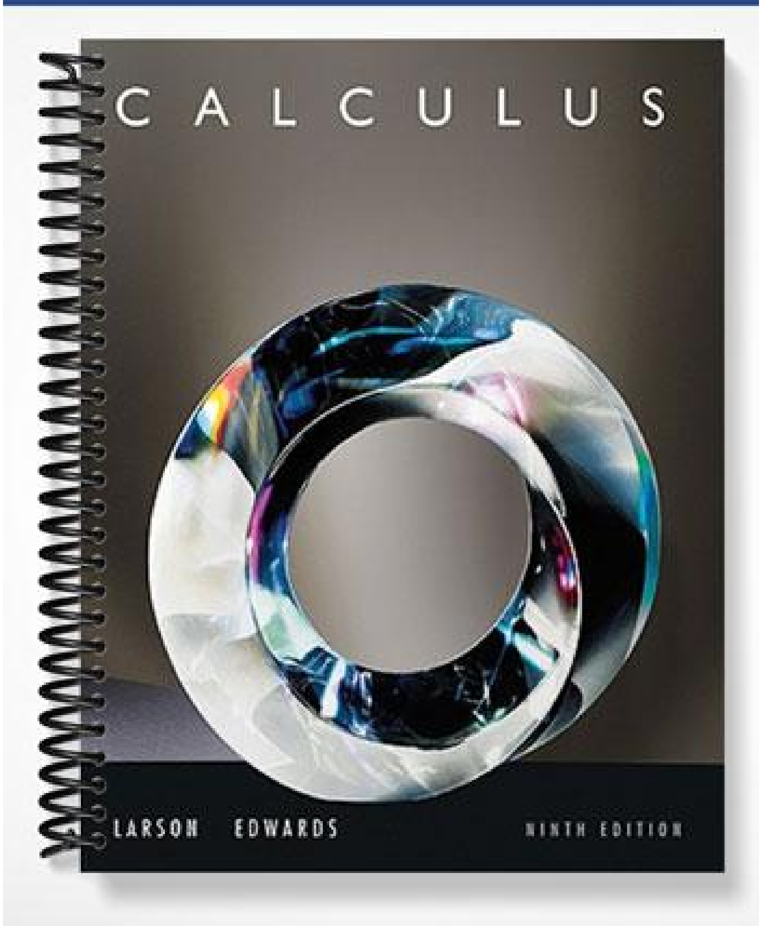
SOLUTIONS MANUAL



Notetaking Guide
Instructor's Annotated Edition

Calculus

NINTH EDITION

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ISBN-13: 978-1-4390-4826-9
ISBN-10: 1-4390-4826-6

Brooks/Cole
10 Davis Drive
Belmont, CA 94002-3098
USA

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Chapter 2 Differentiation

Course Number
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Section 2.1 The Derivative and the Tangent Line Problem

Objective: In this lesson you learned how to find the derivative of a function using the limit definition and understand the relationship between differentiability and continuity.

Important Vocabulary Define each term or concept.

Differentiation The process of finding the derivative of a function.

Differentiable A function is differentiable at x if its derivative exists at x.

I. The Tangent Line Problem (Pages 96–99)

What you should learn
How to find the slope of the tangent line to a curve at a point

Essentially, the problem of finding the tangent line at a point P boils down to the problem of finding the slope of the tangent line at point P. You can approximate this slope using a secant line through the point of tangency $(c, f(c))$ and a second point on the curve $(c + \Delta x, f(c + \Delta x))$. The slope of the secant line through these two points is $m_{\sec} = \dfrac{f(c + \Delta x) - f(c)}{\Delta x}$.

The right side of this equation for the slope of a secant line is called a difference quotient. The denominator Δx is the change in x, and the numerator $\Delta y = f(c+\Delta x) - f(c)$ is the change in y.

The beauty of this procedure is that you can obtain more and more accurate approximations of the slope of the tangent line by choosing points closer and closer to the point of tangency.

If f is defined on an open interval containing c, and if the limit

$\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{f(c+\Delta x) - f(c)}{\Delta x} = m$ exists, then the line passing through $(c, f(c))$ with slope m is ____________ the tangent line to the graph of f at the point $(c, f(c))$ ______________________.

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is also called ____________________ the slope of the graph of f at $x = c$ ____________________.

Example 1: Find the slope of the graph of $f(x) = 9 - \frac{x}{2}$ at the point (4, 7).
– 1/2

Example 2: Find the slope of the graph of $f(x) = 2 - 3x^2$ at the point (–1, –1).
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The definition of a tangent line to a curve does not cover the possibility of a vertical tangent line. If f is continuous at c and

$$\lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = \infty \text{ or } \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = -\infty,$$

the vertical line $x = c$ passing through $(c, f(c))$ is ______ a vertical tangent line ______________ to the graph of f.

II. The Derivative of a Function (Pages 99–101)

> ***What you should learn***
> How to use the limit definition to find the derivative of a function

The ________ derivative of f at x ______________ is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x},$$

provided the limit exists. For all x for which this limit exists, f' is _______ a function of x ________.

The derivative of a function of x gives the ________ slope of the tangent line ________ to the graph of f at the point $(x, f(x))$, provided that the graph has a tangent line at this point.

A function is **differentiable on an open interval (*a*, *b*)** if _____ it is differentiable at every point in the interval __________________.

Example 3: Find the derivative of $f(t) = 4t^2 + 5$.
$f'(t) = 8t$

III. Differentiability and Continuity (Pages 101–103)

> ***What you should learn***
> How to understand the relationship between differentiability and continuity

Name some situations in which a function will not be differentiable at a point.

A graph having a vertical tangent line or a graph with a sharp turn

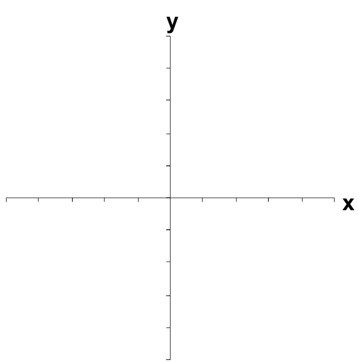
If a function f is differentiable at $x = c$, then ____________ f is continuous at $x = c$ ____________.

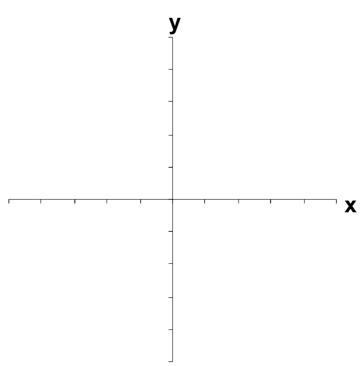
Complete the following statements.

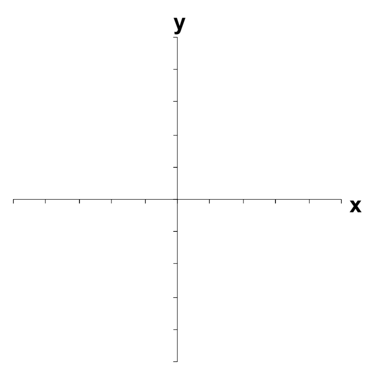
1. If a function is differentiable at $x = c$, then it is continuous at $x = c$. So, differentiability _______ implies _______ continuity.

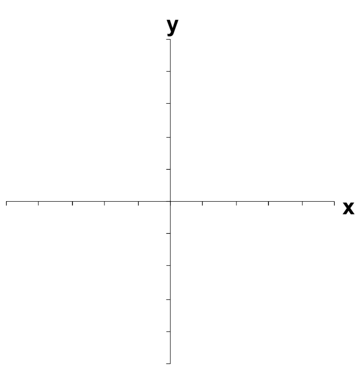
2. It is possible for a function to be continuous at $x = c$ and not be differentiable at $x = c$. So, continuity __________ does not imply ________ differentiability.

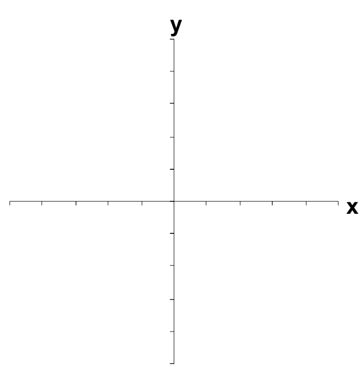
Additional notes

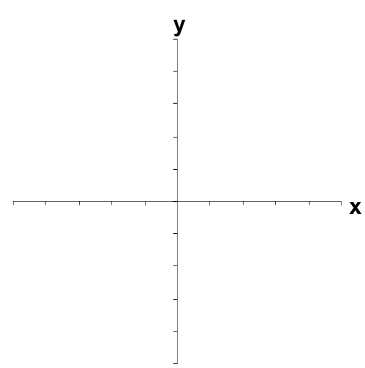












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Section 2.2 Basic Differentiation and Rates of Change

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Objective: In this lesson you learned how to find the derivative of a function using basic differentiation rules.

I. The Constant Rule (Page 107)

What you should learn
How to find the derivative of a function using the Constant Rule

The derivative of a constant function is zero.

If c is a real number, then $\frac{d}{dx}[c] =$ 0.

II. The Power Rule (Pages 108–109)

What you should learn
How to find the derivative of a function using the Power Rule

The **Power Rule** states that if n is a rational number, then the function $f(x) = x^n$ is differentiable and

$\frac{d}{dx}\left[x^n\right] =$ nx^{n-1}. For f to be differentiable at $x = 0$, n must be a number such that x^{n-1} is defined on an interval containing 0.

Also, $\frac{d}{dx}[x] =$ 1.

Example 1: Find the derivative of the function $f(x) = \frac{1}{x^3}$.

$-3/x^4$

Example 2: Find the slope of the graph of $f(x) = x^5$ at $x = 2$.

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III. The Constant Multiple Rule (Pages 110–111)

What you should learn
How to find the derivative of a function using the Constant Multiple Rule

The **Constant Multiple Rule** states that if f is a differentiable function and c is a real number then cf is also differentiable and

$\frac{d}{dx}\left[cf(x)\right] =$ $cf'(x)$.

Informally, the Constant Multiple Rule states that constants can be factored out of the differentiation process, even if the constants appear in the denominator.

Example 3: Find the derivative of $f(x) = \frac{2x}{5}$

2/5

The Constant Multiple Rule and the Power Rule can be combined into one rule. The combination rule is

$\frac{d}{dx}\left[cx^n\right] =$ ______ cnx^{n-1} ______.

Example 4: Find the derivative of $y = \frac{2}{5x^5}$

$-2/x^6$

IV. The Sum and Difference Rules (Page 111)

What you should learn
How to find the derivative of a function using the Sum and Difference Rules

The **Sum and Difference Rules** of Differentiation state that the sum (or difference) of two differentiable functions *f* and *g* is itself differentiable. Moreover, the derivative of $f + g$ (or $f - g$) is the sum (or difference) of the derivatives of *f* and *g*.

That is, $\frac{d}{dx}\left[f(x) + g(x)\right] =$ ______ $f'(x) + g'(x)$ ______

and $\frac{d}{dx}\left[f(x) - g(x)\right] =$ ______ $f'(x) - g'(x)$ ______

Example 5: Find the derivative of $f(x) = 2x^3 - 4x^2 + 3x - 1$

$6x^2 - 8x + 3$

V. Derivatives of Sine and Cosine Functions (Page 112)

What you should learn
How to find the derivative of the sine function and of the cosine function

$\frac{d}{dx}\left[\sin x\right] =$ ______ $\cos x$ ______

$\frac{d}{dx}\left[\cos x\right] =$ ______ $-\sin x$ ______

Example 6: Differentiate the function $y = x^2 - 2\cos x$.
$y' = 2x + 2\sin x$

VI. Rates of Change (Pages 113–114)

> ***What you should learn***
> How to use derivatives to find rates of change

The derivative can also be used to determine the rate of change of one variable with respect to another.

Give some examples of real-life applications of rates of change.

Population growth rates, production rates, water flow rates, velocity, and acceleration.

The function s that gives the position (relative to the origin) of an object as a function of time t is called a position function. The **average velocity** of an object that is moving in a straight line is found as follows.

$$\text{Average velocity} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$$

Example 7: If a ball is dropped from the top of a building that is 200 feet tall, and air resistance is neglected, the height s (in feet) of the ball at time t (in seconds) is given by $s = -16t^2 + 200$. Find the average velocity of the object over the interval [1, 3].
– 64 feet per second

If $s = s(t)$ is the position function for an object moving along a straight line, the (instantaneous) **velocity** of the object at time t is

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = s'(t).$$

In other words, the velocity function is the derivative of the position function. Velocity can be negative, zero, or positive __________. The ____speed____ of an object is the absolute value of its velocity. Speed cannot be ____negative____.

Example 8: If a ball is dropped from the top of a building that is 200 feet tall, and air resistance is neglected, the height s (in feet) of the ball at time t (in seconds) is given by $s(t) = -16t^2 + 200$. Find the velocity of the ball when $t = 3$.

– 96 feet per second

The position function for a free-falling object (neglecting air resistance) under the influence of gravity can be represented by the equation ____$s(t) = 1/2gt^2 + v_0t + s_0$____, where s_0 is the initial height of the object, v_0 is the initial velocity of the object, and g is the acceleration due to gravity. On Earth, the value of g is ____approximately –32 feet per second per second or –9.8 meters per second per second____.

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Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

Objective: In this lesson you learned how to find the derivative of a function using the Product Rule and Quotient Rule.

I. The Product Rule (Pages 119–120)

What you should learn
How to find the derivative of a function using the Product Rule

The product of two differentiable functions *f* and *g* is itself differentiable. The **Product Rule** states that the derivative of the fg is equal to the first function times the derivative of the second, plus the second function times the derivative of the first. That is,

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) .$$

Example 1: Find the derivative of $y = (4x^2 + 1)(2x - 3)$.
$dy/dx = 24x^2 - 24x + 2$

The Product Rule can be extended to cover products that have more than two factors. For example, if *f*, *g*, and *h* are differentiable functions of *x*, then

$$\frac{d}{dx}[f(x)g(x)h(x)] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

Explain the difference between the Constant Multiple Rule and the Product Rule.

The difference between these two rules is that the Constant Multiple Rule deals with the product of a constant and a variable quantity, whereas the Product Rule Deals with the product of two variable quantities.

II. The Quotient Rule (Pages 121–123)

> ***What you should learn***
> How to find the derivative of a function using the Quotient Rule

The quotient f/g of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$. The derivative of f/g is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator. This is called the Quotient Rule, and is given by

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}, \qquad g(x) \neq 0.$$

Example 2: Find the derivative of $y = \dfrac{2x+5}{3x}$.

$-5/(3x^2)$

With the Quotient Rule, it is a good idea to enclose all factors and derivatives in parentheses and to pay special attention to the subtraction required in the numerator.

III. Derivatives of Trigonometric Functions (Pages 123–124)

> ***What you should learn***
> How to find the derivative of a trigonometric function

$\frac{d}{dx}[\tan x] =$ $\sec^2 x$

$\frac{d}{dx}[\cot x] =$ $-\csc^2 x$

$\frac{d}{dx}[\sec x] =$ $\sec x \tan x$

$\frac{d}{dx}[\csc x] =$ ______ $-\csc x \cot x$ ______

Example 3: Differentiate the function $f(x) = \sin x \sec x$.
$f'(x) = \sin x \sec x \tan x + \sec x \cos x$

IV. Higher-Order Derivatives (Page 125)

> ***What you should learn***
> How to find a higher-order derivative of a function

The derivative of $f'(x)$ is the second derivative of $f(x)$ and is denoted by ______ $f''(x)$ ______. The derivative of $f''(x)$ is the ______ third derivative ______ of $f(x)$ and is denoted by f'''.

These are examples of ______ higher-order derivatives ______ of $f(x)$.

The following notation is used to denoted the ______ sixth derivative ______ of the function $y = f(x)$:

$$\frac{d^6y}{dx^6} \quad D_x^6[y] \quad y^{(6)} \quad \frac{d^6}{dx^6}[f(x)] \quad f^{(6)}(x)$$

Example 4: Find $y^{(5)}$ for $y = 2x^7 - x^5$.
$5040x^2 - 120$

Example 5: On the moon, a ball is dropped from a height of 100 feet. Its height s (in feet) above the moon's surface is given by $s = -\frac{27}{10}t^2 + 100$. Find the height, the velocity, and the acceleration of the ball when $t = 5$ seconds.
Height: 32.5 feet above the surface
Velocity: –27 feet per second
Acceleration: –27/5 feet per second squared

Example 6: Find y''' for $y = \sin x$.

$y''' = -\cos x$

Additional notes

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Section 2.4 The Chain Rule

Objective: In this lesson you learned how to find the derivative of a function using the Chain Rule and General Power Rule.

What you should learn
How to find the derivative of a composite function using the Chain Rule

I. The Chain Rule (Pages 130–132)

The Chain Rule, one of the most powerful differentiation rules, deals with ______composite______ functions.

Basically, the Chain Rule states that if y changes dy/du times as fast as u, and u changes du/dx times as fast as x, then y changes ______$(dy/du)(du/dx)$______ times as fast as x.

The **Chain Rule** states that if $y = f(u)$ is a differentiable function of u, and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x, and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$ or, equivalently,

$$\frac{d}{dx}[f(g(x))] = \underline{f'(g(x))g'(x)}.$$

When applying the Chain Rule, it is helpful to think of the composite function $f \circ g$ as having two parts, an *inner part* and an *outer part*. The Chain Rule tells you that the derivative of $y = f(u)$ is the derivative of the ______outer function______ (at the inner function u) *times* the derivative of the ______inner function______. That is, $y' =$ ______$f'(u) \cdot u'$______.

Example 1: Find the derivative of $y = (3x^2 - 2)^5$.
$30x(3x^2 - 2)^4$

II. The General Power Rule (Pages 132–133)

> ***What you should learn***
> How to find the derivative of a function using the General Power Rule

The General Power Rule is a special case of the Chain Rule.

The General Power Rule states that if $y = [u(x)]^n$, where u is a differentiable function of x and n is a rational number, then

$$\frac{dy}{dx} = n[u(x)]^{n-1}\frac{du}{dx}$$ or, equivalently,

$$\frac{d}{dx}\left[u^n\right] = nu^{n-1}u'$$

Example 2: Find the derivative of $y = \dfrac{4}{(2x-1)^3}$.

$$-\frac{24}{(2x-1)^4}$$

III. Simplifying Derivatives (Page 134)

> ***What you should learn***
> How to simplify the derivative of a function using algebra

Example 3: Find the derivative of $y = \dfrac{3x^2}{(1-x^3)^2}$ and simplify.

$y' = 6x(2x^3 + 1) / (1 - x^3)^3$

IV. Trigonometric Functions and the Chain Rule
(Pages 135–136)

What you should learn
How to find the derivative of a trigonometric function using the Chain Rule

Complete each of the following "Chain Rule versions" of the derivatives of the six trigonometric functions.

$\frac{d}{dx}[\sin u] =$ ____(cos u) u'____

$\frac{d}{dx}[\cos u] =$ ____− (sin u) u'____

$\frac{d}{dx}[\tan u] =$ ____($\sec^2 u$) u'____

$\frac{d}{dx}[\cot u] =$ ____− ($\csc^2 u$) u'____

$\frac{d}{dx}[\sec u] =$ ____(sec u tan u) u'____

$\frac{d}{dx}[\csc u] =$ ____− (csc u cot u) u'____

Example 4: Differentiate the function $y = \sec 4x$.
$dy/dx = 4 \sec 4x \tan 4x$

Example 5: Differentiate the function $y = x^2 - \cos(2x + 1)$.

$dy/dx = 2x + 2\sin(2x + 1)$

Additional notes

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Section 2.5 Implicit Differentiation

Objective: In this lesson you learned how to find the derivative of a function using implicit differentiation.

What you should learn
How to distinguish between functions written in implicit form and explicit form

I. Implicit and Explicit Functions (Page 141)

Up to this point in the text, most functions have been expressed in **explicit form** $y = f(x)$, meaning that the variable y is explicitly written as a function of x. However, some functions are only implied by an equation.

Give an example of a function in which y is **implicitly** defined as a function of x.

Answers will vary. For example, $x^2 y = 4$ is in implicit form.

Implicit differentiation is a procedure for taking the derivative of an implicit function when you are unable to solve for y as a function of x.

To understand how to find $\frac{dy}{dx}$ implicitly, realize that the differentiation is taking place with respect to x. This means that when you differentiate terms involving x alone, you can differentiate as usual. However, when you differentiate terms involving y, you must apply the Chain Rule because you are assuming that y is defined implicitly as a differentiable function of x.

Example 1: Differentiate the expression with respect to x:
$4x + y^2$

$$4 + 2y\frac{dy}{dx}$$

What you should learn
How to use implicit differentiation to find the derivative of a function

II. Implicit Differentiation (Pages 142–145)

Consider an equation involving x and y in which y is a differentiable function of x. List the four guidelines for applying implicit differentiation to find dy/dx.

1. Differentiate both sides of the equation with respect to x.

2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.

3. Factor dy/dx out of the left side of the equation.

4. Solve for dy/dx.

Example 2: Find dy/dx for the equation $4y^2 - x^2 = 1$.

$dy/dx = x/(4y)$

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Section 2.6 Related Rates

Objective: In this lesson you learned how to find a related rate.

I. Finding Related Variables (Page 149)

> ***What you should learn***
> How to find a related rate

Another important use of the Chain Rule is to find the rates of change of two or more related variables that are changing with respect to ___time___.

Example 1: The variables x and y are differentiable functions of t and are related by the equation $y = 2x^3 - x + 4$. When $x = 2$, $dx/dt = -1$. Find dy/dt when $x = 2$.
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II. Problem Solving with Related Rates (Pages 150–153)

> ***What you should learn***
> How to use related rates to solve real-life problems

List the guidelines for solving a related-rate problems.

1. Identify all given quantities and quantities to be determined. Make a sketch and label the quantities.

2. Write an equation involving the variables whose rates of change either are given or are to be determined.

3. Using the Chain Rule, implicitly differentiate both sides of the equation with respect to time t.

4. After completing Step 3, substitute into the resulting equation all known values of the variables and their rates of change. Then solve for the required rate of change.

Example 2: Write a mathematical model for the following related-rate problem situation:
The population of a city is decreasing at the rate of 100 people per month.
x = number in population; $dx/dt = -100$ people per month

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