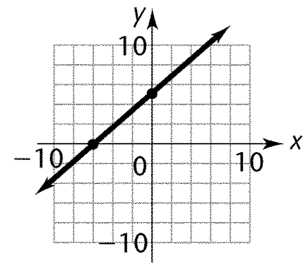
19. slope = $\frac{2}{3}$



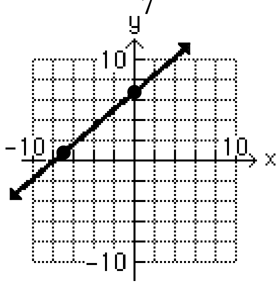
20. slope = $-\frac{4}{3}$



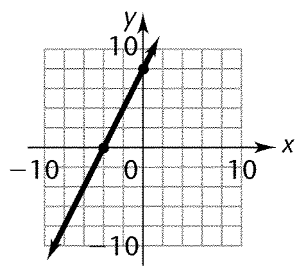
21. slope = $\frac{4}{5}$



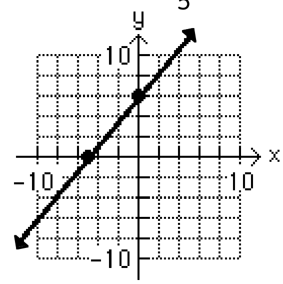
22. slope = $\frac{6}{7}$



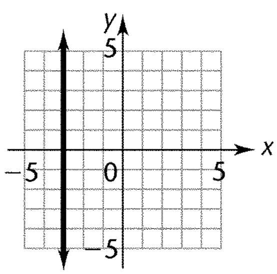
23. slope = 2



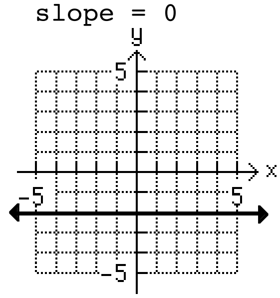
24. slope = $\frac{6}{5}$



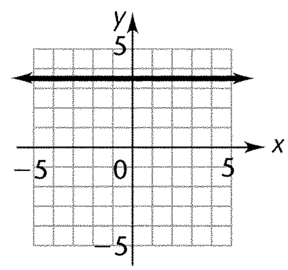
25. slope not defined



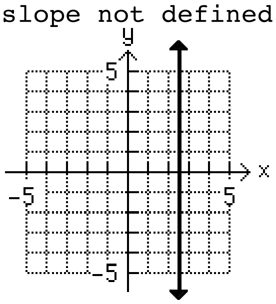
26. slope = 0



27. slope = 0



28. slope not defined



29. $x - y = 0$

30. $x + y = 7$

31. $2x + 3y = -12$

32. $5x - 3y = -18$

33. $y = -2$

34. $x = 3$

35. $y = -2x + 3$

36. $y = 3x - 12$

37. $y = \frac{3}{2}x + \frac{23}{2}$

38. $y = -\frac{4}{5}x - \frac{7}{5}$

39. $y = -\frac{1}{2}x - 4$

40. $y = \frac{4}{3}x - \frac{5}{3}$

41. $y = -4x + 13$

42. $y = -\frac{1}{2}x + \frac{7}{2}$

43. $x = -3$

44. $y = 5$

45. $y = 2$

46. $x = 5$

47. $y = -3x + 4$

48. $y = 2x - 4$

49. $y = -\frac{2}{5}x + 2$ **50.** $y = \frac{1}{2}x$ **51.** $y = -2x + 8$

52. $y = -\frac{1}{3}x + 3$ **53.** $y = -\frac{4}{3}x + \frac{8}{3}$ **54.** $y = \frac{3}{4}x - \frac{5}{2}$

55. $y = 4$ **56.** $y = -2$ **57.** $x = 4$

58. $x = -3$ **59.** $y = \frac{3}{4}x + 3$ **60.** $y = -\frac{5}{4}x - 5$

61. $3x - y = -13$ **62.** $2x + y = -8$ **63.** $3x - y = 9$

64. $3x + 2y = -14$ **65.** $x = 2$ **66.** $y = 3$

67. $x = 3$ **68.** $y = -3$ **69.** $3x - 2y = 15$

70. $3x + 4y = 29$ **71.** $3x - y = 4$ **72.** $5x - 4y = -26$

73. The y intercept of each line is 2.

74. Lines will be parallel. **75.** slope $AB = -\frac{3}{4}$ = slope DC

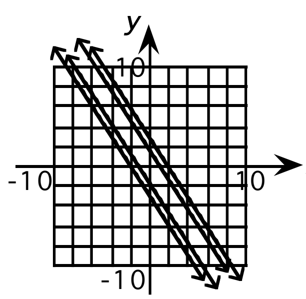
76. $m_{DA} = \frac{4}{3}$; $m_{CB} = \frac{4}{3}$ which shows $DA \parallel CB$.

77. (slope AB)(slope BC) $= \left(-\frac{3}{4}\right)\left(\frac{4}{3}\right) = -1$

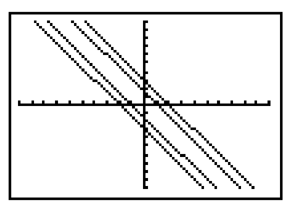
78. $m_{AD} = \frac{4}{3}$; $m_{DC} = -\frac{3}{4}$ from which $m_{AD} \cdot m_{DC} = -1$ which shows $AD \perp DC$.

79. $6x + 9y = -9$ **80.** $8x - 6y = 13$

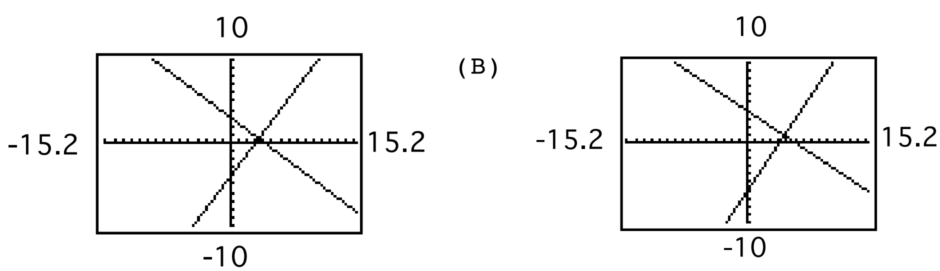
81. (A)



(B)

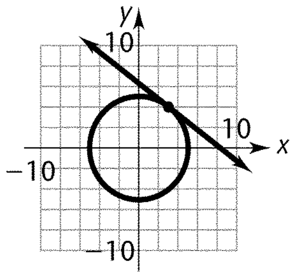


82. (A) (B)

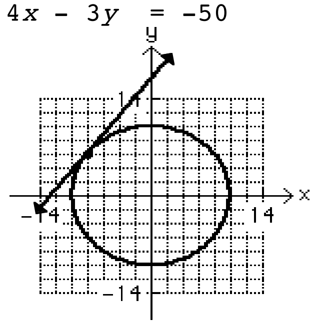


85. $\frac{x}{3} + \frac{y}{5} = 1$, $5x + 3y = 15$ **86.** $7x - 2y = -14$

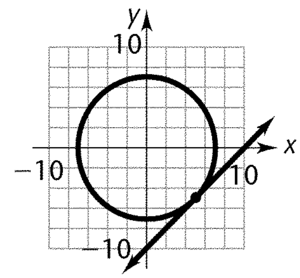
87. $3x + 4y = 25$



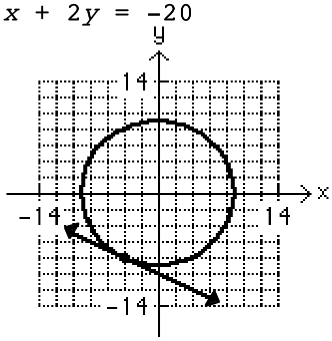
88. $4x - 3y = -50$



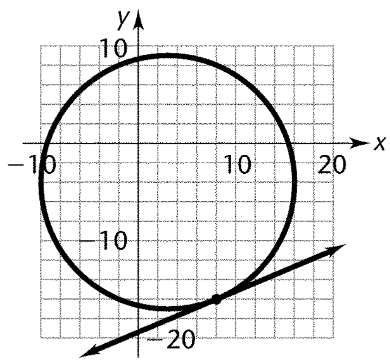
89. $x - y = 10$



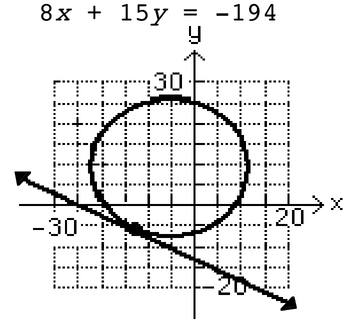
90. $x + 2y = -20$



91. $232 = 5x - 12y$



92. $8x + 15y = -194$



93. (A)

x	0	5,000	10,000	15,000	20,000	25,000	30,000
$212 - 0.0018x = B$	212	203	194	185	176	167	158

(B) The boiling point drops 9°F for each 5,000 foot increase in altitude.

94. (A)

x	0	1	2	3	4	5
A	25	16	7	−2	−11	−20

(B) For every kilometer increase in altitude the air temperature decreases 9°C.

95. The rental charges are \$25 per day plus \$0.25 per mile driven.

96. The installation charges are \$15 for travel to the installation site plus \$0.70 per minute spent at the site.

97. $C(x) = 124 + 0.12x$, 1,050 doughnuts

98. $C(x) = 1{,}200 + 45x$, 80 tables

99. (A) $s = 0.4w$ (B) 8 in (C) 9 lbs

100. (A) $s = 0.4w$ (B) 8 in (C) 9 lbs

101. (A) $F = \frac{9}{5}C + 32$ (B) 68°F, 30°C

102. $R = 1.8K + 0.6$

103. (A) $h = 1.13t + 12.8$ (B) 32.9 hrs

104. (A) $h = 0.25t + 4.75$ (B) 61 hrs

105. (A) $L = 0.281t + 49.2$ (B) 82.9 yrs

106. $N = -0.0251t + 4.76$ (B) $N = 2.25$ people per household

107. 625 ft

108. 1,249 ft

Section 2-4

1. (A) $C = 2{,}147 + 75x$
(B) The rate of change of cost with respect to production is $75.
(C) Increasing production by 1 unit increases cost by $75.

2. (A) $C = 1{,}874 + 46x$
(B) The rate of change of cost with respect to production is $46.
(C) Increasing production by 1 unit increases cost by $46.

3. (A) The rate of change of height with respect to DBH is 4.06 feet per inch.
(B) Increasing DBH by one 1 inch increases height by 4.06 feet.
(C) 73 feet
(D) 17 inches

4. (A) The rate of change of height with respect to DBH is 2.27 feet per inch.
(B) Increasing DBH by one 1 inch increases height by 2.27 feet.
(C) 60 feet
(D) 29 inches

5. (A) Robinson: The rate of change of weight with respect to height is 3.7 pounds per inch.
Miller: The rate of change of weight with respect to height is 3 pounds per inch.
(B) Robinson: 130.2 pounds; Miller: 135 pounds
(C) Robinson: 5'9"; Miller: 5'8″

6. (A) Robinson: The rate of change of weight with respect to height is 4.2 pounds per inch.
Miller: The rate of change of weight with respect to height is 3.1 pounds per inch.
(B) Robinson: 157 pounds; Miller: 155 pounds
(C) Robinson: 5'11"; Miller: 6'

7. $s = 0.75t + 717$, speed increases 0.75 mph for each 1°F change in temperature.

8. $s = 0.6t + 331$, speed increases 0.6 mph for each 1°C change in temperature.

9. $m = -0.31t + 23.3$, 2017

10. $f = -0.18t + 22.9$, 2017

11. (A) $V = 142{,}000 - 7{,}500t$
(B) The tractor’s value is decreasing at the rate of $7,500 per year.
(C) $97,000

12. (A) $V = 154{,}900 - 6{,}800t$
(B) The boat's value is decreasing at the rate of $6,800 per year.
(C) The eighth year

13. (A) $R = 1.4C - 7$
(B) The slope is 1.4, This is the rate of change of retail cost with respect to wholesale cost.
(C) $137

14. (A) $R = 1.5C + 3$
(B) The slope is 1.5, This is the rate of change of retail cost with respect to wholesale cost.
(C) $158

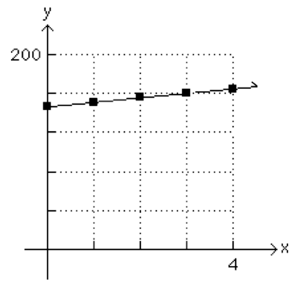
15. (A) $T = -5A + 70$ (B) 14,000 feet

16. (A) $T = 4A + 200$ (B) 226 mph

17. (A) $a = 2{,}880 - 24t$ (B) 24 feet per second

18. (A) $a = 2{,}880 - 16t$ (B) 16 feet per second

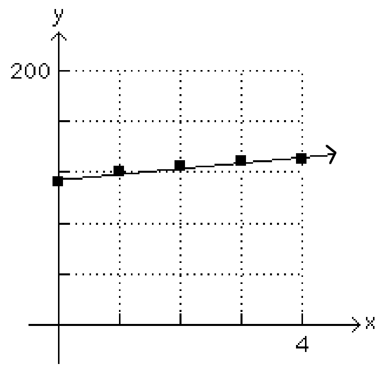
19. (A)



(B) \$192,000

(C) Average price is increasing at the rate of \$4,600 per year.

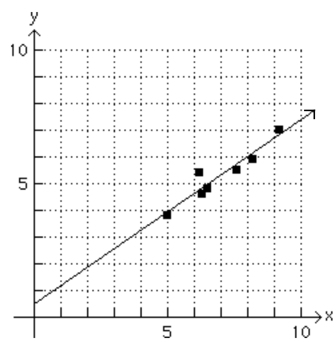
20. (A)



(B) \$158,000

(C) Average price is increasing at the rate of \$4,400 per year.

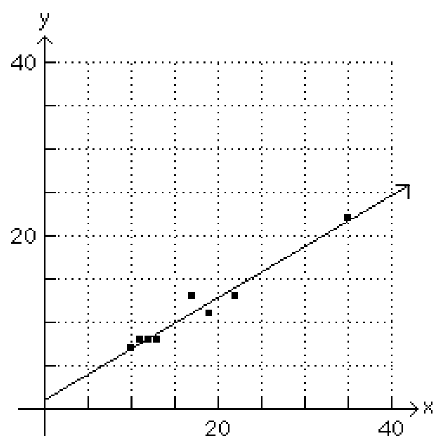
21. (A)



(B) 3.6 million

(C) 4.4 million

22. (A)



(B) 22.0 million

(C) 12.4 million

23. Men: $y = -0.1034x + 51.51$; women: $y = -0.1453x + 58.29$; the models intersect in 2130

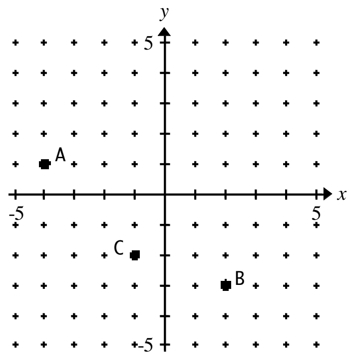
24. Men: $y = -0.28859x + 125.36$; women: $y = -0.39877x + 139.48$; the models intersect in 2096

25. Supply: $y = 0.200x + 0.872$; demand: $y = -0.146x + 3.50$; equilibrium price: \$2.39

26. Supply: $y = 1.53x + 2.85$; demand: $y = -2.21x + 10.7$; equilibrium price: \$6.06

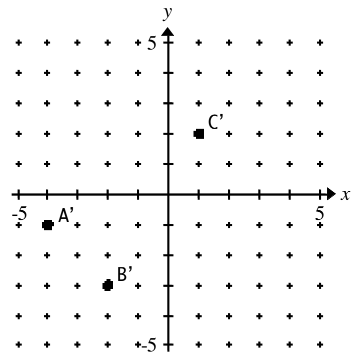
CHAPTER 2 REVIEW

1.



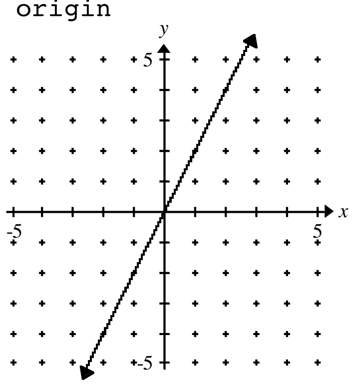
(*2, 1*)

2.

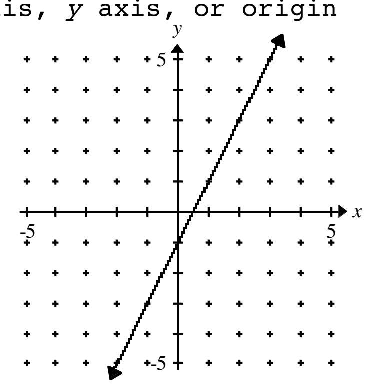


(*2-1*)

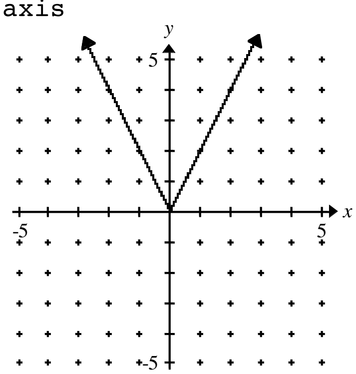
3. (A) Symmetric with respect to the origin



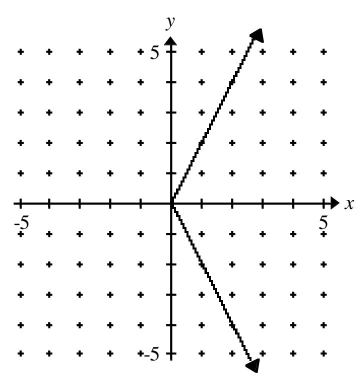
(B) No symmetry with respect to the x axis, y axis, or origin



(C) Symmetric with respect to the y axis



(D) Symmetric with respect to the x axis



(*2-1*)

4. (A) −2 (B) −2, 1 (C) −3, 2 (*2-1*)

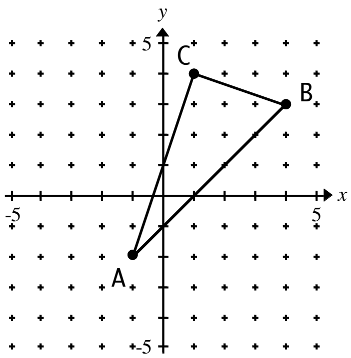
5. (A) $\sqrt{45}$ (B) $m = -\frac{1}{2}$ (C) $m_1 = 2$ (*2-2, 2-3*)

6. (A) $x^2 + y^2 = 7$ (B) $(x - 3)^2 + (y + 2)^2 = 7$ (*2-2*)

7. Center: $C(h, k) = C(-3, 2)$; Radius: $r = \sqrt{5}$ (*2-1*)

8. (A) -10 (B) 11 (C) Both are 10 (*2-2*)

9. (A)



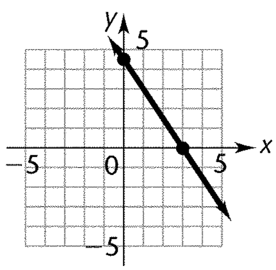
(B) $d(A, B) = \sqrt{50}$, $d(B, C) = \sqrt{10}$, $d(A, C) = 2\sqrt{10}$
Perimeter = 16.56

(C) Since $(\sqrt{50})^2 = (\sqrt{10})^2 + (2\sqrt{10})^2$, that is, $50 = 10 + 40$, the triangle is a right triangle.

(D) Midpoint of AB = (1.5, 0.5),
Midpoint of BC = (2.5, 3.5),
Midpoint of AC = (0, 1) *(2-3)*

10. Rise = −2, run = 5, slope = $-\frac{2}{5} = -0.4$, $2x + 5y = 7$ *(2-3)*

11. slope: $-\frac{3}{2}$



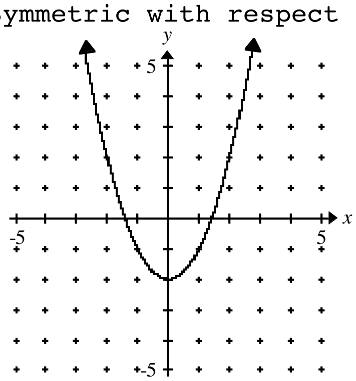
(2-3)

12. $2x + 3y = 12$ *(2-3)*

13. $y = -\frac{2}{3}x + 2$ *(2-3)*

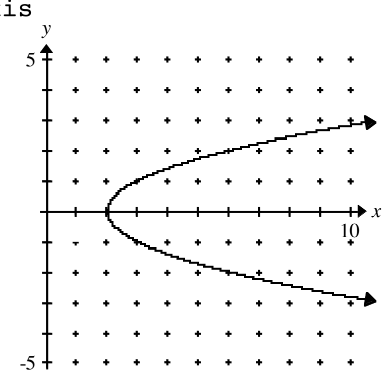
14. *vertical:* $x = -3$, slope not defined; *horizontal:* $y = 4$, slope = 0 *(2-3)*

15. Symmetric with respect to the y axis



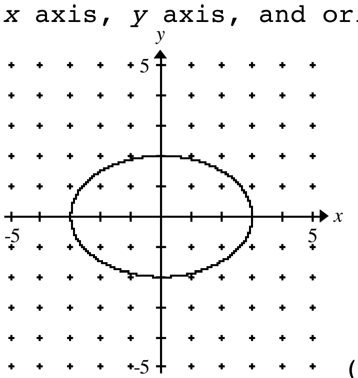
(2-1)

16. Symmetric with respect to the x axis



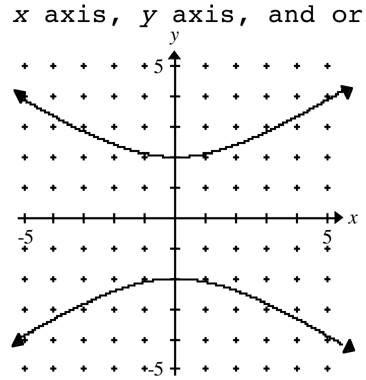
(2-1)

17. Symmetric with respect to the x axis, y axis, and origin



(2-1)

18. Symmetric with respect to the x axis, y axis, and origin



(2-1)

19. The graph is a circle with center (2, 0) and radius 4.
$(x - 2)^2 + y^2 = 4$ (*2-2*)

20. (A) $3x + 2y = -6$ (B) $\sqrt{52}$ (*2-2, 2-3*)

21. (A) $y = -2x - 3$ (B) $y = \frac{1}{2}x + 2$ (*2-3*)

22. $(x - 3)^2 + y^2 = 32$ (*2-2*)

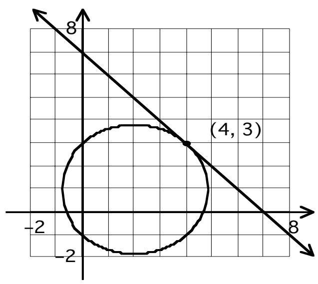
23. Center: $C(h, k) = C(-2, 3)$; Radius $r = \sqrt{16} = 4$ (*2-2*)

24. $x - y = 3$; This is the equation of a line. (*2-2, 2-3*)

25. Perpendicular (*2-3*)

26. Center: (2, 1); radius: $2\sqrt{2}$ (*2-2*)

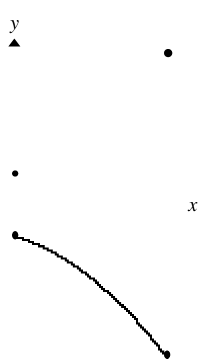
27. $y = -x + 7$



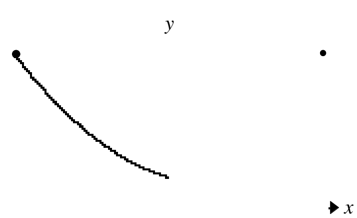
(*2-2, 2-3*)

28. $(x - 4)^2 + (y + 3)^2 = 34$ (*2-2*)

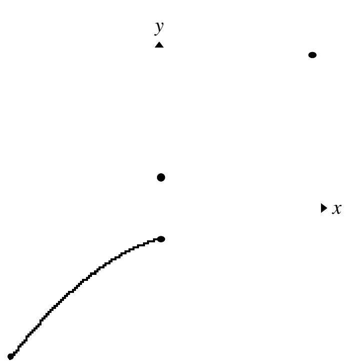
29. (A)



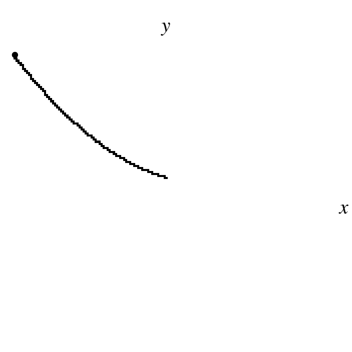
(B)



(C)



(D)



(*2-1*)

30. $b = 5h$ (*2-3*)

31. $h = 0.25b$ (*2-3*)

32. (A) $V = -1{,}250t + 12{,}000$ (B) $V = \$5{,}750$ (2-4)

33. 7,420 CD's (2-4)

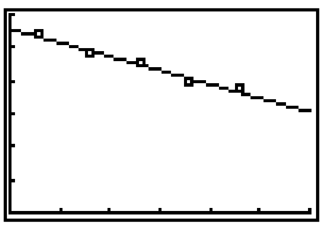
34. (A) The rate of change of height with respect to DBH is 2.9.
(B) Increasing DBH by 1 inch increases height by 2.9 feet.
(C) 39 feet to the nearest foot
(D) $d = 5$ inches (2-4)

35. (A) The rate of change of body surface area with respect to weight is 0.3433.
(B) Increasing the weight by 100 grams increases the BSA by 0.3433(100) = 34.33 square centimeters.
(C) 6,470.5 cm^2 (2-4)

36. $r = 5$ feet (2-2)

37. (A) $H = 0.7(220 - A)$
(B) $H = 140$ beats per minute
(C) $A = 40$ years old (2-4)

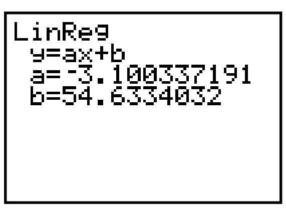
38. (A)



x	12	32	52	72	92
Data	5.41	4.81	4.51	4.00	3.75
$f(x)$	5.32	4.90	4.48	4.06	3.64

(B) 3.302 seconds (2-4)

39. (A)



(B) 45.33% (2-4)

Graphs

ANALYTIC geometry is the study of the relationship between geometric forms, such as circles and lines, and algebraic forms, such as equations and inequalities. The key to this relationship is the Cartesian coordinate system, named after the French mathematician and philosopher René Descartes (1596–1650) who was the first to combine the study of algebra and geometry into a single discipline. We begin by discussing some of the basic tools used to graph equations. Next we cover the relationship between equations of straight lines and their graphs. We conclude by considering models that involve linear equations.

CHAPTER 2

SECTIONS

2-1 Cartesian Coordinate Systems

- Reviewing Cartesian Coordinate Systems
- Graphing: Point by Point
- Using Symmetry as an Aid in Graphing

In Chapter 1 we discussed algebraic methods for solving equations. In this section we show how to find a geometric representation (*graph*) of an equation. Examining the graph of an equation often results in additional insight into the nature of the equation's solutions.

Reviewing Cartesian Coordinate Systems

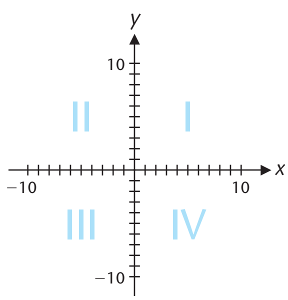


Figure 1
Cartesian coordinate system.

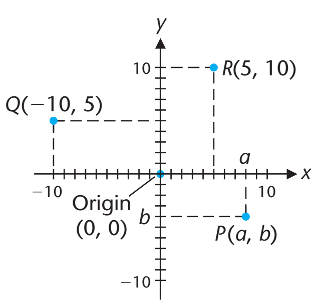


Figure 2
Coordinates in a plane.

Just as a real number line is formed by establishing a one-to-one correspondence between the points on a line and the elements in the set of real numbers, we can form a *real plane* by establishing a one-to-one correspondence between the points in a plane and elements in the set of all ordered pairs of real numbers. This can be done by means of a Cartesian coordinate system.

Recall that to form a **Cartesian** or **rectangular coordinate system,** we select two real number lines, one horizontal and one vertical, and let them cross through their origins, as indicated in Figure 1. Up and to the right are the usual choices for the positive directions. These two number lines are called the **horizontal axis** and the **vertical axis,** or, together, the **coordinate axes.** The horizontal axis is usually referred to as the ***x* axis** and the vertical axis as the ***y* axis,** and each is labeled accordingly. Other labels may be used in certain situations. The coordinate axes divide the plane into four parts called **quadrants,** which are numbered counterclockwise from I to IV (see Fig. 1).

Now we want to assign *coordinates* to each point in the plane. Given an arbitrary point P in the plane, pass horizontal and vertical lines through the point (Fig. 2). The vertical line will intersect the horizontal axis at a point with coordinate a, and the horizontal line will intersect the vertical axis at a point with coordinate b. These two numbers written as the ordered pair* (a, b) form the **coordinates** of the point P. The first coordinate a is called the **abscissa** of P; the second coordinate b is called the **ordinate** of P. The abscissa of Q in Figure 2 is -10, and the ordinate of Q is 5. The coordinates of a point can also be referenced in terms of the axis labels. The ***x* coordinate** of R in Figure 2 is 5, and the ***y* coordinate** of R is 10. The point with coordinates $(0, 0)$ is called the **origin.**

*An **ordered pair** of real numbers is a pair of numbers in which the order is specified. We now use (a, b) as the coordinates of a point in a plane. In Chapter 1 we used (a, b) to represent an interval on a real number line. These concepts are not the same. You must always interpret the symbol (a, b) in terms of the context in which it is used.

The procedure we have just described assigns to each point P in the plane a unique pair of real numbers (a, b). Conversely, if we are given an ordered pair of real numbers (a, b), then, reversing this procedure, we can determine a unique point P in the plane. Thus,

There is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers.

This correspondence is often referred to as the *fundamental theorem of analytic geometry.* Because of this correspondence, we regularly speak of the point (a, b) when we are referring to the point with coordinates (a, b). We also write $P = (a, b)$ to identify the coordinates of the point P. Thus, in Figure 2, referring to Q as the point $(-10, 5)$ and writing $R = (5, 10)$ are both acceptable statements.

› Graphing: Point by Point

Given any set of ordered pairs of real numbers S, the **graph** of S is the set of points in the plane corresponding to the ordered pairs in S. The fundamental theorem of analytic geometry enables us to look at algebraic forms (a set of ordered pairs) geometrically and to look at geometric forms (a graph) algebraically. We begin by considering an algebraic form, an equation in two variables:

$$y = x^2 - 4 \tag{1}$$

A **solution** to equation (1) is an ordered pair of real numbers (a, b) such that

$$b = a^2 - 4$$

The **solution set** of equation (1) is the set of all its solutions. More formally,

$$\text{Solution set of equation (1): } \{(x, y) \mid y = x^2 - 4\}$$

To find a solution to equation (1) we simply replace one of the variables with a number and solve for the other variable. For example, if $x = 2$, then $y = 2^2 - 4 = 0$, and the ordered pair $(2, 0)$ is a solution. Similarly, if $y = 5$, then $5 = x^2 - 4$, $x^2 = 9$, $x = \pm 3$, and the ordered pairs $(3, 5)$ and $(-3, 5)$ are solutions.

Sometimes replacing one variable with a number and solving for the other variable will introduce imaginary numbers. For example, if $y = -5$ in equation (1), then

$$\begin{aligned} -5 &= x^2 - 4 \\ x^2 &= -1 \\ x &= \pm\sqrt{-1} = \pm i \end{aligned}$$

Thus, $(-i, -5)$ and $(i, -5)$ are solutions to $y = x^2 - 4$. However, the coordinates of a point in a rectangular coordinate system must be real numbers.

For that reason, when graphing an equation, we only consider those values of the variables that produce real solutions to the equation.

The **graph of an equation in two variables** is the graph of its solution set. In equation (1), we find that its solution set will have infinitely many elements and its graph will extend off any paper we might choose, no matter how large. Thus, *to sketch the graph of an equation,* we include enough points from its solution set so that the total graph is apparent. This process is called **point-by-point plotting.**

EXAMPLE 1 Graphing an Equation Using Point-by-Point Plotting

Sketch a graph of $y = x^2 - 4$.

SOLUTION

We make up a table of solutions—ordered pairs of real numbers that satisfy the given equation.

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12

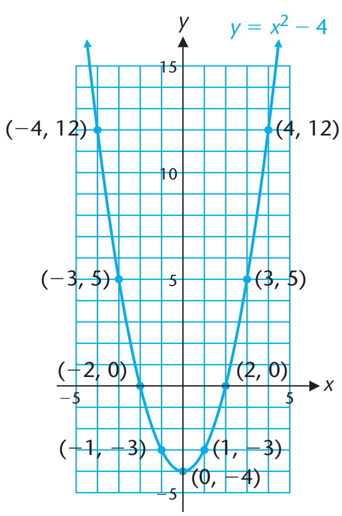


Figure 3

After plotting these solutions, if there are any portions of the graph that are unclear, we plot additional points until the shape of the graph is apparent. Then we join all these plotted points with a smooth curve, as shown in Figure 3. Arrowheads are used to indicate that the graph continues beyond the portion shown here with no significant changes in shape.

The resulting figure is called a *parabola.* Notice that if we fold the paper along the y axis, the right side will match the left side. We say that the graph is *symmetric with respect to the y axis* and call the y axis the *axis of the parabola.* More will be said about parabolas later in the text.

MATCHED PROBLEM 1

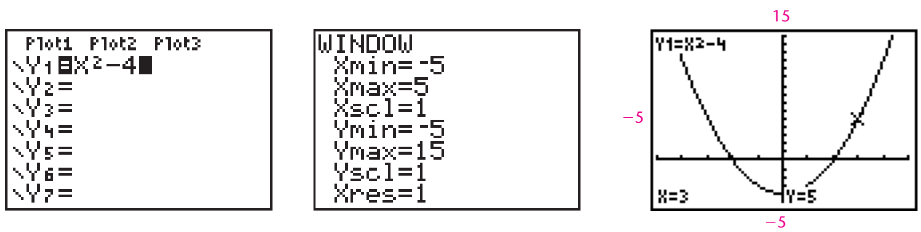
Sketch a graph of $y^2 = x$.

This book contains a number of activities that use an electronic device capable of displaying graphs to emphasize the connection between graphic, numeric, and algebraic viewpoints. The two most common devices are handheld graphing calculators and computers with appropriate software. All these activities are clearly marked and easily omitted if no such device is available.

Technology Connections

To graph the equation in Example 1 on a graphing calculator, we first enter the equation in the calculator's **equation editor*** [Fig. 4(a)]. Using Figure 3 as a guide, we next enter values for the **window variables** [Fig. 4(b)], and then we **graph** the equation [Fig. 4(c)]. The values of the window variables, shown in red in Figure 4(c), are not displayed on the calculator screen. We add them to give you additional insight into the graph.

Compare the graphs in Figure 3 and Figure 4(c). They are similar in shape, but they are not identical. The discrepancy is due to the difference in the axes scales. In Figure 3, one unit on the x axis is equal to one unit on the y axes. In Figure 4(c), one unit on the x axis is equal to about three units on the y axis. We will have more to say about axes scales later in this section.



Enter the equation.
(a)

Enter the window variables.
(b)

Graph the equation.
(c)

› Figure 4

*See the Technology Index for a list of graphing calculator terms used in this book.

››› EXPLORE-DISCUSS 1

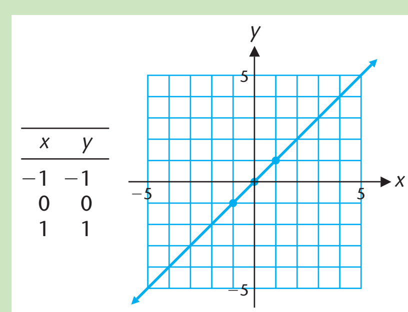
To graph the equation $y = -x^3 + 2x$, we use point-by-point plotting to obtain the graph in Figure 5.

(A) Do you think this is the correct graph of the equation? If so, why? If not, why?

(B) Add points on the graph for $x = -2, -0.5, 0.5$, and 2.

(C) Now, what do you think the graph looks like? Sketch your version of the graph, adding more points as necessary.

(D) Write a short statement explaining any conclusions you might draw from parts A, B, and C.



› Figure 5

Graphs illustrate the relationship between two quantities, one represented by the x coordinates and the other by the y coordinates. If no equation for the graph is available, we can find specific examples of this relationship by estimating coordinates of points on the graph. Example 2 illustrates this process.

EXAMPLE 2 Ozone Levels

The ozone level is measured in parts per billion (ppb). The ozone level during a 12-hour period in a suburb of Milwaukee, Wisconsin, on a particular summer day is given in Figure 6 (*source:* Wisconsin Department of Natural Resources). Use this graph to estimate the following ozone levels to the nearest integer and times to the nearest quarter hour.

(A) The ozone level at 6 P.M.

(B) The highest ozone level and the time when it occurs.

(C) The time(s) when the ozone level is 90 ppb.

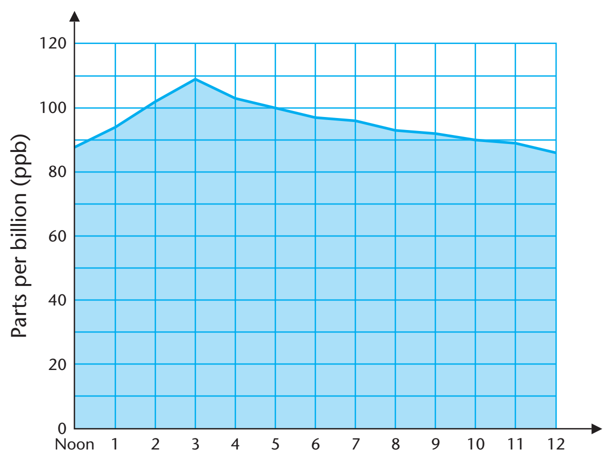


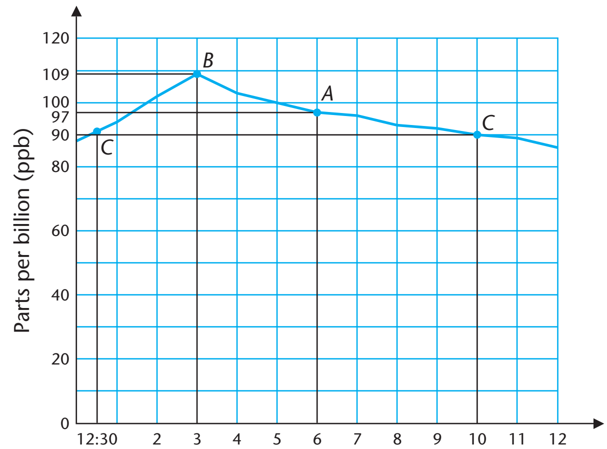
Figure 6
Ozone level.

SOLUTIONS

(A) The ordinate of the point on the graph with abscissa 6 is 97 ppb (see Fig. 7).

(B) The highest ozone level is 109 ppb at 3 P.M.

(C) The ozone level is 90 ppb at about 12:30 P.M. and again at 10 P.M.



› **Figure 7**

MATCHED PROBLEM 2

Use Figure 6 to estimate the following ozone levels to the nearest integer and times to the nearest quarter hour.

(A) The ozone level at 7 P.M.

(B) The time(s) when the ozone level is 100 ppb.

An important aspect of this course, and later in calculus, is the development of tools that can be used to analyze graphs. A particularly useful tool is *symmetry,* which we now discuss.

› Using Symmetry as an Aid in Graphing

We noticed that the graph of $y = x^2 - 4$ in Example 1 is *symmetric with respect to the y axis;* that is, the two parts of the graph coincide if the paper is folded along the y axis. Similarly, we say that a graph is symmetric with respect to the x axis if the parts above and below the x axis coincide when the paper is folded along the x axis. To make the intuitive idea of folding a graph along a line more concrete, we introduce two related concepts—reflection and symmetry.

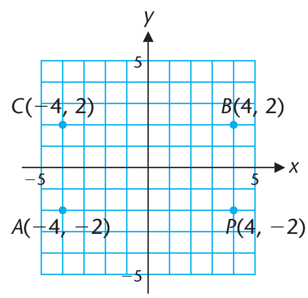
› DEFINITION 1 Reflection

1. The **reflection through the *y* axis** of the point (a, b) is the point $(-a, b)$.
2. The **reflection through the *x* axis** of the point (a, b) is the point $(a, -b)$.
3. The **reflection through the origin** of the point (a, b) is the point $(-a, -b)$.
4. To **reflect a graph** just reflect each point on the graph.

EXAMPLE 3 Reflections

In a Cartesian coordinate system, plot the point $P(4, -2)$ along with its reflection through (A) the y axis, (B) the x axis, (C) and the origin.

SOLUTION



MATCHED PROBLEM 3

In a Cartesian coordinate system, plot the point $P(-3, 5)$ along with its reflection through (A) the x axis, (B) the y axis, and (C) the origin.

DEFINITION 2 Symmetry

A graph is symmetric with respect to

1. **The x axis** if $(a, -b)$ is on the graph whenever (a, b) is on the graph—reflecting the graph through the x axis does not change the graph.
2. **The y axis** if $(-a, b)$ is on the graph whenever (a, b) is on the graph—reflecting the graph through the y axis does not change the graph.
3. **The origin** if $(-a, -b)$ is on the graph whenever (a, b) is on the graph—reflecting the graph through the origin does not change the graph.

Figure 8 illustrates these three types of symmetry.

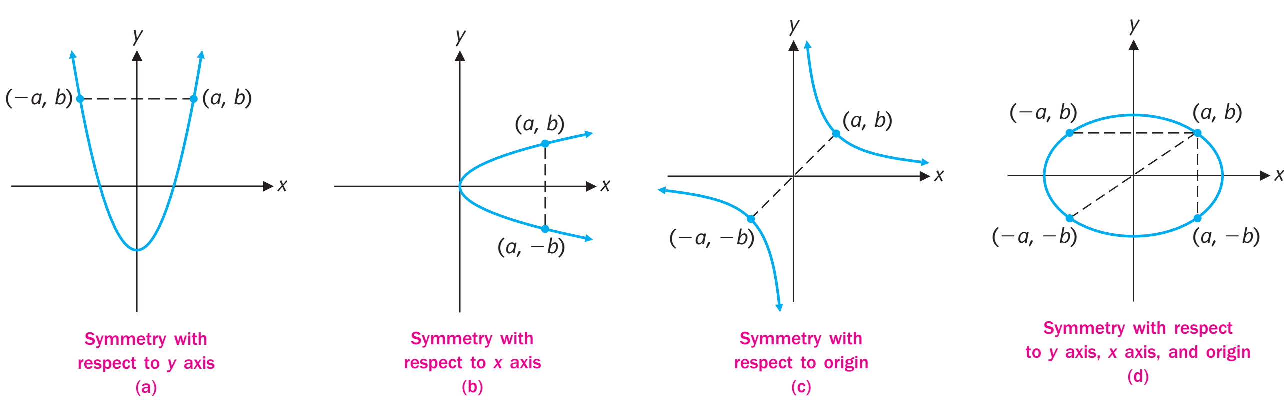


Figure 8 Symmetry.

››› EXPLORE-DISCUSS 2

If a graph possesses two of the three types of symmetry in Definition 1, must it also possess the third? Explain.

Given an equation, if we can determine the symmetry properties of its graph ahead of time, we can save a lot of time and energy in sketching the graph. For example, we know that the graph of $y = x^2 - 4$ in Example 1 is symmetric with respect to the y axis, so we can carefully sketch only the right side of the graph; then reflect the result through the y axis to obtain the whole sketch—the point-by-point plotting is cut in half!

The tests for symmetry are given in Theorem 1. These tests are easily applied and are very helpful aids to graphing. Recall, two equations are equivalent if they have the same solution set.

› THEOREM 1 Tests for Symmetry

Symmetry with respect to the:	Equation is equivalent when:
y axis	x is replaced with $-x$
x axis	y is replaced with $-y$
Origin	x and y are replaced with $-x$ and $-y$

EXAMPLE 4 Using Symmetry as an Aid to Graphing

Test the equation $y = x^3$ for symmetry and sketch its graph.

SOLUTION

Test y Axis
Replace x with $-x$:

$$y = (-x)^3$$
$$y = -x^3$$

Test x Axis
Replace y with $-y$:

$$-y = x^3$$
$$y = -x^3$$

Test Origin
Replace x with $-x$ and y with $-y$:

$$-y = (-x)^3$$
$$-y = -x^3$$
$$y = x^3$$

The only test that produces an equivalent equation is replacing x with $-x$ and y with $-y$. Thus, the only symmetry property for the graph of $y = x^3$ is symmetry with respect to the origin.

GRAPH Note that positive values of x produce positive values for y and negative values of x produce negative values for y. Thus, the graph occurs in the first and third quadrants. First we make a careful sketch in the first quadrant [Fig. 9(a)]. It is easier to perform a reflection through the origin if you first reflect through one axis [Fig. 9(b)] and then through the other axis [Fig. 9(c)]. (See Explore-Discuss 2.)

x	0	1	2
y	0	1	8

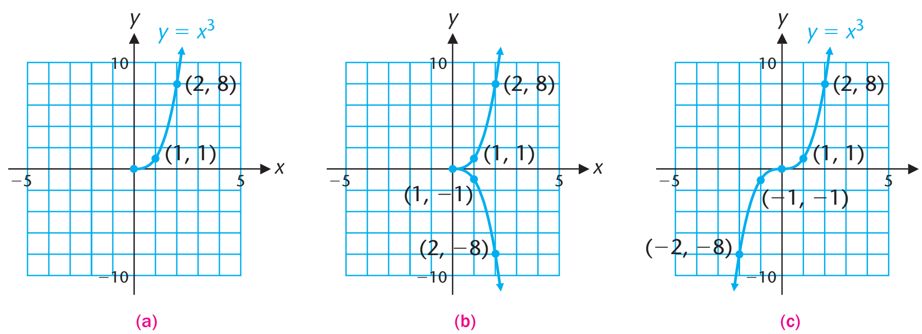


Figure 9

MATCHED PROBLEM 4

Test the equation $y = x$ for symmetry and sketch its graph.

EXAMPLE 5 Using Symmetry as an Aid to Graphing

Test the equation $y = |x|$ for symmetry and sketch its graph.

SOLUTION

Test y Axis
Replace x with $-x$:

$$y = |-x|$$
$$y = |x|$$

Test x Axis
Replace y with $-y$:

$$-y = |x|$$
$$y = -|x|$$

Test Origin
Replace x with $-x$ and y with $-y$:

$$-y = |-x|$$
$$-y = |x|$$
$$y = -|x|$$

Thus, the only symmetry property for the graph of $y = |x|$ is symmetry with respect to the y axis.

GRAPH Since $|x|$ is never negative, this graph occurs in the first and second quadrants. We make a careful sketch in the first quadrant; then reflect this graph through the y axis to obtain the complete sketch shown in Figure 10.

x	0	2	4
y	0	2	4

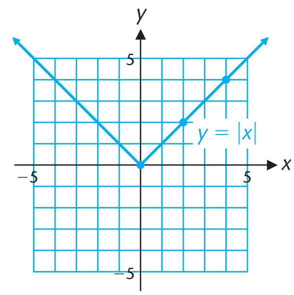


Figure 10

MATCHED PROBLEM 5

Test the equation $y = -|x|$ for symmetry and sketch its graph.

EXAMPLE 6 Using Symmetry as an Aid to Graphing

Test the equation $y^2 - x^2 = 4$ for symmetry and sketch its graph.

SOLUTION

Since $(-x)^2 = x^2$ and $(-y)^2 = y^2$, the equation $y^2 - x^2 = 4$ will be unchanged if x is replaced with $-x$ or if y is replaced with $-y$. Thus, the graph is symmetric with respect to the y axis, the x axis, and the origin. We need to make a careful sketch in only the first quadrant, reflect this graph through the y axis, and then reflect everything through the x axis. To find quadrant I solutions, we solve the equation for either y in terms of x or x in terms of y. We choose to solve for y.

$$y^2 - x^2 = 4$$
$$y^2 = x^2 + 4$$
$$y = \pm\sqrt{x^2 + 4}$$

To obtain the quadrant I portion of the graph, we sketch $y = \sqrt{x^2 + 4}$ for $x = 0, 1, 2, \ldots$. The final graph is shown in Figure 11.

x	0	1	2	3	4
y	2	$\sqrt{5} \approx 2.2$	$\sqrt{8} \approx 2.8$	$\sqrt{13} \approx 3.6$	$\sqrt{20} \approx 4.5$

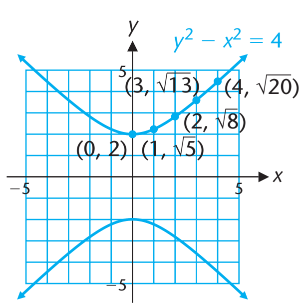


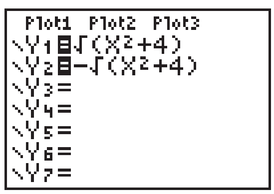
Figure 11

MATCHED PROBLEM 6

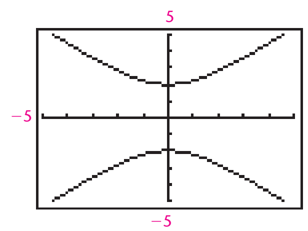
Test the equation $2y^2 - x^2 = 2$ for symmetry and sketch its graph.

Technology Connections

To graph $y^2 - x^2 = 4$ on a graphing calculator, we enter both $\sqrt{x^2 + 4}$ and $-\sqrt{x^2 + 4}$ in the equation editor [Fig. 12(a)] and graph.



(a)

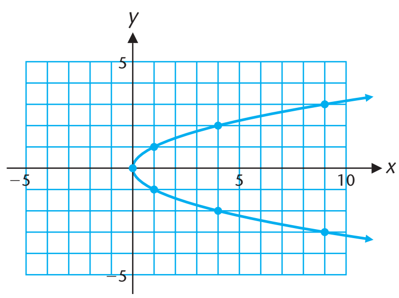


(b)

Figure 12

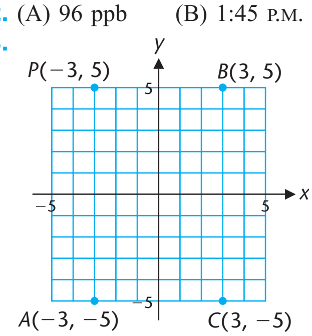
ANSWERS TO MATCHED PROBLEMS

1.

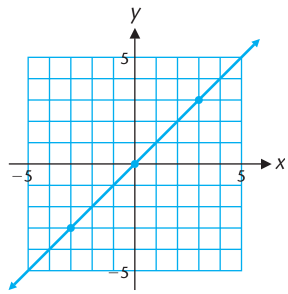


2. (A) 96 ppb (B) 1:45 P.M. and 5 P.M.

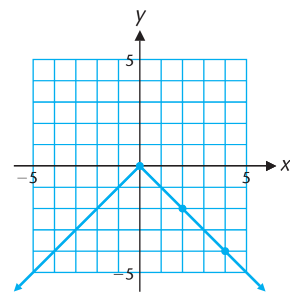
3.



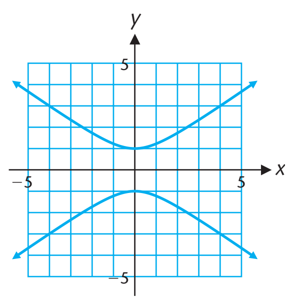
4. Symmetric with respect to the origin



5. Symmetric with respect to the y axis



6. Symmetric with respect to the x axis, the y axis, and the origin



2-1 Exercises

In Problems 1–10, give a verbal description of the indicated subset of the plane in terms of quadrants and axes.

1. $\{(x, y) \mid x = 0\}$

2. $\{(x, y) \mid x > 0, y > 0\}$

3. $\{(x, y) \mid x < 0, y < 0\}$

4. $\{(x, y) \mid y = 0\}$

5. $\{(x, y) \mid x > 0, y < 0\}$

6. $\{(x, y) \mid y < 0, x \neq 0\}$

7. $\{(x, y) \mid x > 0, y \neq 0\}$

8. $\{(x, y) \mid x < 0, y > 0\}$

9. $\{(x, y) \mid xy < 0\}$

10. $\{(x, y) \mid xy > 0\}$

[*Hint:* In Problems 9 and 10, consider two cases.]

In Problems 11–14, plot the given points in a rectangular coordinate system.

11. $(5, 0), (3, -2), (-4, 2), (4, 4)$

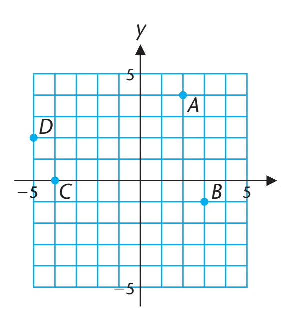
12. $(0, 4), (-3, 2), (5, -1), (-2, -4)$

13. $(0, -2), (-1, -3), (4, -5), (-2, 1)$

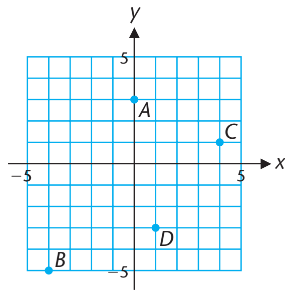
14. $(-2, 0), (3, 2), (1, -4), (-3, 5)$

In Problems 15–18, find the coordinates of points A, B, C, and D and the coordinates of the indicated reflections.

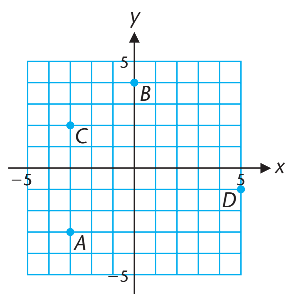
15. Reflect A, B, C, and D through the y axis.



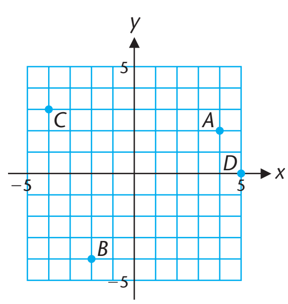
16. Reflect A, B, C, and D through the x axis.



17. Reflect A, B, C, and D through the origin.



18. Reflect A, B, C, and D through the x axis and then through the y axis.



Test each equation in Problems 19–26 for symmetry with respect to the x axis, y axis, and the origin. Sketch the graph of the equation.

19. $y = 2x - 4$ **20.** $y = \frac{1}{2}x + 1$

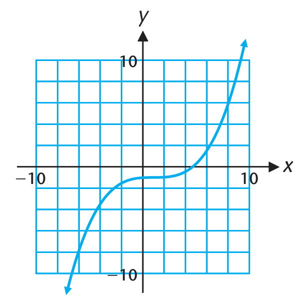
21. $y = \frac{1}{2}x$ **22.** $y = 2x$

23. $|y| = x$ **24.** $|y| = -x$

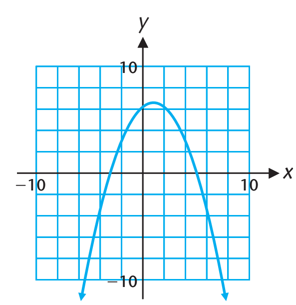
25. $|x| = |y|$ **26.** $y = -x$

In Problems 27–30, use the graph to estimate to the nearest integer the missing coordinates of the indicated points. (Be sure you find all possible answers.)

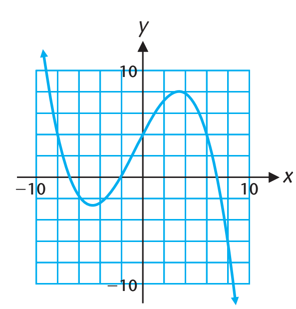
27. (A) (8, ?) (B) (−5, ?) (C) (0, ?) (D) (?, 6) (E) (?, −5) (F) (?, 0)



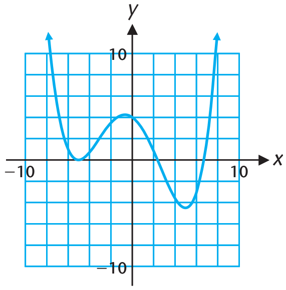
28. (A) (3, ?) (B) (−5, ?) (C) (0, ?) (D) (?, 3) (E) (?, −4) (F) (?, 0)



29. (A) (1, ?) (B) (−8, ?) (C) (0, ?) (D) (?, −6) (E) (?, 4) (F) (?, 0)



30. (A) (6, ?) (B) $(-6, ?)$ (C) (0, ?)
(D) $(?, -2)$ (E) (?, 1) (F) (?, 0)



31. (A) Sketch a graph based on the solutions in the following table.

x	-2	0	2
y	-2	0	2

(B) Sketch a graph based on the solutions in the following table.

x	-1	0	1
y	2	0	-2

(C) Complete the following table for $y = x^3 - 3x$ and sketch a graph of the equation.

x	-2	-1	0	1	2
y					

(D) Write a short statement explaining any conclusions you might draw from parts (A), (B), and (C).

32. (A) Sketch a graph based on the solutions in the following table.

x	-1	1	3
y	4	2	0

(B) Sketch a graph based on the solutions in the following table.

x	0	1	2
y	0	2	4

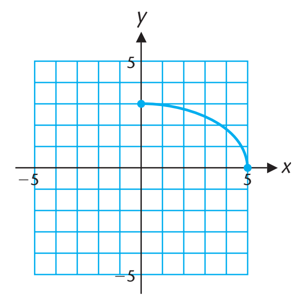
(C) Complete the following table for $y = 3x^2 - x^3$ and sketch a graph of the equation.

x	-1	0	1	2	3
y					

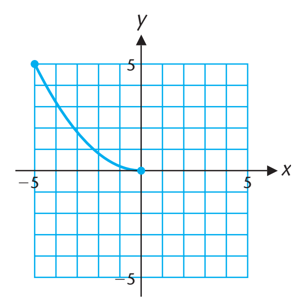
(D) Write a short statement explaining any conclusions you might draw from parts (A), (B), and (C).

The figures in Problems 33 and 34 show a portion of a graph. Extend the given graph to one that exhibits the indicated type of symmetry.

33. (A) x axis only
(B) y axis only
(C) origin only
(D) x axis, y axis, and origin



34. (A) x axis only
(B) y axis only
(C) origin only
(D) x axis, y axis, and origin



Test each equation in Problems 35–40 for symmetry with respect to the x axis, the y axis, and the origin. Do not sketch the graph.

35. $x^2y^2 = 4$ **36.** $x^3y^3 = 8$

37. $x^2 + xy^2 = 3$ **38.** $x + 4xy + 2y^2 = 7$

39. $x^2 + 4y + y^2 = 9$ **40.** $x^2 - 2xy + 3y^2 = 6$

Test each equation in Problems 41–52 for symmetry with respect to the x axis, the y axis, and the origin. Sketch the graph of the equation.

41. $y^2 = x + 2$ **42.** $y^2 = x - 2$

43. $y = x^2 + 1$ **44.** $y + 2 = x^2$

45. $4y^2 - x^2 = 1$ **46.** $4x^2 - y^2 = 1$

47. $y^3 = x$

48. $y = x^4$

49. $y = 0.6x^2 - 4.5$

50. $x = 0.8y^2 - 3.5$

51. $y = x^{2/3}$

52. $y^{2/3} = x$

53. (A) Graph the triangle with vertices $A = (1, 1)$, $B = (7, 2)$, and $C = (4, 6)$.
(B) Now graph the triangle with vertices $A' = (1, -1)$, $B' = (7, -2)$, and $C' = (4, -6)$ in the same coordinate system.
(C) How are these two triangles related? How would you describe the effect of changing the sign of the y coordinate of all the points on a graph?

54. (A) Graph the triangle with vertices $A = (1, 1)$, $B = (7, 2)$, and $C = (4, 6)$.
(B) Now graph the triangle with vertices $A' = (-1, 1)$, $B' = (-7, 2)$, and $C' = (-4, 6)$ in the same coordinate system.
(C) How are these two triangles related? How would you describe the effect of changing the sign of the x coordinate of all the points on a graph?

55. (A) Graph the triangle with vertices $A = (1, 1)$, $B = (7, 2)$, and $C = (4, 6)$.
(B) Now graph the triangle with vertices $A' = (-1, -1)$, $B' = (-7, -2)$, and $C' = (-4, -6)$ in the same coordinate system.
(C) How are these two triangles related? How would you describe the effect of changing the signs of the x and y coordinates of all the points on a graph?

56. (A) Graph the triangle with vertices $A = (1, 2)$, $B = (1, 4)$, and $C = (3, 4)$.
(B) Now graph $y = x$ and the triangle obtained by reversing the coordinates for each vertex of the original triangle: $A' = (2, 1)$, $B' = (4, 1)$, $B' = (4, 3)$.
(C) How are these two triangles related? How would you describe the effect of reversing the coordinates of each point on a graph?

In Problems 57–60, solve for y, producing two equations, and then graph both of these equations in the same viewing window.

57. $2x + y^2 = 3$

58. $x^3 + y^2 = 8$

59. $x^2 - (y + 1)^2 = 4$

60. $(y - 2)^2 - x^2 = 9$

Test each equation in Problems 61–70 for symmetry with respect to the x axis, the y axis, and the origin. Sketch the graph of the equation.

61. $y^3 = |x|$

62. $|y| = x^3$

63. $xy = 1$

64. $xy = -1$

65. $y = 6x - x^2$

66. $y = x^2 - 6x$

67. $y^2 = |x| + 1$

68. $y^2 = 4|x| + 1$

69. $|xy| + 2|y| = 6$

70. $|xy| + |y| = 4$

71. If a graph is symmetric with respect to the x axis and to the origin, must it be symmetric with respect to the y axis? Explain your answer.

72. If a graph is symmetric with respect to the y axis and to the origin, must it be symmetric with respect to the x axis? Explain your answer.

APPLICATIONS

73. **BUSINESS** After extensive surveys, the marketing research department of a producer of popular cassette tapes developed the demand equation

$$n = 10 - p \qquad 5 \le p \le 10$$

where n is the number of units (in thousands) retailers are willing to buy per day at \$$p$ per tape. The company's daily revenue R (in thousands of dollars) is given by

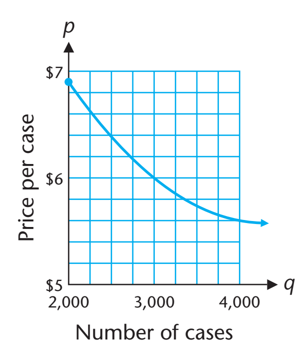
$$R = np = (10 - p)p \qquad 5 \le p \le 10$$

Graph the revenue equation for the indicated values of p.

74. **BUSINESS** Repeat Problem 73 for the demand equation

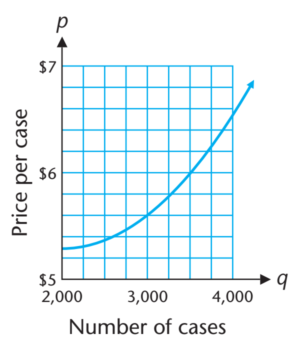
$$n = 8 - p \qquad 4 \le p \le 8$$

75. **PRICE AND DEMAND** The quantity of a product that consumers are willing to buy during some period of time depends on its price. The price p and corresponding weekly demand q for a particular brand of diet soda in a city are shown in the figure. Use this graph to estimate the following demands to the nearest 100 cases.
(A) What is the demand when the price is \$6.00 per case?
(B) Does the demand increase or decrease if the price is increased from \$6.00 to \$6.30 per case? By how much?
(C) Does the demand increase or decrease if the price is decreased from \$6.00 to \$5.70? By how much?
(D) Write a brief description of the relationship between price and demand illustrated by this graph.

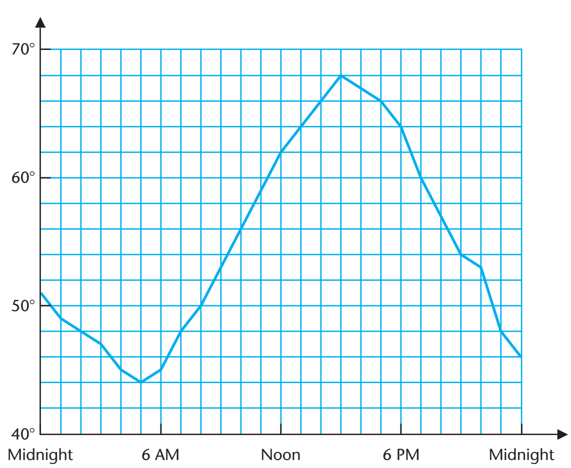


76. **PRICE AND SUPPLY** The quantity of a product that suppliers are willing to sell during some period of time depends on

its price. The price p and corresponding weekly supply q for a particular brand of diet soda in a city are shown in the figure. Use this graph to estimate the following supplies to the nearest 100 cases.
(A) What is the supply when the price is \$5.60 per case?
(B) Does the supply increase or decrease if the price is increased from \$5.60 to \$5.80 per case? By how much?
(C) Does the supply increase or decrease if the price is decreased from \$5.60 to \$5.40 per case? By how much?
(D) Write a brief description of the relationship between price and supply illustrated by this graph.



77. TEMPERATURE The temperature during a spring day in the Midwest is given in the figure. Use this graph to estimate the following temperatures to the nearest degree and times to the nearest hour.
(A) The temperature at 9:00 A.M.
(B) The highest temperature and the time when it occurs.
(C) The time(s) when the temperature is 49°F.

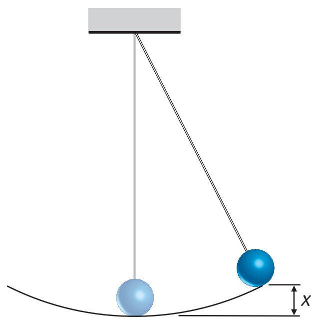


78. TEMPERATURE Use the graph in Problem 77 to estimate the following temperatures to the nearest degree and times to the nearest half hour.
(A) The temperature at 7:00 P.M.
(B) The lowest temperature and the time when it occurs.
(C) The time(s) when the temperature is 52°F.

79. PHYSICS The speed (in meters per second) of a ball swinging at the end of a pendulum is given by

$$v = 0.5\sqrt{2 - x}$$

where x is the vertical displacement (in centimeters) of the ball from its position at rest (see the figure).

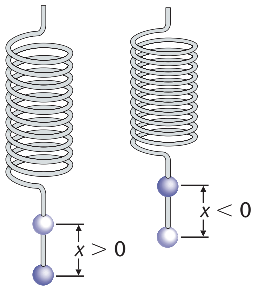


(A) Graph v for $0 \le x \le 2$.
(B) Describe the relationship between this graph and the physical behavior of the ball as it swings back and forth.

80. PHYSICS The speed (in meters per second) of a ball oscillating at the end of a spring is given by

$$v = 4\sqrt{25 - x^2}$$

where x is the vertical displacement (in centimeters) of the ball from its position at rest (positive displacement measured downwards—see the figure).



(A) Graph v for $-5 \le x \le 5$.
(B) Describe the relationship between this graph and the physical behavior of the ball as it oscillates up and down.

2-2 Distance in the Plane

- Distance Between Two Points
- Midpoint of a Line Segment
- Circles

Two basic problems studied in analytic geometry are

1. Given an equation, find its graph.
2. Given a figure (line, circle, parabola, ellipse, etc.) in a coordinate system, find its equation.

The first problem was discussed in Section 2-1. In this section, we introduce some tools that are useful when studying the second problem.

Distance Between Two Points

Given two points P_1 and P_2 in a rectangular coordinate system, we denote the **distance between P_1 and P_2 by $d(P_1, P_2)$.** We begin with an example.

EXAMPLE 1 Distance Between Two Points

Find the distance between the points $P_1 = (1, 2)$ and $P_2 = (4, 6)$.

SOLUTION

Connecting the points P_1, P_2, and $P_3 = (4, 2)$ with straight line segments forms a right triangle (Fig. 1).

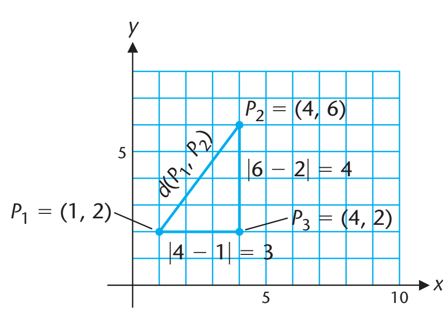


Figure 1

From the figure, we see that lengths of the legs of the triangle are

$$d(P_1, P_3) = |4 - 1| = 3$$

and

$$d(P_3, P_2) = |6 - 2| = 4$$

The length of the hypotenuse is $d(P_1, P_2)$, the distance we are seeking. Applying the Pythagorean theorem (see Appendix B), we have

$$\begin{aligned}[d(P_1, P_2)]^2 &= [d(P_1, P_3)]^2 + [d(P_3, P_2)]^2\\ &= 3^2 + 4^2\\ &= 9 + 16\\ &= 25\end{aligned}$$

Thus,

$$d(P_1, P_2) = \sqrt{25} = 5$$

MATCHED PROBLEM 1

Find the distance between the points $P_1 = (1, 2)$ and $P_2 = (13, 7)$.

The ideas used in Example 1 can be generalized to any two distinct points in the plane. If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points in a rectangular coordinate system (Fig. 2), then

$$\begin{aligned}[d(P_1, P_2)]^2 &= |x_2 - x_1|^2 + |y_2 - y_1|^2\\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \qquad \text{Because } |N|^2 = N^2.\end{aligned}$$

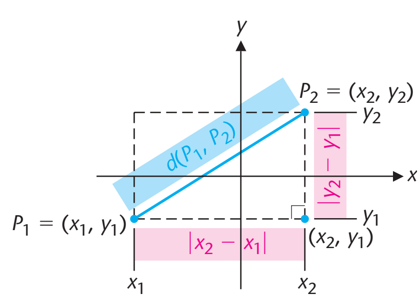


Figure 2 Distance between two points.

Thus,

THEOREM 1 Distance Formula

The distance between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 2

Using the Distance Formula

Find the distance between the points $(-3, 5)$ and $(-2, -8)$.*

SOLUTION

Let $(x_1, y_1) = (-3, 5)$ and $(x_2, y_2) = (-2, -8)$. Then,

$$d = \sqrt{[(-2) - (-3)]^2 + [(-8) - 5]^2}$$
$$= \sqrt{(-2 + 3)^2 + (-8 - 5)^2} = \sqrt{1^2 + (-13)^2} = \sqrt{1 + 169} = \sqrt{170}$$

Notice that if we choose $(x_1, y_1) = (-2, -8)$ and $(x_2, y_2) = (-3, 5)$, then

$$d = \sqrt{[(-3) - (-2)]^2 + [5 - (-8)]^2} = \sqrt{1 + 169} = \sqrt{170}$$

so it doesn't matter which point we designate as P_1 or P_2.

MATCHED PROBLEM 2

Find the distance between the points $(6, -3)$ and $(-7, -5)$.

Midpoint of a Line Segment

EXPLORE-DISCUSS 1

(A) Graph the line segment L joining the points $(1, 2)$ and $(7, 10)$.

(B) Find the average a of the x coordinates of these two points.

(C) Find the average b of the y coordinates of these two points.

(D) Plot the point (a, b). Is it on the line segment L?

(E) Find the distance between $(1, 2)$ and (a, b) and the distance between (a, b) and $(7, 10)$. How would you describe the point (a, b)?

The **midpoint** of the line segment between two points is the point on the line segment that is equidistant from each of the points. A formula for finding the midpoint is given in Theorem 2. The proof is discussed in the exercises.

*We often speak of the point (a, b) when we are referring to the point with coordinates (a, b). This shorthand, though not accurate, causes little trouble, and we will continue the practice.

THEOREM 2 Midpoint Formula

The midpoint of the line segment joining $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The point M is the unique point satisfying

$$d(P_1, M) = d(M, P_2) = \frac{1}{2}d(P_1, P_2)$$

Note that the coordinates of the midpoint are simply the averages of the respective coordinates of the two given points.

EXAMPLE 3 Using the Midpoint Formula

Find the midpoint M of the line segment joining $A = (-3, 2)$ and $B = (4, -5)$. Plot A, B, and M and verify that $d(A, M) = d(M, B) = \frac{1}{2}d(A, B)$.

SOLUTION

We use the midpoint formula with $(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (4, -5)$ to obtain the coordinates of the midpoint M.

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Substitute } x_1 = -3, y_1 = 2, x_2 = 4, \text{ and } y_2 = -5. \\ &= \left(\frac{-3 + 4}{2}, \frac{2 + (-5)}{2}\right) && \text{Simplify.} \\ &= \left(\frac{1}{2}, \frac{-3}{2}\right) \\ &= (0.5, -1.5) \end{aligned}$$

The fraction form of M is probably more convenient for plotting the point. The decimal form is more convenient for computing distances.

We plot the three points (Fig. 3) and compute the distances $d(A, M)$, $D(M, B)$, and $d(A, B)$.

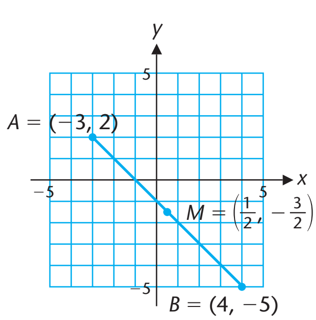


Figure 3

Find the distances $d(A, M)$, $d(M, B)$, and $\frac{1}{2}d(A, B)$:

$$d(A, M) = \sqrt{(-3 - 0.5)^2 + [2 - (-1.5)]^2} = \sqrt{12.25 + 12.25} = \sqrt{24.5}$$
$$d(M, B) = \sqrt{(0.5 - 4)^2 + [-1.5 - (-5)]^2} = \sqrt{12.25 + 12.25} = \sqrt{24.5}$$
$$d(A, B) = \sqrt{(-3 - 4)^2 + [2 - (-5)]^2} = \sqrt{49 + 49} = \sqrt{98}$$
$$\frac{1}{2}d(A, B) = \frac{1}{2}\sqrt{98} = \sqrt{\frac{98}{4}} = \sqrt{24.5} = d(A, M) = d(M, B)$$

This verifies that M is the midpoint of the line segment joining A and B.

MATCHED PROBLEM 3

Find the midpoint M of the line segment joining $A = (4, 1)$ and $B = (-3, -5)$. Plot A, B, and M and verify that $d(A, M) = d(M, B) = \frac{1}{2}d(A, B)$.

EXAMPLE 4 Using the Midpoint Formula

If $M = (1, 1)$ is the midpoint of the line segment joining $A = (-3, -1)$ and $B = (x, y)$, find the coordinates of B.

SOLUTION

From the midpoint formula, we have

$$M = (1, 1) = \left(\frac{-3 + x}{2}, \frac{-1 + y}{2}\right)$$

We equate the corresponding coordinates and solve the resulting equations for x and y:

$$1 = \frac{-3 + x}{2} \qquad 1 = \frac{-1 + y}{2}$$
$$2 = -3 + x \qquad 2 = -1 + y$$
$$\boxed{2 + 3 = -3 + x + 3}^* \qquad \boxed{2 + 1 = -1 + y + 1}$$
$$5 = x \qquad 3 = y$$

Thus, $B = (5, 3)$

MATCHED PROBLEM 4

If $M = (1, -1)$ is the midpoint of the line segment joining $A = (-1, -5)$ and $B = (x, y)$, find the coordinates of B.

*Throughout the book, dashed boxes—called **think boxes**—are used to represent steps that may be performed mentally.

Circles

The distance-between-two-points formula would be helpful if its only use were to find actual distances between points, such as in Example 2. However, its more important use is in finding equations of figures in a rectangular coordinate system. We start with an example.

EXAMPLE 5 Equations and Graphs of Circles

Write an equation for the set of all points that are 5 units from the origin. Graph your equation.

SOLUTION

The distance between a point (x, y) and the origin is

$$d = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

So, an equation for the set of points that are 5 units from the origin is

$$\sqrt{x^2 + y^2} = 5$$

We square both sides of this equation to obtain an equation that does not contain any radicals.

$$x^2 + y^2 = 25$$

Since $(-x)^2 = x^2$ and $(-y)^2 = y^2$, the graph will be symmetric with respect to the x axis, y axis, and origin. We make up a table of solutions, sketch the curve in first quadrant, and use symmetry properties to produce a familiar geometric object—a circle (Fig. 4).

x	y
0	5
3	4
4	3
5	0

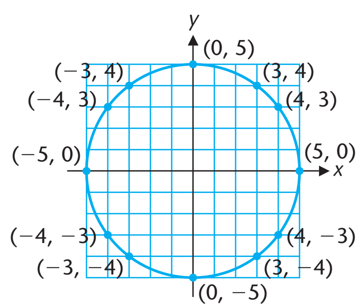


Figure 4

MATCHED PROBLEM 5

Write an equation for the set of all points that are three units from the origin. Graph your equation.

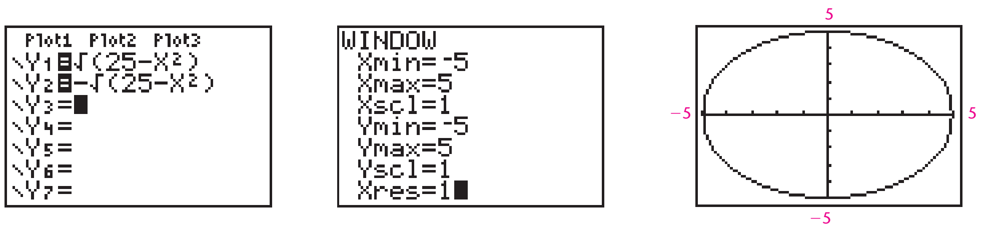
Technology Connections

Refer to Example 5. To graph this circle on a graphing calculator, first we solve $x^2 + y^2 = 25$ for y:

$$x^2 + y^2 = 25$$
$$y^2 = 25 - x^2$$
$$y = \pm\sqrt{25 - x^2}$$

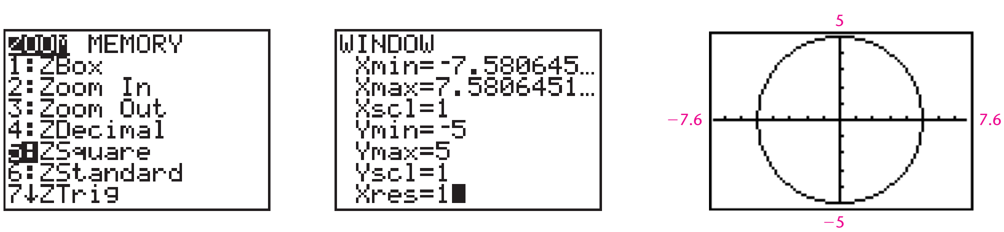
Next we enter $y = \sqrt{25 - x^2}$ and $y = -\sqrt{25 - x^2}$ in the equation editor of a graphing calculator [Fig. 5(a)], enter appropriate window variables [Fig. 5(b)], and graph [Fig. 5(c)].

The graph in Figure 5(a) doesn't look like a circle. (A circle is as wide as it is tall.) This distortion is caused by the difference between axes scales. One unit on the x axis appears to be longer than one unit on the y axis. Most graphing calculators have an option called **ZSquare** under the **zoom menu** [Fig. 6(a)] that automatically adjusts the x axis scale [Fig. 6(b)] to produce a **squared viewing window.** The graph of a circle in a squared viewing window is not distorted [Fig. 6(c)].



(a) (b) (c)

› Figure 5



(a) (b) (c)

› Figure 6

In Example 5, we began with a verbal description of a set of points, produced an algebraic equation that these points must satisfy, constructed a numerical table listing some of these points, and then drew a graphical representation of this set of points. The interplay between verbal, algebraic, numerical, and graphical concepts is one of the central themes of this book.

Now we generalize the ideas introduced in Example 5.

DEFINITION 1 Circle

A **circle** is the set of all points in a plane equidistant from a fixed point. The fixed distance is called the **radius,** and the fixed point is called the **center.**

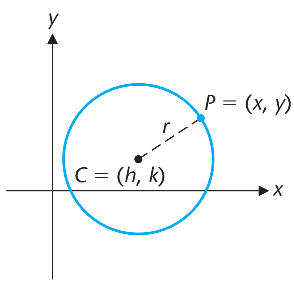


Figure 7
Circle.

Let's find the equation of a circle with radius r ($r > 0$) and center C at (h, k) in a rectangular coordinate system (Fig. 7). The circle consists of all points $P = (x, y)$ satisfying $d(P, C) = r$; that is, all points satisfying

$$\sqrt{(x-h)^2 + (y-k)^2} = r \quad r > 0$$

or, equivalently,

$$(x-h)^2 + (y-k)^2 = r^2 \quad r > 0$$

THEOREM 3 Standard Form of the Equation of a Circle

Circle with radius r and center at (h, k):

$$(x-h)^2 + (y-k)^2 = r^2 \quad r > 0$$

EXAMPLE 6

Equations and Graphs of Circles

Find the equation of a circle with radius 4 and center at $C = (-3, 6)$. Graph the equation.

SOLUTION

$$\begin{aligned} C = (h, k) &= (-3, 6) \text{ and } r = 4 \\ (x-h)^2 + (y-k)^2 &= r^2 \\ [x-(-3)]^2 + (y-6)^2 &= 4^2 \\ (x+3)^2 + (y-6)^2 &= 16 \end{aligned}$$

To graph the equation, plot the center and a few points on the circle (the easiest points to plot are those located 4 units from the center in either in the horizontal or vertical direction), then draw a circle of radius 4 (Fig. 8).

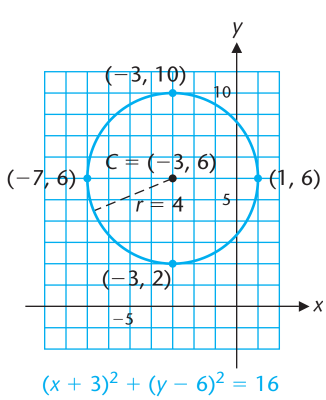


Figure 8

MATCHED PROBLEM 6

Find the equation of a circle with radius 3 and center at $C = (3, -2)$. Graph the equation.

EXAMPLE 7 Finding the Center and Radius of a Circle

Find the center and radius of the circle with equation $x^2 + y^2 + 6x - 4y = 23$.

SOLUTION

We transform the equation into the form $(x - h)^2 + (y - k)^2 = r^2$ by completing the square relative to x and relative to y (see Section 1-5). From this standard form we can determine the center and radius.

$$x^2 + y^2 + 6x - 4y = 23 \quad \text{Group together the terms involving } x \text{ and those involving } y.$$

$$(x^2 + 6x \qquad) + (y^2 - 4y \qquad) = 23 \quad \text{Complete the squares.}$$

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 23 + 9 + 4 \quad \text{Factor each trinomial.}$$

$$(x + 3)^2 + (y - 2)^2 = 36 \quad \text{Write } +3 \text{ as } -(-3) \text{ to identify } h.$$

$$[x - (-3)]^2 + (y - 2)^2 = 6^2$$

Center: $C(h, k) = C(-3, 2)$

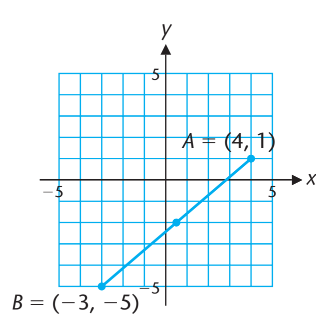
Radius: $r = \sqrt{36} = 6$

MATCHED PROBLEM 7

Find the center and radius of the circle with equation $x^2 + y^2 - 8x + 10y = -25$.

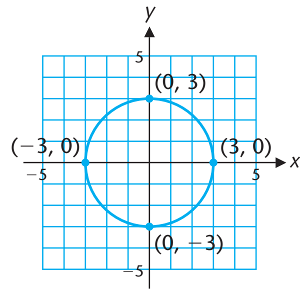
ANSWERS TO MATCHED PROBLEMS

1. 13
2. $\sqrt{173}$
3. $M = (\frac{1}{2}, -2) = (0.5, -2)$; $d(A, B) = \sqrt{85}$; $d(A, M) = \sqrt{21.25} = d(M, B) = \frac{1}{2} d(A, B)$

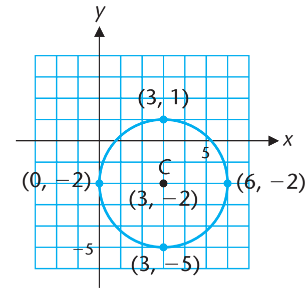


4. $B = (3, 3)$

5. $x^2 + y^2 = 9$



6. $(x - 3)^2 + (y + 2)^2 = 9$



7. $(x - 4)^2 + (y + 5)^2 = 16$; radius: 4, center: $(4, -5)$

2-2 Exercises

In Problems 1–8, find the distance between each pair of points and the midpoint of the line segment joining the points. Leave distance in radical form.

1. (1, 0), (4, 4)

2. (0, 1), (3, 5)

3. (0, −2), (5, 10)

4. (3, 0), (−2, −3)

5. (−6, −4), (3, 4)

6. (−5, 4), (6, −1)

7. (−6, −3), (−2, −1)

8. (−5, −2), (−1, 2)

In Problems 9–16, write the equation of a circle with the indicated center and radius.

9. $C = (0, 0), r = 7$

10. $C = (0, 0), r = 5$

11. $C = (2, 3), r = 6$

12. $C = (5, 6), r = 2$

13. $C = (-4, 1), r = \sqrt{7}$

14. $C = (-5, 6), r = \sqrt{11}$

15. $C = (-3, -4), r = \sqrt{2}$

16. $C = (4, -1), r = \sqrt{5}$

In Problems 17–22, write an equation for the given set of points. Graph your equation.

17. The set of all points that are two units from the origin.

18. The set of all points that are four units from the origin.

19. The set of all points that are one unit from (1, 0).

20. The set of all points that are one unit from (0, −1).

21. The set of all points that are three units from (−2, 1).

22. The set of all points that are two units from (3, −2).

23. Let M be the midpoint of A and B, where

$$A = (a_1, a_2),\ B = (1, 3),\ \text{and } M = (-2, 6).$$

(A) Use the fact that −2 is the average of a_1 and 1 to find a_1.
(B) Use the fact that 6 is the average of a_2 and 3 to find a_2.
(C) Find $d(A, M)$ and $d(M, B)$.

24. Let M be the midpoint of A and B, where

$$A = (-3, 5),\ B = (b_1, b_2),\ \text{and } M = (4, -2).$$

(A) Use the fact that 4 is the average of −3 and b_1 to find b_1.
(B) Use the fact that −2 is the average of 5 and b_2 to find b_2.
(C) Find $d(A, M)$ and $d(M, B)$.

25. Find x such that $(x, 7)$ is 10 units from (−4, 1).

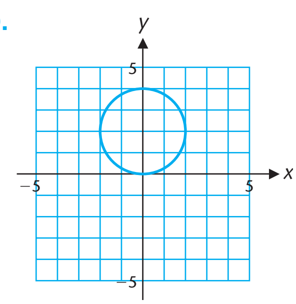
26. Find x such that $(x, 2)$ is 4 units from (3, −3).

27. Find y such that $(2, y)$ is 3 units from (−1, 4).

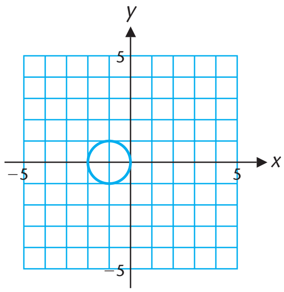
28. Find y such that $(3, y)$ is 13 units from (−9, 2).

In Problems 29–32, write a verbal description of the graph and then write an equation that would produce the graph.

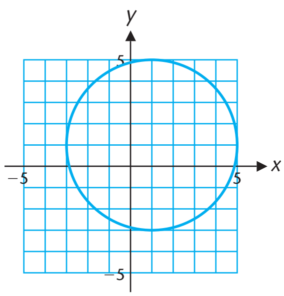
29.



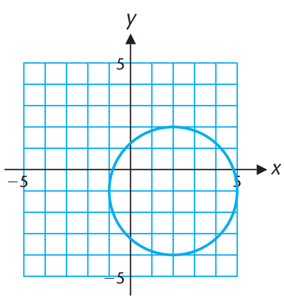
30.



31.



32.



In Problems 33–38, M is the midpoint of A and B. Find the indicated point. Verify that d(A, M) = d(M, B) = $\frac{1}{2}$*d(A, B).*

33. $A = (-4.3, 5.2), B = (9.6, -1.7), M = ?$

34. $A = (2.8, -3.5), B = (-4.1, 7.6), M = ?$

35. $A = (25, 10), M = (-5, -2), B = ?$

36. $M = (2.5, 3.5), B = (12, 10), A = ?$

37. $M = (-8, -6), B = (2, 4), A = ?$

38. $A = (-4, -2), M = (-1.5, -4.5), B = ?$

In Problems 39–48, find the center and radius of the circle with the given equation. Graph the equation.

39. $x^2 + (y + 2)^2 = 9$

40. $(x - 5)^2 + y^2 = 16$

41. $(x + 4)^2 + (y - 2)^2 = 7$

42. $(x - 5)^2 + (y + 7)^2 = 15$

43. $x^2 + 6x + y^2 = 16$

44. $x^2 + y^2 - 8y = 9$

45. $x^2 + y^2 - 6x - 4y = 36$

46. $x^2 + y^2 - 2x - 10y = 55$

47. $3x^2 + 3y^2 + 24x - 18y + 24 = 0$

48. $2x^2 + 2y^2 + 8x + 20y + 30 = 0$

In Problems 49–52, solve for y, producing two equations, and then graph both of these equations in the same viewing window.

49. $x^2 + y^2 = 3$

50. $x^2 + y^2 = 5$

51. $(x + 3)^2 + (y + 1)^2 = 2$

52. $(x - 2)^2 + (y - 1)^2 = 3$

In Problems 53 and 54, show that the given points are the vertices of a right triangle (see the Pythagorean theorem in Appendix B). Find the length of the line segment from the midpoint of the hypotenuse to the opposite vertex.

53. $(-3, 2), (1, -2), (8, 5)$

54. $(-1, 3), (3, 5), (5, 1)$

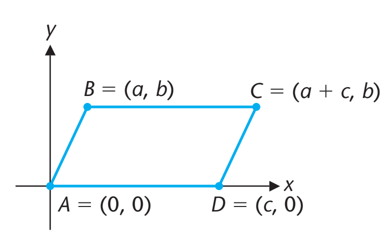
Find the perimeter (to two decimal places) of the triangle with the vertices indicated in Problems 55 and 56.

55. $(-3, 1), (1, -2), (4, 3)$

56. $(-2, 4), (3, 1), (-3, -2)$

57. If $P_1 = (x_1, y_1), P_2 = (x_2, y_2)$ and $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, show that $d(P_1, M) = d(M, P_2) = \frac{1}{2}d(P_1, P_2)$. (This is one step in the proof of Theorem 2).

58. A parallelogram *ABCD* is shown in the figure.
(A) Find the midpoint of the line segment joining *A* and *C*.
(B) Find the midpoint of the line segment joining *B* and *D*.
(C) What can you conclude about the diagonals of the parallelogram?



Find the equation of a circle that has a diameter with the end points given in Problems 59–62.

59. (0, 5), (0, −3)

60. (2, 0), (−8, 0)

61. (7, −3), (1, 7)

62. (−3, 2), (7, −4)

In Problems 63–66, find the equation of circle with the given center whose graph passes through the given point.

63. Center: (0, 0), point on the circle: (3, −9)

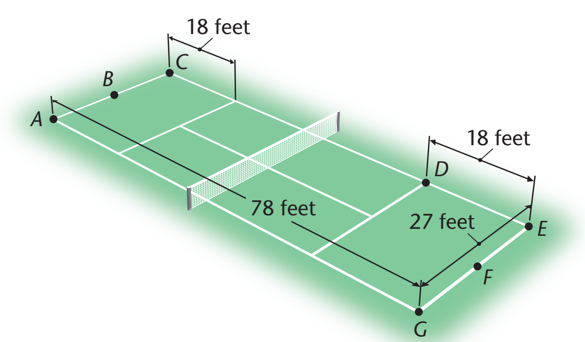
64. Center: (0, 0), point on the circle: (−2, 6)

65. Center: (2, 2), point on the circle: (3, −5)

66. Center: (−5, 4), point on the circle: (2, −3)

APPLICATIONS

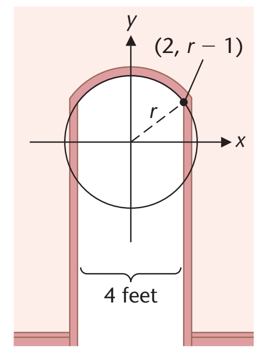
67. SPORTS A singles court for lawn tennis is a rectangle 27 feet wide and 78 feet long (see the figure). Points B and F are the midpoints of the end lines of the court.



(A) Sketch a graph of the court with A at the origin of your coordinate system, C on the positive y axis, and G on the positive x axis. Find the coordinates of points A through G.
(B) Find $d(B, D)$ and $d(F, C)$ to the nearest foot.

68. SPORTS Refer to Problem 67. Find $d(A, D)$ and $d(C, G)$ to the nearest foot.

69. ARCHITECTURE An arched doorway is formed by placing a circular arc on top of a rectangle (see the figure). If the doorway is 4 feet wide and the height of the arc above its ends is 1 foot, what is the radius of the circle containing the arc? [*Hint:* Note that $(2, r - 1)$ must satisfy $x^2 + y^2 = r^2$.]

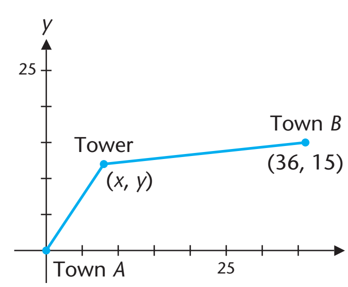


Arched doorway

70. ENGINEERING The cross section of a rivet has a top that is an arc of a circle (see the figure). If the ends of the arc are 12 millimeters apart and the top is 4 millimeters above the ends, what is the radius of the circle containing the arc?

Rivet

★**71. CONSTRUCTION** Town B is located 36 miles east and 15 miles north of town A (see the figure). A local telephone company wants to position a relay tower so that the distance from the tower to town B is twice the distance from the tower to town A.
(A) Show that the tower must lie on a circle, find the center and radius of this circle, and graph.
(B) If the company decides to position the tower on this circle at a point directly east of town A, how far from town A should they place the tower? Compute answer to one decimal place.



★**72. CONSTRUCTION** Repeat Problem 71 if the distance from the tower to town A is twice the distance from the tower to town B.

2-3 Equations of a Line

- Graphing Lines
- Finding the Slope of a Line
- Determining Special Forms of the Equation of a Line
- Connecting Slope to Parallel and Perpendicular Lines

In this section, we consider one of the most basic geometric figures—a line. When we use the term *line* in this book, we mean *straight line*. We will learn how to recognize and graph a line and how to use information concerning a line to find its equation. Adding these important tools to our mathematical toolbox will enable us to use lines as an effective problem-solving tool, as evidenced by the application exercises at the end of this section.

Graphing Lines

With your past experience in graphing equations in two variables, you probably remember that first-degree equations in two variables, such as

$$y = -3x + 5 \qquad 3x - 4y = 9 \qquad y = -\tfrac{2}{3}x$$

have graphs that are lines. This fact is stated in Theorem 1.

THEOREM 1 The Equation of a Line

If A, B, and C are constants, with A and B not both 0, and x and y are variables, then the graph of the equation

$$Ax + By = C \qquad \textbf{Standard Form} \qquad (1)$$

is a line. Any line in a rectangular coordinate system has an equation of this form.

Also, the graph of any equation of the form

$$y = mx + b \qquad (2)$$

where m and b are constants, is a line. Form (2), which we will discuss in detail later, is simply a special case of form (1) for $B \neq 0$. This can be seen by solving form (1) for y in terms of x:

$$y = -\frac{A}{B}x + \frac{C}{B} \qquad B \neq 0$$

To graph either equation (1) or (2), we plot any two points from the solution set and use a straightedge to draw a line through these two points. The points where the line crosses the axes are convenient to use and easy to find. The ***y* intercept*** is the y coordinate of the point where the graph crosses the y axis, and the ***x* intercept** is the x coordinate of the point where the graph crosses the x axis. To find the y intercept, let $x = 0$ and solve for y; to find the x intercept, let $y = 0$ and solve for x. It is often advisable to find a third point as a checkpoint. All three points must lie on the same line or a mistake has been made.

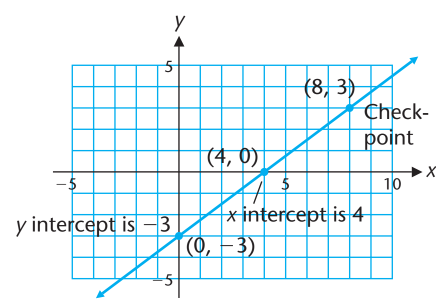
EXAMPLE 1 Using Intercepts to Graph a Line

Graph the equation $3x - 4y = 12$.

SOLUTION

Find intercepts, a third checkpoint (optional), and draw a line through the two (three) points (Fig. 1).

x	0	4	8
y	−3	0	3



› **Figure 1**

MATCHED PROBLEM 1

Graph the equation $4x + 3y = 12$.

*If the x intercept is a and the y intercept is b, then the graph of the line passes through the points $(a, 0)$ and $(0, b)$. It is common practice to refer to both the numbers a and b and the points $(a, 0)$ and $(0, b)$ as the x and y intercepts of the line.

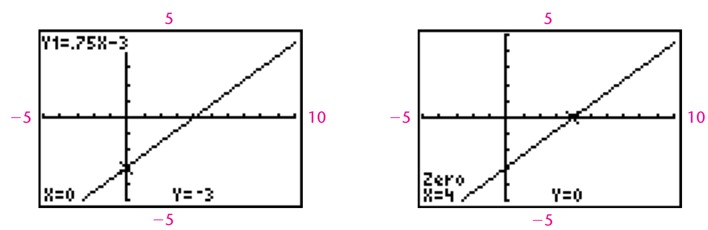
Technology Connections

To solve Example 1 on a graphing calculator, we first solve the equation for y:

$$\begin{aligned} 3x - 4y &= 12 \\ -4y &= -3x + 12 \\ y &= 0.75x - 3 \end{aligned}$$

To find the y intercept of this line, we graph the preceding equation, press **TRACE,** and then enter 0 for x [Fig. 2(a)]. The displayed y value is the y intercept.

The x intercept can be found by using the **zero option** on the CALC menu. After selecting the zero option, you will be asked to provide three x values: a **left bound** (a number less than the zero), a **right bound** (a number greater than the zero), and a **guess** (a number between the left and right bounds). You can enter the three values from the keypad, but most find it easier to use the cursor. The zero or x intercept is displayed at the bottom of the screen [Fig. 2(b)].



(a) y intercept (b) x intercept

Figure 2

Finding the Slope of a Line

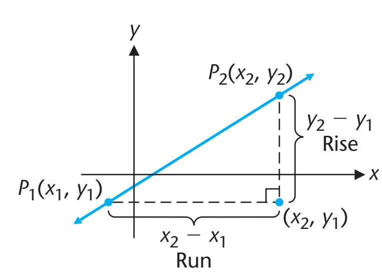
If we take two different points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on a line, then the ratio of the change in y to the change in x as we move from point P_1 to point P_2 is called the **slope** of the line. Roughly speaking, slope is a measure of the "steepness" of a line. Sometimes the change in x is called the **run** and the change in y is called the **rise.**

DEFINITION 1 Slope of a Line

If a line passes through two distinct points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, then its slope m is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2$$

$$= \frac{\text{Vertical change (rise)}}{\text{Horizontal change (run)}}$$

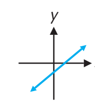
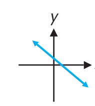
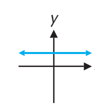
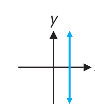


For a horizontal line, y doesn't change as x changes; hence, its slope is 0. On the other hand, for a vertical line, x doesn't change as y changes; hence, $x_1 = x_2$ and its slope is not defined:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{0} \qquad \text{For a vertical line, slope is not defined.}$$

In general, the slope of a line may be positive, negative, 0, or not defined. Each of these cases is interpreted geometrically as shown in Table 1.

Table 1 Geometric Interpretation of Slope

Line	Slope	Example
Rising as x moves from left to right y values are increasing	Positive	
Falling as x moves from left to right y values are decreasing	Negative	
Horizontal y values are constant	0	
Vertical x values are constant	Not defined	

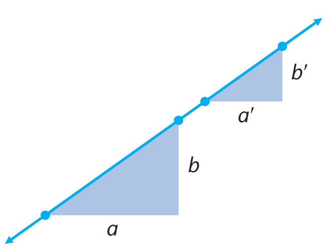
In using the formula to find the slope of the line through two points, it doesn't matter which point is labeled P_1 or P_2, because changing the labeling will change the sign in both the numerator and denominator of the slope formula:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

For example, the slope of the line through the points (3, 2) and (7, 5) is

$$\frac{5 - 2}{7 - 3} = \frac{3}{4} = \frac{-3}{-4} = \frac{2 - 5}{3 - 7}$$

In addition, it is important to note that the definition of slope doesn't depend on the two points chosen on the line as long as they are distinct. This follows from the fact that the ratios of corresponding sides of similar triangles are equal (Fig. 3).



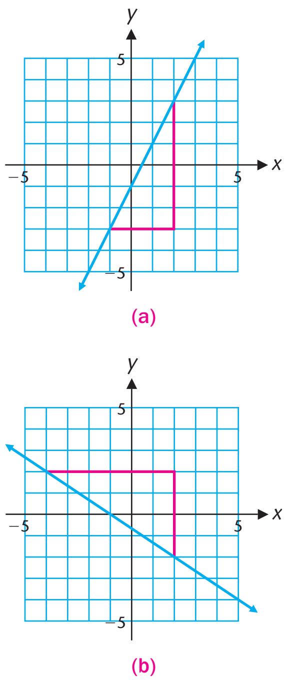
$$m = \frac{b}{a} = \frac{b'}{a'}$$

Figure 3

EXAMPLE 2 Finding Slopes

For each line in Figure 4, find the run, the rise, and the slope. (All the horizontal and vertical line segments have integer lengths.)

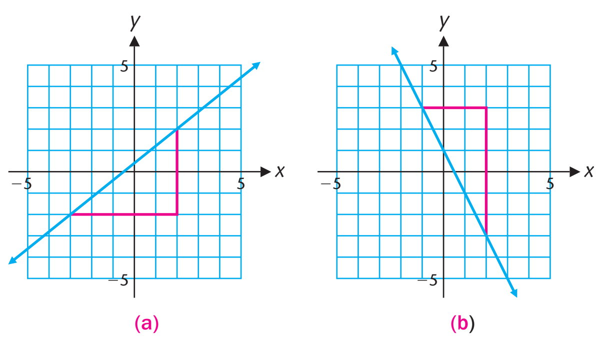
SOLUTION In Figure 4(a), the run is 3, the rise is 6 and the slope is $\frac{6}{3} = 2$. In Figure 4(b), the run is 6, the rise is -4 and the slope is $\frac{-4}{6} = -\frac{2}{3}$.



› Figure 4

MATCHED PROBLEM 2

For each line in Figure 5, find the run, the rise, and the slope. (All the horizontal and vertical line segments have integer lengths.)



› Figure 5

EXAMPLE 3 Finding Slopes

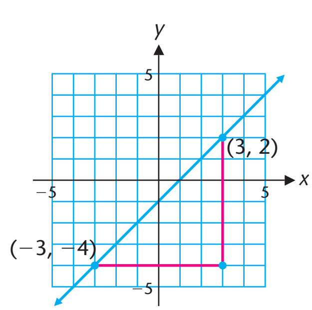
Sketch a line through each pair of points and find the slope of each line.

(A) $(-3, -4), (3, 2)$ (B) $(-2, 3), (1, -3)$

(C) $(-4, 2), (3, 2)$ (D) $(2, 4), (2, -3)$

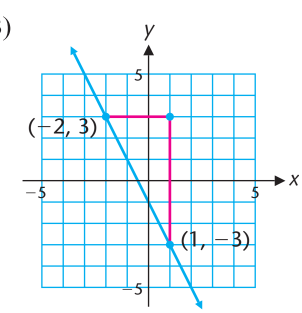
SOLUTIONS

(A)



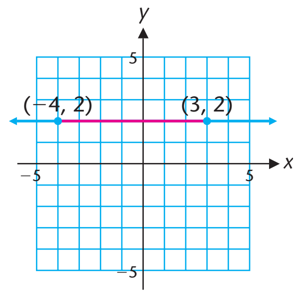
$$m = \frac{2 - (-4)}{3 - (-3)} = \frac{6}{6} = 1$$

(B)



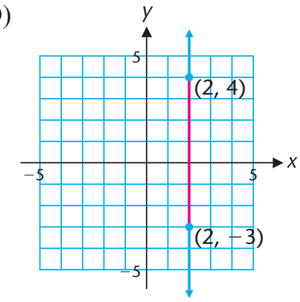
$$m = \frac{-3 - 3}{1 - (-2)} = \frac{-6}{3} = -2$$

(C)



$$m = \frac{2 - 2}{3 - (-4)} = \frac{0}{7} = 0$$

(D)



$$m = \frac{-3 - 4}{2 - 2} = \frac{-7}{0};$$

slope is not defined

MATCHED PROBLEM 3

Find the slope of the line through each pair of points. Do not graph.

(A) $(-3, -3), (2, -3)$ (B) $(-2, -1), (1, 2)$

(C) $(0, 4), (2, -4)$ (D) $(-3, 2), (-3, -1)$

Determining Special Forms of the Equation of a Line

We start by investigating why $y = mx + b$ is called the *slope–intercept form* for a line.

EXPLORE-DISCUSS 1

(A) Graph $y = x + b$ for $b = -5, -3, 0, 3$, and 5 simultaneously in the same coordinate system. Verbally describe the geometric significance of b.

(B) Graph $y = mx - 1$ for $m = -2, -1, 0, 1$, and 2 simultaneously in the same coordinate system. Verbally describe the geometric significance of m.

As you see from the preceding exploration, constants m and b in $y = mx + b$ have special geometric significance, which we now explicitly state.

If we let $x = 0$, then $y = b$ and the graph of $y = mx + b$ crosses the y axis at $(0, b)$. Thus, the constant b is the y intercept. For example, the y intercept of the graph of $y = 2x - 7$ is -7.

We have already seen that the point $(0, b)$ is on the graph of $y = mx + b$. If we let $x = 1$, then it follows that the point $(1, m + b)$ is also on the graph (Fig. 6). Because the graph of $y = mx + b$ is a line, we can use these two points to compute the slope:

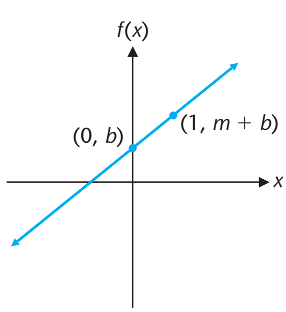


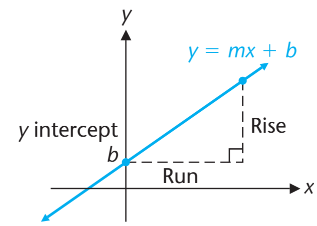
Figure 6

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(m + b) - b}{1 - 0} = m \qquad \begin{matrix}(x_1, y_1) = (0, b) \\ (x_2, y_2) = (1, m + b)\end{matrix}$$

Thus, m is the slope of the line with equation $y = mx + b$.

› THEOREM 2 Slope–Intercept Form

$$y = mx + b$$
$$m = \frac{\text{Rise}}{\text{Run}} = \text{Slope}$$
$$b = y \text{ intercept}$$



EXAMPLE 4 Using the Slope–Intercept Form

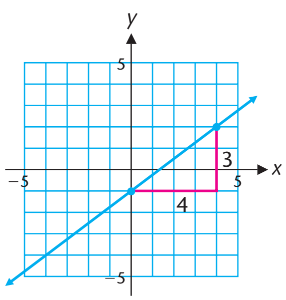
(A) Write the slope–intercept equation of a line with slope $\frac{2}{3}$ and y intercept -5.

(B) Find the slope and y intercept, and graph $y = \frac{3}{4}x - 1$.

SOLUTIONS

(A) Substitute $m = \frac{2}{3}$ and $b = -5$ in $y = mx + b$ to obtain $y = \frac{2}{3}x - 5$.

(B) The y intercept of $y = \frac{3}{4}x - 1$ is -1 and the slope is $\frac{3}{4}$. If we start at the point $(0, -1)$ and move four units to the right (run), then the y coordinate of a point on the line must move up three units (rise) to the point $(4, 2)$. Drawing a line through these two points produces the graph shown in Figure 7.



› Figure 7

MATCHED PROBLEM 4

Write the slope–intercept equation of the line with slope $\frac{5}{4}$ and y intercept -2. Graph the equation.

In Example 4 we found the equation of a line with a given slope and y intercept. It is also possible to find the equation of a line passing through a given point with a given slope or to find the equation of a line containing two given points.

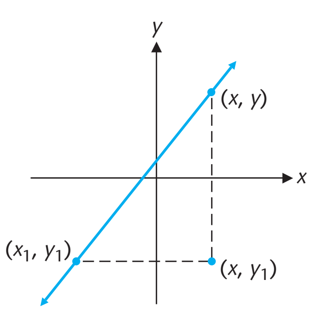
Suppose a line has slope m and passes through the point (x_1, y_1). If (x, y) is any other point on the line (Fig. 8), then

$$\frac{y - y_1}{x - x_1} = m$$

that is,

$$y - y_1 = m(x - x_1) \quad (3)$$

Because the point (x_1, y_1) also satisfies equation (3), we can conclude that equation (3) is an equation of a line with slope m that passes through (x_1, y_1).



› Figure 8

THEOREM 3 Point–Slope Form

An equation of a line with slope m that passes through (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

which is called the **point–slope form** of an equation of a line.

If we are given the coordinates of two points on a line, we can use the given coordinates to find the slope and then use the point–slope form with either of the given points to find the equation of the line.

EXAMPLE 5

Point–Slope Form

(A) Find an equation for the line that has slope $\frac{2}{3}$ and passes through the point $(-2, 1)$. Write the final answer in the form $Ax + By = C$.

(B) Find an equation for the line that passes through the two points $(4, -1)$ and $(-8, 5)$. Write the final answer in the form $y = mx + b$.

SOLUTIONS

(A) If $m = \frac{2}{3}$ and $(x_1, y_1) = (-2, 1)$, then

$$y - y_1 = m(x - x_1)$$ Substitute $y_1 = 1$, $x_1 = -2$, and $m = \frac{2}{3}$.

$$y - 1 = \frac{2}{3}[x - (-2)]$$ Multiply both sides by 3.

$$3(y - 1) = 2(x + 2)$$ Remove parentheses.

$$3y - 3 = 2x + 4$$ Write in standard form.

$$-2x + 3y = 7 \quad \text{or} \quad 2x - 3y = -7$$

(B) First use the slope formula to find the slope of the line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{-8 - 4} = \frac{6}{-12} = -\frac{1}{2}$$ Substitute $x_1 = 4$, $y_1 = -1$, $x_2 = -8$, and $y_2 = 5$ in the slope formula.

Now we choose $(x_1, y_1) = (4, -1)$ and proceed as in part A:

$$y - y_1 = m(x - x_1)$$ Substitute $x_1 = 4$, $y_1 = -1$, and $m = -\frac{1}{2}$.

$$y - (-1) = -\frac{1}{2}(x - 4)$$ Remove parentheses.

$$y + 1 = -\frac{1}{2}x + 2$$ Subtract 1 from both sides.

$$y = -\frac{1}{2}x + 1$$

Verify that choosing $(x_1, y_1) = (-8, 5)$, the other given point, produces the same equation.

MATCHED PROBLEM 5

(A) Find an equation for the line that has slope $-\frac{2}{5}$ and passes through the point $(3, -2)$. Write the final answer in the form $Ax + By = C$.

(B) Find an equation for the line that passes through the two points $(-3, 1)$ and $(7, -3)$. Write the final answer in the form $y = mx + b$.

The simplest equations of lines are those for horizontal and vertical lines. Consider the following two equations:

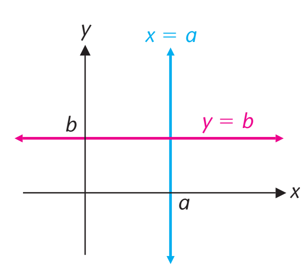
$$x + 0y = a \quad \text{or} \quad x = a \tag{4}$$

$$0x + y = b \quad \text{or} \quad y = b \tag{5}$$

In equation (4), y can be any number as long as $x = a$. Thus, the graph of $x = a$ is a vertical line crossing the x axis at $(a, 0)$. In equation (5), x can be any number as long as $y = b$. Thus, the graph of $y = b$ is a horizontal line crossing the y axis at $(0, b)$. We summarize these results as follows:

THEOREM 4 Vertical and Horizontal Lines

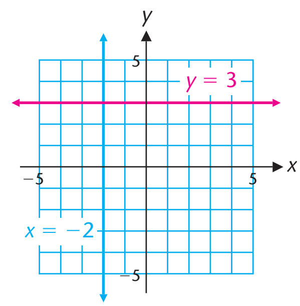
Equation		Graph
$x = a$	(short for $x + 0y = a$)	Vertical line through $(a, 0)$ (Slope is undefined.)
$y = b$	(short for $0x + y = b$)	Horizontal line through $(0, b)$ (Slope is 0.)



EXAMPLE 6 Graphing Horizontal and Vertical Lines

Graph the line $x = -2$ and the line $y = 3$.

SOLUTION



MATCHED PROBLEM 6

Graph the line $x = 4$ and the line $y = -2$.

The various forms of the equation of a line that we have discussed are summarized in Table 2 for convenient reference.

Table 2 Equations of a Line

Standard form	$Ax + By = C$	A and B not both 0
Slope–intercept form	$y = mx + b$	Slope: m; y intercept: b
Point–slope form	$y - y_1 = m(x - x_1)$	Slope: m; Point: (x_1, y_1)
Horizontal line	$y = b$	Slope: 0
Vertical line	$x = a$	Slope: Undefined

››› EXPLORE-DISCUSS 2

Determine conditions on A, B, and C so that the linear equation $Ax + By = C$ can be written in each of the following forms, and discuss the possible number of x and y intercepts in each case.

1. $y = mx + b, m \neq 0$
2. $y = b$
3. $x = a$

Connecting Slope to Parallel and Perpendicular Lines

From geometry, we know that two vertical lines are parallel to each other and that a horizontal line and a vertical line are perpendicular to each other. How can we tell when two nonvertical lines are parallel or perpendicular to each other? Theorem 5, which we state without proof, provides a convenient test.

THEOREM 5 Parallel and Perpendicular Lines

Given two nonvertical lines L_1 and L_2 with slopes m_1 and m_2, respectively, then

$$\begin{aligned} L_1 \parallel L_2 &\quad \text{if and only if} \quad & m_1 &= m_2 \\ L_1 \perp L_2 &\quad \text{if and only if} \quad & m_1 m_2 &= -1 \end{aligned}$$

The symbols $\parallel$ and $\perp$ mean, respectively, "is parallel to" and "is perpendicular to." In the case of perpendicularity, the condition $m_1m_2 = -1$ also can be written as

$$m_2 = -\frac{1}{m_1} \qquad \text{or} \qquad m_1 = -\frac{1}{m_2}$$

Thus,

Two nonvertical lines are perpendicular if and only if their slopes are the negative reciprocals of each other.

EXAMPLE 7 Parallel and Perpendicular Lines

Given the line: L: $3x - 2y = 5$ and the point $P(-3, 5)$, find an equation of a line through P that is

(A) Parallel to L (B) Perpendicular to L

Write the final answers in the slope–intercept form $y = mx + b$.

SOLUTIONS

First, find the slope of L by writing $3x - 2y = 5$ in the equivalent slope–intercept form $y = mx + b$:

$$\begin{aligned} 3x - 2y &= 5 \\ -2y &= -3x + 5 \\ y &= \tfrac{3}{2}x - \tfrac{5}{2} \end{aligned}$$

Thus, the slope of L is $\frac{3}{2}$. The slope of a line parallel to L is the same, $\frac{3}{2}$, and the slope of a line perpendicular to L is $-\frac{2}{3}$. We now can find the equations of the two lines in parts A and B using the point–slope form.

(A) Parallel ($m = \frac{3}{2}$):

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= \tfrac{3}{2}(x + 3) \\ y - 5 &= \tfrac{3}{2}x + \tfrac{9}{2} \\ y &= \tfrac{3}{2}x + \tfrac{19}{2} \end{aligned}$$

(B) Perpendicular ($m = -\frac{2}{3}$):

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Substitute for } x_1, y_1, \text{ and } m. \\ y - 5 &= -\tfrac{2}{3}(x + 3) && \text{Remove parentheses.} \\ y - 5 &= -\tfrac{2}{3}x - 2 && \text{Add 5 to both sides.} \\ y &= -\tfrac{2}{3}x + 3 \end{aligned}$$

MATCHED PROBLEM 7

Given the Line L: $4x + 2y = 3$ and the point $P(2, -3)$, find an equation of a line through P that is

(A) Parallel to L (B) Perpendicular to L

Write the final answers in the slope–intercept form $y = mx + b$.

EXAMPLE 8 Cost Analysis

A hot dog vendor pays $25 per day to rent a pushcart and $1.25 for the ingredients in one hot dog.

(A) Find the cost of selling x hot dogs in 1 day.

(B) What is the cost of selling 200 hot dogs in 1 day?

(C) If the daily cost is $355, how many hot dogs were sold that day?

SOLUTIONS

(A) The rental charge of $25 is the vendor's **fixed cost**—a cost that is accrued every day and does not depend on the number of hot dogs sold. The cost of ingredients does depend on the number sold. The cost of the ingredients for x hot dogs is $\$1.25x$. This is the vendor's **variable cost**—a cost that depends on the number of hot dogs sold. The total cost for selling x hot dogs is

$$C(x) = 1.25x + 25 \qquad \text{Total Cost = Variable Cost + Fixed Cost}$$

(B) The cost of selling 200 hot dogs in 1 day is

$$C(200) = 1.25(200) + 25 = \$275$$

(C) The number of hot dogs that can be sold for \$355 is the solution of the equation

$$
\begin{aligned}
1.25x + 25 &= 355 && \text{Subtract 25 from each side.}\\
1.25x &= 330 && \text{Divide both sides by 1.25.}\\
x &= \frac{330}{1.25} && \text{Simplify.}\\
&= 264 \text{ hot dogs}
\end{aligned}
$$

MATCHED PROBLEM 8

It costs a pretzel vendor \$20 per day to rent a cart and \$0.75 for each pretzel.

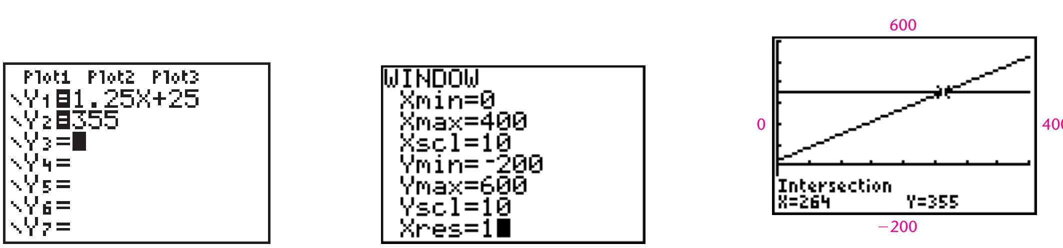
(A) Find the cost of selling x pretzels in 1 day.

(B) What is the cost of selling 150 pretzels in 1 day?

(C) If the daily cost is \$275, how many pretzels were sold that day?

Technology Connections

A graphing calculator can be used to solve equations like $1.25x + 25 = 355$ (see Example 8). First enter both sides of the equation in the equation editor [Fig. 9(a)] and choose window variables [Fig. 9(b)] so that the graphs of both equations appear on the screen. There is no "right" choice for the window variables. Any choice that displays the intersection point will do. (Here is how we chose our window variables: We chose $Y_{max} = 600$ to place the graph of the horizontal line below the top of the window. We chose $Y_{min} = -200$ to place the graph of the x axis above the text displayed at the bottom of the screen. Since x cannot be negative, we chose $X_{min} = 0$. We used trial and error to determine a reasonable choice for X_{max}.) Now choose **intersect** on the CALC menu, and respond to the prompts from the calculator. The coordinates of the intersection point of the two graphs are shown at the bottom of the screen [Fig. 9(c)].

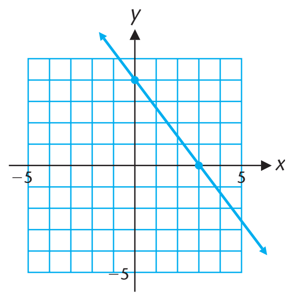


(a) (b) (c)

Figure 9

ANSWERS TO MATCHED PROBLEMS

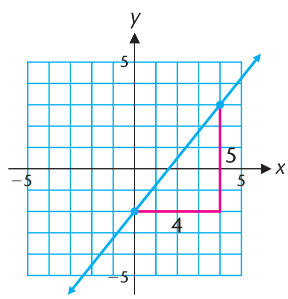
1.



2. (A) Run = 5, rise = 4, slope = $\frac{4}{5} = 0.8$
(B) Run = 3, rise = −6, slope = $\frac{-6}{3} = -2$

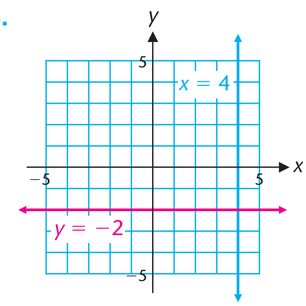
3. (A) $m = 0$ (B) $m = 1$
(C) $m = -4$ (D) m is not defined

4. $y = \frac{5}{4}x - 2$



5. (A) $2x + 5y = -4$ (B) $y = -\frac{2}{5}x - \frac{1}{5}$

6.



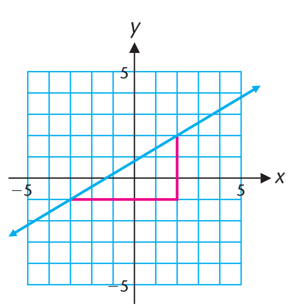
7. (A) $y = -2x + 1$ (B) $y = \frac{1}{2}x - 4$

8. (A) $C(x) = 0.75x + 20$ (B) $132.50 (C) 340 pretzels

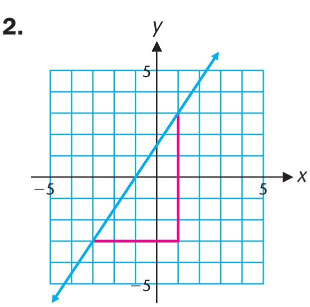
2-3 Exercises

In Problems 1–6, use the graph of each line to find the rise, run, and slope. Write the equation of each line in the standard form $Ax + By = C$, $A \geq 0$. (All the horizontal and vertical line segments have integer lengths.)

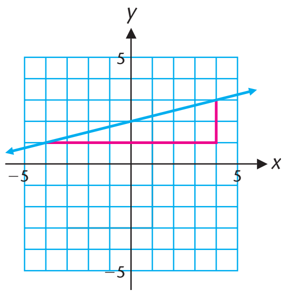
1.



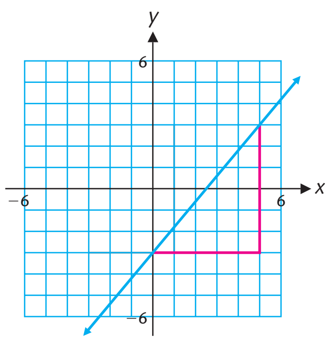
2.



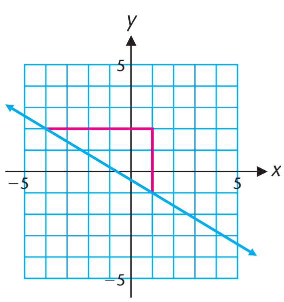
3.



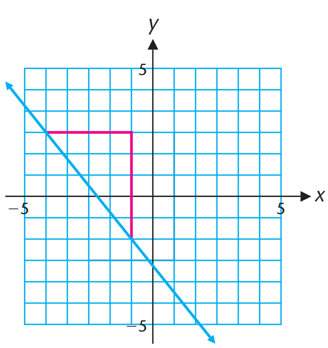
4.



5.

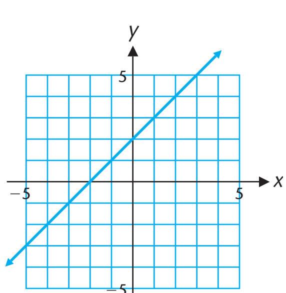


6.

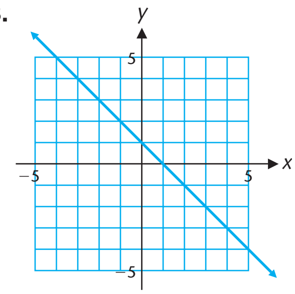


In Problems 7–12, use the graph of each line to find the x intercept, y intercept, and slope, if they exist. Write the equation of each line, using the slope–intercept form whenever possible.

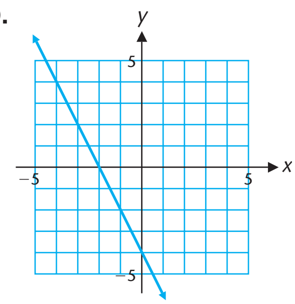
7.



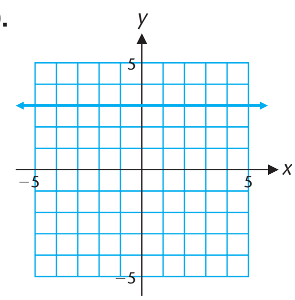
8.



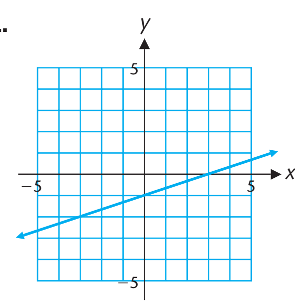
9.



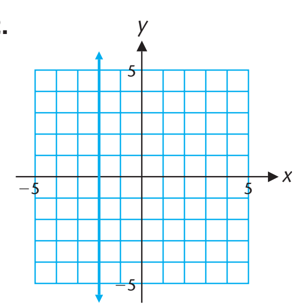
10.



11.



12.



Graph each equation in Problems 13–28, and indicate the slope, if it exists.

13. $y = -\frac{3}{5}x + 4$

14. $y = -\frac{3}{2}x + 6$

15. $y = -\frac{3}{4}x$

16. $y = \frac{2}{3}x - 3$

17. $4x + 2y = 0$

18. $6x - 2y = 0$

19. $2x - 3y = 15$

20. $4x + 3y = 24$

21. $4x - 5y = -24$

22. $6x - 7y = -49$

23. $\frac{y}{8} - \frac{x}{4} = 1$

24. $\frac{y}{6} - \frac{x}{5} = 1$

25. $x = -3$

26. $y = -2$

27. $y = 3.5$

28. $x = 2.5$

In Problems 29–34, find the equation of the line with the indicated slope and y intercept. Write the final answer in the standard form $Ax + By = C$, $A \geq 0$.

29. Slope $= 1$; y intercept $= 0$

30. Slope $= -1$; y intercept $= 7$

31. Slope $= -\frac{2}{3}$; y intercept $= -4$

32. Slope $= \frac{5}{3}$; y intercept $= 6$

33. Slope $= 0$; y intercept $= -2$

34. Slope undefined; x intercept $= 3$

In Problems 35–40, find the equation of the line passing through the given point with the given slope. Write the final answer in the slope–intercept form $y = mx + b$.

Use a graphing calculator to find the intercepts of the line.

35. $(0, 3)$; $m = -2$

36. $(4, 0)$; $m = 3$

37. $(-5, 4)$; $m = \frac{3}{2}$

38. $(2, -3)$; $m = -\frac{4}{5}$

39. $(-2, -3)$; $m = -\frac{1}{2}$

40. $(2, 1)$; $m = \frac{4}{3}$

In Problems 41–46, find the equation of the line passing through the two given points. Write the final answer in the slope–intercept form $y = mx + b$ or in the form $x = c$.

41. $(2, 5)$; $(4, -3)$

42. $(-1, 4)$; $(3, 2)$

43. $(-3, 2)$; $(-3, 5)$

44. $(0, 5)$; $(2, 5)$

45. $(-4, 2)$; $(0, 2)$

46. $(5, -4)$; $(5, 6)$

In Problem 47–60, sketch a graph of the line that contains the indicated point(s) and/or has the indicated slope and/or has the indicated intercepts. Then write the equation of the line in the slope–intercept form $y = mx + b$ or in the form $x = c$.

47. $(0, 4)$; $m = -3$

48. $(2, 0)$; $m = 2$

49. $(-5, 4)$; $m = -\frac{2}{5}$

50. $(-4, -2)$; $m = \frac{1}{2}$

51. $(1, 6)$; $(5, -2)$

52. $(-3, 4)$; $(6, 1)$

53. $(-4, 8)$; $(2, 0)$

54. $(2, -1)$; $(10, 5)$

55. $(-3, 4)$; $(5, 4)$

56. $(0, -2)$; $(4, -2)$

57. $(4, 6)$; $(4, -3)$

58. $(-3, 1)$; $(-3, -4)$

59. x intercept -4; y intercept 3

60. x intercept -4; y intercept -5

In Problems 61–72, write an equation of the line that contains the indicated point and meets the indicated condition(s). Write the final answer in the standard form $Ax + By = C$, $A \geq 0$.

61. $(-3, 4)$; parallel to $y = 3x - 5$

62. $(-4, 0)$; parallel to $y = -2x + 1$

63. $(2, -3)$; perpendicular to $y = -\frac{1}{3}x$

64. $(-2, -4)$; perpendicular to $y = \frac{2}{3}x - 5$

65. $(2, 5)$; parallel to y axis

66. $(7, 3)$; parallel to x axis

67. $(3, -2)$; vertical

68. $(-2, -3)$; horizontal

69. $(5, 0)$; parallel to $3x - 2y = 4$

70. $(3, 5)$; parallel to $3x + 4y = 8$

71. $(0, -4)$; perpendicular to $x + 3y = 9$

72. $(-2, 4)$; perpendicular to $4x + 5y = 0$

73. Discuss the relationship between the graphs of the lines with equation $y = mx + 2$, where m is any real number.

74. Discuss the relationship between the graphs of the lines with equation $y = -0.5x + b$, where b is any real number.

Problems 75–80 refer to the quadrilateral with vertices $A(0, 2)$, $B(4, -1)$, $C(1, -5)$, and $D(-3, -2)$.

75. Show that $AB \parallel DC$.

76. Show that $DA \parallel CB$.

77. Show that $AB \perp BC$.

78. Show that $AD \perp DC$.

79. Find an equation of the perpendicular bisector* of AD.

80. Find an equation of the perpendicular bisector of AB.

81. (A) Graph the following equations in the same coordinate system:

$$3x + 2y = 6 \qquad 3x + 2y = 3$$
$$3x + 2y = -6 \qquad 3x + 2y = -3$$

*The perpendicular bisector of a line segment is a line perpendicular to the segment and passing through its midpoint.

(B) Graph all the equations in part A on a graphing calculator.
(C) From your observations in part A, describe the family of lines obtained by varying C in $Ax + By = C$ while holding A and B fixed.
(D) Verify your conclusions in part B with a proof.

82. (A) Graph the following two equations in the same coordinate system:

$$3x + 4y = 12 \qquad 4x - 3y = 12$$

Graph these equations in a squared viewing window on a graphing calculator.
(B) Graph the following two equations in the same coordinate system:

$$2x + 3y = 12 \qquad 3x - 2y = 12$$

Graph these equations in a squared viewing window on a graphing calculator.
(C) From your observations in parts A and B, describe the apparent relationship of the graphs of $Ax + By = C$ and $Bx - Ay = C$.
(D) Verify your conclusions in part C with a proof.

83. Prove that if a line L has x intercept $(a, 0)$ and y intercept $(0, b)$, then the equation of L can be written in the **intercept form**

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad a, b \neq 0$$

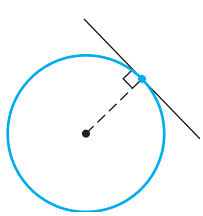
84. Prove that if a line L passes through $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, then the equation of L can be written in the **two-point form**

$$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$$

In Problems 85 and 86, write the equation of the line with the indicated intercepts in intercept form (see Problem 83) and then in the standard form $Ax + By = C, A \geq 0$.

85. $(3, 0)$ and $(0, 5)$ **86.** $(-2, 0)$ and $(0, 7)$

Problems 87–92 are calculus related. Recall that a line tangent to a circle at a point is perpendicular to the radius drawn to that point (see the figure). Find the equation of the line tangent to the circle at the indicated point. Write the final answer in the standard form $Ax + By = C, A \geq 0$. Graph the circle and the tangent line on the same coordinate system.



87. $x^2 + y^2 = 25, (3, 4)$

88. $x^2 + y^2 = 100, (-8, 6)$

89. $x^2 + y^2 = 50, (5, -5)$

90. $x^2 + y^2 = 80, (-4, -8)$

91. $(x - 3)^2 + (y + 4)^2 = 169, (8, -16)$

92. $(x + 5)^2 + (y - 9)^2 = 289, (-13, -6)$

APPLICATIONS

93. BOILING POINT OF WATER At sea level, water boils when it reaches a temperature of 212°F. At higher altitudes, the atmospheric pressure is lower and so is the temperature at which water boils. The boiling point B in degrees Fahrenheit at an altitude of x feet is given approximately by

$$B = 212 - 0.0018x$$

(A) Complete Table 3.

Table 3

x	0	5,000	10,000	15,000	20,000	25,000	30,000
B							

(B) Based on the information in the table, write a brief verbal description of the relationship between altitude and the boiling point of water.

94. AIR TEMPERATURE As dry air moves upward, it expands and cools. The air temperature A in degrees Celsius at an altitude of x kilometers is given approximately by

$$A = 25 - 9x$$

(A) Complete Table 4.

Table 4

x	0	1	2	3	4	5
A						

(B) Based on the information in the table, write a brief verbal description of the relationship between altitude and air temperature.

95. CAR RENTAL A car rental agency computes daily rental charges for compact cars with the equation

$$c = 25 + 0.25x$$

where c is the daily charge in dollars and x is the daily mileage. Translate this algebraic statement into a verbal statement that can be used to explain the daily charges to a customer.

96. INSTALLATION CHARGES A telephone store computes charges for phone installation with the equation

$$c = 15 + 0.7x$$

where c is the installation charge in dollars and x is the time in minutes spent performing the installation. Translate this algebraic statement into a verbal statement that can be used to explain the installation charges to a customer.

97. COST ANALYSIS A doughnut shop has a fixed cost of \$124 per day and a variable cost of \$0.12 per doughnut. Find the total daily cost of producing x doughnuts. How many doughnuts can be produced for a total daily cost of \$250?

98. COST ANALYSIS A small company manufactures picnic tables. The weekly fixed cost is \$1,200 and the variable cost is \$45 per table. Find the total weekly cost of producing x picnic tables. How many picnic tables can be produced for a total weekly cost of \$4,800?

99. PHYSICS Hooke's law states that the relationship between the stretch s of a spring and the weight w causing the stretch is linear (a principle upon which all spring scales are constructed). For a particular spring, a 5-pound weight causes a stretch of 2 inches, while with no weight the stretch of the spring is 0.
(A) Find a linear equation that expresses s in terms of w.
(B) What is the stretch for a weight of 20 pounds?
(C) What weight will cause a stretch of 3.6 inches?

100. PHYSICS The distance d between a fixed spring and the floor is a linear function of the weight w attached to the bottom of the spring. The bottom of the spring is 18 inches from the floor when the weight is 3 pounds and 10 inches from the floor when the weight is 5 pounds.
(A) Find a linear equation that expresses d in terms of w.
(B) Find the distance from the bottom of the spring to the floor if no weight is attached.
(C) Find the smallest weight that will make the bottom of the spring touch the floor. (Ignore the height of the weight.)

101. PHYSICS The two most widespread temperature scales are Fahrenheit* (F) and Celsius† (C). It is known that water freezes at 32°F or 0°C and boils at 212°F or 100°C.
(A) Find a linear equation that expresses F in terms of C.
(B) If a European family sets its house thermostat at 20°C, what is the setting in degrees Fahrenheit? If the outside temperature in Milwaukee is 86°F, what is the temperature in degrees Celsius?

102. PHYSICS Two other temperature scales, used primarily by scientists, are Kelvin‡ (K) and Rankine** (R). Water freezes at 273 K or 492°R and boils at 373 K or 672°R. Find a linear equation that expresses R in terms of K.

103. OCEANOGRAPHY After about 9 hours of a steady wind, the height of waves in the ocean is approximately linearly related to the duration of time the wind has been blowing. During a storm with 50-knot winds, the wave height after 9 hours was found to be 23 feet, and after 24 hours it was 40 feet.
(A) If t is time after the 50-knot wind started to blow and h is the wave height in feet, write a linear equation that expresses height h in terms of time t.
(B) How long will the wind have been blowing for the waves to be 50 feet high?

104. OCEANOGRAPHY Refer to Problem 103. A steady 25-knot wind produces a wave 7 feet high after 9 hours and 11 feet high after 25 hours.
(A) Write a linear equation that expresses height h in terms of time t.
(B) How long will the wind have been blowing for the waves to be 20 feet high?

Express all calculated quantities to three significant digits.

105. DEMOGRAPHICS Life expectancy in the United States has increased from about 49.2 years in 1900 to about 77.3 years in 2000. The growth in life expectancy is approximately linear with respect to time.
(A) If L represents life expectancy and t represents the number of years since 1900, write a linear equation that expresses L in terms of t.
(B) What is the predicted life expectancy in the year 2020?

Express all calculated quantities to three significant digits.

*Invented in 1724 by Daniel Gabriel Fahrenheit (1686–1736), a German physicist.

†Invented in 1742 by Anders Celsius (1701–1744), a Swedish astronomer.

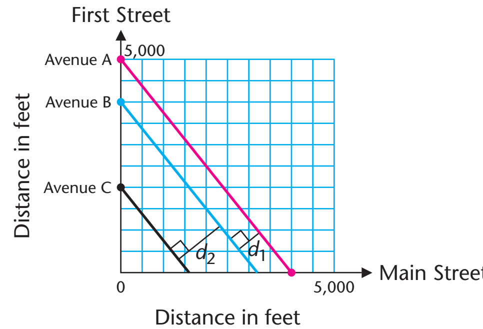
‡Invented in 1848 by Lord William Thompson Kelvin (1824–1907), a Scottish mathematician and physicist. Note that the degree symbol "°" is not used with degrees Kelvin.

**Invented in 1859 by John Maquorn Rankine (1820–1872), a Scottish engineer and physicist.

106. DEMOGRAPHICS The average number of persons per household in the United States has been shrinking steadily for as long as statistics have been kept and is approximately linear with respect to time. In 1900, there were about 4.76 persons per household and in 1990, about 2.5.
(A) If N represents the average number of persons per household and t represents the number of years since 1900, write a linear equation that expresses N in terms of t.
(B) What is the predicted household size in the year 2000?

Express all calculated quantities to three significant digits.

★**107. CITY PLANNING** The design of a new subdivision calls for three parallel streets connecting First Street with Main Street (see the figure). Find the distance d_1 (to the nearest foot) from Avenue A to Avenue B.



★**108. CITY PLANNING** Refer to Problem 107. Find the distance d_2 (to the nearest foot) from Avenue B to Avenue C.

2-4 Linear Equations and Models

- Slope as a Rate of Change
- Linear Models
- Linear Regression

Mathematical modeling is the process of using mathematics to solve real-world problems. This process can be broken down into three steps (Fig. 1):

Step 1. *Construct* the **mathematical model,** a problem whose solution will provide information about the real-world problem.
Step 2. *Solve* the mathematical model.
Step 3. *Interpret* the solution to the mathematical model in terms of the original real-world problem.

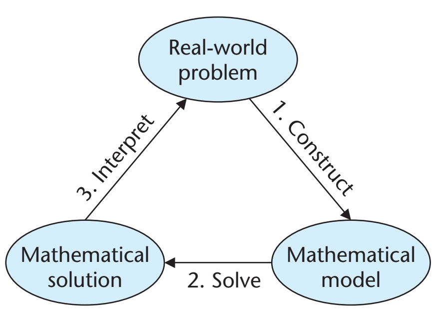


Figure 1

In more complex problems, this cycle may have to be repeated several times to obtain the required information about the real-world problem. In this section, we discuss one of the simplest mathematical models, a linear equation. With the aid of a graphing calculator, we also learn how to analyze a linear model based on real-world data.

Slope as a Rate of Change

If x and y are related by the equation $y = mx + b$, where m and b are constants with $m \neq 0$, then x and y are **linearly related.** If (x_1, y_1) and (x_2, y_2) are two distinct points on this line, then the slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} \qquad (1)$$

In applications, ratio (1) is called the **rate of change** of y with respect to x. Since the slope of a line is unique, *the rate of change of two linearly related variables is constant.* Here are some examples of familiar rates of change: miles per hour, revolutions per minute, price per pound, passengers per plane, etc. If y is distance and x is time, then the rate of change is also referred to as **speed** or **velocity.** If the relationship between x and y is not linear, ratio (1) is called the **average rate of change** of y with respect to x.

EXAMPLE 1 Estimating Body Surface Area

Appropriate doses of medicine for both animals and humans are often based on body surface area (BSA). Since weight is much easier to determine than BSA, veterinarians use the weight of an animal to estimate BSA. The following linear equation expresses BSA for canines in terms of weight*:

$$a = 16.21w + 375.6$$

where a is BSA in square inches and w is weight in pounds.

(A) Interpret the slope of the BSA equation.

(B) What is the effect of a 1-pound increase in weight?

SOLUTIONS

(A) The rate of change BSA with respect to weight is 16.12 square inches per pound.

(B) Since slope is the ratio of rise to run, increasing w by 1 pound (run) increases a by 16.12 square inches (rise).

MATCHED PROBLEM 1

The following linear equation expresses BSA for felines in terms of weight:

$$a = 28.55w + 118.7$$

where a is BSA in square inches and w is weight in pounds.

(A) Interpret the slope of the BSA equation.

(B) What is the effect of a 1-pound increase in weight?

*Based on data from Veterinary Oncology Consultants, PTY LTD.

››› EXPLORE-DISCUSS 1

As illustrated in Example 1(A), the slope m of a line with equation $y = mx + b$ has two interpretations:

1. m is the rate of change of y with respect to x.
2. Increasing x by one unit will change y by m units.

How are these two interpretations related?

› Linear Models

We can use our experience with lines gained in Section 2-3 to construct linear models for applications involving linearly related quantities. This process is best illustrated through examples.

EXAMPLE 2 Business Markup Policy

A sporting goods store sells a fishing rod that cost \$60 for \$82 and a pair of cross-country ski boots that cost \$80 for \$106.

(A) If the markup policy of the store for items that cost more than \$30 is assumed to be linear, find a linear model that express the retail price P in terms of the wholesale cost C.

(B) What is the effect on the price of a \$1 increase in cost for any item costing over \$30?

(C) Use the model to find the retail price for a pair of running shoes that cost \$40.

SOLUTIONS

(A) If price P is linearly related to cost C, then we are looking for the equation of a line whose graph passes through $(C_1, P_1) = (60, 82)$ and $(C_2, P_2) = (80, 106)$. We find the slope, and then use the point–slope form to find the equation.

$$m = \frac{P_2 - P_1}{C_2 - C_1} = \frac{106 - 82}{80 - 60} = \frac{24}{20} = 1.2$$ Substitute $C_1 = 60$, $P_1 = 82$, $C_2 = 80$, and $P_2 = 106$ in the slope formula.

$$P - P_1 = m(C - C_1)$$ Substitute $P_1 = 82$, $C_1 = 60$, and $m = 1.2$ in the point–slope formula.

$$P - 82 = 1.2(C - 60)$$ Remove parentheses.

$$P - 82 = 1.2C - 72$$ Add 82 to both sides.

$$P = 1.2C + 10 \qquad C > 30$$ Linear model

(B) If the cost is increased by \$1, then the price will increase by $1.2(1) = \$1.20$.

(C) $P = 1.2(40) + 10 = \$58$.

MATCHED PROBLEM 2

The sporting goods store in Example 2 is celebrating its twentieth anniversary with a 20% off sale. The sale price of a mountain bike is \$380. What was the presale price of the bike? How much did the bike cost the store?

››› EXPLORE-DISCUSS 2

The wholesale supplier for the sporting goods store in Example 2 offers the store a 15% discount on all items. The store decides to pass on the savings from this discount to the consumer. Which of the following markup policies is better for the consumer?

1. Apply the store's markup policy to the discounted cost.
2. Apply the store's markup policy to the original cost and then reduce this price by 15%.

Support your choice with examples.

EXAMPLE 3 Mixing Antifreeze

Ethylene glycol and propylene glycol are liquids used in antifreeze and deicing solutions. Ethylene glycol is listed as a hazardous chemical by the Environmental Protection Agency, while propylene glycol is generally regarded as safe. Table 1 lists solution concentration percentages and the corresponding freezing points for each chemical.

Table 1

Concentration	Ethylene Glycol	Propylene Glycol
20%	15°F	17°F
50%	−36°F	−28°F

(A) Assume that the concentration and the freezing point for ethylene glycol are linearly related. Construct a linear model for the freezing point.

(B) Interpret the slope in part (A).

(C) What percentage (to one decimal place) of ethylene glycol will result in a freezing point of $-10°F$?

SOLUTIONS

(A) We begin by defining appropriate variables:

Let

p = percentage of ethylene glycol in the antifreeze solution

f = freezing point of the antifreeze solution

From Table 1, we see that (**20**, **15**) and (**50**, **−36**) are two points on the line relating p and f. The slope of this line is

$$m = \frac{f_2 - f_1}{p_2 - p_1} = \frac{15 - (-36)}{20 - 50} = \frac{51}{-30} = -1.7$$

and its equation is

$$f - 15 = -1.7(p - 20)$$
$$f = -1.7p + 49 \quad \text{Linear model}$$

(B) The rate of change of the freezing point with respect to the percentage of ethylene glycol in the antifreeze solution is -1.7 degrees per percentage of ethylene glycol. Increasing the amount of ethylene glycol by 1% will lower the freezing point by 1.7°F.

(C) We must find p when f is $-10°$.

$$f = -1.7p + 49$$
$$-10 = -1.7p + 49 \quad \text{Add } 10 + 1.7p \text{ to both sides.}$$
$$1.7p = 59 \quad \text{Divide both sides by 1.7.}$$
$$p = \frac{59}{1.7} = 34.7\%$$

MATCHED PROBLEM 3

Refer to Table 1.

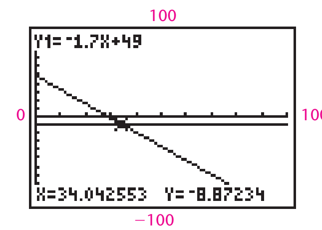
(A) Assume that the concentration and the freezing point for propylene glycol are linearly related. Construct a linear model for the freezing point.

(B) Interpret the slope in part (A).

(C) What percentage (to one decimal place) of propylene glycol will result in a freezing point of $-15°F$?

Technology Connections

Figure 2 shows the solution of Example 3(C) on a graphing calculator.



› **Figure 2**
$y_1 = -1.7x + 49, y_2 = -10$

EXAMPLE 4 Underwater Pressure

The pressure at sea level is 14.7 pounds per square inch. As you descend into the ocean, the pressure increases linearly at a rate of about 0.445 pounds per square foot.

(A) Find the pressure p at a depth of d feet.

(B) If a diver's equipment is rated to be safe up to a pressure of 40 pounds per square foot, how deep (to the nearest foot) is it safe to use this equipment?

SOLUTIONS

(A) Let $p = md + b$. At the surface, $d = 0$ and $p = 14.7$, so $b = 14.7$. The slope m is the given rate of change, $m = 0.445$. Thus, the pressure at a depth of d feet is

$$p = 0.445d + 14.7$$

(B) The safe depth is the solution of the equation

$$0.445d + 14.7 = 40 \qquad \text{Subtract 14.7 from each side.}$$

$$0.445d = 25.3 \qquad \text{Divide both sides by 0.445.}$$

$$d = \frac{25.3}{0.445} \qquad \text{Simplify.}$$

$$\approx 57 \text{ feet}$$

MATCHED PROBLEM 4

The rate of change of pressure in fresh water is 0.432 pounds per square foot. Repeat Example 4 for a body of fresh water.

Technology Connections

Figure 3 shows the solution of Example 4(B) on a graphing calculator.

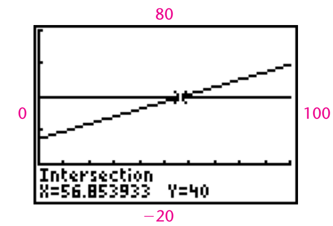


Figure 3
$y_1 = 0.445x + 14.7, y_2 = 40$

Linear Regression

In real-world applications we often encounter numerical data in the form of a table. The very powerful mathematical tool, *regression analysis,* can be used to analyze numerical data. In general, **regression analysis** is a process for finding a function that provides a useful model for a set of data points. Graphs of equations are often called **curves** and regression analysis is also referred to as **curve fitting.** In Example 5, we use a linear model obtained by using *linear regression* on a graphing calculator.

EXAMPLE 5 **Diamond Prices**

Prices for round-shaped diamonds taken from an online trader are given in Table 2.

Table 2 Round-Shaped Diamond Prices

Weight (Carats)	Price
0.5	$1,340
0.6	$1,760
0.7	$2,540
0.8	$3,350
0.9	$4,130
1.0	$4,920

Source: www.tradeshop.com

(A) A linear model for the data in Table 2 is given by

$$p = 7{,}380c - 2{,}530 \qquad (2)$$

where p is the price of a diamond weighing c carats. (We will discuss the source of models like this later in this section.) Plot the points in Table 2 on a Cartesian coordinate system, producing a **scatter plot,** and graph the model on the same axes.

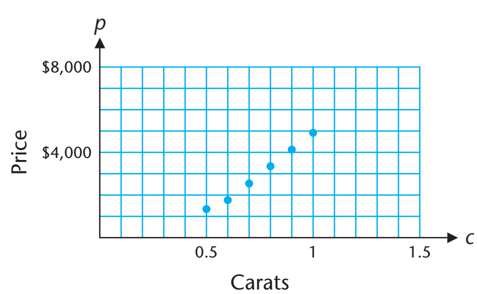
(B) Interpret the slope of the model in equation (2).

(C) Use the model to estimate the cost of a 0.85-carat diamond and the cost of a 1.2-carat diamond. Round answers to the nearest dollar.

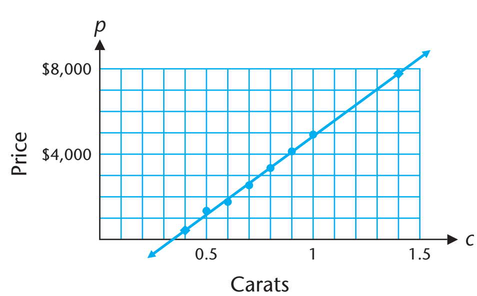
(D) Use the model to estimate the weight of a diamond that sells for $3,000. Round the answer to two significant digits.

SOLUTIONS

(A) A scatter plot is simply a plot of the points in Table 2 [Fig. 4(a)]. To add the graph of the model to the scatter plot, we find any two points that satisfy equation (2) [we choose (0.4, 422) and (1.4, 7,802)]. Plotting these points and drawing a line through them gives us Figure. 4(b).



(a) Scatter plot



(b) Linear model

› **Figure 4**

(B) The rate of change of the price of a diamond with respect to its weight is 7,380. Increasing the weight by 1 carat will increase the price by about $7,380.

(C) The graph of the model [Fig. 4(b)] does not pass through any of the points in the scatter plot, but it comes close to all of them. [Verify this by evaluating equation (2) at $c = 0.5, 0.6, \ldots, 1$.] Thus, we can use equation (2) to approximate points not in Table 2.

$$\begin{aligned} c &= 0.85 & \qquad c &= 1.2 \\ p &= 7{,}380(0.85) - 2{,}530 & \qquad p &= 7{,}380(1.2) - 2{,}530 \\ &= \$3{,}743 & \qquad &= \$6{,}326 \end{aligned}$$

A 0.85-carat diamond will cost about \$3,743 and a 1.2-carat diamond will cost about \$6,326.

(D) To find the weight of a \$3,000 diamond, we solve the following equation for c:

$$\begin{aligned} 7{,}380c - 2{,}530 &= 3{,}000 \\ 7{,}380c &= 3{,}000 + 2{,}530 \\ &= 5{,}530 \\ c &= \frac{5{,}530}{7{,}380} = 0.75 \end{aligned}$$

To two significant digits

A \$3,000 diamond will weigh about 0.75 carats.

MATCHED PROBLEM 5

Prices for emerald-shaped diamonds taken from an online trader are given in Table 3. Repeat Example 5 for this data with the linear model

$$p = 7{,}270c - 2{,}450$$

where p is the price of an emerald-shaped diamond weighing c carats.

Table 3 Emerald-Shaped Diamond Prices

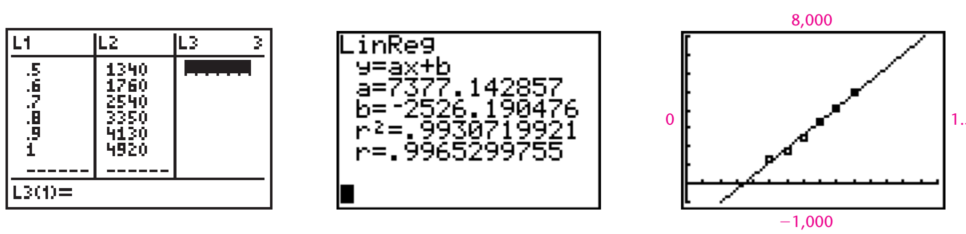
Weight (Carats)	Price
0.5	\$1,350
0.6	\$1,740
0.7	\$2,610
0.8	\$3,320
0.9	\$4,150
1.0	\$4,850

Source: www.tradeshop.com

The model we used in Example 5 was obtained by using a technique called **linear regression** and the model is called the **regression line.** This technique produces a line that is the **best fit** for a given data set. We will not discuss the theory behind this technique, nor the meaning of "best fit." Although you can find a linear regression line by hand, we prefer to leave the calculations to a graphing calculator or a computer. Don't be concerned if you don't have either of these electronic devices. We will supply the regression model in the applications we discuss, as we did in Example 5.

Technology Connections

If you want to use a graphing calculator to construct regression lines, you should consult your user's manual.* The process varies greatly from one calculator to another. Figure 5 shows three of the screens related to the construction of the model in Example 5 on a Texas Instruments TI-83 Plus.



(a) Entering the data. (b) Finding the model. (c) Graphing the data and the model.

Figure 5

*User's manuals for the most popular graphing calculators are readily available on the Internet.

In Example 5, we used the regression line to approximate points that were not given in Table 2, but would fit between points in the table. This process is called **interpolation.** In the next example we use a regression model to approximate points outside the given data set. This process is called **extrapolation** and the approximations are often referred to as **predictions.**

EXAMPLE 6 New Car Sales

Table 4 contains information about new car sales in recent years. The linear regression model for consumer sales is

$$s = 4.85 - 0.13t$$

where s is the numbers of cars (in millions) sold to consumers each year and t is time in years with $t = 0$ corresponding to 2000.

(A) Interpret the slope of the regression line as a rate of change.

(B) Use the regression line to predict the sales in 2015.

Table 4 Sales* of New Cars (millions of vehicles)

	2000	2001	2002	2003	2004
Consumer	4.7	4.6	4.5	4.3	4.2
Business	3.9	3.6	3.4	3.1	3.1

*Includes both purchases and leases.
Source: U.S. Bureau of Transportation Statistics

SOLUTIONS

(A) The slope $m = -0.13$ is the rate of change of sales with respect to time. Since the slope is negative and the sales are given in millions of the units, the sales are decreasing at a rate of $0.13(1{,}000{,}000) = 130{,}000$ cars per year.

(B) If $t = 15$, then

$$s = 4.85 - 0.13(15) = 2.9 \qquad \text{or} \qquad 2{,}900{,}000 \text{ cars}$$

So approximately 2,900,000 new cars will be sold to comsumers in 2015.

MATCHED PROBLEM 6

Repeat Example 6 using the following linear regression model for the number of cars sold to business:

$$b = 4.05 - 0.21t$$

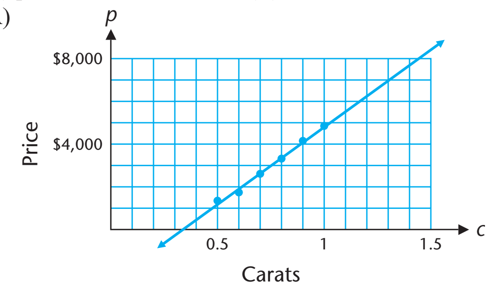
where b is the numbers of cars (in millions) sold to business each year and t is time in years with $t = 0$ corresponding to 2000.

››› EXPLORE-DISCUSS 3

Use the Internet or a library to expand the data in Table 5 to include years after 2004. Compare the actual values for the years you find to the model's predicted values. Discuss the accuracy of predictions given by the regression model as time goes by.

ANSWERS TO MATCHED PROBLEMS

1. (A) The rate of change BSA with respect to weight is 28.55 square inches per pound. (B) Increasing w by 1 pound increases a by 28.55 square inches.
2. Presale price is \$475. Cost is \$387.50
3. (A) $f = -1.5p + 47$ (B) The rate of change of the freezing point with respect to the percentage of propylene glycol in the antifreeze solution is -1.5. Increasing the percentage of propylene glycol by 1% will lower the freezing point by 1.5°F. (C) 41.3%
4. (A) $p = 0.432d + 14.7$ (B) 59 ft
5. (A)



(B) The rate of change of the price of a diamond with respect to the size is 7,270. Increasing the size by 1 carat will increase the price by about \$7,270. (C) \$3,730; \$6,274 (D) 0.75 carats
6. (A) The slope $m = -0.21$ is the rate of change of sales with respect to time. Since the slope is negative and the sales are given in millions of the units, the sales are decreasing at a rate of $0.21(1{,}000{,}000) = 210{,}000$ cars per year. (B) Approximately 900,000 cars will be sold to businesses in 2015.

2-4 Exercises

APPLICATIONS

1. COST ANALYSIS A plant can manufacture 80 golf clubs per day for a total daily cost of \$8,147 and 100 golf clubs per day for a total daily cost of \$9,647.
(A) Assuming that daily cost and production are linearly related, find the total daily cost of producing x golf clubs.
(B) Interpret the slope of this cost equation.
(C) What is the effect of a 1 unit increase in production?

2. COST ANALYSIS A plant can manufacture 50 tennis rackets per day for a total daily cost of \$4,174 and 60 tennis rackets per day for a total daily cost of \$4,634.
(A) Assuming that daily cost and production are linearly related, find the total daily cost of producing x tennis rackets.
(B) Interpret the slope of this cost equation.
(C) What is the effect of a 1 unit increase in production?

3. FORESTRY Forest rangers estimate the height of a tree by measuring the tree's diameter at breast height (DBH) and then using a model constructed for a particular species.* A model for white spruce trees is

$$h = 4.06d + 24.1$$

where d is the DBH in inches and h is the tree height in feet.
(A) Interpret the slope of this model.
(B) What is the effect of a 1-inch increase in DBH?
(C) How tall is a white spruce with a DBH of 12 inches? Round answer to the nearest foot.
(D) What is the DBH of a white spruce that is 100 feet tall? Round answer to the nearest inch.

4. FORESTRY A model for black spruce trees is

$$h = 2.27d + 33.1$$

where d is the DBH in inches and h is the tree height in feet.

*Models in Problems 3 and 4 are based on data found at http://flash.lakeheadu.ca/~fluckai/htdbh04.xls

(A) Interpret the slope of this model.
(B) What is the effect of a 1-inch increase in DBH?
(C) How tall is a black spruce with a DBH of 12 inches? Round answer to the nearest foot.
(D) What is the DBH of a black spruce that is 100 feet tall? Round answer to the nearest inch.

5. In 1983 Dr. J. D. Robinson and Dr. D. R. Miller published the following models for estimating the weight of a woman:

$$\text{Robinson: } w = 108 + 3.7h$$
$$\text{Miller: } w = 117 + 3.0h$$

where w is weight (in pounds) and h is height over 5 feet (in inches).
(A) Interpret the slope of each model.
(B) If a women is 5′6″ tall, what does each model predict her weight to be?
(C) If a women weighs 140 pounds, what does each model predict her height to be?

6. Dr. J. D. Robinson and Dr. D. R. Miller also published the following models for estimating the weight of a man:

$$\text{Robinson: } w = 115 + 4.2h$$
$$\text{Miller: } w = 124 + 3.1h$$

where w is weight (in pounds) and h is height over 5 feet (in inches).
(A) Interpret the slope of each model.
(B) If a man is 5′10″ tall, what does each model predict his weight to be?
(C) If a man weighs 160 pounds, what does each model predict his height to be?

7. SPEED OF SOUND The speed of sound through the air near sea level is linearly related to the temperature of the air. If sound travels at 741 mph at 32°F and at 771 mph at 72°F, construct a linear model relating the speed of sound (s) and the air temperature (t). Interpret the slope of this model.

8. SPEED OF SOUND The speed of sound through the air near sea level is linearly related to the temperature of the air. If sound travels at 337 mps (meters per second) at 10°C and at 343 mps at 20°C, construct a linear model relating the speed of sound (s) and the air temperature (t). Interpret the slope of this model.

9. CIGARETTE SMOKING The percentage of male cigarette smokers in the United States declined from 28.3% in 1990 to 25.2% in 2000. Find a linear model relating percentage of male smokers (m) to years since 1990 (t). Predict the year in which the population of male smokers will fall below 20%.

10. CIGARETTE SMOKING The percentage of female cigarette smokers in the United States declined from 22.9% in 1990 to 21.1% in 2000. Find a linear model relating percentage of female smokers (f) to years since 1990 (t). Predict the year in which the population of female smokers will fall below 18%.

11. BUSINESS—DEPRECIATION A farmer buys a new tractor for \$142,000 and assumes that it will have a trade-in value of \$67,000 after 10 years. The farmer uses a constant rate of depreciation (commonly called **straight-line depreciation**—one of several methods permitted by the IRS) to determine the annual value of the tractor.
(A) Find a linear model for the depreciated value V of the tractor t years after it was purchased.
(B) Interpret the slope of this model.
(C) What is the depreciated value of the tractor after 6 years?

12. BUSINESS—DEPRECIATION A charter fishing company buys a new boat for \$154,900 and assumes that it will have a trade-in value of \$46,100 after 16 years.
(A) Use straight-line depreciation to find a linear model for the depreciated value V of the boat t years after it was purchased.
(B) Interpret the slope of this model.
(C) In which year will the depreciated value of the boat fall below \$100,000?

13. BUSINESS—MARKUP POLICY A drugstore sells a drug costing \$85 for \$112 and a drug costing \$175 for \$238.
(A) If the markup policy of the drugstore is assumed to be linear, write an equation that expresses retail price R in terms of cost C (wholesale price).
(B) What is the slope of the graph of the equation found in part A? Interpret verbally.
(C) What does a store pay (to the nearest dollar) for a drug that retails for \$185?

14. BUSINESS—MARKUP POLICY A clothing store sells a shirt costing \$20 for \$33 and a jacket costing \$60 for \$93.
(A) If the markup policy of the store for items costing over \$10 is assumed to be linear, write an equation that expresses retail price R in terms of cost C (wholesale price).
(B) What is the slope of the equation found in part A? Interpret verbally.
(C) What does a store pay for a suit that retails for \$240?

15. FLIGHT CONDITIONS In stable air, the air temperature drops about 5°F for each 1,000-foot rise in altitude.
(A) If the temperature at sea level is 70°F and a commercial pilot reports a temperature of −20°F at 18,000 feet, write a linear equation that expresses temperature T in terms of altitude A (in thousands of feet).
(B) How high is the aircraft if the temperature is 0°F?

16. FLIGHT NAVIGATION An airspeed indicator on some aircraft is affected by the changes in atmospheric pressure at different altitudes. A pilot can estimate the true airspeed by observing the indicated airspeed and adding to it about 2% for every 1,000 feet of altitude.
(A) If a pilot maintains a constant reading of 200 miles per hour on the airspeed indicator as the aircraft climbs from sea level to an altitude of 10,000 feet, write a linear equation that expresses true airspeed T (miles per hour) in terms of altitude A (thousands of feet).
(B) What would be the true airspeed of the aircraft at 6,500 feet?

17. RATE OF DESCENT—PARACHUTES At low altitudes, the altitude of a parachutist and time in the air are linearly related. A jump at 2,880 ft using the U.S. Army's T-10 parachute system lasts 120 seconds.
(A) Find a linear model relating altitude a (in feet) and time in the air t (in seconds).
(B) The **rate of descent** is the speed at which the jumper falls. What is the rate of descent for a T-10 system?

18. RATE OF DESCENT—PARACHUTES The U.S. Army is considering a new parachute, the ATPS system. A jump at 2,880 ft using the ATPS system lasts 180 seconds.
(A) Find a linear model relating altitude a (in feet) and time in the air t (in seconds).
(B) What is the rate of descent for an ATPS system parachute?

19. REAL ESTATE Table 5 contains recent average and median purchase prices for a house in Texas. A linear regression model for the average purchase price is

$$y = 4.6x + 146$$

where x is years since 2000 and y is average purchase price (in thousands of dollars).
(A) Plot the average price data and the model on the same axes.
(B) Predict the average price in 2010.
(C) Interpret the slope of the model.

Table 5 Texas Real Estate Prices

Year	Average Price (in thousands)	Median Price (in thousands)
2000	\$146	\$112
2001	\$150	\$120
2002	\$156	\$125
2003	\$160	\$128
2004	\$164	\$130

Source: Real Estate Center at Texas A&M University

20. REAL ESTATE A linear regression model for the median purchase price in Table 5 is

$$y = 4.4x + 114$$

where x is years since 2000 and y is median purchase price (in thousands of dollars).
(A) Plot the median price data and the model on the same axes.
(B) Predict the median price in 2010.
(C) Interpret the slope of the model.

21. LICENSED DRIVERS Table 6 contains the state population and the number of licensed drivers in the state (both in millions) for the states with population under 10 million. The regression model for this data is

$$y = 0.69x + 0.5$$

where x is the state population and y is the number of licensed drivers in the state.

Table 6 Licensed Drivers in 2003

State	Population	Licensed Drivers
Alaska	6.5	4.8
Delaware	8.2	5.9
Montana	9.2	7
North Dakota	6.3	4.6
South Dakota	7.6	5.5
Vermont	6.2	5.4
Wyoming	5	3.8

Source: Bureau of Transportation Statistics

(A) Plot the data in Table 6 and the model on the same axes.
(B) If the population of Alabama in 2003 is about 4.5 million, use the model to estimate the number of licensed drivers in Alabama.
(C) If the population of Arizona in 2003 is about 5.6 million, use the model to estimate the number of licensed drivers in Arizona.

22. LICENSED DRIVERS Table 7 contains the state population and the number of licensed drivers in the state (both in millions) for several states with population over 10 million. The regression model for this data is

$$y = 0.59x + 1.06$$

where x is the state population and y is the number of licensed drivers in the state.

Table 7 Licensed Drivers in 2003

State	Population	Licensed Drivers
California	35	22
Florida	17	13
Illinois	13	8
Michigan	10	7
New York	19	11
Ohio	11	8
Pennsylvania	12	8
Texas	22	13

Source: Bureau of Transportation Statistics

(A) Plot the data in Table 7 and the model on the same axes.
(B) If the population of California in 2003 is about 35.5 million, use the model to estimate the number of licensed drivers in California.
(C) If the population of New York in 2003 is about 19.2 million, use the model to estimate the number of licensed drivers in New York.

Problems 23–26 require a graphing calculator or a computer that can calculate the linear regression line for a given data set.

23. OLYMPIC GAMES Find a linear regression model for the men's 100-meter freestyle data given in Table 8, where x is years since 1968 and y is winning time (in seconds). Do the same for the women's 100-meter freestyle data. (Round regression coefficients to four significant digits.) Do these models indicate that the women will eventually catch up with the men? If so, when? Do you think this will actually occur?

Table 8 Winning Times in Olympic Swimming Events

	100-Meter Freestyle Men	100-Meter Freestyle Women	200-Meter Backstroke Men	200-Meter Backstroke Women
1968	52.20	60.0	2:09.60	2:24.80
1972	51.22	58.59	2:02.82	2:19.19
1976	49.99	55.65	1:59.19	2:13.43
1980	50.40	54.79	2:01.93	2:11.77
1984	49.80	55.92	2:00.23	2:12.38
1988	48.63	54.93	1:59.37	2:09.29
1992	49.02	54.65	1:58.47	2:07.06
1996	48.74	54.50	1:58.54	2:07.83
2000	48.30	53.83	1:56.76	2:08.16
2004	48.17	53.84	1:54.76	2:09.16

Source: www.infoplease.com

24. OLYMPIC GAMES Find a linear regression model for the men's 200-meter backstroke data given in Table 8 where x is years since 1968 and y is winning time (in seconds). Do the same for the women's 200-meter backstroke data. (Round regression coefficients to five significant digits.) Do these models indicate that the women will eventually catch up with the men? If so, when? Do you think this will actually occur?

25. SUPPLY AND DEMAND Table 9 contains price–supply data and price–demand data for corn. Find a linear regression model for the price–supply data where x is supply (in billions of bushels) and y is price (in dollars). Do the same for the price–demand data. (Round regression coefficients to three significant digits.) Find the price at which supply and demand are equal. (In economics, this price is referred to as the **equilibrium price.**)

Table 9 Supply and Demand for U.S. Corn

Price ($/bu.)	Supply (Billion bu.)	Price ($/bu.)	Demand (Billion bu.)
2.15	6.29	2.07	9.78
2.29	7.27	2.15	9.35
2.36	7.53	2.22	8.47
2.48	7.93	2.34	8.12
2.47	8.12	2.39	7.76
2.55	8.24	2.47	6.98

Source: www.usda.gov/nass/pubs/histdata.htm

26. SUPPLY AND DEMAND Table 10 contains price–supply data and price–demand data for soybeans. Find a linear regression model for the price–supply data where x is supply (in billions of bushels) and y is price (in dollars). Do the same for the price–demand data. (Round regression coefficients to three significant digits.) Find the equilibrium price for soybeans.

Table 10 Supply and Demand for U.S. Soybeans

Price ($/bu.)	Supply (Billion bu.)	Price ($/bu.)	Demand (Billion bu.)
5.15	1.55	4.93	2.60
5.79	1.86	5.48	2.40
5.88	1.94	5.71	2.18
6.07	2.08	6.07	2.05
6.15	2.15	6.40	1.95
6.25	2.27	6.66	1.85

Source: www.usda.gov/nass/pubs/histdata.htm

CHAPTER 2 Review

2-1 Cartesian Coordinate System

A **Cartesian** or **rectangular coordinate system** is formed by the intersection of a horizontal real number line and a vertical real number line at their origins. These lines are called the **coordinate axes.** The **horizontal axis** is often referred to as the ***x* axis** and the **vertical axis** as the ***y* axis.** These axes divide the plane into four **quadrants.** Each point in the plane corresponds to its **coordinates**—an ordered pair (a, b) determined by passing horizontal and vertical lines through the point. The **abscissa** or ***x* coordinate** a is the coordinate of the intersection of the vertical line with the horizontal axis, and the **ordinate** or ***y* coordinate** b is the coordinate of the intersection of the horizontal line with the vertical axis. The point $(0, 0)$ is called the **origin.** A **solution** of an equation in two variables is an ordered pair of real numbers that makes the equation a true statement. The **solution set** of an equation is the set of all its solutions. The **graph of an equation in two variables** is the graph of its solution set formed using **point-by-point plotting** or with the aid of a graphing utility. The **reflection** of the point (a, b) through the ***y* axis** is the point $(-a, b)$, through the ***x* axis** is the point $(a, -b)$, and through the **origin** is the point $(-a, -b)$. The **reflection** of a graph is the reflection of each point on the graph. If reflecting a graph through the y axis, x axis, or origin does not change its shape, the graph is said to be symmetric with respect to the ***y* axis, *x* axis,** or **origin,** respectively. To **test** an equation for symmetry, determine if the equation is unchanged when y is replaced with $-y$ (x axis symmetry), x is replaced with $-x$ (y axis symmetry), or both x and y are replaced with $-x$ and $-y$ (origin symmetry).

2-2 Distance in the Plane

The **distance** between the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and the **midpoint** of the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The **standard form** for the equation of a **circle** with **radius** r and **center** at (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2, \qquad r > 0$$

2-3 Equations of a Line

The standard form for the equation of a line is $Ax + By = C$, where A, B, and C are constants, A and B not both 0. The ***y* intercept** is the y coordinate of the point where the graph crosses the y axis, and the ***x* intercept** is the x coordinate of the point where the graph crosses the x axis. The slope of the line through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{if } x_1 \neq x_2$$

The slope is not defined for a vertical line where $x_1 = x_2$. Two lines with slopes m_1 and m_2 are parallel if and only if $m_1 = m_2$ and perpendicular if and only if $m_1 m_2 = -1$.

Equations of a Line

Standard form	$Ax + By = C$	A and B not both 0
Slope–intercept form	$y = mx + b$	Slope: m; y intercept: b
Point–slope form	$y - y_1 = m(x - x_1)$	Slope: m; Point: (x_1, y_1)
Horizontal line	$y = b$	Slope: 0
Vertical line x	$x = a$	Slope: Undefined

2-4 Linear Equations and Models

A **mathematical model** is a problem whose solution will provide information about a real-world problem. If $y = mx + b$, then the variables x and y are **linearly related** and the **rate of change** of y with respect to x is the constant m. If x and y are not linearly related, the ratio $(y_2 - y_1)/(x_2 - x_1)$ is called the **average rate of change** of y with respect to x. **Regression analysis** produces a function whose graph is a **curve** that **fits** (approximates) a set of data points. A **scatter plot** is the graph of the points in a data set. **Linear regression** produces a **regression line** that is the **best fit** (in some sense) for a given data set. Graphing calculators or other electronic devices are frequently used to find regression lines.

CHAPTER 2 Review Exercises

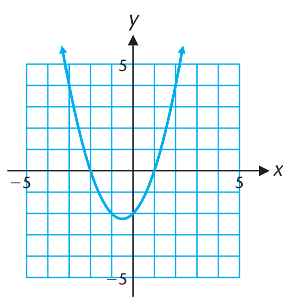
Work through all the problems in this chapter review and check answers in the back of the book. Answers to all review problems are there, and following each answer is a number in italics indicating the section in which that type of problem is discussed. Where weaknesses show up, review appropriate sections in the text.

1. Plot $A = (-4, 1)$, $B = (2, -3)$, and $C = (-1, -2)$ in a rectangular coordinate system.

2. Refer to Problem 1. Plot the reflection of A through the x axis, the reflection of B through the y axis, and the reflection of C through the origin.

3. Test each equation for symmetry with respect to the x axis, y axis, and origin and sketch its graph.
(A) $y = 2x$ (B) $y = 2x - 1$
(C) $y = 2|x|$ (D) $x = 2|y|$

4. Use the following graph to estimate to the nearest integer the missing coordinates of the indicated points. (Be sure you find all possible answers.)
(A) (0, ?) (B) (?, 0) (C) (?, 4)



5. Given the points $A(-2, 3)$ and $B(4, 0)$, find:
(A) Distance between A and B
(B) Slope of the line through A and B
(C) Slope of a line perpendicular to the line through A and B

6. Write the equation of a circle with radius $\sqrt{7}$ and center:
(A) (0, 0) (B) (3, −2)

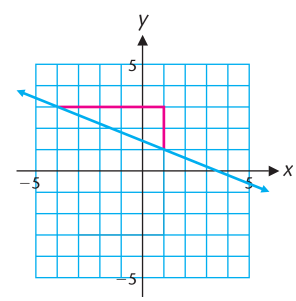
7. Find the center and radius of the circle given by

$$(x + 3)^2 + (y - 2)^2 = 5$$

8. Let M be the midpoint of A and B, where $A = (a_1, a_2)$, $B = (2, -5)$, and $M = (-4, 3)$.
(A) Use the fact that −4 is the average of a_1 and 2 to find a_1.
(B) Use the fact that 3 is the average of a_2 and −5 to find a_2.
(C) Find $d(A, M)$ and $d(M, B)$.

9. (A) Graph the triangle with vertices $A = (-1, -2)$, $B = (4, 3)$, and $C = (1, 4)$.
(B) Find the perimeter to two decimal places.
(C) Use the Pythagorean theorem to determine if the triangle is a right triangle.
(D) Find the midpoint of each side of the triangle.

10. Use the graph of the linear function in the figure to find the rise, run, and slope. Write the equation of the line in the form $Ax + By = C$, where A, B, and C are integers with $A > 0$. (The horizontal and vertical line segments have integer lengths.)



11. Graph $3x + 2y = 9$ and indicate its slope.

12. Write an equation of a line with x intercept 6 and y intercept 4. Write the final answer in the standard form $Ax + By = C$, where A, B, and C are integers.

13. Write the slope–intercept form of the equation of the line with slope $-\frac{2}{3}$ and y intercept 2.

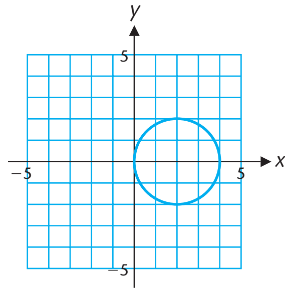
14. Write the equations of the vertical and horizontal lines passing through the point (−3, 4). What is the slope of each?

Test each equation in Problems 15–18 for symmetry with respect to the x axis, y axis, and the origin. Sketch the graph of the equation.

15. $y = x^2 - 2$ **16.** $y^2 = x - 2$

17. $9y^2 + 4x^2 = 36$ **18.** $9y^2 - 4x^2 = 36$

19. Write a verbal description of the graph shown in the figure and then write an equation that would produce the graph.



20. (A) Find an equation of the line through $P(-4, 3)$ and $Q(0, -3)$. Write the final answer in the standard form $Ax + By = C$, where A, B, and C are integers with $A > 0$.
(B) Find $d(P, Q)$.

21. Write the slope–intercept form of the equation of the line that passes through the point $(-2, 1)$ and is
(A) parallel to the line $6x + 3y = 5$
(B) perpendicular to the line $6x + 3y = 5$

22. Find the equation of a circle that passes through the point $(-1, 4)$ with center at $(3, 0)$.

23. Find the center and radius of the circle given by

$$x^2 + y^2 + 4x - 6y = 3$$

24. Find the equation of the set of points equidistant from (3, 3) and (6, 0). What is the name of the geometric figure formed by this set?

25. Are the graphs of $mx - y = b$ and $x + my = b$ parallel, perpendicular, or neither? Justify your answer.

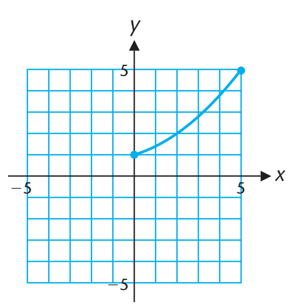
26. Use completing the square to find the center and radius of the circle with equation:

$$x^2 - 4x + y^2 - 2y - 3 = 0$$

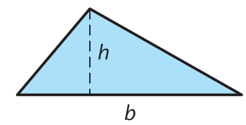
27. Refer to Problem 26. Find the equation of the line tangent to the circle at the point (4, 3). Graph the circle and the line on the same coordinate system.

28. Find the equation of a circle with center $(4, -3)$ whose graph passes through the point (1, 2).

29. Extend the following graph to one that exhibits the indicated symmetry:
(A) x axis only (B) y axis only
(C) origin only (D) x axis, y axis, and origin



Problems 30 and 31 refer to a triangle with base b and height h (see the figure). Write a mathematical expression in terms of b and h for each of the verbal statements in Problems 30 and 31.



30. The base is five times the height.

31. The height is one-fourth of the base.

APPLICATIONS

32. LINEAR DEPRECIATION A computer system was purchased by a small company for \$12,000 and is assumed to have a depreciated value of \$2,000 after 8 years. If the value is depreciated linearly from \$12,000 to \$2,000:
(A) Find the linear equation that relates value V (in dollars) to time t (in years).
(B) What would be the depreciated value of the system after 5 years?

33. COST ANALYSIS A video production company is planning to produce an instructional CD. The producer estimates that it will cost \$24,900 to produce the CD and \$5 per unit to copy and distribute the CD. The budget for this project is \$62,000. How many CDs can be produced without exceeding the budget?

34. FORESTRY Forest rangers estimate the height of a tree by measuring the tree's diameter at breast height (DBH) and then using a model constructed for a particular species. A model for sugar maples is

$$h = 2.9d + 30.2$$

where d is the DBH in inches and h is the tree height in feet.
(A) Interpret the slope of this model.
(B) What is the effect of a 1-inch increase in DBH?
(C) How tall is a sugar maple with a DBH of 3 inches? Round answer to the nearest foot.
(D) What is the DBH of a sugar maple that is 45 feet tall? Round answer to the nearest inch.

35. ESTIMATING BODY SURFACE AREA An important criterion for determining drug dosage for children is the patient's body surface area (BSA). John D. Current published the following useful model for estimating BSA*:

$$\text{BSA} = 1{,}321 + 0.3433 \times \text{Wt}$$

where BSA is given in square centimeters and Wt in grams.
(A) Interpret the slope of this model.
(B) What is the effect of a 100-gram increase in weight?
(C) What is the BSA for a child that weighs 15 kilograms?

*"Body Surface Area in Infants and Children," *The Internet Journal of Anesthesiology,* 1998, Volume 2, Number 2.

★**36.** **ARCHITECTURE** A circular arc forms the top of an entryway with 6-foot vertical sides 8 feet apart. If the top of the arc is 2 feet above the ends, what is the radius of the arc?

37. **SPORTS MEDICINE** The following quotation was found in a sports medicine handout: "The idea is to raise and sustain your heart rate to 70% of its maximum safe rate for your age. One way to determine this is to subtract your age from 220 and multiply by 0.7."
(A) If H is the maximum safe sustained heart rate (in beats per minute) for a person of age A (in years), write a formula relating H and A.
(B) What is the maximum safe sustained heart rate for a 20-year-old?
(C) If the maximum safe sustained heart rate for a person is 126 beats per minute, how old is the person?

38. **DATA ANALYSIS** Winning times in the men's Olympic 400-meter freestyle event in minutes for selected years are given in Table 1. A mathematical model for these data is

$$f(x) = -0.021x + 5.57$$

where x is years since 1900.
(A) Compare the model and the data graphically and numerically.
(B) Estimate (to three decimal places) the winning time in 2008.

Table 1

Year	Time
1912	5.41
1932	4.81
1952	4.51
1972	4.00
1992	3.75

39. **POLITICAL SCIENCE** Association of economic class and party affiliation did not start with Roosevelt's New Deal; it goes back to the time of Andrew Jackson (1767–1845). Paul Lazarsfeld of Columbia University published an article in the November 1950 issue of *Scientific American* in which he discussed statistical investigations of the relationships between economic class and party affiliation. The data in Table 2 are taken from this article.

Table 2 Political Affiliations in 1836

Ward	Average Assessed Value per Person (in $100)	Democratic Votes (%)
12	1.7	51
3	2.1	49
1	2.3	53
5	2.4	36
2	3.6	65
11	3.7	35
10	4.7	29
4	6.2	40
6	7.1	34
9	7.4	29
8	8.7	20
7	11.9	23

(A) Find a linear regression model for the data in the second and third columns of the table, using the average assessed value as the independent variable.
(B) Use the linear regression model to predict (to two decimal places) the percentage of votes for democrats in a ward with an average assessed value of $300.

CHAPTER 2

››› GROUP ACTIVITY Rates of Change

In Section 2-4, we noted that the rate of change of linearly related quantities is always constant. In this activity, we consider some rates of change that are not constant.

1. Average Rate. If you score 90 on your first math exam and 100 on the second exam, then your average exam score for the two exams is $\frac{1}{2}(90 + 100) = 95$. The number 95 is called the **arithmetic average** of 90 and 100. Now suppose you walk uphill at a rate of 3 mph for 5 hours and then turn around and return to your starting point by walking downhill at 6 mph for 2.5 hours. The arithmetic average of the rates for each leg of the trip is $\frac{1}{2}(3 + 6) = 4.5$ mph. On the other hand, you walked a total distance of 30 miles in 7.5 hours so that the rate for the round-trip is $30/7.5 = 4$ mph. Which is your *average rate?* The basic formula $D = RT$ is valid whenever an object travels a distance D at a *constant* rate R for a fixed time T. If the rate is not constant, then this formula can still be used but must be interpreted differently. To be precise, for objects moving at nonconstant rates, **average rate is total distance divided by total time.** Thus, your average rate for the total trip up and down the hill is 4 mph, not 4.5 mph. The formula $R = D/T$ now has two interpretations: $R = D/T$ is the *rate* for an object moving at a constant rate and the *average rate* for an object whose rate is not always the same.

(A) If r is the rate for one leg of a round-trip and s is the rate for the return trip, express the average rate for the round-trip in terms of r and s.

(B) A boat can travel 10 mph in still water. The boat travels 60 miles up a river with a 5 mph current and then returns to its starting point. Find the average rate for the round-trip using the definition of average rate and then check with the formula you found in part A.

(C) Referring to the hill-climbing example discussed earlier, if you walk up the hill at 3 mph, how fast must you walk downhill to average 6 mph for the round-trip? (This is similar to a famous problem communicated to Albert Einstein by Max Wertheimer. See Abraham S. Luchins and Edith H. Luchins, The Einstein–Wertheimer Correspondence on Geometric Proofs and Mathematical Puzzles, *Mathematical Intelligencer 2,* Spring 1990, pp. 40–41. For a discussion of this and other interesting rate–time problems, see Lawrence S. Braden, My Favorite Rate–Time Problems, *Mathematics Teacher,* November 1991, pp. 635–638.)

2. Instantaneous Rate. One of the fundamental concepts of calculus is the *instantaneous rate* of a moving object, which is closely related to the average rate. To introduce this concept, consider the following problem.

A small steel ball dropped from a tower will fall a distance of y feet in x seconds, as given approximately by the formula (from physics)

$$y = 16x^2$$

Figure 1 shows the position of the ball on a number line (positive direction down) at the end of 0, 1, 2, and 3 seconds. Clearly, the ball is not falling at a constant rate.

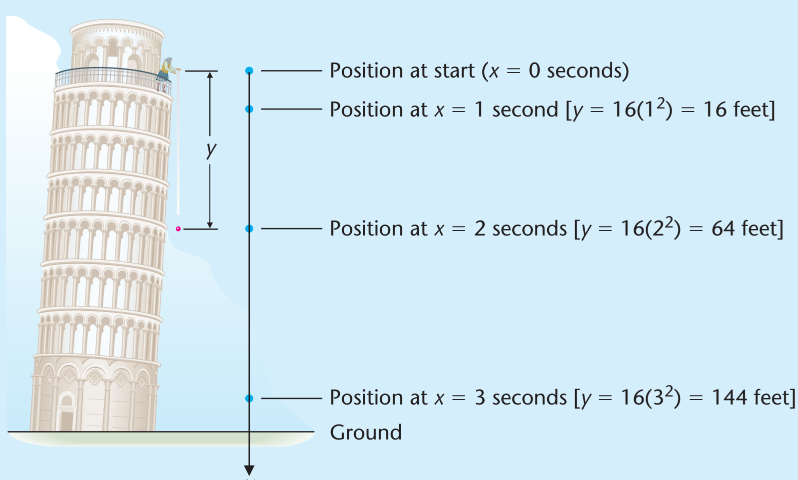


Figure 1 Position of a falling object [*Note:* Positive direction is down.]

(A) What is the average rate that the ball falls during the first second (from $x = 0$ to $x = 1$ second)? During the second second? During the third second?

By definition, average rate involves the distance an object travels over an *interval* of time, as in part A. How can we determine the rate of an object at a *given instant* of time? For example, how fast is the ball falling at exactly 2 seconds after it was released? We will approach this problem from two directions, numerically and algebraically.

(B) Complete the following table of average rates. What number do these average rates appear to approach?

Time interval	[1.9, 2]	[1.99, 2]	[1.999, 2]	[1.9999, 2]
Distance fallen				
Average rate				

(C) Show that the average rate over the time interval $[t, 2]$ is $\frac{64 - 16t^2}{2 - t}$. Simplify this algebraic expression and discuss its values for t very close to 2.

(D) Based on the results of parts B and C, how fast do you think the ball is falling at 2 seconds?