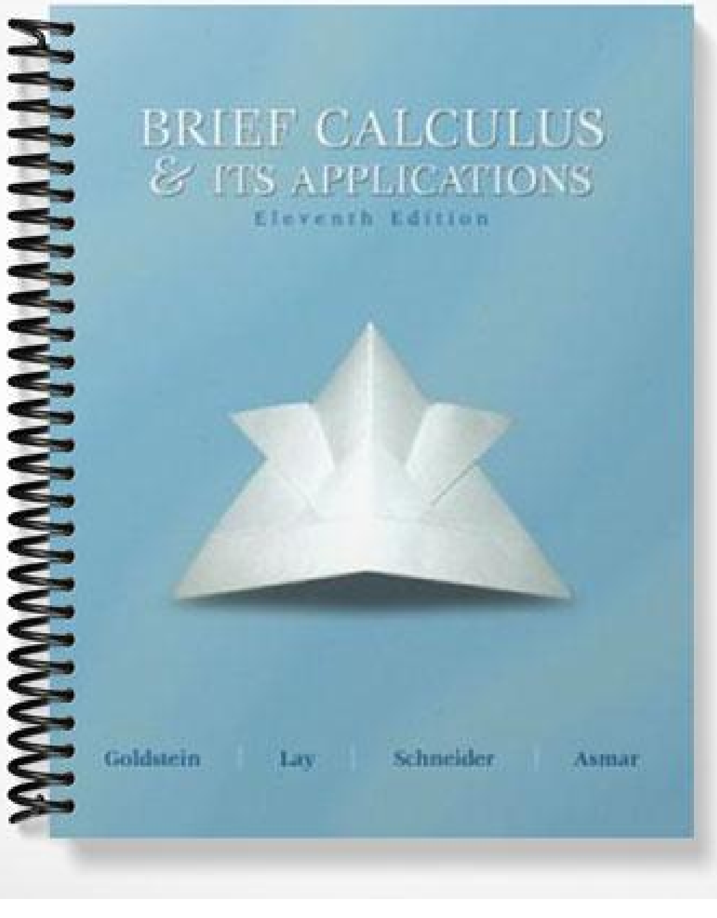
SOLUTIONS MANUAL



Chapter 1

Exercises 1.1

1. $y = 2 - 5x$; y-intercept: $(0, 2)$, slope = -5

2. $y = \dfrac{x+1}{3} = \dfrac{1}{3}x + \dfrac{1}{3}$; y-intercept: $\left(0, \dfrac{1}{3}\right)$, slope = $\dfrac{1}{3}$

3. $3y - 2x - 1 = 0 \Rightarrow y = \dfrac{2}{3}x + \dfrac{1}{3}$; y-intercept: $\left(0, \dfrac{1}{3}\right)$, slope = $\dfrac{2}{3}$

4. $y = 5 \Rightarrow y = 0x + 5$; y-intercept: $(0, 5)$, slope = 0

5. $y = \dfrac{3x-1}{3} = x - \dfrac{1}{3}$; y-intercept: $\left(0, -\dfrac{1}{3}\right)$, slope = 1

6. $2x + 7y = -1 \Rightarrow y = -\dfrac{2}{7}x - \dfrac{1}{7}$; y-intercept: $\left(0, -\dfrac{1}{7}\right)$, slope = $-\dfrac{2}{7}$

7. slope = –7, (5, 0) on line.
Let $(x, y) = (5, 0)$, $m = -7$.
$y - 0 = -7(x - 5)$
$y = -7x + 35$

8. slope = 0; (0, 0) on line.
y-intercept $(0, b) = (0, 0)$, $m = 0$
$y = mx + b$
$y = 0x + 0$
$y = 0$

9. slope = 4; (1, 0) on line.
Let $(x_1, y_1) = (1, 0)$, $m = 4$.
$y - y_1 = m(x - x_1)$
$y - 0 = 4(x - 1)$
$y = 4x - 4$

10. slope = $\dfrac{7}{3}$; (5, –1) on line.
Let $(x_1, y_1) = (5, -1)$, $m = \dfrac{7}{3}$.
$y - y_1 = m(x - x_1)$
$y - (-1) = \dfrac{7}{3}(x - 5)$
$y + 1 = \dfrac{7}{3}x - \dfrac{35}{3}$
$y = \dfrac{7}{3}x - \dfrac{38}{3}$

11. $\left(\dfrac{2}{3}, 5\right)$ and $\left(-\dfrac{5}{6}, -4\right)$ on line.
slope = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4-5}{-\frac{5}{6} - \frac{2}{3}} = \dfrac{-9}{-\frac{9}{6}} = 6$
Let $(x_1, y_1) = \left(\dfrac{2}{3}, 5\right)$, $m = 6$
$y - y_1 = m(x - x_1)$
$y - 5 = 6\left(x - \dfrac{2}{3}\right)$
$y - 5 = 6x - 4$
$y = 6x + 1$

12. $\left(\dfrac{1}{2}, 1\right)$ and (1, 2) on line.
slope = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-1}{1-\frac{1}{2}} = \dfrac{1}{\frac{1}{2}} = 2$
Let $(x_1, y_1) = (1, 2)$, $m = 2$.
$y - y_1 = m(x - x_1)$
$y - 2 = 2(x - 1)$
$y = 2x$

13. (0, 0) and (1, –2) on line.
slope = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-0}{1-0} = -2$
$y - 0 = -2(x - 0)$
$y = -2x$

14. $\left(-\frac{1}{2}, 0\right)$ and (1, 2) on line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{1-\left(-\frac{1}{2}\right)} = \frac{2}{\frac{3}{2}} = \frac{4}{3} = m$$

Let $(x_1, y_1) = \left(-\frac{1}{2}, 0\right)$.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{4}{3}\left(x - \left(-\frac{1}{2}\right)\right)$$
$$y = \frac{4}{3}\left(x + \frac{1}{2}\right)$$
$$y = \frac{4}{3}x + \frac{2}{3}$$

15. Horizontal through (3, –1).
Let $(x_1, y_1) = (3, -1)$, $m = 0$ (horizontal line).

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = 0(x - 3)$$
$$y + 1 = 0$$
$$y = -1$$

16. x-intercept is 2; y-intercept is –2.
The intercepts (2, 0) and (0, –2) are on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-0}{0-2} = 1 = m$$

y-intercept $(0, b) = (0, -2)$

$$y = mx + b$$
$$y = 1x - 2$$
$$y = x - 2$$

17. x-intercept is –1; y-intercept is 1.
The intercepts (–1, 0) and (0, 1) are on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{0-(-1)} = 1$$

y-intercept $(0, b) = (0, 1)$

$$y = mx + b$$
$$y = 1x + 1$$
$$y = x + 1$$

18. Slope = 6; x-intercept is –1.
The x-intercept (–1, 0) is on the line.
Let $(x_1, y_1) = (-1, 0)$, $m = 6$.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 6(x - (-1))$$
$$y = 6(x + 1)$$
$$y = 6x + 6$$

19. Slope = 1; x-intercept is –2.
The x-intercept (–2, 0) is on the line.
Let $(x_1, y_1) = (-2, 0)$, $m = 1$.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 1(x - (-2))$$
$$y = 1(x + 2)$$
$$y = x + 2$$

20. Horizontal through $(-2, \sqrt{2})$.
Let $(x_1, y_1) = (-2, \sqrt{2})$, $m = 0$ (horizontal line).

$$y - y_1 = m(x - x_1)$$
$$y - \sqrt{2} = 0(x - (-2))$$
$$y = \sqrt{2}$$

21. Parallel to $y = 2x$; (2, 0) on line.
slope $= m = 2$, $(x_1, y_1) = (2, 0)$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 2)$$
$$y = 2x - 4$$

22. Parallel to $x + y = 0$; (1, 1) on line.
$y = -x$, slope $= m = -1$, $(x_1, y_1) = (1, 1)$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -1(x - 1)$$
$$y - 1 = -x + 1$$
$$y = -x + 2$$

23. Parallel to $y = -x + 7$; x-intercept is 1

$y = -x + 7$, slope $= m = -1$, $(x_1, y_1) = (1, 0)$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -1(x - 1)$$
$$y = -x + 1$$

24. Parallel to $y - x = 13$; y-intercept is $-1/2$

$y = x + 13$, slope $= m = 1$, $(x_1, y_1) = (0, -1/2)$

$$y - y_1 = m(x - x_1)$$
$$y - \left(-1/2\right) = 1(x - 0)$$
$$y + 1/2 = x$$
$$y = x - 1/2$$

25. Perpendicular to $y = 2x$; (2, 0) on line.

$$\text{slope} = m_1 = 2$$
$$m_1 \cdot m_2 = -1$$
$$2m_2 = -1$$
$$m_2 = -\frac{1}{2} = m$$
$$(x_1, y_1) = (2, 0)$$
$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{1}{2}(x - 2)$$
$$y = -\frac{1}{2}x + 1$$

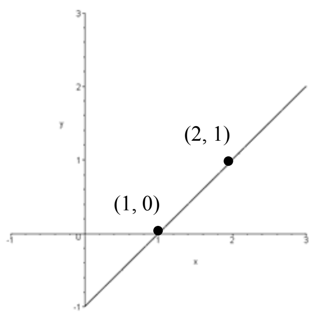
26. Perpendicular to $y = -\frac{1}{2}x + 1$; $(0,0)$ on line.

$$\text{slope} = m_1 = -\frac{1}{2}$$
$$m_1 \cdot m_2 = -1$$
$$-\frac{1}{2}m_2 = -1$$
$$m_2 = 2 = m$$

y-intercept $(0, b) = (0, 0)$

$$y = mx + b$$
$$y = 2x$$

27.



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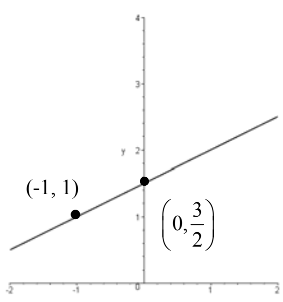

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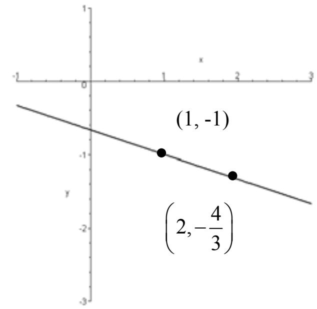

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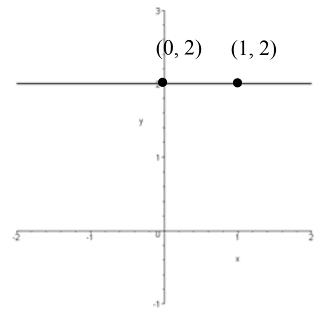


31. (a)–(C) x- and y-intercepts are 1.
(b)–(B) x-intercept is 1, y-intercept is –1.
(c)–(D) x- and y-intercepts are –1.
(d)–(A) x-intercept is –1, y-intercept is 1.

32. Use (4.8, 3.6) and (4.9, 4.8).

$$m = \frac{4.8 - 3.6}{4.9 - 4.8} = 12$$

$y = mx + b$ so

$$6 = 12(5) + b$$
$$b = -54$$

33. $\frac{1}{2} = \frac{j}{4}$, j units in y-direction

$$j = 2$$

34.
$$2 = \frac{j}{\left(\frac{1}{4}\right)}$$
$$j = \frac{1}{2}$$

35.
$$-3 = \frac{j}{.25}$$
$$j = -.75$$

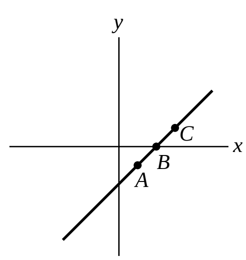
36.
$$.2 = \frac{j}{5}$$
$$j = 1$$

37. Slope = 2, (1, 3) on line.
$x_1 = 1,\ y_1 = 3$
$x = 2$
$y - 3 = 2(2 - 1),\ \ y = 4 - 2 + 3 = 5$
$x = 3$
$y - 3 = 2(3 - 1),\ \ y = 6 - 2 + 3 = 7$
$x = 0$
$y - 3 = 2(0 - 1),\ \ y = 0 - 2 + 3 = 1$

38. Slope = –3, (2, 2) on line.
$x_1 = 2,\ y_1 = 2$
$x = 3$
$y - 2 = -3(3 - 2),\ y = -9 + 6 + 2 = -1$
$x = 4$
$y - 2 = -3(4 - 2),\ y = -12 + 6 + 2 = -4$
$x = 1$
$y - 2 = -3(1 - 2),\ y = -3 + 6 + 2 = 5$

39. a. slope of line through *A* and *B*:
$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{0-(-1)}{2-1} = \frac{1}{1} = 1$$
slope of line through *A* and *C*:
$$m_{AC} = \frac{y_C - y_A}{x_C - x_A} = \frac{1-(-1)}{3-1} = \frac{2}{2} = 1$$
Both lines go through *A* and have the same slope. So points *A*, *B*, and *C* all lie on the same line.

b.

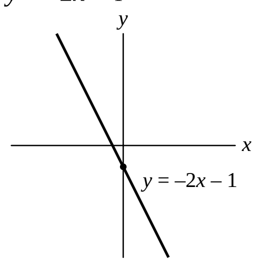


40. First find the slope of $2x + 3y = 0$.
$$2x + 3y = 0$$
$$3y = -2x$$
$$y = -\frac{2}{3}x,\ \text{ so } m_1 = -\frac{2}{3}$$
Now find the slope of the line through (3, 4) and (–1, 2).
$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$$
Since the slopes are not equal, the lines are not parallel.

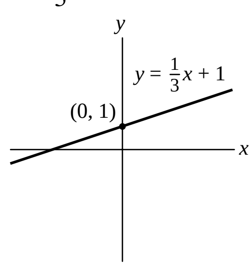
41. l_1

42. l_2

43. Slope = $m = -2$
y-intercept: (0, –1)
$y = mx + b$
$y = -2x - 1$



44. Slope = $m = \frac{1}{3}$
y-intercept: (0, 1)
$y = mx + b$
$y = \frac{1}{3}x + 1$



45. *a* is the *x*-coordinate of the point of intersection of $y = -x + 4$ and $y = 2$. Use substitution to find the *x*-coordinate.
$2 = -x + 4$
$x = 2$
So $a = 2$. $f(a)$ is the *y*-coordinate of the intersection point. So $f(a) = 2$.

46. *a* is the *x*-coordinate of the point of intersection of $y = x$ and $y = \frac{1}{2}x + 1$. Use substitution to find the *x*-coordinate.
$$x = \frac{1}{2}x + 1$$
$$\frac{1}{2}x = 1$$
$$x = 2$$
So $a = 2$. $f(a)$ is the *y*-coordinate of the intersection point. Substituting $x = 2$ into $y = x$ gives $y = 2$.
So $f(a) = 2$.

47. $C(x) = 12x + 1100$

a. $C(10) = 12(10) + 1100 = \$1220$

b. Marginal cost = m = \$12/unit

c. It would cost an additional \$12 to raise the daily production level from 10 units to 11 units.

48. $C(x+1) - C(x)$
$$= \left(12\left(x+1\right)+1100\right) - \left(12x+1100\right)$$
$$= 12x + 12 + 1100 - 12x - 1100$$
$$= \$12$$
\$12 is the marginal cost. It is the additional cost incurred when the production level of this commodity is increased one unit, from x to $x + 1$, per day.

49. Let x be the number of months since January 1, 2004. Then $\left(0, 1.69\right)$ is one point on the line, and the slope is .06. Therefore,

$P(x) = .06x + 1.69$ gives the price of gasoline x months after January 1, 2004.

On April 1, 3 months later, the cost of one gallon of gasoline is:

$P(3) = .06(3) + 1.69 = \$1.87$ /gallon

Therefore, 15 gallons would cost 15 (\$1.87) = \$28.05 on April, 2004.

On September 1, 8 months after January 1, the cost of one gallon of gasoline is:

$P(8) = .06(8) + 1.69 = \$2.17$ /gallon

Therefore, 15 gallons would cost 15 (\$2.17) = \$32.55 on September, 2004.

50. Marginal Cost = 100 $\Rightarrow$ the additional cost for increasing the producing level 1 unit (1 thousand chips) is \$100 thousand dollars. Therefore, the additional cost of increasing the production level by 4 thousand chips is 4 x \$100 thousand = \$400 thousand.

51. The value of the cars in year 0 (2002) is \$125,000. This gives the point (0, 125,000), which is the y-intercept. The depreciation rate is \$25,000 per year, which gives the slope $m = -25{,}000$. The equation of the line in slope-intercept form is
$$y(t) = mt + b$$
$$y(t) = -25{,}000t + 125{,}000$$
Substitute $y(t) = 0$ and solve for t to find the number of years that must pass for the value of the cars to reach zero.
$$0 = -25{,}000t + 125{,}000$$
$$25{,}000t = 125{,}000$$
$$t = 5$$
The value is 0 after 5 years (in 2007).

52. a. The points (5.75, .2) and (6, .18) are on the line. The slope of the line is
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{.18 - .2}{6 - 5.75} = -.08$$
Let $(x_1, y_1) = (6, .18)$. The equation of the line is
$$y - y_1 = m(x - x_1)$$
$$y - .18 = -.08(x - 6)$$
$$y - .18 = -.08x + .48$$
$$y = -.08x + .66$$
$$Q(x) = -.08x + .66$$

b. Let $Q(x) = .1$ (10 employees per 100) and solve for x.
$$.1 = -.08x + .66$$
$$-.56 = -.08x$$
$$x = 7$$
The hourly wage should be \$7.

53. The points (1.97, 1500) and (2.05, 1250) are on the line. The slope of the line is
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1500 - 1250}{1.97 - 2.05} = -3125.$$ Let $(x_1, y_1) = (2.05, 1250)$. The equation of the line is
$$y - y_1 = m(x - x_1)$$
$$y - 1250 = -3125(x - 2.05)$$
$$y - 1250 = -3125x + 6406.25$$
$$y = -3125x + 7656.25$$
$$G(x) = -3125x + 7656.25$$
$$G(2.01) = -3125(2.01) + 7656.25 = 1375 \text{ gallons.}$$

54. Solve for x: $G(x) = -3125x + 7656.25 = 2200$

$\Rightarrow x = \$1.746 \approx \$1.75$ /gallon

55. **a.** $C(x) = mx + b$

$b = \$1500$ (fixed costs)

Total cost of producing 100 rods is \$2200;

$C(100) = m(100) + 1500 = \$2200$

$\Rightarrow m = 7$

$C(x) = 7x + 1500$

b. Marginal cost at $x = 100$ is $m = \$7$/rod

c. Marginal cost = \$7 or

$C(101) - C(100) = 2207 - 2200 = \$7$

56. Each unit sold increases the pay by 5 dollars. The weekly pay is 60 dollars if no units are sold.

57. If the monopolist wants to sell one more unit of goods, then the price per unit must be lowered by 2 cents. No one will pay 7 dollars or more for a unit of goods.

58. x = degrees Fahrenheit, y = degrees Celsius
$0 = 32m / b$ and $100 = 212m / b$ so $b = -32m$ and hence for m, $100 = 212m + -32m$ or
$180m = 100$
$m = \frac{5}{9}$ and $b = -\frac{160}{9}$
Thus, $y = \frac{5}{9}x - \frac{160}{9}$. $y = \frac{5}{9}(98.6) - \frac{160}{9} = 37$
98.6°F corresponds to 37°C.

59. The point (0, 1.5) is on the line and the slope is 6 (ml/min). Let y be the amount of drug in the body x minutes from the start of the infusion. Then

$y - 1.5 = 6(x - 0)$

$y = 6x + 1.5$

60. -2 ml/hour = $-\frac{1}{30}$ ml/min

$y = 6x + 1.5 - \frac{1}{30}x$

$= \frac{179}{30}x + 1.5$

61. $y = x + 7$

$x_1 = 0,\ y_1 = 0 + 7;\ x_2 = 2, y_2 = 2 + 7 = 9$

rate of change over [0,2] = $\frac{9-7}{2-0} = 1$.

$x_1 = -1,\ y_1 = -1 + 7 = 6;\ x_2 = 2, y_2 = 2 + 7 = 9$

rate of change over [-1,2] = $\frac{9-6}{2-(-1)} = 1$.

Slope = m = 1.

62. $y + 2x = 27 \Rightarrow y = -2x + 27$

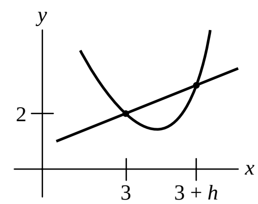
$x_1 = 0,\ y_1 = -2(0) + 27 = 27;\ x_2 = 1, y_2 = -2(1) + 27 = 25$

rate of change over [0,1] = $\frac{25-27}{1-0} = -2$.

$x_1 = 0,\ y_1 = -2(0) + 27 = 27;\ x_2 = 5, y_2 = -2(5) + 27 = 17$

rate of change over [0,1] = $\frac{17-27}{5-0} = -2$. Slope = m = -2.

63. **a–c.**



d. $\frac{f(3+h) - f(3)}{3 + h - 3} = \frac{f(3+h) - f(3)}{h}$

64. Using $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = m$ and the hint,

$\frac{f(x) - f(x_1)}{x - x_1} = m \Rightarrow f(x) - f(x_1) = m(x - x_1)$

$f(x) = m(x - x_1) + f(x_1)$

$= mx + (-mx_1 + f(x_1))$

Let $b = -mx_1 + f(x_1)$. Then $f(x) = mx + b$.

65. $L(x) = -x + 2$, m = -1

$\frac{L(x+h) - L(x)}{h} = \frac{-(x+h) + 2 - (-x+2)}{h}$

$= \frac{-x - h + 2 + x - 2}{h}$

$= -1 = m$

66. $L(x) = \frac{2x+11}{3} = \frac{2}{3}x + \frac{11}{3}$, $m = \frac{2}{3}$

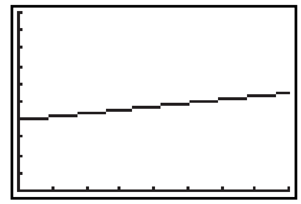
$$\frac{L(x+h)-L(x)}{h} = \frac{\frac{2}{3}(x+h)+\frac{11}{3}-(\frac{2}{3}x+\frac{11}{3})}{h}$$
$$= \frac{\frac{2}{3}x+\frac{2}{3}h+\frac{11}{3}-\frac{2}{3}x-\frac{11}{3}}{h}$$
$$= \frac{2}{3} = m$$

67. Let $y = mx + b$ and $y = m'x + b'$ be two distinct lines. We show that these lines are parallel if and only if $m = m'$. Since two lines are parallel if and only if they have no points in common, it suffices to show that $m = m'$ if and only if the equation $mx + b = m'x + b'$. Suppose $m = m'$. Then $mx + b = m'x + b'$ implies $b = b'$; but since the lines are distinct, $b \neq b'$. Thus if $m = m'$, $mx + b = m'x + b'$ has no solution. If $m \neq m'$, then $x = \frac{b' - b}{m = m'}$ is a solution to $mx + b = m'x + b'$. Thus, $mx + b = m'x + b'$ has no solution in x if and only if $m = m'$, and it follows that two distinct lines are parallel if and only if they have the same slope.

68. Let l_1, l_2, m_1, m_2, a, and b be as in the diagram in the text. Then m_1 is the slope of l_1 and $-m_2$ is the slope of l_2. From the Pythagorean theorem, we have $a^2 + b^2 = (m_1 + m_2)^2$, $1^2 + m_1^2 = a^2$, and $1^2 + m_2^2 = b^2$.
Combining these, we get
$1 + m_1^2 + 1 + m_2^2 = (m_1 + m_2)^2$
$= m_1^2 + 2m_1m_2 + m_2^2$.
Thus, $2 = 2m_1m_2$ and $m_1m_2 = 1$. Since the slope of l_1 is m_1 and the slope of l_2 is $-m_2$, the product of their slopes is therefore -1.

69. **a.** (0, 39.5) and (15, 45.2) on line
slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{45.2 - 39.5}{15 - 0} = \frac{5.7}{15} = .38$
$y - 39.5 = .38(x - 0)$
$y - 39.5 = .38x$
$y = .38x + 39.5$

b.



$[0, 40]$ *by* $[0, 100]$

c. Every year, .38% more of the world population becomes urban.

d. Using the Trace or Evaluate feature on a graphing calculator, the point (10, 43.3) is on the line. Thus, 43.3% of the world population was urban in 1990.

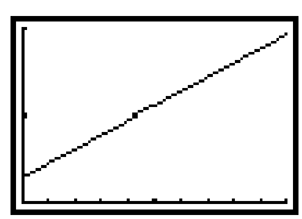
e. Graphing the line $y = 50$ and using the Intersect command, the point (27.63, 50) is on both graphs. In the year $1980 + 27 = 2007$, 50% of the world population will be urban.

f. From the slope, .38% more of the world population becomes urban each year. Thus, in 5 years the percentage of the world population that is urban has increased by 5(.38%) = 1.9%.

70. **a.** (20,000, 729) and (50,000, 1380) on line.
$$= \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{1380 - 729}{50,000 - 20,000}$$
$$= \frac{651}{30,000}$$
slope $= 0.217$

$y - 1380 = .0217(x - 50,000)$
$y - 1380 = .0217x - 1085$
$y = .0217x + 295$

b.



$[0, 75,000]$ *by* $[0, 2,000]$

c. For every increase of \$1 in reported income, the average itemized deductions increase by \$.0217. (Alternatively, an increase of \$100 in reported income corresponds to an average increase of \$2.17 in itemized deductions.)

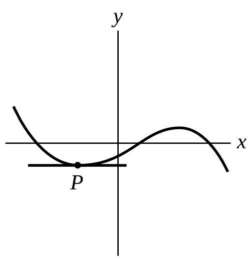
d. Using the Trace or Evaluate feature on a graphing calculator, the point (75,000, 1992.5) is on the line. Thus, the average amount of itemized deductions on a return reporting income of \$75,000 is \$1922.5.

e. Graphing the line $y = 5000$ and using the Intersect command, the point (216,820.28, 5000) is on both lines. An average itemized deduction of \$5000 corresponds to a reported income of \$216,820 (rounded to the nearest dollar).

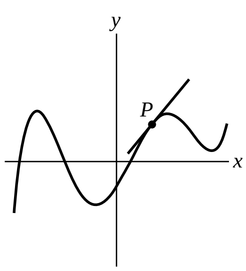
f. An increase of \$15,000 in income level will correspond to an increase of \$15,000(.0217) = \$325.5 in itemized deductions.

Exercises 1.2

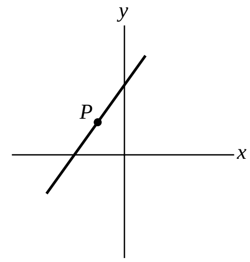
1.



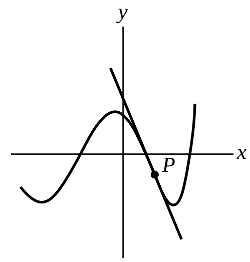
2.



3.



4.



5. $-\frac{4}{3}$

6. 0

7. 1

8. 1

9. Small positive slope

10. Large positive slope

11. Zero slope

12. Large negative slope

13. Zero slope

14. Small negative slope

15.

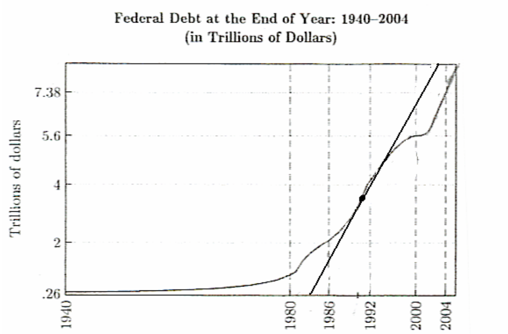


Figure 12. U.S. federal debt.

It appears the points (1984, .26) and (1990, 3.2) are on the line. The slope is

$$m = \frac{3.2 - .26}{1990 - 1984} = .49$$

Therefore, the annual rate of increase of the federal debt in 1990 is approximately \$.49 trillion/year.

16.

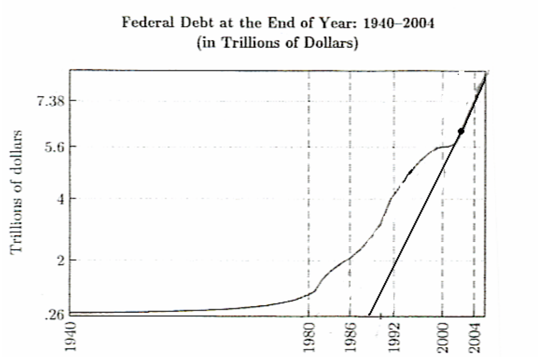


Figure 12. U.S. federal debt.

It appears the points (2000, 4.8) and (2002, 6.49) are on the line. The slope is $m = \frac{6.49 - 4.8}{2002 - 2000}$ $= .845$. Therefore, the annual rate of increase of the federal debt in 2002 is approximately \$.845 trillion/year

17. a. In 1950, dept per capita □ \$1000

In 1990, dept per capita □ \$14000

In 2001, dept per capita □ \$20500

In 2004, dept per capita □ \$24000

b.

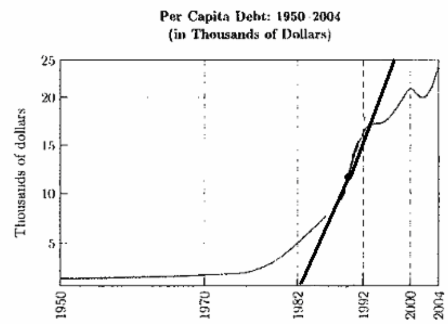


Figure 13. Debt per capita in the U.S.

It appears the points (1982, 0) and (1992, 17.5) are on the line. The slope is

$$m = \frac{17.5 - 0}{1992 - 1982} = 1.75$$

Therefore, the annual rate of increase of the debt per capita in 1990 is approximately \$1.75 thousand/year.

18. a. True, the rate of increase in 1980 >0, the rate of increase in 2000 □ 0 .

b. True, the curve is close to constant up to the mid-1970's and then increases linearly (at a constant rate) from the mid-1970's to the mid-1980's.

For 19–28, note that the slope of the line tangent to the graph of $y = x^2$ at the point (*x*, *y*) is 2*x*.

19. The slope at the point (–2, 4) is 2(–2) = –4.
Let $(x_1, y_1) = (-2, 4)$, $m = -4$.
$y - 4 = -4(x - (-2))$
$y = -4x - 4$

20. At (–.4, .16), 2*x* = 2(–.4) = –.8.
Let $(x_1, y_1) = (-.4, .16)$, $m = -.8$.
$y - .16 = -.8(x - (-.4))$
$y = -.8x - .16$

21. At $\left(\frac{4}{3}, \frac{16}{9}\right)$, slope $= m = 2x = 2\left(\frac{4}{3}\right) = \frac{8}{3}$.

Let $(x_1, y_1) = \left(\frac{4}{3}, \frac{16}{9}\right)$.

$y - \frac{16}{9} = \frac{8}{3}\left(x - \frac{4}{3}\right)$

$y = \frac{8}{3}x - \frac{16}{9}$

22. When $x = -\frac{1}{2}$, slope $= 2\left(-\frac{1}{2}\right) = -1$.

23. When $x = 1.5$, slope $= 2(1.5) = 3$ and $y = (1.5)^2 = 2.25$. Let $(x_1, y_1) = (1.5, 2.25)$, $m = 3$.

$y - 2.25 = 3(x - 1.5)$

$y = 3x - 2.25$

24. When $x = .6$, slope $= 2(.6) = 1.2$ and $y = (.6)^2 = .36$. Let $(x_1, y_1) = (.6, .36)$, $m = 1.2$.

$y - .36 = 1.2(x - .6)$

$y = 1.2x - .36$

25. Set $2x = \frac{5}{3}$

$x = \frac{5}{6}$

When $x = \frac{5}{6}$, $y = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$, so $\left(\frac{5}{6}, \frac{25}{36}\right)$ is the point.

26. Set $2x = -4$

$x = -2$

When $x = -2$, $y = (-2)^2 = 4$, so $(-2, 4)$ is the point.

27. $x + 2y = 4$

$y = -\frac{1}{2}x + 2$

so slope $= m = -\frac{1}{2}$.

Slope of the tangent line is $2x$.

Set $2x = -\frac{1}{2}$

$x = -\frac{1}{4}$

When $x = -\frac{1}{4}$, $y = \left(-\frac{1}{4}\right)^2 = \frac{1}{16}$, so $\left(-\frac{1}{4}, \frac{1}{16}\right)$ is the point.

28. $3x - y = 2$

$y = 3x - 2$

slope = 3

The slope of the tangent line is $2x$.

Set $2x = 3$

$x = \frac{3}{2}$

When $x = \frac{3}{2}$, $y = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$, so $\left(\frac{3}{2}, \frac{9}{4}\right)$ is the point.

29. Slope $= 3x^2$

When $x = 2$, slope $= 3(2)^2 = 12$.

30. Slope $= 3x^2$

When $x = \frac{3}{2}$, slope $= 3\left(\frac{3}{2}\right)^2 = \frac{27}{4}$.

31. Slope $= 3x^2$

When $x = -\frac{1}{2}$, slope $= 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$.

32. When $x = -1$, slope $= 3(-1)^2 = 3$, $y = (-1)^3 = -1$.

Let $(x_1, y_1) = (-1, -1)$.

$y - (-1) = 3(x - (-1))$

$y = 3x + 2$

33. The slope of the line tangent to $y = x^2$ at $x = a$ is $2a$. The slope of $y = 2x - 1$ is 2. Equating these gives: $2a = 2 \Rightarrow a = 1$.

So, $f(a) = (1)^2 = 1$, $f'(1) = 2(1) = 2$

34. The slope of the line tangent to $y = x^2$ at $x = a$ is $2a$. The slope of $y = -x - \frac{1}{4}$ is -1. Equating these gives: $2a = -1 \Rightarrow a = -\frac{1}{2}$.

So, $f(a) = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$, $f'\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) = -1$

35. The graphs of $y = \frac{1}{4}x + 1$ and $y = \frac{1}{2}x$ intersect at $(a, f(a))$. Use substitution to find a.

$\frac{1}{2}x = \frac{1}{4}x + 1;\ \frac{1}{4}x = 1;\ x = 4$

So $a = 4$ and $f(a) = \frac{1}{2}(4) = 2$.

The slope of the tangent line is $\frac{1}{4}$, so $f'(4) = \frac{1}{4}$.

36. The graphs of $y = -\frac{1}{5}x + \frac{6}{5}$ and $y = 2 - x$ intersect at $(a, f(a))$. Use substitution to find a.

$-\frac{1}{5}x + \frac{6}{5} = 2 - x$

$-x + 6 = 10 - 5x;\ 4x = 4;\ x = 1$

So $a = 1$ and $f(a) = 2 - 1 = 1$.

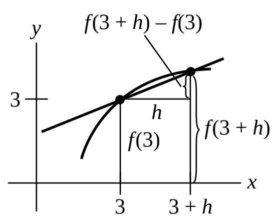
The slope of the tangent line is -1, so $f'(a) = -1$.

37. a. $m = \frac{13-4}{5-2} = 3$

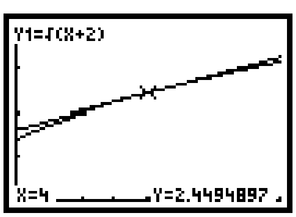
length of d is $13 - 4 = 9$

b. Increase

38.



39.



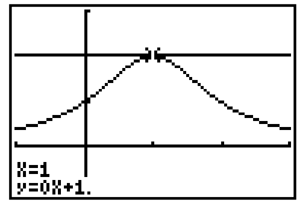
[0, 8] *by* [0, 4]

At $x = 4$, $y \approx 2.45$

$\frac{dy}{dx} \approx .2$

In point-slope form, the tangent line is $y - 2.45 = .2(x - 4)$.

40.



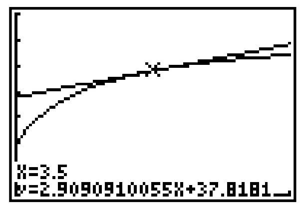
[21, 3] *by* [2.5, 1.5]

At $x = 1$, $y = 1$

$\frac{dy}{dx} = 0$

In point-slope form, the tangent line is $y - 1 = 0(x - 1)$ or $y - 1 = 0$.

41.



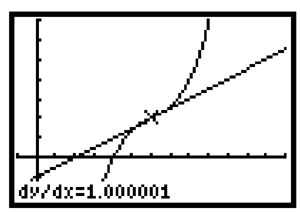
[0, 7] *by* [0, 70]

At $x = 3.5$, $y = 48$

$\frac{dy}{dx} \approx 2.9$

In point-slope form, the tangent line is $y - 48 = 2.9(x - 3.5)$.

42.



[2.5, 6.5] *by* [22, 7]

At $x = 3$, $y = 2$

$\frac{dy}{dx} = 1$

In point-slope form, the tangent line is $y - 2 = x - 3$.

Exercises 1.3

1. $f(x) = 2x - 5,\ f'(x) = 2$

2. $f(x) = -3x + 7,\ f'(x) = -3$

3. $f(x) = x^5,\ f'(x) = 5x^4$

4. $f(x) = x^{19},\ f'(x) = 19x^{18}$

5. $f(x)=x^{\frac{2}{5}},\ f'(x)=\frac{2}{5}x^{-\frac{3}{5}}$

6. $f(x)=\frac{1}{\sqrt{x}}=x^{-\frac{1}{2}},\ f'(x)=-\frac{1}{2}x^{-\frac{3}{2}}$

7. $f(x)=\sqrt[3]{x}=x^{1/3},\ f'(x)=\frac{1}{3}x^{-2/3}$

8. $f(x)=x^{\frac{4}{3}},\ f'(x)=\frac{4}{3}x^{\frac{1}{3}}$

9. $f(x)=\frac{1}{x^2}=x^{-2},\ f'(x)=-2x^{-3}$

10. $f(x)=3^2=9,\ f'(x)=0$

11. $f(x)=x^{-\frac{5}{7}},\ f'(x)=-\frac{5}{7}x^{-\frac{12}{7}}$

12. $f(x)=\frac{1}{x^{-5}}=x^5,\ f'(x)=5x^4$

13. $f(x)=\frac{3}{4},\ f'(x)=0$

14. $f(x)=\frac{1}{\sqrt[7]{x}}=x^{-\frac{1}{7}},\ f'(x)=-\frac{1}{7}x^{-\frac{8}{7}}$

15. $f(x)=\sqrt[3]{x}=x^{-\frac{1}{3}},\ f'(x)=-\frac{1}{3}x^{-\frac{4}{3}}$

16. $f(x)=\sqrt[5]{x}=x^{\frac{1}{5}},\ f'(x)=\frac{1}{5}x^{-\frac{4}{5}}$

17. $f(x)=x^3$ at $x=1$
$f'(x)=3x^2$
$f'(1)=3(1)^2=3$

18. $f(x)=x^5$ at $x=\frac{2}{3}$
$f'(x)=5x^4$
$f'\left(\frac{2}{3}\right)=5\left(\frac{2}{3}\right)^4=\frac{80}{81}$

19. $f(x)=\frac{1}{x}$ at $x=3$
$f(x)=x^{-1}$
$f'(x)=-x^{-2}$
$f'(3)=-\frac{1}{3^2}=-\frac{1}{9}$

20. $f(x)=3^2$ at $x=1$

$f(x)=9$
$f'(x)=0$
$f'(1)=0$

21. $f(x)=4x+11$ at $x=$ -1

$f'(x)=4$
$f'(-1)=4$

22. $f(x)=x^{\frac{3}{2}}$ at $x=4$
$f'(x)=\frac{3}{2}x^{\frac{1}{2}}$
$f'(4)=\frac{3}{2}(4)^{\frac{1}{2}}=\frac{3}{2}\cdot 2=3$

23. $f(x)=\sqrt{x}$ at $x=16$
$f(x)=x^{\frac{1}{2}}$
$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$
$f'(16)=\frac{1}{2\sqrt{16}}=\frac{1}{8}$

24. $f(x)=\frac{1}{x^6}$ at $x=2$
$f(x)=x^{-6}$
$f'(x)=-6x^{-7}=-\frac{6}{x^7}$
$f'(2)=-\frac{6}{2^7}=-\frac{3}{64}$

25. $y=x^4$
slope $=y'=4x^3$
at $x=3,\ y'=4(3)^3=108$

26. $y=x^5$
slope $=y'=5x^4$
at $x=-2,\ y'=5(-2)^4=80$

27. $f(x) = x^2$
$f(-5) = (-5)^2 = 25$
$f'(x) = 2x$
$f'(-5) = 2(-5) = -10$

28. $f(x) = x + 6$
$f(3) = 3 + 6 = 9$
$f'(x) = 1$
$f'(3) = 1$

29. $f(x) = x^{4/3}$
$f(8) = (8)^{4/3} = 16$
$f'(x) = \frac{4}{3}x^{1/3} = \frac{4}{3}\sqrt[3]{x}$
$f'(8) = \frac{4}{3}\sqrt[3]{8} = \frac{8}{3}$

30. $f(x) = \frac{1}{x^2} = x^{-2}$
$f(5) = \frac{1}{5^2} = \frac{1}{25}$
$f'(x) = -2x^{-3} = -\frac{2}{x^3}$
$f'(5) = -\frac{2}{5^3} = -\frac{2}{125}$

31. $f(x) = \frac{1}{x^5} = x^{-5}$
$f(2) = \frac{1}{2^5} = \frac{1}{32}$
$f'(x) = -5x^{-6} = -\frac{5}{x^6}$
$f'(2) = -\frac{5}{2^6} = -\frac{5}{64}$

32. $f(x) = x^{3/2}$
$f(16) = (16)^{3/2} = 64$
$f'(x) = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}$
$f'(16) = \frac{3}{2}\sqrt{16} = \frac{12}{2} = 6$

33. $f(x) = x^2$, $f(-\frac{1}{2}) = (-\frac{1}{2})^2 = \frac{1}{4}$
$f'(x) = 2x$**,** $f'(-\frac{1}{2}) = 2(-\frac{1}{2}) = -1$
point: $\left(-\frac{1}{2}, \frac{1}{4}\right)$, m = -1
$y - \frac{1}{4} = -1\left(x - (-\frac{1}{2})\right)$
$y - \frac{1}{4} = -x - \frac{1}{2}$
$y = -x - \frac{1}{4}$

34. $f(x) = x^3$, $f(-2) = (-2)^3 = -8$
$f'(x) = 3x^2$**,** $f'(-2) = 3(-2)^2 = 12$
point: $(-2, -8)$, $m = 12$
$y - (-8) = 12\left(x - (-2)\right)$
$y + 8 = 12x + 24$
$y = 12x + 16$

35. $f(x) = 3x + 1$, $f(4) = 3(4) + 1 = 13$,
$f'(x) = 3$, $f'(4) = 3$
point: $(4, 13)$, $m = 3$
$y - 13 = 3(x - 4)$
$y - 13 = 3x - 12$
$y = 3x + 1$

36. $f(x) = 5$, $f(-2) = 5$
$f'(x) = 0$, $f'(-2) = 0$
point: $(-2, 5)$, $m = 0$
$y - 5 = 0\left(x - (-2)\right)$
$y - 5 = 0$
$y = 5$

37. $f(x) = \sqrt{x} = x^{1/2}$, $f(9) = 3$
$f'(x) = \frac{1}{2}x^{-1/2}$, $f'(9) = \frac{1}{6}$
point: (9, 3), $m = \frac{1}{6}$
$y - 3 = \frac{1}{6}(x - 9)$; $y - 3 = \frac{1}{6}x - \frac{3}{2}$
$y = \frac{1}{6}x + \frac{3}{2}$

38. $f(x) = \frac{1}{x} = x^{-1}$, $f(.01) = \frac{1}{.01} = 100$

$f'(x) = -x^{-2}$, $f'(.01) = -(.01)^{-2} = -10000$

point: $(.01, 100), m = -10000$

$y - 100 = -10000(x - .01)$

$y - 100 = -10000x + 100$

$y = 10000x + 200$

39. $f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$, $f(4) = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$f'(x) = -\frac{1}{2}x^{-3/2}$, $f'(4) = -\frac{1}{16}$

point: $\left(4, \frac{1}{2}\right)$, $m = -\frac{1}{16}$

$y - \frac{1}{2} = -\frac{1}{16}(x - 4)$

$y - \frac{1}{2} = -\frac{1}{16}x + \frac{1}{4}$

$y = -\frac{1}{16}x + \frac{3}{4}$

40. $f(x) = \frac{1}{x^2} = x^{-2}$, $f(1) = \frac{1}{1^2} = 1$

$f'(x) = -2x^{-3}$, $f'(1) = -2(1)^{-3} = -2$

point: $(1,1)$, $m = -2$

$y - 1 = -2(x - 1)$

$y - 1 = -2x + 2$

$y = -2x + 3$

41. $y - f(a) = f'(a)(x - a)$

$y = f(x) = x^4$

$y' = f'(x) = 4x^3$

$a = 1$, $f(a) = f(1) = 1$

$f'(a) = f'(1) = 4$

$y - 1 = 4(x - 1)$

42. The tangent is perpendicular to $y = 4x + 1$, so the slope of the tangent is $m = -1/4$.

$f(x) = \frac{1}{x} = x^{-1}$, $f'(x) = -x^{-2}$. The slope of the tangent is $f'(a) = -a^{-2}$. Solving $-a^{-2} = -1/4$, $a = \pm 2$. Therefore, $P = \left(2, 1/2\right)$ or $P = \left(-2, -1/2\right)$.

43. The slope of the tangent is $m = 2$.

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$, $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$. The slope of the tangent is $f'(a) = \frac{1}{2}a^{-\frac{1}{2}}$. Solving $\frac{1}{2}a^{-\frac{1}{2}} = 2$, $a = \frac{1}{16}$. Therefore $P = \left(\frac{1}{16}, \frac{1}{4}\right)$. Also, $\frac{1}{4} = 2\left(\frac{1}{16}\right) + b \Rightarrow b = \frac{1}{8}$.

44. The slope of the tangent is $m = a$.

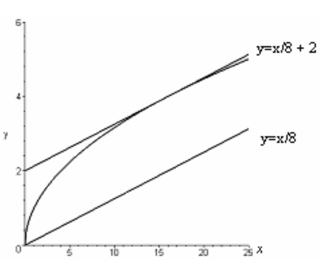
$f(x) = x^3$, $f'(x) = 3x^2$. The slope of the tangent is $f'(-3) = 3(-3)^2 = 27$. Therefore $a = 27$. Also, $-27 = 27(-3) + b \Rightarrow b = 54$.

45. a. The slope of the tangent line is $m = \frac{1}{8}$.

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$, $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$. The slope of the tangent is $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$. Solving $\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{8}$, $x = 16$; $f(16) = \sqrt{16} = 4$.

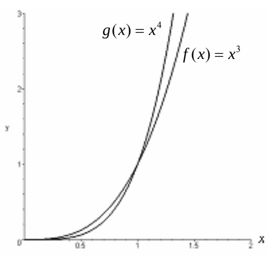
Therefore, the point we are looking for is $(16, 4)$.

b.



46. Slope of graph at point $x = \; y' = -x^{-2} = -\frac{1}{x^2}$. But $-\frac{1}{x^2} < 0$ for all x, so, no, there is no point on the graph where the slope is positive. This implies the graph is always decreasing.

47. a.

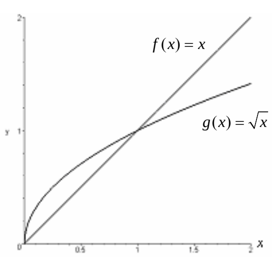


b. $f(x) = x^3;\; f'(x) = 3x^2,\; f'(1) = 3(1)^2 = 3$

$g(x) = x^4;\; g'(x) = 4x^3,\; g'(1) = 4(1)^3 = 4$

Clearly $g'(1)$ is larger. This implies the graph of $g(x)$ is increasing at $x = 1$, and is steeper than $f(x)$ at $x = 1$.

48. a.



b. $f(1) = 1;\; f'(x) = 1 \Rightarrow f'(1) = 1$

$g(1) = \sqrt{1} = 1;\; g'(x) = \frac{1}{2}x^{-\frac{1}{2}} \Rightarrow g'(1) = \frac{1}{2}$

49. $\frac{d}{dx}(x^8) = 8x^7$

50. $\frac{d}{dx}(x^{-3}) = -3x^{-4}$

51. $\frac{d}{dx}(x^{3/4}) = \frac{3}{4}x^{-1/4}$

52. $\frac{d}{dx}(x^{-1/3}) = -\frac{1}{3}x^{-4/3}$

53. $y = 1,\; \frac{d}{dx}(1) = 0$

54. $y = x^{-4},\; \frac{d}{dx}(x^{-4}) = -4x^{-5}$

55. $y = x^{1/5},\; \frac{d}{dx}(x^{1/5}) = \frac{1}{5}x^{-4/5}$

56. $y = \frac{x-1}{3} = \frac{1}{3}x - \frac{1}{3},\; \frac{d}{dx}\left[\frac{x-1}{3}\right] = \frac{1}{3}$

57. The tangent line at $x = b$ is $y = \frac{1}{3}x + 2$, so

$f(6) = \frac{1}{3}(6) + 2 = 4.$

The slope of $y = \frac{1}{3}x + 2$ is $\frac{1}{3}$, so $f'(6) = \frac{1}{3}$.

58. The tangent line at $x = 1$ is $y = 4$, so $f(1) = 4$. The slope of $y = 4$ is 0, so $f'(1) = 0$.

59. $y = f(x) = \sqrt{x} = x^{1/2}$

slope $= f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

The slope of the tangent line $y = \frac{1}{4}x + b$ is $\frac{1}{4}$.

First, find the value of a. Let $\frac{1}{4} = \frac{1}{2\sqrt{a}}$ and solve for a.

$2\sqrt{a} = 4$

$\sqrt{a} = 2$

$a = 4$

When $x = 4$, $f(4) = \sqrt{4} = 2$.

Let $(x_1, y_1) = (4, 2)$.

$y - 2 = \frac{1}{4}(x - 4)$

$y = \frac{1}{4}x - 1 + 2$

$y = \frac{1}{4}x + 1$, so $b = 1$.

60. $y = \frac{1}{x}$

When $x = 2,\ y = \frac{1}{2}$.

slope $= f'(x) = -x^{-2} = -\frac{1}{x^2}$

$f'(2) = -\frac{1}{4}$

To find the equation of the tangent line set $(x_1,\ y_1) = \left(2, \frac{1}{2}\right),\ m = -\frac{1}{4}$.

$y - \frac{1}{2} = -\frac{1}{4}(x-2)$

$y = -\frac{1}{4}x + 1$

To find the value of *a* (which is the *x*-intercept), let $-\frac{1}{4}x + 1 = 0$ and solve for *x*.

$-\frac{1}{4}x + 1 = 0$

$x = 4 = a$

61. At $x = a$, $y = 2.01a - .51$, or $y = 2.02a - .52$, so $.01a = .01$.
$a = 1$, and $y = f(a) = 2.01 - .51 = 1.5$.
$f'(a) = 2$ because the slope of the "smallest" secant line is 2.01.

62. $\frac{f(1+.2) - f(1)}{.2} = \frac{1.1 - .8}{.2} = 1.5$

63. $\frac{f(x+h) - f(x)}{h}$

$= \frac{(x+h)^2 - 2(x+h) + 3 - (x^2 - 2x + 3)}{h}$

$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 3 - x^2 + 2x - 3}{h}$

$= \frac{h(2x+h-2)}{h}$

$= 2x + h - 2$

64. $\frac{f(x+h) - f(x)}{h}$

$= \frac{-2(x+h)^2 + (x+h) + 1 - (-2x^2 + x + 1)}{h}$

$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 1 + 2x^2 - x - 1}{h}$

$= \frac{h(-4x - 2h + 1)}{h}$

$= -4x - 2h + 1$

65. $\frac{f(x+h) - f(x)}{h}$

$= \frac{-(x+h)^2 - (x+h) - 1 - (-x^2 - x - 1)}{h}$

$= \frac{-x^2 - 2xh - h^2 - x - h - 1 + x^2 + x + 1}{h}$

$= \frac{h(-2x - h - 1)}{h}$

$= -2x - h - 1$

66. $\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{2}(x+h)^2 + \frac{1}{2} - (\frac{1}{2}x^2 + \frac{1}{2})}{h}$

$= \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 + \frac{1}{2} - \frac{1}{2}x^2 - \frac{1}{2}}{h}$

$= \frac{h(x + \frac{1}{2}h)}{h}$

$= x + \frac{1}{2}h$

67. $\frac{f(x+h) - f(x)}{h}$

$= \frac{(x+h)^3 + (x+h) + 2 - (x^3 + x + 2)}{h}$

$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 + x + h + 2 - x^3 - x - 2}{h}$

$= \frac{h(3x^2 + 3xh + h^2 + 1)}{h}$

$= 3x^2 + 3xh + h^2 + 1$

68. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{(x+h)^3-2(x+h)^2+\sqrt{5}-(x^3-2x^2+\sqrt{5})}{h}$

$=\dfrac{x^3+3x^2h+3xh^2+h^3-2x^2-4xh-2h^2+\sqrt{5}-x^3+2x^2-\sqrt{5}}{h}$

$=\dfrac{h(3x^2+3xh+h^2-4x-2h)}{h}$

$=3x^2+3xh+h^2-4x-2h$

69. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\dfrac{2}{x+h+2}-\dfrac{2}{x+2}}{h}=\dfrac{\dfrac{2(x+2)-2(x+h+2)}{(x+h+2)(x+2)}}{h}$

$=\dfrac{\dfrac{-2h}{(x+h+2)(x+2)}}{h}=\dfrac{-2}{(x+h+2)(x+2)}$

70. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\dfrac{1}{(x+h)^2+1}-\dfrac{1}{x^2+1}}{h}=\dfrac{\dfrac{(x^2+1)-((x+h)^2+1)}{((x+h)^2+1)(x^2+1)}}{h}$

$=\dfrac{\dfrac{x^2+1-x^2-2xh-h^2-1}{((x+h)^2+1)(x^2+1)}}{h}=\dfrac{\dfrac{h(-2x-h)}{((x+h)^2+1)(x^2+1)}}{h}$

$=\dfrac{-2x-h}{((x+h)^2+1)(x^2+1)}$

71. $f(x)=x^2+1,\ f(1)=1^2+1=2$

$\dfrac{f(1+h)-f(1)}{h}=\dfrac{(1+h)^2+1-2}{h}$

$=\dfrac{h(2+2h)}{h}=2+2h$

h	$2+2h$	**h**	$2+2h$
1	4	-1	0
.1	2.2	-.1	1.8
.01	2.02	-.01	1.98
.001	2.002	-.001	1.998

Conclude: $f'(1)=2$

72. $f(x) = -x^2 + 2,\ f(-1) = -(-1)^2 + 2 = 1$

$$\frac{f(-1+h) - f(-1)}{h} = \frac{-(-1+h)^2 + 2 - 1}{h}$$
$$= \frac{h(2-h)}{h} = 2 - h$$

h	$2-h$	**h**	$2-h$
1	1	-1	3
.1	1.9	-.1	2.1
.01	1.99	-.01	2.01
.001	1.999	-.001	2.001

Conclude: $f'(-1) = 2$

73. $f(x) = x^3,\ f(2) = 2^3 = 8$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^3 - 8}{h}$$
$$= \frac{8 + 12h + 6h^2 + h^3 - 8}{h}$$
$$= \frac{h(12 + 6h + h^2)}{h} = 12 + 6h + h^2$$

h	$12 + 6h + h^2$	**h**	$12 + 6h + h^2$
1	19	-1	7
.1	12.61	-.1	11.41
.01	12.0601	-.01	11.9401
. 001	12.006001	-.001	11.994001

Conclude: $f'(2) = 12$

74. $f(x) = -3x^3 + 1,\ f(-1) = -3(-1)^3 + 1 = 4$

$$\frac{f(-1+h) - f(-1)}{h} = \frac{-3(-1+h)^3 + 1 - 4}{h}$$
$$= \frac{-3(-1 + 3h - 3h^2 + h^3) + 1 - 4}{h}$$
$$= \frac{h(-9 + 9h - 3h^2)}{h} = -9 + 9h - 3h^2$$

h	$-9 + 9h - 3h^2$	***h***	$-9 + 9h - 3h^2$
1	-3	*-1*	*-21*
.1	-8.13	-*.1*	*-9.93*
.01	-8.9103	-*.01*	*-9.0903*
. 001	*-8.991003*	*-.001*	*-9.009003*

Conclude: $f'(-1) = -9$

75. $f(x) = x^2 + 4$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 4 - (x^2 + 4)}{h}$$
$$= \frac{x^2 + 2xh + h^2 + 4 - x^2 - 4}{h} = \frac{h(2x+h)}{h}$$
$$= 2x + h$$

As h approaches 0, the expression $2x + h$ approaches $2x$. Conclude $f'(x) = 2x$.

76. $f(x) = 2x^2 + x$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 + x + h - (2x^2 + x)}{h}$$
$$= \frac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h} = \frac{h(4x + 2h + 1)}{h}$$
$$= 4x + 2h + 1$$

As h approaches 0, the expression $4x + 2h + 1$ approaches $4x + 1$. Conclude $f'(x) = 4x + 1$.

77. $f(x)=-2x^2-1$

$$\frac{f(x+h)-f(x)}{h}=\frac{-2(x+h)^2-1-(-2x^2-1)}{h}$$
$$=\frac{-2x^2-4xh-2h^2-1+2x^2+1}{h}=\frac{h(-4x-2h)}{h}$$
$$=-4x-2h$$

As h approaches 0, the expression $-4x-2h$ approaches $-4x$. Conclude $f'(x)=-4x$.

78. $f(x)=-x^2+\frac{1}{2}x-2$

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{-(x+h)^2+\frac{1}{2}(x+h)-2-(-x^2+\frac{1}{2}x-2)}{h}$$
$$=\frac{-x^2-2xh-h^2+\frac{1}{2}x+\frac{1}{2}h-2+x^2-\frac{1}{2}x+2}{h}$$
$$=\frac{h(-2x-h+\frac{1}{2})}{h}=-2x-h+\frac{1}{2}$$

As h approaches 0, the expression $-2x-h+\frac{1}{2}$ approaches $-2x+\frac{1}{2}$. Conclude $f'(x)=-2x+\frac{1}{2}$.

79. $f(x)=x^3$

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^3-x^3}{h}$$
$$=\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}=\frac{h(3x^2+3xh+h^2)}{h}$$
$$=3x^2+3xh+h^2$$

As h approaches 0, the expression $3x^2+3xh+h$ approaches $3x^2$. Conclude $f'(x)=3x^2$.

80. $f(x)=2x^3+1$

$$\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)^3+1-(2x^3+1)}{h}$$
$$=\frac{2x^3+6x^2h+6xh^2+2h^3+1-2x^3-1}{h}$$
$$=\frac{h(6x^2+6xh+2h^2)}{h}$$
$$=6x^2+6xh+2h^2$$

As h approaches 0, the expression $6x^2+6xh+2h^2$ approaches $6x^2$. Conclude $f'(x)=6x^2$.

81. $f(x)=2x-1$

$$\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)-1-(2x-1)}{h}$$
$$=\frac{2x+2h-1-2x+1}{h}=\frac{2h}{h}=2$$

As h approaches 0, the expression 2 is just 2. Conclude $f'(x)=2$.

82. $f(x)=5$

$$\frac{f(x+h)-f(x)}{h}=\frac{5-5}{h}=0$$

As h approaches 0, the expression 0 is just 0. Conclude $f'(x)=0$.

83. $f(x)=mx+b$

$$\frac{f(x+h)-f(x)}{h}=\frac{m(x+h)+b-(mx+b)}{h}$$
$$=\frac{mx+mh+b-mx-b}{h}=\frac{mh}{h}=m$$

As h approaches 0, the expression m is just m. Conclude $f'(x)=m$.

84. $f(x)=ax^2$

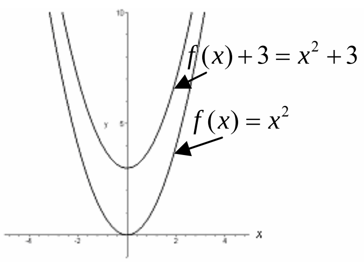
$$\frac{f(x+h)-f(x)}{h}=\frac{a(x+h)^2-ax^2}{h}$$
$$=\frac{ax^2+2axh+ah^2-ax^2}{h}=\frac{h(2ax+ah)}{h}$$
$$=2ax+ah$$

As h approaches 0, the expression $2ax+ah$ is $2ax$. Conclude $f'(x)=2ax$.

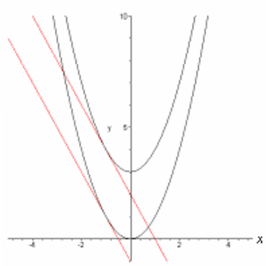
85. The coordinates of A are (4, 5). Reading the graph of the derivative, we see that $f'(4) = \frac{1}{2}$, so the slope of the tangent line is $\frac{1}{2}$. By the point-slope formula, the equation of the tangent line is:
$y - 5 = \frac{1}{2}(x - 4)$

86. The coordinates of P are (2, 1.75). Reading the graph of the derivative, we see that $f'(2) = \frac{1}{2}$, so the slope of the tangent line is $\frac{1}{2}$. By the point-slope formula, the equation of the tangent line is:
$y - 1.75 = \frac{1}{2}(x - 2)$

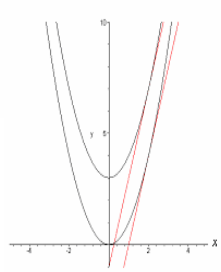
87. a. Let $f(x) = x^2$. Then $f(x) + 3 = x^2 + 3$:



b. Let $x = -1$:



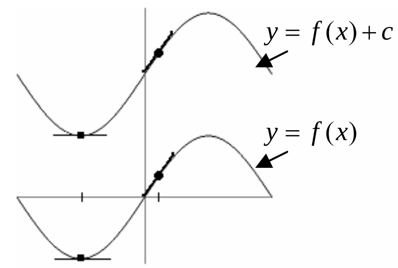
Let $x = 2$:



In both cases the tangent lines are parallel.

c. A vertical shift in a graph does not change its shape. Therefore, the slope at any point x remains the same for any shift in the y-direction.

88.



Observe the tangent lines for the two chosen values of x. For each value of x, the slopes of the lines tangent to $f(x)$ and $f(x)+c$ at x are the same.

89. $f'(0)$, where $f(x) = 2^x$
nDeriv$(2^X$, X, 0) = .69315

90. $f'(1)$, where $f(x) = \frac{1}{1+x^2}$
nDeriv(1/(1+X^2), X, 1) = –.5

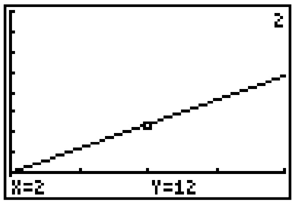
91. $f'(1)$, where $f(x) = \sqrt{1+x^2}$
nDeriv$\left(\sqrt{1+X^2}, X, 1\right)$ = .70711

92. $f'(3)$, where $f(x) = \sqrt{25-x^2}$
nDeriv$\left(\sqrt{25-X^2}, X, 3\right)$ = –.75

93. $f'(2)$, where $f(x) = \frac{x}{1+x}$
nDeriv(X/(1+X), X, 2) = .11111

94. $f'(0)$, where $f(x) = 10^{1+x}$
nDeriv(10 ^ (1 + X), X, 0) = 23.02587

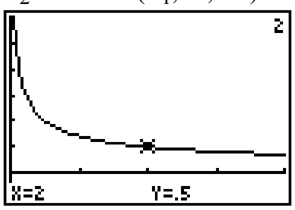
95. $Y_1 = 3X^2 - 5$
$Y_2 = \text{nDeriv}(Y_1, X, X)$



$[0, 4]$ *by* $[25, 40]$

Value of the derivative of Y_1 at $x = 2$ is 12.

96. $Y_1 = \sqrt{2X}$
$Y_2 = \text{nDeriv}(Y_1, X, X)$



$[0, 4]$ *by* $[2.5, 3]$

Value of the derivative of Y_1 at $x = 2$ is 0.5.

97. $f(x) = \sqrt{x},\ (9, 3)$

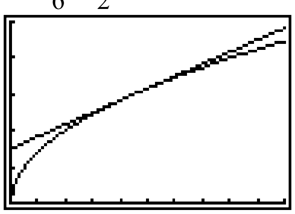
slope $= f'(x) = \frac{1}{2}x^{-1/2}$

At $x = 9$, slope $= \frac{1}{2}(9)^{-1/2} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$y - 3 = \frac{1}{6}(x - 9)$

$y - 3 = \frac{x}{6} - \frac{3}{2}$

$y = \frac{x}{6} + \frac{3}{2}$



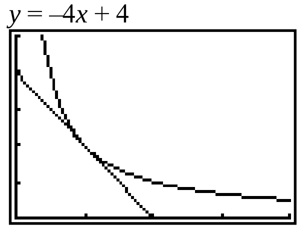
$[0, 20]$ *by* $[0, 5]$

98. $f(x) = \frac{1}{x},\ (.5, 2)$

slope $= f'(x) = -\frac{1}{x^2}$

At $x = .5$, slope $= -\frac{1}{(.5)^2} = -4$

$y - 2 = -4(x - .5)$
$y - 2 = -4x + 2$
$y = -4x + 4$



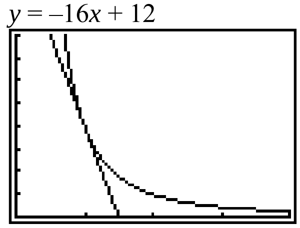
$[0, 2]$ *by* $[0, 5]$

99. $f(x) = \frac{1}{x^2},\ (.5, 4)$

slope $= f'(x) = -\frac{2}{x^3}$

At $x = .5$, slope $= -\frac{2}{(.5)^3} = -16.$

$y - 4 = -16(x - .5)$
$y - 4 = -16x + 8$
$y = -16x + 12$



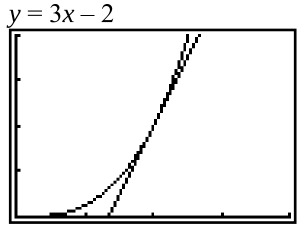
$[0, 2]$ *by* $[0, 8]$

100. $f(x) = x^3,\ (1, 1)$

slope $= f'(x) = 3x^2$

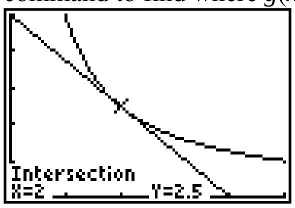
At $x = 1$, slope $= 3(1)^2 = 3.$

$y - 1 = 3(x - 1)$
$y - 1 = 3x - 3$
$y = 3x - 2$



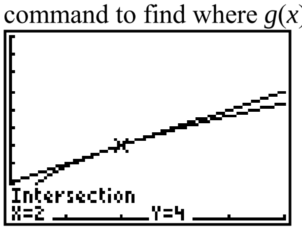
$[0, 2]$ *by* $[0, 2]$

101. $f(x) = \dfrac{5}{x}$, $g(x) = 5 - 1.25x$
Graph the functions and use the Intersect command to find where $g(x)$ is tangent to $f(x)$.



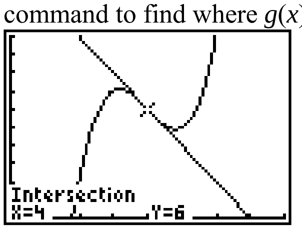
[0, 5] *by* [0, 5]
$a = 2$

102. $f(x) = \sqrt{8x}$, $g(x) = x + 2$
Graph the functions and use the Intersect command to find where $g(x)$is tangent to $f(x)$.



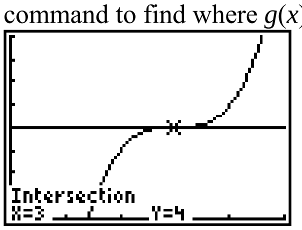
[0, 5] *by* [0, 10]
$a = 2$

103. $f(x) = x^3 - 12x^2 + 46x - 50$, $g(x) = 14 - 2x$
Graph the functions and use the Intersect command to find where $g(x)$ is tangent to $f(x)$.



[0, 8] *by* [0, 10]
$a = 4$

104. $f(x) = (x-3)^3 + 4$, $g(x) = 4$
Graph the functions and use the Intersect command to find where $g(x)$ is tangent to $f(x)$.



[0, 5] *by* [0, 8]
$a = 3$

Exercises 1.4

1. No limit

2. 2

3. 1

4. No limit

5. No limit

6. No limit

7. $\lim_{x\to 1}(1-6x) = 1 - 6(1) = -5$

8. $\lim_{x\to 2} \dfrac{x}{x-2}$ is undefined.

9. $\lim_{x\to 3} \sqrt{x^2+16} = \sqrt{(3)^2+16} = \sqrt{25} = 5$

10. $\lim_{x\to 4}(x^3 - 7) = 4^3 - 7 = 57$

11. $\lim_{x\to 5} \dfrac{x^2+1}{5-x}$ is undefined.

12. $$\lim_{x\to 6}\left(\sqrt{6x} + 3x - \frac{1}{x}\right)(x^2 - 4)$$
$$= \left(\lim_{x\to 6}\sqrt{6x} + \lim_{x\to 6} 3x - \lim_{x\to 6}\frac{1}{x}\right)\left(\lim_{x\to 6} x^2 - \lim_{x\to 6} 4\right)$$
$$= \left(6 + 18 - \frac{1}{6}\right)(36 - 4) = \frac{143}{6}\cdot 32 = \frac{2288}{3}$$

13. $$\lim_{x\to 7}\left(x + \sqrt{x-6}\right)\left(x^2 - 2x + 1\right)$$
$$= \lim_{x\to 7}\left(x + \sqrt{x-6}\right)(x-1)^2$$
$$= \left(\lim_{x\to 7} x + \lim_{x\to 7}\sqrt{x-6}\right)\left(\lim_{x\to 7} x - \lim_{x\to 7} 1\right)^2$$
$$= (7+1)(7-1)^2 = 8\cdot 36 = 288$$

14. $$\lim_{x\to 8}\frac{\sqrt{5x-4} - 1}{3x^2 + 2} = \frac{\lim_{x\to 8}\sqrt{5x-4} - \lim_{x\to 8} 1}{\lim_{x\to 8} 3x^2 + \lim_{x\to 8} 2}$$
$$= \frac{6-1}{192+2} = \frac{5}{194}$$

15. $\lim\limits_{x\to 9}\frac{\sqrt{x^2-5x-36}}{8-3x}$
$=\frac{\left(\lim\limits_{x\to 9}x^2-\lim\limits_{x\to 9}5x-\lim\limits_{x\to 9}36\right)^{1/2}}{\lim\limits_{x\to 9}8-\lim\limits_{x\to 9}3x}$
$=\frac{(81-45-36)^{1/2}}{8-27}=\frac{\sqrt{0}}{-19}=0$

16. $\lim\limits_{x\to 10}(2x^2-15x-50)^{20}$
$=\left(\lim\limits_{x\to 10}2x^2-\lim\limits_{x\to 10}15x-\lim\limits_{x\to 10}50\right)^{20}$
$=(200-150-50)^2=0^{20}=0$

17. $\lim\limits_{x\to 0}\frac{x^2+3x}{x}=\lim\limits_{x\to 0}\frac{x(x+3)}{x}=\lim\limits_{x\to 0}(x+3)=3$

18. $\lim\limits_{x\to 1}\frac{x^2-1}{x-1}=\lim\limits_{x\to 1}\frac{(x-1)(x+1)}{(x-1)}$
$=\lim\limits_{x\to 1}(x+1)=2$

19. $\lim\limits_{x\to 2}\frac{-2x^2+4x}{x-2}=\lim\limits_{x\to 2}\frac{-2x(x-2)}{(x-2)}$
$=\lim\limits_{x\to 2}-2x=-4$

20. $\lim\limits_{x\to 3}\frac{x^2-x-6}{x-3}=\lim\limits_{x\to 3}\frac{(x-3)(x+2)}{(x-3)}$
$=\lim\limits_{x\to 3}(x+2)=5$

21. $\lim\limits_{x\to 4}\frac{x^2-16}{4-x}=\lim\limits_{x\to 4}\frac{(x-4)(x+4)}{-(x-4)}$
$=\lim\limits_{x\to 4}(-x-4)=\lim\limits_{x\to 4}(-x)-\lim\limits_{x\to 4}4$
$=-4-4=-8$

22. $\lim\limits_{x\to 5}\frac{2x-10}{x^2-25}=\lim\limits_{x\to 5}\frac{2(x-5)}{(x-5)(x+5)}$
$=\frac{\lim\limits_{x\to 5}2}{\lim\limits_{x\to 5}(x+5)}=\frac{2}{5+5}=\frac{1}{5}$

23. $\lim\limits_{x\to 6}\frac{x^2-6x}{x^2-5x-6}=\lim\limits_{x\to 6}\frac{x(x-6)}{(x-6)(x+1)}$
$=\frac{\lim\limits_{x\to 6}x}{\lim\limits_{x\to 6}(x+1)}=\frac{6}{6+1}=\frac{6}{7}$

24. $\lim\limits_{x\to 7}\frac{x^3-2x^2+3x}{x^2}=\lim\limits_{x\to 7}\frac{x(x^2-2x+3)}{x^2}$
$=\frac{\lim\limits_{x\to 7}(x^2-2x+3)}{\lim\limits_{x\to 7}x}=\frac{49-14+3}{7}=\frac{38}{7}$

25. $\lim\limits_{x\to 8}\frac{x^2+64}{x-8}$ is undefined.

26. $\lim\limits_{x\to 9}\frac{1}{(x-9)^2}=\frac{\lim\limits_{x\to 9}1}{\lim\limits_{x\to 9}(x-9)^2}$ is undefined.

27. a. $\lim\limits_{x\to 0}(f(x)+g(x))$
$=\lim\limits_{x\to 0}f(x)+\lim\limits_{x\to 0}g(x)$
$=-\frac{1}{2}+\frac{1}{2}=0$

b. $\lim\limits_{x\to 0}(f(x)-2g(x))$
$=\lim\limits_{x\to 0}f(x)-2\cdot\lim\limits_{x\to 0}g(x)$
$=-\frac{1}{2}-2\cdot\frac{1}{2}=-\frac{3}{2}$

c. $\lim\limits_{x\to 0}(f(x)\cdot g(x))$
$=\left[\lim\limits_{x\to 0}f(x)\right]\cdot\left[\lim\limits_{x\to 0}g(x)\right]$
$=\left[-\frac{1}{2}\right]\cdot\left[\frac{1}{2}\right]=-\frac{1}{4}$

d. $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}$
$=\frac{\lim\limits_{x\to 0}f(x)}{\lim\limits_{x\to 0}g(x)}$, since $\lim\limits_{x\to 0}g(x)\neq 0$
$=\frac{-\frac{1}{2}}{\frac{1}{2}}=-1$

28. a. $\lim\limits_{x\to 0}x\left[f(x)\right]^2$
$=\left[\lim\limits_{x\to 0}x\right]\cdot\left[\lim\limits_{x\to 0}f(x)\right]^2$
$=[0]\cdot\left[-\frac{1}{2}\right]^2=0$

b. $\lim_{x\to 0}\left(f(x)+1\right)$

$= \lim_{x\to 0} f(x) + \lim_{x\to 0} 1 = -\frac{1}{2}+1=\frac{1}{2}$

c. $\lim_{x\to 0}\frac{\sqrt{2}}{\sqrt{g(x)}}$

$\lim_{x\to 0}\sqrt{g(x)} = \lim_{x\to 0}\left(g(x)\right)^{\frac{1}{2}} = \left(\lim_{x\to 0} g(x)\right)^{\frac{1}{2}} = \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}} \neq 0.$

So $\lim_{x\to 0}\frac{\sqrt{2}}{\sqrt{g(x)}} = \frac{\sqrt{2}}{\lim_{x\to 0}\sqrt{g(x)}} = \frac{\sqrt{2}}{\frac{1}{\sqrt{2}}} = 2$

d. $\lim_{x\to 0}\frac{1}{f(x)+g(x)}$

$\lim_{x\to 0} f(x)+g(x) = \lim_{x\to 0} f(x) + \lim_{x\to 0} g(x) = -\frac{1}{2}+\frac{1}{2} = 0.$

So $\lim_{x\to 0}\frac{1}{f(x)+g(x)}$ is undefined.

29. $f(x)=x^2+1$

$f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$

$= \lim_{h\to 0}\frac{(3+h)^2+1-(3^2+1)}{h}$

$= \lim_{h\to 0}\frac{9+6h+h^2+1-10}{h} = \lim_{h\to 0}\frac{h^2+6h}{h} = \lim_{h\to 0}(h+6)=6$

30. $f(x)=x^3$

$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$

$= \lim_{h\to 0}\frac{(2+h)^3-2^3}{h} = \lim_{h\to 0}\frac{8+12h+6h^2+h^3-8}{h} = \lim_{h\to 0}\frac{h(12+6h+h^2)}{h} = \lim_{h\to 0}(h^2+6h+12)=12$

31. $f(x)=x^3+3x+1$

$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{h^3+3h+1-1}{h} = \lim_{h\to 0}\frac{h(h^2+3)}{h} = \lim_{h\to 0}(h^2+3)=3$

32. $f(x)=x^2+2x+2$

$f'(0) = \lim_{h\to 0}\frac{(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{h^2+2h+2-2}{h}$

$= \lim_{h\to 0}\frac{h(h+2)}{h} = \lim_{h\to 0}(h+2)=2$

33. $f(x)=\dfrac{1}{2x+5}$

$$f'(3)=\lim_{h\to0}\frac{f(3+h)-f(3)}{h}$$
$$=\lim_{h\to0}\frac{\frac{1}{2(3+h)+5}-\frac{1}{2(3)+5}}{h}$$
$$=\lim_{h\to0}\frac{\frac{1}{6+2h+5}-\frac{1}{11}}{h}=\lim_{h\to0}\frac{\frac{11-(2h+11)}{121+22h}}{h}$$
$$=\lim_{h\to0}\frac{-2h}{121+22h}\cdot\frac{1}{h}=\lim_{h\to0}\frac{-2}{121+22h}=-\frac{2}{121}$$

34. $f(x)=\sqrt{2x-1}$

$$f'(4)=\lim_{h\to0}\frac{f(4+h)-f(4)}{h}$$
$$=\lim_{h\to0}\frac{\sqrt{2(4+h)-1}-\sqrt{2(4)-1}}{h}$$
$$=\lim_{h\to0}\frac{\sqrt{7+2h}-\sqrt{7}}{h}$$
$$=\lim_{h\to0}\frac{\left(\sqrt{7+2h}-\sqrt{7}\right)\left(\sqrt{7+2h}+\sqrt{7}\right)}{h\left(\sqrt{7+2h}+\sqrt{7}\right)}$$
$$=\lim_{h\to0}\frac{7+2h-7}{h\left(\sqrt{7+2h}+\sqrt{7}\right)}$$
$$=\lim_{h\to0}\frac{2}{\sqrt{7+2h}+\sqrt{7}}=\frac{2}{2\sqrt{7}}=\frac{1}{\sqrt{7}}$$

35. $f(x)=\sqrt{5-x}$

$$f'(2)=\lim_{h\to0}\frac{f(2+h)-f(2)}{h}$$
$$=\lim_{h\to0}\frac{\sqrt{3-h}-\sqrt{3}}{h}$$
$$=\lim_{h\to0}\frac{\left(\sqrt{3-h}-\sqrt{3}\right)\left(\sqrt{3-h}+\sqrt{3}\right)}{h\left(\sqrt{3-h}+\sqrt{3}\right)}$$
$$=\lim_{h\to0}\frac{3-h-3}{h\left(\sqrt{3-h}+\sqrt{3}\right)}$$
$$=\lim_{h\to0}-\frac{1}{\sqrt{3-h}+\sqrt{3}}=-\frac{1}{2\sqrt{3}}=-\frac{\sqrt{3}}{6}$$

36. $f(x)=\dfrac{1}{7-2x}$

$$f'(3)=\lim_{h\to0}\frac{f(3+h)-f(3)}{h}$$
$$=\lim_{h\to0}\frac{\frac{1}{7-2(3+h)}-\frac{1}{7-2(3)}}{h}$$
$$=\lim_{h\to0}\frac{\frac{1}{1-2h}-1}{h}=\lim_{h\to0}\frac{1-(1-2h)}{1-2h}\cdot\frac{1}{h}$$
$$=\lim_{h\to0}\frac{2}{1-2h}=2$$

37. $f(x)=3x+1$

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to0}\frac{3(x+h)+1-(3x+1)}{h}$$
$$=\lim_{h\to0}\frac{3x+3h+1-3x-1}{h}=\lim_{h\to0}\frac{3h}{h}$$
$$=\lim_{h\to0}3=3$$

38. $f(x)=-x+11$

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to0}\frac{-(x+h)+11-(-x+11)}{h}$$
$$=\lim_{h\to0}\frac{-x-h+11+x-11}{h}=\lim_{h\to0}\frac{-h}{h}$$
$$=\lim_{h\to0}(-1)=-1$$

39. $f(x) = x + \dfrac{1}{x}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{(x+h) + \dfrac{1}{x+h} - \left(x + \dfrac{1}{x}\right)}{h} = \lim_{h\to 0}\frac{x + h + \dfrac{1}{x+h} - x - \dfrac{1}{x}}{h}$$

$$= \lim_{h\to 0}\frac{h + \dfrac{1}{x+h} - \dfrac{1}{x}}{h} = \lim_{h\to 0}\frac{\dfrac{h(x+h)(x) + x - (x+h)}{(x+h)(x)}}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{hx^2 + h^2x - h}{(x+h)(x)}}{h} = \lim_{h\to 0}\frac{h(x^2 + hx - 1)}{(x+h)(x)}\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\frac{(x^2 + hx - 1)}{(x+h)(x)} = \frac{x^2 - 1}{x^2} = 1 - \frac{1}{x^2}$$

40. $f(x) = \dfrac{1}{x^2}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \lim_{h\to 0}\frac{\dfrac{x^2 - (x+h)^2}{(x+h)^2(x^2)}}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{x^2 - x^2 - 2xh - h^2}{(x+h)^2(x^2)}}{h} = \lim_{h\to 0}\frac{h(-2x - h)}{(x+h)^2(x^2)}\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\frac{-2x - h}{(x+h)^2(x^2)} = \frac{-2x}{(x^2)(x^2)} = \frac{-2}{x^3}$$

41. $f(x) = \dfrac{x}{x+1}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{x+h}{x+h+1} - \dfrac{x}{x+1}}{h} = \lim_{h\to 0}\frac{\dfrac{(x+h)(x+1) - (x)(x+h+1)}{(x+h+1)(x+1)}}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{x^2 + xh + x + h - x^2 - xh - x}{(x+h+1)(x+1)}}{h} = \lim_{h\to 0}\frac{h}{(x+h+1)(x+1)}\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\frac{1}{(x+h+1)(x+1)} = \frac{1}{(x+1)(x+1)} = \frac{1}{(x+1)^2}$$

42. $f(x) = -1 + \dfrac{2}{x-2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{-1 + \frac{2}{x+h-2} - \left(-1 + \frac{2}{x-2}\right)}{h} = \lim_{h \to 0} \frac{\frac{2(x-2) - 2(x+h-2)}{(x+h-2)(x-2)}}{h}$$
$$= \lim_{h \to 0} \frac{\frac{2x-4-2x-2h+4}{(x+h-2)(x-2)}}{h} = \lim_{h \to 0} \frac{-2h}{(x+h-2)(x-2)}\left(\frac{1}{h}\right)$$
$$= \lim_{h \to 0} \frac{-2}{(x+h-2)(x-2)} = \frac{-2}{(x-2)(x-2)} = \frac{-2}{(x-2)^2}$$

43. $f(x) = \dfrac{1}{x^2+1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2+1} - \frac{1}{x^2+1}}{h} = \lim_{h \to 0} \frac{\frac{(x^2+1) - \left((x+h)^2+1\right)}{\left((x+h)^2+1\right)(x^2+1)}}{h}$$
$$= \lim_{h \to 0} \frac{\frac{x^2+1-x^2-2xh-h^2-1}{\left((x+h)^2+1\right)(x^2+1)}}{h} = \lim_{h \to 0} \frac{h(-2x-h)}{\left((x+h)^2+1\right)(x^2+1)}\left(\frac{1}{h}\right)$$
$$= \lim_{h \to 0} \frac{(-2x-h)}{\left((x+h)^2+1\right)(x^2+1)} = \frac{-2x}{(x^2+1)(x^2+1)} = \frac{-2x}{(x^2+1)^2}$$

44. $f(x) = \dfrac{x}{x+2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{\frac{x+h}{x+h+2} - \frac{x}{x+2}}{h} = \lim_{h \to 0} \frac{\frac{(x+h)(x+2) - (x)(x+h+2)}{(x+h+2)(x+2)}}{h}$$
$$= \lim_{h \to 0} \frac{\frac{x^2+xh+2x+2h-x^2-xh-2x}{(x+h+2)(x+2)}}{h} = \lim_{h \to 0} \frac{2h}{(x+h+2)(x+2)}\left(\frac{1}{h}\right)$$
$$= \lim_{h \to 0} \frac{2}{(x+h+2)(x+2)} = \frac{2}{(x+2)(x+2)} = \frac{2}{(x+2)^2}$$

45. $f(x)=\sqrt{x+2}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}=\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\left(\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}\right)$$
$$=\lim_{h\to 0}\frac{(x+h+2)-(x+2)}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)}=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)}$$
$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}=\frac{1}{\sqrt{x+2}+\sqrt{x+2}}=\frac{1}{2\sqrt{x+2}}$$

46. $f(x)=\sqrt{x^2+1}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h}=\lim_{h\to 0}\frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h}\left(\frac{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}\right)$$
$$=\lim_{h\to 0}\frac{(x+h)^2+1-(x^2+1)}{h\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}=\lim_{h\to 0}\frac{x^2+2xh+h^2+1-x^2-1}{h\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}$$
$$=\lim_{h\to 0}\frac{h(2x+h)}{h\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}=\lim_{h\to 0}\frac{(2x+h)}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}$$
$$=\frac{2x}{\sqrt{x^2+1}+\sqrt{x^2+1}}=\frac{x}{\sqrt{x^2+1}}$$

47. $f(x)=\frac{1}{\sqrt{x}}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}=\lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$
$$=\lim_{h\to 0}\left(\frac{1}{h}\right)\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right)$$
$$=\lim_{h\to 0}\left(\frac{1}{h}\right)\frac{x-x-h}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$=\lim_{h\to 0}\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=\frac{-1}{\sqrt{x}\sqrt{x}\left(\sqrt{x}+\sqrt{x}\right)}$$
$$=\frac{-1}{2x\sqrt{x}}=\frac{-1}{2x^{3/2}}$$

48. $f(x) = x\sqrt{x}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{(x+h)\sqrt{x+h}-x\sqrt{x}}{h} = \lim_{h\to 0}\frac{(x+h)\sqrt{x+h}-x\sqrt{x}}{h}\left(\frac{(x+h)\sqrt{x+h}+x\sqrt{x}}{(x+h)\sqrt{x+h}+x\sqrt{x}}\right)$$
$$= \lim_{h\to 0}\frac{(x+h)^2(x+h)-x^2(x)}{h\left((x+h)\sqrt{x+h}+x\sqrt{x}\right)} = \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h\left((x+h)\sqrt{x+h}+x\sqrt{x}\right)}$$
$$= \lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h\left((x+h)\sqrt{x+h}+x\sqrt{x}\right)} = \lim_{h\to 0}\frac{(3x^2+3xh+h^2)}{(x+h)\sqrt{x+h}+x\sqrt{x}} = \frac{3x^2}{2x\sqrt{x}} = \frac{3}{2}x^{\frac{1}{2}}$$

49. $a = 1$ and $f(x) = x^2$

50. $a = 2$ and $f(x) = x^3$

51. $a = 10$ and $f(x) = x^{-1}$

52. $a = 64$ and $f(x) = x^{1/3}$

53. $a = 9$ and $f(x) = \sqrt{x}$

54. $a = 1$ and $f(x) = x^{-1/2} = \dfrac{1}{\sqrt{x}}$

55. $f(x) = x^2;\ f'(x) = 2x$

$$f'(2) = \lim_{h\to 0}\frac{(2+h)^2-4}{h} = 4$$

56. $f(x) = x^3;\ f'(x) = 3x^2$

$$f'(-1) = \lim_{h\to 0}\frac{(-1+h)^3+1}{h} = 3$$

57. $f(x) = \sqrt{x};\ f'(x) = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

$$f'(2) = \lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h} = \frac{1}{2\sqrt{2}}$$

58. $f(x) = \sqrt{x};\ f'(x) = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

$$f'(4) = \lim_{h\to 0}\frac{\sqrt{4+h}-2}{h} = \frac{1}{4}$$

59. $f(x) = x^{1/3};\ f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$$f'(-8) = \lim_{h\to 0}\frac{(-8+h)^{1/3}+2}{h} = \frac{1}{12}$$

60. $f(x) = \frac{1}{x};\ f'(x) = -x = \frac{-1}{x^2}$

$$f'(1) = \lim_{h\to 0}\frac{1}{h}\left[\frac{1}{1+h}-1\right] = -1$$

61. $\lim_{x\to\infty}\frac{1}{x^2} = 0$

62. $\lim_{x\to-\infty}\frac{1}{x^2} = 0$

63. $\lim_{x\to\infty}\frac{1}{x-8} = 0$

64. $\lim_{x\to\infty}\frac{5x+3}{3x-2} = \lim_{x\to\infty}\frac{5+\frac{3}{x}}{3-\frac{2}{x}} = \frac{5}{3}$

65. $\lim_{x\to\infty}\frac{10x+100}{x^2-30} = \lim_{x\to\infty}\frac{10+\frac{100}{x}}{x-\frac{30}{x}} = 0$

66. $\lim_{x\to\infty}\frac{x^2+x}{x^2-1} = \lim_{x\to\infty}\frac{1+\frac{1}{x}}{1-\frac{1}{x^2}} = 1$

67. $\lim_{x\to 0} f(x)$

As x approaches 0 from either side, $f(x)$ approaches $\frac{3}{4}$.

So $\lim_{x\to 0} f(x) = \frac{3}{4}$.

68. $\lim_{x\to\infty} f(x)$

As x increases without bound, $f(x)$ approaches 1.

So $\lim_{x\to\infty} f(x) = 1$

69. $\lim_{x\to 0} xf(x)$

$$= \left[\lim_{x\to 0} x\right]\cdot\left[\lim_{x\to 0} f(x)\right]$$

$$= 0\cdot\frac{3}{4} = 0$$

70. $\lim_{x\to\infty}(1+2f(x))$

$$= \lim_{x\to\infty} 1 + 2\cdot\lim_{x\to\infty} f(x)$$

$$= 1+2\cdot 1 = 3$$

71. $\lim_{x\to\infty}(1-f(x))$

$$= \lim_{x\to\infty} 1 - \lim_{x\to\infty} f(x)$$

$$= 1-1 = 0$$

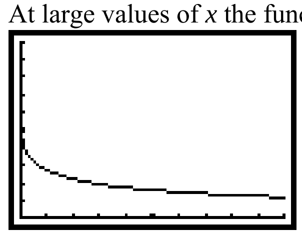
72. $\lim_{x\to 0}[f(x)]^2$

$$= \left[\lim_{x\to 0} f(x)\right]^2$$

$$= \left[\frac{3}{4}\right]^2 = \frac{9}{16}$$

73. $\lim_{x\to\infty}\sqrt{25+x}-\sqrt{x}$

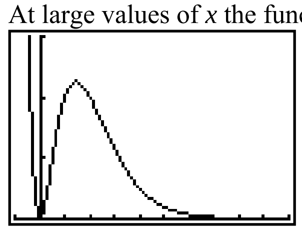
At large values of x the function goes to 0.



[0, 100] *by* [0, 10]

74. $\lim_{x\to\infty}\frac{x^2}{2^x}$

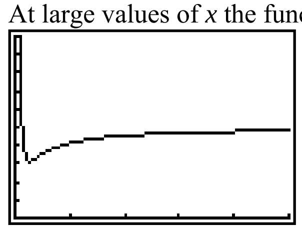
At large values of x the function goes to 0.



[22, 20] *by* [0, 1.5]

75. $\lim_{x\to\infty}\frac{x^2-2x+3}{2x^2+1}$

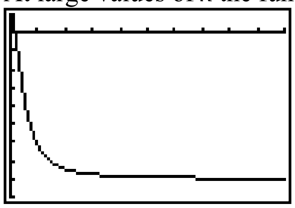
At large values of x the function goes to .5.



[0, 50] *by* [0, 1]

76. $\lim\limits_{x\to\infty} \dfrac{-8x^2+1}{x^2+1}$

At large values of x the function goes to –8.



$[0, 20]$ *by* $[29, 1]$

Exercises 1.5

1. No

2. Yes

3. Yes

4. Yes

5. No

6. No

7. No

8. No

9. Yes

10. Yes

11. No

12. No

13. $f(x) = x^2$

$\lim\limits_{x\to1} f(x) = \lim\limits_{x\to1} x^2 = 1$

$f(1) = 1^2 = 1$

Since $\lim\limits_{x\to1} f(x) = 1 = f(1)$, $f(x)$ is continuous at $x = 1$.

$f'(1) = \lim\limits_{h\to0} \dfrac{f(1+h) - f(1)}{h}$

$= \lim\limits_{h\to0} \dfrac{(1+h)^2 - (1)^2}{h} = \lim\limits_{h\to0} \dfrac{1+2h+h^2-1}{h}$

$= \lim\limits_{h\to0} \dfrac{h(2+h)}{h} = 2$

Therefore, $f(x)$ is continuous and differentiable at $x = 1$.

14. $f(x) = \dfrac{1}{x}$

$\lim\limits_{x\to1} f(x) = \lim\limits_{x\to1} \dfrac{1}{x} = 1$

$f(1) = \dfrac{1}{1} = 1$

Since $\lim\limits_{x\to1} f(x) = 1 = f(1)$, $f(x)$ is continuous at $x = 1$.

$f'(1) = \lim\limits_{h\to0} \dfrac{f(1+h) - f(1)}{h} = \lim\limits_{h\to0} \dfrac{\frac{1}{1+h} - 1}{h}$

$= \lim\limits_{h\to0} \dfrac{1-(1+h)}{1+h} \cdot \dfrac{1}{h} = \lim\limits_{h\to0} -\dfrac{1}{1+h} = -1$

Therefore, $f(x)$ is continuous and differentiable at $x = 1$.

15. $f(x) = \begin{cases} x+2 & \text{for } -1 \le x \le 1 \\ 3x & \text{for } 1 < x < 5 \end{cases}$

$\lim\limits_{x\to1} 3x = 3$

$\lim\limits_{x\to1} (x+2) = 3$

$f(1) = 1 + 2 = 3$

Since $\lim\limits_{x\to1} f(x) = 3 = f(1)$, $f(x)$ is continuous at $x = 1$.

Since the graph of $f(x)$ at $x = 1$ does not have a tangent line, $f(x)$ is not differentiable at $x = 1$.

Therefore, $f(x)$ is continuous but not differentiable at $x = 1$.

16. $f(x) = \begin{cases} x & \text{for } 1 \le x \le 2 \\ x^3 & \text{for } 0 \le x < 1 \end{cases}$

$\lim\limits_{x\to1} x^3 = 1$

$\lim\limits_{x\to1} x = 1$

$f(1) = 1$

Since $\lim\limits_{x\to1} f(x) = 1 = f(1)$, $f(x)$ is continuous at $x = 1$.

Since the graph of $f(x)$ at $x = 1$ does not have a tangent line, $f(x)$ is not differentiable at $x = 1$.

Therefore, $f(x)$ is continuous but not differentiable at $x = 1$.

17. $f(x)=\begin{cases}2x-1 & \text{for } 0\le x\le 1\\ 1 & \text{for } 1<x\end{cases}$

$\lim\limits_{x\to 1} 1 = 1$

$\lim\limits_{x\to 1}(2x-1)=1$

$f(1) = 2(1) - 1 = 1$

Since $\lim\limits_{x\to 1} f(x) = 1 = f(1)$, $f(x)$ is continuous at $x = 1$.

Since the graph of $f(x)$ at $x = 1$ does not have a tangent line, $f(x)$ is not differentiable at $x = 1$. Therefore, $f(x)$ is continuous but not differentiable at $x = 1$.

18. $f(x)=\begin{cases}x & \text{for } x\ne 1\\ 2 & \text{for } x=1\end{cases}$

$\lim\limits_{x\to 1} x = 1$

$f(1) = 2$

Since $\lim\limits_{x\to 1} f(x) = 1 \ne 2 = f(1)$, $f(x)$ is not continuous at $x = 1$. By Theorem 1, since $f(x)$ is not continuous at $x = 1$, it is not differentiable.

19. $f(x)=\begin{cases}\dfrac{1}{x-1} & \text{for } x\ne 1\\ 0 & \text{for } x=1\end{cases}$

$\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1}\dfrac{1}{x-1}$ is undefined. Since $\lim\limits_{x\to 1} f(x)$ does not exist, $f(x)$ is not continuous at $x = 1$. By Theorem 1, since $f(x)$ is not continuous at $x = 1$, it is not differentiable.

20. $f(x)=\begin{cases}x-1 & \text{for } 0\le x<1\\ 1 & \text{for } x=1\\ 2x-2 & \text{for } x>1\end{cases}$

$\lim\limits_{x\to 1}(x-1) = 0 = \lim\limits_{x\to 1}(2x-2)$, but if $f(1) = 1$, so $f(x)$ is not continuous at $x = 1$. Therefore, $f(x)$ is not differentiable at $x = 1$.

21. $\dfrac{x^2-7x+10}{x-5}=\dfrac{(x-5)(x-2)}{x-5}=x-2,$ so define $f(5) = 5 - 2 = 3$.

22. $x^2+x-12=(x+4)(x-3),$ so define $f(-4) = -4 - 3 = -7$.

23. $\dfrac{x^3-5x^2+4}{x^2}$

It is not possible to define $f(x)$ at $x = 0$ and make $f(x)$ continuous.

24. $\dfrac{x^2+25}{x-5}$

It is not possible to define $f(x)$ at $x = 5$ and make $f(x)$ continuous.

25. $\dfrac{(6+x)^2-36}{x}=12+x,$ define $f(0) = 12$.

26. $\dfrac{\sqrt{9+x}-\sqrt{9}}{x}\cdot\dfrac{\sqrt{9+x}+\sqrt{9}}{\sqrt{9+x}+\sqrt{9}}=\dfrac{1}{\sqrt{9+x}+\sqrt{9}},$

define $f(0)=\dfrac{1}{6}$.

27. **a.** The function $T(x)$ is a piecewise-defined function.

For $0 \le x \le 27{,}050$, $T(x) = .15x$.

For $27{,}050 < x \le 65{,}550$, we have

$$\begin{aligned}T(x) &= .15\cdot 27{,}050 + .275\cdot(x-27{,}050)\\ &= .275x - 3381.25\end{aligned}$$

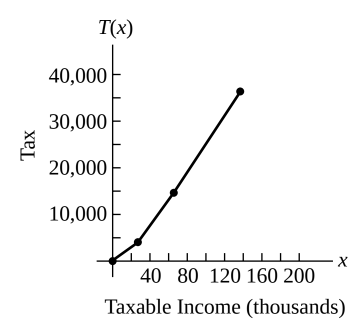
For $65{,}550 < x \le 136{,}750$, we have

$$\begin{aligned}T(x) &= .275\cdot 65{,}550 - 3381.25 + .305(x-65{,}550)\\ &= .305x - 5347.75\end{aligned}$$

All together, the function is

$$T(x)=\begin{cases}.15x \text{ for } 0\le x\le 27{,}050\\ .275x-3381.25 \text{ for } 27{,}050<x\le 65{,}550\\ .305x-5347.75 \text{ for } 65{,}550<x\le 136{,}750\end{cases}$$

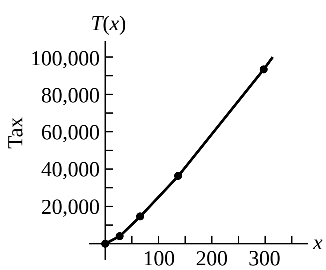
b.



c. $T(65{,}550)$ is the maximum tax you will pay for income below the third tax bracket. $T(27{,}050)$ is the maximum tax for income below the second tax bracket. The maximum tax on the portion of income in the second tax bracket is $T(65{,}550) - T(27{,}050) = 10{,}587.5$ dollars.

28. a. The function $T(x)$ is a piecewise-defined function.
For $0 \le x \le 27{,}050$, $T(x) = .15x$
For $27{,}050 < x \le 65{,}550$, we have
$$T(x) = .15 \cdot 27{,}050 + .275(x - 27{,}050)$$
$$= .275x - 3381.25$$
For $65{,}550 < x \le 136{,}750$, we have
$$T(x) = 43057.50 + .275(65{,}550 - 27{,}050) + .305(x - 65{,}550)$$
$$= .305x - 5347.75$$
For $136{,}750 < x \le 297{,}350$, we have
$$T(x) = .305 \cdot 136{,}750 - 5347.75 + .355(x - 136{,}750)$$
$$= .355x - 12{,}185.25$$
For $297{,}350 < x$, we have
$$T(x) = .355 \cdot 297{,}350 - 12{,}185.25 + .391(x - 297{,}350)$$
$$= .391x - 22{,}889.85$$
All together, we have
$$T(x) = \begin{cases} .15x \text{ for } 0 \le x \le 27{,}050 \\ .275x - 3381.25 \text{ for } 27{,}050 < x \le 65{,}550 \\ .305x - 5347.75 \text{ for } 65{,}550 < x \le 136{,}750 \\ .355x - 12{,}185.25 \text{ for } 136{,}750 < x \le 297{,}350 \\ .391x - 22{,}889.85 \text{ for } x > 297{,}350 \end{cases}$$

b.



c. $T(297{,}350) - T(136{,}750) = 93{,}374 - 36{,}361 = \$57{,}013$

29. a. The function $R(x)$ is a piecewise function
For $0 \le x \le 100$, $R(x) = 2.50 + .07x$.
For $x > 100$, we have
$$R(x) = 2.50 + .07 \cdot 100 + .04(x - 100)$$
$$= 5.50 + .04x$$
All together, we have
$$R(x) = \begin{cases} 2.50 + .07x \text{ for } 0 \le x \le 100 \\ 5.50 + .04x \text{ for } x > 100 \end{cases}$$

b. Let $P(x)$ be the profit on x copies.
For $0 \le x \le 100$, $P(x) = 2.50 + .07x - .03x = 2.50 + .04x$
For $x > 100$, $P(x) = 5.50 + .04x - .03x = 5.50 + .01x$
All together, we have
$$P(x) = \begin{cases} 2.50 + .04x \text{ for } 0 \le x \le 100 \\ 5.50 + .01x \text{ for } x > 100 \end{cases}$$

30. a. For $0 \le x \le 50$, $R(x) = .10x$.
For $x > 50$, $R(x) = .10(50) + .05(x - 50) = 2.50 + .05x$
All together, we have
$$R(x) = \begin{cases} .10x \text{ for } 0 \le x \le 50 \\ 2.50 + .05x \text{ for } x > 50 \end{cases}$$

b. Let $P(x)$ be the profit on x copies.
For $0 \le x \le 50$, $P(x) = .10x - .03x = .07x$
For $x > 50$, $P(x) = 2.50 + 0.5x - .03x = 2.50 + .02x$
All together, we have
$$P(x) = \begin{cases} .07x \text{ for } 0 \le x \le 50 \\ 2.50 + .02x \text{ for } x > 50 \end{cases}$$

31. a. The rate of sales is the slope of the line connecting the points (8, 4) and (10, 10).
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-4}{10-8} = 3$$
The rate of sales between 8 a.m. and 10 a.m. is about $3000 per hour.

b. We need to find the 2-hour period with the greatest slope. Looking at the graph gives 3 possibilities:
8 a.m. – 10 a.m., $m = 3$ (from part a)
12 p.m. – 2 p.m., $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16-12}{14-12} = 2$
6 p.m. – 8 p.m., $m = \frac{22-18}{8-6} = 2$
The interval from 8 a.m. to 10 a.m. has the greatest rate, which is $3000 per hour.

32. a. Find the slope for each 2-hour interval.
Midnight – 2 a.m., $m = \frac{\frac{1}{2} - 0}{2} = \frac{1}{4} = .25$; 2 a.m., – 4 a.m., $m = \frac{1 - \frac{1}{2}}{2} = \frac{1}{4} = .25$
4 a.m.,– 6 a.m., $m = \frac{2-1}{2} = \frac{1}{2} = .5$; 6 a.m.,– 8 a.m., $m = \frac{4-2}{2} = 1$
8 a.m., – 10 a.m., $m = \frac{10-4}{2} = 3$; 10 a.m., – noon, $m = \frac{12-10}{2} = 1$
noon – 2 p.m., $m = \frac{16-12}{2} = 2$; 2 p.m. – 4 p.m., $m = \frac{17-16}{2} = \frac{1}{2} = .5$
4 p.m. – 6 p.m., $m = \frac{18-17}{2} = \frac{1}{2} = .5$; 6 p.m. – 8 p.m., $m = \frac{22-18}{2} = 2$
8 p.m. – 10 p.m., $m = \frac{22.5-22}{2} = \frac{1}{4} = .25$; 10 p.m. – midnight, $m = \frac{23-22.5}{2} = \frac{1}{4} = .25$
The intervals midnight – 2 a.m./w 2 a.m. – 4 a.m., 8 p.m.– 10 p.m., and 10 p.m. – midnight have a sales rate of $250 per hour.
The intervals 4 a.m. – 6 a.m., 2 p.m.– 4 p.m., and 4 p.m. –6 p.m. have a sales rate of $500 per hour.
The intervals 6 a.m. – 8 a.m. and 10 a.m.– noon, have a sales rate of $1000 per hour.
The intervals noon– 2 p.m. and 6 p.m.– 8 p.m. both have a sales rate of $2000 per hour.

b. 4,000 – 0 = $4,000 between midnight and 8 a.m.
10,000 – 4,000 = $6,000 between 8 a.m. and 10 a.m.
The sales between 8 a.m. and 10 a.m. are 50% more than the sales between midnight and 8 a.m.

33. For the function to be continuous, $\lim_{x \to 0} f(x)$ must exist and equal $f(0)$. Therefore
$\lim_{x \to 0} (x + a) = \lim_{x \to 0} 1$, so $a = 1$.

34. For the function to be continuous, $\lim_{x\to 0} f(x)$ must exist and equal $f(0)$. Therefore,

$$\begin{aligned}\lim_{x\to 0} 2(x-a) &= \lim_{x\to 0} x^2+1\\ 2(-a) &= (0)^2+1\\ -2a &= 1\\ a &= -\frac{1}{2}\end{aligned}$$

Exercises 1.6

1. $y = x^3 + x^2$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3+x^2) = \frac{d}{dx}x^3 + \frac{d}{dx}x^2 = 3x^2+2x$$

2. $y = 3x^4$

$$\frac{dy}{dx} = \frac{d}{dx}(3x^4) = 12x^3$$

3. $y = \dfrac{2}{x^2} = 2x^{-2}$

$$\frac{dy}{dx} = \frac{d}{dx}(2x^{-2}) = -4x^{-3} = \frac{-4}{x^3}$$

4. $y = \dfrac{1}{3x^3} = \dfrac{1}{3}x^{-3}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^{-3}\right) = -x^{-4} = \frac{-1}{x^4}$$

5. $y = \dfrac{x}{2} - \dfrac{2}{x} = \dfrac{1}{2}x - 2x^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{2}x - 2x^{-1}\right) = \frac{d}{dx}\left(\frac{1}{2}x\right) - \frac{d}{dx}(2x^{-1})\\ &= \frac{1}{2} + 2x^{-2} = \frac{1}{2} + \frac{2}{x^2}\end{aligned}$$

6. $f(x) = 12 + \dfrac{1}{7^3}$

$$\begin{aligned}\frac{d}{dx}\left(12+\frac{1}{7^3}\right) &= \frac{d}{dx}(12) + \frac{d}{dx}\left(\frac{1}{7^3}\right)\\ &= 0+0 = 0\end{aligned}$$

7. $f(x) = x^4 + x^3 + x$

$$\begin{aligned}\frac{d}{dx}(x^4+x^3+x) &= \frac{d}{dx}x^4 + \frac{d}{dx}x^3 + \frac{d}{dx}x\\ &= 4x^3+3x^2+1\end{aligned}$$

8. $y = 4x^3 - 2x^2 + x + 1$

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^3 - 2x^2 + x + 1\right) = \frac{d}{dx}\left(4x^3\right) - \frac{d}{dx}\left(2x^2\right) + \frac{d}{dx}\left(x\right) + \frac{d}{dx}\left(1\right)$$
$$= 12x^2 - 4x + 1$$

9. $y = (2x+4)^3$

$$\frac{dy}{dx} = \frac{d}{dx}(2x+4)^3 = 3(2x+4)^2\frac{d}{dx}(2x+4)$$
$$= 3(2x+4)^2(2) = 6(2x+4)^2$$

10. $y = (x^2-1)^3$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2-1)^3 = 3(x^2-1)^2\frac{d}{dx}(x^2-1)$$
$$= 3(x^2-1)^2(2x) = 6x(x^2-1)^2$$

11. $y = (x^3+x^2+1)^7$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3+x^2+1)^7 = 7(x^3+x^2+1)^6\frac{d}{dx}(x^3+x^2+1)$$
$$= 7(x^3+x^2+1)^6(3x^2+2x)$$

12. $y = (x^2+x)^{-2}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2+x)^{-2} = -2(x^2+x)^{-3}\frac{d}{dx}(x^2+x)$$
$$= -2(x^2+x)^{-3}(2x+1)$$

13. $y = \frac{4}{x^2} = 4x^{-2}$

$$\frac{dy}{dx} = 4\frac{d}{dx}x^{-2} = -\frac{8}{x^3}$$

14. $y = 4(x^2-6)^{-3}$

$$\frac{dy}{dx} = \frac{d}{dx}4(x^2-6)^{-3} = -12(x^2-6)^{-4}\frac{d}{dx}(x^2-6)$$
$$= -12(x^2-6)^{-4}(2x) = -24x(x^2-6)^{-4}$$

15. $y = 3\sqrt[3]{2x^2+1} = 3(2x^2+1)^{1/3}$

$$\frac{dy}{dx} = \frac{d}{dx}3(2x^2+1)^{1/3} = (2x^2+1)^{-2/3}\frac{d}{dx}(2x^2+1)$$
$$= (2x^2+1)^{-2/3}(4x) = (4x)(2x^2+1)^{-2/3}$$

16. $y = 2\sqrt{x+1} = 2(x+1)^{1/2}$

$$\frac{dy}{dx} = \frac{d}{dx}2(x+1)^{1/2} = (x+1)^{-1/2}\frac{d}{dx}(x+1)$$
$$= (x+1)^{-1/2}(1) = (x+1)^{-1/2}$$

17. $y = 2x + (x+2)^3$

$$\frac{dy}{dx} = \frac{d}{dx}\left(2x + (x+2)^3\right) = \frac{d}{dx}2x + \frac{d}{dx}(x+2)^3$$
$$= 2 + 3(x+2)^2\frac{d}{dx}(x+2) = 2 + 3(x+2)^2(1)$$
$$= 2 + 3(x+2)^2$$

18. $y = (x-1)^3 + (x+2)^4$

$$\frac{dy}{dx} = \frac{d}{dx}\left((x-1)^3 + (x+2)^4\right) = \frac{d}{dx}(x-1)^3 + \frac{d}{dx}(x+2)^4$$
$$= 3(x-1)^2\frac{d}{dx}(x-1) + 4(x+2)^3\frac{d}{dx}(x+2)$$
$$= 3(x-1)^2(1) + 4(x+2)^3(1) = 3(x-1)^2 + 4(x+2)^3$$

19. $y = \dfrac{1}{5x^5} = \dfrac{1}{5}x^{-5}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{5}x^{-5}\right) = (-1)x^{-6} = \frac{-1}{x^6}$$

20. $y = (x^2+1)^2 + 3(x^2-1)^2$

$$\frac{dy}{dx} = \frac{d}{dx}\left((x^2+1)^2 + 3(x^2-1)^2\right) = \frac{d}{dx}(x^2+1)^2 + \frac{d}{dx}3(x^2-1)^2$$
$$= 2(x^2+1)\frac{d}{dx}(x^2+1) + 6(x^2-1)\frac{d}{dx}(x^2-1)$$
$$= 2(x^2+1)(2x) + 6(x^2-1)(2x) = 4x(x^2+1) + 12x(x^2-1)$$

21. $y = \dfrac{1}{x^3+1} = (x^3+1)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3+1)^{-1} = -(x^3+1)^{-2}\frac{d}{dx}(x^3+1)$$
$$= -(x^3+1)^{-2}(3x^2) = -3x^2(x^3+1)^{-2}$$

22. $y = \dfrac{2}{x+1} = 2(x+1)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}2(x+1)^{-1} = -2(x+1)^{-2}\frac{d}{dx}(x+1)$$
$$= -2(x+1)^{-2}(1) = -2(x+1)^{-2}$$

23. $y = x + \frac{1}{x+1} = x + (x+1)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + (x+1)^{-1}\right) = \frac{d}{dx}x + \frac{d}{dx}(x+1)^{-1}$$
$$= 1 + -(x+1)^{-2}\frac{d}{dx}(x+1) = 1 + -(x+1)^{-2}(1)$$
$$= 1 + -(x+1)^{-2}$$

24. $y = 2\sqrt[4]{x^2+1} = 2(x^2+1)^{1/4}$

$$\frac{dy}{dx} = \frac{d}{dx}2(x^2+1)^{1/4} = \frac{1}{2}(x^2+1)^{-3/4}\frac{d}{dx}(x^2+1)$$
$$= \frac{1}{2}(x^2+1)^{-3/4}(2x) = x(x^2+1)^{-3/4}$$

25. $f(x) = 5\sqrt{3x^3+x} = 5(3x^3+x)^{1/2}$

$$\frac{d}{dx}[5(3x^3+x)^{1/2}] = 5\frac{d}{dx}(3x^3+x)^{1/2}$$
$$= 5\cdot\frac{1}{2}(3x^3+x)^{-1/2}\cdot\frac{d}{dx}(3x^3+x)$$
$$= \frac{5(9x^2+1)}{2\sqrt{3x^3+x}} = \frac{45x^2+5}{2\sqrt{3x^3+x}}$$

26. $y = \frac{1}{x^3+x+1} = (x^3+x+1)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3+x+1)^{-1} = -(x^3+x+1)^{-2}\frac{d}{dx}(x^3+x+1)$$
$$= -(x^3+x+1)^{-2}(3x^2+1) = -(3x^2+1)(x^3+x+1)^{-2}$$

27. $y = 3x + \pi^3$

$$\frac{dy}{dx} = \frac{d}{dx}(3x+\pi^3) = \frac{d}{dx}3x + \frac{d}{dx}\pi^3 = 3$$

28. $y = \sqrt{1+x^2} = (1+x^2)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{d}{dx}(1+x^2)^{\frac{1}{2}} = \frac{1}{2}\cdot(1+x^2)^{-\frac{1}{2}}\cdot\frac{d}{dx}(1+x^2) = \frac{1}{2}\cdot(1+x^2)^{-\frac{1}{2}}\cdot 2x$$
$$= \frac{x}{\sqrt{1+x^2}}$$

29. $y = \sqrt{1+x+x^2} = (1+x+x^2)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{d}{dx}(1+x+x^2)^{\frac{1}{2}} = \frac{1}{2}\cdot(1+x+x^2)^{-\frac{1}{2}}\cdot\frac{d}{dx}(1+x+x^2)$$
$$= \frac{1}{2}\cdot(1+x+x^2)^{-\frac{1}{2}}\cdot(1+2x)$$

30. $y = \frac{1}{2x+5} = (2x+5)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}(2x+5)^{-1} = (-1)\cdot(2x+5)^{-2}\cdot\frac{d}{dx}(2x+5)$$
$$= -(2x+5)^{-2}\cdot 2 = -\frac{2}{(2x+5)^2}$$

31. $y = \frac{2}{1-5x} = 2(1-5x)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}[2(1-5x)^{-1}] = 2\cdot\frac{d}{dx}[(1-5x)^{-1}]$$
$$= 2\cdot(-1)(1-5x)^{-2}\cdot\frac{d}{dx}(1-5x)$$
$$= -2(1-5x)^{-2}\cdot(-5) = 10(1-5x)^{-2}$$

32. $y = \frac{7}{\sqrt{1+x}} = 7(1+x)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[7(1+x)^{-\frac{1}{2}}\right] = 7\left(-\frac{1}{2}\right)(1+x)^{-\frac{3}{2}}\cdot\frac{d}{dx}(1+x)$$
$$= 7\cdot\left(-\frac{1}{2}\right)\cdot(1+x)^{-\frac{3}{2}}\cdot 1 = -\frac{7}{2(1+x)^{\frac{3}{2}}}$$

33. $y = \frac{45}{1+x+\sqrt{x}} = 45(1+x+x^{\frac{1}{2}})^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[45(1+x+x^{\frac{1}{2}})^{-1}\right] = 45(-1)\left(1+x+x^{\frac{1}{2}}\right)^{-2}\frac{d}{dx}\left(1+x+x^{\frac{1}{2}}\right)$$
$$= -45\left(1+x+\sqrt{x}\right)^{-2}\left(1+\frac{1}{2}x^{-\frac{1}{2}}\right)$$

34. $y = (1+x+x^2)^{11}$

$$\frac{dy}{dx} = \frac{d}{dx}(1+x+x^2)^{11} = 11(1+x+x^2)^{10}\frac{d}{dx}(1+x+x^2)$$
$$= 11(1+x+x^2)^{10}(1+2x)$$

35. $y = x+1+\sqrt{x+1} = x+1+(x+1)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x+1+(x+1)^{\frac{1}{2}}\right) = \frac{d}{dx}x + \frac{d}{dx}1 + \frac{d}{dx}(x+1)^{\frac{1}{2}}$$
$$= 1 + \frac{1}{2}(x+1)^{-\frac{1}{2}}$$

36. $y = \pi^2 x$

$$\frac{dy}{dx} = \frac{d}{dx}(\pi^2 x) = \pi^2$$

37. $f(x)=\left(\frac{\sqrt{x}}{2}+1\right)^{3/2}$

$$\frac{d}{dx}\left(\frac{\sqrt{x}}{2}+1\right)^{3/2}=\frac{3}{2}\left(\frac{\sqrt{x}}{2}+1\right)^{1/2}\cdot\frac{d}{dx}\left(\frac{\sqrt{x}}{2}+1\right)$$
$$=\frac{3}{2}\left(\frac{\sqrt{x}}{2}+1\right)^{1/2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot x^{-1/2}$$
$$=\frac{3}{2}\left(\frac{\sqrt{x}}{2}+1\right)^{1/2}\left(\frac{1}{4}x^{-1/2}\right)$$
$$=\frac{3}{8\sqrt{x}}\left(\frac{\sqrt{x}}{2}+1\right)^{1/2}$$

38. $y=\left(x-\frac{1}{x}\right)^{-1}=\left(x-x^{-1}\right)^{-1}$

$$\frac{dy}{dx}=\frac{d}{dx}\left[(x-x^{-1})^{-1}\right]=(-1)(x-x^{-1})^{-2}\frac{d}{dx}(x-x^{-1})$$
$$=-(x-x^{-1})^{-2}(1-(-1)x^{-2})=-\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2}=-\frac{x^2+1}{(x^2-1)^2}$$

39. $f(x)=3x^2-2x+1,\ (1, 2)$

slope $=f'(x)=\frac{d}{dx}(3x^2-2x+1)=6x-2$

$f'(1)=6(1)-2=4$

40. $f(x)=x^{10}+1+\sqrt{1-x},\ (0, 2)$

slope $=f'(x)=\frac{d}{dx}\left(x^{10}+1+\sqrt{1-x}\right)$

$=10x^9+\frac{d}{dx}(1-x)^{1/2}$

$=10x^9+\left(\frac{1}{2}(1-x)^{-1/2}\cdot(-1)\right)$

$=10x^9-\frac{1}{2\sqrt{1-x}}$

$f'(0)=10(0)^9-\frac{1}{2\sqrt{1-0}}=-\frac{1}{2}$

41. $y=x^3+3x-8$

slope $=y'=\frac{d}{dx}(x^3+3x-8)=3x^2+3$

$f'(2)=3(2)^2+3=15$

42. $y = x^3 + 3x - 8$

$y' = 3x^2 + 3$, at $x = 2$, $y' = 15$

To find the equation of the tangent line, let $(x_1, y_1) = (2, 6)$ and the slope = 15.

$y - 6 = 15(x - 2)$

$y = 15x - 30 + 6$

$y = 15x - 24$

43. $y = f(x) = (x^2 - 15)^6$

$\text{slope} = \frac{dy}{dx} = \frac{d}{dx}(x^2 - 15)^6$

$= 6(x^2 - 15)^5 \cdot \frac{d}{dx}(x^2 - 15)$

$= 6(x^2 - 15)^5 \cdot 2x = 12x(x^2 - 15)^5$

$\text{slope} = f'(x) = 12x(x^2 - 15)^5$

$f'(4) = 12(4)(16 - 15)^5 = 48$

$f(4) = (4^2 - 15)^6 = 1$

Let $(x_1, y_1) = (4, 1)$, slope = 48.

$y - 1 = 48(x - 4)$

$y = 48x - 192 + 1$

$y = 48x - 191$

44. $y = f(x) = \dfrac{8}{x^2 + x + 2}$

$f(2) = \dfrac{8}{2^2 + 2 + 2} = 1$

$\text{slope} = f'(x) = \frac{d}{dx} 8(x^2 + x + 2)^{-1}$

$= 8(-1)(x^2 + x + 2)^{-2} \cdot \frac{d}{dx}(x^2 + x + 2)$

$= -8(x^2 + x + 2)(2x + 1) = \dfrac{-8(2x + 1)}{(x^2 + x + 2)}$

$f'(2) = \dfrac{-8(4 + 1)}{(4 + 2 + 2)^2} = -\dfrac{40}{64} = -\dfrac{5}{8}$

Let $(x_1, y_1) = (2, 1)$.

$y - 1 = -\frac{5}{8}(x - 2)$

$y = -\frac{5x}{8} + \frac{10}{8} + 1$

$y = -\frac{5x}{8} + \frac{9}{4}$

45. $f(x) = (3x^2 + x - 2)^2$

a. $\frac{d}{dx}(3x^2 + x - 2)^2$

$= 2(3x^2 + x - 2) \cdot \frac{d}{dx}(3x^2 + x - 2)$

$= 2(3x^2 + x - 2)(6x + 1)$

b. $(3x^2 + x - 2)(3x^2 + x - 2)$

$= 9x^4 + 3x^3 - 6x^2 + 3x^3$

$\quad + x^2 - 2x - 6x^2 - 2x + 4$

$= 9x^4 + 6x^3 - 11x^2 - 4x + 4$

$\frac{d}{dx}(9x^4 + 6x^3 - 11x^2 - 4x + 4)$

$= 36x^3 + 18x^2 - 22x - 4$

46. $\frac{d}{dx}[f(x) - g(x)]$

$= \frac{d}{dx} f(x) - \frac{d}{dx} g(x)$ (sum rule)

$= \frac{d}{dx} f(x) + \frac{d}{dx}(-1)g(x)$ (const. mult. rule)

$= \frac{d}{dx} f(x) - \frac{d}{dx} g(x)$

47. $f(1) = .6(1) + 1 = 1.6$, so $g(1) = 3f(1) = 4.8$.

$f'(1) = .6$ (slope of the line), $g'(1) = 3f'(1) = 1.8$

48. $h(1) = f(1) + g(1) = -.4(1) + 2.6 + .26(1) + 1.1$
$= 3.56$

$h'(1) = f'(1) + g'(1) = -.4 + .26 = -.14$

49. $h(5) = 3f(5) + 2g(5) = 3(2) + 2(4) = 14$

$h'(5) = 3f'(5) + 2g'(5) = 3(3) + 2(1) = 11$

50. $f(x) = 2[g(x)]^3$

$f'(x) = 6[g(x)]^2 g'(x)$

$f(3) = 2[g(3)]^3 = 2(2)^3 = 16$

$f'(3) = 6(2)^2 4 = 96$

51. $f(x) = 5\sqrt{g(x)}$

$f'(x) = \dfrac{5}{2\sqrt{g(x)}} g'(x)$

$f(1) = 5\sqrt{g(1)} = 5\sqrt{4} = 10$

$f'(1) = \dfrac{5}{2\sqrt{g(1)}} g'(1) = \dfrac{5}{2\sqrt{4}} 3 = \dfrac{15}{4}$

52. $h(1) = [f(1)]^2 + \sqrt{g(1)} = 1^2 + \sqrt{4} = 3$

$h'(x) = 2[f(x)]f'(x) + \frac{1}{2}[g(x)]^{-\frac{1}{2}} g'(x)$

$h'(1) = 2[f(1)]f'(1) + \frac{1}{2}[g(1)]^{-\frac{1}{2}} g'(1)$

$= 2(1)(-1) + \frac{1}{2}(4)^{-\frac{1}{2}}(4) = -1$

53. $\frac{dy}{dx} = x^2 - 8x + 18 = 3,$ since the slope of $6x - 2y = 1$ is 3.
$x^2 - 8x + 15 = 0$
$(x-5)(x-3) = 0$
$x = 3$ or $x = 5$
Fit 3 and 5 back into y □ points (3, 49) and $\left(5, \frac{161}{3}\right)$.

54. $\frac{dy}{dx} = 3x^2 - 12x - 34 = 2$
$3x^2 - 12x - 36 = 0$
$x^2 - 4x - 12 = 0$
$(x-6)(x+2) = 0$
$x = -2$ or $x = 6$
Put -2 and 6 back into the equation to get the points (-2,27) and (6,-213).

55. $y = f(x)$
slope $= \frac{3-5}{0-4} = \frac{1}{2}$
Let $(x_1, y_1) = (4, 5)$.
$y - 5 = \frac{1}{2}(x-4)$
$y = \frac{1}{2}x - 2 + 5$
$y = \frac{1}{2}x + 3$
$f(4) = \frac{1}{2}(4) + 3 = 2 + 3 = 5$
$f'(4) = \frac{1}{2}$

56. $y = f(x) = \frac{1}{2}x^2 - 4x + 10$
$f(6) = \frac{1}{2}(6)^2 - 4(6) + 10 = 18 - 24 + 10 = 4$
slope $= f'(x) = \frac{d}{dx}\left(\frac{1}{2}x^2 - 4x + 10\right)$
$= \frac{1}{2} \cdot 2x - 4 = x - 4$
$f'(6) = 6 - 4 = 2$
Let $(x_1, y_1) = (6, 4)$.
$y - 4 = 2(x - 6)$
$y = 2x - 12 + 4$
$y = 2x - 8$
To find the value of *b*, let $x = 0$ and solve for *y*.
$y = 2(0) - 8 = -8$

57. a. The sales at the end of January reached $\$120{,}560 \Rightarrow S(1) = \$120{,}560$
Rising at a rate of \$1500/month
$\Rightarrow S'(1) = \$1500$

b. At the end of March, the sales for the month dropped to $\$80{,}000 \Rightarrow S(3) = S(2) - \$80{,}000$
Were falling by about \$200/day
$\Rightarrow S'(3) = \$200(30) = \$6000$

58. a. $S(10) = 3 + \frac{9}{11^2} = \$3.07438$ thousand

$S'(10) = \frac{-18}{11^3} = \$-.013524$ thousand/day

b. Rate of change of sales on January 2: \$-.667 thousand/day (down \$667/day), rate of change of sales on January 10: \$-.013524 (down \$13/day). Although sales are still down on January 10, the rate at which the sales are down is increasing, and since the rates are negative, this implies sales are getting better.

59. a. $S(10) = 3 + \frac{9}{11^2} = \$3.07438$ thousand

$S'(10) = \frac{-18}{11^3} = \$-.013524$ thousand/day

b. $S(11) \approx S(10) + S'(10)$

$= 3.07438 + (-.013524)$
$= \$3.060856$ thousand

$S(11) = 3 + \frac{9}{12^2} = \$3.0625$ thousand

60. a. $T(1) = \frac{24}{5} + \frac{36}{5(3(1)+1)^2} = \$5.25$ thousand

$T'(x) = \frac{d}{dx}\left(\frac{24}{5} + \frac{36}{5}(3x+1)^{-2}\right) = -\frac{72}{5}(3x+1)^{-3}(3)$
$= -\frac{216}{5}(3x+1)^{-3}$

$T'(1) = -\frac{216}{5}(3(1)+1)^{-3} = \$-.675$ thousand/day

$S(1) = 3 + \frac{9}{4} = \$5.25$ thousand

$S'(1) = -\frac{18}{8} = \$-2.25$ thousand/day

b. According to both functions, the sales were \$5250 on January 1. Although, the rate at which sales fell on that date differ. $T(x)$ gives a much smaller rate of sales dropping then $S(x)$.

61. a. When \$8000 is spent on advertising, 1200 computers were sold $\Rightarrow A(8) = 12$, and it was rising at the rate of 50 computers for each \$1000 spent on advertising $\Rightarrow A'(8) = .5$

b. $A(9) \approx A(8) + A'(8) = 12.5$ (hundred) $= 1{,}250$ computers.

62. $S(n)$: The number of video games sold on day n since the item was released.

$S'(n)$: The rate at which the video games are being sold on day n since the item was released.

$S(n) + S'(n)$: The approximate number of video games sold on day $n + 1$ since the item was released.

63. Federal debt at the end of 1999 $= D(4)$

$$D(4) = 4.95 + .402(4) - .1067(4)^2 + .0124(4)^3 - .00024(4)^4 = \$5.58296 \text{ trillion}$$

Rate of increase at the end of 1999 $= D'(4)$

$$D'(x) = .402 - .2134x + .0372x^2 - .00096x^3$$

$$D'(4) = .402 - .2134(4) + .0372(4)^2 - .00096(4)^3 = \$.08216 \text{ trillion/year}$$

64. a. $D(8) = \$6.70296$ trillion

$D(6) = \$5.88816$ trillion

No, the federal debt was not twice as large by the end of 2003 than the end of 2001.

b. $D'(8) = \$.58408$ trillion/year

$D'(6) = \$.25344$ trillion/year

Yes, it is true that by the end of 2003 the federal debt was increasing at a rate that was more than twice the rate at the end of 2001.

Exercises 1.7

1. $f(t) = (t^2+1)^5$

$$\frac{d}{dt}(t^2+1)^5 = 5(t^2+1)^4 \cdot \frac{d}{dt}(t^2+1) = 5(t^2+1)^4(2t) = 10t(t^2+1)^4$$

2. $f(P) = P^3 + 3P^2 - 7P + 2$

$$\frac{d}{dP}(P^3 + 3P^2 - 7P + 2) = 3P^2 + 6P - 7$$

3. $v(t) = 4t^2 + 11\sqrt{t} + 1 = 4t^2 + 11t^{\frac{1}{2}} + 1$

$$\frac{d}{dt}(4t^2 + 11t^{\frac{1}{2}} + 1) = 8t + \frac{11}{2}t^{-\frac{1}{2}}$$

4. $g(y) = y^2 - 2y + 4$

$$\frac{d}{dy}(y^2 - 2y + 4) = 2y - 2$$

5. $y = T^5 - 4T^4 + 3T^2 - T - 1$

$$\frac{dy}{dT} = \frac{d}{dT}(T^5 - 4T^4 + 3T^2 - T - 1) = 5T^4 - 16T^3 + 6T - 1$$

6. $x = 16t^2 + 45t + 10$

$$\frac{dx}{dt} = \frac{d}{dt}(16t^2 + 45t + 10) = 32t + 45$$

7. $$\frac{d}{dP}\left(3P^2 - \frac{1}{2}P + 1\right) = 6P - \frac{1}{2}$$

8. $$\frac{d}{ds}\sqrt{s^2-1} = \frac{d}{ds}(s^2-1)^{\frac{1}{2}} = \frac{1}{2}(s^2-1)^{-\frac{1}{2}}\frac{d}{ds}(s^2-1) = \frac{1}{2}(s^2-1)^{-\frac{1}{2}}(2s)$$

9. $$\frac{d}{dt}(a^2t^2 + b^2t + c^2) = 2a^2t + b^2 + 0 = 2a^2t + b^2$$

10. $$\frac{d}{dP}(T^2 + 3P)^3 = 3(T^2 + 3P)^2\frac{d}{dP}(T^2 + 3P) = 9(T^2 + 3P)^2$$

11. $y = x + 1$

$y' = 1$

$y'' = 0$

12. $y=(x+12)^3$
$y'=3(x+12)^2$
$y''=6(x+12)=6x+72$

13. $y=\sqrt{x}=x^{1/2}$
$y'=\frac{1}{2}x^{-1/2}$
$y''=-\frac{1}{4}x^{-3/2}$

14. $y=100$
$y'=0$
$y''=0$

15. $y=\sqrt{x+1}=(x+1)^{\frac{1}{2}}$
$y'=\frac{1}{2}(x+1)^{-\frac{1}{2}}$
$y''=-\frac{1}{2}\left(\frac{1}{2}\right)(x+1)^{-\frac{3}{2}}=-\frac{1}{4}(x+1)^{-\frac{3}{2}}$

16. $v=2t^2+3t+11$
$v'=4t+3$
$v''=4$

17. $f(r)=\pi r^2$
$f'(r)=2\pi r$
$f''(r)=2\pi$

18. $y=\pi^2+3x^2$
$y'=6x$
$y''=6$

19. $f(P)=(3P+1)^5$
$f'(P)=5(3P+1)^4\cdot\frac{d}{dP}(3P+1)$
$=5(3P+1)^4\cdot 3=15(3P+1)^4$
$f''(P)=60(3P+1)^3\cdot\frac{d}{dP}(3P+1)$
$=60(3P+1)^3\cdot 3=180(3P+1)^3$

20. $T=(1+2t)^2+t^3$
$T'=2\cdot(1+2t)\cdot 2+3t^2=4+8t+3t^2$
$T''=8+6t$

21. $\frac{d}{dx}(2x+7)^2\Big|_{x=1}=\left[2(2x+7)\frac{d}{dx}(2x+7)\right]_{x=1}$
$=[4(2x+7)]_{x=1}=4(2(1)+7)=36$

22. $\frac{d}{dt}\left(t^2+\frac{1}{t+1}\right)\Big|_{t=0}=\frac{d}{dt}\left(t^2+(t+1)^{-1}\right)\Big|_{t=0}$
$=\left[2t+(-1)(t+1)^{-2}\frac{d}{dt}(t+1)\right]\Big|_{t=0}=2t-\frac{1}{(t+1)^2}\Big|_{t=0}$
$=2(0)-\frac{1}{(0+1)^2}=-1$

23. $\frac{d}{dz}(z^2+2z+1)^7\Big|_{z=-1}$
$=\left[7(z^2+2z+1)^6\frac{d}{dz}(z^2+2z+1)\right]\Big|_{z=-1}$
$=7(2z+2)(z^2+2z+1)^6\Big|_{z=-1}$
$=7\left(2(-1)+2\right)\left((-1)^2+2(-1)+1\right)$
$=0$

24. $\frac{d^2}{dx^2}(3x^4+4x^2)\Big|_{x=2}$
$\frac{d}{dx}(3x^4+4x^2)=12x^3+8x$
$\frac{d}{dx}(12x^3+8x)=36x^2+8$
$\frac{d^2}{dx^2}(3x^4+4x^2)\Big|_{x=2}=36(2)^2+8=152$

25. $\frac{d^2}{dx^2}(3x^3-x^2+7x-1)\Big|_{x=2}$
$\frac{d}{dx}(3x^3-x^2+7x-1)=9x^2-2x+7$
$\frac{d}{dx}(9x^2-2x+7)=18x-2$
$\frac{d^2}{dx^2}(3x^3-x^2+7x-1)\Big|_{x=2}=18(2)-2=34$

26. $\frac{d}{dx}\left(\frac{dy}{dx}\right)\Big|_{x=1}$, where $y=x^3+2x-11$
$\frac{dy}{dx}=\frac{d}{dx}(x^3+2x-11)=3x^2+2$
$\frac{d}{dx}(3x^2+2)\Big|_{x=1}=6x\big|_{x=1}=6(1)=6$

27. $f'(1)$ and $f''(1)$, when $f(t)=\dfrac{1}{2+t}$.

$f'(t)=(-1)(2+t)^{-2},\ f'(1)=-(2+1)^{-2}=-\dfrac{1}{9}$

$f''(t)=(-2)(-1)(2+t)^{-3}$,

$f''(1)=2(2+1)^{-3}=\dfrac{2}{3^3}=\dfrac{2}{27}$

28. $g'(0)$ and $g''(0)$, when $g(T)=(T+2)^3$.

$g'(T)=3(T+2)^2$

$g'(0)=3(0+2)^2=12$

$g''(T)=6(T+2)$

$g''(0)=6(0+2)=12$

29. $\left.\dfrac{d}{dt}\left(\dfrac{d\upsilon}{dt}\right)\right|_{t=32}$, where $\upsilon-20t+12$

$\dfrac{d\upsilon}{dt}=20,\ \left.\dfrac{d}{dt}(20)\right|_{t=32}=0$

30. $\dfrac{d}{dt}\left(\dfrac{d\upsilon}{dt}\right)$, where $\upsilon=2t^2+\dfrac{1}{t+1}$

$$\frac{d\upsilon}{dt}=\frac{d}{dt}\left(2t^2+(t+1)^{-1}\right)$$
$$=4t+(-1)(t+1)^{-2}\frac{d}{dt}(t+1)$$
$$=4t-(t+1)^{-2}$$
$$\frac{d}{dt}\left(\frac{d\upsilon}{dt}\right)=\frac{d}{dt}\left(4t-(t+1)^{-2}\right)$$
$$=4-(-2)(t+1)^{-3}\frac{d}{dt}(t+1)$$
$$=4+\frac{2}{(t+1)^3}$$

31. $R=1000+80x-.02x^2$, for $0\le x\le 2000$

$\dfrac{dR}{dx}=80-.04x$

$\left.\dfrac{dR}{dx}\right|_{x=1500}=80-.04(1500)=20$

32. $V=20\left(1-\dfrac{100}{100+t^2}\right),\ 0\le t\le 24$

$V=20-2000(100+t^2)^{-1}$

$\dfrac{dV}{dt}=2000(100+t^2)^{-2}\cdot\dfrac{d}{dt}(100+t^2)$

$=2000(100+t^2)^{-2}\cdot 2t=4000t(100+t^2)^{-2}$

$\left.\dfrac{dV}{dt}\right|_{t=10}=\dfrac{4000(10)}{(100+10^2)^2}=1$

33. a. $f(x)=x^5-x^4+3x$

$f'(x)=5x^4-4x^3+3$

$f''(x)=20x^3-12x^2$

$f'''(x)=60x^2-24x$

b. $f(x)=4x^{5/2}$

$f'(x)=10x^{3/2}$

$f''(x)=15x^{1/2}$

$f'''(x)=\dfrac{15}{2}x^{-1/2}=\dfrac{15}{2\sqrt{x}}$

34. a. $f(t)=t^{10}$

$f'(t)=10t^9$

$f''(t)=90t^8$

$f'''(t)=720t^7$

b. $f(z)=\dfrac{1}{z+5}=(z+5)^{-1}$

$f'(z)=-(z+5)^{-2}$

$f''(z)=2(z+5)^{-3}$

$f'''(z)=-6(z+5)^{-4}=-\dfrac{6}{(z+5)^4}$

35. $s=Tx^2+3xP+T^2$

a. $\dfrac{ds}{dx}=\dfrac{d}{dx}\left(Tx^2+3xP+T^2\right)=2Tx+3P$

b. $\dfrac{ds}{dP}=\dfrac{d}{dP}\left(Tx^2+3xP+T^2\right)=3x$

c. $\dfrac{ds}{dT}=\dfrac{d}{dT}\left(Tx^2+3xP+T^2\right)=x^2+2T$

36. $s=7x^2y\sqrt{z}$

a. $\dfrac{d^2s}{dx^2} = \dfrac{d^2}{dx^2}\left(7x^2y\sqrt{z}\right) = \dfrac{d}{dx}\left(14xy\sqrt{z}\right)$
$= 14y\sqrt{z}$

b. $\dfrac{d^2s}{dy^2} = \dfrac{d^2}{dy^2}\left(7x^2y\sqrt{z}\right) = \dfrac{d}{dx}\left(7x^2\sqrt{z}\right)$
$= 0$

c. $\dfrac{ds}{dz} = \dfrac{d}{dz}\left(7x^2y\sqrt{z}\right) = \dfrac{7}{2}x^2yz^{-\frac{1}{2}} = \dfrac{7x^2y}{2\sqrt{z}}$

37. $C(50) = 5000$: It costs \$5000 to manufacture 50 bicycles in one day.

$C'(50) = 45$: It costs and additional \$45 to make the 51st bicycle.

38. $C(51) \approx C(50) + C'(50) = 5000 + 45 = \$5045$

39. $R(x) = 3x - .01x^2$, $R'(x) = 3 - .02x$

a. $R'(20) = 3 - .02(20) = \$2.6/\text{unit}$

b. $R(x) = 3x - .01x^2 = 200$

$\Rightarrow x = 100$ or $x = 200$ units **.**

40. A → d

B → b

C → a

D → c

41. a. When 1200 chips are produced per day, the revenue is \$22,000 ⇒ $R(12) = 22$, and the marginal revenue is \$.75 per chip ⇒ $R'(12) = \$.075$ thousand / unit (\$.75 per chip = \$75 per unit = \$.075 thousand/unit)

b. Marginal Profit = Marginal Revenue - Marginal Cost

$P'(12) = R'(12) - C'(12) = .075 - .15$
$= -\$.075$ thousand/unit
$= -\$75$/unit

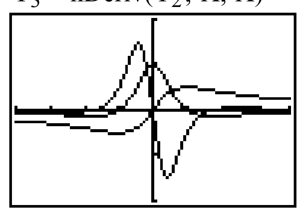
42. $P(13) = R(13) - C(13)$

$R(13) \approx R(12) + R'(12) = 22 + .075$
$= \$22.075$ thousand
$C(13) \approx C(12) + C'(12) = 14 + .15$
$= \$14.15$ thousand
$\Rightarrow P(13) = 22.075 - 14.15 = \$7.925$ thousand

Although cost is increasing at a rate greater than revenue at 1200 chips, it is still profitable to raise the production level to 1300.

43. $f(x) = \dfrac{x}{1 + x^2}$

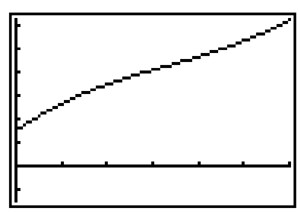
$Y_1 = \dfrac{X}{1 + X^2}$
Y_2 = nDeriv(Y_1, X, X)
Y_3 = nDeriv(Y_2, X, X)



[24, 4] *by* [22, 2]

44. $C(x) = .005x^3 - .5x^2 + 28x + 300$

a.



[0, 60] *by* [2300, 1260]

b. $C(x) = 535$
Graphing the line $y = 535$ and using the Intersect command, the point (10, 535) is on both graphs. A level of production of 10 items has a cost of \$535.

c. $C'(x) = 14$
Graph the derivative:
$Y_1 = .015X^2 - X + 28$ and $Y_2 = 14.$
Using the Intersect command, the points (20, 14) and $\left(46\frac{2}{3}, 14\right)$ are on both graphs.
The marginal cost will be $14 at production levels of 20 items and $46\frac{2}{3}$ items.

Exercises 1.8

1. $f(x) = 4x^2$

a. Over $1 \le x \le 2$,
$$\frac{f(b)-f(a)}{b-a} = \frac{4(2)^2 - 4(1)^2}{2-1} = \frac{16-4}{1} = 12$$
over $1 \le x \le 1.5$,
$$\frac{f(b)-f(a)}{b-a} = \frac{4(1.5)^2 - 4(1)^2}{1.5-1} = \frac{9-4}{.5} = 10$$
over $1 \le x \le 1.1$,
$$\frac{f(b)-f(a)}{b-a} = \frac{4(1.1)^2 - 4(1)^2}{1.1-1} = \frac{4.84-4}{.1}$$
$= 8.4$

b. $f'(x) = 8x$
$f'(1) = 8$

2. $f(x) = -\frac{6}{x}$

a. Over $1 \le x \le 2$,
$$\frac{f(b)-f(a)}{b-a} = \frac{-\frac{6}{2} - \left(-\frac{6}{1}\right)}{2-1} = \frac{-3+6}{1} = 3$$
over $1 \le x \le 1.5$,
$$\frac{f(b)-f(a)}{b-a} = \frac{-\frac{6}{1.5} - \left(-\frac{6}{1}\right)}{1.5-1} = \frac{-4+6}{.5} = 4$$
over $1 \le x \le 1.2$,
$$\frac{f(b)-f(a)}{b-a} = \frac{-\frac{6}{1.2} - \left(-\frac{6}{1}\right)}{1.2-1} = \frac{-5+6}{.2} = 5$$

b. $f'(x) = \frac{6}{x^2}$
$f'(1) = \frac{6}{1^2} = 6$

3. $f(t) = t^2 + 3t - 7$

a. Over $5 \le x \le 6$,
$$\frac{f(b)-f(a)}{b-a} = \frac{6^2 + 3(6) - 7 - (5^2 + 3(5) - 7)}{6-5}$$
$= 36 + 18 - 7 - 25 - 15 + 7 = 14$

b. $f'(t) = 2t + 3$
$f'(5) = 2(5) + 3 = 13$

4. $f(t) = 3t + 2 - \frac{12}{t}$

a. Over $2 \le x \le 3$,
$$\frac{f(b)-f(a)}{b-a} = \frac{3(3) + 2 - \frac{12}{3} - \left(3(2) + 2 - \frac{12}{2}\right)}{3-2}$$
$= 9 + 2 - 4 - 6 - 2 + 6$
$= 5$

b. $f'(t) = 3 + \frac{12}{t^2}$
$f'(2) = 3 + \frac{12}{2^2} = 3 + 3 = 6$

5. a. $f(1) = 14$, $f(5) = 7$
$$\frac{f(b)-f(a)}{b-a} = \frac{7-14}{5-1} = \frac{-7}{4}$$

b. Slope of tangent line at $t = 9$:
$$\frac{10-5}{11-5} = \frac{5}{6}$$
The yield was rising at the rate of $\frac{5}{6}$ percent per year on January 1, 1989.

c. The graph is clearly steeper in 1980 than in 1989, so the percentage yield was rising faster on January 1, 1980.

6. a. $f(20) = 150$, $f(60) = 300$
$$\frac{f(b)-f(a)}{b-a} = \frac{300-150}{60-20} = \frac{150}{40}$$
$= \frac{15}{4}$ acres per year

b. Slope of tangent line at $x = 50$:
$$\frac{400-150}{80-40} = \frac{250}{40} = \frac{25}{4}$$
The mean farm size was increasing at the rate of $\frac{25}{4}$ acres per year on January 1, 1950.

c. The graph is steeper in 1960 than in 1980, so the mean farm size was rising faster on January 1, 1960.

7. $W(t) = .1t^2$
Over $4 \le t \le 5$,
$\frac{f(b)-f(a)}{b-a} = \frac{.1(5)^2 - .1(4)^2}{5-4} = \frac{2.5-1.6}{1}$
$= .9 \frac{\text{grams}}{\text{week}}$
$W'(t) = .2t$
$W'(4) = .2(4) = .8 \frac{\text{grams}}{\text{week}}$

8. $f(t) = -3t^2 + 32t + 100$
Over $3 \le t \le 4$,
$\frac{f(b)-f(a)}{b-a}$
$= \frac{-3(4)^2 + 32(4) + 100 - (-3(3)^2 + 32(3) + 100)}{4-3}$
$= -48 + 128 + 100 + 27 - 96 - 100 = 11$ units/day
$f'(t) = -6t + 32$
$f'(2) = -6(2) + 32 = 20$ units/day

9. $f(t) = 60t + t^2 - \frac{1}{12}t^3$
$f'(t) + 60 + 2t - \frac{1}{4}t^2$
$f'(2) = 60 + 2(2) - \frac{1}{4}(2)^2 = 60 + 4 - 1$
$= 63$ units/hour

10. $f(t) = 5t - \sqrt{t}$
$f'(t) = 5 - \frac{1}{2\sqrt{t}}$
$f'(4) = 5 - \frac{1}{2\sqrt{4}} = 5 - \frac{1}{4} = \frac{19}{4}$ gallons/hour

11. $s(t) = 2t^2 + 4t$

a. $s'(t) = 4t + 4$
$s'(6) = 4(6) + 4 = 28$ km/hr

b. $s(6) = 2(6)^2 + 4(6) = 72 + 24 = 96$ km

c. When does $s'(t) = 6$?
$4t + 4 = 6$
$4t = 2$
$t = \frac{1}{2}$
The object is traveling at the rate of 6 km/hr when $t = \frac{1}{2}$ hr.

12. $s(t) = 50t - \frac{7}{t+1}$
$s'(t) = 50 + \frac{7}{(t+1)^2}$
$s'(0) = 50 + \frac{7}{(0+1)^2} = 57$ km/hr
$s''(t) = \frac{-14}{(t+1)^3}$
$s''(0) = \frac{-14}{(0+1)^3} = -14$ km/hr^2

13. $s(t) = 160t - 16t^2$

a. $s'(t) = 160 - 32t$
$s'(0) = 160 - 32(0) = 160$ ft/sec

b. $s'(2) = 160 - 32(2) = 160 - 64 = 96$ ft/sec

c. $s''(t) = -32$
$s''(3) = -32$ ft/sec^2

d. When will $s(t) = 0$?
$160t - 16t^2 = 0$
$16t(10 - t) = 0$
$t = 0$ or $t = 10$ sec
The rocket will hit the ground after 10 sec.

e. What is $s'(t)$ when $t = 10$?
$s'(10) = 160 - 32(10) = 160 - 320$
$= -160$ ft/sec

14. $s(t) = t^2 + t$

a. When will $s(t) = 20$?
$20 = t^2 + t$
$t^2 + t - 20 = 0$
$(t + 5)(t - 4) = 0$
$t + 5 = 0$ or $t - 4 = 0$
$t = -5$ $t = 4$
t must be positive, so the helicopter takes 4 seconds to rise 20 feet.

b. $s'(t) = 2t + 1$
$s'(4) = 2(4) + 1 = 9$ feet/second
$s''(t) = 2$
$s''(4) = 2$ feet/second^2

15. A. The velocity of the ball after 3 seconds is the first derivative evaluated at $t = 3$, or $s'(3)$.
b

B. To find when the velocity will be 3 feet per second, set $s'(t) = 3$ and solve for t.
d

C. The average velocity during the first 3 seconds can be found from:
$$\frac{f(b) - f(a)}{b - a} = \frac{s(3) - s(0)}{3}$$
f

D. The ball will be 3 feet above the ground when, for some value a, $s(a) = 3$.
e

E. The ball will hit the ground when $s(t) = 0$. Solve for t.
a

F. The ball will be $s(3)$ feet high after 3 seconds.
c

G. The ball travels $s(3) - s(0)$ feet during the first 3 seconds.
g

16. $\frac{f(b) - f(a)}{b - a} = \frac{47.4 - 45}{1.05 - 1} = \frac{2.4}{.05} = 48$ miles/hour
To estimate the speed at time 1 hour, calculate the average speed in a small interval near one hour:
$$\frac{f(b) - f(a)}{b - a} = \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{45.4 - 45}{.01}$$
$$= \frac{.4}{.01} = 40 \text{ miles/hour}$$

17. $s(t) = t^2 + 3t + 2$

a. $s'(t) = 2t + 3$
$s'(6) = 2(6) + 3 = 12 + 3 = 15$ feet/second

b. No; the positive velocity indicates the object is moving away from the reference point.

c. The object is 6 feet from the reference point when $s(t) = 6$.
$s(t) = t^2 + 3t + 2 = 6$
$t^2 + 3t - 4 = 0$
$(t + 4)(t - 1) = 0$
$t + 4 = 0$ or $t - 1 = 0$
$t = -4$ $\quad$ $t = 1$
Time is positive, so $t = 1$ second. The velocity at this time is:
$s'(1) = 2(1) + 3 = 5$ feet/second

18. a. If the car travels at a steady speed, the distance of the car from New York will increase at a constant rate. The distance function will be a straight line with a positive slope.
b

b. If the car is stopped, the value of the distance function will not change, so the function will be a straight line with slope of 0.
c

c. If the car is backing up, its distance function will have a negative slope.
d

d. If the car is accelerating, its velocity is increasing, so the slopes of tangents to the distance curve are increasing.
a

e. If the car is decelerating, its velocity is decreasing, so the slopes of the tangents to the distance curve are decreasing.
f

19. $f(100) = 5000$
$f'(100) = 10$
$f(a + h) - f(a) \approx f'(a) \cdot h$
$f(a + h) \approx f'(a) \cdot h + f(a)$

a. $101 = 100 + 1$
$f(100 + 1) \approx f'(100) \cdot 1 + f(100)$
$\approx 10 + 5000 \approx 5010$

b. $100.5 = 100 + .5$
$f(100 + .5) \approx f'(100) \cdot .5 + f(100)$
$\approx 10 \cdot .5 + 5000 \approx 5005$

c. $99 = 100 + (-1)$
$f(100 + (-1)) \approx f'(100) \cdot (-1) + f(100)$
$\approx 10 \cdot (-1) + 5000 \approx 4990$

d. $98 = 100 + (-2)$
$f(100+(-2)) \approx f'(100)\cdot(-2)+f(100)$
$\approx 10(-2) + 5000 \approx 4980$

e. $99.75 = 100 + (-.25)$
$f(100+(-.25)) \approx f'(100)\cdot(-.25)+f(100)$
$\approx 10(-.25) + 5000 \approx 4997.5$

20. $f(25) = 10$
$f'(25) = -2$
$f(a+h) \approx f'(a)\cdot h + f(a)$

a. $27 = 25 + 2$
$f(25+2) \approx f'(25)\cdot 2 + f(25)$
$\approx -2 \cdot 2 + 10 \approx 6$

b. $26 = 25 + 1$
$f(25+1) \approx f'(25)\cdot 1 + f(25)$
$\approx -2 \cdot 1 + 10 \approx 8$

c. $25.25 = 25 + .25$
$f(25+.25) \approx f'(25)\cdot .25 + f(25)$
$\approx -2 \cdot .25 + 10 \approx 9.5$

d. $24 = 25 + (-1)$
$f(25+(-1)) \approx f'(25)\cdot(-1)+f(25)$
$\approx -2 \cdot (-1) + 10 \approx 12$

e. $23.5 = 25 + (-1.5)$
$f(25+(-1.5)) \approx f'(25)\cdot(-1.5)+f(25)$
$\approx -2 \cdot (-1.5) + 10 \approx 13$

21. $f(4) = 120;\ f'(4) = -5$
Four minutes after it has been poured, the temperature of the coffee is 120°. At that time, its temperature is decreasing by 5° per minute.
At 4.1 minutes:
$4.1 = 4 + .1$
$f(4+.1) \approx f'(4)\cdot .1 + f(4)$
$\approx -5 \cdot .1 + 120 \approx 119.5°$

22. $f(3) = 2;\ f'(3) = -.5$
Three hours after it is injected, the amount of the drug present in the bloodstream is 2 mg. At that time, the concentration of the drug is decreasing by .5 mg/hour.
At 3.5 hours:
$3.5 = 3 + .5$
$f(3+.5) \approx f'(3)\cdot .5 + f(3)$
$\approx -.5 \cdot .5 + 2 \approx 1.75$ mg

23. $f(10{,}000) = 200{,}000;\ f'(10{,}000) = -3$
When the price of a car is \$10,000, 200,000 cars are sold. At that price, the number of cars sold decreases by 3 for each dollar increase in the price.

24. $f(100{,}000) = 3{,}000{,}000;\ f'(100{,}000) = 30$
When \$100,000 is spent on advertising, 3,000,000 toys are sold. For every dollar increase in advertising from that amount, 30 more toys are sold. Or, for every dollar decrease from that amount, 30 fewer toys are sold.

25. $f(12) = 60;\ f'(12) = -2$
When the price of a computer is \$1200, 60,000 computers will be sold. At that price, the number of computers sold decreases by 2000 for every \$100 increase in price.

$f(12.5) \approx f(12) + .5f'(12)$
$= 60 + .5(-2) = 59$
About 59 computers will be sold if the price increases to \$1250.

26. $C(2000) = 50{,}000;\ C'(2000) = 10$
When 2000 radios are manufactured, the cost to manufacture them is \$50,000. For every additional radio manufactured, there is an additional cost of \$10.
At 1998 radios:
$1998 = 2000 + (-2)$
$C(2000+(-2) = C'(2000)\cdot(-2)+C(2000)$
$\approx 10\cdot(-2)+50{,}000 \approx \$49{,}980$

27. $P(100) = 90{,}000;\ P'(100) = 1200$
The profit from manufacturing and selling 100 luxury cars is \$90,000. Each additional car made and sold creates an additional profit of \$1200.
At 99 cars:
$99 = 100 + (-1)$
$f(100+(-1)) \approx f'(100)\cdot(-1)+f(100)$
$\approx 1200(-1)+90{,}000 \approx \$88{,}800$

28. a. $f(100) = 16;\ f'(100) = .25$

The value of the company is \$16 per share 100 days since the company went public. At 100 days since the company went public, he value is increasing at a rate of \$.25/day.

b. $f(101) \approx f(100) + f'(100) = .25 + 16$
$= \$16.25$/share

29. $C(x) = 6x^2 + 2x + 10$

a. $C'(x) = 12x + 2$
$C'(5) = 12(5) + 2 = \$62$ thousand/unit

b. $C(5.25) \approx C'(5)(.25) + C(5)$
$= 62(.25) + 170 = \$185.5$ thousand

c. Solve $6x^2 + 2x + 10 = -x^2 + 39x$
$7x^2 - 37x + 10 = 0$
$$x = \frac{37 \pm \sqrt{(-37)^2 - 4(7)(10)}}{14} = \frac{37 \pm 33}{14}$$
$x = \frac{2}{7}$ or $x = 5$
Since we can't produce 2/7 of an item, the break even point is as $x = 5$ items.

d. $R'(x) = -2x + 39$; $R'(5) = \$29$/unit
$C'(x) = 12x + 2$; $C'(5) = \$62$/unit
No, the company should not increase production beyond $x = 5$ items. The additional cost is greater than the additional revenue generated and the company will lose money.

30. $f(.9) \approx f'(1)(-.1) + f(1)$
$f(x) = (1 + x^2)^{-1}$; $f(1) = .5$
$f'(x) = -(1 + x^2)^{-2}(2x)$; $f'(1) = -.5$
$\Rightarrow f(.9) \approx (-.5)(.1) + .5 = .55$

31. a. $f(7) \approx \$500$ billion

b. $f'(7) \approx \$50$ billion/year

c. $f(t) = 1000$ at $t = 14$, or 1994.

d. $f'(t) = 100$ at $t = 14$, or 1994.

32. a. Find $s(3.5) = 60$ feet.

b. Find $s'(2) = 20$ feet/second.

c. Find $s''(1) = 10$ feet/second^2.

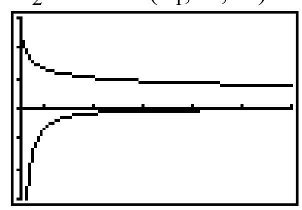
d. Find $s(t) = 120$; $t = 5.5$ seconds.

e. Find $s'(t) = 20$; $t = 7$ seconds.

f. Find maximum $s'(t)$; at $t = 4.5$ seconds, $s'(t) = 30$ feet/second.
$s(4.5) = 90$ feet

33. $f(t) = .36 + .77(t - .5)^{-.36}$

a. Graph:
$Y_1 = .36 + .77(X - .5)^{-.36}$
$Y_2 = \text{nDeriv}(Y_1, X, X)$



[.5, 6] *by* [23, 3]

b. Evaluate at $t = 4$.
$f(4) \approx .85$ seconds

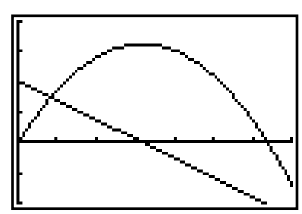
c. Graphing the line $y = .8$ and using the Intersect command, the point (5.23, .8) is on both graphs. After 5 days the judgment time was about .8 seconds.

d. Evaluate $f'(t)$ at $t = 4$.
$f'(4) \approx -.05$ seconds/day

e. Graphing the line $y = -.08$ and $f'(t)$, and using the Intersect command, the point (2.994, –.08) is on both graphs. After 3 days the judgment time was changing at the rate of –.08 seconds per day.

34. $s(t) = 102t - 16t^2$
$s'(t) = 102 - 32t$

a.



[0, 7] *by* [2100, 200]

b. Evaluate $s(t)$ at $t = 2$.
$s(2) = 140$

c. Graphing the line $y = 110$ and using the Intersect command, the point (5, 110) is on both graphs. During the descent, at 5 seconds the ball has a height of 110 feet.

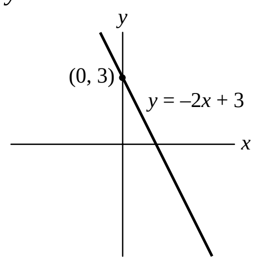
d. $s'(6) = -90$

e. When is $s'(t) = 70$?
Graphing the lines $y = 70$ and $s'(t)$ and using the Intersect command, the point (1, 70) is on both graphs. The velocity is 70 feet/second at 1 second.

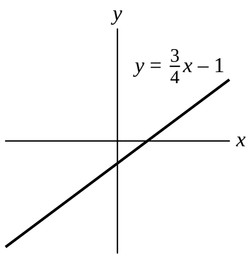
f. When is $s(t) = 0$?
Using the root command, at 6.375
$s'(6.375) = -102$ feet/second.

Chapter 1 Supplementary Exercises

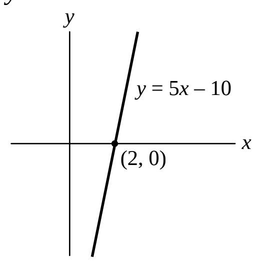
1. Let $(x_1, y_1) = (0, 3)$.
$y - 3 = -2(x - 0)$
$y = 3 - 2x$



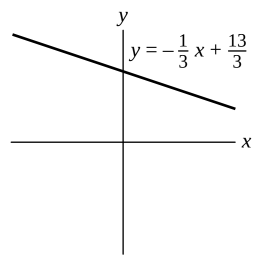
2. Let $(x_1, y_1) = (0, -1)$.
$y - (-1) = \frac{3}{4}(x - 0)$ $y = \frac{3}{4}x - 1$



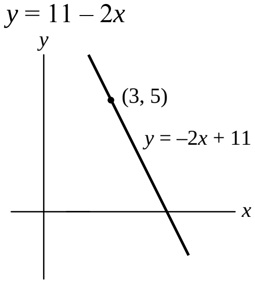
3. Let $(x_1, y_1) = (2, 0)$.
$y - 0 = 5(x - 2)$
$y = 5x - 10$



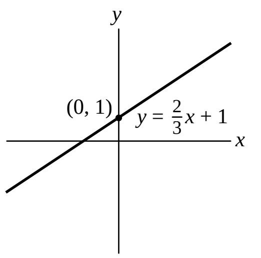
4. Let $(x_1, y_1) = (1, 4)$.
$y - 4 = -\frac{1}{3}(x - 1)$
$y = \frac{13 - x}{3}$



5. $y = -2x$, slope $= -2$
Let $(x_1, y_1) = (3, 5)$.
$y - 5 = -2(x - 3)$
$y = 11 - 2x$



6. $-2x + 3y = 6$
$y = 2 + \frac{2}{3}x$, slope $= \frac{2}{3}$
Let $(x_1, y_1) = (0, 1)$.
$y - 1 = \frac{2}{3}(x - 0)$
$y = \frac{2}{3}x + 1$

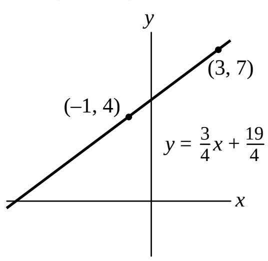


7. slope $= \dfrac{7-4}{3-(-1)} = \dfrac{3}{4}$

Let $(x_1, y_1) = (3, 7)$.

$y - 7 = \dfrac{3}{4}(x-3)$

$y = \dfrac{3}{4}x + \dfrac{19}{4}$

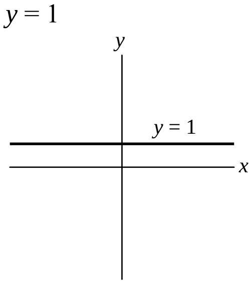


8. slope $= \dfrac{1-1}{5-2} = 0$

Let $(x_1, y_1) = (2, 1)$.

$y - 1 = 0(x-2)$

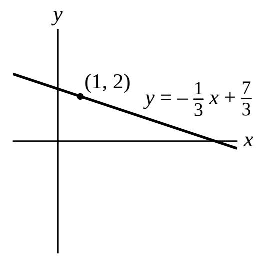
$y = 1$



9. Slope of $y = 3x + 4$ is 3, thus a perpendicular line has slope of $-\dfrac{1}{3}$. The perpendicular line through (1, 2) is

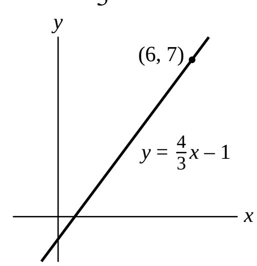
$y - 2 = \left(-\dfrac{1}{3}\right)(x-1)$

$y = -\dfrac{1}{3}x + \dfrac{7}{3}$

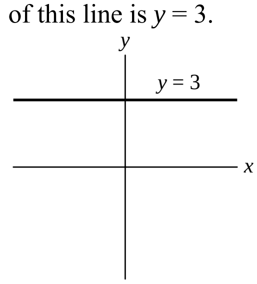


10. Slope of $3x + 4y = 4$ is $-\dfrac{3}{4}$ since $y = -\dfrac{3}{4}x + 1$, thus a perpendicular line has slope of $\dfrac{4}{3}$. The perpendicular line through (6, 7) is

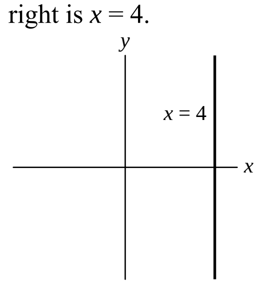
$y - 7 = \dfrac{4}{3}(x-6)$ or $y = \dfrac{4}{3}x - 1$.



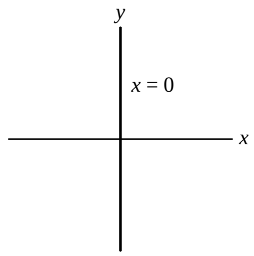
11. The equation of the x-axis is $y = 0$, so the equation of this line is $y = 3$.



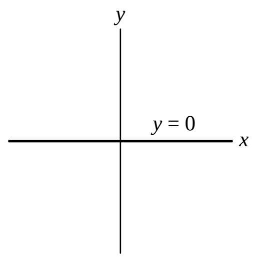
12. The equation of the y-axis is $x = 0$, so 4 units to the right is $x = 4$.



13.



14.



15. $y = x^7 + x^3$; $y' = 7x^6 + 3x^2$

16. $y = 5x^8$; $y' = 40x^7$

17. $y = 6\sqrt{x} = 6x^{1/2}$; $y' = 3x^{-1/2} = \dfrac{3}{\sqrt{x}}$

18. $y = x^7 + 3x^5 + 1$; $y' = 7x^6 + 15x^4$

19. $y = \dfrac{3}{x} = 3x^{-1}$; $y' = -3x^{-2} = -\dfrac{3}{x^2}$

20. $y = x^4 - \dfrac{4}{x} = x^4 - 4x^{-1}$
$y' = 4x^3 + 4x^{-2} = 4x^3 + \dfrac{4}{x^2}$

21. $y = (3x^2 - 1)^8$
$y' = 8(3x^2 - 1)^7(6x) = 48x(3x^2 - 1)^7$

22. $y = \dfrac{3}{4}x^{4/3} + \dfrac{4}{3}x^{3/4}$
$y' = x^{1/3} + x^{-1/4}$

23. $y = \dfrac{1}{5x-1} = (5x-1)^{-1}$
$\dfrac{dy}{dx} = -(5x-1)^{-2}(5) = -\dfrac{5}{(5x-1)^2}$

24. $y = (x^3 + x^2 + 1)^5$
$y' = 5(x^3 + x^2 + 1)^4(3x^2 + 2x)$

25. $y = \sqrt{x^2+1} = (x^2+1)^{1/2}$
$y' = \dfrac{1}{2}(x^2+1)^{-1/2}(2x)$
$= x(x^2+1)^{-1/2} = \dfrac{x}{\sqrt{x^2+1}}$

26. $y = \dfrac{5}{7x^2+1} = 5(7x^2+1)^{-1}$
$\dfrac{dy}{dx} = -5(7x^2+1)^{-2}(14x) = -\dfrac{70x}{(7x^2+1)^2}$

27. $f(x) = \dfrac{1}{\sqrt[4]{x}} = x^{-1/4}$;
$f'(x) = -\dfrac{1}{4}x^{-5/4} = -\dfrac{1}{4x^{5/4}}$

28. $f(x) = (2x+1)^3$
$f'(x) = 3(2x+1)^2(2) = 6(2x+1)^2$

29. $f(x) = 5$; $f'(x) = 0$

30. $f(x) = \dfrac{5x}{2} - \dfrac{2}{5x} = \dfrac{5}{2}x - \dfrac{2}{5}x^{-1}$
$f'(x) = \dfrac{5}{2} + \dfrac{2}{5}x^{-2}$

31. $f(x) = [x^5 - (x-1)^5]^{10}$
$f'(x) = 10[x^5 - (x-1)^5]^9[5x^4 - 5(x-1)^4]$

32. $f(t) = t^{10} - 10t^9$; $f'(t) = 10t^9 - 90t^8$

33. $g(t) = 3\sqrt{t} - \dfrac{3}{\sqrt{t}} = 3t^{1/2} - 3t^{-1/2}$
$g'(t) = \dfrac{3}{2}t^{-1/2} + \dfrac{3}{2}t^{-3/2}$

34. $h(t) = 3\sqrt{2}$; $h'(t) = 0$

35. $f(t) = \dfrac{2}{t-3t^3} = 2(t-3t^3)^{-1}$
$f'(t) = -2(t-3t^3)^{-2}(1-9t^2) = \dfrac{-2(1-9t^2)}{(t-3t^3)^2}$
$= \dfrac{2(9t^2-1)}{(t-3t^3)^2}$

36. $g(P) = 4P^{.7}$; $g'(P) = 2.8P^{-.3}$

37. $h(x)=\frac{3}{2}x^{3/2}-6x^{2/3};\ h'(x)=\frac{9}{4}x^{1/2}-4x^{-1/3}$

38. $f(x)=\sqrt{x+\sqrt{x}}=(x+x^{1/2})^{1/2}$

$f'(x)=\frac{1}{2}(x+x^{1/2})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)$

$=\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)$

39. $f(t)=3t^3-2t^2$

$f'(t)=9t^2-4t$

$f'(2)=36-8=28$

40. $V(r)=15\pi r^2$

$V'(r)=30\pi r$

$V'\left(\frac{1}{3}\right)=10\pi$

41. $g(u)=3u-1$

$g(5)=15-1=14$

$g'(u)=3$

$g'(5)=3$

42. $h(x)=-\frac{1}{2}$

$h(-2)=-\frac{1}{2}$

$h'(x)=0$

$h'(-2)=0$

43. $f(x)=x^{5/2}$

$f'(x)=\frac{5}{2}x^{3/2}$

$f''(x)=\frac{15}{4}x^{1/2}$

$f''(4)=\frac{15}{2}$

44. $g(t)=\frac{1}{4}(2t-7)^4$

$g'(t)=(2t-7)^3(2)=2(2t-7)^3$

$g''(t)=6(2t-7)^2(2)=12(2t-7)^2$

$g''(3)=12[2(3)-7]^2=12$

45. $y=(3x-1)^3-4(3x-1)^2$

slope $=y'=3(3x-1)^2(3)-8(3x-1)(3)$

$=9(3x-1)^2-24(3x-1)$

When $x=0$, slope $=9+24=33$.

46. $y=(4-x)^5$

slope $=y'=5(4-x)^4(-1)=-5(4-x)^4$

When $x=5$, slope $=-5$.

47. $\frac{d}{dx}(x^4-2x^2)=4x^3-4x$

48. $\frac{d}{dt}(t^{5/2}+2t^{3/2}-t^{1/2})=\frac{5}{2}t^{3/2}+3t^{1/2}-\frac{1}{2}t^{-1/2}$

49. $\frac{d}{dP}\left(\sqrt{1-3P}\right)=\frac{d}{dP}(1-3P)^{1/2}$

$=\frac{1}{2}(1-3P)^{-1/2}(-3)=-\frac{3}{2}(1-3P)^{-1/2}$

50. $\frac{d}{dn}(n^{-5})=-5n^{-6}$

51. $\frac{d}{dz}(z^3-4z^2+z-3)\Big|_{z=-2}=(3z^2-8z+1)\Big|_{z=-2}$

$=12+16+1=29$

52. $\frac{d}{dx}(4x-10)^5\Big|_{x=3}=[5(4x-10)^4(4)]\Big|_{x=3}$

$=[20(4x-10)^4]\Big|_{x=3}=320$

53. $\frac{d^2}{dx^2}(5x+1)^4=\frac{d}{dx}[4(5x+1)^3(5)]$

$=60(5x+1)^2(5)=300(5x+1)^2$

54. $\frac{d^2}{dt^2}\left(2\sqrt{t}\right)=\frac{d^2}{dt^2}2t^{1/2}=\frac{d}{dt}t^{-1/2}=-\frac{1}{2}t^{-3/2}$

55. $\frac{d^2}{dt^2}(t^3+2t^2-t)\Big|_{t=-1}=\frac{d}{dt}(3t^2+4t-1)\Big|_{t=-1}$

$=(6t+4)\big|_{t=-1}=-2$

56. $\frac{d^2}{dP^2}(3P+2)\Big|_{P=4}=\frac{d}{dP}3\Big|_{P=4}=0\big|_{P=4}=0$

57. $\frac{d^2y}{dx^2}(4x^{3/2})=\frac{dy}{dx}(6x^{1/2})=3x^{-1/2}$

58. $\frac{d}{dt}\left(\frac{1}{3t}\right) = \frac{d}{dt}\left(\frac{1}{3}t^{-1}\right) = -\frac{1}{3}t^{-2}$ or $-\frac{1}{3t^2}$

$\frac{d}{dt}\left(-\frac{1}{3}t^{-2}\right) = \frac{2}{3}t^{-3}$ or $\frac{2}{3t^3}$

59. $f(x) = x^3 - 4x^2 + 6$

slope $= f'(x) = 3x^2 - 8x$

When $x = 2$, slope $= 3(2)^2 - 8(2) = -4$.

When $x = 2$, $y = 2^3 - 4(2)^2 + 6 = -2$.

Let $(x_1, y_1) = (2, -2)$.

$y - (-2) = -4(x - 2)$

$y = 6 - 4x$

60. $y = \frac{1}{3x-5} = (3x-5)^{-1}$

$y' = -(3x-5)^{-2}(3) = -\frac{3}{(3x-5)^2}$

When $x = 1$, slope $= -\frac{3}{3(1) - 5^2} = -\frac{3}{4}$.

When $x = 1$, $y = \frac{1}{3(1) - 5} = -\frac{1}{2}$.

Let $(x_1, y_1) = \left(1, -\frac{1}{2}\right)$.

$y - \left(-\frac{1}{2}\right) = -\frac{3}{4}(x - 1)$

$y = \frac{1}{4} - \frac{3}{4}x$

61. $y = x^2$

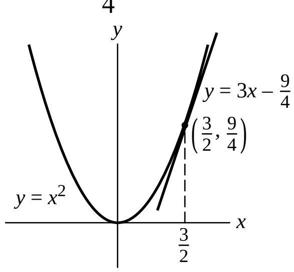
slope $= y' = 2x$

When $x = \frac{3}{2}$, slope $= 2\left(\frac{3}{2}\right) = 3$.

Let $(x_1, y_1) = \left(\frac{3}{2}, \frac{9}{4}\right)$.

$y - \frac{9}{4} = 3\left(x - \frac{3}{2}\right)$

$y = 3x - \frac{9}{4}$



62. $y = x^2$

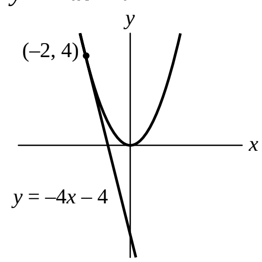
slope $= y' = 2x$

When $x = -2$, slope $= 2(-2) = -4$.

Let $(x_1, y_1) = (-2, 4)$.

$y - 4 = -4(x + 2)$

$y = -4x - 4$



63. $y = 3x^3 - 5x^2 + x + 3$

slope $= y' = 9x^2 - 10x + 1$

When $x = 1$, slope $= 9(1)^2 - 10(1) + 1 = 0$.

When $x = 1$, $y = 3(1)^3 - 5(1)^2 + 1 + 3 = 2$.

Let $(x_1, y_1) = (1, 2)$.

$y - 2 = 0(x - 1)$

$y = 2$

64. $y = (2x^2 - 3x)^3$

slope $= y' = 3(2x^2 - 3x)^2(4x - 3)$

When $x = 2$,

slope $= 3(2(2)^2 - 3(2))^2(4(2) - 3) = 60$.

When $x = 2$, $y = (2(2)^2 - 3(2))^3 = 8$.

Let $(x_1, y_1) = (2, 8)$.

$y - 8 = 60(x - 2)$

$y = 60x - 112$

65. The line has slope –1 and contains the point (5, 0).

$y - 0 = -1(x - 5)$

$y = -x + 5$

$f(2) = -2 + 5 = 3$

$f'(2) = -1$

66. The tangent line contains the points (0, 2) and (a, a^3) and has slope $= 3a^2$. Thus, $\frac{a^3 - 2}{a} = 3a^2$

$a^3 - 2 = 3a^3$

$-2 = 2a^3$

$a = -1$

67. $s'(t) = -32t + 32$
The binoculars will hit the ground when $s(t) = 0$, i.e.,
$s(t) = -16t^2 + 32t + 128 = 0$
$-16(t^2 - 2t - 8) = 0$
$-16(t-4)(t+2) = 0$
$t = 4$ or $t = -2$
$s'(4) = -32(4) + 32 = -96$ feet/sec.
Therefore, when the binoculars hit the ground, they will be falling at the rate of 96 feet/sec.

68. $40t + t^2 - \frac{1}{15}t^3$ tons is the total output of a coal mine after t hours. The rate of output is $40 + 2t - \frac{1}{5}t^2$ tons per hour. At $t = 5$, the rate of output is $40 + 2(5) - \frac{1}{5}(5)^2 = 45$ tons/hour.

69. 11 feet

70. $\frac{s(4) - s(1)}{4-1} = \frac{6-1}{4-1} = \frac{5}{3}$ ft/sec

71. Slope of the tangent line is $\frac{5}{3}$ so $\frac{5}{3}$ ft/sec.

72. $t = 6$, since $s(t)$ is steeper at $t = 6$ than at $t = 5$.

73. $C(x) = .1x^3 - 6x^2 + 136x + 200$

a. $C(21) - C(20)$
$= .1(21)^3 - 6(21)^2 + 136(21) + 200$
$\quad - (.1(20)^3 - 6(20)^2 + 136(20) + 200)$
$= 1336.1 - 1320 = \$16.10$

b. $C'(x) = .3x^2 - 12x + 136$
$C'(20) = .3(20)^2 - 12(20) + 136 = \$16$

74. $f(235) = 4600$
$f'(235) = -100$
$f(a+h) \approx f'(a) \cdot h + f(a)$

a. $237 = 235 + 2$
$f(235+2) \approx f'(235) \cdot 2 + f(235)$
$\approx -100 \cdot 2 + 4600 \approx 4400$ riders

b. $234 = 235 + (-1)$
$f(235 + (-1)) \approx f'(235) \cdot (-1) + f(235)$
$\approx -100 \cdot (-1) + 4600 \approx 4700$ riders

c. $240 = 235 + 5$
$f(235+5) \approx f'(235) \cdot 5 + f(235)$
$\approx -100 \cdot 5 + 4600 \approx 4100$ riders

d. $232 = 235 + (-3)$
$f(235 + (-3)) \approx f'(235) \cdot (-3) + f(235)$
$\approx -100 \cdot (-3) + 4600 \approx 4900$ riders

75. $h(12.5) - h(12) \approx h'(12)(.5) = (1.5)(.5) = .75$ in.

76. $f\left(7 + \frac{1}{2}\right) - f(7) \approx f'(7)\frac{1}{2} = (25.06)\frac{1}{2}$
$= 12.53$
\$12.53 is the additional money earned if the bank paid $7\frac{1}{2}$% interest.

77. $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x+2)(x-2)}{x-2}$
$= \lim_{x\to 2}(x+2) = 2 + 2 = 4$

78. The limit does not exist.

79. The limit does not exist.

80. $\lim_{x\to 5} \frac{x-5}{x^2 - 7x + 2} = \frac{5-5}{25-35+2} = 0$

81. $f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$
If $f(x) = \frac{1}{2x}$, then
$f(5+h) - f(5) = \frac{1}{2(5+h)} - \frac{1}{2(5)}$
$= \frac{1}{2(5+h)} \cdot \frac{5}{5} - \frac{1}{2(5)} \cdot \left(\frac{5+h}{5+h}\right)$
$= \frac{5 - (5+h)}{10(5+h)} = \frac{-h}{10(5+h)}$
Thus,
$f'(5) = \lim_{h\to 0}[f(5+h) - f(5)] \cdot \frac{1}{h}$
$= \lim_{h\to 0} \frac{-h}{10(5+h)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-1}{10(5+h)} = -\frac{1}{50}$

82. $f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$

If $f(x) = x^2 - 2x + 1$, then

$f(3+h) - f(3)$

$= (3+h)^2 - 2(3+h) + 1 - (9 - 6 + 1)$

$= h^2 + 4h.$

Thus,

$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{h^2 + 4h}{h}$

$= \lim_{h \to 0} (h + 4) = 4.$

83. The slope of a secant line at (3, 9)

84. $\frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \frac{\frac{2-2-h}{2(2+h)}}{h} = \frac{-1}{2(2+h)}$

As $h \to 0$, $\frac{-1}{2(2+h)} \to -\frac{1}{4}$.