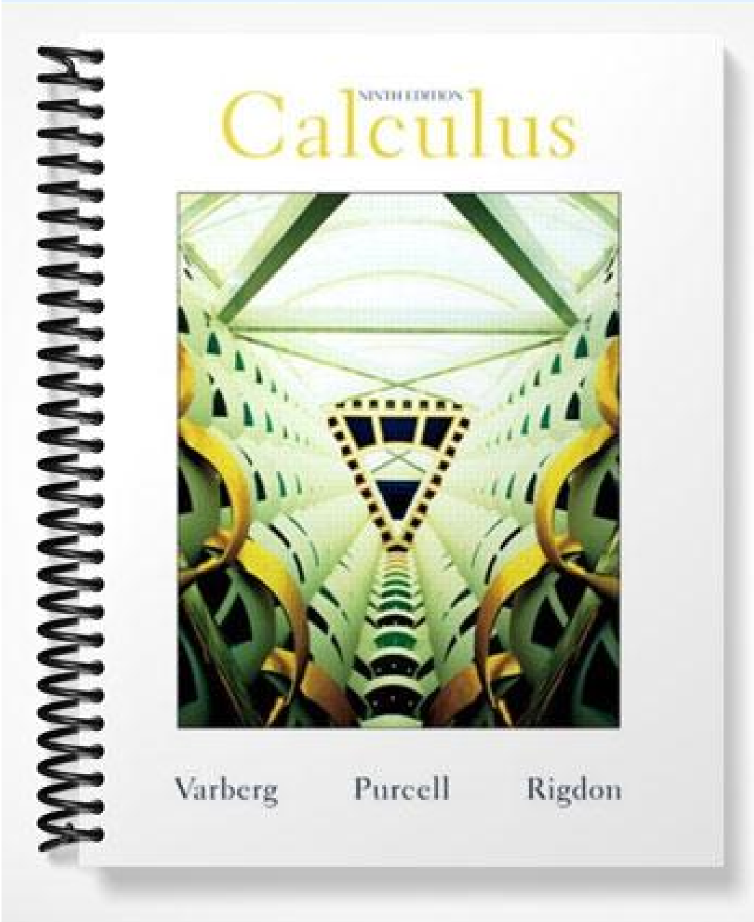
SOLUTIONS MANUAL



CHAPTER 0	Preliminaries

0.1 Concepts Review

1. rational numbers
2. dense
3. If not Q then not P.
4. theorems

Problem Set 0.1

1. $4-2(8-11)+6=4-2(-3)+6$
$=4+6+6=16$

2. $3[2-4(7-12)]=3[2-4(-5)]$
$=3[2+20]=3(22)=66$

3. $-4[5(-3+12-4)+2(13-7)]$
$=-4[5(5)+2(6)]=-4[25+12]$
$=-4(37)=-148$

4. $5[-1(7+12-16)+4]+2$
$=5[-1(3)+4]+2=5(-3+4)+2$
$=5(1)+2=5+2=7$

5. $\frac{5}{7}-\frac{1}{13}=\frac{65}{91}-\frac{7}{91}=\frac{58}{91}$

6. $\frac{3}{4-7}+\frac{3}{21}-\frac{1}{6}=\frac{3}{-3}+\frac{3}{21}-\frac{1}{6}$
$=-\frac{42}{42}+\frac{6}{42}-\frac{7}{42}=-\frac{43}{42}$

7. $\frac{1}{3}\left[\frac{1}{2}\left(\frac{1}{4}-\frac{1}{3}\right)+\frac{1}{6}\right]=\frac{1}{3}\left[\frac{1}{2}\left(\frac{3-4}{12}\right)+\frac{1}{6}\right]$
$=\frac{1}{3}\left[\frac{1}{2}\left(-\frac{1}{12}\right)+\frac{1}{6}\right]$
$=\frac{1}{3}\left[-\frac{1}{24}+\frac{4}{24}\right]$
$=\frac{1}{3}\left(\frac{3}{24}\right)=\frac{1}{24}$

8. $-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)\right]=-\frac{1}{3\left[\frac{2}{5}-\frac{1}{2}\left(\frac{5}{15}-\frac{3}{15}\right)\right]}$
$=-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{2}\left(\frac{2}{15}\right)\right]=-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{15}\right]$
$=-\frac{1}{3}\left(\frac{6}{15}-\frac{1}{15}\right)=-\frac{1}{3}\left(\frac{5}{15}\right)=-\frac{1}{9}$

9. $\frac{14}{21}\left(\frac{2}{5-\frac{1}{3}}\right)^2=\frac{14}{21}\left(\frac{2}{\frac{14}{3}}\right)^2=\frac{14}{21}\left(\frac{6}{14}\right)^2$
$=\frac{14}{21}\left(\frac{3}{7}\right)^2=\frac{2}{3}\left(\frac{9}{49}\right)=\frac{6}{49}$

10. $\frac{\left(\frac{2}{7}-5\right)}{\left(1-\frac{1}{7}\right)}=\frac{\left(\frac{2}{7}-\frac{35}{7}\right)}{\left(\frac{7}{7}-\frac{1}{7}\right)}=\frac{\left(-\frac{33}{7}\right)}{\left(\frac{6}{7}\right)}=-\frac{33}{6}=-\frac{11}{2}$

11. $\frac{\frac{11}{7}-\frac{12}{21}}{\frac{11}{7}+\frac{12}{21}}=\frac{\frac{11}{7}-\frac{4}{7}}{\frac{11}{7}+\frac{4}{7}}=\frac{\frac{7}{7}}{\frac{15}{7}}=\frac{7}{15}$

12. $\frac{\frac{1}{2}-\frac{3}{4}+\frac{7}{8}}{\frac{1}{2}+\frac{3}{4}-\frac{7}{8}}=\frac{\frac{4}{8}-\frac{6}{8}+\frac{7}{8}}{\frac{4}{8}+\frac{6}{8}-\frac{7}{8}}=\frac{\frac{5}{8}}{\frac{3}{8}}=\frac{5}{3}$

13. $1-\frac{1}{1+\frac{1}{2}}=1-\frac{1}{\frac{3}{2}}=1-\frac{2}{3}=\frac{3}{3}-\frac{2}{3}=\frac{1}{3}$

14. $2+\frac{3}{1+\frac{5}{2}}=2+\frac{3}{\frac{2}{2}-\frac{5}{2}}=2+\frac{3}{\frac{7}{2}}$
$=2+\frac{6}{7}=\frac{14}{7}+\frac{6}{7}=\frac{20}{7}$

15. $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$
$=5-3=2$

16. $\left(\sqrt{5}-\sqrt{3}\right)^2=\left(\sqrt{5}\right)^2-2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2$
$=5-2\sqrt{15}+3=8-2\sqrt{15}$

17. $(3x-4)(x+1)=3x^2+3x-4x-4$
$=3x^2-x-4$

18. $(2x-3)^2=(2x-3)(2x-3)$
$=4x^2-6x-6x+9$
$=4x^2-12x+9$

19. $(3x-9)(2x+1)=6x^2+3x-18x-9$
$=6x^2-15x-9$

20. $(4x-11)(3x-7)=12x^2-28x-33x+77$
$=12x^2-61x+77$

21. $(3t^2-t+1)^2=(3t^2-t+1)(3t^2-t+1)$
$=9t^4-3t^3+3t^2-3t^3+t^2-t+3t^2-t+1$
$=9t^4-6t^3+7t^2-2t+1$

22. $(2t+3)^3=(2t+3)(2t+3)(2t+3)$
$=(4t^2+12t+9)(2t+3)$
$=8t^3+12t^2+24t^2+36t+18t+27$
$=8t^3+36t^2+54t+27$

23. $\dfrac{x^2-4}{x-2}=\dfrac{(x-2)(x+2)}{x-2}=x+2,\ x\neq 2$

24. $\dfrac{x^2-x-6}{x-3}=\dfrac{(x-3)(x+2)}{(x-3)}=x+2,\ x\neq 3$

25. $\dfrac{t^2-4t-21}{t+3}=\dfrac{(t+3)(t-7)}{t+3}=t-7,\ t\neq -3$

26. $\dfrac{2x-2x^2}{x^3-2x^2+x}=\dfrac{2x(1-x)}{x(x^2-2x+1)}$
$=\dfrac{-2x(x-1)}{x(x-1)(x-1)}$
$=-\dfrac{2}{x-1}$

27. $\dfrac{12}{x^2+2x}+\dfrac{4}{x}+\dfrac{2}{x+2}$
$=\dfrac{12}{x(x+2)}+\dfrac{4(x+2)}{x(x+2)}+\dfrac{2x}{x(x+2)}$
$=\dfrac{12+4x+8+2x}{x(x+2)}=\dfrac{6x+20}{x(x+2)}$
$=\dfrac{2(3x+10)}{x(x+2)}$

28. $\dfrac{2}{6y-2}+\dfrac{y}{9y^2-1}$
$=\dfrac{2}{2(3y-1)}+\dfrac{y}{(3y+1)(3y-1)}$
$=\dfrac{2(3y+1)}{2(3y+1)(3y-1)}+\dfrac{2y}{2(3y+1)(3y-1)}$
$=\dfrac{6y+2+2y}{2(3y+1)(3y-1)}=\dfrac{8y+2}{2(3y+1)(3y-1)}$
$=\dfrac{2(4y+1)}{2(3y+1)(3y-1)}=\dfrac{4y+1}{(3y+1)(3y-1)}$

29. **a.** $0\cdot 0=0$ **b.** $\dfrac{0}{0}$ is undefined.

c. $\dfrac{0}{17}=0$ **d.** $\dfrac{3}{0}$ is undefined.

e. $0^5=0$ **f.** $17^0=1$

30. If $\dfrac{0}{0}=a$, then $0=0\cdot a$, but this is meaningless because a could be any real number. No single value satisfies $\dfrac{0}{0}=a$.

31.
$$\begin{array}{r} .08\overline{3} \\ 12\overline{)1.000} \\ \underline{96} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

32.

$$\begin{array}{r} .\overline{285714} \\ 7\overline{)2.000000} \\ \underline{1\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 2 \end{array}$$

33.

$$\begin{array}{r} .\overline{142857} \\ 21\overline{)\ 3.000000} \\ \underline{2\,1} \\ 90 \\ \underline{84} \\ 60 \\ \underline{42} \\ 180 \\ \underline{168} \\ 120 \\ \underline{105} \\ 150 \\ \underline{147} \\ 3 \end{array}$$

34.

$$\begin{array}{r} .294117... \\ 17\overline{)5.000000...} \to 0.\overline{2941176470588235} \\ \underline{3\,4} \\ 160 \\ \underline{153} \\ 70 \\ \underline{68} \\ 20 \\ \underline{17} \\ 30 \\ \underline{17} \\ 130 \\ \underline{119} \\ 11 \\ \vdots \end{array}$$

35.

$$\begin{array}{r} 3.\overline{6} \\ 3\overline{)11.0} \\ \underline{9} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

36.

$$\begin{array}{r} .\overline{846153} \\ 13\overline{)11.000000} \\ \underline{10\,4} \\ 60 \\ \underline{52} \\ 80 \\ \underline{78} \\ 20 \\ \underline{13} \\ 70 \\ \underline{65} \\ 50 \\ \underline{39} \\ 11 \end{array}$$

37. $x = 0.123123123...$

$$\begin{aligned} 1000x &= 123.123123... \\ \underline{x} &\underline{= 0.123123...} \\ 999x &= 123 \\ x &= \frac{123}{999} = \frac{41}{333} \end{aligned}$$

38. $x = 0.217171717\ldots$

$$\begin{aligned} 1000x &= 217.171717... \\ \underline{10x} &\underline{= 2.171717...} \\ 990x &= 215 \\ x &= \frac{215}{990} = \frac{43}{198} \end{aligned}$$

39. $x = 2.56565656...$

$$\begin{aligned} 100x &= 256.565656... \\ \underline{x} &\underline{= 2.565656...} \\ 99x &= 254 \\ x &= \frac{254}{99} \end{aligned}$$

40. $x = 3.929292\ldots$

$$\begin{aligned} 100x &= 392.929292... \\ \underline{x} &\underline{= 3.929292...} \\ 99x &= 389 \\ x &= \frac{389}{99} \end{aligned}$$

41. $x = 0.199999...$

$$\begin{array}{r} 100x = 19.99999... \\ \underline{10x = 1.99999...} \\ 90x = 18 \end{array}$$

$$x = \frac{18}{90} = \frac{1}{5}$$

42. $x = 0.399999\ldots$

$$\begin{array}{r} 100x = 39.99999... \\ \underline{10x = 3.99999...} \\ 90x = 36 \end{array}$$

$$x = \frac{36}{90} = \frac{2}{5}$$

43. Those rational numbers that can be expressed by a terminating decimal followed by zeros.

44. $\frac{p}{q} = p\left(\frac{1}{q}\right)$, so we only need to look at $\frac{1}{q}$. If $q = 2^n \cdot 5^m$, then $\frac{1}{q} = \left(\frac{1}{2}\right)^n \cdot \left(\frac{1}{5}\right)^m = (0.5)^n (0.2)^m$. The product of any number of terminating decimals is also a terminating decimal, so $(0.5)^n$ and $(0.2)^m$, and hence their product, $\frac{1}{q}$, is a terminating decimal. Thus $\frac{p}{q}$ has a terminating decimal expansion.

45. Answers will vary. Possible answer: 0.000001, $\frac{1}{\pi^{12}} \approx 0.0000010819...$

46. Smallest positive integer: 1; There is no smallest positive rational or irrational number.

47. Answers will vary. Possible answer: 3.14159101001...

48. There is no real number between 0.9999… (repeating 9's) and 1. 0.9999… and 1 represent the *same* real number.

49. Irrational

50. Answers will vary. Possible answers: $-\pi$ and π, $-\sqrt{2}$ and $\sqrt{2}$

51. $(\sqrt{3}+1)^3 \approx 20.39230485$

52. $\left(\sqrt{2}-\sqrt{3}\right)^4 \approx 0.0102051443$

53. $\sqrt[4]{1.123} - \sqrt[3]{1.09} \approx 0.00028307388$

54. $(3.1415)^{-1/2} \approx 0.5641979034$

55. $\sqrt{8.9\pi^2+1} - 3\pi \approx 0.000691744752$

56. $\sqrt[4]{(6\pi^2-2)\pi} \approx 3.661591807$

57. Let a and b be real numbers with $a < b$. Let n be a natural number that satisfies $1/n < b - a$. Let $S = \{k : k/n > b\}$. Since a nonempty set of integers that is bounded below contains a least element, there is a $k_0 \in S$ such that $k_0/n > b$ but $(k_0 - 1)/n \le b$. Then

$$\frac{k_0 - 1}{n} = \frac{k_0}{n} - \frac{1}{n} > b - \frac{1}{n} > a$$

Thus, $a < \frac{k_0-1}{n} \le b$. If $\frac{k_0-1}{n} < b$, then choose $r = \frac{k_0-1}{n}$. Otherwise, choose $r = \frac{k_0-2}{n}$.

Note that $a < b - \frac{1}{n} < r$.

Given $a < b$, choose r so that $a < r_1 < b$. Then choose r_2, r_3 so that $a < r_2 < r_1 < r_3 < b$, and so on.

58. Answers will vary. Possible answer: $\approx 120 \text{ in}^3$

59. $r = 4000 \text{ mi } \times 5280 \frac{\text{ft}}{\text{mi}} = 21{,}120{,}000 \text{ ft}$

$$\text{equator} = 2\pi r = 2\pi(21{,}120{,}000) \approx 132{,}700{,}874 \text{ ft}$$

60. Answers will vary. Possible answer:

$$70\frac{\text{beats}}{\text{min}} \times 60\frac{\text{min}}{\text{hr}} \times 24\frac{\text{hr}}{\text{day}} \times 365\frac{\text{day}}{\text{year}} \times 20 \text{ yr} = 735{,}840{,}000 \text{ beats}$$

61. $V = \pi r^2 h = \pi\left(\frac{16}{2}\cdot 12\right)^2 (270 \cdot 12)$

$$\approx 93{,}807{,}453.98 \text{ in.}^3$$

volume of one board foot (in inches):

$1 \times 12 \times 12 = 144 \text{ in.}^3$

number of board feet:

$$\frac{93{,}807{,}453.98}{144} \approx 651{,}441 \text{ board ft}$$

62. $V = \pi(8.004)^2(270) - \pi(8)^2(270) \approx 54.3 \text{ ft.}^3$

63. a. If I stay home from work today then it rains. If I do not stay home from work, then it does not rain.

b. If the candidate will be hired then she meets all the qualifications. If the candidate will not be hired then she does not meet all the qualifications.

64. a. If I pass the course, then I got an A on the final exam. If I did not pass the course, thn I did not get an A on the final exam.

b. If I take off next week, then I finished my research paper. If I do not take off next week, then I did not finish my research paper.

65. a. If a triangle is a right triangle, then $a^2 + b^2 = c^2$. If a triangle is not a right triangle, then $a^2 + b^2 \neq c^2$.

b. If the measure of angle *ABC* is greater than 0° and less than 90°, it is acute. If the measure of angle *ABC* is less than 0° or greater than 90°, then it is not acute.

66. a. If angle *ABC* is an acute angle, then its measure is 45°. If angle *ABC* is not an acute angle, then its measure is not 45°.

b. If $a^2 < b^2$ then $a < b$. If $a^2 \geq b^2$ then $a \geq b$.

67. a. The statement, converse, and contrapositive are all true.

b. The statement, converse, and contrapositive are all true.

68. a. The statement and contrapositive are true. The converse is false.

b. The statement, converse, and contrapositive are all false.

69. a. Some isosceles triangles are not equilateral. The negation is true.

b. All real numbers are integers. The original statement is true.

c. Some natural number is larger than its square. The original statement is true.

70. a. Some natural number is not rational. The original statement is true.

b. Every circle has area less than or equal to 9π. The original statement is true.

c. Some real number is less than or equal to its square. The negation is true.

71. a. True; If *x* is positive, then x^2 is positive.

b. False; Take $x = -2$. Then $x^2 > 0$ but $x < 0$.

c. False; Take $x = \frac{1}{2}$. Then $x^2 = \frac{1}{4} < x$

d. True; Let *x* be any number. Take $y = x^2 + 1$. Then $y > x^2$.

e. True; Let *y* be any positive number. Take $x = \frac{y}{2}$. Then $0 < x < y$.

72. a. True; $x + (-x) < x + 1 + (-x) : 0 < 1$

b. False; There are infinitely many prime numbers.

c. True; Let *x* be any number. Take $y = \frac{1}{x} + 1$. Then $y > \frac{1}{x}$.

d. True; $1/n$ can be made arbitrarily close to 0.

e. True; $1/2^n$ can be made arbitrarily close to 0.

73. a. If *n* is odd, then there is an integer *k* such that $n = 2k + 1$. Then

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1$$
$$= 2(2k^2 + 2k) + 1$$

b. Prove the contrapositive. Suppose *n* is even. Then there is an integer *k* such that $n = 2k$. Then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$. Thus n^2 is even.

74. Parts (a) and (b) prove that *n* is odd if and only if n^2 is odd.

75. a. $243 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

b. $124 = 4 \cdot 31 = 2 \cdot 2 \cdot 31$ or $2^2 \cdot 31$

c. $5100 = 2 \cdot 2550 = 2 \cdot 2 \cdot 1275$
$= 2 \cdot 2 \cdot 3 \cdot 425 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 85$
$= 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 17$ or $2^2 \cdot 3 \cdot 5^2 \cdot 17$

76. For example, let $A = b \cdot c^2 \cdot d^3$; then $A^2 = b^2 \cdot c^4 \cdot d^6$, so the square of the number is the product of primes which occur an even number of times.

77. $\sqrt{2} = \frac{p}{q}; 2 = \frac{p^2}{q^2}; 2q^2 = p^2$; Since the prime factors of p^2 must occur an even number of times, $2q^2$ would not be valid and $\frac{p}{q} = \sqrt{2}$ must be irrational.

78. $\sqrt{3} = \frac{p}{q}$; $3 = \frac{p^2}{q^2}$; $3q^2 = p^2$; Since the prime factors of p^2 must occur an even number of times, $3q^2$ would not be valid and $\frac{p}{q} = \sqrt{3}$ must be irrational.

79. Let a, b, p, and q be natural numbers, so $\frac{a}{b}$ and $\frac{p}{q}$ are rational. $\frac{a}{b} + \frac{p}{q} = \frac{aq + bp}{bq}$ This sum is the quotient of natural numbers, so it is also rational.

80. Assume a is irrational, $\frac{p}{q} \neq 0$ is rational, and $a \cdot \frac{p}{q} = \frac{r}{s}$ is rational. Then $a = \frac{q \cdot r}{p \cdot s}$ is rational, which is a contradiction.

81. **a.** $-\sqrt{9} = -3$; rational

b. $0.375 = \frac{3}{8}$; rational

c. $(3\sqrt{2})(5\sqrt{2}) = 15\sqrt{4} = 30$; rational

d. $(1 + \sqrt{3})^2 = 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3}$; irrational

82. **a.** –2 **b.** –2

c. $x = 2.4444...$;
$$\begin{aligned} 10x &= 24.4444... \\ x &= 2.4444... \\ \hline 9x &= 22 \\ x &= \frac{22}{9} \end{aligned}$$

d. 1

e. $n = 1$: $x = 0$, $n = 2$: $x = \frac{3}{2}$, $n = 3$: $x = -\frac{2}{3}$,
$n = 4$: $x = \frac{5}{4}$
The upper bound is $\frac{3}{2}$.

f. $\sqrt{2}$

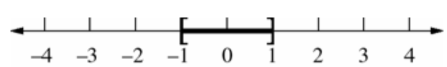
83. **a.** Answers will vary. Possible answer: An example
is $S = \{x : x^2 < 5, x \text{ a rational number}\}$.
Here the least upper bound is $\sqrt{5}$, which is real but irrational.

b. True

0.2 Concepts Review

1. $[-1, 5); (-\infty, -2]$
2. $b > 0; b < 0$
3. (b) and (c)
4. $-1 \leq x \leq 5$

Problem Set 0.2

1. a.


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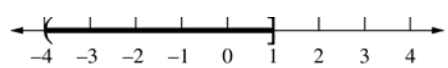

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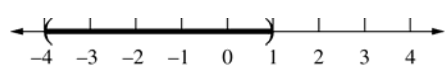

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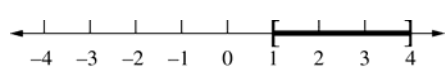
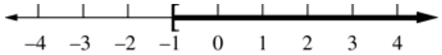
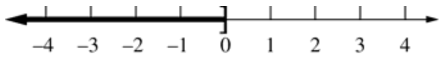


e. 

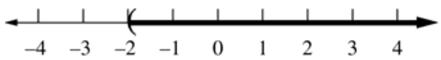
f. 

2. **a.** $(2,7)$ **b.** $[-3,4)$

c. $(-\infty,-2]$ **d.** $[-1,3]$

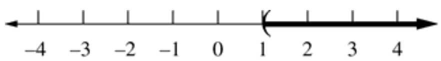
3. $x-7<2x-5$

$-2<x;(-2,\infty)$



4. $3x-5<4x-6$

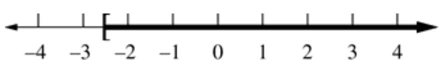
$1<x;(1,\infty)$



5. $7x-2\le 9x+3$

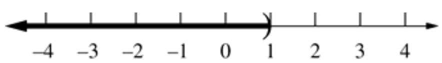
$-5\le 2x$

$x\ge -\frac{5}{2};\left[-\frac{5}{2},\infty\right)$



6. $5x-3>6x-4$

$1>x;(-\infty,1)$



7. $-4<3x+2<5$

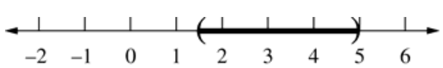
$-6<3x<3$

$-2<x<1;(-2,-1)$

8. $-3<4x-9<11$

$6<4x<20$

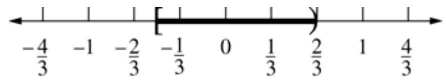
$\frac{3}{2}<x<5;\left(\frac{3}{2},5\right)$



9. $-3<1-6x\le 4$

$-4<-6x\le 3$

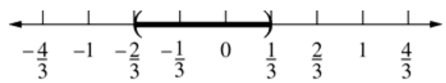
$\frac{2}{3}>x\ge -\frac{1}{2};\left[-\frac{1}{2},\frac{2}{3}\right)$



10. $4<5-3x<7$

$-1<-3x<2$

$\frac{1}{3}>x>-\frac{2}{3};\ \left(-\frac{2}{3},\frac{1}{3}\right)$



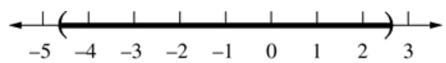
11. $x^2+2x-12<0;$

$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-12)}}{2(1)}=\frac{-2\pm\sqrt{52}}{2}$

$=-1\pm\sqrt{13}$

$\left[x-\left(-1+\sqrt{13}\right)\right]\left[x-\left(-1-\sqrt{13}\right)\right]<0;$

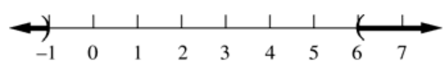
$\left(-1-\sqrt{13},-1+\sqrt{13}\right)$



12. $x^2-5x-6>0$

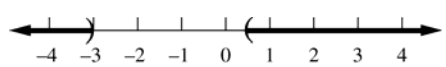
$(x+1)(x-6)>0;$

$(-\infty,-1)\cup(6,\infty)$



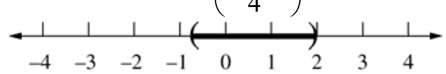
13. $2x^2+5x-3>0;\ (2x-1)(x+3)>0;$

$(-\infty,-3)\cup\left(\frac{1}{2},\infty\right)$

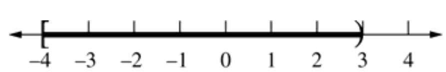


14. $4x^2-5x-6<0$

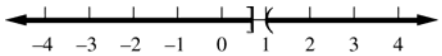
$(4x+3)(x-2)<0;\left(-\frac{3}{4},2\right)$



15. $\frac{x+4}{x-3}\le 0;\quad [-4,3)$



16. $\frac{3x-2}{x-1} \geq 0; \left(-\infty, \frac{2}{3}\right] \cup (1, \infty)$

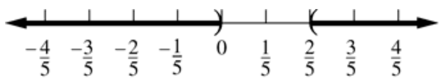


17. $\frac{2}{x} < 5$

$\frac{2}{x} - 5 < 0$

$\frac{2-5x}{x} < 0;$

$(-\infty, 0) \cup \left(\frac{2}{5}, \infty\right)$

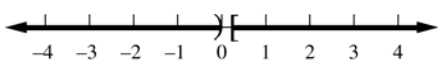


18. $\frac{7}{4x} \leq 7$

$\frac{7}{4x} - 7 \leq 0$

$\frac{7-28x}{4x} \leq 0;$

$(-\infty, 0) \cup \left[\frac{1}{4}, \infty\right)$

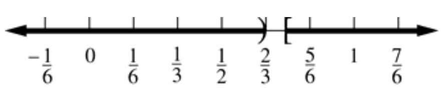


19. $\frac{1}{3x-2} \leq 4$

$\frac{1}{3x-2} - 4 \leq 0$

$\frac{1-4(3x-2)}{3x-2} \leq 0$

$\frac{9-12x}{3x-2} \leq 0; \left(-\infty, \frac{2}{3}\right) \cup \left[\frac{3}{4}, \infty\right)$

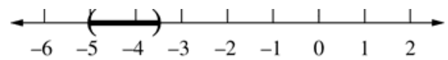


20. $\frac{3}{x+5} > 2$

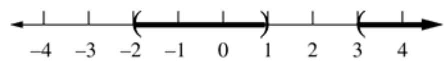
$\frac{3}{x+5} - 2 > 0$

$\frac{3-2(x+5)}{x+5} > 0$

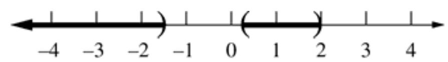
$\frac{-2x-7}{x+5} > 0; \left(-5, -\frac{7}{2}\right)$



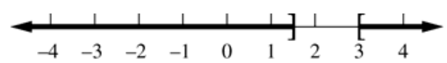
21. $(x+2)(x-1)(x-3) > 0; (-2, 1) \cup (3, 8)$



22. $(2x+3)(3x-1)(x-2) < 0; \left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{1}{3}, 2\right)$

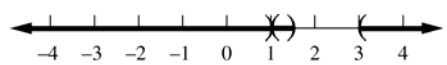


23. $(2x-3)(x-1)^2(x-3) \geq 0; \left(-\infty, \frac{3}{2}\right] \cup [3, \infty)$



24. $(2x-3)(x-1)^2(x-3) > 0;$

$(-\infty, 1) \cup \left(1, \frac{3}{2}\right) \cup (3, \infty)$

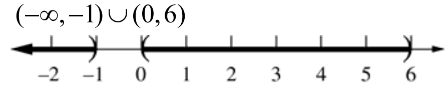


25. $x^3 - 5x^2 - 6x < 0$

$x(x^2 - 5x - 6) < 0$

$x(x+1)(x-6) < 0;$

$(-\infty, -1) \cup (0, 6)$

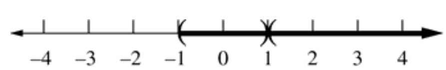


26. $x^3 - x^2 - x + 1 > 0$

$(x^2 - 1)(x - 1) > 0$

$(x+1)(x-1)^2 > 0;$

$(-1, 1) \cup (1, \infty)$



27. **a.** False. **b.** True.

c. False.

28. a. True. **b.** True.

c. False.

29. a. $\Rightarrow$ Let $a < b$, so $ab < b^2$. Also, $a^2 < ab$. Thus, $a^2 < ab < b^2$ and $a^2 < b^2$. $\Leftarrow$ Let $a^2 < b^2$, so $a \neq b$ Then
$0 < (a-b)^2 = a^2 - 2ab + b^2$
$< b^2 - 2ab + b^2 = 2b(b-a)$
Since $b > 0$, we can divide by $2b$ to get $b - a > 0$.

b. We can divide or multiply an inequality by any positive number.
$a < b \Leftrightarrow \frac{a}{b} < 1 \Leftrightarrow \frac{1}{b} < \frac{1}{a}$.

30. (b) and (c) are true.
(a) is false: Take $a = -1, b = 1$.
(d) is false: if $a \le b$, then $-a \ge -b$.

31. a. $3x + 7 > 1$ and $2x + 1 < 3$
$3x > -6$ and $2x < 2$
$x > -2$ and $x < 1$; $(-2, 1)$

b. $3x + 7 > 1$ and $2x + 1 > -4$
$3x > -6$ and $2x > -5$
$x > -2$ and $x > -\frac{5}{2}$; $(-2, \infty)$

c. $3x + 7 > 1$ and $2x + 1 < -4$
$x > -2$ and $x < -\frac{5}{2}$; $\varnothing$

32. a. $2x - 7 > 1$ or $2x + 1 < 3$
$2x > 8$ or $2x < 2$
$x > 4$ or $x < 1$
$(-\infty, 1) \cup (4, \infty)$

b. $2x - 7 \le 1$ or $2x + 1 < 3$
$2x \le 8$ or $2x < 2$
$x \le 4$ or $x < 1$
$(-\infty, 4]$

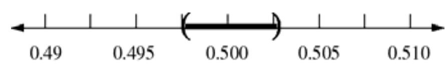
c. $2x - 7 \le 1$ or $2x + 1 > 3$
$2x \le 8$ or $2x > 2$
$x \le 4$ or $x > 1$
$(-\infty, \infty)$

33. a. $(x+1)(x^2 + 2x - 7) \ge x^2 - 1$
$x^3 + 3x^2 - 5x - 7 \ge x^2 - 1$
$x^3 + 2x^2 - 5x - 6 \ge 0$
$(x+3)(x+1)(x-2) \ge 0$
$[-3, -1] \cup [2, \infty)$

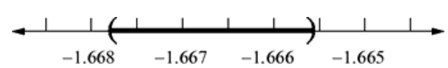
b. $x^4 - 2x^2 \ge 8$
$x^4 - 2x^2 - 8 \ge 0$
$(x^2 - 4)(x^2 + 2) \ge 0$
$(x^2 + 2)(x+2)(x-2) \ge 0$
$(-\infty, -2] \cup [2, \infty)$

c. $(x^2 + 1)^2 - 7(x^2 + 1) + 10 < 0$
$[(x^2 + 1) - 5][(x^2 + 1) - 2] < 0$
$(x^2 - 4)(x^2 - 1) < 0$
$(x+2)(x+1)(x-1)(x-2) < 0$
$(-2, -1) \cup (1, 2)$

34. a. $1.99 < \frac{1}{x} < 2.01$
$1.99x < 1 < 2.01x$
$1.99x < 1$ and $1 < 2.01x$
$x < \frac{1}{1.99}$ and $x > \frac{1}{2.01}$
$\frac{1}{2.01} < x < \frac{1}{1.99}$
$\left(\frac{1}{2.01}, \frac{1}{1.99}\right)$



b. $2.99 < \frac{1}{x+2} < 3.01$
$2.99(x+2) < 1 < 3.01(x+2)$
$2.99x + 5.98 < 1$ and $1 < 3.01x + 6.02$
$x < \frac{-4.98}{2.99}$ and $x > \frac{-5.02}{3.01}$
$-\frac{5.02}{3.01} < x < -\frac{4.98}{2.99}$
$\left(-\frac{5.02}{3.01}, -\frac{4.98}{2.99}\right)$



35. $|x - 2| \ge 5$;
$x - 2 \le -5$ or $x - 2 \ge 5$
$x \le -3$ or $x \ge 7$
$(-\infty, -3] \cup [7, \infty)$

36. $|x+2|<1;$

$-1<x+2<1$

$-3<x<-1$

$(-3,-1)$

37. $|4x+5|\le 10;$

$-10\le 4x+5\le 10$

$-15\le 4x\le 5$

$-\frac{15}{4}\le x\le \frac{5}{4};\left[-\frac{15}{4},\frac{5}{4}\right]$

38. $|2x-1|>2;$

$2x-1<-2$ or $2x-1>2$

$2x<-1$ or $2x>3;$

$x<-\frac{1}{2}$ or $x>\frac{3}{2},\left(-\infty,-\frac{1}{2}\right)\cup\left(\frac{3}{2},\infty\right)$

39. $\left|\frac{2x}{7}-5\right|\ge 7$

$\frac{2x}{7}-5\le -7$ or $\frac{2x}{7}-5\ge 7$

$\frac{2x}{7}\le -2$ or $\frac{2x}{7}\ge 12$

$x\le -7$ or $x\ge 42;$

$(-\infty,-7]\cup[42,\infty)$

40. $\left|\frac{x}{4}+1\right|<1$

$-1<\frac{x}{4}+1<1$

$-2<\frac{x}{4}<0;$

$-8<x<0;\ (-8,0)$

41. $|5x-6|>1;$

$5x-6<-1$ or $5x-6>1$

$5x<5$ or $5x>7$

$x<1$ or $x>\frac{7}{5};(-\infty,1)\cup\left(\frac{7}{5},\infty\right)$

42. $|2x-7|>3;$

$2x-7<-3$ or $2x-7>3$

$2x<4$ or $2x>10$

$x<2$ or $x>5;\ (-\infty,2)\cup(5,\infty)$

43. $\left|\frac{1}{x}-3\right|>6;$

$\frac{1}{x}-3<-6$ or $\frac{1}{x}-3>6$

$\frac{1}{x}+3<0$ or $\frac{1}{x}-9>0$

$\frac{1+3x}{x}<0$ or $\frac{1-9x}{x}>0;$

$\left(-\frac{1}{3},0\right)\cup\left(0,\frac{1}{9}\right)$

44. $\left|2+\frac{5}{x}\right|>1;$

$2+\frac{5}{x}<-1$ or $2+\frac{5}{x}>1$

$3+\frac{5}{x}<0$ or $1+\frac{5}{x}>0$

$\frac{3x+5}{x}<0$ or $\frac{x+5}{x}>0;$

$(-\infty,-5)\cup\left(-\frac{5}{3},0\right)\cup(0,\infty)$

45. $x^2-3x-4\ge 0;$

$x=\frac{3\pm\sqrt{(-3)^2-4(1)(-4)}}{2(1)}=\frac{3\pm 5}{2}=-1,4$

$(x+1)(x-4)=0;(-\infty,-1]\cup[4,\infty)$

46. $x^2-4x+4\le 0;\ x=\frac{4\pm\sqrt{(-4)^2-4(1)(4)}}{2(1)}=2$

$(x-2)(x-2)\le 0; x=2$

47. $3x^2+17x-6>0;$

$x=\frac{-17\pm\sqrt{(17)^2-4(3)(-6)}}{2(3)}=\frac{-17\pm 19}{6}=-6,\frac{1}{3}$

$(3x-1)(x+6)>0;\ (-\infty,-6)\cup\left(\frac{1}{3},\infty\right)$

48. $14x^2+11x-15\le 0;$

$x=\frac{-11\pm\sqrt{(11)^2-4(14)(-15)}}{2(14)}=\frac{-11\pm 31}{28}$

$x=-\frac{3}{2},\frac{5}{7}$

$\left(x+\frac{3}{2}\right)\left(x-\frac{5}{7}\right)\le 0;\left[-\frac{3}{2},\frac{5}{7}\right]$

49. $|x-3|<0.5\Rightarrow 5|x-3|<5(0.5)\Rightarrow |5x-15|<2.5$

50. $|x+2|<0.3\Rightarrow 4|x+2|<4(0.3)\Rightarrow |4x+18|<1.2$

51. $|x-2|<\frac{\varepsilon}{6}\Rightarrow 6|x-2|<\varepsilon\Rightarrow|6x-12|<\varepsilon$

52. $|x+4|<\frac{\varepsilon}{2}\Rightarrow 2|x+4|<\varepsilon\Rightarrow|2x+8|<\varepsilon$

53. $|3x-15|<\varepsilon\Rightarrow|3(x-5)|<\varepsilon$

$\Rightarrow 3|x-5|<\varepsilon$

$\Rightarrow|x-5|<\frac{\varepsilon}{3};\delta=\frac{\varepsilon}{3}$

54. $|4x-8|<\varepsilon\Rightarrow|4(x-2)|<\varepsilon$

$\Rightarrow 4|x-2|<\varepsilon$

$\Rightarrow|x-2|<\frac{\varepsilon}{4};\delta=\frac{\varepsilon}{4}$

55. $|6x+36|<\varepsilon\Rightarrow|6(x+6)|<\varepsilon$

$\Rightarrow 6|x+6|<\varepsilon$

$\Rightarrow|x+6|<\frac{\varepsilon}{6};\delta=\frac{\varepsilon}{6}$

56. $|5x+25|<\varepsilon\Rightarrow|5(x+5)|<\varepsilon$

$\Rightarrow 5|x+5|<\varepsilon$

$\Rightarrow|x+5|<\frac{\varepsilon}{5};\delta=\frac{\varepsilon}{5}$

57. $C=\pi d$

$|C-10|\le 0.02$

$|\pi d-10|\le 0.02$

$\left|\pi\left(d-\frac{10}{\pi}\right)\right|\le 0.02$

$\left|d-\frac{10}{\pi}\right|\le\frac{0.02}{\pi}\approx 0.0064$

We must measure the diameter to an accuracy of 0.0064 in.

58. $|C-50|\le 1.5, \left|\frac{5}{9}(F-32)-50\right|\le 1.5;$

$\frac{5}{9}|(F-32)-90|\le 1.5$

$|F-122|\le 2.7$

We are allowed an error of 2.7° F.

59. $|x-1|<2|x-3|$

$|x-1|<|2x-6|$

$(x-1)^2<(2x-6)^2$

$x^2-2x+1<4x^2-24x+36$

$3x^2-22x+35>0$

$(3x-7)(x-5)>0;$

$\left(-\infty,\frac{7}{3}\right)\cup(5,\infty)$

60. $|2x-1|\ge|x+1|$

$(2x-1)^2\ge(x+1)^2$

$4x^2-4x+1\ge x^2+2x+1$

$3x^2-6x\ge 0$

$3x(x-2)\ge 0$

$(-\infty,0]\cup[2,\infty)$

61. $2|2x-3|<|x+10|$

$|4x-6|<|x+10|$

$(4x-6)^2<(x+10)^2$

$16x^2-48x+36<x^2+20x+100$

$15x^2-68x-64<0$

$(5x+4)(3x-16)<0;$

$\left(-\frac{4}{5},\frac{16}{3}\right)$

62. $|3x-1|<2|x+6|$

$|3x-1|<|2x+12|$

$(3x-1)^2<(2x+12)^2$

$9x^2-6x+1<4x^2+48x+144$

$5x^2-54x-143<0$

$(5x+11)(x-13)<0$

$\left(-\frac{11}{5},13\right)$

63. $|x| < |y| \Rightarrow |x||x| \le |x||y|$ and $|x||y| < |y||y|$ Order property: $x < y \Leftrightarrow xz < yz$ when z is positive.

$\Rightarrow |x|^2 < |y|^2$ Transitivity

$\Rightarrow x^2 < y^2$ $\left(|x|^2 = x^2\right)$

Conversely,

$x^2 < y^2 \Rightarrow |x|^2 < |y|^2$ $\left(x^2 = |x|^2\right)$

$\Rightarrow |x|^2 - |y|^2 < 0$ Subtract $|y|^2$ from each side.

$\Rightarrow (|x| - |y|)(|x| + |y|) < 0$ Factor the difference of two squares.

$\Rightarrow |x| - |y| < 0$ This is the only factor that can be negative.

$\Rightarrow |x| < |y|$ Add $|y|$ to each side.

64. $0 < a < b \Rightarrow a = \left(\sqrt{a}\right)^2$ *and* $b = \left(\sqrt{b}\right)^2$, so

$\left(\sqrt{a}\right)^2 < \left(\sqrt{b}\right)^2$, and, by Problem 63,

$\left|\sqrt{a}\right| < \left|\sqrt{b}\right| \Rightarrow \sqrt{a} < \sqrt{b}$.

65. a. $|a - b| = |a + (-b)| \le |a| + |-b| = |a| + |b|$

b. $|a - b| \ge \big||a| - |b|\big| \ge |a| - |b|$ Use Property 4 of absolute values.

c. $|a + b + c| = |(a + b) + c| \le |a + b| + |c|$
$\le |a| + |b| + |c|$

66. $\left|\frac{1}{x^2+3} - \frac{1}{|x|+2}\right| = \left|\frac{1}{x^2+3} + \left(-\frac{1}{|x|+2}\right)\right|$

$\le \left|\frac{1}{x^2+3}\right| + \left|-\frac{1}{|x|+2}\right|$

$= \left|\frac{1}{x^2+3}\right| + \left|\frac{1}{|x|+2}\right|$

$= \frac{1}{x^2+3} + \frac{1}{|x|+2}$

by the Triangular Inequality, and since

$x^2 + 3 > 0, |x| + 2 > 0 \Rightarrow \frac{1}{x^2+3} > 0, \frac{1}{|x|+2} > 0.$

$x^2 + 3 \ge 3$ and $|x| + 2 \ge 2$, so

$\frac{1}{x^2+3} \le \frac{1}{3}$ and $\frac{1}{|x|+2} \le \frac{1}{2}$, thus,

$\frac{1}{x^2+3} + \frac{1}{|x|+2} \le \frac{1}{3} + \frac{1}{2}$

67. $\left|\frac{x-2}{x^2+9}\right| = \left|\frac{x+(-2)}{x^2+9}\right|$

$\left|\frac{x-2}{x^2+9}\right| \le \left|\frac{x}{x^2+9}\right| + \left|\frac{-2}{x^2+9}\right|$

$\left|\frac{x-2}{x^2+9}\right| \le \frac{|x|}{x^2+9} + \frac{2}{x^2+9} = \frac{|x|+2}{x^2+9}$

Since $x^2 + 9 \ge 9, \frac{1}{x^2+9} \le \frac{1}{9}$

$\frac{|x|+2}{x^2+9} \le \frac{|x|+2}{9}$

$\left|\frac{x-2}{x^2+9}\right| \le \frac{|x|+2}{9}$

68. $|x| \le 2 \Rightarrow \left|x^2 + 2x + 7\right| \le \left|x^2\right| + |2x| + 7$
$\le 4 + 4 + 7 = 15$

and $\left|x^2 + 1\right| \ge 1$ so $\frac{1}{x^2+1} \le 1$.

Thus, $\left|\frac{x^2+2x+7}{x^2+1}\right| = \left|x^2 + 2x + 7\right|\left|\frac{1}{x^2+1}\right|$
$\le 15 \cdot 1 = 15$

69. $\left|x^4 + \frac{1}{2}x^3 + \frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{16}\right|$

$\le \left|x^4\right| + \frac{1}{2}\left|x^3\right| + \frac{1}{4}\left|x^2\right| + \frac{1}{8}|x| + \frac{1}{16}$

$\le 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ since $|x| \le 1$.

So $\left|x^4 + \frac{1}{2}x^3 + \frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{16}\right| \le 1.9375 < 2$.

70. **a.** $x < x^2$

$x - x^2 < 0$

$x(1-x) < 0$

$x < 0 \text{ or } x > 1$

b. $x^2 < x$

$x^2 - x < 0$

$x(x-1) < 0$

$0 < x < 1$

71. $a \neq 0 \Rightarrow$

$0 \le \left(a - \frac{1}{a}\right)^2 = a^2 - 2 + \frac{1}{a^2}$

so, $2 \le a^2 + \frac{1}{a^2}$ or $a^2 + \frac{1}{a^2} \ge 2$.

72. $a < b$

$a + a < a + b$ and $a + b < b + b$

$2a < a + b < 2b$

$a < \frac{a+b}{2} < b$

73. $0 < a < b$

$a^2 < ab$ and $ab < b^2$

$a^2 < ab < b^2$

$a < \sqrt{ab} < b$

74. $\sqrt{ab} \le \frac{1}{2}(a+b) \Leftrightarrow ab \le \frac{1}{4}\left(a^2 + 2ab + b^2\right)$

$\Leftrightarrow 0 \le \frac{1}{4}a^2 - \frac{1}{2}ab + \frac{1}{4}b^2 = \frac{1}{4}\left(a^2 - 2ab + b^2\right)$

$\Leftrightarrow 0 \le \frac{1}{4}(a-b)^2$ which is always true.

75. For a rectangle the area is *ab*, while for a square the area is $a^2 = \left(\frac{a+b}{2}\right)^2$. From Problem 74, $\sqrt{ab} \le \frac{1}{2}(a+b) \Leftrightarrow ab \le \left(\frac{a+b}{2}\right)^2$ so the square has the largest area.

76. $1 + x + x^2 + x^3 + \ldots + x^{99} \le 0$;

$(-\infty, -1]$

77. $\frac{1}{R} \le \frac{1}{10} + \frac{1}{20} + \frac{1}{30}$

$\frac{1}{R} \le \frac{6+3+2}{60}$

$\frac{1}{R} \le \frac{11}{60}$

$R \ge \frac{60}{11}$

$\frac{1}{R} \ge \frac{1}{20} + \frac{1}{30} + \frac{1}{40}$

$\frac{1}{R} \ge \frac{6+4+3}{120}$

$R \le \frac{120}{13}$

Thus, $\frac{60}{11} \le R \le \frac{120}{13}$

78. $A = 4\pi r^2; A = 4\pi(10)^2 = 400\pi$

$\left|4\pi r^2 - 400\pi\right| < 0.01$

$4\pi\left|r^2 - 100\right| < 0.01$

$\left|r^2 - 100\right| < \frac{0.01}{4\pi}$

$-\frac{0.01}{4\pi} < r^2 - 100 < \frac{0.01}{4\pi}$

$\sqrt{100 - \frac{0.01}{4\pi}} < r < \sqrt{100 + \frac{0.01}{4\pi}}$

$\delta \approx 0.00004$ in

0.3 Concepts Review

1. $\sqrt{(x+2)^2 + (y-3)^2}$

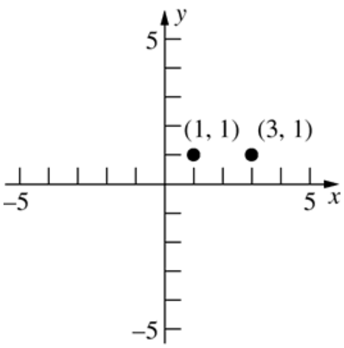
2. $(x + 4)^2 + (y - 2)^2 = 25$

3. $\left(\frac{-2+5}{2}, \frac{3+7}{2}\right) = (1.5, 5)$

4. $\frac{d-b}{c-a}$

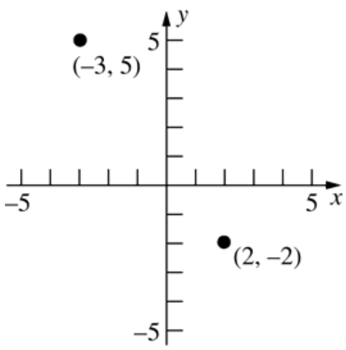
Problem Set 0.3

1.



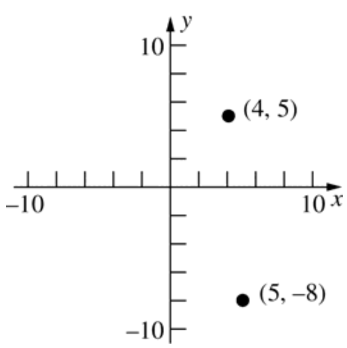
$$d = \sqrt{(3-1)^2 + (1-1)^2} = \sqrt{4} = 2$$

2.



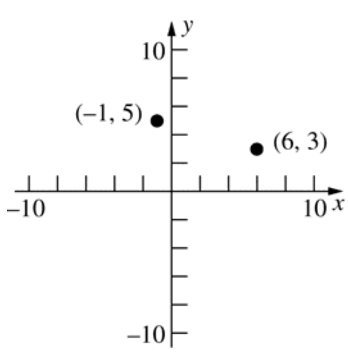
$$d = \sqrt{(-3-2)^2 + (5+2)^2} = \sqrt{74} \approx 8.60$$

3.



$$d = \sqrt{(4-5)^2 + (5+8)^2} = \sqrt{170} \approx 13.04$$

4.



$$d = \sqrt{(-1-6)^2 + (5-3)^2} = \sqrt{49+4} = \sqrt{53}$$
$$\approx 7.28$$

5. $d_1 = \sqrt{(5+2)^2 + (3-4)^2} = \sqrt{49+1} = \sqrt{50}$

$d_2 = \sqrt{(5-10)^2 + (3-8)^2} = \sqrt{25+25} = \sqrt{50}$

$d_3 = \sqrt{(-2-10)^2 + (4-8)^2}$

$= \sqrt{144+16} = \sqrt{160}$

$d_1 = d_2$ so the triangle is isosceles.

6. $a = \sqrt{(2-4)^2 + (-4-0)^2} = \sqrt{4+16} = \sqrt{20}$

$b = \sqrt{(4-8)^2 + (0+2)^2} = \sqrt{16+4} = \sqrt{20}$

$c = \sqrt{(2-8)^2 + (-4+2)^2} = \sqrt{36+4} = \sqrt{40}$

$a^2 + b^2 = c^2$, so the triangle is a right triangle.

7. (–1, –1), (–1, 3); (7, –1), (7, 3); (1, 1), (5, 1)

8. $\sqrt{(x-3)^2 + (0-1)^2} = \sqrt{(x-6)^2 + (0-4)^2}$;

$$x^2 - 6x + 10 = x^2 - 12x + 52$$
$$6x = 42$$
$$x = 7 \Rightarrow (7, 0)$$

9. $\left(\frac{-2+4}{2}, \frac{-2+3}{2}\right) = \left(1, \frac{1}{2}\right)$;

$d = \sqrt{(1+2)^2 + \left(\frac{1}{2} - 3\right)^2} = \sqrt{9 + \frac{25}{4}} \approx 3.91$

10. midpoint of $AB = \left(\frac{1+2}{2}, \frac{3+6}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$

midpoint of $CD = \left(\frac{4+3}{2}, \frac{7+4}{2}\right) = \left(\frac{7}{2}, \frac{11}{2}\right)$

$d = \sqrt{\left(\frac{3}{2} - \frac{7}{2}\right)^2 + \left(\frac{9}{2} - \frac{11}{2}\right)^2}$

$= \sqrt{4+1} = \sqrt{5} \approx 2.24$

11 $(x-1)^2 + (y-1)^2 = 1$

12. $(x+2)^2 + (y-3)^2 = 4^2$

$(x+2)^2 + (y-3)^2 = 16$

13. $(x-2)^2 + (y+1)^2 = r^2$

$(5-2)^2 + (3+1)^2 = r^2$

$r^2 = 9 + 16 = 25$

$(x-2)^2 + (y+1)^2 = 25$

14. $(x-4)^2+(y-3)^2=r^2$

$(6-4)^2+(2-3)^2=r^2$

$r^2=4+1=5$

$(x-4)^2+(y-3)^2=5$

15. center $=\left(\frac{1+3}{2},\frac{3+7}{2}\right)=(2,5)$

radius $=\frac{1}{2}\sqrt{(1-3)^2+(3-7)^2}=\frac{1}{2}\sqrt{4+16}$

$=\frac{1}{2}\sqrt{20}=\sqrt{5}$

$(x-2)^2+(y-5)^2=5$

16. Since the circle is tangent to the *x*-axis, $r=4$.

$(x-3)^2+(y-4)^2=16$

17. $x^2+2x+10+y^2-6y-10=0$

$x^2+2x+y^2-6y=0$

$(x^2+2x+1)+(y^2-6y+9)=1+9$

$(x+1)^2+(y-3)^2=10$

center $=(-1,3)$; radius $=\sqrt{10}$

18. $x^2+y^2-6y=16$

$x^2+(y^2-6y+9)=16+9$

$x^2+(y-3)^2=25$

center $=(0,3)$; radius $=5$

19. $x^2+y^2-12x+35=0$

$x^2-12x+y^2=-35$

$(x^2-12x+36)+y^2=-35+36$

$(x-6)^2+y^2=1$

center $=(6,0)$; radius $=1$

20. $x^2+y^2-10x+10y=0$

$(x^2-10x+25)+(y^2+10y+25)=25+25$

$(x-5)^2+(y+5)^2=50$

center $=(5,-5)$; radius $=\sqrt{50}=5\sqrt{2}$

21. $4x^2+16x+15+4y^2+6y=0$

$4(x^2+4x+4)+4\left(y^2+\frac{3}{2}y+\frac{9}{16}\right)=-15+16+\frac{9}{4}$

$4(x+2)^2+4\left(y+\frac{3}{4}\right)^2=\frac{13}{4}$

$(x+2)^2+\left(y+\frac{3}{4}\right)^2=\frac{13}{16}$

center $=\left(-2,-\frac{3}{4}\right)$; radius $=\frac{\sqrt{13}}{4}$

22. $4x^2+16x+\frac{105}{16}+4y^2+3y=0$

$4(x^2+4x+4)+4\left(y^2+\frac{3}{4}y+\frac{9}{64}\right)$

$=-\frac{105}{16}+16+\frac{9}{16}$

$4(x+2)^2+4\left(y+\frac{3}{8}\right)^2=10$

$(x+2)^2+\left(y+\frac{3}{8}\right)^2=\frac{5}{2}$

center $=\left(-2,-\frac{3}{8}\right)$; radius $=\sqrt{\frac{5}{2}}=\frac{\sqrt{10}}{2}$

23. $\frac{2-1}{2-1}=1$

24. $\frac{7-5}{4-3}=2$

25. $\frac{-6-3}{-5-2}=\frac{9}{7}$

26. $\frac{-6+4}{0-2}=1$

27. $\frac{5-0}{0-3}=-\frac{5}{3}$

28. $\frac{6-0}{0+6}=1$

29. $y-2=-1(x-2)$

$y-2=-x+2$

$x+y-4=0$

30. $y-4=-1(x-3)$

$y-4=-x+3$

$x+y-7=0$

31. $y=2x+3$

$2x-y+3=0$

32. $y=0x+5$

$0x+y-5=0$

33. $m=\dfrac{8-3}{4-2}=\dfrac{5}{2};$

$$y-3=\frac{5}{2}(x-2)$$
$$2y-6=5x-10$$
$$5x-2y-4=0$$

34. $m=\dfrac{2-1}{8-4}=\dfrac{1}{4};$

$$y-1=\frac{1}{4}(x-4)$$
$$4y-4=x-4$$
$$x-4y+0=0$$

35. $3y=-2x+1;\ y=-\dfrac{2}{3}x+\dfrac{1}{3};\ \text{slope}=-\dfrac{2}{3};$

$y\text{-intercept}=\dfrac{1}{3}$

36. $-4y=5x-6$

$$y=-\frac{5}{4}x+\frac{3}{2}$$

slope $=-\dfrac{5}{4}$; y-intercept $=\dfrac{3}{2}$

37. $6-2y=10x-2$

$$-2y=10x-8$$
$$y=-5x+4;$$

slope = –5; y-intercept = 4

38. $4x+5y=-20$

$$5y=-4x-20$$
$$y=-\frac{4}{5}x-4$$

slope $=-\dfrac{4}{5}$; y-intercept $=-4$

39. a. $m=2;$

$$y+3=2(x-3)$$
$$y=2x-9$$

b. $m=-\dfrac{1}{2};$

$$y+3=-\frac{1}{2}(x-3)$$
$$y=-\frac{1}{2}x-\frac{3}{2}$$

c. $2x+3y=6$

$$3y=-2x+6$$
$$y=-\frac{2}{3}x+2;$$
$$m=-\frac{2}{3};$$
$$y+3=-\frac{2}{3}(x-3)$$
$$y=-\frac{2}{3}x-1$$

d. $m=\dfrac{3}{2};$

$$y+3=\frac{3}{2}(x-3)$$
$$y=\frac{3}{2}x-\frac{15}{2}$$

e. $m=\dfrac{-1-2}{3+1}=-\dfrac{3}{4};$

$$y+3=-\frac{3}{4}(x-3)$$
$$y=-\frac{3}{4}x-\frac{3}{4}$$

f. $x=3$ **g.** $y=-3$

40. a. $3x+cy=5$

$$3(3)+c(1)=5$$
$$c=-4$$

b. $c=0$

c. $2x+y=-1$

$$y=-2x-1$$
$$m=-2;$$
$$3x+cy=5$$
$$cy=-3x+5$$
$$y=-\frac{3}{c}x+\frac{5}{c}$$
$$-2=-\frac{3}{c}$$
$$c=\frac{3}{2}$$

d. c must be the same as the coefficient of x, so $c=3$.

e. $y-2=3(x+3);$

perpendicular slope $=-\frac{1}{3};$

$-\frac{1}{3}=-\frac{3}{c}$

$c=9$

41. $m=\frac{3}{2};$

$y+1=\frac{3}{2}(x+2)$

$y=\frac{3}{2}x+2$

42. a. $m=2;$

$kx-3y=10$

$-3y=-kx+10$

$y=\frac{k}{3}x-\frac{10}{3}$

$\frac{k}{3}=2; k=6$

b. $m=-\frac{1}{2};$

$\frac{k}{3}=-\frac{1}{2}$

$k=-\frac{3}{2}$

c. $2x+3y=6$

$3y=-2x+6$

$y=-\frac{2}{3}x+2;$

$m=\frac{3}{2};\ \frac{k}{3}=\frac{3}{2};\ k=\frac{9}{2}$

43. $y = 3(3) – 1 = 8$; (3, 9) is above the line.

44. $(a,0),(0,b);\ m=\frac{b-0}{0-a}=-\frac{b}{a}$

$y=-\frac{b}{a}x+b;\ \frac{bx}{a}+y=b; \frac{x}{a}+\frac{y}{b}=1$

45. $2x+3y=4$

$-3x+y=5$

$2x+3y=4$

$\underline{9x-3y=-15}$

$11x\quad=-11$

$x=-1$

$-3(-1)+y=5$

$y=2$

Point of intersection: (–1, 2)

$3y=-2x+4$

$y=-\frac{2}{3}x+\frac{4}{3}$

$m=\frac{3}{2}$

$y-2=\frac{3}{2}(x+1)$

$y=\frac{3}{2}x+\frac{7}{2}$

46. $4x-5y=8$

$2x+y=-10$

$4x-5y=8$

$\underline{-4x-2y=20}$

$-7y=28$

$y=-4$

$4x-5(-4)=8$

$4x=-12$

$x=-3$

Point of intersection: $(-3,-4);$

$4x-5y=8$

$-5y=-4x+8$

$y=\frac{4}{5}x-\frac{8}{5}$

$m=-\frac{5}{4}$

$y+4=-\frac{5}{4}(x+3)$

$y=-\frac{5}{4}x-\frac{31}{4}$

47. $3x-4y=5$
$2x+3y=9$

$$\begin{aligned}9x-12y&=15\\ \underline{8x+12y}&\underline{=36}\\ 17x\quad&=51\\ x&=3\end{aligned}$$

$$3(3)-4y=5$$
$$-4y=-4$$
$$y=1$$

Point of intersection: (3, 1); $3x-4y=5$;
$$-4y=-3x+5$$
$$y=\frac{3}{4}x-\frac{5}{4}$$
$$m=-\frac{4}{3}$$
$$y-1=-\frac{4}{3}(x-3)$$
$$y=-\frac{4}{3}x+5$$

48. $5x-2y=5$
$2x+3y=6$

$$\begin{aligned}15x-6y&=15\\ \underline{4x+6y}&\underline{=12}\\ 19x\quad&=27\\ x&=\frac{27}{19}\end{aligned}$$

$$2\left(\frac{27}{19}\right)+3y=6$$
$$3y=\frac{60}{19}$$
$$y=\frac{20}{19}$$

Point of intersection: $\left(\frac{27}{19},\frac{20}{19}\right)$;
$$5x-2y=5$$
$$-2y=-5x+5$$
$$y=\frac{5}{2}x-\frac{5}{2}$$
$$m=-\frac{2}{5}$$
$$y-\frac{20}{19}=-\frac{2}{5}\left(x-\frac{27}{19}\right)$$
$$y=-\frac{2}{5}x+\frac{54}{95}+\frac{20}{19}$$
$$y=-\frac{2}{5}x+\frac{154}{95}$$

49. center: $\left(\frac{2+6}{2},\frac{-1+3}{2}\right)=(4,1)$

midpoint $=\left(\frac{2+6}{2},\frac{3+3}{2}\right)=(4,3)$

inscribed circle: radius $=\sqrt{(4-4)^2+(1-3)^2}$
$$=\sqrt{4}=2$$
$$(x-4)^2+(y-1)^2=4$$

circumscribed circle:
radius $=\sqrt{(4-2)^2+(1-3)^2}=\sqrt{8}$
$$(x-4)^2+(y-1)^2=8$$

50. The radius of each circle is $\sqrt{16}=4$. The centers are $(1,-2)$ and $(-9,10)$. The length of the belt is the sum of half the circumference of the first circle, half the circumference of the second circle, and twice the distance between their centers.
$$L=\frac{1}{2}\cdot 2\pi(4)+\frac{1}{2}\cdot 2\pi(4)+2\sqrt{(1+9)^2+(-2-10)^2}$$
$$=8\pi+2\sqrt{100+144}$$
$$\approx 56.37$$

51. Put the vertex of the right angle at the origin with the other vertices at (a, 0) and (0, b). The midpoint of the hypotenuse is $\left(\frac{a}{2},\frac{b}{2}\right)$. The distances from the vertices are
$$\sqrt{\left(a-\frac{a}{2}\right)^2+\left(0-\frac{b}{2}\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}=\frac{1}{2}\sqrt{a^2+b^2},$$
$$\sqrt{\left(0-\frac{a}{2}\right)^2+\left(b-\frac{b}{2}\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}=\frac{1}{2}\sqrt{a^2+b^2},\text{ and}$$
$$\sqrt{\left(0-\frac{a}{2}\right)^2+\left(0-\frac{b}{2}\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}=\frac{1}{2}\sqrt{a^2+b^2},$$
which are all the same.

52. From Problem 51, the midpoint of the hypotenuse, $(4,3,)$, is equidistant from the vertices. This is the center of the circle. The radius is $\sqrt{16+9}=5$. The equation of the circle is
$$(x-4)^2+(y-3)^2=25.$$

53. $x^2 + y^2 - 4x - 2y - 11 = 0$

$(x^2 - 4x + 4) + (y^2 - 2y + 1) = 11 + 4 + 1$

$(x-2)^2 + (y-1)^2 = 16$

$x^2 + y^2 + 20x - 12y + 72 = 0$

$(x^2 + 20x + 100) + (y^2 - 12y + 36)$
$= -72 + 100 + 36$

$(x+10)^2 + (y-6)^2 = 64$

center of first circle: (2, 1)
center of second circle: (–10, 6)

$d = \sqrt{(2+10)^2 + (1-6)^2} = \sqrt{144+25}$
$= \sqrt{169} = 13$

However, the radii only sum to 4 + 8 = 12, so the circles must not intersect if the distance between their centers is 13.

54. $x^2 + ax + y^2 + by + c = 0$

$\left(x^2 + ax + \frac{a^2}{4}\right) + \left(y^2 + by + \frac{b^2}{4}\right)$
$= -c + \frac{a^2}{4} + \frac{b^2}{4}$

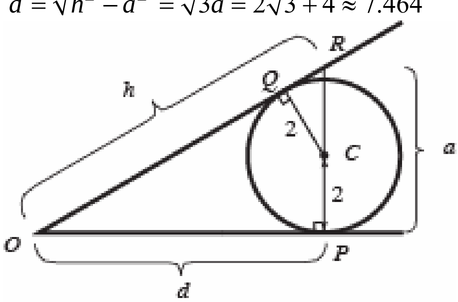
$\left(x + \frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 = \frac{a^2 + b^2 - 4c}{4}$

$\frac{a^2 + b^2 - 4c}{4} > 0 \Rightarrow a^2 + b^2 > 4c$

55. Label the points C, P, Q, and R as shown in the figure below. Let $d = |OP|$, $h = |OR|$, and $a = |PR|$. Triangles ΔOPR and ΔCQR are similar because each contains a right angle and they share angle $\angle QRC$. For an angle of 30°, $\frac{d}{h} = \frac{\sqrt{3}}{2}$ and $\frac{a}{h} = \frac{1}{2} \Rightarrow h = 2a$. Using a property of similar triangles, $|QC|/|RC| = \sqrt{3}/2$,

$\frac{2}{a-2} = \frac{\sqrt{3}}{2} \quad \rightarrow \quad a = 2 + \frac{4}{\sqrt{3}}$

By the Pythagorean Theorem, we have
$d = \sqrt{h^2 - a^2} = \sqrt{3}a = 2\sqrt{3} + 4 \approx 7.464$



56. The equations of the two circles are

$(x-R)^2 + (y-R)^2 = R^2$

$(x-r)^2 + (y-r)^2 = r^2$

Let (a, a) denote the point where the two circles touch. This point must satisfy

$(a-R)^2 + (a-R)^2 = R^2$

$(a-R)^2 = \frac{R^2}{2}$

$a = \left(1 \pm \frac{\sqrt{2}}{2}\right)R$

Since $a < R$, $a = \left(1 - \frac{\sqrt{2}}{2}\right)R.$

At the same time, the point where the two circles touch must satisfy

$(a-r)^2 + (a-r)^2 = r^2$

$a = \left(1 \pm \frac{\sqrt{2}}{2}\right)r$

Since $a > r$, $a = \left(1 + \frac{\sqrt{2}}{2}\right)r.$

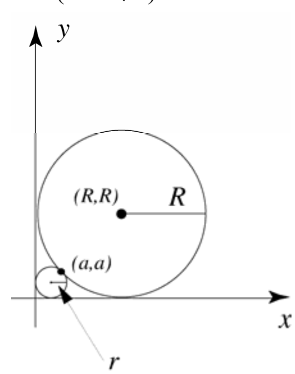
Equating the two expressions for a yields

$\left(1 - \frac{\sqrt{2}}{2}\right)R = \left(1 + \frac{\sqrt{2}}{2}\right)r$

$r = \frac{1 - \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}}R = \frac{\left(1 - \frac{\sqrt{2}}{2}\right)^2}{\left(1 + \frac{\sqrt{2}}{2}\right)\left(1 - \frac{\sqrt{2}}{2}\right)}R$

$r = \frac{1 - \sqrt{2} + \frac{1}{2}}{1 - \frac{1}{2}}R$

$r = (3 - 2\sqrt{2})R \approx 0.1716R$



57. Refer to figure 15 in the text. Given ine l_1 with slope *m*, draw $\triangle ABC$ with vertical and horizontal sides *m*, 1.
Line l_2 is obtained from l_1 by rotating it around the point *A* by $90°$ counter-clockwise. Triangle ABC is rotated into triangle AED.
We read off

$$\text{slope of } l_2 = \frac{1}{-m} = -\frac{1}{m}.$$

58. $2\sqrt{(x-1)^2+(y-1)^2} = \sqrt{(x-3)^2+(y-4)^2}$

$$4(x^2-2x+1+y^2-2y+1)$$
$$= x^2-6x+9+y^2-8y+16$$
$$3x^2-2x+3y^2 = 9+16-4-4;$$
$$3x^2-2x+3y^2 = 17; x^2-\frac{2}{3}x+y^2 = \frac{17}{3};$$
$$\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)+y^2 = \frac{17}{3}+\frac{1}{9}$$
$$\left(x-\frac{1}{3}\right)^2+y^2 = \frac{52}{9}$$

center: $\left(\frac{1}{3}, 0\right)$; radius: $\left(\frac{\sqrt{52}}{3}\right)$

59. Let *a*, *b*, and *c* be the lengths of the sides of the right triangle, with *c* the length of the hypotenuse. Then the Pythagorean Theorem says that $a^2+b^2=c^2$

Thus, $\frac{\pi a^2}{8}+\frac{\pi b^2}{8}=\frac{\pi c^2}{8}$ or

$$\frac{1}{2}\pi\left(\frac{a}{2}\right)^2+\frac{1}{2}\pi\left(\frac{b}{2}\right)^2=\frac{1}{2}\pi\left(\frac{c}{2}\right)^2$$

$\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ is the area of a semicircle with diameter *x*, so the circles on the legs of the triangle have total area equal to the area of the semicircle on the hypotenuse.

From $a^2+b^2=c^2$,

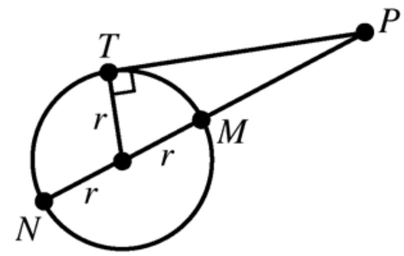
$$\frac{\sqrt{3}}{4}a^2+\frac{\sqrt{3}}{4}b^2=\frac{\sqrt{3}}{4}c^2$$

$\frac{\sqrt{3}}{4}x^2$ is the area of an equilateral triangle with sides of length *x*, so the equilateral triangles on the legs of the right triangle have total area equal to the area of the equilateral triangle on the hypotenuse of the right triangle.

60. See the figure below. The angle at *T* is a right angle, so the Pythagorean Theorem gives

$$(PM+r)^2=(PT)^2+r^2$$
$$\Leftrightarrow (PM)^2+2rPM+r^2=(PT)^2+r^2$$
$$\Leftrightarrow PM(PM+2r)=(PT)^2$$

$PM+2r=PN$ so this gives $(PM)(PN)=(PT)^2$



61. The lengths *A*, *B*, and *C* are the same as the corresponding distances between the centers of the circles:

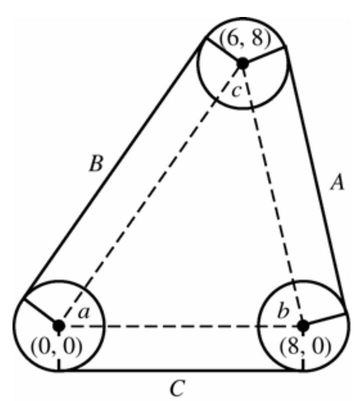
$$A=\sqrt{(-2)^2+(8)^2}=\sqrt{68}\approx 8.2$$
$$B=\sqrt{(6)^2+(8)^2}=\sqrt{100}=10$$
$$C=\sqrt{(8)^2+(0)^2}=\sqrt{64}=8$$

Each circle has radius 2, so the part of the belt around the wheels is

$$2(2\pi-a-\pi)+2(2\pi-b-\pi)+2(2\pi-c-\pi)$$
$$=2[3\pi-(a+b+c)]=2(2\pi)=4\pi$$

Since $a+b+c=\pi$, the sum of the angles of a triangle.

The length of the belt is $\approx 8.2+10+8+4\pi$
≈ 38.8 units.



62 As in Problems 50 and 61, the curved portions of the belt have total length $2\pi r$. The lengths of the straight portions will be the same as the lengths of the sides. The belt will have length $2\pi r+d_1+d_2+\ldots+d_n$.

63. $A = 3, B = 4, C = -6$

$$d = \frac{|3(-3)+4(2)+(-6)|}{\sqrt{(3)^2+(4)^2}} = \frac{7}{5}$$

64. $A = 2, B = -2, C = 4$

$$d = \frac{|2(4)-2(-1)+4)|}{\sqrt{(2)^2+(2)^2}} = \frac{14}{\sqrt{8}} = \frac{7\sqrt{2}}{2}$$

65. $A = 12, B = -5, C = 1$

$$d = \frac{|12(-2)-5(-1)+1|}{\sqrt{(12)^2+(-5)^2}} = \frac{18}{13}$$

66. $A = 2, B = -1, C = -5$

$$d = \frac{|2(3)-1(-1)-5|}{\sqrt{(2)^2+(-1)^2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

67. $2x + 4(0) = 5$

$$x = \frac{5}{2}$$

$$d = \frac{\left|2\left(\frac{5}{2}\right)+4(0)-7\right|}{\sqrt{(2)^2+(4)^2}} = \frac{2}{\sqrt{20}} = \frac{\sqrt{5}}{5}$$

68. $7(0) - 5y = -1$

$$y = \frac{1}{5}$$

$$d = \frac{\left|7(0)-5\left(\frac{1}{5}\right)-6\right|}{\sqrt{(7)^2+(-5)^2}} = \frac{7}{\sqrt{74}} = \frac{7\sqrt{74}}{74}$$

69. $m = \frac{-2-3}{1+2} = -\frac{5}{3};\ m = \frac{3}{5}$; passes through

$$\left(\frac{-2+1}{2}, \frac{3-2}{2}\right) = \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$y - \frac{1}{2} = \frac{3}{5}\left(x + \frac{1}{2}\right)$$

$$y = \frac{3}{5}x + \frac{4}{5}$$

70. $m = \frac{0-4}{2-0} = -2;\ m = \frac{1}{2};$ passes through

$$\left(\frac{0+2}{2}, \frac{4+0}{2}\right) = (1, 2)$$

$$y - 2 = \frac{1}{2}(x-1)$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

$m = \frac{6-0}{4-2} = 3;\ m = -\frac{1}{3};$ passes through

$$\left(\frac{2+4}{2}, \frac{0+6}{2}\right) = (3, 3)$$

$$y - 3 = -\frac{1}{3}(x-3)$$

$$y = -\frac{1}{3}x + 4$$

$$\frac{1}{2}x + \frac{3}{2} = -\frac{1}{3}x + 4$$

$$\frac{5}{6}x = \frac{5}{2}$$

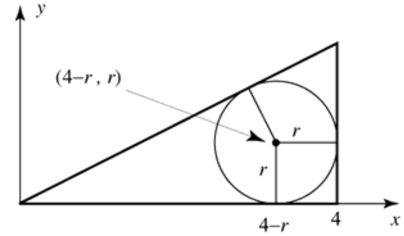
$$x = 3$$

$$y = \frac{1}{2}(3) + \frac{3}{2} = 3$$

center = (3, 3)

71. Let the origin be at the vertex as shown in the figure below. The center of the circle is then $(4-r, r)$, so it has equation $(x-(4-r))^2 + (y-r)^2 = r^2$. Along the side of length 5, the y-coordinate is always $\frac{3}{4}$ times the x-coordinate. Thus, we need to find the value of r for which there is exactly one x-solution to $(x-4+r)^2 + \left(\frac{3}{4}x - r\right)^2 = r^2$.

Solving for x in this equation gives $x = \frac{16}{25}\left(16 - r \pm \sqrt{24\left(-r^2+7r-6\right)}\right)$. There is exactly one solution when $-r^2 + 7r - 6 = 0$, that is, when $r = 1$ or $r = 6$. The root $r = 6$ is extraneous. Thus, the largest circle that can be inscribed in this triangle has radius $r = 1$.



72. The line tangent to the circle at (a,b) will be perpendicular to the line through (a,b) and the center of the circle, which is $(0,0)$. The line through (a,b) and $(0,0)$ has slope

$$m = \frac{0-b}{0-a} = \frac{b}{a}; ax + by = r^2 \Rightarrow y = -\frac{a}{b}x + \frac{r^2}{b}$$

so $ax + by = r^2$ has slope $-\frac{a}{b}$ and is perpendicular to the line through (a,b) and $(0,0)$, so it is tangent to the circle at (a,b).

73. $12a + 0b = 36$
$a = 3$
$3^2 + b^2 = 36$
$b = \pm 3\sqrt{3}$
$3x - 3\sqrt{3}y = 36$
$x - \sqrt{3}y = 12$
$3x + 3\sqrt{3}y = 36$
$x + \sqrt{3}y = 12$

74. Use the formula given for problems 63-66, for $(x,y) = (0,0)$.

$A = m, B = -1, C = B - b; (0,0)$

$$d = \frac{|m(0) - 1(0) + B - b|}{\sqrt{m^2 + (-1)^2}} = \frac{|B-b|}{\sqrt{m^2+1}}$$

75. The midpoint of the side from (0, 0) to (a, 0) is

$$\left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right)$$

The midpoint of the side from (0, 0) to (b, c) is

$$\left(\frac{0+b}{2}, \frac{0+c}{2}\right) = \left(\frac{b}{2}, \frac{c}{2}\right)$$

$$m_1 = \frac{c-0}{b-a} = \frac{c}{b-a}$$

$$m_2 = \frac{\frac{c}{2} - 0}{\frac{b}{2} - \frac{a}{2}} = \frac{c}{b-a}; m_1 = m_2$$

76. See the figure below. The midpoints of the sides are

$$P\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right), Q\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right),$$

$$R\left(\frac{x_3+x_4}{2}, \frac{y_3+y_4}{2}\right), \text{ and}$$

$$S\left(\frac{x_1+x_4}{2}, \frac{y_1+y_4}{2}\right).$$

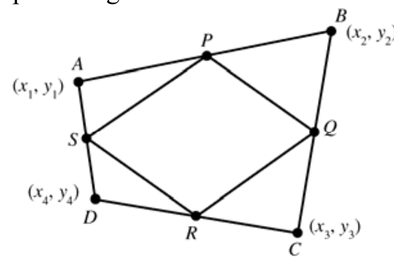
The slope of PS is

$$\frac{\frac{1}{2}[y_1 + y_4 - (y_1 + y_2)]}{\frac{1}{2}[x_1 + x_4 - (x_1 + x_2)]} = \frac{y_4 - y_2}{x_4 - x_2}.$$

The slope of QR is

$$\frac{\frac{1}{2}[y_3 + y_4 - (y_2 + y_3)]}{\frac{1}{2}[x_3 + x_4 - (x_2 + x_3)]} = \frac{y_4 - y_2}{x_4 - x_2}.$$

Thus PS and QR are parallel. The slopes of SR and PQ are both $\frac{y_3 - y_1}{x_3 - x_1}$, so $PQRS$ is a parallelogram.



77. $x^2 + (y-6)^2 = 25$; passes through (3, 2)
tangent line: $3x - 4y = 1$
The dirt hits the wall at $y = 8$.

0.4 Concepts Review

1. y-axis

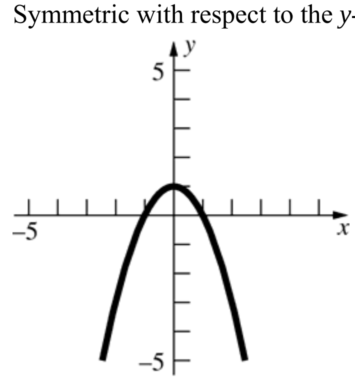
2. $(4, -2)$

3. 8; –2, 1, 4

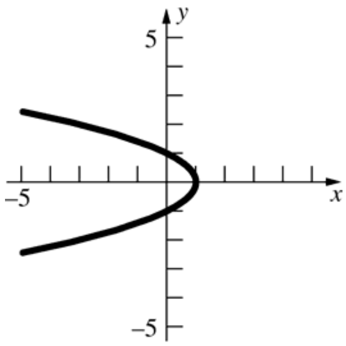
4. line; parabola

Problem Set 0.4

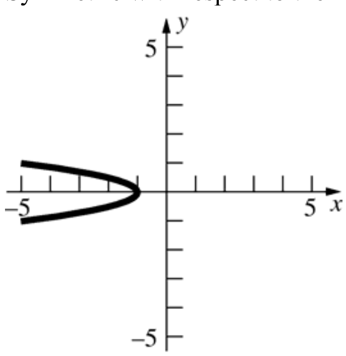
1. $y = -x^2 + 1$; y-intercept = 1; $y = (1 + x)(1 - x)$; x-intercepts = –1, 1
Symmetric with respect to the y-axis



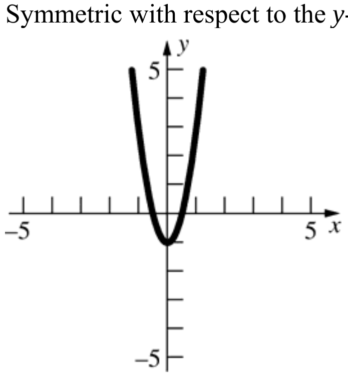
2. $x = -y^2 + 1$; y-intercepts $= -1, 1$;
x-intercept $= 1$.
Symmetric with respect to the x-axis.



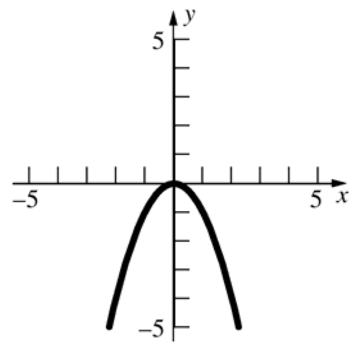
3. $x = -4y^2 - 1$; x-intercept $= -1$
Symmetric with respect to the x-axis



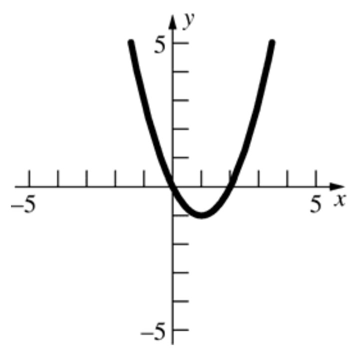
4. $y = 4x^2 - 1$; y-intercept $= -1$
$y = (2x+1)(2x-1)$; x-intercepts $= -\frac{1}{2}, \frac{1}{2}$
Symmetric with respect to the y-axis.



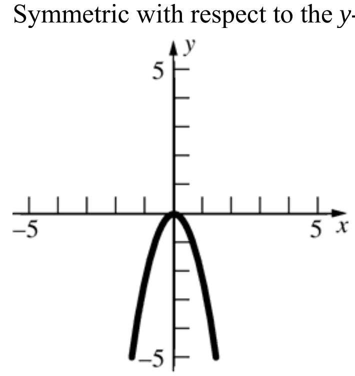
5. $x^2 + y = 0$; $y = -x^2$
x-intercept = 0, y-intercept = 0
Symmetric with respect to the y-axis



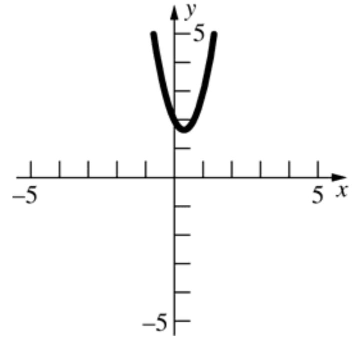
6. $y = x^2 - 2x$; y-intercept = 0
$y = x(2-x)$; x-intercepts = 0, 2



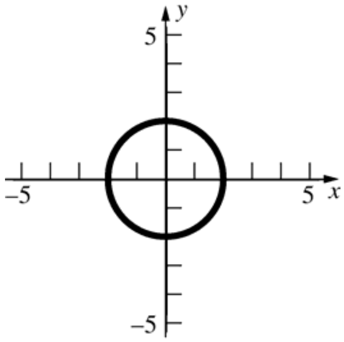
7. $7x^2 + 3y = 0$; $3y = -7x^2$; $y = -\frac{7}{3}x^2$
x-intercept = 0, y-intercept = 0
Symmetric with respect to the y-axis



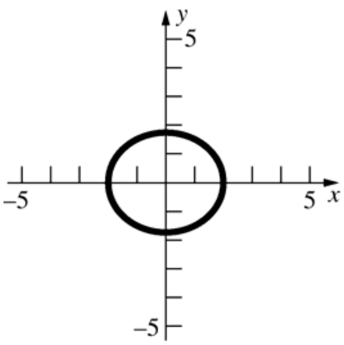
8. $y = 3x^2 - 2x + 2$; y-intercept = 2



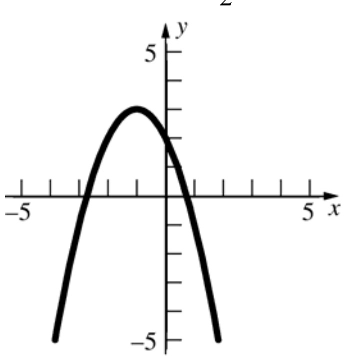
9. $x^2 + y^2 = 4$
x-intercepts = -2, 2; y-intercepts = -2, 2
Symmetric with respect to the x-axis, y-axis, and origin



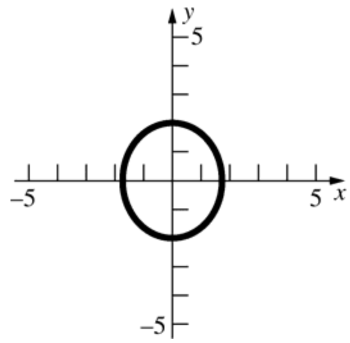
10. $3x^2 + 4y^2 = 12$; y-intercepts $= -\sqrt{3}, \sqrt{3}$
x-intercepts $= -2, 2$
Symmetric with respect to the x-axis, y-axis, and origin



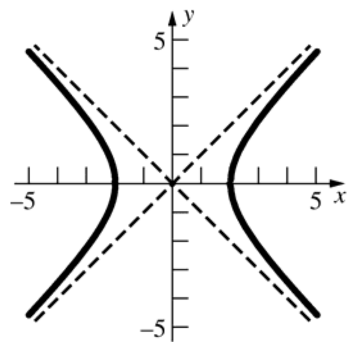
11. $y = -x^2 - 2x + 2$: y-intercept = 2
$x\text{-intercepts} = \dfrac{2 \pm \sqrt{4+8}}{-2} = \dfrac{2 \pm 2\sqrt{3}}{-2} = -1 \pm \sqrt{3}$



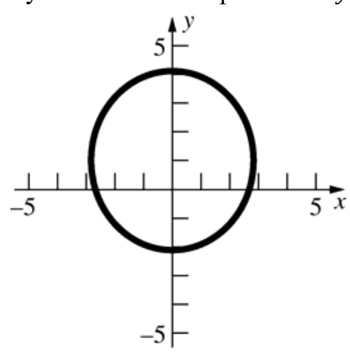
12. $4x^2 + 3y^2 = 12$; y-intercepts $= -2, 2$
x-intercepts $= -\sqrt{3}, \sqrt{3}$
Symmetric with respect to the x-axis, y-axis, and origin



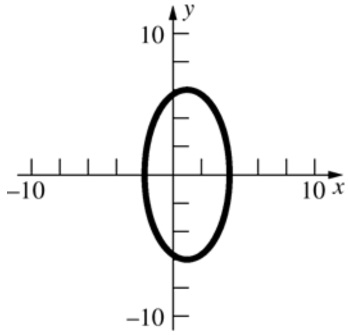
13. $x^2 - y^2 = 4$
x-intercept = -2, 2
Symmetric with respect to the x-axis, y-axis, and origin



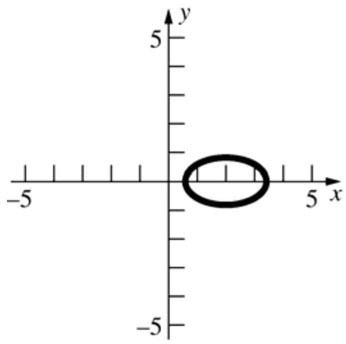
14. $x^2 + (y-1)^2 = 9$; y-intercepts $= -2, 4$
x-intercepts $= -2\sqrt{2}, 2\sqrt{2}$
Symmetric with respect to the y-axis



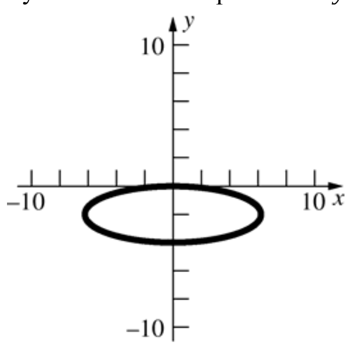
15. $4(x-1)^2 + y^2 = 36;$
$y\text{-intercepts } = \pm\sqrt{32} = \pm 4\sqrt{2}$
x-intercepts = –2, 4
Symmetric with respect to the x-axis



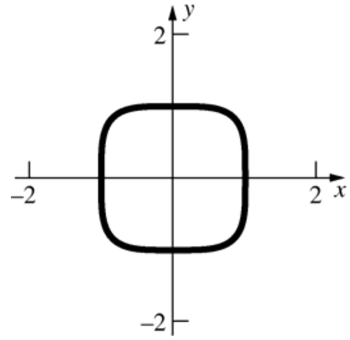
16. $x^2 - 4x + 3y^2 = -2$
$x\text{-intercepts } = 2 \pm \sqrt{2}$
Symmetric with respect to the x-axis



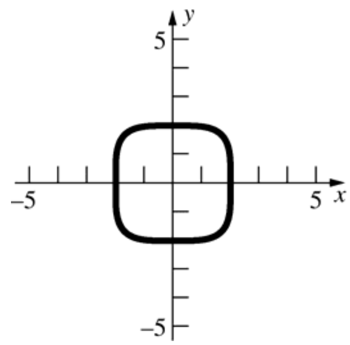
17. $x^2 + 9(y+2)^2 = 36$; y-intercepts = –4, 0
x-intercept = 0
Symmetric with respect to the y-axis



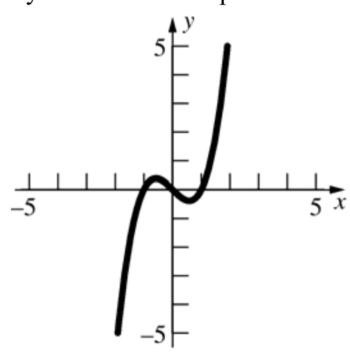
18. $x^4 + y^4 = 1$; $y\text{-intercepts} = -1, 1$
$x\text{-intercepts } = -1, 1$
Symmetric with respect to the x-axis, y-axis, and origin



19. $x^4 + y^4 = 16$; $y\text{-intercepts } = -2, 2$
$x\text{-intercepts } = -2, 2$
Symmetric with respect to the y-axis, x-axis and origin

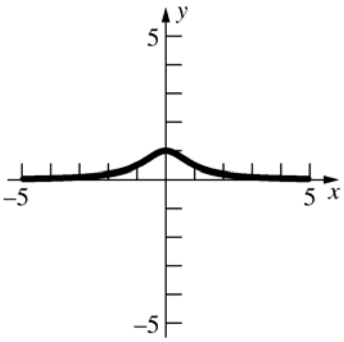


20. $y = x^3 - x$; y-intercepts = 0;
$y = x(x^2 - 1) = x(x+1)(x-1);$
x-intercepts = –1, 0, 1
Symmetric with respect to the origin



21. $y = \dfrac{1}{x^2+1}$; y-intercept = 1

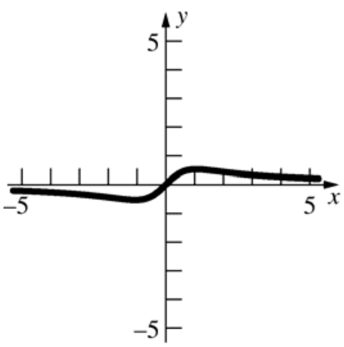
Symmetric with respect to the y-axis



22. $y = \dfrac{x}{x^2+1}$; y-intercept = 0

x-intercept = 0

Symmetric with respect to the origin



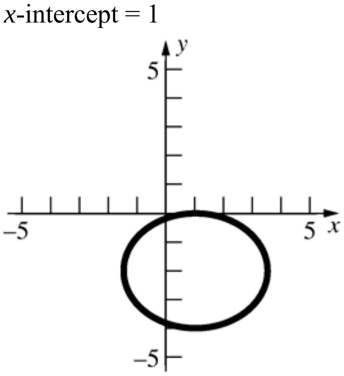
23. $2x^2 - 4x + 3y^2 + 12y = -2$

$2(x^2 - 2x + 1) + 3(y^2 + 4y + 4) = -2 + 2 + 12$

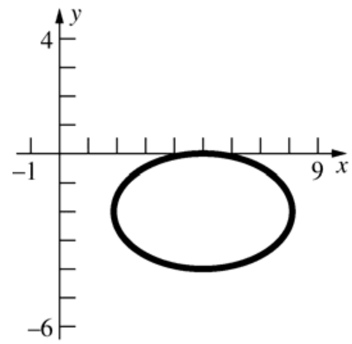
$2(x-1)^2 + 3(y+2)^2 = 12$

y-intercepts = $-2 \pm \dfrac{\sqrt{30}}{3}$

x-intercept = 1

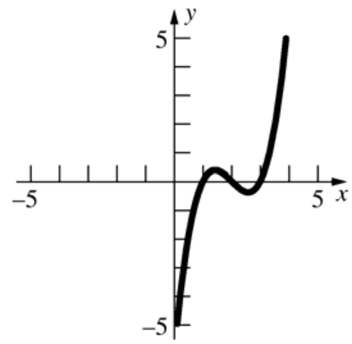


24. $4\left(x-5\right)^2 + 9(y+2)^2 = 36$; x-intercept = 5



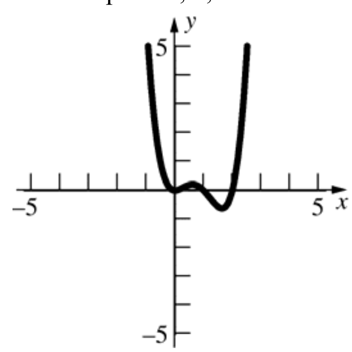
25. $y = (x-1)(x-2)(x-3)$; y-intercept = –6

x-intercepts = 1, 2, 3



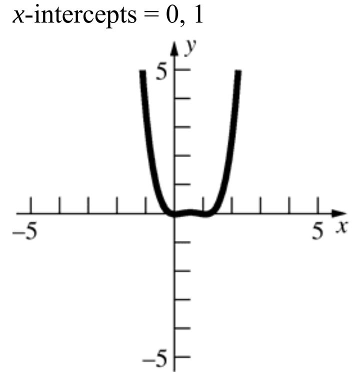
26. $y = x^2(x-1)(x-2)$; y-intercept = 0

x-intercepts = 0, 1, 2



27. $y = x^2(x-1)^2$; y-intercept = 0

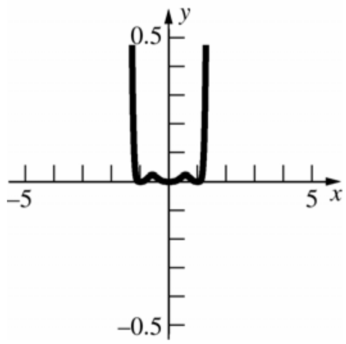
x-intercepts = 0, 1



28. $y = x^4(x-1)^4(x+1)^4$; y-intercept = 0

x-intercepts $= -1, 0, 1$

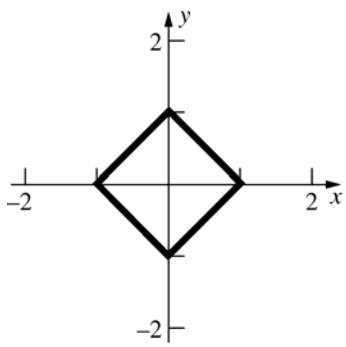
Symmetric with respect to the y-axis



29. $|x| + |y| = 1$; y-intercepts = –1, 1;

x-intercepts = –1, 1

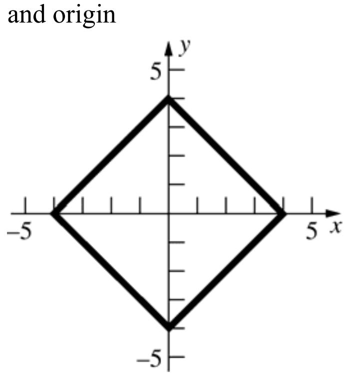
Symmetric with respect to the x-axis, y-axis and origin



30. $|x| + |y| = 4$; y-intercepts = –4, 4;

x-intercepts = –4, 4

Symmetric with respect to the x-axis, y-axis and origin



31.

$$-x + 1 = (x+1)^2$$

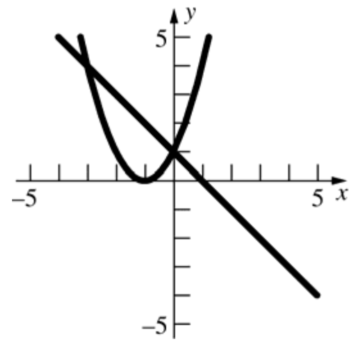
$$-x + 1 = x^2 + 2x + 1$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x = 0, -3$$

Intersection points: (0, 1) and (–3, 4)

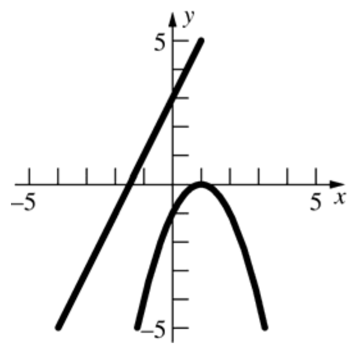


32. $2x + 3 = -(x-1)^2$

$$2x + 3 = -x^2 + 2x - 1$$

$$x^2 + 4 = 0$$

No points of intersection



33. $-2x + 3 = -2(x-4)^2$

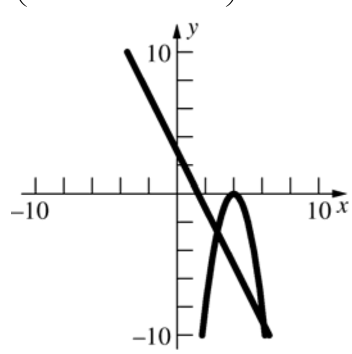
$$-2x + 3 = -2x^2 + 16x - 32$$

$$2x^2 - 18x + 35 = 0$$

$$x = \frac{18 \pm \sqrt{324 - 280}}{4} = \frac{18 \pm 2\sqrt{11}}{4} = \frac{9 \pm \sqrt{11}}{2};$$

Intersection points: $\left(\frac{9-\sqrt{11}}{2}, -6+\sqrt{11}\right)$,

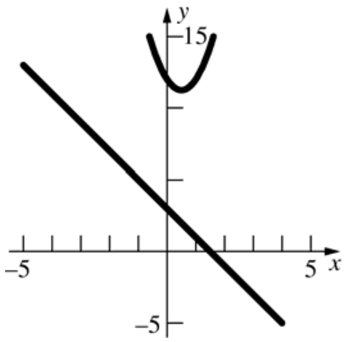
$\left(\frac{9+\sqrt{11}}{2}, -6-\sqrt{11}\right)$



34. $-2x+3=3x^2-3x+12$

$3x^2-x+9=0$

No points of intersection

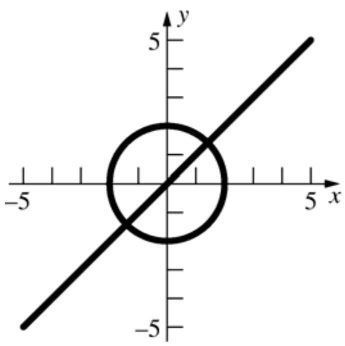


35. $x^2+x^2=4$

$x^2=2$

$x=\pm\sqrt{2}$

Intersection points: $\left(-\sqrt{2},-\sqrt{2}\right),\left(\sqrt{2},\sqrt{2}\right)$



36. $2x^2+3(x-1)^2=12$

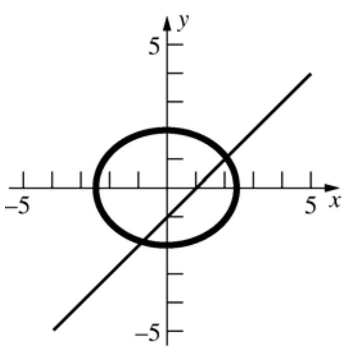
$2x^2+3x^2-6x+3=12$

$5x^2-6x-9=0$

$x=\dfrac{6\pm\sqrt{36+180}}{10}=\dfrac{6\pm6\sqrt{6}}{10}=\dfrac{3\pm3\sqrt{6}}{5}$

Intersection points:

$\left(\dfrac{3-3\sqrt{6}}{5},\dfrac{-2-3\sqrt{6}}{5}\right),\left(\dfrac{3+3\sqrt{6}}{5},\dfrac{-2+3\sqrt{6}}{5}\right)$



37. $y=3x+1$

$x^2+2x+(3x+1)^2=15$

$x^2+2x+9x^2+6x+1=15$

$10x^2+8x-14=0$

$2(5x^2+4x-7)=0$

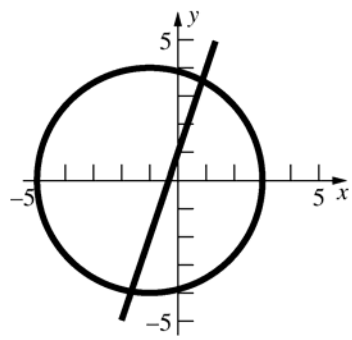
$x=\dfrac{-2\pm\sqrt{39}}{5}\approx-1.65, 0.85$

Intersection points:

$\left(\dfrac{-2-\sqrt{39}}{5},\dfrac{-1-3\sqrt{39}}{5}\right)$ and

$\left(\dfrac{-2+\sqrt{39}}{5},\dfrac{-1+3\sqrt{39}}{5}\right)$

[or roughly (–1.65, –3.95) and (0.85, 3.55)]



38. $x^2+(4x+3)^2=81$

$x^2+16x^2+24x+9=81$

$17x^2+24x-72=0$

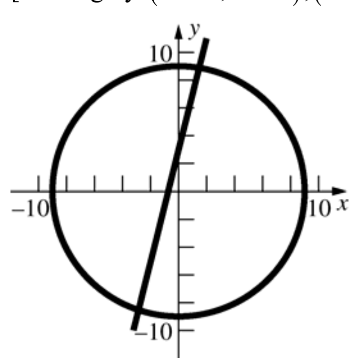
$x=\dfrac{-12\pm\sqrt{38}}{17}\approx-2.88,\ 1.47$

Intersection points:

$\left(\dfrac{-12-\sqrt{38}}{17},\dfrac{3-24\sqrt{38}}{17}\right)$ and

$\left(\dfrac{-12+\sqrt{38}}{17},\dfrac{3+24\sqrt{38}}{17}\right)$

[or roughly $\left(-2.88,-8.52\right),\left(1.47,8.88\right)$]



39. **a.** $y = x^2$; (2)

b. $ax^3 + bx^2 + cx + d$, with $a > 0$: (1)

c. $ax^3 + bx^2 + cx + d$, with $a < 0$: (3)

d. $y = ax^3$, with $a > 0$: (4)

40. $x^2 + y^2 = 13; (-2,-3), (-2,3), (2,-3), (2,3)$

$d_1 = \sqrt{(2+2)^2 + (-3+3)^2} = 4$

$d_2 = \sqrt{(2+2)^2 + (-3-3)^2} = \sqrt{52} = 2\sqrt{13}$

$d_3 = \sqrt{(2-2)^2 + (3+3)^2} = 6$

Three such distances.

41. $x^2 + 2x + y^2 - 2y = 20;\ \left(-2, 1+\sqrt{21}\right),$ $\left(-2, 1-\sqrt{21}\right), \left(2, 1+\sqrt{13}\right), \left(2, 1-\sqrt{13}\right)$

$d_1 = \sqrt{(-2-2)^2 + \left[1+\sqrt{21} - \left(1+\sqrt{13}\right)\right]^2}$

$= \sqrt{16 + \left(\sqrt{21} - \sqrt{13}\right)^2}$

$= \sqrt{50 - 2\sqrt{273}} \approx 4.12$

$d_2 = \sqrt{(-2-2)^2 + \left[1+\sqrt{21} - \left(1-\sqrt{13}\right)\right]^2}$

$= \sqrt{16 + \left(\sqrt{21} + \sqrt{13}\right)^2}$

$= \sqrt{50 + 2\sqrt{273}} \approx 9.11$

$d_3 = \sqrt{(-2+2)^2 + \left[1+\sqrt{21} - \left(1-\sqrt{21}\right)\right]^2}$

$= \sqrt{0 + \left(\sqrt{21} + \sqrt{21}\right)^2} = \sqrt{\left(2\sqrt{21}\right)^2}$

$= 2\sqrt{21} \approx 9.17$

$d_4 = \sqrt{(-2-2)^2 + \left[1-\sqrt{21} - (1+\sqrt{13})\right]^2}$

$= \sqrt{16 + \left(-\sqrt{21} - \sqrt{13}\right)^2}$

$= \sqrt{50 + 2\sqrt{273}} \approx 9.11$

$d_5 = \sqrt{(-2-2)^2 + \left[1-\sqrt{21} - \left(1-\sqrt{13}\right)\right]^2}$

$= \sqrt{16 + \left(\sqrt{13} - \sqrt{21}\right)^2}$

$= \sqrt{50 - 2\sqrt{273}} \approx 4.12$

$d_6 = \sqrt{(2-2)^2 + \left[1+\sqrt{13} - \left(1-\sqrt{13}\right)\right]^2}$

$= \sqrt{0 + \left(\sqrt{13} + \sqrt{13}\right)^2} = \sqrt{\left(2\sqrt{13}\right)^2}$

$= 2\sqrt{13} \approx 7.21$

Four such distances ($d_2 = d_4$ and $d_1 = d_5$).

0.5 Concepts Review

1. domain; range

2. $f(2u) = 3(2u)^2 = 12u^2$; $f(x+h) = 3(x+h)^2$

3. asymptote

4. even; odd; *y*-axis; origin

Problem Set 0.5

1. **a.** $f(1) = 1 - 1^2 = 0$

b. $f(-2) = 1 - (-2)^2 = -3$

c. $f(0) = 1 - 0^2 = 1$

d. $f(k) = 1 - k^2$

e. $f(-5) = 1 - (-5)^2 = -24$

f. $f\left(\frac{1}{4}\right) = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$

g. $f(1+h) = 1 - (1+h)^2 = -2h - h^2$

h. $f(1+h) - f(1) = -2h - h^2 - 0 = -2h - h^2$

i. $f(2+h) - f(2) = 1 - (2+h)^2 + 3$
$= -4h - h^2$

2. **a.** $F(1) = 1^3 + 3 \cdot 1 = 4$

b. $F(\sqrt{2}) = (\sqrt{2})^3 + 3(\sqrt{2}) = 2\sqrt{2} + 3\sqrt{2}$
$= 5\sqrt{2}$

c. $F\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^3 + 3\left(\frac{1}{4}\right) = \frac{1}{64} + \frac{3}{4} = \frac{49}{64}$

d. $F(1+h)=(1+h)^3+3(1+h)$
$=1+3h+3h^2+h^3+3+3h$
$=4+6h+3h^2+h^3$

e. $F(1+h)-1=3+6h+3h^2+h^3$

f. $F(2+h)-F(2)$
$=(2+h)^3+3(2+h)-\left[2^3-3(2)\right]$
$=8+12h+6h^2+h^3+6+3h-14$
$=15h+6h^2+h^3$

3. a. $G(0)=\frac{1}{0-1}=-1$

b. $G(0.999)=\frac{1}{0.999-1}=-1000$

c. $G(1.01)=\frac{1}{1.01-1}=100$

d. $G(y^2)=\frac{1}{y^2-1}$

e. $G(-x)=\frac{1}{-x-1}=-\frac{1}{x+1}$

f. $G\left(\frac{1}{x^2}\right)=\frac{1}{\frac{1}{x^2}-1}=\frac{x^2}{1-x^2}$

4. a. $\Phi(1)=\frac{1+1^2}{\sqrt{1}}=2$

b. $\Phi(-t)=\frac{-t+(-t)^2}{\sqrt{-t}}=\frac{t^2-t}{\sqrt{-t}}$

c. $\Phi\left(\frac{1}{2}\right)=\frac{\frac{1}{2}+\left(\frac{1}{2}\right)^2}{\sqrt{\frac{1}{2}}}=\frac{\frac{3}{4}}{\sqrt{\frac{1}{2}}}\approx 1.06$

d. $\Phi(u+1)=\frac{(u+1)+(u+1)^2}{\sqrt{u+1}}=\frac{u^2+3u+2}{\sqrt{u+1}}$

e. $\Phi(x^2)=\frac{(x^2)+(x^2)^2}{\sqrt{x^2}}=\frac{x^2+x^4}{|x|}$

f. $\Phi(x^2+x)=\frac{(x^2+x)+(x^2+x)^2}{\sqrt{x^2+x}}$
$=\frac{x^4+2x^3+2x^2+x}{\sqrt{x^2+x}}$

5. a. $f(0.25)=\frac{1}{\sqrt{0.25-3}}=\frac{1}{\sqrt{-2.75}}$ is not defined

b. $f(x)=\frac{1}{\sqrt{\pi-3}}\approx 2.658$

c. $f(3+\sqrt{2})=\frac{1}{\sqrt{3+\sqrt{2}-3}}=\frac{1}{\sqrt{\sqrt{2}}}$
$=2^{-0.25}\approx 0.841$

6. a. $f(0.79)=\frac{\sqrt{(0.79)^2+9}}{0.79-\sqrt{3}}\approx -3.293$

b. $f(12.26)=\frac{\sqrt{(12.26)^2+9}}{12.26-\sqrt{3}}\approx 1.199$

c. $f(\sqrt{3})=\frac{\sqrt{(\sqrt{3})^2+9}}{\sqrt{3}-\sqrt{3}}$; undefined

7. a. $x^2+y^2=1$
$y^2=1-x^2$
$y=\pm\sqrt{1-x^2}$; not a function

b. $xy+y+x=1$
$y(x+1)=1-x$
$y=\frac{1-x}{x+1}$; $f(x)=\frac{1-x}{x+1}$

c. $x=\sqrt{2y+1}$
$x^2=2y+1$
$y=\frac{x^2-1}{2}$; $f(x)=\frac{x^2-1}{2}$

d. $x=\frac{y}{y+1}$
$xy+x=y$
$x=y-xy$
$x=y(1-x)$
$y=\frac{x}{1-x}$; $f(x)=\frac{x}{1-x}$

8. The graphs on the left are not graphs of functions, the graphs on the right are graphs of functions.

9. $\dfrac{f(a+h)-f(a)}{h}=\dfrac{[2(a+h)^2-1]-(2a^2-1)}{h}$

$=\dfrac{4ah+2h^2}{h}=4a+2h$

10. $\dfrac{F(a+h)-F(a)}{h}=\dfrac{4(a+h)^3-4a^3}{h}$

$=\dfrac{4a^3+12a^2h+12ah^2+4h^3-4a^3}{h}$

$=\dfrac{12a^2h+12ah^2+4h^3}{h}$

$=12a^2+12ah+4h^2$

11. $\dfrac{g(x+h)-g(x)}{h}=\dfrac{\frac{3}{x+h-2}-\frac{3}{x-2}}{h}$

$=\dfrac{\frac{3x-6-3x-3h+6}{x^2-4x+hx-2h+4}}{h}$

$=\dfrac{-3h}{h(x^2-4x+hx-2h+4)}$

$=-\dfrac{3}{x^2-4x+hx-2h+4}$

12. $\dfrac{G(a+h)-G(a)}{h}=\dfrac{\frac{a+h}{a+h+4}-\frac{a}{a+4}}{h}$

$=\dfrac{\frac{a^2+4a+ah+4h-a^2-ah-4a}{a^2+8a+ah+4h+16}}{h}$

$=\dfrac{4h}{h(a^2+8a+ah+4h+16)}$

$=\dfrac{4}{a^2+8a+ah+4h+16}$

13. **a.** $F(z)=\sqrt{2z+3}$

$2z+3\ \geq\ 0;\ z\geq-\dfrac{3}{2}$

Domain: $\left\{z\in\mathbb{R}:z\geq-\dfrac{3}{2}\right\}$

b. $g(v)=\dfrac{1}{4v-1}$

$4v-1=0;\ v=\dfrac{1}{4}$

Domain: $\left\{v\in\mathbb{R}:v\neq\dfrac{1}{4}\right\}$

c. $\psi(x)=\sqrt{x^2-9}$

$x^2-9\geq0;\ \ x^2\geq9;\ |x|\geq3$

Domain: $\{x\in\mathbb{R}:|x|\geq3\}$

d. $H(y)=-\sqrt{625-y^4}$

$625-y^4\geq0; 625\geq y^4; |y|\leq5$

Domain: $\{y\in\mathbb{R}:|y|\leq5\}$

14. **a.** $f(x)=\dfrac{4-x^2}{x^2-x-6}=\dfrac{4-x^2}{(x-3)(x+2)}$

Domain: $\{x\in\mathbb{R}:x\neq-2,3\}$

b. $G(y)=\sqrt{(y+1)^{-1}}$

$\dfrac{1}{y+1}\geq0;\ y>-1$

Domain: $\{y\in\mathbb{R}:y>-1\}$

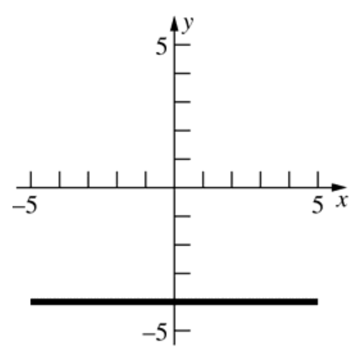
c. $\phi(u)=|2u+3|$

Domain: $\mathbb{R}$ (all real numbers)

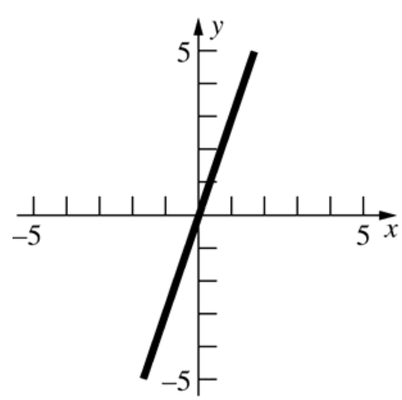
d. $F(t)=t^{2/3}-4$

Domain: $\mathbb{R}$ (all real numbers)

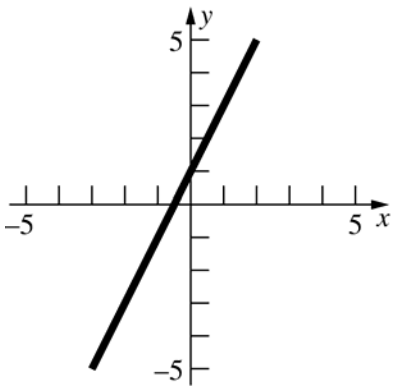
15. $f(x)=-4$; $f(-x)=-4$; even function



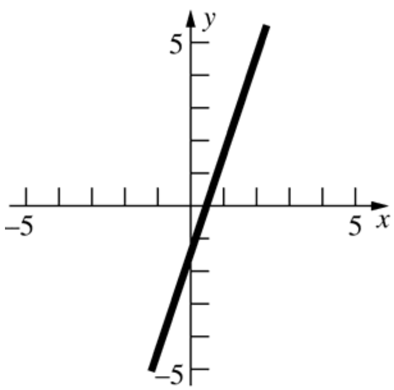
16. $f(x)=3x$; $f(-x)=-3x$; odd function



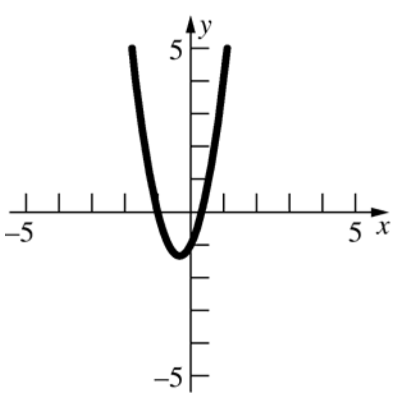
17. $F(x) = 2x + 1$; $F(-x) = -2x + 1$; neither



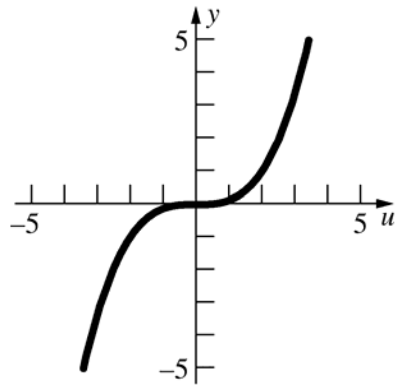
18. $F(x) = 3x - \sqrt{2}; F(-x) = -3x - \sqrt{2};$ neither



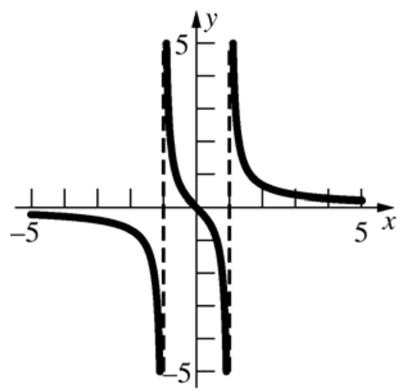
19. $g(x) = 3x^2 + 2x - 1; g(-x) = 3x^2 - 2x - 1;$ neither



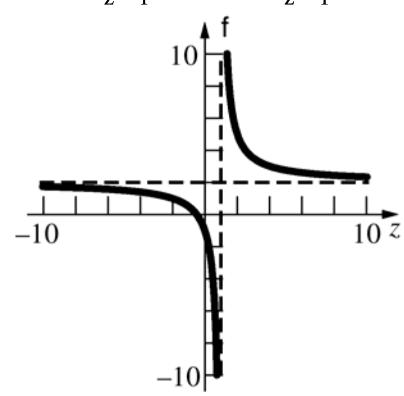
20. $g(u) = \frac{u^3}{8}; g(-u) = -\frac{u^3}{8};$ odd function



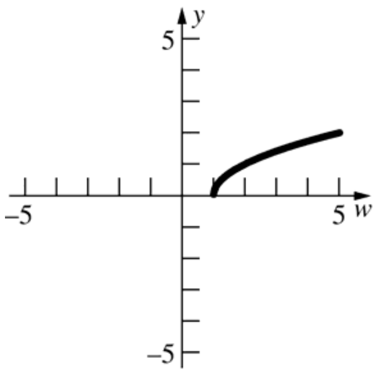
21. $g(x) = \frac{x}{x^2 - 1}; g(-x) = \frac{-x}{x^2 - 1};$ odd



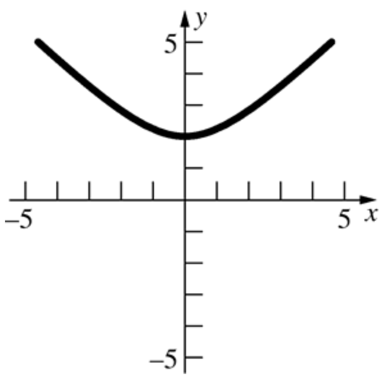
22. $\phi(z) = \frac{2z + 1}{z - 1}; \phi(-z) = \frac{-2z + 1}{-z - 1};$ neither



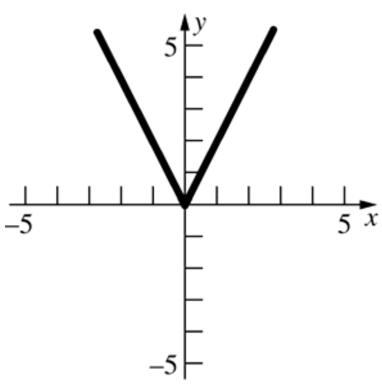
23. $f(w) = \sqrt{w-1};\ f(-w) = \sqrt{-w-1};$ neither



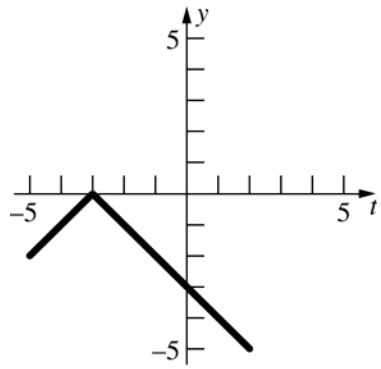
24. $h(x) = \sqrt{x^2+4}; h(-x) = \sqrt{x^2+4};$ even function



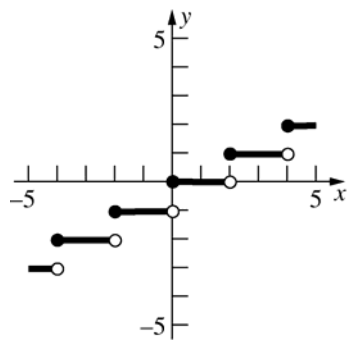
25. $f(x) = |2x|;\ f(-x) = |-2x| = |2x|;$ even function



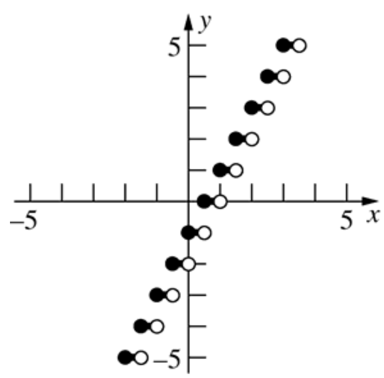
26. $F(t) = -|t+3|; F(-t) = -|-t+3|;$ neither



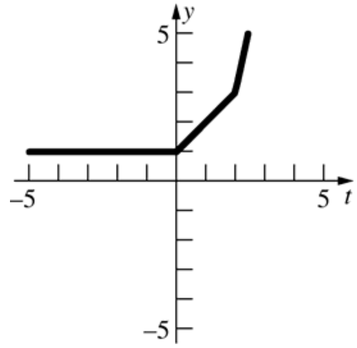
27. $g(x) = \left[\!\left[\frac{x}{2}\right]\!\right]; g(-x) = \left[\!\left[-\frac{x}{2}\right]\!\right];$ neither



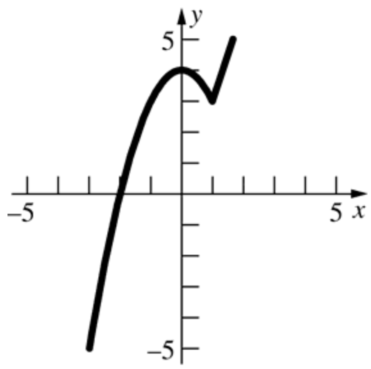
28. $G(x) = [\![2x-1]\!]; G(-x) = [\![-2x+1]\!];$ neither



29. $g(t) = \begin{cases} 1 & \text{if } t \le 0 \\ t+1 & \text{if } 0 < t < 2 \\ t^2 - 1 & \text{if } t \ge 2 \end{cases}$ neither



30. $h(x)=\begin{cases}-x^2+4 & \text{if } x\le 1\\ 3x & \text{if } x>1\end{cases}$ neither



31. $T(x) = 5000 + 805x$
Domain: $\{x \in \text{integers}: 0 \le x \le 100\}$

$u(x)=\frac{T(x)}{x}=\frac{5000}{x}+805$

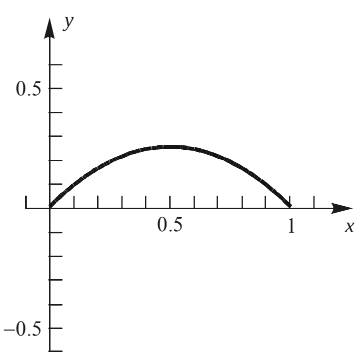
Domain: $\{x \in \text{integers}: 0 < x \le 100\}$

32. **a.** $P(x)=6x-(400+5\sqrt{x(x-4)})$
$=6x-400-5\sqrt{x(x-4)}$

b. $P(200)\approx -190$; $P(1000)\approx 610$

c. ABC breaks even when $P(x) = 0$;
$6x-400-5\sqrt{x(x-4)}=0$; $x\approx 390$

33. $E(x)=x-x^2$



$\frac{1}{2}$ exceeds its square by the maximum amount.

34. Each side has length $\frac{p}{3}$. The height of the triangle is $\frac{\sqrt{3}p}{6}$.

$A(p)=\frac{1}{2}\left(\frac{p}{3}\right)\left(\frac{\sqrt{3}p}{6}\right)=\frac{\sqrt{3}p^2}{36}$

35. Let y denote the length of the other leg. Then

$x^2+y^2=h^2$

$y^2=h^2-x^2$

$y=\sqrt{h^2-x^2}$

$L(x)=\sqrt{h^2-x^2}$

36. The area is

$A(x)=\frac{1}{2}\text{base}\times\text{height}=\frac{1}{2}x\sqrt{h^2-x^2}$

37. **a.** $E(x) = 24 + 0.40x$

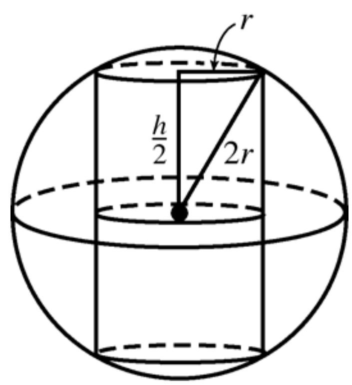
b. $120 = 24 + 0.40x$
$0.40x = 96$; $x = 240$ mi

38. The volume of the cylinder is $\pi r^2 h$, where h is the height of the cylinder. From the figure,

$r^2+\left(\frac{h}{2}\right)^2=(2r)^2$; $\frac{h^2}{4}=3r^2$;

$h=\sqrt{12r^2}=2r\sqrt{3}$.

$V(r)=\pi r^2(2r\sqrt{3})=2\pi r^3\sqrt{3}$



39. The area of the two semicircular ends is $\frac{\pi d^2}{4}$.

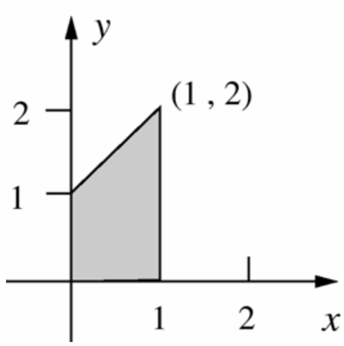
The length of each parallel side is $\frac{1-\pi d}{2}$.

$A(d)=\frac{\pi d^2}{4}+d\left(\frac{1-\pi d}{2}\right)=\frac{\pi d^2}{4}+\frac{d-\pi d^2}{2}$

$=\frac{2d-\pi d^2}{4}$

Since the track is one mile long, $\pi d < 1$, so $d<\frac{1}{\pi}$. Domain: $\left\{d\in\mathbb{R}: 0<d<\frac{1}{\pi}\right\}$

40. **a.** $A(1) = 1(1) + \frac{1}{2}(1)(2-1) = \frac{3}{2}$

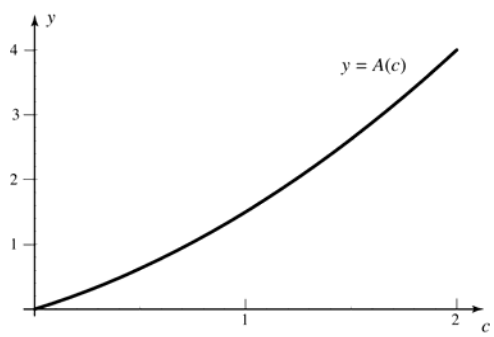


b. $A(2) = 2(1) + \frac{1}{2}(2)(3-1) = 4$

c. $A(0) = 0$

d. $A(c) = c(1) + \frac{1}{2}(c)(c+1-1) = \frac{1}{2}c^2 + c$

e.



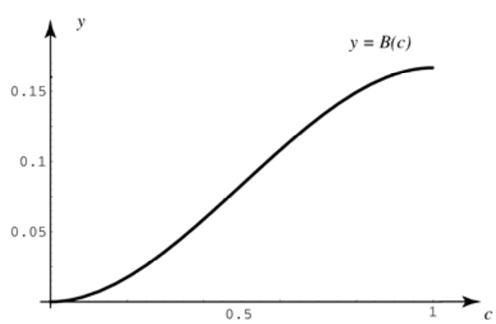
f. Domain: $\{c \in \mathbb{R} : c \geq 0\}$

Range: $\{y \in \mathbb{R} : y \geq 0\}$

41. **a.** $B(0) = 0$

b. $B\left(\frac{1}{2}\right) = \frac{1}{2}B(1) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

c.



42. **a.** $f(x + y) = 2(x + y) = 2x + 2y = f(x) + f(y)$

b. $f(x+y) = (x+y)^2 = x^2 + 2xy + y^2$
$\neq f(x) + f(y)$

c. $f(x + y) = 2(x + y) + 1 = 2x + 2y + 1$
$\neq f(x) + f(y)$

d. $f(x + y) = -3(x + y) = -3x - 3y = f(x) + f(y)$

43. For any x, $x + 0 = x$, so
$f(x) = f(x + 0) = f(x) + f(0)$, hence $f(0) = 0$.
Let m be the value of $f(1)$. For p in N,
$p = p \cdot 1 = 1 + 1 + \ldots + 1$, so
$f(p) = f(1 + 1 + \ldots + 1) = f(1) + f(1) + \ldots + f(1)$
$= pf(1) = pm$.

$1 = p\left(\frac{1}{p}\right) = \frac{1}{p} + \frac{1}{p} + \ldots + \frac{1}{p}$, so

$m = f(1) = f\left(\frac{1}{p} + \frac{1}{p} + \ldots + \frac{1}{p}\right)$

$= f\left(\frac{1}{p}\right) + f\left(\frac{1}{p}\right) + \ldots + f\left(\frac{1}{p}\right) = pf\left(\frac{1}{p}\right)$,

hence $f\left(\frac{1}{p}\right) = \frac{m}{p}$. Any rational number can be written as $\frac{p}{q}$ with p, q in N.

$\frac{p}{q} = p\left(\frac{1}{q}\right) = \frac{1}{q} + \frac{1}{q} + \ldots + \frac{1}{q}$,

so $f\left(\frac{p}{q}\right) = f\left(\frac{1}{q} + \frac{1}{q} + \ldots + \frac{1}{q}\right)$

$= f\left(\frac{1}{q}\right) + f\left(\frac{1}{q}\right) + \ldots + f\left(\frac{1}{q}\right)$

$= pf\left(\frac{1}{q}\right) = p\left(\frac{m}{q}\right) = m\left(\frac{p}{q}\right)$

44. The player has run $10t$ feet after t seconds. He reaches first base when $t = 9$, second base when $t = 18$, third base when $t = 27$, and home plate when $t = 36$. The player is $10t - 90$ feet from first base when $9 \le t \le 18$, hence $\sqrt{90^2 + (10t - 90)^2}$ feet from home plate. The player is $10t - 180$ feet from second base when $18 \le t \le 27$, thus he is $90 - (10t - 180) = 270 - 10t$ feet from third base and $\sqrt{90^2 + (270 - 10t)^2}$ feet from home plate. The player is $10t - 270$ feet from third base when $27 \le t \le 36$, thus he is $90 - (10t - 270) = 360 - 10t$ feet from home plate.

a. $$s = \begin{cases} 10t & \text{if } 0 \le t \le 9 \\ \sqrt{90^2 + (10t - 90)^2} & \text{if } 9 < t \le 18 \\ \sqrt{90^2 + (270 - 10t)^2} & \text{if } 18 < t \le 27 \\ 360 - 10t & \text{if } 27 < t \le 36 \end{cases}$$

b. $$s = \begin{cases} 180 - |180 - 10t| & \text{if } 0 \le t \le 9 \text{ or } 27 < t \le 36 \\ \sqrt{90^2 + (10t - 90)^2} & \text{if } 9 < t \le 18 \\ \sqrt{90^2 + (270 - 10t)^2} & \text{if } 18 < t \le 27 \end{cases}$$

45. a. $f(1.38) \approx 0.2994$
$f(4.12) \approx 3.6852$

b.

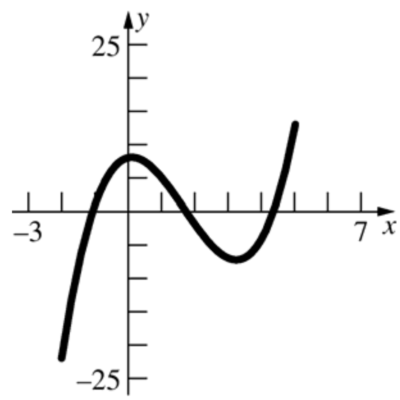
x	$f(x)$
–4	–4.05
–3	–3.1538
–2	–2.375
–1	–1.8
0	–1.25
1	–0.2
2	1.125
3	2.3846
4	3.55

46. a. $f(1.38) \approx -76.8204$
$f(4.12) \approx 6.7508$

b.

x	$f(x)$
–4	–6.1902
–3	0.4118
–2	13.7651
–1	9.9579
0	0
1	–7.3369
2	–17.7388
3	–0.4521
4	4.4378

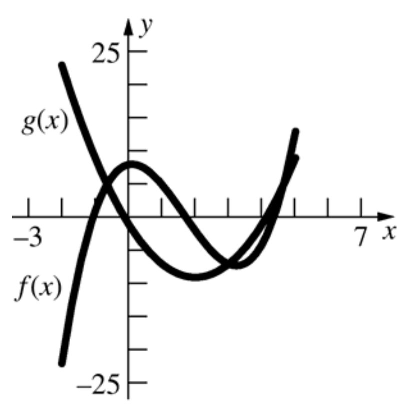
47.



a. Range: $\{y \in \mathbb{R}: -22 \le y \le 13\}$

b. $f(x) = 0$ when $x \approx -1.1, 1.7, 4.3$
$f(x) \ge 0$ on $[-1.1, 1.7] \cup [4.3, 5]$

48.

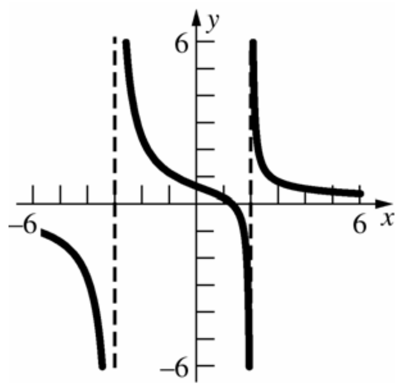


a. $f(x) = g(x)$ at $x \approx -0.6, 3.0, 4.6$

b. $f(x) \geq g(x)$ on $[-0.6, 3.0] \cup [4.6, 5]$

c. $|f(x) - g(x)|$
$= |x^3 - 5x^2 + x + 8 - 2x^2 + 8x + 1|$
$= |x^3 - 7x^2 + 9x + 9|$
Largest value $|f(-2) - g(-2)| = 45$

49.



a. x-intercept: $3x - 4 = 0;\ x = \frac{4}{3}$

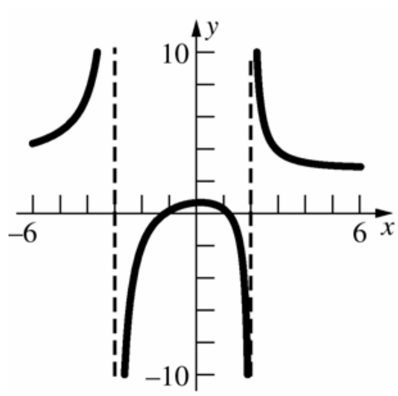
y-intercept: $\frac{3 \cdot 0 - 4}{0^2 + 0 - 6} = \frac{2}{3}$

b. $\mathbb{R}$

c. $x^2 + x - 6 = 0;\ (x + 3)(x - 2) = 0$
Vertical asymptotes at $x = -3$, $x = 2$

d. Horizontal asymptote at $y = 0$

50.



a. x-intercepts:
$3x^2 - 4 = 0; x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2\sqrt{3}}{3}$
y-intercept: $\frac{2}{3}$

b. On $[-6, -3)$, g increases from $g(-6) = \frac{13}{3} \approx 4.3333$ to ∞. On $(2, 6]$, g decreased from ∞ to $\frac{26}{9} \approx 2.8889$. On $(-3, 2)$ the maximum occurs around $x = 0.1451$ with value 0.6748. Thus, the range is $(-\infty, 0.6748] \cup [2.8889, \infty)$.

c. $x^2 + x - 6 = 0;\ (x + 3)(x - 2) = 0$
Vertical asymptotes at $x = -3$, $x = 2$

d. Horizontal asymptote at $y = 3$

0.6 Concepts Review

1. $(x^2 + 1)^3$

2. $f(g(x))$

3. 2; left

4. a quotient of two polynomial functions

Problem Set 0.6

1. a. $(f + g)(2) = (2 + 3) + 2^2 = 9$

b. $(f \cdot g)(0) = (0 + 3)(0^2) = 0$

c. $(g/f)(3) = \frac{3^2}{3+3} = \frac{9}{6} = \frac{3}{2}$

d. $(f \circ g)(1) = f(1^2) = 1 + 3 = 4$

e. $(g \circ f)(1) = g(1 + 3) = 4^2 = 16$

f. $(g \circ f)(-8) = g(-8 + 3) = (-5)^2 = 25$

2. a. $(f - g)(2) = (2^2 + 2) - \frac{2}{2+3} = 6 - \frac{2}{5} = \frac{28}{5}$

b. $(f/g)(1) = \frac{1^2 + 1}{\frac{2}{1+3}} = \frac{2}{\frac{2}{4}} = 4$

c. $g^2(3) = \left[\frac{2}{3+3}\right]^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

d. $(f \circ g)(1) = f\left(\frac{2}{1+3}\right) = \left(\frac{1}{2}\right)^2 + \frac{1}{2} = \frac{3}{4}$

e. $(g \circ f)(1) = g(1^2 + 1) = \frac{2}{2+3} = \frac{2}{5}$

f. $(g \circ g)(3) = g\left(\frac{2}{3+3}\right) = \frac{2}{\frac{1}{3}+3} = \frac{2}{\frac{10}{3}} = \frac{3}{5}$

3. a. $(\Phi + \Psi)(t) = t^3 + 1 + \frac{1}{t}$

b. $(\Phi \circ \Psi)(r) = \Phi\left(\frac{1}{r}\right) = \left(\frac{1}{r}\right)^3 + 1 = \frac{1}{r^3} + 1$

c. $(\Psi \circ \Phi)(r) = \Psi(r^3 + 1) = \frac{1}{r^3 + 1}$

d. $\Phi^3(z) = (z^3 + 1)^3$

e. $(\Phi - \Psi)(5t) = [(5t)^3 + 1] - \frac{1}{5t}$
$= 125t^3 + 1 - \frac{1}{5t}$

f. $((\Phi - \Psi) \circ \Psi)(t) = (\Phi - \Psi)\left(\frac{1}{t}\right)$
$= \left(\frac{1}{t}\right)^3 + 1 - \frac{1}{\frac{1}{t}} = \frac{1}{t^3} + 1 - t$

4. a. $(f \cdot g)(x) = \frac{2\sqrt{x^2 - 1}}{x}$
Domain: $(-\infty, -1] \cup [1, \infty)$

b. $f^4(x) + g^4(x) = \left(\sqrt{x^2 - 1}\right)^4 + \left(\frac{2}{x}\right)^4$
$= (x^2 - 1)^2 + \frac{16}{x^4}$
Domain: $(-\infty, 0) \cup (0, \infty)$

c. $(f \circ g)(x) = f\left(\frac{2}{x}\right) = \sqrt{\left(\frac{2}{x}\right)^2 - 1} = \sqrt{\frac{4}{x^2} - 1}$
Domain: $[-2, 0) \cup (0, 2]$

d. $(g \circ f)(x) = g\left(\sqrt{x^2 - 1}\right) = \frac{2}{\sqrt{x^2 - 1}}$
Domain: $(-\infty, -1) \cup (1, \infty)$

5. $(f \circ g)(x) = f(|1+x|) = \sqrt{|1+x|^2 - 4}$
$= \sqrt{x^2 + 2x - 3}$
$(g \circ f)(x) = g\left(\sqrt{x^2 - 4}\right) = \left|1 + \sqrt{x^2 - 4}\right|$
$= 1 + \sqrt{x^2 - 4}$

6. $g^3(x) = (x^2 + 1)^3 = (x^4 + 2x^2 + 1)(x^2 + 1)$
$= x^6 + 3x^4 + 3x^2 + 1$
$(g \circ g \circ g)(x) = (g \circ g)(x^2 + 1)$
$= g[(x^2 + 1)^2 + 1] = g(x^4 + 2x^2 + 2)$
$= (x^4 + 2x^2 + 2)^2 + 1$
$= x^8 + 4x^6 + 8x^4 + 8x^2 + 5$

7. $g(3.141) \approx 1.188$

8. $g(2.03) \approx 0.000205$

9. $\left[g^2(\pi) - g(\pi)\right]^{1/3} = \left[(11 - 7\pi)^2 - |11 - 7\pi|\right]^{1/3}$
≈ 4.789

10. $[g^3(\pi) - g(\pi)]^{1/3} = [(6\pi - 11)^3 - (6\pi - 11)]^{1/3}$
≈ 7.807

11. a. $g(x) = \sqrt{x},\ f(x) = x + 7$

b. $g(x) = x^{15},\quad f(x) = x^2 + x$

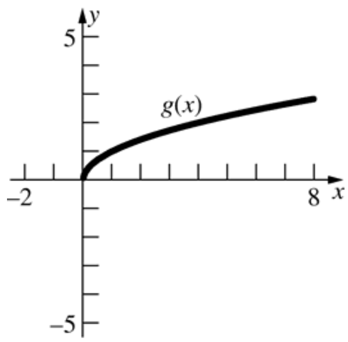
12. a. $f(x) = \frac{2}{x^3},\ g(x) = x^2 + x + 1$

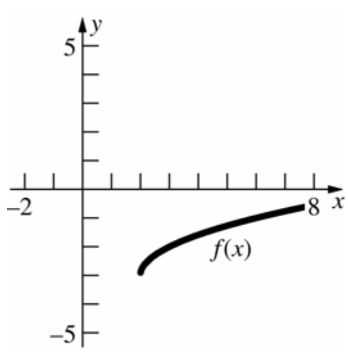
b. $f(x) = \frac{1}{x},\ g(x) = x^3 + 3x$

13. $p = f \circ g \circ h$ if $f(x) = 1/x$, $g(x) = \sqrt{x}$,
$h(x) = x^2 + 1$
$p = f \circ g \circ h$ if $f(x) = 1/\sqrt{x}$, $g(x) = x + 1$,
$h(x) = x^2$

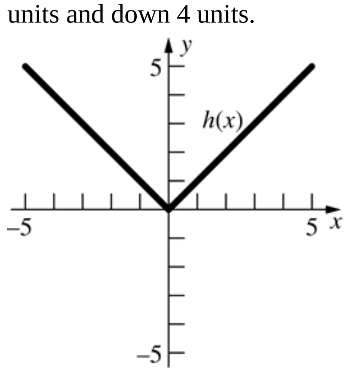
14. $p = f \circ g \circ h \circ l$ if $f(x) = 1/x$, $g(x) = \sqrt{x}$,
$h(x) = x + 1$, $l(x) = x^2$

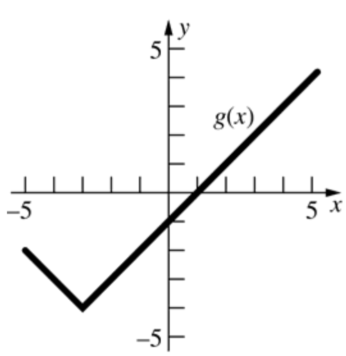
15. Translate the graph of $g(x) = \sqrt{x}$ to the right 2 units and down 3 units.



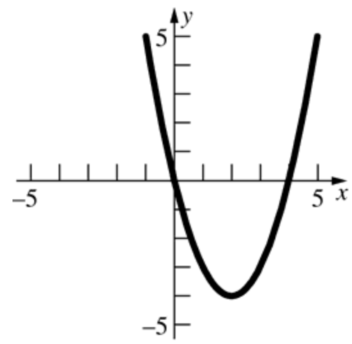


16. Translate the graph of $h(x) = |x|$ to the left 3 units and down 4 units.

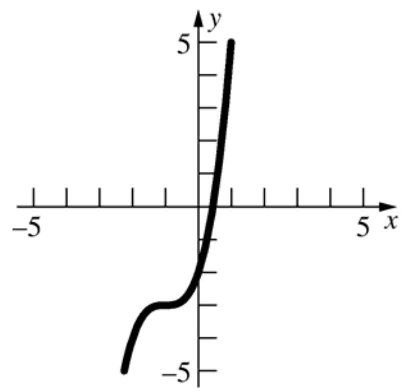




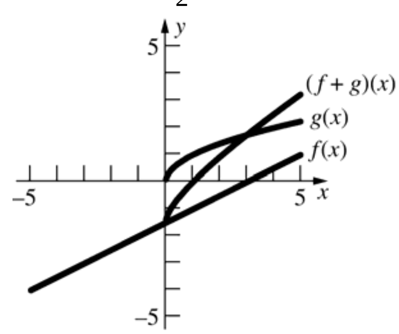
17. Translate the graph of $y = x^2$ to the right 2 units and down 4 units.



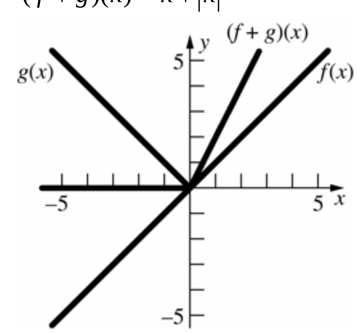
18. Translate the graph of $y = x^3$ to the left 1 unit and down 3 units.



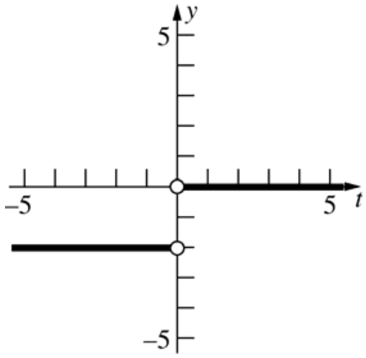
19. $(f+g)(x) = \dfrac{x-3}{2} + \sqrt{x}$



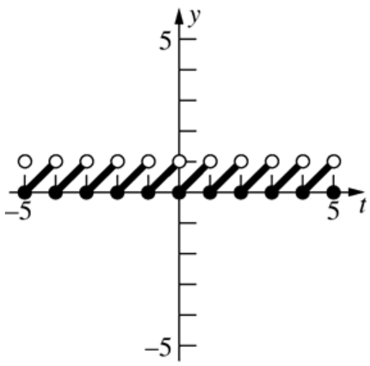
20. $(f+g)(x) = x + |x|$



21. $F(t)=\frac{|t|-t}{t}$



22. $G(t)=t-[\![t]\!]$



23. a. Even;
$(f+g)(-x)=f(-x)+g(-x)=f(x)+g(x)$
$=(f+g)(x)$ if f and g are both even functions.

b. Odd;
$(f+g)(-x)=f(-x)+g(-x)=-f(x)-g(x)$
$=-(f+g)(x)$ if f and g are both odd functions.

c. Even;
$(f\cdot g)(-x)=[f(-x)][g(-x)]$
$=[f(x)][g(x)]=(f\cdot g)(x)$
if f and g are both even functions.

d. Even;
$(f\cdot g)(-x)=[f(-x)][g(-x)]$
$=[-f(x)][-g(x)]=[f(x)][g(x)]$
$=(f\cdot g)(x)$
if f and g are both odd functions.

e. Odd;
$(f\cdot g)(-x)=[f(-x)][g(-x)]$
$=[f(x)][-g(x)]=-[f(x)][g(x)]$
$=-(f\cdot g)(x)$
if f is an even function and g is an odd function.

24. a. $F(x)-F(-x)$ is odd because
$F(-x)-F(x)=-[F(x)-F(-x)]$

b. $F(x)+F(-x)$ is even because
$F(-x)+F(-(-x))=F(-x)+F(x)$
$=F(x)+F(-x)$

c. $\frac{F(x)-F(-x)}{2}$ is odd and $\frac{F(x)+F(-x)}{2}$ is even.
$$\frac{F(x)-F(-x)}{2}+\frac{F(x)+F(-x)}{2}=\frac{2F(x)}{2}=F(x)$$

25. Not every polynomial of even degree is an even function. For example $f(x)=x^2+x$ is neither even nor odd. Not every polynomial of odd degree is an odd function. For example $g(x)=x^3+x^2$ is neither even nor odd.

26. a. Neither

b. PF

c. RF

d. PF

e. RF

f. Neither

27. a. $P=\sqrt{29-3(2+\sqrt{t})+(2+\sqrt{t})^2}$
$=\sqrt{t+\sqrt{t}+27}$

b. When $t = 15$, $P=\sqrt{15+\sqrt{15}+27}\approx 6.773$

28. $R(t)=(120+2t+3t^2)(6000+700t)$
$=2100t^3+19,400t^2+96,000t+720,000$

29. $D(t)=\begin{cases}400t & \text{if } 0<t<1\\ \sqrt{(400t)^2+[300(t-1)]^2} & \text{if } t\ge 1\end{cases}$

$D(t)=\begin{cases}400t & \text{if } 0<t<1\\ \sqrt{250,000t^2-180,000t+90,000} & \text{if } t\ge 1\end{cases}$

30. $D(2.5)\approx 1097$ mi

31. $f(f(x)) = f\left(\frac{ax+b}{cx-a}\right) = \frac{a\left(\frac{ax+b}{cx-a}\right)+b}{c\left(\frac{ax+b}{cx-a}\right)-a}$

$= \frac{a^2x+ab+bcx-ab}{acx+bc-acx+a^2} = \frac{x(a^2+bc)}{a^2+bc} = x$

If $a^2+bc=0$, $f(f(x))$ is undefined, while if $x=\frac{a}{c}$, $f(x)$ is undefined.

32. $f(f(f(x))) = f\left(f\left(\frac{x-3}{x+1}\right)\right) = f\left(\frac{\frac{x-3}{x+1}-3}{\frac{x-3}{x+1}+1}\right)$

$= f\left(\frac{x-3-3x-3}{x-3+x+1}\right) = f\left(\frac{-2x-6}{2x-2}\right) = f\left(\frac{-x-3}{x-1}\right)$

$= \frac{\frac{-x-3}{x-1}-3}{\frac{-x-3}{x-1}+1} = \frac{-x-3-3x+3}{-x-3+x-1} = \frac{-4x}{-4} = x$

If $x=-1$, $f(x)$ is undefined, while if $x=1$, $f(f(x))$ is undefined.

33. a. $f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}}{\frac{1}{x}-1} = \frac{1}{1-x}$

b. $f(f(x)) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$

$= \frac{x}{x-x+1} = x$

c. $f\left(\frac{1}{f(x)}\right) = f\left(\frac{x-1}{x}\right) = \frac{\frac{x-1}{x}}{\frac{x-1}{x}-1} = \frac{x-1}{x-1-x}$

$= 1-x$

34. a. $f(1/x) = \frac{1/x}{\sqrt{1/x}-1} = \frac{1}{\sqrt{x}-x}$

b. $f(f(x)) = f(x/(\sqrt{x}-1)) = \frac{x/(\sqrt{x}-1)}{\sqrt{\frac{x}{\sqrt{x}-1}}-1}$

$= \frac{x}{\sqrt{x(\sqrt{x}-1)}+1-\sqrt{x}}$

35. $(f_1 \circ (f_2 \circ f_3))(x) = f_1((f_2 \circ f_3)(x))$
$= f_1(f_2(f_3(x)))$

$((f_1 \circ f_2) \circ f_3)(x) = (f_1 \circ f_2)(f_3(x))$
$= f_1(f_2(f_3(x)))$
$= (f_1 \circ (f_2 \circ f_3))(x)$

36. $f_1(f_1(x)) = x;$

$f_1(f_2(x)) = \frac{1}{x};$

$f_1(f_3(x)) = 1-x;$

$f_1(f_4(x)) = \frac{1}{1-x};$

$f_1(f_5(x)) = \frac{x-1}{x};$

$f_1(f_6(x)) = \frac{x}{x-1};$

$f_2(f_1(x)) = \frac{1}{x};$

$f_2(f_2(x)) = \frac{1}{\frac{1}{x}} = x;$

$f_2(f_3(x)) = \frac{1}{1-x};$

$f_2(f_4(x)) = \frac{1}{\frac{1}{1-x}} = 1-x;$

$f_2(f_5(x)) = \frac{1}{\frac{x-1}{x}} = \frac{x}{x-1};$

$f_2(f_6(x)) = \frac{1}{\frac{x}{x-1}} = \frac{x-1}{x};$

$f_3(f_1(x)) = 1-x;$

$f_3(f_2(x)) = 1-\frac{1}{x} = \frac{x-1}{x};$

$f_3(f_3(x)) = 1-(1-x) = x;$

$f_3(f_4(x)) = 1-\frac{1}{1-x} = \frac{x}{x-1};$

$f_3(f_5(x)) = 1-\frac{x-1}{x} = \frac{1}{x};$

$f_3(f_6(x)) = 1-\frac{x}{x-1} = \frac{1}{1-x};$

$f_4(f_1(x)) = \frac{1}{1-x};$

$f_4(f_2(x)) = \frac{1}{1-\frac{1}{x}} = \frac{x}{x-1};$

$f_4(f_3(x)) = \frac{1}{1-(1-x)} = \frac{1}{x};$

$f_4(f_4(x)) = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{1-x-1} = \frac{x-1}{x};$

$f_4(f_5(x)) = \frac{1}{1-\frac{x-1}{x}} = \frac{x}{x-(x-1)} = x;$

$f_4(f_6(x)) = \frac{1}{1-\frac{x}{x-1}} = \frac{x-1}{x-1-x} = 1-x;$

$$f_5(f_1(x)) = \frac{x-1}{x};$$

$$f_5(f_2(x)) = \frac{\frac{1}{x}-1}{\frac{1}{x}} = 1-x;$$

$$f_5(f_3(x)) = \frac{1-x-1}{1-x} = \frac{x}{x-1};$$

$$f_5(f_4(x)) = \frac{\frac{1}{1-x}-1}{\frac{1}{1-x}} = \frac{1-(1-x)}{1} = x;$$

$$f_5(f_5(x)) = \frac{\frac{x-1}{x}-1}{\frac{x-1}{x}} = \frac{x-1-x}{x-1} = \frac{1}{1-x};$$

$$f_5(f_6(x)) = \frac{\frac{x}{x-1}-1}{\frac{x}{x-1}} = \frac{x-(x-1)}{x} = \frac{1}{x};$$

$$f_6(f_1(x)) = \frac{x}{x-1};$$

$$f_6(f_2(x)) = \frac{\frac{1}{x}}{\frac{1}{x}-1} = \frac{1}{1-x};$$

$$f_6(f_3(x)) = \frac{1-x}{1-x-1} = \frac{x-1}{x};$$

$$f_6(f_4(x)) = \frac{\frac{1}{1-x}}{\frac{1}{1-x}-1} = \frac{1}{1-(1-x)} = \frac{1}{x};$$

$$f_6(f_5(x)) = \frac{\frac{x-1}{x}}{\frac{x-1}{x}-1} = \frac{x-1}{x-1-x} = 1-x;$$

$$f_6(f_6(x)) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-(x-1)} = x$$

$\circ$	f_1	f_2	f_3	f_4	f_5	f_6
f_1	f_1	f_2	f_3	f_4	f_5	f_6
f_2	f_2	f_1	f_4	f_3	f_6	f_5
f_3	f_3	f_5	f_1	f_6	f_2	f_4
f_4	f_4	f_6	f_2	f_5	f_1	f_3
f_5	f_5	f_3	f_6	f_1	f_4	f_2
f_6	f_6	f_4	f_5	f_2	f_3	f_1

a. $f_3 \circ f_3 \circ f_3 \circ f_3 \circ f_3$
$= ((((f_3 \circ f_3) \circ f_3) \circ f_3) \circ f_3)$
$= (((f_1 \circ f_3) \circ f_3) \circ f_3)$
$= ((f_3 \circ f_3) \circ f_3)$
$= f_1 \circ f_3 = f_3$

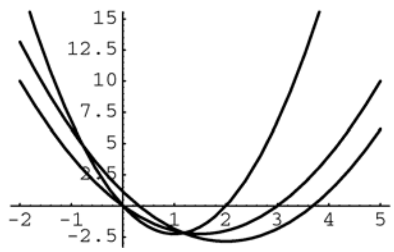
b. $f_1 \circ f_2 \circ f_3 \circ f_4 \circ f_5 \circ f_6$
$= (((((f_1 \circ f_2) \circ f_3) \circ f_4) \circ f_5) \circ f_6)$
$= ((((f_2 \circ f_3) \circ f_4) \circ f_5) \circ f_6)$
$= (f_4 \circ f_4) \circ (f_5 \circ f_6)$
$= f_5 \circ f_2 = f_3$

c. If $F \circ f_6 = f_1$, then $F = f_6$.

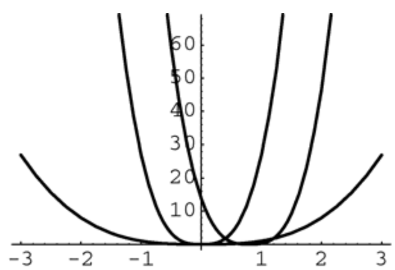
d. If $G \circ f_3 \circ f_6 = f_1$, then $G \circ f_4 = f_1$ so $G = f_5$.

e. If $f_2 \circ f_5 \circ H = f_5$, then $f_6 \circ H = f_5$ so $H = f_3$.

37.



38.



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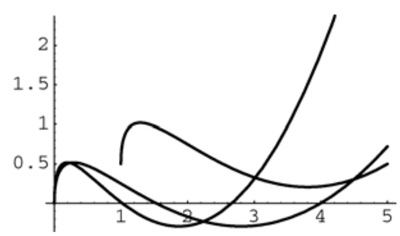
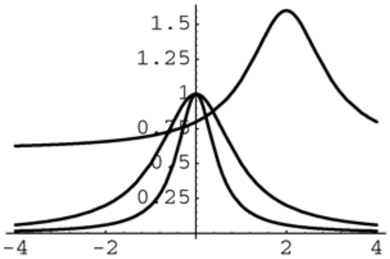
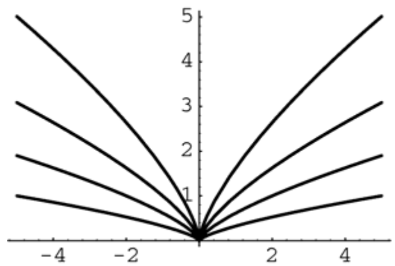


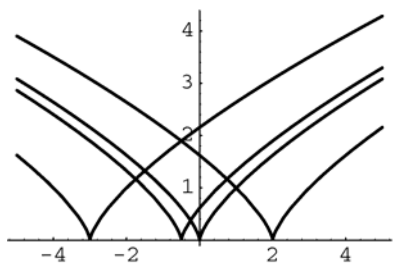
40.



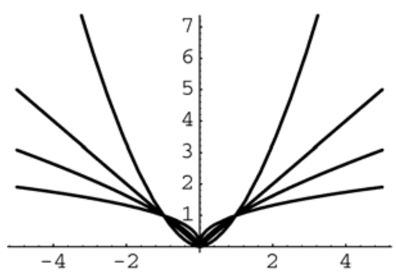
41. a.



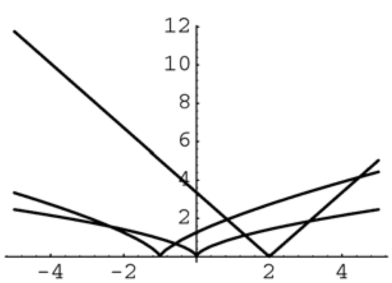
b.



c.



42.



0.7 Concepts Review

1. $(-\infty, \infty)$; $[-1, 1]$

2. 2π; 2π; π

3. odd; even

4. $r = \sqrt{(-4)^2 + 3^2} = 5; \cos\theta = \dfrac{x}{r} = -\dfrac{4}{5}$

Problem Set 0.7

1. a. $30\left(\dfrac{\pi}{180}\right) = \dfrac{\pi}{6}$

b. $45\left(\dfrac{\pi}{180}\right) = \dfrac{\pi}{4}$

c. $-60\left(\dfrac{\pi}{180}\right) = -\dfrac{\pi}{3}$

d. $240\left(\dfrac{\pi}{180}\right) = \dfrac{4\pi}{3}$

e. $-370\left(\dfrac{\pi}{180}\right) = -\dfrac{37\pi}{18}$

f. $10\left(\dfrac{\pi}{180}\right) = \dfrac{\pi}{18}$

2. a. $\dfrac{7}{6}\pi\left(\dfrac{180}{\pi}\right) = 210°$

b. $\dfrac{3}{4}\pi\left(\dfrac{180}{\pi}\right) = 135°$

c. $-\dfrac{1}{3}\pi\left(\dfrac{180}{\pi}\right) = -60°$

d. $\dfrac{4}{3}\pi\left(\dfrac{180}{\pi}\right) = 240°$

e. $-\dfrac{35}{18}\pi\left(\dfrac{180}{\pi}\right) = -350°$

f. $\dfrac{3}{18}\pi\left(\dfrac{180}{\pi}\right) = 30°$

3. a. $33.3\left(\dfrac{\pi}{180}\right) \approx 0.5812$

b. $46\left(\dfrac{\pi}{180}\right) \approx 0.8029$

c. $-66.6\left(\dfrac{\pi}{180}\right) \approx -1.1624$

d. $240.11\left(\dfrac{\pi}{180}\right) \approx 4.1907$

e. $-369\left(\dfrac{\pi}{180}\right) \approx -6.4403$

f. $11\left(\dfrac{\pi}{180}\right) \approx 0.1920$

4. a. $3.141\left(\frac{180}{\pi}\right) \approx 180°$

b. $6.28\left(\frac{180}{\pi}\right) \approx 359.8°$

c. $5.00\left(\frac{180}{\pi}\right) \approx 286.5°$

d. $0.001\left(\frac{180}{\pi}\right) \approx 0.057°$

e. $-0.1\left(\frac{180}{\pi}\right) \approx -5.73°$

f. $36.0\left(\frac{180}{\pi}\right) \approx 2062.6°$

5. a. $\frac{56.4 \tan 34.2°}{\sin 34.1°} \approx 68.37$

b. $\frac{5.34 \tan 21.3°}{\sin 3.1° + \cot 23.5°} \approx 0.8845$

c. $\tan(0.452) \approx 0.4855$

d. $\sin(-0.361) \approx -0.3532$

6. a. $\frac{234.1 \sin(1.56)}{\cos(0.34)} \approx 248.3$

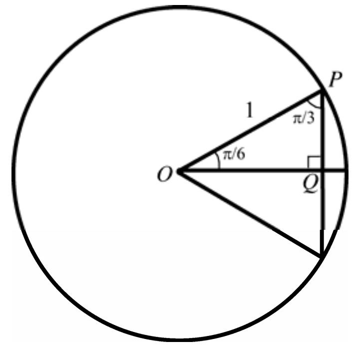
b. $\sin^2(2.51) + \sqrt{\cos(0.51)} \approx 1.2828$

7. a. $\frac{56.3 \tan 34.2°}{\sin 56.1°} \approx 46.097$

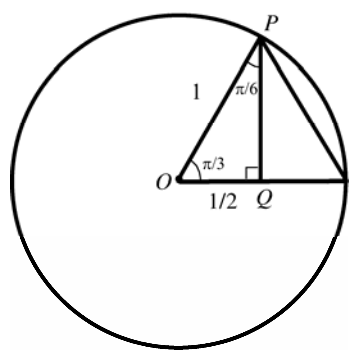
b. $\left(\frac{\sin 35°}{\sin 26° + \cos 26°}\right)^3 \approx 0.0789$

8. Referring to Figure 2, it is clear that $\sin 0 = 0$ and $\cos 0 = 1$. If the angle is $\pi/6$, then the triangle in the figure below is equilateral. Thus, $|PQ| = \frac{1}{2}|OP| = \frac{1}{2}$. This implies that $\sin\frac{\pi}{6} = \frac{1}{2}$. By the Pythagorean Identity, $\cos^2\frac{\pi}{6} = 1 - \sin^2\frac{\pi}{6} = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4}$.

Thus



$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$. The results $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ were derived in the text. If the angle is $\pi/3$ then the triangle in the figure below is equilateral. Thus $\cos\frac{\pi}{3} = \frac{1}{2}$ and by the Pythagorean Identity, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.



Referring to Figure 2, it is clear that $\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$. The rest of the values are obtained using the same kind of reasoning in the second quadrant.

9. a. $\tan\left(\frac{\pi}{6}\right) = \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\sqrt{3}}{3}$

b. $\sec(\pi) = \frac{1}{\cos(\pi)} = -1$

c. $\sec\left(\frac{3\pi}{4}\right) = \frac{1}{\cos\left(\frac{3\pi}{4}\right)} = -\sqrt{2}$

d. $\csc\left(\frac{\pi}{2}\right) = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = 1$

e. $\cot\left(\frac{\pi}{4}\right)=\frac{\cos\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)}=1$

f. $\tan\left(-\frac{\pi}{4}\right)=\frac{\sin\left(-\frac{\pi}{4}\right)}{\cos\left(-\frac{\pi}{4}\right)}=-1$

10. a. $\tan\left(\frac{\pi}{3}\right)=\frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)}=\sqrt{3}$

b. $\sec\left(\frac{\pi}{3}\right)=\frac{1}{\cos\left(\frac{\pi}{3}\right)}=2$

c. $\cot\left(\frac{\pi}{3}\right)=\frac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)}=\frac{\sqrt{3}}{3}$

d. $\csc\left(\frac{\pi}{4}\right)=\frac{1}{\sin\left(\frac{\pi}{4}\right)}=\sqrt{2}$

e. $\tan\left(-\frac{\pi}{6}\right)=\frac{\sin\left(-\frac{\pi}{6}\right)}{\cos\left(-\frac{\pi}{6}\right)}=-\frac{\sqrt{3}}{3}$

f. $\cos\left(-\frac{\pi}{3}\right)=\frac{1}{2}$

11. a. $(1+\sin z)(1-\sin z)=1-\sin^2 z$

$=\cos^2 z=\frac{1}{\sec^2 z}$

b. $(\sec t-1)(\sec t+1)=\sec^2 t-1=\tan^2 t$

c. $\sec t-\sin t\tan t=\frac{1}{\cos t}-\frac{\sin^2 t}{\cos t}$

$=\frac{1-\sin^2 t}{\cos t}=\frac{\cos^2 t}{\cos t}=\cos t$

d. $\frac{\sec^2 t-1}{\sec^2 t}=\frac{\tan^2 t}{\sec^2 t}=\frac{\frac{\sin^2 t}{\cos^2 t}}{\frac{1}{\cos^2 t}}=\sin^2 t$

12. a. $\sin^2 v+\frac{1}{\sec^2 v}=\sin^2 v+\cos^2 v=1$

b. $\cos 3t=\cos(2t+t)=\cos 2t\cos t-\sin 2t\sin t$

$=(2\cos^2 t-1)\cos t-2\sin^2 t\cos t$

$=2\cos^3 t-\cos t-2(1-\cos^2 t)\cos t$

$=2\cos^3 t-\cos t-2\cos t+2\cos^3 t$

$=4\cos^3 t-3\cos t$

c. $\sin 4x=\sin[2(2x)]=2\sin 2x\cos 2x$

$=2(2\sin x\cos x)(2\cos^2 x-1)$

$=2(4\sin x\cos^3 x-2\sin x\cos x)$

$=8\sin x\cos^3 x-4\sin x\cos x$

d. $(1+\cos\theta)(1-\cos\theta)=1-\cos^2\theta=\sin^2\theta$

13. a. $\frac{\sin u}{\csc u}+\frac{\cos u}{\sec u}=\sin^2 u+\cos^2 u=1$

b. $(1-\cos^2 x)(1+\cot^2 x)=(\sin^2 x)(\csc^2 x)$

$=\sin^2 x\left(\frac{1}{\sin^2 x}\right)=1$

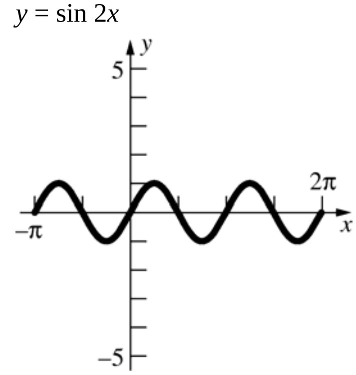
c. $\sin t(\csc t-\sin t)=\sin t\left(\frac{1}{\sin t}-\sin t\right)$

$=1-\sin^2 t=\cos^2 t$

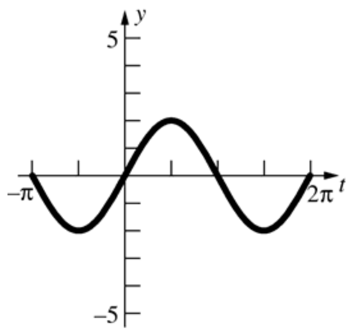
d. $\frac{1-\csc^2 t}{\csc^2 t}=-\frac{\cot^2 t}{\csc^2 t}=-\frac{\frac{\cos^2 t}{\sin^2 t}}{\frac{1}{\sin^2 t}}$

$=-\cos^2 t=-\frac{1}{\sec^2 t}$

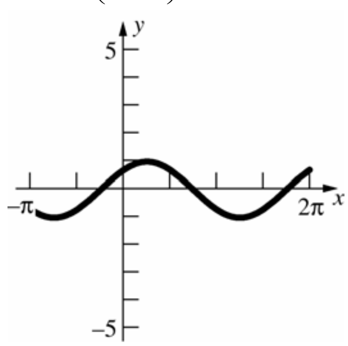
14. a. $y=\sin 2x$



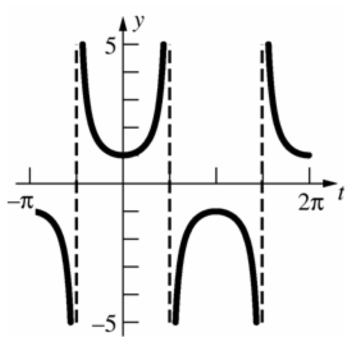
b. $y = 2\sin t$



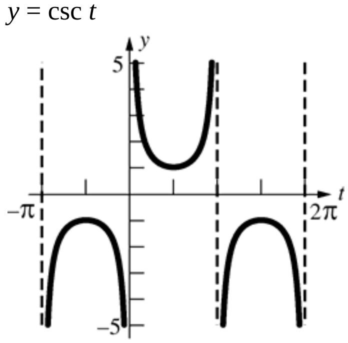
c. $y = \cos\left(x - \frac{\pi}{4}\right)$



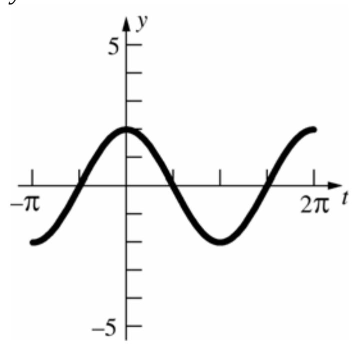
d. $y = \sec t$



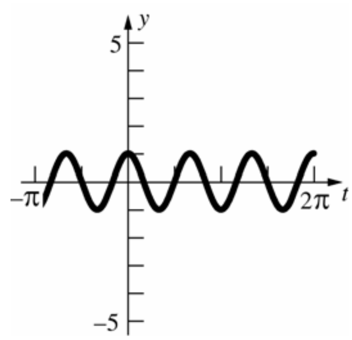
15. a. $y = \csc t$



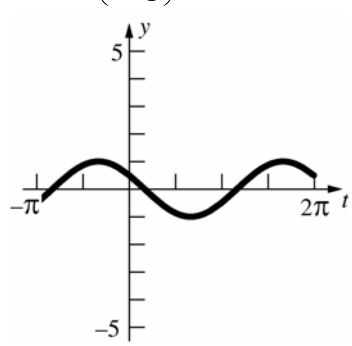
b. $y = 2\cos t$



c. $y = \cos 3t$

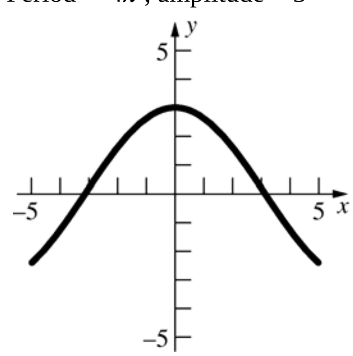


d. $y = \cos\left(t + \frac{\pi}{3}\right)$



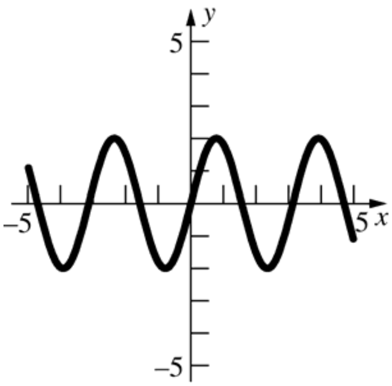
16. $y = 3\cos\frac{x}{2}$

Period = 4π, amplitude = 3



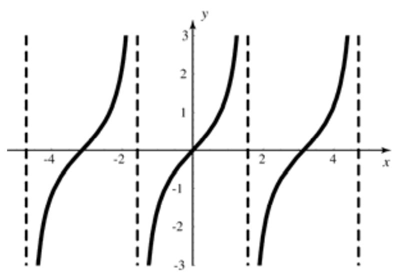
17. $y = 2 \sin 2x$

Period = π, amplitude = 2



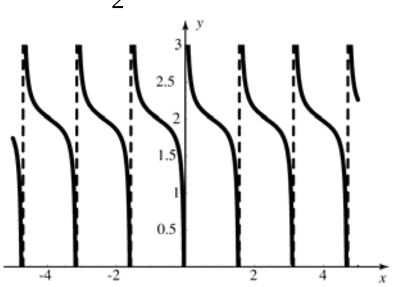
18. $y = \tan x$

Period = π



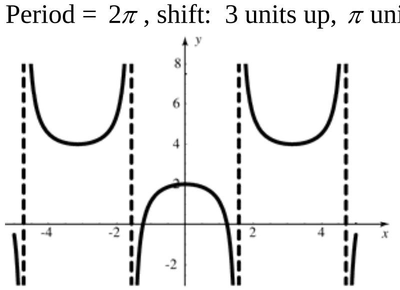
19. $y = 2 + \frac{1}{6}\cot(2x)$

Period = $\frac{\pi}{2}$, shift: 2 units up



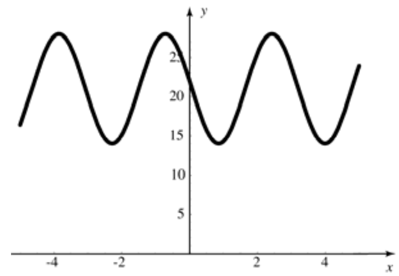
20. $y = 3 + \sec(x - \pi)$

Period = 2π, shift: 3 units up, π units right



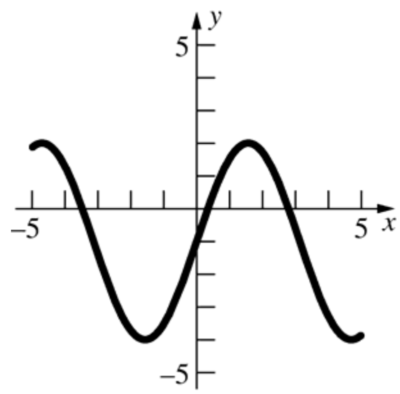
21. $y = 21 + 7\sin(2x + 3)$

Period = π, amplitude = 7, shift: 21 units up, $\frac{3}{2}$ units left



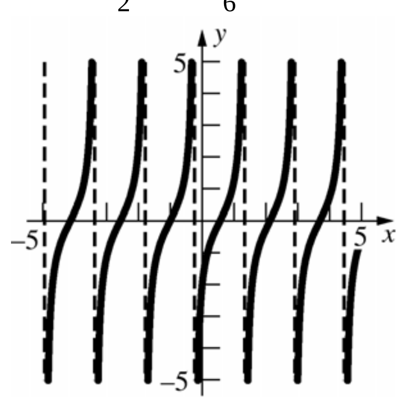
22. $y = 3\cos\left(x - \frac{\pi}{2}\right) - 1$

Period = 2π, amplitude = 3, shifts: $\frac{\pi}{2}$ units right and 1 unit down.



23. $y = \tan\left(2x - \frac{\pi}{3}\right)$

Period = $\frac{\pi}{2}$, shift: $\frac{\pi}{6}$ units right



24. **a.** and **g.**: $y = \sin\left(x + \frac{\pi}{2}\right) = \cos x = -\cos(\pi - x)$

b. and **e.**: $y = \cos\left(x + \frac{\pi}{2}\right) = \sin(x + \pi)$
$= -\sin(\pi - x)$

c. and **f.**: $y = \cos\left(x - \frac{\pi}{2}\right) = \sin x$
$= -\sin(x + \pi)$

d. and **h.**: $y = \sin\left(x - \frac{\pi}{2}\right) = \cos(x + \pi)$
$= \cos(x - \pi)$

25. **a.** $-t \sin(-t) = t \sin t$; even

b. $\sin^2(-t) = \sin^2 t$; even

c. $\csc(-t) = \frac{1}{\sin(-t)} = -\csc t$; odd

d. $|\sin(-t)| = |-\sin t| = |\sin t|$; even

e. $\sin(\cos(-t)) = \sin(\cos t)$; even

f. $-x + \sin(-x) = -x - \sin x = -(x + \sin x)$; odd

26. **a.** $\cot(-t) + \sin(-t) = -\cot t - \sin t$
$= -(\cot t + \sin t)$; odd

b. $\sin^3(-t) = -\sin^3 t$; odd

c. $\sec(-t) = \frac{1}{\cos(-t)} = \sec t$; even

d. $\sqrt{\sin^4(-t)} = \sqrt{\sin^4 t}$; even

e. $\cos(\sin(-t)) = \cos(-\sin t) = \cos(\sin t)$; even

f. $(-x)^2 + \sin(-x) = x^2 - \sin x$; neither

27. $\cos^2\frac{\pi}{3} = \left(\cos\frac{\pi}{3}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

28. $\sin^2\frac{\pi}{6} = \left(\sin\frac{\pi}{6}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

29. $\sin^3\frac{\pi}{6} = \left(\sin\frac{\pi}{6}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

30. $\cos^2\frac{\pi}{12} = \frac{1+\cos 2\left(\frac{\pi}{12}\right)}{2} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2}$
$= \frac{2+\sqrt{3}}{4}$

31. $\sin^2\frac{\pi}{8} = \frac{1-\cos 2\left(\frac{\pi}{8}\right)}{2} = \frac{1-\cos\frac{\pi}{4}}{2} = \frac{1-\frac{\sqrt{2}}{2}}{2}$
$= \frac{2-\sqrt{2}}{4}$

32. **a.** $\sin(x - y) = \sin x \cos(-y) + \cos x \sin(-y)$
$= \sin x \cos y - \cos x \sin y$

b. $\cos(x - y) = \cos x \cos(-y) - \sin x \sin(-y)$
$= \cos x \cos y + \sin x \sin y$

c. $\tan(x - y) = \frac{\tan x + \tan(-y)}{1 - \tan x \tan(-y)}$
$= \frac{\tan x - \tan y}{1 + \tan x \tan y}$

33. $\tan(t + \pi) = \frac{\tan t + \tan\pi}{1 - \tan t \tan\pi} = \frac{\tan t + 0}{1 - (\tan t)(0)}$
$= \tan t$

34. $\cos(x - \pi) = \cos x \cos(-\pi) - \sin x \sin(-\pi)$
$= -\cos x - 0 \cdot \sin x = -\cos x$

35. $s = rt = (2.5 \text{ ft})(2\pi \text{ rad}) = 5\pi$ ft, so the tire goes 5π feet per revolution, or $\frac{1}{5\pi}$ revolutions per foot.
$\left(\frac{1}{5\pi}\frac{\text{rev}}{\text{ft}}\right)\left(60\frac{\text{mi}}{\text{hr}}\right)\left(\frac{1}{60}\frac{\text{hr}}{\text{min}}\right)\left(5280\frac{\text{ft}}{\text{mi}}\right)$
≈ 336 rev/min

36. $s = rt = (2 \text{ ft})(150 \text{ rev})(2\pi \text{ rad/rev}) \approx 1885$ ft

37. $r_1 t_1 = r_2 t_2$; $6(2\pi)t_1 = 8(2\pi)(21)$
$t_1 = 28$ rev/sec

38. $\Delta y = \sin\alpha$ and $\Delta x = \cos\alpha$
$m = \frac{\Delta y}{\Delta x} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

39. a. $\tan\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3}$

b. $\sqrt{3}x + 3y = 6$

$3y = -\sqrt{3}x + 6$

$y = -\frac{\sqrt{3}}{3}x + 2;\ m = -\frac{\sqrt{3}}{3}$

$\tan\alpha = -\frac{\sqrt{3}}{3}$

$\alpha = \frac{5\pi}{6}$

40. $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$

$\tan\theta = \tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 + \tan(-\theta_1)}{1 - \tan\theta_2 \tan(-\theta_1)}$

$= \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1} = \frac{m_2 - m_1}{1 + m_1 m_2}$

41. a. $\tan\theta = \frac{3-2}{1+3(2)} = \frac{1}{7}$

$\theta \approx 0.1419$

b. $\tan\theta = \frac{-1-\frac{1}{2}}{1+\left(\frac{1}{2}\right)(-1)} = -3$

$\theta \approx 1.8925$

c. $2x - 6y = 12 \qquad 2x + y = 0$

$-6y = -2x + 12y = -2x$

$y = \frac{1}{3}x - 2$

$m_1 = \frac{1}{3},\ m_2 = -2$

$\tan\theta = \frac{-2-\frac{1}{3}}{1+\left(\frac{1}{3}\right)(-2)} = -7;\ \theta \approx 1.7127$

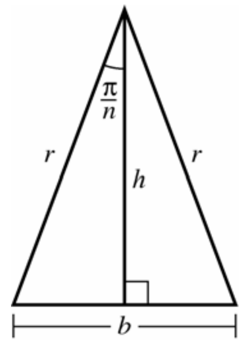
42. Recall that the area of the circle is πr^2. The measure of the vertex angle of the circle is 2π. Observe that the ratios of the vertex angles must equal the ratios of the areas. Thus,

$\frac{t}{2\pi} = \frac{A}{\pi r^2}$, so

$A = \frac{1}{2}r^2 t$.

43. $A = \frac{1}{2}(2)(5)^2 = 25\text{cm}^2$

44. Divide the polygon into n isosceles triangles by drawing lines from the center of the circle to the corners of the polygon. If the base of each triangle is on the perimeter of the polygon, then the angle opposite each base has measure $\frac{2\pi}{n}$. Bisect this angle to divide the triangle into two right triangles (See figure).



$\sin\frac{\pi}{n} = \frac{b}{2r}$ so $b = 2r\sin\frac{\pi}{n}$ and $\cos\frac{\pi}{n} = \frac{h}{r}$ so $h = r\cos\frac{\pi}{n}$.

$P = nb = 2rn\sin\frac{\pi}{n}$

$A = n\left(\frac{1}{2}bh\right) = nr^2\cos\frac{\pi}{n}\sin\frac{\pi}{n}$

45. The base of the triangle is the side opposite the angle t. Then the base has length $2r\sin\frac{t}{2}$ (similar to Problem 44). The radius of the semicircle is $r\sin\frac{t}{2}$ and the height of the triangle is $r\cos\frac{t}{2}$.

$A = \frac{1}{2}\left(2r\sin\frac{t}{2}\right)\left(r\cos\frac{t}{2}\right) + \frac{\pi}{2}\left(r\sin\frac{t}{2}\right)^2$

$= r^2\sin\frac{t}{2}\cos\frac{t}{2} + \frac{\pi r^2}{2}\sin^2\frac{t}{2}$

46. $\cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}\cos\frac{x}{16}$

$= \frac{1}{2}\left[\cos\frac{3}{4}x + \cos\frac{1}{4}x\right]\frac{1}{2}\left[\cos\frac{3}{16}x + \cos\frac{1}{16}x\right]$

$= \frac{1}{4}\left[\cos\frac{3}{4}x + \cos\frac{1}{4}x\right]\left[\cos\frac{3}{16}x + \cos\frac{1}{16}x\right]$

$= \frac{1}{4}\left[\cos\frac{3}{4}x\cos\frac{3}{16}x + \cos\frac{3}{4}x\cos\frac{1}{16}x\right.$

$\left. + \cos\frac{1}{4}x\cos\frac{3}{16}x + \cos\frac{1}{4}x\cos\frac{1}{16}x\right]$

$= \frac{1}{4}\left[\frac{1}{2}\left(\cos\frac{15}{16} + \cos\frac{9}{16}x\right) + \frac{1}{2}\left(\cos\frac{13}{16}x + \cos\frac{11}{16}x\right)\right.$

$\left. + \frac{1}{2}\left(\cos\frac{7}{16}x + \cos\frac{1}{16}x\right) + \frac{1}{2}\left(\cos\frac{5}{16}x + \cos\frac{3}{16}x\right)\right]$

$= \frac{1}{8}\left[\cos\frac{15}{16}x + \cos\frac{13}{16}x + \cos\frac{11}{16}x + \cos\frac{9}{16}x\right.$

$\left. + \cos\frac{7}{16}x + \cos\frac{5}{16}x + \cos\frac{3}{16}x + \cos\frac{1}{16}x\right]$

47. The temperature function is
$T(t) = 80 + 25\sin\left(\frac{2\pi}{12}\left(t - \frac{7}{2}\right)\right)$.
The normal high temperature for November 15th is then $T(10.5) = 67.5$ °F.

48. The water level function is
$F(t) = 8.5 + 3.5\sin\left(\frac{2\pi}{12}(t-9)\right)$.
The water level at 5:30 P.M. is then $F(17.5) \approx 5.12$ ft .

49. As t increases, the point on the rim of the wheel will move around the circle of radius 2.

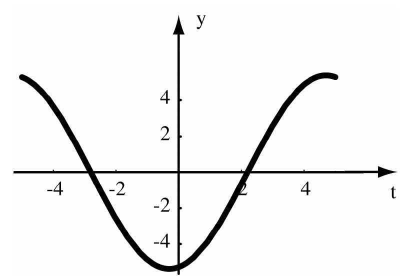
a. $x(2) \approx 1.902$
$y(2) \approx 0.618$
$x(6) \approx -1.176$
$y(6) \approx -1.618$
$x(10) = 0$
$y(10) = 2$
$x(0) = 0$
$y(0) = 2$

b. $x(t) = -2\sin\left(\frac{\pi}{5}t\right), y(t) = 2\cos\left(\frac{\pi}{5}t\right)$

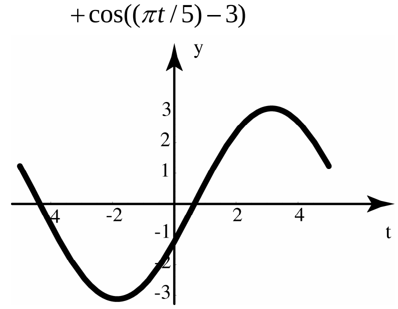
c. The point is at (2, 0) when $\frac{\pi}{5}t = \frac{\pi}{2}$; that is , when $t = \frac{5}{2}$.

50. Both functions have frequency $\frac{2\pi}{10}$. When you add functions that have the same frequency, the sum has the same frequency.

a. $y(t) = 3\sin(\pi t/5) - 5\cos(\pi t/5)$
$+2\sin((\pi t/5) - 3)$



b. $y(t) = 3\cos(\pi t/5 - 2) + \cos(\pi t/5)$
$+\cos((\pi t/5) - 3)$



51. **a.** $C\sin(\omega t+\phi)=(C\cos\phi)\sin\omega t+(C\sin\phi)\cos\omega t$. Thus $A=C\cdot\cos\phi$ and $B=C\cdot\sin\phi$.

b. $A^2+B^2=(C\cos\phi)^2+(C\sin\phi)^2=C^2(\cos^2\phi)+C^2(\sin^2\phi)=C^2$

Also, $\dfrac{B}{A}=\dfrac{C\cdot\sin\phi}{C\cdot\cos\phi}=\tan\phi$

c. $A_1\sin(\omega t+\phi_1)+A_2\sin(\omega t+\phi_2)+A_3(\sin\omega t+\phi_3)$

$=A_1(\sin\omega t\cos\phi_1+\cos\omega t\sin\phi_1)$

$+A_2(\sin\omega t\cos\phi_2+\cos\omega t\sin\phi_2)$

$+A_3(\sin\omega t\cos\phi_3+\cos\omega t\sin\phi_3)$

$=(A_1\cos\phi_1+A_2\cos\phi_2+A_3\cos\phi_3)\sin\omega t$

$+(A_1\sin\phi_1+A_2\sin\phi_2+A_3\sin\phi_3)\cos\omega t$

$=C\sin\left(\omega t+\phi\right)$

where C and ϕ can be computed from

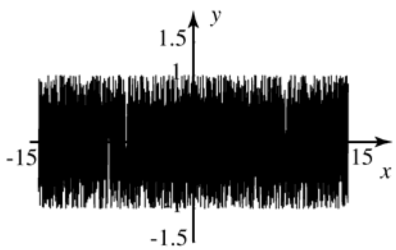
$A=A_1\cos\phi_1+A_2\cos\phi_2+A_3\cos\phi_3$

$B=A_1\sin\phi_1+A_2\sin\phi_2+A_3\sin\phi_3$

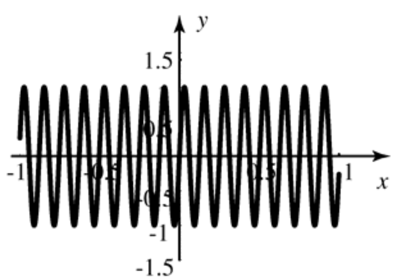
as in part (b).

d. Written response. Answers will vary.

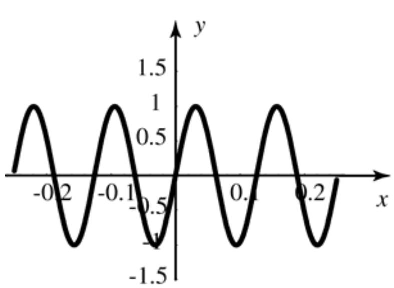
52. **(a.), (b.),** and **(c.)** all look similar to this:



d.

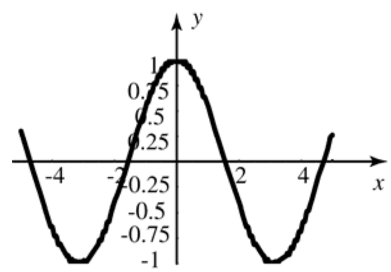


e.

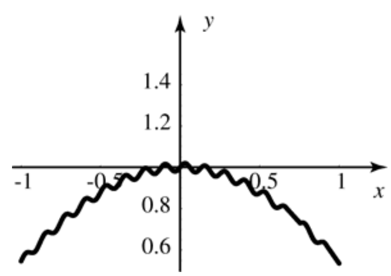


The windows in **(a)-(c)** are not helpful because the function oscillates too much over the domain plotted. Plots in **(d)** or **(e)** show the behavior of the function.

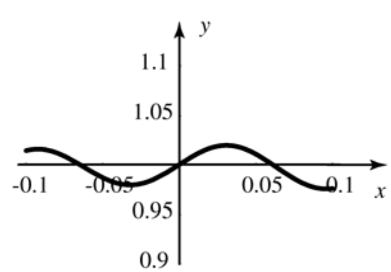
53. **a.**



b.



c.



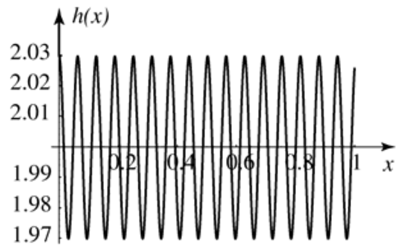
The plot in **(a)** shows the long term behavior of the function, but not the short term behavior, whereas the plot in **(c)** shows the short term behavior, but not the long term behavior. The plot in **(b)** shows a little of each.

54. a. $h(x) = (f \circ g)(x)$

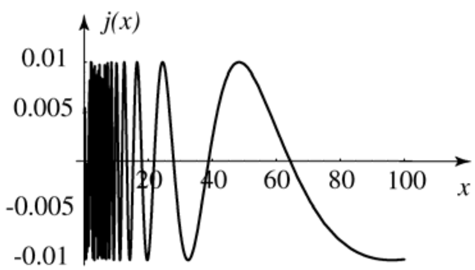
$$= \frac{\frac{3}{100}\cos(100x) + 2}{\left(\frac{1}{100}\right)^2 \cos^2(100x) + 1}$$

$$j(x) = (g \circ f)(x) = \frac{1}{100}\cos\left(100\frac{3x+2}{x^2+1}\right)$$

b.

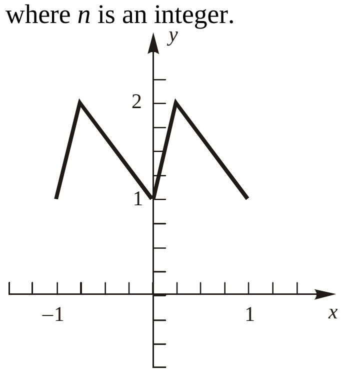


c.



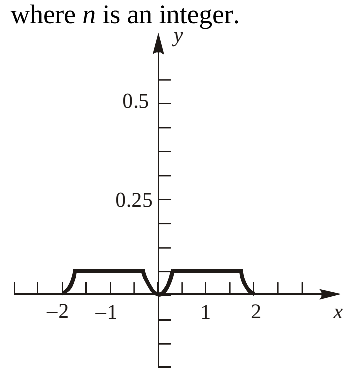
55. $f(x) = \begin{cases} 4(x - [\![x]\!]) + 1 & : x \in \left[n, n + \frac{1}{4}\right) \\ -\frac{4}{3}(x - [\![x]\!]) + \frac{7}{3} & : x \in \left[n + \frac{1}{4}, n+1\right) \end{cases}$

where n is an integer.



56. $f(x) = \begin{cases} (x - 2n)^2, & x \in \left[2n - \frac{1}{4}, 2n + \frac{1}{4}\right] \\ 0.0625, & \text{otherwise} \end{cases}$

where n is an integer.



0.8 Chapter Review

Concepts Test

1. False: p and q must be integers.

2. True: $\frac{p_1}{q_1} - \frac{p_2}{q_2} = \frac{p_1q_2 - p_2q_1}{q_1q_2}$; since $p_1, q_1, p_2,$ and q_2 are integers, so are $p_1q_2 - p_2q_1$ and q_1q_2.

3. False: If the numbers are opposites ($-\pi$ and π) then the sum is 0, which is rational.

4. True: Between any two distinct real numbers there are both a rational and an irrational number.

5. False: 0.999... is equal to 1.

6. True: $(a^m)^n = (a^n)^m = a^{mn}$

7. False: $(a*b)*c = a^{bc}; a*(b*c) = a^{b^c}$

8. True: Since $x \le y \le z$ and $x \ge z, x = y = z$

9. True: If x was not 0, then $\varepsilon = \frac{|x|}{2}$ would be a positive number less than $|x|$.

10. True: $y-x=-(x-y)$ so
$(x-y)(y-x)=(x-y)(-1)(x-y)$
$=(-1)(x-y)^2.$
$(x-y)^2 \geq 0$ for all x and y, so
$-(x-y)^2 \leq 0.$

11. True: $a<b<0$; $a<b$; $\frac{a}{b}>1; \frac{1}{b}<\frac{1}{a}$

12. True: $[a,b]$ and $[b,c]$ share point b in common.

13. True: If (a, b) and (c, d) share a point then $c<b$ so they share the infinitely many points between b and c.

14. True: $\sqrt{x^2}=|x|=-x$ if $x<0$.

15. False: For example, if $x=-3$, then $|-x|=|-(-3)|=|3|=3$ which does not equal x.

16. False: For example, take $x=1$ and $y=-2$.

17. True: $|x|<|y| \Leftrightarrow |x|^4<|y|^4$
$|x|^4=x^4$ and $|y|^4=y^4$, so $x^4<y^4$

18. True: $|x+y|=-(x+y)$
$=-x+(-y)=|x|+|y|$

19. True: If $r=0$, then
$\frac{1}{1+|r|}=\frac{1}{1-r}=\frac{1}{1-|r|}=1.$
For any r, $1+|r| \geq 1-|r|$. Since
$|r|<1, 1-|r|>0$ so $\frac{1}{1+|r|} \leq \frac{1}{1-|r|}$;
also, $-1<r<1$.

If $-1<r<0$, then $|r|=-r$ and
$1-r=1+|r|$, so
$\frac{1}{1+|r|}=\frac{1}{1-r} \leq \frac{1}{1-|r|}.$
If $0<r<1$, then $|r|=r$ and
$1-r=1-|r|$, so
$\frac{1}{1+|r|} \leq \frac{1}{1-r}=\frac{1}{1-|r|}.$

20. True: If $|r|>1$, then $1-|r|<0$. Thus,
since $1+|r| \geq 1-|r|$, $\frac{1}{1-|r|} \leq \frac{1}{1+|r|}$.
If $r>1$, $|r|=r$, and $1-r=1-|r|$, so
$\frac{1}{1-|r|}=\frac{1}{1-r} \leq \frac{1}{1+|r|}.$
If $r<-1, |r|=-r$ and $1-r=1+|r|$,
so $\frac{1}{1-|r|} \leq \frac{1}{1-r}=\frac{1}{1+|r|}.$

21. True: If x and y are the same sign, then
$||x|-|y||=|x-y|$. $|x-y| \leq |x+y|$
when x and y are the same sign, so
$||x|-|y|| \leq |x+y|$. If x and y have opposite signs then either
$||x|-|y||=|x-(-y)|=|x+y|$
$(x>0, y<0)$ or
$||x|-|y||=|-x-y|=|x+y|$
$(x<0, y>0)$. In either case
$||x|-|y||=|x+y|$.
If either $x=0$ or $y=0$, the inequality is easily seen to be true.

22. True: If y is positive, then $x=\sqrt{y}$ satisfies
$x^2=\left(\sqrt{y}\right)^2=y.$

23. True: For every real number y, whether it is positive, zero, or negative, the cube root $x=\sqrt[3]{y}$ satisfies
$x^3=\left(\sqrt[3]{y}\right)^3=y$

24. True: For example $x^2 \leq 0$ has solution [0].

25. True: $x^2+ax+y^2+y=0$
$x^2+ax+\frac{a^2}{4}+y^2+y+\frac{1}{4}=\frac{a^2}{4}+\frac{1}{4}$
$\left(x+\frac{a}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{a^2+1}{4}$
is a circle for all values of a.

26. False: If $a=b=0$ and $c<0$, the equation does not represent a circle.

27. True; $y - b = \frac{3}{4}(x - a)$

$y = \frac{3}{4}x - \frac{3a}{4} + b;$

If $x = a + 4$:

$y = \frac{3}{4}(a + 4) - \frac{3a}{4} + b$

$= \frac{3a}{4} + 3 - \frac{3a}{4} + b = b + 3$

28. True: If the points are on the same line, they have equal slope. Then the reciprocals of the slopes are also equal.

29. True: If $ab > 0$, a and b have the same sign, so (a, b) is in either the first or third quadrant.

30. True: Let $x = \varepsilon / 2$. If $\varepsilon > 0$, then $x > 0$ and $x < \varepsilon$.

31. True: If $ab = 0$, a or b is 0, so (a, b) lies on the x-axis or the y-axis. If $a = b = 0$, (a, b) is the origin.

32. True: $y_1 = y_2$, so (x_1, y_1) and (x_2, y_2) are on the same horizontal line.

33. True: $d = \sqrt{[(a+b) - (a-b)]^2 + (a-a)^2}$

$= \sqrt{(2b)^2} = |2b|$

34. False: The equation of a vertical line cannot be written in point-slope form.

35. True: This is the general linear equation.

36. True: Two non-vertical lines are parallel if and only if they have the same slope.

37. False: The slopes of perpendicular lines are negative reciprocals.

38. True: If a and b are rational and $(a, 0), (0, b)$ are the intercepts, the slope is $-\frac{b}{a}$ which is rational.

39. False: $ax + y = c \Rightarrow y = -ax + c$

$ax - y = c \Rightarrow y = ax - c$

$(a)(-a) \neq -1.$

(unless $a = \pm 1$)

40. True: The equation is $(3 + 2m)x + (6m - 2)y + 4 - 2m = 0$ which is the equation of a straight line unless $3 + 2m$ and $6m - 2$ are both 0, and there is no real number m such that $3 + 2m = 0$ and $6m - 2 = 0$.

41. True: $f(x) = \sqrt{-(x^2 + 4x + 3)}$

$= \sqrt{-(x+3)(x+1)}$

$-(x^2 + 4x + 3) \geq 0$ on $-3 \leq x \leq -1$.

42. False: The domain does not include $n\pi + \frac{\pi}{2}$ where n is an integer.

43. True: The domain is $(-\infty, \infty)$ and the range is $[-6, \infty)$.

44. False: The range is $(-\infty, \infty)$.

45. False: The range $(-\infty, \infty)$.

46. True: If $f(x)$ and $g(x)$ are even functions, $f(x) + g(x)$ is even.

$f(-x) + g(-x) = f(x) + g(x)$

47. True: If $f(x)$ and $g(x)$ are odd functions,

$f(-x) + g(-x) = -f(x) - g(x)$

$= -[f(x) + g(x)]$, so $f(x) + g(x)$ is odd

48. False: If $f(x)$ and $g(x)$ are odd functions,

$f(-x)g(-x) = -f(x)[-g(x)] = f(x)g(x)$, so

$f(x)g(x)$ is even.

49. True: If $f(x)$ is even and $g(x)$ is odd,

$f(-x)g(-x) = f(x)[-g(x)]$

$= -f(x)g(x)$, so $f(x)g(x)$ is odd.

50. False: If $f(x)$ is even and $g(x)$ is odd, $f(g(-x)) = f(-g(x)) = f(g(x))$; while if $f(x)$ is odd and $g(x)$ is even, $f(g(-x)) = f(g(x))$; so $f(g(x))$ is even.

51. False: If $f(x)$ and $g(x)$ are odd functions, $f(g(-x)) = f(-g(x)) = -f(g(x))$, so $f(g(x))$ is odd.

52. True: $f(-x) = \frac{2(-x)^3 + (-x)}{(-x)^2 + 1} = \frac{-2x^3 - x}{x^2 + 1}$

$= -\frac{2x^3 + x}{x^2 + 1}$

53. True: $f(-t)=\frac{(\sin(-t))^2+\cos(-t)}{\tan(-t)\csc(-t)}$

$=\frac{(-\sin t)^2+\cos t}{-\tan t(-\csc t)}=\frac{(\sin t)^2+\cos t}{\tan t\csc t}$

54. False: $f(x) = c$ has domain $(-\infty, \infty)$ and the only value of the range is c.

55. False: $f(x) = c$ has domain $(-\infty, \infty)$, yet the range has only one value, c.

56. True: $g(-1.8)=\left[\!\left[\frac{-1.8}{2}\right]\!\right]=[\![-0.9]\!]=-1$

57. True: $(f\circ g)(x)=(x^3)^2=x^6$

$(g\circ f)(x)=(x^2)^3=x^6$

58. False: $(f\circ g)(x)=(x^3)^2=x^6$

$f(x)\cdot g(x)=x^2x^3=x^5$

59. False: The domain of $\frac{f}{g}$ excludes any values where $g = 0$.

60. True: $f(a) = 0$
Let $F(x) = f(x + h)$, then
$F(a - h) = f(a - h + h) = f(a) = 0$

61. True: $\cot x=\frac{\cos x}{\sin x}$

$\cot(-x)=\frac{\cos(-x)}{\sin(-x)}$

$=\frac{\cos x}{-\sin x}=-\cot x$

62. False: The domain of the tangent function excludes all $n\pi+\frac{\pi}{2}$ where n is an integer.

63. False: The cosine function is periodic, so $\cos s = \cos t$ does not necessarily imply $s = t$; e.g., $\cos 0=\cos\ 2\pi=1$, but $0\neq 2\pi$.

Sample Test Problems

1. a. $\left(n+\frac{1}{n}\right)^n;\left(1+\frac{1}{1}\right)^1=2;\left(2+\frac{1}{2}\right)^2=\frac{25}{4};$

$\left(-2+\frac{1}{-2}\right)^{-2}=\frac{4}{25}$

b. $(n^2-n+1)^2;\left[(1)^2-(1)+1\right]^2=1;$

$\left[(2)^2-(2)+1\right]^2=9;$

$\left[(-2)^2-(-2)+1\right]^2=49$

c. $4^{3/n};4^{3/1}=64;4^{3/2}=8;4^{-3/2}=\frac{1}{8}$

d. $\sqrt[n]{\left|\frac{1}{n}\right|};\sqrt[1]{\left|\frac{1}{1}\right|}=1;\sqrt{\left|\frac{1}{2}\right|}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2};$

$\sqrt[-2]{\left|\frac{1}{-2}\right|}=\sqrt{2}$

2. a. $\left(1+\frac{1}{m}+\frac{1}{n}\right)\left(1-\frac{1}{m}+\frac{1}{n}\right)^{-1}=\frac{1+\frac{1}{m}+\frac{1}{n}}{1-\frac{1}{m}+\frac{1}{n}}$

$=\frac{mn+n+m}{mn-n+m}$

b. $\frac{\frac{2}{x+1}-\frac{x}{x^2-x-2}}{\frac{3}{x+1}-\frac{2}{x-2}}=\frac{\frac{2}{x+1}-\frac{x}{(x-2)(x+1)}}{\frac{3}{x+1}-\frac{2}{x-2}}$

$=\frac{2(x-2)-x}{3(x-2)-2(x+1)}$

$=\frac{x-4}{x-8}$

c. $\frac{(t^3-1)}{t-1}=\frac{(t-1)(t^2+t+1)}{t-1}=t^2+t+1$

3. Let a, b, c, and d be integers.

$\frac{\frac{a}{b}+\frac{c}{d}}{2}=\frac{a}{2b}+\frac{c}{2d}=\frac{ad+bc}{2bd}$ which is rational.

4. $x = 4.1282828\ldots$

$$1000x = 4128.282828\ldots$$
$$10x = 41.282828\ldots$$
$$990x = 4087$$
$$x = \frac{4087}{990}$$

5. Answers will vary. Possible answer:

$\sqrt{\frac{13}{50}} \approx 0.50990...$

6. $\dfrac{\left(\sqrt[3]{8.15 \times 10^4} - 1.32\right)^2}{3.24} \approx 545.39$

7. $\left(\pi - \sqrt{2.0}\right)^{2.5} - \sqrt[3]{2.0} \approx 2.66$

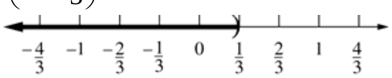
8. $\sin^2(2.45) + \cos^2(2.40) - 1.00 \approx -0.0495$

9. $1 - 3x > 0$

$3x < 1$

$x < \frac{1}{3}$

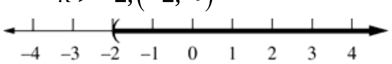
$\left(-\infty, \frac{1}{3}\right)$



10. $6x + 3 > 2x - 5$

$4x > -8$

$x > -2; (-2, \infty)$

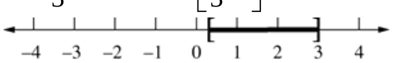


11. $3 - 2x \le 4x + 1 \le 2x + 7$

$3 - 2x \le 4x + 1$ and $4x + 1 \le 2x + 7$

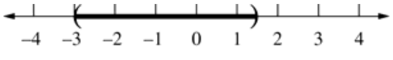
$6x \ge 2$ and $2x \ge 6$

$x \ge \frac{1}{3}$ and $x \le 3$; $\left[\frac{1}{3}, 3\right]$



12. $2x^2 + 5x - 3 < 0; (2x - 1)(x + 3) < 0;$

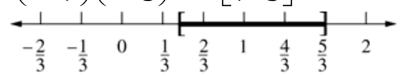
$-3 < x < \frac{1}{2}; \left(-3, \frac{1}{2}\right)$



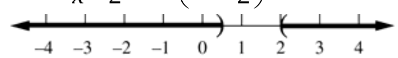
13. $21t^2 - 44t + 12 \le -3; 21t^2 - 44t + 15 \le 0;$

$$t = \frac{44 \pm \sqrt{44^2 - 4(21)(15)}}{2(21)} = \frac{44 \pm 26}{42} = \frac{3}{7}, \frac{5}{3}$$

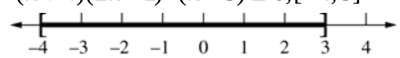
$\left(t - \frac{3}{7}\right)\left(t - \frac{5}{3}\right) \le 0; \left[\frac{3}{7}, \frac{5}{3}\right]$



14. $\frac{2x - 1}{x - 2} > 0; \left(-\infty, \frac{1}{2}\right) \cup (2, \infty)$

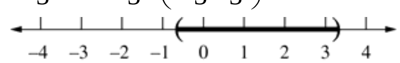


15. $(x + 4)(2x - 1)^2(x - 3) \le 0; [-4, 3]$



16. $|3x - 4| < 6; -6 < 3x - 4 < 6; -2 < 3x < 10;$

$-\frac{2}{3} < x < \frac{10}{3}; \left(-\frac{2}{3}, \frac{10}{3}\right)$



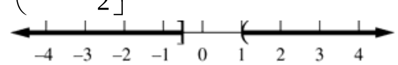
17. $\frac{3}{1 - x} \le 2$

$\frac{3}{1 - x} - 2 \le 0$

$\frac{3 - 2(1 - x)}{1 - x} \le 0$

$\frac{2x + 1}{1 - x} \le 0;$

$\left(-\infty, -\frac{1}{2}\right] \cup (1, \infty)$



18. $|12 - 3x| \ge |x|$

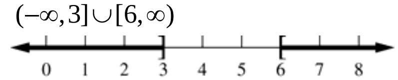
$(12 - 3x)^2 \ge x^2$

$144 - 72x + 9x^2 \ge x^2$

$8x^2 - 72x + 144 \ge 0$

$8(x - 3)(x - 6) \ge 0$

$(-\infty, 3] \cup [6, \infty)$



19. For example, if $x = -2$, $|-(-2)| = 2 \ne -2$

$|-x| \ne x$ for any $x < 0$

20. If $|-x| = x$, then $|x| = x$.

$x \ge 0$

21. $|t-5| = |-(5-t)| = |5-t|$
If $|5-t| = 5-t$, then $5-t \ge 0$.
$t \le 5$

22. $|t-a| = |-(a-t)| = |a-t|$
If $|a-t| = a-t$, then $a-t \ge 0$.
$t \le a$

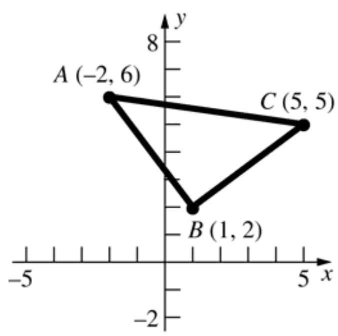
23. If $|x| \le 2$, then
$0 \le |2x^2+3x+2| \le |2x^2| + |3x| + 2 \le 8+6+2 = 16$
also $|x^2+2| \ge 2$ so $\frac{1}{|x^2+2|} \le \frac{1}{2}$. Thus
$$\left|\frac{2x^2+3x+2}{x^2+2}\right| = |2x^2+3x+2|\left|\frac{1}{x^2+2}\right| \le 16\left(\frac{1}{2}\right) = 8$$

24. **a.** The distance between x and 5 is 3.

b. The distance between x and –1 is less than or equal to 2.

c. The distance between x and a is greater than b.

25.



$d(A, B) = \sqrt{(1+2)^2 + (2-6)^2} = \sqrt{9+16} = 5$
$d(B, C) = \sqrt{(5-1)^2 + (5-2)^2} = \sqrt{16+9} = 5$
$d(A, C) = \sqrt{(5+2)^2 + (5-6)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$
$(AB)^2 + (BC)^2 = (AC)^2$, so ΔABC is a right triangle.

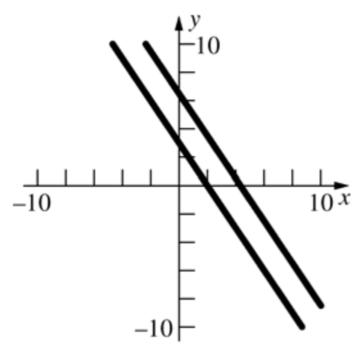
26. midpoint: $\left(\frac{1+7}{2}, \frac{2+8}{2}\right) = (4, 5)$
$d = \sqrt{(4-3)^2 + (5+6)^2} = \sqrt{1+121} = \sqrt{122}$

27. center $= \left(\frac{2+10}{2}, \frac{0+4}{2}\right) = (6, 2)$
radius $= \frac{1}{2}\sqrt{(10-2)^2 + (4-0)^2} = \frac{1}{2}\sqrt{64+16} = 2\sqrt{5}$
circle: $(x-6)^2 + (y-2)^2 = 20$

28. $x^2 + y^2 - 8x + 6y = 0$
$x^2 - 8x + 16 + y^2 + 6y + 9 = 16 + 9$
$(x-4)^2 + (y+3)^2 = 25$;
center $= (4, -3)$, radius = 5

29. $x^2 - 2x + y^2 + 2y = 2$
$x^2 - 2x + 1 + y^2 + 2y + 1 = 2 + 1 + 1$
$(x-1)^2 + (y+1)^2 = 4$
center = (1, –1)
$x^2 + 6x + y^2 - 4y = -7$
$x^2 + 6x + 9 + y^2 - 4y + 4 = -7 + 9 + 4$
$(x+3)^2 + (y-2)^2 = 6$
center = (–3, 2)
$d = \sqrt{(-3-1)^2 + (2+1)^2} = \sqrt{16+9} = 5$

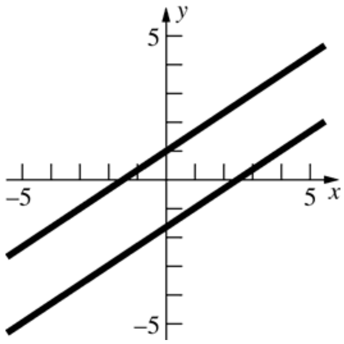
30. **a.** $3x + 2y = 6$
$2y = -3x + 6$
$y = -\frac{3}{2}x + 3$
$m = -\frac{3}{2}$
$y - 2 = -\frac{3}{2}(x-3)$
$y = -\frac{3}{2}x + \frac{13}{2}$



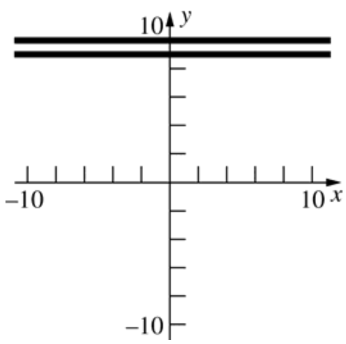
b. $m = \frac{2}{3};$

$$y + 1 = \frac{2}{3}(x - 1)$$

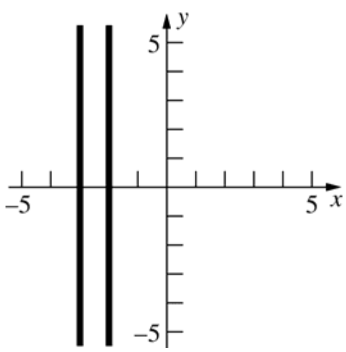
$$y = \frac{2}{3}x - \frac{5}{3}$$



c. $y = 9$



d. $x = –3$



31. a. $m = \frac{3 - 1}{7 + 2} = \frac{2}{9};$

$$y - 1 = \frac{2}{9}(x + 2)$$

$$y = \frac{2}{9}x + \frac{13}{9}$$

b. $3x - 2y = 5$

$$-2y = -3x + 5$$

$$y = \frac{3}{2}x - \frac{5}{2};$$

$$m = \frac{3}{2}$$

$$y - 1 = \frac{3}{2}(x + 2)$$

$$y = \frac{3}{2}x + 4$$

c. $3x + 4y = 9$

$4y = –3x + 9;$

$$y = -\frac{3}{4}x + \frac{9}{4}; \quad m = \frac{4}{3}$$

$$y - 1 = \frac{4}{3}(x + 2)$$

$$y = \frac{4}{3}x + \frac{11}{3}$$

d. $x = –2$

e. contains (–2, 1) and (0, 3); $m = \frac{3 - 1}{0 + 2};$

$y = x + 3$

32. $m_1 = \frac{3 + 1}{5 - 2} = \frac{4}{3}; m_2 = \frac{11 - 3}{11 - 5} = \frac{8}{6} = \frac{4}{3};$

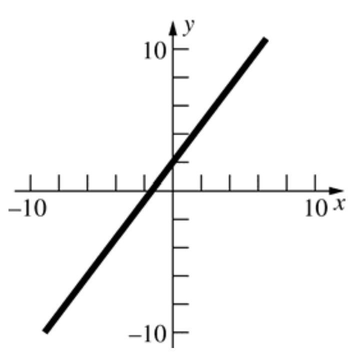
$$m_3 = \frac{11 + 1}{11 - 2} = \frac{12}{9} = \frac{4}{3}$$

$m_1 = m_2 = m_3$, so the points lie on the same line.

33. The figure is a cubic with respect to *y*. The equation is **(b)** $x = y^3$.

34. The figure is a quadratic, opening downward, with a negative *y*-intercept. The equation is **(c)** $y = ax^2 + bx + c$. with $a < 0$, $b > 0$, and $c < 0$.

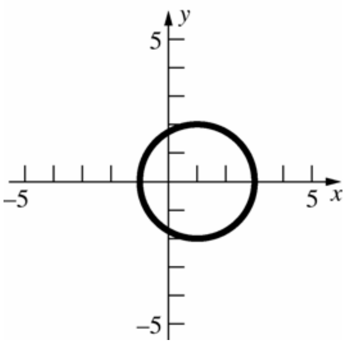
35.



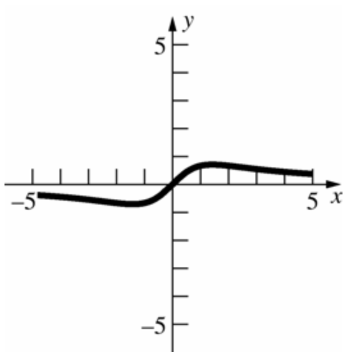
36. $x^2 - 2x + y^2 = 3$

$x^2 - 2x + 1 + y^2 = 4$

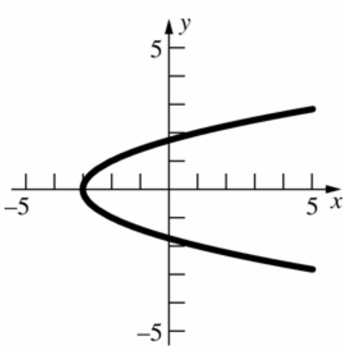
$(x-1)^2 + y^2 = 4$



37.



38.



39. $y = x^2 - 2x + 4$ and $y - x = 4$;

$x + 4 = x^2 - 2x + 4$

$x^2 - 3x = 0$

$x(x-3) = 0$

points of intersection: (0, 4) and (3, 7)

40. $4x - y = 2$

$y = 4x - 2;$

$m = -\frac{1}{4}$

contains $(a,0),(0,b)$;

$\frac{ab}{2} = 8$

$ab = 16$

$b = \frac{16}{a}$

$\frac{b-0}{0-a} = -\frac{b}{a} = -\frac{1}{4};$

$a = 4b$

$a = 4\left(\frac{16}{a}\right)$

$a^2 = 64$

$a = 8$

$b = \frac{16}{8} = 2;\ y = -\frac{1}{4}x + 2$

41. a. $f(1) = \frac{1}{1+1} - \frac{1}{1} = -\frac{1}{2}$

b. $f\left(-\frac{1}{2}\right) = \frac{1}{-\frac{1}{2}+1} - \frac{1}{-\frac{1}{2}} = 4$

c. $f(-1)$ does not exist.

d. $f(t-1) = \frac{1}{t-1+1} - \frac{1}{t-1} = \frac{1}{t} - \frac{1}{t-1}$

e. $f\left(\frac{1}{t}\right) = \frac{1}{\frac{1}{t}+1} - \frac{1}{\frac{1}{t}} = \frac{t}{1+t} - t$

42. a. $g(2) = \frac{2+1}{2} = \frac{3}{2}$

b. $g\left(\frac{1}{2}\right) = \frac{\frac{1}{2}+1}{\frac{1}{2}} = 3$

c. $\frac{g(2+h)-g(2)}{h} = \frac{\frac{2+h+1}{2+h} - \frac{2+1}{2}}{h}$

$= \frac{\frac{2h+6-3h-6}{2(h+2)}}{h} = \frac{-\frac{h}{2(h+2)}}{h} = \frac{-1}{2(h+2)}$

43. a. $\{x \in \mathbb{R} : x \neq -1, 1\}$

b. $\{x \in \mathbb{R} : |x| \leq 2\}$

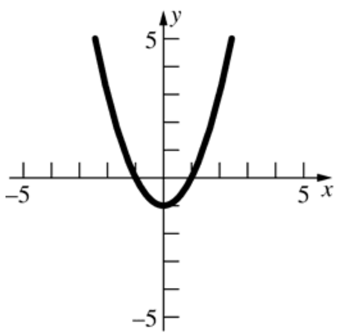
44. a. $f(-x) = \frac{3(-x)}{(-x)^2+1} = -\frac{3x}{x^2+1}$; odd

b. $g(-x) = |\sin(-x)| + \cos(-x)$
$= |-\sin x| + \cos x = |\sin x| + \cos x$; even

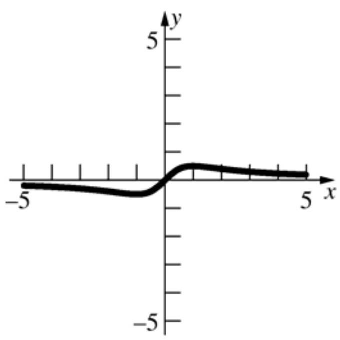
c. $h(-x) = (-x)^3 + \sin(-x) = -x^3 - \sin x$; odd

d. $k(-x) = \frac{(-x)^2+1}{|-x|+(-x)^4} = \frac{x^2+1}{|x|+x^4}$; even

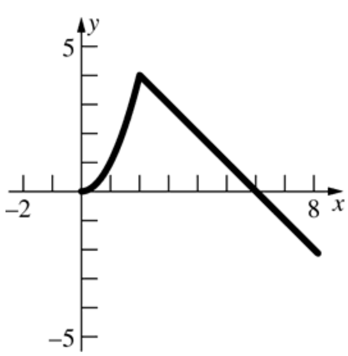
45. a. $f(x) = x^2 - 1$



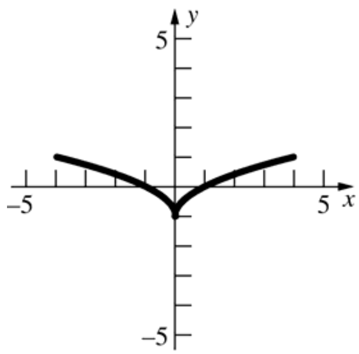
b. $g(x) = \frac{x}{x^2+1}$



c. $h(x) = \begin{cases} x^2 & \text{if } 0 \le x \le 2 \\ 6 - x & \text{if } x > 2 \end{cases}$



46.



47. $V(x) = x(32 - 2x)(24 - 2x)$
Domain [0, 12]

48. a. $(f+g)(2) = \left(2 - \frac{1}{2}\right) + (2^2 + 1) = \frac{13}{2}$

b. $(f \cdot g)(2) = \left(\frac{3}{2}\right)(5) = \frac{15}{2}$

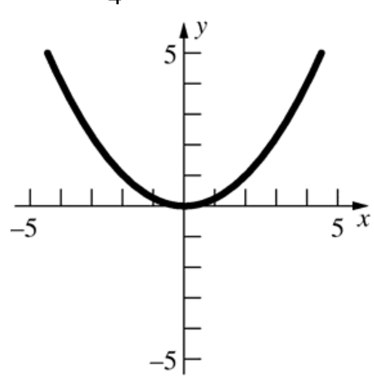
c. $(f \circ g)(2) = f(5) = 5 - \frac{1}{5} = \frac{24}{5}$

d. $(g \circ f)(2) = g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 + 1 = \frac{13}{4}$

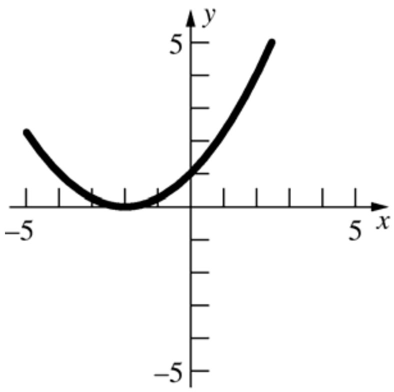
e. $f^3(-1) = \left(-1 + \frac{1}{1}\right)^3 = 0$

f. $f^2(2) + g^2(2) = \left(\frac{3}{2}\right)^2 + (5)^2$
$= \frac{9}{4} + 25 = \frac{109}{4}$

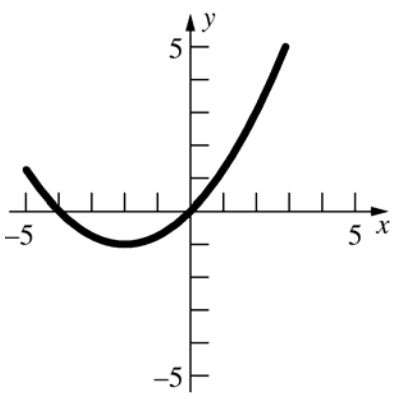
49. a. $y = \frac{1}{4}x^2$



b. $y=\frac{1}{4}(x+2)^2$



c. $y=-1+\frac{1}{4}(x+2)^2$



50. **a.** $(-\infty,16]$

b. $f\circ g=\sqrt{16-x^4}$; domain [–2, 2]

c. $g\circ f=(\sqrt{16-x})^4=(16-x)^2$;
domain $(-\infty,16]$
(note: the simplification
$(\sqrt{16-x})^4=(16-x)^2$ is only true given
the restricted domain)

51. $f(x)=\sqrt{x}, g(x)=1+x, h(x)=x^2, k(x)=\sin x,$
$F(x)=\sqrt{1+\sin^2 x}=f\circ g\circ h\circ k$

52. **a.** $\sin(570°)=\sin(210°)=-\frac{1}{2}$

b. $\cos\left(\frac{9\pi}{2}\right)=\cos\left(\frac{\pi}{2}\right)=0$

c. $\cos\left(-\frac{13\pi}{6}\right)=\cos\left(-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

53. **a.** $\sin(-t)=-\sin t=-0.8$

b. $\sin^2 t+\cos^2 t=1$
$\cos^2 t=1-(0.8)^2=0.36$
$\cos t=-0.6$

c. $\sin 2t=2\sin t\cos t=2(0.8)(-0.6)=-0.96$

d. $\tan t=\frac{\sin t}{\cos t}=\frac{0.8}{-0.6}=-\frac{4}{3}\approx -1.333$

e. $\cos\left(\frac{\pi}{2}-t\right)=\sin t=0.8$

f. $\sin(\pi+t)=-\sin t=-0.8$

54. $\sin 3t=\sin(2t+t)=\sin 2t\cos t+\cos 2t\sin t$
$=2\sin t\cos^2 t+(1-2\sin^2 t)\sin t$
$=2\sin t(1-\sin^2 t)+\sin t-2\sin^3 t$
$=2\sin t-2\sin^3 t+\sin t-2\sin^3 t$
$=3\sin t-4\sin^3 t$

55. $s=rt$
$=9\left(20\frac{\text{rev}}{\text{min}}\right)\left(2\pi\frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right)(1\text{ sec})=6\pi$
$\approx$ 18.85 in.

Review and Preview Problems

1. **a)** $0<2x<4;\ \ 0<x<2$

b) $-6<x<16$

2. **a)** $13<2x<14;\ \ 6.5<x<7$

b) $-4<-x/2<7;\ \ -14<x<8$

3. $x-7=3$ or $x-7=-3$
$x=10$ or $x=4$

4. $x+3=2$ or $x+3=-2$
$x=-1$ or $x=-5$

5. $x-7=3$ or $x-7=-3$
$x=10$ or $x=4$

6. $x-7=d$ or $x-7=-d$
$x=7+d$ or $x=7-d$

7. **a)** $x-7<3$ and $x-7>-3$
$x<10$ and $x>4$
$4<x<10$

b) $x-7 \le 3$ and $x-7 \ge -3$

$x \le 10$ and $x \ge 4$

$4 \le x \le 10$

c) $x-7 \le 1$ and $x-7 \ge -1$

$x \le 8$ and $x \ge 6$

$6 \le x \le 8$

d) $x-7 < 0.1$ and $x-7 > -0.1$

$x < 7.1$ and $x > 6.9$

$6.9 < x < 7.1$

8. a) $x-2 < 1$ and $x-2 > -1$

$x < 3$ and $x > 1$

$1 < x < 3$

b) $x-2 \ge 1$ or $x-2 \le -1$

$x \ge 3$ or $x \le 1$

c) $x-2 < 0.1$ and $x-2 > -0.1$

$x < 2.1$ and $x > 1.9$

$1.9 < x < 2.1$

d) $x-2 < 0.01$ and $x-2 > -0.01$

$x < 2.01$ and $x > 1.99$

$1.99 < x < 2.01$

9. a) $x-1 \ne 0;\ x \ne 1$

b) $2x^2 - x - 1 \ne 0;\ x \ne 1, -0.5$

10. a) $x \ne 0$ **b)** $x \ne 0$

11. a)

$$f(0) = \frac{0-1}{0-1} = 1$$
$$f(0.9) = \frac{0.81-1}{0.9-1} = 1.9$$
$$f(0.99) = \frac{0.9801-1}{0.99-1} = 1.99$$
$$f(0.999) = \frac{0.998001-1}{.999-1} = 1.999$$
$$f(1.001) = \frac{1.002001-1}{1.001-1} = 2.001$$
$$f(1.01) = \frac{1.0201-1}{1.01-1} = 2.01$$
$$f(1.1) = \frac{1.21-1}{1.1-1} = 2.1$$
$$f(2) = \frac{4-1}{2-1} = 3$$

b)

$$g(0) = -1$$
$$g(0.9) = -0.0357143$$
$$g(0.99) = -0.0033557$$
$$g(0.999) = -0.000333556$$
$$g(1.001) = 0.000333111$$
$$g(1.01) = 0.00331126$$
$$g(1.1) = 0.03125$$
$$g(2) = \frac{1}{5}$$

12. a)

$$F(-1) = \frac{1}{-1} = -1$$
$$F(-0.1) = \frac{0.1}{-0.1} = -1$$
$$F(-0.01) = \frac{0.01}{-0.01} = -1$$
$$F(-0.001) = \frac{0.001}{-0.001} = -1$$
$$F(0.001) = \frac{0.001}{0.001} = 1$$
$$F(0.01) = \frac{0.01}{0.01} = 1$$
$$F(0.1) = \frac{0.01}{0.01} = 1$$
$$F(1) = \frac{1}{1} = 1$$

b)

$$G(-1) = 0.841471$$
$$G(-0.1) = 0.998334$$
$$G(-0.01) = 0.999983$$
$$G(-0.001) = 0.99999983$$
$$G(0.001) = 0.99999983$$
$$G(0.01) = 0.999983$$
$$G(0.1) = 0.998334$$
$$G(1) = 0.841471$$

13. $x-5 < 0.1$ and $x-5 > -0.1$

$x < 5.1$ and $x > 4.9$

$4.9 < x < 5.1$

14. $x-5 < \varepsilon$ and $x-5 > -\varepsilon$

$x < 5+\varepsilon$ and $x > 5-\varepsilon$

$5-\varepsilon < x < 5+\varepsilon$

15. a. True. **b.** False: Choose $a = 0$.

c. True. **d.** True

16. $\sin(c+h) = \sin c \cos h + \cos c \sin h$