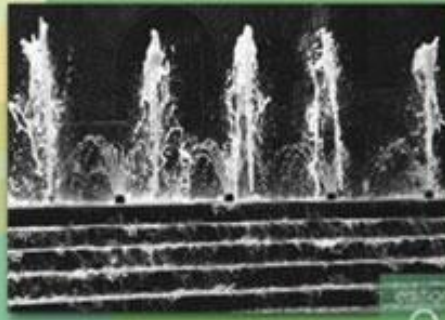


**SOLUTIONS MANUAL**



College Algebra



2

Ratti & McWaters

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# Chapter P Basic Concepts of Algebra

## P.1 The Real Numbers and Their Properties

### P.1 Practice Problems

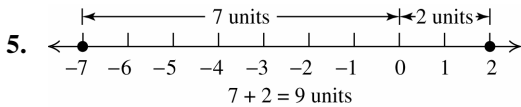
1. a. True
- b. True
- c. True
2.  $A = \{-3, -1, 0, 1, 3\}$ ,  $B = \{-4, -2, 0, 2, 4\}$   
 $A \cap B = \{0\}$ ,  
 $A \cup B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

3.  $A' = \{1, 2, 4, 5, 7, 8\}$

4. a.  $|-10| = 10$

    b.  $|3 - 4| = |-1| = 1$

    c.  $|2(-3) + 7| = |-1| = 1$



$d(-7, 2) = |-7 - 2| = |-9| = 9$

6. a.  $(-3) \cdot 5 + 20 = -15 + 20 = 5$

    b.  $7 - 2 \cdot 3 = 7 - 6 = 1$

    c.  $\frac{9-1}{4} - 5 \cdot 7 = \frac{8}{4} - 5 \cdot 7 = 2 - 35 = -33$

7. a. 2                      b.  $-\frac{1}{7}$

8. a.  $\frac{7}{4} + \frac{3}{8} = \frac{14}{8} + \frac{3}{8} = \frac{17}{8}$

    b.  $\frac{8}{3} - \frac{2}{5} = \frac{40}{15} - \frac{6}{15} = \frac{34}{15}$

    c.  $\frac{9}{14} \cdot \frac{7}{3} = \frac{\cancel{9}^3}{\cancel{14}_2} \cdot \frac{\cancel{7}^1}{\cancel{3}_1} = \frac{3}{2}$

    d.  $\frac{\frac{5}{8}}{\frac{15}{16}} = \frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \cdot \frac{16}{15} = \frac{\cancel{5}^1}{\cancel{15}_3} \cdot \frac{\cancel{16}^2}{\cancel{8}_1} = \frac{2}{3}$

9. a.  $(3 - 2) \div 3 + 3 = 1 \div 3 + 3 = \frac{10}{3}$

    b.  $7 - \frac{-1}{|3|} = 7 + \frac{1}{3} = \frac{22}{3}$

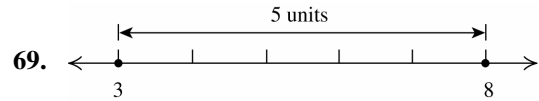
10.  $^{\circ}\text{C} = \frac{39}{3} + 4 = 13 + 4 = 17^{\circ}\text{C}$

11. Since replacing  $x$  with  $-2$  results in a denominator of 0,  $-2$  is not in the domain. Thus, the domain is  $(-\infty, -2) \cup (-2, \infty)$ .

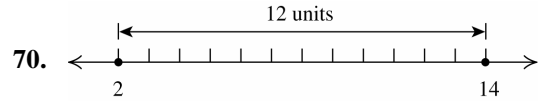
### P.1 A Exercises: Basic Skills and Concepts

1. Whole numbers are formed by adding the number zero to the set of natural numbers.
2. The number  $-3$  is an integer, but it is also a rational number and a real number.
3. If  $a < b$ , then  $a$  is to the left of  $b$  on the number line.
4. If a real number is not a rational number, it is an irrational number.
5. True
6. False.  $\frac{-5}{2} = -2\frac{1}{2}$
7.  $0.\bar{3}$ , repeating
8.  $0.\bar{6}$ , repeating
9.  $-0.8$ , terminating
10.  $-0.25$ , terminating
11.  $0.\overline{27}$ , repeating
12.  $0.\bar{3}$ , repeating
13.  $3.\overline{16}$ , repeating
14.  $2.\overline{73}$ , repeating
15. rational
16. rational
17. rational
18. rational
19. rational
20. rational
21. irrational
22. irrational
23. rational
24. rational
25.  $3 > -2$
26.  $-3 < -2$

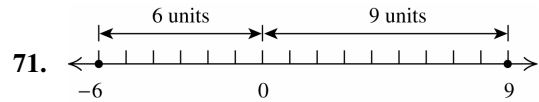
27.  $\frac{1}{2} \geq \frac{1}{2}$       28.  $x < x + 1$   
 29.  $5 \leq 2x$       30.  $x - 1 > 2$   
 31.  $-x > 0$       32.  $x < 0$   
 33.  $2x + 7 \leq 14$       34.  $2x + 3 \leq 5$   
 35.  $=$       36.  $<$   
 37.  $<$       38.  $=$   
 39.  $\{1, 2, 3\}$       40.  $\{1, 2, 3, 4, 5\}$   
 41.  $\{3, 4, 5, 6, 7\}$       42.  $\{8, 9, 10, 11\}$   
 43.  $\{-3, -2, -1\}$       44.  $\{0, 1, 2, 3, 4\}$   
 45.  $A \cup B = \{-4, -3, -2, 0, 1, 2, 3, 4\}$   
 46.  $A \cap B = \{0, 2, 4\}$   
 47.  $A' = \{-3, -1, 1, 3\}$   
 48.  $C' = \{1, 3, 4\}$   
 49.  $(B \cap C) = \{-3, 0, 2\}$   
 $A' \cup (B \cap C) = \{-3, -1, 0, 1, 2, 3\}$   
 50.  $B' = \{-4, -2, -1\}$   
 $A \cap B' = \{-4, -2\}$   
 51.  $(A \cup B)' = \{-1\}$   
 52.  $(A \cap B)' = \{-4, -3, -2, -1, 1, 3\}$   
 53.  $(A \cup B) \cap C = \{-4, -3, -2, 0, 2\}$   
 54.  $(A \cup B) \cup C = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$   
 55. 20      56. 12  
 57. -4      58. -17  
 59.  $\frac{5}{7}$       60.  $\frac{3}{5}$   
 61.  $5 - \sqrt{2}$       62.  $5 - \sqrt{2}$   
 63. 1      64. -1  
 65. 12      66. 2  
 67. 3      68. 3



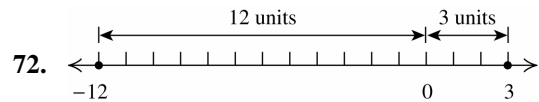
$$d(3, 8) = |3 - 8| = |-5| = 5$$



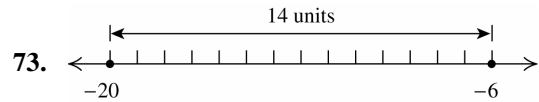
$$d(2, 14) = |2 - 14| = |-12| = 12$$



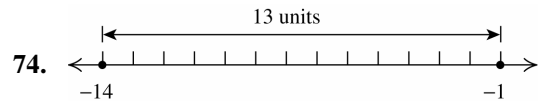
$$d(-6, 9) = |-6 - 9| = |-15| = 15$$



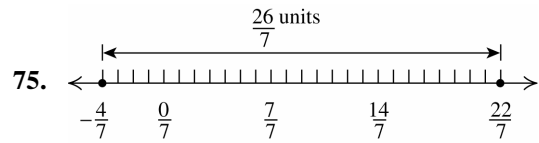
$$d(-12, 3) = |-12 - 3| = |-15| = 15$$



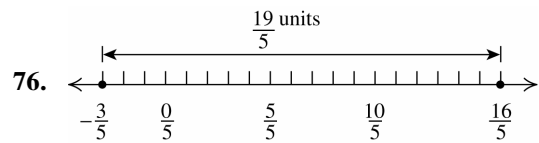
$$d(-20, -6) = |-20 - (-6)| = |-14| = 14$$



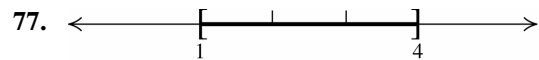
$$d(-14, -1) = |-14 - (-1)| = |-13| = 13$$



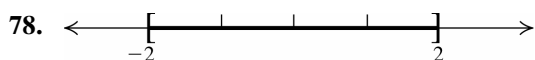
$$d\left(\frac{22}{7}, -\frac{4}{7}\right) = \left|\frac{22}{7} - \left(-\frac{4}{7}\right)\right| = \left|\frac{26}{7}\right| = \frac{26}{7}$$



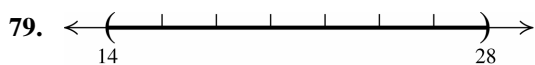
$$d\left(\frac{16}{5}, -\frac{3}{5}\right) = \left|\frac{16}{5} - \left(-\frac{3}{5}\right)\right| = \left|\frac{19}{5}\right| = \frac{19}{5}$$



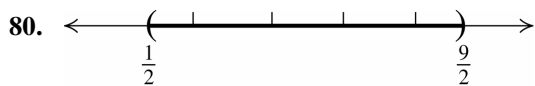
$$1 \leq x \leq 4$$



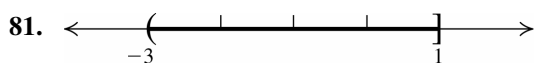
$$-2 \leq x \leq 2$$



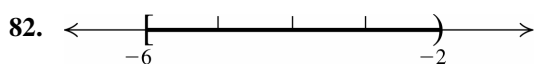
$$14 < x < 28$$



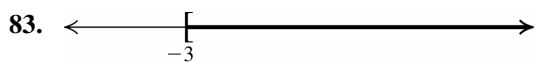
$$\frac{1}{2} < x < \frac{9}{2}$$



$$-3 < x \leq 1$$



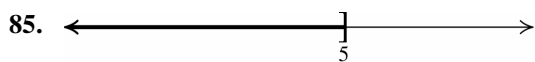
$$-6 \leq x < -2$$



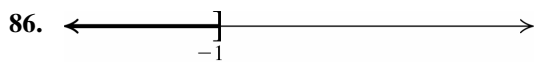
$$x \geq -3$$



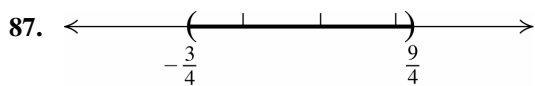
$$x \geq 0$$



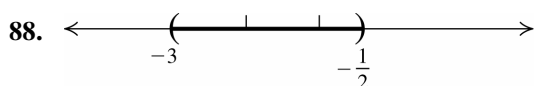
$$x \leq 5$$



$$x \leq -1$$



$$-\frac{3}{4} < x < \frac{9}{4}$$



$$-3 < x < -\frac{1}{2}$$

89.  $4(x+1) = 4x+4$

90.  $(-3)(2-x) = -6+3x$

91.  $5(x-y+1) = 5x-5y+5$

92.  $2(3x+5-y) = 6x+10-2y$

93. additive inverse:  $-5$ ; reciprocal:  $\frac{1}{5}$

94. additive inverse:  $\frac{2}{3}$ ; reciprocal:  $-\frac{3}{2}$

95. additive inverse:  $0$ ; no reciprocal

96. additive inverse:  $-1.7$ ; reciprocal:  $\frac{10}{17}$

97. additive inverse

98. additive inverse

99. multiplicative identity

100. multiplicative identity

101. associative property of multiplication

102. associative property of multiplication

103. multiplicative inverse

104. multiplicative inverse

105. additive identity

106. additive identity

107. associative property of addition

108. commutative, associative property of addition

109.  $\frac{3}{5} + \frac{4}{3} = \frac{9}{15} + \frac{20}{15} = \frac{29}{15}$

110.  $\frac{7}{10} + \frac{3}{4} = \frac{14}{20} + \frac{15}{20} = \frac{29}{20}$

111.  $\frac{6}{5} + \frac{5}{7} = \frac{42}{35} + \frac{25}{35} = \frac{67}{35}$

112.  $\frac{9}{2} + \frac{5}{12} = \frac{54}{12} + \frac{5}{12} = \frac{59}{12}$

113.  $\frac{5}{6} + \frac{3}{10} = \frac{25}{30} + \frac{9}{30} = \frac{34}{30} = \frac{17}{15}$

114.  $\frac{8}{15} + \frac{2}{9} = \frac{24}{45} + \frac{10}{45} = \frac{34}{45}$

115.  $\frac{5}{8} - \frac{9}{10} = \frac{25}{40} - \frac{36}{40} = -\frac{11}{40}$

116.  $\frac{7}{8} - \frac{1}{5} = \frac{35}{40} - \frac{8}{40} = \frac{27}{40}$

117.  $\frac{5}{9} - \frac{7}{11} = \frac{55}{99} - \frac{63}{99} = -\frac{8}{99}$
118.  $\frac{5}{8} - \frac{7}{12} = \frac{15}{24} - \frac{14}{24} = \frac{1}{24}$
119.  $\frac{2}{5} - \frac{1}{2} = \frac{4}{10} - \frac{5}{10} = -\frac{1}{10}$
120.  $\frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \frac{1}{12}$
121.  $\frac{3}{4} \cdot \frac{8}{27} = \frac{2}{9}$
122.  $\frac{9}{7} \cdot \frac{14}{27} = \frac{2}{3}$
123.  $\frac{\frac{8}{5}}{\frac{16}{15}} = \frac{8}{5} \div \frac{16}{15} = \frac{8}{5} \cdot \frac{15}{16} = \frac{3}{2}$
124.  $\frac{\frac{5}{6}}{\frac{15}{6}} = \frac{5}{6} \div \frac{15}{6} = \frac{5}{6} \cdot \frac{6}{15} = \frac{1}{3}$
125.  $\frac{\frac{7}{8}}{\frac{21}{16}} = \frac{7}{8} \div \frac{21}{16} = \frac{7}{8} \cdot \frac{16}{21} = \frac{2}{3}$
126.  $\frac{\frac{3}{10}}{\frac{7}{15}} = \frac{3}{10} \div \frac{7}{15} = \frac{3}{10} \cdot \frac{15}{7} = \frac{9}{14}$
127.  $5 \cdot \frac{3}{10} - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1$
128.  $2 \cdot \frac{7}{2} - \frac{3}{2} = \frac{14}{2} - \frac{3}{2} = \frac{11}{2}$
129.  $3 \cdot \frac{2}{15} - \frac{1}{3} = \frac{2}{5} - \frac{1}{3} = \frac{6}{15} - \frac{5}{15} = \frac{1}{15}$
130.  $2 \cdot \frac{5}{3} - \frac{3}{2} = \frac{10}{3} - \frac{3}{2} = \frac{20}{6} - \frac{9}{6} = \frac{11}{6}$
131.  $2(x + y) - 3y = 2(3 + (-5)) - 3(-5)$   
 $= 2(-2) - (-15) = -4 + 15 = 11$
132.  $-2(x + y) + 5y = -2(3 + (-5)) + 5(-5)$   
 $= -2(-2) + (-25)$   
 $= 4 + (-25) = -21$
133.  $3|x| - 2|y| = 3|3| - 2|-5| = 3(3) - 2(5)$   
 $= 9 - 10 = -1$
134.  $7|x - y| = 7|3 - (-5)| = 7|8| = 7(8) = 56$
135.  $\frac{x - 3y}{2} + xy = \frac{3 - 3(-5)}{2} + 3(-5)$   
 $= \frac{3 - (-15)}{2} + (-15)$   
 $= \frac{18}{2} + (-15) = 9 + (-15) = -6$
136.  $\frac{y + 3}{x} - xy = \frac{-5 + 3}{3} - 3(-5)$   
 $= \frac{-2}{3} - (-15) = \frac{43}{3}$
137.  $\frac{2(1 - 2x)}{y} - (-x)y = \frac{2(1 - 2(3))}{-5} - (-3)(-5)$   
 $= \frac{2(-5)}{-5} - 15 = 2 - 15$   
 $= -13$
138.  $\frac{3(2 - x)}{y} - (1 - xy) = \frac{3(2 - 3)}{-5} - (1 - 3(-5))$   
 $= \frac{3(-1)}{-5} - (1 - (-15))$   
 $= \frac{3}{5} - 16 = -\frac{77}{5}$
139.  $\frac{\frac{14}{x} + \frac{1}{2}}{\frac{-y}{4}} = \frac{\frac{14}{3} + \frac{1}{2}}{\frac{-(-5)}{4}} = \frac{31}{6} \cdot \frac{4}{5} = \frac{62}{15}$
140.  $\frac{\frac{4}{-y} + \frac{8}{x}}{\frac{y}{2}} = \frac{\frac{4}{-(-5)} + \frac{8}{3}}{\frac{-5}{2}} = \left(\frac{4}{5} + \frac{8}{3}\right) \cdot \left(-\frac{2}{5}\right)$   
 $= \frac{52}{15} \cdot \left(-\frac{2}{5}\right) = -\frac{104}{75}$
141. The denominator cannot equal 0  $\Rightarrow$   
 $x - 1 \neq 0 \Rightarrow x \neq 1$ , so the domain is  
 $(-\infty, 1) \cup (1, \infty)$ .
142. The denominator cannot equal 0  $\Rightarrow$   
 $1 - x \neq 0 \Rightarrow x \neq 1$ , so the domain is  
 $(-\infty, 1) \cup (1, \infty)$ .
143. The denominator cannot equal 0  $\Rightarrow$  the  
domain is  $(-\infty, 0) \cup (0, \infty)$ .

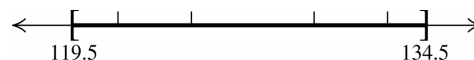
144. The denominator cannot equal 0  $\Rightarrow$   
 $x + 7 \neq 0 \Rightarrow x \neq -7$ , so the domain is  
 $(-\infty, -7) \cup (-7, \infty)$ .
145. The denominator cannot equal 0  $\Rightarrow$   
 $x \neq 0, x + 1 \neq 0 \Rightarrow x \neq -1$  so the domain is  
 $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$ .
146. The denominator cannot equal 0  $\Rightarrow$   
 $x \neq 0, x - 5 \neq 0 \Rightarrow x \neq 5$  so the domain is  
 $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$ .
147. The denominator cannot equal 0  $\Rightarrow$   
 $(x - 1)(x + 2) \neq 0 \Rightarrow x \neq 1, x \neq -2$  so the  
 domain is  $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$ .
148. The denominator cannot equal 0  $\Rightarrow$   
 $x(x + 3) \neq 0 \Rightarrow x \neq 0, x \neq -3$  so the domain is  
 $(-\infty, -3) \cup (-3, 0) \cup (0, \infty)$ .
149. The denominator cannot equal 0  $\Rightarrow$   
 $x \neq 0, 2 - x \neq 0 \Rightarrow x \neq 2$  so the domain is  
 $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$ .
150. The denominator cannot equal 0  $\Rightarrow$   
 $x - 3 \neq 0$  or  $3 - x \neq 0 \Rightarrow x \neq 3$  so the domain  
 is  $(-\infty, 3) \cup (3, \infty)$ .
151. There are no real numbers for which  
 $x^2 + 1 = 0$ , so the domain is  $(-\infty, \infty)$ .
152. There are no real numbers for which  
 $x^2 + 2 = 0$ , so the domain is  $(-\infty, \infty)$ .

### P.1 B Exercises: Applying the Concepts

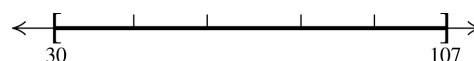
153. a. people who own either MP3 players or  
 people who own DVD players.  
 b. people who own both MP3 players and  
 DVD players.
154. a.  $A = \{2005 \text{ Lexus SC 430}\}$   
 b.  $B = \{2005 \text{ Lexus SC 430, 2005 Lincoln}$   
 Town Car}  
 c.  $C = \{2005 \text{ Lexus SC 430, 2005 Lincoln}$   
 Town Car, 2009 Audi A3}

- d.  $A \cap B = \{2005 \text{ Lexus SC 430}\}$   
 e.  $B \cap C = \{2005 \text{ Lexus SC 430, 2005}$   
 Lincoln Town Car}  
 f.  $A \cup B = \{2005 \text{ Lexus SC 430, 2005}$   
 Lincoln Town Car}  
 g.  $A \cup C = \{2005 \text{ Lexus SC 430, 2005}$   
 Lincoln Town Car, 2009 Audi A3}

155.  $119.5 \leq x \leq 134.5$



156.  $30 \leq x \leq 107$



157. a.  $|124 - 120| = 4$

b.  $|137 - 120| = 17$

c.  $|114 - 120| = |-6| = 6$

158. a.  $|15 - 14| = 1$

b.  $|17.5 - 15| = 2.5$

c.  $|15 - 15| = 0$

159. The pressure cannot be a negative number, so  
 the domain is  $[0, \infty)$ .
160. The height must be more than zero, so the  
 domain is  $(0, \infty)$ .
161. Five feet = 60 inches and five feet, 10 inches =  
 70 inches, so the domain is  $[60, 70]$ .
162. The worker works between 0 and 40 hours per  
 week, so the domain is  $[0, 40]$ .
163. Let  $x$  = the number of calories from broccoli.  
 Then we have  
 $522.5 - 55x = 0 \Rightarrow 522.5 = 55x \Rightarrow 9.5 = x$   
 The number of grams of broccoli is  
 $9.5 \times 100 = 950$  grams.
164. Let  $x$  = the number of orders of french fries.  
 The number of calories lost from broccoli is  
 $6 \times 55 = 330$ . Then we have  
 $165x - 330 = 0 \Rightarrow 165x = 330 \Rightarrow x = 2$   
 So, Carmen will have to eat 2 orders of french  
 fries.



## P.2 Integer Exponents and Scientific Notation

### P.2 Practice Exercises

1. a.  $2^3 = 8$

b.  $(3a)^2 = (3a)(3a) = 9a^2$

c.  $\left(\frac{1}{2}\right)^4 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

2. a.  $2^{-1} = \frac{1}{2}$

b.  $\left(\frac{4}{5}\right)^0 = 1$

c.  $\left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

3. a.  $x^2 \cdot 3x^7 = 3x^9$

b.  $\frac{1}{4^{-2}} = 4^2 = 16$

4. a.  $\frac{3^4}{3^0} = 3^{4-0} = 3^4 = 81$

b.  $\frac{5}{5^{-2}} = 5^{1-(-2)} = 5^3 = 125$

c.  $\frac{2x^3}{3x^{-4}} = \frac{2x^{3-(-4)}}{3} = \frac{2x^7}{3}$

5. a.  $(7^{-5})^0 = 7^{(-5)(0)} = 7^0 = 1$

b.  $(7^0)^{-5} = 7^{(0)(-5)} = 7^0 = 1$

c.  $(x^{-1})^8 = x^{(-1)(8)} = x^{-8} = \frac{1}{x^8}$

d.  $(x^{-2})^{-5} = x^{(-2)(-5)} = x^{10}$

6. a.  $\left(\frac{1}{2}x\right)^{-1} = \left(\frac{1}{2}\right)^{-1} (x)^{-1} = 2\left(\frac{1}{x}\right) = \frac{2}{x}$

b.  $(5x^{-1})^2 = 5^2 (x^{-1})^2 = 5^2 x^{(-1)(2)}$   
 $= 25x^{-2} = \frac{25}{x^2}$

c.  $(xy^2)^3 = x^3 y^{2(3)} = x^3 y^6$

d.  $(x^{-2}y)^{-3} = x^{(-2)(-3)} y^{-3} = \frac{x^6}{y^3}$

7. a.  $\left(\frac{1}{3}\right)^2 = \frac{1^2}{3^2} = \frac{1}{9}$

b.  $\left(\frac{10}{7}\right)^{-2} = \left(\frac{7}{10}\right)^2 = \frac{7^2}{10^2} = \frac{49}{100}$

8. a.  $(2x^4)^{-2} = \left(\frac{1}{2x^4}\right)^2 = \frac{1}{4x^8}$

b.  $\frac{x^2(-y)^3}{(xy^2)^3} = \frac{-x^2y^3}{x^3y^6} = -\frac{1}{xy^3}$

9.  $732,000 = 7.32 \times 10^5$

10.  $\frac{1.3 \times 10^{10}}{3 \times 10^8} \approx 0.433 \times 10^2 \approx \$43$  per person

### P.2 A Exercises: Basic Skills and Concepts

- In the expression  $7^2$ , the number 2 is called the exponent.
- In the expression  $-3^7$ , the base is 3.
- The number  $\frac{1}{4^{-2}}$  simplifies to be the positive integer 16.
- The power-of-a-product rule allows us to rewrite  $(5a)^3$  as  $5^3 a^3$ .
- False.  $(-11)^{10} = 11^{10}$
- False. When  $(x^2)^3$  is simplified, the expression becomes  $x^6$ .
- Exponent: 3, base: 17
- Exponent: 2, base: 10
- Exponent: 0, base: 9
- Exponent: 0, base: -2
- Exponent: 5, base: -5

12. Exponent: 2, base:  $-99$
13. Exponent: 3, base: 10
14. Exponent: 7, base:  $-3$
15. Exponent: 2, base:  $a$
16. Exponent: 3, base:  $-b$
17.  $6^1 = 6$
18.  $3^4 = 3 \times 3 \times 3 \times 3 = 81$
19.  $7^0 = 1$
20.  $(-8)^0 = 1$
21.  $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$
22.  $(3^2)^3 = 3^{2 \cdot 3} = 3^6 = 729$
23.  $(3^2)^{-2} = 3^{2(-2)} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$
24.  $(7^2)^{-1} = 7^{2(-1)} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$
25.  $(5^{-2})^3 = \left(\frac{1}{5^2}\right)^3 = \left(\frac{1}{25}\right)^3 = \frac{1}{15,625}$
26.  $(5^{-1})^3 = \left(\frac{1}{5}\right)^3 = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$
27.  $(4^{-3}) \cdot (4^5) = 4^{(-3+5)} = 4^2 = 16$
28.  $(7^{-2}) \cdot (7^3) = 7^{(-2+3)} = 7^1 = 7$
29.  $3^0 + 10^0 = 1 + 1 = 2$
30.  $5^0 - 9^0 = 1 - 1 = 0$
31.  $3^{-2} + \left(\frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$
32.  $5^{-2} + \left(\frac{1}{5}\right)^2 = \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^2 = \frac{1}{25} + \frac{1}{25} = \frac{2}{25}$
33.  $\frac{2^{11}}{2^{10}} = 2^{(11-10)} = 2^1 = 2$
34.  $\frac{3^6}{3^8} = 3^{(6-8)} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
35.  $\frac{(5^3)^4}{5^{12}} = \frac{5^{12}}{5^{12}} = 1$
36.  $\frac{(9^5)^2}{9^8} = \frac{9^{10}}{9^8} = 9^{(10-8)} = 9^2 = 81$
37.  $\frac{2^5 \cdot 3^{-2}}{2^4 \cdot 3^{-3}} = 2^{(5-4)} \cdot 3^{(-2-(-3))} = 2^1 \cdot 3^1 = 6$
38.  $\frac{4^{-2} \cdot 5^3}{4^{-3} \cdot 5} = 4^{(-2-(-3))} \cdot 5^{(3-1)}$   
 $= 4^1 \cdot 5^2 = 4 \cdot 25 = 100$
39.  $\frac{-5^{-2}}{2^{-1}} = -\frac{1}{5^2} \cdot 2^1 = -\frac{1}{25} \cdot 2 = -\frac{2}{25}$
40.  $\frac{-7^{-2}}{3^{-1}} = -\frac{1}{7^2} \cdot 3^1 = -\frac{1}{49} \cdot 3 = -\frac{3}{49}$
41.  $\left(\frac{11}{7}\right)^{-2} = \frac{1}{\left(\frac{11}{7}\right)^2} = \frac{1}{\frac{121}{49}} = \frac{49}{121}$
42.  $\left(\frac{13}{5}\right)^{-2} = \frac{1}{\left(\frac{13}{5}\right)^2} = \frac{1}{\frac{169}{25}} = \frac{25}{169}$
43.  $x^4 y^0 = x^4 \cdot 1 = x^4$
44.  $x^{-1} y^0 = \frac{1}{x} \cdot 1 = \frac{1}{x}$
45.  $x^{-1} y = \frac{1}{x} \cdot y = \frac{y}{x}$
46.  $x^2 y^{-2} = x^2 \cdot \frac{1}{y^2} = \frac{x^2}{y^2}$
47.  $-8x^{-1} = -8 \cdot \frac{1}{x} = -\frac{8}{x}$
48.  $(-8x)^{-1} = -\frac{1}{8x}$
49.  $x^{-1}(3y^0) = \frac{1}{x} \cdot 3(1) = \frac{3}{x}$
50.  $x^{-3}(3y^2) = \frac{1}{x^3} \cdot 3y^2 = \frac{3y^2}{x^3}$

51.  $x^{-1}y^{-2} = \frac{1}{xy^2}$

52.  $x^{-3}y^{-2} = \frac{1}{x^3y^2}$

53.  $(x^{-3})^4 = x^{-12} = \frac{1}{x^{12}}$

54.  $(x^{-5})^2 = x^{-10} = \frac{1}{x^{10}}$

55.  $(x^{-11})^{-3} = x^{(-11) \cdot (-3)} = x^{33}$

56.  $(x^{-4})^{-12} = x^{(-4) \cdot (-12)} = x^{48}$

57.  $-3(xy)^5 = -3x^5y^5$

58.  $-8(xy)^6 = -8x^6y^6$

59.  $4(xy^{-1})^2 = 4x^2y^{-2} = \frac{4x^2}{y^2}$

60.  $6(x^{-1}y)^3 = 6x^{-3}y^3 = \frac{6y^3}{x^3}$

61.  $3(x^{-1}y)^{-5} = 3x^{(-1) \cdot (-5)}y^{-5} = \frac{3x^5}{y^5}$

62.  $-5(xy^{-1})^{-6} = -5x^{-6}y^{(-1) \cdot (-6)} = -\frac{5y^6}{x^6}$

63.  $\frac{(x^3)^2}{(x^2)^5} = \frac{x^{3 \cdot 2}}{x^{2 \cdot 5}} = \frac{x^6}{x^{10}} = x^{6-10} = x^{-4} = \frac{1}{x^4}$

64.  $\frac{x^2}{(x^3)^4} = \frac{x^2}{x^{3 \cdot 4}} = \frac{x^2}{x^{12}} = x^{2-12} = x^{-10} = \frac{1}{x^{10}}$

65.  $\left(\frac{2xy}{x^2}\right)^3 = \frac{2^3x^3y^3}{x^{2 \cdot 3}} = \frac{8x^3y^3}{x^6} = 8x^{3-6}y^3$   
 $= 8x^{-3}y^3 = \frac{8y^3}{x^3}$

66.  $\left(\frac{5xy}{x^3}\right)^4 = \frac{5^4x^4y^4}{x^{3 \cdot 4}} = \frac{625x^4y^4}{x^{12}} = 625x^{4-12}y^4$   
 $= 625x^{-8}y^4 = \frac{625y^4}{x^8}$

67.  $\left(\frac{-3x^2y}{x}\right)^5 = \frac{(-3)^5x^{2 \cdot 5}y^5}{x^5} = \frac{-243x^{10}y^5}{x^5}$   
 $= -243x^{10-5}y^5 = -243x^5y^5$

68.  $\left(\frac{-2xy^2}{y}\right)^3 = \frac{(-2)^3x^3y^{2 \cdot 3}}{y^3} = \frac{-8x^3y^6}{y^3}$   
 $= -8x^3y^{6-3} = -8x^3y^3$

69.  $\left(\frac{-3x}{5}\right)^{-2} = \frac{(-3)^{-2}x^{-2}}{5^{-2}} = \frac{\frac{1}{9} \cdot \frac{1}{x^2}}{\frac{1}{25}} = \frac{\frac{1}{9x^2}}{\frac{1}{25}} = \frac{25}{9x^2}$

70.  $\left(\frac{-5y}{3}\right)^{-4} = \frac{(-5)^{-4}y^{-4}}{3^{-4}} = \frac{\frac{1}{5^4} \cdot \frac{1}{y^4}}{\frac{1}{3^4}} = \frac{\frac{1}{625y^4}}{\frac{1}{81}}$   
 $= \frac{81}{625y^4}$

71.  $\left(\frac{4x^{-2}}{xy^5}\right)^3 = \frac{4^3x^{-2 \cdot 3}}{x^3y^{5 \cdot 3}} = \frac{64x^{-6}}{x^3y^{15}} = \frac{64}{x^9y^{15}}$

72.  $\left(\frac{3x^2y}{y^3}\right)^5 = \frac{3^5x^{2 \cdot 5}y^5}{y^{3 \cdot 5}} = \frac{243x^{10}y^5}{y^{15}} = \frac{243x^{10}}{y^{10}}$

73.  $\frac{x^3y^{-3}}{x^{-2}y} = x^{3-(-2)}y^{-3-1} = x^5y^{-4} = \frac{x^5}{y^4}$

74.  $\frac{x^2y^{-2}}{x^{-1}y^2} = x^{2-(-1)}y^{-2-2} = x^3y^{-4} = \frac{x^3}{y^4}$

75.  $\frac{27x^{-3}y^5}{9x^{-4}y^7} = 3x^{-3-(-4)}y^{5-7} = 3x^1y^{-2} = \frac{3x}{y^2}$

76.  $\frac{15x^5y^{-2}}{3x^7y^{-3}} = 5x^{5-7}y^{-2-(-3)} = 5x^{-2}y^1 = \frac{5y}{x^2}$

77.  $\frac{5a^{-2}bc^2}{a^4b^{-3}c^2} = 5a^{-2-4}b^{1-(-3)}c^{2-2}$   
 $= 5a^{-6}b^4 \cdot 1 = \frac{5b^4}{a^6}$

78.  $\frac{(-3)^2a^5(bc)^2}{(-2)^3a^2b^3c^4} = \frac{9a^5b^2c^2}{-8a^2b^3c^4} = -\frac{9}{8}a^{5-2}b^{2-3}c^{2-4}$   
 $= -\frac{9}{8}a^3b^{-1}c^{-2} = -\frac{9a^3}{8bc^2}$

$$\begin{aligned}
 79. \left( \frac{xy^{-3}z^{-2}}{x^2y^{-4}z^3} \right)^{-3} &= \frac{x^{-3}y^{(-3)(-3)}z^{(-2)(-3)}}{x^{2(-3)}y^{(-4)(-3)}z^{3(-3)}} \\
 &= \frac{x^{-3}y^9z^6}{x^{-6}y^{12}z^{-9}} \\
 &= x^{-3-(-6)}y^{9-12}z^{6-(-9)} \\
 &= x^3y^{-3}z^{15} = \frac{x^3z^{15}}{y^3}
 \end{aligned}$$

$$\begin{aligned}
 80. \left( \frac{xy^{-2}z^{-1}}{x^{-5}yz^{-8}} \right)^{-1} &= \frac{x^{-1}y^{(-2)(-1)}z^{(-1)(-1)}}{x^{(-5)(-1)}y^{-1}z^{(-8)(-1)}} = \frac{x^{-1}y^2z^1}{x^5y^{-1}z^8} \\
 &= x^{-1-5}y^{2-(-1)}z^{1-8} = x^{-6}y^3z^{-7} = \frac{y^3}{x^6z^7}
 \end{aligned}$$

$$81. 125 = 1.25 \times 10^2$$

$$82. 247 = 2.47 \times 10^2$$

$$83. 850,000 = 8.5 \times 10^5$$

$$84. 205,000 = 2.05 \times 10^5$$

$$85. 0.007 = 7 \times 10^{-3}$$

$$86. 0.0019 = 1.9 \times 10^{-3}$$

$$87. 0.00000275 = 2.75 \times 10^{-6}$$

$$88. 0.0000038 = 3.8 \times 10^{-6}$$

### P.2 B Exercises: Applying the Concepts

$$89. 135 \text{ ft}^3 \times 2^3 = 1080 \text{ ft}^3$$

$$90. 675 \text{ in.}^3 \times 3^3 = 18,225 \text{ in.}^3$$

$$91. \text{ a. } (2x)^2 = 4x^2 = 4A = 2^2A$$

$$\text{ b. } (3x)^2 = 9x^2 = 9A = 3^2A$$

$$92. \text{ a. } \pi \left( \frac{2d}{2} \right)^2 = \pi d^2 = 4\pi \left( \frac{d}{2} \right)^2 = 2^2A$$

$$\text{ b. } \pi \left( \frac{3d}{2} \right)^2 = 9\pi \left( \frac{d}{2} \right)^2 = 9\pi \left( \frac{d}{2} \right)^2 = 3^2A$$

$$93. F = Sw^2 = 25,000(0.25)^2 = 1562.5 \text{ lb}$$

$$\begin{aligned}
 94. F = S\pi \left( \frac{d}{2} \right)^2 &= 10,000(3.14) \left( \frac{1.5}{2} \right)^2 \\
 &\approx 17,662.5 \text{ lbs.}
 \end{aligned}$$

Celestial Body	Equatorial Diameter (km)	Scientific Notation
Earth	12,700	$1.27 \times 10^4$ km
Moon	3480	$3.48 \times 10^3$ km
Sun	1,390,000	$1.39 \times 10^6$ km
Jupiter	134,000	$1.34 \times 10^5$ km
Mercury	4800	$4.8 \times 10^3$ km

$$96. 37,600,000,000,000,000,000,000,000 = 3.76 \times 10^{22} \text{ atoms}$$

$$97. 602,000,000,000,000,000,000,000 = 6.02 \times 10^{20} \text{ atoms}$$

$$98. 0.00000000000000000000266 = 2.66 \times 10^{-20} \text{ kg}$$

$$99. 0.000000000000000000000167 = 1.67 \times 10^{-21} \text{ kg}$$

$$100. 380,000,000 = 3.8 \times 10^8 \text{ m}$$

$$101. 5,980,000,000,000,000,000,000,000,000 = 5.98 \times 10^{24} \text{ kg}$$

## P.3 Polynomials

### P.3 Practice Problems

$$1. 16(7)^2 + 15(7) = 889 \text{ ft}$$

$$\begin{aligned}
 2. (7x^3 + 2x^2 - 5) + (-2x^3 + 3x^2 + 2x + 1) \\
 = 5x^3 + 5x^2 + 2x - 4
 \end{aligned}$$

$$\begin{aligned}
 3. (3x^4 - 5x^3 + 2x^2 + 7) - (-2x^4 + 3x^2 + x - 5) \\
 = 5x^4 - 5x^3 - x^2 - x + 12
 \end{aligned}$$

$$4. -2x^3(4x^2 + 2x - 5) = -8x^5 - 4x^4 + 10x^3$$

$$\begin{aligned}
 5. (5x^2 + 2x)(-2x^2 + x - 7) \\
 = 5x^2(-2x^2 + x - 7) + 2x(-2x^2 + x - 7) \\
 = -10x^4 + 5x^3 - 35x^2 - 4x^3 + 2x^2 - 14x \\
 = -10x^4 + x^3 - 33x^2 - 14x
 \end{aligned}$$

$$\begin{aligned}
 6. \text{ a. } (4x - 1)(x + 7) &= 4x^2 + 28x - x - 7 \\
 &= 4x^2 + 27x - 7
 \end{aligned}$$

$$\begin{aligned}
 \text{ b. } (3x - 2)(2x - 5) &= 6x^2 - 15x - 4x + 10 \\
 &= 6x^2 - 19x + 10
 \end{aligned}$$

$$7. (3x+2)^2 = (3x)^2 + 2(3x)(2) + 2^2 \\ = 9x^2 + 12x + 4$$

$$8. (1-2x)(1+2x) = 1 - 4x^2$$

$$9. x^2(y+x) = x^2y + x^3$$

$$18. (-x^3 + 2x - 4) - (x^3 + 3x^2 - 7x + 2) \\ = (-x^3 - x^3) - 3x^2 + (2x - (-7x)) + (-4 - 2) \\ = -2x^3 - 3x^2 + 9x - 6$$

$$19. (-2x^4 + 3x^2 - 7x) - (8x^4 + 6x^3 - 9x^2 - 17) \\ = (-2x^4 - 8x^4) - 7x - 6x^3 \\ \quad + (3x^2 - (-9x^2)) - (-17) \\ = -10x^4 - 6x^3 + 12x^2 - 7x + 17$$

### P.3 A Exercises: Basic Skills and Concepts

- The polynomial  $-3x^7 + 2x^2 - 9x + 4$  has leading coefficient -3 and degree 7.
- When a polynomial is written so that the exponents in each term decrease from left to right, it is said to be in standard form.
- When a polynomial in  $x$  of degree 3 is added to a polynomial in  $x$  of degree 4, the resulting polynomial has degree 4.
- When a polynomial in  $x$  of degree 3 is multiplied by a polynomial in  $x$  of degree 4, the resulting polynomial has degree 7.
- True
- True. This is true if  $A$  or  $B$  or both are zero.
- a polynomial;  $x^2 + 2x + 1$
- not a polynomial
- not a polynomial
- a polynomial;  $x^7 + 3x^5 + 3x^4 - 2x + 1$
- degree: 1; terms:  $7x, 3$
- degree: 2; terms:  $-3x^2, 7$
- degree: 4; terms:  $-x^4, x^2, 2x, -9$
- degree: 7; terms:  $9x^7, 2x^3, x, -21$
- $(x^3 + 2x^2 - 5x + 3) + (-x^3 + 2x - 4) \\ = (x^3 + (-x^3)) + 2x^2 + (-5x + 2x) + (3 - 4) \\ = 2x^2 - 3x - 1$
- $(x^3 - 3x + 1) + (x^3 - x^2 + x - 3) \\ = (x^3 + x^3) - x^2 + (-3x + x) + (1 - 3) \\ = 2x^3 - x^2 - 2x - 2$
- $(2x^3 - x^2 + x - 5) - (x^3 - 4x + 3) \\ = (2x^3 - x^3) - x^2 + (x - (-4x)) + (-5 - 3) \\ = x^3 - x^2 + 5x - 8$
- $(-x^3 + 2x - 4) - (x^3 + 3x^2 - 7x + 2) \\ = (-x^3 - x^3) - 3x^2 + (2x - (-7x)) + (-4 - 2) \\ = -2x^3 - 3x^2 + 9x - 6$
- $(-2x^4 + 3x^2 - 7x) - (8x^4 + 6x^3 - 9x^2 - 17) \\ = (-2x^4 - 8x^4) - 7x - 6x^3 \\ \quad + (3x^2 - (-9x^2)) - (-17) \\ = -10x^4 - 6x^3 + 12x^2 - 7x + 17$
- $3(x^2 - 2x + 2) + 2(5x^2 - x + 4) \\ = 3x^2 - 6x + 6 + 10x^2 - 2x + 8 \\ = 13x^2 - 8x + 14$
- $-2(3x^2 + x + 1) + 6(-3x^2 - 2x - 2) \\ = -6x^2 - 2x - 2 - 18x^2 - 12x - 12 \\ = -24x^2 - 14x - 14$
- $2(5x^2 - x + 3) - 4(3x^2 + 7x + 1) \\ = 10x^2 - 2x + 6 - 12x^2 - 28x - 4 \\ = -2x^2 - 30x + 2$
- $(3y^3 - 4y + 2) + (2y + 1) - (y^3 - y^2 + 4) \\ = 3y^3 - 4y + 2 + 2y + 1 - y^3 + y^2 - 4 \\ = 2y^3 + y^2 - 2y - 1$
- $(5y^2 + 3y - 1) - (y^2 - 2y + 3) + (2y^2 + y + 5) \\ = 5y^2 + 3y - 1 - y^2 + 2y - 3 + 2y^2 + y + 5 \\ = 6y^2 + 6y + 1$
- $6x(2x + 3) = 12x^2 + 18x$
- $7x(3x - 4) = 21x^2 - 28x$
- $(x+1)(x^2 + 2x + 2) \\ = x(x^2 + 2x + 2) + 1(x^2 + 2x + 2) \\ = x^3 + 2x^2 + 2x + x^2 + 2x + 2 \\ = x^3 + 3x^2 + 4x + 2$
- $(x-5)(2x^2 - 3x + 1) \\ = x(2x^2 - 3x + 1) - 5(2x^2 - 3x + 1) \\ = 2x^3 - 3x^2 + x - 10x^2 + 15x - 5 \\ = 2x^3 - 13x^2 + 16x - 5$
- $(3x-2)(x^2 - x + 1) \\ = 3x(x^2 - x + 1) - 2(x^2 - x + 1) \\ = 3x^3 - 3x^2 + 3x - 2x^2 + 2x - 2 \\ = 3x^3 - 5x^2 + 5x - 2$

30.  $(2x+1)(x^2-3x+4)$   
 $= 2x(x^2-3x+4) + 1(x^2-3x+4)$   
 $= 2x^3 - 6x^2 + 8x + x^2 - 3x + 4$   
 $= 2x^3 - 5x^2 + 5x + 4$
31.  $(x+1)(x+2) = x(x+2) + 1(x+2)$   
 $= x^2 + 2x + x + 2 = x^2 + 3x + 2$
32.  $(x+2)(x+3) = x(x+3) + 2(x+3)$   
 $= x^2 + 3x + 2x + 6$   
 $= x^2 + 5x + 6$
33.  $(3x+2)(3x+1) = 9x^2 + 3x + 6x + 2$   
 $= 9x^2 + 9x + 2$
34.  $(x+3)(2x+5) = 2x^2 + 5x + 6x + 15$   
 $= 2x^2 + 11x + 15$
35.  $(-4x+5)(x+3) = -4x^2 - 12x + 5x + 15$   
 $= -4x^2 - 7x + 15$
36.  $(-2x+1)(x-5) = -2x^2 + 10x + x - 5$   
 $= -2x^2 + 11x - 5$
37.  $(3x-2)(2x-1) = 6x^2 - 3x - 4x + 2$   
 $= 6x^2 - 7x + 2$
38.  $(x-1)(5x-3) = 5x^2 - 3x - 5x + 3$   
 $= 5x^2 - 8x + 3$
39.  $(2x-3a)(2x+5a) = 4x^2 + 10ax - 6ax - 15a^2$   
 $= 4x^2 + 4ax - 15a^2$
40.  $(5x-2a)(x+5a) = 5x^2 + 25ax - 2ax - 10a^2$   
 $= 5x^2 + 23ax - 10a^2$
41.  $(x+2)^2 - x^2 = x^2 + 4x + 4 - x^2 = 4x + 4$
42.  $(x-3)^2 - x^2 = x^2 - 6x + 9 - x^2 = -6x + 9$
43.  $(x+3)^3 - x^3 = x^3 + 9x^2 + 27x + 27 - x^3$   
 $= 9x^2 + 27x + 27$
44.  $(x-2)^3 - x^3 = x^3 - 6x^2 + 12x - 8 - x^3$   
 $= -6x^2 + 12x - 8$
45.  $(4x+1)^2 = 16x^2 + 8x + 1$
46.  $(3x+2)^2 = 9x^2 + 12x + 4$
47.  $(3x+1)^3 = (3x)^3 + 3(3x)^2(1) + 3(3x)(1^2) + 1^3$   
 $= 27x^3 + 27x^2 + 9x + 1$
48.  $(2x+3)^3 = (2x)^3 + 3(2x)^2(3) + 3(2x)(3^2) + 3^3$   
 $= 8x^3 + 36x^2 + 54x + 27$
49.  $(5-2x)(5+2x) = 25 - 4x^2$
50.  $(3-4x)(3+4x) = 9 - 16x^2$
51.  $\left(x + \frac{3}{4}\right)^2 = x^2 + 2\left(\frac{3}{4}\right)x + \left(\frac{3}{4}\right)^2$   
 $= x^2 + \frac{3}{2}x + \frac{9}{16}$
52.  $\left(x + \frac{2}{5}\right)^2 = x^2 + 2\left(\frac{2}{5}\right)x + \left(\frac{2}{5}\right)^2$   
 $= x^2 + \frac{4}{5}x + \frac{4}{25}$
53.  $(2x-3)(x^2-3x+5)$   
 $= 2x(x^2-3x+5) - 3(x^2-3x+5)$   
 $= 2x^3 - 6x^2 + 10x - 3x^2 + 9x - 15$   
 $= 2x^3 - 9x^2 + 19x - 15$
54.  $(x-2)(x^2-4x-3)$   
 $= x(x^2-4x-3) - 2(x^2-4x-3)$   
 $= x^3 - 4x^2 - 3x - 2x^2 + 8x + 6$   
 $= x^3 - 6x^2 + 5x + 6$
55.  $(1+y)(1-y+y^2)$   
 $= 1(1-y+y^2) + y(1-y+y^2)$   
 $= 1 - y + y^2 + y - y^2 + y^3$   
 $= y^3 + 1$
56.  $(y+4)(y^2-4y+16)$   
 $= y(y^2-4y+16) + 4(y^2-4y+16)$   
 $= y^3 - 4y^2 + 16y + 4y^2 - 16y + 64$   
 $= y^3 + 64$
57.  $(x-6)(x^2+6x+36)$   
 $= x(x^2+6x+36) - 6(x^2+6x+36)$   
 $= x^3 + 6x^2 + 36x - 6x^2 - 36x - 216$   
 $= x^3 - 216$
58.  $(x-1)(x^2+x+1)$   
 $= x(x^2+x+1) - 1(x^2+x+1)$   
 $= x^3 + x^2 + x - x^2 - x - 1 = x^3 - 1$

$$\begin{aligned} 59. (x+2y)(3x+5y) &= 3x^2 + 5xy + 6xy + 10y^2 \\ &= 3x^2 + 11xy + 10y^2 \end{aligned}$$

$$\begin{aligned} 60. (2x+y)(7x+2y) &= 14x^2 + 4xy + 7xy + 2y^2 \\ &= 14x^2 + 11xy + 2y^2 \end{aligned}$$

$$\begin{aligned} 61. (2x-y)(3x+7y) &= 6x^2 + 14xy - 3xy - 7y^2 \\ &= 6x^2 + 11xy - 7y^2 \end{aligned}$$

$$\begin{aligned} 62. (x-3y)(2x+5y) &= 2x^2 + 5xy - 6xy - 15y^2 \\ &= 2x^2 - xy - 15y^2 \end{aligned}$$

$$\begin{aligned} 63. (x-y)^2(x+y)^2 &= (x-y)(x-y)(x+y)(x+y) \\ &= [(x-y)(x+y)][(x-y)(x+y)] \\ &= (x^2 - y^2)(x^2 - y^2) \\ &= x^4 - 2x^2y^2 + y^4 \end{aligned}$$

$$\begin{aligned} 64. (2x+y)^2(2x-y)^2 &= (2x+y)(2x+y)(2x-y)(2x-y) \\ &= [(2x-y)(2x+y)][(2x-y)(2x+y)] \\ &= (4x^2 - y^2)(4x^2 - y^2) \\ &= 16x^4 - 8x^2y^2 + y^4 \end{aligned}$$

$$\begin{aligned} 65. (x+y)(x-2y)^2 &= (x+y)(x^2 - 4xy + 4y^2) \\ &= x(x^2 - 4xy + 4y^2) + y(x^2 - 4xy + 4y^2) \\ &= x^3 - 4x^2y + 4xy^2 + x^2y - 4xy^2 + 4y^3 \\ &= x^3 - 3x^2y + 4y^3 \end{aligned}$$

$$\begin{aligned} 66. (x-y)(x+2y)^2 &= (x-y)(x^2 + 4xy + 4y^2) \\ &= x(x^2 + 4xy + 4y^2) - y(x^2 + 4xy + 4y^2) \\ &= x^3 + 4x^2y + 4xy^2 - x^2y - 4xy^2 - 4y^3 \\ &= x^3 + 3x^2y - 4y^3 \end{aligned}$$

$$\begin{aligned} 67. (x-2y)^3(x+2y) &= (x^3 + 3x^2(-2y) + 3x(-2y)^2 + (-2y)^3) \\ &\quad \cdot (x+2y) \\ &= (x^3 - 6x^2y + 12xy^2 - 8y^3)(x+2y) \\ &= x(x^3 - 6x^2y + 12xy^2 - 8y^3) \\ &\quad + 2y(x^3 - 6x^2y + 12xy^2 - 8y^3) \\ &= x^4 - 6x^3y + 12x^2y^2 - 8xy^3 \\ &\quad + 2x^3y - 12x^2y^2 + 24xy^3 - 16y^4 \\ &= x^4 - 4x^3y + 16xy^3 - 16y^4 \end{aligned}$$

$$\begin{aligned} 68. (2x+y)^3(2x-y) &= ((2x)^3 + 3(2x)^2y + 3(2x)y^2 + y^3)(2x-y) \\ &= (8x^3 + 12x^2y + 6xy^2 + y^3)(2x-y) \end{aligned}$$

$$\begin{aligned} &= 2x(8x^3 + 12x^2y + 6xy^2 + y^3) \\ &\quad - y(8x^3 + 12x^2y + 6xy^2 + y^3) \\ &= 16x^4 + 24x^3y + 12x^2y^2 + 2xy^3 \\ &\quad - 8x^3y - 12x^2y^2 - 6xy^3 - y^4 \\ &= 16x^4 + 16x^3y - 4xy^3 - y^4 \end{aligned}$$

### P.3 B Exercises: Applying the Concepts

$$69. -0.025(6^2) + 0.44(6) + 4.28 = \$6.02 \text{ in 2003}$$

(six years after 1997)

$$70. 0.035(4^2) + 0.15(4) + 5.17 = 6.33$$

In 1997 (four years after 1993), theater grosses were 6.33 billion dollars.

$$71. 0.1(40^2) + 40 + 50 = \$250.00$$

$$72. (15-10)^2 + 5(15) = 5^2 + 75$$

$$= 25 + 75 = \$100.00$$

$$73. d = 16(5^2) + 20(5) = 16(25) + 100 = 500 \text{ feet}$$

$$74. d = 16(2^2) + 10(2) = 16(4) + 20 = 84 \text{ feet}$$

$$75. \text{ a. } -x + 22.50$$

$$\begin{aligned} \text{ b. } (30)(22.5) + 10x(22.5 - x) \\ &= 30(22.5) + 22.5(10x) - 10x^2 \\ &= 675 + 225x - 10x^2 \\ &= -10x^2 + 225x + 675 \end{aligned}$$

$$76. \text{ a. } 10n + 250$$

$$\begin{aligned} \text{ b. } (50 - 2n)(10n + 250) \\ &= 50(10n) + 50(250) - \\ &\quad 2n(10n) - 2n(250) \\ &= 500n + 12,500 - 20n^2 - 500n \\ &= -20n^2 + 12,500 \end{aligned}$$

### P.4 Factoring Polynomials

#### P.4 Practice Exercises

$$1. \text{ a. } 6x^5 + 14x^3 = 2x^3(3x^2 + 7)$$

$$\text{ b. } 7x^5 + 21x^4 + 35x^2 = 7x^2(x^3 + 3x^2 + 5)$$

$$2. \text{ a. } x^2 + 6x + 8 = (x+4)(x+2)$$

$$\text{ b. } x^2 - 3x - 10 = (x-5)(x+2)$$

3. a.  $x^2 + 4x + 4 = (x + 2)^2$   
 b.  $9x^2 - 6x + 1 = (3x - 1)^2$
4. a.  $x^2 - 16 = (x - 4)(x + 4)$   
 b.  $4x^2 - 25 = (2x - 5)(2x + 5)$
5.  $x^4 - 81 = (x^2 - 9)(x^2 + 9)$   
 $= (x - 3)(x + 3)(x^2 + 9)$
6. a.  $x^3 - 125 = (x - 5)(x^2 + 5x + 25)$   
 b.  $27x^3 + 8 = (3x + 2)(9x^2 - 6x + 4)$
7. Following the reasoning in Example 7, in four years, the company will have invested the initial 12 million dollars, plus an additional 4 million dollars. Thus, the total investment is 16 million dollars. To find the profit or loss with 16 million dollars invested, let  $x = 16$  in the profit-loss polynomial:  
 $(0.012)(x - 14)(x^2 + 14x + 196)$   
 $= (0.012)(16 - 14)(16^2 + 14 \cdot 16 + 196)$   
 $= 16.224$   
 The company will have made a profit of 16.224 million dollars in four years.
8. a.  $5x^2 + 11x + 2 = (5x + 1)(x + 2)$   
 b.  $9x^2 - 9x + 2 = (3x - 2)(3x - 1)$
9. a.  $x^3 + 3x^2 + x + 3 = x^2(x + 3) + 1(x + 3)$   
 $= (x + 3)(x^2 + 1)$   
 b.  $28x^3 - 20x^2 - 7x + 5$   
 $= 4x^2(7x - 5) - (7x - 5)$   
 $= (7x - 5)(4x^2 - 1)$   
 $= (7x - 5)(2x + 1)(2x - 1)$   
 c.  $x^2 - y^2 - 2y - 1 = x^2 - (y^2 + 2y + 1)$   
 $= x^2 - (y + 1)^2$   
 $= (x - (y + 1))(x + (y + 1))$   
 $= (x - y - 1)(x + y + 1)$

## P.4 A Exercises: Basic Skills and Concepts

- The polynomials  $x + 2$  and  $x - 2$  are called factors of the polynomial  $x^2 - 4$ .
- The polynomial  $3y$  is the greatest common monomial factor of the polynomial  $3y^2 + 6y$ .
- The GCF of the polynomial  $10x^3 + 30x^2$  is  $10x^2$ .
- A polynomial that cannot be factored as a product of two polynomials (excluding constant polynomials  $\pm 1$ ) is said to be irreducible.
- True
- True
- $8x - 24 = 8(x - 3)$
- $5x + 25 = 5(x + 5)$
- $-6x^2 + 12x = -6x(x - 2)$
- $-3x^2 + 21 = -3(x^2 - 7)$
- $7x^2 + 14x^3 = 7x^2(1 + 2x)$
- $9x^3 - 18x^4 = 9x^3(1 - 2x)$
- $x^4 + 2x^3 + x^2 = x^2(x^2 + 2x + 1)$
- $x^4 - 5x^3 + 7x^2 = x^2(x^2 - 5x + 7)$
- $3x^3 - x^2 = x^2(3x - 1)$
- $2x^3 + 2x^2 = 2x^2(x + 1)$
- $8ax^3 + 4ax^2 = 4ax^2(2x + 1)$
- $ax^4 - 2ax^2 + ax = ax(x^3 - 2x + 1)$
- $x^3 + 3x^2 + x + 3 = x^2(x + 3) + 1(x + 3)$   
 $= (x + 3)(x^2 + 1)$
- $x^3 + 5x^2 + x + 5 = x^2(x + 5) + 1(x + 5)$   
 $= (x + 5)(x^2 + 1)$
- $x^3 - 5x^2 + x - 5 = x^2(x - 5) + 1(x - 5)$   
 $= (x - 5)(x^2 + 1)$

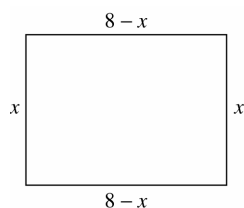


22.  $x^3 - 7x^2 + x - 7 = x^2(x - 7) + 1(x - 7)$   
 $= (x - 7)(x^2 + 1)$
23.  $6x^3 + 4x^2 + 3x + 2 = 2x^2(3x + 2) + 1(3x + 2)$   
 $= (3x + 2)(2x^2 + 1)$
24.  $3x^3 + 6x^2 + x + 2 = 3x^2(x + 2) + 1(x + 2)$   
 $= (x + 2)(3x^2 + 1)$
25.  $12x^7 + 4x^5 + 3x^4 + x^2$   
 $= 4x^5(3x^2 + 1) + x^2(3x^2 + 1)$   
 $= (3x^2 + 1)(4x^5 + x^2)$   
 $= x^2(3x^2 + 1)(4x^3 + 1)$
26.  $3x^7 + 3x^5 + x^4 + x^2 = 3x^5(x^2 + 1) + x^2(x^2 + 1)$   
 $= (x^2 + 1)(3x^5 + x^2)$   
 $= x^2(x^2 + 1)(3x^3 + 1)$
27.  $x^2 + 7x + 12 = (x + 3)(x + 4)$
28.  $x^2 + 8x + 15 = (x + 3)(x + 5)$
29.  $x^2 - 6x + 8 = (x - 4)(x - 2)$
30.  $x^2 - 9x + 14 = (x - 7)(x - 2)$
31.  $x^2 - 3x - 4 = (x - 4)(x + 1)$
32.  $x^2 - 5x - 6 = (x - 6)(x + 1)$
33. irreducible
34. irreducible
35.  $2x^2 + x - 36 = (2x + 9)(x - 4)$
36.  $2x^2 + 3x - 27 = (2x + 9)(x - 3)$
37.  $6x^2 + 17x + 12 = (2x + 3)(3x + 4)$
38.  $8x^2 - 10x - 3 = (2x - 3)(4x + 1)$
39.  $3x^2 - 11x - 4 = (3x + 1)(x - 4)$
40.  $5x^2 + 7x + 2 = (5x + 2)(x + 1)$
41. irreducible
42. irreducible
43.  $x^2 + 6x + 9 = (x + 3)^2$
44.  $x^2 + 8x + 16 = (x + 4)^2$
45.  $9x^2 + 6x + 1 = (3x + 1)^2$
46.  $36x^2 + 12x + 1 = (6x + 1)^2$
47.  $25x^2 - 20x + 4 = (5x - 2)^2$
48.  $64x^2 + 32x + 4 = (8x + 2)^2$
49.  $49x^2 + 42x + 9 = (7x + 3)^2$
50.  $9x^2 + 24x + 16 = (3x + 4)^2$
51.  $x^2 - 64 = (x - 8)(x + 8)$
52.  $x^2 - 121 = (x - 11)(x + 11)$
53.  $4x^2 - 1 = (2x - 1)(2x + 1)$
54.  $9x^2 - 1 = (3x - 1)(3x + 1)$
55.  $16x^2 - 9 = (4x - 3)(4x + 3)$
56.  $25x^2 - 49 = (5x - 7)(5x + 7)$
57.  $x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1)$
58.  $x^4 - 81 = (x^2 - 9)(x^2 + 9)$   
 $= (x - 3)(x + 3)(x^2 + 9)$
59.  $20x^4 - 5 = 5(4x^4 - 1) = 5(2x^2 - 1)(2x^2 + 1)$   
 $= 5(\sqrt{2}x - 1)(\sqrt{2}x + 1)(2x^2 + 1)$
60.  $12x^4 - 75 = 3(4x^4 - 25) = 3(2x^2 - 5)(2x^2 + 5)$   
 $= 3(\sqrt{2}x + 5)(\sqrt{2}x - 5)(2x^2 + 5)$
61.  $x^3 + 64 = x^3 + 4^3 = (x + 4)(x^2 - 4x + 16)$
62.  $x^3 + 125 = x^3 + 5^3 = (x + 5)(x^2 - 5x + 25)$
63.  $x^3 - 27 = x^3 - 3^3 = (x - 3)(x^2 + 3x + 9)$
64.  $x^3 - 216 = x^3 - 6^3 = (x - 6)(x^2 + 6x + 36)$
65.  $8 - x^3 = 2^3 - x^3 = (2 - x)(4 + 2x + x^2)$
66.  $27 - x^3 = 3^3 - x^3 = (3 - x)(9 + 3x + x^2)$
67.  $8x^3 - 27 = (2x)^3 - 3^3 = (2x - 3)(4x^2 + 6x + 9)$

68.  $8x^3 - 125 = (2x)^3 - 5^3$   
 $= (2x - 5)(4x^2 + 10x + 25)$
69.  $40x^3 + 5 = 5(8x^3 + 1) = 5((2x)^3 + 1^3)$   
 $= 5(2x + 1)(4x^2 - 2x + 1)$
70.  $7x^3 + 56 = 7(x^3 + 8) = 7(x^3 + 2^3)$   
 $= 7(x + 2)(x^2 - 2x + 4)$
71.  $1 - 16x^2 = (1 + 4x)(1 - 4x)$
72.  $4 - 25x^2 = (2 - 5x)(2 + 5x)$
73.  $x^2 - 6x + 9 = (x - 3)^2$
74.  $x^2 - 8x + 16 = (x - 4)^2$
75.  $4x^2 + 4x + 1 = (2x + 1)^2$
76.  $16x^2 + 8x + 1 = (4x + 1)^2$
77.  $2x^2 - 8x - 10 = 2(x^2 - 4x - 5)$   
 $= 2(x - 5)(x + 1)$
78.  $5x^2 - 10x - 40 = 5(x^2 - 2x - 8)$   
 $= 5(x - 4)(x + 2)$
79.  $2x^2 + 3x - 20 = (2x - 5)(x + 4)$
80.  $2x^2 - 7x - 30 = (2x + 5)(x - 6)$
81.  $x^2 - 24x + 36$  is irreducible.
82.  $x^2 - 20x + 25$  is irreducible.
83.  $3x^5 + 12x^4 + 12x^3 = 3x^3(x^2 + 4x + 4)$   
 $= 3x^3(x + 2)^2$
84.  $2x^5 + 16x^4 + 32x^3 = 2x^3(x^2 + 8x + 16)$   
 $= 2x^3(x + 4)^2$
85.  $9x^2 - 1 = (3x - 1)(3x + 1)$
86.  $16x^2 - 25 = (4x - 5)(4x + 5)$
87.  $16x^2 + 24x + 9 = (4x + 3)^2$
88.  $4x^2 + 20x + 25 = (2x + 5)^2$
89.  $x^2 + 15$  is irreducible.
90.  $x^2 + 24$  is irreducible.
91.  $45x^3 + 8x^2 - 4x = x(45x^2 + 8x - 4)$   
 $= x(5x + 2)(9x - 2)$
92.  $25x^3 + 40x^2 - 9x = x(25x^2 + 40x - 9)$   
 $= x(5x + 9)(5x - 1)$
93.  $ax^2 - 7a^2x - 8a^3 = a(x^2 - 7ax - 8a^2)$   
 $= a(x - 8a)(x + a)$
94.  $ax^2 - 10a^2x - 24a^3 = a(x^2 - 10ax - 24a^2)$   
 $= a(x - 12a)(x + 2a)$
95.  $x^2 - 16a^2 + 8x + 16 = (x^2 + 8x + 16) - 16a^2$   
 $= (x + 4)^2 - 16a^2$   
 $= (x + 4 - 4a)(x + 4 + 4a)$
96.  $x^2 - 36a^2 + 6x + 9 = (x^2 + 6x + 9) - 36a^2$   
 $= (x + 3)^2 - 36a^2$   
 $= (x + 3 - 6a)(x + 3 + 6a)$
97.  $3x^5 + 12x^4y + 12x^3y^2 = 3x^3(x^2 + 4xy + 4y^2)$   
 $= 3x^3(x + 2y)^2$
98.  $4x^5 + 24x^4y + 36x^3y^2 = 4x^3(x^2 + 6xy + 9y^2)$   
 $= 4x^3(x + 3y)^2$
99.  $x^2 - 25a^2 + 4x + 4 = (x^2 + 4x + 4) - 25a^2$   
 $= (x + 2)^2 - 25a^2$   
 $= (x + 2 - 5a)(x + 2 + 5a)$
100.  $x^2 - 16a^2 + 4x + 4 = (x^2 + 4x + 4) - 16a^2$   
 $= (x + 2)^2 - 16a^2$   
 $= (x + 2 - 4a)(x + 2 + 4a)$
101.  $18x^6 + 12x^5y + 2x^4y^2 = 2x^4(9x^2 + 6xy + y^2)$   
 $= 2x^4(3x + y)^2$
102.  $12x^6 + 12x^5y + 3x^4y^2 = 3x^4(4x^2 + 4xy + y^2)$   
 $= 3x^4(2x + y)^2$

**P.4 B Exercises: Applying the Concepts**

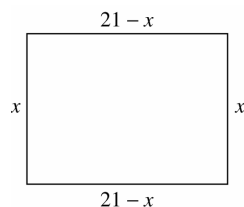
103.



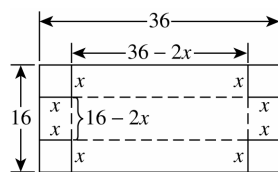
If one side of the garden is  $x$  feet, and the perimeter is 16 feet, then  $\frac{16-2x}{2} = 8-x$  gives the other dimension of the rectangle. So, the area of the garden is  $x(8-x)$ .

104.

If one side of the tray is  $x$  inches, and the perimeter is 42 inches, then  $\frac{42-2x}{2} = 21-x$  gives the other dimension of the tray. So, the area of the tray is  $x(21-x)$ .



Exercise 104



Exercises 105–106

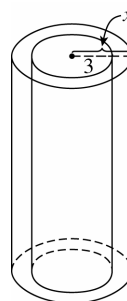
105. If  $x$  = the length of the cut corner, then  $36-2x$  = the length of the box, and  $16-2x$  = the width of the box. The height of the box is  $x$ . So,  

$$v = x(36-2x)(16-2x) = x(2(18-x))(2(8-x)) = 4x(18-x)(8-x)$$

106. 
$$\begin{aligned} S.A. &= (36-2x)(16-2x) + 2x(36-2x) \\ &\quad + 2x(16-2x) \\ &= 576 - 104x + 4x^2 + 72x - 4x^2 + 32x - 4x^2 \\ &= 576 - 4x^2 = 4(144 - x^2) \\ &= 4(12-x)(12+x) \end{aligned}$$

107. The area of the outside circle =  $4\pi \text{ cm}^2$ . The area of the inside circle =  $\pi x^2 \text{ cm}^2$ . So the area of the disk = area of outside circle - area of inside circle =  $4\pi - \pi x^2 = \pi(4-x^2) = \pi(2-x)(2+x) \text{ cm}^2$ .

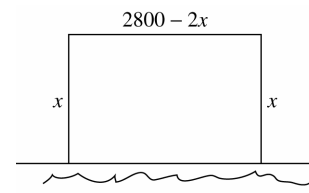
108.



The volume of the inside cylinder is  $3^2\pi \cdot 8 = 72\pi \text{ ft}^3$ , and the volume of the outside cylinder is  $x^2\pi \cdot 8 = 8\pi x^2 \text{ ft}^3$ . So the volume between the cylinders is  $8\pi x^2 - 72\pi = 8\pi(x^2 - 9) = 8\pi(x-3)(x+3) \text{ ft}^3$ .

109. If one side of the fence is  $x$  feet, and the rancher needs a total of 2800 feet of fencing, then the width of the fence is  $2800-2x$ . So, the area of the pen is

$$\begin{aligned} x(2800-2x) &= 2800x - 2x^2 = \\ &= 2x(1400-x) \text{ ft}^2. \end{aligned}$$



110. The area of the figure = the area of the rectangle plus the area of the circle. Find the length of the rectangle as follows: The

circumference of the circle =  $2\pi\left(\frac{x}{2}\right) = \pi x$ .

The perimeter of the figure is 48, so the length of the rectangle is  $\frac{48-\pi x}{2}$ . The area of the

circle is  $\frac{\pi x^2}{4}$  and the area of the rectangle is

$$x\left(24 - \frac{\pi x}{2}\right) = 24x - \frac{\pi x^2}{2}.$$

So the area of the figure is

$$\frac{\pi x^2}{4} + \left(24x - \frac{\pi x^2}{2}\right) = \frac{1}{4}(96x - \pi x^2) =$$

$$\frac{1}{4}x(96 - \pi x) \text{ in.}^2$$

## P.5 Rational Expressions

### P.5 Practice Problems

$$1. \frac{0.05(1-0.1)}{0.95(0.1)+(0.05)(1-0.1)} \approx 0.32$$

The likelihood that a student who tests positive is a nonuser when the test that is used is 95% accurate is about 32%.

$$2. \text{ a. } \frac{2x^3 + 8x^2}{3x^2 + 12x} = \frac{2x^2(x+4)}{3x(x+4)} = \frac{2x}{3}$$

$$\text{ b. } \frac{x^2 - 4}{x^2 + 4x + 4} = \frac{(x-2)(x+2)}{(x+2)^2} = \frac{x-2}{x+2}$$

$$3. \frac{\frac{x^2 - 2x - 3}{7x^3 + 28x^2}}{\frac{4x + 4}{2x^4 + 8x^3}} = \frac{x^2 - 2x - 3}{7x^3 + 28x^2} \div \frac{4x + 4}{2x^4 + 8x^3}$$

$$= \frac{x^2 - 2x - 3}{7x^3 + 28x^2} \cdot \frac{2x^4 + 8x^3}{4x + 4}$$

$$= \frac{(x-3)(x+1)}{7x^2(x+4)} \cdot \frac{2x^3(x+4)}{4(x+1)}$$

$$= \frac{x(x-3)}{14}$$

$$4. \text{ a. } \frac{5x + 22}{x^2 - 36} + \frac{2(x+10)}{x^2 - 36} = \frac{5x + 22 + 2x + 20}{x^2 - 36}$$

$$= \frac{7x + 42}{x^2 - 36}$$

$$= \frac{7(x+6)}{(x-6)(x+6)} = \frac{7}{x-6}$$

$$\text{ b. } \frac{4x+1}{x^2+x-12} - \frac{3x+4}{x^2+x-12} = \frac{(4x+1)-(3x+4)}{x^2+x-12}$$

$$= \frac{x-3}{x^2+x-12}$$

$$= \frac{x-3}{(x+4)(x-3)}$$

$$= \frac{1}{x+4}$$

$$5. \text{ a. } \frac{2x}{x+2} + \frac{3x}{x-5}$$

$$= \frac{2x(x-5)}{(x+2)(x-5)} + \frac{3x(x+2)}{(x+2)(x-5)}$$

$$= \frac{2x^2 - 10x + 3x^2 + 6x}{(x+2)(x-5)} = \frac{5x^2 - 4x}{(x+2)(x-5)}$$

$$= \frac{x(5x-4)}{(x+2)(x-5)}$$

$$\text{ b. } \frac{5x}{x-4} - \frac{2x}{x+3}$$

$$= \frac{5x(x+3)}{(x-4)(x+3)} - \frac{2x(x-4)}{(x-4)(x+3)}$$

$$= \frac{5x^2 + 15x - 2x^2 + 8x}{(x-4)(x+3)}$$

$$= \frac{3x^2 + 23x}{(x-4)(x+3)} = \frac{x(3x+23)}{(x-4)(x+3)}$$

$$6. \text{ a. } \frac{x^2 + 3x}{x^2(x+2)^2(x-2)}, \frac{4x^2 + 1}{3x(x-2)^2}$$

$$\text{LCD} = 3x^2(x+2)^2(x-2)^2$$

$$\text{ b. } \frac{2x-1}{x^2-25}, \frac{3-7x^2}{x^2+4x-5}$$

$$x^2 - 25 = (x-5)(x+5)$$

$$x^2 + 4x - 5 = (x-1)(x+5)$$

$$\text{LCD} = (x-1)(x-5)(x+5)$$

$$7. \text{ a. } \frac{4}{x^2 - 4x + 4} + \frac{x}{x^2 - 4}$$

$$= \frac{4}{(x-2)^2} + \frac{x}{(x-2)(x+2)}$$

$$= \frac{4(x+2)}{(x-2)^2(x+2)} + \frac{x(x-2)}{(x-2)^2(x+2)}$$

$$= \frac{(4x+8) + (x^2-2x)}{(x-2)^2(x+2)} = \frac{x^2 + 2x + 8}{(x-2)^2(x+2)}$$

$$\text{ b. } \frac{2x}{3(x-5)^2} - \frac{6x}{2(x^2-5x)}$$

$$= \frac{2x}{3(x-5)^2} - \frac{6x}{2x(x-5)}$$

$$= \frac{2x(2x)}{3 \cdot 2x(x-5)^2} - \frac{6x(3)(x-5)}{3 \cdot 2x(x-5)^2}$$

$$= \frac{4x^2 - 18x^2 + 90x}{6x(x-5)^2} = \frac{-14x^2 + 90x}{6x(x-5)^2}$$

$$= \frac{2x(-7x+45)}{6x(x-5)^2} = \frac{-7x+45}{3(x-5)^2}$$

$$8. \frac{\frac{5}{3x} + \frac{1}{3}}{\frac{x^2 - 25}{3x}} = \frac{\left(\frac{5}{3x} + \frac{1}{3}\right) \cdot 3x}{\left(\frac{x^2 - 25}{3x}\right) \cdot 3x} = \frac{5 + x}{x^2 - 25}$$

$$= \frac{x + 5}{(x - 5)(x + 5)} = \frac{1}{x - 5}$$

$$9. \frac{5x}{3x - \frac{4}{2 - \frac{1}{3}}} = \frac{5x}{3x - \frac{4}{\frac{5}{3}}} = \frac{5x}{3x - 4\left(\frac{3}{5}\right)} = \frac{5x}{3x - \frac{12}{5}}$$

$$= \frac{5x \cdot 5}{\left(3x - \frac{12}{5}\right) \cdot 5} = \frac{25x}{15x - 12}$$

$$11. \frac{2x - 6}{9 - x^2} = \frac{2(x - 3)}{(3 - x)(3 + x)} = \frac{-2(3 - x)}{(3 - x)(3 + x)}$$

$$= -\frac{2}{3 + x}, x \neq -3, x \neq 3$$

$$12. \frac{15 + 3x}{x^2 - 25} = \frac{3(5 + x)}{(x - 5)(x + 5)} = \frac{3}{x - 5},$$

$$x \neq -5, x \neq 5$$

$$13. \frac{2x - 1}{1 - 2x} = \frac{-1(1 - 2x)}{1 - 2x} = -1, x \neq \frac{1}{2}$$

$$14. \frac{2 - 5x}{5x - 2} = \frac{-1(5x - 2)}{5x - 2} = -1, x \neq \frac{2}{5}$$

$$15. \frac{x^2 - 6x + 9}{4x - 12} = \frac{(x - 3)^2}{4(x - 3)} = \frac{x - 3}{4}, x \neq 3$$

$$16. \frac{x^2 - 10x + 25}{3x - 15} = \frac{(x - 5)^2}{3(x - 5)} = \frac{x - 5}{3}, x \neq 5$$

$$17. \frac{7x^2 + 7x}{x^2 + 2x + 1} = \frac{7x(x + 1)}{(x + 1)^2} = \frac{7x}{x + 1}, x \neq -1$$

$$18. \frac{4x^2 + 12x}{x^2 + 6x + 9} = \frac{4x(x + 3)}{(x + 3)^2} = \frac{4x}{x + 3}, x \neq -3$$

$$19. \frac{x^2 - 11x + 10}{x^2 + 6x - 7} = \frac{(x - 10)(x - 1)}{(x + 7)(x - 1)} = \frac{x - 10}{x + 7},$$

$$x \neq -7, x \neq 1$$

$$20. \frac{x^2 + 2x - 15}{x^2 - 7x + 12} = \frac{(x + 5)(x - 3)}{(x - 4)(x - 3)} = \frac{x + 5}{x - 4},$$

$$x \neq 3, x \neq 4$$

$$21. \frac{6x^4 + 14x^3 + 4x^2}{6x^4 - 10x^3 - 4x^2} = \frac{2x^2(3x^2 + 7x + 2)}{2x^2(3x^2 - 5x - 2)}$$

$$= \frac{2x^2(3x + 1)(x + 2)}{2x^2(3x + 1)(x - 2)}$$

$$= \frac{x + 2}{x - 2}, x \neq -\frac{1}{3}, x \neq 0, x \neq 2$$

$$22. \frac{3x^3 + x^2}{3x^4 - 11x^3 - 4x^2} = \frac{x^2(3x + 1)}{x^2(3x^2 - 11x - 4)}$$

$$= \frac{x^2(3x + 1)}{x^2(3x + 1)(x - 4)}$$

$$= \frac{1}{x - 4},$$

$$x \neq -\frac{1}{3}, x \neq 0, x \neq 4$$

### P.5 A Exercises: Basic Skills and Concepts

- The least common denominator for two rational expressions is the polynomial of least degree that contains each denominator as a factor.
- The first step in finding the LCD for two rational expressions is to factor the denominators completely.
- If the denominators for two rational expressions are  $x^2 + 2x$  and  $x^2 - x - 2$ , then the LCD is  $x(x + 2)(x - 2)(x + 1)$ .
- A rational expression that contains another rational expression in its numerator or denominator is called a complex rational expression or (complex fraction).

5. False

6. True

$$7. \frac{2x + 2}{x^2 + 2x + 1} = \frac{2(x + 1)}{(x + 1)^2} = \frac{2}{x + 1}, x \neq -1$$

$$8. \frac{3x - 6}{x^2 - 4x + 4} = \frac{3(x - 2)}{(x - 2)^2} = \frac{3}{x - 2}, x \neq 2$$

$$9. \frac{3x + 3}{x^2 - 1} = \frac{3(x + 1)}{(x - 1)(x + 1)} = \frac{3}{x - 1}, x \neq -1, x \neq 1$$

$$10. \frac{10 - 5x}{4 - x^2} = \frac{5(2 - x)}{(2 - x)(2 + x)} = \frac{5}{2 + x},$$

$$x \neq -2, x \neq 2$$

$$23. \frac{x-3}{2x+4} \cdot \frac{10x+20}{5x-15} = \frac{x-3}{2(x+2)} \cdot \frac{10(x+2)}{5(x-3)} = 1$$

$$24. \frac{6x+4}{2x-8} \cdot \frac{x-4}{9x+6} = \frac{2(3x+2)}{2(x-4)} \cdot \frac{x-4}{3(3x+2)} = \frac{1}{3}$$

$$25. \frac{2x+6}{4x-8} \cdot \frac{x^2+x-6}{x^2-9} = \frac{2(x+3)}{4(x-2)} \cdot \frac{(x+3)(x-2)}{(x+3)(x-3)} = \frac{x+3}{2(x-3)}$$

$$26. \frac{25x^2-9}{4-2x} \cdot \frac{4-x^2}{10x-6} = \frac{(5x+3)(5x-3)}{2(2-x)} \cdot \frac{(2-x)(2+x)}{2(5x-3)} = \frac{(5x+3)(2+x)}{4}$$

$$27. \frac{x^2-7x}{x^2-6x-7} \cdot \frac{x^2-1}{x^2} = \frac{x(x-7)}{(x+1)(x-7)} \cdot \frac{(x+1)(x-1)}{x^2} = \frac{x-1}{x}$$

$$28. \frac{x^2-9}{x^2-6x+9} \cdot \frac{5x-15}{x+3} = \frac{(x-3)(x+3)}{(x-3)(x-3)} \cdot \frac{5(x-3)}{x+3} = 5$$

$$29. \frac{x^2-x-6}{x^2+3x+2} \cdot \frac{x^2-1}{x^2-9} = \frac{(x-3)(x+2)}{(x+1)(x+2)} \cdot \frac{(x-1)(x+1)}{(x-3)(x+3)} = \frac{x-1}{x+3}$$

$$30. \frac{x^2+2x-8}{x^2+x-20} \cdot \frac{x^2-16}{x^2+5x+4} = \frac{(x-2)(x+4)}{(x-4)(x+5)} \cdot \frac{(x-4)(x+4)}{(x+1)(x+4)} = \frac{(x-2)(x+4)}{(x+1)(x+5)}$$

$$31. \frac{2-x}{x+1} \cdot \frac{x^2+3x+2}{x^2-4} = \frac{-1(x-2)}{x+1} \cdot \frac{(x+1)(x+2)}{(x-2)(x+2)} = -1$$

$$32. \frac{3-x}{x+5} \cdot \frac{x^2+8x+15}{x^2-9} = \frac{-1(x-3)}{x+5} \cdot \frac{(x+3)(x+5)}{(x-3)(x+3)} = -1$$

$$33. \frac{x+2}{6} \div \frac{4x+8}{9} = \frac{x+2}{6} \cdot \frac{9}{4(x+2)} = \frac{3}{8}$$

$$34. \frac{x+3}{20} \div \frac{4x+12}{9} = \frac{x+3}{20} \cdot \frac{9}{4(x+3)} = \frac{9}{80}$$

$$35. \frac{x^2-9}{x} \div \frac{2x+6}{5x^2} = \frac{(x-3)(x+3)}{x} \cdot \frac{5x^2}{2(x+3)} = \frac{5x(x-3)}{2}$$

$$36. \frac{x^2-1}{3x} \div \frac{7x-7}{x^2+x} = \frac{(x-1)(x+1)}{3x} \cdot \frac{x(x+1)}{7(x-1)} = \frac{(x+1)^2}{21}$$

$$37. \frac{x^2+2x-3}{x^2+8x+16} \div \frac{x-1}{3x+12} = \frac{(x-1)(x+3)}{(x+4)^2} \cdot \frac{3(x+4)}{x-1} = \frac{3(x+3)}{x+4}$$

$$38. \frac{x^2+5x+6}{x^2+6x+9} \div \frac{x^2+3x+2}{x^2+7x+12} = \frac{(x+2)(x+3)}{(x+3)(x+3)} \cdot \frac{(x+3)(x+4)}{(x+1)(x+2)} = \frac{x+4}{x+1}$$

$$39. \left( \frac{x^2-9}{x^3+8} \div \frac{x+3}{x^3+2x^2-x-2} \right) \left( \frac{1}{x^2-1} \right) = \frac{(x-3)(x+3)}{(x+2)(x^2-2x+4)} \cdot \frac{(x^2-1)(x+2)}{x+3} \cdot \frac{1}{x^2-1} = \frac{x-3}{x^2-2x+4}$$

$$40. \left( \frac{x^2-25}{x^2-3x-4} \div \frac{x^2+3x-10}{x^2-1} \right) \left( \frac{x-2}{x-5} \right) = \frac{(x-5)(x+5)}{(x-4)(x+1)} \cdot \frac{(x-1)(x+1)}{(x+5)(x-2)} \cdot \frac{x-2}{x-5} = \frac{x-1}{x-4}$$

$$41. \frac{x}{5} + \frac{3}{5} = \frac{x+3}{5}$$

$$42. \frac{7}{4} - \frac{x}{4} = \frac{7-x}{4}$$

$$43. \frac{x}{2x+1} + \frac{4}{2x+1} = \frac{x+4}{2x+1}$$

$$44. \frac{2x}{7x-3} + \frac{x}{7x-3} = \frac{3x}{7x-3}$$

$$45. \frac{x^2}{x+1} - \frac{x^2-1}{x+1} = \frac{x^2 - (x^2-1)}{x+1} = \frac{1}{x+1}$$

$$46. \frac{2x+7}{3x+2} - \frac{x-2}{3x+2} = \frac{(2x+7)-(x-2)}{3x+2} = \frac{x+9}{3x+2}$$

$$47. \frac{4}{3-x} + \frac{2x}{x-3} = \frac{4}{-1(x-3)} + \frac{2x}{x-3} \\ = \frac{-4+2x}{x-3} = \frac{2(x-2)}{x-3}$$

$$48. \frac{-2}{1-x} + \frac{2-x}{x-1} = \frac{-2}{1-x} + \frac{2-x}{-1(1-x)} \\ = \frac{-2}{1-x} - \frac{2-x}{1-x} = \frac{-4+x}{1-x} \\ = \frac{x-4}{1-x} \text{ or } -\frac{x-4}{x-1}$$

$$49. \frac{5x}{x^2+1} + \frac{2x}{x^2+1} = \frac{7x}{x^2+1}$$

$$50. \frac{x}{2(x-1)^2} + \frac{3x}{2(x-1)^2} = \frac{4x}{2(x-1)^2} = \frac{2x}{(x-1)^2}$$

$$51. \frac{7x}{2(x-3)} + \frac{x}{2(x-3)} = \frac{8x}{2(x-3)} = \frac{4x}{x-3}$$

$$52. \frac{4x}{4(x+5)^2} + \frac{8x}{4(x+5)^2} = \frac{12x}{4(x+5)^2} = \frac{3x}{(x+5)^2}$$

$$53. \frac{x}{x^2-4} - \frac{2}{x^2-4} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$$

$$54. \frac{5x}{x^2-1} - \frac{5}{x^2-1} = \frac{5x-5}{(x-1)(x+1)} \\ = \frac{5(x-1)}{(x-1)(x+1)} = \frac{5}{x+1}$$

$$55. \frac{x-2}{2x+1} - \frac{x}{2x-1} \\ = \frac{(x-2)(2x-1)}{(2x+1)(2x-1)} - \frac{x(2x+1)}{(2x+1)(2x-1)} \\ = \frac{(2x^2-5x+2)-(2x^2+x)}{(2x+1)(2x-1)} \\ = \frac{-6x+2}{(2x+1)(2x-1)} = \frac{2(1-3x)}{(2x+1)(2x-1)}$$

$$56. \frac{2x-1}{4x+1} - \frac{2x}{4x-1} \\ = \frac{(2x-1)(4x-1)}{(4x+1)(4x-1)} - \frac{2x(4x+1)}{(4x+1)(4x-1)}$$

$$= \frac{(8x^2-6x+1)-(8x^2+2x)}{(4x+1)(4x-1)}$$

$$= \frac{1-8x}{(4x+1)(4x-1)}$$

$$57. \frac{-x}{x+2} + \frac{x-2}{x} - \frac{x}{x-2} \\ = \frac{-x^2(x-2)}{x(x+2)(x-2)} + \frac{(x-2)^2(x+2)}{x(x+2)(x-2)} \\ - \frac{x^2(x+2)}{x(x+2)(x-2)} \\ = \frac{(-x^3+2x^2)}{x(x+2)(x-2)} + \frac{(x^3-2x^2-4x+8)}{x(x+2)(x-2)} \\ + \frac{-(x^3+2x^2)}{x(x+2)(x-2)} \\ = \frac{-x^3-2x^2-4x+8}{x(x+2)(x-2)} = -\frac{x^3+2x^2+4x-8}{x(x+2)(x-2)}$$

$$58. \frac{3x}{x-1} + \frac{x+1}{x} - \frac{2x}{x+1} \\ = \frac{3x^2(x+1)}{x(x-1)(x+1)} + \frac{(x+1)^2(x-1)}{x(x-1)(x+1)} \\ - \frac{2x^2(x-1)}{x(x-1)(x+1)} \\ = \frac{3x^3+3x^2}{x(x-1)(x+1)} + \frac{x^3+x^2-x-1}{x(x-1)(x+1)} \\ - \frac{2x^3-2x^2}{x(x-1)(x+1)} \\ = \frac{2x^3+6x^2-x-1}{x(x-1)(x+1)}$$

In exercises 59–66, to find the LCD, first factor each denominator and then multiply each prime factor the greatest number of times it appears as a factor.

$$59. \quad 3x-6 = 3(x-2) \text{ and } 4x-8 = 4(x-2) \Rightarrow \\ \text{LCD} = 3 \cdot 4(x-2) = 12(x-2)$$

$$60. \quad 7+21x = 7(1+3x) \text{ and } 3+9x = 3(1+3x) \Rightarrow \\ \text{LCD} = 7 \cdot 3(1+3x) = 21(1+3x)$$

$$61. \quad 4x^2-1 = (2x+1)(2x-1) \text{ and } \\ (2x+1)^2 = (2x+1)(2x+1) \Rightarrow \\ \text{LCD} = (2x-1)(2x+1)^2$$

$$62. \quad (3x-1)^2 = (3x-1)(3x-1) \text{ and } 9x^2-1 = \\ (3x-1)(3x+1) \Rightarrow \text{LCD} = (3x-1)^2(3x+1)$$

$$63. \quad x^2 + 3x + 2 = (x+1)(x+2) \text{ and } x^2 - 1 = (x-1)(x+1) \Rightarrow \text{LCD} = (x-1)(x+1)(x+2)$$

$$64. \quad x^2 - x - 6 = (x-3)(x+2) \text{ and } x^2 - 9 = (x-3)(x+3) \Rightarrow \text{LCD} = (x-3)(x+3)(x+2)$$

$$65. \quad x^2 - 5x + 4 = (x-1)(x-4) \text{ and } x^2 + x - 2 = (x-1)(x+2) \Rightarrow \text{LCD} = (x-1)(x-4)(x+2)$$

$$66. \quad x^2 - 2x - 3 = (x-3)(x+1) \text{ and } x^2 + 3x + 2 = (x+1)(x+2) \Rightarrow \text{LCD} = (x-3)(x+1)(x+2)$$

$$67. \quad \frac{5}{x-3} + \frac{2x}{x^2-9} = \frac{5}{x-3} + \frac{2x}{(x-3)(x+3)} = \frac{5(x+3)}{(x-3)(x+3)} + \frac{2x}{(x-3)(x+3)} = \frac{5x+15+2x}{(x-3)(x+3)} = \frac{7x+15}{(x-3)(x+3)}$$

$$68. \quad \frac{3x}{x-1} + \frac{x}{x^2-1} = \frac{3x}{x-1} + \frac{x}{(x-1)(x+1)} = \frac{3x(x+1)}{(x-1)(x+1)} + \frac{x}{(x-1)(x+1)} = \frac{3x^2+3x+x}{(x-1)(x+1)} = \frac{3x^2+4x}{(x-1)(x+1)} = \frac{x(3x+4)}{(x-1)(x+1)}$$

$$69. \quad \frac{2x}{x^2-4} - \frac{x}{x+2} = \frac{2x}{(x-2)(x+2)} - \frac{x}{x+2} = \frac{2x}{(x-2)(x+2)} - \frac{x(x-2)}{(x+2)(x-2)} = \frac{2x - (x^2 - 2x)}{(x-2)(x+2)} = \frac{-x^2 + 4x}{(x-2)(x+2)} = \frac{x(4-x)}{(x-2)(x+2)}$$

$$70. \quad \frac{3x-1}{x^2-16} - \frac{2x+1}{x-4} = \frac{3x-1}{(x-4)(x+4)} - \frac{2x+1}{x-4} = \frac{3x-1}{(x-4)(x+4)} - \frac{(2x+1)(x+4)}{(x-4)(x+4)} = \frac{3x-1 - (2x^2+9x+4)}{(x-4)(x+4)} = \frac{-2x^2-6x-5}{(x-4)(x+4)} = -\frac{2x^2+6x+5}{(x-4)(x+4)}$$

$$71. \quad \frac{x-2}{x^2+3x-10} + \frac{x+3}{x^2+x-6} = \frac{x-2}{(x-2)(x+5)} + \frac{x+3}{(x-2)(x+3)} = \frac{1}{x+5} + \frac{1}{x-2} = \frac{x+5}{(x+5)(x-2)} + \frac{x+5}{(x+5)(x-2)} = \frac{2x+3}{(x-2)(x+5)}$$

$$72. \quad \frac{x+3}{x^2-x-2} + \frac{x-1}{x^2+2x+1} = \frac{x+3}{(x-2)(x+1)} + \frac{x-1}{(x+1)(x+1)} = \frac{(x+3)(x+1)}{(x-2)(x+1)(x+1)} + \frac{(x-1)(x-2)}{(x+1)(x+1)(x-2)} = \frac{(x^2+4x+3) + (x^2-3x+2)}{(x-2)(x+1)(x+1)} = \frac{2x^2+x+5}{(x-2)(x+1)(x+1)}$$

$$73. \quad \frac{2x-3}{9x^2-1} + \frac{4x-1}{(3x-1)^2} = \frac{2x-3}{(3x-1)(3x+1)} + \frac{4x-1}{(3x-1)^2} = \frac{(2x-3)(3x-1)}{(3x+1)(3x-1)^2} + \frac{(4x-1)(3x+1)}{(3x+1)(3x-1)^2} = \frac{(6x^2-11x+3) + (12x^2+x-1)}{(3x+1)(3x-1)^2} = \frac{18x^2-10x+2}{(3x+1)(3x-1)^2} = \frac{2(9x^2-5x+1)}{(3x+1)(3x-1)^2}$$



$$\begin{aligned}
74. \quad & \frac{3x+1}{(2x+1)^2} + \frac{x+3}{4x^2-1} \\
&= \frac{3x+1}{(2x+1)^2} + \frac{x+3}{(2x-1)(2x+1)} \\
&= \frac{(3x+1)(2x-1) + (x+3)(2x+1)}{(2x+1)^2(2x-1)} + \frac{(x+3)(2x+1)}{(2x-1)(2x+1)^2} \\
&= \frac{(6x^2-x-1) + (2x^2+7x+3)}{(2x-1)(2x+1)^2} \\
&= \frac{8x^2+6x+2}{(2x-1)(2x+1)^2} = \frac{2(4x^2+3x+1)}{(2x-1)(2x+1)^2}
\end{aligned}$$

$$\begin{aligned}
75. \quad & \frac{x-3}{x^2-25} - \frac{x-3}{x^2+9x+20} \\
&= \frac{x-3}{(x-5)(x+5)} - \frac{x-3}{(x+4)(x+5)} \\
&= \frac{(x-3)(x+4) - (x-3)(x+5)}{(x-5)(x+5)(x+4)} \\
&= \frac{(x^2+x-12) - (x^2-8x+15)}{(x-5)(x+5)(x+4)} \\
&= \frac{9x-27}{(x-5)(x+5)(x+4)} \\
&= \frac{9(x-3)}{(x-5)(x+5)(x+4)}
\end{aligned}$$

$$\begin{aligned}
76. \quad & \frac{2x}{x^2-16} - \frac{2x-7}{x^2-7x+12} \\
&= \frac{2x}{(x-4)(x+4)} - \frac{2x-7}{(x-4)(x-3)} \\
&= \frac{2x(x-3) - (2x-7)(x+4)}{(x-4)(x+4)(x-3)} \\
&= \frac{(2x^2-6x) - (2x^2+x-28)}{(x-4)(x+4)(x-3)} \\
&= \frac{-7x+28}{(x-4)(x+4)(x-3)} \\
&= \frac{-7(x-4)}{(x-4)(x+4)(x-3)} \\
&= \frac{7}{(x+4)(x-3)}
\end{aligned}$$

$$\begin{aligned}
77. \quad & \frac{3}{x^2-4} + \frac{1}{2-x} - \frac{1}{2+x} \\
&= -\frac{3}{4-x^2} + \frac{1}{2-x} - \frac{1}{2+x} \\
&= -\frac{3}{(2-x)(2+x)} + \frac{1}{2-x} - \frac{1}{2+x} \\
&= \frac{-3}{(2-x)(2+x)} + \frac{1(2+x)}{(2-x)(2+x)} - \frac{1(2-x)}{(2+x)(2-x)} \\
&= \frac{-3+(2+x)-(2-x)}{(2-x)(2+x)} = \frac{2x-3}{(2-x)(2+x)}
\end{aligned}$$

$$\begin{aligned}
78. \quad & \frac{2}{5+x} + \frac{5}{x^2-25} + \frac{7}{5-x} \\
&= \frac{2}{5+x} - \frac{5}{25-x^2} + \frac{7}{5-x} \\
&= \frac{2}{5+x} - \frac{5}{(5-x)(5+x)} + \frac{7}{5-x} \\
&= \frac{2(5-x) - 5 + (35+7x)}{(5+x)(5-x)} = \frac{10-2x-5+(35+7x)}{(5+x)(5-x)} \\
&= \frac{5x+40}{(5+x)(5-x)} = \frac{5(x+8)}{(5+x)(5-x)} + \frac{7(5+x)}{(5-x)(5+x)}
\end{aligned}$$

$$\begin{aligned}
79. \quad & \frac{x+3a}{x-5a} - \frac{x+5a}{x-3a} \\
&= \frac{(x+3a)(x-3a)}{(x-5a)(x-3a)} - \frac{(x+5a)(x-5a)}{(x-3a)(x-5a)} \\
&= \frac{(x^2-9a^2) - (x^2-25a^2)}{(x-5a)(x-3a)} \\
&= \frac{16a^2}{(x-5a)(x-3a)}
\end{aligned}$$

$$\begin{aligned}
80. \quad & \frac{3x-a}{2x-a} - \frac{2x+a}{3x+a} \\
&= \frac{(3x-a)(3x+a)}{(2x-a)(3x+a)} - \frac{(2x+a)(2x-a)}{(3x+a)(2x-a)} \\
&= \frac{(9x^2-a^2) - (4x^2-a^2)}{(2x-a)(3x+a)} \\
&= \frac{5x^2}{(2x-a)(3x+a)}
\end{aligned}$$

$$81. \frac{1}{x+h} - \frac{1}{x} = \frac{x}{x(x+h)} - \frac{1(x+h)}{x(x+h)} \\ = \frac{x - (x+h)}{x(x+h)} = -\frac{h}{x(x+h)}$$

$$82. \frac{1}{(x+h)^2} - \frac{1}{x^2} = \frac{x^2}{x^2(x+h)^2} - \frac{1(x+h)^2}{x^2(x+h)^2} \\ = \frac{x^2 - (x+h)^2}{x^2(x+h)^2} \\ = \frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2} \\ = \frac{-2xh - h^2}{x^2(x+h)^2} = -\frac{h(2x+h)}{x^2(x+h)^2}$$

$$83. \frac{\frac{2}{x}}{\frac{3}{x^2}} = \frac{2}{x} \cdot \frac{x^2}{3} = \frac{2x}{3}$$

$$84. \frac{\frac{-6}{x}}{\frac{2}{x^3}} = \frac{-6}{x} \cdot \frac{x^3}{2} = -3x^2$$

$$85. \frac{\frac{1}{x}}{1 - \frac{1}{x}} = \frac{1}{x} \div \left(1 - \frac{1}{x}\right) = \frac{1}{x} \div \left(\frac{x-1}{x}\right) \\ = \frac{1}{x} \cdot \frac{x}{x-1} = \frac{1}{x-1}$$

$$86. \frac{\frac{1}{x^2}}{\frac{1}{x^2} - 1} = \frac{1}{x^2} \div \left(\frac{1}{x^2} - 1\right) = \frac{1}{x^2} \div \left(\frac{1-x^2}{x^2}\right) \\ = \frac{1}{x^2} \cdot \frac{x^2}{1-x^2} = \frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)}$$

We use Method 2 for exercises 87–90.

$$87. \frac{\frac{1}{x} - 1}{\frac{1}{x} + 1} = \frac{\left(\frac{1}{x} - 1\right)x}{\left(\frac{1}{x} + 1\right)x} = \frac{1-x}{1+x}$$

$$88. \frac{2 - \frac{2}{x}}{1 + \frac{2}{x}} = \frac{\left(2 - \frac{2}{x}\right)x}{\left(1 + \frac{2}{x}\right)x} = \frac{2x-2}{x+2} = \frac{2(x-1)}{x+2}$$

$$89. \frac{\frac{1}{x} - x}{1 - \frac{1}{x^2}} = \frac{\left(\frac{1}{x} - x\right)x^2}{\left(1 - \frac{1}{x^2}\right)x^2} = \frac{x-x^3}{x^2-1} = \frac{x(1-x^2)}{x^2-1} \\ = -x, x \neq 0$$

$$90. \frac{x - \frac{1}{x}}{\frac{1}{x^2} - 1} = \frac{\left(x - \frac{1}{x}\right)x^2}{\left(\frac{1}{x^2} - 1\right)x^2} = \frac{x^3 - x}{1-x^2} = \frac{x(x^2-1)}{1-x^2} \\ = -x, x \neq 0$$

$$91. x - \frac{x}{x + \frac{1}{2}} = x - \frac{x}{\frac{2x+1}{2}} = x - \left(x \cdot \frac{2}{2x+1}\right) \\ = x - \frac{2x}{2x+1} = \frac{x(2x+1)}{2x+1} - \frac{2x}{2x+1} \\ = \frac{2x^2 + x - 2x}{2x+1} = \frac{2x^2 - x}{2x+1} \\ = \frac{x(2x-1)}{2x+1}$$

$$92. x + \frac{x}{x + \frac{1}{2}} = x + \frac{x}{\frac{2x+1}{2}} = x + \left(x \cdot \frac{2}{2x+1}\right) \\ = x + \frac{2x}{2x+1} = \frac{x(2x+1)}{2x+1} + \frac{2x}{2x+1} \\ = \frac{2x^2 + x + 2x}{2x+1} = \frac{2x^2 + 3x}{2x+1} \\ = \frac{x(2x+3)}{2x+1}$$

$$93. \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \left(\frac{1}{x+h} - \frac{1}{x}\right) \div h \\ = \left(\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}\right) \div h \\ = \frac{x - (x+h)}{x(x+h)} \div h = \frac{-h}{x(x+h)} \div h \\ = \frac{-h}{x(x+h)} \cdot \frac{1}{h} = -\frac{1}{x(x+h)}, \\ x \neq -h, x \neq 0, h \neq 0$$

$$\begin{aligned}
 94. \quad & \frac{1}{(x+h)^2} - \frac{1}{x^2} \\
 &= \frac{h}{\left(\frac{1}{(x+h)^2} - \frac{1}{x^2}\right)} \div h \\
 &= \left(\frac{x^2}{x^2(x+h)^2} - \frac{(x+h)^2}{x^2(x+h)^2}\right) \div h \\
 &= \frac{x^2 - (x+h)^2}{x^2(x+h)^2} \div h \\
 &= \frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2} \cdot \frac{1}{h} \\
 &= \frac{-2xh - h^2}{x^2(x+h)^2} \cdot \frac{1}{h} = \frac{-h(2x+h)}{x^2(x+h)^2} \cdot \frac{1}{h} \\
 &= -\frac{(2x+h)}{x^2(x+h)^2}, x \neq -h, x \neq 0, h \neq 0
 \end{aligned}$$

$$\begin{aligned}
 95. \quad & \frac{1}{x-a} + \frac{1}{x+a} \\
 &= \frac{1}{x-a} - \frac{1}{x+a} \\
 &= \left(\frac{1}{x-a} + \frac{1}{x+a}\right) \div \left(\frac{1}{x-a} - \frac{1}{x+a}\right) \\
 &= \frac{(x+a) + (x-a)}{(x-a)(x+a)} \div \frac{(x+a) - (x-a)}{(x-a)(x+a)} \\
 &= \frac{2x}{(x-a)(x+a)} \cdot \frac{(x-a)(x+a)}{2a} = \frac{x}{a}, \\
 & x \neq a, x \neq -a
 \end{aligned}$$

$$\begin{aligned}
 96. \quad & \frac{1}{x-a} + \frac{1}{x+a} \\
 &= \left(\frac{1}{x-a} + \frac{1}{x+a}\right) \div \left(\frac{x}{x-a} - \frac{a}{x+a}\right) \\
 &= \frac{(x+a) + (x-a)}{(x-a)(x+a)} \div \frac{x(x+a) - a(x-a)}{(x-a)(x+a)} \\
 &= \frac{(x+a) + (x-a)}{(x-a)(x+a)} \div \frac{x^2 + ax - ax + a^2}{(x-a)(x+a)} \\
 &= \frac{2x}{(x-a)(x+a)} \cdot \frac{(x-a)(x+a)}{x^2 + a^2} = \frac{2x}{x^2 + a^2}
 \end{aligned}$$

### P.5 B Exercises: Applying the Concepts

97.  $d = 4 \Rightarrow r = 2$ . Substituting 2 for  $r$  in the formula gives  $\frac{125.6}{(3.14)(2^2)} = \frac{125.6}{12.56} = 10$  cm.

$$\begin{aligned}
 98. \quad & \frac{13.5 - 2(1.5^2)}{6(1.5)} = \frac{13.5 - 2(2.25)}{9.0} = \frac{13.5 - 4.5}{9.0} \\
 &= \frac{9.0}{9.0} = 1 \text{ foot}
 \end{aligned}$$

99.a. Originally the citrus extract is 3 out of 100

gallons or  $\frac{3}{100}$ . When  $x$  gallons of water are added to the mixture, the total number of gallons in the mixture is  $100 + x$ . There are still 3 gallons of citrus extract, so the fraction is  $\frac{3}{100+x}$ .

b. If  $x = 50$ , then  $\frac{3}{100+x} = \frac{3}{150} = 0.02 = 2\%$

100. a. The reservoir is 0.75% acid, so if the reservoir is half-full, there are  $0.0075 \times 200,000 = 1500$  gallons of acid. When  $x$  gallons of water are added, there are 1500 gallons of acid out of  $200,000 + x$  gallons of water, or  $\frac{1500}{200,000+x}$ .

b. If  $x = 100,000$ , then  $\frac{1500}{200,000+x} = \frac{1500}{200,000+100,000} = \frac{1500}{300,000} = 0.005 = 0.5\%$

101. a. The volume of a cylinder is given by  $V = 120 = \pi r^2 h \Rightarrow h = \frac{120}{\pi r^2}$ . The cost of each base =  $5(\pi x^2)$ ; since there are two bases, the total cost of the bases is  $10\pi x^2$ . The cost of the side is given by  $2\pi r h$ ; since  $r = x$  and  $h = \frac{120}{\pi r^2}$ , this gives  $2\pi x \left(\frac{120}{\pi x^2}\right) = \frac{240}{x}$ . So the total cost for the container is  $10\pi x^2 + \frac{240}{x} = \frac{10\pi x^3 + 240}{x}$ .

b.  $\frac{10(3.14)(2^3) + 240}{2} \approx 245.6$  cents or \$2.46

102. a. The area of the base =  $x^2$ . The height of the box is given by  $\frac{2.25}{x^2}$ , so the total area of the sides of the container =  
 $4x\left(\frac{2.25}{x^2}\right) = \frac{9}{x}$ . The cost of the container is  
 $40\left(x^2 + \frac{9}{x}\right) = \frac{40x^3 + 360}{x}$ .
- b.  $\frac{40(1.5^3) + 360}{1.5} = \frac{495}{1.5} = 330$  cents = \$3.30

## P.6 Rational Exponents and Radicals

### P.6 Practice Problems

1. a.  $\sqrt{144} = \sqrt{12^2} = 12$
- b.  $\sqrt{\frac{1}{49}} = \sqrt{\left(\frac{1}{7}\right)^2} = \frac{1}{7}$
- c.  $\sqrt{\frac{4}{64}} = \sqrt{\frac{2^2}{8^2}} = \sqrt{\left(\frac{2}{8}\right)^2} = \frac{2}{8} = \frac{1}{4}$
2. a.  $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$
- b.  $\sqrt{6}\sqrt{8} = \sqrt{6 \cdot 8} = \sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$
- c.  $\sqrt{12x^2} = \sqrt{4 \cdot 3 \cdot x^2} = \sqrt{4}\sqrt{3}\sqrt{x^2} = 2|x|\sqrt{3}$
- d.  $\sqrt{\frac{20y^3}{27x^2}} = \frac{\sqrt{20y^3}}{\sqrt{27x^2}} = \frac{\sqrt{4 \cdot 5 \cdot y^2 \cdot y}}{\sqrt{9 \cdot 3 \cdot x^2}} = \frac{\sqrt{4}\sqrt{5}\sqrt{y^2}\sqrt{y}}{\sqrt{9}\sqrt{3}\sqrt{x^2}} = \frac{2|y|\sqrt{5y}}{3|x|\sqrt{3}}$
3. Using Bernoulli's equation, the velocity of the water is  
 $v = \sqrt{2gh} \approx \sqrt{2(10 \text{ m/s}^2)(25 \text{ m})}$   
 $\approx 22.36 \text{ m/s}$   
 $\approx (22.36 \text{ m/s})(39.37 \text{ in./m})(60 \text{ s/min})$   
 $\approx 52,819 \text{ in./min}$   
 The area of the hole made by the bullet is  
 $\pi\left(\frac{0.240}{2}\right)^2$  sq. in., so the rate of the water

leaving the water tower is

$$R = \pi\left(\frac{0.240}{2}\right)^2 (52,819 \text{ in.}^3/\text{min})$$

$$\approx 2389 \text{ in.}^3/\text{min} \times 0.004 \text{ gal/in.}^3$$

$$\approx 9.6 \text{ gal/min}$$

The initial rate at which water flows from the punctured hole is approximately 9.6 gallons per minute.

4. a.  $\frac{7}{\sqrt{8}} = \frac{7}{2\sqrt{2}} = \frac{7\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{7\sqrt{2}}{2 \cdot 2} = \frac{7\sqrt{2}}{4}$
- b.  $\frac{3}{1-\sqrt{7}} = \frac{3(1+\sqrt{7})}{(1-\sqrt{7})(1+\sqrt{7})} = \frac{3(1+\sqrt{7})}{1-7}$   
 $= \frac{3(1+\sqrt{7})}{-6} = -\frac{1+\sqrt{7}}{2}$
- c.  $\frac{x}{\sqrt{x}-3} = \frac{x(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)} = \frac{x(\sqrt{x}+3)}{x-9}$   
 $= \frac{x\sqrt{x}+3x}{x-9}$
5. a.  $\sqrt[3]{-8} = -2$       b.  $\sqrt[5]{32} = 2$
- c.  $\sqrt[4]{81} = 3$
- d.  $\sqrt[6]{-4}$  is not a real number
6. a.  $\sqrt[3]{72} = \sqrt[3]{8 \cdot 9} = \sqrt[3]{8} \cdot \sqrt[3]{9} = 2\sqrt[3]{9}$
- b.  $\sqrt[4]{48a^2} = \sqrt[4]{16 \cdot 3a^2} = 2\sqrt[4]{3a^2}$
7. a.  $3\sqrt{12} + 7\sqrt{3} = 3\sqrt{4 \cdot 3} + 7\sqrt{3}$   
 $= 6\sqrt{3} + 7\sqrt{3} = 13\sqrt{3}$
- b.  $2\sqrt[3]{135x} - 3\sqrt[3]{40x} = 2\sqrt[3]{27 \cdot 5x} - 3\sqrt[3]{8 \cdot 5x}$   
 $= 2 \cdot 3\sqrt[3]{5x} - 3 \cdot 2\sqrt[3]{5x}$   
 $= 6\sqrt[3]{5x} - 6\sqrt[3]{5x} = 0$
8.  $\sqrt{3}\sqrt[5]{2} = \sqrt[2]{3}\sqrt[5]{2} = \sqrt[2 \cdot 5]{3^5 \cdot 2^2} = \sqrt[10]{3^5 2^2}$   
 $= \sqrt[10]{2^2 \cdot 3^5} = \sqrt[10]{972}$
9. a.  $\left(\frac{1}{4}\right)^{1/2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$
- b.  $(125)^{1/3} = \sqrt[3]{125} = 5$
- c.  $(-32)^{1/5} = \sqrt[5]{-32} = -2$

10. a.  $(25)^{2/3} = \sqrt[3]{25^2} = \sqrt[3]{5^4} = 5\sqrt[3]{5}$   
 b.  $-36^{3/2} = -(\sqrt{36})^3 = -(6)^3 = -216$   
 c.  $16^{-5/2} = \frac{1}{(\sqrt{16})^5} = \frac{1}{4^5} = \frac{1}{1024}$   
 d.  $(-36)^{-1/2} = \frac{1}{\sqrt{-36}}$   
 Not a real number
11. a.  $4x^{1/2} \cdot 3x^{1/5} = 12x^{1/2+1/5} = 12x^{7/10}$   
 b.  $\frac{25x^{-1/4}}{5x^{1/3}} = 5x^{-1/4-1/3} = 5x^{-7/12} = \frac{5}{x^{7/12}}$   
 c.  $(x^{2/3})^{-1/5} = x^{-2/15} = \frac{1}{x^{2/15}}$
12.  $x(x+3)^{-1/2} + (x+3)^{1/2}$   
 $= \frac{x}{(x+3)^{1/2}} + (x+3)^{1/2}$   
 $= \frac{x}{(x+3)^{1/2}} + \frac{(x+3)^{1/2}(x+3)^{1/2}}{(x+3)^{1/2}}$   
 $= \frac{x+(x+3)}{(x+3)^{1/2}} = \frac{2x+3}{(x+3)^{1/2}}$   
 $= \frac{(2x+3)(x+3)^{1/2}}{(x+3)^{1/2}(x+3)^{1/2}} = \frac{(2x+3)(x+3)^{1/2}}{x+3}$
13. a.  $\sqrt[6]{x^4} = x^{4/6} = x^{2/3} = \sqrt[3]{x^2}$   
 b.  $\sqrt[4]{25}\sqrt{5} = \sqrt[4]{5^2}\sqrt{5} = 5^{2/4} \cdot 5^{1/2} = 5^{1/2} \cdot 5^{1/2} = 5$   
 c.  $\sqrt[3]{\sqrt{x^{12}}} = \sqrt[3]{x^{12/2}} = \sqrt[3]{x^6} = (x^6)^{1/3} = x^2$
5. False. For all real  $x$ ,  $\sqrt{x^2} = |x|$ .  
 6. True  
 7.  $\sqrt{64} = \sqrt{8^2} = 8$   
 8.  $\sqrt{100} = \sqrt{10^2} = 10$   
 9.  $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$       10.  $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$   
 11.  $\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$   
 12.  $\sqrt[3]{-216} = \sqrt[3]{(-6)^3} = -6$   
 13.  $\sqrt[3]{-\frac{1}{8}} = \sqrt[3]{\left(-\frac{1}{2}\right)^3} = -\frac{1}{2}$   
 14.  $\sqrt[3]{\frac{27}{64}} = \sqrt[3]{\left(\frac{3}{4}\right)^3} = \frac{3}{4}$       15.  $\sqrt{(-3)^2} = \sqrt{9} = 3$   
 16.  $-\sqrt{4(-3)^2} = -\sqrt{4(9)} = -\sqrt{36} = -6$   
 17. There is no real number  $x$  such that  $x^4 = -16$ .  
 18. There is no real number  $x$  such that  $x^6 = -64$ .  
 19.  $-\sqrt[5]{-1} = -\sqrt[5]{(-1)^5} = -(-1) = 1$   
 20.  $\sqrt[7]{-1} = \sqrt[7]{(-1)^7} = -1$   
 21.  $\sqrt[5]{(-7)^5} = -7$   
 22.  $-\sqrt[5]{(-4)^5} = -(-4) = 4$   
 23.  $\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$   
 24.  $\sqrt{125} = \sqrt{25 \cdot 5} = \sqrt{25} \cdot \sqrt{5} = 5\sqrt{5}$   
 25.  $\sqrt{18x^2} = \sqrt{9 \cdot 2x^2} = \sqrt{9} \cdot \sqrt{2} \cdot \sqrt{x^2} = 3x\sqrt{2}$   
 26.  $\sqrt{27x^2} = \sqrt{9 \cdot 3x^2} = \sqrt{9} \cdot \sqrt{3} \cdot \sqrt{x^2} = 3x\sqrt{3}$   
 27.  $\sqrt{9x^3} = \sqrt{9 \cdot x^2 \cdot x} = \sqrt{9} \cdot \sqrt{x^2} \cdot \sqrt{x} = 3x\sqrt{x}$   
 28.  $\sqrt{8x^3} = \sqrt{4 \cdot 2 \cdot x^2 \cdot x}$   
 $= \sqrt{4} \cdot \sqrt{2} \cdot \sqrt{x^2} \cdot \sqrt{x} = 2x\sqrt{2x}$   
 29.  $\sqrt{6x}\sqrt{3x} = \sqrt{18x^2} = \sqrt{9 \cdot 2 \cdot x^2}$   
 $= \sqrt{9} \cdot \sqrt{2} \cdot \sqrt{x^2} = 3x\sqrt{2}$

### P.6 A Exercises: Basic Skills and Concepts

- Any positive number has two square roots.
- To rationalize the denominator of an expression with denominator  $2\sqrt{5}$ , multiply the numerator and denominator by  $\sqrt{5}$ .
- Radicals that have the same index and the same radicand are called like radicals.
- The radical notation for  $7^{1/3}$  is  $\sqrt[3]{7}$ .

30.  $\sqrt{5x}\sqrt{10x} = \sqrt{50x^2} = \sqrt{25 \cdot 2 \cdot x^2}$   
 $= \sqrt{25} \sqrt{2} \sqrt{x^2} = 5x\sqrt{2}$
31.  $\sqrt{15x}\sqrt{3x^2} = \sqrt{45x^3} = \sqrt{9 \cdot 5 \cdot x^2 \cdot x}$   
 $= \sqrt{9} \sqrt{5} \sqrt{x^2} \sqrt{x} = 3x\sqrt{5x}$
32.  $\sqrt{2x^2}\sqrt{18x} = \sqrt{36x^3} = \sqrt{36 \cdot x^2 \cdot x}$   
 $= \sqrt{36} \sqrt{x^2} \sqrt{x} = 6x\sqrt{x}$
33.  $\sqrt[3]{-8x^3} = \sqrt[3]{-8} \sqrt[3]{x^3} = -2x$
34.  $\sqrt[3]{64x^3} = \sqrt[3]{64} \sqrt[3]{x^3} = 4x$
35.  $\sqrt[3]{-x^6} = -x^2$
36.  $\sqrt[3]{-27x^6} = -3x^2$
37.  $\sqrt{\frac{5}{32}} = \frac{\sqrt{5}}{\sqrt{32}} = \frac{\sqrt{5}}{\sqrt{16 \cdot 2}} = \frac{\sqrt{5}}{4\sqrt{2}}$   
 $= \frac{\sqrt{5}\sqrt{2}}{4\sqrt{2}\sqrt{2}} = \frac{\sqrt{10}}{8}$
38.  $\sqrt{\frac{3}{50}} = \frac{\sqrt{3}}{\sqrt{50}} = \frac{\sqrt{3}}{\sqrt{25 \cdot 2}} = \frac{\sqrt{3}}{5\sqrt{2}}$   
 $= \frac{\sqrt{3}\sqrt{2}}{5\sqrt{2}\sqrt{2}} = \frac{\sqrt{6}}{10}$
39.  $\sqrt[3]{\frac{8}{x^3}} = \frac{\sqrt[3]{8}}{\sqrt[3]{x^3}} = \frac{2}{x}$
40.  $\sqrt[3]{\frac{7}{8x^3}} = \frac{\sqrt[3]{7}}{\sqrt[3]{8x^3}} = \frac{\sqrt[3]{7}}{2x}$
41.  $\sqrt[4]{\frac{2}{x^5}} = \frac{\sqrt[4]{2}}{\sqrt[4]{x^5}} = \frac{\sqrt[4]{2}}{\sqrt[4]{x^4 \cdot x}} = \frac{\sqrt[4]{2}}{\sqrt[4]{x^4} \sqrt[4]{x}} = \frac{\sqrt[4]{2}}{x\sqrt[4]{x}}$   
 $= \frac{\sqrt[4]{2}\sqrt[4]{x^3}}{x\sqrt[4]{x}\sqrt[4]{x^3}} = \frac{\sqrt[4]{2x^3}}{x\sqrt[4]{x^4}} = \frac{\sqrt[4]{2x^3}}{x^2}$
42.  $-\sqrt[4]{\frac{5}{16x^5}} = -\frac{\sqrt[4]{5}}{\sqrt[4]{16x^5}} = -\frac{\sqrt[4]{5}}{\sqrt[4]{16}\sqrt[4]{x^5}}$   
 $= -\frac{\sqrt[4]{5}}{2\sqrt[4]{x^4}\sqrt[4]{x}} = -\frac{\sqrt[4]{5}}{2x\sqrt[4]{x}}$   
 $= -\frac{\sqrt[4]{5}\sqrt[4]{x^3}}{2x\sqrt[4]{x}\sqrt[4]{x^3}} = -\frac{\sqrt[4]{5x^3}}{2x\sqrt[4]{x^4}} = -\frac{\sqrt[4]{5x^3}}{2x^2}$
43.  $\sqrt{\sqrt{x^8}} = \sqrt{x^4} = x^2$
44.  $\sqrt{\frac{4x^4}{25y^6}} = \frac{\sqrt{4x^4}}{\sqrt{25y^6}} = \frac{2x^2}{5y^3}$
45.  $\sqrt{\frac{9x^6}{y^4}} = \frac{\sqrt{9x^6}}{\sqrt{y^4}} = \frac{3x^3}{y^2}$
46.  $\sqrt[5]{x^{15}y^5} = \sqrt[5]{x^{15}} \sqrt[5]{y^5} = x^3y$
47.  $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$
48.  $7\sqrt{2} - \sqrt{2} = 6\sqrt{2}$
49.  $6\sqrt{5} - \sqrt{5} + 4\sqrt{5} = 9\sqrt{5}$
50.  $5\sqrt{7} + 3\sqrt{7} - 2\sqrt{7} = 6\sqrt{7}$
51.  $\sqrt{98x} - \sqrt{32x} = 7\sqrt{2x} - 4\sqrt{2x} = 3\sqrt{2x}$
52.  $\sqrt{45x} + \sqrt{20x} = 3\sqrt{5x} + 2\sqrt{5x} = 5\sqrt{5x}$
53.  $\sqrt[3]{24} - \sqrt[3]{81} = \sqrt[3]{8 \cdot 3} - \sqrt[3]{27 \cdot 3}$   
 $= 2\sqrt[3]{3} - 3\sqrt[3]{3} = -\sqrt[3]{3}$
54.  $\sqrt[3]{54} + \sqrt[3]{16} = \sqrt[3]{27 \cdot 2} + \sqrt[3]{8 \cdot 2}$   
 $= 3\sqrt[3]{2} + 2\sqrt[3]{2} = 5\sqrt[3]{2}$
55.  $\sqrt[3]{3x} - 2\sqrt[3]{24x} + \sqrt[3]{375x}$   
 $= \sqrt[3]{3x} - 2\sqrt[3]{8 \cdot 3x} + \sqrt[3]{125 \cdot 3x}$   
 $= \sqrt[3]{3x} - 2\sqrt[3]{8} \sqrt[3]{3x} + \sqrt[3]{125} \sqrt[3]{3x}$   
 $= \sqrt[3]{3x} - 2 \cdot 2\sqrt[3]{3x} + 5\sqrt[3]{3x}$   
 $= \sqrt[3]{3x} - 4\sqrt[3]{3x} + 5\sqrt[3]{3x} = 2\sqrt[3]{3x}$
56.  $\sqrt[3]{16x} + 3\sqrt[3]{54x} - \sqrt[3]{2x}$   
 $= \sqrt[3]{8 \cdot 2x} + 3\sqrt[3]{27 \cdot 2x} - \sqrt[3]{2x}$   
 $= \sqrt[3]{8} \sqrt[3]{2x} + 3\sqrt[3]{27} \sqrt[3]{2x} - \sqrt[3]{2x}$   
 $= 2\sqrt[3]{2x} + 3 \cdot 3\sqrt[3]{2x} - \sqrt[3]{2x}$   
 $= 2\sqrt[3]{2x} + 9\sqrt[3]{2x} - \sqrt[3]{2x} = 10\sqrt[3]{2x}$
57.  $\sqrt{2x^5} - 5\sqrt{32x} + \sqrt{18x^3}$   
 $= \sqrt{2x^4x} - 5\sqrt{16 \cdot 2x} + \sqrt{9 \cdot 2x^2x}$   
 $= x^2\sqrt{2x} - 5 \cdot 4\sqrt{2x} + 3x\sqrt{2x}$   
 $= x^2\sqrt{2x} - 20\sqrt{2x} + 3x\sqrt{2x}$   
 $= \sqrt{2x}(x^2 + 3x - 20)$

$$\begin{aligned}
 58. \quad & 2\sqrt{50x^5} + 7\sqrt{2x^3} - 3\sqrt{72x} \\
 &= 2\sqrt{25 \cdot 2x^4x} + 7\sqrt{2x^2x} - 3\sqrt{36 \cdot 2x} \\
 &= 2 \cdot 5x^2\sqrt{2x} + 7x\sqrt{2x} - 3 \cdot 6\sqrt{2x} \\
 &= 10x^2\sqrt{2x} + 7x\sqrt{2x} - 18\sqrt{2x} \\
 &= \sqrt{2x}(10x^2 + 7x - 18)
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & \sqrt{48x^5y} - 4y\sqrt{3x^3y} + y\sqrt{3xy^3} \\
 &= \sqrt{16 \cdot 3x^4xy} - 4y\sqrt{3x^2xy} + y\sqrt{3xy^2y} \\
 &= 4x^2\sqrt{3xy} - 4xy\sqrt{3xy} + y^2\sqrt{3xy} \\
 &= \sqrt{3xy}(4x^2 - 4xy + y^2) = \sqrt{3xy}(2x - y)^2
 \end{aligned}$$

$$\begin{aligned}
 60. \quad & x\sqrt{8xy} + 4\sqrt{2xy^3} - \sqrt{18x^5y} \\
 &= x\sqrt{4 \cdot 2xy} + 4\sqrt{2xy^2y} - \sqrt{9 \cdot 2x^4xy} \\
 &= 2x\sqrt{2xy} + 4y\sqrt{2xy} - 3x^2\sqrt{2xy} \\
 &= \sqrt{2xy}(2x + 4y - 3x^2)
 \end{aligned}$$

$$61. \quad \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$62. \quad \frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

$$63. \quad \frac{7}{\sqrt{15}} = \frac{7\sqrt{15}}{\sqrt{15}\sqrt{15}} = \frac{7\sqrt{15}}{15}$$

$$64. \quad \frac{-3}{\sqrt{8}} = \frac{-3\sqrt{8}}{\sqrt{8}\sqrt{8}} = -\frac{3 \cdot 2\sqrt{2}}{8} = -\frac{6\sqrt{2}}{8} = -\frac{3\sqrt{2}}{4}$$

$$65. \quad \frac{1}{\sqrt{2} + x} = \frac{1(\sqrt{2} - x)}{(\sqrt{2} + x)(\sqrt{2} - x)} = \frac{\sqrt{2} - x}{2 - x^2}$$

$$66. \quad \frac{1}{\sqrt{5} + 2x} = \frac{1(\sqrt{5} - 2x)}{(\sqrt{5} + 2x)(\sqrt{5} - 2x)} = \frac{\sqrt{5} - 2x}{5 - 4x^2}$$

$$\begin{aligned}
 67. \quad & \frac{3}{2 - \sqrt{3}} = \frac{3(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{3(2 + \sqrt{3})}{2^2 - \sqrt{3}^2} \\
 &= \frac{3(2 + \sqrt{3})}{4 - 3} = 3(2 + \sqrt{3})
 \end{aligned}$$

$$\begin{aligned}
 68. \quad & \frac{-5}{1 - \sqrt{2}} = \frac{-5(1 + \sqrt{2})}{(1 - \sqrt{2})(1 + \sqrt{2})} = \frac{-5(1 + \sqrt{2})}{1^2 - \sqrt{2}^2} \\
 &= \frac{-5(1 + \sqrt{2})}{1 - 2} = 5(1 + \sqrt{2})
 \end{aligned}$$

$$\begin{aligned}
 69. \quad & \frac{1}{\sqrt{3} + \sqrt{2}} = \frac{1(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} \\
 &= \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}^2 - \sqrt{2}^2} = \frac{\sqrt{3} - \sqrt{2}}{3 - 2} \\
 &= \sqrt{3} - \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 70. \quad & \frac{6}{\sqrt{5} - \sqrt{3}} = \frac{6(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} \\
 &= \frac{6(\sqrt{5} + \sqrt{3})}{\sqrt{5}^2 - \sqrt{3}^2} = \frac{6(\sqrt{5} + \sqrt{3})}{5 - 3} \\
 &= \frac{6(\sqrt{5} + \sqrt{3})}{2} = 3(\sqrt{5} + \sqrt{3})
 \end{aligned}$$

$$\begin{aligned}
 71. \quad & \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}} = \frac{(\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2})}{(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})} \\
 &= \frac{\sqrt{5}^2 - 2\sqrt{10} + \sqrt{2}^2}{\sqrt{5}^2 - \sqrt{2}^2} = \frac{7 - 2\sqrt{10}}{5 - 2}
 \end{aligned}$$

$$\begin{aligned}
 72. \quad & \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}} = \frac{(\sqrt{7} + \sqrt{3})(\sqrt{7} + \sqrt{3})}{(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})} \\
 &= \frac{\sqrt{7}^2 + 2\sqrt{21} + \sqrt{3}^2}{\sqrt{7}^2 - \sqrt{3}^2} \\
 &= \frac{10 + 2\sqrt{21}}{4} = \frac{5 + \sqrt{21}}{2}
 \end{aligned}$$

$$\begin{aligned}
 73. \quad & \frac{\sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\
 &= \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} - \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})(\sqrt{x+h} - \sqrt{x})} \\
 &= \frac{(\sqrt{x+h})^2 - 2\sqrt{x}\sqrt{x+h} + \sqrt{x}^2}{(\sqrt{x+h})^2 - \sqrt{x}^2} \\
 &= \frac{x+h - 2\sqrt{x(x+h)} + x}{x+h+x} \\
 &= \frac{2x+h - 2\sqrt{x(x+h)}}{h}
 \end{aligned}$$

$$74. \frac{a\sqrt{x}+3}{a\sqrt{x}-3} = \frac{(a\sqrt{x}+3)(a\sqrt{x}+3)}{(a\sqrt{x}-3)(a\sqrt{x}+3)}$$

$$= \frac{a^2\sqrt{x}^2+6a\sqrt{x}+9}{(a\sqrt{x})^2-9}$$

$$= \frac{a^2x+6a\sqrt{x}+9}{a^2x-9}$$

$$75. 25^{1/2} = \sqrt{25} = 5$$

$$76. 144^{1/2} = \sqrt{144} = 12$$

$$77. (-8)^{1/3} = \sqrt[3]{-8} = -2$$

$$78. (-27)^{1/3} = \sqrt[3]{-27} = -3$$

$$79. 8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{(2^3)^2} = \sqrt[3]{2^6} = 2^2 = 4$$

$$80. 16^{3/2} = \sqrt{16^3} = \sqrt{(4^2)^3} = \sqrt{4^6} = 4^3 = 64$$

$$81. -25^{-3/2} = -\frac{1}{25^{3/2}} = -\frac{1}{\sqrt{25^3}} = -\frac{1}{\sqrt{(5^2)^3}}$$

$$= -\frac{1}{\sqrt{5^6}} = -\frac{1}{5^3} = -\frac{1}{125}$$

$$82. -9^{-3/2} = -\frac{1}{9^{3/2}} = -\frac{1}{\sqrt{9^3}}$$

$$= -\frac{1}{\sqrt{(3^2)^3}} = -\frac{1}{\sqrt{3^6}} = -\frac{1}{3^3} = -\frac{1}{27}$$

$$83. \left(\frac{9}{25}\right)^{-3/2} = \left(\frac{25}{9}\right)^{3/2} = \sqrt{\left(\frac{25}{9}\right)^3} = \frac{\sqrt{25^3}}{\sqrt{9^3}}$$

$$= \frac{\sqrt{(5^2)^3}}{\sqrt{(3^2)^3}} = \frac{\sqrt{5^6}}{\sqrt{3^6}} = \frac{5^3}{3^3} = \frac{125}{27}$$

$$84. \left(\frac{1}{27}\right)^{-2/3} = 27^{2/3} = \sqrt[3]{27^2} = \sqrt[3]{(3^3)^2}$$

$$= \sqrt[3]{3^6} = 3^2 = 9$$

$$85. x^{1/2} \cdot x^{2/5} = x^{1/2+2/5} = x^{9/10}$$

$$86. x^{3/5} \cdot 5x^{2/3} = 5x^{3/5+2/3} = 5x^{19/15}$$

$$87. x^{3/5} \cdot x^{-1/2} = x^{3/5-1/2} = x^{1/10}$$

$$88. x^{5/3} \cdot x^{-3/4} = x^{5/3-3/4} = x^{11/12}$$

$$89. (8x^6)^{2/3} = 8^{2/3} x^{6(2/3)} = (2^3)^{2/3} x^{6(2/3)}$$

$$= 2^2 x^4 = 4x^4$$

$$90. (16x^3)^{1/2} = 16^{1/2} x^{3(1/2)} = 4x^{3/2}$$

$$91. (27x^6y^3)^{-2/3} = 27^{-2/3} x^{6(-2/3)} y^{3(-2/3)}$$

$$= 3^{3(-2/3)} x^{6(-2/3)} y^{3(-2/3)}$$

$$= 3^{-2} x^{-4} y^{-2}$$

$$= \frac{1}{3^2 x^4 y^2} = \frac{1}{9x^4 y^2}$$

$$92. (16x^4y^6)^{-3/2} = 16^{-3/2} x^{4(-3/2)} y^{6(-3/2)}$$

$$= 4^{2(-3/2)} x^{4(-3/2)} y^{6(-3/2)}$$

$$= 4^{-3} x^{-6} y^{-9}$$

$$= \frac{1}{4^3 x^6 y^9} = \frac{1}{64x^6 y^9}$$

$$93. \frac{15x^{3/2}}{3x^{1/4}} = 5x^{3/2-1/4} = 5x^{5/4}$$

$$94. \frac{20x^{5/2}}{4x^{2/3}} = 5x^{5/2-2/3} = 5x^{11/6}$$

$$95. \left(\frac{x^{-1/4}}{y^{-2/3}}\right)^{-12} = \frac{x^{(-1/4)(-12)}}{y^{(-2/3)(-12)}} = \frac{x^3}{y^8}$$

$$96. \left(\frac{27x^{-5/2}}{y^{-3}}\right)^{-1/3} = \frac{27^{-1/3} x^{(-5/2)(-1/3)}}{y^{-3(-1/3)}}$$

$$= \frac{(3^3)^{-1/3} x^{5/6}}{y}$$

$$= \frac{3^{-1} x^{5/6}}{y} = \frac{x^{5/6}}{3y}$$

$$97. \sqrt[4]{3^2} = 3^{2/4} = 3^{1/2}$$

$$98. \sqrt[4]{5^2} = 5^{2/4} = 5^{1/2}$$

$$99. \sqrt[3]{x^9} = x^{9/3} = x^3$$

$$100. \sqrt[3]{x^{12}} = x^{12/3} = x^4$$

$$101. \sqrt[3]{x^6y^9} = x^{6/3}y^{9/3} = x^2y^3$$

$$102. \sqrt[3]{8x^3y^{12}} = \sqrt[3]{2^3x^3y^{12}} = 2^{3(1/3)}x^{3(1/3)}y^{12(1/3)}$$

$$= 2xy^4$$



$$103. \sqrt[4]{9}\sqrt{3} = \sqrt[4]{3^2}\sqrt{3} = 3^{2/4} \cdot 3^{1/2} \\ = 3^{1/2} \cdot 3^{1/2} = 3^{1/2+1/2} = 3^1 = 3$$

$$104. \sqrt[4]{49}\sqrt{7} = \sqrt[4]{7^2}\sqrt{7} = 7^{2/4} \cdot 7^{1/2} = 7^{1/2} \cdot 7^{1/2} \\ = 7^{1/2+1/2} = 7^1 = 7$$

$$105. \sqrt[3]{\sqrt{x^{10}}} = \sqrt{x^{10/3}} = (x^{10/3})^{1/2} \\ = x^{(10/3)(1/2)} = x^{5/3}$$

$$106. \sqrt[3]{\sqrt[3]{64x^6}} = \sqrt[3]{\sqrt[3]{4^3x^6}} = \sqrt[3]{4^{3/3}x^{6/3}} = \sqrt[3]{4x^2} \\ = 4^{1/2}(x^2)^{1/2} = 2x^{2(1/2)} = 2x$$

$$107. \sqrt[3]{2} \cdot \sqrt[5]{3} = 3 \cdot \sqrt[3]{2^5} \cdot 3 \cdot \sqrt[5]{3^3} = 1^5 \sqrt[3]{32} \cdot 1^5 \sqrt[5]{27} \\ = 1^5 \sqrt[3]{32 \cdot 27} = 1^5 \sqrt[3]{864}$$

$$108. \sqrt[3]{3} \cdot \sqrt[4]{2} = 3 \cdot \sqrt[4]{3^4} \cdot 3 \cdot \sqrt[4]{2^3} = 1^2 \sqrt[4]{81} \cdot 1^2 \sqrt[4]{8} \\ = 1^2 \sqrt[4]{81 \cdot 8} = 1^2 \sqrt[4]{648}$$

$$109. \sqrt[3]{x^2} \cdot \sqrt[4]{x^3} = 3 \cdot 4 \sqrt[4]{(x^2)^4} \cdot 3 \cdot 4 \sqrt[4]{(x^3)^3} = 1^2 \sqrt{x^8} \cdot 1^2 \sqrt{x^9} \\ = 1^2 \sqrt{x^8 \cdot x^9} = 1^2 \sqrt{x^{17}} = 1^2 \sqrt{x^{12} \cdot x^5} \\ = 1^2 \sqrt{x^{12}} \cdot 1^2 \sqrt{x^5} = x^2 \sqrt{x^5}$$

$$110. \sqrt[5]{x^2} \cdot \sqrt[7]{x^5} = 5 \cdot 7 \sqrt[7]{(x^2)^7} \cdot 5 \cdot 7 \sqrt[7]{(x^5)^5} \\ = 3^5 \sqrt{x^{14}} \cdot 3^5 \sqrt{x^{25}} = 3^5 \sqrt{x^{14} \cdot x^{25}} \\ = 3^5 \sqrt{x^{39}} = 3^5 \sqrt{x^{35} \cdot x^4} \\ = 3^5 \sqrt{x^{35}} \cdot 3^5 \sqrt{x^4} = x^3 \sqrt{x^4}$$

$$111. \sqrt[4]{2m^2n} \cdot \sqrt[12]{5m^5n^2} \\ = 4 \cdot 3 \sqrt[3]{(2m^2n)^3} \cdot 1^2 \sqrt[12]{5m^5n^2} \\ = 1^2 \sqrt[3]{2^3(m^2)^3 n^3} \cdot 1^2 \sqrt[12]{5m^5n^2} \\ = 1^2 \sqrt[3]{8m^6n^3} \cdot 1^2 \sqrt[12]{5m^5n^2} \\ = 1^2 \sqrt[3]{8m^6n^3} \cdot 5m^5n^2 \\ = 1^2 \sqrt[3]{40m^{6+5}n^{3+2}} = 1^2 \sqrt[3]{40m^{11}n^5}$$

$$112. \sqrt{3xy} \cdot \sqrt[3]{9x^2y^2} \\ = 2 \cdot 3 \sqrt{(3xy)^3} \cdot 2 \cdot 3 \sqrt{(9x^2y^2)^2} \\ = \sqrt[3]{3^3x^3y^3} \cdot \sqrt{(3^2)^2(x^2)^2(y^2)^2} \\ = \sqrt[3]{3^3x^3y^3} \cdot \sqrt[3]{3^4x^4y^4}$$

$$= \sqrt[6]{3^3x^3y^3 \cdot 3^4x^4y^4} \\ = \sqrt[6]{3^{3+4}x^{3+4}y^{3+4}} = \sqrt[6]{3^7x^7y^7} \\ = 3xy \sqrt[6]{3xy}$$

$$113. \sqrt[5]{x^4y^3} \cdot \sqrt[6]{x^3y^5} \\ = 5 \cdot 6 \sqrt{(x^4y^3)^6} \cdot 5 \cdot 6 \sqrt{(x^3y^5)^5} \\ = 3^0 \sqrt{(x^4)^6(y^3)^6} \cdot 3^0 \sqrt{(x^3)^5(y^5)^5} \\ = 3^0 \sqrt{x^{24}y^{18}} \cdot 3^0 \sqrt{x^{15}y^{25}} \\ = 3^0 \sqrt{x^{24}y^{18} \cdot x^{15}y^{25}} = 3^0 \sqrt{x^{24+15}y^{18+25}} \\ = 3^0 \sqrt{x^{39}y^{43}} = 3^0 \sqrt{x^{30+9}y^{30+13}} \\ = 3^0 \sqrt{x^{30}x^9y^{30}y^{13}} = xy \sqrt[30]{x^9y^{13}}$$

$$114. \sqrt[5]{3a^4b^2} \cdot \sqrt[10]{7a^2b^6} \\ = 2 \cdot 5 \sqrt{(3a^4b^2)^2} \cdot 1^0 \sqrt[10]{7a^2b^6} \\ = 1^0 \sqrt{3^2(a^4)^2(b^2)^2} \cdot 1^0 \sqrt[10]{7a^2b^6} \\ = 1^0 \sqrt{9a^8b^4} \cdot 1^0 \sqrt[10]{7a^2b^6} \\ = 1^0 \sqrt{9a^8b^4} \cdot 7a^2b^6 = 1^0 \sqrt{63a^{8+2}b^{6+4}} \\ = 1^0 \sqrt{63a^{10}b^{10}} = ab \sqrt[10]{63}$$

### P.6 B Exercises: Applying the Concepts

115. Substituting 692 for  $A$  in  $\sqrt{\frac{4A}{3}}$  gives

$$\sqrt{\frac{4(692)}{3}} = \sqrt{\frac{2768}{1.73}} = \sqrt{1600} = 40 \text{ cm.}$$

116. Substituting \$1,210,000 for  $P$  and \$1,411,344

for  $S$  in  $r = \sqrt{\frac{S}{P}} - 1$  gives

$$r = \sqrt{\frac{1,411,344}{1,210,000}} - 1 = \sqrt{1.1664} - 1 = 1.08 - 1 \\ = 0.08 = 8\%.$$

117. Substituting 6 for  $w$  in  $V = \sqrt{\frac{w}{1.5}}$  gives

$$V = \sqrt{\frac{6}{1.5}} = \sqrt{4} = 2 \text{ cm/sec.}$$

118. Substituting 2 for  $m$ , 18 for  $r$ , and 49 for  $F$  in

$$V = \sqrt{\frac{rF}{m}} \text{ gives } V = \sqrt{\frac{(18)(49)}{2}} = \sqrt{\frac{882}{2}}$$

$$= \sqrt{441} = 21 \text{ m/sec.}$$

119. Substituting 1058 for  $W$  and 2 for  $R$  in

$$I = \sqrt{\frac{W}{R}} \text{ gives } I = \sqrt{\frac{1058}{2}} = \sqrt{529} = 23 \text{ amp.}$$

120. Substituting  $2.19 \times 10^{19}$  for  $V$  in  $r = \left(\frac{3V}{4\pi}\right)^{1/3}$

gives

$$r = \left(\frac{3 \times 2.19 \times 10^{19}}{4\pi}\right)^{1/3} \approx (0.5228 \times 10^{19})^{1/3}$$

$$\approx (5.23 \times 10^{18})^{1/3} = 5.23^{1/3} \times 10^6$$

$$\approx 1.74 \times 10^6 \text{ m}$$

121. Substituting 16.25 for  $h$  in

$$P = 14.7(0.5)^{h/3.25} \text{ gives}$$

$$P = 14.7(0.5)^{16.25/3.25} = 14.7(0.5)^5 =$$

$$14.7(0.03125) \approx 0.46 \text{ lb/sq in.}$$

122. Substituting 3 for  $r$  in  $F = 42r^{3/2}$  gives

$$F = 42(3^{3/2}) = 42(5.2) \approx 218 \text{ lb.}$$

## P.7 Topics in Geometry

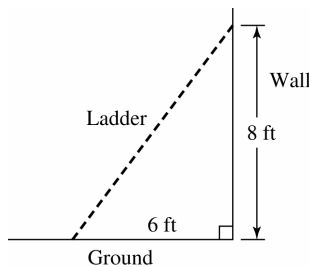
### P.7 Practice Problems

1. Use the Pythagorean Theorem,  $c^2 = a^2 + b^2$  to find the length of the hypotenuse.

$$c^2 = 8^2 + 6^2 \Rightarrow c^2 = 64 + 36 = 100 \Rightarrow c = 10$$

The hypotenuse has length 10.

2.



$$c^2 = a^2 + b^2 \Rightarrow c^2 = 8^2 + 6^2 = 64 + 36 = 100 \Rightarrow$$

$$c = 10$$

A 10-foot ladder is required.

3. To determine if the triangle is a right triangle, see if the lengths of the sides make  $c^2 = a^2 + b^2$  true. The longest side has length 6, so  $c = 6$ .

$$6^2 \stackrel{?}{=} 2^2 + 5^2$$

$$36 \stackrel{?}{=} 4 + 25$$

$$36 \neq 29$$

Therefore, the triangle is not a right triangle.

### P.7 A Exercises: Basic Skills and Concepts

1. A triangle with two sides of equal length is an isosceles triangle.
2. A triangle with all three sides of equal length is an equilateral triangle.
3. Two triangles are similar if and only if they have equal corresponding angles and their corresponding sides are proportional.
4. Two triangles are congruent if and only if they have equal corresponding angles and sides.
5. True
6. True

In exercises 7–16, use the Pythagorean Theorem,  $c^2 = a^2 + b^2$  to find the length of the missing side.

7.  $c^2 = 7^2 + 24^2 = 49 + 576 = 625 \Rightarrow$   
 $c = \sqrt{625} = 25$

8.  $c^2 = 5^2 + 12^2 = 25 + 144 = 169 \Rightarrow$   
 $c = \sqrt{169} = 13$

9.  $c^2 = 9^2 + 12^2 = 81 + 144 = 225 \Rightarrow$   
 $c = \sqrt{225} = 15$

10.  $c^2 = 10^2 + 24^2 = 100 + 576 = 676 \Rightarrow$   
 $c = \sqrt{676} = 26$

11.  $c^2 = 3^2 + 3^2 = 9 + 9 = 18 \Rightarrow$   
 $c = \sqrt{18} = 3\sqrt{2}$

12.  $c^2 = 2^2 + 4^2 = 4 + 16 = 20 \Rightarrow$   
 $c = \sqrt{20} = 2\sqrt{5}$

13.  $17^2 = 15^2 + b^2 \Rightarrow 289 = 225 + b^2 \Rightarrow$   
 $64 = b^2 \Rightarrow 8 = b$

14.  $37^2 = 35^2 + b^2 \Rightarrow 1369 = 1225 + b^2 \Rightarrow 144 = b^2 \Rightarrow 12 = b$

15.  $50^2 = 14^2 + b^2 \Rightarrow 2500 = 196 + b^2 \Rightarrow 2304 = b^2 \Rightarrow 48 = b$

16.  $26^2 = 10^2 + b^2 \Rightarrow 676 = 100 + b^2 \Rightarrow 576 = b^2 \Rightarrow 24 = b$

In exercises 17–20, the formula for the area of a rectangle with length  $l$  and width  $w$  is  $A = lw$ . The formula for the perimeter is  $P = 2(l + w)$ .

17.  $A = 3(5) = 15, P = 2(3 + 5) = 2(8) = 16$

18.  $A = 4(3) = 12, P = 2(3 + 4) = 2(7) = 14$

19.  $A = 10\left(\frac{1}{2}\right) = 5, P = 2\left(10 + \frac{1}{2}\right) = 2\left(\frac{21}{2}\right) = 21$

20.  $A = 16\left(\frac{1}{4}\right) = 4, P = 2\left(16 + \frac{1}{4}\right) = 2\left(\frac{65}{4}\right) = \frac{65}{2}$

In exercises 21–24, the formula for the area of a triangle with base  $b$  and height  $h$  is  $A = \frac{1}{2}bh$ .

21.  $A = \frac{1}{2}(3)(4) = 6$

22.  $A = \frac{1}{2}(6)(1) = 3$

23.  $A = \frac{1}{2}\left(\frac{1}{2}\right)(12) = 3$

24.  $A = \frac{1}{2}\left(\frac{1}{4}\right)\left(\frac{2}{3}\right) = \frac{1}{12}$

In exercises 25–28, the formula for the area of a circle with radius  $r$  is  $A = \pi r^2$ . The formula for the circumference is  $C = 2\pi r$ .

25.  $A = 3.14(1^2) = 3.14, C = 2(3.14)(1) = 6.28$

26.  $A = 3.14(3^2) = 3.14(9) = 28.26, C = 2(3.14)(3) = 18.84$

27.  $A = 3.14\left(\frac{1}{2}\right)^2 = 3.14\left(\frac{1}{4}\right) = 0.785, C = 2(3.14)\left(\frac{1}{2}\right) = 3.14$

28.  $A = 3.14\left(\frac{3}{4}\right)^2 = 3.14\left(\frac{9}{16}\right) = 1.76625, C = 2(3.14)\left(\frac{3}{4}\right) = 4.71$

In exercises 29–32, the formula for the volume of a box with length  $l$ , width  $w$ , and height  $h$  is  $V = lwh$ .

29.  $V = (1)(1)(5) = 5$

30.  $V = (2)(7)(4) = 56$

31.  $V = \left(\frac{1}{2}\right)(10)(3) = 15$

32.  $V = \left(\frac{1}{3}\right)\left(\frac{3}{7}\right)(14) = 2$

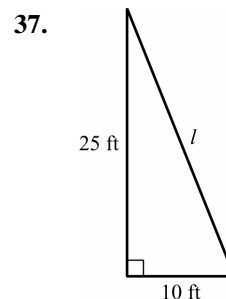
**P.7 B Exercises: Applying the Concepts**

33. To find the length of the garden, use the Pythagorean theorem:  $20^2 = 12^2 + l^2 \Rightarrow l = 16$  feet.  $A = 12(16) = 192$  sq ft

34. To find the length of the mural, use the Pythagorean theorem:  $10^2 = 6^2 + l^2 \Rightarrow l = 8$  feet.  $P = 2(6 + 8) = 28$  feet.

35. To find the height of the monitor, use the Pythagorean theorem:  $15^2 = 12^2 + h^2 \Rightarrow h = 9$  inches.

36. The distance from home plate to second base is the diagonal of the square:  $d^2 = 90^2 + 90^2 = 16,200 \Rightarrow d = \sqrt{16,200} = 90\sqrt{2} \approx 127$  feet.



The ladder is the hypotenuse of the right triangle formed by the side of the house and the ground, so use the Pythagorean theorem to find its length:

$l^2 = 25^2 + 10^2 = 725 \Rightarrow l = \sqrt{725} = 5\sqrt{29} \approx 27$  ft.

38. The ramp is the hypotenuse of the right triangle formed by the floor and the truck, so use the Pythagorean theorem to find its length:

$$l^2 = 4^2 + 8^2 = 80 \Rightarrow l = \sqrt{80} = 4\sqrt{5} \approx 8.94 \text{ ft}$$

39.  $P = 2(120 + 53) = 346 \text{ yd}$

40. Let  $w$  = the width of the pool. Then  $w + 1860$  = the length of the pool. Using the formula for perimeter of a rectangle, we have

$$4400 = 2(w + 1860 + w) \Rightarrow$$

$$4400 = 2(2w + 1860) \Rightarrow 4400 = 4w + 3720 \Rightarrow$$

$$680 = 4w \Rightarrow w = 170 \text{ ft. So the length} = 170$$

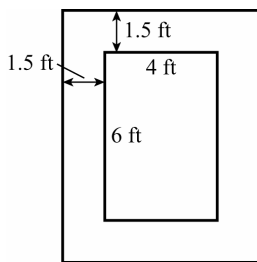
$$+ 1860 = 2030 \text{ ft. The area}$$

$$= (170)(2030) = 345,100 \text{ square feet.}$$

41.  $C = 2\pi r = \pi d = 3.14(787) \approx 2471 \text{ feet}$

42.  $C = 2\pi r = \pi d = 3.14(26) \approx 81.64 \text{ inches}$

43. The width of the border is  $4 + 2(1.5) = 7$  feet and the length of the border is  $6 + 2(1.5) = 9$  feet. The area of the border = the total area – the area of the fishpond.



The total area =  $(7)(9) = 63$  square feet; the area of the fishpond =  $(4)(6) = 24$  square feet. So the area of the border =  $63 - 24 = 39$  sq ft.

44. To find the length of the garden, use the Pythagorean theorem:  $13^2 = 5^2 + l^2 \Rightarrow$

$$144 = l^2 \Rightarrow 12 = l.$$

$$\text{The perimeter} = 2(12 + 5) = 34 \text{ feet.}$$

45.  $V = 2(2.5)(3) = 15 \text{ ft}^3$

46. The radius of the semicircular regions =  $185/2 = 92.5$  ft. So, the total area of the two semicircles =

$$\pi(92.5^2) = 3.14(8556.25) \approx 26,867 \text{ ft}^2. \text{ The}$$

$$\text{width of the rectangular region} = 740 - 185 = 555 \text{ ft. So, the area of the rectangular region} =$$

$$(555)(185) = 102,675 \text{ ft}^2. \text{ The total area} =$$

$$102,675 \text{ ft}^2 + 26,867 \text{ ft}^2 = 129,542 \text{ ft}^2.$$

## Chapter P: Review Exercises

1. a.  $\{4\}$

b.  $\{0, 4\}$

c.  $\{-5, 0, 4\}$

d.  $\{-5, 0, 0.2, 0.3\bar{1}, \frac{1}{2}, 4\}$

e.  $\{\sqrt{7}, \sqrt{12}\}$

f.  $\{-5, 0, 0.2, 0.3\bar{1}, \frac{1}{2}, \sqrt{7}, \sqrt{12}, 4\}$

2. commutative property of addition

3. distributive property

4. commutative property of multiplication

5. multiplicative identity

6.  $-2 \leq x < 3$

7.  $x \leq 1$

8.  $[1, 4)$

9.  $(0, \infty)$

10.  $\frac{-12}{|2|} = \frac{-12}{2} = -6$

11.  $|2 - |-3|| = |2 - 3| = |-1| = 1$

12.  $|-5 - |-7|| = |-5 - 7| = |-12| = 12$

13.  $|1 - \sqrt{15}| = \sqrt{15} - 1$

14.  $-5^0 = -1$

15.  $(-4)^3 = -64$

16.  $\frac{2^{18}}{2^{17}} = 2^{18-17} = 2^1 = 2$

17.  $2^4 - 5 \cdot 3^2 = 16 - 5 \cdot 9 = 16 - 45 = -29$

18.  $(2^3)^2 = 8^2 = 64$

19.  $-25^{1/2} = -5$
20.  $\left(\frac{1}{16}\right)^{1/4} = \frac{1^{1/4}}{16^{1/4}} = \frac{1}{(2^4)^{1/4}} = \frac{1}{2^{4(1/4)}} = \frac{1}{2}$
21.  $8^{4/3} = (2^3)^{4/3} = 2^{3(4/3)} = 2^4 = 16$
22.  $(-27)^{2/3} = ((-3)^3)^{2/3} = (-3)^{3(2/3)} = (-3)^2 = 9$
23.  $\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \frac{(6^2)^{3/2}}{(5^2)^{3/2}} = \frac{6^{2(3/2)}}{5^{2(3/2)}} = \frac{6^3}{5^3} = \frac{216}{125}$
24.  $81^{-3/2} = \frac{1}{81^{3/2}} = \frac{1}{(9^2)^{3/2}} = \frac{1}{9^{2(3/2)}} = \frac{1}{9^3} = \frac{1}{729}$
25.  $2 \cdot 5^3 = 2 \cdot 125 = 250$
26.  $3^2 \cdot 2^5 = 9 \cdot 32 = 288$
27.  $\frac{21 \times 10^6}{3 \times 10^7} = \frac{7}{10} = 0.7$
28.  $\sqrt[4]{10,000} = \sqrt[4]{10^4} = 10$
29.  $\sqrt{5^2} = 5$
30.  $\sqrt{(-9)^2} = \sqrt{81} = 9$
31.  $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$
32.  $3^{1/2} \cdot 27^{1/2} = 3^{1/2} \cdot (3^3)^{1/2} = 3^{1/2} \cdot 3^{3/2} = 3^{1/2+3/2} = 3^2 = 9$
33.  $64^{-1/3} = \frac{1}{64^{1/3}} = \frac{1}{(4^3)^{1/3}} = \frac{1}{4}$
34.  $\sqrt{5} \sqrt{20} = \sqrt{5} \sqrt{4 \cdot 5} = \sqrt{5} \cdot 2\sqrt{5} = 2\sqrt{5 \cdot 5} = 2\sqrt{25} = 2 \cdot 5 = 10$
35.  $(\sqrt{3} + 2)(\sqrt{3} - 2) = (\sqrt{3})^2 - 2^2 = 3 - 4 = -1$
36.  $\left(\frac{1}{16}\right)^{-3/2} = 16^{3/2} = (4^2)^{3/2} = 4^{2(3/2)} = 4^3 = 64$
37.  $\frac{3^{-2} \cdot 7^0}{18^{-1}} = \frac{\frac{1}{3^2} \cdot 1}{\frac{1}{18}} = \frac{1}{9} \cdot 18 = 2$
38.  $4(3+7) + 2(4) = 4(10) + 8 = 48$
39.  $\frac{2-6\sqrt{4}}{4(2)} = \frac{2-6(2)}{8} = \frac{2-12}{8} = \frac{-10}{8} = -\frac{5}{4}$
40.  $\frac{|6|}{6} + \frac{|-3|}{-3} = \frac{6}{6} + \frac{3}{-3} = 1 + (-1) = 0$
41.  $16^{-1/2} = \frac{1}{16^{1/2}} = \frac{1}{4}$
42.  $\frac{x^{-6}}{x^{-9}} = x^{-6-(-9)} = x^3$
43.  $(x^{-2})^{-5} = x^{(-2)(-5)} = x^{10}$
44.  $\frac{x^{-3}}{y^{-2}} = \frac{y^2}{x^3}$
45.  $\left(\frac{x^{-3}}{y^{-1}}\right)^{-2} = \frac{x^{-3(-2)}}{y^{-1(-2)}} = \frac{x^6}{y^2}$
46.  $\left(\frac{xy^3}{x^5y}\right)^{-2} = \frac{x^{-2}y^{3(-2)}}{x^{5(-2)}y^{-2}} = \frac{x^{-2}y^{-6}}{x^{-10}y^{-2}} = x^{-2-(-10)}y^{-6-(-2)} = x^8y^{-4} = \frac{x^8}{y^4}$
47.  $(16x^{-2/3}y^{-4/3})^{3/2} = 16^{3/2}x^{(-2/3)(3/2)}y^{(-4/3)(3/2)} = 4^{2(3/2)}x^{-1}y^{-2} = \frac{4^3}{xy^2} = \frac{64}{xy^2}$
48.  $(49x^{-4/3}y^{2/3})^{-3/2} = 7^{2(-3/2)}x^{(-4/3)(-3/2)}y^{(2/3)(-3/2)} = 7^{-3}x^2y^{-1} = \frac{x^2}{7^3y} = \frac{x^2}{343y}$

$$49. \left(\frac{64y^{-9/2}}{x^{-3}}\right)^{-2/3} = \frac{4^{3(-2/3)}y^{(-9/2)(-2/3)}}{x^{-3(-2/3)}} \\ = \frac{4^{-2}y^3}{x^2} = \frac{y^3}{4^2x^2} = \frac{y^3}{16x^2}$$

$$50. (2x^{2/3})(5x^{3/4}) = 10x^{2/3+3/4} = 10x^{17/12}$$

$$51. (7x^{1/4})(3x^{3/2}) = 21x^{1/4+3/2} = 21x^{7/4}$$

$$52. \frac{32x^{2/3}}{8x^{1/4}} = 4x^{(2/3)-(1/4)} = 4x^{5/12}$$

$$53. \frac{x^5(2x)^3}{4x^3} = \frac{x^5(2^3)x^3}{4x^3} = \frac{8x^{5+3}}{4x^3} = 2x^{8-3} = 2x^5$$

$$54. \frac{(64x^4y^4)^{1/2}}{4y^2} = \frac{64^{1/2}x^{4(1/2)}y^{4(1/2)}}{4y^2} \\ = \frac{8x^2y^2}{4y^2} = 2x^2$$

$$55. \left(\frac{x^2y^{4/3}}{x^{1/3}y}\right)^6 = \frac{x^{2(6)}y^{(4/3)(6)}}{x^{(1/3)(6)}y^6} = \frac{x^{12}y^8}{x^2y^6} \\ = x^{12-2}y^{8-6} = x^{10}y^2$$

$$56. 7\sqrt{3} + 3\sqrt{75} = 7\sqrt{3} + 3\sqrt{25 \cdot 3} \\ = 7\sqrt{3} + 3\sqrt{25}\sqrt{3} \\ = 7\sqrt{3} + 3(5)\sqrt{3} \\ = 7\sqrt{3} + 15\sqrt{3} = 22\sqrt{3}$$

$$57. \frac{\sqrt{64}}{\sqrt{11}} = \frac{8}{\sqrt{11}} = \frac{8\sqrt{11}}{\sqrt{11} \cdot \sqrt{11}} = \frac{8\sqrt{11}}{11}$$

$$58. \sqrt{180x^2} = \sqrt{36 \cdot 5x^2} = \sqrt{36} \cdot \sqrt{5} \cdot \sqrt{x^2} = 6x\sqrt{5}$$

$$59. 7\sqrt{6} - 3\sqrt{24} = 7\sqrt{6} - 3\sqrt{4 \cdot 6} \\ = 7\sqrt{6} - 3\sqrt{4}\sqrt{6} = 7\sqrt{6} - 3(2)\sqrt{6} \\ = 7\sqrt{6} - 6\sqrt{6} = \sqrt{6}$$

$$60. 7\sqrt[3]{54} + \sqrt[3]{128} = 7\sqrt[3]{27 \cdot 2} + \sqrt[3]{64 \cdot 2} \\ = 7\sqrt[3]{27}\sqrt[3]{2} + \sqrt[3]{64}\sqrt[3]{2} \\ = 7(3)\sqrt[3]{2} + 4\sqrt[3]{2} \\ = 21\sqrt[3]{2} + 4\sqrt[3]{2} = 25\sqrt[3]{2}$$

$$61. \sqrt{2x}\sqrt{6x} = \sqrt{12x^2} = \sqrt{4 \cdot 3x^2} \\ = \sqrt{4}\sqrt{3}\sqrt{x^2} = 2x\sqrt{3}$$

$$62. \sqrt{75x^2} = \sqrt{3 \cdot 25x^2} = \sqrt{3}\sqrt{25}\sqrt{x^2} = 5x\sqrt{3}$$

$$63. \frac{\sqrt{100x^3}}{\sqrt{4x}} = \sqrt{\frac{100x^3}{4x}} = \sqrt{25x^2} = 5x$$

$$64. 5\sqrt{2x} - 2\sqrt{8x} = 5\sqrt{2x} - 2\sqrt{4 \cdot 2x} \\ = 5\sqrt{2x} - 2\sqrt{4}\sqrt{2x} \\ = 5\sqrt{2x} - 2(2)\sqrt{2x} \\ = 5\sqrt{2x} - 4\sqrt{2x} = \sqrt{2x}$$

$$65. 4\sqrt[3]{135} + \sqrt[3]{40} = 4\sqrt[3]{27 \cdot 5} + \sqrt[3]{8 \cdot 5} \\ = 4\sqrt[3]{27}\sqrt[3]{5} + \sqrt[3]{8}\sqrt[3]{5} \\ = 4(3)\sqrt[3]{5} + 2\sqrt[3]{5} \\ = 12\sqrt[3]{5} + 2\sqrt[3]{5} = 14\sqrt[3]{5}$$

$$66. y\sqrt[3]{56x} - \sqrt[3]{189xy^3} = y\sqrt[3]{8 \cdot 7x} - \sqrt[3]{27 \cdot 7xy^3} \\ = y\sqrt[3]{8}\sqrt[3]{7x} - \sqrt[3]{27}\sqrt[3]{7x}\sqrt[3]{y^3} \\ = 2y\sqrt[3]{7x} - 3y\sqrt[3]{7x} \\ = -y\sqrt[3]{7x}$$

$$67. \frac{7}{\sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

$$68. \frac{4}{9-\sqrt{6}} = \frac{4(9+\sqrt{6})}{(9-\sqrt{6})(9+\sqrt{6})} = \frac{36+4\sqrt{6}}{9^2-(\sqrt{6})^2} \\ = \frac{36+4\sqrt{6}}{81-6} = \frac{36+4\sqrt{6}}{75}$$

$$69. \frac{1-\sqrt{3}}{1+\sqrt{3}} = \frac{(1-\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{1-2\sqrt{3}+3}{1^2-(\sqrt{3})^2} \\ = \frac{4-2\sqrt{3}}{1-3} = \frac{4-2\sqrt{3}}{-2} = -2+\sqrt{3}$$

$$70. \frac{2\sqrt{5}+3}{3-4\sqrt{5}} = \frac{(2\sqrt{5}+3)(3+4\sqrt{5})}{(3-4\sqrt{5})(3+4\sqrt{5})} \\ = \frac{6\sqrt{5}+40+9+12\sqrt{5}}{3^2-(4\sqrt{5})^2} = \frac{49+18\sqrt{5}}{9-80} \\ = \frac{49+18\sqrt{5}}{-71} = -\frac{49+18\sqrt{5}}{71}$$

$$71. 3.7 \times (6.23 \times 10^{12}) = 23.051 \times 10^{12} \\ = 2.3051 \times 10^{13}$$

72.  $\frac{3.19 \times 10^{-9}}{0.02 \times 10^{-3}} = 159.5 \times 10^{-9-(-3)}$   
 $= 159.5 \times 10^{-6} = 0.0001595$
73.  $(x^3 - 6x^2 + 4x - 2) + (3x^3 - 6x^2 + 5x - 4)$   
 $= 4x^3 - 12x^2 + 9x - 6$
74.  $(10x^3 + 8x^2 - 7x - 3) - (5x^3 - x^2 + 4x - 9)$   
 $= 10x^3 + 8x^2 - 7x - 3 - 5x^3 + x^2 - 4x + 9$   
 $= 5x^3 + 9x^2 - 11x + 6$
75.  $(4x^4 + 3x^3 - 5x^2 + 9) + (5x^4 + 8x^3 - 7x^2 + 5)$   
 $= 9x^4 + 11x^3 - 12x^2 + 14$
76.  $(8x^4 + 4x^3 + 3x^2 + 5) + (7x^4 + 3x^3 + 8x^2 - 3)$   
 $= 15x^4 + 7x^3 + 11x^2 + 2$
77.  $(x - 12)(x - 3) = x^2 - 3x - 12x + 36$   
 $= x^2 - 15x + 36$
78.  $(x - 7)^2 = (x - 7)(x - 7) = x^2 - 7x - 7x + 49$   
 $= x^2 - 14x + 49$
79.  $(x^5 - 2)(x^5 + 2) = (x^5)^2 - 2^2 = x^{10} - 4$
80.  $(4x - 3)(4x + 3) = (4x)^2 - 3^2$   
 $= 4^2x^2 - 9 = 16x^2 - 9$
81.  $(2x + 5)(3x - 11) = 6x^2 - 22x + 15x - 55$   
 $= 6x^2 - 7x - 55$
82.  $(3x - 6)(x^2 + 2x + 4)$   
 $= 3x^3 + 6x^2 + 12x - 6x^2 - 12x - 24$   
 $= 3x^3 - 24$
83.  $x^2 - 3x - 10 = (x - 5)(x + 2)$
84.  $x^2 + 10x + 9 = (x + 1)(x + 9)$
85.  $24x^2 - 38x - 11 = (6x - 11)(4x + 1)$
86.  $15x^2 + 33x + 18 = 3(5x^2 + 11x + 6)$   
 $= 3(5x + 6)(x + 1)$
87.  $x(x + 11) + 5(x + 11) = (x + 11)(x + 5)$
88.  $x^3 - x^2 + 2x - 2 = x^3 + 2x - x^2 - 2$   
 $= x(x^2 + 2) - 1(x^2 + 2)$   
 $= (x^2 + 2)(x - 1)$
89.  $x^4 - x^3 + 7x - 7 = x^4 + 7x - x^3 - 7$   
 $= x(x^3 + 7) - 1(x^3 + 7)$   
 $= (x^3 + 7)(x - 1)$
90.  $9x^2 + 24x + 16 = (3x + 4)^2$
91.  $10x^2 + 23x + 12 = (5x + 4)(2x + 3)$
92.  $8x^2 + 18x + 9 = (4x + 3)(2x + 3)$
93.  $12x^2 + 7x - 12 = (4x - 3)(3x + 4)$
94.  $x^4 - 4x^2 = x^2(x^2 - 4) = x^2(x - 2)(x + 2)$
95.  $4x^2 - 49 = (2x - 7)(2x + 7)$
96.  $16x^2 - 81 = (4x - 9)(4x + 9)$
97.  $x^2 + 12 + 36 = (x + 6)^2$
98.  $x^2 - 10x + 100$  is irreducible
99.  $64x^2 + 48x + 9 = (8x + 3)^2$
100.  $8x^3 - 1 = (2x - 1)(4x^2 + 2x + 1)$
101.  $8x^3 + 27 = (2x + 3)(4x^2 - 6x + 9)$
102.  $7x^3 - 7 = 7(x^3 - 1) = 7(x - 1)(x^2 + x + 1)$
103.  $x^3 + 5x^2 - 16x - 80$   
 $= x^3 - 16x + 5x^2 - 80$   
 $= x(x^2 - 16) + 5(x^2 - 16)$   
 $= (x^2 - 16)(x + 5) = (x - 4)(x + 4)(x + 5)$
104.  $x^3 + 6x^2 - 9x - 54$   
 $= x^3 - 9x + 6x^2 - 54$   
 $= x(x^2 - 9) + 6(x^2 - 9)$   
 $= (x^2 - 9)(x + 6) = (x - 3)(x + 3)(x + 6)$
105.  $\frac{4}{x - 9} - \frac{10}{9 - x} = \frac{4}{x - 9} + \frac{10}{x - 9} = \frac{14}{x - 9}$

$$\begin{aligned}
 106. \quad & \frac{2}{x^2-3x+2} + \frac{6}{x^2-1} \\
 &= \frac{2}{(x-2)(x-1)} + \frac{6}{(x-1)(x+1)} \\
 &= \frac{2(x+1)}{(x-2)(x-1)(x+1)} + \frac{6(x-2)}{(x-2)(x-1)(x+1)} \\
 &= \frac{2x+2+6x-12}{(x-2)(x-1)(x+1)} \\
 &= \frac{8x-10}{(x-2)(x-1)(x+1)} \\
 &= \frac{2(4x-5)}{(x-2)(x-1)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 107. \quad & \frac{3x+5}{x^2+14x+48} - \frac{3x-2}{x^2+10x+16} \\
 &= \frac{3x+5}{(x+6)(x+8)} - \frac{3x-2}{(x+2)(x+8)} \\
 &= \frac{(3x+5)(x+2) - (3x-2)(x+6)}{(x+6)(x+8)(x+2)} \\
 &= \frac{3x^2+11x+10 - (3x^2+16x-12)}{(x+2)(x+6)(x+8)} \\
 &= \frac{-5x+22}{(x+2)(x+6)(x+8)}
 \end{aligned}$$

$$\begin{aligned}
 108. \quad & \frac{x+7}{x^2-7x+6} - \frac{5-3x}{x^2-2x-24} \\
 &= \frac{x+7}{(x-1)(x-6)} - \frac{5-3x}{(x-6)(x+4)} \\
 &= \frac{(x+7)(x+4) - (5-3x)(x-1)}{(x-1)(x-6)(x+4)} \\
 &= \frac{x^2+11x+28 - (5x-5-3x^2+3x)}{(x-1)(x-6)(x+4)} \\
 &= \frac{4x^2+3x+33}{(x-1)(x-6)(x+4)}
 \end{aligned}$$

$$\begin{aligned}
 109. \quad & \frac{x-1}{x+1} - \frac{x+1}{x-1} = \frac{(x-1)^2}{(x+1)(x-1)} - \frac{(x+1)^2}{(x+1)(x-1)} \\
 &= \frac{x^2-2x+1 - (x^2+2x+1)}{(x+1)(x-1)} \\
 &= -\frac{4x}{(x+1)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 110. \quad & \frac{x^3-1}{3x^2-3} \cdot \frac{6x+6}{x^2+x+1} \\
 &= \frac{(x-1)(x^2+x+1)}{3(x^2-1)} \cdot \frac{6(x+1)}{x^2+x+1} \\
 &= \frac{\cancel{(x-1)} \cancel{(x^2+x+1)}}{3 \cancel{(x-1)} \cancel{(x+1)}} \cdot \frac{6 \cancel{(x+1)}}{\cancel{x^2+x+1}} = 2
 \end{aligned}$$

$$\begin{aligned}
 111. \quad & \frac{x-1}{2x-3} \cdot \frac{4x^2-9}{2x^2-x-1} \\
 &= \frac{\cancel{x-1}}{2 \cancel{x-3}} \cdot \frac{(2x+3) \cancel{(2x-3)}}{(2x+1) \cancel{(x-1)}} = \frac{2x+3}{2x+1}
 \end{aligned}$$

$$\begin{aligned}
 112. \quad & \frac{x^2+2x-8}{x^2+5x+6} \cdot \frac{x+2}{x+4} = \frac{(x-2) \cancel{(x+4)}}{\cancel{(x+2)}(x+3)} \cdot \frac{\cancel{x+2}}{\cancel{x+4}} \\
 &= \frac{x-2}{x+3}
 \end{aligned}$$

$$\begin{aligned}
 113. \quad & \frac{x^2-4}{4x^2-9} \cdot \frac{2x^2-3x}{2x+4} \\
 &= \frac{(x-2) \cancel{(x+2)}}{\cancel{(2x-3)}(2x+3)} \cdot \frac{x \cancel{(2x-3)}}{2 \cancel{(x+2)}} \\
 &= \frac{x(x-2)}{2(2x+3)}
 \end{aligned}$$

$$\begin{aligned}
 114. \quad & \frac{3x^2-17x+10}{x^2-4x-5} \cdot \frac{x^2+3x+2}{x^2+x-2} \\
 &= \frac{(3x-2) \cancel{(x-5)}}{\cancel{(x-5)} \cancel{(x+1)}} \cdot \frac{\cancel{(x+1)} \cancel{(x+2)}}{\cancel{(x+2)}(x-1)} \\
 &= \frac{3x-2}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 115. \quad & \frac{\frac{1}{x^2} - x}{\frac{1}{x^2} + x} = \frac{\frac{1-x^3}{x^2}}{\frac{1+x^3}{x^2}} = \frac{1-x^3}{x^2} \cdot \frac{x^2}{1+x^3} = \frac{1-x^3}{1+x^3} \\
 &= \frac{(1-x)(1+x+x^2)}{(1+x)(1-x+x^2)}
 \end{aligned}$$

$$\begin{aligned}
 116. \quad & \frac{\frac{x}{x-2} + x}{\frac{1}{x^2-4}} = \frac{\frac{x+x(x-2)}{x-2}}{\frac{1}{x^2-4}} = \frac{x^2-x}{x-2} \cdot \frac{x^2-4}{1} \\
 &= \frac{x(x-1) \cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = x(x-1)(x+2)
 \end{aligned}$$



$$\begin{aligned}
 117. \quad \frac{\frac{x}{x-3} + x}{\frac{x}{3-x} - x} &= \frac{\frac{x+x(x-3)}{x-3}}{\frac{x-x(3-x)}{3-x}} \\
 &= \frac{x^2-2x}{x-3} \cdot \frac{3-x}{x^2-2x} \\
 &= \frac{x^2-2x}{x-3} \cdot \left(-\frac{x-3}{x^2-2x}\right) = -1
 \end{aligned}$$

$$\begin{aligned}
 118. \quad \frac{x+2-\frac{18}{x-5}}{x-1-\frac{12}{x-5}} &= \frac{\left(x+2-\frac{18}{x-5}\right)(x-5)}{\left(x-1-\frac{12}{x-5}\right)(x-5)} \\
 &= \frac{x(x-5)+2(x-5)-18}{x(x-5)-1(x-5)-12} \\
 &= \frac{x^2-5x+2x-10-18}{x^2-5x-x+5-12} \\
 &= \frac{x^2-3x-28}{x^2-6x-7} \\
 &= \frac{(x-7)(x+4)}{(x-7)(x+1)} = \frac{x+4}{x+1}
 \end{aligned}$$

119. Using the Pythagorean Theorem, we have

$$c^2 = 20^2 + 21^2 \Rightarrow c^2 = 841 \Rightarrow c = 29$$

120. Let  $x$  = the length of the other leg. Then  $x + 1$  = the length of the hypotenuse. Using the Pythagorean Theorem, we have

$$\begin{aligned}
 (x+1)^2 &= 5^2 + x^2 \Rightarrow \\
 x^2 + 2x + 1 &= 5^2 + x^2 \Rightarrow 2x + 1 = 25 \Rightarrow \\
 2x &= 24 \Rightarrow x = 12
 \end{aligned}$$

So the length of the other leg is 12 in., and the length of the hypotenuse is 13 in.

121. First, find the length of the missing leg:

$$20^2 = 12^2 + x^2 \Rightarrow 400 = 144 + x^2 \Rightarrow$$

$$256 = x^2 \Rightarrow 16 = x. \text{ Area} = \frac{1}{2}bh \Rightarrow$$

$$A = \frac{1}{2}(12)(16) = 96. \text{ Perimeter} = \text{the sum of}$$

the sides  $\Rightarrow P = 12 + 16 + 20 = 48$ .

$$122. \quad V = \frac{1}{3}(7)(60) = 140$$

123. First, find the length of the missing side:

$$10^2 = 6^2 + x^2 \Rightarrow 100 = 36 + x^2 \Rightarrow$$

$64 = x^2 \Rightarrow 8 = x$ . So the area =  $(6)(8) = 48$  square feet.

$$124. \quad \frac{4.2(2)}{10-2} = \frac{8.4}{8} = 1.05 \text{ grams}$$

$$125. \quad \sqrt{7920(0.7) + 0.7^2} = \sqrt{5544 + 0.49} \approx 74.46 \text{ miles}$$

126. 2% of 1,000,000 is 20,000. So there are 20,000 gallons of arsenic to start. If  $x$  gallons of water are added, then the percent of arsenic in the water is  $\frac{20,000}{1,000,000 + x}$ .

$$127. \quad \frac{36-3(2^2)}{8(2)} = \frac{36-3(4)}{16} = \frac{36-12}{16} = \frac{24}{16} = 1.5 \text{ ft}$$

$$128. \quad 16(9^2) + 25(9) = 16(81) + 225 = 1296 + 225 = 1521 \text{ feet}$$

## Chapter P: Practice Test

$$1. \quad |7 - |-3|| = |7 - 3| = |4| = 4$$

$$2. \quad |\sqrt{2} - 100| = 100 - \sqrt{2}$$

$$3. \quad \frac{5-3(-3)}{2} + (5)(-3) = \frac{5+9}{2} - 15 = -8$$

$$4. \quad x \neq -9, x \neq 3 \Rightarrow (-\infty, -9) \cup (-9, 3) \cup (3, \infty)$$

$$5. \quad \left(\frac{-3x^2y}{x}\right)^3 = \frac{(-3)^3 x^{2(3)} y^3}{x^3} = \frac{-27x^6 y^3}{x^3} = -27x^{6-3} y^3 = -27x^3 y^3$$

$$6. \quad \sqrt[3]{-8x^6} = \sqrt[3]{(-2)^3 x^6} = -2x^2$$

$$7. \quad \sqrt{75x} - \sqrt{27x} = \sqrt{3 \cdot 25x} - \sqrt{3 \cdot 9x} = 5\sqrt{3x} - 3\sqrt{3x} = 2\sqrt{3x}$$

$$8. \quad -16^{-3/2} = -\frac{1}{16^{3/2}} = -\frac{1}{(4^2)^{3/2}} = -\frac{1}{4^{2(3/2)}} = -\frac{1}{4^3} = -\frac{1}{64}$$

$$\begin{aligned}
 9. \quad & \left( \frac{x^{-2}y^4}{25x^3y^3} \right)^{-1/2} \\
 & = \left( \frac{25x^3y^3}{x^{-2}y^4} \right)^{1/2} = \frac{25^{1/2}(x^3)^{1/2}(y^3)^{1/2}}{(x^{-2})^{1/2}(y^4)^{1/2}} \\
 & = \frac{5x^{3/2}y^{3/2}}{x^{-1}y^2} = 5x^{(3/2)-(-1)}y^{(3/2)-2} \\
 & = 5x^{5/2}y^{-1/2} = \frac{5x^{5/2}}{y^{1/2}} = \frac{5x^{5/2}y^{1/2}}{y}
 \end{aligned}$$

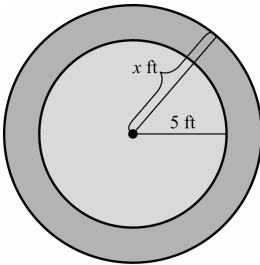
$$\begin{aligned}
 10. \quad & \frac{5}{1-\sqrt{3}} = \frac{5(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} \\
 & = \frac{5+5\sqrt{3}}{1-3} = -\frac{5+5\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & 4(x^2 - 3x + 2) + 3(5x^2 - 2x + 1) \\
 & = 4x^2 - 12x + 8 + 15x^2 - 6x + 3 \\
 & = 19x^2 - 18x + 11
 \end{aligned}$$

$$12. \quad (x-2)(5x-1) = 5x^2 - 11x + 2$$

$$13. \quad (x^2 + 3y^2)^2 = x^4 + 6x^2y^2 + 9y^4$$

14.



The total area is  $\pi x^2$  square feet. The area of the rug is  $25\pi$  square feet. So the area of the border = the total area - the area of the rug =  $\pi x^2 - 25\pi = \pi(x^2 - 25) = \pi(x-5)(x+5)$ .

$$15. \quad x^2 - 5x + 6 = (x-3)(x-2)$$

$$16. \quad 9x^2 + 12x + 4 = (3x+2)^2$$

$$17. \quad 8x^3 - 27 = (2x-3)(4x^2 + 6x + 9)$$

$$\begin{aligned}
 18. \quad & \frac{6-3x}{x^2-4} - \frac{12}{2x+4} \\
 & = \frac{6-3x}{(x-2)(x+2)} - \frac{12}{2(x+2)} \\
 & = \frac{2(6-3x)}{2(x-2)(x+2)} - \frac{12(x-2)}{2(x-2)(x+2)} \\
 & = \frac{12-6x-(12x-24)}{2(x-2)(x+2)} = \frac{-18x+36}{2(x-2)(x+2)} \\
 & = \frac{-18(x-2)}{2(x-2)(x+2)} = -\frac{9}{x+2}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{1}{x^2} = \frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x^2}{(3x-1)(3x+1)} \\
 & 9 - \frac{1}{x^2} = \frac{9x^2-1}{x^2} = \frac{1}{x^2} \cdot \frac{x^2}{(3x-1)(3x+1)} \\
 & = \frac{1}{(3x-1)(3x+1)}
 \end{aligned}$$

$$20. \quad c^2 = 35^2 + 12^2 \Rightarrow c^2 = 1369 \Rightarrow c = 37$$

# Chapter 1 Equations and Inequalities

## 1.1 Linear Equations in One Variable

### 1.1 Practice Problems

1. a. Both sides of the equation  $\frac{x}{3} - 7 = 5$  are defined for all real numbers, so the domain is  $(-\infty, \infty)$ .

b. The left side of the equation  $\frac{2}{2-x} = 4$  is not defined if  $x = 2$ . The right side of the equation is defined for all real numbers, so the domain is  $(-\infty, 2) \cup (2, \infty)$ .

c. The left side of the equation  $\sqrt{x-1} = 0$  is not defined if  $x < 1$ . The right side of the equation is defined for all real numbers, so the domain is  $[1, \infty)$ .

2.  $\frac{2}{3} - \frac{3}{2}x = \frac{1}{6} - \frac{7}{3}x$

To clear the fractions, multiply both sides of the equation by the LCD, 6.

$$\begin{aligned} 4 - 9x &= 1 - 14x \\ 4 - 9x + 14x &= 1 - 14x + 14x \\ 4 + 5x &= 1 \\ 4 + 5x - 4 &= 1 - 4 \\ 5x &= -3 \\ \frac{5x}{5} &= \frac{-3}{5} \\ x &= -\frac{3}{5} \end{aligned}$$

Solution set:  $\left\{-\frac{3}{5}\right\}$

3.  $3x - [2x - 6(x+1)] = -1$

$$\begin{aligned} 3x - (2x - 6x - 6) &= -1 \\ 3x - (-4x - 6) &= -1 \\ 3x + 4x + 6 &= -1 \\ 7x + 6 &= -1 \\ 7x + 6 - 6 &= -1 - 6 \\ 7x &= -7 \\ \frac{7x}{7} &= \frac{-7}{7} \\ x &= -1 \end{aligned}$$

Solution set:  $\{-1\}$

4.  $2(3x-6)+5=12-(x+5)$

$$\begin{aligned} 6x - 12 + 5 &= 12 - x - 5 \\ 6x - 7 &= 7 - x \\ 6x - 7 + x &= 7 - x + x \\ 7x - 7 &= 7 \\ 7x - 7 + 7 &= 7 + 7 \\ 7x &= 14 \\ \frac{7x}{7} &= \frac{14}{7} \Rightarrow x = 2 \end{aligned}$$

Solution set:  $\{2\}$

5.  $\frac{3}{x} - \frac{1}{2x} = \frac{7}{2} + \frac{3}{4x}$

To clear the fractions, multiply both sides of the equation by the LCD,  $4x$ .

$$\begin{aligned} 12 - 2 &= 14x + 3 \\ 10 &= 14x + 3 \\ 10 - 3 &= 14x + 3 - 3 \\ 7 &= 14x \\ \frac{7}{14} &= \frac{14x}{14} \Rightarrow \frac{1}{2} = x \end{aligned}$$

Solution set:  $\left\{\frac{1}{2}\right\}$

6.  $\frac{1}{x-2} - \frac{1}{x+2} = \frac{4}{x^2-4}$

To clear the fractions, multiply both sides of the equation by the LCD,  $(x-2)(x+2)$ .

$$(x+2) - (x-2) = 4 \Rightarrow 4 = 4$$

The equation  $4 = 4$  is equivalent to the original equation and is always true for all values of  $x$  in its domain. The domain of  $x$  is all real numbers except  $-2$  and  $2$ . Therefore, the original equation is an identity.

Solution set:  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

7.  $\frac{t}{t-4} - \frac{4}{t-4} = 8$

To clear the fractions, multiply both sides of the equation by the LCD,  $t-4$ .

$$\begin{aligned} t - 4 &= 8(t - 4) \\ t - 4 &= 8t - 32 \\ t - 4 + 4 &= 8t - 32 + 4 \\ t &= 8t - 28 \\ t - 8t &= 8t - 28 - 8t \\ -7t &= -28 \\ \frac{-7t}{-7} &= \frac{-28}{-7} \Rightarrow t = 4 \end{aligned}$$

(continued on next page)

(continued from page 40)

Check:

$$\frac{4}{4-4} - \frac{4}{4-4} = 8 \Rightarrow \frac{4}{0} - \frac{4}{0} = 8$$

Since dividing by zero is undefined, reject 4 as an extraneous solution. Therefore, there is no number  $t$  that satisfies the equation, and the equation is inconsistent.

Solution set:  $\emptyset$

$$8. \quad F = \frac{9}{5}C + 32$$

$$50 = \frac{9}{5}C + 32$$

$$50 - 32 = \frac{9}{5}C + 32 - 32$$

$$18 = \frac{9}{5}C$$

$$18 \cdot \frac{5}{9} = \frac{5}{9} \cdot \frac{9}{5}C$$

$$10 = C$$

Thus, 50°F converts to 10°C.

$$9. \quad P = 2l + 2w$$

Subtract  $2l$  from both sides.

$$P - 2l = 2w$$

Now, divide both sides by 2.

$$\frac{P - 2l}{2} = w$$

10. Following the reasoning in example 10, we have  $x + 2x = 3x$  is the maximum extended length (in feet) of the cord.

$$3x + 7 + 10 = 120$$

$$3x + 17 = 120$$

$$3x + 17 - 17 = 120 - 17$$

$$3x = 103$$

$$\frac{3x}{3} = \frac{103}{3} \Rightarrow x \approx 34.3$$

The cord should be no longer than 34.3 feet.

### 1.1 A Exercises: Basic Skills and Concepts

- The domain of the variable in an equation is the set of all real number for which both sides of the equation are defined.
- Standard form for a linear equation in  $x$  is of the form  $ax + b = 0$ .
- Two equations with the same solution sets are called equivalent.
- A conditional equation is one that is not true for some values of the variables.
- True

6. True

7. a. Substitute 0 for  $x$  in the equation

$$x - 2 = 5x + 6:$$

$$0 - 2 = 5(0) + 6 \Rightarrow -2 \neq 6$$

So, 0 is not a solution of the equation.

- b. Substitute  $-2$  for  $x$  in the equation

$$x - 2 = 5x + 6:$$

$$-2 - 2 = 5(-2) + 6 \Rightarrow -4 = -10 + 6 \Rightarrow$$

$$-4 = -4$$

So,  $-2$  is a solution of the equation.

8. a. Substitute  $-1$  for  $x$  in the equation

$$8x + 3 = 14x - 1:$$

$$8(-1) + 3 = 14(-1) - 1 \Rightarrow -8 + 3 = -14 - 1 \Rightarrow$$

$$-5 \neq -15$$

So,  $-1$  is not a solution of the equation.

- b. Substitute  $2/3$  for  $x$  in the equation

$$8x + 3 = 14x - 1:$$

$$8\left(\frac{2}{3}\right) + 3 = 14\left(\frac{2}{3}\right) - 1 \Rightarrow \frac{16}{3} + 3 = \frac{28}{3} - 1 \Rightarrow$$

$$\frac{25}{3} = \frac{25}{3}$$

So,  $2/3$  is a solution of the equation.

9. a. Substitute 4 for  $x$  in the equation

$$\frac{2}{x} = \frac{1}{3} + \frac{1}{x+2}:$$

$$\frac{2}{4} = \frac{1}{3} + \frac{1}{4+2} \Rightarrow \frac{1}{2} = \frac{1}{3} + \frac{1}{6} \Rightarrow \frac{1}{2} = \frac{1}{2}$$

So, 4 is a solution of the equation.

- b. Substitute 1 for  $x$  in the equation

$$\frac{2}{x} = \frac{1}{3} + \frac{1}{x+2}:$$

$$\frac{2}{1} = \frac{1}{3} + \frac{1}{1+2} \Rightarrow 2 = \frac{1}{3} + \frac{1}{3} \Rightarrow 2 \neq \frac{2}{3}$$

So, 1 is not a solution of the equation.

10. a. Substitute  $1/2$  for  $x$  in the equation

$$(x-3)(2x+1) = 0:$$

$$\left(\frac{1}{2}-3\right)\left(2\cdot\frac{1}{2}+1\right) = 0 \Rightarrow \left(-\frac{5}{2}\right)(2) = 0 \Rightarrow$$

$$-5 \neq 0$$

So,  $1/2$  is not a solution of the equation.

- b. Substitute 3 for  $x$  in the equation

$$(x-3)(2x+1) = 0:$$

$$(3-3)(2\cdot 3+1) = 0 \Rightarrow (0)(7) = 0 \Rightarrow 0 = 0$$

So, 3 is a solution of the equation.

11. a. The equation  $2x + 3x = 5x$  is an identity, so every real number is a solution of the equation. Thus 157 is a solution of the equation. This can be checked by substituting 157 for  $x$  in the equation:

$$2(157) + 3(157) = 5(157) \Rightarrow \\ 314 + 471 = 785 \Rightarrow 785 = 785$$

- b. The equation  $2x + 3x = 5x$  is an identity, so every real number is a solution of the equation. Thus  $-2046$  is a solution of the equation. This can be checked by substituting  $-2046$  for  $x$  in the equation:

$$2(-2046) + 3(-2046) = 5(-2046) \\ -4092 - 6138 = -10,230 \\ -10,230 = -10,230$$

12. Both sides of the equation  $(2 - x) - 4x = 7 - 3(x + 4)$  are defined for all real numbers, so the domain is  $(-\infty, \infty)$ .
13. The left side of the equation  $\frac{y}{y-1} = \frac{3}{y+2}$  is not defined if  $y = 1$ , and the right side of the equation is not defined if  $y = -2$ . The domain is  $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$ .
14. The left side of the equation  $\frac{1}{y} = 2 + \sqrt{y}$  is not defined if  $y = 0$ . The right side of the equation is not defined if  $y < 0$ , so the domain is  $(0, \infty)$ .
15. The left side of the equation  $\frac{3x}{(x-3)(x-4)} = 2x + 9$  is not defined if  $x = 3$  or  $x = 4$ . The right side is defined for all real numbers. So, the domain is  $(-\infty, 3) \cup (3, 4) \cup (4, \infty)$ .
16. The left side of the equation  $\frac{1}{\sqrt{x}} = x^2 - 1$  is not defined if  $x \leq 0$ . The right side of the equation is defined for all real numbers. So the domain is  $(0, \infty)$ .
17. Substitute 0 for  $x$  in  $2x + 3 = 5x + 1$ . Because  $3 \neq 1$ , the equation is not an identity.
18. When the like terms on the right side of the equation  $3x + 4 = 6x + 2 - (3x - 2)$  are

collected, the equation becomes  $3x + 4 = 3x + 4$ , which is an identity.

19. When the terms on the left side of the equation  $\frac{1}{x} + \frac{1}{2} = \frac{2+x}{2x}$  are collected, the equation becomes  $\frac{2+x}{2x} = \frac{2+x}{2x}$ , which is an identity.
20. The right side of the equation  $\frac{1}{x+3} = \frac{1}{x} + \frac{1}{3}$  is not defined for  $x = 0$ , while the left side is defined for  $x = 0$ . Therefore, the equation is not an identity.

In exercises 21–46, solve the equations using the procedures listed on page 86: eliminate fractions, simplify, isolate the variable term, combine terms, isolate the variable term, and check the solution.

21.  $3x + 5 = 14$   
 $3x + 5 - 5 = 14 - 5$   
 $3x = 9$   
 $\frac{3x}{3} = \frac{9}{3}$   
 $x = 3$   
 Solution set:  $\{3\}$
22.  $2x - 17 = 7$   
 $2x - 17 + 17 = 7 + 17$   
 $2x = 24$   
 $\frac{2x}{2} = \frac{24}{2}$   
 $x = 12$   
 Solution set:  $\{12\}$
23.  $-10x + 12 = 32$   
 $-10x + 12 - 12 = 32 - 12$   
 $-10x = 20$   
 $\frac{-10x}{-10} = \frac{20}{-10}$   
 $x = -2$   
 Solution set:  $\{-2\}$
24.  $-2x + 5 = 6$   
 $-2x + 5 - 5 = 6 - 5$   
 $-2x = 1$   
 $\frac{-2x}{-2} = \frac{1}{-2} \Rightarrow x = -\frac{1}{2}$   
 Solution set:  $\left\{-\frac{1}{2}\right\}$
25.  $3 - y = -4$   
 $3 - y - 3 = -4 - 3$   
 $-y = -7 \Rightarrow y = 7$   
 Solution set:  $\{7\}$

$$\begin{aligned}
 26. \quad & 2 - 7y = 23 \\
 & 2 - 7y - 2 = 23 - 2 \\
 & \quad -7y = 21 \\
 & \quad \frac{-7y}{-7} = \frac{21}{-7} \Rightarrow y = -3
 \end{aligned}$$

Solution set:  $\{-3\}$

$$\begin{aligned}
 27. \quad & 7x + 7 = 2(x + 1) \\
 & 7x + 7 = 2x + 2 \\
 & 7x + 7 - 7 = 2x + 2 - 7 \\
 & \quad 7x = 2x - 5 \\
 & 7x - 2x = 2x - 5 - 2x \\
 & \quad 5x = -5 \\
 & \quad \frac{5x}{5} = \frac{-5}{5} \Rightarrow x = -1
 \end{aligned}$$

Solution set:  $\{-1\}$

$$\begin{aligned}
 28. \quad & 3(x + 2) = 4 - x \\
 & 3x + 6 = 4 - x \\
 & 3x + 6 + x = 4 - x + x \\
 & \quad 4x + 6 = 4 \\
 & 4x + 6 - 6 = 4 - 6 \\
 & \quad 4x = -2 \\
 & \quad \frac{4x}{4} = \frac{-2}{4} \Rightarrow x = -\frac{1}{2}
 \end{aligned}$$

Solution set:  $\left\{-\frac{1}{2}\right\}$

$$\begin{aligned}
 29. \quad & 3(2 - y) + 5y = 3y \\
 & 6 - 3y + 5y = 3y \\
 & \quad 6 + 2y = 3y \\
 & 6 + 2y - 2y = 3y - 2y \Rightarrow 6 = y
 \end{aligned}$$

Solution set:  $\{6\}$

$$\begin{aligned}
 30. \quad & 9y - 3(y - 1) = 6 + y \\
 & 9y - 3y + 3 = 6 + y \\
 & \quad 6y + 3 = 6 + y \\
 & 6y + 3 - y = 6 + y - y \\
 & \quad 5y + 3 = 6 \\
 & 5y + 3 - 3 = 6 - 3 \\
 & \quad 5y = 3 \\
 & \quad \frac{5y}{5} = \frac{3}{5} \Rightarrow y = \frac{3}{5}
 \end{aligned}$$

Solution set:  $\left\{\frac{3}{5}\right\}$

$$\begin{aligned}
 31. \quad & 4y - 3y + 7 - y = 2 - (7 - y) \\
 & \text{Distribute } -1 \text{ to clear the parentheses.} \\
 & \quad 7 = 2 - 7 + y \\
 & \quad 7 = -5 + y \\
 & 7 + 5 = -5 + y + 5 \Rightarrow 12 = y
 \end{aligned}$$

Solution set:  $\{12\}$

$$\begin{aligned}
 32. \quad & 3(y - 1) = 6y - 4 + 2y - 4y \\
 & \quad 3y - 3 = 4y - 4 \\
 & 3y - 3 + 3 = 4y - 4 + 3 \\
 & \quad 3y = 4y - 1 \\
 & 3y - 4y = 4y - 1 - 4y \\
 & \quad -y = -1 \Rightarrow y = 1
 \end{aligned}$$

Solution set:  $\{1\}$

$$\begin{aligned}
 33. \quad & 3(x - 2) + 2(3 - x) = 1 \\
 & \quad 3x - 6 + 6 - 2x = 1 \Rightarrow x = 1
 \end{aligned}$$

Solution set:  $\{1\}$

$$\begin{aligned}
 34. \quad & 2x - 3 - (3x - 1) = 6 \\
 & \text{Distribute } -1 \text{ to clear the parentheses.} \\
 & \quad 2x - 3 - 3x + 1 = 6 \\
 & \quad -x - 2 = 6 \\
 & -x - 2 + 2 = 6 + 2 \\
 & \quad -x = 8 \Rightarrow x = -8
 \end{aligned}$$

Solution set:  $\{-8\}$

$$\begin{aligned}
 35. \quad & 2x + 3(x - 4) = 7x + 10 \\
 & 2x + 3x - 12 = 7x + 10 \\
 & \quad 5x - 12 = 7x + 10 \\
 & 5x - 12 + 12 = 7x + 10 + 12 \\
 & \quad 5x = 7x + 22 \\
 & 5x - 7x = 7x + 22 - 7x \\
 & \quad -2x = 22 \\
 & \quad \frac{-2x}{-2} = \frac{22}{-2} \Rightarrow x = -11
 \end{aligned}$$

Solution set:  $\{-11\}$

$$\begin{aligned}
 36. \quad & 3(2 - 3x) - 4x = 3x - 10 \\
 & 6 - 9x - 4x = 3x - 10 \\
 & \quad 6 - 13x = 3x - 10 \\
 & 6 - 13x - 6 = 3x - 10 - 6 \\
 & \quad -13x = 3x - 16 \\
 & -13x - 3x = 3x - 16 - 3x \\
 & \quad -16x = -16 \\
 & \quad \frac{-16x}{-16} = \frac{-16}{-16} \Rightarrow x = 1
 \end{aligned}$$

Solution set:  $\{1\}$

$$\begin{aligned}
 37. \quad & 4[x + 2(3 - x)] = 2x + 1 \\
 & \text{Distribute } 2 \text{ to clear the inner parentheses.} \\
 & \quad 4[x + 6 - 2x] = 2x + 1 \\
 & \text{Combine like terms within the brackets.} \\
 & \quad 4[6 - x] = 2x + 1
 \end{aligned}$$

Distribute 4 to clear the brackets.

$$24 - 4x = 2x + 1$$

$$24 - 4x - 24 = 2x + 1 - 24$$

$$-4x = 2x - 23$$

$$-4x - 2x = -23$$

$$-6x = -23$$

$$\frac{-6x}{-6} = \frac{-23}{-6} \Rightarrow x = \frac{23}{6}$$

Solution set:  $\left\{\frac{23}{6}\right\}$

38.  $3 - [x - 3(x + 2)] = 4$   
 Distribute  $-3$  to clear the parentheses.

$$3 - [x - 3x - 6] = 4$$

Combine like terms in the brackets.

$$3 - [-2x - 6] = 4$$

Distribute  $-1$  to clear the brackets.

$$3 + 2x + 6 = 4$$

$$2x + 9 = 4$$

$$2x + 9 - 9 = 4 - 9$$

$$2x = -5$$

$$\frac{2x}{2} = \frac{-5}{2} \Rightarrow x = -\frac{5}{2}$$

Solution set:  $\left\{-\frac{5}{2}\right\}$

39.  $3(4y - 3) = 4[y - (4y - 3)]$   
 Distribute  $3$  on the left side and  $-1$  on the right side to clear parentheses.

$$12y - 9 = 4[y - 4y + 3]$$

Combine like terms in the brackets.

$$12y - 9 = 4[-3y + 3]$$

Distribute  $4$  to clear the brackets.

$$12y - 9 = -12y + 12$$

$$12y - 9 + 9 = -12y + 12 + 9$$

$$12y = -12y + 21$$

$$12y + 12y = -12y + 21 + 12y$$

$$24y = 21$$

$$\frac{24y}{24} = \frac{21}{24} \Rightarrow y = \frac{21}{24} = \frac{7}{8}$$

Solution set:  $\left\{\frac{7}{8}\right\}$

40.  $5 - (6y + 9) + 2y = 2(y + 1)$   
 Distribute  $-1$  on the left and  $2$  on the right to clear the parentheses.

$$5 - 6y - 9 + 2y = 2y + 2$$

$$-4 - 4y = 2y + 2$$

$$-4 - 4y + 4 = 2y + 2 + 4$$

$$-4y = 2y + 6$$

$$-4y - 2y = 2y + 6 - 2y$$

$$-6y = 6$$

$$\frac{-6y}{-6} = \frac{6}{-6}$$

$$y = -1$$

Solution set:  $\{-1\}$

41.  $2x - 3(2 - x) = (x - 3) + 2x + 1$   
 Distribute  $-3$  on the left to clear the parentheses.

$$2x - 6 + 3x = x - 3 + 2x + 1$$

$$5x - 6 = 3x - 2$$

$$5x - 6 + 6 = 3x - 2 + 6$$

$$5x = 3x + 4$$

$$5x - 3x = 3x + 4 - 3x$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2} \Rightarrow x = 2$$

Solution set:  $\{2\}$

42.  $5(x - 3) - 6(x - 4) = -5$   
 Distribute  $5$  to clear the first set of parentheses. Distribute  $-6$  to clear the second set of parentheses.

$$5x - 15 - 6x + 24 = -5$$

$$-x + 9 = -5$$

$$-x + 9 - 9 = -5 - 9$$

$$-x = -14 \Rightarrow x = 14$$

Solution set:  $\{14\}$

43.  $\frac{2x + 1}{9} - \frac{x + 4}{6} = 1$

To clear the fractions, multiply both sides of the equation by the least common denominator,  $36$ .

$$36\left(\frac{2x + 1}{9} - \frac{x + 4}{6}\right) = 36(1)$$

$$4(2x + 1) - 6(x + 4) = 36$$

$$8x + 4 - 6x - 24 = 36$$

$$2x - 20 = 36$$

$$2x - 20 + 20 = 36 + 20$$

$$2x = 56$$

$$\frac{2x}{2} = \frac{56}{2} \Rightarrow x = 28$$

Solution set:  $\{28\}$

44.  $\frac{2 - 3x}{7} + \frac{x - 1}{3} = \frac{3x}{7}$

To clear the fractions, multiply both sides of the equation by the least common denominator,  $21$ .

$$21\left(\frac{2 - 3x}{7} + \frac{x - 1}{3}\right) = 21\left(\frac{3x}{7}\right)$$

$$3(2 - 3x) + 7(x - 1) = 3(3x)$$

$$6 - 9x + 7x - 7 = 9x$$

$$-1 - 2x = 9x$$

$$-1 - 2x + 2x = 9x + 2x$$

$$-1 = 11x$$

$$\frac{-1}{11} = \frac{11x}{11} \Rightarrow x = -\frac{1}{11}$$

Solution set:  $\left\{-\frac{1}{11}\right\}$

45. 
$$\frac{1-x}{4} + \frac{5x+1}{2} = 3 - \frac{2(x+1)}{8}$$

To clear the fractions, multiply both sides by the least common denominator, 8.

$$8\left(\frac{1-x}{4} + \frac{5x+1}{2}\right) = 8\left(3 - \frac{2(x+1)}{8}\right)$$

Distribute the 8 on both sides.

$$8\left(\frac{1-x}{4}\right) + 8\left(\frac{5x+1}{2}\right) = 8(3) - 8\left(\frac{2(x+1)}{8}\right)$$

$$2(1-x) + 4(5x+1) = 8(3) - 2(x+1)$$

Simplify by collecting like terms and combining constants.

$$2 - 2x + 20x + 4 = 24 - 2x - 2$$

$$18x + 6 = 22 - 2x$$

$$18x + 6 + 2x = 22 - 2x + 2x$$

$$20x + 6 = 22$$

$$20x + 6 - 6 = 22 - 6$$

$$20x = 16$$

$$\frac{20x}{20} = \frac{16}{20}$$

$$x = \frac{16}{20} = \frac{4}{5}$$

Solution set:  $\left\{\frac{4}{5}\right\}$

46. 
$$\frac{x+4}{3} + 2x - \frac{1}{2} = \frac{3x+2}{6}$$

To clear the fractions, multiply both sides of the equation by the least common denominator, 6.

$$6\left(\frac{x+4}{3} + 2x - \frac{1}{2}\right) = 6\left(\frac{3x+2}{6}\right)$$

Distribute the 6 on both sides.

$$6\left(\frac{x+4}{3}\right) + 6(2x) - 6\left(\frac{1}{2}\right) = 6\left(\frac{3x+2}{6}\right)$$

$$2(x+4) + 12x - 3 = 3x + 2$$

Simplify by collecting like terms and combining constants.

$$2x + 8 + 12x - 3 = 3x + 2$$

$$14x + 5 = 3x + 2$$

$$14x + 5 - 3x = 3x + 2 - 3x$$

$$11x + 5 = 2$$

$$11x + 5 - 5 = 2 - 5$$

$$11x = -3$$

$$\frac{11x}{11} = \frac{-3}{11} \Rightarrow x = -\frac{3}{11}$$

Solution set:  $\left\{-\frac{3}{11}\right\}$

In exercises 47–64, be sure to check each exercise for extraneous solutions. We will show the checks only for those exercises with extraneous solutions.

47. 
$$\frac{2}{x} - 3 = \frac{5}{x}$$

Multiply both sides of the equation by the LCD,  $x$ .

$$2 - 3x = 5$$

$$2 - 3x - 2 = 5 - 2$$

$$-3x = 3$$

$$\frac{-3x}{-3} = \frac{3}{-3} \Rightarrow x = -1$$

Solution set:  $\{-1\}$

48. 
$$\frac{4}{x} + 3 = \frac{10}{x}$$

Multiply both sides of the equation by the LCD,  $x$ .

$$4 + 3x = 10$$

$$4 + 3x - 4 = 10 - 4$$

$$3x = 6$$

$$\frac{3x}{3} = \frac{6}{3} \Rightarrow x = 2$$

Solution set:  $\{2\}$

49. 
$$\frac{1}{x} + \frac{1}{3} = 4$$

Multiply both sides of the equation by the LCD,  $3x$ .

$$3 + x = 12x$$

$$3 + x - x = 12x - x$$

$$3 = 11x$$

$$\frac{3}{11} = \frac{11x}{11} \Rightarrow x = \frac{3}{11}$$

Solution set:  $\left\{\frac{3}{11}\right\}$

50. 
$$\frac{2}{x} - 3 = \frac{1}{2}$$

Multiply both sides of the equation by the LCD,  $2x$ .

$$4 - 6x = x$$

$$4 - 6x + 6x = x + 6x$$

$$4 = 7x$$

$$\frac{4}{7} = \frac{7x}{7} \Rightarrow x = \frac{4}{7}$$

Solution set:  $\left\{\frac{4}{7}\right\}$



$$51. \frac{1+x}{x} = \frac{1}{x} + 1$$

Multiply both sides of the equation by the LCD,  $x$ .

$$1+x = 1+x \Rightarrow 0 = 0$$

The equation  $0 = 0$  is equivalent to the original equation and is always true for all values of  $x$  in its domain. The domain of  $x$  is all real numbers except 0.

Solution set:  $(-\infty, 0) \cup (0, \infty)$

$$52. \frac{x-1}{x-2} - 1 = \frac{1}{x-2}$$

Multiply both sides of the equation by the LCD,  $x-2$ .

$$x-1 - (x-2) = 1 \Rightarrow 1 = 1$$

The equation  $1 = 1$  is equivalent to the original equation and is always true for all values of  $x$  in its domain. The domain of  $x$  is all real numbers except 2.

Solution set:  $(-\infty, 2) \cup (2, \infty)$

$$53. \frac{1}{3x} + \frac{1}{2x} = \frac{1}{6} - \frac{1}{x}$$

Multiply both sides of the equation by the LCD,  $6x$ .

$$2+3 = x-6$$

$$5 = x-6$$

$$5+6 = x-6+6$$

$$11 = x$$

Solution set:  $\{11\}$

$$54. \frac{5}{2x} - \frac{4}{7} = \frac{3}{14} - \frac{2}{x}$$

Multiply both sides of the equation by the LCD,  $14x$ .

$$35-8x = 3x-28$$

$$35-8x+8x = 3x-28+8x$$

$$35 = 11x-28$$

$$35+28 = 11x-28+28$$

$$63 = 11x$$

$$\frac{63}{11} = \frac{11x}{11} \Rightarrow x = \frac{63}{11}$$

Solution set:  $\left\{\frac{63}{11}\right\}$

$$55. \frac{2}{x-1} = \frac{3}{x+1}$$

Multiply both sides of the equation by the LCD,  $(x-1)(x+1)$ .

$$2(x+1) = 3(x-1)$$

$$2x+2 = 3x-3$$

$$2x+2-2x = 3x-3-2x$$

$$2 = x-3$$

$$2+3 = x-3+3$$

$$5 = x$$

Solution set:  $\{5\}$

$$56. \frac{3}{y+2} = \frac{4}{y-1}$$

Multiply both sides of the equation by the LCD,  $(y+2)(y-1)$ .

$$3(y-1) = 4(y+2)$$

$$3y-3 = 4y+8$$

$$3y-3-3y = 4y+8-3y$$

$$-3 = y+8$$

$$-3-8 = y+8-8$$

$$-11 = y$$

Solution set:  $\{-11\}$

$$57. \frac{1}{3-m} + \frac{7}{2m+3} = 0$$

Multiply both sides of the equation by the LCD,  $(3-m)(2m+3)$ .

$$(2m+3) + 7(3-m) = 0$$

$$2m+3+21-7m = 0$$

$$-5m+24 = 0$$

$$-5m = -24$$

$$m = \frac{24}{5}$$

Solution set:  $\left\{\frac{24}{5}\right\}$

$$58. \frac{t}{t-2} = -\frac{2}{3}$$

Multiply both sides of the equation by the LCD,  $3(t-2)$ .

$$3t = -2(t-2)$$

$$3t = -2t+4$$

$$3t+2t = -2t+4+2t$$

$$5t = 4$$

$$t = \frac{4}{5}$$

Solution set:  $\left\{\frac{4}{5}\right\}$

$$59. \frac{2}{x+1} + 3 = \frac{8}{x+1}$$

Multiply both sides of the equation by the LCD,  $x + 1$ .

$$2 + 3(x+1) = 8$$

$$2 + 3x + 3 = 8$$

$$5 + 3x = 8$$

$$5 + 3x - 5 = 8 - 5$$

$$3x = 3 \Rightarrow x = 1$$

Solution set:  $\{1\}$

$$60. \frac{3}{x+2} = \frac{5}{x+2} - 4$$

Multiply both sides of the equation by the LCD,  $x + 2$ .

$$3 = 5 - 4(x+2)$$

$$3 = 5 - 4x - 8$$

$$3 = -3 - 4x$$

$$3 + 3 = -3 - 4x + 3$$

$$6 = -4x$$

$$\frac{6}{-4} = \frac{-4x}{-4} \Rightarrow x = -\frac{6}{4} = -\frac{3}{2}$$

Solution set:  $\left\{-\frac{3}{2}\right\}$

$$61. \frac{5x}{x-1} = \frac{5}{x-1} + 3$$

Multiply both sides of the equation by the LCD,  $x - 1$ .

$$5x = 5 + 3(x-1)$$

$$5x = 5 + 3x - 3$$

$$5x = 2 + 3x$$

$$5x - 3x = 2 + 3x - 3x$$

$$2x = 2$$

$$x = 1$$

Check:

$$\frac{5(1)}{1-1} = \frac{5}{1-1} + 3 \Rightarrow \frac{5}{0} = \frac{5}{0} + 3$$

Since dividing by zero is undefined, reject 1 as an extraneous solution. Therefore, there is no number  $x$  that satisfies the equation.

Solution set:  $\emptyset$

$$62. \frac{3x}{x+1} - 3 + \frac{3}{2(x+1)} = 0$$

Multiply both sides of the equation by the LCD,  $2(x + 1)$ .

$$2(3x) - 3 \cdot 2(x+1) + 3 = 0$$

$$6x - 6x - 6 + 3 = 0$$

$$-3 = 0$$

Since the last equation is false, there is no solution. Solution set:  $\emptyset$

$$63. \frac{2}{x-2} + \frac{3}{2x-4} = \frac{1}{3} + \frac{5}{6x-12}$$

Factor the denominators to find the LCD:

$2x - 4 = 2(x - 2)$  and  $6x - 12 = 6(x - 2)$ . The LCD is  $6(x - 2)$ .

$$\begin{aligned} 6(x-2)\left(\frac{2}{x-2}\right) + 6(x-2)\left(\frac{3}{2(x-2)}\right) \\ = 6(x-2)\left(\frac{1}{3}\right) + 6(x-2)\left(\frac{5}{6(x-2)}\right) \end{aligned}$$

$$6(2) + 3(3) = 2(x-2) + 5$$

$$12 + 9 = 2x - 4 + 5$$

$$21 = 2x + 1$$

$$21 - 1 = 2x + 1 - 1$$

$$20 = 2x$$

$$10 = x$$

Solution set:  $\{10\}$

$$64. \frac{1}{2x+2} + \frac{1}{6} = \frac{5}{18} - \frac{1}{3x+3}$$

Factor the denominators to find the LCD:

$2x + 2 = 2(x + 1)$ ,  $3x + 3 = 3(x + 1)$ , and  $18 = 2 \cdot 3 \cdot 3$ . The LCD is  $18(x + 1)$ .

$$\begin{aligned} 18(x+1)\left(\frac{1}{2(x+1)}\right) + 18(x+1)\left(\frac{1}{6}\right) \\ = 18(x+1)\left(\frac{5}{18}\right) - 18(x+1)\left(\frac{1}{3(x+1)}\right) \end{aligned}$$

$$9 + 3(x+1) = 5(x+1) - 6$$

$$9 + 3x + 3 = 5x + 5 - 6$$

$$12 + 3x = 5x - 1$$

$$12 + 3x - 3x = 5x - 1 - 3x$$

$$12 = 2x - 1$$

$$12 + 1 = 2x - 1 + 1$$

$$13 = 2x \Rightarrow \frac{13}{2} = x$$

Solution set:  $\left\{\frac{13}{2}\right\}$

65. To solve  $d = rt$  for  $r$ , divide both sides of the equation by  $t$ .  $r = \frac{d}{t}$ .

66. To solve  $F = ma$  for  $a$ , divide both sides of the equation by  $m$ .  $a = \frac{F}{m}$ .

67. To solve  $C = 2\pi r$  for  $r$ , divide both sides of the equation by  $2\pi$ .  $r = \frac{C}{2\pi}$ .

68. To solve  $A = 2\pi rx + \pi r^2$  for  $x$ , subtract  $\pi r^2$  from both sides.

$$A - \pi r^2 = 2\pi rx + \pi r^2 - \pi r^2$$

$$A - \pi r^2 = 2\pi rx$$

Divide both sides by  $2\pi r$ .

$$\frac{A - \pi r^2}{2\pi r} = \frac{2\pi rx}{2\pi r}$$

$$\frac{A - \pi r^2}{2\pi r} = x$$

69. To solve  $I = \frac{E}{R}$  for  $R$ , multiply both sides by  $R$ .

$$RI = R\left(\frac{E}{R}\right) \Rightarrow RI = E$$

Divide both sides by  $I$ .

$$\frac{RI}{I} = \frac{E}{I} \Rightarrow R = \frac{E}{I}$$

70. To solve  $A = P(1 + rt)$  for  $t$ , distribute  $P$ .

$$A = P + Prt$$

Subtract  $P$  from both sides.

$$A - P = P + Prt - P$$

$$A - P = Prt$$

Divide both sides by  $Pr$ .

$$\frac{A - P}{Pr} = \frac{Prt}{Pr}$$

$$\frac{A - P}{Pr} = t$$

71. To solve  $A = \frac{(a+b)h}{2}$  for  $h$ , multiply both sides by 2.

$$2A = (a+b)h$$

Divide both sides by  $(a+b)$ .

$$\frac{2A}{a+b} = \frac{(a+b)h}{a+b} \Rightarrow \frac{2A}{a+b} = h$$

72. To solve  $T = a + (n-1)d$  for  $d$ , subtract  $a$  from both sides.

$$T - a = a + (n-1)d - a$$

$$T - a = (n-1)d$$

Divide both sides by  $(n-1)$ .

$$\frac{T - a}{n-1} = \frac{(n-1)d}{n-1} \Rightarrow \frac{T - a}{n-1} = d$$

73. To solve  $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$  for  $u$ , clear the fractions

by multiplying both sides by the least common denominator,  $fu v$ .

$$fu v \left( \frac{1}{f} \right) = fu v \left( \frac{1}{u} + \frac{1}{v} \right)$$

$$fu v \left( \frac{1}{f} \right) = fu v \left( \frac{1}{u} \right) + fu v \left( \frac{1}{v} \right)$$

Simplify.

$$uv = fv + fu$$

Subtract  $fu$  from both sides.

$$uv - fu = fv + fu - fu$$

$$uv - fu = fv$$

Factor the left side.

$$u(v - f) = fv$$

Divide both sides by  $v - f$ .

$$\frac{u(v - f)}{v - f} = \frac{fv}{v - f} \Rightarrow u = \frac{fv}{v - f}$$

74. To solve  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$  for  $R_2$ ,

clear the fractions by multiplying both sides by the least common denominator,  $R \cdot R_1 \cdot R_2$ .

$$R \cdot R_1 \cdot R_2 \left( \frac{1}{R} \right) = R \cdot R_1 \cdot R_2 \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$R \cdot R_1 \cdot R_2 \left( \frac{1}{R} \right) = R \cdot R_1 \cdot R_2 \left( \frac{1}{R_1} \right)$$

$$+ R \cdot R_1 \cdot R_2 \left( \frac{1}{R_2} \right)$$

Simplify.

$$R_1 R_2 = RR_2 + RR_1$$

Subtract  $RR_2$  from both sides.

$$R_1 R_2 - RR_2 = RR_2 + RR_1 - RR_2$$

$$R_1 R_2 - RR_2 = RR_1$$

Factor the left side.

$$R_2(R_1 - R) = RR_1$$

Divide both sides by  $(R_1 - R)$ .

$$\frac{R_2(R_1 - R)}{(R_1 - R)} = \frac{RR_1}{(R_1 - R)} \Rightarrow R_2 = \frac{RR_1}{(R_1 - R)}$$

75. To solve  $y = mx + b$  for  $m$ , subtract  $b$  from both sides.

$$y - b = mx + b - b \Rightarrow y - b = mx$$

Divide both sides by  $x$ .

$$\frac{y - b}{x} = \frac{mx}{x} \Rightarrow \frac{y - b}{x} = m$$

76. To solve  $ax + by = c$  for  $y$ , subtract  $ax$  from both sides.

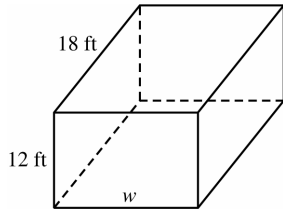
$$ax + by - ax = c - ax \Rightarrow by = c - ax$$

Divide both sides by  $b$ .

$$\frac{by}{b} = \frac{c - ax}{b} \Rightarrow y = \frac{c - ax}{b}$$

1.1 B Exercises: Applying the Concepts

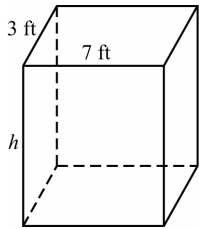
77. The formula for volume is  $V = lwh$ .  
Substitute 2808 for  $V$ , 18 for  $l$ , and 12 for  $h$ .  
Solve for  $w$ .



$$\begin{aligned} 2808 &= 18 \cdot 12 \cdot w \\ 2808 &= 216w \\ \frac{2808}{216} &= \frac{216w}{216} \\ 13 &= w \end{aligned}$$

The width of the pool is 13 ft.

78. The formula for volume is  $V = lwh$ .  
Substitute 168 for  $V$ , 7 for  $l$ , and 3 for  $w$ .  
Solve for  $h$ .



$$\begin{aligned} 168 &= 7 \cdot 3 \cdot h \\ 168 &= 21h \\ \frac{168}{21} &= \frac{21h}{21} \\ 8 &= h \end{aligned}$$

The hole must be 8 ft deep.

79. The formula for circumference of a circle is  $C = 2\pi r$ . Substitute  $114\pi$  for  $C$ . Solve for  $r$ .

$$114\pi = 2\pi r \Rightarrow \frac{114\pi}{2\pi} = \frac{2\pi r}{2\pi} \Rightarrow 57 = r$$

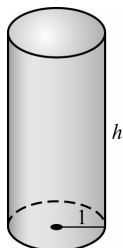
The radius is 57 cm.

80. The formula for perimeter of a rectangle is  $P = 2l + 2w$ . Substitute 28 for  $P$  and 5 for  $w$ .  
Solve for  $l$ .

$$\begin{aligned} 28 &= 2l + 2(5) \\ 28 &= 2l + 10 \\ 28 - 10 &= 2l + 10 - 10 \\ 18 &= 2l \\ \frac{18}{2} &= \frac{2l}{2} \\ 9 &= l \end{aligned}$$

The length is 9 m.

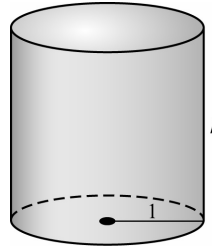
81. The formula for surface area of a cylinder is  $S = 2\pi rh + 2\pi r^2$ . Substitute  $6\pi$  for  $S$  and 1 for  $r$ . Solve for  $h$ .



$$\begin{aligned} 6\pi &= 2\pi(1)h + 2\pi(1^2) \\ 6\pi &= 2\pi h + 2\pi \\ 6\pi - 2\pi &= 2\pi h + 2\pi - 2\pi \\ 4\pi &= 2\pi h \\ \frac{4\pi}{2\pi} &= \frac{2\pi h}{2\pi} \Rightarrow 2 = h \end{aligned}$$

The height is 2 m.

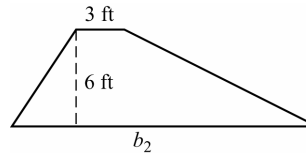
82. The formula for volume of a cylinder is  $V = \pi r^2 h$ . Substitute  $148\pi$  for  $V$  and 2 for  $r$ .  
Solve for  $h$ .



$$\begin{aligned} 148\pi &= \pi \cdot 2^2 \cdot h \\ 148\pi &= 4\pi h \\ \frac{148\pi}{4\pi} &= \frac{4\pi h}{4\pi} \\ 37 &= h \end{aligned}$$

The height of the can is 37 cm.

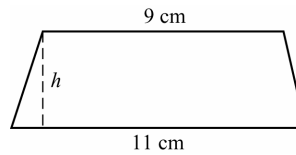
83. The formula for area of a trapezoid is  $A = \frac{1}{2}h(b_1 + b_2)$ . Substitute 66 for  $A$ , 6 for  $h$ , and 3 for  $b_1$ . Solve for  $b_2$ .



$$\begin{aligned} 66 &= \frac{1}{2} \cdot 6(3 + b_2) \\ 66 &= 3(3 + b_2) \\ 66 &= 9 + 3b_2 \\ 66 - 9 &= 9 + 3b_2 - 9 \\ 57 &= 3b_2 \\ \frac{57}{3} &= \frac{3b_2}{3} \\ 19 &= b_2 \end{aligned}$$

The length of the second base is 19 ft.

84. The formula for area of a trapezoid is  $A = \frac{1}{2}h(b_1 + b_2)$ . Substitute 35 for  $A$ , 9 for  $b_1$ , and 11 for  $b_2$ . Solve for  $h$ .



$$\begin{aligned} 35 &= \frac{1}{2}h(9 + 11) \\ 35 &= \frac{1}{2}h(20) \\ 35 &= 10h \\ \frac{35}{10} &= \frac{10h}{10} \Rightarrow 3.5 = h \end{aligned}$$

The height of the trapezoid is 3.5 cm.

85. Substitute 920 for  $A$ , 500 for  $P$  and 0.06 for  $r$  into the formula  $A = P + Prt$ . Solve for  $t$ .

$$\begin{aligned} 920 &= 500 + 500 \cdot 0.06t \\ 920 &= 500 + 30t \\ 920 - 500 &= 500 + 30t - 500 \\ 420 &= 30t \\ \frac{420}{30} &= \frac{30t}{30} \Rightarrow 14 = t \end{aligned}$$

It will take 14 years until the amount of money available is \$920.

86. Substitute 2482 for  $A$ , 10 for  $t$  and 0.07 for  $r$  into the formula  $A = P + Prt$ . Solve for  $P$ .

$$2482 = P + P \cdot 0.07 \cdot 10$$

$$2482 = P + 0.7P$$

$$2482 = 1.7P$$

$$\frac{2482}{1.7} = \frac{1.7P}{1.7} \Rightarrow 1460 = P$$

\$1460 was invested.

87. Substitute 427 for  $M$  and 302 for  $b$  into the formula  $M = \frac{m-b}{4}$ . Solve for  $m$ .

$$427 = \frac{m-302}{4}$$

$$4(427) = 4\left(\frac{m-302}{4}\right)$$

$$1708 = m - 302$$

$$1708 + 302 = m - 302 + 302 \Rightarrow 2010 = m$$

The gross monthly income must be \$2010.

88. Substitute 67,181 for  $S$  and 8 for  $b$  into the formula  $S = 48,917 + 1080b + 1604a$ . Solve for  $a$ .

$$67181 = 48917 + 1080(8) + 1604a$$

$$67181 = 48917 + 8640 + 1604a$$

$$67181 = 57557 + 1604a$$

$$67181 - 57557 = 1604a$$

$$9624 = 1604a$$

$$\frac{9624}{1604} = \frac{1604a}{1604} \Rightarrow 6 = a$$

She worked 6 years after receiving her degree.

89. Substitute 170 for  $P$  into the formula  $P = 200 - 0.02q$ . Solve for  $q$ .

$$170 = 200 - 0.02q$$

$$170 - 200 = 200 - 0.02q - 200$$

$$-30 = -0.02q$$

$$\frac{-30}{-0.02} = \frac{-0.02q}{-0.02} \Rightarrow 1500 = q$$

Note that the solution must fall between 100 and 2000 cameras. 1500 cameras must be ordered.

90. Substitute 200 for  $V_2$ , 600 for  $V_1$ , and 400 for

$P_1$  into the formula  $V_2 = \frac{V_1 P_1}{P_2}$ . Solve for  $V_2$ .

$$200 = \frac{600 \cdot 400}{P_2}$$

$$200 = \frac{240,000}{P_2}$$

$$200P_2 = P_2 \left( \frac{240,000}{P_2} \right)$$

$$200P_2 = 240,000$$

$$\frac{200P_2}{200} = \frac{240,000}{200} \Rightarrow P_2 = 1200$$

The new pressure,  $P_2$ , is 1200 millimeters of mercury.

91. Substitute 950 for  $V$ , 100,000 for  $R_1$ , and 100 for  $R_2$  into the formula  $V = \alpha \frac{R_1}{R_2}$ . Solve for  $\alpha$ .

$$950 = \alpha \frac{100,000}{100}$$

$$\left( \frac{100}{100,000} \right) 950 = \alpha \left( \frac{100,000}{100} \right) \left( \frac{100}{100,000} \right)$$

$$0.95 = \alpha$$

The current gain,  $\alpha$ , is 0.95.

92. Substitute 37,000 for  $Q$ , 1500 for  $L$ , and 3200 for  $I$  into the formula  $Q = \frac{A - I}{L}$ . Solve for  $A$ .

$$37,000 = \frac{A - 3200}{1500}$$

$$1500(37,000) = 1500 \left( \frac{A - 3200}{1500} \right)$$

$$55,500,000 = A - 3200$$

$$55,500,000 + 3200 = A - 3200 + 3200$$

$$55,503,200 = A$$

The current assets are \$55,503,200.

93. Substitute 1247.65 for  $P$ , 0.1391 for  $r$ , and  $567/365$  for  $t$  into the formula  $A = P + Prt$ . Solve for  $A$ . Note that the formula calls for  $t$  to be given in years while the problem gives  $t$  in days.

$$A = 1247.65 + 1247.65 \cdot 0.1391 \cdot \frac{567}{365}$$

$$A = 1517.24$$

The amount resulting will be \$1517.24.

94. Substitute 3264 for  $A$ , 2400 for  $P$  and 4 for  $t$  into the formula  $A = P + Prt$ . Solve for  $r$ .

$$3264 = 2400 + 2400 \cdot 4r$$

$$3264 = 2400 + 9600r$$

$$3264 - 2400 = 2400 + 9600r - 2400$$

$$864 = 9600r$$

$$\frac{864}{9600} = \frac{9600r}{9600} \Rightarrow 0.09 = r$$

The interest rate is 0.09 or 9%.

95. Joe can run  $\frac{1}{8}$  mile in one minute, while Dick can run  $\frac{1}{12}$  mile in one minute. Let  $x$  = the number of minutes until Joe will lead Dick by exactly one lap (one mile). Then,
- $$\frac{1}{8}x - \frac{1}{12}x = 1.$$
- Multiply both sides of the equation by the LCD, 24.
- $$3x - 2x = 24 \Rightarrow x = 24$$
- Joe will lead Dick by exactly one lap (one mile) in 24 minutes.

96. Joe can run  $\frac{1}{8}$  mile in one minute, while Dick can run  $\frac{1}{12}$  mile in one minute. Let  $x$  = the number of minutes until Joe and Dick meet. Then,
- $$\frac{1}{8}x + \frac{1}{12}x = 1$$
- Multiply both sides of the equation by the LCD, 24.
- $$3x + 2x = 24$$
- $$5x = 24 \Rightarrow x = 4.8$$
- They will meet in 4.8 seconds.

### 1.1 C Exercises: Beyond the Basics

In exercises 97–106, be sure to check each exercise for extraneous solutions. We will show the checks only for those exercises with extraneous solutions.

97.  $\frac{x}{x-2} - \frac{1}{x+2} = 1$
- Multiply both sides of the equation by the LCD,  $(x-2)(x+2)$ .
- $$x(x+2) - 1(x-2) = (x+2)(x-2)$$
- $$x^2 + 2x - x + 2 = x^2 - 4$$
- $$x + 2 = -4$$
- $$x = -6$$
- Solution set:  $\{-6\}$
98.  $4 - \frac{2-x}{x+1} = \frac{5x+3}{x+2}$
- Multiply both sides of the equation by the LCD,  $(x+1)(x+2)$ .
- $$4(x+1)(x+2) - (2-x)(x+2) = (5x+3)(x+1)$$

$$4(x^2 + 3x + 2) - (4 - x^2) = 5x^2 + 8x + 3$$

$$4x^2 + 12x + 8 - 4 + x^2 = 5x^2 + 8x + 3$$

$$5x^2 + 12x + 4 = 5x^2 + 8x + 3$$

$$12x + 4 = 8x + 3$$

$$4x = -1 \Rightarrow x = -\frac{1}{4}$$

Solution set:  $\left\{-\frac{1}{4}\right\}$

99.  $\frac{1}{x-3} - \frac{4}{x+3} = \frac{6}{x^2-9}$
- Multiply both sides of the equation by the LCD,  $(x-3)(x+3) = x^2 - 9$ .
- $$(x+3) - 4(x-3) = 6$$
- $$x+3 - 4x+12 = 6$$
- $$-3x+15 = 6$$
- $$-3x = -9$$
- $$x = 3$$

Check:

$$\frac{1}{3-3} - \frac{4}{3+3} = \frac{6}{3^2-9} \Rightarrow \frac{1}{0} - \frac{4}{6} = \frac{6}{0}$$

Since dividing by zero is undefined, reject 3 as an extraneous solution. Therefore, there is no number  $x$  that satisfies the equation.

Solution set:  $\emptyset$

100.  $\frac{1+x}{1-x} - \frac{1-x}{1+x} = \frac{1}{1-x^2}$
- Multiply both sides of the equation by the LCD,  $(1-x)(1+x) = 1-x^2$
- $$(1+x)(1+x) - (1-x)(1-x) = 1$$
- $$x^2 + 2x + 1 - (x^2 - 2x + 1) = 1$$
- $$4x = 1$$
- $$x = \frac{1}{4}$$

Solution set:  $\left\{\frac{1}{4}\right\}$

101.  $\frac{5}{x+2} - \frac{8}{x^2-x-6} = \frac{4}{x-3}$
- Multiply both sides of the equation by the LCD,  $(x+2)(x-3) = x^2 - x - 6$ .
- $$5(x-3) - 8 = 4(x+2)$$
- $$5x - 15 - 8 = 4x + 8$$
- $$5x - 23 = 4x + 8$$
- $$5x = 4x + 31$$
- $$x = 31$$
- Solution set:  $\{31\}$

$$102. \frac{2x-1}{x+2} + \frac{x+1}{2-x} = \frac{x^2+3x+1}{x^2-4}$$

Multiply both sides of the equation by the

LCD,  $-(x+2)(2-x) = x^2 - 4$

$$\begin{aligned} -(2x-1)(2-x) + [-(x+1)(x+2)] \\ = x^2 + 3x + 1 \\ (2x^2 - 5x + 2) + (-x^2 - 3x - 2) = x^2 + 3x + 1 \\ x^2 - 8x = x^2 + 3x + 1 \\ -8x = 3x + 1 \\ -11x = 1 \\ x = -\frac{1}{11} \end{aligned}$$

Solution set:  $\left\{-\frac{1}{11}\right\}$

$$103. \frac{x+3}{x-1} - \frac{5}{x+1} = \frac{x^2-3x+11}{x^2-1}$$

Multiply both sides of the equation by the

LCD,  $(x-1)(x+1) = x^2 - 1$ .

$$\begin{aligned} (x+3)(x+1) - 5(x-1) = x^2 - 3x + 11 \\ x^2 + 4x + 3 - 5x + 5 = x^2 - 3x + 11 \\ x^2 - x + 8 = x^2 - 3x + 11 \\ -x + 8 = -3x + 11 \\ 2x + 8 = 11 \\ 2x = 3 \\ x = \frac{3}{2} \end{aligned}$$

Solution set:  $\left\{\frac{3}{2}\right\}$

$$104. \frac{x}{x-3} + \frac{x^2-2}{9-x^2} = \frac{1}{x+3}$$

Multiply both sides of the equation by the

LCD,  $-(x-3)(x+3) = 9 - x^2$ .

$$\begin{aligned} -x(x+3) + x^2 - 2 = -(x-3) \\ -x^2 - 3x + x^2 - 2 = -x + 3 \\ -3x - 2 = -x + 3 \\ -3x = -x + 5 \\ -2x = 5 \\ x = -\frac{5}{2} \end{aligned}$$

Solution set:  $\left\{-\frac{5}{2}\right\}$

$$105. \frac{1}{x-4} - \frac{1}{x-3} = \frac{1}{x-2} - \frac{1}{x-1}$$

Multiply both sides of the equation by the

LCD  $(x-1)(x-2)(x-3)(x-4)$ .

$$\begin{aligned} (x-1)(x-2)(x-3) - (x-1)(x-2)(x-4) \\ = (x-1)(x-3)(x-4) - (x-2)(x-3)(x-4) \\ (x^3 - 6x^2 + 11x - 6) - (x^3 - 7x^2 + 14x - 8) \\ = (x^3 - 8x^2 + 19x - 12) - (x^3 - 9x^2 + 26x - 24) \\ x^2 - 3x + 2 = x^2 - 7x + 12 \\ -3x + 2 = -7x + 12 \\ 4x + 2 = 12 \\ 4x = 10 \\ x = \frac{10}{4} = \frac{5}{2} \end{aligned}$$

Solution set:  $\left\{\frac{5}{2}\right\}$

$$106. \frac{2}{x+1} + \frac{4}{x^2-1} = \frac{1}{x-1}$$

Multiply both sides of the equation by the

LCD,  $(x+1)(x-1) = x^2 - 1$

$$\begin{aligned} 2(x-1) + 4 = 1(x+1) \\ 2x - 2 + 4 = x + 1 \\ 2x + 2 = x + 1 \\ x + 2 = 1 \\ x = -1 \end{aligned}$$

Check:

$$\frac{2}{-1+1} + \frac{4}{(-1)^2+1} = \frac{1}{-1-1} \Rightarrow \frac{2}{0} + \frac{4}{2} = \frac{1}{-2}$$

Since dividing by zero is undefined, reject  $-1$  as an extraneous solution. Therefore, there is no number  $x$  that satisfies the equation.

Solution set:  $\emptyset$

107. a. The solution set of  $x^2 = x$  is  $\{0, 1\}$ , while the solution set of  $x = 1$  is  $\{1\}$ . Therefore, the equations are not equivalent.

b. The solution set of  $x^2 = 9$  is  $\{-3, 3\}$ , while the solution set of  $x = 3$  is  $\{3\}$ . Therefore, the equations are not equivalent.

c. The solution set of  $x^2 - 1 = x - 1$  is  $\{0, 1\}$ , while the solution set of  $x = 0$  is  $\{0\}$ . Therefore, the equations are not equivalent.

- d. The equation  $\frac{x}{x-2} = \frac{2}{x-2}$  is an inconsistent equation, so its solution set is  $\emptyset$ . The solution set of  $x = 2$  is  $\{2\}$ . Therefore, the equations are not equivalent.
- 108.** In the division step, there is division by zero because  $x = 1$ . The remainder of the argument is invalid due to the division by zero.
- 109.** First, solve  $7x + 2 = 16$ . Subtracting 2 from both sides, we have  $7x = 14$ . Then divide both sides by 7; we obtain  $x = 2$ . Now substitute 2 for  $x$  in  $3x - 1 = k$ . This becomes  $3(2) - 1 = k$ , so  $k = 5$ .
- 110.** Substitute 9 for  $y$  in the equation  $\frac{5}{y-4} = \frac{6}{y+k}$  and then solve for  $k$ .
- $$\frac{5}{9-4} = \frac{6}{9+k}$$
- $$1 = \frac{6}{9+k}$$
- $$(9+k)(1) = (9+k)\left(\frac{6}{9+k}\right)$$
- $$9+k = 6$$
- $$9+k-9 = 6-9 \Rightarrow k = -3$$
- 111.** If  $k = -2$  then the equation becomes  $\frac{3}{y-2} = \frac{4}{y-2}$ , which is inconsistent. Note that two fractions with the same denominator are not equal if the numerators are not equal.
- 112.** Because the numerators are the same, we can set the denominators equal to each other in order to create an identity.
- $$x^2 - 9 = (x-3)(x+k)$$
- When the left side is factored, it becomes  $(x-3)(x+3)$ , so  $k = 3$ .
- 113.** To solve  $a(a+x) = b^2 - bx$  for  $x$ , distribute  $a$  on the left side of the equation.
- $$a^2 + ax = b^2 - bx$$
- Add  $bx$  to both sides.
- $$a^2 + ax + bx = b^2 - bx + bx$$
- $$a^2 + ax + bx = b^2$$
- Subtract  $a^2$  from both sides.
- $$a^2 + ax + bx - a^2 = b^2 - a^2$$
- $$ax + bx = b^2 - a^2$$
- Factor both sides. The right side is the difference of two squares.
- $$x(a+b) = (b-a)(b+a)$$
- Divide both sides by  $(a+b)$ .
- $$\frac{x(a+b)}{(a+b)} = \frac{(b-a)(b+a)}{(a+b)}$$
- $$x = b - a$$
- 114.** To solve  $9 + a^2x - ax = 6x + a^2$  for  $x$ , subtract  $6x$  from both sides.
- $$9 + a^2x - ax - 6x = 6x + a^2 - 6x$$
- $$9 + a^2x - ax - 6x = a^2$$
- Subtract 9 from both sides.
- $$9 + a^2x - ax - 6x - 9 = a^2 - 9$$
- $$a^2x - ax - 6x = a^2 - 9$$
- Factor out  $x$  the left side.
- $$x(a^2 - a - 6) = a^2 - 9$$
- Divide both sides by  $(a^2 - a - 6)$ .
- $$\frac{x(a^2 - a - 6)}{(a^2 - a - 6)} = \frac{a^2 - 9}{(a^2 - a - 6)}$$
- $$x = \frac{a^2 - 9}{(a^2 - a - 6)}$$
- Factor the numerator and denominator on the right side to simplify the fraction.
- $$x = \frac{(a+3)(a-3)}{(a+2)(a-3)}$$
- $$x = \frac{a+3}{a+2}$$
- 115.** To solve  $\frac{ax}{b} - \frac{bx}{a} = \frac{(a+b)^2}{ab}$  for  $x$ , multiply both sides by the common denominator,  $ab$ .
- $$ab\left(\frac{ax}{b} - \frac{bx}{a}\right) = ab\left(\frac{(a+b)^2}{ab}\right)$$
- $$a^2x - b^2x = (a+b)^2$$
- Factor the left side.
- $$x(a^2 - b^2) = (a+b)^2$$
- Divide both sides by  $(a^2 - b^2)$ .
- $$\frac{x(a^2 - b^2)}{(a^2 - b^2)} = \frac{(a+b)^2}{(a^2 - b^2)} \Rightarrow x = \frac{(a+b)^2}{(a^2 - b^2)}$$
- Factor the numerator and denominator on the right side. Then simplify.
- $$x = \frac{(a+b)(a+b)}{(a-b)(a+b)}$$
- $$x = \frac{a+b}{a-b}$$



116. To solve  $\frac{2}{3} - \frac{x}{3b} - \frac{2x+b}{2a} + \frac{6b^2+a^2}{6ab} = 0$  for  $x$ , multiply both sides by the common denominator,  $6ab$ .

$$6ab \left( \frac{2}{3} - \frac{x}{3b} - \frac{2x+b}{2a} + \frac{6b^2+a^2}{6ab} \right) = 6ab(0)$$

$$4ab - 2ax - 3b(2x+b) + 6b^2 + a^2 = 0$$

$$4ab - 2ax - 6bx - 3b^2 + 6b^2 + a^2 = 0$$

$$4ab - 2ax - 6bx + 3b^2 + a^2 = 0$$

Subtract  $(a^2 + 4ab + 3b^2)$  from both sides.

$$\begin{aligned} 4ab - 2ax - 6bx + 3b^2 + a^2 \\ - (4ab + 3b^2 + a^2) \\ = 0 - (a^2 + 4ab + 3b^2) \\ - 2ax - 6bx = -(a^2 + 4ab + 3b^2) \end{aligned}$$

Factor both sides.

$$-2x(a+3b) = -(a+3b)(a+b)$$

Divide both sides by  $-2(a+3b)$

and simplify.

$$\begin{aligned} \frac{-2x(a+3b)}{-2(a+3b)} &= \frac{-(a+3b)(a+b)}{-2(a+3b)} \\ x &= \frac{a+b}{2} \end{aligned}$$

117. To solve  $\frac{b(bx-1)}{a} - \frac{a(1+ax)}{b} = 1$  for  $x$ , multiply both sides by the common denominator,  $ab$ .

$$ab \left( \frac{b(bx-1)}{a} - \frac{a(1+ax)}{b} \right) = ab(1)$$

$$b(b(bx-1)) - a(a(1+ax)) = ab$$

$$b^2(bx-1) - a^2(1+ax) = ab$$

$$b^3x - b^2 - a^2 - a^3x = ab$$

Add  $b^2 + a^2$  to both sides.

$$\begin{aligned} b^3x - b^2 - a^2 - a^3x + b^2 + a^2 \\ = ab + b^2 + a^2 \\ b^3x - a^3x = a^2 + ab + b^2 \end{aligned}$$

Factor both sides.

$$x(b-a)(b^2+ab+a^2) = a^2+ab+b^2$$

Divide both sides by  $(b-a)(b^2+ab+a^2)$  and then simplify.

$$\begin{aligned} \frac{x(b-a)(b^2+ab+a^2)}{(b-a)(b^2+ab+a^2)} &= \\ \frac{a^2+ab+b^2}{(b-a)(b^2+ab+a^2)} \\ x &= \frac{1}{b-a} \end{aligned}$$

118. To solve  $\frac{x-2a}{b} - \frac{3}{2} = \frac{b-x}{2a} + \frac{1}{2}$  for  $x$ , multiply both sides by the common denominator,  $2ab$ .

$$2ab \left( \frac{x-2a}{b} - \frac{3}{2} \right) = 2ab \left( \frac{b-x}{2a} + \frac{1}{2} \right)$$

Distribute and collect like terms.

$$2a(x-2a) - ab(3) = b(b-x) + ab(1)$$

$$2ax - 4a^2 - 3ab = b^2 - bx + ab$$

Add  $bx$  to both sides.

$$2ax - 4a^2 - 3ab + bx = b^2 - bx + ab + bx$$

$$2ax + bx - 4a^2 - 3ab = b^2 + ab$$

Add  $4a^2 + 3ab$  to both sides.

$$\begin{aligned} 2ax + bx - 4a^2 - 3ab \\ + 4a^2 + 3ab = b^2 + ab + 4a^2 + 3ab \\ 2ax + bx = b^2 + 4ab + 4a^2 \end{aligned}$$

Factor both sides.

$$x(2a+b) = (b+2a)(b+2a)$$

Divide both sides by  $2a+b$ .

$$\begin{aligned} \frac{x(2a+b)}{2a+b} &= \frac{(b+2a)(b+2a)}{2a+b} \\ x &= 2a+b \end{aligned}$$

## 1.1 Critical Thinking

119. If  $x$  represents the amount the pawn shop owner paid for the first watch and the owner made a profit of 10%, then  $1.1x = 499$ , so  $x = 453.64$ . If  $y$  represents the amount the pawn shop owner paid for the second watch and the owner lost 10%, then  $0.9y = 499$ , so  $y = 554.44$ . Together the two watches cost  $\$453.64 + \$554.44 = \$1008.08$ . But the pawn shop owner sold the two watches for  $\$998$ , so there was a loss. The amount of loss is  $(1008.08 - 998)/1008.08 = 10.08/1008.08 \approx 0.01 = 1\%$ . The answer is (C).

120. Let  $x$  represent the amount of gasoline used in July. Then  $0.8x$  represents the amount of gasoline used in August. Let  $y$  represent the price of gasoline in July. Then  $1.2y$  represents the cost of gasoline in August. The cost of gasoline used in July is  $xy$  (amount  $\times$  price), and the cost of gasoline used in August is  $0.8x \times 1.2y = 0.96xy$ . So the cost of gasoline used in August is 96% of the cost of gasoline used in July, which is a decrease of 4%. The answer is (D).

## 1.2 Applications of Linear Equations

### 1.2 Practice Problems

1. Let  $w$  = the width of the rectangle. Then  $2w + 5$  = the length of the rectangle.

$$P = 2l + 2w, \text{ so we have}$$

$$28 = 2(2w + 5) + 2w$$

$$28 = 4w + 10 + 2w$$

$$28 = 6w + 10$$

$$18 = 6w$$

$$3 = w$$

The width of the rectangle is 3 m and the length is  $2(3) + 5 = 11$  m

2. Let  $x$  = the amount invested in stocks. Then  $15,000 - x$  = the amount invested in bonds.

$$x = 3(15,000 - x)$$

$$x = 45,000 - 3x$$

$$4x = 45,000$$

$$x = 11,250$$

Tyrick invested \$11,250 in stocks and  $\$15,000 - \$11,250 = \$3,750$  in bonds.

3. Let  $x$  = the amount of capital. Then  $\frac{x}{5}$  = the amount invested at 5%,  $\frac{x}{6}$  = the amount invested at 8%, and

$$x - \left(\frac{x}{5} + \frac{x}{6}\right) = \frac{19x}{30} = \text{the amount invested at 10\%}.$$

Principal	Rate	Time	Interest
$\frac{x}{5}$	0.05	1	$0.05\left(\frac{x}{5}\right)$
$\frac{x}{6}$	0.08	1	$0.08\left(\frac{x}{6}\right)$
$\frac{19x}{30}$	0.1	1	$0.1\left(\frac{19x}{30}\right)$

The total interest is \$130, so

$$0.05\left(\frac{x}{5}\right) + 0.08\left(\frac{x}{6}\right) + 0.1\left(\frac{19x}{30}\right) = 130$$

$$0.3x + 0.4x + 1.9x = 3900 \quad \text{Multiply by the LCD, 30.}$$

$$2.6x = 3900$$

$$x = 1500$$

The total capital is \$1500.

4. Let  $x$  = the length of the bridge. Then  $x + 130$  = the distance the train travels.

$$rt = d, \text{ so } 25(21) = x + 130 \Rightarrow 525 = x + 130 \Rightarrow 395 = x$$

The bridge is 395 m long.

5. The initial separation between the ship and the aircraft is 955 miles.

Let  $t$  = time elapsed when ship and aircraft meet. Then  $32t$  = the distance the ship traveled and  $350t$  = distance the aircraft traveled.

$$32t + 350t = 955$$

$$382t = 955$$

$$t = 2.5$$

The aircraft and ship meet after 2.5 hours.

6. Let  $x$  = the amount of time they worked together to complete the job. Then  $\frac{1}{45}x$  = the portion of the job done by Jim and  $\frac{1}{30}x$  = the portion of the job done by Anita.
- $$\frac{1}{45}x + \frac{1}{30}x = 1$$
- $$2x + 3x = 90 \quad \text{Multiply by the LCD, 90.}$$
- $$5x = 90$$
- $$x = 18$$

It took them 18 minutes to wash the car together.

7. Let  $x$  = the amount of 40% sulfuric acid.

Acid	Amount of acid	% acid	Amount of pure acid
40%	$x$	0.4	$0.4x$
20%	$50 - x$	0.2	$0.2(50 - x)$
25%	50	0.25	$0.25(50)$

$$0.4x + 0.2(50 - x) = 0.25(50)$$

$$0.4x + 10 - 0.2x = 12.5$$

$$0.2x + 10 = 12.5$$

$$0.2x = 2.5$$

$$x = 12.5$$

12.5 gallons of 40% sulfuric acid solution should be added.

### 1.2 A Exercises: Basic Skills and Concepts

- If the sides of a rectangle are  $a$  and  $b$  units, the perimeter of the rectangle is  $2a + 2b$  units.
- The formula for simple interest is  $I = Prt$ , where  $P$  = dollars borrowed (the principal),  $r$  = the interest rate per year, and  $t$  = the number of years.
- The distance  $d$  traveled by an object moving at rate  $r$  for time  $t$  is given by  $d = rt$ .
- The portion of a job completed per unit of time is called the rate of work.
- False. The interest  $I = (100)(0.05)(3)$ .
- False. Since the rate is given in feet per second, the time must also be converted to seconds. 15 minutes =  $15(60) = 900$  seconds. Therefore,  $d = 60(900)$  feet.

- $0.065x$
- $\frac{5}{6}x$
- $\$22,000 - x$
- $\frac{229.50 - 72}{x}$
- a. Natasha:  $\frac{t}{4}$       b. Natasha's brother:  $\frac{t}{5}$
- a. Jermaine:  $\frac{t}{3}$       b. Ralph:  $\frac{t}{2}$
- a.  $10 - x$       b.  $4.60x$   
c.  $7.40(10 - x)$
- a.  $15 - x$       b.  $6.25(15 - x)$   
c.  $5.50x$
- a.  $8 + x$       b.  $0.8 + x$
- a.  $1 + x$       b.  $\left(\frac{.25}{1+x} \times 100\right)\%$

### 1.2 B Exercises: Applying the Concepts

- Let  $x$  = one number. Then  $3x$  = the other number.  
 $x + 3x = 28 \Rightarrow 4x = 28 \Rightarrow x = 7$ ,  $3x = 3(7) = 21$   
The numbers are 7 and 21.
- Let  $x$  = the first even integer. Then  $x + 2$  = the second even integer, and  $x + 4$  = the third even integer.  
 $x + (x + 2) + (x + 4) = 42$   
 $3x + 6 = 42$   
 $3x = 36 \Rightarrow x = 12$   
 $x + 2 = 14$ ,  $x + 4 = 16$   
The numbers are 12, 14, and 16.
- Let  $w$  = the width of the rectangle.  
Then  $2w - 5$  = the length of the rectangle.  
 $2w + 2(2w - 5) = 80$   
 $2w + 4w - 10 = 80$   
 $6w - 10 = 80$   
 $6w = 90$   
 $w = 15$ ,  $2w - 5 = 25$   
The width of the rectangle is 15 ft and its length is 25 feet.

20. Let  $l$  = the length of the rectangle.

Then  $3 + \frac{1}{2}l$  = the width of the rectangle.

$$2l + 2\left(3 + \frac{1}{2}l\right) = 36$$

$$2l + 6 + l = 36$$

$$3l + 6 = 36$$

$$3l = 30$$

$$l = 10, 3 + \frac{1}{2}l = 8$$

The length of the rectangle is 10 ft and its width is 8 ft.

21. Let  $x$  = the cost of the less expensive land. Then  $x + 23,000$  = the cost of the more expensive land. Together they cost \$147,000, so

$$x + (x + 23,000) = 147,000$$

$$2x + 23,000 = 147,000$$

$$2x = 124,000 \Rightarrow x = 62,000$$

The less expensive piece of land costs \$62,000 and the more expensive piece of land costs \$62,000 + \$23,000 = \$85,000.

22. Let  $x$  = the amount the assistant manager earns. Then  $x + 450$  = the amount the manager earns. Together they earn \$3700, so

$$x + (x + 450) = 3700$$

$$2x + 450 = 3700$$

$$2x = 3250 \Rightarrow x = 1625$$

The assistant manager earns \$1625, and the manager earns \$1625 + \$450 = \$2075.

23. Let  $x$  = the lottery ticket sales in July. Then  $1.10x$  = the lottery ticket sales in August. A total of 1113 tickets were sold, so

$$x + 1.10x = 1113$$

$$2.10x = 1113 \Rightarrow x = 530$$

530 tickets were sold in July, and  $1.10(530) = 583$  tickets were sold in August.

24. Let  $x$  = Jan's commission in March. Then  $15 + 0.5x$  = Jan's commission in February. She earned a total of \$633, so

$$x + (15 + 0.5x) = 633$$

$$1.5x + 15 = 633 \Rightarrow 1.5x = 618 \Rightarrow x = 412$$

Jan's commission was \$412 in March and  $15 + 0.5(412) = \$221$  in February.

25. Let  $x$  = the amount the younger son receives. Then  $4x$  = the amount the older son receives. Together they receive \$225,000, so

$$x + 4x = 225,000 \Rightarrow 5x = 225,000 \Rightarrow$$

$$x = 45,000$$

The younger son will receive \$45,000, and the older son will receive  $4(\$45,000) = \$180,000$ .

26. Let  $x$  = the amount Kevin kept for himself. Then  $x/2$  = the amount he gave his daughter, and  $x/4$  = the amount he gave his dad. He won \$735,000, so

$$x + \frac{x}{2} + \frac{x}{4} = 735,000$$

$$4\left(x + \frac{x}{2} + \frac{x}{4}\right) = 4(735,000)$$

$$4x + 2x + x = 2,940,000$$

$$7x = 2,940,000 \Rightarrow x = 420,000$$

Kevin kept \$420,000 for himself. He gave  $\$420,000/2 = \$210,000$  to his daughter and  $\$420,000/4 = \$105,000$  to his dad.

27. a. Let  $x$  = the number of points needed to average 75.

$$\frac{87 + 59 + 73 + x}{4} = 75$$

$$219 + x = 300$$

$$x = 81$$

You need to score 81 in order to average 75.

b. 
$$\frac{87 + 59 + 73 + 2x}{5} = 75$$

$$219 + 2x = 375$$

$$2x = 156$$

$$x = 78$$

You need to score 78 in order to average 75 if the final carries double weight.

28. Let  $x$  = the amount invested in real estate. Then  $4200 - x$  = the amount invested in a savings and loan.

Investment	Principal	Rate	Time	Interest
Real estate	$x$	0.15	1	$0.15x$
Savings	$4200 - x$	0.08	1	$0.08(4200 - x)$

The total income was \$448, so

$$0.15x + 0.08(4200 - x) = 448$$

$$0.15x + 336 - 0.08x = 448$$

$$0.07x + 336 = 448$$

$$0.07x = 112 \Rightarrow x = 1600$$

So, the real estate agent invested \$1600 in real estate and  $4200 - 1600 = \$2600$  in a savings and loan.

29. Let  $x$  = the amount invested in a tax shelter.  
Then  $7000 - x$  = the amount invested in a bank.

Investment	Principal	Rate	Time	Interest
Tax shelter	$x$	0.09	1	$0.09x$
Bank	$7000 - x$	0.06	1	$0.06(7000 - x)$

The total interest was \$540, so  
 $0.09x + 0.06(7000 - x) = 540$   
 $0.09x + 420 - 0.06x = 540$   
 $0.03x + 420 = 540$   
 $0.03x = 120 \Rightarrow x = 4000$

Mr. Mostafa invested \$4000 in a tax shelter and  $7000 - 4000 = \$3000$  in a bank.

30. Let  $x$  = the amount invested at 6%. Then  $4900 - x$  = the amount invested at 8%

Principal	Rate	Time	Interest
$x$	0.06	1	$0.06x$
$4900 - x$	0.08	1	$0.08(4900 - x)$

The amount of interest for each investment is equal, so  
 $0.06x = 0.08(4900 - x)$   
 $0.06x = 392 - 0.08x$   
 $0.14x = 392 \Rightarrow x = 2800$

Ms. Jordan invested \$2800 at 6% and \$2100 at 8%. The amount of interest she earned on each investment is \$168, so she earned \$336 in all.

31. Let  $x$  = the amount to be invested at 8%.

Principal	Rate	Time	Interest
5000	0.05	1	250
$x$	0.08	1	$0.08x$
$5000 + x$	0.06	1	$0.06(5000 + x)$

The amount of interest for the total investment is the sum of the interest earned on the individual investments, so

$0.06(5000 + x) = 250 + 0.08x$   
 $300 + 0.06x = 250 + 0.08x$   
 $50 + 0.06x = 0.08x$   
 $50 = 0.02x \Rightarrow 2500 = x$

So, \$2500 must be invested at 8%.

32. Let  $x$  = the selling price. Then  $x - 480$  = the profit. So  $x - 480 = 0.2x \Rightarrow -480 = -0.8x \Rightarrow 600 = x$ . The selling price is \$600.

33. There is a profit of \$2 on each shaving set. They want to earn  $\$40,000 + \$30,000 = \$70,000$ . Let  $x$  = the number of shaving sets to be sold. Then  $2x$  = the amount of profit for  $x$  shaving sets. So,  $2x = 70,000 \Rightarrow x = 35,000$ . They must sell 35,000 shaving sets.

34. Let  $x$  = Angelina's rate in meters per minute. Then  $15 + x$  = Harry's rate in meters per minute.

	Rate	Distance	Time
Angelina	$x$	100	$\frac{100}{x}$
Harry	$15 + x$	150	$\frac{150}{15 + x}$

The times are equal, so

$\frac{100}{x} = \frac{150}{15 + x}$   
 $100(15 + x) = 150x$   
 $1500 + 100x = 150x$   
 $1500 = 50x \Rightarrow 30 = x$

So, Angelina jogged at 30 meters per minute. Harry biked at  $15 + 30 = 45$  meters per minute.

35. Let  $x$  = the time the second car travels. Then  $1 + x$  = the time the first car travels. So,

	Rate	Time	Distance
First car	50	$1 + x$	$50(1 + x)$
Second car	70	$x$	$70x$

The distances are equal, so

$50(1 + x) = 70x$   
 $50 + 50x = 70x$   
 $50 = 20x \Rightarrow 2.5 = x$

So, it will take the second car 2.5 hours to overtake the first car.

36. Let  $x$  = the time the planes travel. So,

	Rate	Time	Distance
First plane	470	$x$	$470x$
Second plane	430	$x$	$430x$

The planes are 2250 km apart, so  
 $470x + 430x = 2250 \Rightarrow 900x = 2250 \Rightarrow x = 2.5$   
 So, the planes will be 2250 km apart at 2.5 hours.

37. At 20 miles per hour, it will take Lucas two minutes to bike the remaining  $\frac{2}{3}$  of a mile.

$$\left(\frac{20 \text{ mi}}{1 \text{ hr}} = \frac{20 \text{ mi}}{60 \text{ min}} = \frac{1 \text{ mi}}{3 \text{ min}}\right) \text{ So his brother}$$

will have to bike 1 mile in 2 minutes:

$$\frac{1 \text{ mi}}{2 \text{ min}} = \frac{30 \text{ mi}}{60 \text{ min}} = \frac{30 \text{ mi}}{1 \text{ hr}}$$

38. Driving at 40 miles per hour, it will take Karen's husband  $\frac{45}{40}$  hours or 1 hour and 7.5 minutes to get to the airport. Driving at 60 miles per hour, it will take Karen 45 minutes to get to the airport. Her husband has already driven for 15 minutes, so it will take him an additional 52.5 minutes to get to the airport. Karen will get there before he does.

39. Let  $x$  = the rate the slower car travels. Then  $x + 7$  = the rate the faster car travels. So,

	Rate	Time	Distance
First car	$x$	3	$3x$
Second car	$x + 7$	3	$3(x + 7)$

The planes are 621 miles apart, so

$$3x + 3(x + 7) = 621$$

$$3x + 3x + 21 = 621$$

$$6x + 21 = 621 \Rightarrow 6x = 600 \Rightarrow x = 100$$

One car is traveling at 100 miles per hour, and the other car is traveling at 107 miles per hour.

40. Let  $x$  = the distance to Aya's friend's house.

	Rate	Distance	Time
go	16	$x$	$\frac{x}{16}$
return	80	$x$	$\frac{x}{80}$

She traveled for a total of 3 hours, so

$$\frac{x}{16} + \frac{x}{80} = 3$$

$$80\left(\frac{x}{16} + \frac{x}{80}\right) = 80(3)$$

$$5x + x = 240 \Rightarrow 6x = 240 \Rightarrow x = 40$$

So, her friend lives 40 km away.

41. Let  $x$  = the amount of time it takes both pumps to drain the pool together. The old pump drains  $\frac{1}{6}$  of the pool in one hour, so it will drain  $\frac{x}{6}$  of the pool. The new pump drains  $\frac{1}{4}$  of the pool in one hour, so it will drain  $\frac{x}{4}$  of the pool. So,

$$\frac{x}{6} + \frac{x}{4} = 1$$

$$12\left(\frac{x}{6} + \frac{x}{4}\right) = 12(1)$$

$$2x + 3x = 12$$

$$5x = 12 \Rightarrow x = \frac{12}{5} = 2.4$$

It will take the two pumps 2.4 hours or 2 hours and 24 minutes to drain the pool working together.

42. Let  $x$  = the time needed for the pool to drain. Then  $\frac{x}{3}$  = the time needed for the first drain to empty the pool alone, and  $\frac{x}{7}$  = the time needed for the second drain to empty the pool alone. So,

$$\frac{x}{3} + \frac{x}{7} = 1$$

$$21\left(\frac{x}{3} + \frac{x}{7}\right) = 21(1)$$

$$7x + 3x = 21$$

$$10x = 21 \Rightarrow x = 2.1$$

It will take 2.1 hours or 2 hours and 6 minutes to empty the pool using the two drains together.

43. Let  $x$  = the amount of time it takes both blowers to fill the blimp together. The first blower fills  $\frac{1}{6}$  of the blimp in one hour, so it will fill  $\frac{x}{6}$  of the blimp. The second blower fills  $\frac{1}{9}$  of the blimp in one hour, so it will fill  $\frac{x}{9}$  of the blimp. So,

$$\frac{x}{6} + \frac{x}{9} = 1$$

$$36\left(\frac{x}{6} + \frac{x}{9}\right) = 36(1)$$

$$6x + 4x = 36$$

$$10x = 36 \Rightarrow x = 3.6$$

It will take the two blowers 3.6 hours or 3 hours and 36 minutes to fill the blimp working together.

44. The first shredder completes  $\frac{5}{9}$  of the job in 10 hours, so it will take  $\frac{9}{5} \times 10 = 18$  hours to complete the entire job. When the second shredder is added, there is still  $\frac{4}{9}$  of the job to be completed. If  $x$  = the number of hours the second shredder takes to complete the entire job, then  $\frac{3}{x}$  = the portion of the job that the second shredder can complete in three hours. The first shredder can complete  $\frac{3}{18} = \frac{1}{6}$  of the job in 3 hours. So,

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$$\begin{aligned}\frac{1}{6} + \frac{3}{x} &= \frac{4}{9} \\ 18x\left(\frac{1}{6} + \frac{3}{x}\right) &= 18x\left(\frac{4}{9}\right) \\ 3x + 54 &= 8x \\ 54 &= 5x \Rightarrow 10.8 = x\end{aligned}$$

The second shredder takes 10.8 hours or 10 hours and 48 minutes to complete the entire job alone.

45. Let  $x$  = the time for the new sorter to complete the job alone. Then  $2x$  = the time for the old sorter to complete the job alone. So,  $8/x$  is the portion of the job done by the new sorter and  $8/(2x) = 4/x$  is the portion of the job done by the old sorter.

$$\begin{aligned}\frac{8}{x} + \frac{4}{x} &= 1 \\ x\left(\frac{8}{x} + \frac{4}{x}\right) &= x(1) \\ 8 + 4 &= x \Rightarrow 12 = x\end{aligned}$$

It will take 12 hours for the new sorter to complete the job working alone.

46. Let  $x$  = the time needed for the son to work by himself. So  $6/x$  = the amount of the field the son can plow by himself in 6 days. The farmer can plow  $6/15$  of the field by himself in 6 days.

$$\begin{aligned}\frac{6}{x} + \frac{6}{15} &= 1 \\ 15x\left(\frac{6}{x} + \frac{6}{15}\right) &= 15x(1) \\ 90 + 6x &= 15x \\ 90 &= 9x \Rightarrow 10 = x\end{aligned}$$

It will take the son 10 days to plow the field by himself.

47. Let  $x$  = the amount of time they worked together to complete the job. Then  $x/9$  = the portion of the job done by a professor and  $x/6$  = the portion of the job done by a student.

$$\begin{aligned}3\left(\frac{1}{9}x\right) + 2\left(\frac{1}{6}x\right) &= 1 \\ \frac{x}{3} + \frac{x}{3} &= 1 \\ 2x &= 3 \\ x &= 1.5\end{aligned}$$

The job will take 1.5 hours.

48. Let  $x$  = the number of quarts of milk to be drained. Then also  $x$  = number of quarts of buttermilk to be added.

Amount (in quarts) of buttermilk in final mixture	=	Amount (in quarts) of buttermilk in original mixture	-	Amount (in quarts) of buttermilk drained from original mixture	+	Amount (in quarts) of buttermilk added.
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(Note that two gallons equals 8 quarts.)

$$\begin{aligned}0.05(8) &= 0.02(8) - 0.02x + x \\ 0.4 &= 0.16 + 0.98x \\ 0.24 &= 0.98x \Rightarrow \frac{0.24}{0.98} = \frac{12}{49} = x\end{aligned}$$

So,  $12/49$  of a quart of buttermilk must be added.

49. Let  $x$  = the number of grams of pure gold to be added. Then  $120 + x$  = the number of grams in the new alloy.

Number of grams of gold in the new alloy	=	Number of grams of gold in the original alloy	+	Number of grams of pure gold added.
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$$0.8(120 + x) = 0.75(120) + x \Rightarrow 96 + 0.8x = 90 + x \Rightarrow 6 + 0.8x = x \Rightarrow 6 = 0.2x \Rightarrow 30 = x$$

So, 30 grams of pure gold must be added.

50. Let  $x$  = the amount (in kg) of coffee with 35% chicory. Then  $500 - x$  = the amount (in kg) of coffee with 15% chicory. We can use the following table to organize the information.

Coffee Type	Amount of coffee	% chicory	Amount of chicory
35% chicory	$x$	0.35	$0.35x$
15% chicory	$500 - x$	0.15	$0.15(500 - x)$
18% chicory blend	500	0.18	$0.18(500)$

The amount of chicory in the final mixture is equal to the sum of the chicory in the two ingredients, so

$$\begin{aligned} 0.35x + 0.15(500 - x) &= 0.18(500) \\ 0.35x + 75 - 0.15x &= 90 \\ 0.20x + 75 &= 90 \\ 0.20x &= 15 \Rightarrow x = 75 \end{aligned}$$

So, there are 75 kg of 35% chicory coffee and  $500 - 75 = 425$  kg of the 15% chicory coffee.

51. Let  $x$  = the amount of 60-40 solder to be added.

Solder	Amount of solder	% tin	Amount of tin
60-40	$x$	0.6	$0.6x$
40-60	$600 - x$	0.4	$0.4(600 - x)$
55-45	600	0.55	$0.55(600)$

$$\begin{aligned} 0.6x + 0.4(600 - x) &= 0.55(600) \\ 0.6x + 240 - 0.4x &= 330 \\ 0.2x + 240 &= 330 \\ 0.2x &= 90 \\ x &= 450 \end{aligned}$$

450 g of 60-40 solder must be added to 150 g of 40-60 solder.

52. Let  $x$  = the amount of 90 proof whiskey.

Whiskey	Amount of whiskey	% alcohol	Amount of alcohol
90 proof	$x$	0.45	$0.45x$
70 proof	$36 - x$	0.35	$0.35(36 - x)$
85 proof	36	0.425	$0.425(36)$

$$\begin{aligned} 0.45x + 0.35(36 - x) &= 0.425(36) \\ 0.45x + 12.6 - 0.35x &= 15.3 \\ 0.1x + 12.6 &= 15.3 \\ 0.1x &= 2.7 \Rightarrow x = 27 \end{aligned}$$

27 gallons of 90 proof whiskey must be mixed with 9 gallons of 70 proof whiskey.

53. Let  $x$  = the amount of 60% boric acid

Solution	Amount of solution	% boric acid	Amount of boric acid
60%	$x$	0.6	$0.6x$
8%	7.5	0.08	$0.08(7.5)$
20%	$x + 7.5$	0.2	$0.2(x + 7.5)$

$$\begin{aligned} 0.6x + 0.08(7.5) &= 0.2(x + 7.5) \\ 0.6x + 0.6 &= 0.2x + 1.5 \\ 0.4x &= 0.9 \Rightarrow x = 2.25 \end{aligned}$$

2.25 liters of 60% boric acid solution are needed.

54. Let  $x$  = the number of pounds of cashews. Then  $3x$  = the number of pounds of almonds, and  $100 - 4x$  = the number of pounds of pecans. We can use the following table to organize the information.

Type of Nut	Number of pounds	Price per pound	Total cost
Cashews	$x$	1.00	$x$
Almonds	$3x$	0.50	$0.50(3x)$
Pecans	$100 - 4x$	0.75	$0.75(100 - 4x)$
Mixture	100	0.70	$0.70(100)$

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The cost of the mixture equals the sum of the total cost of each of the nuts, so

$$\begin{aligned} x + 0.50(3x) + 0.75(100 - 4x) &= 0.70(100) \\ x + 1.5x + 75 - 3x &= 70 \\ -0.5x + 75 &= 70 \\ -0.5x &= -5 \Rightarrow x = 10 \end{aligned}$$

So, there are 10 lbs of cashews, 30 lbs of almonds and 60 lbs of pecans.

55. Let  $x$  = the number of dimes. Then  $3x$  = the number of nickels, and  $4x$  = the number of quarters.

Coin	Number of coins	Value of each coin	Total
Nickels	$3x$	0.05	$0.15x$
Dimes	$x$	0.10	$0.10x$
Quarters	$4x$	0.25	$x$

$$\begin{aligned} 0.15x + 0.10x + x &= 96.25 \\ 1.25x &= 96.25 \Rightarrow x = 77 \end{aligned}$$

So, there are 77 dimes,  $3(77) = 231$  nickels, and  $4(77) = 308$  quarters.

56. Let  $x$  = the total number of gumdrops. Then

$12 + \frac{x}{2}$  = the number of red gumdrops, and

$19 + \frac{1}{2}\left(12 + \frac{x}{2}\right)$  = the number of green gumdrops. So,

$$\begin{aligned} \left(12 + \frac{x}{2}\right) + \left(19 + \frac{1}{2}\left(12 + \frac{x}{2}\right)\right) &= x \\ \left(12 + \frac{x}{2}\right) + \left(19 + 6 + \frac{x}{4}\right) &= x \\ \left(12 + \frac{x}{2}\right) + \left(25 + \frac{x}{4}\right) &= x \\ 37 + \frac{3x}{4} &= x \\ 4\left(37 + \frac{3x}{4}\right) &= 4x \\ 148 + 3x &= 4x \Rightarrow 148 = x \end{aligned}$$

There are 148 gumdrops in total. There are

$$12 + \frac{148}{2} = 86 \text{ red gumdrops and}$$

$$19 + \frac{86}{2} = 62 \text{ green gumdrops.}$$

57. Let  $x$  = Eric's grandfather's age now. Then  $x - 57$  = Eric's age now.  $x + 5$  = Eric's grandfather's age five years from now, and  $(x - 57) + 5 = x - 52$  = Eric's age five years from now. So,

$$\begin{aligned} x + 5 &= 4(x - 52) \\ x + 5 &= 4x - 208 \\ x + 213 &= 4x \Rightarrow 213 = 3x \Rightarrow 71 = x \end{aligned}$$

Eric's grandfather is 71 years old now.

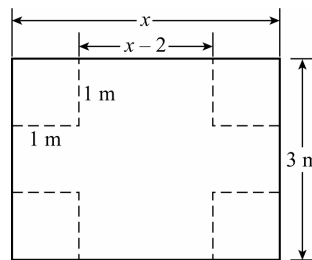
58. Let  $x$  = the total acreage bought. Then,  $7200/x$  = the cost per acre, and  $7200/x + 30$  = the selling price per acre. So,

$$\begin{aligned} \frac{3x}{4}\left(\frac{7200}{x} + 30\right) &= 7200 \\ 5400 + \frac{90x}{4} &= 7200 \\ 4\left(5400 + \frac{90x}{4}\right) &= 4(7200) \\ 21,600 + 90x &= 28,800 \\ 90x &= 7200 \Rightarrow x = 80 \end{aligned}$$

The real estate agent bought 80 acres and sold

$$\frac{3}{4}(80) = 60 \text{ acres.}$$

59. Let  $x$  = the length of the tin rectangle. Then  $x - 2$  = the length of the box.



The formula for volume is  $V = lwh$ , so

$$(x - 2)(1)(1) = 2 \Rightarrow x - 2 = 2 \Rightarrow x = 4$$

The length of the tin rectangle is 4 m.

60. Suppose Mr. Kaplan invests  $P$  dollars at  $x\%$  and  $0.5P$  at  $2x\%$ .

Principal	Rate	Time	Interest
$P$	$\frac{x}{100}$	1	$\frac{Px}{100}$
$0.5P$	$\frac{2x}{100}$	1	$\frac{Px}{100}$
$1.5P$	0.08	1	$0.08(1.5P)$

(continued on next page)

(continued from page 62)

$$\frac{Px}{100} + \frac{Px}{100} = 0.08(1.5P)$$

$$\frac{2Px}{100} = 0.12P$$

$$2Px = 12P \Rightarrow x = 6$$

So, the interest rates are 6% and 12%.

**1.2 C Exercises: Beyond the Basics**

61. Let  $x$  = the average speed for the second half of the trip.

	Rate	Distance	Time
1 <sup>st</sup> half	75	$D$	$\frac{D}{75}$
2 <sup>nd</sup> half	$x$	$D$	$\frac{D}{x}$
Whole trip	60	$2D$	$\frac{2D}{60}$

So,

$$\frac{D}{75} + \frac{D}{x} = \frac{2D}{60}$$

$$300x \left( \frac{D}{75} + \frac{D}{x} \right) = 300x \left( \frac{2D}{60} \right)$$

$$4Dx + 300D = 5x(2D)$$

$$4Dx + 300D = 10Dx$$

$$300D = 6Dx \Rightarrow 50 = x$$

The average speed for the second half of the drive is 50 mph.

62. First we need to compute how much time it will take for Davinder and Mikhael to meet. Let  $t$  = the time it will take for them to meet. So,

	Rate	Time	Distance
Davinder	3.7	$t$	$3.7t$
Mikhail	4.3	$t$	$4.3t$

$$3.7t + 4.3t = 2 \Rightarrow 8t = 2 \Rightarrow t = 0.25$$

They will be walking for 0.25 hour until they meet.

The dog starts with Davinder. Let  $t_{d1}$  = the amount of time it takes for the dog to meet Mikhail. So,

	Rate	Time	Distance
dog	6	$t_{d1}$	$6t_{d1}$
Mikhail	4.3	$t_{d1}$	$4.3t_{d1}$

$$6t_{d1} + 4.3t_{d1} = 2$$

$$10.3t_{d1} = 2$$

$$t = 0.19$$

The dog meets Mikhail for the first time when they have walked for 0.19 hour. The dog will have traveled 1.14 mi.

While the dog has been running towards Mikhail, Davinder has continued to walk. During the 0.19 hour, he walked 0.70 mi, so now he and the dog are  $1.14 - 0.70 = 0.44$  mi apart. Let  $t_{d2}$  = the time it takes the dog to meet Davinder.

	Rate	Time	Distance
dog	6	$t_{d2}$	$6t_{d2}$
Davinder	3.7	$t_{d2}$	$3.7t_{d2}$

$$6t_{d2} + 3.7t_{d2} = 0.44$$

$$9.7t_{d2} = 0.44$$

$$t = 0.05$$

So, the dog meets Davinder when they have walked for another 0.05 hour. The dog will have traveled 0.3 mi.

They have now walked for  $0.19 + 0.05 = 0.24$  hr. Since Davinder and Mikhail don't meet until they have walked for 0.25 hours, the dog must walk for 0.01 hr more. In that time, the dog will travel 0.06 mi. So in total the dog will travel  $1.14 + 0.3 + 0.06 = 1.5$  mi.

63. Let  $x$  = the number of liters of water in the original mixture. Then  $5x$  = the number of liters of alcohol in the original mixture, and  $6x$  = the total number of liters in the original mixture.

$x + 5$  = the number of liters of water in the new mixture. Then  $6x + 5$  = the total number of liters in the new mixture. Since the ratio of alcohol to water in the new mixture is 5:2, then the amount of alcohol in the new mixture is  $\frac{5}{7}$  of the total mixture or  $\frac{5}{7}(6x + 5)$ . There

was no alcohol added, so the amount of alcohol in the original mixture equals the amount of alcohol in the new mixture. This gives

$$\frac{5}{7}(6x + 5) = 5x$$

$$5(6x + 5) = 35x$$

$$30x + 25 = 35x \Rightarrow 25 = 5x \Rightarrow 5 = x$$

So, there were 5 liters of water in the original mixture and 25 liters of alcohol.

64. Let  $x$  = the amount of each alloy. There are 13 parts in the first alloy and 8 parts in the second alloy. We can use the following table to organize the information:

	Total	Zinc	Copper
Alloy 1	$x$	$\frac{5x}{13}$	$\frac{8x}{13}$
Alloy 2	$x$	$\frac{5x}{8}$	$\frac{3x}{8}$
Total	$2x$	$\frac{5x}{13} + \frac{5x}{8}$	$\frac{8x}{13} + \frac{3x}{8}$

The amount of zinc in the new mixture is  $\frac{5x}{13} + \frac{5x}{8} = \frac{105x}{104}$ , and the amount of copper in the new mixture is  $\frac{8x}{13} + \frac{3x}{8} = \frac{103x}{104}$ . So, the ratio of zinc to copper in the new mixture is  $\frac{105x}{104} : \frac{103x}{104}$  or 105:103.

65. Let  $x$  = Democratus' age now. Then  $x/6$  = the number of years as a boy,  $x/8$  = the number of years as a youth, and  $x/2$  = the number of years as a man. He has spent 15 years as a mature adult. So,
- $$\frac{x}{6} + \frac{x}{8} + \frac{x}{2} + 15 = x$$
- $$24\left(\frac{x}{6} + \frac{x}{8} + \frac{x}{2} + 15\right) = 24x$$
- $$4x + 3x + 12x + 360 = 24x$$
- $$19x + 360 = 24x$$
- $$360 = 5x \Rightarrow 72 = x$$
- Democratus is 72 years old.
66. Let  $x$  = the man's age now. When the woman is  $x$  years old, the man will be  $119 - x$  years old. So the difference in their ages is  $(119 - x) - x = 119 - 2x$  years. So the woman's age now is  $x - (119 - 2x) = 3x - 119$ . When the man was  $3x - 119$  years old, she was  $\frac{3x - 119}{2}$  years old. Since the difference in their ages is  $119 - 2x$ , we have

$$(3x - 119) - \frac{3x - 119}{2} = 119 - 2x$$

$$6x - 238 - 3x + 119 = 238 - 4x$$

$$3x - 119 = 238 - 4x$$

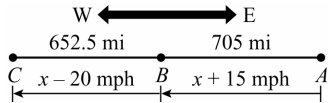
$$7x - 119 = 238$$

$$7x = 357 \Rightarrow x = 51$$

- So the man is now 51 years old. Check by verifying the facts in the problem. When she is 51 years old, he will be  $119 - 51 = 68$  years old. The difference in their ages is  $68 - 51 = 17$  years. So she is  $51 - 17 = 34$  years old now. When he was 34 years old, she was 17 years old, which is  $1/2$  of 34.
67. There are 180 minutes from 3 p.m. to 6 p.m. So, the number of minutes before 6 p.m. plus 50 minutes plus  $4 \times$  the number of minutes before 6 p.m. equals 180 minutes. Let  $x$  = the number of minutes before 6 p.m. So,
- $$x + 50 + 4x = 180 \Rightarrow 5x + 50 = 180 \Rightarrow 5x = 130 \Rightarrow x = 26$$
- So it is 26 minutes before 6 p.m. or 5:34 p.m. Check this by verifying that  $26 + 50 = 76$  minutes before 6 p.m. is the same time as  $4(26) = 104$  minutes after 3 p.m. Seventy-six minutes before 6 p.m. is 4:44 p.m., while 104 minutes after 3 p.m. is also 4:44 p.m.
68. Let  $x$  = the number of minutes pipe B is open. Pipe A is open for 18 minutes, so it fills  $18/24$  or  $3/4$  of the tank. Pipe B fills  $x/32$  of the tank. So,
- $$\frac{3}{4} + \frac{x}{32} = 1 \Rightarrow 32\left(\frac{3}{4} + \frac{x}{32}\right) = 32(1) \Rightarrow 24 + x = 32 \Rightarrow x = 8$$
- Pipe B should be turned off after 8 minutes.
69. a. Because of the head wind, the plane flies at 140 mph from Atlanta to Washington and 160 mph from Washington to Atlanta. Let  $x$  = the distance the plane flew before turning back. So,
- |      | Rate | Distance | Time    |
|------|------|----------|---------|
| to   | 140  | $x$      | $x/140$ |
| from | 160  | $x$      | $x/160$ |
- $$\frac{x}{140} + \frac{x}{160} = 1.5$$
- $$160x + 140x = 1.5(140)(160)$$
- $$300x = 33,600 \Rightarrow x = 112$$
- The plane flew 112 miles before turning back.

- b. The plane traveled 224 miles in 1.5 hours,  
so the average speed is  $\frac{224}{1.5} = 149.33$  mph.

70. Let  $x$  = the airspeed of the plane. Because of the wind, the actual speed of the plane between airports A and B is  $x + 15$ . The actual speed of the plane between airports B and C is  $x - 20$ .



	Rate	Distance	Time
A to B	$x + 15$	705	$\frac{705}{x + 15}$
B to C	$x - 20$	652.5	$\frac{652.5}{x - 20}$

The times are the same, so we have

$$\begin{aligned} \frac{705}{x + 15} &= \frac{652.5}{x - 20} \\ 705(x - 20) &= 652.5(x + 15) \\ 705x - 14,100 &= 652.5x + 9787.5 \\ 52.5x - 14,100 &= 9787.5 \\ 52.5x &= 23887.5 \\ x &= 455 \end{aligned}$$

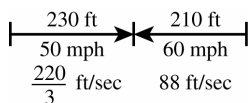
The airspeed of the plane is 455 mph.

71. The ends of the trains are 440 feet apart when the trains first meet. Because the speed is in miles per hour, but the distance is measured in feet, convert the speed to feet per hour. The distance is relatively short, so we need to convert the feet per hour to feet per second.

$$\frac{50 \text{ mi}}{\text{hr}} \cdot \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{220}{3} \text{ feet per second.}$$

$$\frac{60 \text{ mi}}{\text{hr}} \cdot \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 88 \text{ feet per second.}$$

	Rate	Time	Distance
Train A	$\frac{220}{3}$	$t$	$\frac{220t}{3}$
Train B	88	$t$	$88t$



The total distance is 440 feet, so we have

$$\begin{aligned} \frac{220t}{3} + 88t &= 440 \\ 220t + 264t &= 1320 \\ 484t &= 1320 \\ t &= 2.7 \end{aligned}$$

The ends of the trains will pass each other after 2.7 seconds.

72. Call the distance between the two points D. The speed to go from point A to point B is  $x$ . Then the time needed to go from point A to point B is  $D/x$ . Similarly, we have the time needed to go from point B to point A is  $D/y$ . The total distance traveled is  $2D$  and the total time is  $\frac{D}{x} + \frac{D}{y}$ . So the average speed is

$$\begin{aligned} \frac{2D}{\frac{D}{x} + \frac{D}{y}} &= \frac{2D}{\frac{Dy + Dx}{xy}} = \frac{2Dxy}{D(x + y)} \\ &= \frac{2xy}{x + y} = \frac{2}{\frac{1}{x} + \frac{1}{y}} \end{aligned}$$

To extend this, we know that each distance is the same, D. Using the same reasoning as before, we have that the times are  $D/x$ ,  $D/y$ , and  $D/z$ , respectively. The total distance traveled is  $3D$ . So the total time traveled is

$$\begin{aligned} \frac{D}{x} + \frac{D}{y} + \frac{D}{z} \text{ and the average speed is} \\ \frac{3D}{\frac{D}{x} + \frac{D}{y} + \frac{D}{z}} = \frac{3}{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}} = \frac{3xyz}{x + y + z} \end{aligned}$$

### 1.2 Critical Thinking

73. In one day, 1.5 men can do 1 job. Let  $x$  = the amount of time it takes one man to do the job.

$$\text{Then } \frac{1}{x} + \frac{5}{x} = 1 \Rightarrow 1.5 = x$$

It takes one man 1.5 days to do the job.

74. Let  $x$  = the amount of time Chris takes to do the job alone. Then

$$\begin{aligned} \frac{3}{6} + \frac{3}{8} + \frac{3}{x} &= 1 \Rightarrow 12x + 9x + 72 = 24x \Rightarrow \\ 21x + 72 &= 24x \Rightarrow 72 = 3x \Rightarrow 24 = x \end{aligned}$$

It would take Chris 24 hours to do the job alone. Therefore, in three hours, he did  $1/8$  of the job and earned \$100.

### 1.3 Complex Numbers

#### 1.3 Practice Problems

1. a.  $-1 + 2i$   
real part:  $-1$ , imaginary part:  $2$
  - b.  $-\frac{1}{3} - 6i$   
real part:  $-\frac{1}{3}$ , imaginary part:  $-6$
  - c.  $8 = 8 + 0i$   
real part:  $8$ , imaginary part:  $0$
2. Let  $z = (1 - 2a) + 3i$  and let  $w = 5 - (2b - 5)i$ .  
Then  

$$\begin{aligned} \operatorname{Re}(z) &= \operatorname{Re}(w) & \operatorname{Im}(z) &= \operatorname{Im}(w) \\ 1 - 2a &= 5 & 3 &= -(2b - 5) \\ -2a &= 4 & 3 &= -2b + 5 \\ a &= -2 & -2 &= -2b \\ & & 1 &= b \end{aligned}$$
  3. a.  $(1 - 4i) + (3 + 2i) = 4 - 2i$
  - b.  $(4 + 3i) - (5 - i) = -1 + 4i$
  - c.  $(3 - \sqrt{-9}) - (5 - \sqrt{-64}) = (3 - 3i) - (5 - 8i)$   
 $= -2 + 5i$
  4. a.  $(2 - 6i)(1 + 4i) = 2 + 8i - 6i - 24i^2$   
 $= 2 + 2i + 24 = 26 + 2i$
  - b.  $-3i(7 - 5i) = -21i + 15i^2 = -15 - 21i$
  5. a.  $(-3 + \sqrt{-4})^2 = (-3 + 2i)^2$   
 $= (-3)^2 + 2(-3)(2i) + (2i)^2$   
 $= 9 - 12i + 4i^2 = 9 - 12i - 4$   
 $= 5 - 12i$
  - b.  $(5 + \sqrt{-2})(4 + \sqrt{-8}) = (5 + i\sqrt{2})(4 + 2i\sqrt{2})$   
 $= 20 + 10i\sqrt{2} + 4i\sqrt{2} + 4i^2$   
 $= 20 + 14i\sqrt{2} - 4$   
 $= 16 + 14i\sqrt{2}$
  6. a.  $z = 1 + 6i \Rightarrow \bar{z} = 1 - 6i$   
 $z\bar{z} = (1 + 6i)(1 - 6i) = 1 - 36i^2 = 1 + 36 = 37$
  - b.  $z = -2i \Rightarrow \bar{z} = 2i$   
 $z\bar{z} = (-2i)(2i) = -4i^2 = 4$

$$7. \text{ a. } \frac{2}{1-i} = \frac{2}{1-i} \cdot \frac{1+i}{1+i} = \frac{2+2i}{1-i^2} = \frac{2+2i}{1+1} = \frac{2+2i}{2} = 1+i$$

$$\text{ b. } \frac{-3i}{4+\sqrt{-25}} = \frac{-3i}{4+5i} = \frac{-3i}{4+5i} \cdot \frac{4-5i}{4-5i} = \frac{-12i+15i^2}{16-25i^2} = \frac{-15-12i}{16+25} = \frac{-15-12i}{41} = -\frac{15}{41} - \frac{12i}{41}$$

$$8. \ Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(1+2i)(2-3i)}{(1+2i)+(2-3i)} = \frac{2-3i+4i-6i^2}{3-i} = \frac{2+i+6}{3-i} = \frac{8+i}{3-i} = \frac{8+i}{3-i} \cdot \frac{3+i}{3+i} = \frac{24+8i+3i+i^2}{9-i^2} = \frac{24+11i-1}{9+1} = \frac{23+11i}{10} = \frac{23}{10} + \frac{11}{10}i$$

#### 1.3 A Exercises: Basic Skills and Concepts

1. We define  $i = \sqrt{-1}$ , so that  $i^2 = -1$ .
2. A complex number in the form  $a + bi$  is said to be in standard form.
3. For  $b > 0$ ,  $\sqrt{-b} = i\sqrt{b}$ .
4. The conjugate of  $a + bi$  is  $a - bi$ , and the conjugate of  $a - bi$  is  $a + bi$ .
5. True
6. True

In exercises 7–10, to find the real numbers  $x$  and  $y$  that make the equation true, set the real parts of the equation equal to each other and then set the imaginary parts of the equation equal to each other.

7.  $2 + xi = y + 3i$ , so  $x = 3$  and  $y = 2$ .
8.  $x - 2i = 7 + yi$ , so  $x = 7$  and  $y = -2$ .
9.  $x - \sqrt{-16} = 2 + yi$ .  $\sqrt{-16} = 4i$ , so the equation becomes  $x - 4i = 2 + yi$ .  
 $x = 2$  and  $y = -4$ .
10.  $3 + yi = x - \sqrt{-25}$ .  $\sqrt{-25} = 5i$ , so the equation becomes  $3 + yi = x - 5i$ .  
 $x = 3$  and  $y = -5$ .
11.  $(5 + 2i) + (3 + i) = (5 + 3) + (2 + 1)i = 8 + 3i$

12.  $(6 + i) + (1 + 2i) = (6 + 1) + (1 + 2)i = 7 + 3i$
13.  $(4 - 3i) - (5 + 3i) = (4 - 5) + (-3 - 3)i$   
 $= -1 - 6i$
14.  $(3 - 5i) - (3 + 2i) = (3 - 3) + (-5 - 2)i = -7i$
15.  $(-2 - 3i) + (-3 - 2i) = [-2 + (-3)] + (-3 - 2)i$   
 $= -5i - 5i$
16.  $(-5 - 3i) + (2 - i) = (-5 + 2) + (-3 - 1)i$   
 $= -3 - 4i$
17.  $3(5 + 2i) = 3(5) + 3(2i) = 15 + 6i$
18.  $4(3 + 5i) = 4(3) + 4(5i) = 12 + 20i$
19.  $-4(2 - 3i) = -4(2) - 4(-3i) = -8 + 12i$
20.  $-7(3 - 4i) = -7(3) - 7(-4i) = -21 + 28i$
21.  $3i(5 + i) = 3i(5) + 3i(i) = 15i + 3i^2$ . Because  
 $i^2 = -1$ ,  $3i^2 = -3$ .  
 So,  $15i + 3i^2 = 15i - 3 = -3 + 15i$ .
22.  $2i(4 + 3i) = 2i(4) + 2i(3i) = 8i + 6i^2$ . Because  
 $i^2 = -1$ ,  $6i^2 = -6$ . So  $8i + 6i^2 = -6 + 8i$ .
23.  $4i(2 - 5i) = 4i(2) + 4i(-5i) = 8i - 20i^2$ .  
 Because  $i^2 = -1$ ,  $-20i^2 = (-20)(-1) = 20$ .  
 So,  $8i - 20i^2 = 8i + 20 = 20 + 8i$ .
24.  $-3i(5 - 2i) = -3i(5) - 3i(-2i) = -15i + 6i^2$ .  
 Because  $i^2 = -1$ ,  $6i^2 = -6$ .  
 So,  $-15i + 6i^2 = -15i - 6 = -6 - 15i$ .
25.  $(3 + i)(2 + 3i) = 3 \cdot 2 + 3 \cdot 3i + i \cdot 2 + i \cdot 3i$   
 $= 6 + 9i + 2i + 3i^2$   
 $= 6 + 11i + 3(-1)$   
 $= 3 + 11i$
26.  $(4 + 3i)(2 + 5i) = 4 \cdot 2 + 4 \cdot 5i + 3i \cdot 2 + 3i \cdot 5i$   
 $= 8 + 20i + 6i + 15i^2$   
 $= 8 + 26i + 15(-1) = -7 + 26i$
27.  $(2 - 3i)(2 + 3i)$   
 $= 2 \cdot 2 + 2 \cdot 3i + (-3i) \cdot 2 + (-3i) \cdot 3i$   
 $= 4 + 6i - 6i - 9i^2$   
 $= 4 - 9(-1) = 4 + 9 = 13$
28.  $(4 - 3i)(4 + 3i)$   
 $= 4 \cdot 4 + 4 \cdot 3i + (-3i) \cdot 4 + (-3i) \cdot 3i$   
 $= 16 + 12i - 12i - 9i^2$   
 $= 16 - 9(-1) = 16 + 9 = 25$
29.  $(3 + 4i)(4 - 3i)$   
 $= 3 \cdot 4 + 3 \cdot (-3i) + 4i \cdot 4 + (4i) \cdot (-3i)$   
 $= 12 - 9i + 16i - 12i^2$   
 $= 12 + 7i - 12(-1)$   
 $= 12 + 7i + 12 = 24 + 7i$
30.  $(-2 + 3i)(-3 + 10i)$   
 $= (-2) \cdot (-3) + (-2) \cdot (10i)$   
 $+ 3i \cdot (-3) + (3i) \cdot (10i)$   
 $= 6 - 20i - 9i + 30i^2$   
 $= 6 - 29i + 30(-1)$   
 $= 6 - 29i - 30 = -24 - 29i$
31.  $(\sqrt{3} - 12i)^2$   
 $= (\sqrt{3} - 12i)(\sqrt{3} - 12i)$   
 $= \sqrt{3} \cdot \sqrt{3} + \sqrt{3} \cdot (-12i)$   
 $- 12i \cdot \sqrt{3} + (-12i) \cdot (-12i)$   
 $= 3 - 12\sqrt{3}i - 12\sqrt{3}i + 144i^2$   
 $= 3 - 24\sqrt{3}i + 144(-1)$   
 $= 3 - 24\sqrt{3}i - 144$   
 $= -141 - 24\sqrt{3}i$
32.  $(-\sqrt{5} - 13i)^2$   
 $= (-\sqrt{5} - 13i)(-\sqrt{5} - 13i)$   
 $= (-\sqrt{5}) \cdot (-\sqrt{5}) + (-\sqrt{5}) \cdot (-13i)$   
 $- 13i \cdot (-\sqrt{5}) + (-13i) \cdot (-13i)$   
 $= 5 + 13\sqrt{5}i + 13\sqrt{5}i + 169i^2$   
 $= 5 + 26\sqrt{5}i + 169(-1)$   
 $= 5 + 26\sqrt{5}i - 169$   
 $= -164 + 26\sqrt{5}i$
33.  $(2 - \sqrt{-16})(3 + 5i)$   
 $= (2 - 4i)(3 + 5i)$   
 $= 2 \cdot 3 + 2 \cdot 5i - 4i \cdot 3 - 4i \cdot 5i$   
 $= 6 + 10i - 12i - 20i^2$   
 $= 6 - 2i - 20(-1)$   
 $= 6 - 2i + 20$   
 $= 26 - 2i$

34.  $(5-2i)(3+\sqrt{-25})$   
 $= (5-2i)(3+5i)$   
 $= 5 \cdot 3 + 5 \cdot 5i - 2i \cdot 3 - 2i \cdot 5i$   
 $= 15 + 25i - 6i - 10i^2$   
 $= 15 + 19i - 10(-1)$   
 $= 15 + 19i + 10$   
 $= 25 + 19i$
35. If  $z = 2 - 3i$  then  $\bar{z} = 2 + 3i$ , and  
 $z\bar{z} = (2-3i)(2+3i) = 4 - 9i^2 = 4 + 9 = 13$ .
36. If  $z = 4 + 5i$  then  $\bar{z} = 4 - 5i$ , and  
 $z\bar{z} = (4+5i)(4-5i) = 16 - 25i^2 = 16 + 25 = 41$ .
37. If  $z = \frac{1}{2} - 2i$  then  $\bar{z} = \frac{1}{2} + 2i$ , and  
 $z\bar{z} = \left(\frac{1}{2} - 2i\right)\left(\frac{1}{2} + 2i\right) = \frac{1}{4} - 4i^2 = \frac{1}{4} + 4 = \frac{17}{4}$
38. If  $z = \frac{2}{3} + \frac{1}{2}i$  then  $\bar{z} = \frac{2}{3} - \frac{1}{2}i$ , and  
 $z\bar{z} = \left(\frac{2}{3} + \frac{1}{2}i\right)\left(\frac{2}{3} - \frac{1}{2}i\right) = \frac{4}{9} - \frac{1}{4}i^2 = \frac{4}{9} + \frac{1}{4} = \frac{25}{36}$
39. If  $z = \sqrt{2} - 3i$  then  $\bar{z} = \sqrt{2} + 3i$ , and  
 $z\bar{z} = (\sqrt{2} - 3i)(\sqrt{2} + 3i) = 2 - 9i^2 = 2 + 9 = 11$ .
40. If  $z = \sqrt{5} + \sqrt{3}i$  then  $\bar{z} = \sqrt{5} - \sqrt{3}i$ , and  
 $z\bar{z} = (\sqrt{5} + \sqrt{3}i)(\sqrt{5} - \sqrt{3}i) = 5 - 3i^2$   
 $= 5 + 3 = 8$
41. The denominator is  $-i$ , so its conjugate is  $i$ .  
 Multiply the numerator and denominator by  $i$ .  
 $\frac{5}{-i} = \frac{5i}{-i \cdot i} = \frac{5i}{1} = 5i$
42. The denominator is  $-3i$ , so its conjugate is  $3i$ .  
 Multiply the numerator and denominator by  $3i$ .  
 $\frac{2}{-3i} = \frac{2(3i)}{-3i \cdot 3i} = \frac{6i}{9} = \frac{2}{3}i$
43. The denominator is  $1+i$ , so its conjugate is  $1-i$ . Multiply the numerator and denominator by  $1-i$ .  
 $\frac{-1}{1+i} = \frac{-1(1-i)}{(1+i)(1-i)} = \frac{-1+i}{1+1}$   
 $= \frac{-1+i}{2} = -\frac{1}{2} + \frac{1}{2}i$
44. The denominator is  $2-i$ , so its conjugate is  $2+i$ . Multiply the numerator and denominator by  $2+i$ .  
 $\frac{1}{2-i} = \frac{1(2+i)}{(2-i)(2+i)} = \frac{2+i}{4+1} = \frac{2+i}{5}$   
 $= \frac{2}{5} + \frac{1}{5}i$
45. The denominator is  $2+i$ , so its conjugate is  $2-i$ . Multiply the numerator and denominator by  $2-i$ .  
 $\frac{5i}{2+i} = \frac{5i(2-i)}{(2+i)(2-i)} = \frac{10i - 5i^2}{4+1}$   
 $= \frac{10i+5}{5} = 1+2i$
46. The denominator is  $2-i$ , so its conjugate is  $2+i$ . Multiply the numerator and denominator by  $2+i$ .  
 $\frac{3i}{2-i} = \frac{3i(2+i)}{(2-i)(2+i)} = \frac{6i+3i^2}{4+1}$   
 $= \frac{-3+6i}{5} = -\frac{3}{5} + \frac{6}{5}i$
47. The denominator is  $1+i$ , so its conjugate is  $1-i$ . Multiply the numerator and denominator by  $1-i$ .  
 $\frac{2+3i}{1+i} = \frac{(2+3i)(1-i)}{(1+i)(1-i)} = \frac{2-2i+3i-3i^2}{1+1}$   
 $= \frac{2+i+3}{2} = \frac{5+i}{2} = \frac{5}{2} + \frac{1}{2}i$
48. The denominator is  $4+i$ , so its conjugate is  $4-i$ . Multiply the numerator and denominator by  $4-i$ .  
 $\frac{3+5i}{4+i} = \frac{(3+5i)(4-i)}{(4+i)(4-i)} = \frac{12-3i+20i-5i^2}{16+1}$   
 $= \frac{12+17i+5}{17} = \frac{17+17i}{17} = 1+i$
49. The denominator is  $4-7i$ , so its conjugate is  $4+7i$ . Multiply the numerator and denominator by  $4+7i$ .  
 $\frac{2-5i}{4-7i} = \frac{(2-5i)(4+7i)}{(4-7i)(4+7i)} = \frac{8+14i-20i-35i^2}{16+49}$   
 $= \frac{8-6i+35}{65} = \frac{43-6i}{65} = \frac{43}{65} - \frac{6}{65}i$

50. The denominator is  $1 - 3i$ , so its conjugate is  $1 + 3i$ . Multiply the numerator and denominator by  $1 + 3i$ .

$$\begin{aligned}\frac{3+5i}{1-3i} &= \frac{(3+5i)(1+3i)}{(1-3i)(1+3i)} = \frac{3+9i+5i+15i^2}{1+9} \\ &= \frac{3+14i-15}{10} = \frac{-12+14i}{10} = -\frac{6}{5} + \frac{7}{5}i\end{aligned}$$

51. The denominator is  $1 + i$ , so its conjugate is  $1 - i$ . Multiply the numerator and denominator by  $1 - i$ .

$$\begin{aligned}\frac{2+\sqrt{-4}}{1+i} &= \frac{(2+2i)}{(1+i)} = \frac{(2+2i)(1-i)}{(1+i)(1-i)} \\ &= \frac{2-2i+2i-2i^2}{1+1} = \frac{2-2i^2}{2} = \frac{2+2}{2} = 2\end{aligned}$$

52. The denominator is  $3 + 2i$ , so its conjugate is  $3 - 2i$ . Multiply the numerator and denominator by  $3 - 2i$ .

$$\begin{aligned}\frac{5-\sqrt{-9}}{3+2i} &= \frac{5-3i}{3+2i} = \frac{(5-3i)(3-2i)}{(3+2i)(3-2i)} \\ &= \frac{15-10i-9i+6i^2}{9+4} = \frac{15-19i+6i^2}{13} \\ &= \frac{15-19i-6}{13} = \frac{9-19i}{13} = \frac{9}{13} - \frac{19}{13}i\end{aligned}$$

53. The denominator is  $2 - 3i$ , so its conjugate is  $2 + 3i$ . Multiply the numerator and denominator by  $2 + 3i$ .

$$\begin{aligned}\frac{-2+\sqrt{-25}}{2-3i} &= \frac{-2+5i}{2-3i} = \frac{(-2+5i)(2+3i)}{(2-3i)(2+3i)} \\ &= \frac{-4-6i+10i+15i^2}{4+9} \\ &= \frac{-4+4i-15}{13} = \frac{-19+4i}{13} \\ &= -\frac{19}{13} + \frac{4}{13}i\end{aligned}$$

54. The denominator simplifies to  $5 - 3i$ , so its conjugate is  $5 + 3i$ . Multiply the numerator and denominator by  $5 + 3i$ .

$$\begin{aligned}\frac{-5-\sqrt{-4}}{5-\sqrt{-9}} &= \frac{-5-2i}{5-3i} = \frac{(-5-2i)(5+3i)}{(5-3i)(5+3i)} \\ &= \frac{-25-15i-10i-6i^2}{25+9} \\ &= \frac{-25-25i+6}{34} = \frac{-19-25i}{34} \\ &= -\frac{19}{34} - \frac{25}{34}i\end{aligned}$$

### 1.3 B Exercises: Applying the Concepts

55.  $Z_1 = 4 + 3i$  and  $Z_2 = 5 - 2i$ .

So,  $Z_1 + Z_2 = (4 + 3i) + (5 - 2i) = 9 + i$ .

56.  $I = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(4+3i)(5-2i)}{(4+3i)+(5-2i)}$   
 $= \frac{20-8i+15i-6i^2}{9+i} = \frac{20+7i+6}{9+i} = \frac{26+7i}{9+i}$

Now simplify the fraction by multiplying the numerator and denominator by  $9 - i$ .

$$\begin{aligned}\frac{26+7i}{9+i} &= \frac{(26+7i)(9-i)}{(9+i)(9-i)} \\ &= \frac{234-26i+63i-7i^2}{81+1} \\ &= \frac{234+37i+7}{82} = \frac{241+37i}{82} \\ &= \frac{241}{82} + \frac{37}{82}i\end{aligned}$$

57.  $Z = \frac{V}{I}$ ,  $I = 7 + 5i$ ,  $V = 35 + 70i$ . Then,

$Z = \frac{35+70i}{7+5i}$ . Simplify the fraction by multiplying the numerator and denominator by  $7 - 5i$ .

$$\begin{aligned}\frac{35+70i}{7+5i} &= \frac{(35+70i)(7-5i)}{(7+5i)(7-5i)} \\ &= \frac{245-175i+490i-350i^2}{49+25} \\ &= \frac{245+315i+350}{74} = \frac{595+315i}{74} \\ &= \frac{595}{74} + \frac{315}{74}i\end{aligned}$$

58.  $Z = \frac{V}{I}$ ,  $I = 7 + 4i$ ,  $V = 45 + 88i$ . Then,

$Z = \frac{45+88i}{7+4i}$ . Simplify the fraction by multiplying the numerator and denominator by  $7 - 4i$ .

$$\begin{aligned}\frac{45+88i}{7+4i} &= \frac{(45+88i)(7-4i)}{(7+4i)(7-4i)} \\ &= \frac{315-180i+616i-352i^2}{49+16} \\ &= \frac{315+436i+352}{65} = \frac{667+436i}{65} \\ &= \frac{667}{65} + \frac{436}{65}i\end{aligned}$$



59.  $Z = \frac{V}{I}$ ,  $Z = 5 - 7i$ ,  $I = 2 + 5i$ . Then,

$$\begin{aligned} V &= ZI = (5 - 7i)(2 + 5i) \\ &= 10 + 25i - 14i - 35i^2 \\ &= 10 + 11i + 35 = 45 + 11i \end{aligned}$$

60.  $Z = \frac{V}{I}$ ,  $Z = 7 - 8i$ ,  $I = \frac{1}{3} + \frac{1}{6}i$ . Then,

$$\begin{aligned} V &= ZI = (7 - 8i)\left(\frac{1}{3} + \frac{1}{6}i\right) \\ &= \frac{7}{3} + \frac{7}{6}i - \frac{8}{3}i - \frac{8}{6}i^2 \\ &= \frac{7}{3} + \frac{7}{6}i - \frac{16}{6}i + \frac{4}{3} \\ &= \frac{11}{3} - \frac{9}{6}i = \frac{11}{3} - \frac{3}{2}i \end{aligned}$$

61.  $Z = \frac{V}{I}$ ,  $V = 12 + 10i$ ,  $Z = 12 + 6i$ . Then,

$I = \frac{V}{Z} = \frac{12 + 10i}{12 + 6i}$ . Simplify the fraction by multiplying the numerator and denominator by  $12 - 6i$ .

$$\begin{aligned} \frac{12 + 10i}{12 + 6i} &= \frac{(12 + 10i)(12 - 6i)}{(12 + 6i)(12 - 6i)} \\ &= \frac{144 - 72i + 120i - 60i^2}{144 + 36} \\ &= \frac{144 + 48i + 60}{180} \\ &= \frac{204 + 48i}{180} = \frac{17}{15} + \frac{4}{15}i \end{aligned}$$

62.  $Z = \frac{V}{I}$ ,  $V = 29 + 18i$ ,  $Z = 25 + 6i$ . Then,

$I = \frac{V}{Z} = \frac{29 + 18i}{25 + 6i}$ . Simplify the fraction by multiplying the numerator and denominator by  $25 - 6i$ .

$$\begin{aligned} \frac{29 + 18i}{25 + 6i} &= \frac{(29 + 18i)(25 - 6i)}{(25 + 6i)(25 - 6i)} \\ &= \frac{725 - 174i + 450i - 108i^2}{625 + 36} \\ &= \frac{725 + 276i + 108}{661} \\ &= \frac{833 + 276i}{661} = \frac{833}{661} + \frac{276}{661}i \end{aligned}$$

### 1.3 C Exercises: Beyond the Basics

63. To find  $i^{17}$ , first divide 17 by 4. The remainder is 1, so  $i^{17} = i^1 = i$ .

64. To find  $i^{125}$ , first divide 125 by 4. The remainder is 1, so  $i^{125} = i^1 = i$ .

65. To find  $i^{-7}$ , first rewrite it as  $\frac{1}{i^7}$ . Then divide 7 by 4. The remainder is 3, so  $\frac{1}{i^7} = \frac{1}{i^3}$ . Simplify the fraction by multiplying the numerator and denominator by  $i$ .

$$\frac{1}{i^3} \cdot \frac{i}{i} = \frac{i}{i^4} = i.$$

66. To find  $i^{-24}$ , first rewrite it as  $\frac{1}{i^{24}}$ . Then divide 24 by 4. The remainder is 0, so  $\frac{1}{i^{24}} = \frac{1}{i^0} = 1$ .

67. To find  $i^{10}$ , first divide 10 by 4. The remainder is 2, so  $i^{10} = i^2 = -1$ . So  $i^{10} + 7 = -1 + 7 = 6$ .

68.  $i^3 = -i$ , so  $9 + i^3 = 9 - i$ .

69. To find  $i^5$ , first divide 5 by 4. The remainder is 1, so  $i^5 = i$ . So  $3i^5 = 3i$ .  $i^3 = -i$ , so  $-2i^3 = 2i$ . Then,  $3i^5 - 2i^3 = 3i + 2i = 5i$ .

70. To find  $i^6$ , first divide 6 by 4. The remainder is 2, so  $i^6 = i^2 = -1$ . So  $5i^6 = -5$ .  $i^4 = 1$ , so  $-3i^4 = -3$ . Then,  $5i^6 - 3i^4 = -5 - 3 = -8$ .

71.  $i^3 = -i$ , so  $2i^3 = -2i$ .  $i^4 = 1$ , so  $1 + i^4 = 1 + 1 = 2$ . Then  $2i^3(1 + i^4) = -2i(2) = -4i$ .

72. To find  $i^5$ , first divide 5 by 4. The remainder is 1, so  $i^5 = i$ . So  $5i^5 = 5i$ .  $i^3 = -i$ , so  $i^3 - i = -i - i = -2i$ . Then,  $5i^5(i^3 - i) = 5i(-2i) = -10i^2 = 10$ .

$$73. \frac{1}{a+bi} = \frac{1(a-bi)}{(a+bi)(a-bi)} = \frac{a-bi}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i$$

$$74. z = a + bi, \text{ so } \operatorname{Re}(z) = a. \frac{z + \bar{z}}{2} = \frac{(a+bi) + (a-bi)}{2} = \frac{2a}{2} = a = \operatorname{Re}(z).$$

$$75. z = a + bi, \text{ so } \operatorname{Im}(z) = b. \frac{z - \bar{z}}{2i} = \frac{(a+bi) - (a-bi)}{2i} = \frac{2bi}{2i} = b = \operatorname{Im}(z).$$

$$76. \operatorname{Re}\left(\frac{z}{z+w}\right) = \operatorname{Re}\left(\frac{a+bi}{(a+bi)+(c+di)}\right) = \operatorname{Re}\left(\frac{a+bi}{(a+c)+(b+d)i}\right) = \operatorname{Re}\left(\frac{(a+bi)[(a+c)-(b+d)i]}{[(a+c)+(b+d)i][(a+c)-(b+d)i]}\right)$$

$$= \frac{a(a+c) - bi(b+d)i}{(a+c)^2 + (b+d)^2} = \frac{a(a+c) + b(b+d)}{(a+c)^2 + (b+d)^2}$$

Note that when we simplify the fraction, in the numerator we multiply only the first and last terms because we need only the real terms. Multiplying the inside and outside terms give the imaginary terms

$$\operatorname{Re}\left(\frac{w}{z+w}\right) = \operatorname{Re}\left(\frac{c+di}{(a+bi)+(c+di)}\right) = \operatorname{Re}\left(\frac{c+di}{(a+c)+(b+d)i}\right) = \operatorname{Re}\left(\frac{(c+di)[(a+c)-(b+d)i]}{[(a+c)+(b+d)i][(a+c)-(b+d)i]}\right)$$

$$= \frac{c(a+c) - di(b+d)i}{(a+c)^2 + (b+d)^2} = \frac{c(a+c) + d(b+d)}{(a+c)^2 + (b+d)^2}$$

$$\text{So, } \operatorname{Re}\left(\frac{z}{z+w}\right) + \operatorname{Re}\left(\frac{w}{z+w}\right) = \frac{a(a+c) + b(b+d)}{(a+c)^2 + (b+d)^2} + \frac{c(a+c) + d(b+d)}{(a+c)^2 + (b+d)^2} = \frac{(a+c)^2 + (b+d)^2}{(a+c)^2 + (b+d)^2} = 1.$$

$$77. z\bar{z} = (a+bi)(a-bi) = a^2 + b^2$$

$$a^2 + b^2 = 0 \text{ if and only if } a = 0 \text{ and } b = 0.$$

$$\text{So, } z = 0 + 0i = 0.$$

$$\frac{2-3i}{9-7i} = \frac{2-3i}{9-7i} \cdot \frac{9+7i}{9+7i} = \frac{18+14i-27i-21i^2}{81-49i^2}$$

$$= \frac{18-13i+21}{81+49} = \frac{39-13i}{130} = \frac{3}{10} - \frac{1}{10}i$$

$$78. \frac{x}{i} + y = 3 + i \Rightarrow \frac{x}{i} = i \Rightarrow x = i^2 \Rightarrow x = -1 \text{ and } y = 3.$$

$$\frac{1-2i}{5-5i} = \frac{3}{10} - \frac{1}{10}i = \frac{2-3i}{9-7i} \Rightarrow \frac{1-2i}{5-5i} = \frac{2-3i}{9-7i}$$

$$79. x - \frac{y}{i} = 4i + 1 \Rightarrow x = 1$$

$$-\frac{y}{i} = 4i \Rightarrow -y = 4i^2 \Rightarrow -y = -4 \Rightarrow y = 4$$

$$83. \frac{1+i}{1-i} \div \frac{2+i}{1+2i} = \frac{1+i}{1-i} \cdot \frac{1+2i}{2+i} = \frac{1+2i+i+2i^2}{2+i-2i-i^2}$$

$$= \frac{1+3i-2}{2-i+1} = \frac{-1+3i}{3-i}$$

$$= \frac{-1+3i}{3-i} \cdot \frac{3+i}{3+i} = \frac{-3-i+9i+3i^2}{9-i^2}$$

$$= \frac{-3+8i-3}{9+1} = \frac{-6+8i}{10} = -\frac{3}{5} + \frac{4}{5}i$$

$$80. \frac{x+yi}{i} = 5-7i \Rightarrow x+yi = i(5-7i)$$

$$\Rightarrow x+yi = 7+5i \Rightarrow x=7, y=5.$$

$$81. \frac{5x+yi}{2-i} = 2+i \Rightarrow 5x+yi = (2+i)(2-i)$$

$$\Rightarrow 5x+yi = 5 \Rightarrow x=1, y=0.$$

$$82. \frac{1-2i}{5-5i} = \frac{1-2i}{5-5i} \cdot \frac{5+5i}{5+5i} = \frac{5+5i-10i-10i^2}{25-25i^2}$$

$$= \frac{5-5i+10}{25+25} = \frac{15-5i}{50} = \frac{3}{10} - \frac{1}{10}i$$

$$84. (1+3i)z + (2+4i) = 7-3i$$

$$(1+3i)z = 7-3i - (2+4i) = 5-7i$$

$$z = \frac{5-7i}{1+3i} = \frac{5-7i}{1+3i} \cdot \frac{1-3i}{1-3i}$$

$$= \frac{5-15i-7i+21i^2}{1-9i^2}$$

$$= \frac{5-22i-21}{1+9} = \frac{-16-22i}{10}$$

$$= -\frac{8}{5} - \frac{11}{5}i$$

85.  $z = 2 - 3i$ ,  $w = 1 + 2i$

$$\text{a. } \overline{(zw)} = \overline{((2-3i)(1+2i))} = \overline{(2+4i-3i-6i^2)}$$

$$= \overline{(8+i)} = 8-i$$

$$(\overline{z})(\overline{w}) = (2+3i)(1-2i) = 2-4i+3i-6i^2$$

$$= 8-i$$

$$\text{b. } \frac{z}{w} = \frac{2-3i}{1+2i} = \frac{2-3i}{1+2i} \cdot \frac{1-2i}{1-2i}$$

$$= \frac{2-4i-3i+6i^2}{1-4i^2}$$

$$= \frac{-4-7i}{5} = -\frac{4}{5} - \frac{7}{5}i$$

$$\overline{\left(\frac{z}{w}\right)} = -\frac{4}{5} + \frac{7}{5}i$$

$$\frac{\overline{z}}{\overline{w}} = \frac{2+3i}{1-2i} = \frac{2+3i}{1-2i} \cdot \frac{1+2i}{1+2i}$$

$$= \frac{2+4i+3i+6i^2}{1-4i^2}$$

$$= \frac{-4+7i}{5} = -\frac{4}{5} + \frac{7}{5}i$$

### 1.3 Critical Thinking

86. a. True. Every real number  $a$  can be written as a complex number  $a + 0i$ .
- b. False.
- c. False. A complex number with the form  $a + 0i$  does not have an imaginary component.
- d. True
- e. True.  $(a+bi)(a-bi) = a^2 + b^2$ . There is no imaginary component.
- f. True

### 1.3 Group Project

If  $i = 0$ , then  $i^2 = 0^2 \Rightarrow -1 = 0$ , which is a contradiction. If  $i < 0$ , then  $i \cdot i > 0 \cdot i$  (since  $i$  is negative)  $\Rightarrow i^2 > 0 \Rightarrow -1 > 0$ , a contradiction.

If  $i > 0$ , then  $i \cdot i > 0 \cdot i \Rightarrow i^2 > 0 \Rightarrow -1 > 0$ , a contradiction. Thus, the set of complex numbers does not have the ordering properties of the set of real numbers.

## 1.4 Quadratic Equations

### 1.4 Practice Problems

$$\text{1. } x^2 + 25x = -84$$

$$x^2 + 25x + 84 = 0$$

$$(x+4)(x+21) = 0$$

$$x+4=0 \quad | \quad x+21=0$$

$$x=-4 \quad | \quad x=-21$$

Solution set:  $\{-21, -4\}$ 

$$\text{2. } 2m^2 = 5m$$

$$2m^2 - 5m = 0$$

$$m(2m-5) = 0$$

$$m=0 \quad | \quad 2m-5=0$$

$$\quad \quad \quad 2m=5$$

$$\quad \quad \quad m=\frac{5}{2}$$

Solution set:  $\left\{0, \frac{5}{2}\right\}$ 

$$\text{3. } x^2 - 6x = -9$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x-3=0$$

$$x=3$$

Solution set:  $\{3\}$ 

$$\text{4. } (x+2)^2 = 5$$

$$x+2 = \pm\sqrt{5}$$

$$x = -2 \pm \sqrt{5}$$

Solution set:  $\{-2-\sqrt{5}, -2+\sqrt{5}\}$ 

$$\text{5. } x^2 - 6x + 7 = 0$$

$$x^2 - 6x = -7$$

$$x^2 - 6x + 9 = -7 + 9$$

$$(x-3)^2 = 2$$

$$x-3 = \pm\sqrt{2}$$

$$x = 3 \pm \sqrt{2}$$

Solution set:  $\{3-\sqrt{2}, 3+\sqrt{2}\}$

6.  $4x^2 - 24x + 25 = 0$

$$4x^2 - 24x = -25$$

$$x^2 - 6x = -\frac{25}{4}$$

$$x^2 - 6x + 9 = -\frac{25}{4} + 9$$

$$(x-3)^2 = \frac{11}{4}$$

$$x-3 = \pm \frac{\sqrt{11}}{2} \Rightarrow x = 3 \pm \frac{\sqrt{11}}{2}$$

Solution set:  $\left\{ 3 - \frac{\sqrt{11}}{2}, 3 + \frac{\sqrt{11}}{2} \right\}$

7.  $6x^2 - x - 2 = 0$

$a = 6, b = -1, c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-2)}}{2(6)}$$

$$= \frac{1 \pm \sqrt{49}}{12} = \frac{1 \pm 7}{12} = \frac{-6}{12} = -\frac{1}{2} \text{ or } \frac{8}{12} = \frac{2}{3}$$

Solution set:  $\left\{ -\frac{1}{2}, \frac{2}{3} \right\}$

8. a.  $4x^2 + 9 = 0$

$4x^2 = -9$

$$x^2 = -\frac{9}{4} \Rightarrow x = \pm \sqrt{-\frac{9}{4}} = \pm \frac{3}{2}i$$

Solution set:  $\left\{ -\frac{3}{2}i, \frac{3}{2}i \right\}$

b.  $x^2 = 4x - 13$

$x^2 - 4x + 13 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

Solution set:  $\{2 - 3i, 2 + 3i\}$

9. a.  $9x^2 - 6x + 1 = 0 \Rightarrow a = 9, b = -6, c = 1$

So,  $D = (-6)^2 - 4(9)(1) = 36 - 36 = 0$ .

Therefore, there is one real root.

b.  $x^2 - 5x + 3 = 0 \Rightarrow a = 1, b = -5, c = 3$

So,  $D = (-5)^2 - 4(1)(3) = 25 - 12 = 13 > 0$

Therefore, there are two unequal real roots.

c.  $2x^2 - 3x + 4 = 0 \Rightarrow a = 2, b = -3, c = 4$

So,  $D = (-3)^2 - 4(2)(4) = 9 - 32 = -23 < 0$

Therefore, there are two nonreal complex solutions.

10. Let  $x$  = the frontage of the building.Then  $5x$  = the depth of the building and $5x - 45$  = the depth of the rear portion.

$$x(5x - 45) = 2100$$

$$5x^2 - 45x = 2100$$

$$5x^2 - 45x - 2100 = 0$$

$a = 5, b = -45, c = -2100$

$$x = \frac{-(-45) \pm \sqrt{(-45)^2 - 4(5)(-2100)}}{2(5)}$$

$$= \frac{45 \pm \sqrt{44,025}}{10} \approx -16.48 \text{ or } 25.48$$

Reject the negative solution.

$$5x = 5 \cdot 25.482 = 127.41$$

The building is approximately 25.48 ft by 127.41 ft.

11.  $\Phi = \frac{\text{length}}{\text{width}} \Rightarrow \frac{1 + \sqrt{5}}{2} = \frac{x}{36}$

$$x = 36 \left( \frac{1 + \sqrt{5}}{2} \right) = 18 + 18\sqrt{5} \approx 58.25 \text{ ft}$$

### 1.4 A Exercises: Basic Skills and Concepts

- Any equation of the form  $ax^2 + bx + c = 0$  with  $a \neq 0$ , is called a quadratic equation.
- If  $P(x)$ ,  $D(x)$ , and  $Q(x)$  are polynomials, and  $P(x) = D(x)Q(x)$ , then the solutions of  $P(x) = 0$  are the solutions of  $Q(x) = 0$  together with the solutions of  $D(x) = 0$ .
- If you complete the square in the quadratic equation  $ax^2 + bx + c = 0$ , you get the quadratic formula for the solutions:  
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- If  $b^2 - 4ac < 0$ , the quadratic equation has two nonreal complex solutions; if  $b^2 - 4ac = 0$ , the equation has one real solution; if  $b^2 - 4ac > 0$ , the equation has two unequal real solutions.
- True

6. False. We form a perfect square by adding  $\left(\frac{k}{2}\right)^2$ .
7.  $(-6)^2 + 4(-6) - 12 = 36 - 24 - 12 = 0$ , so  $-6$  is a solution of the equation.
8.  $9^2 - 8(9) - 9 = 81 - 72 - 9 = 0$ , so  $9$  is a solution of the equation.
9.  $3\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) - 6 = 3\left(\frac{4}{9}\right) + \frac{14}{3} - 6$   
 $= \frac{4}{3} + \frac{14}{3} - 6 = 0$ , so  $2/3$  is a solution of the equation.
10.  $2\left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) - 3 = 2\left(\frac{1}{4}\right) + \frac{5}{2} - 3$   
 $= \frac{1}{2} + \frac{5}{2} - 3 = 0$ , so  $-1/2$  is a solution of the equation.
11.  $(2 - \sqrt{3})^2 - 4(2 - \sqrt{3}) + 1 =$   
 $(4 - 4\sqrt{3} + 3) - 8 + 4\sqrt{3} + 1 = 0$ , so  $2 - \sqrt{3}$  is a solution of the equation.
12.  $(3 + 2\sqrt{2})^2 - 6(3 + 2\sqrt{2}) + 1 =$   
 $17 + 12\sqrt{2} - 18 - 12\sqrt{2} + 1 = 0$ , so  $3 + 2\sqrt{2}$  is a solution of the equation.
13.  $4(2 + 3i)^2 - 8(2 + 3i) + 13 =$   
 $4(-5 + 12i) - 16 - 24i + 13$   
 $= -20 + 48i - 16 - 24i + 13 = -23 + 24i \neq 0$ , so  $2 + 3i$  is not a solution of the equation.
14.  $(5 - 2i)^2 - 6(5 - 2i) + 13 =$   
 $25 - 20i + 4i^2 - 30 + 12i + 13 = -8i + 4 \neq 0$ , so  $5 - 2i$  is not a solution of the equation.
15.  $k(1)^2 + 1 - 3 = 0 \Rightarrow k - 2 = 0 \Rightarrow k = 2$
16.  $k(\sqrt{7})^2 + \sqrt{7} - 3 = 0 \Rightarrow 7k + \sqrt{7} - 3 = 0 \Rightarrow$   
 $7k = 3 - \sqrt{7} \Rightarrow k = \frac{3 - \sqrt{7}}{7}$
17.  $x^2 - 5x = 0 \Rightarrow x(x - 5) = 0 \Rightarrow$   
 $x = 0$  or  $x - 5 = 0 \Rightarrow x = 0$  or  $x = 5$
18.  $x^2 - 5x + 4 = 0$   
 $(x - 4)(x - 1) = 0$   
 $x - 4 = 0$  or  $x - 1 = 0 \Rightarrow x = 4$  or  $x = 1$
19.  $x^2 + 5x = 14$   
 $x^2 + 5x - 14 = 0$   
 $(x + 7)(x - 2) = 0$   
 $x + 7 = 0$  or  $x - 2 = 0$   
 $x = -7$  or  $x = 2$
20.  $x^2 - 11x = 12$   
 $x^2 - 11x - 12 = 0$   
 $(x - 12)(x + 1) = 0$   
 $x - 12 = 0$  or  $x + 1 = 0$   
 $x = 12$  or  $x = -1$
21.  $x^2 = 5x + 6$   
 $x^2 - 5x - 6 = 0$   
 $(x - 6)(x + 1) = 0$   
 $x - 6 = 0$  or  $x + 1 = 0$   
 $x = 6$  or  $x = -1$
22.  $x = x^2 - 12$   
 $0 = x^2 - x - 12$   
 $0 = (x - 4)(x + 3)$   
 $x - 4 = 0$  or  $x + 3 = 0$   
 $x = 4$  or  $x = -3$
23.  $2x^2 + 5x - 3 = 0$   
 $(2x - 1)(x + 3) = 0$   
 $2x - 1 = 0$  or  $x + 3 = 0$   
 $x = \frac{1}{2}$  or  $x = -3$
24.  $2x^2 - 9x + 10 = 0$   
 $(2x - 5)(x - 2) = 0$   
 $2x - 5 = 0$  or  $x - 2 = 0$   
 $x = \frac{5}{2}$  or  $x = 2$
25.  $3y^2 + 5y + 2 = 0$   
 $(3y + 2)(y + 1) = 0$   
 $3y + 2 = 0$  or  $y + 1 = 0$   
 $y = -\frac{2}{3}$  or  $y = -1$
26.  $6x^2 + 11x + 4 = 0$   
 $(3x + 4)(2x + 1) = 0$   
 $3x + 4 = 0$  or  $2x + 1 = 0$   
 $x = -\frac{4}{3}$  or  $x = -\frac{1}{2}$

27.  $5x^2 + 12x + 4 = 0$   
 $(5x + 2)(x + 2) = 0$   
 $5x + 2 = 0$  or  $x + 2 = 0$   
 $x = -\frac{2}{5}$  or  $x = -2$
28.  $3x^2 - 2x - 5 = 0$   
 $(3x - 5)(x + 1) = 0$   
 $3x - 5 = 0$  or  $x + 1 = 0$   
 $x = \frac{5}{3}$  or  $x = -1$
29.  $2x^2 + x = 15$   
 $2x^2 + x - 15 = 0$   
 $(2x - 5)(x + 3) = 0$   
 $2x - 5 = 0$  or  $x + 3 = 0 \Rightarrow x = \frac{5}{2}$  or  $x = -3$
30.  $6x^2 = 1 - x$   
 $6x^2 + x - 1 = 0$   
 $(3x - 1)(2x + 1) = 0$   
 $3x - 1 = 0$  or  $2x + 1 = 0$   
 $x = \frac{1}{3}$  or  $x = -\frac{1}{2}$
31.  $12x^2 - 10x = 12$   
 $12x^2 - 10x - 12 = 0$   
 $2(6x^2 - 5x - 6) = 0$   
 $6x^2 - 5x - 6 = 0$   
 $(3x + 2)(2x - 3) = 0$   
 $3x + 2 = 0$  or  $2x - 3 = 0$   
 $x = -\frac{2}{3}$  or  $x = \frac{3}{2}$
32.  $-x^2 + 10x + 1200 = 0$   
 $x^2 - 10x - 1200 = 0$   
 $(x - 40)(x + 30) = 0$   
 $x - 40 = 0$  or  $x + 30 = 0$   
 $x = 40$  or  $x = -30$
33.  $18x^2 - 45x = -7$   
 $18x^2 - 45x + 7 = 0$   
 $(3x - 7)(6x - 1) = 0$   
 $3x - 7 = 0$  or  $6x - 1 = 0 \Rightarrow x = \frac{7}{3}$  or  $x = \frac{1}{6}$
34.  $18x^2 + 57x + 45 = 0$   
 $3(6x^2 + 19x + 15) = 0$   
 $6x^2 + 19x + 15 = 0$   
 $(3x + 5)(2x + 3) = 0$   
 $3x + 5 = 0$  or  $2x + 3 = 0$   
 $x = -\frac{5}{3}$  or  $x = -\frac{3}{2}$
35.  $4x^2 - 10x - 750 = 0$   
 $2(2x^2 - 5x - 375) = 0$   
 $2x^2 - 5x - 375 = 0$   
 $(2x + 25)(x - 15) = 0$   
 $2x + 25 = 0$  or  $x - 15 = 0$   
 $x = -\frac{25}{2}$  or  $x = 15$
36.  $12x^2 + 43x + 36 = 0$   
 $(3x + 4)(4x + 9) = 0$   
 $3x + 4 = 0$  or  $4x + 9 = 0$   
 $x = -\frac{4}{3}$  or  $x = -\frac{9}{4}$
37.  $3x^2 = 48 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$
38.  $2x^2 = 50 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$
39.  $x^2 + 1 = 5 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$
40.  $2x^2 - 1 = 17 \Rightarrow 2x^2 = 18 \Rightarrow$   
 $x^2 = 9 \Rightarrow x = \pm 3$
41.  $x^2 + 5 = 1 \Rightarrow x^2 = -4 \Rightarrow x = \pm 2i$
42.  $4x^2 + 9 = 0$   
 $4x^2 = -9$   
 $x^2 = -\frac{9}{4} \Rightarrow x = \pm \frac{3}{2}i$
43.  $(x - 1)^2 = 16$   
 $x - 1 = -4$  or  $x - 1 = 4 \Rightarrow x = -3$  or  $x = 5$
44.  $(2x - 3)^2 = 25 \Rightarrow$   
 $2x - 3 = -5$  or  $2x - 3 = 5 \Rightarrow x = -1$  or  $x = 4$
45.  $(3x - 2)^2 + 16 = 0$   
 $(3x - 2)^2 = -16$   
 $3x - 2 = -4i$  or  $3x - 2 = 4i$   
 $3x = 2 - 4i$  or  $3x = 2 + 4i$   
 $x = \frac{2}{3} - \frac{4}{3}i$  or  $x = \frac{2}{3} + \frac{4}{3}i$

46.  $(2x+3)^2 + 25 = 0$   
 $(2x+3)^2 = -25$   
 $2x+3 = -5i$  or  $2x+3 = 5i$   
 $2x = -3-5i$  or  $2x = -3+5i$   
 $x = -\frac{3}{2} - \frac{5}{2}i$  or  $x = -\frac{3}{2} + \frac{5}{2}i$
47. To complete the square, find  $1/2$  of the coefficient of the  $x$ -term,  $4/2 = 2$ , and then square the answer.  $2^2 = 4$ .
48. To complete the square, find  $1/2$  of the coefficient of the  $y$ -term,  $10/2 = 5$ , and then square the answer.  $5^2 = 25$ .
49. To complete the square, find  $1/2$  of the coefficient of the  $x$ -term,  $6/2 = 3$ , and then square the answer.  $3^2 = 9$ .
50. To complete the square, find  $1/2$  of the coefficient of the  $y$ -term,  $8/2 = 4$ , and then square the answer.  $4^2 = 16$ .
51. To complete the square, find  $1/2$  of the coefficient of the  $x$ -term and then square the answer.  $\left(\frac{7}{2}\right)^2 = \frac{49}{4}$ .
52. To complete the square, find  $1/2$  of the coefficient of the  $x$ -term and then square the answer.  $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ .
53. To complete the square, find  $1/2$  of the coefficient of the  $x$ -term,  $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$  and then square the answer.  $\left(\frac{1}{6}\right)^2 = \frac{1}{36}$ .
54. To complete the square, find  $1/2$  of the coefficient of the  $x$ -term,  $\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$  and then square the answer.  $(3/4)^2 = 9/16$ .
55. To complete the square, find  $1/2$  of the coefficient of the  $x$ -term and then square the answer.  $(a/2)^2 = a^2/4$ .
56. To complete the square, find  $1/2$  of the coefficient of the  $x$ -term,  
 $\frac{1}{2} \cdot \frac{2a}{3} = \frac{a}{3}$ , and then square the answer.  
 $\left(\frac{a}{3}\right)^2 = \frac{a^2}{9}$ .
57.  $x^2 + 2x - 5 = 0$   
 $x^2 + 2x = 5$   
 Now, complete the square.  
 $x^2 + 2x + 1 = 5 + 1$   
 $(x+1)^2 = 6$   
 $x+1 = \pm\sqrt{6} \Rightarrow x = -1 \pm \sqrt{6}$
58.  $x^2 + 6x = -7$   
 Now, complete the square.  
 $x^2 + 6x + 9 = -7 + 9$   
 $(x+3)^2 = 2 \Rightarrow x+3 = \pm\sqrt{2} \Rightarrow x = -3 \pm \sqrt{2}$
59.  $x^2 - 3x - 1 = 0$   
 $x^2 - 3x = 1$   
 Now, complete the square.  
 $x^2 - 3x + \frac{9}{4} = 1 + \frac{9}{4}$   
 $\left(x - \frac{3}{2}\right)^2 = \frac{13}{4}$   
 $x - \frac{3}{2} = \pm\frac{\sqrt{13}}{2}$   
 $x = \frac{3}{2} \pm \frac{\sqrt{13}}{2} = \frac{3 \pm \sqrt{13}}{2}$
60.  $x^2 - x - 3 = 0$   
 $x^2 - x = 3$   
 Now, complete the square.  
 $x^2 - x + \frac{1}{4} = 3 + \frac{1}{4} \Rightarrow \left(x - \frac{1}{2}\right)^2 = \frac{13}{4}$   
 $x - \frac{1}{2} = \pm\sqrt{\frac{13}{4}} = \pm\frac{\sqrt{13}}{2} \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{13}}{2}$
61.  $2r^2 + 3r = 9$   
 $r^2 + \frac{3}{2}r = \frac{9}{2}$   
 $r^2 + \frac{3}{2}r + \frac{9}{16} = \frac{9}{2} + \frac{9}{16}$   
 $\left(r + \frac{3}{4}\right)^2 = \frac{81}{16}$   
 $r + \frac{3}{4} = \pm\frac{9}{4} \Rightarrow r = \frac{-3 \pm 9}{4} = \frac{3}{2}$  or  $-3$

- 62.**  $3k^2 - 5k + 1 = 0$   
 $3k^2 - 5k = -1$   
 $k^2 - \frac{5}{3}k = -\frac{1}{3}$   
 Now, complete the square.  
 $k^2 - \frac{5}{3}k + \frac{25}{36} = -\frac{1}{3} + \frac{25}{36}$   
 $\left(k - \frac{5}{6}\right)^2 = \frac{13}{36}$   
 $k - \frac{5}{6} = \pm\sqrt{\frac{13}{36}} = \pm\frac{\sqrt{13}}{6}$   
 $k = \frac{5}{6} \pm \frac{\sqrt{13}}{6} = \frac{5 \pm \sqrt{13}}{6}$
- 63.**  $z^2 - 2z + 2 = 0$   
 $z^2 - 2z = -2$   
 Now, complete the square.  
 $z^2 - 2z + 1 = -2 + 1$   
 $(z-1)^2 = -1 \Rightarrow z-1 = \pm i \Rightarrow z = 1 \pm i$
- 64.**  $x^2 - 6x + 11 = 0$   
 $x^2 - 6x = -11$   
 Now, complete the square.  
 $x^2 - 6x + 9 = -11 + 9$   
 $(x-3)^2 = -2$   
 $x-3 = \pm i\sqrt{2} \Rightarrow x = 3 \pm i\sqrt{2}$
- 65.**  $2x^2 - 20x + 49 = -7$   
 $2x^2 - 20x = -56$   
 $x^2 - 10x = -28$   
 Now, complete the square.  
 $x^2 - 10x + 25 = -28 + 25$   
 $(x-5)^2 = -3$   
 $x-5 = \pm i\sqrt{3} \Rightarrow x = 5 \pm i\sqrt{3}$
- 66.**  $4y^2 + 4y + 5 = 0$   
 $4y^2 + 4y = -5$   
 $y^2 + y = -\frac{5}{4}$   
 Now, complete the square.  
 $y^2 + y + \frac{1}{4} = -\frac{5}{4} + \frac{1}{4}$   
 $\left(y + \frac{1}{2}\right)^2 = -1$   
 $y + \frac{1}{2} = \pm i \Rightarrow y = -\frac{1}{2} \pm i$
- 67.**  $5x^2 - 6x = 4x^2 + 6x - 3$   
 $x^2 - 12x = -3$   
 Now, complete the square.  
 $x^2 - 12x + 36 = -3 + 36$   
 $(x-6)^2 = 33$   
 $x-6 = \pm\sqrt{33} \Rightarrow x = 6 \pm \sqrt{33}$
- 68.**  $x^2 + 7x - 5 = x - x^2$   
 $2x^2 + 6x - 5 = 0$   
 $2x^2 + 6x = 5$   
 $x^2 + 3x = \frac{5}{2}$   
 Now, complete the square.  
 $x^2 + 3x + \frac{9}{4} = \frac{5}{2} + \frac{9}{4}$   
 $\left(x + \frac{3}{2}\right)^2 = \frac{19}{4}$   
 $x + \frac{3}{2} = \pm\sqrt{\frac{19}{4}} = \pm\frac{\sqrt{19}}{2}$   
 $x = -\frac{3}{2} \pm \frac{\sqrt{19}}{2} = \frac{-3 \pm \sqrt{19}}{2}$
- 69.**  $5y^2 + 10y + 4 = 2y^2 + 3y + 1$   
 $3y^2 + 7y = -3$   
 $y^2 + \frac{7}{3}y = -1$   
 Now, complete the square.  
 $y^2 + \frac{7}{3}y + \frac{49}{36} = -1 + \frac{49}{36}$   
 $\left(y + \frac{7}{6}\right)^2 = \frac{13}{36}$   
 $y + \frac{7}{6} = \pm\sqrt{\frac{13}{36}} = \pm\frac{\sqrt{13}}{6}$   
 $y = -\frac{7}{6} \pm \frac{\sqrt{13}}{6} = \frac{-7 \pm \sqrt{13}}{6}$
- 70.**  $3x^2 - 1 = 5x^2 - 3x - 5$   
 $4 = 2x^2 - 3x$   
 $2 = x^2 - \frac{3}{2}x$   
 Now, complete the square.  
 $2 + \frac{9}{16} = x^2 - \frac{3}{2}x + \frac{9}{16}$   
 $\frac{41}{16} = \left(x - \frac{3}{4}\right)^2$

(continued on next page)



(continued from page 77)

$$\begin{aligned}\pm\sqrt{\frac{41}{16}} &= x - \frac{3}{4} \Rightarrow \pm\frac{\sqrt{41}}{4} = x - \frac{3}{4} \\ x &= \frac{3}{4} \pm \frac{\sqrt{41}}{4} = \frac{3 \pm \sqrt{41}}{4}\end{aligned}$$

In exercises 71–86, use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\begin{aligned}71. \quad x^2 + 2x - 4 = 0 &\Rightarrow a = 1, b = 2, c = -4 \\ x &= \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 16}}{2} \\ &= \frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}\end{aligned}$$

$$\begin{aligned}72. \quad m^2 + 3m + 2 = 0 &\Rightarrow a = 1, b = 3, c = 2 \\ m &= \frac{-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)} = \frac{-3 \pm \sqrt{9 - 8}}{2} \\ &= \frac{-3 \pm \sqrt{1}}{2} = \frac{-3 \pm 1}{2} \\ m &= \frac{-3 + 1}{2} = \frac{-2}{2} = -1 \text{ or} \\ m &= \frac{-3 - 1}{2} = \frac{-4}{2} = -2\end{aligned}$$

$$\begin{aligned}73. \quad 6x^2 = 7x + 5 &\Rightarrow 6x^2 - 7x - 5 = 0 \Rightarrow \\ a = 6, b = -7, c = -5 \\ x &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-5)}}{2(6)} \\ &= \frac{7 \pm \sqrt{49 + 120}}{12} = \frac{7 \pm \sqrt{169}}{12} = \frac{7 \pm 13}{12} \\ x &= \frac{7 + 13}{12} = \frac{20}{12} = \frac{5}{3} \text{ or} \\ x &= \frac{7 - 13}{12} = \frac{-6}{12} = -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}74. \quad t^2 + 7 = 4t &\Rightarrow t^2 - 4t + 7 = 0 \Rightarrow \\ a = 1, b = -4, c = 7 \\ t &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)} \\ &= \frac{4 \pm \sqrt{16 - 28}}{2} = \frac{4 \pm \sqrt{-12}}{2} \\ &= \frac{4 \pm 2i\sqrt{3}}{2} = 2 \pm i\sqrt{3}\end{aligned}$$

$$\begin{aligned}75. \quad 3z^2 - 2z = 7 &\Rightarrow 3z^2 - 2z - 7 = 0 \Rightarrow \\ a = 3, b = -2, c = -7 \\ z &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-7)}}{2(3)} \\ &= \frac{2 \pm \sqrt{4 + 84}}{6} = \frac{2 \pm \sqrt{88}}{6} = \frac{2 \pm 2\sqrt{22}}{6} \\ &= \frac{1 \pm \sqrt{22}}{3}\end{aligned}$$

$$\begin{aligned}76. \quad 6y^2 + 11y = 10 &\Rightarrow 6y^2 + 11y - 10 = 0 \Rightarrow \\ a = 6, b = 11, c = -10 \\ y &= \frac{-11 \pm \sqrt{11^2 - 4(6)(-10)}}{2(6)} \\ &= \frac{-11 \pm \sqrt{121 + 240}}{12} = \frac{-11 \pm \sqrt{361}}{12} \\ &= \frac{-11 \pm 19}{12} \\ y &= \frac{8}{12} = \frac{2}{3} \text{ or } y = \frac{-30}{12} = -\frac{5}{2}\end{aligned}$$

$$\begin{aligned}77. \quad 3p^2 + 8p + 4 = 0 &\Rightarrow a = 3, b = 8, c = 4 \\ p &= \frac{-8 \pm \sqrt{8^2 - 4(3)(4)}}{2(3)} \\ &= \frac{-8 \pm \sqrt{64 - 48}}{6} = \frac{-8 \pm \sqrt{16}}{6} = \frac{-8 \pm 4}{6} \\ p &= \frac{-4}{6} = -\frac{2}{3} \text{ or } p = \frac{-12}{6} = -2\end{aligned}$$

$$\begin{aligned}78. \quad 8(x^2 - x) = x^2 - 3 &\Rightarrow 8x^2 - 8x = x^2 - 3 \Rightarrow \\ 7x^2 - 8x + 3 = 0 &\Rightarrow a = 7, b = -8, c = 3 \\ x &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(7)(3)}}{2(7)} \\ &= \frac{8 \pm \sqrt{64 - 84}}{14} = \frac{8 \pm \sqrt{-20}}{14} \\ &= \frac{8 \pm 2i\sqrt{5}}{14} = \frac{4 \pm i\sqrt{5}}{7} \\ x &= \frac{4}{7} + \frac{\sqrt{5}}{7}i \text{ or } x = \frac{4}{7} - \frac{\sqrt{5}}{7}i\end{aligned}$$

$$\begin{aligned}79. \quad x^2 = 5(x - 1) &\Rightarrow x^2 = 5x - 5 \Rightarrow \\ x^2 - 5x + 5 = 0 &\Rightarrow a = 1, b = -5, c = 5 \\ x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(5)}}{2(1)} \\ &= \frac{5 \pm \sqrt{25 - 20}}{2} = \frac{5 \pm \sqrt{5}}{2}\end{aligned}$$