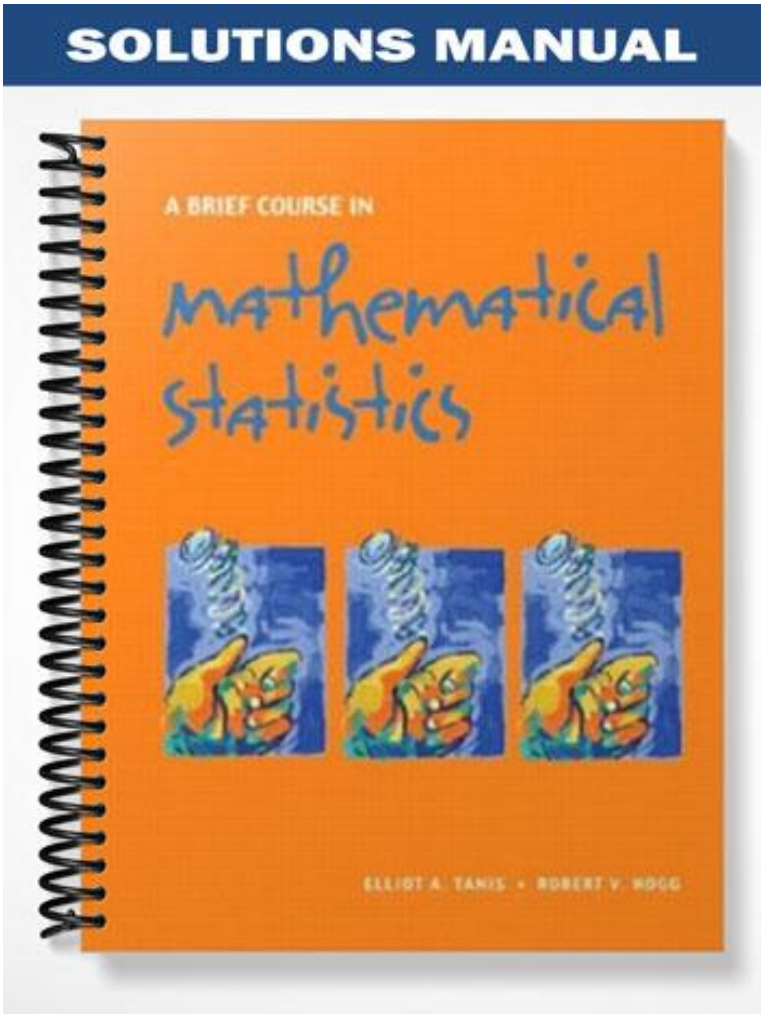
SOLUTIONS MANUAL



INSTRUCTOR'S

SOLUTIONS MANUAL

A BRIEF COURSE IN

MATHEMATICAL STATISTICS

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Contents

Preface

This solutions manual provides answers for the even-numbered exercises in *A Brief Course in Mathematical Statistics* by Elliot A. Tanis and Robert V. Hogg. Complete solutions are given for most of these exercises. You, the instructor, may decide how many of these answers you want to make available to your students. Note that the answers for the odd-numbered exercises are given in the textbook.

All of the figures in this manual were generated using *Maple*, a computer algebra system. Most of the figures were generated and many of the solutions, especially those involving data, were solved using procedures that were written by Zaven Karian from Denison University. We thank him for providing these. These procedures are available free of charge for your use. They are available on a "CD" that can be downloaded from Prentice-Hall. Short descriptions of these procedures are provided on the "Maple Card" on the "CD". Complete descriptions of these procedures are given in *Probability and Statistics: Explorations with MAPLE*, second edition, 1999, written by Zaven Karian and Elliot Tanis, published by Prentice Hall (ISBN 0-13-021536-8).

Our hope is that this solutions manual will be helpful to each of you in your teaching.

If you find an error or wish to make a suggestion, send these to Elliot Tanis at `tanis@hope.edu` and he will post corrections on his web page, http://www.math.hope.edu/tanis/.

E.A.T.
R.V.H.

Chapter 1

Probability

1.1 Basic Concepts

1.1-2 **(a)** $O = \{$HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT$\}$;

(b) **(i)** $5/16$, **(ii)** 0, **(iii)** $11/16$, **(iv)** $4/16$, **(v)** $4/16$, **(vi)** $9/16$, **(vii)** $4/16$.

1.1-4 **(a)** $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$;

(b)
$$\begin{aligned} A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.4 &= P(A \cap B') + 0.3 \\ P(A \cap B) &= 0.1; \end{aligned}$$

(c) $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.3 = 0.7$.

1.1-6 **(a)**
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ 0.7 &= 0.4 + 0.5 - P(A \cap B) \\ P(A \cap B) &= 0.2; \end{aligned}$$

(b)
$$\begin{aligned} P(A' \cup B') = P[(A \cap B)'] &= 1 - P(A \cap B) \\ &= 1 - 0.2 \\ &= 0.8. \end{aligned}$$

1.1-8 **(a)** $O = \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$;

(b) **(i)** $1/10$; **(ii)** $5/10$.

1.1-10 $P(A) = \dfrac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \dfrac{\sqrt{3}}{2}$.

1.1-12
$$\begin{aligned} A \cup B \cup C &= A \cup (B \cup C) \\ P(A \cup B \cup C) &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) \\ &\quad + P(A \cap B \cap C). \end{aligned}$$

1.2 Methods of Enumeration

1.2-2 $(4)(3)(2) = 24$.

1.2-4 **(a)** $(4)(5)(2) = 40$; **(b)** $(2)(2)(2) = 8$.

1.2-6 $O = \{$ FFF, FFRF, FRFF, RFFF, FFRRF, FRFRF, RFFRF, FRRFF, RFRFF, RRFFF, RRR, RRFR, RFRR, FRRR, RRFFR, RFRFR, FRRFR, RFFRR, FRFRR, FFRRR $\}$ so there are 20 possibilities.

1.2-8
$$\begin{aligned}\binom{n-1}{r} + \binom{n-1}{r-1} &= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} \\ &= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}.\end{aligned}$$

1.2-10
$$0 = (1-1)^n = \sum_{r=0}^{n} \binom{n}{r}(-1)^r(1)^{n-r} = \sum_{r=0}^{n}(-1)^r\binom{n}{r}.$$
$$2^n = (1+1)^n = \sum_{r=0}^{n} \binom{n}{r}(1)^r(1)^{n-r} = \sum_{r=0}^{n}\binom{n}{r}.$$

1.3 Conditional Probability

1.3-2 **(a)** $\dfrac{1041}{1456}$;

(b) $\dfrac{392}{633}$;

(c) $\dfrac{649}{823}$.

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.3-4 **(a)** $P(\text{HH}) = \dfrac{13}{52}\cdot\dfrac{12}{51} = \dfrac{1}{17}$;

(b) $P(\text{HC}) = \dfrac{13}{52}\cdot\dfrac{13}{51} = \dfrac{13}{204}$;

(c) $P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$
$$= \frac{12}{52}\cdot\frac{4}{51} + \frac{1}{52}\cdot\frac{3}{51} = \frac{51}{52\cdot 51} = \frac{1}{52}.$$

1.3-6 **(a)** $\dfrac{8}{14}\cdot\dfrac{7}{13} = \dfrac{56}{182}$;

(b) $\dfrac{6}{14}\cdot\dfrac{5}{13} = \dfrac{30}{182}$;

(c) $2\left(\dfrac{8}{14}\cdot\dfrac{6}{13}\right) = \dfrac{96}{182}$ or $1 - \left[\dfrac{56}{182} + \dfrac{30}{182}\right] = \dfrac{96}{182}$.

1.3-8
$$\frac{\binom{2}{0}\binom{8}{5}}{\binom{10}{5}}\cdot\frac{2}{5} + \frac{\binom{2}{1}\binom{8}{4}}{\binom{10}{5}}\cdot\frac{1}{5} = \frac{1}{5}.$$

1.3-10 **(a)** It doesn't matter because $P(B_1) = \dfrac{1}{18}$, $P(B_5) = \dfrac{1}{18}$, $P(B_{18}) = \dfrac{1}{18}$;

(b) $P(B) = \dfrac{2}{18} = \dfrac{1}{9}$ on each draw.

1.3-12 $\dfrac{3}{5} \cdot \dfrac{5}{8} + \dfrac{2}{5} \cdot \dfrac{4}{8} = \dfrac{23}{40}$.

1.4 Independent Events

1.4-2 **(a)**
$$\begin{aligned} P(A \cap B) &= P(A)P(B) = (0.3)(0.6) = 0.18; \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72. \end{aligned}$$

(b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{0.6} = 0.$

1.4-4
$$\begin{aligned} P[A \cap (B \cap C)] &= P[A \cap B \cap C] \\ &= P(A)P(B)P(C) \\ &= P(A)P(B \cap C). \end{aligned}$$

$$\begin{aligned} P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C). \end{aligned}$$

$$\begin{aligned} P[A' \cap (B \cap C')] &= P(A' \cap C' \cap B) \\ &= P(B)[P(A' \cap C') \,|\, B] \\ &= P(B)[1 - P(A \cup C \,|\, B)] \\ &= P(B)[1 - P(A \cup C)] \\ &= P(B)P[(A \cup C)'] \\ &= P(B)P(A' \cap C') \\ &= P(B)P(A')P(C') \\ &= P(A')P(B)P(C') \\ &= P(A')P(B \cap C') \end{aligned}$$

$$\begin{aligned} P[A' \cap B' \cap C'] &= P[(A \cup B \cup C)'] \\ &= 1 - P(A \cup B \cup C) \\ &= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + \\ &\qquad P(B)P(C) - PA)P(B)P(C) \\ &= [1 - P(A)][1 - P(B)][1 - P(C)] \\ &= P(A')P(B')P(C'). \end{aligned}$$

1.4-6 $\dfrac{1}{6} \cdot \dfrac{2}{6} \cdot \dfrac{3}{6} + \dfrac{1}{6} \cdot \dfrac{4}{6} \cdot \dfrac{3}{6} + \dfrac{5}{6} \cdot \dfrac{2}{6} \cdot \dfrac{3}{6} = \dfrac{2}{9}$.

1.4-8 **(a)** $\dfrac{3}{4} \cdot \dfrac{3}{4} = \dfrac{9}{16}$;

(b) $\dfrac{1}{4} \cdot \dfrac{3}{4} + \dfrac{3}{4} \cdot \dfrac{2}{4} = \dfrac{9}{16}$;

(c) $\dfrac{2}{4} \cdot \dfrac{1}{4} + \dfrac{2}{4} \cdot \dfrac{4}{4} = \dfrac{10}{16}$.