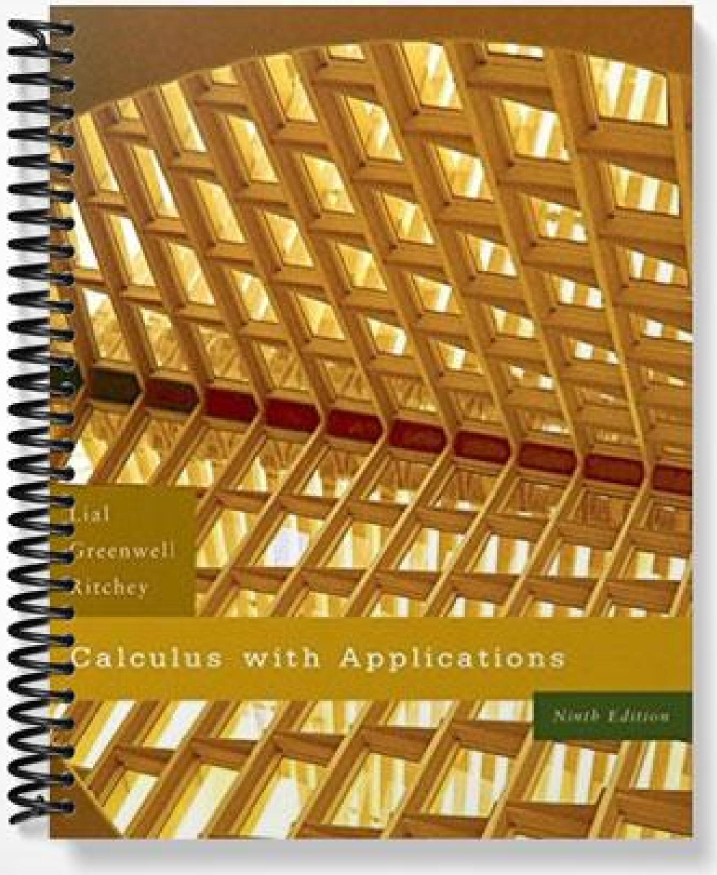
SOLUTIONS MANUAL



Chapter 2

NONLINEAR FUNCTIONS

2.1 Properties of Functions

1. The x-value of 82 corresponds to two y-values, 93 and 14. In a function, each value of x must correspond to exactly one value of y.
The rule is not a function.

2. Each x-value corresponds to exactly one y-value.
The rule is a function.

3. Each x-value corresponds to exactly one y-value.
The rule is a function.

4. 9 corresponds to 3 and -3, 4 corresponds to 2 and -2, and 1 corresponds to -1 and 1.
The rule is not a function.

5. $y = x^3 + 2$

Each x-value corresponds to exactly one y-value.
The rule is a function.

6. $y = \sqrt{x}$

Each x-value corresponds to exactly one y-value.
The rule is a function.

7. $x = |y|$

Each value of x (except 0) corresponds to two y-values.
The rule is not a function.

8. $x = y^2 + 4$

Solve the rule for y.

$$y^2 = x - 4 \quad \text{or} \quad y = \pm\sqrt{x-4}$$

Each value of x (greater than 4) corresponds to two y-values

$y = \sqrt{x-4}$ and $y = -\sqrt{x-4}$.

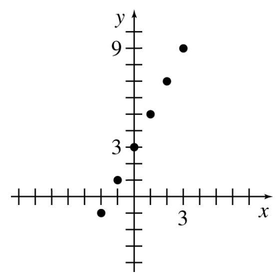
The rule is not a function.

9. $y = 2x + 3$

x	-2	-1	0	1	2	3
y	-1	1	3	5	7	9

Pairs: $(-2,-1),\ (-1,1),\ (0,3),$
$(1,5),\ (2,7),\ (3,9)$

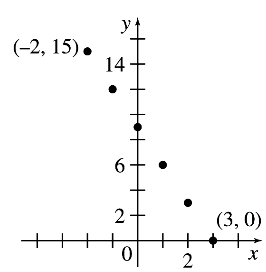
Range: $\{-1,\ 1,\ 3,\ 5,\ 7,\ 9\}$



10. $y = -3x + 9$

x	-2	-1	0	1	2	3
y	15	12	9	6	3	0

Pairs: $(-2,15),\ (-1,12),\ (0,9),$
$(1,6),\ (2,3),\ (3,0)$
Range: $\{0,3,6,9,12,15\}$



11. $2y - x = 5$

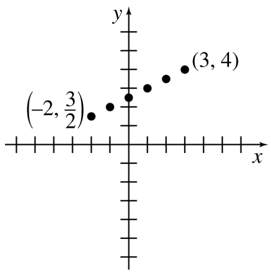
$$2y = 5 + x$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

x	-2	-1	0	1	2	3
y	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4

Pairs: $(-2,\frac{3}{2}),\ (-1,2),\ (0,\frac{5}{2}), (1,3),\ (2,\frac{7}{2}),\ (3,4)$

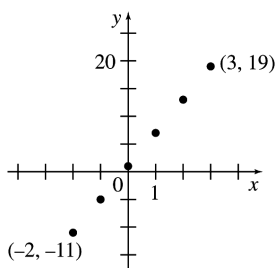
Range: $\{\frac{3}{2},\ 2,\ \frac{5}{2}\ 3,\ \frac{7}{2},\ 4\}$



12. $6x - y = -1$
$-y = -6x - 1$
$y = 6x + 1$

x	-2	-1	0	1	2	3
y	-11	-5	1	7	13	19

Pairs: $(-2,-11),\ (-1,-5),\ (0,1),$
$(1,7),\ (2,13),\ (3,19)$
Range: $\{-11,-5,1,7,13,19\}$

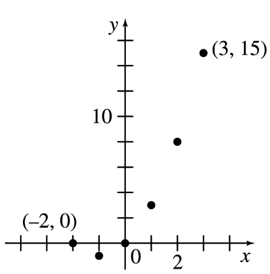


13. $y = x(x+2)$

x	-2	-1	0	1	2	3
y	0	-1	0	3	8	15

Pairs: $(-2,0),\ (-1,-1),\ (0,0),$
$(1,3),\ (2,8), (3,15)$

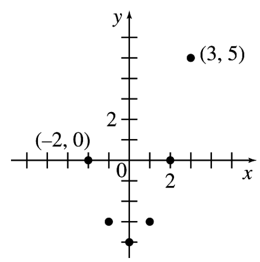
Range: $\{-1,\ 0,\ 3,\ 8,\ 15\}$



14. $y = (x-2)(x+2)$

x	-2	-1	0	1	2	3
y	0	-3	-4	-3	0	5

Pairs: $(-2,0),\ (-1,-3),\ (0,-4),$
$(1,-3),\ (2,0),\ (3,5)$
Range: $\{-4,-3,0,5\}$

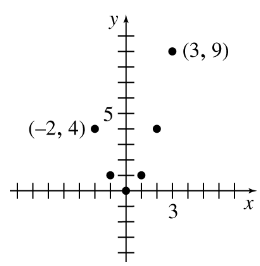


15. $y = x^2$

x	-2	-1	0	1	2	3
y	4	1	0	1	4	9

Pairs: $(-2,4),\ (-1,1),\ (0,0),$
$(1,1),\ (2,4),\ (3,9)$

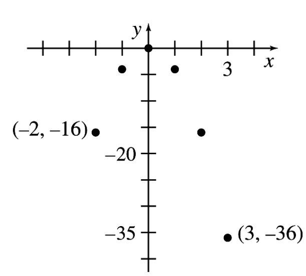
Range: $\{0,\ 1,\ 4,\ 9\}$



16. $y = -4x^2$

x	-2	-1	0	1	2	3
y	-16	-4	0	-4	-16	-36

Pairs: $(-2,-16),\ (-1,-4),\ (0,0),$
$(1,-4),\ (2,-16),\ (3,-36)$
Range: $\{-36,-16,-4,0\}$

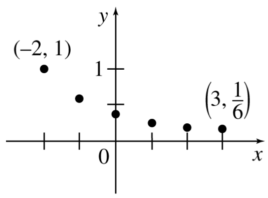


17. $y = \dfrac{1}{x+3}$

x	-2	-1	0	1	2	3
y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$

Pairs: $(-2,1), (-1,\frac{1}{2}), (0,\frac{1}{3}), (1,\frac{1}{4}), (2,\frac{1}{5}), (3,\frac{1}{6})$

Range: $\{1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{6}\}$

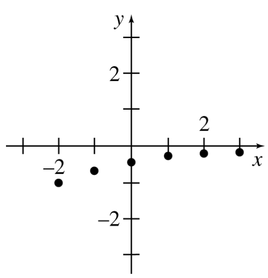


18. $y = \dfrac{-2}{x+4}$

x	-2	-1	0	1	2	3
y	-1	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{2}{5}$	$-\frac{1}{3}$	$-\frac{2}{7}$

Pairs: $(-2,-1),\ (-1,-\frac{2}{3}),\ (0,-\frac{1}{2}),$
$(1,-\frac{2}{5}),\ (2,-\frac{1}{3}),\ (3,-\frac{2}{7})$

Range: $\{-1,-\frac{2}{3},-\frac{1}{2},-\frac{2}{5},-\frac{1}{3},-\frac{2}{7}\}$

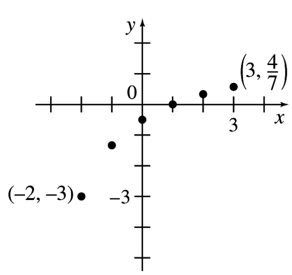


19. $y = \dfrac{2x-2}{x+4}$

x	-2	-1	0	1	2	3
y	-3	$-\frac{4}{3}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{4}{7}$

Pairs:$(-2,-3),\ (-1,-\frac{4}{3}),\ (0,-\frac{1}{2}), (1,0),\ (2,\frac{1}{3}),\ (3,\frac{4}{7})$

Range: $\{-3,\ -\frac{4}{3},\ -\frac{1}{2},\ 0,\ \frac{1}{3},\ \frac{4}{7}\}$

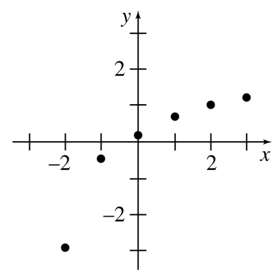


20. $y = \dfrac{2x+1}{x+3}$

x	-2	-1	0	1	2	3
y	-3	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{4}$	1	$\frac{7}{6}$

Pairs: $(-2,-3),\ (-1,-\frac{1}{2}),\ (0,\frac{1}{3}),$
$(1,\frac{3}{4}),\ (2,1),\ (3,\frac{7}{6})$

Range: $\{-3,-\frac{1}{2},\frac{1}{3},\frac{3}{4},1,\frac{7}{6}\}$



21. $f(x) = 2x$

x can take on any value, so the domain is the set of real numbers, $(-\infty,\infty)$.

22. $f(x) = 2x+3$

x can take on any value, so the domain is the set of real numbers, which is written $(-\infty,\infty)$.

23. $f(x) = x^4$

x can take on any value, so the domain is the set of real numbers, $(-\infty,\infty)$.

24. $f(x) = (x+3)^2$

x can take on any value, so the domain is the set of real numbers, $(-\infty,\infty)$.

25. $f(x) = \sqrt{4-x^2}$

For $f(x)$ to be a real number, $4-x^2 \geq 0$.
Solve $4-x^2=0$.

$$(2-x)(2+x)=0$$
$$x=2 \quad \text{or} \quad x=-2$$

The numbers form the intervals $(-\infty,-2),\ (-2,2),$ and $(2,\infty)$.
Only values in the interval $(-2,2)$ satisfy the inequality. The domain is $[-2,2]$.

26. $f(x) = |3x - 6|$

x can take on any value, so the domain is the set of real numbers, $(-\infty, \infty)$.

27. $f(x) = (x-3)^{1/2} = \sqrt{x-3}$

For $f(x)$ to be a real number,

$$\begin{aligned} x - 3 &\geq 0 \\ x &\geq 3. \end{aligned}$$

The domain is $[3, \infty)$.

28. $f(x) = (3x+5)^{1/2} = \sqrt{3x+5}$

For $f(x)$ to be a real number,

$$\begin{aligned} 3x + 5 &\geq 0 \\ 3x &\geq -5 \\ \frac{1}{3}(3x) &\geq \frac{1}{3}(-5) \\ x &\geq -\frac{5}{3}. \end{aligned}$$

In interval notation, the domain is $[-\frac{5}{3}, \infty)$.

29. $f(x) = \dfrac{2}{1-x^2} = \dfrac{2}{(1-x)(1+x)}$

Since division by zero is not defined,
$(1-x) \cdot (1+x) \neq 0$.
When $(1-x)(1+x) = 0$,

$$\begin{aligned} 1 - x = 0 \quad &\text{or} \quad 1 + x = 0 \\ x = 1 \quad &\text{or} \quad x = -1. \end{aligned}$$

Thus, x can be any real number except ± 1.
The domain is

$$(-\infty, -1) \cup (-1, 1) \cup (1, \infty).$$

30. $f(x) = \dfrac{-8}{x^2 - 36}$

In order for $f(x)$ to be a real number, $x^2 - 36$ cannot be equal to 0.

When $x^2 - 36 = 0$,

$$\begin{aligned} x^2 &= 36 \\ x = 6 \quad &\text{or} \quad x = -6. \end{aligned}$$

Thus, the domain is any real number except 6 or -6. In interval notation, the domain is

$$(-\infty, -6) \cup (-6, 6) \cup (6, \infty).$$

31. $f(x) = -\sqrt{\dfrac{2}{x^2-16}} = -\sqrt{\dfrac{2}{(x-4)(x+4)}}$.

$(x-4)\cdot(x+4) > 0$, since $(x-4)\cdot(x+4) < 0$ would produce a negative radicand and $(x-4)\cdot(x+4) = 0$ would lead to division by zero.

Solve $(x-4)\cdot(x+4) = 0$.
$x - 4 = 0$ or $x + 4 = 0$
$x = 4$ or $x = -4$

Use the values -4 and 4 to divide the number line into 3 intervals, $(-\infty, -4)$, $(-4, 4)$ and $(4, \infty)$. Only the values in the intervals $(-\infty, -4)$ and $(4, \infty)$ satisfy the inequality.
The domain is

$$(-\infty, -4) \cup (4, \infty).$$

32. $f(x) = -\sqrt{\dfrac{5}{x^2+36}}$

x can take on any value. No choice for x will produce a zero in the denominator. Also, no choice for x will produce a negative number under the radical. The domain is $(-\infty, \infty)$.

33. $f(x) = \sqrt{x^2 - 4x - 5} = \sqrt{(x-5)(x+1)}$

See the method used in Exercise 25.

$$(x-5)(x+1) \geq 0$$

when $x \geq 5$ and when $x \leq -1$.
The domain is $(-\infty, -1] \cup [5, \infty)$.

34. $f(x) = \sqrt{15x^2 + x - 2}$

The expression under the radical must be nonnegative.

$$\begin{aligned} 15x^2 + x - 2 &\geq 0 \\ (5x+2)(3x-1) &\geq 0 \end{aligned}$$

Solve $(5x+2)(3x-1) = 0$.

$$\begin{aligned} 5x + 2 = 0 \quad &\text{or} \quad 3x - 1 = 0 \\ 5x = -2 \quad & \quad 3x = 1 \\ x = -\frac{2}{5} \quad &\text{or} \quad x = \frac{1}{3} \end{aligned}$$

Use these numbers to divide the number line into 3 intervals, $\left(-\infty, -\frac{2}{5}\right)$, $\left(-\frac{2}{5}, \frac{1}{3}\right)$, and $\left(\frac{1}{3}, \infty\right)$.
Only values in the intervals $\left(-\infty, -\frac{2}{5}\right)$ and $\left(\frac{1}{3}, \infty\right)$ satisfy the inequality. The domain is $\left(-\infty, -\frac{2}{5}\right] \cup \left[\frac{1}{3}, \infty\right)$.

35. $f(x) = \dfrac{1}{\sqrt{3x^2 + 2x - 1}} = \dfrac{1}{\sqrt{(3x-1)(x+1)}}$

$(3-2)(x+1) > 0$, since the radicand cannot be negative and the denominator of the function cannot be zero.
Solve $(3-1)(x+1) = 0$.

$$\begin{aligned} 3 - 1 = 0 \quad &\text{or} \quad x + 1 = 0 \\ x = \tfrac{1}{3} \quad &\text{or} \quad x = -1 \end{aligned}$$

Use the values -1 and $\frac{1}{3}$ to divide the number line into 3 intervals, $(-\infty, -1)$, $(-1, 4)$ and $(\frac{1}{3}, \infty)$.
Only the values in the intervals $(-\infty, -\frac{1}{3})$ and $(\frac{1}{3}, \infty)$ satisfy the inequality.
The domain is $(-\infty, -1) \cup (\frac{1}{3}, \infty)$.

36. $f(x) = \sqrt{\dfrac{x^2}{3-x}}$

For $f(x)$ to be a real number,

$$\begin{aligned} x^2 \geq 0 \quad &\text{and} \quad 3 - x > 0 \\ -\infty < x < \infty \quad &\text{and} \quad x < 3 \\ & \quad x < 3. \end{aligned}$$

The domain is $(-\infty, 3)$.

37. By reading the graph, the domain is all numbers greater than or equal to -5 and less than 4. The range is all numbers greater than or equal to -2 and less than or equal to 6.
Domain: $[-5, 4)$; range: $[-2, 6]$

38. By reading the graph, the domain is all numbers greater than or equal to -5. The range is all numbers greater than or equal to 0.
Domain: $[-5, \infty)$ Range: $[0, \infty)$

39. By reading the graph, x can take on any value, but y is less than or equal to 12.
Domain: $(-\infty, \infty)$; range: $(-\infty, 12]$

40. By reading the graph, both x and y can take on any values.
Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

41. $f(x) = 3x^2 - 4x + 1$

(a) $$\begin{aligned} f(4) &= 3(4)^2 - 4(4) + 1 \\ &= 48 - 16 + 1 \\ &= 33 \end{aligned}$$

(b) $$\begin{aligned} f\left(-\frac{1}{2}\right) &= 3\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + 1 \\ &= \frac{3}{4} + 2 + 1 \\ &= \frac{15}{4} \end{aligned}$$

(c) $$\begin{aligned} f(a) &= 3(a)^2 - 4(a) + 1 \\ &= 3a^2 - 4a + 1 \end{aligned}$$

(d) $$\begin{aligned} f\left(\frac{2}{m}\right) &= 3\left(\frac{2}{m}\right)^2 - 4\left(\frac{2}{m}\right) + 1 \\ &= \frac{12}{m^2} - \frac{8}{m} + 1 \\ &\text{or} \quad \frac{12 - 8m + m^2}{m^2} \end{aligned}$$

(e) $$\begin{aligned} f(x) &= 1 \\ 3x^2 - 4x + 1 &= 1 \\ 3x^2 - 4x &= 0 \\ x(3x - 4) &= 0 \\ x = 0 \quad \text{or} \quad x &= \frac{4}{3} \end{aligned}$$

42. $f(x) = (x+3)(x-4)$

(a) $f(4) = (4+3)(4-4) = (7)(0) = 0$

(b) $$\begin{aligned} f\left(-\frac{1}{2}\right) &= \left(-\frac{1}{2} + 3\right)\left(-\frac{1}{2} - 4\right) \\ &= \left(\frac{5}{2}\right)\left(-\frac{9}{2}\right) = -\frac{45}{4} \end{aligned}$$

(c) $f(a) = [(a) + 3][(a) - 4] = (a+3)(a-4)$

(d) $$\begin{aligned} f\left(\frac{2}{m}\right) &= \left(\frac{2}{m} + 3\right)\left(\frac{2}{m} - 4\right) \\ &= \left(\frac{2 + 3m}{m}\right)\left(\frac{2 - 4m}{m}\right) \\ &= \frac{(2 + 3m)(2 - 4m)}{m^2} \\ &\text{or} \quad \frac{2(2 + 3m)(1 - 2m)}{m^2} \end{aligned}$$

(e) $$\begin{aligned} f(x) &= 1 \\ (x+3)(x-4) &= 1 \\ x^2 - x - 12 &= 1 \\ x^2 - x - 13 &= 0 \end{aligned}$$

$$x = \frac{1 \pm \sqrt{1 + 52}}{2}$$

$$x = \frac{1 \pm \sqrt{53}}{2}$$

$x \approx -3.140$ or $x \approx 4.140$

43. $f(x) = \dfrac{2x+1}{x-2}$

(a) $f(4) = \dfrac{2(4)+1}{4-2} = \dfrac{9}{2}$

(b) $f\left(-\dfrac{1}{2}\right) = \dfrac{2(-\frac{1}{2})+1}{-\frac{1}{2}-2} = \dfrac{-1+1}{\frac{5}{2}} = \dfrac{0}{\frac{5}{2}} = 0$

(c) $f(a) = \dfrac{2(a)+1}{(a)-2} = \dfrac{2a+1}{a-2}$

(d) $f\left(\dfrac{2}{m}\right) = \dfrac{2\left(\frac{2}{m}\right)+1}{\frac{2}{m}-2} = \dfrac{\frac{4}{m}+\frac{m}{m}}{\frac{2}{m}-\frac{2m}{m}} = \dfrac{\frac{4+m}{m}}{\frac{2-2m}{m}} = \dfrac{4+m}{m}\cdot\dfrac{m}{2-2m} = \dfrac{4+m}{2-2m}$

(e)
$$f(x) = 1$$
$$\frac{2x+1}{x-2} = 1$$
$$2x+1 = x-2$$
$$x = -3$$

44. $f(x) = \dfrac{3x+2}{2x-4}$

(a) $f(4) = \dfrac{3(4)+2}{2(4)-4} = \dfrac{12+2}{8-4} = \dfrac{14}{4} = \dfrac{7}{2}$

(b) $f\left(-\dfrac{1}{2}\right) = \dfrac{3\left(-\frac{1}{2}\right)+2}{2\left(-\frac{1}{2}\right)-4} = \dfrac{-\frac{3}{2}+2}{-1-4} = \dfrac{-\frac{3}{2}+\frac{4}{2}}{-5} = \dfrac{\frac{1}{2}}{-5} = -\dfrac{1}{10}$

(c) $f(a) = \dfrac{3(a)+2}{2(a)-4} = \dfrac{3a+2}{2a-4}$

(d) $f\left(\dfrac{2}{m}\right) = \dfrac{3\left(\frac{2}{m}\right)+2}{2\left(\frac{2}{m}\right)-4} = \dfrac{\frac{6}{m}+\frac{2m}{m}}{\frac{4}{m}-\frac{4m}{m}} = \dfrac{\frac{6+2m}{m}}{\frac{4-4m}{m}} = \dfrac{6+2m}{m}\cdot\dfrac{m}{4-4m} = \dfrac{6+2m}{4-4m} = \dfrac{3+m}{2-2m}$

(e)
$$f(x) = 1$$
$$\frac{3x+2}{2x-4} = 1$$
$$3x+2 = 2x-4$$
$$x = -6$$

45. The domain is all real numbers between the endpoints of the curve, or $[-2, 4]$.
The range is all real numbers between the minimum and maximum values of the function or $[0, 4]$.

(a) $f(-2) = 0$

(b) $f(0) = 4$

(c) $f\left(\dfrac{1}{2}\right) = 3$

(d) From the graph, $f(x) = 1$ when $x = -1.5,\ 1.5,$ or 2.5.

46. The domain is all real numbers between the end points of the curve, or $[-2, 4]$.
The range is all real numbers between the minimum and maximum values of the function or $[0, 5]$.

(a) $f(-2) = 5$ **(b)** $f(0) = 0$

(c) $f\left(\dfrac{1}{2}\right) = 1$

(d) From the graph, if $f(x) = 1,\ x = -0.2,\ 0.5,\ 1.2,$ or 2.8.

47. The domain is all real numbers between the endpoints of the curve, or $[-2, 4]$.
The range is all real numbers between the minimum and maximum values of the function or $[-3, 2]$.

(a) $f(-2) = -3$

(b) $f(0) = -2$

(c) $f\left(\dfrac{1}{2}\right) = -1$

(d) From the graph, $f(x) = 1$ when $x = 2.5$.

48. The domain is all real numbers between the end points of the curve, or $[-2, 4]$.
The range is all real numbers between the minimum and maximum values of the function, or in this case, $\{3\}$.

(a) $f(-2) = 3$

(b) $f(0) = 3$

(c) $f\left(\dfrac{1}{2}\right) = 3$

(d) From the graph, $f(x)$ is 1 nowhere.

49. $f(x) = 6x^2 - 2$

$$\begin{aligned} f(t+1) &= 6(t+1)^2 - 2 \\ &= 6(t^2 + 2t + 1) - 2 \\ &= 6t^2 + 12t + 6 - 2 \\ &= 6t^2 + 12t + 4 \end{aligned}$$

50.

$$\begin{aligned} f(x) &= 6x^2 - 2 \\ f(2-r) &= 6(2-r)^2 - 2 \\ &= 6(4 - 4r + r^2) - 2 \\ &= 24 - 24r + 6r^2 - 2 \\ &= 6r^2 - 24r + 22 \end{aligned}$$

51. $g(r+h)$

$$\begin{aligned} &= (r+h)^2 - 2(r+h) + 5 \\ &= r^2 + 2hr + h^2 - 2r - 2h + 5 \end{aligned}$$

52. $g(z-p)$

$$\begin{aligned} &= (z-p)^2 - 2(z-p) + 5 \\ &= z^2 - 2zp + p^2 - 2z + 2p + 5 \end{aligned}$$

53.

$$\begin{aligned} g\left(\frac{3}{q}\right) &= \left(\frac{3}{q}\right)^2 - 2\left(\frac{3}{q}\right) + 5 \\ &= \frac{9}{q^2} - \frac{6}{q} + 5 \\ \text{or} \quad & \frac{9 - 6q + 5q^2}{q^2} \end{aligned}$$

54.

$$\begin{aligned} g\left(-\frac{5}{z}\right) &= \left(-\frac{5}{z}\right)^2 - 2\left(-\frac{5}{z}\right) + 5 \\ &= \frac{25}{z^2} + \frac{10}{z} + 5 \\ &= \frac{25}{z^2} + \frac{10z}{z^2} + \frac{5z^2}{z^2} \\ &= \frac{25 + 10z + 5z^2}{z^2} \end{aligned}$$

55. A vertical line drawn anywhere through the graph will intersect the graph in only one place. The graph represents a function.

56. A vertical line drawn anywhere through the graph will intersect the graph in only one place. The graph represents a function.

57. A vertical line drawn through the graph may intersect the graph in two places. The graph does not represent a function.

58. A vertical line drawn through the graph may intersect the graph in two or more places. The graph does not represent a function.

59. A vertical line drawn anywhere through the graph will intersect the graph in only one place. The graph represents a function.

60. A vertical line is not a function since the one x-value in the domain corresponds to more than one, in fact, infinitely many y-values. The graph does not represent a function.

61. $f(x) = 2x + 1$

(a)

$$\begin{aligned} f(x+h) &= 2(x+h) + 1 \\ &= 2x + 2h + 1 \end{aligned}$$

(b) $f(x+h) - f(x)$

$$\begin{aligned} &= \frac{2x + 2h + 1}{2x + 1} \\ &= 2x + 2h + 1 - 2x - 1 \\ &= 2h \end{aligned}$$

(c) $\dfrac{f(x+h) - f(x)}{h}$

$$\begin{aligned} &= \frac{\frac{2x+2h+1}{2x+1}}{h} \\ &= \frac{2x + 2h + 1 - 2x - 1}{h} \\ &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

62. $f(x) = x^2 - 3$

(a)

$$\begin{aligned} f(x+h) &= (x+h)^2 - 3 \\ &= (x^2 + 2xh + h^2) - 3 \\ &= x^2 + 2xh + h^2 - 3 \end{aligned}$$

(b) $f(x+h) - f(x)$

$$\begin{aligned} &= (x^2 + 2xh + h^2 - 3) \\ &\quad - (x^2 - 3) \\ &= x^2 + 2xh + h^2 - 3 - x^2 + 3 \\ &= 2xh + h^2 \end{aligned}$$

(c)

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{2xh + h^2}{h} \\ &= \frac{2x + h}{h} \\ &= 2x + h \end{aligned}$$

63. $f(x) = 2x^2 - 4x - 5$

(a) $f(x+h)$
$$= 2(x+h)^2 - 4(x+h) - 5$$
$$= 2(x^2 + 2hx + h^2) - 4x - 4h - 5$$
$$= 2x^2 + 4hx + 2h^2 - 4x - 4h - 5$$

(b) $f(x+h) - f(x)$
$$= 2x^2 + 4hx + 2h^2 - 4x - 4h - 5 - (2x^2 - 4x - 5)$$
$$= 2x^2 + 4hx + 2h^2 - 4x - 4h - 5 - 2x^2 + 4x + 5$$
$$= 4hx + 2h^2 - 4h$$

(c) $\dfrac{f(x+h) - f(x)}{h}$
$$= \frac{4hx + 2h^2 - 4h}{h}$$
$$= \frac{h(4x + 2h - 4)}{h}$$
$$= 4x + 2h - 4$$

64. $f(x) = -4x^2 + 3x + 2$

(a) $f(x+h)$
$$= -4(x+h)^2 + 3(x+h) + 2$$
$$= -4(x^2 + 2hx + h^2) + 3x + 3h + 2$$
$$= -4x^2 - 8hx - 4h^2 + 3x + 3h + 2$$

(b) $f(x+h) - f(x)$
$$= -4x^2 - 8hx - 4h^2 + 3x + 3h + 2 - (-4x^2 + 3x + 2)$$
$$= -4x^2 - 8hx - 4h^2 + 3x + 3h + 2 + 4x^2 - 3x - 2$$
$$= -8hx - 4h^2 + 3h$$

(c)
$$\frac{f(x+h) - f(x)}{h} = \frac{-8hx - 4h^2 + 3h}{h}$$
$$= \frac{h(-8x - 4h + 3)}{h}$$
$$= -8x - 4h + 3$$

65. $f(x) = \dfrac{1}{x}$

(a) $f(x+h) = \dfrac{1}{x+h}$

(b) $f(x+h) - f(x)$
$$= \frac{1}{x+h} - \frac{1}{x}$$
$$= \left(\frac{x}{x}\right)\frac{1}{x+h} - \frac{1}{x}\left(\frac{x+h}{x+h}\right)$$
$$= \frac{x - (x+h)}{x(x+h)}$$
$$= \frac{-h}{x(x+h)}$$

(c) $\dfrac{f(x+h) - f(x)}{h}$
$$= \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$
$$= \frac{\frac{1}{x+h}\left(\frac{x}{x}\right) - \frac{1}{x}\left(\frac{x+h}{x+h}\right)}{h}$$
$$= \frac{\frac{x-(x+h)}{(x+h)x}}{h}$$
$$= \frac{1}{h}\left[\frac{x - x - h}{(x+h)x}\right]$$
$$= \frac{1}{h}\left[\frac{-h}{(x+h)x}\right]$$
$$= \frac{-1}{x(x+h)}$$

66. $f(x) = -\dfrac{1}{x^2}$

(a)
$$f(x+h) = -\frac{1}{(x+h)^2}$$
$$= -\frac{1}{x^2 + 2xh + h^2}$$

(b)
$$f(x+h) - f(x) = -\frac{1}{x^2 + 2xh + h^2} - \left(-\frac{1}{x^2}\right)$$
$$= -\frac{1}{x^2 + 2xh + h^2} + \frac{1}{x^2}$$
$$= -\frac{x^2}{x^2(x^2 + 2xh + h^2)} + \frac{(x^2 + 2xh + h^2)}{x^2(x^2 + 2xh + h^2)}$$
$$= \frac{-x^2 + x^2 + 2xh + h^2}{x^2(x^2 + 2xh + h^2)}$$
$$= \frac{2xh + h^2}{x^2(x^2 + 2xh + h^2)}$$

(c) $$\frac{f(x+h)-f(x)}{h} = \frac{\frac{h(2x+h)}{x^2(x^2+2xh+h^2)}}{h} = \frac{2x+h}{x^2(x^2+2xh+h^2)}$$

67. $$\begin{aligned} f(x) &= 3x \\ f(-x) &= 3(-x) \\ &= -(3x) \\ &= -f(x) \end{aligned}$$

The function is odd.

68. $$\begin{aligned} f(x) &= 5x \\ f(-x) &= 5(-x) \\ &= -(5x) \\ &= -f(x) \end{aligned}$$

The function is odd.

69. $$\begin{aligned} f(x) &= 2x^2 \\ f(-x) &= 2(-x)^2 \\ &= 2x^2 \\ &= f(x) \end{aligned}$$

The function is even.

70. $$\begin{aligned} f(x) &= x^2-3 \\ f(-x) &= (-x)^2-3 \\ &= x^2-3 \\ &= f(x) \end{aligned}$$

The function is even.

71. $$\begin{aligned} f(x) &= \frac{1}{x^2+4} \\ f(-x) &= \frac{1}{(-x)^2+4} \\ &= \frac{1}{x^2+4} \\ &= f(x) \end{aligned}$$

The function is even.

72. $$\begin{aligned} f(x) &= x^3+x \\ f(-x) &= (-x)^3+(-x) \\ &= -x^3-x \\ &= -(x^3+x) \\ &= -f(x) \end{aligned}$$

The function is odd.

73. $$\begin{aligned} f(x) &= \frac{x}{x^2-9} \\ f(-x) &= \frac{-x}{(-x)^2-9} \\ &= -\frac{x}{x^2-9} \\ &= -f(x) \end{aligned}$$

The function is odd.

74. $$\begin{aligned} f(x) &= |x-2| \\ f(-x) &= |-x-2| \\ &= |-(x+2)| \\ &= |x+2| \end{aligned}$$

$f(-x)$ does not equal either $f(x)$ or $f(-x)$. The function is neither even nor odd.

75. **(a)** The independent variable is the years.

(b) The dependent variable is the number of Internet users.

(c) $f(2003) = 719$ million users.

(d) The domain is $1995 \le x \le 2006$.
The range is $16{,}000{,}000 \le y \le 1{,}043{,}000{,}000$.

76. If x is a whole number of days, the cost of renting a saw in dollars is $S(x) = 28x + 8$. For x in whole days and a fraction of a day, substitute the next whole number for x in $28x + 8$, because a fraction of a day is charged as a whole day.

(a) $S\left(\frac{1}{2}\right) = S(1) = 28(1) + 8 = 36$

The cost is \$36.

(b) $S(1) = 28(1) + 8 = 36$

The cost is \$36.

(c) $S\left(1\frac{1}{4}\right) = S(2) = 28(2) + 8 = 56 + 8 = 64$

The cost is \$64.

(d) $S\left(3\frac{1}{2}\right) = S(4) = 28(4) + 8 = 112 + 8 = 120$

The cost is \$120.

(e) $S(4) = 28(4) + 8 = 112 + 8 = 120$

The cost is \$120.

(f) $S\left(4\frac{1}{10}\right) = S(5) = 28(5) + 8 = 140 + 8 = 148$

The cost is \$148.

(g) $S\left(4\frac{9}{10}\right) = S(5) = 28(5) + 8 = 140 + 8 = 148$

The cost is \$148.

(h) To continue the graph, continue the horizontal bars up and to the right.

(i) The independent variable is x, the number of full and partial days.

(j) The dependent variable is S, the cost of renting a saw.

77. If x is a whole number of days, the cost of renting a car is given by

$$C(x) = 54x + 44.$$

For x in whole days plus a fraction of a day, substitute the next whole number for x in $54x + 44$ because a fraction of a day is charged as a whole day.

(a) $C\left(\frac{3}{4}\right) = C(1) = 54(1) + 44 = \$98$

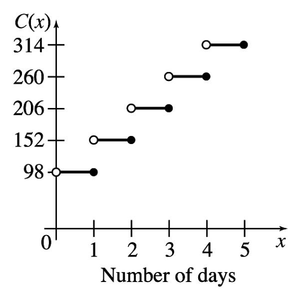
(b) $C\left(\frac{9}{10}\right) = C(1) = \$98$

(c) $C(1) = \$98$

(d) $C\left(1\frac{5}{8}\right) = C(2) = 54(2) + 44 = \$152$

(e) $C(2.4) = C(3) = 54(3) + 44 = \$206$

(f)



(g) Yes, C is a function.

(h) No, C is not a linear function.

78. **(a)** The curve in the graph crosses the point with x-coordinate 17:37 and y-coordinate of approximately 140. So, at time 17 hours, 37 minutes the whale reaches a depth of about 140 m.

(b) The curve in the graph crosses the point with x-coordinate 17:39 and y-coordinate of approximately 240. So, at time 17 hours, 39 minutes the whale reaches a depth of about 240 m.

79. **(a)** **(i)** $y = f(5) = 19.7(5)^{0.753} \approx 66$ kcal/day

(ii) $y = f(25) = 19.7(25)^{0.753} \approx 222$ kcal/day

(b) Since 1 pound equals 0.454 kg, then $x = g(z) = 0.454z$ is the number of kilograms equal in weight to z pounds.

80. **(a)(i)** By the given function f, a muskrat weighing 800 g expends

$$f(800) = 0.01(800)^{0.88} \approx 3.6, \text{ or approximately}$$

3.6 kcal/km when swimming at the surface of the water.

(ii) A sea otter weighing 20,000 g expends

$$f(20{,}000) = 0.01(20{,}000)^{0.88} \approx 61, \text{ or approximately}$$

61 kcal/km when swimming at the surface of the water.

(b) If z is the number of kilogram of an amimal's weight, then $x = g(z) = 1000z$ is the number of grams since 1 kilogram equals 1000 grams.

81. **(a)** In the graph, the curves representing wood and coal intersect approximately at the point $(1880, 50)$. So, in 1880 use of wood and coal were both about 50% of the global energy consumption.

(b) In the graph, the curves representing oil and coal intersect approximately at the point $(1965, 35)$. So, in 1965 use of oil and coal were both about 35% of the global energy consumption.

82. **(a)** $P = 2L + 2W$

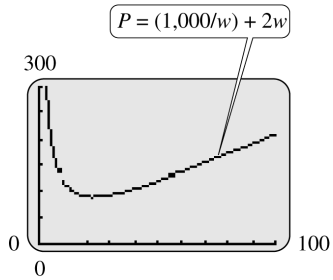
However, $LW = 500$, so $L = \dfrac{500}{W}$.

$$P(W) = 2\left(\frac{500}{W}\right) + 2W$$

$$P(W) = \frac{1000}{W} + 2W$$

(b) Since $L = \frac{500}{W}$, $W \neq 0$ but W could be any positive value. Therefore, the domain of P is $0 < W < \infty$.

(c)



83. **(a)** Let $w =$ the width of the field;
$l =$ the length.

The perimeter of the field is 6000 ft, so

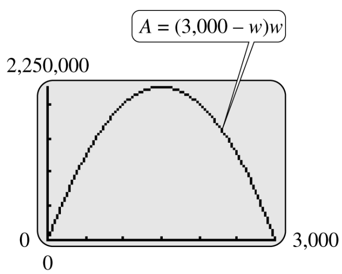
$$2l + 2w = 6000$$
$$l + w = 3000$$
$$l = 3000 - w.$$

Thus, the area of the field is given by

$$A = lw$$
$$A = (3000 - w)w.$$

(b) Since $l = 3000 - w$ and w cannot be negative, $0 \le w \le 3000$.
The domain of A is $0 \le w \le 3000$.

(c)



2.2 Quadratic Functions; Translation and Reflection

3. The graph of $y = x^2 - 3$ is the graph of $y = x^2$ translated 3 units downward.
This is graph d.

4. The graph of $y = (x - 3)^2$ is the graph of $y = x^2$ translated 3 units to the right.
This is graph f.

5. The graph of $y = (x - 3)^2 + 2$ is the graph of $y = x^2$ translated 3 units to the right and 2 units upward.
This is graph a.

6. The graph of $y = (x+3)^2+2$ is the graph of $y = x^2$ translated 3 units to the left and 2 units upward.
This is graph b.

7. The graph of $y = -(3 - x)^2 + 2$ is the same as the graph of $y = -(x - 3)^2 + 2$. This is the graph of $y = x^2$ reflected in the x-axis, translated 3 units to the right, and translated 2 units upward.
This is graph c.

8. The graph of $y = -(x + 3)^2 + 2$ is the graph of $y = x^2$ reflected in the x-axis, translated three units to the left, and two units upward.
This is graph e.

9. $y = x^2 + 5x + 6$
$y = (x + 3)(x + 2)$

Set $y = 0$ to find the x-intercepts.

$$0 = (x + 3)(x + 2)$$
$$x = -3, \ x = -2$$

The x-intercepts are -3 and -2.
Set $x = 0$ to find the y-intercept.

$$y = 0^2 + 5(0) + 6$$
$$y = 6$$

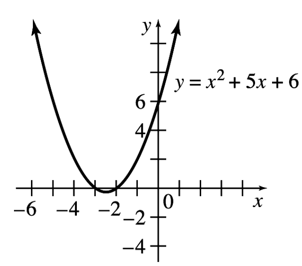
The y-intercept is 6.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-5}{2} = -\frac{5}{2}.$$

Substitute to find the y-coordinate.

$$y = (-\frac{5}{2})^2 + 5(-\frac{5}{2}) + 6 = \frac{25}{4} - \frac{25}{2} + 6 = -\frac{1}{4}$$

The vertex is $(-\frac{5}{2}, -\frac{1}{4})$.
The axis is $x = -\frac{5}{2}$, the vertical line through the vertex.



10. $y = x^2 + 4x - 5$
$= (x+5)(x-1)$

Set $y = 0$ to find the x-intercepts.

$$0 = (x+5)(x-1)$$
$$x = -5 \text{ or } x = 1$$

The x-intercepts are -5 and 1.
Set $x = 0$ to find the y-intercept.

$$y = 0^2 + 4(0) - 5$$
$$y = -5$$

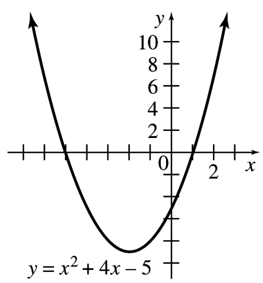
The y-intercept is -5.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-4}{2} = -2.$$

Substitute to find the y-coordinate.

$$\begin{aligned} y &= (-2)^2 + 4(-2) - 5 \\ &= 4 - 8 - 5 \\ &= -9 \end{aligned}$$

The vertex is $(-2, -9)$.
The axis is $x = -2$, the vertical line through the vertex.



11. $y = -2x^2 - 12x - 16$
$= -2(x^2 + 6x + 8)$
$= -2(x+4)(x+2)$

Let $y = 0$.

$$0 = -2(x+4)(x+2)$$
$$x = -4, \ x = -2$$

-4 and -2 are the x-intercepts.
Let $x = 0$.

$$y = -2(0)^2 + 12(0) - 16$$

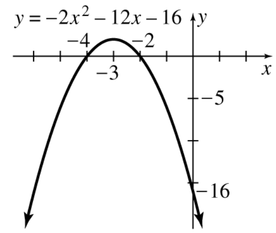
-16 is the y-intercept.

Vertex: $x = \dfrac{-b}{2a} = \dfrac{12}{-4} = -3$

$$\begin{aligned} y &= -2(-3)^2 - 12(-3) - 16 \\ &= -18 + 36 - 16 = 2 \end{aligned}$$

The vertex is $(-3, 2)$.

The axis is $x = -3$, the vertical line through the vertex.



12. $y = -3x^2 - 6x + 4$

Let $y = 0$.

The equation

$$0 = -3x^2 - 6x + 4$$

cannot be solved by factoring.
Use the quadratic formula.

$$\begin{aligned} 0 &= -3x^2 - 6x + 4 \\ x &= \frac{6 \pm \sqrt{6^2 - 4(-3)(4)}}{2(-3)} \\ &= \frac{6 \pm \sqrt{36+48}}{-6} \\ &= \frac{6 \pm \sqrt{84}}{-6} = \frac{6 \pm 2\sqrt{21}}{-6} \\ &= -1 \pm \frac{\sqrt{21}}{3} \end{aligned}$$

The x-intercepts are $-1 + \frac{\sqrt{21}}{3} \approx 0.53$ and

$-1 - \frac{\sqrt{21}}{3} \approx -2.53$

Let $x = 0$.

$$y = -3(0)^2 - 6(0) + 4$$
$$y = 4$$

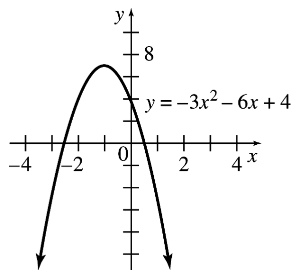
4 is the y-intercept.

Vertex: $x = \dfrac{-b}{2a} = \dfrac{6}{2(-3)} = \dfrac{6}{-6} = -1$

$$\begin{aligned} y &= -3(-1)^2 - 6(-1) + 4 \\ &= -3 + 6 + 4 \\ &= 7 \end{aligned}$$

The vertex is $(-1, 7)$.

The axis is $x = -1$, the vertical line through the vertex.



13. $y = 2x^2 + 8x - 8$

Let $y = 0$.

$$2x^2 + 8x - 8 = 0$$
$$x^2 + 4x - 4 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-4)}}{2(1)} = \frac{-4 \pm \sqrt{32}}{2} = \frac{-4 \pm 4\sqrt{2}}{2} = -2 \pm 2\sqrt{2}$$

The x-intercepts are $-2 \pm 2\sqrt{2} \approx 0.83$ or -4.83.
Let $x = 0$.

$$y = 2(0)^2 + 8(0) - 8 = -8$$

The y-intercept is -8.

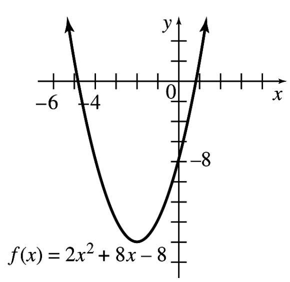
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = -\frac{8}{4} = -2.$$

If $x = -2$,

$$y = 2(-2)^2 + 8(-2) - 8 = 8 - 16 - 8 = -16.$$

The vertex is $(-2, -16)$.
The axis is $x = -2$.



14. $f(x) = -x^2 + 6x - 6$

Let $f(x) = 0$.

$0 = -x^2 + 6x - 6$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-6)}}{2(-1)} = \frac{-6 \pm \sqrt{36 - 24}}{-2} = \frac{-6 \pm \sqrt{12}}{-2} = \frac{-6 \pm 2\sqrt{3}}{-2} = 3 \pm \sqrt{3}$$

The x-intercepts are $3 + \sqrt{3} \approx 4.73$ and $3 - \sqrt{3} \approx 1.27$.

Let $x = 0$.

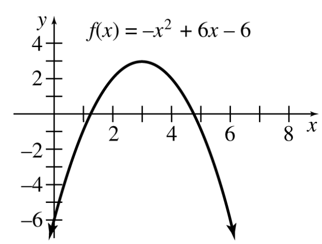
$$y = -0^2 + 6(0) - 6$$

-6 is the y-intercept.

Vertex: $x = \frac{-b}{2a} = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$

$$y = -3^2 + 6(3) - 6 = -9 + 18 - 6 = 3$$

The vertex is $(3, 3)$.
The axis is $x = 3$.



15. $f(x) = 2x^2 - 4x + 5$

Let $f(x) = 0$.

$0 = 2x^2 - 4x + 5$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(5)}}{2(2)} = \frac{4 \pm \sqrt{16 - 40}}{4} = \frac{4 \pm \sqrt{-24}}{4}$$

Since the radicand is negative, there are no x-intercepts.

Let $x = 0$.

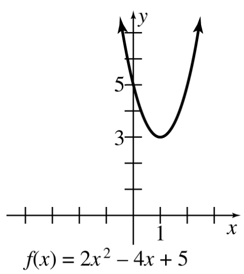
$$y = 2(0)^2 - 4(0) + 5$$
$$y = 5$$

5 is the y-intercept.

Vertex: $x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$

$$y = 2(1)^2 - 4(1) + 5 = 2 - 4 + 5 = 3$$

The vertex is $(1, 3)$.
The axis is $x = 1$.



16. $f(x) = \frac{1}{2}x^2 + 6x + 24$

Let $f(x) = 0$.

$$0 = \frac{1}{2}x^2 + 6x + 24$$

Multiply by 2 to clear fractions.

$$0 = x^2 + 12x + 48$$

Use the quadratic formula.

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(48)}}{2(1)} = \frac{-12 \pm \sqrt{144 - 192}}{2} = \frac{-12 \pm \sqrt{-48}}{2}$$

Since the radicand is negative, there are no x-intercepts.
Let $x = 0$.

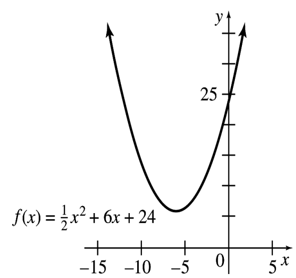
$$y = \frac{1}{2}(0)^2 + 6(0) + 24$$
$$y = 24$$

24 is the y-intercept.

Vertex: $x = \frac{-b}{2a} = \frac{-6}{2\left(\frac{1}{2}\right)} = \frac{-6}{1} = -6$

$$y = \frac{1}{2}(-6)^2 + 6(-6) + 24$$
$$= 18 - 36 + 24 = 6$$

The vertex is $(-6, 6)$.
The axis $x = -6$.



17. $f(x) = -2x^2 + 16x - 21$

Let $f(x) = 0$
Use the quadratic formula.

$$x = \frac{-16 \pm \sqrt{16^2 - 4(-2)(-21)}}{2(-2)} = \frac{-16 \pm \sqrt{88}}{-4} = \frac{-16 \pm 2\sqrt{22}}{-4} = 4 \pm \frac{\sqrt{22}}{2}$$

The x-intercepts are $4 + \frac{\sqrt{22}}{2} \approx 6.35$ and $4 - \frac{\sqrt{22}}{2} \approx 1.65$.

Let $x = 0$.

$$y = -2(0)^2 + 16(0) - 21$$
$$y = -21$$

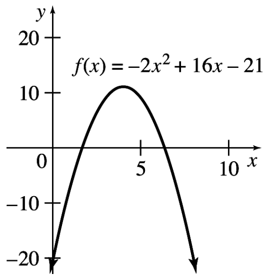
-21 is the y-intercept.

Vertex: $x = \frac{-b}{2a} = \frac{-16}{2(-2)} = \frac{-16}{-4} = 4$

$$y = -2(4)^2 + 16(4) - 21$$
$$= -32 + 64 - 21 = 11$$

The vertex is $(4, 11)$.

The axis is $x = 4$.



18. $f(x) = \dfrac{3}{2}x^2 - x - 4$

Let $f(x) = 0$.

$$0 = \frac{3}{2}x^2 - x - 4$$

Multiply by 2 to clear fractions, then solve.

$$\begin{aligned} 0 &= 3x^2 - 2x - 8 \\ &= (3x + 4)(x - 2) \\ x &= -\frac{4}{3}, 2 \end{aligned}$$

The x-intercepts are $-\frac{4}{3}$ and 2.

Let $x = 0$.

$$y = \frac{3}{2}(0)^2 - 0 - 4$$

$$y = -4$$

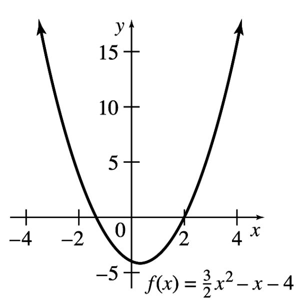
The y-intercept is -4.

Vertex: $x = \dfrac{-b}{2a} = \dfrac{-(-1)}{2\left(\frac{3}{2}\right)} = \dfrac{1}{3}$

$$y = \frac{3}{2}\left(\frac{1}{3}\right)^2 - \frac{1}{3} - 4 = \frac{1}{6} - \frac{2}{6} - \frac{24}{6} = -\frac{25}{6}$$

The vertex is $\left(\frac{1}{3}, -\frac{25}{6}\right)$.

The axis $x = \frac{1}{3}$.



19. $y = \dfrac{1}{3}x^2 - \dfrac{8}{3}x + \dfrac{1}{3}$

Let $y = 0$.

$$0 = \frac{1}{3}x^2 - \frac{8}{3}x + \frac{1}{3}$$

Multiply by 3.

$0 = x^2 - 8x + 1$

$$\begin{aligned} x &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(1)}}{2(1)} \\ &= \frac{8 \pm \sqrt{64 - 4}}{2} = \frac{8 \pm \sqrt{60}}{2} \\ &= \frac{8 \pm 2\sqrt{15}}{2} = 4 \pm \sqrt{15} \end{aligned}$$

The x-intercepts are $4 + \sqrt{15} \approx 7.87$ and $4 - \sqrt{15} \approx 0.13$.
Let $x = 0$.

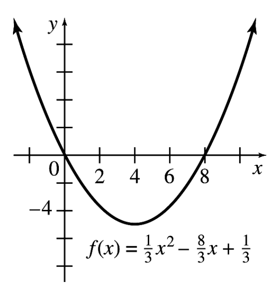
$$y = \frac{1}{3}(0)^2 - \frac{8}{3}(0) + \frac{1}{3}$$

$\frac{1}{3}$ is the y-intercept.

Vertex: $x = \dfrac{-b}{2a} = \dfrac{-\left(-\frac{8}{3}\right)}{2\left(\frac{1}{3}\right)} = \dfrac{\frac{8}{3}}{\frac{2}{3}} = 4$

$$\begin{aligned} y &= \frac{1}{3}(4)^2 - \frac{8}{3}(4) + \frac{1}{3} \\ &= \frac{16}{3} - \frac{32}{3} + \frac{1}{3} = -\frac{15}{3} = -5 \end{aligned}$$

The vertex is $(4, -5)$.
The axis is $x = 4$.



20. $y = -\frac{1}{2}x^2 - x - \frac{7}{2}$

Let $y = 0$.

$$0 = -\frac{1}{2}x^2 - x - \frac{7}{2}$$

Multiply by -2 to clear fractions.

$$0 = x^2 + 2x + 7$$

Use the quadratic formula.

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{4 - 4(1)(7)}}{2} \\ &= \frac{-2 \pm \sqrt{-24}}{2} \end{aligned}$$

Since the radicand is negative, there are no x-intercepts.

Let $x = 0$.

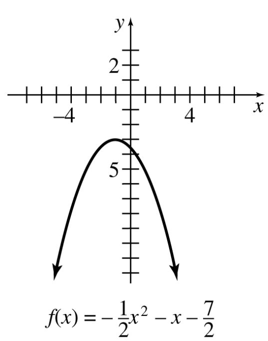
$$y = -\frac{1}{2}(0)^2 - 0 - \frac{7}{2} = -\frac{7}{2}$$

$-\frac{7}{2}$ is the y-intercept.

Vertex: $\frac{-b}{2a} = \frac{-(-1)}{2\left(-\frac{1}{2}\right)} = \frac{1}{-1} = -1$

$$\begin{aligned} y &= -\frac{1}{2}(-1)^2 - (-1) - \frac{7}{2} \\ &= -\frac{1}{2} + 1 - \frac{7}{2} \\ &= -3 \end{aligned}$$

The vertex is $(-1, -3)$.
The axis is $x = -1$.



21. The graph of $y = \sqrt{x+2} - 4$ is the graph of $y = \sqrt{x}$ translated 2 units to the left and 4 units downward.
This is graph d.

22. The graph of $y = \sqrt{x-2} - 4$ is the graph of $y = \sqrt{x}$ translated 2 units to the right and 4 units downward.
This is graph a.

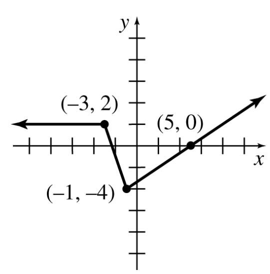
23. The graph of $y = \sqrt{-x+2} - 4$ is the graph of $y = \sqrt{-(x-2)} - 4$, which is the graph of $y = \sqrt{x}$ reflected in the y-axis, translated 2 units to the right, and translated 4 units downward.
This is graph c.

24. The graph of $y = \sqrt{-x-2} - 4$ is the same as the graph of $y = \sqrt{-(x+2)} - 4$. This is the graph of $y = \sqrt{x}$ reflected in the y-axis, translated 2 units to the left, and translated 4 units downward.
This is graph b.

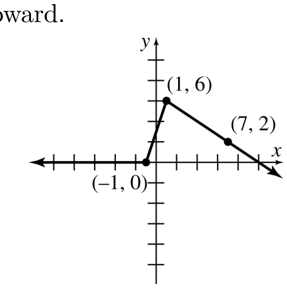
25. The graph of $y = -\sqrt{x+2} - 4$ is the graph of $y = \sqrt{x}$ reflected in the x-axis, translated 2 units to the left, and translated 4 units downward.
This is graph e.

26. The graph of $y = -\sqrt{x-2} - 4$ is the graph of $y = \sqrt{x}$ reflected in the x-axis, translated 2 units to the right, and translated 4 units downward.
This is graph f.

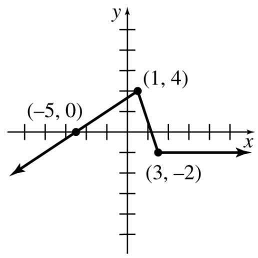
27. The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected in the x-axis.



28. The graph of $y = f(x-2) + 2$ is the graph of $y = f(x)$ translated 2 units to the right and 2 units upward.

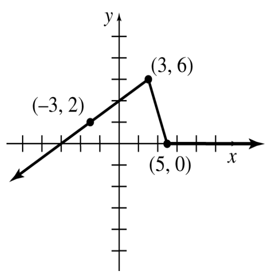


29. The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected in the y-axis.



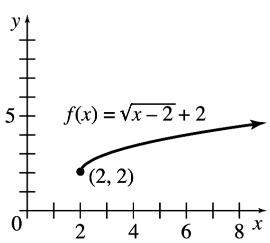
30. $y = f(2 - x) + 2$
$y = f[-(x - 2)] + 2$

This is the graph of $y = f(x)$ reflected in the y-axis, translated 2 units to the right, and translated 2 units upward.



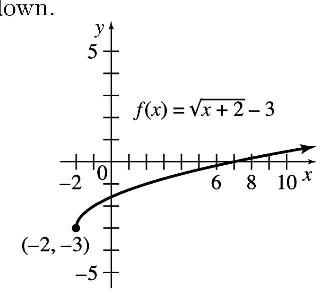
31. $f(x) = \sqrt{x - 2} + 2$

Translate the graph of $f(x) = \sqrt{x}$ 2 units right and 2 units up.



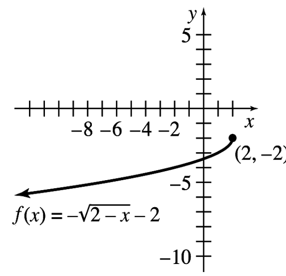
32. $f(x) = \sqrt{x + 2} - 3$

Translate the graph of $f(x) = \sqrt{x}$ 2 units left and 3 units down.



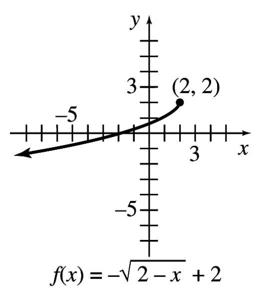
33. $f(x) = -\sqrt{2 - x} - 2$
$= -\sqrt{-(x - 2)} - 2$

Reflect the graph of $f(x)$ vertically and horizontally.
Translate the graph 2 units right and 2 units down.

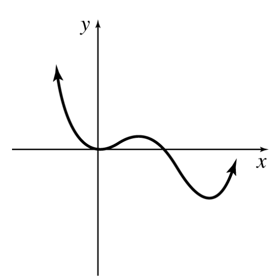


34. $f(x) = -\sqrt{2 - x} + 2 = -\sqrt{-(x - 2)} + 2$

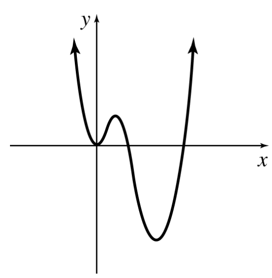
Reflect the graph of $f(x)$ vertically and horizontally.
Translate the graph 2 units right and 2 units up.



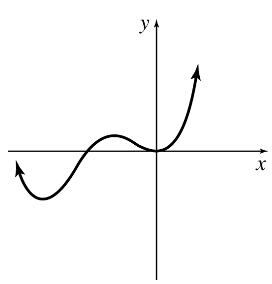
35. If $0 < a < 1$, the graph of $f(ax)$ will be flatter and wider than the graph of $f(x)$.
Multiplying x by a fraction makes the y-values less than the original y-values.



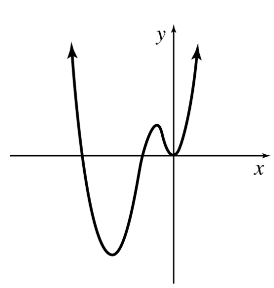
36. If $1 < a$, the graph of $f(ax)$ will be taller and thinner than the graph of $f(x)$. Multiplying x by a constant greater than 1 pairs x-values of smaller absolute value with y-values of points for which the x-values have larger absolute value.



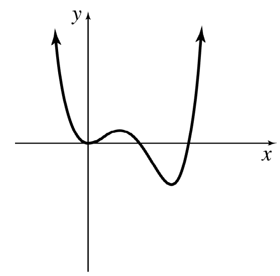
37. If $-1 < a < 0$, the graph of $f(ax)$ will be reflected horizontally, since a is negative. It will be flatter because multiplying x by a fraction decreases the corresponding y-values.



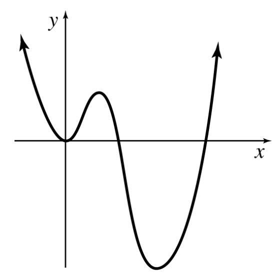
38. If $a < -1$, the graph of $f(ax)$ will be reflected horizontally, since a is negative. It will also be taller and thinner.



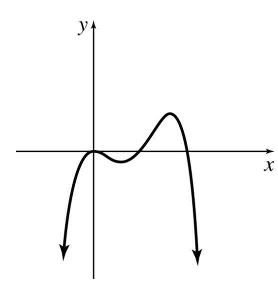
39. If $0 < a < 1$, the graph of $af(x)$ will be flatter and wider than the graph of $f(x)$. Each y-value is only a fraction of the height of the original y-values.



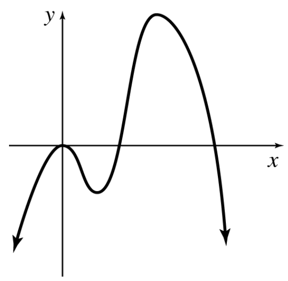
40. If $1 < a$ (or $a > 1$), the graph of $af(x)$ will be taller than the graph of $f(x)$. The absolute value of the y-value will be larger than the original y-values, while the x-values will remain the same.



41. If $-1 < a < 0$, the graph will be reflected vertically, since a will be negative. Also, because a is a fraction, the graph will be flatter because each y-value will only be a fraction of its original height.



42. If $a < -1$, the graph of $af(x)$ will be reflected vertically. It will also be taller than the graph of $f(x)$ since the absolute value of each y-value will be larger than the original y-values, while the x-values stay the same.



43. **(a)** Since the graph of $y = f(x)$ is reflected vertically to obtain the graph of $y = -f(x)$, the x-intercept is unchanged. The x-intercept of the graph of $y = f(x)$ is r.

(b) Since the graph of $y = f(x)$ is reflected horizontally to obtain the graph of $y = f(-x)$, the x-intercept of the graph of $y = f(-x)$ is $-r$.

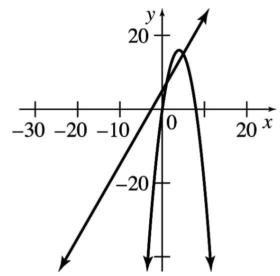
(c) Since the graph of $y = f(x)$ is reflected both horizontally and vertically to obtain the graph of $y = -f(-x)$, the x-intercept of the graph of $y = -f(-x)$ is $-r$.

44. **(a)** Since the graph of $y = f(x)$ is reflected vertically to obtain the graph of $y = -f(x)$, the y-intercept of the graph of $y = -f(x)$ is $-b$.

(b) Since the graph of $y = f(x)$ is reflected horizontally to obtain the graph of $y = f(-x)$, the y-intercept is unchanged. The y-intercept of the graph of $y = f(-x)$ is b.

(c) Since the graph of $y = f(x)$ is reflected both horizontally and vertically to obtain the graph of $y = -f(-x)$, the y-intercept of the graph of $y = -f(-x)$ is $-b$.

45. **(a)**



(b) Break-even quantities are values of x = number of widgets for which revenue and cost are equal. Set $R(x) = C(x)$ and solve for x.

$$\begin{aligned} -x^2 + 8x &= 2x + 5 \\ x^2 - 6x + 5 &= 0 \\ (x-5)(x-1) &= 0 \\ x - 5 = 0 \quad \text{or} \quad x - 1 &= 0 \\ x = 5 \quad \text{or} \quad x &= 1 \end{aligned}$$

So, the break-even quantities are 1 and 5. The minimum break-even quantity is $x = 1$.

(c) The maximum revenue occurs at the vertex of R. Since $R(x) = -x^2 + 8x$, then the x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{8}{2(-1)} = 4.$$

So, the maximum revenue is

$$R(4) = -4^2 + 8(4) = 16.$$

(d) The maximum profit is the maximum difference $R(x) - C(x)$. Since

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -x^2 + 8x - (2x + 5) \\ &= -x^2 + 6x - 5 \end{aligned}$$

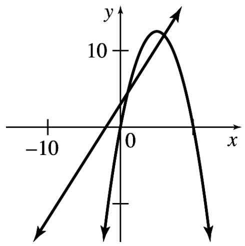
is a quadratic function, we can find the maximum profit by finding the vertex of P. This occurs at

$$x = -\frac{b}{2a} = \frac{-6}{2(-1)} = 3.$$

Therefore, the maximum profit is

$$P(3) = -(3)^2 + 6(3) - 5 = 4.$$

46. **(a)**



(b) Break-even quantities are values of x = number of widgets for which revenue and cost are equal. Set $R(x) = C(x)$ and solve for x.

$$\begin{aligned} -\frac{x^2}{2} + 5x &= \frac{3}{2}x + 3 \\ -x^2 + 10x &= 3x + 6 \\ x^2 - 7x + 6 &= 0 \\ (x-1)(x-6) &= 0 \\ x - 1 = 0 \quad &\text{or} \quad x - 6 = 0 \\ x = 1 \quad &\text{or} \quad x = 6 \end{aligned}$$

So, the break-even quantities are 1 and 6. The minimum break-even quantity is $x = 1$.

(c) The maximum revenue occurs at the vertex of R. Since $R(x) = -\frac{x^2}{2} + 5x$, then the x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-5}{2\left(-\frac{1}{2}\right)} = 5.$$

So, the maximum revenue is

$$R(8) = -\frac{5^2}{2} + 5(5) = 12.5.$$

(d) The maximum profit is the maximum difference $R(x) - C(x)$. Since

$$\begin{aligned} P(x) = R(x) - C(x) &= -\frac{x^2}{2} + 5x - \left(\frac{3}{2}x + 3\right) \\ &= -\frac{x^2}{2} + \frac{7}{2}x - 3 \end{aligned}$$

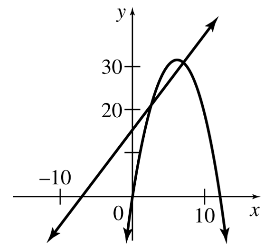
is a quadratic function, we can find the maximum profit by finding the vertex of P. This occurs at

$$x = -\frac{b}{2a} = \frac{-\frac{7}{2}}{2\left(-\frac{1}{2}\right)} = \frac{7}{2}.$$

Therefore, the maximum profit is

$$P\left(\frac{7}{2}\right) = \frac{-\left(\frac{7}{2}\right)^2}{2} + \frac{7}{2}\left(\frac{7}{2}\right) - 3 = 3.125.$$

47. **(a)**



(b) Break-even quantities are values of x = number of widgets for which revenue equals cost.
Set $R(x) = C(x)$ and solve for x.

$$\begin{aligned} -\frac{4}{5}x^2 + 10x &= 2x + 15 \\ \frac{4}{5}x^2 - 8x + 15 &= 0 \\ 4x^2 - 40x + 75 &= 0 \\ 4x^2 - 10x - 30x + 75 &= 0 \\ 2x(2x-5) - 15(2x-5) &= 0 \\ (2x-5)(2x-15) &= 0 \\ 2x - 5 = 0 \quad &\text{or} \quad 2x - 15 = 0 \\ x = 2.5 \quad &\text{or} \quad x = 7.5 \end{aligned}$$

So, the break-even quantities are 2.5 and 7.5 with $x = 2.5$ the minimum break-even quantity.

(c) The maximum revenue occurs at the vertex of R. Since $R(x) = -\frac{4}{5}x^2 + 10x$, then the x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{10}{2\left(\frac{-4}{5}\right)} = 6.25.$$

So, the maximum revenue is

$$R(6.25) = 31.25.$$

(d) The maximum profit is the maximum difference $R(x) - C(x)$. Since

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -\frac{4}{5}x^2 + 10x - (2x + 15) \\ &= -\frac{4}{5}x^2 + 8x - 15 \end{aligned}$$

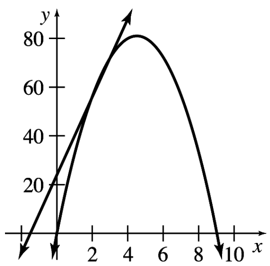
is a quadratic function, we can find the maximum profit by finding the vertex of P. This occurs at

$$x = -\frac{b}{2a} = -\frac{8}{2\left(\frac{-4}{5}\right)} = 5.$$

Therefore, the maximum profit is

$$P(5) = -\frac{4}{5}5^2 + 8(5) - 15 = 5.$$

48. **(a)**



(b) Break-even quantities are values of x for which revenue equals cost.
Set $R(x) = C(x)$ and solve for x.

$$\begin{aligned}-4x^2 + 36x &= 16x + 24\\ 4x^2 - 20x + 24 &= 0\\ 4(x-2)(x-3) &= 0\\ x - 2 = 0 \quad \text{or} \quad x - 3 &= 0\\ x = 2 \quad \text{or} \quad x &= 3\end{aligned}$$

So, the break-even quantites are 2 and 3. The minimum break-even quantity is $x = 2$.

(c) The maximum revenue occurs at the vertex of R. Since $R(x) = -4x^2 + 36x$, then x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-36}{2(-4)} = \frac{9}{2}.$$

So, the maximum revenue is

$$R(5) = -4\left(\frac{9}{2}\right)^2 + 36\left(\frac{9}{2}\right) = 81.$$

(d) The maximum profit is the maximum difference $R(x) - C(x)$.

$$\begin{aligned}\text{Since} \quad P(x) &= R(x) - C(x)\\ &= -4x^2 + 36x - (16x + 24)\\ &= -4x^2 + 20x - 24\end{aligned}$$

is a quadratic function, we can find the maximum profit by finding the vertex of P. This occurs at

$$x = -\frac{b}{2a} = -\frac{20}{2(-4)} = \frac{5}{2} = 2.5.$$

Therefore, the maximum profit is

$$P\left(\frac{5}{2}\right) = -4\left(\frac{5}{2}\right)^2 + 20\left(\frac{5}{2}\right) - 24 = 1.$$

49. $R(x) = 8000 + 70x - x^2$
$\quad = -x^2 + 70x + 8000$

The maximum revenue occurs at the vertex.

$$x = \frac{-b}{2a} = \frac{-70}{2(-1)} = 35$$

$$\begin{aligned}y &= 8000 + 70(35) - (35)^2\\ &= 8000 + 2450 - 1225\\ &= 9225\end{aligned}$$

The vertex is $(35, 9225)$.
The maximum revenue of \$9225 is realized when 35 seats are left unsold.

50. **(a)** The revenue is

$R(x) =$ (Price per ticket)
$\quad\cdot$ (Number of people flying).

Number of people flying $= 100 - x$
Price per ticket $= 200 + 4x$

$$\begin{aligned}R(x) &= (200 + 4x)(100 - x)\\ &= 20{,}000 + 200x - 4x^2\end{aligned}$$

(b) $R(x) = -4x^2 + 200x + 20{,}000$

x-intercepts:

$$\begin{aligned}0 &= (200 + 4x)(100 - x)\\ x &= -50 \quad \text{or} \quad x = 100\end{aligned}$$

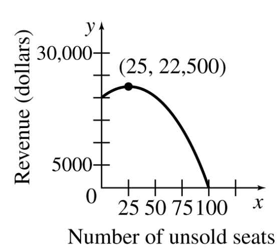
y-intercept:

$$\begin{aligned}y &= -4(0)^2 + 200(0) + 20{,}000\\ &= 20{,}000\end{aligned}$$

Vertex: $x = \dfrac{-b}{2a} = \dfrac{-200}{-8} = 25$

$y = -4(25)^2 + 200(25) + 20{,}000 = 22{,}500$

This is a parabola which opens downward. The vertex is at (25, 22,500).



(c) The maximum revenue occurs at the vertex, (25, 22,500).
This will happen when $x = 25$, or there are 25 unsold seats.

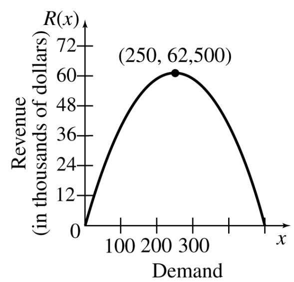
(d) The maximum revenue is \$22,500, as seen from the graph.

51. $p = 500 - x$

(a) The revenue is

$$\begin{aligned} R(x) &= px \\ &= (500 - x)(x) \\ &= 500x - x^2. \end{aligned}$$

(b)



(c) From the graph, the vertex is halfway between $x = 0$ and $x = 500$, so $x = 250$ units corresponds to maximum revenue. Then the price is

$$\begin{aligned} p &= 500 - x \\ &= 500 - 250 = \$250. \end{aligned}$$

Note that price, p, cannot be read directly from the graph of

$$R(x) = 500x - x^2.$$

(d) $$\begin{aligned} R(x) &= 500x - x^2 \\ &= -x^2 + 500x \end{aligned}$$

Find the vertex.

$$x = \frac{-b}{2a} = \frac{-500}{2(-1)} = 250$$

$$\begin{aligned} y &= -(250)^2 + 500(250) \\ &= 62{,}500 \end{aligned}$$

The vertex is (250, 62,500).
The maximum revenue is \$62,500.

52. Let $x =$ the number of weeks to wait.

(a) Income per pound (in cents):

$$80 - 4x$$

(b) Yield in pounds per tree:

$$100 + 5x$$

(c) Revenue per tree (in cents):

$$\begin{aligned} R(x) &= (100 + 5x)(80 - 4x) \\ R(x) &= 8000 - 20x^2 \end{aligned}$$

(d) Find the vertex.

$$x = \frac{-b}{2a} = \frac{0}{-20} = 0$$

$$y = 8000 - 20(0)^2 = 8000$$

The vertex is $(0, 8000)$.
To produce maximum revenue, wait 0 weeks. Pick the peaches now.

(e) $R(0) = 8000 - 20(0)^2 = 8000$

or the maximum revenue is 8000 cents per tree or \$80.00 per tree.

53. Let $x =$ the number of \$25 increases.

(a) Rent per apartment: $800 + 25x$

(b) Number of apartments rented: $80 - x$

(c) Revenue:
$$\begin{aligned} R(x) &= (\text{number of apartments rented}) \\ &\quad \times (\text{rent per apartment}) \\ &= (80 - x)(800 + 25x) \\ &= -25x^2 + 1200x + 64{,}000 \end{aligned}$$

(d) Find the vertex:

$$x = \frac{-b}{2a} = \frac{-1200}{2(-25)} = 24$$

$$\begin{aligned} y &= -25(24)^2 + 1200(24) + 64{,}000 \\ &= 78{,}400 \end{aligned}$$

The vertex is $(24, 78{,}400)$. The maximum revenue occurs when $x = 24$.

(e) The maximum revenue is the y-coordinate of the vertex, or \$78,400.

54. $S(x) = -\frac{1}{4}(x - 10)^2 + 40$ for $0 \le x \le 10$

(a) $S(0) = -\frac{1}{4}(0 - 10)^2 + 40 = -25 + 40 = 15$

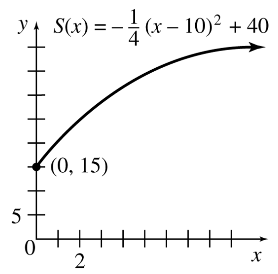
The increase in sales is \$15,000.

(b) For \$10,000, $x = 10$.

$$\begin{aligned} S(10) &= -\frac{1}{4}(10 - 10)^2 + 40 \\ S(10) &= 40 \end{aligned}$$

The increase in sales is \$40,000.

(c) The graph of $S(x) = -\frac{1}{4}(x-10)^2 + 40$ is the graph of $y = -\frac{1}{4}x^2$ translated 10 units to the right and 40 units upward.



55. $S(x) = 1 - 0.058x - 0.076x^2$

(a)

$$\begin{aligned} 0.50 &= 1 - 0.058x - 0.076x^2 \\ 0.076x^2 + 0.058x - 0.50 &= 0 \\ 76x^2 + 58x - 500 &= 0 \\ 38x^2 + 29x - 250 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-29 \pm \sqrt{(29)^2 - 4(38)(-250)}}{2(38)} \\ &= \frac{-29 \pm \sqrt{38{,}841}}{76} \end{aligned}$$

$\frac{-29-\sqrt{38{,}841}}{76} \approx -2.97$ and $\frac{-29+\sqrt{38{,}841}}{76} \approx 2.21$

We ignore the negative value.
The value $x = 2.2$ represents 2.2 decades or 22 years, and 22 years after 65 is 87.
The median length of life is 87 years.

(b) If nobody lives, $S(x) = 0$.

$$\begin{aligned} 1 - 0.058x - 0.076x^2 &= 0 \\ 76x^2 + 58x - 1000 &= 0 \\ 38x^2 + 29x - 500 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-29 \pm \sqrt{(29)^2 - 4(38)(-500)}}{2(38)} \\ &= \frac{-29 \pm \sqrt{76{,}841}}{76} \end{aligned}$$

$\frac{-29-\sqrt{76{,}841}}{76} \approx -4.03$ and $\frac{-29+\sqrt{76{,}841}}{76} \approx 3.27$

We ignore the negative value.
The value $x = 3.3$ represents 3.3 decades or 33 years, and 33 years after 65 is 98.
Virtually nobody lives beyond 98 years.

56. (a) When $t = 14, L = 2.024$, so the length of the crown is 2.024 mm at 14 weeks. When $t = 24$, $L = 6.104$, so the length is 6.104 mm at 24 weeks.

(b) The vertex occurs at

$$\begin{aligned} t &= \frac{-b}{2a} = \frac{-0.788}{2(-0.01)} \\ &\approx 39.4. \end{aligned}$$

When $t = 39.4, L \approx 8.48$, so the maximum length is 8.48 mm and occurs 39.4 weeks after conception.

57. (a) The vertex of the quadratic function $y = 0.057x - 0.001x^2$ is at

$$x = -\frac{b}{2a} = -\frac{0.057}{2(-0.001)} = 28.5.$$

Since the coefficient of the leading term, -0.001, is negative, then the graph of the function opens downward, so a maximum is reached at 28.5 weeks of gestation.

(b) The maximum splenic artery resistance reached at the vertex is

$$\begin{aligned} y &= 0.057(28.5) - 0.001(28.5)^2 \\ &\approx 0.81. \end{aligned}$$

(c) The splenic artery resistance equals 0, when $y = 0$.

$$\begin{aligned} 0.057x - 0.001x^2 &= 0 && \text{Substitute in the expression in } x \text{ for } y. \\ x(0.057 - 0.001x) &= 0 && \text{Factor.} \\ x = 0 \text{ or } 0.057 - 0.001x &= 0 && \text{Set each factor equal to 0.} \\ x &= \frac{0.057}{0.001} = 57 \end{aligned}$$

So, the splenic artery resistance equals 0 at 0 weeks or 57 weeks of gestation.
No, this is not reasonable because at $x = 0$ or 57 weeks, the fetus does not exist.

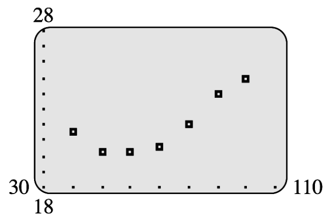
58. The vertex of the quadratic function $y = 10.5x^2 - 125x + 17{,}186$ is at

$$x = \frac{-b}{2a} = \frac{-(-125)}{2(10.5)} \approx 66.$$

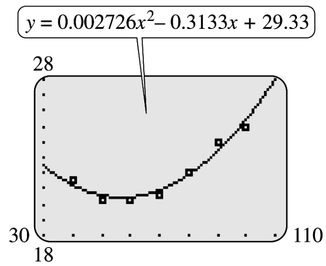
Since the coefficient of the leading term, 10.5, is positive, the graph of the function opens upward, so a minimum of first year enrollments is reached when $x \approx 6$, which is the year 1999.

For 2004, $x = 11$. The domain of $f(x)$ is [0, 11].

59. **(a)**



(c) $y = 0.002726x^2 - 0.3113x + 29.33$



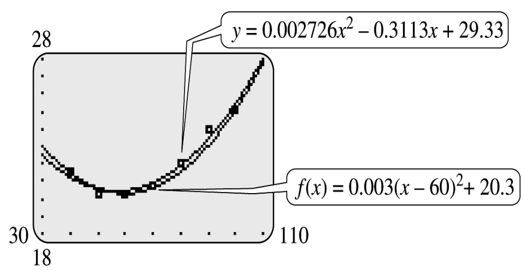
(d) Given that $(h, k) = (60, 20.3)$, the equation has the form

$$y = a(x - 60)^2 + 20.3.$$

Since $(100, 25.1)$ is also on the curve.

$$\begin{aligned} 25.1 &= a(100 - 60)^2 + 20.3 \\ 4.8 &= 1600a \\ a &= 0.003 \end{aligned}$$

(e)



The two graphs are very close.

60. **(a)** The vertex of the quadratic function $y = 0.016391x^2 - 1.2375x + 90.901$ is at

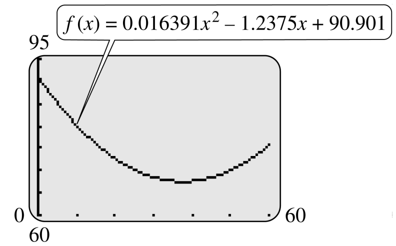
$$x = \frac{-b}{2a} = \frac{-(-1.2375)}{2(0.016391)} \approx 38.$$

This corresponds to the year 1988.

(b) When $x = 38$,

$$\begin{aligned} y &= 0.016391(38)^2 - 1.2375(38) + 90.901 \\ &\approx 67.54. \end{aligned}$$

(c)



61. $f(x) = 60.0 - 2.28x + 0.0232x^2$

$$\frac{-b}{2a} = -\frac{-2.28}{2(0.0232)} \approx 49.1$$

The minimum value occurs when $x \approx 49.1$. The age at which the accident rate is a minimum is 49 years. The minimum rate is

$$\begin{aligned} f(49.1) &= 60.0 - 2.28(49.1) + 0.0232(49.1)^2 \\ &= 60.0 - 111.948 + 55.930792 \\ &\approx 3.98. \end{aligned}$$

62. $$\begin{aligned} h &= 32t - 16t^2 \\ &= -16t^2 + 32t \end{aligned}$$

(a) Find the vertex.

$$x = \frac{-b}{2a} = \frac{-32}{-32} = 1$$

$$\begin{aligned} y &= -16(1)^2 + 32(1) \\ &= 16 \end{aligned}$$

The vertex is $(1, 16)$, so the maximum height is 16 ft.

(b) When the object hits the ground, $h = 0$, so

$$\begin{aligned} 32t - 16t^2 &= 0 \\ 16t(2 - t) &= 0 \\ t = 0 \quad \text{or} \quad t &= 2. \end{aligned}$$

When $t = 0$, the object is thrown upward. When $t = 2$, the object hits the ground; that is, after 2 sec.

63. $y = 0.056057x^2 + 1.06657x$

(a) If $x = 25$ mph,

$$\begin{aligned} y &= 0.056057(25)^2 + 1.06657(25) \\ y &\approx 61.7. \end{aligned}$$

At 25 mph, the stopping distance is approximately 61.7 ft.

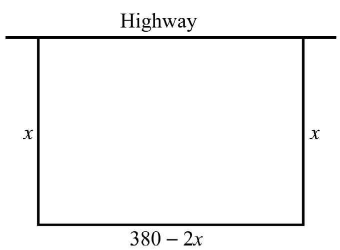
(b)
$$0.056057x^2 + 1.06657x = 150$$
$$0.056057x^2 + 1.06657x - 150 = 0$$

$$x = \frac{-1.06657 \pm \sqrt{(1.06657)^2 - 4(0.056057)(-150)}}{2(0.056057)}$$

$x \approx 43.08$ or $x \approx -62.11$

We ignore the negative value.
To stop within 150 ft, the fastest speed you can drive is 43 mph.

64. Let $x =$ the width.
Then $380 - 2x =$ the length.



$$\text{Area} = x(380 - 2x) = -2x^2 + 380x$$

Find the vertex:

$$x = \frac{-b}{2a} = \frac{-380}{-4} = 95$$

$$y = -2(95)^2 + 380(95) = 18{,}050$$

The graph of the area function is a parabola with vertex (95, 18,050).
The maximum area of 18,050 sq ft occurs when the width is 95 ft and the length is

$$380 - 2x = 380 - 2(95) = 190 \text{ ft.}$$

65. Let $x =$ the length of the lot and
$y =$ the width of the lot.

The perimeter is given by

$$\begin{aligned} P &= 2x + 2y \\ 380 &= 2x + 2y \\ 190 &= x + y \\ 190 - x &= y. \end{aligned}$$

Area $= xy$ (quantity to be maximized)

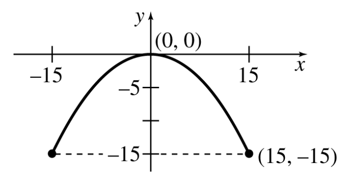
$$\begin{aligned} A &= x(190 - x) \\ &= 190x - x^2 \\ &= -x^2 + 190x \end{aligned}$$

Find the vertex: $\frac{-b}{2a} = \frac{-190}{-2} = 95$

$$\begin{aligned} y &= -(95)^2 + 190(95) \\ &= 9025 \end{aligned}$$

This is a parabola with vertex $(95, 9025)$ that opens downward. The maximum area is the value of A at the vertex, or 9025 sq ft.

66. Draw a sketch of the arch with the vertex at the origin.



Since the arch is a parabola that opens downward, the equation of the parabola is the form $y = a(x - h)^2 + k$, where the vertex $(h, k) = (0, 0)$ and $a < 0$. That is, the equation is of the form $y = ax^2$.
Since the arch is 30 meters wide at the base and 15 meters high, the points $(15, -15)$ and $(-15, -15)$ are on the parabola. Use $(15, -15)$ as one point on the parabola.

$$\begin{aligned} -15 &= a(15)^2 \\ a &= \frac{-15}{15^2} = -\frac{1}{15} \end{aligned}$$

So, the equation is

$$y = -\frac{1}{15}x^2.$$

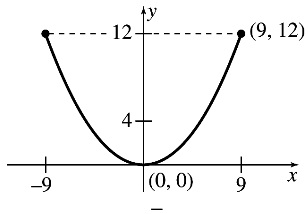
Ten feet from the ground (the base) is at $y = -5$.
Substitute -5 for y and solve for x.

$$\begin{aligned} -5 &= -\frac{1}{15}x^2 \\ x^2 &= -5(-15) = 75 \\ x &= \pm\sqrt{75} = \pm 5\sqrt{3} \end{aligned}$$

The width of the arch ten feet from the ground is then

$$\begin{aligned} 5\sqrt{3} - (-5\sqrt{3}) &= 10\sqrt{3} \text{ meters} \\ &\approx 17.32 \text{ meters.} \end{aligned}$$

67. Sketch the culvert on the xy-axes as a parabola that opens upward with vertex at $(0, 0)$.



The equation is of the form $y = ax^2$. Since the culvert is 18 ft wide at 12 ft from its vertex, the points $(9, 12)$ and $(-9, 12)$ are on the parabola. Use $(9, 12)$ as one point on the parabola.

$$\begin{aligned} 12 &= a(9)^2 \\ 12 &= 81a \\ \frac{12}{81} &= a \\ \frac{4}{27} &= a \end{aligned}$$

Thus,

$$y = \frac{4}{27}x^2.$$

To find the width 8 feet from the top, find the points with

$$y\text{-value } = 12 - 8 = 4.$$

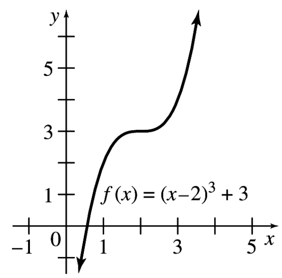
Thus,

$$\begin{aligned} 4 &= \frac{4}{27}x^2 \\ 108 &= 4x^2 \\ 27 &= x^2 \\ x^2 &= 27 \\ x &= \pm\sqrt{27} \\ x &= \pm 3\sqrt{3}. \end{aligned}$$

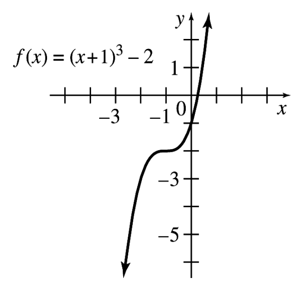
The width of the culvert is $3\sqrt{3} + \left|-3\sqrt{3}\right| = 6\sqrt{3}$ ft ≈ 10.39 ft.

2.3 Polynomial and Rational Functions

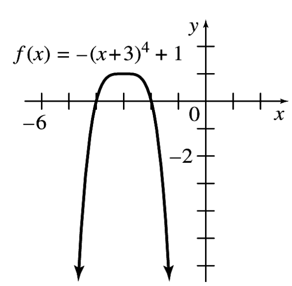
3. The graph of $f(x) = (x-2)^3 + 3$ is the graph of $y = x^3$ translated 2 units to the right and 3 units upward.



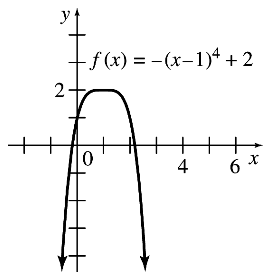
4. The graph of $f(x) = (x+1)^3 - 2$ is the graph of $y = x^3$ translated 1 unit to the left and 2 units downward.



5. The graph of $f(x) = -(x+3)^4 + 1$ is the graph of $y = x^4$ reflected horizontally, translated 3 units to the left, and translated 1 unit upward.



6. The graph of $f(x) = -(x-1)^4 + 2$ is the graph of $y = x^4$ reflected in the x-axis, and translated 1 unit to the right and 2 units upward.



7. The graph of $y = x^3 - 7x - 9$ has the right end up, the left end down, at most two turning points, and a y-intercept of -9.
This is graph d.

8. The graph of $y = -x^3 + 4x^2 + 3x - 8$ has the right end down, the left end up, at most two turning points, and a y-intercept of -8.
The is graph c.

9. The graph of $y = -x^3 - 4x^2 + x + 6$ has the right end down, the left end up, at most two turning points, and a y-intercept of 6.
This is graph e.

10. The graph of $y = 2x^3 + 4x + 5$ has the right end up, the left end down, at most two turning points, and a y-intercept of 5.
This is graph b.

11. The graph of $y = x^4 - 5x^2 + 7$ has both ends up, at most three turning points, and a y-intercept of 7.
This is graph i.

12. The graph of $y = x^4 + 4x^3 - 20$ has both ends up, at most three turning points, and a y-intercept of -20.
This is graph f.

13. The graph of $y = -x^4 + 2x^3 + 10x + 15$ has both ends down, at most three turning points, and a y-intercept of 15.
This is graph g.

14. The graph of $y = 0.7x^5 - 2.5x^4 - x^3 + 8x^2 + x + 2$ has the right end up, the left end down, at most four turning points, and a y-intercept of 2.
This is graph h.

15. The graph of $y = -x^5 + 4x^4 + x^3 - 16x^2 + 12x + 5$ has the right end down, the left end up, at most four turning points, and a y-intercept of 5.
This is graph a.

16. The graph of $y = \frac{2x^2+3}{x^2-1}$ has the lines with equations $x = 1$ and $x = -1$ as vertical asymptotes, the line with equation $y = 2$ as a horizontal asymptote, and a y-intercept of -3.
This is graph b.

17. The graph of $y = \frac{2x^2+3}{x^2+1}$ has no vertical asymptote, the line with equation $y = 2$ as a horizontal asymptote, and a y-intercept of 3.
This is graph d.

18. The graph of $y = \frac{-2x^2-3}{x^2-1}$ has the lines with equations $x = 1$ and $x = -1$ as vertical asymptotes, the line with equation $y = -2$ as a horizontal asymptote, and a y-intercept of 3.
This is graph a.

19. The graph $y = \frac{-2x^2-3}{x^2+1}$ has no vertical asymptote, the line with equation $y = -2$ as a horizontal asymptote, and a y-intercept of -3.
This is graph e.

20. The graph of $y = \frac{2x^2+3}{x^3-1}$ has the line with equation $x = 1$ as a vertical asymptote, the x-axis as a horizontal asymptote, and a y-intercept of -3.
This is graph c.

21. The right end is up and the left end is up. There are three turning points.
The degree is an even integer equal to 4 or more. The x^n term has a $+$ sign.

22. The graph has the right end up, the left end down, and four turning points. The degree is an odd integer equal to 5 or more. The x^n term has a $+$ sign.

23. The right end is up and the left end is down. There are four turning points. The degree is an odd integer equal to 5 or more. The x^n term has a $+$ sign.

24. Both ends are down, and there are five turning points. The degree is an even integer equal to 6 or more. The x^n term has a $-$ sign.

25. The right end is down and the left end is up. There are six turning points. The degree is an odd integer equal to 7 or more. The x^n term has a $-$ sign.

26. The right end is up, the left end is down, and there are six turning points. The degree is an odd integer equal to 7 or more. The x^n term has a + sign.

27. $y = \dfrac{-4}{x+2}$

The function is undefined for $x = -2$, so the line $x = -2$ is a vertical asymptote.

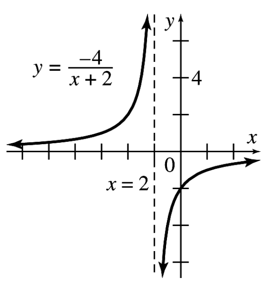
x	-102	-12	-7	-5	-3	-1	8	98
$x+2$	-100	-10	-5	-3	-1	1	10	100
y	0.04	0.4	0.8	1.3	4	-4	-0.4	-0.04

The graph approaches $y = 0$, so the line $y = 0$ (the x-axis) is a horizontal asymptote.
Asymptotes: $y = 0,\ x = -2$

x-intercept:
none, because the x-axis is an asymptote

y-intercept:

-2, the value when $x = 0$

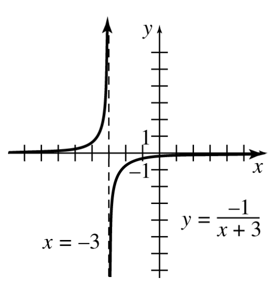


28. $y = \dfrac{-1}{x+3}$

A vertical asymptote occurs when $x + 3 = 0$ or when $x = -3$, since this value makes the denominator 0.

x	-6	-5	-4	-2	-1	0
$x+3$	-3	-2	-1	1	2	3
y	$\frac{1}{3}$	$\frac{1}{2}$	1	-1	$-\frac{1}{2}$	$-\frac{1}{3}$

As $|x|$ gets larger, $\frac{-1}{x+3}$ approaches 0, so $y = 0$ is a horizontal asymptote.
Asymptotes: $y = 0,\ x = -3$



x-intercept:
none, since the x-axis is an asymptote
y-intercept:

$-\frac{1}{3}$, the value when $x = 0$

29. $y = \dfrac{2}{3+2x}$

$3 + 2x = 0$ when $2x = -3$ or $x = -\frac{3}{2}$, so the line $x = -\frac{3}{2}$ is a vertical asymptote.

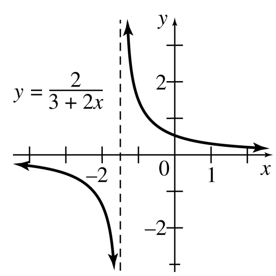
x	-51.5	-6.5	-2	-1	3.5	48.5
$3+2x$	-100	-10	-1	1	10	100
y	-0.02	-0.2	-2	2	0.2	0.02

The graph approaches $y = 0$, so the line $y = 0$ (the x-axis) is a horizontal asymptote.
Asymptote: $y = 0,\ x = -\frac{3}{2}$

x-intercept:
none, since the x-axis is an asymptote

y-intercept:

$\frac{2}{3}$, the value when $x = 0$



30. $y = \dfrac{8}{5-3x}$

Undefined for

$$\begin{aligned} 5 - 3x &= 0 \\ 3x &= 5 \\ x &= \frac{5}{3} \end{aligned}$$

Since $x = \frac{5}{3}$ causes the denominator to equal 0, $x = \frac{5}{3}$ is a vertical asymptote.

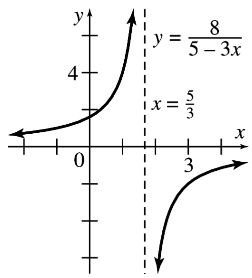
x	-1	0	1	2	3	4
$5-3x$	8	5	2	-1	-4	-7
y	0.5	0.8	2	-4	-1	-0.571

The graph approaches $y = 0$, so the line $y = 0$ is a horizontal asymptote.
Asymptotes: $y = 0,\ x = \frac{5}{3}$

x-intercept:
none, since the x-axis is an asymptote

y-intercept:

$\frac{8}{5}$, the value when $x = 0$



31. $y = \dfrac{2x}{x-3}$

$x - 3 = 0$ when $x = 3$, so the line $x = 3$ is a vertical asymptote.

x	-97	-7	-1	1	2	2.5
$2x$	-194	-14	-2	2	4	5
$x-3$	-100	-10	-4	-2	-1	-0.5
y	1.94	1.4	0.5	-1	-4	-10

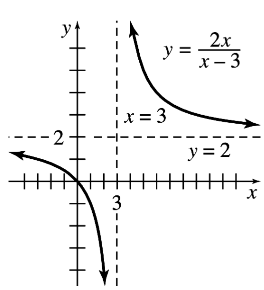
x	3.5	4	5	7	11	103
$2x$	7	8	10	14	22	206
$x-3$	0.5	1	2	4	8	100
y	14	8	5	3.5	2.75	2.06

As x gets larger,

$$\frac{2x}{x-3} \approx \frac{2x}{x} = 2.$$

Thus, $y = 2$ is a horizontal asymptote.
Asymptotes: $y = 2$, $x = 3$
x-intercept:
0, the value when $y = 0$

y-intercept:
0, the value when $x = 0$



32. $y = \dfrac{4x}{3-2x}$

Since $x = \frac{3}{2}$ causes the denominator to equal 0, $x = \frac{3}{2}$ is a vertical asymptote.

x	-3	-2	-1	0	1
$4x$	-12	-8	-4	0	4
$3-2x$	9	7	5	3	1
y	-1.33	-1.14	-0.8	0	4

x	2	3	4
$4x$	8	12	16
$3-2x$	-1	-3	-5
y	-8	-4	-3.2

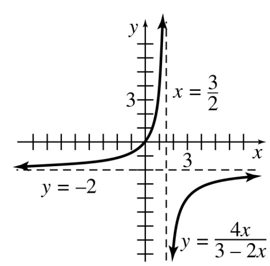
As x gets larger,

$$\frac{4x}{3-2x} \approx \frac{4x}{-2x} = -2.$$

Thus, the line $y = -2$ is a horizontal asymptote.
Asymptotes: $y = -2$, $x = \frac{3}{2}$

x-intercept:
0, the value when $y = 0$

y-intercept:
0, the value when $x = 0$



33. $y = \dfrac{x+1}{x-4}$

$x - 4 = 0$ when $x = 4$, so $x = 4$ is a vertical asymptote.

x	-96	-6	-1	0	3
$x+1$	-95	-5	0	1	4
$x-4$	-100	-10	-5	-4	-1
y	0.95	0.5	0	-0.25	-4

x	3.5	4.5	5	14	104
$x+1$	4.5	5.5	6	15	105
$x-4$	-0.5	0.5	1	10	100
y	-9	11	6	1.5	1.05

As x gets larger,

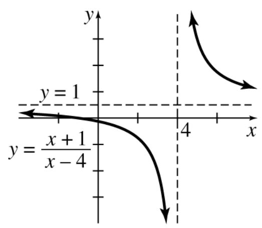
$$\frac{x+1}{x-4} \approx \frac{x}{x} = 1.$$

Thus, $y = 1$ is a horizontal asymptote.
Asymptotes: $y = 1,\ x = 4$

x-intercept:
-1, the value when $y = 0$

y-intercept:

$-\frac{1}{4}$, the value when $x = 0$



34. $y = \dfrac{x-4}{x+1}$

Since $x = -1$ causes the denominator to equal 0, $x = -1$ is a vertical asymptote.

x	-4	-3	-2	0	1
$x-4$	-8	-7	-6	-4	-3
$x+1$	-3	-2	-1	1	2
y	2.67	3.5	6	-4	-1.5

x	2	3	4
$x-4$	-2	-1	0
$x+1$	3	4	5
y	-0.67	-0.25	0

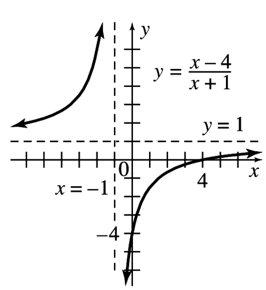
As x gets larger,

$$\frac{x-4}{x+1} \approx \frac{x}{x} = 1.$$

Thus, the line $y = 1$ is a horizontal asymptote.
Asymptotes: $y = 1, x = -1$
x-intercept: 4, the value when $y = 0$

y-intercept:

-4, the value when $x = 0$



35. $y = \dfrac{3-2x}{4x+20}$

$4x+20 = 0$ when $4x = -20$ or $x = -5$, so the line $x = -5$ is a vertical asymptote.

x	-8	-7	-6	-4	-3	-2
$3-2x$	-26	-23	-20	-14	-11	-8
$4x+20$	-12	-8	-4	4	8	12
y	2.17	2.88	5	-3.5	-1.38	-0.67

As x gets larger,

$$\frac{3-2x}{4x+20} \approx \frac{-2x}{4x} = -\frac{1}{2}.$$

Thus, the line $y = -\frac{1}{2}$ is a horizontal asymptote.

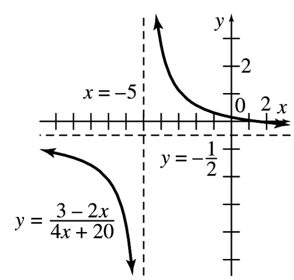
Asymptotes: $x = -5,\ y = -\frac{1}{2}$

x-intercept:

$\frac{3}{2}$, the value when $y = 0$

y-intercept:

$\frac{3}{20}$, the value when $x = 0$



36. $y = \dfrac{6-3x}{4x+12}$

$4x+12 = 0$ when $4x = -12$ or $x = -3$, so $x = -3$ is a vertical asymptote.

x	-6	-5	-4	-2	-1	0
$6-3x$	24	21	18	12	9	6
$4x+12$	-12	-8	-4	4	8	12
y	-2	-2.625	-4.5	3	1.125	0.5

As x gets larger,

$$\frac{6-3x}{4x+12} \approx \frac{-3x}{4x} = -\frac{3}{4}.$$

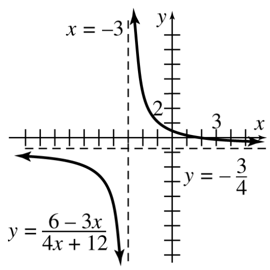
The line $y = -\frac{3}{4}$ is a horizontal asymptote.

Asymptotes: $y = -\frac{3}{4},\ x = -3$

x-intercept:
2, the value when $y = 0$

y-intercept:

$\frac{1}{2}$, the value when $x = 0$



37. $y = \dfrac{-x-4}{3x+6}$

$3x + 6 = 0$ when $3x = -6$ or $x = -2$, so the line $x = -2$ is a vertical asymptote.

x	-5	-4	-3	-1	0	1
$-x-4$	1	0	-1	-3	-4	-5
$3x+6$	-9	-6	-3	3	6	9
y	-0.11	0	0.33	-1	-0.67	-0.56

As x gets larger,

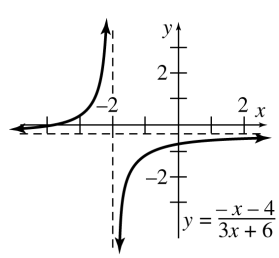
$$\frac{-x-4}{3x+6} \approx \frac{-x}{3x} = -\frac{1}{3}.$$

The line $y = -\frac{1}{3}$ is a horizontal asymptote.
Asymptotes: $y = -\frac{1}{3},\ x = -2$

x-intercept:
-4, the value when $y = 0$

y-intercept:

$-\frac{2}{3}$, the value when $x = 0$



38. $y = \dfrac{-2x+5}{x+3}$

$x + 3 = 0$ when $x = -3$, so $x = -3$ is a vertical asymptote.

x	-6	-5	-4	-2	-1	0
$-2x+5$	17	15	13	9	7	5
$x+3$	-3	-2	-1	1	2	3
y	-5.67	-7.5	-13	9	3.5	1.67

As x gets larger,

$$\frac{-2x+5}{x+3} \approx \frac{-2x}{x} = -2.$$

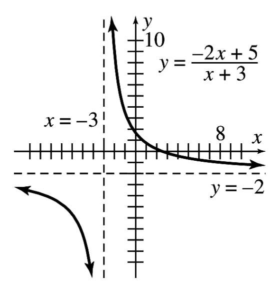
The line $y = -2$ is a horizontal asymptote.

Asymptotes: $y = -2,\ x = -3$

x-intercept:
$\frac{5}{2}$, the value when $y = 0$

y-intercept:

$\frac{5}{3}$, the value when $x = 0$

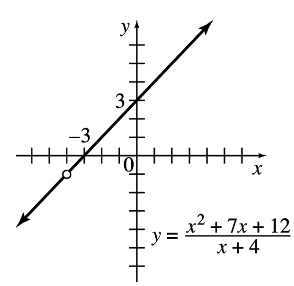


39. $y = \dfrac{x^2+7x+12}{x+4}$

$= \dfrac{(x+3)(x+4)}{x+4}$

$= x + 3,\ x \neq -4$

There are no asymptotes, but there is a hole at $x = -4$.
x-intercept: -3, the value when $y = 0$.
y-intercept: 3, the value when $x = 0$.

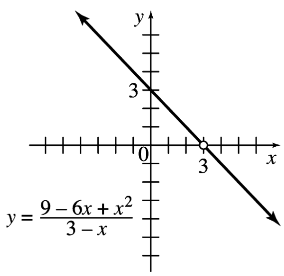


40. $y = \dfrac{9-6x+x^2}{3-x}$

$= \dfrac{(3-x)(3-x)}{3-x}$

$= 3-x,\ x \neq 3$

There are no asymptotes, but there is a hole at $x = 3$.
There is no x-intercept, since $3 - x = 0$ implies $x = 3$, but there is a hole at $x = 3$.
y-intercept: 3, the value when $x = 0$.



41. For a vertical asymptote at $x = 1$, put $x - 1$ in the denominator. For a horizontal asymptote at $y = 2$, the degree of the numerator must equal the degree of the denominator and the quotient of their leading terms must equal 2. So, $2x$ in the numerator would cause y to approach 2 as x gets larger.
So, one possible answer is $y = \dfrac{2x}{x-1}$.

42. For a vertical asymptote at $x = -2$, put $x + 2$ in the denominator. For a horizontal asymptote at $y = 0$, the only condition is that the degree of the numerator is less than the degree of the denominator. If the degree of the denominator is 1, then put a constant in the numerator to make y approach 0 as x gets larger. So, one possible solution is $y = \frac{3}{x+2}$.

43. $f(x) = (x-1)(x-2)(x+3)$,
$g(x) = x^3 + 2x^2 - x - 2$,
$h(x) = 3x^3 + 6x^2 - 3x - 6$

(a) $f(1) = (0)(-1)(4) = 0$

(b) $f(x)$ is zero when $x = 2$ and when $x = -3$.

(c) $g(-1) = (-1)^3 + 2(-1)^2 - (-1) - 2$
$= -1 + 2 + 1 - 2 = 0$
$g(1) = (1)^3 + 2(1)^2 - (1) - 2$
$= 1 + 2 - 1 - 2$
$= 0$
$g(-2) = (-2)^3 + 2(-2)^2 - (-2) - 2$
$= -8 + 8 + 2 - 2$
$= 0$

(d) $g(x) = [x - (-1)](x-1)[x-(-2)]$
$g(x) = (x+1)(x-1)(x+2)$

(e) $h(x) = 3g(x)$
$= 3(x+1)(x-1)(x+2)$

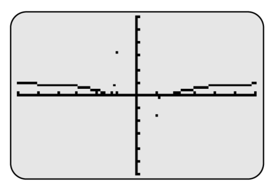
(f) If f is a polynomial and $f(a) = 0$ for some number a, then one factor of the polynomial is $x - a$.

44. Graph

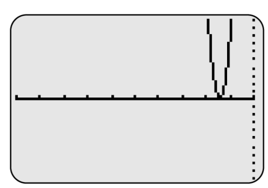
$$f(x) = \frac{x^7 - 4x^5 - 3x^4 + 4x^3 + 12x^2 - 12}{x^7}$$

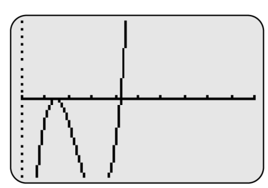
using a graphing calculator with the indicated viewing windows.

(a) There appear to be two x-intercepts, one at $x = -1.4$ and one at $x = 1.4$.



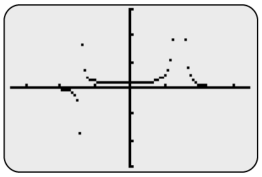
(b) There appear to be three x-intercepts, one at $x = -1.414$, one at $x = 1.414$, and one at $x = 1.442$.



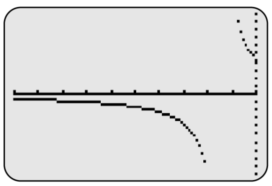


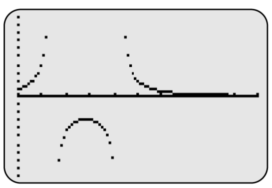
45. $f(x) = \dfrac{1}{x^5 - 2x^3 - 3x^2 + 6}$

(a) Two vertical asymptotes appear, one at $x = -1.4$ and one at $x = 1.4$.



(b) Three vertical asymptotes appear, one at $x = -1.414$, one at $x = 1.414$, and one at $x = 1.442$.





46. $\overline{C}(x) = \dfrac{600}{x + 20}$

(a)
$$\overline{C}(10) = \frac{600}{10 + 20} = \frac{600}{30} = \$20$$

$$\overline{C}(20) = \frac{600}{20 + 20} = \frac{600}{40} = \$15$$

$$\overline{C}(50) = \frac{600}{50 + 20} = \frac{600}{70} \approx \$8.57$$

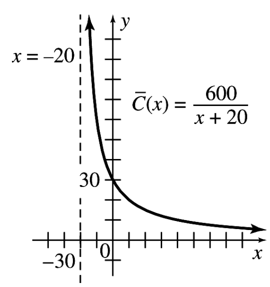
$$\overline{C}(75) = \frac{600}{75 + 20} = \frac{600}{95} \approx \$6.32$$

$$\overline{C}(100) = \frac{600}{100 + 20} = \frac{600}{120} = \$5$$

(b) $(0, \infty)$ would be a more reasonable domain for average cost than $[0, \infty)$. If zero were included in the domain, there would be no units produced. It is not reasonable to discuss the average cost per unit of zero units.

(c) The graph has a vertical asymptote at $x = -20$, a horizontal asymptote at $y = 0$ (the x-axis), and y-intercept $\frac{600}{20} \approx 30$.

(d)



47. $\overline{C}(x) = \dfrac{220{,}000}{x + 475}$

(a) If $x = 25$,
$$\overline{C}(25) = \frac{220{,}000}{25 + 475} = \frac{220{,}000}{500} = 440.$$

If $x = 50$,
$$\overline{C}(50) = \frac{220{,}000}{50 + 475} = \frac{220{,}000}{525} \approx 419.$$

If $x = 100$,
$$\overline{C}(100) = \frac{220{,}000}{100 + 475} = \frac{220{,}000}{575} \approx 383.$$

If $x = 200$,
$$\overline{C}(200) = \frac{220{,}000}{200 + 475} = \frac{220{,}000}{675} \approx 326.$$

If $x = 300$,
$$\overline{C}(300) = \frac{220{,}000}{300 + 475} = \frac{220{,}000}{775} \approx 284.$$

If $x = 400$,

$$\overline{C}(400) = \frac{220{,}000}{400 + 475} = \frac{220{,}000}{875} \approx 251.$$

(b) A vertical asymptote occurs when the denominator is 0.

$$x + 475 = 0$$
$$x = -475$$

A horizontal asymptote occurs when $\overline{C}(x)$ approaches a value as x gets larger. In this case, $\overline{C}(x)$ approaches 0.
The asymptotes are $x = -475$ and $y = 0$.

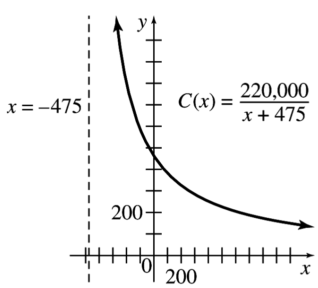
(c) x-intercepts:

$0 = \dfrac{220{,}000}{x + 475}$; no such x, so no x-intercepts

y-intercepts:

$$\overline{C}(0) = \frac{220{,}000}{0 + 475} \approx 463.2$$

(d) Use the following ordered pairs:
$(25, 440)$, $(50, 419)$, $(100, 383)$,
$(200, 326)$, $(300, 284)$, $(400, 251)$.



48. $y = x(100 - x)(x^2 + 500)$,

$x =$ tax rate;
$y =$ tax revenue in hundreds of thousands of dollars.

(a) $x = 10$

$$\begin{aligned} y &= 10(100 - 10)(10^2 + 500) \\ &= 10(90)(600) \\ &= \$54 \text{ billion} \end{aligned}$$

(b) $x = 40$

$$\begin{aligned} y &= 40(100 - 40)(40^2 + 500) \\ &= 40(60)(2100) \\ &= \$504 \text{ billion} \end{aligned}$$

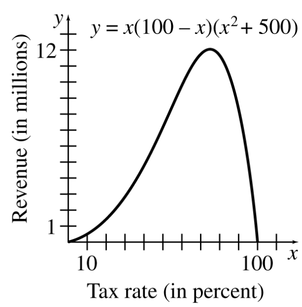
(c) $x = 50$

$$\begin{aligned} y &= 50(100 - 50)(50^2 + 500) \\ &= 50(50)(3000) \\ &= \$750 \text{ billion} \end{aligned}$$

(d) $x = 80$

$$\begin{aligned} y &= 80(100 - 80)(80^2 + 500) \\ &= 80(20)(6900) \\ &= \$1104 \text{ billion} \end{aligned}$$

(e)



49. Quadratic functions with roots at $x = 0$ and $x = 100$ are of the form $f(x) = ax(100 - x)$.
$f_1(x)$ has a maximum of 100, which occurs at the vertex. The x-coordinate of the vertex lies between the two roots.
The vertex is $(50, 100)$.

$$\begin{aligned} 100 &= a(50)(100 - 50) \\ 100 &= a(50)(50) \\ \frac{100}{2500} &= a \\ \frac{1}{25} &= a \end{aligned}$$

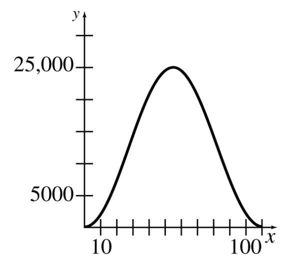
$$f_1(x) = \frac{1}{25}x(100 - x) \text{ or } \frac{x(100 - x)}{25}$$

$f_2(x)$ has a maximum of 250, occurring at $(50, 250)$.

$$\begin{aligned} 250 &= a(50)(100 - 50) \\ 250 &= a(50)(50) \\ \frac{250}{2500} &= a \\ \frac{1}{10} &= a \end{aligned}$$

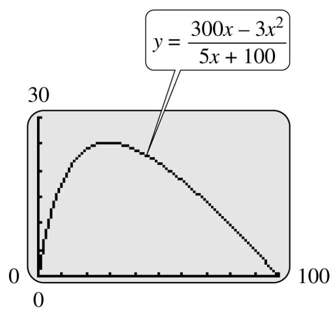
$$f_2(x) = \frac{1}{10}x(100 - x) \text{ or } \frac{x(100 - x)}{10}$$

$$f_1(x) \cdot f_2(x) = \left[\frac{x(100-x)}{25}\right] \cdot \left[\frac{x(100-x)}{10}\right]$$
$$= \frac{x^2(100-x)^2}{250}$$



$f(x) = \dfrac{x^2\,(100-x)^2}{250}$

50. **(a)**



(b) The tax rate for maximum revenue is 29.0%. The maximum revenue is $25.2 million.

51. $y = \dfrac{6.7x}{100-x}$,

Let x = percent of pollutant;
y = cost in thousands.

(a) $x = 50$

$$y = \frac{6.7(50)}{100-50} = 6.7$$

The cost is $6700.

$x = 70$

$$y = \frac{6.7(70)}{100-70} \approx 15.6$$

The cost is $15,600.

$x = 80$

$$y = \frac{6.7(80)}{100-80} = 26.8$$

The cost is $26,800.

$x = 90$

$$y = \frac{6.7(90)}{100-90} = 60.3$$

The cost is $60,300.

$x = 95$

$$y = \frac{6.7(95)}{100-95}$$

The cost is $127,300.

$x = 98$

$$y = \frac{6.7(98)}{100-98} = 328.3$$

The cost is $328,300.

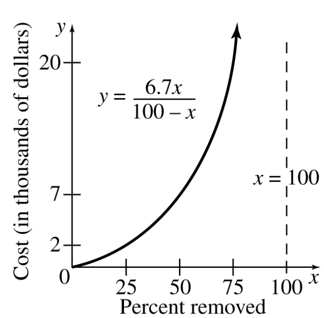
$x = 99$

$$y = \frac{6.7(99)}{100-99} = 663.3$$

The cost is $663,300.

(b) No, because $x = 100$ makes the denominator zero, so $x = 100$ is a vertical asymptote.

(c)



52. $y = \dfrac{6.5x}{102 - x}$

y = percent of pollutant;
x = cost in thousands of dollars.

(a) $x = 0$

$$y = \frac{6.5(0)}{102 - 0} = \frac{0}{102} = \$0$$

$x = 50$

$$y = \frac{6.5(50)}{102 - 50} = \frac{325}{52} = 6.25$$

$6.25(1000) = \$6250$

$x = 80$

$$y = \frac{6.5(80)}{102 - 80} = \frac{520}{22} = 23.636$$

$$(23.636)(1000) = \$23{,}636 \approx \$24{,}000$$

$x = 90$

$$y = \frac{6.5(90)}{102 - 90} = \frac{585}{12} = 48.75$$

$$(48.75)(1000) = \$48{,}750 \approx \$48{,}800$$

$x = 95$

$$y = \frac{6.5(95)}{102 - 95} = \frac{617.5}{7} = 88.214$$

$$(88.214)(1000) = 88{,}214 \approx \$88{,}000$$

$x = 99$

$$y = \frac{6.5(99)}{102 - 99} = \frac{643.5}{3} = 214.5$$

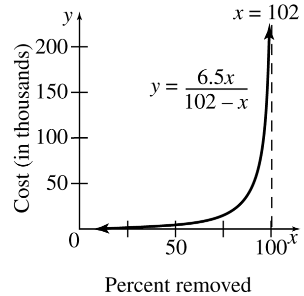
$$(214.500)(1000) = 214{,}500 \approx \$214{,}500$$

$x = 100$

$$y = \frac{6.5(100)}{102 - 100} = \frac{650}{2} = 325$$

$(325)(1000) = \$325{,}000$

(b)



53. **(a)** $a = \dfrac{k}{d}$

$k = ad$

d	a	$k = ad$
36.000	9.37	337.32
36.125	9.34	337.4075
36.250	9.31	337.4875
36.375	9.27	337.19625
36.500	9.24	337.26
36.625	9.21	337.31625
36.750	9.18	337.365
36.875	9.15	337.40625
37.000	9.12	337.44

We find the average of the nine values of k by adding them and dividing by 9. This gives 337.35, or, rounding to the nearest integer, $k = 337$. Therefore,

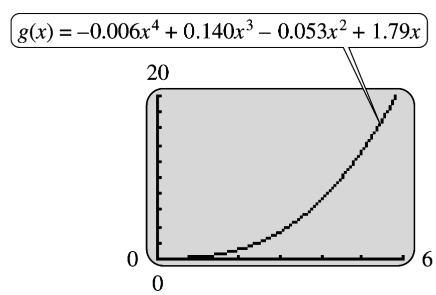
$$a = \frac{337}{d}.$$

(b) When $d = 40.50$,

$$a = \frac{337}{40.50} \approx 8.32.$$

The strength for 40.50 diopter lenses is 8.32 mm of arc.

54. **(a)**



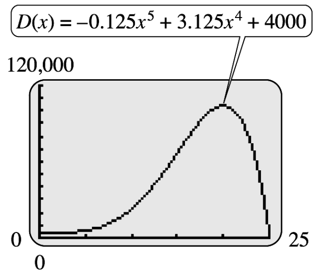
(b) Because the leading coefficient is negative and the degree of the polynomial is even, the graph will have right end down, so it cannot keep increasing forever.

55. $D(x) = -0.125x^5 + 3.125x^4 + 4000$

(a)

x	0	5	10	15
$D(x)$	4000	5563	22,750	67,281

x	20	25
$D(x)$	104,000	4000

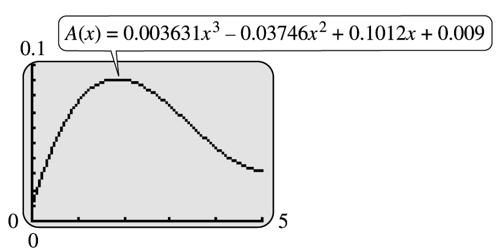


(b) $D(x)$ increases from $x = 0$ to $x = 20$. This corresponds to an increasing population from 1905 to 1925. $D(x)$ does not change much from $x = 0$ to $x = 5$. This corresponds to a relatively stable population from 1905 to 1910.
$D(x)$ decreases from $x = 20$ to $x = 25$. This corresponds to a decreasing population from 1925 to 1930.

56. $A(x) = 0.003631x^3 - 0.03746x^2 + 0.1012x + 0.009$

(a)

x	0	1	2	3	4	5
$A(x)$	0.009	0.076	0.091	0.073	0.047	0.032



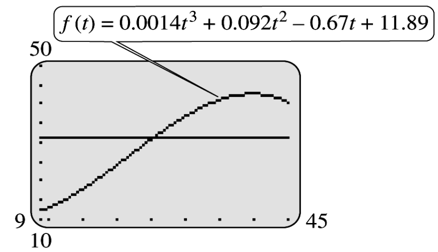
(b) The peak of the curve comes at about $x = 2$ hours.

(c) The curve rises to a y-value of 0.08 at about $x = 1.1$ hours and stays at or above that level until about $x = 2.7$ hours.

57. $f(t) = -0.0014t^3 + 0.092t^2 - 0.67t + 11.89$

(a)

x	9	18	27	36	45
$A(x)$	12.29	21.47	33.31	41.68	40.47



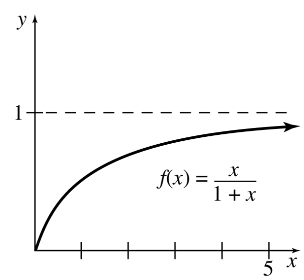
(b) The graph of $f(t)$ intersects the graph of $g(t) = 31$ at about $t = 25$, which corresponds to the year 1985.

58. $f(x) = \dfrac{\lambda x}{1 + (ax)^b}$

(a) A reasonable domain for the function is $[0, \infty)$. Populations are not measured using negative numbers and they may get extremely large.

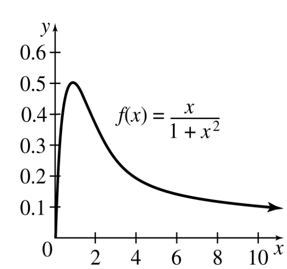
(b) If $\lambda = a = b = 1$, the function becomes

$$f(x) = \frac{x}{1 + x}.$$



(c) If $\lambda = a = 1$ and $b = 2$, the function becomes

$$f(x) = \frac{x}{1 + x^2}.$$



(d) As seen from the graphs, when b increases, the population of the next generation, $f(x)$, gets smaller when the current generation, x, is larger.

59. **(a)** A reasonable domain for the function is $[0, \infty)$. Populations are not measured using negative numbers, and they may get extremely large.

(b) $f(x) = \dfrac{Kx}{A + x}$

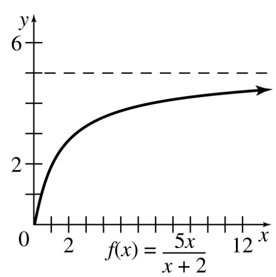
When $K = 5$ and $A = 2$,

$$f(x) = \frac{5x}{2 + x}.$$

The graph has a horizontal asymptote at $y = 5$ since

$$\frac{5x}{2 + x} \approx \frac{5x}{x} = 5$$

as x gets larger.



(c) $f(x) = \dfrac{Kx}{A + x}$

As x gets larger,

$$\frac{Kx}{A + x} \approx \frac{Kx}{x} = K.$$

Thus, $y = K$ will always be a horizontal asymptote for this function.

(d) K represents the maximum growth rate. The function approaches this value asymptotically, showing that although the growth rate can get very close to K, it can never reach the maximum, K.

(e) $f(x) = \dfrac{Kx}{A + x}$

Let $A = x$, the quantity of food present.

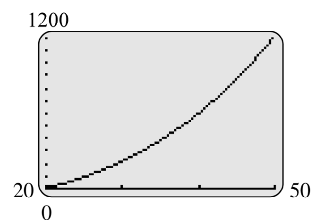
$$f(x) = \frac{Kx}{A + x} = \frac{Kx}{2x} = \frac{K}{2}$$

K is the maximum growth rate, so $\frac{K}{2}$ is half the maximum. Thus, A represents the quantity of food for which the growth rate is half of its maximum.

60. **(a)** When $c = 30, w = \frac{30^3}{100} - \frac{1500}{30} = 220$, so the brain weights 220 g when its circumference measures 30 cm. When $c = 40, w = \frac{40^3}{100} - \frac{1500}{40} = 602.5$, so the brain weighs 602.5 g when its circumference is 40 cm. When $c = 50, w = \frac{50^3}{100} - \frac{1500}{50} = 1220$, so the brain weighs 1220 g when its circumference is 50 cm.

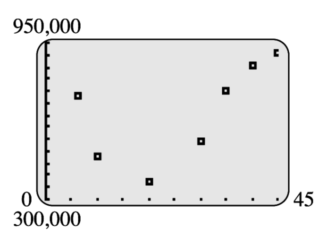
(b) Set the window of a graphing calculator so you can trace to the positive x-intercept of the function. Using a "root" or "zero" program, this x-intercept is found to be approximately 19.68. Notice in the graph that positive c values less than 19.68 correspond to negative w values. Therefore, the answer is $c < 19.68$.

(c)

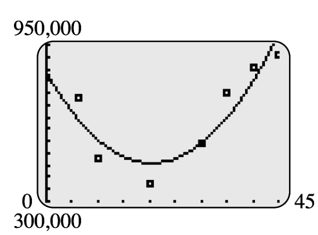


(d) One method is to graph the line $y = 700$ on the graph found in part (c) and use an "intercept" program to find the point of intersection of the two graphs. This point has the approximate coordinates $(41.9, 700)$. Therefore, an infant has a brain weighing 700 g when the circumference measures 41.9 cm.

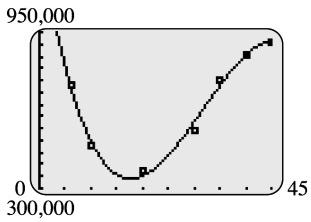
61. **(a)**



(b) $y = 890.37x^2 - 36{,}370x + 830{,}144$



(c) $y = -52.954x^3 + 5017.88x^2 - 127{,}714x + 1{,}322{,}606$



62. (a) Using a graphing calculator with the given data, for the 4.0-foot pendulum and for

$n = 1$,

$$4.0 = k(2.22)^1$$
$$k = 1.80;$$

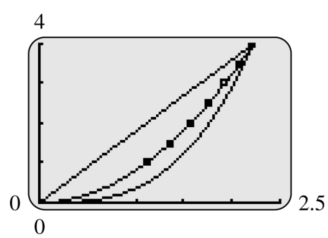
$n = 2$,

$$4.0 = k(2.22)^2$$
$$k = 0.812;$$

$n = 3$

$$4.0 = k(2.22)^3$$
$$k = 0.366.$$

(b)



From the graphs, $L = 0.812T^2$ is the best fit.

(c) $5.0 = 0.812T^2$

$$T^2 = \frac{5.0}{0.812}$$
$$T = \sqrt{\frac{5.0}{0.812}}$$
$$T \approx 2.48$$

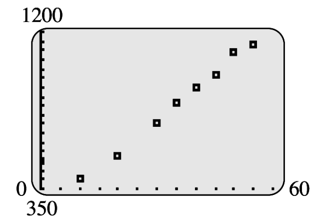
The period will be 2.48 seconds.

(d) $L = 0.812T^2$

If L doubles, T^2 doubles, and T increases by a factor of $\sqrt{2}$.

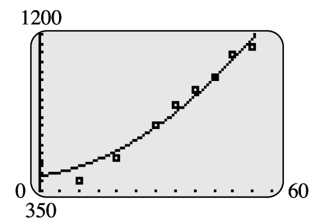
(e) $L \approx 0.822T^2$ which is very close to $L = 0.812T^2$.

63. (a)



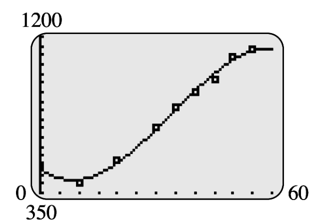
(b) $f(x) = 0.19327x^2 + 3.0039x + 431.30$

(c)



(d) $f(x) = -0.010883x^3 + 1.1079x^2 - 16.432x + 485.45$

(e)



2.4 Exponential Functions

1.

number of folds	1	2	3	4	5	...	10	...	50
layers of paper	2	4	8	16	32	...	1024	...	2^{50}

$2^{50} = 1.125899907 \times 10^{15}$

2. 500 sheets are 2 inches high

$$\frac{500}{2 \text{ in.}} = \frac{2^{50}}{x \text{ in.}}$$
$$x = \frac{2 \cdot 2^{50}}{500}$$
$$= 4.503599627 \times 10^{12} \text{ in.}$$
$$= 71{,}079{,}539.57 \text{ mi}$$

3. The graph of $y = 3^x$ is the graph of an exponential function $y = a^x$ with $a > 1$.
This is graph e.

4. The graph of $y = 3^{-x}$ is the graph of $y = 3^x$ reflected in the y-axis.
This is graph d.

5. The graph of $y = \left(\frac{1}{3}\right)^{1-x}$ is the graph of $y = (3^{-1})^{1-x}$ or $y = 3^{x-1}$. This is the graph of $y = 3^x$ translated 1 unit to the right.
This is graph c.

6. The graph of $y = 3^{x+1}$ is the graph of $y = 3^x$ translated 1 unit to the left.
This is graph f.

7. The graph of $y = 3(3)^x$ is the same as the graph of $y = 3^{x+1}$. This is the graph of $y = 3^x$ translated 1 unit to the left.
This is graph f.

8. The graph of $y = \left(\frac{1}{3}\right)^x$ is the graph of $y = (3^{-1})^x = 3^{-x}$. This is the graph of $y = 3^x$ reflected in the y-axis.
This is graph d.

9. The graph of $y = 2 - 3^{-x}$ is the same as the graph of $y = -3^{-x} + 2$. This is the graph of $y = 3^x$ reflected in the x-axis, reflected in the y-axis, and translated up 2 units.
This is graph a.

10. The graph of $y = -2 + 3^{-x}$ is the same as the graph of $y = 3^{-x} - 2$. This is the graph of $y = 3^x$ reflected in the y-axis and translated 2 units downward.
This is graph b.

11. The graph of $y = 3^{x-1}$ is the graph of $y = 3^x$ translated 1 unit to the right.
This is graph c.

13.
$$\begin{aligned} 2^x &= 32 \\ 2^x &= 2^5 \\ x &= 5 \end{aligned}$$

14.
$$\begin{aligned} 4^x &= 64 \\ 4^x &= 4^3 \\ x &= 3 \end{aligned}$$

15.
$$\begin{aligned} 3^x &= \frac{1}{81} \\ 3^x &= \frac{1}{3^4} \\ 3^x &= 3^{-4} \\ x &= -4 \end{aligned}$$

16.
$$\begin{aligned} e^x &= \frac{1}{e^5} \\ e^x &= e^{-5} \\ x &= -5 \end{aligned}$$

17.
$$\begin{aligned} 4^x &= 8^{x+1} \\ (2^2)^x &= (2^3)^{x+1} \\ 2^{2x} &= 2^{3x+3} \\ 2x &= 3x + 3 \\ -x &= 3 \\ x &= -3 \end{aligned}$$

18.
$$\begin{aligned} 25^x &= 125^{x+2} \\ (5^2)^x &= (5^3)^{x+2} \\ 5^{2x} &= 5^{3x+6} \\ 2x &= 3x + 6 \\ -6 &= x \end{aligned}$$

19.
$$\begin{aligned} 16^{x+3} &= 64^{2x-5} \\ (2^4)^{x+3} &= (2^6)^{2x-5} \\ 2^{4x+12} &= 2^{12x-30} \\ 4x + 12 &= 12x - 30 \\ 42 &= 8x \\ \frac{21}{4} &= x \end{aligned}$$

20.
$$\begin{aligned} (e^3)^{-2x} &= e^{-x+5} \\ e^{-6x} &= e^{-x+5} \\ -6x &= -x + 5 \\ -5x &= 5 \\ x &= -1 \end{aligned}$$

21.
$$\begin{aligned} e^{-x} &= (e^4)^{x+3} \\ e^{-x} &= e^{4x+12} \\ -x &= 4x + 12 \\ -5x &= 12 \\ x &= -\frac{12}{5} \end{aligned}$$

22.
$$\begin{aligned} 2^{|x|} &= 8 \\ 2^{|x|} &= 2^3 \\ |x| &= 3 \\ x = 3 \quad &\text{or} \quad x = -3 \end{aligned}$$

23.
$$\begin{aligned} 5^{-|x|} &= \frac{1}{25} \\ 5^{-|x|} &= 5^{-2} \\ -|x| &= -2 \\ |x| &= 2 \\ x = 2 \quad &\text{or} \quad x = -2 \end{aligned}$$

24.
$$\begin{aligned}2^{x^2-4x} &= \left(\frac{1}{16}\right)^{x-4}\\ 2^{x^2-4x} &= (2^{-4})^{x-4}\\ 2^{x^2-4x} &= 2^{-4x+16}\\ x^2-4x &= -4x+16\\ x^2-16 &= 0\\ (x+4)(x-4) &= 0\\ x=-4 \quad &\text{or} \quad x=4\end{aligned}$$

25.
$$\begin{aligned}5^{x^2+x} &= 1\\ 5^{x^2+x} &= 5^0\\ x^2+x &= 0\\ x(x+1) &= 0\\ x=0 \quad \text{or} \quad x+1&=0\\ x=0 \quad \text{or} \quad x&=-1\end{aligned}$$

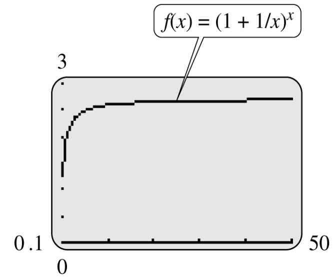
26.
$$\begin{aligned}8^{x^2} &= 2^{x+4}\\ (2^3)^{x^2} &= 2^{x+4}\\ 2^{3x^2} &= 2^{x+4}\\ 3x^2 &= x+4\\ 3x^2-x-4 &= 0\\ (3x-4)(x+1) &= 0\\ x=\frac{4}{3} \quad &\text{or} \quad x=-1\end{aligned}$$

27.
$$\begin{aligned}27^x &= 9^{x^2+x}\\ (3^3)^x &= (3^2)^{x^2+x}\\ 3^{3x} &= 3^{2x^2+2x}\\ 3x &= 2x^2+2x\\ 0 &= 2x^2-x\\ 0 &= x(2x-1)\\ x=0 \quad \text{or} \quad 2x-1&=0\\ x=0 \quad \text{or} \quad x&=\frac{1}{2}\end{aligned}$$

28.
$$\begin{aligned}e^{x^2+5x+6} &= 1\\ e^{x^2+5x+6} &= e^0\\ x^2+5x+6 &= 0\\ (x+3)(x+2) &= 0\\ x+3=0 \quad \text{or} \quad x+2&=0\\ x=-3 \quad \text{or} \quad x&=-2\end{aligned}$$

30. 4 and 6 cannot be easily written as powers of the same base, so the equation $4^x = 6$ cannot be solved using this approach.

32.



$f(x)$ approaches $e \approx 2.71828$.

33. $A = P\left(1+\frac{r}{m}\right)^{tm}$, $P = 10{,}000$, $r = 0.04$, $t = 5$

(a) annually, $m = 1$

$$\begin{aligned}A &= 10{,}000\left(1+\frac{0.04}{1}\right)^{5(1)}\\ &= 10{,}000(1.04)^5\\ &= \$12{,}166.53\end{aligned}$$

$$\begin{aligned}\text{Interest} &= \$12{,}166.53 - \$10{,}000\\ &= \$2166.53\end{aligned}$$

(b) semiannually, $m = 2$

$$\begin{aligned}A &= 10{,}000\left(1+\frac{0.04}{2}\right)^{5(2)}\\ &= 10{,}000(1.02)^{10}\\ &= \$12{,}189.94\end{aligned}$$

$$\begin{aligned}\text{Interest} &= \$12{,}189.94 - \$10{,}000\\ &= \$2189.94\end{aligned}$$

(c) quarterly, $m = 4$

$$\begin{aligned}A &= 10{,}000\left(1+\frac{0.04}{4}\right)^{5(4)}\\ &= 10{,}000(1.01)^{20}\\ &= \$12{,}201.90\end{aligned}$$

$$\begin{aligned}\text{Interest} &= \$12{,}201.90 - \$10{,}000\\ &= \$2201.90\end{aligned}$$

(d) monthly, $m = 12$

$$\begin{aligned}A &= 10{,}000\left(1+\frac{0.04}{12}\right)^{5(12)}\\ &= 10{,}000(1.00\overline{3})^{60}\\ &= \$12{,}209.97\end{aligned}$$

$$\begin{aligned}\text{Interest} &= \$12{,}209.97 - \$10{,}000\\ &= \$2209.97\end{aligned}$$

34. $A = P\left(1 + \frac{r}{m}\right)^{tm}$, $P = 26{,}000,\ r = 0.06,\ t = 4$

(a) annually, $m = 1$

$$\begin{aligned} A &= 26{,}000\left(1 + \frac{0.06}{1}\right)^{4(1)} \\ &= 26{,}000(1.06)^4 \\ &= \$32{,}824.40 \end{aligned}$$

$$\begin{aligned} \text{Interest} &= \$32{,}824.40 - \$26{,}000 \\ &= \$6824.40 \end{aligned}$$

(b) semiannually, $m = 2$

$$\begin{aligned} A &= 26{,}000\left(1 + \frac{0.06}{2}\right)^{4(2)} \\ &= 26{,}000(1.03)^8 \\ &= \$32.936.02 \end{aligned}$$

$$\begin{aligned} \text{Interest} &= \$32.936.02 - \$26{,}000 \\ &= \$6936.02 \end{aligned}$$

(c) quarterly, $m = 4$

$$\begin{aligned} A &= 26{,}000\left(1 + \frac{0.06}{4}\right)^{4(4)} \\ &= 26{,}000(1.015)^{16} \\ &= \$32{,}993.62 \end{aligned}$$

$$\begin{aligned} \text{Interest} &= \$32{,}993.62 - \$26{,}000 \\ &= \$6993.62 \end{aligned}$$

(d) monthly, $m = 12$

$$\begin{aligned} A &= 26{,}000\left(1 + \frac{0.06}{12}\right)^{4(12)} \\ &= 26{,}000(1.005)^{48} \\ &= \$33{,}032.72 \end{aligned}$$

$$\begin{aligned} \text{Interest} &= \$33{,}032.72 - \$26{,}000 \\ &= \$7032.72 \end{aligned}$$

35. For 6% compounded annually for 2 years,

$$\begin{aligned} A &= 18{,}000(1 + 0.06)^2 \\ &= 18{,}000(1.06)^2 \\ &= 20{,}224.80 \end{aligned}$$

For 5.9% compounded monthly for 2 years,

$$\begin{aligned} A &= 18{,}000\left(1 + \frac{0.059}{12}\right)^{12(2)} \\ &= 18{,}000\left(\frac{12.059}{12}\right)^{24} \\ &= 20{,}248.54 \end{aligned}$$

The 5.9% investment is better. The additional interest is

$$\$20{,}248.54 - \$20{,}224.80 = \$23.74.$$

36. $A = P\left(1 + \frac{r}{m}\right)^{tm}$, $P = 5000,\ A = \$7500, t = 5$

(a) $m = 1$

$$\begin{aligned} 7500 &= 5000\left(1 + \frac{r}{1}\right)^{5(1)} \\ \frac{3}{2} &= (1 + r)^5 \\ \left(\frac{3}{2}\right)^{1/5} - 1 &= r \\ 0.084 &\approx r \end{aligned}$$

The interest rate is about 8.4%.

(b) $m = 4$

$$\begin{aligned} 7500 &= 5000\left(1 + \frac{r}{4}\right)^{5(4)} \\ \frac{3}{2} &= \left(1 + \frac{r}{4}\right)^{20} \\ \left(\frac{3}{2}\right)^{1/20} - 1 &= \frac{r}{4} \\ 4\left[\left(\frac{3}{2}\right)^{1/20} - 1\right] &\approx r \\ 0.082 &= r \end{aligned}$$

The interest rate is about 8.2%.

37. $A = Pe^{rt}$

(a) $r = 3\%$

$$A = 10e^{0.03(3)} = \$10.94$$

(b) $r = 4\%$

$$A = 10e^{0.04(3)} = \$11.27$$

(c) $r = 5\%$

$$A = 10e^{0.05(3)} = \$11.62$$

38. $P = \$25{,}000,\ r = 5.5\%$

Use the formula for continuous compounding,

$$A = Pe^{rt}.$$

(a) $t = 1$

$$A = 25{,}000e^{0.055(1)} = \$26{,}413.52$$

(b) $t = 5$

$$A = 25{,}000e^{0.055(5)} = \$32{,}913.27$$

(c) $t = 10$

$$A = 25{,}000e^{0.055(10)} = \$43{,}331.33$$

39. $$1200 = 500\left(1+\frac{r}{4}\right)^{(14)(4)}$$
$$\frac{1200}{500} = \left(1+\frac{r}{4}\right)^{56}$$
$$2.4 = \left(1+\frac{r}{4}\right)^{56}$$
$$1+\frac{r}{4} = (2.4)^{1/56}$$
$$4 + r = 4(2.4)^{1/56}$$
$$r = 4(2.4)^{1/56} - 4$$
$$r \approx 0.0630$$

The required interest rate is 6.30%.

40. **(a)** $$30{,}000 = 10{,}500\left(1+\frac{r}{4}\right)^{(4)(12)}$$
$$\frac{300}{105} = \left(1+\frac{r}{4}\right)^{48}$$
$$1+\frac{r}{4} = \left(\frac{300}{105}\right)^{1/48}$$
$$4 + r = 4\left(\frac{300}{105}\right)^{1/48}$$
$$r = 4\left(\frac{300}{105}\right)^{1/48} - 4$$
$$r \approx 0.0884$$

The required interest rate is 8.84%.

(b) $$30{,}000 = 10{,}500e^{12r}$$
$$\frac{300}{105} = e^{12r}$$
$$12r = \ln\left(\frac{300}{105}\right)$$
$$r = \frac{\ln\left(\frac{300}{105}\right)}{12}$$
$$r \approx 0.0875$$

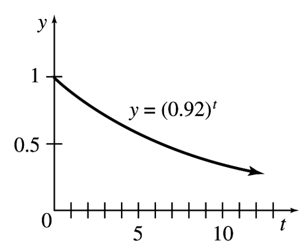
The required interest rate is 8.75%.

41. $y = (0.92)^t$

(a)

t	y
0	$(0.92)^0 = 1$
1	$(0.92)^1 = 0.92$
2	$(0.92)^2 \approx 0.85$
3	$(0.92)^3 \approx 0.78$
4	$(0.92)^4 \approx 0.72$
5	$(0.92)^5 \approx 0.66$
6	$(0.92)^6 \approx 0.61$
7	$(0.92)^7 \approx 0.56$
8	$(0.92)^8 \approx 0.51$
9	$(0.92)^9 \approx 0.47$
10	$(0.92)^{10} \approx 0.43$

(b)



(c) Let x = the cost of the house in 10 years.

Then, $0.43x = 165{,}000$
$x \approx 383{,}721.$

In 10 years, the house will cost about \$384,000.

(d) Let x = the cost of the book in 8 years.

Then, $0.51x = 50$
$x \approx 98$

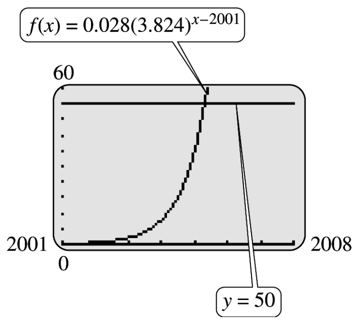
In 8 years, the textbook will cost about \$98.

42. **(a)** $f(x) = f_0 a^{x-2001}$
$f_0 = 0.027733$

Use the point $(2005, 0.027733a^4)$ to find a.

$$a^4 = \frac{5.9330}{0.027733}$$
$$a = \sqrt[4]{\frac{5.9330}{0.027733}}$$
$$\approx 3.824$$
$$f(x) = 0.028(3.824)^{x-2001}$$

(b)



$f(x) = 50$ when $x \approx 2006.58$, or about the year 2007.

(c) $f(x) = f_0 a^{x-2002}$
$f_0 = 0.029947$

Use the point $(2005, 3.31656a^3)$ to find a.

$$3.31656 = 0.029947a^3$$
$$a^3 = \frac{3.31656}{0.029947}$$
$$a = \sqrt[3]{\frac{3.31656}{0.029947}}$$
$$\approx 4.802$$
$$f(x) = 0.02995(4.802)^{x-2002}$$

(d) XM: Annual increase $= 3.824 - 1 = 2.824$ $= 282.4\%$
Sirius: Annual increase $= 4.801 - 1 = 3.801$ $= 380.1\%$

Sirius is increasing more rapidly.

43. $A = P\left(1 + \dfrac{r}{m}\right)^{tm}$

$$A = 1000\left(1 + \frac{j}{2}\right)^{5(2)} = 1000\left(1 + \frac{j}{2}\right)^{10}$$

This represents the amount in Bank X on January 1, 1985.

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$
$$= \left[1000\left(1 + \frac{j}{2}\right)^{10}\right]\left(1 + \frac{k}{4}\right)^{3(4)}$$
$$= 1000\left(1 + \frac{j}{2}\right)^{10}\left(1 + \frac{k}{4}\right)^{12}$$

This represents the amount in Bank Y on January 1, 1988, \$1990.76.

$$A = P\left(1 + \frac{r}{m}\right)^{tm} = 1000\left(1 + \frac{k}{4}\right)^{8\cdot 4}$$
$$= 1000\left(1 + \frac{k}{4}\right)^{32}$$

This represents the amount he could have had from January 1, 1980, to January 1, 1988, at a rate of k per annum compounded quarterly, \$2203.76.

So,

$$1000\left(1 + \frac{j}{2}\right)^{10}\left(1 + \frac{k}{4}\right)^{12} = 1990.76$$
$$\text{and} \quad 1000\left(1 + \frac{k}{4}\right)^{32} = 2203.76.$$
$$\left(1 + \frac{k}{4}\right)^{32} = 2.20376$$
$$1 + \frac{k}{4} = (2.20376)^{1/32}$$
$$1 + \frac{k}{4} = 1.025$$
$$\frac{k}{4} = 0.025$$
$$k = 0.1 \quad \text{or} \quad 10\%$$

Substituting, we have

$$\begin{aligned}1000\left(1+\frac{j}{2}\right)^{10}\left(1+\frac{1.0}{4}\right)^{12} &= 1990.76\\ 1000\left(1+\frac{j}{2}\right)^{10}(1.025)^{12} &= 1990.76\\ \left(1+\frac{j}{2}\right)^{10} &= 1.480\\ 1+\frac{j}{2} &= (1.480)^{1/10}\\ 1+\frac{j}{2} &= 1.04\\ \frac{j}{2} &= 0.04\\ j &= 0.08 \quad \text{or} \quad 8\%.\end{aligned}$$

The ratio $\frac{k}{j} = \frac{0.1}{0.08} = 1.25$, is choice (a).

44. $A(t) = 2600e^{0.017t}$

(a) 1970: $t = 20$

$$\begin{aligned}A(20) &= 2600e^{0.017(20)}\\ &= 2600e^{0.34}\\ &\approx 3650\end{aligned}$$

The function gives a population of about 3650 million in 1970.
This is very close to the actual population of about 3700 million.

(b) 2000: $t = 50$

$$\begin{aligned}A(50) &= 2600e0^{0.017(50)}\\ &= 2600e^{0.85}\\ &\approx 6083\end{aligned}$$

The function gives a population of 6083 million in 2000.

(c) 2010: $t = 60$

$$\begin{aligned}&= 2600e^{0.017(60)}\\ &= 2600e^{1.02}\\ &= 7210\end{aligned}$$

From the function, we estimate that the world population in 2010 will be 7210 million.

45. $f(x) = 500 \cdot 2^{3x}$

(a) After 1 hour:

$$f(1) = 500 \cdot 2^{3(1)} = 500 \cdot 8 = 4000 \text{ bacteria}$$

(b) initially:

$$f(0) = 500 \cdot 2^{3(0)} = 500 \cdot 1 = 500 \text{ bacteria}$$

(c) The bacteria double every $3x = 1$ hour, or every $\frac{1}{3}$ hour.

(d) When does $f(x) = 32{,}000$?

$$\begin{aligned}32{,}000 &= 500 \cdot 2^{3x}\\ 64 &= 2^{3x}\\ 2^6 &= 2^{3x}\\ 6 &= 3x\\ x &= 2\end{aligned}$$

The number of bacteria will increase to 32,000 in 2 hours.

46. (a) Hispanic population:

$$\begin{aligned}h(t) &= 37.79(1.021)^t\\ h(5) &= 37.79(1.021)^5\\ &\approx 41.93\end{aligned}$$

The projected Hispanic population in 2005 is 41.93 million, which is slightly less than the actual value of 42.69 million.

(b) Asian population:

$$\begin{aligned}h(t) &= 11.14(1.023)^t\\ h(5) &= 11.14(1.023)^5\\ &\approx 12.48\end{aligned}$$

The projected Asian population in 2005 is 12.48 million, which is very close to the actual value of 12.69 million.

(c) Annual Hispanic percent increase:

$1.021 - 1 = 0.021 = 2.1\%$

Annual Asian percent increase:

$1.023 - 1 = 0.023 = 2.3\%$

The Asian population is growing at a slightly faster rate.

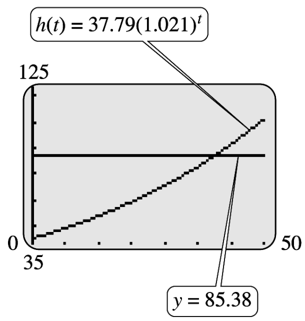
(d) Black population:

$$\begin{aligned}b(t) &= 0.5116t + 35.43\\ b(5) &= 0.5116(5) + 35.43\\ &\approx 37.99\end{aligned}$$

The projected Black population in 2005 is 37.99 million, which is extremely close to the actual value of 37.91 million.

(e) Hispanic population:
Double the actual 2005 value is

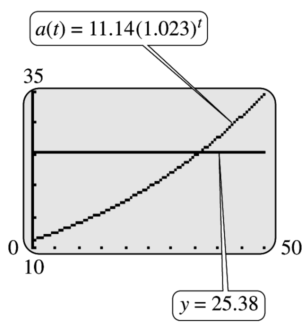
$$2(42.69) = 85.38 \text{ million.}$$



The doubling point is reached when $t \approx 39$, or in the year 2039.

Asian population:
Double the actual 2005 value is

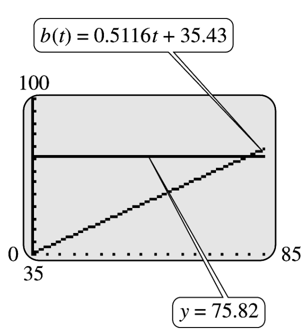
$$2(12.69) = 25.38 \text{ million.}$$



The doubling point is reached when $t \approx 36$, or in the 2036.

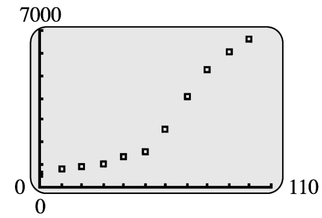
Black population:
Double the actual 2005 value is

$$2(37.91) = 7582 \text{ million.}$$



The doubling point is reached when $t \approx 79$, or in the year 2079.

47. (a)



The emissions appear to grow exponentially.

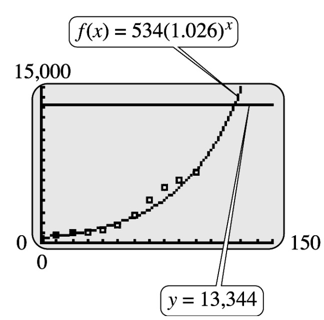
(b) $f(x) = f_0 a^x$
$f_0 = 534$

Use the point $(100, 6672)$ to find a.

$$\begin{aligned} 6672 &= 534a^{100} \\ a^{100} &= \frac{6672}{534} \\ a &= \sqrt[100]{\frac{6672}{534}} \\ &\approx 1.026 \\ f(x) &= 534(1.026)^x \end{aligned}$$

(c) $1.026 - 1 = 0.026 = 2.6\%$

(d) Double the 2000 value is $2(6672) = 13{,}344$.



The doubling point is reached when $x \approx 125.4$. The first year in which emissions equal or exceed that threshold is 2026.

48. $Q(t) = 1000(5^{-0.3t})$

(a) $$\begin{aligned} Q(6) &= 1000[5^{-0.3(6)}] \\ &= 1000(5^{-1.8}) \\ &= 55 \end{aligned}$$

The amount present in 6 months will be 55 grams.

(b) $$\begin{aligned} 8 &= 1000(5^{-0.3t}) \\ \frac{1}{125} &= 5^{-0.3t} \\ 5^{-3} &= 5^{-0.3t} \\ -3 &= -0.3t \\ 10 &= t \end{aligned}$$

It will take 10 months to reduce the substance to 8 grams.

49. (a) When $x = 0,\ P = 1013.$
When $x = 10,000,\ P = 265.$
First we fit $P = ae^{kx}$.

$$\begin{aligned} 1013 &= ae^0 \\ a &= 1013 \\ P &= 1013e^{kx} \\ 265 &= 1013e^{k(10,000)} \\ \frac{265}{1013} &= e^{10,000k} \\ 10,000k &= \ln\left(\frac{265}{1013}\right) \\ k &= \frac{\ln\left(\frac{265}{1013}\right)}{10,000} \approx -1.34 \times 10^{-4} \end{aligned}$$

Therefore $P = 1013e^{(-1.34\times 10^{-4})x}$.

Next we fit $P = mx + b$.
We use the points $(0, 1013)$ and $(10,000, 265)$.

$$\begin{aligned} m &= \frac{265 - 1013}{10,000 - 0} = -0.0748 \\ b &= 1013 \end{aligned}$$

Therefore $P = -0.0748x + 1013$.

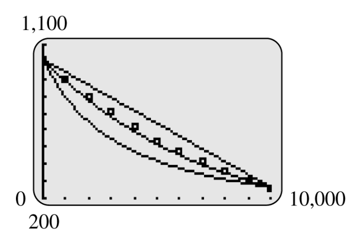
Finally, we fit $P = \frac{1}{ax+b}$.

$$\begin{aligned} 1013 &= \frac{1}{a(0) + b} \\ b &= \frac{1}{1013} \approx 9.87 \times 10^{-4} \\ P &= \frac{1}{ax + \frac{1}{1013}} \\ 265 &= \frac{1}{10,000a + \frac{1}{1013}} \\ \frac{1}{265} &= 10,000a + \frac{1}{1013} \\ 10,000a &= \frac{1}{265} - \frac{1}{1013} \\ a &= \frac{\frac{1}{265} - \frac{1}{1013}}{10,000} \approx 2.79 \times 10^{-7} \end{aligned}$$

Therefore,

$$P = \frac{1}{(2.79 \times 10^{-7})x + (9.87 \times 10^{-4})}.$$

(b)



$P = 1013e^{(-1.34\times 10^{-4})x}$ is the best fit.

(c) $$\begin{aligned} P(1500) &= 1013e^{-1.34\times 10^{-4}(1500)} \approx 829 \\ P(11,000) &= 1013e^{-1.34\times 10^{-4}(11,000)} \approx 232 \end{aligned}$$

We predict that the pressure at 1500 meters will be 829 millibars, and at 11,000 meters will be 232 millibars.

(d) Using exponential regression, we obtain $P = 1038(0.99998661)^x$ which differs slightly from the function found in part (b) which can be rewritten as

$$P = 1013(0.99998660)^x.$$

50. (a) $y = mx + b$
$b = 0.275$

Use the point $(21, 291)$ to find m.

$$\begin{aligned} 291 &= m(21) + 0.275 \\ 290.725 &= 21m \\ m &= \frac{290.725}{21} \\ &\approx 13.844 \\ y &= 13.844x + 0.275 \end{aligned}$$

$$\begin{aligned} y &= ax^2 + b \\ b &= 0.275 \end{aligned}$$

Use the point $(21, 291)$ to find a.

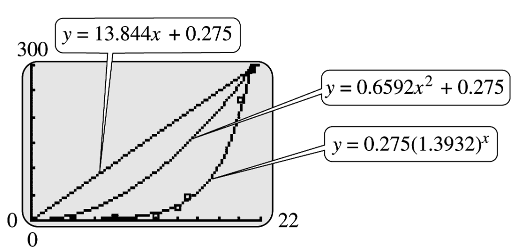
$$\begin{aligned} 291 &= a(21)^2 + 0.275 \\ 290.725 &= 441a \\ m &= \frac{290.725}{441} \\ &\approx 0.6592 \\ y &= 0.6592x^2 + 0.275 \end{aligned}$$

$$\begin{aligned} y &= ab^x \\ a &= 0.275 \end{aligned}$$

Use the point $(21, 291)$ to find b.

$$\begin{aligned} 291 &= 0.275b^{21} \\ b^{21} &= \frac{291}{0.275} \\ b &= \sqrt[21]{\frac{291}{0.275}} \\ &\approx 1.3932 \\ y &= 0.275(1.3932)^x \end{aligned}$$

(b)



$y = 0.275(1.3932)^x$ is the best fit.

(c) $y = 0.275(1.3932)^{25} \approx 1095.6$ million

(d) The regression equation is $y = 0.247(1.3942)^x$. This is close to the function found in part (b).

2.5 Logarithmic Functions

1. $5^3 = 125$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is

$$\log_5 125 = 3.$$

2. $7^2 = 49$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is

$$\log_7 49 = 2.$$

3. $3^4 = 81$

The equation in logarithmic form is

$$\log_3 81 = 4.$$

4. $2^7 = 128$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is

$$\log_2 128 = 7.$$

5. $3^{-2} = \frac{1}{9}$

The equation in logarithmic form is

$$\log_3 \frac{1}{9} = -2.$$

6. $\left(\frac{4}{5}\right)^{-2} = \frac{25}{16}$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is

$$\log_{4/5} \frac{25}{16} = -2.$$

7. $\log_2 32 = 5$

Since $y = \log_a x$ means $a^y = x$, the equation in exponential form is

$$2^5 = 32.$$

8. $\log_3 81 = 4$

Since $y = \log_a x$ means $a^y = x$, the equation in exponential form is

$$3^4 = 81.$$

9. $\ln \frac{1}{e} = -1$

The equation in exponential form is

$$e^{-1} = \frac{1}{e}.$$

10. $\log_2 \frac{1}{8} = -3$

The equation in exponential form is

$$2^{-3} = \frac{1}{8}.$$

11.
$$\begin{aligned} \log\ 100{,}000 &= 5 \\ \log_{10}\ 100{,}000 &= 5 \\ 10^5 &= 100{,}000 \end{aligned}$$

When no base is written, $\log_{10}$ is understood.

12.
$$\begin{aligned} \log\ 0.001 &= -3 \\ \log_{10}\ 0.001 &= -3 \\ 10^{-3} &= 0.001 \end{aligned}$$

When no base is written, $\log_{10}$ is understood.

13. Let $\log_8\ 64 = x$.

$$\begin{aligned} \text{Then,}\quad 8^x &= 64 \\ 8^x &= 8^2 \\ x &= 2. \end{aligned}$$

Thus, $\log_8\ 64 = 2$.

14. Let $\log_9\ 81 = x$.

$$\begin{aligned} \text{Then,}\ 9^x &= 81 \\ 9^x &= 9^2 \\ x &= 2. \end{aligned}$$

Thus, $\log_9\ 81 = 2$.

15.
$$\begin{aligned} \log_4\ 64 &= x \\ 4^x &= 64 \\ 4^x &= 4^3 \\ x &= 3 \end{aligned}$$

16. $\log_3\ 27 = x$

$$\begin{aligned} 3^x &= 27 \\ 3^x &= 3^3 \\ x &= 3 \end{aligned}$$

17. $\log_2 \frac{1}{16} = x$

$$\begin{aligned} 2^x &= \frac{1}{16} \\ 2^x &= 2^{-4} \\ x &= -4 \end{aligned}$$

18. $\log_3 \frac{1}{81} = x$

$$\begin{aligned} 3^x &= \frac{1}{81} \\ 3^x &= 3^{-4} \\ x &= -4 \end{aligned}$$

19. $\log_2 \sqrt[3]{\frac{1}{4}} = x$

$$\begin{aligned} 2^x &= \left(\frac{1}{4}\right)^{1/3} \\ 2^x &= \left(\frac{1}{2^2}\right)^{1/3} \\ 2^x &= 2^{-2/3} \\ x &= -\frac{2}{3} \end{aligned}$$

20. $\log_8 \sqrt[4]{\frac{1}{2}} = x$

$$\begin{aligned} 8^x &= \sqrt[4]{\frac{1}{2}} = \left(\frac{1}{2}\right)^{1/4} \\ (2^3)^x &= 2^{-1/4} \\ 3x &= -\frac{1}{4} \\ x &= -\frac{1}{12} \end{aligned}$$

21. $\ln\ e = x$

Recall that ln means $\log_e$.

$$\begin{aligned} e^x &= e \\ x &= 1 \end{aligned}$$

22. $\ln\ e^3 = x$

Recall that ln means $\log_e$.

$$\begin{aligned} e^x &= e^3 \\ x &= 3 \end{aligned}$$

23.
$$\begin{aligned} \ln\ e^{5/3} &= x \\ e^x &= e^{5/3} \\ x &= \frac{5}{3} \end{aligned}$$

24.
$$\begin{aligned} \ln\ 1 &= x \\ e^x &= 1 \\ e^x &= e^0 \\ x &= 0 \end{aligned}$$

25. The logarithm to the base 3 of 4 is written $\log_3\ 4$. The subscript denotes the base.

27. $\log_5 (3k) = \log_5 3 + \log_5 k$

28. $\log_9 (4m) = \log_9 4 + \log_9 m$

29. $\log_3 \dfrac{3p}{5k}$

$$\begin{aligned} &= \log_3 3p - \log_3 5k \\ &= (\log_3 3 + \log_3 p) - (\log_3 5 + \log_3 k) \\ &= 1 + \log_3 p - \log_3 5 - \log_3 k \end{aligned}$$

30. $\log_7 \dfrac{15p}{7y}$

$$\begin{aligned} &= \log_7 15p - \log_7 7y \\ &= (\log_7 15 + \log_7 p) - (\log_7 7 + \log_7 y) \\ &= \log_7 15 + \log_7 p - \log_7 7 - \log_7 y \\ &= \log_7 15 + \log_7 p - 1 - \log_7 y \end{aligned}$$

31. $\ln \dfrac{3\sqrt{5}}{\sqrt[3]{6}}$

$$\begin{aligned} &= \ln 3\sqrt{5} - \ln \sqrt[3]{6} \\ &= \ln 3 \cdot 5^{1/2} - \ln 6^{1/3} \\ &= \ln 3 + \ln 5^{1/2} - \ln 6^{1/3} \\ &= \ln 3 + \frac{1}{2} \ln 5 - \frac{1}{3} \ln 6 \end{aligned}$$

32. $\ln \dfrac{9\sqrt[3]{5}}{\sqrt[4]{3}}$

$$\begin{aligned} &= \ln 9\sqrt[3]{5} - \ln \sqrt[4]{3} \\ &= \ln 9 \cdot 5^{1/3} - \ln 3^{1/4} \\ &= \ln 9 + \ln 5^{1/3} - \ln 3^{1/4} \\ &= \ln 9 + \frac{1}{3} \ln 5 - \frac{1}{4} \ln 3 \end{aligned}$$

33.
$$\begin{aligned} \log_b 32 &= \log_b 2^5 \\ &= 5 \log_b 2 \\ &= 5a \end{aligned}$$

34. $\log_b 18$

$$\begin{aligned} &= \log_b (2 \cdot 9) \\ &= \log_b (2 \cdot 3^2) \\ &= \log_b 2 + \log_b 3^2 \\ &= \log_b 2 + 2 \log_b 3 \\ &= a + 2c \end{aligned}$$

35.
$$\begin{aligned} \log_b 72b &= \log_b 72 + \log_b b \\ &= \log_b 72 + 1 \\ &= \log_b 2^3 \cdot 3^3 + 1 \\ &= \log_b 2^3 + \log_b 3^2 + 1 \\ &= 3 \log_b 2 + 2 \log_b 3 + 1 \\ &= 3a + 2c + 1 \end{aligned}$$

36.
$$\begin{aligned} \log_b (9b^2) &= \log_b 9 + \log_b b^2 \\ &= \log_b 3^2 + \log_b b^2 \\ &= 2 \log_b 3 + 2 \log_b b \\ &= 2c + 2(1) \\ &= 2c + 2 \end{aligned}$$

37.
$$\begin{aligned} \log_5 30 &= \frac{\ln 30}{\ln 5} \\ &\approx \frac{3.4012}{1.6094} \\ &\approx 2.113 \end{aligned}$$

38.
$$\begin{aligned} \log_{12} 210 &= \frac{\ln 210}{\ln 12} \\ &\approx 2.152 \end{aligned}$$

39.
$$\begin{aligned} \log_{1.2} 0.95 &= \frac{\ln 0.95}{\ln 1.2} \\ &\approx -0.281 \end{aligned}$$

40.
$$\begin{aligned} \log_{2.8} 0.12 &= \frac{\ln 0.12}{\ln 2.8} \\ &\approx -2.059 \end{aligned}$$

41.
$$\begin{aligned} \log_x 36 &= -2 \\ x^{-2} &= 36 \\ (x^{-2})^{-1/2} &= 36^{-1/2} \\ x &= \frac{1}{6} \end{aligned}$$

42.
$$\begin{aligned} \log_9 27 &= m \\ 9^m &= 27 \\ (3^2)^m &= 3^3 \\ 3^{2m} &= 3^3 \\ 2m &= 3 \\ m &= \frac{3}{2} \end{aligned}$$

43.
$$\begin{aligned} \log_8 16 &= z \\ 8^z &= 16 \\ (2^3)^z &= 2^4 \\ 2^{3z} &= 2^4 \\ 3z &= 4 \\ z &= \frac{4}{3} \end{aligned}$$

44. $\log_y 8 = \frac{3}{4}$

$$\begin{aligned} y^{3/4} &= 8 \\ (y^{3/4})^{4/3} &= 8^{4/3} \\ y &= (8^{1/3})^4 \\ &= 2^4 = 16 \end{aligned}$$

45. $\log_r 5 = \frac{1}{2}$

$$\begin{aligned} r^{1/2} &= 5 \\ (r^{1/2})^2 &= 5^2 \\ r &= 25 \end{aligned}$$

46. $\log_4 (5x+1) = 2$

$$\begin{aligned} 4^2 &= 5x+1 \\ 16 &= 5x+1 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

47. $\log_5 (9x-4) = 1$

$$\begin{aligned} 5^1 &= 9x-4 \\ 9 &= 9x \\ 1 &= x \end{aligned}$$

48. $\log_4 x - \log_4 (x+3) = -1$

$$\begin{aligned} \log_4 \frac{x}{x+3} &= -1 \\ 4^{-1} &= \frac{x}{x+3} \\ \frac{1}{4} &= \frac{x}{x+3} \\ 4x &= x+3 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

49. $\log_9 m - \log_9 (m-4) = -2$

$$\begin{aligned} \log_9 \frac{m}{m-4} &= -2 \\ 9^{-2} &= \frac{m}{m-4} \\ \frac{1}{81} &= \frac{m}{m-4} \\ m-4 &= 81m \\ -4 &= 80m \\ -0.05 &= m \end{aligned}$$

This value is not possible since $\log_9 (-0.05)$ does not exist.
Thus, there is no solution to the original equation.

50. $\log (x+5) + \log (x+2) = 1$

$$\begin{aligned} \log [(x+5)(x+2)] &= 1 \\ (x+5)(x+2) &= 10^1 \\ x^2+7x+10 &= 10 \\ x^2+7x &= 0 \\ x(x+7) &= 0 \\ x = 0 \quad \text{or} \quad x &= -7 \end{aligned}$$

$x = -7$ is not a solution of the original equation because if $x = -7, x+5$ and $x+2$ would be negative, and the domain of $y = \log x$ is $(0, \infty)$. Therefore, $x = 0$.

51. $\log_3 (x-2) + \log_3 (x+6) = 2$

$$\begin{aligned} \log_3 [(x-2)(x+6)] &= 2 \\ (x-2)(x+6) &= 3^2 \\ x^2+4x-12 &= 9 \\ x^2+4x-21 &= 0 \\ (x+7)(x-3) &= 0 \\ x = -7 \quad \text{or} \quad x &= 3 \end{aligned}$$

$x = -7$ does not check in the original equation.
The only solution is 3.

52. $\log_3(x^2+17) - \log_3(x+5) = 1$

$$\begin{aligned} \log_3 \frac{x^2+17}{x+5} &= 1 \\ 3^1 &= \frac{x^2+17}{x+5} \\ 3x+15 &= x^2+17 \\ 0 &= x^2-3x+2 \\ 0 &= (x-1)(x-2) \end{aligned}$$

$x = 1$ or $x = 2$

53. $\log_2(x^2-1) - \log_2(x+1) = 2$

$$\begin{aligned} \log_2 \frac{x^2-1}{x+1} &= 2 \\ 2^2 &= \frac{x^2-1}{x+1} \\ 4 &= \frac{(x-1)(x+1)}{x+1} \\ 4 &= x-1 \\ x &= 5 \end{aligned}$$

54.

$$\begin{aligned} 2^x &= 6 \\ \ln 2^x &= \ln 6 \\ x \ln 2 &= \ln 6 \\ x &= \frac{\ln 6}{\ln 2} \approx 2.585 \end{aligned}$$

55.

$$\begin{aligned} 5^x &= 12 \\ x \log 5 &= \log 12 \\ x &= \frac{\log 12}{\log 5} \\ &\approx 1.544 \end{aligned}$$

56.

$$\begin{aligned} e^{k-1} &= 6 \\ \ln e^{k-1} &= \ln 6 \\ (k-1) \ln e &= \ln 6 \\ k-1 &= \frac{\ln 6}{\ln e} \\ k-1 &= \frac{\ln 6}{1} \\ k &= 1 + \ln 6 \\ &\approx 2.792 \end{aligned}$$

57.

$$\begin{aligned} e^{2y} &= 15 \\ \ln e^{2y} &= \ln 15 \\ 2y \ln e &= \ln 15 \\ 2y(1) &= \ln 15 \\ y &= \frac{\ln 15}{2} \\ &\approx 1.354 \end{aligned}$$

58.

$$\begin{aligned} 2e^{5a+12} &= 10 \\ e^{5a+12} &= 5 \\ \ln e^{5a+12} &= \ln 5 \\ (5a+12) \ln e &= \ln 5 \\ 5a + 12 &= \ln 5 \\ a &= \frac{\ln 5 - 12}{5} \\ a &\approx -2.078 \end{aligned}$$

59.

$$\begin{aligned} 10e^{3z-7} &= 100 \\ \ln 10e^{3z-7} &= \ln 100 \\ \ln 10 + \ln e^{3z-7} &= \ln 100 \\ \ln 10 + (3z-7) \ln e &= \ln 100 \\ 3z - 7 &= \ln 100 - \ln 10 \\ 3z &= \ln 100 - \ln 10 + 7 \\ z &= \frac{\ln 100 - \ln 10 + 7}{3} \\ &\approx 3.101 \end{aligned}$$

60.

$$\begin{aligned} 5(0.10)^x &= 4(0.12)^x \\ \ln[5(0.10)^x] &= \ln[4(0.12)^x] \\ \ln 5 + x \ln 0.10 &= \ln 4 + x \ln 0.12 \\ x(\ln 0.12 - \ln 0.10) &= \ln 5 - \ln 4 \\ x &= \frac{\ln 5 - \ln 4}{\ln 0.12 - \ln 0.10} \\ &\approx 1.224 \end{aligned}$$

61.

$$\begin{aligned} 1.5(1.05)^x &= 2(1.01)^x \\ \ln[1.5(1.05)^x] &= \ln[2(1.01)^x] \\ \ln 1.5 + x \ln 1.05 &= \ln 2 + x \ln 1.01 \\ x(\ln 1.05 - \ln 1.01) &= \ln 2 - \ln 1.5 \\ x &= \frac{\ln 2 - \ln 1.5}{\ln 1.05 - \ln 1.01} \\ &\approx 7.407 \end{aligned}$$

62. $f(x) = \log(5 - x)$

$$\begin{aligned} 5 - x &> 0 \\ -x &> -5 \\ x &< 5 \end{aligned}$$

The domain of f is $x < 5$.

63. $f(x) = \ln(x^2 - 9)$

Since the domain of $f(x) = \ln x$ is $(0, \infty)$, the domain of $f(x) = \ln(x^2 - 9)$ is the set of all real numbers x for which

$$x^2 - 9 > 0.$$

To solve this quadratic inequality, first solve the corresponding quadratic equation.

$$\begin{aligned} x^2 - 9 &= 0 \\ (x+3)(x-3) &= 0 \\ x + 3 = 0 \quad &\text{or} \quad x - 3 = 0 \\ x = -3 \quad &\text{or} \quad x = 3 \end{aligned}$$

These two solutions determine three intervals on the number line: $(-\infty, -3), (-3, 3)$, and $(3, \infty)$.

If $x = -4$, $(-4+2)(-4-2) > 0$.
If $x = 0$, $(0+2)(0-2) \not> 0$.
If $x = 4$, $(4+2)(4-2) > 0$.

The domain is $x < -3$ or $x > 3$, which is written in interval notation as $(-\infty, -3) \cup (3, \infty)$.

64. $\log A - \log B = 0$

$$\begin{aligned} \log \frac{A}{B} &= 0 \\ \frac{A}{B} &= 10^0 = 1 \\ A &= B \\ A - B &= 0 \end{aligned}$$

Thus, solving $\log A - \log B = 0$ is equivalent to solving $A - B = 0$.

65. Let $m = \log_a \frac{x}{y}$, $n = \log_a x$, and $p = \log_a y$.
Then $a^m = \frac{x}{y}$, $a^n = x$, and $a^p = y$.
Substituting gives

$$a^m = \frac{x}{y} = \frac{a^n}{a^p} = a^{n-p}.$$

So $m = n - p$.
Therefore,

$$\log_a \frac{x}{y} = \log_a x - \log_a y.$$

66. Let $m = \log_a x^r$ and $n = \log_a x$.
Then, $a^m = x^r$ and $a^n = x$.
Substituting gives

$$a^m = x^r = (a^n)^r = a^{nr}.$$

Therefore, $m = nr$, or

$$\log_a x^r = r \log_a x.$$

67. From Example 7, the doubling time t in years when $m = 1$ is given by

$$t = \frac{\ln 2}{\ln(1+r)}.$$

(a) Let $r = 0.03$.

$$t = \frac{\ln 2}{\ln 1.03} = 23.4 \text{ years}$$

(b) Let $r = 0.06$.

$$t = \frac{\ln 2}{\ln 1.06} = 11.9 \text{ years}$$

(c) Let $r = 0.08$.

$$t = \frac{\ln 2}{\ln 1.08} = 9.0 \text{ years}$$

(d) Since $0.001 \le 0.03 \le 0.05$, for $r = 0.03$, we use the rule of 70.

$$\frac{70}{100r} = \frac{70}{100(0.03)} = 23.3 \text{ years}$$

Since $0.05 \le 0.06 \le 0.12$, for $r = 0.06$, we use the rule of 72.

$$\frac{72}{100r} = \frac{72}{100(0.06)} = 12 \text{ years}$$

For $r = 0.08$, we use the rule of 72.

$$\frac{72}{100(0.08)} = 9 \text{ years}$$

68. (a) $t = \frac{\ln 2}{\ln(1+r)}$

$$t = \frac{\ln 2}{\ln(1+0.07)}$$

$$t \approx 10.24$$

It will take 11 years for the compound amount to be at least double.

(b) $t = \frac{\ln 3}{\ln(1+0.07)}$

$$t \approx 16.24$$

It will take 17 years for the compound amount to at least triple.

(c) The rule of 72 gives

$$\frac{72}{100(0.07)} = 10.29$$

years as the doubling time.

69. $A = Pe^{rt}$

$$1200 = 500e^{r \cdot 14}$$
$$2.4 = e^{14r}$$
$$\ln(2.4) = \ln e^{14r}$$
$$\ln(2.4) = 14r$$
$$\frac{\ln(2.4)}{14} = r$$
$$0.0625 \approx r$$

The interest rate should be 6.25%.

70.

r (sec)	0.001	0.02	0.05	0.08	0.12
$\frac{\ln 2}{\ln(1+r)}$	693.5	35	14.2	9.01	6.12
$\frac{70}{100r}$	700	35	14	8.75	5.83
$\frac{72}{100r}$	720	36	14.4	9	6

For $0.001 \le r < 0.05$, the Rule of 70 is more accurate. For $0.05 < r \le 0.12$, the Rule of 72 is more accurate. At $r = 0.05$, the two are equally accurate.

71. After x years at Humongous Enterprises, your salary would be 45,000 $(1+0.04)^x$ or 45,000 $(1.04)^x$.
After x years at Crabapple Inc., your salary would be 30,000 $(1+0.06)^x$ or 30,000 $(1.06)^x$.
First we find when the salaries would be equal.

$$45{,}000(1.04)^x = 30{,}000(1.06)^x$$
$$\frac{(1.04)^x}{(1.06)^x} = \frac{30{,}000}{45{,}000}$$
$$\left(\frac{1.04}{1.06}\right)^x = \frac{2}{3}$$
$$\log\left(\frac{1.04}{1.06}\right)^x = \log\left(\frac{2}{3}\right)$$
$$x\log\left(\frac{1.04}{1.06}\right) = \log\left(\frac{2}{3}\right)$$
$$x = \frac{\log\left(\frac{2}{3}\right)}{\log\left(\frac{1.04}{1.06}\right)}$$
$$x \approx 21.29$$
$$2009 + 21.29 = 2030.29$$

Therefore, on July 1, 2031, the job at Crabapple, Inc., will pay more.

72. If the number N is proportional to $m^{-0.6}$, where m is the mass, then $N = km^{-0.6}$, for some constant of proportionality k.
Taking the common log of both sides, we have

$$\begin{aligned}\log N &= \log(km^{-0.6})\\ &= \log k + \log m^{-0.6}\\ &= \log k - 0.6\log m.\end{aligned}$$

This is a linear equation in $\log m$. Its graph is a straight line with slope -0.6 and vertical intercept $\log k$.

73. (a) The total number of individuals in the community is 50 + 50, or 100.

Let $P_1 = \frac{50}{100} = 0.5,\ P_2 = 0.5$.

$$\begin{aligned}H &= -1[P_1\ \ln\ P_1 + P_2\ \ln\ P_2]\\ &= -1[0.5\ \ln\ 0.5 + 0.5\ \ln\ 0.5]\\ &\approx 0.693\end{aligned}$$

(b) For 2 species, the maximum diversity is $\ln 2$.

(c) Yes, $\ln 2 \approx 0.693$.

74. $H = -[P_1\ \ln\ P_1 + P_2\ \ln\ P_2 + P_3\ \ln\ P_3 + P_4\ \ln\ P_4]$

$H = -[0.521\ \ln\ 0.521 + 0.324\ \ln\ 0.324 + 0.081\ \ln\ 0.081 + 0.074\ \ln 0.074]$

$H = 1.101$

75. (a) 3 species, $\frac{1}{3}$ each:

$$P_1 = P_2 = P_3 = \frac{1}{3}$$

$$\begin{aligned}H &= -(P_1\ln P_1 + P_2\ln P_2 + P_3\ln P_3)\\ &= -3\left(\frac{1}{3}\ln\frac{1}{3}\right)\\ &= -\ln\frac{1}{3}\\ &\approx 1.099\end{aligned}$$

(b) 4 species, $\frac{1}{4}$ each:

$$P_1 = P_2 = P_3 = P_4 = \frac{1}{4}$$

$$\begin{aligned}H &= (P_1\ln P_1 + P_2\ln P_2 + P_3\ln P_3 + P_4\ln P_4)\\ &= -4\left(\frac{1}{4}\ln\frac{1}{4}\right)\\ &= -\ln\frac{1}{4}\\ &\approx 1.386\end{aligned}$$

(c) Notice that

$$-\ln\frac{1}{3} = \ln(3^{-1})^{-1} = \ln 3 \approx 1.099$$

and

$$-\ln\frac{1}{4} = \ln(4^{-1})^{-1} = \ln 4 \approx 1.386$$

by Property c of logarithms, so the populations are at a maximum index of diversity.

76.
$$\begin{aligned}mX + N &= m\ \log_b\ x + \log_b\ n\\ &= \log_b\ x^m + \log_b\ n\\ &= \log_b\ nx^m\\ &= \log_b\ y\\ &= Y\end{aligned}$$

Thus, $Y = mX + N$.

77. $C(t) = C_0 e^{-kt}$

When $t = 0,\ C(t) = 2$, and when $t = 3,\ C(t) = 1$.

$$\begin{aligned} 2 &= C_0 e^{-k(0)} \\ C_0 &= 2 \\ 1 &= 2e^{-3k} \\ \frac{1}{2} &= e^{-3k} \\ -3k &= \ln \frac{1}{2} = \ln 2^{-1} = -\ln 2 \\ k &= \frac{\ln 2}{3} \\ T &= \frac{1}{k} \ln \frac{C_2}{C_1} \\ T &= \frac{1}{\frac{\ln 2}{3}} \ln \frac{5\, C_1}{C_1} \\ T &= \frac{3 \ln 5}{\ln 2} \\ T &\approx 7.0 \end{aligned}$$

The drug should be given about every 7 hours.

78. **(a)** From the given graph, when $x = 0.3$ kg $y \approx 4.3$ ml/ min, and when $x = 0.7$ kg $y \approx 7.8$ ml/ min.

(b) If $y = ax^b$, then

$$\begin{aligned} \ln y &= \ln (ax^b) \\ &= \ln a + b \ln x. \end{aligned}$$

Thus, there is a linear relationship between $\ln y$ and $\ln x$.

(c)

$$\begin{aligned} 4.3 &= a(0.3)^b \\ 7.8 &= a(0.7)^b \\ \frac{4.3}{7.8} &= \frac{a(0.3)^b}{a(0.7)^b} \\ \frac{4.3}{7.8} &= \left(\frac{0.3}{0.7}\right)^b \\ \ln \left(\frac{4.3}{7.8}\right) &= \ln \left(\frac{0.3}{0.7}\right)^b \\ \ln \left(\frac{4.3}{7.8}\right) &= b \ln \left(\frac{0.3}{0.7}\right) \\ b &= \frac{\ln \left(\frac{4.3}{7.8}\right)}{\ln \left(\frac{0.3}{0.7}\right)} \\ b &\approx 0.7028 \end{aligned}$$

Substituting this value into $4.3 = a(0.3)^b$,

$$\begin{aligned} 4.3 &= a(0.3)^{0.7028} \\ a &= \frac{4.3}{(0.3)^{0.7028}} \approx 10.02. \end{aligned}$$

Therefore, $y = 10.02x^{0.7028}$.

(d) If $x = 0.5$,

$$\begin{aligned} y &= 10.02(0.5)^{0.7028} \\ &\approx 6.16. \end{aligned}$$

We predict that the oxygen consumption for a guinea pig weighing 0.5 kg will be about 6.16 ml/min.

79. **(a)** $h(t) = 37.79(1.021)^t$

Double the 2005 population is $2(42.69) = 85.38$ million

$$\begin{aligned} 85.38 &= 37.79(1.021)^t \\ \frac{85.38}{37.79} &= (1.021)^t \\ \log_{1.021} \left(\frac{85.38}{37.79}\right) &= t \\ t &= \frac{\ln \left(\frac{85.38}{37.79}\right)}{\ln 1.021} \\ &\approx 39.22 \end{aligned}$$

The Hispanic population is estimated to double their 2005 population in 2039.

(b) $h(t) = 11.14(1.023)^t$

Double the 2005 population is $2(12.69) = 25.38$ million

$$\begin{aligned} 25.38 &= 11.14(1.023)^t \\ \frac{25.38}{11.14} &= (1.023)^t \\ \log_{1.023} \left(\frac{25.38}{11.14}\right) &= t \\ t &= \frac{\ln \left(\frac{25.38}{11.14}\right)}{\ln 1.023} \\ &\approx 36.21 \end{aligned}$$

The Asian population is estimated to double their 2005 population in 2036.

80. $N(r) = -5000 \ln r$

(a) $N(0.9) = -5000 \ln(0.9) \approx 530$

(b) $N(0.5) = -5000 \ln(0.5) \approx 3500$

(c) $N(0.3) = -5000 \ln(0.3) \approx 6000$

(d) $N(0.7) = -5000 \ln(0.7) \approx 1800$

(e) $-5000 \ln r = 1000$

$$\begin{aligned} \ln r &= \frac{1000}{-5000} \\ \ln r &= -\frac{1}{5} \\ r &= e^{-1/5} \\ r &\approx 0.8 \end{aligned}$$

81. $C = B \log_2\left(\frac{s}{n} + 1\right)$

$$\begin{aligned} \frac{C}{B} &= \log_2\left(\frac{s}{n} + 1\right) \\ 2^{C/B} &= \frac{s}{n} + 1 \\ \frac{s}{n} &= 2^{C/B} - 1 \end{aligned}$$

82. Decibel rating: $10 \log \frac{I}{I_0}$

(a) Intensity, $I = 115I_0$

$$\begin{aligned} &10 \log\left(\frac{115I_0}{I_0}\right) \\ &= 10 \cdot \log 115 \\ &\approx 21 \end{aligned}$$

(b) $I = 9{,}500{,}000I_0$

$$\begin{aligned} 10 \log\left(\frac{9.5 \times 10^6 I_0}{I_0}\right) &= 10 \log 9.5 \times 10^6 \\ &\approx 70 \end{aligned}$$

(c) $I = 1{,}200{,}000{,}000I_0$

$$\begin{aligned} 10 \log\left(\frac{1.2 \times 10^9 I_0}{I_0}\right) &= 10 \log 1.2 \times 10^9 \\ &\approx 91 \end{aligned}$$

(d) $I = 895{,}000{,}000{,}000I_0$

$$\begin{aligned} 10 \log\left(\frac{8.95 \times 10^{11} I_0}{I_0}\right) &= 10 \log 8.95 \times 10^{11} \\ &\approx 120 \end{aligned}$$

(e) $I = 109{,}000{,}000{,}000{,}000I_0$

$$\begin{aligned} 10 \log\left(\frac{1.09 \times 10^{14} I_0}{I_0}\right) &= 10 \log 1.09 \times 10^{14} \\ &\approx 140 \end{aligned}$$

(f) $I_0 = 0.0002$ microbars

$$\begin{aligned} 1{,}200{,}000{,}000I_0 &= 1{,}200{,}000{,}000(0.0002) \\ &= 240{,}000 \text{ microbars} \\ 895{,}000{,}000{,}000I_0 &= 895{,}000{,}000{,}000(0.0002) \\ &= 179{,}000{,}000 \text{ microbars} \end{aligned}$$

83. Let I_1 be the intensity of the sound whose decibel rating is 85.

(a)
$$\begin{aligned} 10 \log \frac{I_1}{I_0} &= 85 \\ \log \frac{I_1}{I_0} &= 8.5 \\ \log I_1 - \log I_0 &= 8.5 \\ \log I_1 &= 8.5 + \log I_0 \end{aligned}$$

Let I_2 be the intensity of the sound whose decimal rating is 75.

$$\begin{aligned} 10 \log \frac{I_2}{I_0} &= 75 \\ \log \frac{I_2}{I_0} &= 7.5 \\ \log I_2 - \log I_0 &= 7.5 \\ \log I_0 &= \log I_2 - 7.5 \end{aligned}$$

Substitute for I_0 in the equation for $\log I_1$.

$$\begin{aligned} \log I_1 &= 8.5 + \log I_0 \\ &= 8.5 + \log I_2 - 7.5 \\ &= 1 + \log I_2 \end{aligned}$$

$$\begin{aligned} \log I_1 - \log I_2 &= 1 \\ \log \frac{I_1}{I_2} &= 1 \end{aligned}$$

Then $\frac{I_1}{I_2} = 10$, so $I_2 = \frac{1}{10} I_1$. This means the intensity of the sound that had a rating of 75 decibels is $\frac{1}{10}$ as intense as the sound that had a rating of 85 decibels.

84. $R(I) = \log \frac{I}{I_0}$

(a) $R(1{,}000{,}000\ I_0)$

$$\begin{aligned} &= \log \frac{1{,}000{,}000\ I_0}{I_0} \\ &= \log 1{,}000{,}000 \\ &= 6 \end{aligned}$$

(b) $R(100{,}000{,}000\ I_0)$

$$\begin{aligned} &= \log \frac{100{,}000{,}000\ I_0}{I_0} \\ &= \log 100{,}000{,}000 \\ &= 8 \end{aligned}$$

(c) $$\begin{aligned} R(I) &= \log \frac{I}{I_0} \\ 6.7 &= \log \frac{I}{I_0} \\ 10^{6.7} &= \frac{I}{I_0} \\ I &\approx 5{,}000{,}000 I_0 \end{aligned}$$

(d) $$\begin{aligned} R(I) &= \log \frac{I}{I_0} \\ 8.1 &= \log \frac{I}{I_0} \\ 10^{8.1} &= \frac{I}{I_0} \\ I &\approx 126{,}000{,}000 I_0 \end{aligned}$$

(e) $$\frac{1985 \text{ quake}}{1999 \text{ quake}} = \frac{126{,}000{,}000 I_0}{5{,}000{,}000 I_0} \approx 25$$

The 1985 earthquake had an amplitude more than 25 times that of the 1999 earthquake.

(f) $R(E) = \frac{2}{3} \log \frac{E}{E_0}$

For the 1999 earthquake

$$\begin{aligned} 6.7 &= \frac{2}{3} \log \frac{E}{E_0} \\ 10.05 &= \log \frac{E}{E_0} \\ \frac{E}{E_0} &= 10^{10.05} \\ E &= 10^{10.05} E_0 \end{aligned}$$

For the 1985 earthquake,

$$\begin{aligned} 8.1 &= \frac{2}{3} \log \frac{E}{E_0} \\ 12.15 &= \log \frac{E}{E_0} \\ \frac{E}{E_0} &= 10^{12.15} \\ E &= 10^{12.15} E_0 \end{aligned}$$

The ratio of their energies is

$$\frac{10^{12.15} E_0}{10^{10.05} E_0} = 10^{2.1} \approx 126$$

The 1985 earthquake had an energy about 126 times that of the 1999 earthquake.

(g) Find the energy of a magnitude 6.7 earthquake. Using the formula from part f,

$$\begin{aligned} 6.7 &= \frac{2}{3} \log \frac{E}{E_0} \\ \log \frac{E}{E_0} &= 10.05 \\ \frac{E}{E_0} &= 10^{10.05} \\ E &= E_0 10^{10.05} \end{aligned}$$

For an earthquake that releases 15 times this much energy, $E = E_0(15)10^{10.05}$.

$$\begin{aligned} R(E_0(15)10^{10.05}) &= \frac{2}{3} \log \left(\frac{E_0(15)10^{10.05}}{E_0} \right) \\ &= \frac{2}{3} \log(15 \cdot 10^{10.05}) \\ &\approx 7.5 \end{aligned}$$

So, it's true that a magnitude 7.5 earthquake releases 15 times more energy than one of magnitude 6.7.

85. pH $= -\log\,[\mathrm{H}^+]$

(a) For pure water:

$$\begin{aligned} 7 &= -\log\,[\mathrm{H}^+] \\ -7 &= \log\,[\mathrm{H}^+] \\ 10^{-7} &= [\mathrm{H}^+] \end{aligned}$$

For acid rain:

$$\begin{aligned} 4 &= -\log\,[\mathrm{H}^+] \\ -4 &= \log\,[\mathrm{H}^+] \\ 10^{-4} &= [\mathrm{H}^+] \\ \frac{10^{-4}}{10^{-7}} &= 10^3 = 1000 \end{aligned}$$

The acid rain has a hydrogen ion concentration 1000 times greater than pure water.

(b) For laundry solution:

$$\begin{aligned} 11 &= -\log\,[\mathrm{H}^+] \\ 10^{-11} &= [\mathrm{H}^+] \end{aligned}$$

For black coffee:

$$\begin{aligned} 5 &= -\log\,[\mathrm{H}^+] \\ 10^{-5} &= [\mathrm{H}^+] \\ \frac{10^{-5}}{10^{-11}} &= 10^6 = 1{,}000{,}000 \end{aligned}$$

The coffee has a hydrogen ion concentration 1,000,000 times greater than the laundry mixture.

2.6 Applications: Growth and Decay; Mathematics of Finance

2. y_0 represents the initial quantity; k represents the rate of growth or decay.

4. The half-life of a quantity is the time period for the quantity to decay to one-half of the initial amount.

5. Assume that $y = y_0 e^{kt}$ represents the amount remaining of a radioactive substance decaying with a half-life of T. Since $y = y_0$ is the amount of the substance at time $t = 0$, then $y = \frac{y_0}{2}$ is the amount at time $t = T$. Therefore, $\frac{y_0}{2} = y_0 e^{kT}$, and solving for k yields

$$\begin{aligned} \frac{1}{2} &= e^{kT} \\ \ln\left(\frac{1}{2}\right) &= kT \\ k &= \frac{\ln\left(\frac{1}{2}\right)}{T} \\ &= \frac{\ln(2^{-1})}{T} \\ &= -\frac{\ln 2}{T}. \end{aligned}$$

6. Assume that $y = y_0 e^{kt}$ is the amount left of a radioactive substance decaying with a half-life of T. From Exercise 5, we know $k = \frac{-\ln 2}{T}$, so

$$y = y_0 e^{(-\ln 2/T)t} = y_0 e^{-(t/T)\ln 2} = y_0 e^{\ln(2^{-t/T})}$$
$$= y_0 2^{-t/T} = y_0 \left[\left(\frac{1}{2}\right)^{-1}\right]^{-t/T} = y_0 \left(\frac{1}{2}\right)^{t/T}$$

7. $r = 4\%$ compounded quarterly, $m = 4$

$$\begin{aligned} r_E &= \left(1 + \frac{r}{m}\right)^m - 1 \\ &= \left(1 + \frac{0.04}{4}\right)^4 - 1 \\ &\approx 0.0406 \\ &\approx 4.06\% \end{aligned}$$

8. $r = 6\%$ compounded monthly, $m = 12$

$$\begin{aligned} r_E &= \left(1 + \frac{0.06}{12}\right)^{12} - 1 \\ &\approx 0.0617 \\ &= 6.17\% \end{aligned}$$

9. $r = 8\%$ compounded continuously

$$\begin{aligned} r_E &= e^r - 1 \\ &= e^{0.08} - 1 \\ &= 0.0833 \\ &= 8.33\% \end{aligned}$$

10. $r = 5\%$ compounded continuously

$$\begin{aligned} r_E &= e^r - 1 \\ &= e^{0.05} - 1 \\ &\approx 0.0513 \\ &= 5.13\% \end{aligned}$$

11. $A = \$10,000,\ r = 6\%,\ m = 4,\ t = 8$

$$\begin{aligned} P &= A\left(1 + \frac{r}{m}\right)^{-tm} \\ &= 10,000\left(1 + \frac{0.06}{4}\right)^{-8(4)} \\ &\approx \$6209.93 \end{aligned}$$

12. $A = \$45,678.93,\ r = 7.2\%,\ m = 12,\ t = 11$ months

$$\begin{aligned} P &= A\left(1 + \frac{r}{m}\right)^{-tm} \\ &= 45,678.93\left(1 + \frac{0.072}{12}\right)^{-(11/12)(12)} \\ &\approx \$42,769.89 \end{aligned}$$

13. $A = \$7300,\ r = 5\%$ compounded continuously, $t = 3$

$$\begin{aligned} A &= Pe^{rt} \\ P &= \frac{A}{e^{rt}} \\ &= \frac{7300}{e^{0.5(3)}} \\ &\approx \$6283.17 \end{aligned}$$

14. $A = \$25,000,\ r = 4.6\%$ compounded continuously, $t = 8$

$$\begin{aligned} A &= Pe^{rt} \\ P &= \frac{A}{e^{rt}} \\ &= \frac{25,000}{e^{0.046(8)}} \\ &\approx \$17,302.93 \end{aligned}$$

15. $r = 9\%$ compounded semiannually

$$r_E = \left(1 + \frac{0.09}{2}\right)^2 - 1$$
$$\approx 0.0920$$
$$\approx 9.20\%$$

16. $r = 6.2\%,\ m = 4$

$$r_E = \left(1 + \frac{0.062}{4}\right)^4 - 1$$
$$\approx 0.0635$$
$$= 6.35\%$$

17. $r = 6\%$ compounded monthly

$$r_E = \left(1 + \frac{0.06}{12}\right)^{12} - 1$$
$$\approx 0.0617$$
$$\approx 6.17\%$$

18. $A = \$20{,}000,\ t = 4,\ r = 6.5\%,\ m = 12$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$
$$20{,}000 = P\left(1 + \frac{0.065}{12}\right)^{12 \cdot 4}$$
$$\frac{20{,}000}{(1.0541\overline{6})^{48}} = P$$
$$\$15{,}431.86 = P$$

19. **(a)** $A = \$307{,}000,\ t = 3,\ r = 6\%,\ m = 2$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$
$$307{,}000 = P\left(1 + \frac{0.06}{2}\right)^{3(2)}$$
$$307{,}000 = P(1.03)^6$$
$$\frac{307{,}000}{(1.03)^6} = P$$
$$\$257{,}107.67 = P$$

(b) Interest $= 307{,}000 - 257{,}107.67$
$= \$49{,}892.33$

(c) $P = \$200{,}000$
$A = 200{,}000(1.03)^6$
$= 238{,}810.46$

The additional amount needed is

$307{,}000 - 238{,}810.46$
$= \$68{,}189.54.$

20. $A = \$40{,}000,\ t = 5$

(a) $r = 0.064,\ m = 4$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$
$$40{,}000 = P\left(1 + \frac{0.064}{1}\right)^{5(4)}$$
$$\frac{40{,}000}{(1.016)^{20}} = P$$
$$\$29{,}119.63 = P$$

(b) Interest $= \$40{,}000 - \$29{,}119.63$
$= \$10{,}880.37$

(c) $P = \$20{,}000$

$$A = 20{,}000\left(1 + \frac{0.064}{4}\right)^{5(4)}$$
$$= \$27{,}472.88$$

The amount needed will be

$\$40{,}000 - \$27{,}472.88 = \$12{,}527.12.$

21. $P = \$60{,}000$

(a) $r = 8\%$ compounded quarterly:

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$
$$= 60{,}000\left(1 + \frac{0.08}{4}\right)^{5(4)}$$
$$\approx \$89{,}156.84$$

$r = 7.75\%$ compounded continuously

$$A = Pe^{rt}$$
$$= 60{,}000e^{0.0775(5)}$$
$$\approx \$88{,}397.58$$

Linda will earn more money at 8% compounded quarterly.

(b) She will earn \$759.26 more.

(c) $r = 8\%,\ m = 4:$

$$r_E = \left(1 + \frac{r}{m}\right)^m - 1$$
$$= \left(1 + \frac{0.08}{4}\right)^4 - 1$$
$$\approx 0.0824$$
$$= 8.24\%$$

$r = 7.75\%$ compounded continuously:

$$\begin{aligned} r_E &= e^r - 1 \\ &= e^{0.0775} - 1 \\ &\approx 0.0806 \\ &= 8.06\% \end{aligned}$$

(d) $A = \$80{,}000$

$$\begin{aligned} A &= Pe^{rt} \\ 80{,}000 &= 60{,}000e^{0.0775t} \\ \frac{4}{3} &= e^{0.0775t} \\ \ln\ \frac{4}{3} &= \ln\ e^{0.0775t} \\ \ln\ 4 - \ln\ 3 &= 0.0775t \\ \frac{\ln\ 4 - \ln\ 3}{0.0775} &= t \\ 3.71 &= t \end{aligned}$$

\$60,000 will grow to \$80,000 in about 3.71 years.

(e) $60{,}000\left(1 + \frac{0.08}{4}\right)^{4x} \geq 80{,}000$

$$\begin{aligned} (1.02)^{4x} &\geq \frac{80{,}000}{60{,}000} \\ (1.02)^{4x} &\geq \frac{4}{3} \\ \log\ (1.02)^{4x} &\geq \log\ \left(\frac{4}{3}\right) \\ 4x\ \log\ (1.02) &\geq \log\ \left(\frac{4}{3}\right) \\ x &\geq \frac{\log\ \left(\frac{4}{3}\right)}{4\ \log\ (1.02)} \approx 3.63 \end{aligned}$$

It will take about 3.63 years.

22. (a)

$$\begin{aligned} 11{,}000 &= 5000\left(1 + \frac{0.063}{4}\right)^{4t} \\ \frac{11}{5} &= (1.01575)^{4t} \\ \ln\ \left(\frac{11}{5}\right) &= 4t\ \ln\ 1.01575 \\ t &= \frac{\ln\ \left(\frac{11}{5}\right)}{4\ \ln\ 1.01575} \\ t &\approx 12.61 \end{aligned}$$

Since the interest is only added at the end of the quarter, it will take 12.75 years.

(b)

$$\begin{aligned} 11{,}000 &= 5000e^{0.063t} \\ \frac{11}{5} &= e^{0.063t} \\ 0.063t &= \ln\ \left(\frac{11}{5}\right) \\ t &= \frac{\ln\ \left(\frac{11}{5}\right)}{0.063} \\ t &\approx 12.52 \end{aligned}$$

It will take about 12.52 years.

23. The figure is not correct.

$(1 + 0.09)(1 + 0.08)(1 + 0.07) = 1.2596$

This is a 25.96% increase.

24. The statement is not correct.

$$\begin{aligned} (1 + r)^{15} &= 1.94 + 1 \\ 1 + r &= (2.94)^{1/15} \\ r &= (2.94)^{1/15} - 1 \\ r &\approx 0.0745 \end{aligned}$$

The required annual percent increase is 7.45%.

25. $S(x) = 1000 - 800e^{-x}$

(a)

$$\begin{aligned} S(0) &= 1000 - 800e^0 \\ &= 1000 - 800 \\ &= 200 \end{aligned}$$

(b)

$$\begin{aligned} S(x) &= 500 \\ 500 &= 1000 - 800e^{-x} \\ -500 &= -800e^{-x} \\ \frac{5}{8} &= e^{-x} \\ \ln\ \frac{5}{8} &= \ln\ e^{-x} \\ -\ln\ \frac{5}{8} &= x \\ 0.47 &\approx x \end{aligned}$$

Sales reach 500 in about $\frac{1}{2}$ year.

(c) Since $800e^{-x}$ will never actually be zero, $S(x) = 1000 - 800e^{-x}$ will never be 1000.

(d) Graphing the function $y = S(x)$ on a graphing calculator will show that there is a horizontal asymptote at $y = 1000$. This indicates that the limit on sales is 1000 units.

26. $S(x) = 5000 - 4000e^{-x}$

(a) $S(0) = 5000 - 4000e^0$
$= 1000$

Since $S(x)$ represents sales in thousands, in year 0 the sales are \$1,000,000.

(b) Let $S(x) = 4500$.

$$\begin{aligned} 4500 &= 5000 - 4000e^{-x} \\ -500 &= -4000e^{-x} \\ 0.125 &= e^{-x} \\ -\ln 0.125 &= x \\ x &\approx 2 \end{aligned}$$

It will take about 2 years for sales to reach \$4,500,000.

(c) Graphing the function $y = S(x)$ on a graphing calculator will show that there is a horizontal asymptote at $y = 5000$. Since this represents \$5000 thousand or \$5,000,000, the limit on sales is \$5,000,000.

27. **(a)** $P = P_0e^{kt}$

When $t = 1650$, $P = 470$.
When $t = 2005$, $P = 6451$.

$$\begin{aligned} 470 &= P_0e^{1650k} \\ 6451 &= P_0e^{2005k} \\ \frac{6451}{470} &= \frac{P_0e^{2005k}}{P_0e^{1650k}} \\ \frac{6451}{470} &= e^{355k} \\ 355k &= \ln\left(\frac{6451}{470}\right) \\ k &= \frac{\ln\left(\frac{6451}{470}\right)}{355} \\ k &\approx 0.007378 \end{aligned}$$

Substitute this value into $470 = P_0e^{1650k}$ to find P_0.

$$\begin{aligned} 470 &= P_0e^{1650(0.007378)} \\ P_0 &= \frac{470}{e^{1650(0.007378)}} \\ P_0 &\approx 0.002427 \end{aligned}$$

Therefore, $P(t) = 0.002427e^{0.007378t}$.

(b) $P(1) = 0.002427e^{0.007378} \approx 0.002445$ million or 2445

The exponential equation gives a world population of only 2445 in the year 1.

(c) No, the answer in part (b) is too small. Exponential growth does not accurately describe population growth for the world over a long period of time.

28. **(a)** The function giving the number after t hours is

$$y = y_02^{t/12}$$

(b) For 10 days, $t = 10 \cdot 24$ or 240.

$$\begin{aligned} y &= (1)2^{240/12} \\ &= 2^{20} \\ &= 1{,}048{,}576 \end{aligned}$$

For 15 days, $t = 15 \cdot 24$ or 360.

$$\begin{aligned} y &= (1)2^{360/12} \\ &= 2^{30} \\ &= 1{,}073{,}741{,}824 \end{aligned}$$

29. From 1960 to 2005 is an interval of $t = 45$ years.

$$\begin{aligned} P &= P_0e^{rt} \\ 1500 &= 59e^{45r} \\ \frac{1500}{59} &= e^{45r} \\ 45r &= \ln\frac{1500}{59} \\ r &= \frac{1}{45}\ln\frac{1500}{59} \\ &\approx 0.0719 \end{aligned}$$

This is an annual increase of 7.19%.

30. $y = y_0e^{kt}$
$y = 40{,}000$, $y_0 = 25{,}000$, $t = 10$

(a)

$$\begin{aligned} 40{,}000 &= 25{,}000e^{k(10)} \\ 1.6 &= e^{10k} \\ \ln 1.6 &= 10k \\ 0.047 &= k \end{aligned}$$

The equation is

$$y = 25{,}000e^{0.047t}.$$

(b)

$$\begin{aligned} y &= 60{,}000 \\ 60{,}000 &= 25{,}000e^{0.047t} \\ 2.4 &= e^{0.047t} \\ \ln 2.4 &= 0.047t \\ 18.6 &= t \end{aligned}$$

There will be 60,000 bacteria in about 18.6 hours.

31. $y = y_0e^{kt}$

(a) $y = 20{,}000,\ y_0 = 50{,}000,\ t = 9$

$$\begin{aligned} 20{,}000 &= 50{,}000e^{9k} \\ 0.4 &= e^{9k} \\ \ln\ 0.4 &= 9k \\ -0.102 &= k \end{aligned}$$

The equation is

$$y = 50{,}000e^{-0.102t}.$$

(b) $\frac{1}{2}(50{,}000) = 25{,}000$

$$\begin{aligned} 25{,}000 &= 50{,}000e^{-0.102t} \\ 0.5 &= e^{-0.102t} \\ \ln\ 0.5 &= -0.102t \\ 6.8 &= t \end{aligned}$$

Half the bacteria remain after about 6.8 hours.

32. $f(t) = 500e^{0.1t}$

(a)
$$\begin{aligned} f(t) &= 3000 \\ 3000 &= 500e^{0.1t} \\ 6 &= e^{0.1t} \\ \ln\ 6 &= 0.1t \\ 17.9 &\approx t \end{aligned}$$

It will take 17.9 days.

(b) If $t = 0$ corresponds to January 1, the date January 17 should be placed on the product. January 18 would be more than 17.9 days.

33. Use $y = y_0e^{-kt}$.

When $t = 5,\ y = 0.37y_0$.

$$\begin{aligned} 0.37y_0 &= y_0e^{-5k} \\ 0.37 &= e^{-5k} \\ -5k &= \ln\ (0.37) \\ k &= \frac{\ln\ (0.37)}{-5} \\ k &\approx 0.1989 \end{aligned}$$

34. **(a)** From the graph, the risks of chromosomal abnormality per 1000 at ages 20, 35, 42, and 49 are 2, 5, 29, and 125, respectively.

(Note: It is difficult to read the graph accurately. If you read different values from the graph, your answers to parts (b)-(e) may differ from those given here.)

(b) $y = Ce^{kt}$

When $t = 20,\ y = 2$, and when $t = 35,\ y = 5$.

$$\begin{aligned} 2 &= Ce^{20k} \\ 5 &= Ce^{35k} \\ \frac{5}{2} &= \frac{Ce^{35k}}{Ce^{20k}} \\ 2.5 &= e^{15k} \\ 15k &= \ln\ 2.5 \\ k &= \frac{\ln\ 2.5}{15} \\ k &\approx 0.061 \end{aligned}$$

(c) $y = Ce^{kt}$

When $t = 42,\ y = 29$, and when $t = 49,\ y = 125$.

$$\begin{aligned} 29 &= Ce^{42k} \\ 125 &= Ce^{49k} \\ \frac{125}{29} &= \frac{Ce^{49k}}{Ce^{42k}} \\ \frac{125}{29} &= e^{7k} \\ 7k &= \ln\left(\frac{125}{29}\right) \\ k &= \frac{\ln\left(\frac{125}{29}\right)}{7} \\ k &\approx 0.21 \end{aligned}$$

(d) Since the values of k are different, we cannot assume the graph is of the form $y = Ce^{kt}$.

(e) The results are summarized in the following table.

n	Value of k for [20, 35]	Value of k for [42, 49]
2	0.0011	0.0023
3	2.6×10^{-5}	3.4×10^{-5}
4	6.8×10^{-7}	5.5×10^{-7}

The value of n should be somewhere between 3 and 4.

35.
$$\begin{aligned} A(t) &= A_0 e^{kt} \\ 0.60\ A_0 &= A_0 e^{(-\ln\ 2/5600)t} \\ 0.60 &= e^{(-\ln\ 2/5600)t} \\ \ln\ 0.60 &= -\frac{\ln\ 2}{5600}t \\ \frac{5600(\ln\ 0.60)}{-\ln\ 2} &= t \\ 4127 &\approx t \end{aligned}$$

The sample was about 4100 years old.

36.
$$\begin{aligned} \frac{1}{2}A_0 &= A_0 e^{-0.053t} \\ \frac{1}{2} &= e^{-0.053t} \\ \ln\ \frac{1}{2} &= -0.053t \\ \ln\ 1 - \ln\ 2 &= -0.053t \\ \frac{0 - \ln\ 2}{-0.053} &= t \\ 13 &\approx t \end{aligned}$$

The half-life of plutonium 241 is about 13 years.

37.
$$\begin{aligned} \frac{1}{2}\ A_0 &= A_0 e^{-0.00043t} \\ \frac{1}{2} &= e^{-0.00043t} \\ \ln\ \frac{1}{2} &= -0.00043t \\ \ln\ 1 - \ln\ 2 &= -0.00043t \\ \frac{0 - \ln\ 2}{-0.00043} &= t \\ 1612 &\approx t \end{aligned}$$

The half-life of radium 226 is about 1600 years.

38. **(a)**
$$\begin{aligned} A(t) &= A_0 \left(\frac{1}{2}\right)^{t/13} \\ A(100) &= 4.0 \left(\frac{1}{2}\right)^{100/13} \\ A(100) &\approx 0.0193 \end{aligned}$$

After 100 years, about 0.0193 gram will remain.

(b)
$$\begin{aligned} 0.1 &= 4.0 \left(\frac{1}{2}\right)^{t/13} \\ \frac{0.1}{4.0} &= \left(\frac{1}{2}\right)^{t/13} \\ \ln\ 0.025 &= \frac{t}{13}\ \ln\ \left(\frac{1}{2}\right) \\ t &= \frac{13\ \ln\ 0.025}{\ln\ \left(\frac{1}{2}\right)} \\ t &\approx 69.19 \end{aligned}$$

It will take 69 years.

39. **(a)**
$$\begin{aligned} A(t) &= A_0 \left(\frac{1}{2}\right)^{t/1620} \\ A(100) &= 4.0 \left(\frac{1}{2}\right)^{100/1620} \\ A(100) &\approx 3.8 \end{aligned}$$

After 100 years, about 3.8 grams will remain.

(b)
$$\begin{aligned} 0.1 &= 4.0 \left(\frac{1}{2}\right)^{t/1620} \\ \frac{0.1}{4} &= \left(\frac{1}{2}\right)^{t/1620} \\ \ln\ 0.025 &= \frac{t}{1620} \ln \frac{1}{2} \\ t &= \frac{1620\ \ln\ 0.025}{\ln\ \left(\frac{1}{2}\right)} \\ t &\approx 8600 \end{aligned}$$

The half-life is about 8600 years.

40. **(a)** $y = y_0 e^{kt}$

When $t = 0,\ y = 500$, so $y_0 = 500$.
When $t = 3,\ y = 386$.

$$386 = 500e^{3k}$$
$$\frac{386}{500} = e^{3k}$$
$$e^{3k} = 0.772$$
$$3k = \ln\ 0.772$$
$$k = \frac{\ln\ 0.772}{3}$$
$$k \approx -0.0863$$
$$y = 500e^{-0.0863t}$$

(b) $$\frac{1}{2}\,y_0 = y_0 e^{-0.0863t}$$
$$\ln\ \frac{1}{2} = -0.0863t$$
$$t = \frac{\ln\ \left(\frac{1}{2}\right)}{-0.0863}$$
$$t \approx 8.0$$

The half-life is about 8.0 days.

41. **(a)** $y = y_0 e^{kt}$

When $t = 0,\ y = 25.0$, so $y_0 = 25.0$.
When $t = 50,\ y = 19.5$.

$$19.5 = 25.0e^{50k}$$
$$\frac{19.5}{25.0} = e^{50k}$$
$$50k = \ln\ \left(\frac{19.5}{25.0}\right)$$
$$k = \frac{\ln\ \left(\frac{19.5}{25.0}\right)}{50}$$
$$k \approx -0.00497$$
$$y = 25.0e^{-0.00497t}$$

(b) $$\frac{1}{2}y_0 = y_0 e^{-0.00497t}$$
$$\frac{1}{2} = e^{-0.00497t}$$
$$-0.00497t = \ln\ \left(\frac{1}{2}\right)$$
$$t = \frac{\ln\ \left(\frac{1}{2}\right)}{-0.00497}$$
$$t \approx 139$$

The half-life is about 139 days.

42. $y = 40e^{-0.004t}$

(a) $t = 180$

$$y = 40e^{-0.004(180)} = 40e^{-0.72}$$
$$\approx 19.5 \text{ watts}$$

(b) $$20 = 40e^{-0.0004t}$$
$$\frac{1}{2} = e^{-0.004t}$$
$$\ln\ \frac{1}{2} = -0.004t$$
$$\frac{\ln\ 1 - \ln\ 2}{-0.004} = t$$
$$173 \approx t$$

It will take about 173 days.

(c) The power will never be completely gone. The power will approach 0 watts but will never be exactly 0.

43. $A(t) = A_0 \left(\frac{1}{2}\right)^{t/5600}$

$$A(43{,}000) = A_0 \left(\frac{1}{2}\right)^{43{,}000/5600}$$
$$\approx 0.005A_0$$

About 0.5% of the original carbon 14 was present.

44. $P(t) = 100e^{-0.1t}$

(a) $P(4) = 100e^{-0.1(4)} \approx 67\%$

(b) $P(10) = 100e^{-0.1(10)} \approx 37\%$

(c) $$10 = 100e^{-0.1t}$$
$$0.1 = e^{-0.1t}$$
$$\ln\ (0.1) = -0.1t$$
$$\frac{-\ln\ (0.1)}{0.1} = t$$
$$23 \approx t$$

It would take about 23 days.

(d) $$1 = 100e^{-0.1t}$$
$$0.01 = e^{-0.1t}$$
$$\ln\ (0.01) = -0.1t$$
$$\frac{-\ln\ (0.01)}{0.1} = t$$
$$46 \approx t$$

It would take about 46 days.

45. **(a)** Let $t =$ the number of degrees Celsius.

$y = y_0 \cdot e^{kt}$
$y_0 = 10$ when $t = 0°$.
To find k, let $y = 11$ when $t = 10°$.

$$\begin{aligned} 11 &= 10e^{10k} \\ e^{10k} &= \frac{11}{10} \\ 10k &= \ln\ 1.1 \\ k &= \frac{\ln\ 1.1}{10} \\ &\approx 0.0095 \end{aligned}$$

The equation is

$$y = 10e^{0.0095t}.$$

(b) Let $y = 15$; solve for t.

$$\begin{aligned} 15 &= 10e^{0.0095t} \\ \ln\ 1.5 &= 0.0095t \\ t &= \frac{\ln\ 1.5}{0.0095} \\ &\approx 42.7 \end{aligned}$$

15 grams will dissolve at 42.7°C.

46. $t = 9,\ T_0 = 18,\ C = 5,\ k = .6$

$$f(t) = T_0 + Ce^{-kt}$$

$$\begin{aligned} f(t) &= 18 + 5e^{-0.6(9)} \\ &= 18 + 5e^{-5.4} \\ &\approx 18.02 \end{aligned}$$

The temperature is about 18.02°.

47. $$f(t) = T_0 + Ce^{-kt}$$

$$\begin{aligned} 25 &= 20 + 100e^{-0.1t} \\ 5 &= 100e^{-0.1t} \\ e^{-0.1t} &= 0.05 \\ -0.1t &= \ln\ 0.05 \\ t &= \frac{\ln\ 0.05}{-0.1} \\ &\approx 30 \end{aligned}$$

It will take about 30 min.

48. $C = -14.6,\ k = 0.6,\ T_0 = 18°,$

$f(t) = 10°$
$f(t) = T_0 + Ce^{-kt}$

$$\begin{aligned} f(t) &= 18 + (-14.6)e^{-0.6t} \\ -8 &= -14.6e^{-0.6t} \\ 0.5479 &= e^{-0.6t} \\ \ln\ 0.5479 &= -0.6t \\ \frac{-\ln\ 0.5479}{0.6} &= t \\ 1 &\approx t \end{aligned}$$

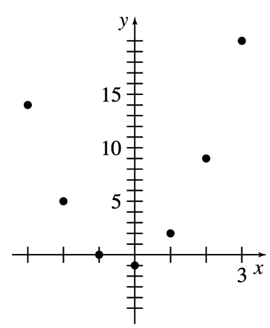
It would take about 1 hour for the pizza to thaw.

Chapter 2 Review Exercises

5. $y = (2x - 1)(x + 1)$
$\quad = 2x^2 + x - 1$

x	-3	-2	-1	0	1	2	3
y	14	5	0	-1	2	9	20

Pairs: $(-3, 14), (-2, 5), (-1, 0), (0, -1), (1, 2), (2, 9), (3,\ 20)$
Range: $\{-1, 0, 2, 5, 9, 14, 20\}$

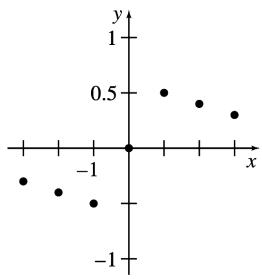


6. $y = \dfrac{x}{x^2 + 1}$

x	-3	-2	-1	0	1	2	3
y	$-\frac{3}{10}$	$-\frac{2}{5}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{10}$

Pairs: $\left(-3, -\frac{3}{10}\right), \left(-2, -\frac{2}{5}\right), \left(-1, -\frac{1}{2}\right),$
$(0, 0), \left(1, \frac{1}{2}\right), \left(2, \frac{2}{5}\right), \left(3, \frac{3}{10}\right)$

Range: $\left\{-\frac{1}{2}, -\frac{2}{5}, -\frac{3}{10}, 0, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}\right\}$



7. $f(x) = 5x^2 - 3$ and $g(x) = -x^2 + 4x + 1$

(a) $f(-2) = 5(-2)^2 - 3 = 17$

(b) $g(3) = -(3)^2 + 4(3) + 1 = 4$

(c) $f(-k) = 5(-k)^2 - 3 = 5k^2 - 3$

(d) $$\begin{aligned} g(3m) &= -(3m)^2 + 4(3m) + 1 \\ &= -9m^2 + 12m + 1 \end{aligned}$$

(e) $$\begin{aligned} f(x+h) &= 5(x+h)^2 - 3 \\ &= 5(x^2 + 2xh + h^2) - 3 \\ &= 5x^2 + 10xh + 5h^2 - 3 \end{aligned}$$

(f) $$\begin{aligned} g(x+h) &= -(x+h)^2 + 4(x+h) + 1 \\ &= -(x^2 + 2xh + h^2) + 4x + 4h + 1 \\ &= -x^2 - 2xh - h^2 + 4x + 4h + 1 \end{aligned}$$

(g) $$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{5(x+h)^2 - 3 - (5x^2 - 3)}{h} \\ &= \frac{5(x^2 + 2hx + h^2) - 3 - 5x^2 + 3}{h} \\ &= \frac{5x^2 + 10hx + 5h^2 - 5x^2}{h} \\ &= \frac{10hx + 5h^2}{h} \\ &= 10x + 5h \end{aligned}$$

(h) $$\begin{aligned} &\frac{g(x+h) - g(x)}{h} \\ &= \frac{-(x+h)^2 + 4(x+h) + 1 - (-x^2 + 4x + 1)}{h} \\ &= \frac{-(x^2 + 2xh + h^2) + 4x + 4h + 1 + x^2 - 4x - 1}{h} \\ &= \frac{-x^2 - 2xh - h^2 + 4h + x^2}{h} \\ &= \frac{-2xh - h^2 + 4h}{h} \\ &= -2x - h + 4 \end{aligned}$$

8. $f(x) = 2x^2 + 5$ and $g(x) = 3x^2 + 4x - 1$

(a) $f(-3) = 2(-3)^2 + 5 = 23$

(b) $g(2) = 3(2)^2 + 4(2) - 1 = 19$

(c) $f(3m) = 2(3m)^2 + 5 = 18m^2 + 5$

(d) $g(-k) = 3(-k)^2 + 4(-k) - 1 = 3k^2 - 4k - 1$

(e) $$\begin{aligned} f(x+h) &= 2(x+h)^2 + 5 \\ &= 2(x^2 + 2xh + h^2) + 5 \\ &= 2x^2 + 4xh + 2h^2 + 5 \end{aligned}$$

(f) $$\begin{aligned} g(x+h) &= 3(x+h)^2 + 4(x+h) - 1 \\ &= 3(x^2 + 2xh + h^2) + 4x + 4h - 1 \\ &= 3x^2 + 6xh + 3h^2 + 4x + 4h - 1 \end{aligned}$$

(g) $$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{2(x+h)^2 + 5 - (2x^2 + 5)}{h} \\ &= \frac{2(x^2 + 2xh + h^2) + 5 - 2x^2 - 5}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 + 5 - 2x^2 - 5}{h} \\ &= \frac{4xh + 2h^2}{h} \\ &= 4x + 2h \end{aligned}$$

(h) $$\begin{aligned} &\frac{g(x+h) - g(x)}{h} \\ &= \frac{3(x+h)^2 + 4(x+h) - 1 - (3x^2 + 4x - 1)}{h} \\ &= \frac{3(x^2 + 2xh + h^2) + 4x + 4h - 1 - 3x^2 - 4x + 1}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 + 4x + 4h - 1 - 3x^2 - 4x + 1}{h} \\ &= \frac{6xh + 3h^2 + 4h}{h} \\ &= 6x + 3h + 4 \end{aligned}$$

9. $y = \ln(x + 7)$

$$\begin{aligned} x + 7 &> 0 \\ x &> -7 \end{aligned}$$

Domain: $(-7, \infty)$.

10. $y = \ln(x^2 - 16)$

$$\begin{aligned} x^2 - 16 &> 0 \\ x^2 &> 16 \end{aligned}$$

$$x > 4 \quad \text{or} \quad x < -4$$

Domain: $(-\infty, -4) \cup (4, \infty)$

11. $y = \dfrac{3x-4}{x}$

$x \neq 0$

Domain: $(-\infty, 0) \cup (0, \infty)$

12. $y = \dfrac{\sqrt{x-2}}{2x+3}$

$x - 2 \geq 0$ and $2x + 3 \neq 0$

$x \geq 2$ $\qquad 2x \neq -3$

$x \neq -\dfrac{3}{2}$

Domain: $[2, \infty)$

13. $y = 2x^2 + 3x - 1$

The graph is a parabola.
Let $y = 0$.

$$0 = 2x^2 + 3x - 1$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{9+8}}{4}$$
$$= \frac{-3 \pm \sqrt{17}}{4}$$

The x-intercepts are $\frac{-3+\sqrt{17}}{4} \approx 0.28$ and $\frac{-3-\sqrt{17}}{4} \approx -1.48$.

Let $x = 0$.

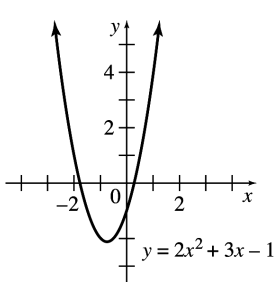
$$y = 2(0)^2 + 3(0) - 1$$

-1 is the y-intercept.

Vertex: $x = \dfrac{-b}{2a} = \dfrac{-3}{2(2)} = -\dfrac{3}{4}$

$$y = 2\left(-\frac{3}{4}\right)^2 + 3\left(-\frac{3}{4}\right) - 1$$
$$= \frac{9}{8} - \frac{9}{4} - 1$$
$$= -\frac{17}{8}$$

The vertex is $\left(-\frac{3}{4}, -\frac{17}{8}\right)$.



14. $y = -\dfrac{1}{4}x^2 + x + 2$

The graph is a parabola.
Let $y = 0$.

$$0 = -\frac{1}{4}x^2 + x + 2$$

Multiply by 4.

$$0 = -x^2 + 4x + 8$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(8)}}{2(-1)}$$
$$= \frac{-4 \pm \sqrt{48}}{-2}$$
$$= 2 \pm 2\sqrt{3}$$

The x-intercepts are $2 + 2\sqrt{3} \approx 5.46$ and $2 - 2\sqrt{3} \approx -1.46$.
Let $x = 0$.

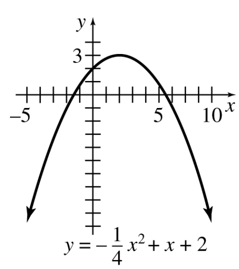
$$y = -\frac{1}{4}(0)^2 + 0 + 2$$

$y = 2$ is the y-intercept.

Vertex: $x = \dfrac{-b}{2a} = \dfrac{-1}{2\left(-\frac{1}{4}\right)} = 2$

$$y = -\frac{1}{4}(2)^2 + 2 + 2$$
$$= -1 + 4$$
$$= 3$$

The vertex is $(2, 3)$.



15. $y = -x^2 + 4x + 2$

Let $y = 0$.

$$0 = -x^2 + 4x + 2$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(2)}}{2(-1)}$$
$$= \frac{-4 \pm \sqrt{24}}{-2}$$
$$= 2 \pm \sqrt{6}$$

The x-intercepts are $2+\sqrt{6}\approx 4.45$ and $2-\sqrt{6}\approx -0.45$.
Let $x=0$.

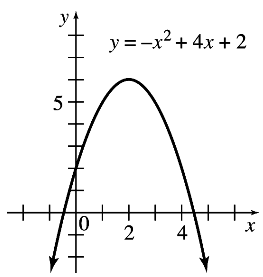
$$y=-0^2+4(0)+2$$

2 is the y-intercept.

$$\text{Vertex: } x=\frac{-b}{2a}=\frac{-4}{2(-1)}=\frac{-4}{-2}=2$$

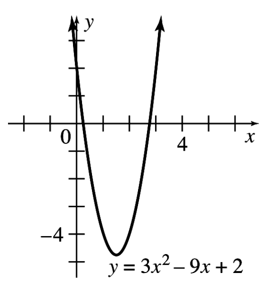
$$y=-2^2+4(2)+2=6$$

The vertex is $(2,6)$.



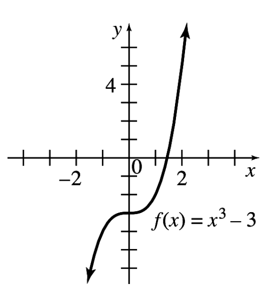
16. $y=3x^2-9x+2$

x-intercepts: 0.24 and 2.76
y-intercept: 2
Vertex: $\left(\frac{3}{2},-\frac{19}{4}\right)$



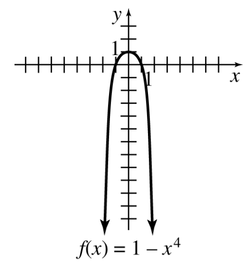
17. $f(x)=x^3-3$

Translate the graph of $f(x)=x^3$ 3 units down.



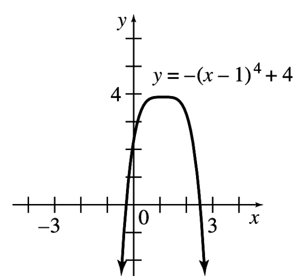
18. $f(x)=1-x^4$
$\quad\quad=-x^4+1$

Reflect the graph of $y=x^4$ vertically then translate 1 unit upward.



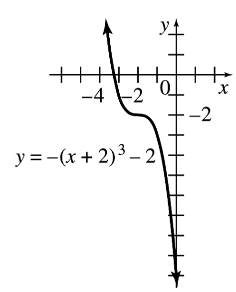
19. $y=-(x-1)^4+4$

Translate the graph of $y=x^4$ 1 unit to the right and reflect vertically. Translate 4 units upward.



20. $y=-(x+2)^3-2$

Translate the graph of $y=x^3$ 2 units to the left and reflect vertically. Translate 2 units downward.



21. $f(x) = \dfrac{8}{x}$

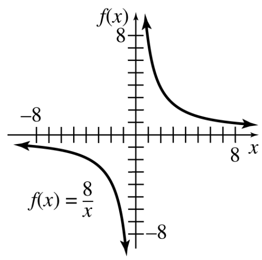
Vertical asymptote: $x = 0$

Horizontal asymptote:

$\frac{8}{x}$ approaches zero as x gets larger.

$y = 0$ is an asymptote.

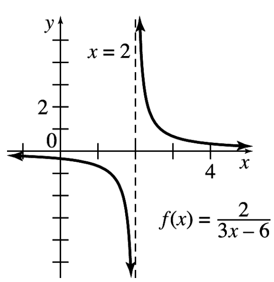
x	-4	-3	-2	-1	1	2	3	4
y	-2	-2.7	-4	-8	8	4	2.7	2



22. $f(x) = \dfrac{2}{3x-6}$

Vertical asymptote: $3x - 6 = 0$ or $x = 2$
Horizontal asymptote: $y = 0$, since $\frac{2}{3x-6}$ approaches zero as x gets larger.

x	0	1	3	4	5
y	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{9}$



23. $f(x) = \dfrac{4x-2}{3x+1}$

Vertical asymptote:

$$3x + 1 = 0$$
$$x = -\frac{1}{3}$$

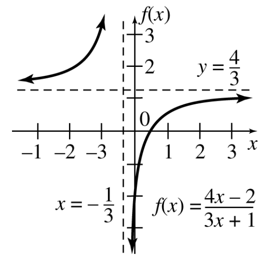
Horizontal asymptote:

As x gets larger,

$$\frac{4x-2}{3x-1} \approx \frac{4x}{3x} = \frac{4}{3}.$$

$y = \frac{4}{3}$ is an asymptote.

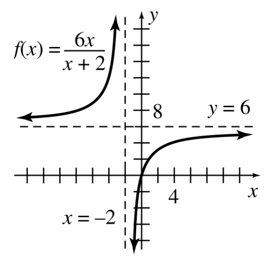
x	-3	-2	-1	0	1	2	3
y	1.75	2	3	-2	0.5	0.86	1



24. $f(x) = \dfrac{6x}{x+2}$

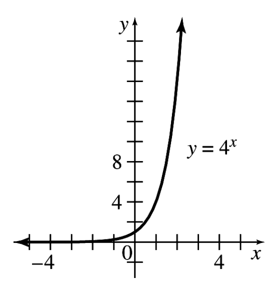
Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 6$

x	-5	-4	-3	-1	0	1	2
y	10	12	18	-6	0	2	3



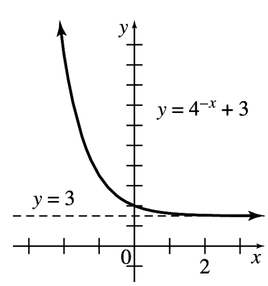
25. $y = 4^x$

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16



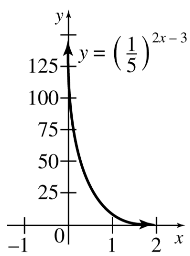
26. $y = 4^{-x} + 3$

x	-2	-1	0	1	2
y	19	7	4	$\frac{13}{4}$	$\frac{49}{16}$



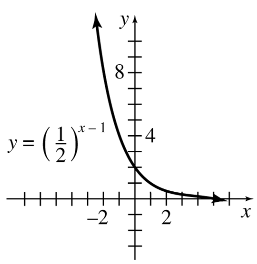
27. $y = \left(\frac{1}{5}\right)^{2x-3}$

x	0	1	2
y	125	5	$\frac{1}{5}$



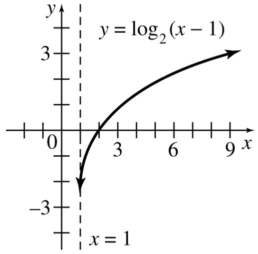
28. $y = \left(\frac{1}{2}\right)^{x-1}$

x	−2	−1	0	1	2
y	8	4	2	1	$\frac{1}{2}$



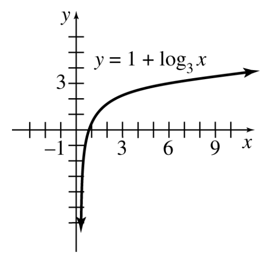
29. $y = \log_2 (x-1)$
$2^y = x - 1$
$x = 1 + 2^y$

x	2	3	5	9
y	0	1	2	3



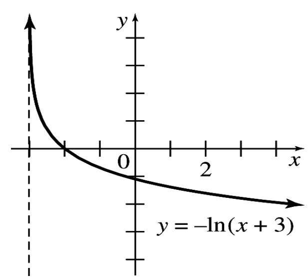
30. $y = 1 + \log_3 x$
$y - 1 = \log_3 x$
$3^{y-1} = x$

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
y	−1	0	1	2	3



31. $y = -\ln (x+3)$
$-y = \ln (x+3)$
$e^{-y} = x + 3$
$e^{-y} - 3 = x$

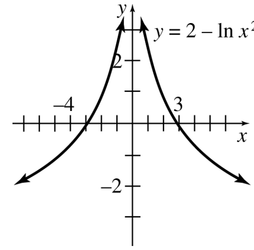
x	−2.63	−2	−0.28	4.39
y	1	0	−1	−2



32. $y = 2 - \ln x^2$

x	−4	−3	−2	−1
y	−0.8	−0.2	0.6	2

x	1	2	3	4
y	2	0.6	−0.2	−0.8



33. $$\begin{aligned} 2^{x+2} &= \frac{1}{8} \\ 2^{x+2} &= \frac{1}{2^3} \\ 2^{x+2} &= 2^{-3} \\ x+2 &= -3 \\ x &= -5 \end{aligned}$$

34. $$\begin{aligned} \left(\frac{9}{16}\right)^x &= \frac{3}{4} \\ \left(\frac{3}{4}\right)^{2x} &= \left(\frac{3}{4}\right)^1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

35. $$\begin{aligned} 9^{2y+3} &= 27^y \\ (3^2)^{2y+3} &= (3^3)^y \\ 3^{4y+6} &= 3^{3y} \\ 4y+6 &= 3y \\ y &= -6 \end{aligned}$$

36. $$\begin{aligned} \frac{1}{2} &= \left(\frac{b}{4}\right)^{1/4} \\ \left(\frac{1}{2}\right)^4 &= \frac{b}{4} \\ 4\left(\frac{1}{2}\right)^4 &= b \\ 4\left(\frac{1}{16}\right) &= b \\ \frac{1}{4} &= b \end{aligned}$$

37. $3^5 = 243$

The equation in logarithmic form is

$$\log_3\ 243 = 5.$$

38. $5^{1/2} = \sqrt{5}$

The equation in logarithmic form is

$$\log_5\ \sqrt{5} = \frac{1}{2}.$$

39. $e^{0.8} = 2.22554$

The equation in logarithmic form is

$$\ln 2.22554 = 0.8.$$

40. $10^{1.07918} = 12$

The equation in logarithmic form is

$$\log_{10}\ 12 = 1.07918.$$

41. $\log_2\ 32 = 5$

The equation in exponential form is

$$2^5 = 32.$$

42. $\log_9\ 3 = \frac{1}{2}$

The equation in exponential form is

$$9^{1/2} = 3.$$

43. $\ln\ 82.9 = 4.41763$

The equation in exponential form is

$$e^{4.41763} = 82.9.$$

44. $\log\ 3.21 = 0.50651$

The equation in exponential form is

$$10^{0.50651} = 3.21.$$

Recall that $\log x$ means $\log_{10} x$.

45. $$\begin{aligned} \log_3\ 81 &= x \\ 3^x &= 81 \\ 3^x &= 3^4 \\ x &= 4 \end{aligned}$$

46. $$\begin{aligned} \log_{32}\ 16 &= x \\ 32^x &= 16 \\ 2^{5x} &= 2^4 \\ 5x &= 4 \\ x &= \frac{4}{5} \end{aligned}$$

47. $$\begin{aligned} \log_4\ 8 &= x \\ 4^x &= 8 \\ (2^2)^x &= 2^3 \\ 2x &= 3 \\ x &= \frac{3}{2} \end{aligned}$$

48. $$\begin{aligned} \log_{100} 1000 &= x \\ 100^x &= 1000 \\ (10^2)^x &= 10^3 \\ 2x &= 3 \\ x &= \frac{3}{2} \end{aligned}$$

49. $\log_5 3k + \log_5 7k^3$
$= \log_5 3k(7k^3)$
$= \log_5 (21k^4)$

50. $\log_3 2y^3 - \log_3 8y^2$
$= \log_3 \frac{2y^3}{8y^2}$
$= \log_3 \frac{y}{4}$

51. $4 \log_3 y - 2 \log_3 x$
$= \log_3 y^4 - \log_3 x^2$
$= \log_3 \left(\frac{y^4}{x^2}\right)$

52. $3 \log_4 r^2 - 2 \log_4 r$
$= \log_4 (r^2)^3 - \log_4 r^2$
$= \log_4 \left(\frac{r^6}{r^2}\right)$
$= \log_4 (r^4)$

53.
$$\begin{aligned} 6^p &= 17 \\ \ln 6^p &= \ln 17 \\ p \ln 6 &= \ln 17 \\ p &= \frac{\ln 17}{\ln 6} \\ &\approx 1.581 \end{aligned}$$

54.
$$\begin{aligned} 3^{z-2} &= 11 \\ \ln 3^{z-2} &= \ln 11 \\ (z-2) \ln 3 &= \ln 11 \\ z - 2 &= \frac{\ln 11}{\ln 3} \\ z &= \frac{\ln 11}{\ln 3} + 2 \\ &\approx 4.183 \end{aligned}$$

55.
$$\begin{aligned} 2^{1-m} &= 7 \\ \ln 2^{1-m} &= \ln 7 \\ (1-m) \ln 2 &= \ln 7 \\ 1 - m &= \frac{\ln 7}{\ln 2} \\ -m &= \frac{\ln 7}{\ln 2} - 1 \\ m &= 1 - \frac{\ln 7}{\ln 2} \\ &\approx -1.807 \end{aligned}$$

56.
$$\begin{aligned} 12^{-k} &= 9 \\ \ln 12^{-k} &= \ln 9 \\ -k \ln 12 &= \ln 9 \\ k &= -\frac{\ln 9}{\ln 12} \\ &\approx -0.884 \end{aligned}$$

57.
$$\begin{aligned} e^{-5-2x} &= 5 \\ \ln e^{-5-2x} &= \ln 5 \\ (-5-2x) \ln e &= \ln 5 \\ (-5-2x) \cdot 1 &= \ln 5 \\ -2x &= \ln 5 + 5 \\ x &= \frac{\ln 5 + 5}{-2} \\ &\approx -3.305 \end{aligned}$$

58.
$$\begin{aligned} e^{3x-1} &= 14 \\ \ln (e^{3x-1}) &= \ln 14 \\ 3x - 1 &= \ln 14 \\ 3x &= 1 + \ln 14 \\ x &= \frac{1 + \ln 14}{3} \\ &\approx 1.213 \end{aligned}$$

59.
$$\begin{aligned} \left(1 + \frac{m}{3}\right)^5 &= 15 \\ \left[\left(1 + \frac{m}{3}\right)^5\right]^{1/5} &= 15^{1/5} \\ 1 + \frac{m}{3} &= 15^{1/5} \\ \frac{m}{3} &= 15^{1/5} - 1 \\ m &= 3(15^{1/5} - 1) \\ &\approx 2.156 \end{aligned}$$

60.
$$\begin{aligned} \left(1 + \frac{2p}{5}\right)^2 &= 3 \\ 1 + \frac{2p}{5} &= \pm\sqrt{3} \\ 5 + 2p &= \pm 5\sqrt{3} \\ 2p &= -5 \pm 5\sqrt{3} \\ p &= \frac{-5 \pm 5\sqrt{3}}{2} \\ p &= \frac{-5 + 5\sqrt{3}}{2} \approx 1.830 \\ \text{or} \quad p &= \frac{-5 - 5\sqrt{3}}{2} \approx -6.830 \end{aligned}$$

61. $\log_k 64 = 6$
$$k^6 = 64$$
$$k^6 = 2^6$$
$$k = 2$$

62. $\log_3(2x+5) = 5$
$$3^5 = 2x+5$$
$$243 = 2x+5$$
$$238 = 2x$$
$$x = 119$$

63. $\log(4p+1) + \log p = \log 3$
$$\log[p(4p+1)] = \log 3$$
$$\log(4p^2+p) = \log 3$$
$$4p^2+p = 3$$
$$4p^2+p-3 = 0$$
$$(4p-3)(p+1) = 0$$
$$4p-3 = 0 \quad \text{or} \quad p+1 = 0$$
$$p = \frac{3}{4} \qquad\qquad p = -1$$

p cannot be negative, so $p = \frac{3}{4}$.

64. $\log_2(5m-2) - \log_2(m+3) = 2$
$$\log_2 \frac{5m-2}{m+3} = 2$$
$$\frac{5m-2}{m+3} = 2^2$$
$$5m-2 = 4(m+3)$$
$$5m-2 = 4m+12$$
$$m = 14$$

65. $f(x) = a^x; a > 0, a \neq 1$

(a) The domain is $(-\infty, \infty)$.

(b) The range is $(0, \infty)$.

(c) The y-intercept is 1.

(d) The graph has no discontinuities.

(e) The x-axis, $y = 0$, is a horizontal asymptote.

(f) The function is increasing if $a > 1$.

(g) The function is decreasing if $0 < a < 1$.

66. $f(x) = \log_a x;\ a > 0,\ a \neq 1$

(a) The domain is $(0, \infty)$.

(b) The range is $(-\infty, \infty)$.

(c) The x-intercept is 1.

(d) There are no discontinuities.

(e) The y-axis, $x = 0$, is a vertical asymptote.

(f) f is increasing if $a > 1$.

(g) f is decreasing if $0 < a < 1$.

68. **(a)** For x in the interval $0 < x \leq 1$, the renter is charged the fixed cost of \$60 and 1 day's rent of \$60 so
$$C\left(\tfrac{3}{4}\right) = \$40 + \$60(1)$$
$$= \$40 + \$60$$
$$= \$100.$$

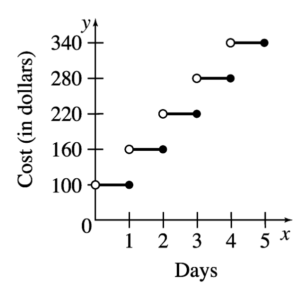
(b) $C\left(\frac{9}{10}\right) = \$40 + \$60(1)$
$$= \$40 + \$60$$
$$= \$100.$$

(c) $C(1) = \$40 + \$60(1)$
$$= \$40 + \$60$$
$$= \$100.$$

(d) For x in the interval $1 < x \leq 2$, the renter is charged the fixed cost of \$40 and 2 days rent of \$120, so
$$C\left(1\tfrac{5}{8}\right) = \$40 + \$60(2)$$
$$= \$40 + \$120$$
$$= \$160.$$

(e) For x in the interval $2 < x \leq 3$ the renter is charged the fixed cost of \$40 and 3 days rent of \$180. So
$$C\left(2\tfrac{1}{9}\right) = \$40 + \$60(3)$$
$$= \$40 + \$180$$
$$= \$220.$$

(f)



(g) The independent variable is the number of days, or x.

(h) The dependent variable is the cost, or $C(x)$.

69. $y = \dfrac{7x}{100 - x}$

(a) $y = \dfrac{7(80)}{100 - 80} = \dfrac{560}{20} = 28$

The cost is \$28,000.

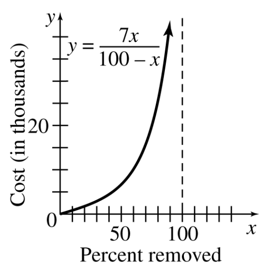
(b) $y = \dfrac{7(50)}{100 - 50} = \dfrac{350}{50} = 7$

The cost is \$7000.

(c) $\dfrac{7(90)}{100 - 90} = \dfrac{630}{10} = 63$

The cost is \$63,000.

(d) Plot the points $(80, 28)$, $(50, 7)$, and $(90, 63)$.



(e) No, because all of the pollutant would be removed when $x = 100$, at which point the denominator of the function would be zero.

70. $P = \$6902,\ r = 6\%,\ t = 8,\ m = 2$

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$
$$A = 6902\left(1 + \frac{0.06}{2}\right)^{8(2)}$$
$$= 6902(1.03)^{16}$$
$$= \$11,075.68$$

$$\text{Interest} = A - P$$
$$= \$11,075.68 - \$6902$$
$$= \$4173.68$$

71. $P = \$2781.36,\ r = 4.8\%,\ t = 6,\ m = 4$

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$
$$A = 2781.36\left(1 + \frac{0.048}{4}\right)^{(6)(4)}$$
$$= 2781.36(1.012)^{24}$$
$$= \$3703.31$$

$$\text{Interest} = \$3703.31 - \$2781.36$$
$$= \$921.95$$

72. \$1000 deposited at 6% compounded semiannually.

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$

To double:

$$2(1000) = 1000\left(1 + \frac{0.06}{2}\right)^{t\cdot 2}$$
$$2 = 1.03^{2t}$$
$$\ln 2 = 2t \ln 1.03$$
$$t = \frac{\ln 2}{2 \ln 1.03}$$
$$\approx 12 \text{ years}$$

To triple:

$$3(1000) = 1000\left(1 + \frac{0.06}{2}\right)^{t\cdot 2}$$
$$3 = 1.03^{2t}$$
$$\ln 3 = 2t \ln 1.03$$
$$t = \frac{\ln 3}{2 \ln 1.03}$$
$$\approx 19 \text{ years}$$

73. \$2100 deposited at 4% compounded quarterly.

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$

To double:

$$2(2100) = 2100\left(1 + \frac{0.04}{4}\right)^{t\cdot 4}$$
$$2 = 1.01^{4t}$$
$$\ln 2 = 4t \ln 1.01$$
$$t = \frac{\ln 2}{4 \ln 1.01}$$
$$\approx 17.4$$

Because interest is compounded quarterly, round the result up to the nearest quarter, which is 17.5 years or 70 quarters.

To triple:

$$3(2100) = 2100\left(1 + \frac{0.04}{4}\right)^{t\cdot 4}$$
$$3 = 1.01^{4t}$$
$$\ln 3 = 4t \ln 1.01$$
$$t = \frac{\ln 3}{4 \ln 1.01}$$
$$\approx 27.6$$

Because interest is compounded quarterly, round the result up to the nearest quarter, which is 27.75 years or 111 quarters.

74. $P = \$12{,}104,\ r = 6.2\%,\ t = 2$

$$\begin{aligned} A &= Pe^{rt} \\ &= 12,104e^{0.062(2)} \\ &= \$13,701.92 \end{aligned}$$

75. $P = \$12{,}104,\ r = 6.2\%,\ t = 4$

$$\begin{aligned} A &= Pe^{rt} \\ A &= 12{,}104e^{0.062(4)} \\ &= 12{,}104e^{0.248} \\ &= \$15{,}510.79 \end{aligned}$$

76. $A = \$1500,\ r = 0.06,\ t = 9$

$$\begin{aligned} A &= Pe^{rt} \\ &= 1500e^{0.06(9)} \\ &= 1500e^{0.54} \\ &= \$2574.01 \end{aligned}$$

77. $P = \$12{,}000,\ r = 0.05,\ t = 8$

$$\begin{aligned} A &= 12{,}000e^{0.05(8)} \\ &= 12{,}000e^{0.40} \\ &= \$17{,}901.90 \end{aligned}$$

78. $r = 7\%,\ m = 4$

$$\begin{aligned} r_E &= \left(1 + \frac{r}{m}\right)^m = 1 \\ &= \left(1 + \frac{0.07}{4}\right)^4 - 1 \\ &= 0.0719 = 7.19\% \end{aligned}$$

79. $r = 6\%,\ m = 12$

$$\begin{aligned} r_E &= \left(1 + \frac{r}{m}\right)^m - 1 \\ &= \left(1 + \frac{0.06}{12}\right)^{12} - 1 \\ &= 0.0617 = 6.17\% \end{aligned}$$

80. $r = 5\%$ compounded continuously

$$\begin{aligned} r_E &= e^r - 1 \\ &= e^{0.05} - 1 \\ &= 0.0513 = 5.13\% \end{aligned}$$

81. $A = \$2000,\ r = 6\%,\ t = 5,\ m = 1$

$$\begin{aligned} P &= A\left(1 + \frac{r}{m}\right)^{-tm} \\ &= 2000\left(1 + \frac{0.06}{1}\right)^{-5(1)} \\ &= 2000(1.06)^{-5} \\ &= \$1494.52 \end{aligned}$$

82. $A = \$10{,}000,\ r = 8\%,\ m = 2,\ t = 6$

$$\begin{aligned} P &= A\left(1 + \frac{r}{m}\right)^{-tm} \\ &= 10{,}000\left(1 + \frac{0.08}{2}\right)^{-2(6)} \\ &= 10{,}000(1.04)^{-12} \\ &= \$6245.97 \end{aligned}$$

83. $r = 7\%,\ t = 8,\ m = 2,\ P = 10{,}000$

$$\begin{aligned} A &= P\left(1 + \frac{r}{m}\right)^{tm} \\ &= 10{,}000\left(1 + \frac{0.07}{2}\right)^{8(2)} \\ &= 10{,}000(1.035)^{16} \\ &= \$17{,}339.86 \end{aligned}$$

84. $P = \$1,\ r = 0.08$

$$\begin{aligned} A &= Pe^{rt},\ A = 3(1) \\ 3 &= 1e^{0.08t} \\ \ln\ 3 &= 0.08t \\ \frac{\ln\ 3}{0.08} &= t \\ 13.7 &= t \end{aligned}$$

It would take about 13.7 years.

85. $P = \$6000,\ A = \$8000,\ t = 3$

$$\begin{aligned} A &= Pe^{rt} \\ 8000 &= 6000e^{3r} \\ \frac{4}{3} &= e^{3r} \\ \ln\ 4 - \ln\ 3 &= 3r \\ r &= \frac{\ln\ 4 - \ln\ 3}{3} \\ r &\approx 0.0959 \text{ or about } 9.59\% \end{aligned}$$

86. $P = A\left(1+\frac{r}{m}\right)^{-tm}$

$$P = 25{,}000\left(1+\frac{0.06}{12}\right)^{-3(12)}$$
$$= 25{,}000(1.005)^{-36}$$
$$= \$20{,}891.12$$

87. **(a)** $n = 1000-(p-50)(10),\ p \geq 50$
$$= 1000 - 10p + 500$$
$$= 1500 - 10p$$

(b) $R = pn$
$R = p(1500-10p)$

(c) $p \geq 50$

Since n cannot be negative,

$$1500 - 10p \geq 0$$
$$-10p \geq -1500$$
$$p \leq 150.$$

Therefore, $50 \leq p \leq 150$.

(d) Since $n = 1500 - 10p$,

$$10p = 1500 - n$$
$$p = 150 - \frac{n}{10}.$$

$$R = pn$$
$$R = \left(150 - \frac{n}{10}\right)n$$

(e) Since she can sell at most 1000 tickets, $0 \leq n \leq 1000$.

(f) $R = -10p^2 + 1500p$

$$\frac{-b}{2a} = \frac{-1500}{2(-10)} = 75$$

The price producing maximum revenue is \$75.

(g) $R = -\frac{1}{10}n^2 + 150n$

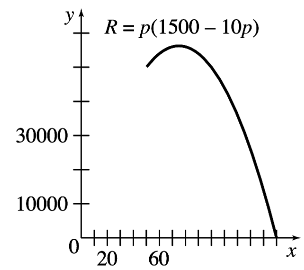
$$\frac{-b}{2a} = \frac{-150}{2\left(-\frac{1}{10}\right)} = 750$$

The number of tickets producing maximum revenue is 750.

(h)
$$R(p) = -10p^2 + 1500p$$
$$R(75) = -10(75)^2 + 1500(75)$$
$$= -56{,}250 + 112{,}500$$
$$= 56{,}250$$

The maximum revenue is \$56,250.

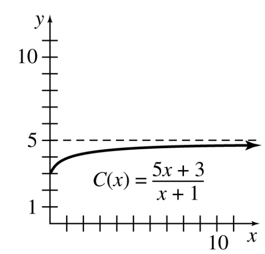
(i)



(j) The revenue starts at \$50,000 when the price is \$50, rises to a maximum of \$56,250 when the price is \$75, and falls to 0 when the price is \$150.

88. $C(x) = \frac{5x+3}{x+1}$

(a)



(b) $C(x+1) - C(x)$
$$= \frac{5(x+1)+3}{(x+1)+1} - \frac{5x+3}{x+1}$$
$$= \frac{5x+8}{x+2} - \frac{5x+3}{x+1}$$
$$= \frac{(5x+8)(x+1)-(5x+3)(x+2)}{(x+2)(x+1)}$$
$$= \frac{5x^2+13x+8-5x^2-13x-6}{(x+2)(x+1)}$$
$$= \frac{2}{(x+2)(x+1)}$$

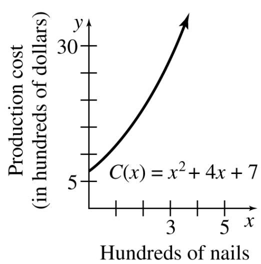
(c) $A(x) = \frac{C(x)}{x} = \frac{\frac{5x+3}{x+1}}{x}$
$$= \frac{5x+3}{x(x+1)}$$

(d) $A(x+1) - A(x)$

$$= \frac{5(x+1)+3}{(x+1)[(x+1)+1]} - \frac{5x+3}{x(x+1)}$$
$$= \frac{5x+8}{(x+1)(x+2)} - \frac{5x+3}{x(x+1)}$$
$$= \frac{x(5x+8)-(5x+3)(x+2)}{x(x+1)(x+2)}$$
$$= \frac{5x^2+8x-5x^2-13x-6}{x(x+1)(x+2)}$$
$$= \frac{-5x-6}{x(x+1)(x+2)}$$

89. $C(x) = x^2 + 4x + 7$

(a)



(b) $C(x+1) - C(x)$
$$= (x+1)^2 + 4(x+1) + 7 - (x^2+4x+7)$$
$$= x^2 + 2x + 1 + 4x + 4 + 7 - x^2 - 4x - 7$$
$$= 2x + 5$$

(c) $A(x) = \frac{C(x)}{x} = \frac{x^2+4x+7}{x}$
$$= x + 4 + \frac{7}{x}$$

(d) $A(x+1) - A(x)$
$$= (x+1) + 4 + \frac{7}{x+1} - \left(x + 4 + \frac{7}{x}\right)$$
$$= x + 1 + 4 + \frac{7}{x+1} - x - 4 - \frac{7}{x}$$
$$= 1 + \frac{7}{x+1} - \frac{7}{x}$$
$$= 1 + \frac{7x - 7(x+1)}{x(x+1)}$$
$$= 1 + \frac{7x - 7x - 7}{x(x+1)}$$
$$= 1 - \frac{7}{x(x+1)}$$

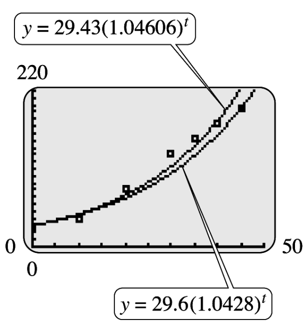
90. (a) $y = a^t$

Let $y_0 = 29.6$, the value of y at $t = 0$. Use the point $(45, 195.3)$ to find a.

$$195.3 = 29.6a^{45}$$
$$a^{45} = \frac{195.3}{29.6}$$
$$a = \sqrt[45]{\frac{195.3}{29.6}}$$
$$\approx 1.0428$$
$$y = 29.6(1.0428)^t$$

(b) $y = 29.43(1.04606)^t$

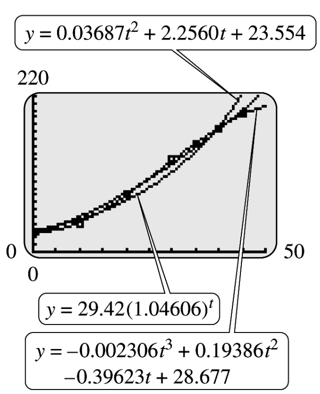
(c)



Yes, the answers are close to each other.

(d) $y = 0.03687x^2 + 2.2560x + 23.554$
$y = -0.002306x^3 + 0.19386x^2 - 0.39623x + 28.677$

(e)



91. $F(x) = -\frac{2}{3}x^2 + \frac{14}{3}x + 96$

The maximum fever occurs at the vertex of the parabola.

$$x = \frac{-b}{2a} = \frac{-\frac{14}{3}}{-\frac{4}{3}} = \frac{7}{2}$$

$$\begin{aligned} y &= -\frac{2}{3}\left(\frac{7}{2}\right)^2 + \frac{14}{3}\left(\frac{7}{2}\right) + 96 \\ &= -\frac{2}{3}\left(\frac{49}{4}\right) + \frac{49}{3} + 96 \\ &= -\frac{49}{6} + \frac{49}{3} + 96 \\ &= -\frac{49}{6} + \frac{98}{6} + \frac{576}{6} = \frac{625}{6} \approx 104.2 \end{aligned}$$

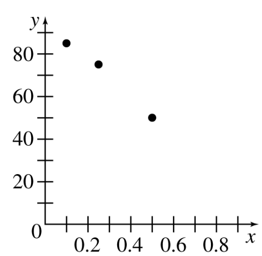
The maximum fever occurs on the third day. It is about 104.2°F.

92. **(a)** $100\% - 87.5\% = 12.5\%$

$$12.5\% = 0.125 = \frac{125}{1000} = \frac{1}{8}$$

The fraction let in is 1 over the SPF rating.

(b)



(c) $\text{UVB} = 1 - \frac{1}{\text{SPF}}$

(d) $1 - \frac{1}{8} = 87.5\%$

$1 - \frac{1}{4} = 75.0\%$

The increase is 12.5%.

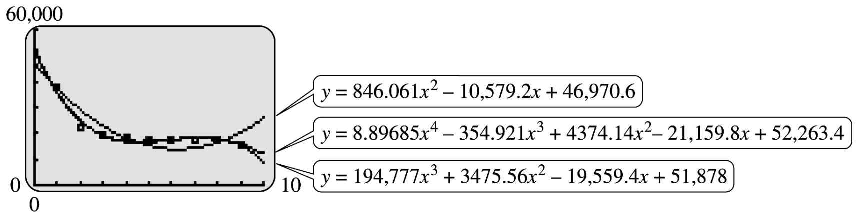
(e) $1 - \frac{1}{30} = 96.\overline{6}\%$

$1 - \frac{1}{15} = 93.\overline{3}\%$

The increase is $3.\overline{3}\%$ or about 3.3%.

(f) The increase in percent protection decreases to zero.

93. (a)



(b) Quadratic:

$y = 846.061x^2 - 10{,}579.2x + 46{,}970.6$

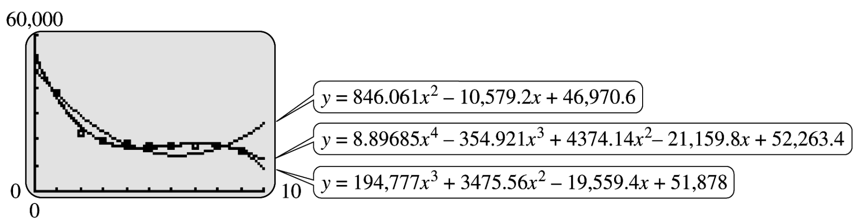
Cubic:

$y = -194.777x^3 + 3475.56x^2 - 19{,}558.4x + 51{,}879$

Quartic:

$y = 8.89685x^4 - 354.921x^3 + 4374.14x^2 - 21{,}159.8x + 52{,}263.4$

(c)



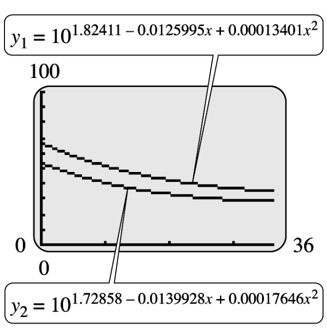
(d) Use the functions found in part (b), with $x = 11$.

Quadratic: 32,973
Cubic: -1969
Quartic: 6635

94. (a) The first three years of infancy corresponds to 0 months to 36 months, so the domain is [0, 36].

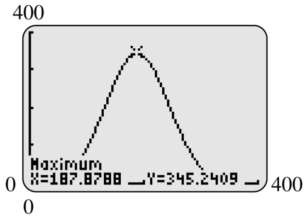
(b) In both cases, the graph of the quadratic function in the exponent opens upward and the x coordinate of the vertex is greater than 36 ($x \approx 47$ for the awake infants and $x \approx 40$ for the sleeping infants). So the quadratic functions are both decreasing over this time. Therefore, both respiratory rates are decreasing.

(c)



(d) When $x = 12$, the waking respiratory rate is $y \approx 49.23$ breaths per minute, and the sleeping respiratory rate is $y \approx 38.55$. Therefore, for a 1-year-old infant in the 95th percentile, the waking respiratory rate is approximately $49.23 - 38.55 \approx 10.7$ breaths per minute higher.

95.



This function has a maximum value at $x \approx 187.9$. At $x \approx 187.9, y \approx 345$. The largest girth for which this formula gives a reasonable answer is 187.9 cm. The predicted mass of a polar bear with this girth is 345 kg.

96. $y = 17{,}000,\ y_0 = 15{,}000,\ t = 4$

(a)

$$\begin{aligned} y &= y_0 e^{kt} \\ 17{,}000 &= 15{,}000e^{4k} \\ \frac{17}{15} &= e^{4k} \\ \ln\left(\frac{17}{15}\right) &= 4k \\ \frac{0.125}{4} &= k \\ 0.0313 &= k \end{aligned}$$

So, $y = 15{,}000e^{0.0313t}$.

(b)

$$\begin{aligned} 45{,}000 &= 15{,}000e^{0.0313t} \\ 3 &= e^{0.0313t} \\ \ln 3 &= 0.0313t \\ \frac{\ln 3}{0.0313} &= k \\ 35 &= k \end{aligned}$$

It would take about 35 years.

97. $p(t) = \dfrac{1.79 \cdot 10^{11}}{(2026.87 - t)^{0.99}}$

(a) $p(2005) \approx 8.441$ billion

This is about 1.964 billion more than the estimate of 6.477 billion.

(b) $p(2020) \approx 26.56$ billion
$p(2025) \approx 96.32$ billion

98. $I(x) \geq 1$
$I(x) = 10e^{-0.3x}$

$$\begin{aligned} 10e^{-0.3x} &\geq 1 \\ e^{-0.3x} &\geq 0.1 \\ -0.3x &\geq \ln 0.1 \\ x &\leq \frac{\ln 0.1}{-0.3} \approx 7.7 \end{aligned}$$

The greatest depth is about 7.7 m.

99. Graph

$$y = c(t) = e^{-t} - e^{-2t}$$

on a graphing calculator and locate the maximum point. A calculator shows that the x-coordinate of the maximum point is about 0.69, and the y-coordinate is exactly 0.25. Thus, the maximum concentration of 0.25 occurs at about 0.69 minutes.

100. $g(t) = \dfrac{c}{a} + \left(g_0 - \dfrac{c}{a}\right) e^{-at}$

(a) If $g_0 = 0.08, c = 0.1$, and $a = 1.3$, the function becomes

$$g(t) = \frac{0.1}{1.3} + \left(0.08 - \frac{0.1}{1.3}\right) e^{-1.3t}.$$

Graph this function on a graphing calculator. Use a window with Xmin = 0, since this represents the time when the drug is first injected. A good choice for the viewing window is [0, 5] by [0.07, 0.11], Xscl = 0.5, Yscl = 0.01.
From the graph, we see that the maximum value of g for $t \geq 0$ occurs at $t = 0$, the time when the drug is first injected.

The maximum amount of glucose in the bloodstream, given by $G(0)$, is 0.08 gram.

(b) From the graph, we see that the amount of glucose in the bloodstream decreases from the initial value of 0.08 gram, so it will never increase to 0.1 gram. We can also reach this conclusion by graphing $y_1 = G(t)$ and $y_2 = 0.1$ on the same screen with the window given in (a) and observing that the graphs of y_1 and y_2 do not intersect.

(c) From the graph, we see that as t increases, the graph of $y_1 = G(t)$ becomes almost horizontal, and $G(t)$ approaches approximately 0.0769. Note that

$$\frac{c}{a} = \frac{0.1}{1.3} \approx 0.0769.$$

The amount of glucose in the bloodstream after a long time approaches 0.0769 grams.

101. $y = y_0 e^{-kt}$

(a)
$$\begin{aligned} 100{,}000 &= 128{,}000e^{-k(5)} \\ 128{,}000 &= 100{,}000e^{5k} \\ \frac{128}{100} &= e^{5k} \\ \ln\left(\frac{128}{100}\right) &= 5k \\ 0.05 &\approx k \end{aligned}$$

$y = 100{,}000e^{-0.05t}$

(b)
$$\begin{aligned} 70{,}000 &= 100{,}000e^{-0.05t} \\ \frac{7}{10} &= e^{-0.05t} \\ \ln\frac{7}{10} &= -0.05t \\ 7.1 &\approx t \end{aligned}$$

It will take about 7.1 years.

102. $t = (1.26 \times 10^9)\dfrac{\ln\left[1 + 8.33\left(\frac{A}{K}\right)\right]}{\ln\ 2}$

(a) $A = 0,\ K > 0$

$$\begin{aligned} t &= (1.26 \times 10^9)\frac{\ln\left[1 + 8.33(0)\right]}{\ln\ 2} \\ &= (1.26 \times 10^9)(0) = 0 \text{ years} \end{aligned}$$

(b)
$$\begin{aligned} t &= (1.26 \times 10^9)\frac{\ln\left[1 + 8.33(0.212)\right]}{\ln\ 2} \\ &= (1.26 \times 10^9)\frac{\ln\ 2.76596}{\ln\ 2} \\ &= 1{,}849{,}403{,}169 \end{aligned}$$
or about 1.85×10^9 years

(c) As r increases, t increases, but at a slower and slower rate. As r decreases, t decreases at a faster and faster rate.

103. **(a)** Since the speed in one direction is $v + w$ and in the other direction is $v - w$, the time in one direction is $\frac{d}{v+w}$ and in the other direction is $\frac{d}{v-w}$. So the total time is $\frac{d}{v+w} + \frac{d}{v-w}$.

(b) The average speed is the total distance divided by the total time. So

$$v_{aver} = \frac{2d}{\frac{d}{v+w} + \frac{d}{v-w}}.$$

(c)
$$\begin{aligned} &\frac{2d}{\frac{d}{v+w} + \frac{d}{v-w}} \\ &= \frac{2d}{\frac{d}{v+w} + \frac{d}{v-w}} \cdot \frac{(v+w)(v-w)}{(v+w)(v-w)} \\ &= \frac{2d(v^2 - w^2)}{d(v-w) + d(v+w)} \\ &= \frac{2d(v^2 - w^2)}{dv - dw + dv + dw} \\ &= \frac{2d(v^2 - w^2)}{2dv} \\ &= \frac{v^2 - w^2}{v} = v - \frac{w^2}{v} \end{aligned}$$

(d) $v_{aver} = v - \dfrac{w^2}{v}$

v_{aver} will be greatest when $w = 0$.

104. **(a)** $x = 0.9$ means the speed is 10% slower on the return trip.
$x = 1.1$ means the speed is 10% faster on the return trip.

(b)
$$\begin{aligned} v_{aver} &= \frac{2d}{\frac{d}{v} + \frac{d}{xv}} = \frac{2d}{\frac{d}{v} + \frac{d}{xv}} \cdot \frac{xv}{xv} \\ &= \frac{2dxv}{dx + d} = \frac{2dxv}{d(x+1)} \\ &= \frac{2xv}{x+1} = \left(\frac{2x}{x+1}\right)v \end{aligned}$$

(c) The formula for v_{aver} is a rational function with a horizontal asymptote at $v_{aver} = 2v$. This means that as the return velocity becomes greater and greater, the average velocity approaches twice the velocity on the first part of the trip, and can never exceed twice that velocity.

105. **(a)**
$$\begin{aligned} P &= kD^1 \\ 164.8 &= k(30.1) \\ k &= \frac{164.8}{30.1} \approx 5.48 \end{aligned}$$

For $n = 1,\ P = 5.48D$.

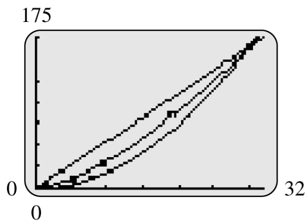
$$\begin{aligned} P &= kD^{1.5} \\ 164.8 &= k(30.1)^{1.5} \\ k &= \frac{164.8}{(30.1)^{1.5}} \approx 1.00 \end{aligned}$$

For $n = 1.5,\ P = 1.00D^{1.5}$.

$$P = kD^2$$
$$164.8 = k(30.1)^2$$
$$k = \frac{164.8}{(30.1)^2} \approx 0.182$$

For $n = 2,\ P = 0.182D^2$.

(b)



$P = 1.00D^{1.5}$ appears to be the best fit.

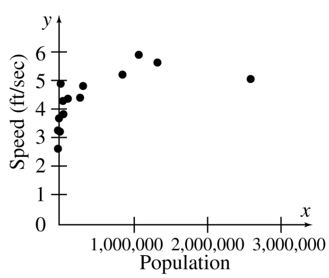
(c) $P = 1.00(39.5)^{1.5} \approx 248.3$ years

(d) We obtain

$$P = 1.00D^{1.5}.$$

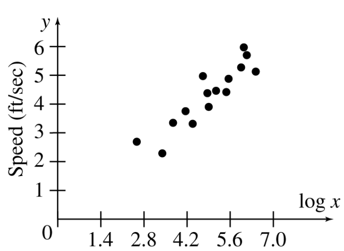
This is the same as the function found in part (b).

106. **(a)**



(b) Using a graphing calculator, $r = 0.63$.

(c)



Yes, the data are now more linear than in part (a).

(d) Using a graphing calculator, $r = 0.91$. r is closer to 1.

(e) Using a graphing calculator,

$$Y = 0.873 \log x - 0.0255.$$

Extended Application: Characteristics of the Monkeyface Prickleback

1. $L_t = L_x\,(1 - e^{-kt})$
$L_t = 71.5(1 - e^{-0.1t})$
$L_4 = 71.5(1 - e^{-0.4}) \approx 23.6$
$L_{11} = 71.5(1 - e^{-1.1}) \approx 47.7$
$L_{17} = 71.5(1 - e^{-1.7}) \approx 58.4$

The estimates are low.

2. $W = aL^b$
$W = 0.01289L^{2.9}$

If $L = 25,\ W = 0.01289(25)^{2.9} \approx 146.0$
If $L = 40,\ W = 0.01289(40)^{2.9} \approx 570.5$
If $L = 60,\ W = 0.01289(60)^{2.9} \approx 1848.8$

Compared to the curve, the answers are reasonable.