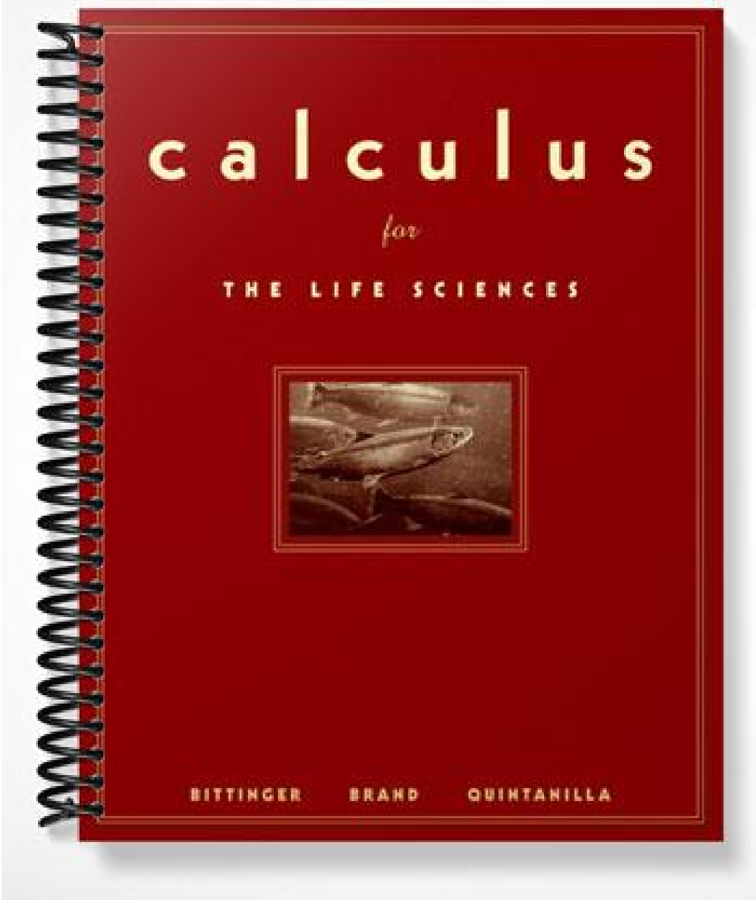
SOLUTIONS MANUAL



Chapter 2

Differentiation

Exercise Set 2.1

1. The function is not continuous at $x = 1$ since the limit from the left of $x = 1$ is not equal to the limit from the right of $x = 1$ and therefore the limit of the function at $x = 1$ does not exist.

2. The function is not continuous at $x = -2$ since the limit from the left of $x = -2$ is not equal to the limit from the right of $x = -2$ and therefore the limit of the function at $x = -2$ does not exist.

3. The function is continuous at every point in the given plot. Note that the graph can be traced without a jump from one point to another.

4. The function is not continout at $x = -2$ since the value of the function at $x = -2$ is undefined.

5. **a)** As we approach the x-value of 1 from the right we notice that the y-value is approaching a value of -1. Thus, $\lim_{x\to 1^+} f(x) = -1$. As we approach the x-value of 1 from the left we notice that the y-value is approaching a value of 2. Thus, $\lim_{x\to 1^-} f(x) = 2$. Since $\lim_{x\to 1^+} f(x) \neq \lim_{x\to 1^-} f(x)$ then $\lim_{x\to 1} f(x)$ does not exist.

b) Reading the value from the graph $f(1) = -1$.

c) Since the $\lim_{x\to 1^-} f(x)$ does not exist, then $f(x)$ is not continuous at $x = 1$.

d) As we approach the x-value of -2 from the right we notice that the y-value is approaching a value of 3. Thus, $\lim_{x\to -2^+} f(x) = 3$. As we approach the x-value of -2 from the left we notice that the y-value is approaching a value of 3. Thus, $\lim_{x\to -2^-} f(x) = 3$. Since $\lim_{x\to -2^+} f(x) = \lim_{x\to -2^-} f(x) = 3$ then $\lim_{x\to -2} f(x) = 3$.

e) Reading the value from the graph $f(-2) = 3$.

f) Since $\lim_{x\to -2} f(x) = 3$ and $f(-2) = 3$, then $f(x)$ is continuous at $x = -2$.

6. **a)** $\lim_{x\to 1^+} g(x) = -2$, $\lim_{x\to 1^-} g(x) = -2$, therefore $\lim_{x\to 1} f(x) = -2$

b) $g(1) = -2$

c) Since $\lim_{x\to 1} g(x) = -1 = g(1) = -2$, then $g(x)$ is continuous at $x = 1$

d) $\lim_{x\to -2^+} g(x) = 4$, $\lim_{x\to -2^-} g(x) = -3$, thus $\lim_{x\to -2} g(x)$ does not exist

e) $g(-2) = -3$

f) Since $\lim_{x\to -2} g(x) \neq g(-2)$, then the function is not continuous at $x = -2$

7. **a)** As we approach the x-value of 1 from the right we notice that the y-value is approaching a value of 2. Thus, $\lim_{x\to 1^+} h(x) = 2$. As we approach the x-value of 1 from the left we notice that the y-value is approaching a value of 2. Thus, $\lim_{x\to 1^-} h(x) = 2$. Since $\lim_{x\to 1^+} h(x) = \lim_{x\to 1^-} h(x) = 2$ then $\lim_{x\to 1} h(x) = 2$.

b) Reading the value from the graph $h(1) = 2$.

c) Since the $\lim_{x\to 1} h(x) = 2$ and $= h(1) = 2$ then $h(x)$ is continuous at $x = 1$.

d) As we approach the x-value of -2 from the right we notice that the y-value is approaching a value of 3. Thus, $\lim_{x\to -2^+} h(x) = 0$. As we approach the x-value of -2 from the left we notice that the y-value is approaching a value of 0. Thus, $\lim_{x\to -2^-} h(x) = 0$. Since $\lim_{x\to -2^+} h(x) = \lim_{x\to -2^-} h(x) = 0$ then $\lim_{x\to -2} h(x) = 0$.

e) Reading the value from the graph $h(-2) = 0$.

f) Since $\lim_{x\to -2} h(x) = 0$ and $h(-2) = 0$, then $h(x)$ is continuous at $x = -2$.

8. **a)** $\lim_{x\to 1^+} t(x) \approx 0.25$, $\lim_{x\to 1^-} t(x) \approx 0.25$, therefore $\lim_{x\to 1} t(x) \approx 0.25$

b) $t(1) =\approx 0.25$

c) Since $\lim_{x\to 1} t(x) = t(1) \approx 0.25$, then $t(x)$ is continuous at $x = 1$

d) $\lim_{x\to -2^+} t(x) = \text{undefined}$, $\lim_{x\to -2^-} t(x) = \text{undefined}$, thus $\lim_{x\to -2} t(x)$ does not exists

e) $t(-2) = \text{undefined}$

f) Since $\lim_{x\to -2} t(x)$ does not exist, then the function is not continuous at $x = -2$

9. **a)** As we approach the x value of 1 from the right we find that the y value is approching 3. Thus $\lim_{x\to 1^+} f(x) = 3$

b) As we approach the x value of 1 from the left, we find that the y value is approching 3. Thus $\lim_{x\to 1^-} f(x) = 3$

c) Since $\lim_{x\to 1^+} f(x) = 3$ and $\lim_{x\to 1^-} f(x) = 3$ then $\lim_{x\to 1} f(x) = 3$

d) From the given conditions $f(1) = 2$

e) $f(x)$ is not continuous at $x = 1$ since $\lim_{x\to 3} f(x) \neq f(1)$

f) $f(x)$ is continuous at $x = 2$ since $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = 2 = f(2)$

10. **a)** $\lim_{x \to -2^+} f(x) = 0$

b) $\lim_{x \to -2^+} f(x) = 0$

c) $\lim_{x \to -2} f(x) = 0$

d) $f(-2) = 3$

e) $f(x)$ is not continuous at $x = -2$

f) $f(x)$ is continuous at $x = 1$

11. **a)** True. The values of y as we approch $x = 0$ from the right is the same as the value of the function at $x = 0$, which is 0

b) True. The values of y as we approch $x = 0$ from the left is the same as the value of the function at $x = 0$, which is 0

c) True. Since $\lim_{x \to 0^+} f(x) = 0$ and $\lim_{x \to 0^-} f(x) = 0$

d) False. Since $\lim_{x \to 3^+} f(x) = 3$ and $\lim_{x \to 3^-} f(x) = 1$

e) True. Since $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 0$

f) False. Since $\lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x)$

g) True. Since $\lim_{x \to 0} f(x) = 0 = f(0)$

h) False. Since $\lim_{x \to 3} f(x)$ does not exist

12. **a)** True. Since $\lim_{x \to 2^+} g(x) = 1 = g(2)$

b) False. Since $\lim_{x \to 2^-} g(x) = -1 \neq 1 = g(2)$

c) False. Since $\lim_{x \to 2^+} g(x) = 1$ and $\lim_{x \to 2^-} g(x) = -1$

d) False. Since $\lim_{x \to 2^+} g(x) \neq \lim_{x \to 2^-} g(x)$

e) False. Since $\lim_{x \to 2} g(x)$ does not exist

13. **a)** False. As we approach the x value of -2 from the right we find that the y value is approching 2.

b) True. As we approach the x value of -2 from the left, we find that the y value is approching 0.

c) False. Since $\lim_{x \to -2^+} f(x) = 1$ and $\lim_{x \to 1^-} f(x) = 0$

d) False. Since $\lim_{x \to -2^+} f(x) \neq \lim_{x \to -2^-} f(x)$

e) False. Since $\lim_{x \to -2} f(x)$ does not exist

f) True. Since $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 0$

g) True. The graph indicate a point (solid dot) at $(0, 2)$

h) False. Since $\lim_{x \to -2^+} f(x) \neq \lim_{x \to -2^-} f(x)$

i) False. Since $\lim_{x \to 0} f(x) \neq f(0)$

j) True. Since $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x) = f(-1)$

14. **a)** False. Since $\lim_{x \to 2^-} f(x) = 0$

b) False. Since $\lim_{x \to 2^+} f(x) = 3$

c) False.

d) False. Since $\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$

e) True. Since $\lim_{x \to 4^+} f(x) \neq \lim_{x \to 4^-} f(x) = 3$

f) False. Since $\lim_{x \to 4} f(x) = 3$ and $f(4) = -1$

g) False. Since $\lim_{x \to 4} f(x) \neq f(4)$

h) True. Since $\lim_{x \to 0} f(x) = 4$ and $f(0) = 4$

i) True. Since $\lim_{x \to 3} f(x) = 3 = \lim_{x \to 5} f(x)$

j) False. Since $\lim_{x \to 2} f(x)$ does not exist

15. **a)** True. As we approach the x value of 0 from the right we find that the y value is approching 0, which is the value of the function at $x = 0$.

b) False. As we approach the x value of 0 from the left, we find that the y value is approching 2 instead of 0.

c) False. Since $\lim_{x \to 0^+} f(x) = 0$ and $\lim_{x \to 0^-} f(x) = 2$

d) True. Since $\lim_{x \to 2^+} f(x) = 4 = \lim_{x \to 2^-} f(x)$

e) False. Since $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

f) True. Since $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = 4$

g) False. Since $\lim_{x \to 0} f(x)$ does not exist

h) True. Since $\lim_{x \to 2} f(x) = 4 = f(2)$

16. **a)** True. $\lim_{x \to 0^+} g(x) = 0 = g(0)$

b) True. $\lim_{x \to 0^-} g(x) = 0 = g(0)$

c) True.

d) True. $\lim_{x \to 0} g(x) = 0$

e) True. $\lim_{x \to 0} g(x) = 0 = g(0)$

17. The function p is not continuous at $x = 1$ since the $\lim_{x \to 1} p(x)$ does not exist. p is continuous at $x = 1.5$ since $\lim_{x \to 1.5} p(x) = 0.6 = p(1.5)$. p is not continuous at $x = 1$ since the $\lim_{x \to 2} p(x)$ does not exist. p is continuous at $x = 2.01$ since $\lim_{x \to 2.01} p(x) = 0.8 = p(2.01)$.

18. p is continuous at $x = 2.99$ since $\lim_{x \to 2.99} p(x) = 0.8 = p(2.99)$. p is not continuous at $x = 3$ since the $\lim_{x \to 3} p(x)$ does not exist. p is continuous at $x = 3.04$ since $\lim_{x \to 3.04} p(x) = 1 = p(3.04)$. p is not continuous at $x = 4$ since the $\lim_{x \to 4} p(x)$ does not exist.

19. $\lim_{x \to 1^-} p(x) = 0.4$, $\lim_{x \to 1^+} p(x) = 0.6$, therefore $\lim_{x \to 1} p(x)$ does not exist

20. $\lim_{x \to 1^-} p(x) = 0.6$, $\lim_{x \to 2^+} p(x) = 0.8$, therefore $\lim_{x \to 2} p(x)$ does not exist

21. $\lim_{x \to 2.6^-} p(x) = 0.8$, $\lim_{x \to 2.6^+} p(x) = 0.8$, therefore $\lim_{x \to 2.6} p(x) = 0.8$

22. $\lim_{x \to 3} p(x)$ does not exist since $\lim_{x \to 3^-} p(x) = 0.8$ and $\lim_{x \to 3^+} p(x) = 1$

23. $\lim_{x \to 3.4} p(x) = 1$ since $\lim_{x \to 3.4^-} p(x) = 1$, $\lim_{x \to 3.4^+} p(x) = 1$

24. C is continuous at $x = 0.1$ since $\lim_{x\to 0.1} C(x) = 2.3 = C(0.1)$. C is not continuous at $x = 0.2$ since the $\lim_{x\to 0.2} C(x)$ does not exist. C is continuous at $x = 0.25$ since $\lim_{x\to 0.25} C(x) = 3 = C(0.25)$. C is continuous at $x = 0.267$ since $\lim_{x\to 0.267} C(x) = 3 = C(0.267)$.

25. If we continue the pattern used for the taxi fare function, we see that for $x = 2.3$, which falls in the range of 2.2 and 2.4 miles, the fare will be \$5.60, for $x = 5$, which falls between the range 2.4 and 2.6 miles, the fare is \$5.90. For $x = 2.6$ and $x = 3$ we need to be careful since they act as a boundary of two possible fares. Therefore C is continuous at $x = 2.3$ since $\lim_{x\to 2.3} C(x) = 5.60 = C(2.3)$. C is continuous at $x = 2.5$ since $\lim_{x\to 2.5} C(x) = 5.90 = C(2.5)$. C is not continuous at $x = 2.6$ since $\lim_{x\to 2.6^-} C(x) = 5.90$ and $\lim_{x\to 2.6^+} C(x) = 6.20$ thus, $\lim_{x\to 2.6} C(x)$ does not exist. C is not continuous at $x = 3$ since $\lim_{x\to 3^-} C(x)6.50$ and $\lim_{x\to 3^+} C(x) = 6.80$ thus, $\lim_{x\to 3} C(x)$ does not exist.

26. $\lim_{x\to 1/4^-} C(x) = \$2.60$, $\lim_{x\to 1/4^+} C(x) = \$2.60$, therefore $\lim_{x\to 1/4} C(x) = \$2.60$

27. $\lim_{x\to 0.2^-} C(x) = \$2.30$, $\lim_{x\to 0.2^+} C(x) = \$2.60$, therefore $\lim_{x\to 0.2} C(x)$ does not exist

28. $\lim_{x\to 0.6^-} C(x) = \$2.90$, $\lim_{x\to 0.6^+} C(x) = \$3.20$, therefore $\lim_{x\to 0.6} C(x)$ does not exist

29. $\lim_{x\to 0.5^-} C(x) = \$2.90$, $\lim_{x\to 0.5^+} C(x) = \$2.90$, therefore $\lim_{x\to 0.5} C(x) = \$2.90$

30. $\lim_{x\to 0.4^-} C(x) = \$2.60$, $\lim_{x\to 0.4^+} C(x) = \$2.90$, therefore $\lim_{x\to 0.4} C(x)$ does not exist

31. The population function, $p(t)$, is discontinuous at $t^* = 0.5$, $t^* = 0.75$, $t^* = 1.25$, $t^* = 1.5$, and at $t^* = 1.75$ since at these points the population function has a "jump" which means that the $\lim_{t\to t^*} p(t)$ does not exist

32. There was a jump in the population at $t = 0.5$, $t = 0.75$, $t = 1.5$, and $t = 1.75$ due to births. While there was a decline in the populaion at $t = 1.25$ due to deaths.

33. $\lim_{t\to 1.5^+} p(t) = 12$

34. $\lim_{t\to 1.5^-} p(t) = 11$

35. The population function, $p(t)$, is discontinuous at $t^* = 0.1$, $t^* = 0.3$, $t^* = 0.4$, $t^* = 0.5$, $t^* = 0.6$, and at $t^* = 0.8$ since at these points the population function has a "jump" which means that the $\lim_{t\to t^*} p(t)$ does not exist

36. There was a jump in the population at $t = 0.1$, $t = 0.4$, $t = 0.5$, and $t = 0.6$ due to births. While there was a decline in the populaion at $t = 0.3$ and $t = 0.8$ due to deaths.

37. $\lim_{t\to 0.6^+} p(t) = 35$

38. $\lim_{t\to 0.6^-} p(t) = 33$

39. From the graph, the "I've got it" experience seems to occur after spending 20 hours on the task.

40. Once the task mastered one should get 100 correct trials out of 100 trials.

41. $\lim_{t\to 20^+} N(t) = 100$, $\lim_{t\to 20^-} N(t) = 30$, therefore $\lim_{t\to} N(t)$ does not exist

42. $\lim_{t\to 30^-} N(t) = 100$, $\lim_{t\to 30^+} N(t) = 100$, therefore $\lim_{t\to 30} N(t) = 100$

43. $N(t)$ is discontinuous at $t = 20$ since $\lim_{t\to 20} N(t)$ does not exist. $N(t)$ is continuous at $t = 30$ since $\lim_{t\to 30} N(t) = 100 = N(30)$

44. $N(t)$ is discontinuous at $t = 10$ since $\lim_{t\to 10} N(t)$ does not exist. $N(t)$ is continuous at $t = 26$ since $\lim_{t\to 26} N(t) = 100 = N(26)$

45. A function may not be continuous if the function is not defined at one of the points in the domain, it also may not be continuous if the limit at a point does not exist, it also may not be continuous if the limit at a point is different than the value of the function at that point.
NOTE: See the graphs on page 77.

46. $f(x)$ is continuous by C1 and C2, $\lim_{x\to 3} f(x) = 19$

47. $f(x)$ is continuous by C1 and C2, $\lim_{x\to 1} f(x) = 0$

48. $g(x)$ is continuous by C4, $\lim_{x\to -1} g(x) = \dfrac{1}{2}$

49. $g(x)$ is continuous by C4, $\lim_{x\to 1} g(x) = 1$

50. $tan\ x$ is continuous by C5, $\lim_{x\to \frac{\pi}{4}} tan\ x = 1$

51. $cot\ x$ is continuous by C5, $\lim_{x\to \frac{\pi}{3}} cot\ x = \dfrac{1}{\sqrt{3}}$

52. $sec\ x$ is continuous by C5, $\lim_{x\to \frac{\pi}{6}} sec\ x = \dfrac{2}{\sqrt{3}}$

53. $csc\ x$ is continuous by C5, $\lim_{x\to \frac{\pi}{4}} csc\ x = \sqrt{2}$

54. $f(x)$ is continuous by C5, $\lim_{x\to 3} f(x) = \sqrt{19}$

55. $f(x)$ is continuous by C5, $\lim_{x\to \frac{\pi}{3}} f(x) = \dfrac{\sqrt[4]{3}}{\sqrt{2}}$

56. $g(x)$ is continuous by C3, $\lim_{x\to \frac{\pi}{4}} f(x) = \dfrac{1}{2}$

57. $g(x)$ is continuous by C5, $ds\lim_{x\to \frac{\pi}{6}} g(x) = -\frac{1}{2}$

58. Limit approaches 0.92857

59. Limit approaches 0

60. Limit approaches 0

61. Limit approaches 1

62. Limit approaches 0

63. Limit does not exist

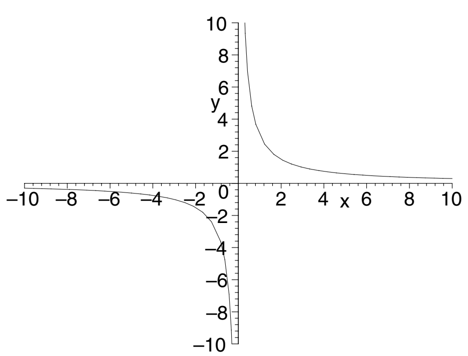
Exercise Set 2.2

1. $x^2 - 3$ is a continuous function (it is a polynomial). Therefore, we can use direct substitution

$$\begin{aligned}\lim_{x\to 1}(x^2-3) &= (1)^2-3\\ &= 1-3\\ &= -2\end{aligned}$$

2. $\lim_{x\to 1}(x^2+4) = (1)^2+4 = 1+4 = 5$

3. The function $f(x) = \frac{3}{x}$ is not continuous at $x - 0$ since the denominator equals zero. There are no algebraic simplifications that can be done to the function. To find the limit, we can either plug points that are approaching 0 from the right and the left and detrmine the limit from each side, or we can use the graph of the function to determine the limit (if it exists). Looking at the graph, we see that as x approches 0 from the left the y values are becoming more and more negative, and as x approches 0 from the right, the y values are becoming more and more positive. Therefore, since $\lim_{x\to 0^+}\frac{3}{x} \neq \lim_{x\to 0^-}\frac{3}{x}$ then $\lim_{x\to 0}\frac{3}{x}$ does not exist.



4. $\lim_{x\to 0^+}\frac{-4}{x}$ does not exist since the limit from the left of $x = 0$ does not equal the limit from the right of $x = 0$

5. $2x+5$ is a continuous function (it is a polynomial). Therefore, we can use direct substitution

$$\begin{aligned}\lim_{x\to 3}(2x+5) &= 2(3)+5\\ &= 6+5\\ &= 11\end{aligned}$$

6. $\lim_{x\to 4}(5-3x) = 5-3(4) = 5-12 = -7$

7. The function $\frac{x^2-25}{x+5}$ is discontinuous at $x = -5$, but it can be simplified algebraically.

$$\frac{x^2-25}{x+5} = \frac{(x-5)(x+5)}{x+5} = x-5$$

Therfore, $\lim_{x\to -5}\frac{x^2-25}{x+5} = \lim_{x\to -5}(x-5) = -5-5 = -10$

8. $\lim_{x\to -4}\frac{x^2-16}{x+4} = \frac{(x-4)(x+4)}{x+4} = \lim_{x\to -4}(x-4)$
$= -4-4 = -8$

9. Since $\frac{5}{x}$ is continuous at $x = -2$ we can use direct substitution.
$\lim_{x\to -2}\frac{5}{x} = \frac{5}{-2} = -\frac{5}{2}$

10. $\lim_{x\to -5}\frac{-2}{x} = \frac{-2}{-5} = \frac{2}{5}$

11. The function $\frac{x^2+x-6}{x-2}$ is discontinuous at $x = 2$, but it can be simplified algebraically. The limit is then computed as follows:

$$\begin{aligned}\lim_{x\to 2}\frac{x^2+x-6}{x-2} &= \lim_{x\to 2}\frac{(x-2)(x+3)}{x-2}\\ &= \lim_{x\to 2}(x+3)\\ &= 2+3\\ &= 5\end{aligned}$$

12.

$$\begin{aligned}\lim_{x\to -4}\frac{x^2-x-20}{x+4} &= \lim_{x\to -4}\frac{(x-5)(x+4)}{x+4}\\ &= \lim_{x\to -4}(x-5)\\ &= -4-5\\ &= -9\end{aligned}$$

13. Since $\sqrt[3]{x^2-17}$ is continuous at $x = 5$ we can use direct substitution

$$\begin{aligned}\lim_{x\to 5}\sqrt[3]{x^2-17} &= \sqrt[3]{5^2-17}\\ &= \sqrt[3]{25-17}\\ &= \sqrt[3]{8}\\ &= 2\end{aligned}$$

14. $\lim_{x\to 2}\sqrt{x^2+5} = \sqrt{2^2+5} = \sqrt{4+5} = \sqrt{9} = 3$

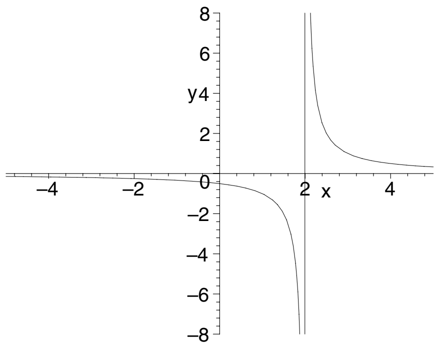
15. $\lim_{x\to \frac{\pi}{4}}(x + sin\ x) = \frac{\pi}{4} + sin\ \frac{\pi}{4} = \frac{\pi}{4} + \frac{1}{\sqrt{2}}$

16. $\lim_{x\to \frac{\pi}{6}}(cos\ x + tan\ x) = \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}$

17. $\lim_{x\to 0}\frac{1+sin\ x}{1-sin\ x} = \frac{1+0}{1-0} = 1$

18. $\lim_{x\to 0}\frac{1+cos\ x}{cos\ x} = \frac{1+1}{1} = 2$

19. Using the graph of $\frac{1}{x-2}$ we find that the limit as x approaches 2 does not exist since the limit from the left of $x = 2$ does not equal the limit from the right of $x = 2$



20. The limit to the left of $x = 1$ is ∞ and the limit to the right of $x = 1$ is ∞. Thus, $\lim_{x\to 1} \frac{1}{(x-1)^2}$ does not exist

21. Since $\frac{3x^2-4x+2}{7x^2-5x+3}$ is continuous at $x = 2$ we can use direct substitution

$$\begin{aligned}\lim_{x\to 2} \frac{3x^2-4x+2}{7x^2-5x+3} &= \frac{3(2)^2-4(2)+2}{7(2)^2-5(2)+3} \\ &= \frac{12-8+2}{28-10+3} \\ &= \frac{6}{21} \\ &= \frac{2}{7}\end{aligned}$$

22.

$$\begin{aligned}\lim_{x\to -1} \frac{4x^2+5x-7}{3x^2-2x+1} &= \frac{4(-1)^2+5(-1)-7}{3(-1)^2-2(-1)+1} \\ &= \frac{4-5-7}{3+2+1} \\ &= \frac{-8}{6} \\ &= -\frac{4}{3}\end{aligned}$$

23. The function $\frac{x^2+x-6}{x^2-4}$ is discontinuous at $x = 2$. But we can simplify it algebraically first then find the limit as follows

$$\begin{aligned}\lim_{x\to 2} \frac{x^2+x-6}{x^2-4} &= \lim_{x\to 2} \frac{(x-2)(x+3)}{(x-2)(x+2)} \\ &= \lim_{x\to 2} \frac{(x+3)}{(x+2)} \\ &= \frac{2+3}{2+2} \\ &= \frac{5}{4}\end{aligned}$$

24.

$$\begin{aligned}\lim_{x\to 4} \frac{x^2-16}{x^2-x-12} &= \lim_{x\to 4} \frac{(x-4)(x+4)}{(x-4)(x+3)} \\ &= \lim_{x\to 2} \frac{(x+4)}{(x+3)} \\ &= \frac{4+4}{4+3} \\ &= \frac{8}{7}\end{aligned}$$

25. Since we have a limit in terms of h, we can treat x as a constant. To evaluate the limit we can use direct substitution (we have a polynomial in h, which is continuous for all values of h).
$\lim_{h\to 0}(6x^2 + 6xh + 2h^2)$
$= 6x^2 + 6x(0) + 2(0)^2$
$= 6x^2 + 0 + 0$
$= 6x^2$

26. $\lim_{h\to 0}(10x + 5h) = 10x + 5(0) = 10x$

27. Since we have a limit in terms of h, we can treat x as a constant. Since $\frac{-2x-h}{x^2(x+h)^2}$ is continuous at $h = 0$ we can use direct substitution

$$\begin{aligned}\lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} &= \frac{-2x-0}{x^2(x+0)^2} \\ &= \frac{-2x}{x^2(x)^2} \\ &= \frac{-2x}{x^4} \\ &= \frac{-2}{x^3}\end{aligned}$$

28. $\lim_{h\to 0} \frac{-5}{x(x+h)} = \frac{-5}{x(x+0)} = \frac{-5}{x^2}$

29. $\lim_{x\to 0} \frac{tan\ x}{x} = \lim_{x\to 0} \frac{sin\ x}{x}\ cos\ x = 1 \cdot 1 = 1$ Recall that $\lim_{x\to 0} \frac{sin\ x}{x} = 1$.

30. $\lim_{x\to 0} xcsc\ x = \lim_{x\to 0} \frac{x}{sin\ x} = \frac{1}{1} = 1$

31. $\lim h \to 0 \frac{sin\ x\ sin\ h}{h} = sin\ x \lim_{h\to 0} \frac{sin\ h}{h}$
$= sin\ x \cdot 1 = sin\ x$

32. $\lim_{h\to 0} \frac{sin\ x(cos\ h-1)}{h} = sin\ x \lim_{h\to 0} \frac{cos\ h-1}{h} = sin\ x \cdot 0 = 0$

33.

$$\begin{aligned}\lim_{x\to 0} \frac{x^2+3x}{x-2x^4} &= \lim_{x\to 0} \frac{x(x+3)}{x(1-2x^3)} \\ &= \lim_{x\to 0} \frac{(x+3)}{(1-2x)} \\ &= \frac{(0+3)}{(1-0)} \\ &= \frac{3}{1} = 3\end{aligned}$$

34. $\lim_{x\to 0} \frac{x^2-2x}{x^2+3x} = \lim_{x\to 0} \frac{x(1-2x^2)}{(x+3)} = \frac{0}{3} = 0$

35.

$$\begin{aligned}\lim_{x\to 0} \frac{x\sqrt{x}}{x+x^2} &= \lim_{x\to 0} \frac{x\sqrt{x}}{x(1+x)} \\ &= \lim_{x\to 0} \frac{\sqrt{x}}{(1+x)} \\ &= \frac{\sqrt{0}}{(1+0)} = 0\end{aligned}$$

36. $\lim_{x\to 0}\dfrac{x+x^2}{x\sqrt{x}} = \lim_{x\to 0}\dfrac{x(1+x)}{x\sqrt{x}}$
$= \lim_{x\to 0}\dfrac{1+x}{\sqrt{x}} = \dfrac{1}{\sqrt{x}} + \sqrt{x}$ The limit does not exist

37.

$$\begin{aligned}\lim_{x\to 2}\frac{x-2}{x^2-x-2} &= \lim_{x\to 2}\frac{x-2}{(x-2)(x+1)}\\ &= \lim_{x\to 2}\frac{1}{(x+1)}\\ &= \frac{1}{(2+1)} = \frac{1}{3}\end{aligned}$$

38.

$$\begin{aligned}\lim_{x\to -1}\frac{x^2-1}{x+1} &= \lim_{x\to 1}\frac{(x-1)(x+1)}{x+1}\\ &= \lim_{x\to -1}(x-1)\\ &= (-1-1) = -2\end{aligned}$$

39.

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-9}{2x-6} &= \lim_{x\to 3}\frac{(x-3)(x+3)}{2(x-3)}\\ &= \lim_{x\to 3}\frac{(x+3)}{2}\\ &= \frac{(3+3)}{2}\\ &= \frac{6}{2} = 3\end{aligned}$$

40.

$$\begin{aligned}\lim_{x\to -2}\frac{3x^2+5x-2}{x^2-3x-10} &= \lim_{x\to -2}\frac{(x+2)(3x-1)}{(x+2)(x-5)}\\ &= \lim_{x\to -2}\frac{(3x-1)}{(x-5)}\\ &= \frac{(3(-2)-1)}{(-2-5)}\\ &= \frac{-7}{-7} = 1\end{aligned}$$

41.

$$\begin{aligned}\frac{a^2-4}{\sqrt{a^2+5}-3} &= \frac{a^2-4}{\sqrt{a^2+5}-3}\cdot\frac{\sqrt{a^2+5}+3}{\sqrt{a^2+5}+3}\\ &= \frac{(a^2-4)(\sqrt{a^2+5}+3)}{(a^2+5-9)}\\ &= \frac{(a^2-4)(\sqrt{a^2+5}+3)}{(a^2-4)}\\ &= \sqrt{a^2+5}+3\end{aligned}$$

Thus, $\lim_{a\to -2}(\sqrt{a^2+5}+3) = 6$

42.

$$\begin{aligned}\frac{\sqrt{x}-1}{x-1} &= \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}\\ &= \frac{1}{\sqrt{x}+1}\end{aligned}$$

Thus, $\lim_{x\to 1}\dfrac{1}{\sqrt{x}+1} = \dfrac{1}{\sqrt{2}}$

43.

$$\begin{aligned}\frac{\sqrt{3-x}-\sqrt{3}}{x} &= \frac{\sqrt{3-x}-\sqrt{3}}{x}\cdot\frac{\sqrt{3-x}+\sqrt{3}}{\sqrt{3-x}+\sqrt{3}}\\ &= \frac{3-x-3}{x(\sqrt{3-x}+\sqrt{3})}\\ &= \frac{-1}{\sqrt{3-x}+\sqrt{3}}\end{aligned}$$

Thus, $\lim_{x\to 0}\dfrac{-1}{\sqrt{3-x}+\sqrt{3}} = \dfrac{-1}{2\sqrt{3}}$

44.

$$\begin{aligned}\frac{\sqrt{4+x}-\sqrt{4-x}}{x} &= \frac{\sqrt{4+x}-\sqrt{4-x}}{x}\cdot\frac{\sqrt{4+x}+\sqrt{4-x}}{\sqrt{4+x}+\sqrt{4-x}}\\ &= \frac{4+x-(4-x)}{x(\sqrt{4+x}+\sqrt{4-x})}\\ &= \frac{2}{\sqrt{4+x}+\sqrt{4-x}}\end{aligned}$$

Thus, $\lim_{x\to 0}\dfrac{2}{\sqrt{4+x}+\sqrt{4-x}} = \dfrac{1}{2}$

45. Limit approaches $\frac{3}{4}$

46.

$$\begin{aligned}\frac{\sqrt{7+2x}-\sqrt{7}}{x} &= \frac{\sqrt{7+2x}-\sqrt{7}}{x}\cdot\frac{\sqrt{7+2x}+\sqrt{7}}{\sqrt{7+2x}+\sqrt{7}}\\ &= \frac{7+2x-7}{x(\sqrt{7=2x}+\sqrt{7})}\\ &= \frac{2}{\sqrt{3-x}+\sqrt{3}}\end{aligned}$$

Thus, $\lim_{x\to 0}\dfrac{2}{\sqrt{7+2x}+\sqrt{7}} = \dfrac{1}{\sqrt{7}}$

47.

$$\begin{aligned}\frac{2-\sqrt{x}}{4-x} &= \frac{2-\sqrt{x}}{4-x}\cdot\frac{2+\sqrt{x}}{2+\sqrt{x}}\\ &= \frac{4-x}{(4-x)(2+\sqrt{x})}\\ &= \frac{1}{2+\sqrt{x}}\end{aligned}$$

Thus, $\lim_{x\to 4}\dfrac{1}{2+\sqrt{x}} = \dfrac{1}{4}$

48.

$$\begin{aligned}\frac{7-\sqrt{49-x^2}}{x} &= \frac{7-\sqrt{49-x^2}}{x}\cdot\frac{7+\sqrt{49-x^2}}{7+\sqrt{49-x^2}}\\ &= \frac{49-(49-x^2)}{x(7+\sqrt{49-x^2})}\\ &= \frac{x}{7+\sqrt{49-x^2}}\end{aligned}$$

Thus, $\lim_{x\to 0}\dfrac{x}{7+\sqrt{49-x^2}} = 0$

Exercise Set 2.3

1. **a)** First we obtain the expression for $f(x+h)$ with $f(x)=7x^2$

$$\begin{aligned} f(x+h) &= 7(x+h)^2 \\ &= 7(x^2+2xh+h^2) \\ &= 7x^2+14xh+7h^2 \end{aligned}$$

Then

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(7x^2+14xh+7h^2)-7x^2}{h} \\ &= \frac{14xh+7h^2}{h} \\ &= \frac{h(14x+7h)}{h} \\ &= 14x+7h \end{aligned}$$

b) For $x=4$ and $h=2$,

$$14x+7h = 14(4)+7(2) = 56+14 = 70$$

For $x=4$ and $h=1$,

$$14x+7h = 14(4)+7(1) = 56+7 = 63$$

For $x=4$ and $h=0.1$,

$$14x+7h = 14(4)+7(0.1) = 56+0.7 = 56.7$$

For $x=4$ and $h=0.01$,

$$14x+7h = 14(4)+7(0.01) = 56+0.07 = 56.07$$

2. **a)**

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(5x^2+10xh+5h^2)-5x^2}{h} \\ &= \frac{10xh+5h^2}{h} \\ &= \frac{h(10x+5h)}{h} \\ &= 10x+5h \end{aligned}$$

b) For $x=4$ and $h=2$,

$$10x+5h = 10(4)+5(2) = 40+10 = 50$$

For $x=4$ and $h=1$,

$$10x+5h = 10(4)+5(1) = 40+5 = 45$$

For $x=4$ and $h=0.1$,

$$10x+5h = 10(4)+5(0.1) = 40+0.5 = 40.5$$

For $x=4$ and $h=0.01$,

$$10x+5h = 10(4)+5(0.01) = 40+0.05 = 40.05$$

3. **a)** First we obtain the expression for $f(x+h)$ with $f(x)=-7x^2$

$$\begin{aligned} f(x+h) &= -7(x+h)^2 \\ &= -7(x^2+2xh+h^2) \\ &= -7x^2-14xh-7h^2 \end{aligned}$$

Then

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(-7x^2-14xh-7h^2)-(-7x^2)}{h} \\ &= \frac{-14xh-7h^2}{h} \\ &= \frac{h(-14x-7h)}{h} \\ &= -14x-7h \end{aligned}$$

b) For $x=4$ and $h=2$,

$$-14x-7h = -14(4)-7(2) = -56-14 = -70$$

For $x=4$ and $h=1$,

$$-14x-7h = -14(4)-7(1) = -56-7 = -63$$

For $x=4$ and $h=0.1$,

$$-14x-7h = -14(4)-7(0.1) = -56-0.7 = -56.7$$

For $x=4$ and $h=0.01$,

$$-14x-7h = -14(4)-7(0.01) = -56-0.07 = -56.07$$

4. **a)**

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(-5x^2-10xh-5h^2)-(-5x^2)}{h} \\ &= \frac{-10xh-5h^2}{h} \\ &= \frac{h(-10x-5h)}{h} \\ &= -10x-5h \end{aligned}$$

b) For $x=4$ and $h=2$,

$$-10x-5h = -10(4)-5(2) = -40-10 = -50$$

For $x=4$ and $h=1$,

$$-10x-5h = -10(4)-5(1) = -40-5 = -45$$

For $x=4$ and $h=0.1$,

$$-10x-5h = -10(4)-5(0.1) = -40-0.5 = -40.5$$

For $x=4$ and $h=0.01$,

$$-10x-5h = -10(4)-5(0.01) = -40-0.05 = -40.05$$

5. **a)** First we obtain the expression for $f(x+h)$ with $f(x)=7x^3$

$$\begin{aligned} f(x+h) &= 7(x+h)^3 \\ &= 7(x^3+3x^2h+3xh^2+h^3) \\ &= 7x^3+21x^2h+21xh^2+7h^3 \end{aligned}$$

Then

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(7x^3+21x^2h+21xh^2+7h^3)-7x^3}{h}\\ &= \frac{21x^2h+21xh^2+7h^3}{h}\\ &= \frac{h(21x^2+21xh+7h^2}{h}\\ &= 21x^2+21xh+7h^2\end{aligned}$$

b) For $x=4$ and $h=2$,

$$\begin{aligned}21x^2+21xh+7h^2 &= 21(4)^2+21(4)(2)+7(2)^2\\ &= 336+168+28\\ &= 532\end{aligned}$$

For $x=4$ and $h=1$,

$$\begin{aligned}21x^2+21xh+7h^2 &= 21(4)^2+21(4)(1)+7(1)^2\\ &= 336+84+7\\ &= 427\end{aligned}$$

For $x=4$ and $h=0.1$,

$$\begin{aligned}21x^2+21xh+7h^2 &= 21(4)^2+21(4)(0.1)^2\\ &= 336+8.4+0.07\\ &= 344.47\end{aligned}$$

For $x=4$ and $h=0.01$,

$$\begin{aligned}21x^2+21xh+7h^2 &= 21(4)^2+21(4)(0.01)+7(0.01)^2\\ &= 336+0.84+0.0007\\ &= 336.8407\end{aligned}$$

6. a)

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(5x^3+15x^2h+15xh^2+5h^3)-5x^3}{h}\\ &= \frac{15x^2h+15xh^2+5h^3}{h}\\ &= \frac{h(15x^2+15xh+5h^2}{h}\\ &= 15x^2+15xh+5h^2\end{aligned}$$

b) For $x=4$ and $h=2$,

$$\begin{aligned}15x^2+15xh+5h^2 &= 15(4)^2+15(4)(2)+5(2)^2\\ &= 240+120+20\\ &= 380\end{aligned}$$

For $x=4$ and $h=1$,

$$\begin{aligned}15x^2+15xh+5h^2 &= 15(4)^2+15(4)(1)+5(1)^2\\ &= 240+60+5\\ &= 305\end{aligned}$$

For $x=4$ and $h=0.1$,

$$\begin{aligned}15x^2+15xh+5h^2 &= 15(4)^2+15(4)(0.1)+5(0.1)^2\\ &= 240+6+0.05\\ &= 246.05\end{aligned}$$

For $x=4$ and $h=0.01$,

$$\begin{aligned}15x^2+15xh+5h^2 &= 15(4)^2+15(4)(0.01)+5(0.01)^2\\ &= 240+.6+0.0005\\ &= 240.6005\end{aligned}$$

7. a) First we obtain the expression for $f(x+h)$ with $f(x)=\frac{5}{x}$

$$f(x+h) = \frac{5}{(x+h)}$$

Then

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{5}{(x+h)}-\frac{5}{x}}{h}\\ &= \frac{\frac{5}{(x+h)}\cdot x(x+h)-\frac{5}{x}\cdot x(x+h)}{\frac{h}{1}\cdot x(x+h)}\\ &= \frac{5x-5(x+h)}{hx(x+h)}\\ &= \frac{-5h}{hx(x+h)}\\ &= \frac{-5}{x(x+h)}\end{aligned}$$

b) For $x=4$ and $h=2$,

$$\frac{-5}{x(x+h)}=\frac{-5}{4(4+2)}=\frac{-5}{26}\approx -0.208$$

For $x=4$ and $h=1$,

$$\frac{-5}{x(x+h)}=\frac{-5}{4(4+1)}=\frac{-5}{20}\approx -0.25$$

For $x=4$ and $h=0.1$,

$$\frac{-5}{x(x+h)}=\frac{-5}{4(4+0.1)}=\frac{-5}{16.4}\approx -0.305$$

For $x=4$ and $h=0.01$,

$$\frac{-5}{x(x+h)}=\frac{-5}{4(4+0.01)}=\frac{-5}{16.04}\approx -0.312$$

8. a)

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{4}{(x+h)}-\frac{4}{x}}{h}\\ &= \frac{\frac{4}{(x+h)}\cdot x(x+h)-\frac{4}{x}\cdot x(x+h)}{\frac{h}{1}\cdot x(x+h)}\\ &= \frac{4x-4(x+h)}{hx(x+h)}\\ &= \frac{-4h}{hx(x+h)}\\ &= \frac{-4}{x(x+h)}\end{aligned}$$

b) For $x = 4$ and $h = 2$,

$$\frac{-4}{x(x+h)} = \frac{-4}{4(4+2)} = \frac{-1}{6} \approx -0.167$$

For $x = 4$ and $h = 1$,

$$\frac{-4}{x(x+h)} = \frac{-4}{4(4+1)} = \frac{-1}{5} \approx -0.2$$

For $x = 4$ and $h = 0.1$,

$$\frac{-4}{x(x+h)} = \frac{-4}{4(4+0.1)} = \frac{-1}{4.1} \approx -0.244$$

For $x = 4$ and $h = 0.01$,

$$\frac{-4}{x(x+h)} = \frac{-4}{4(4+0.01)} = \frac{-4}{4.04} \approx -0.249$$

9. a) First we obtain the expression for $f(x+h)$ with $f(x) = -2x + 5$

$$\begin{aligned} f(x+h) &= -2(x+h)+5 \\ &= -2x-2h+5 \end{aligned}$$

Then

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(-2x-2h+5)-(-2x+5)}{h} \\ &= \frac{-2h}{h} \\ &= -2 \end{aligned}$$

b) Since the difference quotient is a constant, then the value of the difference quotient will be -2 for all the values of x and h.

10. a)

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(2x+2h+3)-(2x+3)}{h} \\ &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

b) Since the difference quotient is a constant, then the value of the difference quotient will be 2 for all the values of x and h.

11. a) First we obtain the expression for $f(x+h)$ with $f(x) = x^2 - x$

$$\begin{aligned} f(x+h) &= (x+h)^2-(x+h) \\ &= x^2 = 2xh + h^2 - x - h \end{aligned}$$

Then

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(x^2+2xh+h^2-x-h)-(x^2-x)}{h} \\ &= \frac{2xh+h^2-h}{h} \\ &= \frac{h(2x+h-1)}{h} \\ &= 2x+h-1 \end{aligned}$$

b) For $x = 4$ and $h = 2$,

$$2x+h-1 = 2(4)+2-1 = 8+2-1 = 9$$

For $x = 4$ and $h = 1$,

$$2x+h-1 = 2(4)+2-1 = 8+1-1 = 8$$

For $x = 4$ and $h = 0.1$,

$$2x+h-1 = 2(4)+0.1-1 = 8+0.1-1 = 7.1$$

For $x = 4$ and $h = 0.01$,

$$2x+h-1 = 2(4)+2-1 = 8+0.01-1 = 7.01$$

12. a)

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(x^2+2xh+h^2+x+h)-(x^2+x)}{h} \\ &= \frac{2xh+h^2+h}{h} \\ &= \frac{h(2x+h+1)}{h} \\ &= 2x+h-1 \end{aligned}$$

b) For $x = 4$ and $h = 2$,

$$2x+h+1 = 2(4)+2+1 = 8+2+1 = 11$$

For $x = 4$ and $h = 1$,

$$2x+h+1 = 2(4)+2+1 = 8+1+1 = 10$$

For $x = 4$ and $h = 0.1$,

$$2x+h+1 = 2(4)+0.1+1 = 8+0.1+1 = 9.1$$

For $x = 4$ and $h = 0.01$,

$$2x+h+1 = 2(4)+2+1 = 8+0.01+1 = 9.01$$

13. a) For the average growth rate during the first year we use the points $(0, 7.9)$ and $(12, 22.4)$

$$\begin{aligned} \frac{y_2-y_1}{x_2-x_1} &= \frac{22.4-7.9}{12-0} \\ &= \frac{14.5}{12} \\ &\approx 1.20834 \text{ pounds per month} \end{aligned}$$

b) For the average growth rate during the second year we use the points $(12, 22.4)$ and $(24, 27.8)$

$$\begin{aligned} \frac{y_2-y_1}{x_2-x_1} &= \frac{27.8-22.4}{24-12} \\ &= \frac{5.4}{12} \\ &= 0.45 \text{ pounds per month} \end{aligned}$$

c) For the average growth rate during the third year we use the points $(24, 27.8)$ and $(36, 31.5)$

$$\begin{aligned} \frac{y_2-y_1}{x_2-x_1} &= \frac{31.5-27.8}{36-24} \\ &= \frac{3.7}{12} \\ &\approx 0.30834 \text{ pounds per month} \end{aligned}$$

d) For the average growth rate during his first three years we use the points $(0, 7.9)$ and $(36, 31.5)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{31.5 - 7.9}{36 - 0} \\ &= \frac{23.6}{36} \\ &\approx 1.967 \text{ pounds per month}\end{aligned}$$

e) The graph indicates that the highest growth rate out of the first three years of a boy's life happens at birth (that is were the graph is the steepest).

14. a) $\frac{20.5-7.9}{9-0} = 1.4$ pounds per month

b) $\frac{17.4-7.9}{6-0} \approx 1.583$ pounds per month

c) $\frac{13.2-7.9}{3-0} \approx 1.767$ pounds per month

d) Based on the the calculated average growth rates above, we can estimate the average growth rate to be larger than 1.767 for the first few weeks of a typical boy's life.

15. a) For the average growth rate between ages 12 and 18 months

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{25.9 - 22.4}{18 - 12} \\ &= \frac{3.5}{6} \\ &\approx 0.583 \text{ pounds per month}\end{aligned}$$

b) For the average growth rate between ages 12 and 14 (we use the point at 15 months)

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{24.5 - 22.4}{15 - 12} \\ &= \frac{2.1}{3} \\ &= 0.7 \text{ pounds per month}\end{aligned}$$

c) For the average growth rate between ages 12 and 13 (we can approximate the value of y when $x = 13$ by reading it from the graph)

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &\approx \frac{23.23 - 22.4}{15 - 12} \\ &\approx \frac{0.83}{1} \\ &\approx 0.83 \text{ pounds per month}\end{aligned}$$

d) The average growth of a typical boy when he is 12 months old is about 0.9 pounds per month

16. a) $\frac{98.6-98.6}{10-1} = 0$ degrees per day. Using this rate of change we would not be able to conclude the person was sick.

b) $\frac{99.5-98.6}{2-1} = 0.9$ degrees per day
$\frac{101-99.5}{3-2} = 1.5$ degrees per day
$\frac{102-101}{4-3} = 1$ degrees per day
$\frac{102.5-102}{5-4} = 0.5$ degrees per day
$\frac{102.5-102.5}{6-5} = 0$ degrees per day
$\frac{102.4-102.5}{7-6} = -0.1$ degrees per day
$\frac{102-102.4}{8-7} = -0.4$ degrees per day
$\frac{100-102}{9-8} = -2$ degrees per day
$\frac{98.6-100}{10-9} = -1.4$ degrees per day
$\frac{98.6-98.6}{11-10} = 0$ degrees per day

c) The temperature began to rise on day 1, reached the peak on day 5, began to subside on day 6, and was back to normal on day 10

d) By examining the graph, we notice an increase in temperaure after day 1 which reaches a maximum value of 102.5^o on day 5 and continues at the max temp through day 6 at which time the temperature begins to drop until it reaches the normal level of 98.6^o on day 10.

17. a) Average rate of change from $t = 0$ to $t = 8$

$$\begin{aligned}\frac{N_2 - N_1}{t_2 - t_1} &= \frac{10 - 0}{8 - 0} \\ &= \frac{10}{8} = 1.25 \text{ words per minute}\end{aligned}$$

Average rate of change from $t = 8$ to $t = 16$

$$\begin{aligned}\frac{N_2 - N_1}{t_2 - t_1} &= \frac{20 - 10}{16 - 8} \\ &= \frac{10}{8} = 1.25 \text{ words per minute}\end{aligned}$$

Average rate of change from $t = 16$ to $t = 24$

$$\begin{aligned}\frac{N_2 - N_1}{t_2 - t_1} &= \frac{25 - 20}{24 - 16} \\ &= \frac{5}{8} = 0.625 \text{ words per minute}\end{aligned}$$

Average rate of change from $t = 24$ to $t = 32$

$$\begin{aligned}\frac{N_2 - N_1}{t_2 - t_1} &= \frac{25 - 25}{32 - 24} \\ &= \frac{0}{8} = 0 \text{ words per minute}\end{aligned}$$

Average rate of change from $t = 32$ to $t = 36$

$$\begin{aligned}\frac{N_2 - N_1}{t_2 - t_1} &= \frac{25 - 25}{36 - 32} \\ &= \frac{0}{4} = 0 \text{ words per minute}\end{aligned}$$

b) The rate of change becomes 0 after 24 minutes because the number of words memorized does not change and remains at 25 words, that means that there is no change in the number of words memorized after 24 minutes.

18. a) $s(2) = 10(2)^2 = 10(4) = 40$ miles.
$s(5) = 10(5)^2 = 10(25) = 250$ miles

b) $s(5) - s(2) = 250 - 40 = 210$ miles. This represents the distance travels between $t = 2$ and $t = 5$ seconds.

c) $\frac{250-40}{5-2} = \frac{210}{3} = 70$ miles per hour

19. a) When $t = 3$, $s = 16(3)^2 = 16(9) = 144$ feet

b) When $t = 5$, $s = 16(5)^2 = 16(25) = 400$ feet

c) Average velocity $= \frac{400-144}{5-3} = \frac{256}{2} = 128$ feet per second

20. **a)** $\frac{30970-30680}{20} = 14.5$ miles per gallon

b) $\frac{30970-30680}{20} = 14.5$ miles per gallon

21. **a)** Population A: The average growth rate $= \frac{500-0}{4-0} = \frac{500}{4} = 125$ million per year
Population B: The average growth rate $= \frac{500-0}{4-0} =$ 125 million per year

b) We would not detect the fact that the population grow at different rates. The calculation shows the populations growing at the same average growth rate, since for either population we used the same points to calculate the average growth rate $(0,0)$, and $(4,500)$.

c) Population A:
Between $t=0$ and $t=1$, Average Growth Rate $= \frac{290-0}{1-0} = 290$ million people per year
Between $t=1$ and $t=2$, Average Growth Rate $= \frac{250-290}{2-1} = -40$ million people per year
Between $t=2$ and $t=3$, Average Growth Rate $= \frac{200-250}{3-2} = -50$ million people per year
Between $t=3$ and $t=4$, Average Growth Rate $= \frac{500-200}{4-3} = 300$ million people per year

Population B:
Between $t=0$ and $t=1$, Average Growth Rate $= \frac{125-0}{1-0} = 125$ million people per year
Between $t=1$ and $t=2$, Average Growth Rate $= \frac{250-125}{2-1} = 125$ million people per year
Between $t=2$ and $t=3$, Average Growth Rate $= \frac{375-250}{3-2} = 125$ million people per year
Between $t=3$ and $t=4$, Average Growth Rate $= \frac{400-375}{4-3} = 125$ million people per year

d) It is clear from part (c) that the first population has different growing rates depending on which interval of time we choose. Therefore, the statement "the population grew by 125 million each year" does not convey how population went through periods were the population increased and periods were the population decreased.

22. In the period between 1850 and 1860 the deer population is decreasing which corresponds to a negative rate of change. In the period between 1890 and 1960 the deer population is increasing which corresponds to a positive rate of change.

23. The rate of change in the period between 1800 and 1850 is similar to that of 1930 to 1950 is the sense that they both exhibit steady increase in the population. The drastic drop in the population shortly after 1850 is similar to the drop in population seen near 1975.

24.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{m(x+h)+b-(mx+b)}{h} \\ &= \frac{mx+mh+b-mx-b}{h} \\ &= \frac{mh}{h} \\ &= m\end{aligned}$$

25.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{a(x+h)^2+b(x+h)+c}{h} - \\ &\quad \frac{(ax^2+bx+c)}{h} \\ &= \frac{ax^2+2axh+ah^2+bx+bh+c}{h} \\ &\quad \frac{-ax^2+bx+c}{h} \\ &= \frac{2axh+ah^2+bh}{h} \\ &= 2ax+ah+b \\ &= a(2x+h)+b\end{aligned}$$

26.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{a(x+h)^3+b(x+h)^2-(ax^3+bx^2)}{h} \\ &= \frac{ax^3+3ax^2h+3axh^2+h^3}{h} + \\ &\quad \frac{bx^2+2bxh+bh^2-ax^3-bx^2}{h} \\ &= \frac{3ax^2h+3axh^2+h^3+2bxh+bh^2}{h} \\ &= 3ax^2+3axh+h^2+2bx+bh \\ &= (3ax^2+2bx)+h(3ax+b)\end{aligned}$$

27.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{x+h}-\sqrt{x}}{h} \\ &= \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \\ &= \frac{x+h-x}{h\sqrt{x+h}+\sqrt{x}} \\ &= \frac{1}{\sqrt{x+h}+\sqrt{x}}\end{aligned}$$

28.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^4-x^4}{h} \\ &= \frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h} \\ &= \frac{4x^3h+6x^2h^2+4xh^3+h^4}{h} \\ &= 4x^3+6x^2h+4xh^2+h^3\end{aligned}$$

29.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} \\ &= \frac{x^2-(x+h)^2}{hx^2(x+h)^2} \\ &= \frac{x^2-x^2-2xh-h^2}{hx^2(x+h)^2} \\ &= \frac{-2x-h}{x^2(x+h)^2} \\ &= -\frac{2x+h}{x^2(x+h)^2}\end{aligned}$$

30.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{1-(x+h)}-\frac{1}{1-x}}{h}\\ &= \frac{(1-x)-(1-(x+h))}{h(1-x)(1-(x+h))}\\ &= \frac{1-x-1+x+h}{h(1-x)(1-(x+h))}\\ &= \frac{1}{(1-x)(1-(x+h))}\end{aligned}$$

31.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{(x+h)}{(1+x+h)}-\frac{x}{1+x}}{h}\\ &= \frac{(x+h)(1+x)-x(1+x+h)}{h(1+x)(1+x+h)}\\ &= \frac{x^2+x+xh+h-x-x^2-xh}{h(1+x)(1+x+h)}\\ &= \frac{1}{(1+x)(1+x+h)}\end{aligned}$$

32.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{3-2(x+h)}-\sqrt{3-2x}}{h}\\ &= \frac{3-2(x+h)-(3-2x)}{h(\sqrt{3-2(x+h)}+\sqrt{3-2x})}\\ &= \frac{-2}{\sqrt{3-2(x+h)}+\sqrt{3-2x}}\end{aligned}$$

33.

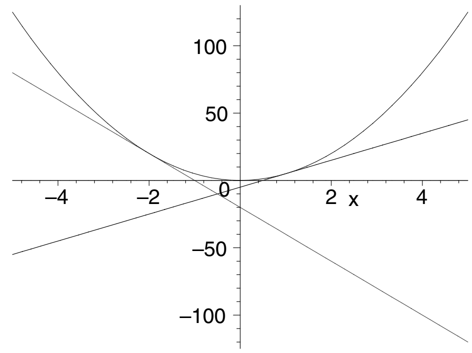
$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}\\ &= \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\\ &= \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\,\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\\ &= \frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\\ &= \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\end{aligned}$$

34.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{2(x+h)}{x+h-1}-\frac{2x}{x-1}}{h}\\ &= \frac{2(x+h)(x-1)-2x(x+h-1)}{h(x-1)(x+h-1)}\\ &= \frac{2x^2-2x+2xh-2h-2x^2-2xh+2x}{h(x-1)(x+h-1)}\\ &= \frac{-2}{(x-1)(x+h-1)}\end{aligned}$$

Exercise Set 2.4

1. a-b) $f(x)=5x^2$

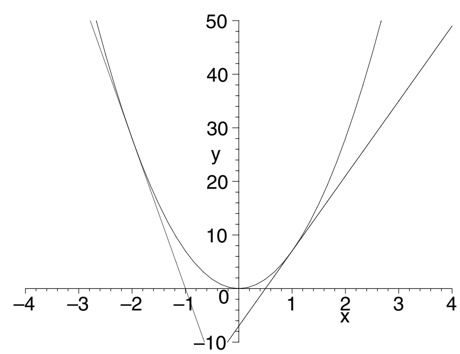


c)

$$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{5(x+h)^2-5x^2}{h}\\ &= \lim_{h\to 0}\frac{5x^2+10xh+5h^2-5x^2}{h}\\ &= \lim_{h\to 0}\frac{h(10x+5h)}{h}\\ &= \lim_{h\to 0}10x+5h\\ &= 10x\end{aligned}$$

d) $f'(-2)=10(-2)=-20$
$f'(0)=10(0)=0$
$f'(1)=10(1)=10$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

2. a-b) $f(x)=7x^2$

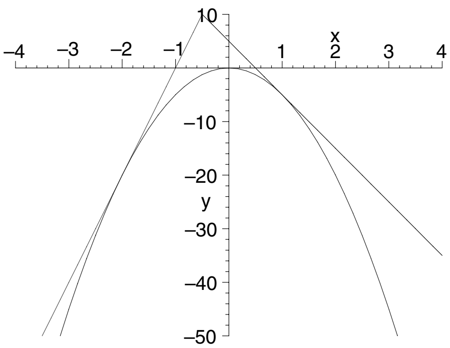


c)

$$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{7(x+h)^2-7x^2}{h}\\ &= \lim_{h\to 0}\frac{7x^2+14xh+7h^2-7x^2}{h}\\ &= \lim_{h\to 0}\frac{h(14x+7h)}{h}\\ &= \lim_{h\to 0}14x+7h\\ &= 14x\end{aligned}$$

d) $f'(-2) = 14(-2) = -28$
$f'(0) = 14(0) = 0$
$f'(1) = 14(1) = 14$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

3. a-b) $f(x) = -5x^2$

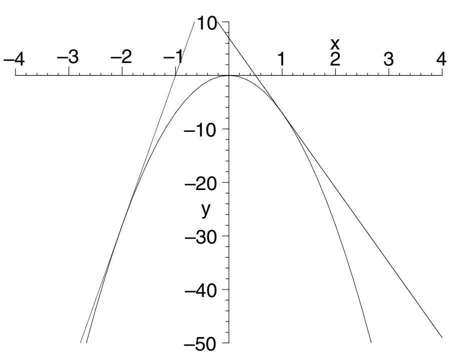


c)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{-5(x+h)^2-(-5x^2)}{h} \\ &= \lim_{h\to 0} \frac{-5x^2-10xh-5h^2-(-5x^2)}{h} \\ &= \lim_{h\to 0} \frac{h(-10x-5h)}{h} \\ &= \lim_{h\to 0} -10x-5h \\ &= -10x \end{aligned}$$

d) $f'(-2) = -10(-2) = 20$
$f'(0) = -10(0) = 0$
$f'(1) = -10(1) = -10$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

4. a-b) $f(x) = -7x^2$

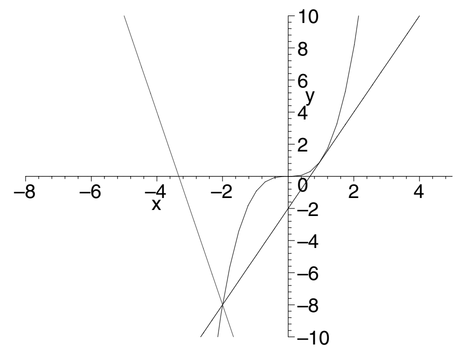


c)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{-7(x+h)^2-(-7x^2)}{h} \\ &= \lim_{h\to 0} \frac{-7x^2-14xh-7h^2-(-7x^2)}{h} \\ &= \lim_{h\to 0} \frac{h(-14x-7h)}{h} \\ &= \lim_{h\to 0} -14x-7h \\ &= -14x \end{aligned}$$

d) $f'(-2) = -14(-2) = 28$
$f'(0) = -14(0) = 0$
$f'(1) = -14(1) = -14$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

5. a-b) $f(x) = x^3$

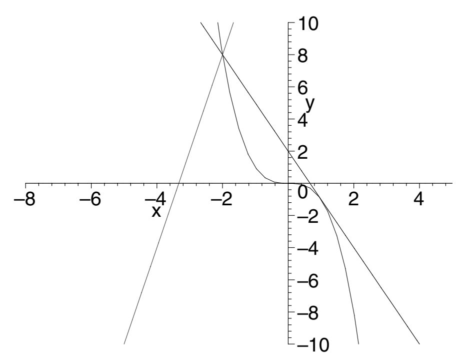


c)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^3-x^3}{h} \\ &= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} \\ &= \lim_{h\to 0} \frac{h(3x^2+3xh+h^2)}{h} \\ &= \lim_{h\to 0} 3x^2+3xh+h^2 \\ &= 3x^2 \end{aligned}$$

d) $f'(-2) = 3(-2)^2 = 12$
$f'(0) = 3(0)^2 = 0$
$f'(1) = 3(1)^2 = 3$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

6. a-b) $f(x) = -x^3$



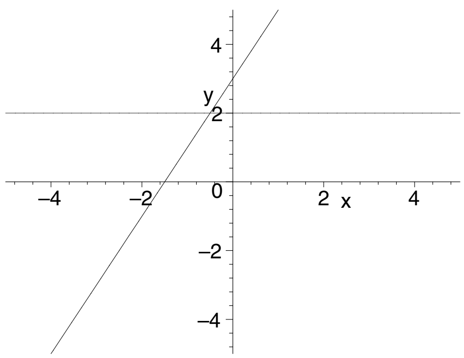
c)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{-(x+h)^3-(-x^3)}{h} \end{aligned}$$

$$\begin{aligned}
&= \lim_{h\to 0} \frac{-x^3 - 3x^2h - 3xh^2 - h^3 - (-x^3)}{h} \\
&= \lim_{h\to 0} \frac{h(-3x^2 - 3xh - h^2)}{h} \\
&= \lim_{h\to 0} -3x^2 - 3xh - h^2 \\
&= -3x^2
\end{aligned}$$

d) $f'(-2) = -3(-2)^2 = -12$
$f'(0) = -3(0)^2 = 0$
$f'(1) = -3(1)^2 = -3$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

7. a-b) $f(x) = 2x + 3$

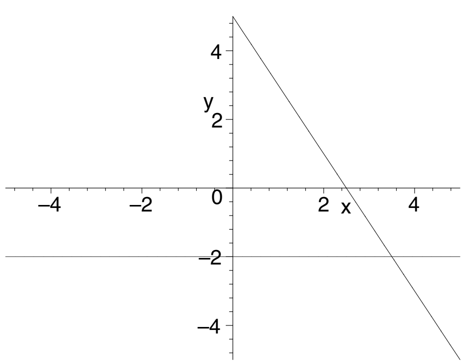


c)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{2(x+h) + 3 - (2x+3)}{h} \\
&= \lim_{h\to 0} \frac{2x + 2h + 3 - 2x - 3}{h} \\
&= \lim_{h\to 0} \frac{2h}{h} \\
&= 2
\end{aligned}$$

d) $f'(-2) = 2$
$f'(0) = 2$
$f'(1) = 2$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

8. a-b) $f(x) = -2x + 5$

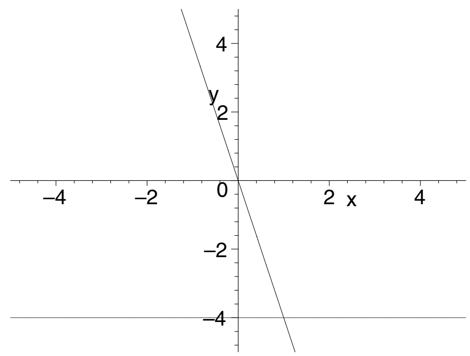


c)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{-2(x+h) + 5 - (-2x+5)}{h} \\
&= \lim_{h\to 0} \frac{-2x - 2h + 5 + 2x - 5}{h} \\
&= \lim_{h\to 0} \frac{-2h}{h} \\
&= -2
\end{aligned}$$

d) $f'(-2) = -2$
$f'(0) = -2$
$f'(1) = -2$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

9. a-b) $f(x) = -4x$

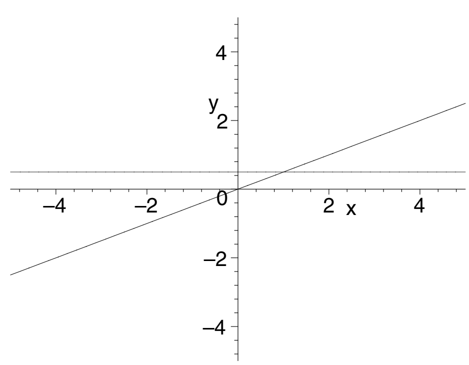


c)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{-4(x+h) - (-4x)}{h} \\
&= \lim_{h\to 0} \frac{-4x - 4h + 4x}{h} \\
&= \lim_{h\to 0} \frac{-4h}{h} \\
&= -4
\end{aligned}$$

d) $f'(-2) = -4$
$f'(0) = -4$
$f'(1) = -4$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

10. a-b) $f(x) = \frac{1}{2}x$



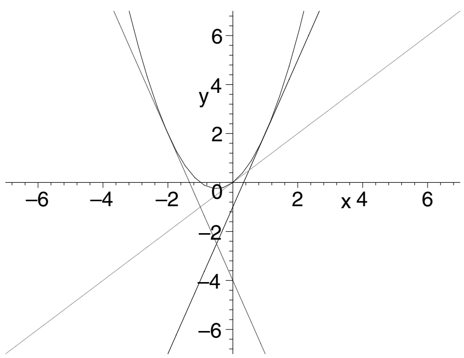
c)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$
\begin{aligned}
&= \lim_{h\to 0} \frac{\frac{1}{2}(x+h) - \frac{1}{2}x}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{2}x - \frac{1}{2}h - \frac{1}{2}x}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{2}h}{h} \\
&= \frac{1}{2}
\end{aligned}
$$

d) $f'(-2) = \frac{1}{2}$
$f'(0) = \frac{1}{2}$
$f'(1) = \frac{1}{2}$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

11. a-b) $f(x) = x^2 + x$

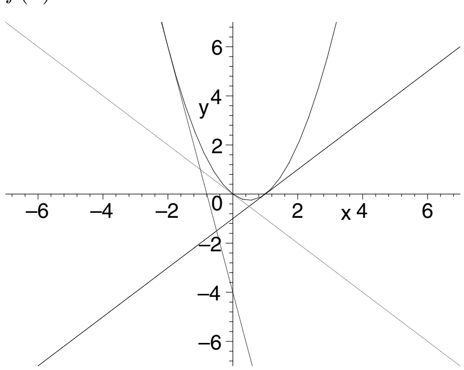


c)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^2 + (x+h) - (x^2+x)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2 + h}{h} \\
&= \lim_{h\to 0} \frac{h(2x+h+1)}{h} \\
&= 2x + 1
\end{aligned}
$$

d) $f'(-2) = 2(-2) + 1 = -3$
$f'(0) = 2(0) + 1 = 1$
$f'(1) = 2(1) + 1 = 3$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

12. a-b) $f(x) = x^2 - x$

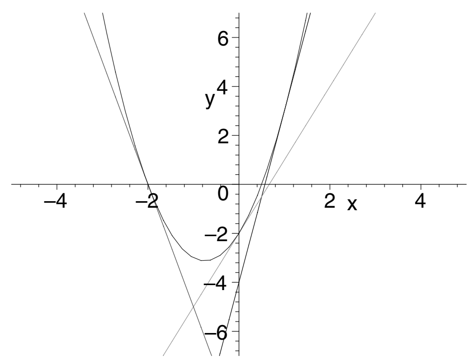


c)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^2 - (x+h) - (x^2-x)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2 - h}{h} \\
&= \lim_{h\to 0} \frac{h(2x+h-1)}{h} \\
&= 2x - 1
\end{aligned}
$$

d) $f'(-2) = 2(-2) - 1 = -5$
$f'(0) = 2(0) - 1 = -1$
$f'(1) = 2(1) - 1 = 1$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

13. a-b) $f(x) = 2x^2 + 3x - 2$

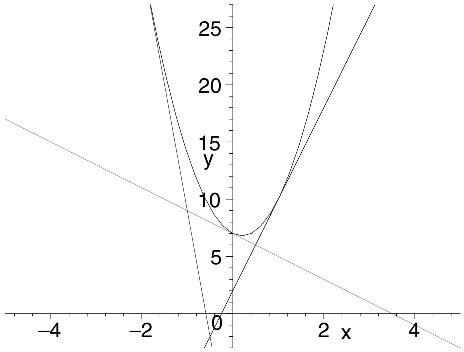


c)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) - 2 - (2x^2+3x-2)}{h} \\
&= \lim_{h\to 0} \frac{4xh + 2h^2 + 3h}{h} \\
&= \lim_{h\to 0} \frac{h(4x+3)}{h} \\
&= 4x + 3
\end{aligned}
$$

d) $f'(-2) = 4(-2) + 3 = -5$
$f'(0) = 4(0) + 3 = 3$
$f'(1) = 4(1) + 3 = 7$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

14. a-b) $f(x) = 5x^2 - 2x + 7$

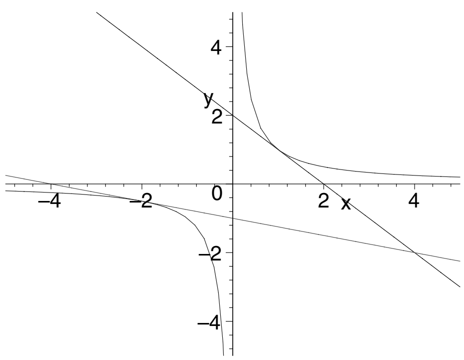


c)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{5(x+h)^2-2(x+h)+7-(5x^2-2x+7)}{h}\\
&= \lim_{h\to 0}\frac{10xh+5h^2-2h}{h}\\
&= \lim_{h\to 0}\frac{h(10x+5h-2)}{h}\\
&= 10x-2
\end{aligned}
$$

d) $f'(-2) = 10(-2) - 2 = -22$
$f'(0) = 10(0) - 2 = -2$
$f'(1) = 10(1) - 2 = 8$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

15. a-b) $f(x) = \frac{1}{x}$

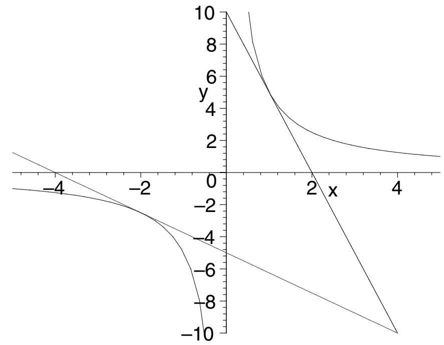


c)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}\\
&= \lim_{h\to 0}\frac{\frac{1}{(x+h)}\cdot x(x+h)-\frac{1}{x}\cdot x(x+h)}{h\cdot x(x+h)}\\
&= \lim_{h\to 0}\frac{x-(x+h)}{hx(x+h)}\\
&= \lim_{h\to 0}\frac{-h)}{hx(x+h)}\\
&= \lim_{h\to 0}\frac{-1}{x(x+h)}\\
&= -\frac{1}{x^2}
\end{aligned}
$$

d) $f'(-2) = -\frac{1}{(-2)^2} = -\frac{1}{4}$
$f'(0) =$ does not exist
$f'(1) = -\frac{1}{(1)^2} = -1$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

16. a-b) $f(x) = \frac{5}{x}$



c)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\frac{5}{(x+h)}-\frac{5}{x}}{h}\\
&= \lim_{h\to 0}\frac{\frac{5}{(x+h)}\cdot x(x+h)-\frac{5}{x}\cdot x(x+h)}{h\cdot x(x+h)}\\
&= \lim_{h\to 0}\frac{5x-5(x+h)}{hx(x+h)}\\
&= \lim_{h\to 0}\frac{-5h)}{hx(x+h)}\\
&= \lim_{h\to 0}\frac{-5}{x(x+h)}\\
&= -\frac{5}{x^2}
\end{aligned}
$$

d) $f'(-2) = -\frac{5}{(-2)^2} = -\frac{5}{4}$
$f'(0) =$ does not exist
$f'(1) = -\frac{5}{(1)^2} = -5$. These slopes are in agreement with the slopes of the tangent lines drawn in part (b).

17. $f(x) = mx$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{m(x+h)-mx}{h}\\
&= \lim_{h\to 0}\frac{mx+mh-mx}{h}\\
&= \lim_{h\to 0}\frac{mh}{h}\\
&= m
\end{aligned}
$$

18. $f(x) = ax^2 + bx + c$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{a(x+h)^2+b(x+h)+c-(ax^2+bx+c)}{h}
\end{aligned}
$$

$$\begin{aligned} &= \lim_{h\to 0} \frac{ax^2+2axh+ah^2+bx+bh-ax^2-bx-c}{h} \\ &= \lim_{h\to 0} \frac{2axh+ah^2+bh}{h} \\ &= \lim_{h\to 0} \frac{h(2ax+ah+b)}{h} \\ &= \lim_{h\to 0} 2ax+ah+b \\ &= 2ax+b \end{aligned}$$

19. $f(x)=x^2$. From Example 3, $f'(x)=2x$. For the point $(3,9)$ we have $f'(3)=2(3)=6=m$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-9 &= 6(x-3) \\ y &= 6x-18+9 \\ y &= 6x-9 \end{aligned}$$

For the point $(-1,1)$ we have $f'(-1)=2(-1)=-2$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-1 &= -2(x-(-1)) \\ y-1 &= -2x-2 \\ y &= -2x-2+1 \\ y &= -2x-1 \end{aligned}$$

For the point $(10,100)$ we have $f'(10)=2(10)=20$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-100 &= 20(x-10) \\ y &= 20x-200+100 \\ y &= 20x-100 \end{aligned}$$

20. $f(x)=x^3$. From Example 4, $f'(x)=3x^2$. For the point $(-2,-8)$ we have $f'(-2)=12=m$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y+8 &= 12(x+2) \\ y &= 12x+24-8 \\ y &= 12x+16 \end{aligned}$$

For the point $(0,0)$ we have $f'(0)=0$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-0 &= 0(x-0) \\ y &= 0 \end{aligned}$$

For the point $(4,64)$ we have $f'(4)=48$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-64 &= 48(x-4) \\ y &= 48x-192+64 \\ y &= 48x-128 \end{aligned}$$

21. From Exercise 14, $f'(x)=-\frac{5}{x^2}$. For the point $(1,5)$ we have $f'(1)=-\frac{5}{1^2}=-5=m$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-5 &= -5(x-1) \\ y &= -5x+5+5 \\ y &= -5x+10 \end{aligned}$$

For the point $(-1,-5)$ we have $f'(-1)=-\frac{5}{(-1)^2}=-5$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-5) &= -5(x-(-1)) \\ y+5 &= -5(x+1) \\ y &= -5x-5-5 \\ y &= -5x-10 \end{aligned}$$

For the point $(100,0.05)$, which can be rewritten as $(100,\frac{1}{20})$, we have $f'(10)=-\frac{5}{100^2}=-\frac{1}{2000}$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-\frac{1}{20} &= -\frac{1}{2000}(x-100) \\ y &= -\frac{x}{2000}+\frac{1}{20}+\frac{1}{20} \\ y &= -\frac{x}{2000}+\frac{2}{20} \\ y &= -\frac{x}{2000}+\frac{1}{10} \end{aligned}$$

22. $f(x)=\frac{2}{x}$. Using the difference quotient we find

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{\frac{2}{(x+h)}-\frac{2}{x}}{h} \\ &= \lim_{h\to 0}\frac{\frac{2}{(x+h)}\cdot x(x+h)-\frac{2}{x}\cdot x(x+h)}{h\cdot x(x+h)} \\ &= \lim_{h\to 0}\frac{2x-2(x+h)}{hx(x+h)} \\ &= \lim_{h\to 0}\frac{-2h)}{hx(x+h)} \\ &= \lim_{h\to 0}\frac{-2}{x(x+h)} \\ &= -\frac{2}{x^2} \end{aligned}$$

For the point $(-1,-2)$ we have $f'(-1)=-\frac{2}{(-1)^2}=-2=m$. So the equation of the tangent line is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-2) &= -2(x-(-1)) \\ y+2 &= -2(x+1) \\ y &= -2x-2-2 \\ y &= -2x-4 \end{aligned}$$

For the point $(2,1)$ we have $f'(2) = -\frac{2}{2^2} = -\frac{1}{2}$. So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= -\frac{1}{2}(x-2) \\ y &= -\frac{x}{2} + 1 + 1 \\ y &= -\frac{x}{2} + 2 \end{aligned}$$

For the point $(10, \frac{1}{5})$ we have $f'(10) = -\frac{2}{10^2} = -\frac{1}{50}$. So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \frac{1}{5} &= -\frac{1}{50}(x-10) \\ y &= -\frac{x}{50} + \frac{1}{5} + \frac{1}{5} \\ y &= -\frac{x}{50} + \frac{2}{5} \end{aligned}$$

23. First, let us find the expression for $f'(x)$.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{4 - (x+h)^2 - (4 - x^2)}{h} \\ &= \lim_{h\to 0} \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h} \\ &= \lim_{h\to 0} \frac{-2xh - h^2}{h} \\ &= \lim_{h\to 0} \frac{h(-2x - h)}{h} \\ &= \lim_{h\to 0} (-2x - h) \\ &= -2x \end{aligned}$$

For the point $(-1,3)$ we have $f'(-1) = -2(-1) = 2 = m$. So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= 2(x - (-1)) \\ y - 3 &= -2x + 2 \\ y &= 2x + 2 + 3 \\ y &= 2x + 5 \end{aligned}$$

For the point $(0,4)$ we have $f'(0) = -2(0) = 0$. So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= 0(x - 0) \\ y &= 0 + 4 \\ y &= 4 \end{aligned}$$

For the point $(5,-21)$ we have $f'(5) = -2(5) = -10$. So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-21) &= -10(x - 5) \\ y + 21 &= -10x + 50 \\ y &= -10x + 50 - 21 \\ y &= -10x + 29 \end{aligned}$$

24.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h} \\ &= \lim_{h\to 0} \frac{2xh + h^2 - 2h}{h} \\ &= \lim_{h\to 0} \frac{h(2x + h - 2)}{h} \\ &= \lim_{h\to 0} (2x + h - 2) \\ &= 2x - 2 \end{aligned}$$

For the point $(-2,8)$ we have $f'(-2) = -6 = m$. So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 8 &= -6(x - (-2)) \\ y - 8 &= -6x - 12 \\ y &= -6x - 12 + 8 \\ y &= -6x - 4 \end{aligned}$$

For the point $(1,-1)$ we have $f'(1) = 0$. So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-1) &= 0(x - 0) \\ y &= 0 - 1 \\ y &= -1 \end{aligned}$$

For the point $(4,8)$ we have $f'(4) = 6$. So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 8 &= 6(x - 4) \\ y &= 6x - 24 + 8 \\ y &= 6x - 16 \end{aligned}$$

25. The function is not differentiable at x_0 since it is discontinuous, x_3 since it has a corner, x_4 since it has a corner, x_6 since it has a corner, and x_{12} since it has a vertical tangent.

26. The function is not differentiable at x_2, x_4, x_5, x_7 and x_8.

27. The function is not differentiable at integer values of x since the function is not continuous at integer values of x.

28. The function is not differentiable at values of x of the form $x = 0.2n$ where $n = 1, 2, 3, \cdots$.

29. The function is differentiable for all values in the domain.

30. The function is not differentiable in the year 1850, 1860, 1865, 1880, 1910, 1960, 1975, and 1980.

31. The function is differentiable for all values in the domain.

32. L_4 and L_6 are the only lines that appear to be tangent since the others intersect the curve and are no "tangent" to it.

33. As the points Q get closer to P the secant lines are getting closer to the tangent line at point P.

34.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h} \\ &= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} \\ &= \lim_{h\to 0} 4x^3 + 6x^2h + 4xh^2 + h^3 \\ &= 4x^3 \end{aligned}$$

35.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{x^2+2xh+h^2} - \frac{1}{x^2}}{h} \\ &= \lim_{h\to 0} \frac{x^2 - x^2 - 2xh - h^2}{hx^2(x^2+2xh+h^2)} \\ &= \lim_{h\to 0} \frac{-h(2x-h)}{hx^2(x^2+2xh+h^2)} \\ &= \lim_{h\to 0} \frac{-2x+h}{x^2(x^2+2xh+h^2)} \\ &= \frac{-2x}{x^2(x^2)} \\ &= \frac{-2}{x^3} \end{aligned}$$

36.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{1-x-h} - \frac{1}{1-x}}{h} \\ &= \lim_{h\to 0} \frac{1-x-1+x+h}{h(1-x)(1-x-h)} \\ &= \lim_{h\to 0} \frac{1}{(1-x)(1-x-h)} \\ &= \frac{1}{(1-x)^2} \end{aligned}$$

37.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{x+h}{1+x+h} - \frac{x}{1+x}}{h} \\ &= \lim_{h\to 0} \frac{(x+h)(1+x) - x(1+x+h)}{h(1+x)(1+x+h)} \\ &= \lim_{h\to 0} \frac{x + x^2 + h + hx - x - x^2 - xh}{h(1+x)(1+x+h)} \\ &= \lim_{h\to 0} \frac{h}{h(1+x)(1+x+h)} \\ &= \lim_{h\to 0} \frac{1}{(1+x)(1+x+h)} \\ &= \frac{1}{(1+x)^2} \end{aligned}$$

38.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\ &= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h\to 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$

39.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \\ &= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \\ &= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\ &= \lim_{h\to 0} \frac{x-x-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\ &= \lim_{h\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\ &= \frac{-1}{2\sqrt{x^3}} \end{aligned}$$

40.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{3(x+h)}{x+h+5} - \frac{3x}{x+5}}{h} \\ &= \lim_{h\to 0} \frac{(3x+3h)(x+5) - 3x(x+h+5)}{h(x+5)(x+h+5)} \\ &= \lim_{h\to 0} \frac{3x^2 + 15x + 3xh + 15h - 3x^2 - 3xh - 15x}{h(x+5)(x+h+5)} \\ &= \lim_{h\to 0} \frac{15h}{h(x+5)(x+h+5)} \\ &= \lim_{h\to 0} \frac{15}{(x+5)(x+h+5)} \\ &= \frac{15}{(x+5)^2} \end{aligned}$$

41. The function $f(x)$ will is not differentiable at $x = -3$.

42. $f'(3) = 6$, $f'(-1) = -2$, $f'(10) = 20$

43. $f'(-2) = 12$, $f'(0) = 0$, $f'(4) = 48$

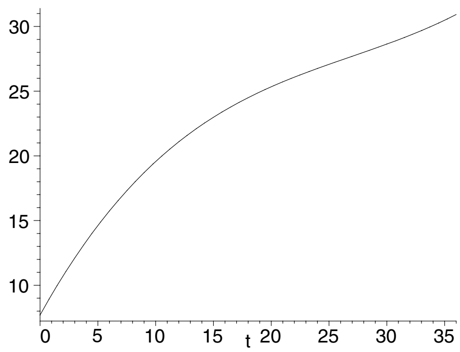
44. $f'(1) = -5$, $f'(-1) = -5$, $f'(100) = \frac{-1}{2000}$

45. $f'(-1) = -2$, $f'(2) = \frac{-1}{2}$, $f'(10) = \frac{-2}{100}$

46. $f'(-1) = 2$, $f'(0) = 0$, $f'(5) = -10$

47. $f'(-2) = -6$, $f'(1) = 0$, $f'(4) = 6$

48. **a)**



b) $y = 0.41x + 16.067$

c) Rate of change is 0.41

d)
- $(12, w(12)$ and $(24, w(24))$:
  $y = 0.472416x+15.318$, Rate of change $= 0.47241$
- $(12, w(12)$ and $(18, w(18))$:
  $y = 0.5685x + 14.165$, Rate of change $= 0.0.5685$
- $(12, w(12)$ and $(15, w(15))$:
  $y = 0.6326x + 13.395$, Rate of change $= 0.0.6326$

e) The slope of the tangent line at $(12, w(12))$ appears to be approaching 0.71

Exercise Set 2.5

1. $\frac{dy}{dx} = 7x^{7-1} = 7x^6$

2. $\frac{dy}{dx} = 8x^7$

3. $\frac{dy}{dx} = 3 \cdot 2x^{2-1} = 6x$

4. $\frac{dy}{dx} = 20x^3$

5. $\frac{dy}{dx} = 4 \cdot 3x^{3-1} = 12x^2$

6. $\frac{dy}{dx} = 10cos(x)$

7. $\frac{dy}{dx} = 3 \cdot \frac{2}{3}x^{2/3-1} = 2x^{-1/3}$

8. $\frac{dy}{dx} = -2 \cdot \frac{1}{2}x^{-1/2} = -x^{-1/2}$

9. Rewrite as $y = x^{3/4}$, $\frac{dy}{dx} = \frac{3}{4}x^{3/4-1} = \frac{3}{4}x^{-1/4}$

10. Rewrite as $y = x^{4/7}$, $\frac{dy}{dx} = \frac{4}{7}x^{-3/7}$

11. $\frac{dy}{dx} = 4cos\ x$

12. $\frac{dy}{dx} = -3sin\ x$

13. $\frac{dy}{dx} = cos\ x - 1x^{-2} = cos\ x - \frac{1}{x^2}$

14. $\frac{dy}{dx} = \frac{d}{dx}(x^{1/2} - x^{-1/2}) = \frac{1}{2}x^{1/2-1} + \frac{1}{2}x^{-1/2-1}$
$= \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$ or $\frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$

15. $\frac{dy}{dx} = 2(2x+1)^{2-1}(2) = 4(2x+1)$

16. $\frac{dy}{dx} = 2(3x-2)(3) = 6(3x-2)$

17. $f'(x) = 0.25(3.2x^{3.2-1}) = 0.8x^{2.2}$

18. $f'(x) = 0.32(12.5x^{11.5} = 4x^{11.5}$

19. $f'(x) = 10cos\ x + 12sin\ x$

20. $f'(x) = \sqrt{5}\ cos\ x$

21. $f'(x) = -\sqrt[3]{9}\ sin\ x$

22. $f'(x) = \frac{-2}{x^2} - \frac{1}{2}$

23. Rewrite as $f(x) = 5x^{-1} + \frac{x}{5}$, $f'(x) = -5x^{-2} + \frac{1}{5}$

24. $f'(x) = -2x^{-5/3} + \frac{3}{4}x^{-1/4} + \frac{84}{5}x^{1/5} - \frac{8}{3}x^{-4}$

25. $f'(x) = -x^{-1/2} - x^{-3/4} - \frac{1}{2}x^{-5/4} + \frac{7}{2}x^{-3/2}$

26. $f(x) = x^{7/2}$
$f'(x) = \frac{7}{2}x^{5/2}$

27. $f(x) = x^{18/5} - x^{13/5}$
$f'(x) = \frac{18}{5}x^{13/5} - \frac{13}{5}x^{8/5}$

28. $f(x) = 2 + x^{-1}$
$f'(x) = -x^{-2}$

29. $f(x) = 3x^{-1} - 4x^{-2}$
$f'(x) = -3x^{-2} + 8x^{-3}$

30. $f(x) = x + 2 + 3x^{-1} + 4x^{-2}$
$f'(x) = 1 - 3x^{-2} - 8x^{-3}$

31. $f(x) = 4x + 3 - 2x^{-1}$
$f'(x) = 4 + 2x^{-2}$

32. $g(x) = 2x^{-1} - 4x^{-2} + 6x^{-3}$
$g'(x) = -2x^{-2} + 8x^{-3} - 18x^{-4}$

33. $p(x) = 6x^{-4} - 2x^{-3}$
$p'(x) = -24x^{-5} + 6x^{-4}$

34. $r(x) = 12cos\ x + 8sin\ x - 5$
$r'(x) = -12sin\ x + 8cos\ x$

35. $s(x) = 3\sqrt{2}cos\ x - 2\sqrt{2}sin\ x$
$s'(x) = -3\sqrt{2}sin\ x - 2\sqrt{2}cos\ x$

36. $f'(x) = \frac{3}{2}cos\ x + \frac{5}{8}sin\ x$

37. $q'(x) = 8\left(\frac{\sqrt{5}}{3}\right) cos\ x + 8\left(\frac{\sqrt[3]{5}}{7}\right) sin\ x$

38. $W(x) = \frac{x}{3} + \frac{2}{3} - x^{-1} + \frac{4}{3}cos\ x$
$W'(x) = \frac{1}{3} + x^{-2} - \frac{4}{3}sin\ x$

39. $U(x) = sin\ x - 2x^{1/2} + 3x^{-1/2}$
$U'(x) = cos\ x - x^{-1/2} - \frac{3}{2}x^{-3/2}$

40. $f'(x) = 2cos\ x$
The slops at $(0,0)$ is $f'(0) = cos(0) = 1$.
So the equation of the tangent lilne is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= 1(x - 0) \\ y &= x \end{aligned}$$

41. $f'(x) = -3sin\ x$
The slope at $(0,4)$ is $f'(0) = 0$.
So the equation of the tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= 0(x - 0) \\ y - 4 &= 0 \\ y &= 4 \end{aligned}$$

42. $f'(x) = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2}$
The slope at $(4,10)$ is
$f'(4) = \frac{3}{2}(4)^{1/2} + \frac{1}{2}(4)^{-1/2} = 3.25$
So the equation of the line is

$$\begin{aligned} y - 10 &= 3.25(x - 4) \\ y - 10 &= = 3.25x - 13 \\ y &= 3.25x - 3 \end{aligned}$$

43. $f'(x) = \frac{4}{3}x^{1/3} + \frac{2}{3}x^{-2/3}$
The slope at $(8,20)$ is
$f'(8) = \frac{4}{3}(8)^{1/3} + \frac{2}{3}(8)^{-2/3} = \frac{17}{6}$
So the equation of the line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 20 &= \frac{17}{6}(x - 8) \\ y &= \frac{17}{6}x - \frac{68}{3} + 20 \\ y &= \frac{17}{6}x - \frac{8}{3} \end{aligned}$$

44. $y = x^{1/12}$
$\frac{dy}{dx} = \frac{1}{12}x^{-11/12}$

45. Rewrite $y = \left(\frac{x^{2/3}}{x^{3/2}}\right)^{1/3} = x^{-5/18}$
$\frac{dy}{dx} = \frac{-5}{18}x^{-23/8}$

46. $4sin\ x$
$\frac{dy}{dx} = 4cos\ x$

47. Use the trigonometric identity $1+tan^2 x = sec^2 x$ to rewrite $y = 3sec^2 x\ cos^3 x = 3cos\ x$
$\frac{dy}{dx} = -3sin\ x$

48. $y = \frac{1}{\sqrt{2}}sin\ x + \frac{1}{\sqrt{2}}cos\ x$
$\frac{dy}{dx} = \frac{1}{\sqrt{2}}(cos\ x - sin\ x)$

49. Use the sum identity for cosine to rewrite $y = \frac{\sqrt{3}}{2}cos\ x - \frac{1}{2}sin\ x$
$\frac{dy}{dx} = \frac{-\sqrt{3}}{2}sin\ x - \frac{1}{2}cos\ x$

50. $\frac{dy}{dx} = 2\left(x + \frac{1}{2}\right)(1 - x^{-2})$
The slope at $(2, \frac{25}{4})$ is $\frac{dy}{dx}|_{x=2} = \frac{25}{4}$
The equation of the line is

$$\begin{aligned} y - \frac{25}{4} &= \frac{25}{4}(x - 2) \\ y &= \frac{25}{4}x - \frac{25}{4} \end{aligned}$$

51. $\frac{dy}{dx} = 2\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)(\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x^3}})$
The slope at $(1,0)$ is $\frac{dy}{dx}|_{x=1} = 0$
The equation of the line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= 0(x - 1) \\ y &= 0 \end{aligned}$$

52. Let $D(x) = f(x) - g(x)$. Then

$$\begin{aligned} \frac{D(x+h) - D(x)}{h} &= \frac{[f(x+h) - g(x+h)] - [f(x) - g(x)]}{h} \\ &= \frac{f(x+h) - f(x)}{h} - \frac{g(x+h) - g(x)}{h} \end{aligned}$$

$$\begin{aligned} D'(x) &= \lim_{h\to 0}\frac{D(x+h) - D(x)}{h} \\ &= \lim_{h\to 0}\left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}\right] \\ &= \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} + \lim_{h\to 0}\frac{g(x+h) - g(x)}{h} \\ &= f'(x) - g'(x) \end{aligned}$$

53. Let $f(x) = cos\ x$ then

$$\begin{aligned} f'(x) &= lim_{h\to 0}\frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0}\frac{cos(x+h) - cos(x)}{h} \\ &= \lim_{h\to 0}\frac{cos\ xcos\ h - sin\ xsin\ h - cos\ x}{h} \\ &= \lim_{h\to 0} cos\ x\left(\frac{cos\ h - 1}{h}\right) - sin\ x\left(\frac{sin\ h}{h}\right) \\ &= cos\ x \cdot \lim_{h\to 0}\frac{cos\ h - 1}{h} - sin\ x \cdot \lim_{h\to 0}\frac{sin\ h}{h} \\ &= cos\ x \cdot 0 - sin\ x \cdot 1 \\ &= -sin\ x \end{aligned}$$

54. The value of the slope of the tangent lines of the sine function correspond to whether the cosine function is has positive or negative values. For a positive slope of the tangent line means the cosine has a positive values at the aprticular x value, and vice versa.

55. Left to the student

56. Left to the student

57. The tangent line is horizontal at $x = \pm\ 0.6922$

58. The tangent line is horizontal at $x = 0$

59. The tangent line is horizontal at $x = -0.3456$ and at $x = 1.9289$

60. The tangent line is horizontal at $x = 1.9384$, $x = 3.6613$ and at $x = 5.6498$

Exercise Set 2.6

1. **a)** $v(t) = \frac{ds(t)}{dt} = 3t^2 + 1$

b) $a(t) = \frac{dv(t)}{dt} = 6t$

c) $v(4) = 3(4)^2 + 1 = 48 + 1 = 49$ feet per second
$a(4) = 6(4) = 24$ feet per squared seconds

2. **a)** $v(t) = \frac{ds}{dt} = 3$ Note that the velocity is constant and does not depent on time

b) $a(t) = \frac{dv}{dt} = 0$ Note that the velocity is constant and does not depent on time

c) $v(2) = 3$ miles per hour
$a(2) = 0$ miles per squared hours

d) Uniform motion means that the velocity and acceleration are constants namely, the velocity is the slope of the linear distance function and the acceleration is zero.

3. **a)** $v(t) = \frac{ds(t)}{dt} = -20t + 2$

b) $a(t) = \frac{dv(t)}{dt} = -20$

c) $v(1) = -20(1) + 2 = -20 + 2 = 18$ feet per second
$a(1) = -20$ feet per squared seconds

4. **a)** $v(t) = \frac{ds}{dt} = 2t - \frac{1}{2}$

b) $a(t) = \frac{dv}{dt} = 2$

c) $v(1) = 2(1) - \frac{1}{2} = 2 - \frac{1}{2} = \frac{3}{2} = 1.5$ feet per second
$a(1) = 2$ feet per squared seconds

5. **a)** $v(t) = \frac{ds(t)}{dt} = 5 + 2cos\ t$

b) $a(t) = \frac{dv(t)}{dt} = -2sin\ t$

c) Find $v(\frac{\pi}{4})$ and $a(\frac{\pi}{4})$

$$\begin{aligned} v(\frac{\pi}{4}) &= 5 + 2cos(\frac{\pi}{4}) \\ &= 5 + 2 \cdot \frac{\sqrt{2}}{2} \\ &= 5 + \sqrt{2} \\ &\approx 6.414 \ \ m/sec \end{aligned}$$

$$\begin{aligned} a(\frac{\pi}{4}) &= -2sin(\frac{\pi}{4}) \\ &= 2 \cdot \frac{\sqrt{2}}{2} \\ &= \sqrt{2} \\ &\approx 1.414 \ \ m/sec^2 \end{aligned}$$

d) Find t when $v(t) = 3$

$$\begin{aligned} 3 &= 5 + 2cos\ t \\ -2 &= 2cos\ t \\ -1 &= cos\ t \\ t &= cos^{-1}(-1) \\ &= \pi + 2n\pi \end{aligned}$$

6. **a)** $v(t) = 3 + sin\ t$

b) $a(t) = cos\ t$

c) Find $v(\frac{\pi}{3})$ and $a(\frac{\pi}{3})$

$$\begin{aligned} v(\frac{\pi}{3}) &= 3 + sin(\frac{\pi}{3}) \\ &= 3 + \frac{\sqrt{3}}{2} \\ &\approx 3.866 \ \ m/sec \end{aligned}$$

$$\begin{aligned} a(\frac{\pi}{3}) &= cos(\frac{\pi}{3}) \\ &= \frac{1}{2} = 0.5 \ \ m/sec^2 \end{aligned}$$

d) Find t when $v(t) = 2$

$$\begin{aligned} 2 &= 3 + sin\ t \\ -1 &= sin\ t \\ t &= sin^{-1}(-1) \\ &= \frac{3\pi}{2} + 2n\pi \end{aligned}$$

7. **a)** $\frac{dN}{da} = -2a + 300$

b) Since a is counted in thousands we need to find

$$\begin{aligned} N(10) &= -(10)^2 + 300(10) + 6 \\ &= -100 + 3000 + 6 \\ &= 2996 \end{aligned}$$

There will be 2996 units sold after spending \$10000 on advertsing

c) At $a = 10$, $\frac{DN}{da} = -2(10) + 300 = -20 + 300 = 280$ units per thousand dollars spent on advertising

d) The rate of change of the number of units sold depends on the amount spent on advertising according to the equation $\frac{dN}{da} = -2a + 300$ which means that for every a thousand dollars spent on advertising, the change in the units solds is $-2a+300$ units. If \$10000 is spent on advertising, then there will be a 280 unit increase in the number of units sold.

8. **a)** $\frac{dw}{dt} = 1.82 - 0.1192t + 0.002274t^2$

b) $w(0) = 8.15 + 1.82(0) - 0.0596(0)^2 + 0.000758(0)^3 = 8.15$ pounds

c) $\frac{dw}{dt}|_{t-0} = 1.82 - 0.1192(0) + 0.002274(0)^2 = 1.82$ pounds per month

d) $w(12) = 8.15 + 1.82(12) - 0.0596(12)^2 + 0.000758(12)^3 = 22.72$ pounds

e) $\frac{dw}{dt}|_{t=12} = 1.82 - 0.1192(12) + 0.002274(12)^2 = 0.717$ pounds per month

f) Average rate of change $= \frac{w(12)-w(0)}{12} = \frac{22.72-8.15}{12} = 1.21$ pounds per month

g)

$$\begin{aligned} 1.82 - 0.1192t + 0.002274t^2 &= 1.21 \\ 0.002274t^2 - 0.1192t + 0.61 &= 0 \end{aligned}$$

Using the quadratic formula we get that $t = 5.747$ months.

9. **a)** $\frac{dw}{dt} = 1.61 - 0.0968t + 0.0018t^2$

b) $w(0) = 7.60 + 1.61(0) - 0.0484(0)^2 + 0.0006(0)^3 = 7.60$ pounds

c) $\frac{dw}{dt}|_{t=0} = 1.61 - 0.0968(0) + 0.0018(0)^2 = 1.61$ pounds per month

d) $w(12) = 7.60 + 1.61(12) - 0.0484(12)^2 + 0.0006(12)^3 = 20.987$ pounds

e) $\frac{dw}{dt}|_{12} = 1.61 - 0.0968(12) + 0.0018(12)^2 = 0.708$ pounds per month

f) Average rate of change $= \frac{w(12)-w(0)}{12} = \frac{20.987-7.6}{12} = 1.1156$ pounds per month

g)

$$\begin{aligned}
1.61 - 0.0968t + 0.0018t^2 &= 1.1156 \\
0.0018t^2 - 0.0968t + 0.4944 &= 0 \\
\frac{0.0968 + \sqrt{(0.0968)^2 - 4(0.0018)(0.4944)}}{2(0.0018)} &= t \\
5.7147 &= t
\end{aligned}$$

10. **a)** $\frac{dD}{dF} = 2$ feet per degrees

b) For every increase of one degree there will be an increase of 2 feet in the stopping distance of an object on glare ice

11. **a)** $\frac{dC}{dr} = 2\pi$

b) For every increase of one centimeter of the radius the healing wound circumference increases by 2π centimeters

12. **a)** $\frac{dA}{dr} = 2\pi r$

b) For every increase of one centimeter of the radius the circular area of a healing wound increases by $2\pi r$ squared centimeters

13. **a)** $\frac{dT}{dt} = -0.2t + 1.2$

b) At $t = 1.5$

$$\begin{aligned}
T(1.5) &= -0.1(1.5)^2 + 1.2(1.5) + 98.6 \\
&= 100.2 \text{ degrees}
\end{aligned}$$

c) $\frac{dT}{dt}|_{t=1.5} = -0.2(1.5) + 1.2 = 0.9$ degrees per day

d) The sign of $T'(t)$ is significant because it indicates whether the rate of change in temperature is an increase (if positive) or a decrease (if negative). That is, whether the fever is increasing or decreasing during the illness

14. **a)** $\frac{dB}{dx} = 0.1x - 0.9x^2$

b) The sensitivity is affected by the following rule: for every increase in the dosage of x cubic centimeters the blood pressure is changed by an amount equal to $0.1x - 0.9x^2$

15. **a)** $\frac{dT}{dW} = 1.31W^{0.31}$

b) For every increase of W in body weight the territorial area of an animal increases by an amount equal to $1.31W^{0.31}$

16. **a)** $\frac{dH}{dW} = 1.41W^{0.41}$

b) For every increase of W in body weight the home range of an animal increases by an amount equal to $1.41W^{0.41}$

17. First rewrite $R(Q)$ as follows $R(Q) = \frac{k}{2}Q^2 - \frac{1}{3}Q^3$

a) $\frac{dR}{dQ} = \frac{k}{2}(2Q) - \frac{1}{3}(3Q^2) = kQ - Q^2$

b) For every increase Q in the dosage there will be a change in the reaction of the body to that dosage change equal to the amount $kQ - Q^2$

18. **a)** $\frac{dP}{dt} = 4000t$

b) $P(10) = 100000 + 2000(10)^2 = 300000$ people

c) $\frac{dP}{dt}|_{t=10} = 4000(10) = 40000$ people per year

d) The population of the city will reach 300000 people after 10 years.
The population growth rate at 10 years will be 40000 people per year, that is, the population will increase by 40000 for the next year

19. **a)** $\frac{dA}{dt} = 0.08$

b) The rate of change for the median age of women at first marriage is constant at 0.08. That is, each year the median age of women at first marriage in increasing by 0.08 years

20. **a)** $\frac{dV}{dh} = 1.22(\frac{1}{2}h^{-1/2}) = \frac{0.61}{\sqrt{h}}$

b) $V(40000) = 1.22\sqrt{40000} = 244$ miles

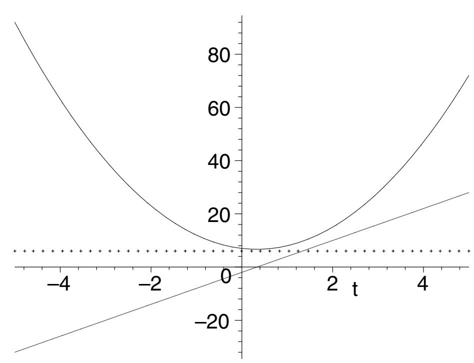
c) $\frac{dV}{dh}|_{h=40000} = \frac{0.61}{\sqrt{40000}} = 0.00305$ miles per feet

d) At a height of 40000 feet the view is 244 miles.
At a height of 40000, for every one foot increase in height corresponds to an increase in the view of 0.00305 miles

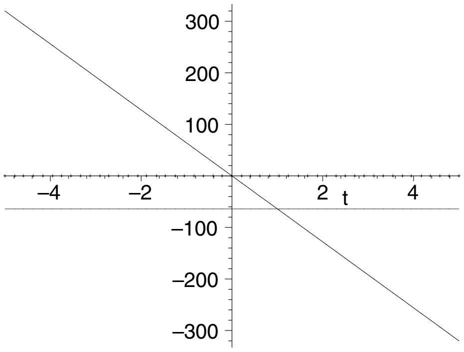
21. The average rate of change of a function is the value of the difference quotient evaluated over a period of time, while the instantaneous rate of change is the value of the slope of the tangent line at that particular instant.

22. The derivative of a function at a point x can be thought of as the slope of the tangent line at that point. It could also be thought of as the instantaneous rate of change of teh function at that point.

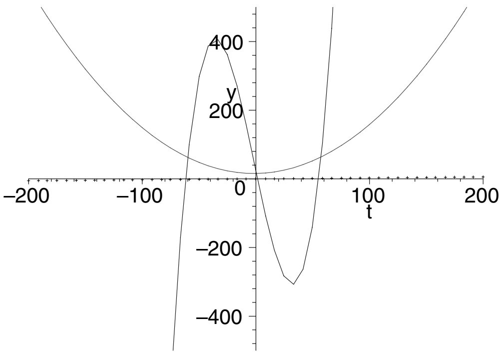
23.



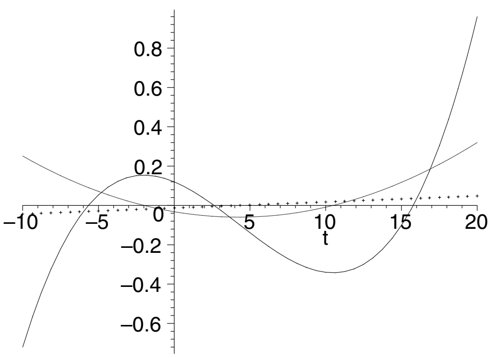
24.



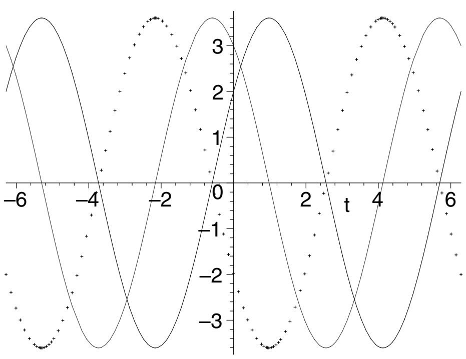
25.



26.

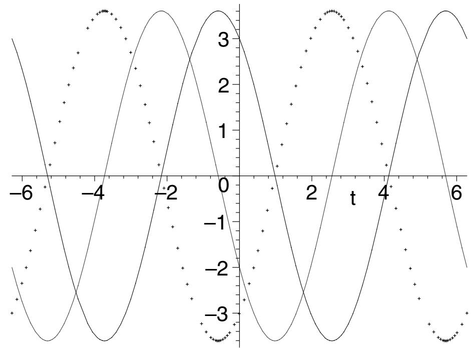


27. **a)**



b) The acceleration function is the same as the product of the position function times -1. That is it is a reflection of the position function about the t axis.

28. **a)**



b) The acceleration function is the same as the product of the position function times -1. That is it is a reflection of the position function about the t axis.

Exercise Set 2.7

1. Method One: $x^3 \cdot x^8 = x^{3+8} = x^{11}$. $\frac{dy}{dx} = 11x^{10}$
Method Two (product rule):

$$\begin{aligned} \frac{dy}{dx} &= x^3 \cdot 8x^7 + 3x^2 \cdot x^8 \\ &= 8x^{10} + 3x^{10} \\ &= 11x^{10} \end{aligned}$$

2. Method One: $x^4 \cdot x^4 = x^{4+9} = x^{13}$. $\frac{dy}{dx} = 13x^{12}$
Method Two (product rule):

$$\begin{aligned} \frac{dy}{dx} &= x^4 \cdot 9x^8 + 4x^3 \cdot x^9 \\ &= 9x^{12} + 4x^{12} \\ &= 13x^{12} \end{aligned}$$

3. Method One: $x\sqrt{x} = x^{3/2}$, $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$
Method Two (product rule):

$$\begin{aligned} \frac{dy}{dx} &= x(\frac{1}{2}x^{-1/2}) + (1)x^{1/2} \\ &= \frac{1}{2}x^{1/2} + x^{1/2} \\ &= \frac{3}{2}x^{1/2} \end{aligned}$$

4. Method One: $y = x^{8/3}$, $\frac{dy}{dx} = \frac{8}{3}x^{5/3}$
Method Two (product rule)

$$\begin{aligned} \frac{dy}{dx} &= x^2(\frac{2}{3}x^{-1/3} + 2x(x^{2/3}) \\ &= \frac{2}{3}x^{5/3} + 2x^{5/3} \\ &= \frac{8}{3}x^{5/3} \end{aligned}$$

5. Method One: $\frac{x^8}{x^5} = x^3$, $\frac{dy}{dx} = 3x^2$
Method Two (quotient rule):

$$\begin{aligned} \frac{dy}{dx} &= \frac{x^5(8x^7) - 5x^4(x^8)}{(x^5)^2} \\ &= \frac{8x^{12} - 5x^{12}}{x^{10}} \\ &= \frac{3x^{12}}{x^{10}} \\ &= 3x^2 \end{aligned}$$

6. Method One: $\frac{x^5}{x^8} = x^{-3}$, $\frac{dy}{dx} = -3x^{-4}$
Method Two (quotient rule):

$$\begin{aligned} \frac{dy}{dx} &= \frac{x^8(5x^4) - x^5(8x^7)}{(x^8)^2} \\ &= \frac{5x^{12} - 8x^{12}}{x^{16}} \\ &= \frac{-3x^{12}}{x^{16}} \\ &= -3x^{-4} \end{aligned}$$

7. Method One: $y = x^2 - 25$, $\frac{dy}{dx} = 2x$
Method Two (product rule):

$$\begin{aligned} \frac{dy}{dx} &= (x+5)(1) + (1)(x-5) \\ &= x + 5 + x - 5 \\ &= 2x \end{aligned}$$

8. Method One: $f(x) = \frac{(x+3)(x-3)}{x+3} = (x-3)$, $f'(x) = 1$
Methods Two (quotient rule):

$$\begin{aligned} f'(x) &= \frac{(x+3)(2x) - (x^2-9)(1)}{(x+3)^2} \\ &= \frac{2x^2 + 6x - x^2 + 9}{x^2 + 6x + 9} \\ &= \frac{x^2 + 6x + 9}{x^2 + 6x + 9} \\ &= 1 \end{aligned}$$

9.

$$\begin{aligned} y &= (8x^5 - 3x^2 + 20)(8x^4 - 3x^{1/2}) \\ \frac{dy}{dx} &= (8x^5 - 3x^2 + 20)(32x^3 - \frac{3}{2}x^{-1/2}) + \\ &\quad (40x^4 - 6x)(8x^4 - 3x^{1/2}) \\ &= (8x^5 - 3x^2 + 20)(32x^3 - \frac{3}{2\sqrt{x}}) + \\ &\quad (40x^4 - 6x)(8x^4 - 3\sqrt{x}) \end{aligned}$$

10.

$$\begin{aligned} f(x) &= (7x^6 + 4x^3 - 50)(9x^{10} - 7x^{1/2}) \\ f'(x) &= (7x^6 + 4x^3 - 50)(90x^9 - \frac{7}{2}x^{-1/2}) + \\ &\quad (42x^5 + 12x^2)(9x^{10} - 7x^{1/2}) \\ &= (7x^6 + 4x^3 - 50)(90x^9 - \frac{7}{2\sqrt{x}}) + \\ &\quad (42x^5 + 12x^2)(9x^{10} - 7\sqrt{x}) \end{aligned}$$

11. $f(x) = (x^{1/2} - x^{1/3})(2x+3)$

$$\begin{aligned} f'(x) &= (\frac{1}{2}x^{-1/2} - \frac{1}{3}x^{-2/3})(2x+3) + (\sqrt{x} - \sqrt[3]{x})(2) \\ &= 2(\sqrt{x} - \sqrt[3]{x}) + (2x+3)\left(\frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}}\right) \end{aligned}$$

12.

$$\begin{aligned} f'(x) &= (\sqrt[3]{x} + 2x)sec^2x + tan\ x\left(\frac{1}{3}x^{-2/3} + 2\right) \\ &= (\sqrt[3]{x} + 2x)sec^2x + tan\ x\left(\frac{1}{3\sqrt[3]{x^2}} + 2\right) \end{aligned}$$

13. $f(x) = x^{1/2}tan\ x$

$$\begin{aligned} f'(x) &= x^{1/2}sec^2x + tan\ x\left(\frac{1}{2}x^{-1/2}\right) \\ &= \sqrt{x}\ sec^2x + \left(\frac{1}{2\sqrt{x}}\right)tan\ x \end{aligned}$$

14.

$$\begin{aligned} g'(x) &= \sqrt[3]{x}\ sec\ x\ tan\ x + sec\ x\left(\frac{1}{3}x^{-2/3}\right) \\ &= \sqrt[3]{x}\ sec\ x\ tan\ x + \frac{sec\ x}{3\sqrt[3]{x^2}} \end{aligned}$$

15. $(2t+3)^2 = (2t+3)(2t+3) = 4t^2 + 12t + 9$
$f'(t) = 8t + 12$

16. $r(t) = (5t-4)^2 = (5t-4)(5t-4)$

$$\begin{aligned} r'(t) &= (5t-4)(5) + (5)(5t-4) \\ &= 25t - 20 + 25t - 20 \\ &= 50t - 40 \end{aligned}$$

17. $g(x) = (0.02x^2 + 1.3x - 11.7)(4.1x + 11.3)$

$$\begin{aligned} g'(x) &= (0.02x^2 + 1.3x - 11.7)(4.1) + \\ &\quad (0.04x + 1.3)(4.1x + 11.3) \end{aligned}$$

18. $g(x) = (3.12x^2 + 10.2x - 5.01)(2.9x^2 + 4.3x - 2.1)$

$$\begin{aligned} g'(x) &= (3.12x^2 + 10.2x - 5.01)(5.8x + 4.3) + \\ &\quad (6.24x + 10.2)(2.9x^2 + 4.3x - 2.1) \end{aligned}$$

19. $g(x) = sec\ x\ csc\ x$

$$\begin{aligned} g'(x) &= sec\ x(-csc\ x\ cot\ x) + csc\ x(sec\ x\ tan\ x) \\ &= \frac{-1}{sin^2x} + \frac{1}{cos^2x} \\ &= sec^2x - csc^2x \end{aligned}$$

20.

$$\begin{aligned} p'(x) &= cot\ x(-csc\ x\ cot\ x) + csc\ x(-csc^2x) \\ &= -csc\ x\ cot^2x - csc^3x \\ &= -csc\ x(cot^2x + csc^2x) \end{aligned}$$

21. $q(x) = \frac{sin\ x}{1 + cos\ x}$

$$\begin{aligned} q'(x) &= \frac{(1 + cos\ x)(cos\ x) - sin\ x(-sin\ x)}{(1 + cos\ x)^2} \\ &= \frac{cos\ x + cos^2x + sin^2x}{(1 + cos\ x)^2} \\ &= \frac{cos\ x + 1}{(1 + cos\ x)^2} \\ &= \frac{1}{1 + cos\ x} \end{aligned}$$

22.

$$\begin{aligned} f'(x) &= \frac{sin\ x(sin\ x) - (1-cos\ x)(cos\ x)}{sin^2 x} \\ &= \frac{sin^2 x - cos\ x + cos^2 x}{sin^2 x} \\ &= \frac{1-cos\ x}{1-cos^2 x} \\ &= \frac{1}{1+cos\ x} \end{aligned}$$

23. $s(t) = tan^2 t$
$s'(t) = 2tan\ t\ sec^2 t$

24.

$$\begin{aligned} s'(t) &= 2sec\ t(sec\ t\ tan\ t) \\ &= 2sec^2 t\ tan\ t \end{aligned}$$

25. $f(x) = (x+2x^{-1})(x^2-3)$

$$\begin{aligned} f'(x) &= (x+2x^{-1})(2x) + (1-2x^{-2})(x^2-3) \\ &= 2x^2 + 4 + x^2 - 3 - 2 + 6x^{-2} \\ &= 3x^2 - 1 + 6x^{-2} \\ &= 3x^2 - 1 + \frac{6}{x^2} \end{aligned}$$

26. $y = 4x^4 - x^3 - 20x^2 + 5x$
$\frac{dy}{dx} = 16x^3 - 3x^2 - 40x + 5$

27. You could use the quotient rule, but a better technique to use would be to rewrite the function as follows:$q(x) = \frac{3}{5}x^2 - \frac{6}{5}x + \frac{4}{5}$ then

$$q'(x) = \frac{6}{5}x - \frac{6}{5}$$

28. $q(x) = \frac{4}{3}x^3 + \frac{2}{3}x^2 - \frac{5}{3}x + \frac{13}{3}$

$$q'(x) = 4x^2 + \frac{4}{3}x - \frac{5}{3}$$

29. $y = \frac{x^2+3x-4}{2x-1}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(2x-1)(2x+3) - (x^2+3x-4)(2)}{(2x-1)^2} \\ &= \frac{2x^2+6x-2x-3-2x^2-6x+8}{(2x-1)^2} \\ &= \frac{-2x+5}{(2x-1)^2} \end{aligned}$$

30. $y = \frac{(x-1)(x-3)}{(x-1)(x+1)} = \frac{x-3}{x+1}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+1)(1) - (x-3)(1)}{(x+1)^2} \\ &= \frac{x+1-x+3}{(x+1)^2} \end{aligned}$$

31. $w = \frac{3t-1}{t^2-2t+6}$

$$\begin{aligned} \frac{dw}{dt} &= \frac{(t^2-2t+6)(3) - (3t-1)(2t-2)}{(t^2-2t+6)^2} \\ &= \frac{3t^2-6t+18-[6t^2-6t-2t+2]}{(t^2-2t+6)^2} \\ &= \frac{-3t^2+2t+16}{(t^2-2t+6)^2} \end{aligned}$$

32. $q = \frac{t^2+7t-1}{2t^2-3t-7}$

$$\begin{aligned} \frac{dq}{dt} &= \frac{(2t^2-3t-7)(2t+7) - (t^2+7t-1)(4t-3)}{(2t^2-3t-7)^2} \\ &= \frac{11t^2-38t-52}{(2t^2-3t-7)^2} \end{aligned}$$

33. $f(x) = \frac{x}{\frac{1}{x}+1} = \frac{x^2}{1+x}$

$$\begin{aligned} f'(x) &= \frac{(1+x)(2x) - (x^2)(1)}{(1+x)^2} \\ &= \frac{2x+2x^2-x^2}{(1+x)^2} \\ &= \frac{2x+x^2}{(1+x)^2} \end{aligned}$$

34. $f(x) = \frac{\frac{1}{x}}{x+\frac{1}{x}} = \frac{1}{x^2+1}$

$$\begin{aligned} f'(x) &= \frac{(x^2+1)(0) - 1(2x)}{(x^2+1)^2} \\ &= \frac{-2x}{(x^2+1)^2} \end{aligned}$$

35. $y = \frac{tan\ t}{1+sec\ t}$ Which can be rewritten as $y = \frac{\frac{sin\ t}{cos\ t}}{1+\frac{1}{cos\ t}}$ multiplying every term by $cos\ t$ to clear the fractions gives $y = \frac{sin\ t}{cos\ t+1}$ which has a derivative of $\frac{dy}{dx} = \frac{1}{1+cos\ t}$ (see problem 21 for details)

36. Rewrite $y = \frac{cos\ t}{sin\ t+1}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(sin\ t+1)(-sin\ t) - cos\ t(cos\ t)}{(sin\ t+1)^2} \\ &= \frac{-sin^2 t - sin\ t - cos^2 t}{(sin\ t+1)^2} \\ &= \frac{-(sin\ t+1)}{(sin\ t+1)^2} \\ &= \frac{-1}{sin\ t+1} \end{aligned}$$

37. $w = \frac{tan\ x + x\ sin\ x}{\sqrt{x}}$

$$\begin{aligned} \frac{dw}{dx} &= \frac{\sqrt{x}(sec^2 x + xcos\ x + sin\ x) - \frac{(tan\ x + x\ sin\ x)}{2\sqrt{x}}}{(\sqrt{x})^2} \\ &= \frac{2x(sec^2 x + xcos\ x + sin\ x) - tan\ x - x\ sin\ x}{2x^{3/2}} \\ &= \frac{2xsec^2 x + 2x^2 cos\ x + x\ sin\ x - tan\ x}{2x^{3/2}} \end{aligned}$$

38.

$$\begin{aligned}\frac{dw}{dx} &= \frac{\sqrt{x}(-sinx + x^2cscxcotx - 2xcscx) - \frac{cosx - x^2cscx}{2\sqrt{x}}}{(\sqrt{x})^2} \\ &= \frac{-sin\ x}{\sqrt{x}} - x^{3/2}csc\ x\ cot\ x + 2x^{5/2}csc\ x \\ &\quad -\frac{cos\ x}{2x^{3/2}} - \frac{\sqrt{x}csc\ x}{2}\end{aligned}$$

39. $y = \frac{1+t^{1/2}}{1-t^{1/2}}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(1-t^{1/2})(\frac{1}{2t^{1/2}}) - (1+t^{1/2})(\frac{-1}{2t^{1/2}})}{(1-t^{1/2})^2} \\ &= \frac{1-t^{1/2}+1+t^{1/2}}{2t^{1/2}(1-\sqrt{t})^2} \\ &= \frac{2}{2\sqrt{t}(1-\sqrt{t})^2} \\ &= \frac{1}{\sqrt{t}(1-\sqrt{t})^2}\end{aligned}$$

40. Rewrite $y = \frac{2x-3x^2}{15x^2+9}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(15x^2+9)(2-6x) - (2x-3x^2)(30x)}{(15x^2+9)^2} \\ &= \frac{30x^2 - 90x^3 + 18 - 54x - 60x^2 - 90x^3}{(15x^2+9)^2} \\ &= \frac{-150x^3 - 60x^2 - 54x + 18}{(15x^2+9)^2}\end{aligned}$$

41. $f(t) = t\ sin\ t\ tan\ t$

$$\begin{aligned}f'(t) &= (t\ sin\ t)(sec^2t) + (tan\ t)(t\ cos\ t + sin\ t) \\ &= t\ sin\ t\ sec^2t + t\ tan\ t\ cos\ t + sin\ t\ tan\ t \\ &= t\ sin\ t\ \frac{1}{cos^2t} + t\ \frac{sin\ t}{cos\ t}cos\ t + sin\ t\ tan\ t \\ &= t\ tan\ t\ sec\ t + t\ sin\ t + sin\ t\ tan\ t\end{aligned}$$

42.

$$\begin{aligned}g'(t) &= (t\ csc\ t)(-sin\ t) + (1 + cos\ t)(-t\ csc\ t\ cot\ t + csc\ t) \\ &= -t - t\ csc\ t\ cot\ t + csc\ t - t\ cot^2t + cot\ t\end{aligned}$$

43. - 84. Left to the student

85. **a)** $f(x) = \frac{x}{x+1}$

$$\begin{aligned}f'(x) &= \frac{(x+1)(1) - x(1)}{(x+1)^2} \\ &= \frac{x+1-x}{(x+1)^2} \\ &= \frac{1}{(x+1)^2}\end{aligned}$$

b) $g(x) = \frac{-1}{x+1}$

$$\begin{aligned}g'(x) &= \frac{(x+1)(0) - (-1)(1)}{(x+1)^2} \\ &= \frac{1}{(x+1)^2} \\ &= \frac{1}{(x+1)^2}\end{aligned}$$

c) Since the graphs of both functions are similar then the average rate of change for the functions will be the same (that is why the answers in part (a) and part (b) are equal).

86. **a)** $f(x) = \frac{x^2}{x^2-1}$

$$\begin{aligned}f'(x) &= \frac{(x^2-1)(2x) - x^2(2x)}{(x^2-1)^2} \\ &= \frac{2x^3 - 2x - 2x^3}{(x^2-1)^2} \\ &= \frac{-2x}{(x^2-1)^2}\end{aligned}$$

b) $g(x) = \frac{1}{x^2-1}$

$$\begin{aligned}g'(x) &= \frac{(x^2-1)(0) - (1)(2x)}{(x^2-1)^2} \\ &= \frac{0-2x}{(x^2-1)^2} \\ &= \frac{-2x}{(x^2-1)^2}\end{aligned}$$

c) Since the graphs of both functions are similar then the average rate of change for the functions will be the same (that is why the answers in part (a) and part (b) are equal).

87. $f(x) = sin^2x + cos^2x$

a)

$$\begin{aligned}f'(x) &= 2sin\ x\ cos\ x + 2cos\ x(-sin\ x) \\ &= 2sin\ x\ cos\ x - 2sin\ x\ cos\ x \\ &= 0\end{aligned}$$

b) Since $sin^2x + cos^2x = 1$ (fundamental trigonometric identity) we would expect the derivative to be zero since we are taking a derivative of a constant, which is always zero.

88. **a)** $f'(x) = 2tan\ x\ sec^2x$

b) $g'(x) = 2sec\ x\ (sec\ x\ tan\ x) = 2tan\ xsec^2x$

c) The answers in parts a) and b) are the same. The reason is that $tan^2x + 1 = sec^2x$ and therefore when we take the derivative we get the same result since the derivative of 1 is zero.

89. $y = \frac{8}{x^2+4}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x^2+4)(0) - 8(2x)}{(x^2+4)^2} \\ &= \frac{0-16x}{(x^2+4)^2} \\ &= \frac{-16x}{(x^2+4)^2}\end{aligned}$$

For the point $(0,2)$, $\frac{dy}{dx}|_{x=0} = m = \frac{-16(0)}{(0^2+4)^2} = 0$. The tangent line is

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ y - 2 &= 0(x - 0) \\ y - 2 &= 0 \\ y &= 2\end{aligned}$$

For the point $(-2,-1)$, $\frac{dy}{dx}|_{x=-2} = m = \frac{-16(-2)}{((-2)^2+4)^2} = \frac{32}{8} = 4$. The tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1) &= 4(x - (-2)) \\ y - 1 &= 4x + 8 \\ y &= 4x + +8 + 1 \\ y &= 4x + 9 \end{aligned}$$

90. $y = \frac{4x}{1+x^2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(1+x^2)(4) - 4x(2x)}{(1+x^2)^2} \\ &= \frac{4 + 4x^2 - 4x^2}{(1+x^2)^2} \\ &= \frac{4}{(1+x^2)^2} \end{aligned}$$

For the point $(0,0)$, $\frac{dy}{dx}|_{x=0} = m = 4$. The tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= 4(x - 0) \\ y &= 4 \end{aligned}$$

For the point $(-1,-2)$, $\frac{dy}{dx}|_{x=-1} = 1$. The tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-2) &= 1(x - (-1)) \\ y + 2 &= x + 1 \\ y &= x - 1 \end{aligned}$$

91. $y = \frac{\sqrt{x}}{x+1} = \frac{x^{1/2}}{x+1}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+1)(\frac{1}{2}x^{-1/2}) - x^{1/2}(1)}{(x+1)^2} \\ &= \frac{\frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2} - x^{1/2}}{(x+1)^2} \\ &= \frac{\frac{1}{2}x^{-1/2}}{(x+1)^2} \\ &= \frac{1}{2\sqrt{x}(x+1)^2} \end{aligned}$$

When $x = 1$, $y = \frac{\sqrt{1}}{1+1} = \frac{1}{2}$, and $\frac{dy}{dx}_{x=1} = m = \frac{1}{2\sqrt{1}(1+1)^2} = \frac{1}{8}$. The tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \frac{1}{2} &= \frac{1}{8}(x - 1) \\ y - \frac{1}{2} &= \frac{1}{8}x - \frac{1}{8} \\ y &= \frac{1}{8}x - \frac{1}{8} + \frac{1}{2} \\ y &= \frac{1}{8}x - \frac{1}{8} + \frac{4}{8} \\ y &= \frac{1}{8}x + \frac{3}{8} \end{aligned}$$

When $x = \frac{1}{4}$, $y = \frac{\sqrt{\frac{1}{4}}}{1+\frac{1}{4}} = \frac{2}{5}$, and $\frac{dy}{dx}|_{x=\frac{1}{4}} = \frac{1}{2\sqrt{\frac{1}{4}}(1+\frac{1}{4})^2} = \frac{16}{25}$. The tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \frac{2}{5} &= \frac{16}{25}(x - \frac{1}{4}) \\ y - \frac{2}{5} &= \frac{16}{25}x - \frac{4}{25} \\ y &= \frac{16}{25}x - \frac{4}{25} + \frac{2}{5} \\ y &= \frac{16}{25}x - \frac{4}{25} + \frac{10}{25} \\ y &= \frac{16}{25}x + \frac{6}{25} \end{aligned}$$

92. $y = \frac{x^2+3}{x-1}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x-1)(2x) - (x^2+3)(1)}{(x-1)^2} \\ &= \frac{2x^2 - 2x - x^2 - 3}{(x-1)^2} \\ &= \frac{(x-3)(x+1)}{(x-1)^2} \end{aligned}$$

When $x = 2$, $y = \frac{2^2+3}{2-1} = 7$, and $\frac{dy}{dx}|_{x=2} = -1$. The tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 7 &= -1(x - 2) \\ y - 7 &= -x + 2 \\ y &= -x + 9 \end{aligned}$$

When $x = 3$, $y = \frac{3^2+3}{3-1} = 6$ and $\frac{dy}{dx}|_3 = 0$. The tangent line is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 6 &= 0(x - 3) \\ y - 6 &= 0 \\ y &= 6 \end{aligned}$$

93. $y = x\ sin\ x$

$$\frac{dy}{dx} = x\ cos\ x + sin\ x$$

When $x = \frac{\pi}{4}$

$$\begin{aligned} \frac{dy}{dx}|_{x=\frac{\pi}{4}} &= m \\ &= \frac{\pi}{4} cos(\frac{\pi}{4}) + sin(\frac{\pi}{4}) \\ &= \frac{\pi}{4}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}(\pi+4)}{8} \end{aligned}$$

The tangent line is

$$y - y_1 = m(x - x_1)$$

$$\begin{aligned}
y - \frac{\sqrt{2}\pi}{8} &= \sqrt{2}\left(\frac{\pi+4}{8}\right)(x - \frac{\pi}{4})\\
y &= \sqrt{2}\left(\frac{\pi+4}{8}\right)x - \frac{\sqrt{2}\pi}{4}\left(\frac{\pi+4}{8}\right) + \frac{\sqrt{2}\pi}{8}\\
y &= \sqrt{2}\left(\frac{\pi+4}{8}\right)x - \frac{\pi^2}{16\sqrt{2}} - \frac{\sqrt{2}\pi}{8} + \frac{\sqrt{2}\pi}{8}\\
y &= \sqrt{2}\left(\frac{\pi+4}{8}\right)x - \frac{\pi^2}{16\sqrt{2}}
\end{aligned}$$

94. $y = x\ tan\ x$

$$\frac{dy}{dx} = x\ sec^2x + tan\ x$$

When $x = \frac{\pi}{4}$

$$\begin{aligned}
\frac{dy}{dx}|_{x=\frac{\pi}{4}} &= m\\
&= \frac{\pi}{4}sec^2(\frac{\pi}{4}) + tan(\frac{\pi}{4})\\
&= \frac{\pi}{4}(\frac{1}{2}) + 1\\
&= \frac{\pi+8}{8}
\end{aligned}$$

The tangent line is

$$\begin{aligned}
y - y_1 &= m(x - x_1)\\
y - \frac{\pi}{4} &= \left(\frac{\pi+8}{8}\right)(x - \frac{\pi}{4})\\
y &= \left(\frac{\pi+8}{8}\right)x - \frac{\pi}{4}\left(\frac{\pi+8}{8}\right) + \frac{\pi}{4}\\
y &= \left(\frac{\pi+8}{8}\right)x - \frac{\pi^2}{32}
\end{aligned}$$

95. **a)** $T(t) = \frac{4t}{t^2+1} + 98.6$

$$\begin{aligned}
\frac{dT}{dt} &= \frac{(t^2+1)(4) - 4t(2t)}{(t^2+1)^2} + 0\\
&= \frac{4t^2 + 4 - 8t^2}{(t^2+1)^2}\\
&= \frac{-4t^2+4}{(t^2+1)^2}
\end{aligned}$$

b) When $t = 2$ hours

$$\begin{aligned}
T &= \frac{4(2)}{2^2+1} + 98.6\\
&= \frac{8}{5} + 98.6\\
&= 100.2 \text{ degrees}
\end{aligned}$$

c) When $t = 2$ hours

$$\begin{aligned}
\frac{dT}{dt} &= \frac{-4(2)^2+4}{(2^2+1)^2}\\
&= \frac{-12}{5}\\
&= -2.4 \text{ degrees per hour}
\end{aligned}$$

96. **a)** $P(t) = 10 + \frac{50t}{2t^2+9}$

$$\begin{aligned}
\frac{dP}{dt} &= \frac{(2t^2+9)(50) - (50t)(4t)}{(2t^2+9)^2}\\
&= \frac{-100t^2+450}{(2t^2+9)^2}
\end{aligned}$$

b) When $t = 8$ years

$$\begin{aligned}
P &= 10 + \frac{50(8)}{2(8)^2+9}\\
&= 10 + \frac{400}{137}\\
&\approx 12.920
\end{aligned}$$

The population will reach approximately 12920 people after 8 years

c) When $t = 12$

$$\begin{aligned}
\frac{dP}{dt} &= \frac{-100(12)^2+450}{(2(12)^2+9)^2}\\
&= \frac{-14400}{88209} \approx -0.163\\
&\approx -163 \text{ people per year}
\end{aligned}$$

97. **a)** Since $\frac{s(t)}{100} = tan\ t$ then $s(t) = 100\ tan\ t$

b) $\frac{d\ s(t)}{dt} = 100sec^2t$

c)

$$\begin{aligned}
100sec^2t &= 200\\
sec^2t &= 2\\
\frac{1}{cos^2t} &= 2\\
cos^2t &= \frac{1}{2}\\
cos\ t &= \pm\frac{1}{\sqrt{2}}\\
t &= \pm\frac{\pi}{4}2n\pi\\
&= \frac{\pi}{4} + \frac{n\pi}{2}
\end{aligned}$$

98. **a)**

$$\begin{aligned}
v(t) &= s'(t)\\
&= \frac{(\sqrt{t}+1)(-3\ sin\ t) - (3\ cos\ t)(\frac{1}{2\sqrt{t}})}{(\sqrt{t}+1)^2}
\end{aligned}$$

b)

$$\begin{aligned}
v(1) &= \frac{(\sqrt{1}+1)(-3\ sin\ 1) - (3\ cos\ 1)(\frac{1}{2\sqrt{1}})}{(\sqrt{1}+1)^2}\\
&= -1.465\ in/sec
\end{aligned}$$

NOTE: the negative sign indicates a downward movement.

c)

$$\begin{aligned} v(1) &= \frac{(\sqrt{\frac{\pi}{3}}+1)(-3\ sin\ \frac{\pi}{3})-(3\ cos\ \frac{\pi}{3})(\frac{1}{2\sqrt{\frac{\pi}{3}}})}{(\sqrt{\frac{\pi}{3}}+1)^2} \\ &= -1.487\ in/sec \end{aligned}$$

99. $g(x)=\frac{(x^2+1)tan\ x}{(x^2-1)}$

$$\begin{aligned} g'(x) &= \frac{(x^2-1)sec^2x(x^2+1)+2x\ tan\ x}{(x^2-1)^2}- \\ &\quad \frac{2x(x^2+1)tan\ x}{(x^2-1)^2} \\ &= \frac{(x^4-1)sec^2x}{(x^2-1)^2}+\frac{2x\ tan\ x}{(x^2-1)^2}- \\ &\quad \frac{2x\ tan\ x}{(x^2-1)^2}-\frac{2x^3tan\ x}{(x^2-1)^2} \\ &= \frac{(x^4-1)sec^2x}{(x^2-1)^2}-\frac{2x^3\ tan\ x}{(x^2-1)^2} \\ &= \frac{sec^2x(x^4-2x^3\ sin\ x\ cos\ x-1)}{(x^2-1)^2} \\ &= \frac{sec^2x(x^4-x^3\ sin(2x)-1)}{(x^2-1)^2} \end{aligned}$$

100.

$$\begin{aligned} g'(t) &= \frac{(t^3+1)[(-csc\ x\ cot\ x)(t^3-1)+csc\ x(3t^2)]}{(t^3+1)^2}- \\ &\quad \frac{3t^2(t^3-1)csc\ x}{(t^3+1)^2} \\ &= \frac{[(-csc\ x\ cot\ x)(t^3-1)+csc\ x(3t^2)]}{(t^3+1)}- \\ &\quad \frac{3t^2(t^3-1)csc\ x}{(t^3+1)^2} \end{aligned}$$

101. $s(t)=\frac{tan\ t}{t\ cos\ t}$

$$\begin{aligned} s'(t) &= \frac{t\ cos\ tsec^2t-tan\ t(-tsin\ t+cos\ t)}{(t\ cos\ t)^2} \\ &= \frac{t\ sec\ t+t\ sin\ t\ tan\ t-sin\ t}{t^2\ cos^2t} \\ &= \frac{sec^2t(t\ sec\ t+t\ sin\ t\ tan\ t-sin\ t}{t^2} \\ &= \frac{t\ sec^3t+t\ sin\ t\ tan\ tsec^2t-sin\ t\ sec^2t}{t^2} \\ &= \frac{sec\ t(t\ sec^2t+t\ sin\ t\ tan\ t\ sec\ t-sin\ t\ sec\ t)}{t^2} \\ &= \frac{sec\ t(t\ sec^2t+t\ tan^2\ t-tan\ t)}{t^2} \\ &= \frac{sec\ t(t\ sec^2t+tan\ t(t\ tan\ t-1))}{t^2} \end{aligned}$$

102. Rewrite $f(x)=\frac{x\ sin\ x-cos\ x}{x\ sin\ x+cos\ x}$,

$$\begin{aligned} f'(x) &= \frac{(x\ sin\ x+cos\ x)(x\ cos\ x+sin\ x+sin\ x)}{(x\ sin\ x+cos\ x)^2} \\ &\quad -\frac{(x\ sin\ x-cos\ x)(x\ cos\ x+sin\ x-sin\ x)}{(x\ sin\ x+cos\ x)^2} \\ &= \frac{2x\ sin^2x+x\ cos^2x}{(x\ sin\ x+cos\ x)^2} \end{aligned}$$

103.

$$\begin{aligned} g'(x) &= \frac{(x+cosx)[-xsinx+cosx(xcosx+sinx)-sec^2x]}{(x+cos\ x)^2} \\ &\quad -\frac{(1-sinx)[xsinxcosx-tanx]}{(x+cosx)^2} \\ &= \frac{-x^2sin^2x+x^2cos^2x+xcosxsinx-xsec^2x}{(x+cosx)^2} \\ &\quad -\frac{xcosxsin^2x+xcos^3x+cos^2xsinx-secx}{(x+cosx)^2} \\ &\quad -\frac{xsinxcosx+tanx+xsin^2xcosx-sinxtanx}{(x+cosx)^2} \\ &= \frac{x\ cos^2x+(x^2+sin\ x)cos^2x-x\ sec^2x}{(x+cos\ x)^2} \\ &\quad +\frac{-x^2sin^2x-sec\ x-sin\ x\ tan\ x+tan\ x}{(x+cos\ x)^2} \end{aligned}$$

104. $f(x)=\frac{x^{1/2}sin\ x-x^{3/2}cos\ x}{x^2+2x+3}$

$$\begin{aligned} f'(x) &= \frac{(x^2+2x+3)[x^{1/2}cos\ x+\frac{sin\ x}{2\sqrt{x}}]}{(x^2+2x+3)^2} \\ &\quad -\frac{(-x^{3/2}sin\ x+\frac{3}{2}x^{1/2}cos\ x)}{(x^2+2x+3)^2} \\ &\quad \frac{(\sqrt{x}sin\ x-x^{3/2}cos\ x)(2x+2)}{(x^2+2x+3)^2} \end{aligned}$$

105. Let $y=(x-1)(x-2)(x-3)$

a)

$$\begin{aligned} \frac{dy}{dx} &= (x-1)[(x-2)(1)+(x-3)(1)]+[(x-2)(x-3)](1) \\ &= (x-1)(x-2)+(x-1)(x-3)+(x-2)(x-3) \\ &= 3x^2-12x+12 \end{aligned}$$

b) $y=(2x+1)(3x-5)(-x+3)$

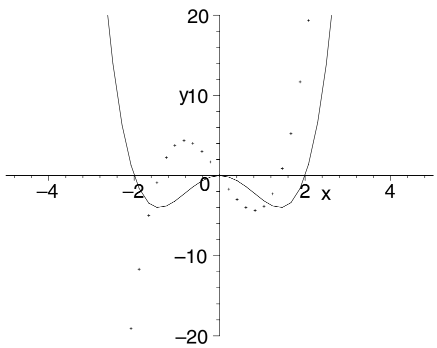
$$\begin{aligned} \frac{dy}{dx} &= (2x+1)(3x-5)(-1)+(2x+3)(-x+3)(3) \\ &\quad +(3x-5)(-x+3)(2) \\ &= -(2x+1)(3x-5)+3(2x+1)(-x+3) \\ &\quad +2(3x-5)(-x+3) \\ &= -18x^2+50x-16 \end{aligned}$$

c) The derivative of a product of three functions is the sum of all possible combinations consisting of the product of two functions and the derivative of the third function.

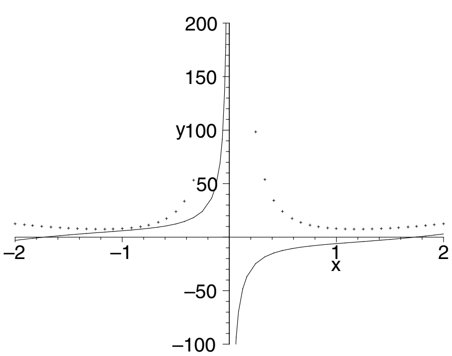
d) The derivative of more than three function is the sum of all possible combinations consisting of the product of three of the functions and the derivative of the fourth function.
Let $y = x(x+1)(2x+3)(-x+1) = -2x^4 - 3x^3 + 2x^2 + 3x$ which has a derivative $y' = -8x^3 - 9x^2 + 4x + 3$. Let us use the rule to find the derivative of y:

$$\begin{aligned} y' &= -x(x+1)(2x+3) + x(2x+3)(-x+1) + \\ &\quad 2x(x+1)(-x+1) + (x+1)(2x+3)(-x+1) \\ &= -2x^3 - 5x^2 - 3x - 2x^3 - x^2 + 3x \\ &\quad -2x^3 + 2x - 2x^3 - 3x^2 + 2x + 3 \\ &= -8x^3 - 9x^2 + 4x + 3 \end{aligned}$$

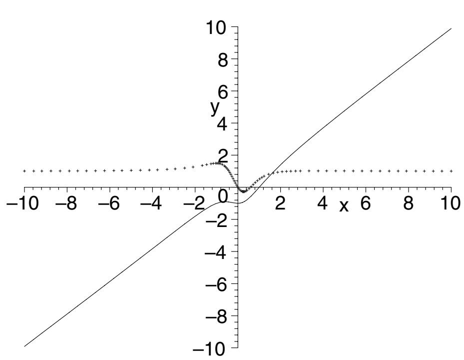
106. $(\pm\ 1.414, -4)$



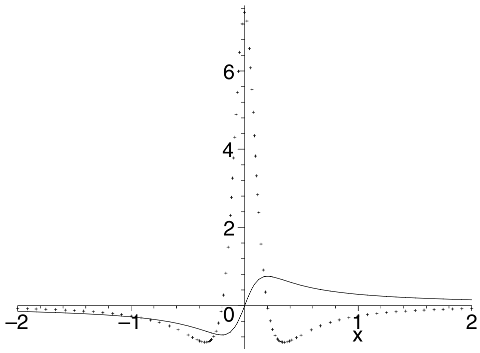
107. No horizontal tangent lines



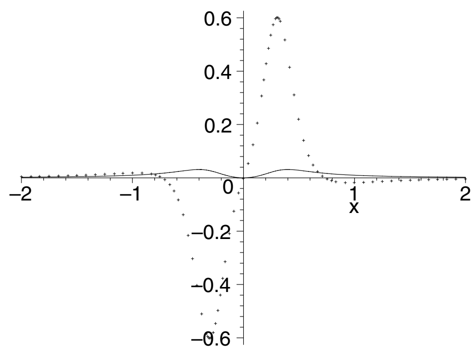
108. $(0, -1)$



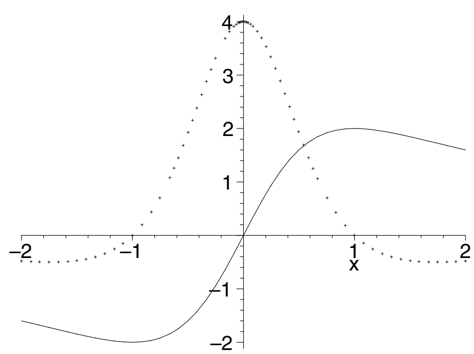
109. $(0.2, 0.75)$ and $(-0.2, -0.75)$



110. $(0, 0)$ and $(\pm\ 0.213, 0.0164)$



111. $(1, 2)$ and $(-1, -2)$



112. $y = \frac{4-4x^2}{(x^2+1)^2}$ seems to be the correct derivative of $y = \frac{4x}{x^2+1}$

Exercise Set 2.8

1. $y = (2x+1)^2$
Method One (chain rule):

$$\begin{aligned} \frac{dy}{dx} &= 2(2x+1)(2) \\ &= 4(2x+1) \\ &= 8x+4 \end{aligned}$$

Method Two (product rule): $y = (2x+1)(2x+1)$

$$\begin{aligned} \frac{dy}{dx} &= (2x+1)(2) + (2x+1)(2) \\ &= 4x+2+4x+2 \\ &= 8x+4 \end{aligned}$$

Method Three (expand first):

$$\begin{aligned} y &= 4x^2+4x+1 \\ \frac{dy}{dx} &= 4(2x)+4 \\ &= 8x+4 \end{aligned}$$

2. $y=(3-2x)^2$
Method One:

$$\begin{aligned} \frac{dy}{dx} &= 2(3-2x)(-2) \\ &= -4(3-2x) \\ &= -12+8x \end{aligned}$$

Method Two:

$$\begin{aligned} y &= (3-2x)(3-2x) \\ \frac{dy}{dx} &= (3-2x)(-2)+(3-2x)(-2) \\ &= -6+4x-6+4x \\ &= -12+8x \end{aligned}$$

Method Three:

$$\begin{aligned} y &= 9-12x+4x^2 \\ \frac{dy}{dx} &= -12+4x \end{aligned}$$

3. $y=(1-x)^{55}$

$$\begin{aligned} \frac{dy}{dx} &= 55(1-x)^{54}(-1) \\ &= -55(1-x)^{54} \end{aligned}$$

4. $y=tan^2x$, $\frac{dy}{dx}=2\ tan\ x\ sec^2x$

5. sec^2x

$$\begin{aligned} \frac{dy}{dx} &= 2\ sec\ x\ sec\ x\ tan\ x \\ &= 2\ tan\ x\ sec^2x \end{aligned}$$

6. $y=(2x+3)^{10}$
$\frac{dy}{dx}=10(2x+3)^9(2)=20(2x+3)^9$

7. $y=\sqrt{1-3x}=(1-3x)^{1/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(1-3x)^{-1/2}(-3) \\ &= \frac{-3}{2\sqrt{1-3x}} \end{aligned}$$

8. $y=\sqrt[3]{x^2+1}=(x^2+1)^{1/3}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{3}(x^2+1)^{-2/3}(2x) \\ &= \frac{2x}{3\sqrt[3]{(x^2+1)^2}} \end{aligned}$$

9. $y=\frac{2}{3x^2+1}=2(3x^2+1)^{-1}$

$$\begin{aligned} \frac{dy}{dx} &= 2(-1)(3x^2+1)^{-2}(6x) \\ &= \frac{-12x}{(3x^2+1)^2} \end{aligned}$$

10. $y=3(2x+4)^{-1/2}$

$$\begin{aligned} \frac{dy}{dx} &= 3(-\frac{1}{2}(2x+4)^{-3/2}(2)) \\ &= \frac{-3}{\sqrt{(2x+4)^3}} \end{aligned}$$

11. $s(t)=t(2t+3)^{1/2}$

$$\begin{aligned} s'(t) &= t\left(\frac{1}{2}(2t+3)^{-1/2}(2)\right)+(2t+3)^{1/2}(1) \\ &= \frac{t}{\sqrt{2t+3}}+\sqrt{2t+3} \end{aligned}$$

12.

$$\begin{aligned} s'(t) &= t^2\left(\frac{1}{3}(3t+4)^{-2/3}(3)\right)+2t(3t+4)^{1/3} \\ &= \frac{t^2}{\sqrt[3]{3t+4}}+2t\ \sqrt[3]{3t+4} \end{aligned}$$

13. $s(t)=sin\left(\frac{\pi}{6}\ t+\frac{\pi}{3}\right)$

$$\begin{aligned} s'(t) &= cos\left(\frac{\pi}{6}\ t+\frac{\pi}{3}\right)\cdot\frac{\pi}{6} \\ &= \frac{\pi}{6}\ cos\left(\frac{\pi}{6}\ t+\frac{\pi}{3}\right) \end{aligned}$$

14. $s'(t)=-sin(3t-4)(3)=-3sin(3t-4)$

15. $g(x)=(1+x^3)^3-(1+x^3)^4$

$$\begin{aligned} g'(x) &= 3(1+x^3)^2(3x^2)-4(1+x^3)^3(3x^2) \\ &= 9x^2(1+x^3)^2-12x^2(1+x^3)^3 \end{aligned}$$

16.

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(1+sec\ x)^{-1/2}(sec\ x\ tan\ x) \\ &= \frac{sec\ x\ tan\ x}{2\sqrt{1+sec\ x}} \end{aligned}$$

17. $y=\sqrt{1-csc\ x}=(1-csc\ x)^{1/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(1-csc\ x)^{-1/2}(-(-csc\ x\ cot\ x)) \\ &= \frac{csc\ x\ cot\ x}{2\sqrt{1-csc\ x}} \end{aligned}$$

18.

$$\begin{aligned} g'(x) &= \frac{1}{2}x^{-1/2}+3(x-3)^2 \\ &= \frac{1}{2\sqrt{x}}+3(x-3)^2 \end{aligned}$$

19. $g(x) = (2x-1)^{1/3} + (4-x)^2$

$$\begin{aligned} g'(x) &= \frac{1}{3}(2x-1)^{-2/3}(2) + 2(4-x)(-1) \\ &= \frac{2}{3\sqrt[3]{(2x-1)^2}} - 2(4-x) \end{aligned}$$

20.

$$\begin{aligned} \frac{dy}{dx} &= x^2(-sin\ x) + 2x\ cos\ x - 2x\ cos\ x \\ &\quad +2sin\ x - 2(-csc^2 x) \\ &= -x^2 sin\ x + 2sin\ x + 2csc^2 x \end{aligned}$$

21. $y = x^3 sin\ x + 5x\ cos\ x + 4sec\ x$

$$\begin{aligned} \frac{dy}{dx} &= x^3 cos\ x + 3x^2 sin\ x + 5x(-sin\ x) \\ &\quad +5cos\ x + 4sec\ x\ tan\ x \\ &= x^3 cos\ x + 3x^2 sin\ x - 5x\ sin\ x \\ &\quad +5cos\ x + 4sec\ x\ tan\ x \end{aligned}$$

22.

$$\begin{aligned} f'(x) &= (2x+3)^{1/2}(2x+3) + \frac{1}{2}(2x+3)^{-1/2}(2)(x^2+3x+1) \\ &= (2x+3)^{3/2} + \frac{x^2+3x+1}{\sqrt{2x+3}} \end{aligned}$$

23. $y = (x^2+x^3)^{1/2}(2x^2+3x+5)$

$$\begin{aligned} \frac{dy}{dx} &= (x^2+x^3)^{1/2}(4x+3) \\ &\quad +\frac{1}{2}(x^2+x^3)^{-1/2}(2x+3x^2)(2x^2+3x+5) \\ &= (4x+3)\sqrt{x^2+x^3} + \frac{(2x+3x^2)(2x^2+3x+5)}{2\sqrt{x^2+x^3}} \end{aligned}$$

24. $\frac{dy}{dx} = -csc(1/x)(-1/x^2) = \frac{csc(1/x)}{x^2}$

25. $f(t) = cos\sqrt{t}$

$$\begin{aligned} f'(t) &= -sin\sqrt{t}\left(\frac{1}{2}t^{-1/2}\right) \\ &= \frac{-sin\sqrt{t}}{2\sqrt{t}} \end{aligned}$$

26.

$$\begin{aligned} f'(t) &= cos\sqrt{t}\left(\frac{1}{2}t^{-1/2}\right) \\ &= \frac{cos\sqrt{t}}{2\sqrt{t}} \end{aligned}$$

27. $f(x) + (3x+2)(2x+5)^{1/2}$

$$\begin{aligned} f'(x) &= (3x+2)\left(\frac{1}{2}(2x+5)^{-1/2}(2)\right) + 3(2x+5)^{1/2} \\ &= \frac{3x+2}{\sqrt{2x+5}} + 3\sqrt{2x+5} \end{aligned}$$

28.

$$\begin{aligned} f'(x) &= (5x-2)\left(\frac{1}{2}(3x+4)^{-1/2}(3)\right) + 5(3x+4)^{1/2} \\ &= \frac{3(5x-2)}{2\sqrt{3x+4}} + 5\sqrt{3x+5} \end{aligned}$$

29.

$$\begin{aligned} \frac{dy}{dx} &= cos(cos\ x)(-sin\ x) \\ &= -sin\ x\ cos(cos\ x) \end{aligned}$$

30. $\frac{dy}{dx} = sec^2(sin\ x)(cos\ x)$

31. $y = (cos(4t))^{1/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(cos(4t))^{-1/2}(-sin(4t)(4)) \\ &= \frac{-2sin(4t)}{\sqrt{cos(4t)}} \end{aligned}$$

32.

$$\begin{aligned} f'(x) &= \frac{(x^3-1)^5[4(x^2+3)^3(2x)]}{(x^3-1)^{10}} - \\ &\quad \frac{(x^2+3)^4[5(x^3-1)^4(3x^2)]}{(x^3-1)^{10}} \\ &= \frac{8x(x^3-1)^5(x^2+3)^3 - 15x^2(x^2+3)^4(x^3-1)^4}{(x^3-1)^{10}} \end{aligned}$$

33. $f(x) = \frac{(x^3+2x^2+3x-1)^3}{(2x^4+1)^2}$

$$\begin{aligned} f'(x) &= \frac{(2x^4+1)^2[3(x^3+2x+3x-1)^2(3x^2+4x+3)]}{(2x^4+1)^4} - \\ &\quad \frac{(x^3+2x^2+3x-1)^3[2(2x^4+1)(8x^3)]}{(2x^4+1)^4} \\ &= \frac{3(x^3+2x^2+3x-1)^2(2x^4+1)^2(3x^2+4x+3)}{(2x^4+1)^4} - \\ &\quad \frac{16x^3(x^3+2x^2+3x-1)^3(2x^4+1)}{(2x^4+1)^4} \end{aligned}$$

34. $y = \left(\frac{2x+3}{3x-5}\right)^{1/3}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{3}\left(\frac{2x+3}{3x-5}\right)^{-2/3}\left[\frac{(3x-5)(2)-(2x+3)(3)}{(3x-5)^2}\right] \\ &= \frac{6x-10-6x-9}{3(3x-5)^{4/3}\sqrt[3]{(3x-5)^2}} \\ &= \frac{-19}{3(3x-5)^{4/3}\sqrt[3]{(3x-5)^2}} \end{aligned}$$

35. $y = \left(\frac{3x-4}{5x+3}\right)^{1/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}\left(\frac{3x-4}{5x+3}\right)^{-1/2}\left[\frac{(5x+3)(3)-(3x-4)(5)}{(5x+3)^2}\right] \\ &= \frac{15x+9-15x+20}{2\sqrt{3x-4}(5x+3)^{3/2}} \\ &= \frac{29}{2\sqrt{3x-4}(5x+3)^{3/2}} \end{aligned}$$

36.

$$\begin{aligned} f'(x) &= sec^2(x\sqrt{x-1})\left(x[\frac{1}{2}(x-1)^{-1/2}] + (x-1)^{1/2}(1)\right) \\ &= \frac{x\ sec^2(x\sqrt{x-1})}{2\sqrt{x-1}} + \sqrt{x-1}\ sec^2(x\sqrt{x-1}) \end{aligned}$$

37. $r(x) = x(0.01x^2 + 2.391x - 8.51)^5$

$$\begin{aligned} r'(x) &= x[5(0.01x^2 + 2.391x - 8.51)^4(0.02x + 2.391)] + \\ &\quad (0.01x^2 + 2.391x - 8.51)^5(1) \\ &= 5x(0.02x + 2.391)(0.01x^2 + 2.391x - 8.51)^4 + \\ &\quad (0.01x^2 + 2.391x - 8.51)^5 \\ &= (0.01x^2 + 2.391x - 8.51)^4(5x(0.02x + 2.391) + \\ &\quad (0.01x^2 + 2.391x - 8.51)) \\ &= (0.01x^2 + 2.391x - 8.51)^4(0.1x^2 + 11.955x + \\ &\quad 0.01x^2 + 2.391x - 8.51) \\ &= (0.01x^2 + 2.391x - 8.51)^4(0.11x^2 + \\ &\quad 14.346x - 8.51) \end{aligned}$$

38. $r(x) = (3.21x - 5.87)^3(2.36x - 5.45)^5$

$$\begin{aligned} r'(x) &= (3.21x - 5.87)^3[5(2.36x - 5.45)^4(2.36)] + \\ &\quad (2.36x - 5.45)^5[3(3.21x - 5.87)^2(3.21)] \\ &= (2.36x - 5.45)^4(3.21x - 5.87)^2(11.8) \\ &\quad (3.21x - 5.87) + 9.63(2.36x - 5.45)) \\ &= (2.36x - 5.45)^4(3.21x - 5.87)^2(60.6x - 121.75) \end{aligned}$$

39. $y = (cot\ 5x - cos\ 5x)^{1/5}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{5}(cot\ 5x - cos\ 5x)^{-4/5}(-csc^2(5x)(5) + sin(5x)(5)) \\ &= \frac{sin\ 5x - csc\ 5x}{(cot\ 5x - cos\ 5x)^{4/5}} \end{aligned}$$

40.

$$\begin{aligned} \frac{dy}{dx} &= -csc\ x - cos(cos^2 x)(2cos\ x(-sin\ x)) \\ &= -csc\ x + 2sin\ x\ cos\ x\ cos(cos^2 x) \end{aligned}$$

41. $y = sin(sec^4(x^2))$

$$\begin{aligned} \frac{dy}{dx} &= cos(sec^4(x^2)) \cdot 4sec^3(x^2)(sec\ x^2\ tan\ x^2) \cdot 2x \\ &= 8x\ sec^4(x^2)\ tan(x^2)\ cos(sec^4(x^2)) \end{aligned}$$

42. $y = ((2x+3)^{1/2} + 1)^{1/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}((2x+3)^{1/2} + 1)^{-1/2}\ \left(\frac{1}{2}(2x+3)^{-1/2}(2)\right) \\ &= \frac{1}{2\sqrt{\sqrt{2x+3}+1}\sqrt{2x+3}} \end{aligned}$$

43. $y = ((x^2+2)^{1/4} + 1)^{1/3}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{3}((x^2+2)^{1/4} + 1)^{-2/3}\ \left(\frac{1}{4}(x^2+2)^{-3/4}(2x)\right) \\ &= \frac{x}{6\sqrt[4]{(x^2+2)^3}\sqrt[3]{(\sqrt[4]{x^2+2}+1)^2}} \end{aligned}$$

44.

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+sinx)^2(1) - x[2(x+xsinx)(1+xcosx+sinx)]}{(x+sinx)^4} \\ &= \frac{(x+sinx)^2 + (-2x^2 - 2x^2 sinx)(1+xcosx+sinx)}{(x+sin\ x)^4} \end{aligned}$$

45. $y = \frac{sin^3 x}{x^2+5}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x^2+5)3sin^2 x\ cos\ x - sin^3 x(2x)}{(x^2+5)^2} \\ &= \frac{3x^2\ sin^2 x\ cos\ x + 15sin^2 x\ cos\ x - 2x\ sin^3 x}{(x^2+5)^2} \\ &= \frac{sin^2 x(3x^2\ cos\ x - 2x\ sin\ x + 15\ cos\ x}{(x^2+5)^2} \end{aligned}$$

46.

$$\begin{aligned} \frac{dy}{dx} &= 2\ tan(\sqrt{t+2})sec^2(\sqrt{t+2}) \cdot \frac{1}{2\sqrt{t+2}} \\ &= \frac{tan(\sqrt{t+2})\ sec^2(\sqrt{t+2})}{\sqrt{t+2}} \end{aligned}$$

47. $f(x) = cot^3(x\ sin(2x+4))$

$$\begin{aligned} f'(x) &= 3cot^2(x\ sin(2x+4))(-csc^2(x\ sin(2x+4)) \times \\ &\quad [x\ cos(2x+4)(2) + sin(2x+4)] \\ &= -3cot^2(x\ sin(2x+4))csc^2(x\ sin(2x+4)) \times \\ &\quad [2x\ cos(2x+4) + sin(2x+4)] \end{aligned}$$

48.

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2\sqrt{2+cos^2 t}} \cdot 2\ cos\ t\ (-sin\ t) \\ &= \frac{-cos\ t\ sin\ t}{\sqrt{2+cos^2 t}} \end{aligned}$$

49. $y = \sqrt{sec^4 x + x}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2\sqrt{sec^4 x + x}} \cdot (4sec^3 x(sec\ x\ tan\ x) + 1) \\ &= \frac{2\ sec^4\ x\ tan\ x + 1}{\sqrt{sec^4 x + x}} \end{aligned}$$

50.

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2\sqrt{x + csc\ x}} \cdot (1 - csc\ x\ cot\ x) \\ &= \frac{1 - csc\ x\ cot\ x}{2\sqrt{x + csc\ x}} \end{aligned}$$

51. $y = \sqrt{u} = u^{1/2}, \quad u = x^2 - 1$

$$\frac{dy}{du} = \frac{1}{2}u^{1/2-1} = \frac{1}{2}u^{-1/2} = \frac{1}{2\sqrt{u}}$$

$$\frac{du}{dx} = 2x$$

$$\begin{aligned} \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} &= \frac{1}{2\sqrt{u}} \cdot 2x \\ &= \frac{2x}{2\sqrt{x^2-1}} \\ &= \frac{x}{\sqrt{x^2-1}} \end{aligned}$$

52. $y = \frac{15}{u^3} = 15u^{-3}$, $u = 2x+1$

$$\frac{dy}{du} = -45u^{-4}, \text{ or } \frac{-45}{u^4}$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{-45}{u^4} \cdot 2 = \frac{-90}{(2x+1)^4}$$

53. $y = u^{50}$, $u = 4x^3 - 2x^2$

$$\frac{dy}{du} = 50u^{49}$$

$$\frac{du}{dx} = 12x^2 - 4x$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= 50u^{49}(12x^2 - 4x) \\ &= 50(4x^3 - 2x^2)^{49}(12x^2 - 4x)\end{aligned}$$

54. $y = \frac{u+1}{u-1}$, $u = 1 + \sqrt{x} = 1 + x^{1/2}$

$$\begin{aligned}\frac{dy}{du} &= \frac{(u-1)(1) - 1 \cdot (u+1)}{(u-1)^2} \\ &= \frac{u-1-u-1}{(u-1)^2} \\ &= \frac{-2}{(u-1)^2}\end{aligned}$$

$$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$$

$$\begin{aligned}\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} &= \frac{-2}{(u-1)^2} \cdot \frac{1}{2}x^{-1/2} \\ &= \frac{-2}{(1 + x^{1/2} - 1)^2} \cdot \frac{1}{2}x^{-1/2} \\ &= \frac{-1}{(x^{1/2})^2} \cdot x^{-1/2} \\ &= \frac{-1}{x} \cdot x^{-1/2} \\ &= -x^{-3/2}\end{aligned}$$

55. $y = u(u+1)$, $u = x^3 - 2x$

$$\begin{aligned}\frac{dy}{du} &= u \cdot 1 + 1 \cdot (u+1) \\ &= u + u + 1 \\ &= 2u + 1\end{aligned}$$

$$\frac{du}{dx} = 3x^2 - 2$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} = (2u+1)(3x^2 - 2) \\ &= [2(x^3 - 2x) + 1](3x^2 - 2) \\ &= (2x^3 - 4x + 1)(3x^2 - 2)\end{aligned}$$

56. $y = (u+1)(u-1)$, $u = x^3 + 1$

$$\begin{aligned}\frac{dy}{du} &= (u+1)(1) + 1 \cdot (u-1) \\ &= u + 1 + u - 1 \\ &= 2u\end{aligned}$$

$$\frac{du}{dx} = 3x^2$$

$$\begin{aligned}\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} &= 2u \cdot 3x^2 \\ &= 2(x^3+1)(3x^2) \\ &= 6x^2(x^3+1)\end{aligned}$$

57. $y = \sqrt{x^2 + 3x} = (x^2 + 3x)^{1/2}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2}(x^2 + 3x)^{-1/2}(2x+3) \\ &= \frac{2x+3}{2\sqrt{x^2+3x}}\end{aligned}$$

When $x = 1$, $$\begin{aligned}\frac{dy}{dx} &= \frac{2 \cdot 1 + 3}{2\sqrt{1^2 + 3 \cdot 1}} \\ &= \frac{2+3}{2\sqrt{4}} \\ &= \frac{5}{2 \cdot 2} \\ &= \frac{5}{4}\end{aligned}$$

Thus, at $(1, 2)$, $m = \frac{5}{4}$. We use point-slope equation.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ y - 2 &= \frac{5}{4}(x-1) \\ y - 2 &= \frac{5}{4}x - \frac{5}{4} \\ y &= \frac{5}{4}x + \frac{3}{4}\end{aligned}$$

58. $y = (x^3 - 4x)^{10}$

$$\frac{dy}{dx} = 10(x^3 - 4x)^9(3x^2 - 4)$$

When $x = 2$, $$\begin{aligned}\frac{dy}{dx} &= 10(2^3 - 4 \cdot 2)^9(3 \cdot 2^2 - 4) \\ &= 10(8-8)^9(12-4) \\ &= 10 \cdot 0 \cdot 8 \\ &= 0\end{aligned}$$

Use the point-slope equation:

$$\begin{aligned}y - 0 &= 0(x-2) \\ y &= 0\end{aligned}$$

59. $y = x\sqrt{2x+3} = x(2x+3)^{1/2}$

$$\begin{aligned}\frac{dy}{dx} &= x \cdot \frac{1}{2}(2x+3)^{-1/2}(2) + 1 \cdot (2x+3)^{1/2} \\ &= \frac{x}{\sqrt{2x+3}} + \sqrt{2x+3}\end{aligned}$$

When $x = 3$, $\dfrac{dy}{dx} = \dfrac{3}{\sqrt{2\cdot 3+3}} + \sqrt{2\cdot 3+3}$

$= \dfrac{3}{\sqrt{9}} + \sqrt{9}$

$= \dfrac{3}{3} + 3$

$= 1 + 3 = 4$

Thus, at $(3,9)$, $m = 4$. We use point-slope equation.

$$y - y_1 = m(x - x_1)$$
$$y - 9 = 4(x-3)$$
$$y - 9 = 4x - 12$$
$$y = 4x - 3$$

60. $y - \left(\dfrac{2x+3}{x-1}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{2x+3}{x-1}\right)^2\left[\frac{(x-1)(2)-1\cdot(2x+3)}{(x-1)^2}\right]$$
$$= 3\left(\frac{2x+3}{x-1}\right)^2\left(\frac{2x-2-2x-3}{(x-1)^2}\right)$$
$$= 3\left(\frac{2x+3}{x-1}\right)^2\left(\frac{-5}{(x-1)^2}\right)$$

When $x = 2, \dfrac{dy}{dx} = 3\left(\dfrac{2\cdot 2+3}{2-1}\right)^2\left(\dfrac{-5}{(2-1)^2}\right)$

$= 3(7^2)(-5) = -735$

Use the point-slope equation:

$$y - 343 = -735(x-2)$$
$$y - 343 = -735x + 1470$$
$$y = -735x + 1813$$

61. $f(x) = sin^2 x$

$$\frac{dy}{dx} = 2\ sin\ x\ cos\ x$$

When $x = -\frac{\pi}{6}$, $\frac{dy}{dx} = 2sin(\frac{-\pi}{6})cos(\frac{-\pi}{6}) = \frac{-\sqrt{3}}{2}$

Use the point-slope equation:

$$y - \tfrac{1}{4} = \tfrac{-\sqrt{3}}{2}(x - (-\tfrac{\pi}{6}))$$
$$y - \tfrac{1}{4} = -\tfrac{\sqrt{3}}{2}x - \tfrac{\sqrt{3}}{12}$$
$$y = -\tfrac{\sqrt{3}}{2}x - \tfrac{\sqrt{3}}{12} + \tfrac{1}{4}$$
$$y = \tfrac{1}{12}(-6\sqrt{3}x - \sqrt{3}\pi + 3)$$

62. $f(x) = x\ sin\ 2x$

$$\frac{dy}{dx} = 2x\ cos\ 2x + sin\ 2x$$

When $x = \pi$, $\frac{dy}{dx} = 2\pi\ cos\ 2\pi + sin\ 2\pi = 2\pi$

Use the point-slope equation:

$$y - 0 = 2\pi(x - \pi)$$
$$y = 2\pi x - 2\pi^2$$

63. $f(x) = \frac{x^2}{(1+x)^5}$

a)

$$f'(x) = \frac{(1+x)^5(2x) - x^2(5(1+x)^4)}{(1+x)^{10}}$$
$$= \frac{(1+x)^4[2x + 2x^2 - 5x^2]}{(1+x)^{10}}$$
$$= \frac{2x - 3x^2}{(1+x)^6}$$

b)

$$f'(x) = x^2[-5(1+x)^{-6}(1)] + 2x(1+x)^{-5}$$
$$= \frac{-5x^2}{(1+x)^6} + \frac{2x}{(1+x)^5}$$
$$= \frac{-5x^2 + 2x(1+x)}{(1+x)^6}$$
$$= \frac{2x - 3x^2}{(1+x)^6}$$

c) The results in the previous parts are the same.

64. a) $g(x) = (x^3 + 5x)^2$

$$g'(x) = 2(x^3+5x)(3x^2+5)$$
$$= 2(3x^5 + 20x^3 + 25x)$$
$$= 6x^5 + 40x^3 + 50x$$

b) $g(x) = x^6 + 10x^4 + 25x^2$

$$g'(x) = 6x^5 + 40x^3 + 50x$$

c) The answers are the same

65. Using the Chain Rule:

Let $y = f(u)$. Then

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$
$$= 3u^2(8x^3)$$

$= 3(2x^4+1)^2(8x^3)$ Substituting $2x^4 + 1$ for u

When $x = -1$, $\dfrac{dy}{dx} = 3[2(-1)^4 + 1]^2[8(-1)^3]$

$= 3(2+1)^2(-8)$

$= 3\cdot 3^2(-8)$

$= -216$

Finding $f(g(x))$:

$f\circ g(x) = f(g(x)) = f(2x^4+1) = (2x^4+1)^3$

Then $(f\circ g)'(x) = 3(2x^4+1)^2(8x^3)$ and

$(f\circ g)'(-1) = -216$ as above.

66. Using the Chain Rule:

Let $y = f(u)$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\cdot\frac{du}{dx}\\ &= \frac{(u-1)(1)-(1)(u+1)}{(u-1)^2}\cdot\frac{1}{2}x^{-1/2}\\ &= \frac{u-1-u-1}{(u-1)^2}\cdot\frac{1}{2}x^{-1/2}\\ &= \frac{-2}{(u-1)^2}\cdot\frac{1}{2\sqrt{x}}\\ &= \frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}\end{aligned}$$

When $x = 4$, $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{4}(\sqrt{4}-1)^2} = \dfrac{-1}{2\cdot 1^2} = -\dfrac{1}{2}$.

Finding $f(g(x))$:

$$f\circ g(x) = f(g(x)) = f(\sqrt{x}) = \frac{\sqrt{x}+1}{\sqrt{x}-1} = \frac{x^{1/2}+1}{x^{1/2}-1}$$

$$\begin{aligned}\text{Then } (f\circ g)'(x) &= \frac{(x^{1/2}-1)\left(\frac{1}{2}x^{-1/2}\right)-\frac{1}{2}x^{-1/2}(x^{1/2}+1)}{(x^{1/2}-1)^2}\\ &= \frac{\frac{1}{2}-\frac{1}{2}x^{-1/2}-\frac{1}{2}-\frac{1}{2}x^{-1/2}}{(x^{1/2}-1)^2}\\ &= \frac{-x^{-1/2}}{(x^{1/2}-1)^2}\\ &= \frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}\end{aligned}$$

and $(f\circ g)'(4) = -\dfrac{1}{2}$ as above.

67. Using the Chain Rule:

Let $y = f(u) = \sqrt[3]{u} = u^{1/3}$. Then

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\cdot\frac{du}{dx}\\ &= \frac{1}{3}u^{-2/3}\cdot(-6x)\\ &= -2x\cdot u^{-2/3}\\ &= -2x(1-3x^2)^{-2/3} \quad \text{Substituting } 1-3x^2 \text{ for } u\end{aligned}$$

$$\begin{aligned}\text{When } x = 2,\quad \frac{dy}{dx} &= 2\cdot 2(1-3\cdot 2^2)^{-2/3}\\ &= -4(-11)^{-2/3} \approx -0.8087\end{aligned}$$

Finding $f(g(x))$:

$f\circ g(x) = f(g(x)) = f(1-3x^2) = \sqrt[3]{1-3x^2}$, or $(1-3x^2)^{1/3}$

Then $(f\circ g)'(x) = \dfrac{1}{3}(1-3x^2)^{-2/3}(-6x) =$ $-2x(1-3x^2)^{-2/3}$ and

$(f\circ g)'(2) = -4(-11)^{-2/3} \approx -0.8087$ as above.

68. Using the Chain Rule:

Let $y = f(u)$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\cdot\frac{du}{dx}\\ &= 10u^4\cdot\frac{(4+x)(-1)-(1)(3-x)}{(4+x)^2}\\ &= 10u^4\cdot\frac{-4-x-3+x}{(4+x)^2}\\ &= 10\left(\frac{3-x}{4+x}\right)^4\cdot\frac{-7}{(4+x)^2}\end{aligned}$$

$$\begin{aligned}\text{When } x = -10,\ \frac{dy}{dx} &= 10\left(\frac{3+10}{4-10}\right)^4\cdot\frac{-7}{(4-10)^2}\\ &= 10\left(\frac{13}{-6}\right)^4\left(\frac{-7}{(-6)^2}\right)\\ &= \frac{-70\cdot 13^4}{6^6} \approx -42.8513\end{aligned}$$

Finding $f(g(x))$:

$$f\circ g(x) = f(g(x)) = 2\left(\frac{3-x}{4+x}\right)^5$$

$$\begin{aligned}\text{Then } (f\circ g)'(x) &= 2\cdot 5\left(\frac{3-x}{4+x}\right)^4\cdot\frac{4+x(-1)-(1)(3-x)}{(4+x)^2}\\ &= 10\left(\frac{3-x}{4+x}\right)^4\cdot\frac{-4-x-3+x}{(4+x)^2}\\ &= 10\left(\frac{3-x}{4+x}\right)^4\cdot\frac{-7}{(4+x)^2}\end{aligned}$$

and $(f\circ g)'(-10) = 10\left(\dfrac{3+10}{4-10}\right)^4\cdot\dfrac{-7}{(4-10)^2} =$ $\dfrac{-70\cdot 13^4}{6^6} \approx -42.8513$ as above.

69. $A = 1000(1+i)^3$

a)

$$\begin{aligned}\frac{dA}{di} &= 1000(3(1+i)^2)\\ &= 3000(1+i)^2\end{aligned}$$

b) $\frac{dA}{di}$ represents the rate at which the amount of investment is changing with respect to an annual interest rate i.

70. **a)**

$$\begin{aligned}\frac{dA}{di} &= 1000\left(20(1+\frac{i}{4})^{19}(\frac{1}{4})\right)\\ &= 5000(1+\frac{i}{4})^{19}\end{aligned}$$

b) $\frac{dA}{di}$ represents the rate at which the amount of investment is changing with respect to an quarterly componded interest rate i.

71. $D = 0.85A(c+25)$, $c = (140-y)\dfrac{w}{72x}$

a) To find D as a function of c, we substitute 5 for A in the formula for D.

$$D = 0.85A(c+25)$$

$$\begin{aligned} &= 0.85(5)(c+25) \\ &= 4.25(c+25) \\ &= 4.25(c+25) \\ &= 4.25c+106.25 \end{aligned}$$

To find c as a function of w, we substitute 45 for y and 0.6 for x in the formula for c.

$$\begin{aligned} c &= (140-45)\frac{w}{72(0.6)} \\ &= 95\cdot\frac{w}{43.2} \\ &\approx 2.199w \end{aligned}$$

b) $\dfrac{dD}{dc} = 4.25$

c) $\dfrac{dc}{dw} = 2.199$

d) First we find $D\circ c(w)$.

$$\begin{aligned} D\circ c(w) &= D(c(w)) \\ &= 4.25(2.199w)+106.25 \\ &= 9.34575w+106.25 \end{aligned}$$

Then we have

$$\frac{dD}{dw} = 9.34575 \approx 9.346.$$

e) $\dfrac{dD}{dw}$ represents the rate of change of the dosage with respect to the patient's weight. For each additional kilogram of weight, the dosage is increased by about 9.35 mg.

72. $D = A(c+25)$, $c = (140-y)\dfrac{w}{72x}$

a) To find D as a function of c, we substitute 5 for A in the formula for D.

$$\begin{aligned} D &= A(c+25) \\ &= 5(c+25) \\ &= 5c+125 \end{aligned}$$

To find c as a function of w, we substitute 45 for y and 0.6 for x in the formula for c.

$$\begin{aligned} c &= (140-45)\frac{w}{72(0.6)} \\ &= 95\cdot\frac{w}{43.2} \\ &\approx 2.199w \end{aligned}$$

b) $\dfrac{dD}{dc} = 5$

c) $\dfrac{dc}{dw} = 2.199$

d) First we find $D\circ c(w)$.

$$\begin{aligned} D\circ c(w) &= D(c(w)) \\ &= 5(2.199w)+125 \\ &= 10.995w+125 \end{aligned}$$

Then we have

$$\frac{dD}{dw} = 10.995.$$

e) $\dfrac{dD}{dw}$ represents the rate of change of the dosage with respect to the patient's weight. For each additional kilogram of weight, the dosage is increased by about 11 mg.

73. a) January 2009 corresponds to $t = 52$

$$\begin{aligned} C'(t) &= 0.74+0.02376t-1.0814\pi\ cos(2\pi t) \\ C'(52) &= 0.74+0.02376(52)-1.0814\pi cos(104\pi) \\ &= -1.4218\ ppmv/yr \end{aligned}$$

b) July 2009 corresponds to $t = 52.5$

$$\begin{aligned} C'(52.5) &= 0.74+0.02376(52.5)-1.0814\pi cos(105\pi) \\ &= 5.3847\ ppmv/yr \end{aligned}$$

74. $T'(t) = \frac{0.82\pi}{12}\ cos\left(\frac{\pi}{12}[t+2]\right)$

$$\begin{aligned} T'(8) &= \frac{0.82\pi}{12}\ cos\left(\frac{\pi}{12}[8+2]\right) \\ &= -0.1859\ degrees/hr \end{aligned}$$

75. $y = ((x^2+4)^8+3\sqrt{x})^4$

$$\begin{aligned} \frac{dy}{dx} &= 4((x^2+4)^8+3\sqrt{x})^3[8(x^2+4)^7(2x)+\frac{3}{2\sqrt{x}}] \\ &= 4((x^2+4)^8+3\sqrt{x})^3\left(16x(x^2+4)^7+\frac{3}{2\sqrt{x}}\right) \end{aligned}$$

76.

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}((x^5+x+1)^3+7\ sec^2x)^{-1/2}\times \\ &\quad [3(x^5+x+1)^2(5x^4+1)+14\ sec\ x\ sec\ x\ tan\ x] \\ &= \frac{3(x^5+x+1)^2(5x^4+1)+14\ sec^2x\ tan\ x}{2\sqrt{(x^5+x+1)^3+7\ sec^2x}} \end{aligned}$$

77. Let $y = sin(sin(sin\ x))$ then

$$\begin{aligned} \frac{dy}{dx} &= cos(sin(sin\ x))\cdot cos(sin\ x)\cdot cos\ x \\ &= cos\ x\ cos(sin\ x)\ cos(sin(sin\ x)) \end{aligned}$$

78. Let $y = cos(sec(sin\ 2x))$ then

$$\begin{aligned} \frac{dy}{dx} &= -sin(sec(sin\ 2x))\cdot sec(sin\ 2x)\ tan(sin\ 2x)\cdot 2cos\ 2x \\ &= -2cos\ 2xsec(sin\ 2x)\ tan(sin\ 2x)\ sin(sec(sin\ 2x)) \end{aligned}$$

79. Let $y = tan(cot(sec\ 3x))$ then

$$\begin{aligned} \frac{dy}{dx} &= sec^2(cot(sec\ 3x))\cdot -csc^2(sec\ 3x)\cdot 3\ sec\ 3x\ tan\ 3x \\ &= -3\ sec\ 3x\ tan\ 3xcsc^2(sec\ 3x)\ sec^2(cot(sex\ 3x)) \end{aligned}$$

80. Let $y = csc(cos(sec(\sqrt{x^2+1})))$ then

$$\begin{aligned} \frac{dy}{dx} &= -csc(cos(sec(\sqrt{x^2+1})))\ cot(cos(sex(\sqrt{x^2+1})))\times \\ &\quad -sin(sec(\sqrt{x^2+1}))\cdot sec(\sqrt{x^2+1})\ tan(\sqrt{x^2+1})\times \\ &\quad \frac{2x}{2\sqrt{x^2+1}} \end{aligned}$$

81. $y = \left(sin\left(\frac{3\pi}{2}+3\right)\right)^{1/5}$ is a constant, which means $\frac{dy}{dx} = 0$.

82. The derivative of a constant is 0

83.

$$\begin{aligned} sin(a+x) &= sin\ a\ cos\ x + cos\ a\ sin\ x \\ \frac{d}{dx}(sin(a+x)) &= \frac{d}{dx}(sin\ a\ cos\ x + cos\ a\ sin\ x) \\ &= -sin\ a\ sin\ x + cos\ a\ cos\ x \\ &= cos\ a\ cos\ x - sin\ a\ sin\ x \\ &= cos(a+x) \end{aligned}$$

84.

$$\begin{aligned} cos(a+x) &= cos\ a\ cos\ x - sin\ a\ sin\ x \\ \frac{d}{dx}(cos(a+x)) &= \frac{d}{dx}(cos\ a\ cos\ x - sin\ a\ sin\ x) \\ &= -cos\ a\ sin\ x + sin\ a\ cos\ x \\ &= sin\ a\ cos\ x - cos\ a\ sin\ x \\ &= sin(a+x) \end{aligned}$$

85. Let $Q(x) = \frac{N(x)}{D(x)}$. Then we can write

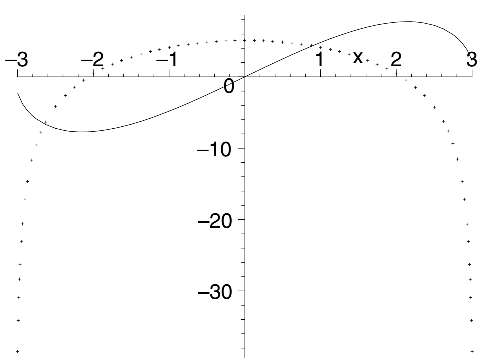
$$Q(x) = N(x) \cdot [D(x)]^{-1}$$

using the property of negative exponents. Now we use the product differentiation rule

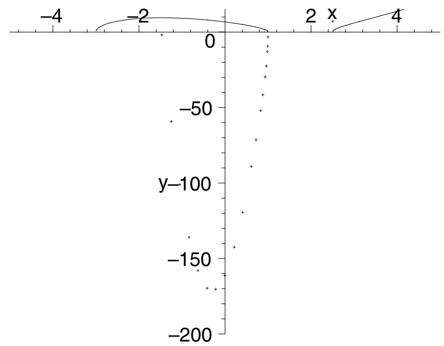
$$\begin{aligned} Q'(x) &= N(x) \cdot -1[D(x)]^{-2} \cdot D'(x) + [D(x)]^{-1} \cdot N'(x) \\ &= \frac{-N(x) \cdot D'(x)}{[D(x)]^2} + \frac{N'(x)}{D(x)} \\ &= \frac{-N(x) \cdot D'(x)}{[D(x)]^2} + \frac{N'(x) \cdot D(x)}{[D(x)]^2} \\ &= \frac{N'(x) \cdot D(x) - N(x) \cdot D'(x)}{[D(x)]^2} \end{aligned}$$

86. One might use the following: Composition of functions could be expressed as a process through which one function is evaluated using another function that falls in the original function's domain.

87. $(-2.145, -7.728)$ and $(2.145, 7.728)$



88. $(-1.475, 9.488)$



89.

$$\begin{aligned} f'(x) &= x \cdot \frac{-2x}{2\sqrt{4-x^2}} + \sqrt{4-x^2} \\ &= \frac{-x^2}{\sqrt{4-x^2}} + \frac{4-x^2}{\sqrt{4-x^2}} \\ &= \frac{4-2x^2}{\sqrt{4-x^2}} \end{aligned}$$

90.

$$\begin{aligned} f'(x) &= \frac{4 \cdot \sqrt{x-10} - 4x \cdot \frac{1}{2\sqrt{x-10}}}{x-10} \\ &= \frac{4(x-10) - 2x}{(x-10)^{3/2}} \\ &= \frac{2x-40}{(x+10)^{3/2}} \end{aligned}$$

Exercise Set 2.9

1. $y = 3x + 5$

$$\begin{aligned} \frac{dy}{dx} &= 3 \\ \frac{d^2y}{dx^2} &= 0 \end{aligned}$$

2. $y = -4x + 7$

$$\begin{aligned} \frac{dy}{dx} &= -4 \\ \frac{d^2y}{dx^2} &= 0 \end{aligned}$$

3. $y = -3(2x+2)^{-1}$

$$\begin{aligned} \frac{dy}{dx} &= 3(2x+2)^{-2}(2) = 6(2x+2)^{-2} = \frac{6}{(2x+2)^2} \\ \frac{d^2y}{dx^2} &= -12(2x+2)^{-3}(2) = \frac{-24}{(2x+2)^3} \end{aligned}$$

4. $y = -(3x-4)^{-1}$

$$\begin{aligned} \frac{dy}{dx} &= (3x-4)^{-2}(3) = \frac{3}{(3x-4)^2} \\ \frac{d^2y}{dx^2} &= 3(-2(3x-4)^{-3}(3)) = \frac{-18}{(3x-4)^3} \end{aligned}$$

5. $y = (2x+1)^{1/3}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{3}(2x+1)^{-2/3}(2) = \frac{2}{3(2x+1)^{2/3}} \\ \frac{d^2y}{dx^2} &= \frac{2}{3}(-\frac{2}{3}(2x+1)^{-5/3}(2)) = \frac{-8}{9(2x+1)^{5/2}} \end{aligned}$$

6. $f(x) = (3x+2)^{-3}$

$$\begin{aligned} f'(x) &= -3(3x+2)^{-4}(3) = -9(3x+2)^{-4} \\ f''(x) &= 36(3x+2)^{-5}(3) = \frac{108}{(3x+2)^5} \end{aligned}$$

7. $f(x) = (4-3x)^{-4}$

$$\begin{aligned} f'(x) &= -4(4-3x)^{-5}(-3) = 12(4-3x)^{-5} \\ f''(x) &= -60(4-3x)^{-6}(-3) = \frac{180}{(4-3x)^6} \end{aligned}$$

8. $y = \sqrt{x-1} = (x-1)^{1/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(x-1)^{1/2-1}\cdot 1 \\ &= \frac{1}{2}(x-1)^{-1/2} \\ \frac{d^2y}{dx^2} &= \frac{1}{2}\cdot\Big(-\frac{1}{2}\Big)(x-1)^{-1/2-1}\cdot 1 \\ &= -\frac{1}{4}(x-1)^{-3/2} \\ &= -\frac{1}{4(x-1)^{3/2}} \\ &= -\frac{1}{4\sqrt{(x-1)^3}} \end{aligned}$$

9. $y = \sqrt{x+1} = (x+1)^{1/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(x+1)^{-1/2}\cdot 1 \\ &= \frac{1}{2}(x+1)^{-1/2} \\ \frac{d^2y}{dx^2} &= -\frac{1}{4}(x+1)^{-3/2}\cdot 1 \\ &= -\frac{1}{4}(x+1)^{-3/2} \\ &= -\frac{1}{4(x+1)^{3/2}} \\ &= -\frac{1}{4\sqrt{(x+1)^3}} \end{aligned}$$

10. $f(x) = (3x+2)^{10}$

$$\begin{aligned} f'(x) &= 10(3x+2)^9(3) = 30(3x+2)^9 \\ f''(x) &= 270(3x+2)^8(3) = 810(3x+2)^8 \end{aligned}$$

11. $f(x) = (2x+9)^{16}$

$$\begin{aligned} f'(x) &= 16(2x+9)^{15}(2) = 32(2x+9)^{15} \\ f''(x) &= 480(2x+9)^{14}(2) = 960(2x+9)^{14} \end{aligned}$$

12. $g(x) = tan(2x)$

$$\begin{aligned} g'(x) &= sec^2(2x)(2) = 2\ sec^2(2x) \\ g''(x) &= 2(2\ sec(2x)\ sec(2x)\ tan(2x)(2)) \\ &= 8\ sec^2(2x)\ tan(2x) \end{aligned}$$

13. $g(x) = sec(3x+1)$

$$\begin{aligned} g'(x) &= sec(3x+1)\ tan(3x+1)(3) \\ &= 3\ sec(3x+1)\ tan(3x+1) \\ g''(x) &= 3\ sec(3x+1)\ sec(3x+1)(3) + \\ &\quad tan(3x+1)(3\ sec(3x+1)\ tan(3x+1)(3) \\ &= 9\ sec^3(3x+1) + 9\ sec(3x+1)\ tan^2(3x+1) \\ &= 9\ sec(3x+1)[sec^2(3x+1) + tan^2(3x+1)] \end{aligned}$$

14. $f(x) = 13x^2 + 2x + 7 - csc\ x$

$$\begin{aligned} f'(x) &= 26x + 2 - (-csc\ x\ cot\ x) \\ &= 26x + 2 + csc\ x\ cot\ x \\ f''(x) &= 26 + csc\ x(-csc^2 x) + cot\ x(-csc\ x\ cot\ x) \\ &= 26 - csc^3 x - csc\ x\ cot^2 x \end{aligned}$$

15. $f(x) = sec(2x+3) + 4x^2 + 3x - 7$

$$\begin{aligned} f'(x) &= sec(2x+3)\ tan(2x+3)(2) + 8x + 3 \\ &= 2\ sec(2x+3)\ tan(2x+3) + 8x + 3 \\ f''(x) &= 2\ sec(2x+3)\ sec^2(2x+3)(2) + \\ &\quad tan(2x+3)\ sec(2x+3)\ tan(2x+3)(2) + 8 \\ &= 4\ sec^3(2x+3) + 2\ sec(2x+3)\ tan^2(2x+3) + 8 \end{aligned}$$

16. $g(x) = mx + b$

$$\begin{aligned} g'(x) &= m \\ g''(x) &= 0 \end{aligned}$$

17. $y = ax^2 + bx + c$

$$\begin{aligned} \frac{dy}{dx} &= a\cdot 2x + b + 0 \\ &= 2ax + b \\ \frac{d^2y}{dx^2} &= 2a + 0 \\ &= 2a \end{aligned}$$

18. $y = \sqrt[3]{2x+4} = (2x+4)^{1/3}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{3}(2x+4)^{-2/3}(2) \\ &= \frac{2}{3}(2x+4)^{-2/3} \\ \frac{d^2y}{dx^2} &= -\frac{2}{3}\cdot\frac{2}{3}(2x+4)^{-5/3}(2) \\ &= -\frac{8}{9}(2x+4)^{-5/3} \\ &= -\frac{8}{9(2x+4)^{5/3}} \\ &= -\frac{8}{9\sqrt[3]{(2x+4)^5}} \end{aligned}$$

19. $y = \sqrt[4]{(x^2+1)^3} = (x^2+1)^{3/4}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{3}{4}(x^2+1)^{-1/4}(2x)\\ &= \frac{3}{2}x(x^2+1)^{-1/4}\\ \frac{d^2y}{dx^2} &= \frac{3}{2}\left[x\left(-\frac{1}{4}\right)(x^2+1)^{-5/4}(2x) + 1\cdot(x^2+1)^{-1/4}\right]\\ &= -\frac{3x^2}{4(x^2+1)^{5/4}} + \frac{3}{2(x^2+1)^{1/4}}\\ &= -\frac{3x^2}{4\sqrt[4]{(x^2+1)^5}} + \frac{3}{2\sqrt[4]{x^2+1}}\end{aligned}$$

20. $y = 13x^{3.2} + 12x^{1.2} - 5x^{-0.25}$

$$\begin{aligned}\frac{dy}{dx} &= 41.6x^{2.2} + 14.4x^{0.2} - 1.25x^{-1.25}\\ \frac{d^2y}{dx^2} &= 91.52x^{1.2} + 2.88x^{-0.8} + 1.5625x^{2.25}\end{aligned}$$

21. $f(x) = (4x+3)\ cos\ x$

$$\begin{aligned}f'(x) &= (4x+3)\ (-sin\ x) + 4\ cos\ x\\ &= -(4x+3)\ sin\ x + 4\ cos\ x\end{aligned}$$

22. $s(t) = sin(at)$

$$\begin{aligned}s'(t) &= cos(at)\ a = a\ cos(at)\\ s''(t) &= a(-sin(at)\ a)\\ &= -a^2\ sin(at)\end{aligned}$$

23. $s(t) = cos(at+b)$

$$\begin{aligned}s'(t) &= -sin(at+b)\ a = -a\ sin(at+b)\\ s''(t) &= -a(cos(at+b)\ a)\\ &= -a^2\ cos(at+b)\end{aligned}$$

24. $y = x^{5/2} + x^{3/2} - x^{1/2}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{5}{2}x^{3/2} + \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}\\ \frac{d^2y}{dx^2} &= \frac{15}{4}x^{1/2} + \frac{3}{4}x^{-1/2} + \frac{1}{4}x^{-3/2}\end{aligned}$$

25. $y = \frac{(t^2+3)^{1/2}}{7} + (3t^2+1)^{1/3}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{7}\cdot\frac{1}{2}(t^2+3)^{-1/2}(2t) + \frac{1}{3}(3t^2+1)^{-2/3}(6t)\\ &= \frac{t}{7}(t^2+3)^{-1/2} + 2t(3t^2+1)^{-2/3}\\ \frac{d^2y}{dx^2} &= \frac{t}{7}(\frac{-1}{2}(t^2+3)^{-3/2}(2t)) + (2)(t^2+3)^{-1/2} +\\ &\quad 2t(\frac{-2}{3}(3t^2+1)^{-5/3}(6t)) + (2)(3t^2+1)^{-2/3}\\ &= \frac{-t^2}{7(t^2+3)^{3/2}} + \frac{2}{(t^2+3)^{1/2}}\\ &\quad -\frac{8t^2}{(3t^2+1)^{5/3}} + \frac{2}{3t^2+1}\end{aligned}$$

26. $y = \frac{1}{\pi^2}\ tan\ \pi t + \frac{4}{\pi^2}\ sec\ \pi t$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\pi^2}sec^2\pi t\cdot\pi + \frac{4}{\pi^2}\ sec\ \pi t\ tan\ \pi t\cdot\pi\\ &= \frac{1}{\pi}\ sec^2\pi t + \frac{4}{\pi}\ sec\ \pi t\ tan\ \pi t\\ \frac{d^2y}{dx^2} &= \frac{1}{\pi}(2\ sec\ \pi t\ sec\ \pi t\ tan\ \pi t\cdot\pi) +\\ &\quad \frac{4}{\pi}(sec\ \pi t\ sec^2\pi t\cdot\pi + sec\ \pi t\ tan\ \pi t\ tan\ \pi t\cdot\pi)\\ &= 2\ sec^2\pi t\ tan\ \pi t + 4\ sec^3\pi t + 4\ sec\ \pi t\ tan^2\pi t\end{aligned}$$

27. $y = x^4$

$$\begin{aligned}\frac{dy}{dx} &= 4x^3\\ \frac{d^2y}{dx^2} &= 4\cdot 3x^2\\ &= 12x^2\\ \frac{d^3y}{dx^3} &= 12\cdot 2x\\ &= 24x\\ \frac{d^4y}{dy^4} &= 24\end{aligned}$$

28. $y = x^5$

$$\begin{aligned}\frac{dy}{dx} &= 5x^4\\ \frac{d^2y}{dx^2} &= 20x^3\\ \frac{d^3y}{dx^3} &= 60x^2\\ \frac{d^4y}{dy^4} &= 120x\end{aligned}$$

29. $y = x^6 - x^3 + 2x$

$$\begin{aligned}\frac{dy}{dx} &= 6x^5 - 3x^2 + 2\\ \frac{d^2y}{dx^2} &= 30x^4 - 6x\\ \frac{d^3y}{dx^3} &= 120x^3 - 6\\ \frac{d^4y}{dx^4} &= 360x^2\\ \frac{d^5y}{dx^5} &= 720x\end{aligned}$$

30. $y = x^7 - 8x^2 + 2$

$$\begin{aligned}\frac{dy}{dx} &= 7x^6 - 16x\\ \frac{d^2y}{dx^2} &= 42x^5 - 16\\ \frac{d^3y}{dx^3} &= 210x^4\\ \frac{d^4y}{dx^4} &= 840x^3\\ \frac{d^5y}{dx^5} &= 2520x^2\\ \frac{d^6y}{dx^6} &= 5040x\end{aligned}$$

31. $y = (x^2-5)^{10}$

$$\begin{aligned}\frac{dy}{dx} &= 10(x^2-5)^9 \cdot 2x \\ &= 20x(x^2-5)^9 \\ \frac{d^2y}{dx^2} &= 20x \cdot 9(x^2-5)^8 \cdot 2x + 20(x^2-5)^9 \\ &= 360x^2(x^2-5)^8 + 20(x^2-5)^9 \\ &= 20(x^2-5)^8[18x^2 + (x^2-5)] \\ &= 20(x^2-5)^8(19x^2-5)\end{aligned}$$

32. $y = x^k$

$$\begin{aligned}\frac{dy}{dx} &= kx^{k-1} \\ \frac{d^2y}{dx^2} &= k(k-1)x^{k-2} \\ \frac{d^3y}{dx^3} &= k(k-1)(k-2)x^{k-3} \\ \frac{d^4y}{dx^4} &= k(k-1)(k-2)(k-3)x^{k-4} \\ \frac{d^5y}{dx^5} &= k(k-1)(k-2)(k-3)(k-4)x^{k-5}\end{aligned}$$

33. $y = sec(2x+3)$

$$\begin{aligned}\frac{dy}{dx} &= sec(2x+3)\ tan(2x+3)\ (2) \\ &= 2\ sec(2x+3)\ tan(2x+3) \\ \frac{d^y}{dx^2} &= 2\ sec(2x+3)\ sec^2(2x+3)(2) + \\ &\quad [2\ sec(2x+3)\ tan(2x+3)(2)]\ tan(2x+3) \\ &= 4\ sec^3(2x+3) + 4\ sec(2x+3)\ tan^2(2x+3) \\ \frac{d^3y}{dx^3} &= 4(3sec^2(2x+3)\ sec(2x+3)\ tan(2x+3)(2)) + \\ &\quad 4\ sec(2x+3)[2\ tan(2x+3)\ sec^2(2x+3)(2)] + \\ &\quad sec(2x+3)\ tan(2x+3)\ (2)\ tan^2(2x+3) \\ &= 40\ sec^3(2x+3)\ tan(2x+3) + \\ &\quad 8\ sec(2x+3)\ tan^3(2x+3)\end{aligned}$$

34. $y = cot(3x-1)$

$$\begin{aligned}\frac{dy}{dx} &= -3csc^2(3x-1) \\ \frac{d^2y}{dx^2} &= -3[2\ csc(3x-1)\ (-csc(3x-1)\ cot(3x-1)\ (3))] \\ &= 18\ csc^2(3x-1)\ cot(3x-1) \\ \frac{d^3y}{dx^3} &= 18\ csc^2(3x-1)(-csc^2(3x-1)(3)) + \\ &\quad 18[2\ csc^2(3x-1)cot(3x-1)(3))]cot(3x-1) \\ &= -54\ csc^4(3x-1) - 108\ csc^2(3x-1)\ cot^2(3x-1)\end{aligned}$$

35. $s(t)10\ cos(3t+2) - 4\ sin(3t+2)$

$$\begin{aligned}v(t) &= 10[-sin(3t+2)(3)] - 4[cos(3t+2)(3) \\ &= -30\ sin(3t+2) - 12\ cos(3t+2) \\ a(t) &= -30[cos(3t+2)(3)] - 12[-sin(3t+2)(3)] \\ &= -90\ cos(3t+2) - 36\ sin(3t+2) \\ &= 9[10\ cos(3t+2) - 4\ sin(3t+2)] \\ &= 9\ s(t)\end{aligned}$$

36. $s(t) = 6.8\ tan(2.6t-1)$

$$\begin{aligned}v(t) &= 6.8\ sec^2(2.6t-1)(2.6) = 17.68\ sec^2(2.6t-1) \\ a(t) &= 17.68[2\ sec(2.6t-1)\ sec(2.6t-1)\ tan(2.6t-1)(2.6)] \\ &= 91.936\ sec^2(2.6t-1)\ tan(2.6t-1)\end{aligned}$$

37. $s(t) = t^3 + t^2 + 2t$

$$\begin{aligned}v(t) &= s'(t) = 3t^2 + 2t + 2 \\ a(t) &= s''(t) = 6t + 2\end{aligned}$$

38. $s(t) = t^4 + t^2 + 3t$

$$\begin{aligned}v(t) &= s'(t) = 4t^3 + 2t + 3 \\ a(t) &= s''(t) = 12t^2 + 2\end{aligned}$$

39. $w(t) = 0.000758t^3 - 0.0596t^2 - 1.82t + 8.15$
The acceleration of a function that depends on time is the second derivative of the function with respect to time.

$$\begin{aligned}w'(t) &= 0.002274t^2 - 0.1192t + 1.82 \\ w''(t) &= 0.004548t - 0.1192\end{aligned}$$

40. $w(t) = 0.0006t^3 - 0.0484t^2 - 1.61t + 7.60$

$$\begin{aligned}w'(t) &= 0.0018t^2 - 0.0968t + 1.61 \\ w''(t) &= 0.0036t - 0.068\end{aligned}$$

41. $P(t)100000(1 + 0.6t + t^2)$

$$\begin{aligned}P'(t) &= 100000(0.6 + 2t) \\ P''(t) &= 100000(2) \\ &= 200000\end{aligned}$$

42. $P(t)100000(1 + 0.4t + t^2)$

$$\begin{aligned}P'(t) &= 100000(0.4 + 2t) \\ P''(t) &= 100000(2) \\ &= 200000\end{aligned}$$

43. $y = \frac{x}{(x-1)^{1/2}}$

$$\begin{aligned}y' &= \frac{\sqrt{x-1}(1) - x \cdot \frac{1}{2\sqrt{x-1}}}{x-1} \\ &= \frac{2(x-1) - x}{2(x-1)\sqrt{x-1}} \\ &= \frac{x-2}{2(x-1)^{3/2}}\end{aligned}$$

$$\begin{aligned}y'' &= \frac{2(x-1)^{3/2}(1) - (x-2)\left[2 \cdot \frac{3}{2}(x-1)^{1/2}\right]}{4(x-1)^3} \\ &= \frac{2(x-1)^{3/2} - 3(x-2)(x-1)^{1/2}}{4(x-1)^3} \\ &= \frac{(x-1)^{1/2}[2(x-1) - 3(x-2)]}{(x-1)^3} \\ &= \frac{4-x}{(x-1)^{5/2}}\end{aligned}$$

$$\begin{aligned} y''' &= \frac{4(x-1)^{5/2}(-1)-(4-x)\left[4\cdot\frac{5}{2}(x-1)^{3/2}\right]}{16(x-1)^5} \\ &= \frac{(x-1)^{3/2}[4(x-1)-10(4-x)]}{16(x-1)^5} \\ &= \frac{4x-4-40+10x}{16(x-1)^{7/2}} \\ &= \frac{3x-18}{16(x-1)^{7/2}} \end{aligned}$$

44. $=\frac{\sqrt{x}-1}{\sqrt{x}+1}$

$$\begin{aligned} y' &= \frac{(\sqrt{x}+1)(\frac{1}{2\sqrt{x}})-(\sqrt{x}-1)(\frac{1}{2\sqrt{x}})}{(\sqrt{x}+1)^2} \\ &= \frac{1}{\sqrt{x}(\sqrt{x}+1)^2} \\ &= [\sqrt{x}(\sqrt{x}+1)^2]^{-1} \end{aligned}$$

$$\begin{aligned} y'' &= -1[\sqrt{x}(\sqrt{x}+1)^2]^{-2}[\sqrt{x}[2(\sqrt{x}+1)(\frac{1}{2\sqrt{x}})]+ \\ &\quad -1[\sqrt{x}(\sqrt{x}+1)^2]^{-2}\frac{1}{2\sqrt{x}}(\sqrt{x}+1)^2] \\ &= -[\sqrt{x}(\sqrt{x}+1)^2]^{-2}\left(\frac{3}{2}\sqrt{x}+2+\frac{1}{2\sqrt{x}}\right) \end{aligned}$$

$$\begin{aligned} y''' &= -[\sqrt{x}(\sqrt{x}+1)^2]^{-2}[\frac{3}{2}x^{-1/2}+\frac{1}{4}x^{-3/2}]+ \\ &\quad 2[\sqrt{x}(\sqrt{x}+1)^2]^{-3}[\sqrt{x}[2(\sqrt{x}+1)(\frac{1}{2\sqrt{x}})]+ \\ &\quad -1[\sqrt{x}(\sqrt{x}+1)^2]^{-2}\frac{1}{2\sqrt{x}}] \\ &= -[\sqrt{x}(\sqrt{x}+1)^2]^{-2}[\frac{3}{2}x^{-1/2}+\frac{1}{4}x^{-3/2}]+ \\ &\quad 2[\sqrt{x}(\sqrt{x}+1)^2]^{-3}[\sqrt{x}(x+1)]-[\sqrt{x}(\sqrt{x}+1)^2]^{-2}\frac{1}{2\sqrt{x}}] \end{aligned}$$

45. $f(x)=\frac{x}{x-1}$

$$\begin{aligned} f'(x) &= \frac{(x-1)(1)-x(1)}{(x-1)^2} \\ &= \frac{-1}{(x-1)^2} \\ &= -(x-1)^{-2} \end{aligned}$$

$$\begin{aligned} f''(x) &= -(-2(x-1)^{-3}) \\ &= \frac{2}{(x-1)^3} \end{aligned}$$

46. $f(x)=(1+x^2)^{-1}$

$$\begin{aligned} f'(x) &= -1(1+x^2)^{-2}(2x) \\ &= -2x(1+x^2)^{-2} \end{aligned}$$

$$\begin{aligned} f''(x) &= -2x[-2(1+x^2)^{-3}(2x)]+(-2)(1+x^2)^{-2} \\ &= \frac{-8x^2}{(1+x^2)^3}+\frac{4}{(1+x^2)^2} \end{aligned}$$

47. $y=sin\ x$

a) $\frac{dy}{dx}=cos\ x$
b) $\frac{d^2y}{dx^2}=-sin\ x$
c) $\frac{d^3y}{dx^3}=-cos\ x$
d) $\frac{d^4y}{dx^4}=sin\ x$
e) $\frac{d^8y}{dx^8}=sin\ x$
f) $\frac{d^{10}y}{dx^{10}}=-sin\ x$
g) $\frac{d^{837}y}{dx^{837}}=cos\ x$

48. $y=cos\ x$

a) $\frac{dy}{dx}=-sin\ x$
b) $\frac{d^2y}{dx^2}=-cos\ x$
c) $\frac{d^3y}{dx^3}=sin\ x$
d) $\frac{d^4y}{dx^4}=cos\ x$
e) $\frac{d^8y}{dx^8}=cos\ x$
f) $\frac{d^{11}y}{dx^{10}}=sin\ x$
g) $\frac{d^{523}y}{dx^{523}}=sin\ x$

49. Functions that have the form $f(x)=Asin\ x+Bcos\ x$ where A and B are constants, will satisfy the condition of their second derivative being the negative of the original function.

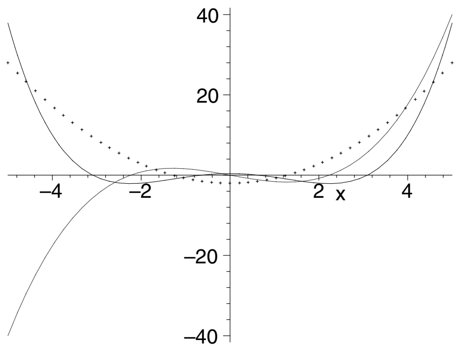
50. $f(x)=\frac{x-1}{x+2}$

$$\begin{aligned} f'(x) &= \frac{(x+2)(1)-(x-1)(1)}{(x+2)^2} \\ &= \frac{3}{(x+2)^2}=3(x+2)^{-2} \\ f''(x) &= -6(x+2)^{-3}=\frac{-6}{(x+2)^3} \\ f'''(x) &= 18(x+2)^{-4}=\frac{18}{(x+2)^4} \\ f^{(4)(x)} &= -72(x+2)^{-5}=\frac{-72}{(x+2)^5} \\ f^{(5)(x)} &= 360(x+2)^{-6}=\frac{360}{(x+2)^6} \end{aligned}$$

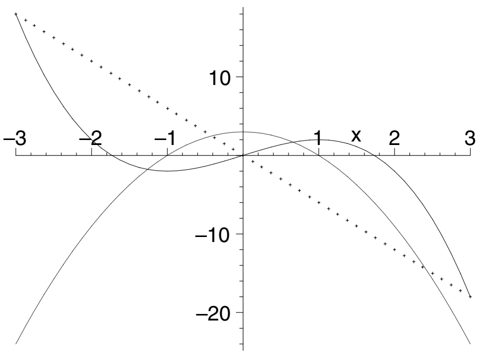
51. $f(x)=\frac{x+3}{x-2}$

$$\begin{aligned} f'(x) &= \frac{(x-2)(1)-(x+3)(1)}{(x-2)^2} \\ &= \frac{-5}{(x-2)^2}=-5(x-2)^{-2} \\ f''(x) &= 10(x-2)^{-3}=\frac{10}{(x-2)^3} \\ f'''(x) &= -30(x-2)^{-4}=\frac{-30}{(x-2)^4} \\ f^{(4)}(x) &= 120(x-2)^{-5}=\frac{120}{(x-2)^5} \\ f^{(5)}(x) &= -600(x-2)^{-6}=\frac{-600}{(x-2)^6} \end{aligned}$$

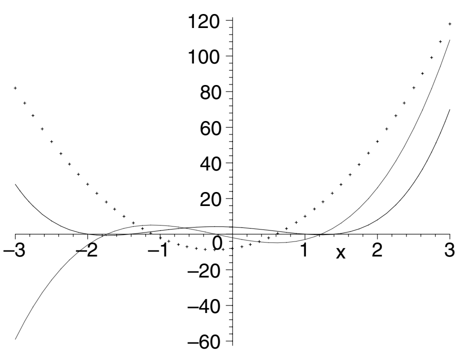
52.



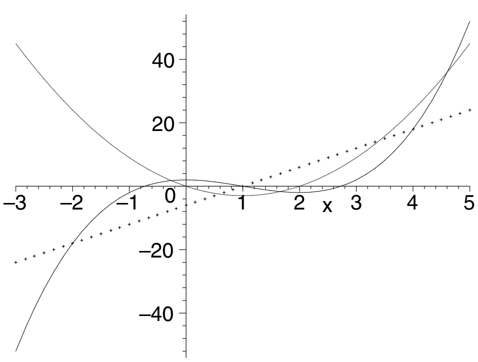
53.



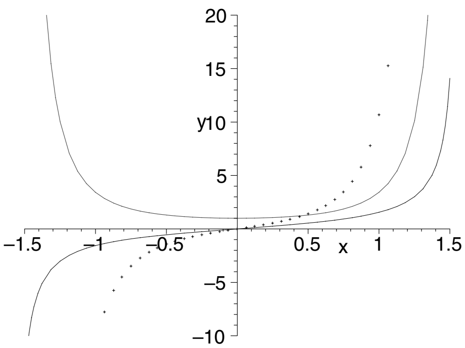
54.



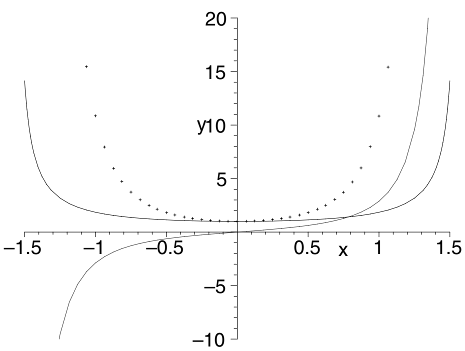
55.



56.



57.



Chapter Review Exercises

1. a)

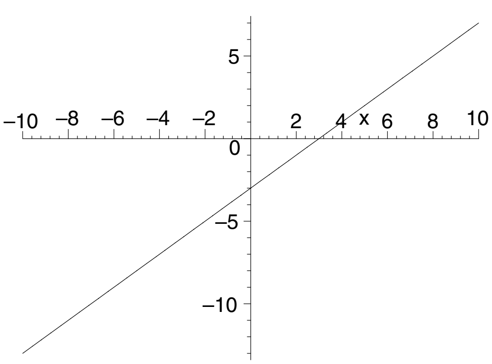
x	f(x)
-8	-11
-7.5	-10.5
-7.1	-10.1
-7.01	-10.01
-7.001	-10.001
-7.0001	-10.0001
-6	-9
-6.5	-9.5
-6.9	-9.9
-6.99	-9.99
-6.999	-9.999
-6.9999	-9.9999

b) $\lim_{x\to -7^-} f(x) = -10$

$\lim_{x\to -7^+} f(x) = -10$

$\lim_{x\to -7} f(x) = -10$

2. From the graph below, we can see that $\lim_{x\to -7} f(x) = -10$



3. $f(x) = \frac{(x+7)(x-3)}{(x+7)} = (x-3)$, so $\lim_{x\to -7} f(x) = -10$

4. $\lim_{x\to -2} \frac{8}{x} = -4$

5. $\lim_{x\to 1}(2x^4 - 3x^2 + x + 4) = 2(1) - 3(1) + (1) + 4 = 4$

6. $\lim_{x\to 6} \frac{(x-6)(x+11)}{(x-6)} = 17$

7. $\lim_{x\to 4} \sqrt{x^2+9} = \sqrt{4^2+9} = \sqrt{25} = 5$

8. The function is not continuous at $x = -2$

9. The function is continuous

10. $\lim_{x\to 1} g(x) = -4$

11. $g(1) = -4$

12. Yes, $g(x)$ is continuoous at $x = 1$

13. $\lim_{x\to -2} g(x)$ does not exist

14. $g(-2) = -2$

15. No, $g(x)$ is not continuous at $x = -2$

16. $f(2) = 8$, $f(-1) = 2)$, average rate of change is

$$\frac{8-2}{2-(-1)} = \frac{6}{3} = 2$$

17.

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{-3(x+h)+5-(-3x+5)}{h} \\ &= \frac{-3x-3h+5+3x-5}{h} \\ &= \frac{-3h}{h} \\ &= -3 \end{aligned}$$

18.

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{2(x+h)^2-3-(2x^2-3)}{h} \\ &= \frac{2x^2+4xh+2h^2-3-2x^2+3}{h} \\ &= \frac{4xh+2h^2}{h} \\ &= 4x+2h \end{aligned}$$

19. $\frac{dy}{dx} = 2x+3$
The slope at $(-1,-2)$ is

$$\begin{aligned} m &= \frac{dy}{dx}|_{x=-1} \\ &= 2(-1)+3 \\ &= 1 \end{aligned}$$

Now we use the slope-point equation

$$\begin{aligned} y-(-2) &= 1(x-(-1)) \\ y &= x+1-2 \\ y &= x-1 \end{aligned}$$

20. $\frac{dy}{dx} = -2x+8$

$$\begin{aligned} -2x+8 &= 0 \\ -2x &= -8 \\ x = 4 & \end{aligned}$$

$$\begin{aligned} y &= -(4)^2+8(4)-11 \\ &= -16+32-11 \\ &= 5 \end{aligned}$$

The slope of the tangent line is horizontal at $(4,5)$

21. $\frac{dy}{dx} = 10x-49$

$$\begin{aligned} 10x-49 &= 1 \\ 10x &= 50 \\ x = 5 & \end{aligned}$$

$$\begin{aligned} y &= 5(5)^2-49(5)+12 \\ &= 125-245+12 \\ &= -108 \end{aligned}$$

The slope of the tangent line is 1 at $(5,-108)$

22. $\frac{dy}{dx} = 4\cdot 5x^4 = 20x^4$

23. $y = 3x^{1/3}$
$\frac{dy}{dx} = 3\cdot\frac{1}{3}x^{-2/3} = \frac{1}{x^{2/3}}$

24. $y = -8x^{-8}$
$\frac{dy}{dx} = -64x^{-9} = \frac{-64}{x^9}$

25. $y = 21x^{4/3}$
$\frac{dy}{dx} = 21\cdot\frac{4}{3}x^{1/3} = 28x^{1/3}$

26. $y = sec\ 5x$
$\frac{dy}{dx} = 5\ sec\ 5x\ tan\ 5x$

27. $y = x\ cot(x^2)$

$$\begin{aligned} \frac{dy}{dx} &= x\ (-csc^2(x^2)(2x)) + cot(x^2) \\ &= -2x^2\ csc^2(x^2) + cot(x^2) \end{aligned}$$

28. $y = 2.3\sqrt{0.4x+5.3} + 0.01\ sin(0.17x-0.31)$

$$\begin{aligned} \frac{dy}{dx} &= 2.3\frac{1}{2\sqrt{0.4x+5.3}}(0.4) + 0.01\ cos(0.17x-0.31)(0.17) \\ &= \frac{0.46}{\sqrt{0.4x+5.3}} + 0.0017\ cos(0.17x-0.31) \end{aligned}$$

29. $f(x) = \frac{1}{6}x^6 + 8x^4 - 5x$

$$\begin{aligned} f'(x) &= \frac{1}{6}\cdot 6x^5 + 8\cdot 4x^3 - 5 \\ &= x^5 + 32x^3 - 5 \end{aligned}$$

30. $y = x^2+1$
$\frac{dy}{dx} = 2x$

31. $y = \frac{x^2+8}{8-x}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(8-x)(2x)-(x^2+8)(-1)}{(8-x)^2} \\ &= \frac{16x-2x^2+x^2+8}{(8-x)^2} \\ &= \frac{-x^2+16x+8}{(8-x)^2} \end{aligned}$$

32. $y = \frac{tan\ x}{x}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{x \cdot sec^2 x - tan\ x \cdot (1)}{x^2} \\ &= \frac{x\ sec^2 x - tan\ x}{x^2} \end{aligned}$$

33. $y = (sin^2 x + x)^{1/2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(sin^2 x + x)^{-1/2}(2\ sin\ x\ cos\ x + 1) \\ &= \frac{2\ sin\ x\ cos\ x + 1}{2\sqrt{sin^2 x + x}} \end{aligned}$$

34. $f(x) = tan^2(x\ cos\ x)$

$$\begin{aligned} f'(x) &= 2\ tan(x\ cos\ x)\ sec^2(x\ cos\ x)(x(-sin\ x) + cos\ x(1)) \\ &= 2\ sec^2(x\ cos\ x)\ tan(x\ cos\ x)(cos\ xx\ sin\ x) \end{aligned}$$

35. $f(x) = cot(x - cos\ x)$

$$\begin{aligned} f'(x) &= -csc^2(x - cos\ x)(1 - (-sin\ x)) \\ &= -csc^2(x - cos\ x)(1 + sin\ x) \end{aligned}$$

36. $f(x) = x^2(4x+3)^{3/4}$

$$\begin{aligned} f'(x) &= x^2[\frac{3}{4}(4x+3)^{-1/4}(4)] + 2x(4x+3)^{3/4} \\ &= \frac{3x^2}{(4x+3)^{1/4}} + 2x(4x+3)^{3/4} \end{aligned}$$

37. $y = x^3 - 2x^{-1}$

$$\begin{aligned} \frac{dy}{dx} &= 3x^2 + 2x^{-2} \\ \frac{d^2y}{dx^2} &= 6x - 4x^{-3} \\ \frac{d^3y}{dx^3} &= 6 + 12x^{-4} \\ \frac{d^4y}{dx^4} &= -48x^{-5} \\ \frac{d^5y}{dx^5} &= 240x^{-6} = \frac{240}{x^6} \end{aligned}$$

38. $y = \frac{1}{42}x^7 - 10x^3 + 13x^2 + 28x - 5 + cos\ x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{6}x^6 - 30x^2 + 26x + 28 - sin\ x \\ \frac{d^2y}{dx^2} &= x^5 - 60x + 26 - cos\ x \\ \frac{d^3y}{dx^3} &= 5x^4 - 60 + sin\ x \\ \frac{d^4y}{dx^4} &= 20x^3 + cos\ x \end{aligned}$$

39. $s(t) = t + t^4$

a) $v(t) = s'(t) = 1 + 4t^3$

b) $a(t) = v'(t) = 12t^2$

c) $v(2) = 1 + 4(2)^3 = 1 + 4(8) = 33$
$a(2) = 12(2)^2 = 12(4) = 48$

40. **a)** $v(t) = 0.036t^2 - 1.85 - \frac{0.002\pi}{15}\ sin(\frac{\pi}{15}\ t)$

b) $a(t) = 0.072t - \frac{0.002\pi^2}{225}\ cos(\frac{\pi}{15}\ t)$

c) $v(2.5) = -1.62521$
$a(2.5) = 0.17992$

41. **a)** Amplitude $= \frac{135-1}{2} = 67$, period $= \frac{1}{2} = \frac{2\pi}{b} \to b = 4\pi$, mid-line$= 67 + 1 = 68$ and the capsule begins at the top means we are using a cosine model. Therefore $h(t) = 67\ cos(4\pi t) + 68$

b) Five minutes after reaching the bottom corrspond to $t = 20\ min\ = \frac{1}{3}$ hours since it is five minutes after half a period.

$$\begin{aligned} h\left(\frac{1}{12}\right) &= 67\ cos\left(4\pi\frac{1}{3}\right) + 68 \\ &= 34.5\ ft \end{aligned}$$

c) $h'(t) = -67\ sin(4\pi t)(4\pi) = -268\pi\ sin(4\pi t)$

$$\begin{aligned} h'\left(\frac{1}{3}\right) &= -268\pi\ sin\left(\frac{4\pi}{3}\right) \\ &= 729.1473\ ft/hr \end{aligned}$$

42. **a)** $P'(t) = 100t$

b) $P(20) = 10000 + 50(400) = 30000$ people

c) $P'(20) = 100(20) = 2000$ people/year

43.

$$\begin{aligned} (f \circ g)(x) &= (1-2x)^2 + 5 \\ &= 1 - 4x + 4x^2 + 5 \\ &= 4x^2 - 4x + 6 \end{aligned}$$

$$\begin{aligned} (g \circ f)(x) &= 1 - 2(x^2 + 5) \\ &= 1 - 2x^2 - 10 \\ &= -2x^2 - 9 \end{aligned}$$

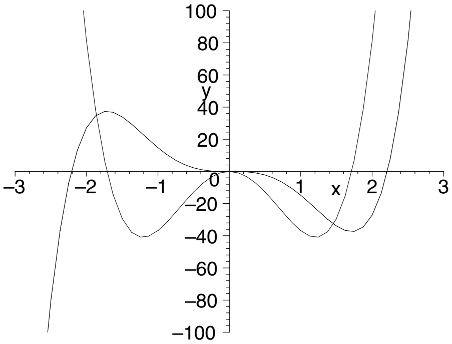
44.

$$\begin{aligned} \frac{dy}{dx} &= \frac{(1+x^3)[(x)(\frac{3}{2\sqrt{1+3x}}) + \sqrt{1+3x}(1)] - x\sqrt{1+3x}(3x^2)}{(1+x^3)^2} \\ &= \frac{3x(1+x^3) + 2(1+3x)(1+x^3) - 6x^3(1+3x)}{2\sqrt{1+3x}(1+x^3)^2} \\ &= \frac{3x + 3x^4 + 2 + 2x^3 + 6x + 6x^4 - 6x^3 - 18x^4}{2\sqrt{1+3x}(1+x^3)^2} \\ &= \frac{-9x^4 - 4x^3 + 9x + 2}{2\sqrt{1+3x}(1+x^3)^2} \end{aligned}$$

45. Limit approaches $-\frac{1}{4}$

46. Limit approaches $\frac{1}{6}$

47.



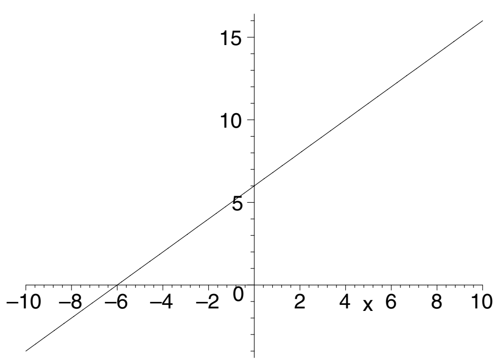
Chapter 2 Test

1. **a)**

x	f(x)
5	11
5.7	11.7
5.9	11.9
5.99	11.99
5.999	11.999
5.9999	11.9999
7	13
6.5	12.5
6.1	12.1
6.01	12.01
6.001	12.001
6.0001	12.0001
6.00001	12.00001

b) $\lim_{x\to 6^-} f(x) = 12$
$\lim_{x\to 6^+} f(x) = 12$
$\lim_{x\to 6} f(x) = 12$

2. From the graph below, we can see that $\lim_{x\to 6} f(x) = 12$



3. $f(x) = \frac{(x-6)(x+6)}{(x-6)} = (x+6)$, so $\lim_{x\to 6} f(x) = 12$

4. $\lim_{x\to -5} f(x)$ does not exist

5. $\lim_{x\to -4} f(x) = 0$

6. $\lim_{x\to -3} f(x)$ does not exist

7. $\lim_{x\to -2} f(x) = 2$

8. $\lim_{x\to -1} f(x) = 4$

9. $\lim_{x\to 1} f(x) = 1$

10. $\lim_{x\to 2} f(x) = 1$

11. $\lim_{x\to 3} f(x) = 1$

12. Function is continuous

13. Function is not continuous at $x = 3$

14. $\lim_{x\to 3} f(x)$ does not exist

15. $f(3) = 1$

16. No, the function is not continuous at $x = 3$

17. $\lim_{x\to 4} f(x) = 3$

18. $f(4) = 3$

19. Yes, the function is continupus at $x = 4$

20. $\lim_{x\to 1}(3x^4 - 2x^2 + 5 = 3(1) - 2(1) + 5 = 6$

21. $\lim_{x\to 2^+} \dfrac{x-2}{x(x-2)(x+2)} =$
$\lim_{x\to 2^+} \dfrac{1}{x(x+2)} = \dfrac{1}{8}$

22. $\lim_{x\to 0} \dfrac{7}{x}$ does not exist

23. $f(x) = 2x^2 + 3x - 9$, $\frac{f(x+h)-f(x)}{h}$

$$
\begin{aligned}
&= \frac{2(x+h)^2 + 3(x+h) - 9 - (2x^2 + 3x - 9)}{h} \\
&= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 9 - 2x^2 - 3x + 9}{h} \\
&= \frac{4xh + 2h^2 + 3h}{h} \\
&= 4x + 3 + 2h
\end{aligned}
$$

24. First find $\frac{dy}{dx}$

$$
\begin{aligned}
\frac{dy}{dx} &= 1 + 4(-1x^{-2}) \\
&= 1 - \frac{4}{x^2}
\end{aligned}
$$

Next find the slope at $(4, 5)$

$$
\begin{aligned}
m &= 1 - \frac{4}{(4)^2} \\
&= 1 - \frac{1}{4} \\
&= \frac{3}{4}
\end{aligned}
$$

Finally use the point-slope equation

$$
\begin{aligned}
y - 5 &= \frac{3}{4}(x - 4) \\
y &= \frac{3}{4}x - 3 + 5 \\
y &= \frac{3}{4}x + 2
\end{aligned}
$$

25. $\frac{dy}{dx} = 3x^2 - 6x$. The tangent line is horizontal when $\frac{dy}{dx} = 0$ so

$$\begin{aligned} 3x^2 - 6x &= 0 \\ 3x(x-2) &= 0 \\ \text{either} & \\ 3x &= 0 \\ x &= 0 \\ \text{or} & \\ x - 2 &= 0 \\ x &= 2 \end{aligned}$$

When $x = 0$, $y = 0^3 - 3(0)^2 = 0$, we have the point $(0, 0)$
When $x = 2$, $y = 2^3 - 3(2)^2 = 8 - 12 = -4$, we have the point $(2, -4)$

26. $\frac{dy}{dx} = 10 \cdot \frac{1}{2\sqrt{x}} = \frac{5}{\sqrt{x}}$

27. $y = -10x^{-1}$
$\frac{dy}{dx} = -10 \cdot -1x^{-2} = \frac{10}{x^2}$

28. $\frac{dy}{dx} = \frac{5}{4}x^{1/4}$

29. $y = -0.5x^2 + 0.61x + 90$
$\frac{dy}{dx} = -0.5(2x) + 0.61 = -x + 0.61$

30. $\frac{dy}{dx} = (sec^2\ 2x)(2) = 2\ sec^2\ 2x$

31. $f(x) = \frac{x}{5-x}$

$$\begin{aligned} f'(x) &= \frac{(5-x)(1) - x(-1)}{(5-x)^2} \\ &= \frac{5 - x + x}{(5-x)^2} \\ &= \frac{5}{(5-x)^2} \end{aligned}$$

32. $\frac{dy}{dx} = -5(x^5 - 4x^3 + x)^{-6}(5x^4 - 12x^2 + 1)$
$= \frac{-5(5x^4 - 12x^2 + 1)}{(x^5 - 4x^3 + x)^6}$

33. $f(x) = x\sqrt{x^2 + 5}$

$$\begin{aligned} f'(x) &= x \cdot \frac{2x}{2\sqrt{x^2+5}} + \sqrt{x^2+5} \\ &= \frac{x^2}{\sqrt{x^2+5}} + \sqrt{x^2+5} \cdot \frac{\sqrt{x^2+5}}{\sqrt{x^2+5}} \\ &= \frac{x^2}{\sqrt{x^2+5}} + \frac{x^2+5}{\sqrt{x^2+5}} \\ &= \frac{2x^2+5}{\sqrt{x^2+5}} \end{aligned}$$

34. $\frac{dy}{dx} = -sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} = \frac{-sin(\sqrt{x})}{2\sqrt{x}}$

35. $f(x) = tan\ 2x\ sec\ 3x$

$$\begin{aligned} f'(x) &= tan\ 2x[(sec\ 3x\ tan\ 3x)(3)] + [(sec^2\ 2x)(2)]sec\ 3x \\ &= sec\ 3x(3\ tan\ 2x\ tan\ 3x + 2\ sec^2\ 2x) \end{aligned}$$

36.

$$\begin{aligned} f'(x) &= \frac{(cos^2 x - x)(2\ tan\ x\ sec^2 x)}{(cos^2 x - x)^2} - \\ & \quad \frac{(tan^2 x)(-2cos\ x\ sin\ x - 1)}{(cos^2 x - x)^2} \\ &= \frac{2\ tan\ x - 2x\ tan\ x sec^2 x + 2sin^2 x\ tan\ x + tan^2 x}{(cos^2 x - x)^2} \\ &= \frac{tan\ x(2 - 2x\ sec^2 x + 2sin^2 x + tan\ x)}{(cos^2 x - x)^2} \end{aligned}$$

37. $f(x) = sin(cos(x^2))$

$$\begin{aligned} f'(x) &= cos(cos(x^2)) \cdot -sin(x^2) \cdot 2x \\ &= -2x\ sin(x^2)\ cos(cos(x^2)) \end{aligned}$$

38.

$$\begin{aligned} f'(x) &= 2x + cos(2x\sqrt{x+2}) \left(\cdot 2x \cdot \frac{1}{2\sqrt{x+2}} + (2)\sqrt{x+2} \right) \\ &= 2x + \left(\frac{x}{\sqrt{x+2}} + 2\sqrt{x+2} \right) cos(2x\sqrt{x+2}) \end{aligned}$$

39. $y = x^4 - 3x^2$

$$\begin{aligned} \frac{dy}{dx} &= 4x^3 - 6x \\ \frac{d^2y}{dx^2} &= 12x^2 - 6 \\ \frac{d^3y}{dx^3} &= 24x \end{aligned}$$

40. **a)** $v(t) = \frac{2t\ cos\ 2t - sin\ 2t}{t^2}$

b)

$$\begin{aligned} a(t) &= \frac{t^2[2t(-sin\ 2t)(2) + 2\ cos\ 2t - (cos\ 2t)(2)]}{t^4} - \\ & \quad \frac{[(2t\ cos\ 2t - sin\ 2t)(2t)]}{t^4} \\ &= \frac{-4t^3\ sin\ 2t - 4t^2\ cos\ 2t - 2t\ sin\ 2t}{t^4} \\ &= \frac{2(1 - 2t^2)sin\ 2t - 4t\ cos\ 2t}{t^4} \end{aligned}$$

c) $s(\frac{7\pi}{6}) = \frac{sin(\frac{7\pi}{3})}{\frac{7\pi}{6}} = \frac{3\sqrt{3}}{7\pi}$
$v(\frac{7\pi}{6}) = \frac{\frac{7\pi}{3}cos(\frac{7\pi}{3}) - sin(\frac{7\pi}{3})}{\frac{49\pi^2}{36}} = \frac{6(3\sqrt{3} - 7\pi)}{49\pi^2}$
$a(\frac{7\pi}{6}) = \frac{291 - 2(\frac{7\pi}{6})^2 sin(\frac{7\pi}{3}) - \frac{14\pi}{3}cos(\frac{7\pi}{3})}{(\frac{7\pi}{6})^4} = \frac{-12(-18\sqrt{3} + 42\pi + 49\sqrt{3}\pi^2)}{343\pi^3}$

41. $M = -0.001t^3 + 0.1t^2$

a) $M'(t) = -0.003t^2 + 0.2t$

b) $M(10) = 0.001(10)^3 + 0.1(10)^2 = -1 + 10 = 9$ words

c) $M'(10) = -0.003(10)^2 + 0.2(10) = -0.3 + 2 = 1.7$ words/minutes

42.

$$\begin{aligned}(f \circ g)(x) &= (2x^3)^2 - (2x^3) \\ &= 4x^6 - 2x^3\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= 2(x^2 - x)^3 \\ &= 2(x^6 - 3x^5 + 3x^4 - x^3) \\ &= 2x^6 - 6x^5 + 6x^4 - 2x^3\end{aligned}$$

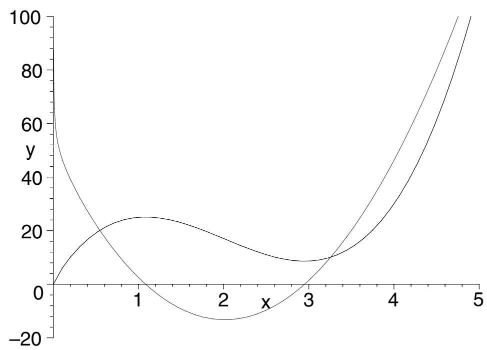
43. $y = (1-3x)^{2/3}(1+3x)^{1/3}$

$$\begin{aligned}\frac{dy}{dx} &= (1-3x)^{2/3}\left(\frac{1}{3}(1+3x)^{-2/3}(3)\right) + \\ &\quad \left(\frac{2}{3}(1-3x)^{-1/3}(-3)\right)(1+3x)^{1/3} \\ &= \left(\frac{1-3x}{1+3x}\right)^{2/3} - 2\left(\frac{1+3x}{1-3x}\right)^{1/3}\end{aligned}$$

44. $\lim\limits_{x\to 3} \dfrac{(x-3)(x^2+3x+9)}{x-3} = \lim\limits_{x\to 3}(x^2+3x+9) = 27$

45. $\lim\limits_{t\to 0} \dfrac{tan\ t + sin\ t}{t} = \lim\limits_{t\to 0}\left(\dfrac{tan\ t}{t} + \dfrac{sin\ t}{t}\right) =$
$\lim\limits_{t\to 0}\left(\dfrac{tan\ t}{t}\right) + \lim\limits_{t\to 0}\left(\dfrac{sin\ t}{t}\right) = 1 + 1 = 2$

46. $(1.0836, 25.1029)$ and $(2.9503, 8.6247)$



47. Limit approaches 0

Technology Connection

- **Page 73:**
  1. 3
  2. -0.2
- **Page 87:**
  1. 53
  2. 2.8284
  3. Limit does not exist
- **Page 95:**
  1. $f(x+h) = 64, \quad f(x) + h = 38$
  2. $f(x+h) = 218.78, \quad f(x) + h = 208.1$
- **Page 109:**
  1. $f'(x) = -\frac{3}{x^2}$, $f'(-2) = \frac{-3}{4}$, $f'(-1/2) = -12$
  2. $y = -\frac{3}{4}x - 3, \quad y = -12x - 12$
- **112:**
  1. $f'(20) = 60, \quad f'(37) = 26, \quad f'(50) = 0,$ $f'(90) = -80$
  2. $f'(-5) = -96, \quad f'(0) = -11, \quad f'(7) = 24,$ $f'(12) = -11, \quad f'(15) = -56$
  3. $f'(-2) = -36, \quad f'(0) = 0, \quad f'(2) = 12,$ $f'(4) = 0, \quad f'(6.3) = -43.47$
  4. $f'(-2)$ does not exist, $f'(-1.3) = 0.4079$, $f'(-0.5) = 1.8074$, $f'(0) = 2$, $f'(1) = 1.1547$, $f'(2)$ does not exist
  5. $x = 3$ is not in the domain of the function $x\sqrt{4-x}$
- **Page 118:**
  1. $f'(24) = 152, \quad f'(138) = -76, \quad f'(150) = -100,$ $f'(190) = -180$
  2. $f'(-5) = -96, \quad f'(0) = -11, \quad f'(7) = 24,$ $f'(12) = -11, \quad f'(15) = -56$
  3. $f'(-2) = -36, \quad f'(0) = 0, \quad f'(2) = 12,$ $f'(4) = 0, \quad f'(6.3) = -43.47$
  4. $f'(0) = 1.61, \quad f'(12) = 0.7076, \quad f'(24) = 0.3236,$ $f'(6.3) = 0.458$
- **Page 137:**
  1. Part "c" is the correct derivative
  2. Left to the student
  3. Left to the student

Extended Life Science Connection

1. $f(x) = \sqrt{4+x}$

$$\begin{aligned}f'(x) &= \frac{1}{2\sqrt{4+x}} \\ f'(0) &= \frac{1}{2\sqrt{4+0}} \\ &= \frac{1}{2\sqrt{4}} \\ &= 0.25\end{aligned}$$

2. **a)**

h	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
$F_+(h)$	0.242641	0.24621	0.24808	0.24903

b)

h	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
$F_-(h)$	0.258342	0.25403	0.25198	0.25098

3. **a)** Forward difference:: $\frac{350-264}{(2/7)} = 301$ Backward difference: $\frac{264-167}{(2/7)} = 339.5$

b) Forward difference:: $\frac{306-264}{(1/7)} = 294$ Backward difference: $\frac{264-219}{(1/7)} = 315$ It seems that the answers from part a) are more accurate since they use points closer to the point under consideration (March 19) and therefore represent the slope of the tangent line more closely than points away from the point corresponding to March 19.

4.

$$\begin{aligned}
F_c &= \frac{1}{2}[F_+(h)+F_-(h)] \\
&= \frac{1}{2}\left(\frac{f(h)-f(0)}{h}+\frac{f(0)-f(-h)}{h}\right) \\
&= \frac{1}{2}\left(f(h)-f(0)+f(0)-f(-h)\right) \\
&= \frac{1}{2}\left(\frac{f(h)-f(-h)}{h}\right) \\
&= \frac{f(h)-f(-h)}{2h}
\end{aligned}$$

5. Since $F_c(h)$ cover more of the function domain containing $x=0$ than does $F_+(h)$ or $F_-(h)$, then it makes sense that $F_c(h)$ be closer to $f'(0)$ than either of them.

6. Using the results from question 2 parts a) and b) and the definition of F_c we get

h	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
$F_c(h)$	0.25049	0.25012	0.25003	0.250005

7. The answers in Exercise 6 are closer to $f'(0)=0.25$ than those from Exercise 2. The value of $h=\frac{1}{16}$ using the central difference quotient gives the closest value to 0.25. This is expected since the central difference quotient is the most accurate out of the three difference quotients and $\frac{1}{16}$ is the smallest value of h to be considered.

8. **a)** $F_c(1/7)=\frac{f(1/7)-f(-1/7)}{2(1/7)}=\frac{306-219}{(2/7)}=304.5$

b) $F_c(2/7)=\frac{f(2/7)-f(-2/7)}{2(2/7)}=\frac{350-167}{(4/7)}=320.25$

c) The answer from part a) gives the more accurate rate of change.

9. $F_c(4/7)=\frac{f(4/7)-f(-4/7)}{2(4/7)}=\frac{7739-7053}{(8/7)}=600.25$

$F_c(4/7)=\frac{f(2/7)-f(-2/7)}{2(2/7)}=\frac{7628-7296}{(4/7)}=581$

10. **a)** $f(x)=x^3+x$
$f'(x)=3x^2+1,\ f'(0)=1$
Close to answer given by nDeriv, which is 1.000001

b) $f(x)=1000x^3+x$
$f'(x)=3000x^2,\ f'(0)=1$
Close to answer given by nDeriv, which is 1.001

c) $f(x)=1000000x^3+x$
$f'(x)=3000000x^2+1,\ f'(0)=1$
Does not match the answer from nDeriv, which is 2.

d) $f(x)=1000000000x^3+x$
$f'(x)=3000000000x^2+1,\ f'(0)=1$
Does not match the answer from nDeriv, which is 1001.

e) $f(x)=\mid x\mid$
$f'(x)$ does not exist at $x=0$
Does not match the answer given by nDeriv, which is 0.