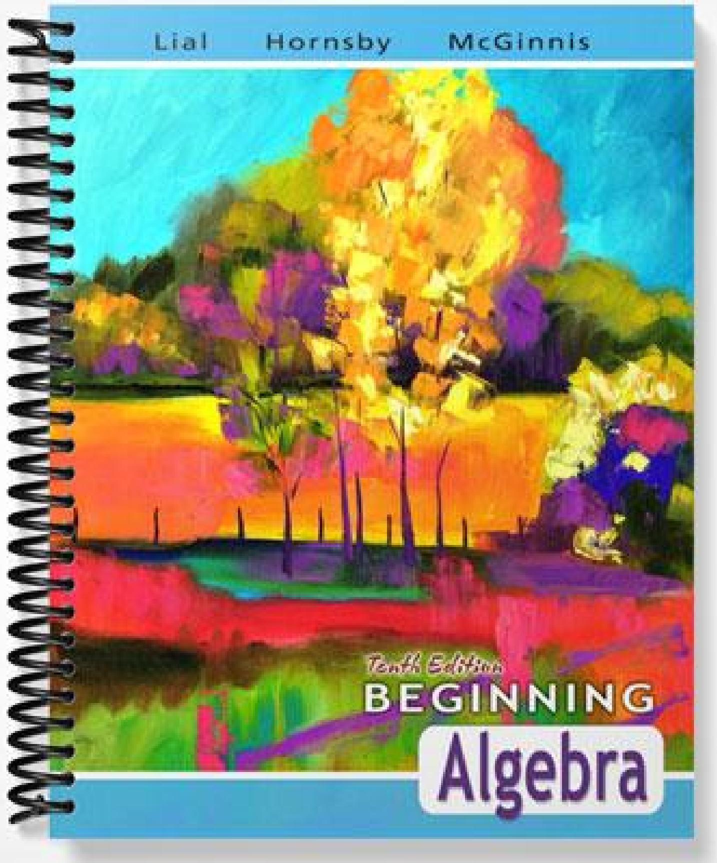
SOLUTIONS MANUAL



CHAPTER 2 LINEAR EQUATIONS AND INEQUALITIES IN ONE VARIABLE

2.1 The Addition Property of Equality

2.1 Classroom Examples

1. Note: When solving equations we will write "Add 5" as a shorthand notation for "Add 5 to each side" and "Subtract 5" as a notation for "Subtract 5 from each side."

$$\begin{aligned} x-12&=-3 && \textit{Given}\\ x-12+12&=-3+12 && \textit{Add 12.}\\ x&=9 && \textit{Combine like terms.}\end{aligned}$$

We check by substituting 9 for x in the *original* equation.

Check: $x-12=-3$ *Original equation*
$9-12=-3$? *Let x = 9.*
$-3=-3$ *True*

Since a true statement results, $\{9\}$ is the solution set.

2.
$$\begin{aligned} m-4.1&=-6.3\\ m-4.1+4.1&=-6.3+4.1 && \textit{Add 4.1.}\\ m&=-2.2\end{aligned}$$

Check $m=-2.2$: $-6.3=-6.3$ *True*

This is a shorthand notation for showing that if we substitute -2.2 for m, both sides are equal to -6.3, and hence a true statement results. In practice, this is what you will do, especially if you're using a calculator.

The solution set is $\{-2.2\}$.

3.
$$\begin{aligned} -22&=x+16\\ -22-16&=x+16-16 && \textit{Subtract 16.}\\ -38&=x\end{aligned}$$

Check $x=-38$: $-22=-22$ *True*

The solution set is $\{-38\}$.

4.
$$\begin{aligned} \frac{7}{2}m+1&=\frac{9}{2}m\\ \frac{7}{2}m+1-\frac{7}{2}m&=\frac{9}{2}m-\frac{7}{2}m && \textit{Subtract } \tfrac{7}{2}m.\\ 1&=\frac{2}{2}m && \textit{Combine terms.}\\ 1&=m\end{aligned}$$

Check $m=1$: $\frac{9}{2}=\frac{9}{2}$ *True*

The solution set is $\{1\}$.

5.
$$\begin{aligned} 9r+4r+6-2&=9r+4+3r && \textit{Given}\\ 13r+4&=12r+4 && \textit{Combine terms.}\\ 13r+4-12r&=12r+4-12r && \textit{Subtract 12r.}\\ r+4&=4 && \textit{Combine terms.}\\ r+4-4&=4-4 && \textit{Subtract 4.}\\ r&=0 && \textit{Combine terms.}\end{aligned}$$

Check $r=0$: $4=4$ *True*

The solution set is $\{0\}$.

6.
$$\begin{aligned} 4(x+1)-(3x+5)&=1 && \textit{Given}\\ 4(x+1)-1(3x+5)&=1 && -a=-1a\\ 4x+4-3x-5&=1 && \textit{Distributive property}\\ x-1&=1 && \textit{Combine terms.}\\ x-1+1&=1+1 && \textit{Add 1.}\\ x&=2\end{aligned}$$

Check $r=2$: $1=1$ *True*

The solution set is $\{2\}$.

2.1 Section Exercises

1. Equations that have exactly the same solution sets are **equivalent equations**.

A.
$$\begin{aligned} x+2&=6\\ x+2-2&=6-2 && \textit{Subtract 2.}\\ x&=4\end{aligned}$$

So $x+2=6$ and $x=4$ *are* equivalent equations.

B.
$$\begin{aligned} 10-x&=5\\ 10-x-10&=5-10 && \textit{Subtract 10.}\\ -x&=-5\\ -1(-x)&=-1(-5) && \textit{Multiply by } -1.\\ x&=5\end{aligned}$$

So $10-x=5$ and $x=-5$ *are not* equivalent equations.

C. Subtract 3 from both sides to get $x=6$, so $x+3=9$ and $x=6$ *are* equivalent equations.

D. Subtract 4 from both sides to get $x=4$. The second equation is $x=-4$, so $4+x=8$ and $x=-4$ *are not* equivalent equations.

2. **(a)** $5x + 8 - 4x + 7$

This is an expression, not an equation, since there is no equals sign. It can be simplified by rearranging terms and then combining like terms.

$$\begin{aligned} 5x + 8 - 4x + 7 &= 5x - 4x + 8 + 7 \\ &= x + 15 \end{aligned}$$

(b) $-6y + 12 + 7y - 5$

This is an expression, not an equation, since there is no equals sign. It can be simplified by rearranging terms and then combining like terms.

$$\begin{aligned} -6y + 12 + 7y - 5 &= -6y + 7y + 12 - 5 \\ &= y + 7 \end{aligned}$$

(c) $5x + 8 - 4x = 7$

This is an equation because of the equals sign.

$$\begin{aligned} 5x + 8 - 4x &= 7 \\ x + 8 &= 7 \\ x &= -1 \end{aligned}$$

(d) $-6y + 12 + 7y = -5$

This is an equation because of the equals sign.

$$\begin{aligned} -6y + 12 + 7y &= -5 \\ y + 12 &= -5 \\ y &= -17 \end{aligned}$$

3. The addition property of equality says that the same number (or expression) added to each side of an equation results in an equivalent equation. *Example*: $-x$ can be added to each side of $2x + 3 = x - 5$ to get the equivalent equation $x + 3 = -5$.

4. Check by replacing the variable(s) in the *original* equation with the proposed solution. A true statement will result if the proposed solution is correct.

For Exercises 5–36, all solutions should be checked by substituting into the original equation. Checks will be shown here for only a few of the exercises.

5.
$$\begin{aligned} x - 4 &= 8 \\ x - 4 + 4 &= 8 + 4 \\ x &= 12 \end{aligned}$$

Check this solution by replacing x with 12 in the original equation.

$$\begin{aligned} x - 4 &= 8 \\ 12 - 4 &= 8 \ ? \quad \textit{Let } x = 12. \\ 8 &= 8 \quad \textit{True} \end{aligned}$$

Because the final statement is true, $\{12\}$ is the solution set.

6.
$$\begin{aligned} x - 8 &= 9 \\ x - 8 + 8 &= 9 + 8 \\ x &= 17 \end{aligned}$$

Check $x = 17$:

$$\begin{aligned} 17 - 8 &= 9 \ ? \quad \textit{Let } x = 17. \\ 9 &= 9 \quad \textit{True} \end{aligned}$$

Thus, $\{17\}$ is the solution set.

7.
$$\begin{aligned} x - 12 &= 19 \\ x - 12 + 12 &= 19 + 12 \\ x &= 31 \end{aligned}$$

Check $x = 31$:

$$\begin{aligned} 31 - 12 &= 19 \ ? \quad \textit{Let } x = 31. \\ 19 &= 19 \quad \textit{True} \end{aligned}$$

Thus, $\{31\}$ is the solution set.

8.
$$\begin{aligned} x - 15 &= 25 \\ x - 15 + 15 &= 25 + 15 \\ x &= 40 \end{aligned}$$

Checking yields a true statement, so $\{40\}$ is the solution set.

9.
$$\begin{aligned} x - 5 &= -8 \\ x - 5 + 5 &= -8 + 5 \\ x &= -3 \end{aligned}$$

Checking yields a true statement, so $\{-3\}$ is the solution set.

10.
$$\begin{aligned} x - 7 &= -9 \\ x - 7 + 7 &= -9 + 7 \\ x &= -2 \end{aligned}$$

Checking yields a true statement, so $\{-2\}$ is the solution set.

11.
$$\begin{aligned} r + 9 &= 13 \\ r + 9 - 9 &= 13 - 9 \\ r &= 4 \end{aligned}$$

Checking yields a true statement, so $\{4\}$ is the solution set.

12.
$$\begin{aligned} x + 6 &= 10 \\ x + 6 - 6 &= 10 - 6 \\ x &= 4 \end{aligned}$$

Checking yields a true statement, so $\{4\}$ is the solution set.

13.
$$\begin{aligned} x + 26 &= 17 \\ x + 26 - 26 &= 17 - 26 \\ x &= -9 \end{aligned}$$

Checking yields a true statement, so $\{-9\}$ is the solution set.

14.
$$\begin{aligned} x + 45 &= 24 \\ x + 45 - 45 &= 24 - 45 \\ x &= -21 \end{aligned}$$

Checking yields a true statement, so $\{-21\}$ is the solution set.

15.
$$\begin{aligned} 7 + r &= -3 \\ r + 7 &= -3 \\ r + 7 - 7 &= -3 - 7 \\ r &= -10 \end{aligned}$$

The solution set is $\{-10\}$.

16.
$$\begin{aligned} 8 + k &= -4 \\ k + 8 &= -4 \\ k + 8 - 8 &= -4 - 8 \\ k &= -12 \end{aligned}$$

The solution set is $\{-12\}$.

17.
$$\begin{aligned} 2 &= p + 15 \\ 2 - 15 &= p + 15 - 15 \\ -13 &= p \end{aligned}$$

The solution set is $\{-13\}$.

18.
$$\begin{aligned} 3 &= z + 17 \\ 3 - 17 &= z + 17 - 17 \\ -14 &= z \end{aligned}$$

The solution set is $\{-14\}$.

19.
$$\begin{aligned} -2 &= x - 12 \\ -2 + 12 &= x - 12 + 12 \\ 10 &= x \end{aligned}$$

The solution set is $\{10\}$.

20.
$$\begin{aligned} -6 &= x - 21 \\ -6 + 21 &= x - 21 + 21 \\ 15 &= x \end{aligned}$$

The solution set is $\{15\}$.

21.
$$\begin{aligned} x - 8.4 &= -2.1 \\ x - 8.4 + 8.4 &= -2.1 + 8.4 \\ x &= 6.3 \end{aligned}$$

The solution set is $\{6.3\}$.

22.
$$\begin{aligned} x - 15.5 &= -5.1 \\ x - 15.5 + 15.5 &= -5.1 + 15.5 \\ x &= 10.4 \end{aligned}$$

The solution set is $\{10.4\}$.

23.
$$\begin{aligned} t + 12.3 &= -4.6 \\ t + 12.3 - 12.3 &= -4.6 - 12.3 \\ t &= -16.9 \end{aligned}$$

The solution set is $\{-16.9\}$.

24.
$$\begin{aligned} x + 21.5 &= -13.4 \\ x + 21.5 - 21.5 &= -13.4 - 21.5 \\ x &= -34.9 \end{aligned}$$

The solution set is $\{-34.9\}$.

25.
$$\begin{aligned} 3x + 7 &= 2x + 4 \\ 3x + 7 - 2x &= 2x + 4 - 2x \\ x + 7 &= 4 \\ x + 7 - 7 &= 4 - 7 \\ x &= -3 \end{aligned}$$

The solution set is $\{-3\}$.

26.
$$\begin{aligned} 9x - 1 &= 8x + 4 \\ 9x - 1 - 8x &= 8x + 4 - 8x \\ x - 1 &= 4 \\ x - 1 + 1 &= 4 + 1 \\ x &= 5 \end{aligned}$$

The solution set is $\{5\}$.

27.
$$\begin{aligned} 8t + 6 &= 7t + 6 \\ 8t + 6 - 7t &= 7t + 6 - 7t \\ t + 6 &= 6 \\ t + 6 - 6 &= 6 - 6 \\ t &= 0 \end{aligned}$$

The solution set is $\{0\}$.

28.
$$\begin{aligned} 13t + 9 &= 12t + 9 \\ 13t + 9 - 12t &= 12t + 9 - 12t \\ t + 9 &= 9 \\ t + 9 - 9 &= 9 - 9 \\ t &= 0 \end{aligned}$$

The solution set is $\{0\}$.

29.
$$\begin{aligned} -4x + 7 &= -5x + 9 \\ -4x + 7 + 5x &= -5x + 9 + 5x \\ x + 7 &= 9 \\ x + 7 - 7 &= 9 - 7 \\ x &= 2 \end{aligned}$$

The solution set is $\{2\}$.

30.
$$\begin{aligned} -6x + 3 &= -7x + 10 \\ -6x + 3 + 7x &= -7x + 10 + 7x \\ x + 3 &= 10 \\ x + 3 - 3 &= 10 - 3 \\ x &= 7 \end{aligned}$$

The solution set is $\{7\}$.

31.
$$\begin{aligned}\frac{2}{5}w-6&=\frac{7}{5}w\\ \frac{2}{5}w-6-\frac{2}{5}w&=\frac{7}{5}w-\frac{2}{5}w \quad \textit{Subtract } \frac{2}{5}w.\\ -6&=\frac{5}{5}w\\ -6&=w\end{aligned}$$

The solution set is $\{-6\}$.

32.
$$\begin{aligned}-\frac{2}{7}z+2&=\frac{5}{7}z\\ -\frac{2}{7}z+2+\frac{2}{7}z&=\frac{5}{7}z+\frac{2}{7}z \quad \textit{Add } \frac{2}{7}z.\\ 2&=\frac{7}{7}z\\ 2&=z\end{aligned}$$

The solution set is $\{2\}$.

33.
$$\begin{aligned}5.6x+2&=4.6x\\ 5.6x+2-4.6x&=4.6x-4.6x\\ 1.0x+2&=0\\ x+2-2&=0-2\\ x&=-2\end{aligned}$$

The solution set is $\{-2\}$.

34.
$$\begin{aligned}9.1x-5&=8.1x\\ 9.1x-5-8.1x&=8.1x-8.1x\\ 1.0x-5&=0\\ x-5+5&=0+5\\ x&=5\end{aligned}$$

The solution set is $\{5\}$.

35.
$$\begin{aligned}3p&=2p\\ 3p-2p&=2p-2p\\ p&=0\end{aligned}$$

The solution set is $\{0\}$.

36.
$$\begin{aligned}8b&=7b\\ 8b-7b&=7b-7b\\ b&=0\end{aligned}$$

The solution set is $\{0\}$.

37.
$$\begin{aligned}1.2y-4&=0.2y-4\\ 1.2y-4-0.2y&=0.2y-4-0.2y\\ 1.0y-4&=-4\\ y-4+4&=-4+4\\ y&=0\end{aligned}$$

The solution set is $\{0\}$.

38.
$$\begin{aligned}7.7r+6&=6.7r+6\\ 7.7r+6-6.7r&=6.7r+6-6.7r\\ 1.0r+6&=6\\ r+6-6&=6-6\\ r&=0\end{aligned}$$

The solution set is $\{0\}$.

39.
$$\begin{aligned}\frac{1}{2}x+2&=-\frac{1}{2}x\\ \frac{1}{2}x+\frac{1}{2}x+2&=-\frac{1}{2}x+\frac{1}{2}x\\ x+2&=0\\ x+2-2&=0-2\\ x&=-2\end{aligned}$$

The solution set is $\{-2\}$.

40.
$$\begin{aligned}\frac{1}{5}x-7&=-\frac{4}{5}x\\ \frac{1}{5}x-7+\frac{4}{5}x&=-\frac{4}{5}x+\frac{4}{5}x\\ \frac{5}{5}x-7&=0\\ x-7+7&=0+7\\ x&=7\end{aligned}$$

The solution set is $\{7\}$.

41.
$$\begin{aligned}3x+7-2x&=0\\ x+7&=0\\ x+7-7&=0-7\\ x&=-7\end{aligned}$$

The solution set is $\{-7\}$.

42.
$$\begin{aligned}5x+4-4x&=0\\ x+4&=0\\ x+4-4&=0-4\\ x&=-4\end{aligned}$$

The solution set is $\{-4\}$.

43.
$$\begin{aligned}4x+30-3x&=0\\ x+30&=0\\ x+30-30&=0-30\\ x&=-30\end{aligned}$$

The solution set is $\{-30\}$.

44. Equations **A** $x^2-5x+6=0$ and **B** $x^3=x$ are *not* linear equations in one variable because they cannot be written in the form $Ax+B=C$. Note that in a linear equation the exponent on the variable must be 1.

45. A sample answer might be, "A linear equation in one variable is an equation that can be written using only one variable term with the variable to the first power."

46. If $A=0$, the equation $Ax+B=C$ becomes $B=C$; with no variable, it is not a linear equation.

47.
$$\begin{aligned} 5t+3+2t-6t &= 4+12 \\ (5+2-6)t+3 &= 16 \\ t+3-3 &= 16-3 \\ t &= 13 \end{aligned}$$

Check $t=13$: $16=16$ *True*

The solution set is $\{13\}$.

48.
$$\begin{aligned} 4x+3x-6-6x &= 10+3 \\ (4+3-6)x-6 &= 13 \\ x-6+6 &= 13+6 \\ x &= 19 \end{aligned}$$

Check $x=19$: $13=13$ *True*

The solution set is $\{19\}$.

49.
$$\begin{aligned} 6x+5+7x+3 &= 12x+4 \\ 13x+8 &= 12x+4 \\ 13x+8-12x &= 12x+4-12x \\ x+8 &= 4 \\ x+8-8 &= 4-8 \\ x &= -4 \end{aligned}$$

Check $x=-4$: $-44=-44$ *True*

The solution set is $\{-4\}$.

50.
$$\begin{aligned} 4x-3-8x+1 &= -5x+9 \\ -4x-2 &= -5x+9 \\ -4x-2+5x &= -5x+9+5x \\ x-2 &= 9 \\ x-2+2 &= 9+2 \\ x &= 11 \end{aligned}$$

Check $x=11$: $-46=-46$ *True*

The solution set is $\{11\}$.

51.
$$\begin{aligned} 5.2q-4.6-7.1q &= -0.9q-4.6 \\ -1.9q-4.6 &= -0.9q-4.6 \\ -1.9q-4.6+0.9q &= -0.9q-4.6+0.9q \\ -1.0q-4.6 &= -4.6 \\ -1.0q-4.6+4.6 &= -4.6+4.6 \\ -q &= 0 \\ q &= 0 \end{aligned}$$

Check $q=0$: $-4.6=-4.6$ *True*

The solution set is $\{0\}$.

52.
$$\begin{aligned} -4.0x+2.7-1.6x &= -4.6x+2.7 \\ -5.6x+2.7 &= -4.6x+2.7 \\ -5.6x+2.7+4.6x &= -4.6x+2.7+4.6x \\ -1.0x+2.7 &= 2.7 \\ -x+2.7-2.7 &= 2.7-2.7 \\ -x &= 0 \\ x &= 0 \end{aligned}$$

Check $x=0$: $2.7=2.7$ *True*

The solution set is $\{0\}$.

53.
$$\begin{aligned} \frac{5}{7}x+\frac{1}{3} &= \frac{2}{5}-\frac{2}{7}x+\frac{2}{5} \\ \frac{5}{7}x+\frac{1}{3} &= \frac{4}{5}-\frac{2}{7}x \\ \frac{5}{7}x+\frac{2}{7}x+\frac{1}{3} &= \frac{4}{5}-\frac{2}{7}x+\frac{2}{7}x && \textit{Add } \tfrac{2}{7}x. \\ \frac{7}{7}x+\frac{1}{3} &= \frac{4}{5} && \textit{Combine like terms.} \\ 1x+\frac{1}{3}-\frac{1}{3} &= \frac{4}{5}-\frac{1}{3} && \textit{Subtract } \tfrac{1}{3}. \\ x &= \frac{12}{15}-\frac{5}{15} && \textit{LCD = 15} \\ x &= \tfrac{7}{15} \end{aligned}$$

Check $x=\frac{7}{15}$: $\frac{2}{3}=\frac{2}{3}$ *True*

The solution set is $\left\{\frac{7}{15}\right\}$.

54.
$$\begin{aligned} \frac{6}{7}s-\frac{3}{4} &= \frac{4}{5}-\frac{1}{7}s+\frac{1}{6} \\ \frac{6}{7}s-\frac{3}{4} &= \frac{24}{30}-\frac{1}{7}s+\frac{5}{30} && \textit{LCD = 30} \\ \frac{6}{7}s-\frac{3}{4} &= \frac{29}{30}-\frac{1}{7}s && \textit{Add.} \\ \frac{6}{7}s-\frac{3}{4}+\frac{1}{7}s &= \frac{29}{30}-\frac{1}{7}s+\frac{1}{7}s && \textit{Add } \tfrac{1}{7}s. \\ \frac{7}{7}s-\frac{3}{4} &= \frac{29}{30} && \textit{Combine like terms.} \\ 1s-\frac{3}{4}+\frac{3}{4} &= \frac{29}{30}+\frac{3}{4} && \textit{Add } \tfrac{3}{4}. \\ s &= \frac{58}{60}+\frac{45}{60} && \textit{LCD = 60} \\ s &= \tfrac{103}{60} \end{aligned}$$

Check $s=\frac{103}{60}$: $\frac{101}{140}=\frac{101}{140}$ *True*

The solution set is $\left\{\frac{103}{60}\right\}$.

55.
$$\begin{aligned} (5y+6)-(3+4y) &= 10 \\ 5y+6-3-4y &= 10 && \textit{Distributive property} \\ y+3 &= 10 && \textit{Combine terms.} \\ y+3-3 &= 10-3 && \textit{Subtract 3.} \\ y &= 7 \end{aligned}$$

Check $y=7$: $10=10$ *True*

The solution set is $\{7\}$.

56.
$$\begin{aligned} (8r-3)-(7r+1) &= -6 \\ 8r-3-7r-1 &= -6 \\ r-4 &= -6 \\ r-4+4 &= -6+4 \\ r &= -2 \end{aligned}$$

Check $r=-2$: $-6=-6$ *True*

The solution set is $\{-2\}$.

57. $$\begin{aligned} 2(p+5)-(9+p)&=-3 \\ 2p+10-9-p&=-3 \\ p+1&=-3 \\ p+1-1&=-3-1 \\ p&=-4 \end{aligned}$$

Check $p=-4$: $-3=-3$ *True*

The solution set is $\{-4\}$.

58. $$\begin{aligned} 4(k-6)-(3k+2)&=-5 \\ 4k-24-3k-2&=-5 \\ k-26&=-5 \\ k-26+26&=-5+26 \\ k&=21 \end{aligned}$$

Check $k=21$: $-5=-5$ *True*

The solution set is $\{21\}$.

59. $$\begin{aligned} -6(2b+1)+(13b-7)&=0 \\ -12b-6+13b-7&=0 \\ b-13&=0 \\ b-13+13&=0+13 \\ b&=13 \end{aligned}$$

Check $b=13$: $0=0$ *True*

The solution set is $\{13\}$.

60. $$\begin{aligned} -5(3w-3)+(1+16w)&=0 \\ -15w+15+1+16w&=0 \\ w+16&=0 \\ w+16-16&=0-16 \\ w&=-16 \end{aligned}$$

Check $w=-16$: $0=0$ *True*

The solution set is $\{-16\}$.

61. $$\begin{aligned} 10(-2x+1)&=-19(x+1) \\ -20x+10&=-19x-19 \\ -20x+10+19x&=-19x-19+19x \\ -x+10&=-19 \\ -x+10-10&=-19-10 \\ -x&=-29 \\ x&=29 \end{aligned}$$

Check $x=29$: $-570=-570$ *True*

The solution set is $\{29\}$.

62. $$\begin{aligned} 2(2-3r)&=-5(r-3) \\ 4-6r&=-5r+15 \\ 4-6r+5r&=-5r+15+5r \\ 4-r&=15 \\ 4-r-4&=15-4 \\ -r&=11 \\ r&=-11 \end{aligned}$$

Check $r=-11$: $70=70$ *True*

The solution set is $\{-11\}$.

63. $$\begin{aligned} -2(8p+2)-3(2-7p)-2(4+2p)&=0 \\ -16p-4-6+21p-8-4p&=0 \\ p-18&=0 \\ p-18+18&=0+18 \\ p&=18 \end{aligned}$$

Check $p=18$: $0=0$ *True*

The solution set is $\{18\}$.

64. $$\begin{aligned} -5(1-2z)+4(3-z)-7(3+z)&=0 \\ -5+10z+12-4z-21-7z&=0 \\ -z-14&=0 \\ -z-14+z&=0+z \\ -14&=z \end{aligned}$$

Check $z=-14$: $0=0$ *True*

The solution set is $\{-14\}$.

65. $$\begin{aligned} 4(7x-1)+3(2-5x)-4(3x+5)&=-6 \\ 28x-4+6-15x-12x-20&=-6 \\ x-18&=-6 \\ x-18+18&=-6+18 \\ x&=12 \end{aligned}$$

Check $x=12$: $-6=-6$ *True*

The solution set is $\{12\}$.

66. $$\begin{aligned} 9(2m-3)-4(5+3m)-5(4+m)&=-3 \\ 18m-27-20-12m-20-5m&=-3 \\ m-67&=-3 \\ m-67+67&=-3+67 \\ m&=64 \end{aligned}$$

Check $m=64$: $-3=-3$ *True*

The solution set is $\{64\}$.

67. Answers will vary. One example is $x-6=-8$.

68. Answers will vary. One example is $x+\frac{1}{2}=1$.

69. "Three times a number is 17 more than twice the number."

$$\begin{aligned} 3x&=2x+17 \\ 3x-2x&=2x+17-2x \\ x&=17 \end{aligned}$$

The number is 17 and $\{17\}$ is the solution set.

70. "One added to three times a number is three less than four times the number."

$$\begin{aligned} 1+3x&=4x-3 \\ 1+3x-3x&=4x-3-3x \\ 1&=x-3 \\ 1+3&=x-3+3 \\ 4&=x \end{aligned}$$

The number is 4 and $\{4\}$ is the solution set.

71. "If six times a number is subtracted from seven times the number, the result is -9."

$$7x - 6x = -9$$
$$x = -9$$

The number is -9 and $\{-9\}$ is the solution set.

72. "If five times a number is added to three times the number, the result is the sum of seven times the number and 9."

$$\begin{aligned} 5x + 3x &= 7x + 9 \\ 8x &= 7x + 9 \\ 8x - 7x &= 7x + 9 - 7x \\ x &= 9 \end{aligned}$$

The number is 9 and $\{9\}$ is the solution set.

73. $\frac{2}{3}\left(\frac{3}{2}\right) = \frac{2 \cdot 3}{3 \cdot 2} = 1$

74. $\frac{5}{6}\left(\frac{6}{5}\right) = \frac{5 \cdot 6}{6 \cdot 5} = 1$

75. $$\begin{aligned} -\frac{5}{4}\left(-\frac{4}{5}x\right) &= -\frac{5}{4}\left(-\frac{4}{5}\right)x \\ &= \frac{5 \cdot 4}{4 \cdot 5}x \\ &= 1x = x \end{aligned}$$

76. $$\begin{aligned} -\frac{9}{7}\left(-\frac{7}{9}x\right) &= -\frac{9}{7}\left(-\frac{7}{9}\right)x \\ &= \frac{9 \cdot 7}{7 \cdot 9}x \\ &= 1x = x \end{aligned}$$

77. $$\begin{aligned} 9\left(\frac{r}{9}\right) &= 9\left(\frac{1}{9}r\right) \\ &= 9\left(\frac{1}{9}\right)r \\ &= 1r = r \end{aligned}$$

78. $$\begin{aligned} 6\left(\frac{t}{6}\right) &= 6\left(\frac{1}{6}t\right) \\ &= 6\left(\frac{1}{6}\right)t \\ &= 1t = t \end{aligned}$$

2.2 The Multiplication Property of Equality

2.2 Classroom Examples

1. $$\begin{aligned} 8x &= 20 \\ \frac{8x}{8} &= \frac{20}{8} && \textit{Divide by 8.} \\ x &= \tfrac{20}{8} = \tfrac{5}{2} && \textit{Write in lowest terms.} \end{aligned}$$

Check $x = \frac{5}{2}$: $20 = 20$ *True*

The solution set is $\left\{\frac{5}{2}\right\}$.

2. $$\begin{aligned} -0.7x &= 5.04 \\ \frac{-0.7x}{-0.7} &= \frac{5.04}{-0.7} && \textit{Divide by } -0.7. \\ x &= -7.2 \end{aligned}$$

Check $x = -7.2$: $5.04 = 5.04$ *True*

The solution set is $\{-7.2\}$.

3. $$\begin{aligned} \frac{x}{4} &= -6 \\ \frac{1}{4}x &= -6 \\ 4 \cdot \frac{1}{4}x &= 4(-6) && \textit{Multiply by 4, the reciprocal of } \tfrac{1}{4}. \\ p &= -24 \end{aligned}$$

Check $p = -24$: $-6 = -6$ *True*

The solution set is $\{-24\}$.

4. $$\begin{aligned} -\frac{2}{3}h &= -12 \\ -\frac{3}{2}\left(-\frac{2}{3}h\right) &= -\frac{3}{2}(-12) && \textit{Multiply by } -\tfrac{3}{2}. \\ 1 \cdot h &= -\frac{3}{2} \cdot \frac{-12}{1} && \textit{Multiplicative inverse property} \\ h &= 18 && \textit{Multiplicative identity property; multiply fractions.} \end{aligned}$$

Check $h = 18$: $-12 = -12$ *True*

The solution set is $\{18\}$.

5. $$\begin{aligned} -p &= 7 \\ -1 \cdot p &= 7 && -p = -1 \cdot p \\ (-1)(-1) \cdot p &= (-1)(7) && \textit{Multiply by } -1. \\ 1 \cdot p &= -7 \\ p &= -7 \end{aligned}$$

Check $p = -7$: $7 = 7$ *True*

The solution set is $\{-7\}$.

6. $$\begin{aligned} 4r - 9r &= 20 \\ -5r &= 20 && \textit{Combine terms.} \\ \frac{-5r}{-5} &= \frac{20}{-5} && \textit{Divide by } -5. \\ r &= -4 \end{aligned}$$

Check $r = -4$: $20 = 20$ *True*

The solution set is $\{-4\}$.

2.2 Section Exercises

1. The multiplication property of equality says that the same nonzero number (or expression) multiplied on each side of the equation results in an equivalent equation. *Example*: Multiplying each side of $7x = 4$ by $\frac{1}{7}$ gives the equivalent equation $x = \frac{4}{7}$.

2. If you multiply both sides of an equation by 0, you will obtain the equation $0 = 0$. While this equation is true, the variable has been lost and we are unable to solve the equation.

3. Choice **C** doesn't require the use of the multiplicative property of equality. After the equation is simplified, the variable x is alone on the left side.

$$\begin{aligned} 5x - 4x &= 7 \\ x &= 7 \end{aligned}$$

4. **(a)** multiplication property of equality; to get x alone on the left side of the equation, multiply each side by $\frac{1}{3}$ (or divide each side by 3).

(b) addition property of equality; to get x alone on the left side of the equation, add -3 (or subtract 3) on each side.

(c) multiplication property of equality; to get x alone on the left side of the equation, multiply each side by -1 (or divide each side by -1).

(d) addition property of equality; to get x alone on the right side of the equation, add -6 (or subtract 6) on each side.

5. To get x alone on the left side, divide by 4, the coefficient of x.

6. To find the solution of $-x = 5$, multiply each side by -1, or use the rule "If $-x = a$, then $x = -a$."

7. $\frac{2}{3}x = 8$

To get just x on the left side, multiply both sides of the equation by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$.

8. $\frac{4}{5}x = 6$

To get just x on the left side, multiply both sides of the equation by the reciprocal of $\frac{4}{5}$, which is $\frac{5}{4}$.

9. $\dfrac{x}{10} = 3$

This equation is equivalent to $\frac{1}{10}x = 3$. To get just x on the left side, multiply both sides of the equation by the reciprocal of $\frac{1}{10}$, which is 10.

10. $\dfrac{x}{100} = 8$

This equation is equivalent to $\frac{1}{100}x = 8$. To get just x on the left side, multiply both sides of the equation by the reciprocal of $\frac{1}{100}$, which is 100.

11. $-\frac{9}{2}x = -4$

To get just x on the left side, multiply both sides of the equation by the reciprocal of $-\frac{9}{2}$, which is $-\frac{2}{9}$.

12. $-\frac{8}{3}x = -11$

To get just x on the left side, multiply both sides of the equation by the reciprocal of $-\frac{8}{3}$, which is $-\frac{3}{8}$.

13. $-x = 0.36$

This equation is equivalent to $-1x = 0.36$. To get just x on the left side, multiply both sides of the equation by the reciprocal of -1, which is -1.

14. $-x = 0.29$

This equation is equivalent to $-1x = 0.29$. To get just x on the left side, multiply both sides of the equation by the reciprocal of -1, which is -1.

15. $6x = 5$

To get just x on the left side, divide both sides of the equation by the coefficient of x, which is 6.

16. $7x = 10$

To get just x on the left side, divide both sides of the equation by the coefficient of x, which is 7.

17. $-4x = 13$

To get just x on the left side, divide both sides of the equation by the coefficient of x, which is -4.

18. $-13x = 6$

To get just x on the left side, divide both sides of the equation by the coefficient of x, which is -13.

19. $0.12x = 48$

To get just x on the left side, divide both sides of the equation by the coefficient of x, which is 0.12.

20. $0.21x = 63$

To get just x on the left side, divide both sides of the equation by the coefficient of x, which is 0.21.

21. $-x = 23$

This equation is equivalent to $-1x = 23$. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is -1.

22. $-x = 49$

This equation is equivalent to $-1x = 49$. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is -1.

23.
$$\begin{aligned} 5x &= 30 \\ \frac{5x}{5} &= \frac{30}{5} \quad \textit{Divide by 5.} \\ 1x &= 6 \\ x &= 6 \end{aligned}$$

Check $x = 6$: $30 = 30$ *True*

The solution set is $\{6\}$.

24. $7x = 56$

$$\frac{7x}{7} = \frac{56}{7} \quad \textit{Divide by 7.}$$
$$x = 8$$

Check $x = 8$: $56 = 56$ *True*

The solution set is $\{8\}$.

25. $2m = 15$

$$\frac{2m}{2} = \frac{15}{2} \quad \textit{Divide by 2.}$$
$$m = \tfrac{15}{2}$$

Check $m = \frac{15}{2}$: $15 = 15$ *True*

The solution set is $\left\{\frac{15}{2}\right\}$.

26. $3m = 10$

$$\frac{3m}{3} = \frac{10}{3} \quad \textit{Divide by 3.}$$
$$m = \tfrac{10}{3}$$

Check $m = \frac{10}{3}$: $10 = 10$ *True*

The solution set is $\left\{\frac{10}{3}\right\}$.

27. $3a = -15$

$$\frac{3a}{3} = \frac{-15}{3} \quad \textit{Divide by 3.}$$
$$a = -5$$

Check $a = -5$: $-15 = -15$ *True*

The solution set is $\{-5\}$.

28. $5k = -70$

$$\frac{5k}{5} = \frac{-70}{5} \quad \textit{Divide by 5.}$$
$$k = -14$$

Check $k = -14$: $-70 = -70$ *True*

The solution set is $\{-14\}$.

29. $-3x = 12$

$$\frac{-3x}{-3} = \frac{12}{-3} \quad \textit{Divide by } -3.$$
$$x = -4$$

Check $x = -4$: $12 = 12$ *True*

The solution set is $\{-4\}$.

30. $-4x = 36$

$$\frac{-4x}{-4} = \frac{36}{-4} \quad \textit{Divide by } -4.$$
$$x = -9$$

Check $x = -9$: $36 = 36$ *True*

The solution set is $\{-9\}$.

31. $10t = -36$

$$\frac{10t}{10} = \frac{-36}{10} \quad \textit{Divide by 10.}$$
$$t = -\tfrac{36}{10} = -\tfrac{18}{5} \quad \textit{Lowest terms}$$

Check $t = -\frac{18}{5}$: $-36 = -36$ *True*

The solution set is $\left\{-\frac{18}{5}\right\}$.

32. $4s = -34$

$$\frac{4s}{4} = \frac{-34}{4} \quad \textit{Divide by 4.}$$
$$s = -\tfrac{34}{4} = -\tfrac{17}{2} \quad \textit{Lowest terms}$$

Check $s = -\frac{17}{2}$: $-34 = -34$ *True*

The solution set is $\left\{-\frac{17}{2}\right\}$.

33. $-6x = -72$

$$\frac{-6x}{-6} = \frac{-72}{-6} \quad \textit{Divide by } -6.$$
$$x = 12$$

Check $x = 12$: $-72 = -72$ *True*

The solution set is $\{12\}$.

34. $-8x = -64$

$$\frac{-8x}{-8} = \frac{-64}{-8} \quad \textit{Divide by } -8.$$
$$x = 8$$

Check $x = 8$: $-64 = -64$ *True*

The solution set is $\{8\}$.

35. $2r = 0$

$$\frac{2r}{2} = \frac{0}{2} \quad \textit{Divide by 2.}$$
$$r = 0$$

Check $r = 0$: $0 = 0$ *True*

The solution set is $\{0\}$.

36. $5x = 0$

$$\frac{5x}{5} = \frac{0}{5} \quad \textit{Divide by 5.}$$
$$x = 0$$

Check $x = 0$: $0 = 0$ *True*

The solution set is $\{0\}$.

37. $0.2t = 8$

$$\frac{0.2t}{0.2} = \frac{8}{0.2}$$
$$t = 40$$

Check $t = 40$: $8 = 8$ *True*

The solution set is $\{40\}$.

38. $0.9x = 18$

$$\frac{0.9x}{0.9} = \frac{18}{0.9}$$

$$x = 20$$

Check $x = 20$: $18 = 18$ *True*

The solution set is $\{20\}$.

39. $-2.1m = 25.62$

$$\frac{-2.1m}{-2.1} = \frac{25.62}{-2.1}$$

$$m = -12.2$$

Check $m = -12.2$: $25.62 = 25.62$ *True*

The solution set is $\{-12.2\}$.

40. $-3.9a = 31.2$

$$\frac{-3.9a}{-3.9} = \frac{31.2}{-3.9}$$

$$a = -8$$

Check $a = -8$: $31.2 = 31.2$ *True*

The solution set is $\{-8\}$.

41. $\frac{1}{4}x = -12$

$4 \cdot \frac{1}{4}x = 4(-12)$ *Multiply by 4.*

$1x = -48$

$x = -48$

Check $x = -48$: $-12 = -12$ *True*

The solution set is $\{-48\}$.

42. $\frac{1}{5}p = -3$

$5 \cdot \frac{1}{5}p = 5(-3)$ *Multiply by 5.*

$p = -15$

Check $p = -15$: $-3 = -3$ *True*

The solution set is $\{-15\}$.

43. $\frac{z}{6} = 12$

$\frac{1}{6}z = 12$

$6 \cdot \frac{1}{6}z = 6 \cdot 12$

$z = 72$

Check $z = 72$: $12 = 12$ *True*

The solution set is $\{72\}$.

44. $\frac{x}{5} = 15$

$\frac{1}{5}x = 15$

$5 \cdot \frac{1}{5}x = 5 \cdot 15$

$x = 75$

Check $x = 75$: $15 = 15$ *True*

The solution set is $\{75\}$.

45. $\frac{x}{7} = -5$

$\frac{1}{7}x = -5$

$7\left(\frac{1}{7}x\right) = 7(-5)$

$x = -35$

Check $x = -35$: $-5 = -5$ *True*

The solution set is $\{-35\}$.

46. $\frac{k}{8} = -3$

$\frac{1}{8}k = -3$

$8\left(\frac{1}{8}k\right) = 8(-3)$

$k = -24$

Check $k = -24$: $-3 = -3$ *True*

The solution set is $\{-24\}$.

47. $\frac{2}{7}p = 4$

$\frac{7}{2}\left(\frac{2}{7}p\right) = \frac{7}{2}(4)$ *Multiply by the reciprocal of $\frac{2}{7}$.*

$p = 14$

Check $p = 14$: $4 = 4$ *True*

The solution set is $\{14\}$.

48. $\frac{3}{8}x = 9$

$\left(\frac{8}{3}\right)\left(\frac{3}{8}x\right) = \left(\frac{8}{3}\right)(9)$ *Multiply by the reciprocal of $\frac{3}{8}$.*

$x = 24$

Check $x = 24$: $9 = 9$ *True*

The solution set is $\{24\}$.

49. $-\frac{5}{6}t = -15$

$-\frac{6}{5}\left(-\frac{5}{6}t\right) = -\frac{6}{5}(-15)$ *Multiply by the reciprocal of $-\frac{5}{6}$.*

$t = 18$

Check $t = 18$: $-15 = -15$ *True*

The solution set is $\{18\}$.

50. $-\frac{3}{4}k = -21$

$-\frac{4}{3}\left(-\frac{3}{4}k\right) = -\frac{4}{3}(-21)$ *Multiply by the reciprocal of $-\frac{3}{4}$.*

$k = 28$

Check $k = 28$: $-21 = -21$ *True*

The solution set is $\{28\}$.

51. $$-\frac{7}{9}c = \frac{3}{5}$$

$$-\frac{9}{7}\left(-\frac{7}{9}c\right) = -\frac{9}{7} \cdot \frac{3}{5}$$ *Multiply by the reciprocal of $-\frac{7}{9}$.*

$$c = -\tfrac{27}{35}$$

Check $c = -\frac{27}{35}$: $\frac{3}{5} = \frac{3}{5}$ *True*

The solution set is $\left\{-\frac{27}{35}\right\}$.

52. $$-\frac{5}{6}d = \frac{4}{9}$$

$$\left(-\frac{6}{5}\right)\left(-\frac{5}{6}d\right) = \left(-\frac{6}{5}\right)\left(\frac{4}{9}\right)$$ *Multiply by the reciprocal of $-\frac{5}{6}$.*

$$d = -\frac{2 \cdot 3 \cdot 4}{5 \cdot 3 \cdot 3} = -\frac{8}{15}$$

Check $d = -\frac{8}{15}$: $\frac{4}{9} = \frac{4}{9}$ *True*

The solution set is $\left\{-\frac{8}{15}\right\}$.

53. $$-x = 12$$
$$-1 \cdot (-x) = -1 \cdot 12$$ *Multiply by –1.*
$$x = -12$$

Check $x = -12$: $12 = 12$ *True*

The solution set is $\{-12\}$.

54. $$-t = 14$$
$$-1 \cdot (-t) = -1 \cdot 14$$ *Multiply by –1.*
$$t = -14$$

Check $t = -14$: $14 = 14$ *True*

The solution set is $\{-14\}$.

55. $$-x = -\tfrac{3}{4}$$
$$-1 \cdot (-x) = -1 \cdot \left(-\tfrac{3}{4}\right)$$
$$x = \tfrac{3}{4}$$

Check $x = \frac{3}{4}$: $-\frac{3}{4} = -\frac{3}{4}$ *True*

The solution set is $\left\{\frac{3}{4}\right\}$.

56. $$-x = -\tfrac{1}{2}$$
$$-1 \cdot (-x) = -1 \cdot \left(-\tfrac{1}{2}\right)$$
$$x = \tfrac{1}{2}$$

Check $x = \frac{1}{2}$: $-\frac{1}{2} = -\frac{1}{2}$ *True*

The solution set is $\left\{\frac{1}{2}\right\}$.

57. $$-0.3x = 9$$
$$\frac{-0.3x}{-0.3} = \frac{9}{-0.3}$$ *Divide by −0.3.*
$$x = -30$$

Check $x = -30$: $9 = 9$ *True*

The solution set is $\{-30\}$.

58. $$-0.5x = 20$$
$$\frac{-0.5x}{-0.5} = \frac{20}{-0.5}$$ *Divide by −0.5.*
$$x = -40$$

Check $x = -40$: $20 = 20$ *True*

The solution set is $\{-40\}$.

59. $$4x + 3x = 21$$
$$7x = 21$$
$$\frac{7x}{7} = \frac{21}{7}$$
$$x = 3$$

Check $x = 3$: $21 = 21$ *True*

The solution set is $\{3\}$.

60. $$9x + 2x = 121$$
$$11x = 121$$
$$\frac{11x}{11} = \frac{121}{11}$$
$$x = 11$$

Check $x = 11$: $121 = 121$ *True*

The solution set is $\{11\}$.

61. $$3r - 5r = 10$$
$$-2r = 10$$
$$\frac{-2r}{-2} = \frac{10}{-2}$$
$$r = -5$$

Check $r = -5$: $10 = 10$ *True*

The solution set is $\{-5\}$.

62. $$9p - 13p = 24$$
$$-4p = 24$$
$$\frac{-4p}{-4} = \frac{24}{-4}$$
$$p = -6$$

Check $p = -6$: $24 = 24$ *True*

The solution set is $\{-6\}$.

63. $$5m + 6m - 2m = 63$$
$$9m = 63$$
$$\frac{9m}{9} = \frac{63}{9}$$
$$m = 7$$

Check $m = 7$: $63 = 63$ *True*

The solution set is $\{7\}$.

64. $$\begin{aligned} 11r - 5r + 6r &= 168 \\ 12r &= 168 \\ \frac{12r}{12} &= \frac{168}{12} \\ r &= 14 \end{aligned}$$

Check $r = 14$: $168 = 168$ *True*

The solution set is $\{14\}$.

65. $$\begin{aligned} -6x + 4x - 7x &= 0 \\ -9x &= 0 \\ \frac{-9x}{-9} &= \frac{0}{-9} \\ x &= 0 \end{aligned}$$

Check $x = 0$: $0 = 0$ *True*

The solution set is $\{0\}$.

66. $$\begin{aligned} -5x + 4x - 8x &= 0 \\ -9x &= 0 \\ \frac{-9x}{-9} &= \frac{0}{-9} \\ x &= 0 \end{aligned}$$

Check $x = 0$: $0 = 0$ *True*

The solution set is $\{0\}$.

67. $$\begin{aligned} 9w - 5w + w &= -3 \\ 5w &= -3 \\ \frac{5w}{5} &= \frac{-3}{5} \\ w &= -\tfrac{3}{5} \end{aligned}$$

Check $w = -\frac{3}{5}$: $-3 = -3$ *True*

The solution set is $\left\{-\frac{3}{5}\right\}$.

68. $$\begin{aligned} 10x - 6x + 3x &= -4 \\ 7x &= -4 \\ \frac{7x}{7} &= \frac{-4}{7} \\ x &= -\tfrac{4}{7} \end{aligned}$$

Check $x = -\frac{4}{7}$: $-4 = -4$ *True*

The solution set is $\left\{-\frac{4}{7}\right\}$.

69. $$\begin{aligned} \frac{1}{3}x - \frac{1}{4}x + \frac{1}{12}x &= 3 \\ \left(\frac{1}{3} - \frac{1}{4} + \frac{1}{12}\right)x &= 3 && \textit{Distributive property} \\ \left(\frac{4}{12} - \frac{3}{12} + \frac{1}{12}\right)x &= 3 && \textit{LCD = 12} \\ \tfrac{1}{6}x &= 3 && \textit{Lowest terms} \\ 6\left(\tfrac{1}{6}x\right) &= 6(3) && \textit{Multiply by 6.} \\ x &= 18 \end{aligned}$$

Check $x = 18$: $6 - 4.5 + 1.5 = 3$ *True*

The solution set is $\{18\}$.

70. $$\begin{aligned} \frac{2}{5}x + \frac{1}{10}x - \frac{1}{20}x &= 18 \\ \left(\frac{2}{5} + \frac{1}{10} - \frac{1}{20}\right)x &= 18 && \textit{Distributive property} \\ \left(\frac{8}{20} + \frac{2}{20} - \frac{1}{20}\right)x &= 18 && \textit{LCD = 20} \\ \tfrac{9}{20}x &= 18 && \textit{Add fractions.} \\ \tfrac{20}{9}\left(\tfrac{9}{20}x\right) &= \tfrac{20}{9}(18) && \textit{Multiply by } \tfrac{20}{9}. \\ x &= 40 \end{aligned}$$

Check $x = 40$: $16 + 4 - 2 = 18$ *True*

The solution set is $\{40\}$.

71. Answers will vary. One example is

$$\tfrac{3}{2}x = -6.$$

72. Answers will vary. One example is

$$100x = 17.$$

73. "When a number is multiplied by 4, the result is 6."

$$\begin{aligned} 4x &= 6 \\ \frac{4x}{4} &= \frac{6}{4} \\ x &= \tfrac{3}{2} \end{aligned}$$

The number is $\frac{3}{2}$ and $\left\{\frac{3}{2}\right\}$ is the solution set.

74. "When a number is multiplied by -4, the result is 10."

$$\begin{aligned} -4x &= 10 \\ \frac{-4x}{-4} &= \frac{10}{-4} \\ x &= -\tfrac{10}{4} = -\tfrac{5}{2} \end{aligned}$$

The number is $-\frac{5}{2}$ and $\left\{-\frac{5}{2}\right\}$ is the solution set.

75. "When a number is divided by -5, the result is 2."

$$\begin{aligned} \frac{x}{-5} &= 2 \\ (-5)\left(-\tfrac{1}{5}x\right) &= (-5)(2) \\ x &= -10 \end{aligned}$$

The number is -10 and $\{-10\}$ is the solution set.

76. "If twice a number is divided by 5, the result is 4."

$$\begin{aligned} \frac{2x}{5} &= 4 \\ \left(\tfrac{5}{2}\right)\left(\tfrac{2}{5}x\right) &= \left(\tfrac{5}{2}\right)\left(\tfrac{4}{1}\right) \\ x &= 10 \end{aligned}$$

The number is 10 and $\{10\}$ is the solution set.

77. $-(3m+5)$
$= -1(3m+5)$
$= -1(3m) - 1(5)$
$= -3m - 5$

78. $-4(-1+6x)$
$= -4(-1) - 4(6x)$
$= 4 - 24x$

79. $4(-5+2p) - 3(p-4)$
$= 4(-5) + 4(2p) - 3(p) - 3(-4)$
$= -20 + 8p - 3p + 12$
$= -20 + 12 + 8p - 3p$
$= -8 + 5p$

80. $2(4k-7) - 4(-k+3)$
$= 2(4k) + 2(-7) - 4(-k) - 4(3)$
$= 8k - 14 + 4k - 12$
$= 8k + 4k - 14 - 12$
$= 12k - 26$

81. $4x + 5 + 2x = 7x$
$6x + 5 = 7x$ *Combine terms.*
$5 = x$ *Subtract 6x.*

Check $x = 5$: $20 + 5 + 10 = 35$ *True*

The solution set is $\{5\}$.

82. $2x + 5x - 3x + 4 = 3x + 2$
$4x + 4 = 3x + 2$ *Combine terms.*
$x + 4 = 2$ *Subtract 3x.*
$x = -2$ *Subtract 4.*

Check $x = -2$:

$-4 - 10 + 6 + 4 = -6 + 2$?
$-4 = -4$ *True*

The solution set is $\{-2\}$.

2.3 More on Solving Linear Equations

2.3 Classroom Examples

1. *Step 1*
$-3x - 5x - 6 + 11 = 2x - 5$
$-8x + 5 = 2x - 5$ *Combine terms.*

Step 2
$-8x + 5 + 5 = 2x - 5 + 5$ *Add 5.*
$-8x + 10 = 2x$
$-8x + 10 + 8x = 2x + 8x$ *Add 8x.*
$10 = 10x$

Step 3
$\frac{10}{10} = \frac{10x}{10}$ *Divide by 10.*
$1 = x$

Step 4
Check $x = 1$: $-3 = -3$ *True*

The solution set is $\{1\}$.

2. *Step 1*
$11 + 3(a+1) = 5a + 16$
$11 + 3a + 3 = 5a + 16$ *Distributive property*
$3a + 14 = 5a + 16$ *Combine terms.*

Step 2
$3a + 14 - 14 = 5a + 16 - 14$ *Subtract 14.*
$3a = 5a + 2$
$3a - 5a = 5a + 2 - 5a$ *Subtract 5a.*
$-2a = 2$

Step 3
$\frac{-2a}{-2} = \frac{2}{-2}$ *Divide by −2.*
$a = -1$

Step 4
Check $a = -1$: $11 = 11$ *True*

The solution set is $\{-1\}$.

3. *Step 1*
$4x - (x+7) = 9$
$4x - x - 7 = 9$ *Distributive property*
$3x - 7 = 9$

Step 2
$3x - 7 + 7 = 9 + 7$ *Add 7.*
$3x = 16$

Step 3
$\frac{3x}{3} = \frac{16}{3}$ *Divide by 3.*
$x = \frac{16}{3}$

Step 4
Check $x = \frac{16}{3}$: $9 = 9$ *True*

The solution set is $\left\{\frac{16}{3}\right\}$.

4. *Step 1*
$2 - 3(2+6z) = 4(z+1) + 14$
$2 - 6 - 18z = 4z + 4 + 14$ *Dist. prop.*
$-4 - 18z = 4z + 18$

Step 2
$-4 - 18z + 4 = 4z + 18 + 4$ *Add 4.*
$-18z = 4z + 22$
$-18z - 4z = 4z + 22 - 4z$ *Subtract 4z.*
$-22z = 22$

Step 3
$\frac{-22z}{-22} = \frac{22}{-22}$ *Divide by −22.*
$z = -1$

Step 4
Check $z = -1$:
$2 - 3(-4) = 4(0) + 14$?
$14 = 14$ *True*

The solution set is $\{-1\}$.

5. $$\frac{1}{3}x - \frac{5}{12} = \frac{3}{4} + \frac{1}{2}x$$

The LCD of all the fractions in the equation is 12, so multiply each side by 12 to clear the fractions.

$$12\left(\frac{1}{3}x - \frac{5}{12}\right) = 12\left(\frac{3}{4} + \frac{1}{2}x\right)$$

$$12\left(\frac{1}{3}x\right) - 12\left(\frac{5}{12}\right) = 12\left(\frac{3}{4}\right) + 12\left(\frac{1}{2}x\right)$$

Distributive property

$$4x - 5 = 9 + 6x$$

Step 1
Like terms are combined.

Step 2

$$4x - 6x - 5 = 9 + 6x - 6x \quad \textit{Subtract 6x.}$$
$$-2x - 5 = 9$$
$$-2x - 5 + 5 = 9 + 5 \quad \textit{Add 5.}$$
$$-2x = 14$$

Step 3

$$\frac{-2x}{-2} = \frac{14}{-2} \quad \textit{Divide by } -2.$$
$$x = -7$$

Step 4
Check $x = -7$: $-\frac{7}{3} - \frac{5}{12} = \frac{3}{4} - \frac{7}{2}$?

$\frac{-28-5}{12} = \frac{3-14}{4}$?

$-\frac{33}{12} = -\frac{11}{4}$ *True*

The solution set is $\{-7\}$.

6. $$0.5(2 - 3x) = 4.5 - 0.1(x + 7)$$

To clear decimals, multiply both sides by 10.

$$10[0.5(2 - 3x)] = 10[4.5 - 0.1(x + 7)]$$
$$5(2 - 3x) = 45 - 1(x + 7)$$

Step 1

$$10 - 15x = 45 - x - 7$$
$$10 - 15x = 38 - x$$

Step 2

$$10 - 15x + x = 38 - x + x$$
$$10 - 14x = 38$$
$$10 - 10 - 14x = 38 - 10$$
$$-14x = 28$$

Step 3

$$\frac{-14x}{-14} = \frac{28}{-14}$$
$$x = -2$$

Step 4
Check $x = -2$: $0.5(8) = 4.5 - 0.1(5)$?

$4 = 4$ *True*

The solution set is $\{-2\}$.

7. $$3x - x + 10 = 2x - 4 + 14$$
$$2x + 10 = 2x + 10$$
$$2x + 10 - 10 = 2x + 10 - 10 \quad \textit{Subtract 10.}$$
$$2x = 2x$$
$$2x - 2x = 2x - 2x \quad \textit{Subtract 2x.}$$
$$0 = 0 \quad \textit{True}$$

The variable x has "disappeared," and a *true* statement has resulted. The original equation is an identity. This means that for every real number value of x, the equation is true. Thus, the solution set is $\{$all real numbers$\}$.

8. $$3x + 8 = 6(x - 1) - 3x$$
$$3x + 8 = 6x - 6 - 3x \quad \textit{Distributive property}$$
$$3x + 8 = 3x - 6 \quad \textit{Combine terms.}$$
$$3x - 8 - 3x = 3x - 6 - 3x \quad \textit{Subtract 3x.}$$
$$8 = -6 \quad \textit{False}$$

The variable x has "disappeared," and a *false* statement has resulted. This means that for every real number value of x, the equation is false. Thus, the equation has ***no solution*** and its solution set is the **empty set**, or **null set**, symbolized $\emptyset$.

9. If one number is represented by x, then twice that number is represented by $2x$, and 5 more than twice that number is represented by $2x + 5$.

2.3 Section Exercises

1. *Step 1*: Clear parentheses and combine like terms, as needed.

Step 2: Use the addition property to get all variable terms on one side of the equation and all numbers on the other. Then combine like terms.

Step 3: Use the multiplication property to get the equation in the form $x =$ a number.

Step 4: Check the solution. Examples will vary.

2. No, it is incorrect to divide by a variable. If $-3x$ is added to both sides, the equation becomes $4x = 0$, so $x = 0$ and $\{0\}$ is the correct solution set.

3. Equations **A**, **B**, and **C** each have $\{$all real numbers$\}$ for their solution set. However, equation **D** gives

$$3x = 2x$$
$$3x - 2x = 2x - 2x$$
$$x = 0.$$

The only solution of this equation is 0, so the correct choice is **D**.

4. If the equation has decimals as coefficients, multiply each side of the equation by the power of 10 that makes all decimals into integers.

If the equation has fractions as coefficients, multiply each side of the equation by the LCD of all fractions in the equation.

5. $3x+8=5x+10$
$-2x+8=10$ *Subtract 5x.*
$-2x=2$ *Subtract 8.*
$x=-1$ *Divide by −2.*

Check $x=-1$: $5=5$ *True*

The solution set is $\{-1\}$.

6. $10p+6=12p-4$
$-2p+6=-4$ *Subtract 12p.*
$-2p=-10$ *Subtract 6.*
$p=5$ *Divide by −2.*

Check $p=5$: $56=56$ *True*

The solution set is $\{5\}$.

7. $12h-5=11h+5-h$
$12h-5=10h+5$ *Combine terms.*
$2h-5=5$ *Subtract 10h.*
$2h=10$ *Add 5.*
$h=5$ *Divide by 2.*

Check $h=5$: $55=55$ *True*

The solution set is $\{5\}$.

8. $-4x-1=-5x+1+3x$
$-4x-1=-2x+1$ *Combine terms.*
$-2x-1=1$ *Add 2x.*
$-2x=2$ *Add 1.*
$x=-1$ *Divide by −2.*

Check $x=-1$: $3=3$ *True*

The solution set is $\{-1\}$.

9. $3(4x+2)+5x=30-x$
$12x+6+5x=30-x$ *Distributive property*
$17x+6=30-x$ *Combine terms.*
$18x+6=30$ *Add 1x.*
$18x=24$ *Subtract 6.*
$x=\frac{24}{18}=\frac{4}{3}$ *Divide by 18.*

Check $x=\frac{4}{3}$: $\frac{86}{3}=\frac{86}{3}$ *True*

The solution set is $\left\{\frac{4}{3}\right\}$.

10. $5(2m+3)-4m=8m+27$
$10m+15-4m=8m+27$ *Distributive property*
$6m+15=8m+27$ *Combine terms.*
$6m=8m+12$ *Subtract 15.*
$-2m=12$ *Subtract 8m.*
$m=-6$ *Divide by −2.*

Check $m=-6$: $-21=-21$ *True*

The solution set is $\{-6\}$.

11. $-2p+7=3-(5p+1)$
$-2p+7=3-5p-1$ *Distributive property*
$-2p+7=-5p+2$ *Combine terms.*
$3p+7=2$ *Add 5p.*
$3p=-5$ *Subtract 7.*
$p=-\frac{5}{3}$

Check $p=-\frac{5}{3}$: $\frac{31}{3}=\frac{31}{3}$ *True*

The solution set is $\left\{-\frac{5}{3}\right\}$.

12. $4x+9=3-(x-2)$
$4x+9=3-x+2$ *Distributive property*
$4x+9=-x+5$ *Combine terms.*
$5x+9=5$ *Add 1x.*
$5x=-4$ *Subtract 9.*
$x=-\frac{4}{5}$ *Divide by 5.*

Check $x=-\frac{4}{5}$: $\frac{29}{5}=\frac{29}{5}$ *True*

The solution set is $\left\{-\frac{4}{5}\right\}$.

13. $6(3w+5)=2(10w+10)$
$18w+30=20w+20$
$18w=20w-10$ *Subtract 30.*
$-2w=-10$ *Subtract 20w.*
$w=5$ *Divide by −2.*

Check $w=5$: $120=120$ *True*

The solution set is $\{5\}$.

14. $4(2x-1)=-6(x+3)$
$8x-4=-6x-18$
$14x-4=-18$ *Add 6x.*
$14x=-14$ *Add 4.*
$x=-1$ *Divide by 14.*

Check $x=-1$: $-12=-12$ *True*

The solution set is $\{-1\}$.

15. $6(4x-1)=12(2x+3)$
$24x-6=24x+36$
$-6=36$ *Subtract 24x.*

The variable has "disappeared," and the resulting equation is false. Therefore, the equation has no solution set, symbolized by $\emptyset$.

16. $6(2x+8)=4(3x-6)$

$12x+48=12x-24$

$48=-24$ *Subtract 12x.*

Since $48=-24$ is a false statement, the equation has no solution set, symbolized by $\emptyset$.

17. $3(2x-4)=6(x-2)$

$6x-12=6x-12$

$-12=-12$ *Subtract 6x.*

$0=0$ *Add 12.*

The variable has "disappeared." Since the resulting statement is a *true* one, *any* real number is a solution. We indicate the solution set as {all real numbers}.

18. $3(6-4x)=2(-6x+9)$

$18-12x=-12x+18$

$18=18$ *Add 12x.*

Since $18=18$ is a true statement, the solution set is {all real numbers}.

19. $7r-5r+2=5r-r$

$2r+2=4r$ *Combine terms.*

$2=2r$ *Subtract 2r.*

$1=r$ *Divide by 2.*

Check $r=1$: $4=4$ *True*

The solution set is $\{1\}$.

20. $9p-4p+6=7p-3p$

$5p+6=4p$ *Combine terms.*

$p+6=0$ *Subtract 4p.*

$p=-6$ *Subtract 6.*

Check $p=-6$: $-24=-24$ *True*

The solution set is $\{-6\}$.

21. $11x-5(x+2)=6x+5$

$11x-5x-10=6x+5$

$6x-10=6x+5$

$-10=5$ *Subtract 6x.*

The variable has "disappeared," and the resulting equation is false. Therefore, the equation has no solution set, symbolized by $\emptyset$.

22. $6x-4(x+1)=2x+4$

$6x-4x-4=2x+4$

$2x-4=2x+4$

$-4=4$ *Subtract 2x.*

Since $-4=4$ is a false statement, the equation has no solution set, symbolized by $\emptyset$.

23. $\frac{3}{5}t-\frac{1}{10}t=t-\frac{5}{2}$

The least common denominator of all the fractions in the equation is 10.

$10\left(\frac{3}{5}t-\frac{1}{10}t\right)=10\left(t-\frac{5}{2}\right)$ *Multiply both sides by 10.*

$10\left(\frac{3}{5}t\right)+10\left(-\frac{1}{10}t\right)=10t+10\left(-\frac{5}{2}\right)$ *Distributive property*

$6t-t=10t-25$

$5t=10t-25$

$-5t=-25$ *Subtract 10t.*

$\frac{-5t}{-5}=\frac{-25}{-5}$ *Divide by −5.*

$t=5$

Check $t=5$: $\frac{5}{2}=\frac{5}{2}$ *True*

The solution set is $\{5\}$.

24. $-\frac{2}{7}r+2r=\frac{1}{2}r+\frac{17}{2}$

The least common denominator of all the fractions in the equation is 14, so multiply both sides by 14 and solve for r.

$14\left(-\frac{2}{7}r+2r\right)=14\left(\frac{1}{2}r+\frac{17}{2}\right)$

$-4r+28r=7r+119$

$24r=7r+119$

$17r=119$

$r=\frac{119}{17}=7$

Check $r=7$: $12=12$ *True*

The solution set is $\{7\}$.

25. $-\frac{1}{4}(x-12)+\frac{1}{2}(x+2)=x+4$

The LCD of all the fractions is 4.

$4\left[-\frac{1}{4}(x-12)+\frac{1}{2}(x+2)\right]=4(x+4)$ *Multiply by 4.*

$4\left(-\frac{1}{4}\right)(x-12)+4\left(\frac{1}{2}\right)(x+2)=4x+16$ *Distributive property*

$(-1)(x-12)+2(x+2)=4x+16$ *Multiply.*

$-x+12+2x+4=4x+16$ *Distributive property*

$x+16=4x+16$

$-3x+16=16$

$-3x=0$

$\frac{-3x}{-3}=\frac{0}{-3}$ *Divide by −3.*

$x=0$

Check $x=0$: $4=4$ *True*

The solution set is $\{0\}$.

26. $\frac{1}{9}(p+18)+\frac{1}{3}(2p+3)=p+3$

The least common denominator of all the fractions in the equation is 9, so multiply both sides by 9 and solve for p.

$$\begin{aligned} 9\left[\tfrac{1}{9}(p+18)+\tfrac{1}{3}(2p+3)\right] &= 9(p+3) \\ p+18+3(2p+3) &= 9p+27 \\ p+18+6p+9 &= 9p+27 \\ 7p+27 &= 9p+27 \\ -2p+27 &= 27 \\ -2p &= 0 \\ \frac{-2p}{-2} &= \frac{0}{-2} \\ p &= 0 \end{aligned}$$

Check $p=0$: $3=3$ *True*

The solution set is $\{0\}$.

27. $\frac{2}{3}k-\left(k-\frac{1}{2}\right)=\frac{1}{6}(k-51)$

The least common denominator of all the fractions in the equation is 6, so multiply both sides by 6 and solve for k.

$$\begin{aligned} 6\left[\tfrac{2}{3}k-\left(k-\tfrac{1}{2}\right)\right] &= 6\left[\tfrac{1}{6}(k-51)\right] \\ 6\left(\tfrac{2}{3}k\right)-6\left(k-\tfrac{1}{2}\right) &= 6\left[\tfrac{1}{6}(k-51)\right] \\ &\quad \textit{Distributive property} \\ 4k-6k+3 &= 1(k-51) \\ -2k+3 &= k-51 \\ -3k+3 &= -51 \\ -3k &= -54 \\ k &= 18 \end{aligned}$$

Check $k=18$: $-\frac{11}{2}=-\frac{11}{2}$ *True*

The solution set is $\{18\}$.

28. $-\frac{5}{6}q-(q-1)=\frac{1}{4}(-q+80)$

The least common denominator is 12.

$$\begin{aligned} 12\left[-\tfrac{5}{6}q-(q-1)\right] &= 12\left[\tfrac{1}{4}(-q+80)\right] \\ &\quad \textit{Multiply by 12.} \\ 12\left(-\tfrac{5}{6}q\right)-12(q-1) &= 12\left[\tfrac{1}{4}(-q+80)\right] \\ -10q-12q+12 &= 3(-q+80) \\ -22q+12 &= -3q+240 \\ -19q+12 &= 240 \\ -19q &= 228 \\ q &= \tfrac{228}{-19} = -12 \end{aligned}$$

Check $q=-12$: $23=23$ *True*

The solution set is $\{-12\}$.

29. $0.20(60)+0.05x=0.10(60+x)$

To eliminate the decimal in 0.20 and 0.10, we need to multiply the equation by 10. But to eliminate the decimal in 0.05, we need to multiply by 100, so we choose 100.

$$\begin{aligned} 100[0.20(60)+0.05x] &= 100[0.10(60+x)] \\ &\quad \textit{Multiply by 100.} \\ 100[0.20(60)]+100(0.05x) &= 100[0.10(60+x)] \\ &\quad \textit{Distributive property} \\ 20(60)+5x &= 10(60+x) \\ &\quad \textit{Multiply.} \\ 1200+5x &= 600+10x \\ 1200-5x &= 600 \\ -5x &= -600 \\ x &= \tfrac{-600}{-5} = 120 \end{aligned}$$

Check $x=120$: $18=18$ *True*

The solution set is $\{120\}$.

30.

$$\begin{aligned} 0.30(30)+0.15x &= 0.20(30+x) \\ 100[0.30(30)+0.15x] &= 100[0.20(30+x)] \\ &\quad \textit{Multiply both sides by 100.} \\ 30(30)+15x &= 20(30+x) \\ 900+15x &= 600+20x \\ 900-5x &= 600 \\ -5x &= -300 \\ x &= 60 \end{aligned}$$

Check $x=60$: $18=18$ *True*

The solution set is $\{60\}$.

31. $1.00x+0.05(12-x)=0.10(63)$

To clear the equation of decimals, we multiply both sides by 100.

$$\begin{aligned} 100[1.00x+0.05(12-x)] &= 100[0.10(63)] \\ 100(1.00x)+100[0.05(12-x)] &= (100)(0.10)(63) \\ 100x+5(12-x) &= 10(63) \\ 100x+60-5x &= 630 \\ 95x+60 &= 630 \\ 95x &= 570 \\ x &= \tfrac{570}{95} = 6 \end{aligned}$$

Check $x=6$: $6.3=6.3$ *True*

The solution set is $\{6\}$.

32.
$$0.92x + 0.98(12 - x) = 0.96(12)$$
$$100[0.92x + 0.98(12 - x)] = 100[0.96(12)]$$
Multiply both sides by 100.
$$92x + 98(12 - x) = 96(12)$$
$$92x + 1176 - 98x = 1152$$
$$-6x + 1176 = 1152$$
$$-6x = -24$$
$$x = \tfrac{-24}{-6} = 4$$

Check $x = 4$: $11.52 = 11.52$ *True*

The solution set is $\{4\}$.

33.
$$0.6(10{,}000) + 0.8x = 0.72(10{,}000 + x)$$
$$100[0.6(10{,}000)] + 100(0.8x) = 100[0.72(10{,}000 + x)]$$
Multiply by both sides by 100.
$$60(10{,}000) + 80x = 72(10{,}000 + x)$$
$$600{,}000 + 80x = 720{,}000 + 72x$$
$$600{,}000 + 8x = 720{,}000$$
$$8x = 120{,}000$$
$$x = \tfrac{120{,}000}{8} = 15{,}000$$

Check $x = 15{,}000$: $18{,}000 = 18{,}000$ *True*

The solution set is $\{15{,}000\}$.

34.
$$0.2(5000) + 0.3x = 0.25(5000 + x)$$
$$100[0.2(5000)] + 100(0.3x) = 100[0.25(5000 + x)]$$
Multiply both sides by 100.
$$20(5000) + 30x = 25(5000 + x)$$
$$100{,}000 + 30x = 125{,}000 + 25x$$
$$5x + 100{,}000 = 125{,}000$$
$$5x = 25{,}000$$
$$x = \tfrac{25{,}000}{5} = 5000$$

Check $x = 5000$: $2500 = 2500$ *True*

The solution set is $\{5000\}$.

35.
$$10(2x - 1) = 8(2x + 1) + 14$$
$$20x - 10 = 16x + 8 + 14$$
$$20x - 10 = 16x + 22$$
$$4x - 10 = 22$$
$$4x = 32$$
$$x = 8$$

Check $x = 8$: $150 = 150$ *True*

The solution set is $\{8\}$.

36.
$$9(3k - 5) = 12(3k - 1) - 51$$
$$27k - 45 = 36k - 12 - 51$$
$$27k - 45 = 36k - 63$$
$$-45 = 9k - 63$$
$$18 = 9k$$
$$2 = k$$

Check $k = 2$: $9 = 9$ *True*

The solution set is $\{2\}$.

37.
$$-(4x + 2) - (-3x - 5) = 3$$
$$-1(4x + 2) - 1(-3x - 5) = 3$$
$$-4x - 2 + 3x + 5 = 3$$
$$-x + 3 = 3$$
$$-x = 0$$
$$x = 0$$

Check $x = 0$: $3 = 3$ *True*

The solution set is $\{0\}$.

38.
$$-(6k - 5) - (-5k + 8) = -3$$
$$-1(6k - 5) - 1(-5k + 8) = -3$$
$$-6k + 5 + 5k - 8 = -3$$
$$-k - 3 = -3$$
$$-k = 0$$
$$k = 0$$

Check $k = 0$: $-3 = -3$ *True*

The solution set is $\{0\}$.

39.
$$\frac{1}{2}(x + 2) + \frac{3}{4}(x + 4) = x + 5$$
To clear fractions, multiply both sides by the LCD, which is 4.
$$4\left[\tfrac{1}{2}(x + 2) + \tfrac{3}{4}(x + 4)\right] = 4(x + 5)$$
$$4\left(\tfrac{1}{2}\right)(x + 2) + 4\left(\tfrac{3}{4}\right)(x + 4) = 4x + 20$$
$$2(x + 2) + 3(x + 4) = 4x + 20$$
$$2x + 4 + 3x + 12 = 4x + 20$$
$$5x + 16 = 4x + 20$$
$$x + 16 = 20$$
$$x = 4$$

Check $x = 4$: $9 = 9$ *True*

The solution set is $\{4\}$.

40.
$$\frac{1}{3}(x + 3) + \frac{1}{6}(x - 6) = x + 3$$
To clear fractions, multiply both sides by the LCD, which is 6.
$$6\left[\tfrac{1}{3}(x + 3) + \tfrac{1}{6}(x - 6)\right] = 6(x + 3)$$
$$6\left(\tfrac{1}{3}\right)(x + 3) + 6\left(\tfrac{1}{6}\right)(x - 6) = 6(x + 3)$$
$$2(x + 3) + 1(x - 6) = 6(x + 3)$$
$$2x + 6 + x - 6 = 6x + 18$$
$$3x = 6x + 18$$
$$-3x = 18$$
$$x = \tfrac{18}{-3} = -6$$

Check $x = -6$: $-3 = -3$ *True*

The solution set is $\{-6\}$.

41. $0.1(x+80)+0.2x=14$

To eliminate the decimals, multiply both sides by 10.

$$\begin{aligned} 10[0.1(x+80)+0.2x] &= 10(14) \\ 1(x+80)+2x &= 140 \\ x+80+2x &= 140 \\ 3x+80 &= 140 \\ 3x &= 60 \\ x &= 20 \end{aligned}$$

Check $x=20$: $14=14$ *True*

The solution set is $\{20\}$.

42. $0.3(x+15)+0.4(x+25)=25$

To clear the decimals, multiply both sides by 10.

$$\begin{aligned} 3(x+15)+4(x+25) &= 250 \\ 3x+45+4x+100 &= 250 \\ 7x+145 &= 250 \\ 7x &= 105 \\ x &= 15 \end{aligned}$$

Check $x=15$: $25=25$ *True*

The solution set is $\{15\}$.

43.

$$\begin{aligned} 4(x+8) &= 2(2x+6)+20 \\ 4x+32 &= 4x+12+20 \\ 4x+32 &= 4x+32 \\ 4x &= 4x \\ 0 &= 0 \end{aligned}$$

Since $0=0$ is a *true* statement, the solution set is {all real numbers}.

44.

$$\begin{aligned} 4(x+3) &= 2(2x+8)-4 \\ 4x+12 &= 4x+16-4 \\ 4x+12 &= 4x+12 \\ 12 &= 12 \end{aligned}$$

Since $12=12$ is a *true* statement, the solution set is {all real numbers}.

45.

$$\begin{aligned} 9(v+1)-3v &= 2(3v+1)-8 \\ 9v+9-3v &= 6v+2-8 \\ 6v+9 &= 6v-6 \\ 9 &= -6 \end{aligned}$$

Because $9=-6$ is a *false* statement, the equation has no solution set, symbolized by $\emptyset$.

46.

$$\begin{aligned} 8(t-3)+4t &= 6(2t+1)-10 \\ 8t-24+4t &= 12t+6-10 \\ 12t-24 &= 12t-4 \\ -24 &= -4 \end{aligned}$$

Because $-24=-4$ is a *false* statement, the equation has no solution set, symbolized by $\emptyset$.

47. The sum of q and the other number is 11. To find the other number, you would subtract q from 11, so an expression for the other number is $11-q$.

48. The sum of r and the other number is 26. To find the other number, you would subtract r from 26, so an expression for the other number is $26-r$.

49. The product of k and the other number is 9. To find the other number, you would divide 9 by k, so an expression for the other number is $\dfrac{9}{k}$.

50. The product of m and the other number is -3. To find the other number, you would divide -3 by m, so an expression for the other number is $\dfrac{-3}{m}$.

51. An expression for the total number of yards is $x+7$.

52. An expression for the total number of yards is $y+4$.

53. If a baseball player gets 65 hits in one season, and h of the hits are in one game, then $65-h$ of the hits came in the rest of the games.

54. If a hockey player scores 42 goals in one season, and n of the goals are in one game, then $42-n$ of the goals came in the rest of the games.

55. If Monica is a years old now, then 12 years from now, she will be $a+12$ years old. Two years ago, she was $a-2$ years old.

56. If Chandler is b years old now, then three years ago he was $b-3$ years old. Five years from now, he will be $b+5$ years old.

57. Since the value of each quarter is 25 cents, the value of r quarters is $25r$ cents.

58. Since the value of each dime is 10 cents, the value of y dimes is $10y$ cents.

59. Since each bill is worth 5 dollars, the number of bills is $\dfrac{t}{5}$.

60. Since each bill is worth 20 dollars, the number of bills is $\dfrac{v}{20}$.

61. Since each adult ticket costs b dollars, the cost of 3 adult tickets is $3b$. Since each child's ticket costs d dollars, the cost of 2 children's tickets is $2d$. Therefore, the total cost is $3b+2d$ (dollars).

62. Since each adult ticket costs c dollars, the cost of 4 adult tickets is $4c$. Since each child's ticket costs f dollars, the cost of 6 children's tickets is $6f$. Therefore, the total cost is $4c+6f$ (dollars).

63. "A number added to -6" is written

$$-6+x.$$

64. "The sum of a number and twice the number" is written

$$x + 2x.$$

65. "A number decreased by 9" is written

$$x - 9.$$

66. "The difference of -5 and a number" is written

$$-5 - x.$$

67. "The quotient of -6 and a nonzero number" is written

$$\frac{-6}{x}.$$

68. "A number divided by 17" is written

$$\frac{x}{17}.$$

69. "The product of 12 and the difference of a number and 9" is written

$$12(x - 9).$$

70. "The quotient of 9 more than a number and 6 less than the number" is written

$$\frac{x + 9}{x - 6}.$$

Summary Exercises on Solving Linear Equations

1. $a + 2 = -3$

$a = -5$ *Subtract 2.*

Check $a = -5$: $-3 = -3$ *True*

The solution set is $\{-5\}$.

2. $2m + 8 = 16$

$2m = 8$ *Subtract 8.*

$m = 4$ *Divide by 2.*

Check $m = 4$: $16 = 16$ *True*

The solution set is $\{4\}$.

3. $12.5k = -63.75$

$k = \frac{-63.75}{12.5}$ *Divide by 12.5.*

$= -5.1$

Check $k = -5.1$: $-63.75 = -63.75$ *True*

The solution set is $\{-5.1\}$.

4. $-x = -12$

$x = 12$ *Multiply by -1.*

Check $x = 12$: $-12 = -12$ *True*

The solution set is $\{12\}$.

5. $\frac{4}{5}x = -20$

$x = \left(\frac{5}{4}\right)(-20)$ *Multiply by $\frac{5}{4}$.*

$= -25$

Check $x = -25$: $-20 = -20$ *True*

The solution set is $\{-25\}$.

6. $7m - 5m = -12$

$2m = -12$ *Combine terms.*

$m = -6$ *Divide by 2.*

Check $m = -6$: $-12 = -12$ *True*

The solution set is $\{-6\}$.

7. $5x - 9 = 4(x - 3)$

$5x - 9 = 4x - 12$ *Distributive property*

$x - 9 = -12$ *Subtract 4x.*

$x = -3$ *Add 9.*

Check $x = -3$: $-24 = -24$ *True*

The solution set is $\{-3\}$.

8. $\frac{a}{-2} = 8$

$-\frac{1}{2}a = 8$ *Equivalent form*

$a = -2(8)$ *Multiply by -2.*

$= -16$

Check $a = -16$: $8 = 8$ *True*

The solution set is $\{-16\}$.

9. $-3(m - 4) + 2(5 + 2m) = 29$

$-3m + 12 + 10 + 4m = 29$

$m + 22 = 29$

$m = 7$

Check $m = 7$: $29 = 29$ *True*

The solution set is $\{7\}$.

10. $\frac{2}{3}x + 8 = \frac{1}{4}x$

To clear fractions, multiply both sides by the LCD, which is 12.

$12\left(\frac{2}{3}x + 8\right) = 12\left(\frac{1}{4}x\right)$

$8x + 96 = 3x$

$5x + 96 = 0$

$5x = -96$

$x = -\frac{96}{5}$

Check $x = -\frac{96}{5}$: $-\frac{24}{5} = -\frac{24}{5}$ *True*

The solution set is $\left\{-\frac{96}{5}\right\}$.

11. $0.08x + 0.06(x+9) = 1.24$

To eliminate the decimals, multiply both sides by 100.

$$\begin{aligned} 100[0.08x + 0.06(x+9)] &= 100(1.24) \\ 8x + 6(x+9) &= 124 \\ 8x + 6x + 54 &= 124 \\ 14x + 54 &= 124 \\ 14x &= 70 \\ x &= 5 \end{aligned}$$

Check $x = 5$: $0.4 + 0.84 = 1.24$ *True*

The solution set is $\{5\}$.

12. $x - 16.2 = 7.5$

$x = 23.7$ *Add 16.2.*

Check $x = 23.7$: $7.5 = 7.5$ *True*

The solution set is $\{23.7\}$.

13.

$$\begin{aligned} 4x + 2(3 - 2x) &= 6 \\ 4x + 6 - 4x &= 6 \\ 6 &= 6 \end{aligned}$$

Since $6 = 6$ is a *true* statement, the solution set is {all real numbers}.

14. $-0.3x + 2.1(x-4) = -6.6$

To eliminate the decimals, multiply both sides by 10.

$$\begin{aligned} 10[-0.3x + 2.1(x-4)] &= 10(-6.6) \\ -3x + 21(x-4) &= -66 \\ -3x + 21x - 84 &= -66 \\ 18x - 84 &= -66 \\ 18x &= 18 \\ x &= 1 \end{aligned}$$

Check $x = 1$: $-0.3 - 6.3 = -6.6$ *True*

The solution set is $\{1\}$.

15. $-x = 6$

$x = -6$ *Multiply by −1.*

Check $x = -6$: $6 = 6$ *True*

The solution set is $\{-6\}$.

16.

$$\begin{aligned} 3(m+5) - 1 + 2m &= 5(m+2) \\ 3m + 15 - 1 + 2m &= 5m + 10 \\ 5m + 14 &= 5m + 10 \\ 14 &= 10 \end{aligned}$$

Because $14 = 10$ is a *false* statement, the equation has no solution set, symbolized by $\emptyset$.

17.

$$\begin{aligned} 7m - (2m - 9) &= 39 \\ 7m - 2m + 9 &= 39 \\ 5m + 9 &= 39 \\ 5m &= 30 \\ m &= 6 \end{aligned}$$

Check $m = 6$: $39 = 39$ *True*

The solution set is $\{6\}$.

18.

$$\begin{aligned} 7(p-2) + p &= 2(p+2) \\ 7p - 14 + p &= 2p + 4 \\ 8p - 14 &= 2p + 4 \\ 6p - 14 &= 4 \\ 6p &= 18 \\ p &= 3 \end{aligned}$$

Check $p = 3$: $10 = 10$ *True*

The solution set is $\{3\}$.

19.

$$\begin{aligned} -2t + 5t - 9 &= 3(t-4) - 5 \\ -2t + 5t - 9 &= 3t - 12 - 5 \\ 3t - 9 &= 3t - 17 \\ -9 &= -17 \end{aligned}$$

Because $-9 = -17$ is a *false* statement, the equation has no solution set, symbolized by $\emptyset$.

20. $-6z = -14$

$z = \frac{-14}{-6}$ *Divide by −6.*

$= \frac{7}{3}$

Check $z = \frac{7}{3}$: $-14 = -14$ *True*

The solution set is $\left\{\frac{7}{3}\right\}$.

21. $0.2(50) + 0.8r = 0.4(50 + r)$

To eliminate the decimals, multiply both sides by 10.

$$\begin{aligned} 10[0.2(50) + 0.8r] &= 10[0.4(50 + r)] \\ 2(50) + 8r &= 4(50 + r) \\ 100 + 8r &= 200 + 4r \\ 100 + 4r &= 200 \\ 4r &= 100 \\ r &= 25 \end{aligned}$$

Check $r = 25$: $10 + 20 = 30$ *True*

The solution set is $\{25\}$.

22. $2.3x + 13.7 = 1.3x + 2.9$

To eliminate the decimals, multiply both sides by 10.

$$\begin{aligned} 10[2.3x + 13.7] &= 10[1.3x + 2.9] \\ 23x + 137 &= 13x + 29 \\ 10x + 137 &= 29 \\ 10x &= -108 \\ x &= -10.8 \end{aligned}$$

Check $x = -10.8$: $-11.14 = -11.14$ *True*

The solution set is $\{-10.8\}$.

23.

$$\begin{aligned} 2(3 + 7x) - (1 + 15x) &= 2 \\ 6 + 14x - 1 - 15x &= 2 \\ -x + 5 &= 2 \\ -x &= -3 \\ x &= 3 \end{aligned}$$

Check $x = 3$: $48 - 46 = 2$ *True*

The solution set is $\{3\}$.

24. $6q - 9 = 12 + 3q$
$3q - 9 = 12$
$3q = 21$
$q = 7$

Check $q = 7$: $33 = 33$ *True*

The solution set is $\{7\}$.

25. $2(4 + 3r) = 3(r + 1) + 11$
$8 + 6r = 3r + 3 + 11$
$8 + 6r = 3r + 14$
$8 + 3r = 14$
$3r = 6$
$r = 2$

Check $r = 2$: $20 = 20$ *True*

The solution set is $\{2\}$.

26. $r + 9 + 7r = 4(3 + 2r) - 3$
$8r + 9 = 12 + 8r - 3$
$8r + 9 = 8r + 9$
$9 = 9$

Since $9 = 9$ is a *true* statement, the solution set is {all real numbers}.

27. $\frac{1}{4}x - 4 = \frac{3}{2}x + \frac{3}{4}x$

To clear fractions, multiply both sides by the LCD, which is 4.

$4\left(\frac{1}{4}x - 4\right) = 4\left(\frac{3}{2}x + \frac{3}{4}x\right)$
$x - 16 = 6x + 3x$
$x - 16 = 9x$
$-16 = 8x$
$x = -2$

Check $x = -2$: $-4.5 = -3 - 1.5$ *True*

The solution set is $\{-2\}$.

28. $0.6(100 - x) + 0.4x = 0.5(92)$

To eliminate the decimals, multiply both sides by 10.

$10[0.6(100 - x) + 0.4x] = 10[0.5(92)]$
$6(100 - x) + 4x = 5(92)$
$600 - 6x + 4x = 460$
$600 - 2x = 460$
$-2x = -140$
$x = 70$

Check $x = 70$: $1.8 + 2.8 = 4.6$ *True*

The solution set is $\{70\}$.

29. $\frac{3}{4}(a - 2) - \frac{1}{3}(5 - 2a) = -2$

To clear fractions, multiply both sides by the LCD, which is 12.

$12\left[\frac{3}{4}(a - 2) - \frac{1}{3}(5 - 2a)\right] = 12(-2)$
$9(a - 2) - 4(5 - 2a) = -24$
$9a - 18 - 20 + 8a = -24$
$17a - 38 = -24$
$17a = 14$
$a = \frac{14}{17}$

Check $a = \frac{14}{17}$: $-\frac{15}{17} - \frac{19}{17} = -2$ *True*

The solution set is $\left\{\frac{14}{17}\right\}$.

30. $2 - (m + 4) = 3m + 8$
$2 - m - 4 = 3m + 8$
$-m - 2 = 3m + 8$
$-4m - 2 = 8$
$-4m = 10$
$m = \frac{10}{-4} = -\frac{5}{2}$

Check $m = -\frac{5}{2}$: $\frac{1}{2} = \frac{1}{2}$ *True*

The solution set is $\left\{-\frac{5}{2}\right\}$.

2.4 An Introduction to Applications of Linear Equations

2.4 Classroom Examples

1. *Step 2*
Let $x =$ the number.

Step 3

If 5 is	added to	the product of 9 and a number,	the result is	19 less than the number.
↓	↓	↓	↓	↓
5	$+$	$9x$	$=$	$x - 19$

Step 4
Solve the equation.

$5 + 9x = x - 19$
$5 + 9x - 5 = x - 19 - 5$ *Subtract 5.*
$9x = x - 24$
$9x - x = x - 24 - x$ *Subtract x.*
$8x = -24$
$\frac{8x}{8} = \frac{-24}{8}$ *Divide by 8.*
$x = -3$

Step 5
The number is -3.

Step 6
9 times -3 is -27. 5 added to -27 is -22, which is 19 less than -3, so -3 is the number.

2. *Step 2*
Let $x =$ the number of medals Norway won.
Let $x + 6 =$ the number of medals the U.S. won.

Step 3

The total	is	the number of medals Norway won	plus	the number of medals the U.S. won.
↓	↓	↓	↓	↓
44	$=$	x	$+$	$(x+6)$

Step 4
Solve this equation.

$$\begin{aligned} 44 &= 2x + 6 \\ 44 - 6 &= 2x + 6 - 6 \\ 38 &= 2x \\ \frac{38}{2} &= \frac{2x}{2} \\ 19 &= x \end{aligned}$$

Step 5
Norway won 19 medals and the U.S. won $19 + 6 = 25$ medals.

Step 6
25 is 6 more than 19 and the sum of 19 and 25 is 44.

3. *Step 2*
Let $x =$ the number of orders for muffins.
Then $\frac{1}{6}x =$ the number of orders for croissants.

Step 3

The total	is	orders for muffins	plus	orders for croissants.
↓	↓	↓	↓	↓
56	$=$	x	$+$	$\frac{1}{6}x$

Step 4
Solve this equation.

$$\begin{aligned} 56 &= 1x + \tfrac{1}{6}x && \textit{x = 1x} \\ 56 &= \tfrac{6}{6}x + \tfrac{1}{6}x && \textit{LCD = 6} \\ 56 &= \tfrac{7}{6}x && \textit{Combine like terms.} \\ \tfrac{6}{7}(56) &= \tfrac{6}{7}\left(\tfrac{7}{6}x\right) && \textit{Multiply by } \tfrac{6}{7}. \\ 48 &= x \end{aligned}$$

Step 5
The number of orders for muffins was 48 and the number of orders for croissants was $\frac{1}{6}(48) = 8$.

Step 6
One-sixth of 48 is 8 and the sum of 48 and 8 is 56.

4. *Step 2*
Let $x =$ the number of members.
Then $2x =$ the number of nonmembers.
(If each member brought two nonmembers, there would be twice as many nonmembers as members.)

Step 3

Number of members	plus	Number of nonmembers	is	the total in attendance
↓	↓	↓	↓	↓
x	$+$	$2x$	$=$	27

Step 4
Solve this equation.

$$\begin{aligned} x + 2x &= 27 \\ 3x &= 27 \\ \frac{3x}{3} &= \frac{27}{3} \\ x &= 9 \end{aligned}$$

Step 5
There were 9 members and $2 \cdot 9 = 18$ nonmembers.

Step 6
18 is twice as much as 9 and the sum of 9 and 18 is 27.

5. *Step 2*
Let $x =$ the length of the middle-sized piece.
Then $x + 10 =$ the length of the longest piece and $x - 5 =$ the length of the shortest piece.

Step 3

Length of shortest piece	plus	length of middle-sized piece
↓	↓	↓
$x - 5$	$+$	x

plus	length of longest piece	is	total length of pipe.
↓	↓	↓	↓
$+$	$x + 10$	$=$	50

Step 4
Solve this equation.

$$\begin{aligned} (x-5) + x + (x+10) &= 50 \\ 3x + 5 &= 50 \\ 3x &= 45 \\ \frac{3x}{3} &= \frac{45}{3} \\ x &= 15 \end{aligned}$$

Step 5
The middle-sized piece is 15 inches long, the longest piece is $15 + 10 = 25$ inches long, and the shortest piece is $15 - 5 = 10$ inches long.

Step 6
Since 25 inches is 10 inches longer than 15 inches, 15 inches is 5 inches longer than 10 inches, and $15 + 25 + 10 = 50$ inches (the length of the pipe), the answers are correct.

6. *Step 2*
Let $x =$ the degree measure of the angle.
Then $90 - x =$ the degree measure of its complement, and $180 - x =$ the degree measure of its supplement.

Step 3

complement	plus	supplement	equals	174
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$90 - x$	$+$	$180 - x$	$=$	174

Step 4
$$\begin{aligned} 270 - 2x &= 174 \\ -2x &= -96 \quad \textit{Subtract 270.} \\ x &= 48 \quad \textit{Divide by } -2. \end{aligned}$$

Step 5
The measure of the angle is $48°$.

Step 6
The complement of $48°$ is $90° - 48° = 42°$ and the supplement is $180° - 48° = 132°$. The sum of $42°$ and $132°$ is $174°$.

7. *Step 2*
Let $x =$ the lesser page number.
Then $x + 1 =$ the greater page number.

Step 3
Because the sum of the page numbers is 569, an equation is
$$x + (x + 1) = 569$$

Step 4
$$\begin{aligned} 2x + 1 &= 569 \quad \textit{Combine like terms.} \\ 2x &= 568 \quad \textit{Subtract 1.} \\ x &= 284 \quad \textit{Divide by 2.} \end{aligned}$$

Step 5
The lesser page number is 284, and the greater page number is $284 + 1 = 285$.

Step 6
285 is one more than 284 and the sum of 284 and 285 is 569.

8. Let $x =$ the lesser even integer.
Then $x + 2 =$ the greater even integer.

From the given information, we have
$$6 \cdot x + (x + 2) = 86.$$

Solve this equation.
$$\begin{aligned} 7x + 2 &= 86 \\ 7x &= 84 \\ x &= 12 \end{aligned}$$

The lesser even integer is 12 and the greater consecutive even integer is $12 + 2 = 14$. Six times 12 is 72 and 72 plus 14 is 86.

2.4 Section Exercises

1. Choice **D**, $6\frac{1}{2}$, is *not* a reasonable answer in an applied problem that requires finding the number of cars on a dealer's lot, since you cannot have $\frac{1}{2}$ of a car. The number of cars must be a whole number.

2. Choice **D**, 25, is *not* a reasonable answer since finding the number of hours a light bulb is on during a day cannot be more than 24.

3. Choice **A**, -10, is *not* a reasonable answer since distance cannot be negative.

4. Choice **C**, -5, is *not* a reasonable answer since time cannot be negative.

The applied problems in this section should be solved by using the six-step method shown in the text. These steps will only be listed in a few of the solutions, but all of the solutions are based on this method.

5. *Step 2*
Let $x =$ the unknown number. Then $5x + 2$ represents "2 is added to five times a number," and $4x + 5$ represents "5 more than four times a number."

Step 3 $\quad 5x + 2 = 4x + 5$

Step 4
$$\begin{aligned} 5x + 2 &= 4x + 5 \\ x + 2 &= 5 \\ x &= 3 \end{aligned}$$

Step 5
The number is 3.

Step 6
Check that 3 is the correct answer by substituting this result into the words of the original problem. 2 added to five times a number is $2 + 5(3) = 17$ and 5 more than four times the number is $5 + 4(3) = 17$. The values are equal, so the number 3 is the correct answer.

6. *Step 2*
Let $x =$ the unknown number. Then $8 + 4x$ represents "four times a number added to 8," and $5 + 3x$ represents "three times the number added to 5."

Step 3 $\quad 8 + 4x = 5 + 3x$

Step 4
$$\begin{aligned} 8 + 4x &= 5 + 3x \\ 8 + x &= 5 \\ x &= -3 \end{aligned}$$

Step 5
The number is -3.

Step 6
Check that -3 is the correct answer by substituting this result into the words of the original problem. Four times a number is added to 8 is $8 + 4(-3) = -4$ and three times the number added to 5 is $5 + 3(-3) = -4$. The values are equal, so the number -3 is the correct answer.

7. *Step 2*
Let $x =$ the unknown number. Then $x - 2$ is two subtracted from the number, $3(x-2)$ is triple the difference, and $x + 6$ is six more than the number.

Step 3 $3(x-2) = x + 6$

Step 4
$$\begin{aligned} 3x - 6 &= x + 6 \\ 2x - 6 &= 6 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

Step 5
The number is 6.

Step 6
Check that 6 is the correct answer by substituting this result into the words of the original problem. Two subtracted from the number is $6 - 2 = 4$. Triple this difference is $3(4) = 12$, which is equal to 6 more than the number, since $6 + 6 = 12$.

8. *Step 2*
Let $x =$ the unknown number. Then $x + 3$ is 3 is added to a number, $2(x+3)$ is this sum is doubled, and $x + 2$ is 2 more than the number.

Step 3 $2(x+3) = x + 2$

Step 4
$$\begin{aligned} 2x + 6 &= x + 2 \\ x + 6 &= 2 \\ x &= -4 \end{aligned}$$

Step 5
The number is -4.

Step 6
Check that -4 is the correct answer by substituting this result into the words of the original problem. Three added to the number is -1, double this value is -2. Two more than -4 is also -2, so the number is -4.

9. *Step 2*
Let $x =$ the unknown number. Then $3x$ is three times the number, $x + 7$ is 7 more than the number, $2x$ is twice the number, and $-11 - 2x$ is the difference between -11 and twice the number.

Step 3 $3x + (x + 7) = -11 - 2x$

Step 4
$$\begin{aligned} 4x + 7 &= -11 - 2x \\ 6x + 7 &= -11 \\ 6x &= -18 \\ x &= -3 \end{aligned}$$

Step 5
The number is -3.

Step 6
Check that -3 is the correct answer by substituting this result into the words of the original problem. The sum of three times a number and 7 more than the number is $3(-3) + (-3 + 7) = -5$ and the difference between -11 and twice the number is $-11 - 2(-3) = -5$. The values are equal, so the number -3 is the correct answer.

10. *Step 2*
Let $x =$ the unknown number. Then $2x + 4$ is 4 is added to twice the number, $2(2x+4)$ is the sum multiplied by 2, and $3x + 4$ is the number is multiplied by 3 and 4 is added to the product.

Step 3 $2(2x+4) = 3x + 4$

Step 4
$$\begin{aligned} 4x + 8 &= 3x + 4 \\ x + 8 &= 4 \\ x &= -4 \end{aligned}$$

Step 5
The number is -4.

Step 6
Check that -4 is the correct answer by substituting this result into the words of the original problem.

Twice the number is $2(-4) = -8$. Four added to twice the number is $-8 + 4 = -4$. This sum multiplied by 2 is $2(-4) = -8$. The number multiplied by 3 is $3(-4) = -12$. Four added to this product is $-12 + 4 = -8$. Because both results are -8, the answer, -4, checks.

11. Let $x =$ the number of drive-in movie screens in New York.
Then $x + 11 =$ the number of drive-in movie screens in California.
Since the total number of screens was 107, we can write the equation
$$x + (x + 11) = 107.$$
Solve this equation.
$$\begin{aligned} 2x + 11 &= 107 \\ 2x &= 96 \\ x &= 48 \end{aligned}$$

Since $x = 48$, $x + 11 = 48 + 11 = 59$.

There were 48 drive-in movie screens in New York and 59 in California. Since $48 + 59 = 107$, this answer checks.

12. Let $x =$ the number of *Dallas* viewers.
Then $x + 9 =$ the number of *M*A*S*H* viewers.
Since the total number of viewers is 91 (all numbers in millions), we can write the equation

$$x + (x + 9) = 91.$$

Solve this equation.

$$\begin{aligned} 2x + 9 &= 91 \\ 2x &= 82 \\ x &= 41 \end{aligned}$$

Since $x = 41$, $x + 9 = 50$.

There were 41 million *Dallas* viewers and 50 million *M*A*S*H* viewers. Since $41 + 50 = 91$, this answer checks.

13. Let $x =$ the number of Democrats.
Then $x + 11 =$ the number of Republicans.

The number of Democrats	plus	the number of Republicans
↓	↓	↓
x	$+$	$(x + 11)$

equals	the number of members of the Senate.
↓	↓
$=$	99

Solve the equation.

$$\begin{aligned} x + (x + 11) &= 99 \\ 2x + 11 &= 99 \\ 2x &= 88 \\ x &= 44 \end{aligned}$$

There were 44 Democrats and $44 + 11 = 55$ Republicans.

14. Let $x =$ the number of Democrats.
Then $x + 30 =$ the number of Republicans.

Since the total number of Democrats and Republicans was 434, we can write the equation

$$x + (x + 30) = 434.$$

Solve this equation.

$$\begin{aligned} 2x + 30 &= 434 \\ 2x &= 404 \\ x &= 202 \end{aligned}$$

Since $x = 202$, $x + 30 = 232$.

There were 232 Republicans and 202 Democrats. Since $232 + 202 = 434$, this answer checks.

15. Let $x =$ revenue from ticket sales for Bruce Springsteen and the E Street Band.
Then $x - 35.4 =$ revenue from ticket sales for Céline Dion.
Since the total revenue from ticket sales was \$196.4 (all numbers in millions), we can write the equation

$$x + (x - 35.4) = 196.4.$$

Solve this equation.

$$\begin{aligned} 2x - 35.4 &= 196.4 \\ 2x &= 231.8 \\ x &= 115.9 \end{aligned}$$

Since $x = 115.9$, $x - 35.4 = 80.5$.

Bruce Springsteen and the E Street Band took in \$115.9 million and Céline Dion took in \$80.5 million. Since 80.5 is 35.4 less than 115.9 and $115.9 + 80.5 = 196.4$, this answer checks.

16. Let $x =$ the number of Toyota Camry sales.
Then $x - 40 =$ the number of Honda Accord sales.
Since the total number of sales was 814 (all numbers in thousands), we can write the equation

$$x + (x - 40) = 814.$$

Solve this equation.

$$\begin{aligned} 2x - 40 &= 814 \\ 2x &= 854 \\ x &= 427 \end{aligned}$$

Since $x = 427$, $x - 40 = 387$.

There were 427 thousand Toyota Camry sales and 387 thousand Honda Accord sales. Since 387 is 40 less than 427 and $427 + 387 = 814$, this answer checks.

17. Let $x =$ the number of games the Suns lost.
Then $3x + 2 =$ the number of games the Suns won.
Since the total number of games played was 82, we can write the equation

$$x + (3x + 2) = 82.$$

Solve this equation.

$$\begin{aligned} 4x + 2 &= 82 \\ 4x &= 80 \\ x &= 20 \end{aligned}$$

Since $x = 20$, $3x + 2 = 62$.

The Suns won 62 games and lost 20 games. Since $62 + 20 = 82$, this answer checks.

18. Let $x =$ the number of games the White Sox lost.
Then $2x - 27 =$ the number of games the White Sox won.
Since the total number of games played was 162, we can write the equation

$$x + (2x - 27) = 162.$$

Solve this equation.

$$\begin{aligned} 3x - 27 &= 162 \\ 3x &= 189 \\ x &= 63 \end{aligned}$$

Since $x = 63$, $2x - 27 = 99$.

The White Sox won 99 games and lost 63 games. Since $99 + 63 = 162$, this answer checks.

19. Let $x =$ the value of the 1945 nickel.
Then $\frac{8}{7}x =$ the value of the 1950 nickel.
The total value of the two coins is \$15.00, so

$$x + \tfrac{8}{7}x = 15.$$

Solve this equation. First multiply both sides by 7 to clear fractions.

$$\begin{aligned} 7\left(x + \tfrac{8}{7}x\right) &= 7(15) \\ 7x + 8x &= 105 \\ 15x &= 105 \\ x &= 7 \quad \textit{Divide by 15.} \end{aligned}$$

Since $x = 7$, $\frac{8}{7}x = \frac{8}{7}(7) = 8$.

The value of the 1945 Philadelphia nickel is \$7.00 and the value of the 1950 Denver nickel is \$8.00.

20. Let $x =$ the population of China.
Then $\frac{4}{5}x =$ the population of India.
The combined population of the two countries was 2.4 billion, so

$$x + \tfrac{4}{5}x = 2.4.$$

Solve this equation.

$$\begin{aligned} 1x + \tfrac{4}{5}x &= 2.4 \\ \tfrac{9}{5}x &= 2.4 \\ \tfrac{5}{9}\left(\tfrac{9}{5}x\right) &= \tfrac{5}{9}\left(\tfrac{24}{10}\right) \\ x &= \tfrac{4}{3} \quad [\approx 1.3] \end{aligned}$$

Since $x = \frac{4}{3}$, $\frac{4}{5}x = \frac{4}{5}\left(\frac{4}{3}\right) = \frac{16}{15} \quad [\approx 1.1]$.

To the nearest tenth of a billion, the population of China was 1.3 billion and the population of India was 1.1 billion.

21. Let $x =$ the number of kg of onions.
Then $6.6x =$ the number of kg of grilled steak.
The total weight of these two ingredients was 617.6 kg, so

$$x + 6.6x = 617.6.$$

Solve this equation.

$$\begin{aligned} 1x + 6.6x &= 617.6 \\ 7.6x &= 617.6 \\ x &= \tfrac{617.6}{7.6} \approx 81.3 \end{aligned}$$

Since $x = \frac{617.6}{7.6}$, $6.6x = 6.6\left(\frac{617.6}{7.6}\right) \approx 536.3$.

To the nearest tenth of a kilogram, 81.3 kg of onions and 536.3 kg of grilled steak were used to make the taco.

22. Let $x =$ the number of calories burned doing weight training.
Then $\frac{12}{5}x =$ the number of calories burned doing aerobics.
The total number of calories burned was 374, so

$$x + \tfrac{12}{5}x = 374.$$

$$\begin{aligned} 1x + \tfrac{12}{5}x &= 374 \\ \tfrac{17}{5}x &= 374 \\ \tfrac{5}{17}\left(\tfrac{17}{5}x\right) &= \tfrac{5}{17}(374) \\ x &= \frac{5}{\cancel{17}_{1}} \cdot \frac{\cancel{374}^{22}}{1} = 110 \end{aligned}$$

Since $x = 110$, $\frac{12}{5}x = \frac{12}{5}(110) = 264$.

He will burn 110 calories doing weight training and 264 calories doing aerobics.

23. Let $x =$ the number of CDs sold.
Then $\frac{8}{5}x =$ the number of DVDs sold.

The total number of CDs and DVDs sold was 273, so

$$x + \tfrac{8}{5}x = 273.$$

Solve this equation.

$$\begin{aligned} 1x + \tfrac{8}{5}x &= 273 \\ \tfrac{13}{5}x &= 273 \\ \tfrac{5}{13}\left(\tfrac{13}{5}x\right) &= \tfrac{5}{13}(273) \\ x &= \frac{5}{\cancel{13}_{1}} \cdot \frac{\cancel{273}^{21}}{1} = 105 \end{aligned}$$

Since $x = 105$, $\frac{8}{5}x = \frac{8}{5}(105) = 168$.

There were 168 DVDs sold.

24. Let $x =$ the number of pounds of nickel.
Then $3x =$ the number of pounds of copper.

The total number of pounds would be 560, so

$$x + 3x = 560.$$

Solve this equation.

$$4x = 560$$
$$x = \tfrac{560}{4} = 140$$

Since $x = 140$, $3x = 3(140) = 420$. To make 560 pounds of five-cent coins, use 420 pounds of copper.

25. Let $x =$ the number of ounces of rye flour.
Then $4x =$ the number of ounces of whole wheat flour.

The total number of ounces would be 32, so

$$x + 4x = 32.$$

Solve this equation.

$$5x = 32$$
$$x = \tfrac{32}{5} = 6.4$$

Since $x = 6.4$, $4x = 4(6.4) = 25.6$. To make a loaf of bread weighing 32 oz, use 6.4 oz of rye flour and 25.6 oz of whole wheat flour.

26. Let $x =$ the number of milligrams of inert ingredients.
Then $9x =$ the number of milligrams of active ingredients.

The total number of milligrams would be 250, so

$$x + 9x = 250.$$

Solve this equation.

$$10x = 250$$
$$x = \tfrac{250}{10} = 25$$

Since $x = 25$, $9x = 9(25) = 225$. In a single 250-mg caplet, there would be 25 mg of inert ingredients and 225 mg of active ingredients.

27. Let $x =$ the number of tickets booked on United Airlines.
Then $x + 7 =$ the number of tickets booked on American Airlines, and
$2x + 4 =$ the number of tickets booked on Southwest Airlines.

The total number of tickets booked was 55, so

$$x + (x + 7) + (2x + 4) = 55.$$

Solve this equation.

$$4x + 11 = 55$$
$$4x = 44$$
$$x = \tfrac{44}{4} = 11$$

Since $x = 11$, $x + 7 = 11 + 7 = 18$, and $2x + 4 = 2(11) + 4 = 26$. He booked 18 tickets on American, 11 tickets on United, and 26 tickets on Southwest.

28. Let $x =$ the number of hours making telephone calls.
Then $x + 0.5 =$ the number of hours writing e-mails, and
$2x =$ the number of hours attending meetings.

The total number of hours is 7.5, so

$$x + (x + 0.5) + 2x = 7.5.$$

Solve this equation.

$$4x + 0.5 = 7.5$$
$$4x = 7$$
$$x = \tfrac{7}{4} = 1.75$$

Since $x = 1.75$, $x + 0.5 = 1.75 + 0.5 = 2.25$, and $2x = 2(1.75) = 3.5$. In her 7.5 hour-day, she spent 1.75 hours making telephone calls, 2.25 hours writing e-mails, and 3.5 hours attending meetings.

29. Let $x =$ the length of the shortest piece.
Then $x + 5 =$ the length of the middle piece, and
$x + 9 =$ the length of the longest piece.

The total length is 59 inches, so

$$x + (x + 5) + (x + 9) = 59.$$

Solve this equation.

$$3x + 14 = 59$$
$$3x = 45$$
$$x = 15$$

Since $x = 15$, $x + 5 = 20$, and $x + 9 = 24$.

The shortest piece should be 15 inches, the middle piece should be 20 inches, and the longest piece should be 24 inches. The answer checks since

$$15 + 20 + 24 = 59.$$

30. Let $x =$ the number of bronze medals.
Then $x + 10 =$ the number of silver medals, and
$x + 6 =$ the number of gold medals.

The total number of medals earned by the United States was 103, so

$$x + (x + 10) + (x + 6) = 103.$$

Solve this equation.

$$3x + 16 = 103$$
$$3x = 87$$
$$x = 29$$

Since $x = 29$, $x + 10 = 39$, and $x + 6 = 35$.

The United States earned 35 gold medals, 39 silver medals, and 29 bronze medals. The answer checks since

$$35 + 39 + 29 = 103.$$

31. Let $x =$ the distance of Mercury from the sun.
Then $x + 31.2 =$ the distance of Venus from the sun, and
$x + 57 =$ the distance of Earth from the sun.

Since the total of the distances from these three planets is 196.2 (all distances in millions of miles), we can write the equation

$$x + (x + 31.2) + (x + 57) = 196.2.$$

Solve this equation.

$$\begin{aligned} 3x + 88.2 &= 196.2 \\ 3x &= 108 \\ x &= 36 \end{aligned}$$

Mercury is 36 million miles from the sun, Venus is $36 + 31.2 = 67.2$ million miles from the sun, and Earth is $36 + 57 = 93$ million miles from the sun. The answer checks since

$$36 + 67.2 + 93 = 196.2.$$

32. Let $x =$ the number of satellites of Saturn.
Then $x + 16 =$ the number of satellites of Jupiter, and $x - 20 =$ the number of satellites of Uranus.

Since the total number of satellites is 137,

$$x + (x + 16) + (x - 20) = 137.$$

Solve this equation.

$$\begin{aligned} 3x - 4 &= 137 \\ 3x &= 141 \\ x &= \tfrac{141}{3} = 47 \end{aligned}$$

Since $x = 47$, $x + 16 = 63$, and $x - 20 = 27$.

Uranus has 27 known satellites.

33. Let $x =$ the measure of angles A and B.
Then $x + 60 =$ the measure of angle C.

The sum of the measures of the angles of any triangle is 180°, so

$$x + x + (x + 60) = 180.$$

Solve this equation.

$$\begin{aligned} 3x + 60 &= 180 \\ 3x &= 120 \\ x &= 40 \end{aligned}$$

Angles A and B have measures of 40 degrees, and angle C has a measure of $40 + 60 = 100$ degrees. The answer checks since

$$40 + 40 + 100 = 180.$$

34. Let $x =$ the measure of angles B and C.
Then $x + 141 =$ the measure of angle A.

The sum of the measures of the angles of any triangle is 180°, so

$$(x + 141) + x + x = 180.$$

Solve this equation.

$$\begin{aligned} 3x + 141 &= 180 \\ 3x &= 39 \\ x &= 13 \end{aligned}$$

Since $x = 13$, $x + 141 = 154$.

The measure of angle A is $154°$; the measures of angles B and C are both 13°. The answer checks since

$$154 + 13 + 13 = 180.$$

35. Subtract one of the numbers (m) from the sum (k) to express the other number. The other number is $k - m$.

36. The product of two numbers is r and one number is s $(s \neq 0)$, so you must divide to find the other number. The other number is $\frac{r}{s}$.

37. An angle cannot have its supplement equal to its complement. The sum of an angle and its supplement equals 180°, while the sum of an angle and its complement equals 90°. If we try to solve the equation

$$90 - x = 180 - x,$$

we will get

$$\begin{aligned} 90 - x + x &= 180 - x + x \\ 90 &= 180 \quad \textit{False} \end{aligned}$$

so this equation has no solution.

38. If x is the angle, then $180 - x$ is its supplement. Since they are equal,

$$\begin{aligned} x &= 180 - x \\ 2x &= 180 \\ x &= 90. \end{aligned}$$

There is an angle that is equal to its supplement. Its measure is 90°.

Similarly, for the complement,

$$\begin{aligned} x &= 90 - x \\ 2x &= 90 \\ x &= 45. \end{aligned}$$

There is an angle that is equal to its complement. Its measure is 45°.

39. If x represents an integer, the next smaller consecutive integer is 1 less than x, that is, $x - 1$.

40. If x represents an integer, the next smaller even integer is 2 less than x, that is, $x - 2$.

41. Let $x =$ the measure of the angle.
Then $90 - x =$ the measure of its complement.

The "complement is four times its measure" can be written as

$$90 - x = 4x.$$

Solve this equation.

$$90 = 5x$$
$$x = \frac{90}{5} = 18$$

The measure of the angle is $18°$. The complement is $90° - 18° = 72°$, which is four times $18°$.

42. Let $x =$ the measure of the angle.
Then $180 - x =$ the measure of its supplement.

The "supplement is three times its measure" can be written as

$$180 - x = 3x.$$

Solve this equation.

$$180 = 4x$$
$$x = \frac{180}{4} = 45$$

The measure of the angle is $45°$. The supplement is $180° - 45° = 135°$, which is three times $45°$.

43. Let $x =$ the measure of the angle. Then
$90 - x =$ the measure of its complement, and
$180 - x =$ the measure of its supplement.

Its supplement	measures	39°	more than	twice its complement.
↓	↓	↓	↓	↓
$180 - x$	$=$	39	$+$	$2(90 - x)$

Solve the equation.

$$180 - x = 39 + 2(90 - x)$$
$$180 - x = 39 + 180 - 2x$$
$$180 - x = 219 - 2x$$
$$x + 180 = 219$$
$$x = 39$$

The measure of the angle is $39°$. The complement is $90° - 39° = 51°$. Now $39°$ more than twice its complement is $39° + 2(51°) = 141°$, which is the supplement of $39°$ since $180° - 39° = 141°$.

44. Let $x =$ the measure of the angle. Then
$180 - x =$ the measure of its supplement, and
$90 - x =$ the measure of its complement.

Angle's supplement		three times		angle's complement	less 38°
↓		↓		↓	↓
$180 - x$	$=$	3	$\cdot$	$(90 - x)$	$- 38$

$$180 - x = 270 - 3x - 38$$
$$180 - x = 232 - 3x$$
$$180 + 2x = 232$$
$$2x = 52$$
$$x = 26$$

The measure of the angle is $26°$.

45. Let $x =$ the measure of the angle. Then
$180 - x =$ the measure of its supplement, and
$90 - x =$ the measure of its complement.

The difference between the measure of its supplement and		three times the measure of its complement	is	10°.
↓	↓	↓	↓	↓
$(180 - x)$	$-$	$3(90 - x)$	$=$	10

Solve the equation.

$$(180 - x) - 3(90 - x) = 10$$
$$180 - x - 270 + 3x = 10$$
$$2x - 90 = 10$$
$$2x = 100$$
$$x = 50$$

The measure of the angle is $50°$. The supplement is $180° - 50° = 130°$ and the complement is $90° - 50° = 40°$. The answer checks since $130° - 3(40°) = 10°$.

46. Let $x =$ the measure of the angle. Then
$90 - x =$ the measure of its complement, and
$180 - x =$ the measure of its supplement.

The sum of the measures of its complement and supplement is $160°$ can be written as

$$(90 - x) + (180 - x) = 160.$$

Solve the equation.

$$-2x + 270 = 160$$
$$-2x = -110$$
$$x = 55$$

The measure of the angle is $55°$. The sum of the measures of its complement $(90° - 55° = 35°)$ and its supplement $(180° - 55° = 125°)$ is $160°$ $(35° + 125° = 160°)$.

47. Let $x =$ the number on the first locker.
Then $x + 1 =$ the number on the next locker.

Since the numbers have a sum of 137, we can write the equation

$$x + (x + 1) = 137.$$

Solve the equation.

$$\begin{aligned} 2x + 1 &= 137 \\ 2x &= 136 \\ x &= \tfrac{136}{2} = 68 \end{aligned}$$

Since $x = 68$, $x + 1 = 69$.

The lockers have numbers 68 and 69. Since $68 + 69 = 137$, this answer checks.

48. Let $x =$ the number on the first check.
Then $x + 1 =$ the number on the second check.

Since the sum of the numbers is 357, we can write the equation

$$x + (x + 1) = 357.$$

Solve this equation.

$$\begin{aligned} 2x + 1 &= 357 \\ 2x &= 356 \\ x &= \tfrac{356}{2} = 178 \end{aligned}$$

Since $x = 178$, $x + 1 = 179$.

The checkbook check numbers are 178 and 179. Since $178 + 179 = 357$, this answer checks.

49. Let $x =$ the lesser even integer.
Then $x + 2 =$ the greater even integer.

The lesser added to three times the greater gives a sum of 46 can be written as

$$\begin{aligned} x + 3(x + 2) &= 46. \\ x + 3x + 6 &= 46 \\ 4x + 6 &= 46 \\ 4x &= 40 \\ x &= 10 \end{aligned}$$

Since $x = 10$, $x + 2 = 12$.

The integers are 10 and 12. This answer checks since $10 + 3(12) = 46$.

50. Let $x =$ the lesser odd integer.
Then $x + 2 =$ the greater odd integer.

Twice the greater is 17 more than the lesser can be written as

$$\begin{aligned} 2(x + 2) &= x + 17. \\ 2x + 4 &= x + 17 \\ x + 4 &= 17 \\ x &= 13 \end{aligned}$$

Since $x = 13$, $x + 2 = 15$.

The integers are 13 and 15. This answer checks since $2(15) = 13 + 17$.

51. Because the two pages are back-to-back, they must have page numbers that are consecutive integers.

Let $x =$ the lesser page number.
Then $x + 1 =$ the greater page number.

$$\begin{aligned} x + (x + 1) &= 203 \\ 2x + 1 &= 203 \\ 2x &= 202 \\ x &= 101 \end{aligned}$$

Since $x = 101$, $x + 1 = 102$.

The page numbers are 101 and 102. This answer checks since the sum is 203.

52. Let $x =$ the first house number.
Then $x + 2 =$ the second house number.

Since the sum of the numbers is 58, we have

$$x + (x + 2) = 58.$$

Solve the equation.

$$\begin{aligned} 2x + 2 &= 58 \\ 2x &= 56 \\ x &= 28 \end{aligned}$$

Since $x = 28$, $x + 2 = 30$.

The house numbers are 28 and 30. Since $28 + 30 = 58$, this answer checks.

53. Let $x =$ the lesser integer.
Then $x + 1 =$ the greater integer.

$$\begin{aligned} x + 3(x + 1) &= 43 \\ x + 3x + 3 &= 43 \\ 4x + 3 &= 43 \\ 4x &= 40 \\ x &= 10 \end{aligned}$$

Since $x = 10$, $x + 1 = 11$.

The integers are 10 and 11. This answer checks since $10 + 3(11) = 43$.

54. Let $x =$ the lesser integer.
Then $x + 1 =$ the greater integer.

$$\begin{aligned} 5x + 3(x + 1) &= 59 \\ 5x + 3x + 3 &= 59 \\ 8x + 3 &= 59 \\ 8x &= 56 \\ x &= 7 \end{aligned}$$

Since $x = 7$, $x + 1 = 8$.

The integers are 7 and 8. This answer checks since $5(7) + 3(8) = 35 + 24 = 59$.

55. Let $x =$ the first even integer.
Then $x + 2 =$ the second even integer, and
$x + 4 =$ the third even integer.

$$\begin{aligned} x + (x+2) + (x+4) &= 60 \\ 3x + 6 &= 60 \\ 3x &= 54 \\ x &= 18 \end{aligned}$$

Since $x = 18$, $x + 2 = 20$, and $x + 4 = 22$.

The first even integer is 18. This answer checks since $18 + 20 + 22 = 60$.

56. Let $x =$ the first odd integer.
Then $x + 2 =$ the second odd integer, and
$x + 4 =$ the third odd integer.

$$\begin{aligned} x + (x+2) + (x+4) &= 69 \\ 3x + 6 &= 69 \\ 3x &= 63 \\ x &= 21 \end{aligned}$$

Since $x = 21$, $x + 2 = 23$, and $x + 4 = 25$.

The third odd integer is 25. This answer checks since $21 + 23 + 25 = 69$.

57. Let $x =$ the first odd integer.
Then $x + 2 =$ the second odd integer, and
$x + 4 =$ the third odd integer.

$$\begin{aligned} 2[(x+4) - 6] &= [x + 2(x+2)] - 23 \\ 2(x-2) &= x + 2x + 4 - 23 \\ 2x - 4 &= 3x - 19 \\ -4 &= x - 19 \\ 15 &= x \end{aligned}$$

Since $x = 15$, $x + 2 = 17$, and $x + 4 = 19$.

The integers are 15, 17, and 19.

58. Let $x =$ the first even integer.
Then $x + 2 =$ the second even integer, and
$x + 4 =$ the third even integer.

$$\begin{aligned} x + (x+4) &= 3(x+2) - 22 \\ x + x + 4 &= 3x + 6 - 22 \\ 2x + 4 &= 3x - 16 \\ -x + 4 &= -16 \\ -x &= -20 \\ x &= 20 \end{aligned}$$

Since $x = 20$, $x + 2 = 22$, and $x + 4 = 24$.

The integers are 20, 22, and 24.

59. Let $x =$ the amount of Head Start funding in the first year (in billions of dollars).
Then $x + 0.13 =$ the amount of funding in the second year, and
$(x + 0.13) + 0.10 = x + 0.23 =$ the amount of funding in the third year.

The total funding was 19.98 billion dollars, so

$$x + (x + 0.13) + (x + 0.23) = 19.98.$$

Solve this equation.

$$\begin{aligned} 3x + 0.36 &= 19.98 \\ 3x &= 19.62 \\ x &= \tfrac{19.62}{3} = 6.54 \end{aligned}$$

Since $x = 6.54$, $x + 0.13 = 6.67$, and $x + 0.23 = 6.77$.

The Head Start funding was 6.54 billion dollars in the first year, 6.67 billion dollars in the second year, and 6.77 billion dollars in the third year.

60. Let $x =$ the number of injuries in boatbuilding.
Then $x + 12 =$ the number of injuries in amusement parks and arcades, and
$(x + 12) + 30 = x + 42 =$ the number of injuries in iron foundries.

$$\begin{aligned} x + (x+12) + (x+42) &= 387 \\ 3x + 54 &= 387 \\ 3x &= 333 \\ x &= 111 \end{aligned}$$

Since $x = 111$, $x + 12 = 123$, and $x + 42 = 153$.

In this group of 3000 workers, there were 111 injuries in boatbuilding, 123 in amusement parks and arcades, and 153 in iron foundries.

61. $L = 6$ and $W = 4$, so $LW = 6 \cdot 4 = 24$.

62. $r = 25$ and $t = 4.5$, so $rt = 25(4.5) = 112.5$.

63. $L = 8$ and $W = 2$, so
$2L + 2W = 2(8) + 2(2) = 16 + 4 = 20$.

64. $p = 4000$, $r = 0.04$, and $t = 2$, so
$prt = 4000(0.04)(2) = 320$.

65. $B = 27$ and $h = 6$, so $\frac{1}{2}Bh = \frac{1}{2}(27)(6) = 81$.

66. $h = 10,\ b = 4$, and $B = 12$,
so $\frac{1}{2}h(b + B) = \frac{1}{2}(10)(4 + 12) = 5(16) = 80$.

2.5 Formulas and Applications from Geometry

2.5 Classroom Examples

1.
$$\begin{aligned} P &= 2L + 2W \\ 126 &= 2L + 2(25) && \textit{Let } P = 126 \textit{ and } W = 25. \\ 126 &= 2L + 50 \\ 76 &= 2L && \textit{Subtract 50.} \\ 38 &= L && \textit{Divide by 2.} \end{aligned}$$

2. The fence will enclose the perimeter of the rectangular field, so use the formula for the perimeter of a rectangle. Find the length of the field by substituting $P = 800$ and $W = L - 50$ into the formula and solving for L.

$$\begin{aligned} P &= 2L + 2W \\ 800 &= 2L + 2(L - 50) \\ 800 &= 2L + 2L - 100 \\ 900 &= 4L \\ 225 &= L \end{aligned}$$

Since $L = 225$, $W = 225 - 50 = 175$. The dimensions of the field are 225 m by 175 m.

3. Let $s =$ the length of the shortest side, in inches;
$s + 5 =$ the length of the medium side, and,
$(s + 5) + 1 = s + 6 =$ the length of the longest side.

The perimeter is 32 inches, so

$$\begin{aligned} s + (s + 5) + (s + 6) &= 32. \\ 3s + 11 &= 32 \\ 3s &= 21 \\ s &= 7 \end{aligned}$$

Since $s = 7$, $s + 5 = 12$, and $s + 6 = 13$. The lengths of the sides are 7, 12, and 13 inches. The perimeter is $7 + 12 + 13 = 32$, as required.

4. Use the formula for the area of a triangle.

$$\begin{aligned} A &= \tfrac{1}{2}bh \\ 120 &= \tfrac{1}{2}b(24) && \textit{Let } A = 120,\ h = 24. \\ 120 &= 12b \\ 10 &= b \end{aligned}$$

The length of the base is 10 meters.

5. The sum of the measures of the two angles is 180° because together they form a straight angle.

$$\begin{aligned} (6x + 29) + (x + 11) &= 180 \\ 7x + 40 &= 180 \\ 7x &= 140 \\ x &= 20 \end{aligned}$$

If $x = 20$, $6x + 29 = 6(20) + 29 = 149$ and $x + 11 = 20 + 11 = 31$.

The measures of the angles are 149° and 31°.

6. Solve $I = prt$ for t.

$$\begin{aligned} I &= prt \\ \frac{I}{pr} &= \frac{prt}{pr} && \textit{Divide by pr.} \\ \frac{I}{pr} &= t, \quad \text{or} \quad t = \frac{I}{pr} \end{aligned}$$

7. Solve $S = 2\pi rh + 2\pi r^2$ for h.

$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ S - 2\pi r^2 &= 2\pi rh && \textit{Subtract } 2\pi r^2. \\ \frac{S - 2\pi r^2}{2\pi r} &= h, && \textit{Divide by } 2\pi r. \\ \text{or} \quad h &= \frac{S - 2\pi r^2}{2\pi r} \end{aligned}$$

8. Solve $A = p + prt$ for t.

$$\begin{aligned} A &= p + prt \\ A - p &= p + prt - p && \textit{Subtract p.} \\ A - p &= prt \\ \frac{A - p}{pr} &= \frac{prt}{pr} && \textit{Divide by pr.} \\ \frac{A - p}{pr} &= t, \quad \text{or} \quad t = \frac{A - p}{pr} \end{aligned}$$

9. Solve $A = \frac{1}{2}h(b + B)$ for h.

$$\begin{aligned} A &= \tfrac{1}{2}h(b + B) \\ 2A &= h(b + B) && \textit{Multiply by 2.} \\ \frac{2A}{b + B} &= h && \textit{Divide by } b + B. \end{aligned}$$

2.5 Section Exercises

1. **(a)** The perimeter of a plane geometric figure is the distance around the figure. It can be found by adding up the lengths of all the sides. Perimeter is a one-dimensional (linear) measurement, so it is given in linear units (inches, centimeters, feet, etc.).

(b) The area of a plane geometric figure is the measure of the surface covered or enclosed by the figure. Area is a two-dimensional measurement, so it is given in square units (square centimeters, square feet, etc.).

2. **(a)** *Area* is given in *square* feet.

(b) *Perimeter* is given in *linear* yards.

(c) *Volume* is given in *cubic* meters.

3. You would need to be given 4 values in a formula with 5 variables to find the value of any one variable.

4. Using the rule for multiplication of fractions and writing C as $\dfrac{C}{1}$, we see that

$$\frac{9}{5}C = \frac{9}{5}\cdot\frac{C}{1} = \frac{9C}{5}.$$

5. Carpeting for a bedroom covers the surface of the bedroom floor, so *area* would be used.

6. Sod for a lawn covers the surface of the lawn, so *area* would be used.

7. To measure fencing for a yard, use *perimeter* since you would need to measure the lengths of the sides of the yard.

8. The baseboards of a living room go around the edges of the room. The amount of baseboard needed will be the sum of the lengths of the sides of the room, so *perimeter* would be used.

9. Tile for a bathroom covers the surface of the bathroom floor, so *area* would be used.

10. Fertilizer for a garden covers the surface of the garden, so *area* would be used.

11. To determine the cost for replacing a linoleum floor with a wood floor, use *area* since you need to know the measure of the surface covered by the wood.

12. Determining the cost for planting rye grass in a lawn for the winter requires finding the amount of surface to be covered, so *area* would be used.

In Exercises 13–32, substitute the given values into the formula and then solve for the remaining variable.

13. $P = 2L + 2W;\ L = 8,\ W = 5$

$$\begin{aligned} P &= 2L + 2W \\ &= 2(8) + 2(5) \\ &= 16 + 10 \\ P &= 26 \end{aligned}$$

14. $P = 2L + 2W;\ L = 6,\ W = 4$

$$\begin{aligned} P &= 2L + 2W \\ &= 2(6) + 2(4) \\ &= 12 + 8 \\ P &= 20 \end{aligned}$$

15. $A = \frac{1}{2}bh;\ b = 8, h = 16$

$$\begin{aligned} A &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}(8)(16) \\ A &= 64 \end{aligned}$$

16. $A = \frac{1}{2}bh;\ b = 10,\ h = 14$

$$\begin{aligned} A &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}(10)(14) \\ &= (5)(14) \\ A &= 70 \end{aligned}$$

17. $P = a + b + c;\ P = 12, a = 3, c = 5$

$$\begin{aligned} P &= a + b + c \\ 12 &= 3 + b + 5 \\ 12 &= b + 8 \\ 4 &= b \end{aligned}$$

18. $P = a + b + c;\ P = 15, a = 3, b = 7$

$$\begin{aligned} P &= a + b + c \\ 15 &= 3 + 7 + c \\ 15 &= 10 + c \\ 5 &= c \end{aligned}$$

19. $d = rt;\ d = 252,\ r = 45$

$$\begin{aligned} d &= rt \\ 252 &= 45t \\ \frac{252}{45} &= \frac{45t}{45} \\ 5.6 &= t \end{aligned}$$

20. $d = rt;\ d = 100,\ t = 2.5$

$$\begin{aligned} d &= rt \\ 100 &= r(2.5) \\ 100 &= 2.5r \\ \frac{100}{2.5} &= \frac{2.5r}{2.5} \\ 40 &= r \end{aligned}$$

21. $I = prt;\ p = 7500,\ r = 0.035,\ t = 6$

$$\begin{aligned} I &= prt \\ &= (7500)(0.035)(6) \\ I &= 1575 \end{aligned}$$

22. $I = prt;\ p = 5000,\ r = 0.025,\ t = 7$

$$\begin{aligned} I &= prt \\ &= (5000)(0.025)(7) \\ I &= 875 \end{aligned}$$

23. $A = \frac{1}{2}h(b + B);\ A = 91, h = 7,\ b = 12$

$$\begin{aligned} A &= \tfrac{1}{2}h(b + B) \\ 91 &= \tfrac{1}{2}(7)(12 + B) \\ 182 &= (7)(12 + B) \\ 12 + B &= \tfrac{1}{7}(182) \\ B &= 26 - 12 = 14 \end{aligned}$$

24. $A = \frac{1}{2}h(b+B);\ A = 75, b = 19, B = 31$

$$\begin{aligned} A &= \tfrac{1}{2}h(b+B) \\ 75 &= \tfrac{1}{2}h(19+31) \\ 150 &= h(50) \\ h &= \tfrac{150}{50} = 3 \end{aligned}$$

25. $C = 2\pi r;\ C = 16.328,\ \pi = 3.14$

$$\begin{aligned} C &= 2\pi r \\ 16.328 &= 2(3.14)r \\ 16.328 &= 6.28r \\ 2.6 &= r \end{aligned}$$

26. $C = 2\pi r;\ C = 8.164,\ \pi = 3.14$

$$\begin{aligned} C &= 2\pi r \\ 8.164 &= 2(3.14)r \\ 8.164 &= 6.28r \\ 1.3 &= r \end{aligned}$$

27. $C = 2\pi r;\ C = 20\pi$

$$\begin{aligned} C &= 2\pi r \\ 20\pi &= 2\pi r \\ 10 &= r \qquad \textit{Divide by } 2\pi. \end{aligned}$$

28. $C = 2\pi r;\ C = 100\pi$

$$\begin{aligned} C &= 2\pi r \\ 100\pi &= 2\pi r \\ 50 &= r \qquad \textit{Divide by } 2\pi. \end{aligned}$$

29. $A = \pi r^2;\ r = 4,\ \pi = 3.14$

$$\begin{aligned} A &= \pi r^2 \\ &= 3.14(4)^2 \\ &= 3.14(16) \\ A &= 50.24 \end{aligned}$$

30. $A = \pi r^2;\ r = 12,\ \pi = 3.14$

$$\begin{aligned} A &= \pi r^2 \\ &= 3.14(12)^2 \\ &= 3.14(144) \\ A &= 452.16 \end{aligned}$$

31. $S = 2\pi rh;\ S = 120\pi, h = 10$

$$\begin{aligned} S &= 2\pi rh \\ 120\pi &= 2\pi r(10) \\ 120\pi &= 20\pi r \\ 6 &= r \qquad \textit{Divide by } 20\pi. \end{aligned}$$

32. $S = 2\pi rh;\ S = 720\pi, h = 30$

$$\begin{aligned} S &= 2\pi rh \\ 720\pi &= 2\pi r(30) \\ 720\pi &= 60\pi r \\ 12 &= r \qquad \textit{Divide by } 60\pi. \end{aligned}$$

In Exercises 33–38, substitute the given values into the formula and then solve for V.

33. $V = LWH;\ L = 10, W = 5, H = 3$

$$\begin{aligned} V &= LWH \\ &= (10)(5)(3) \\ V &= 150 \end{aligned}$$

34. $V = LWH;\ L = 12,\ W = 8,\ H = 4$

$$\begin{aligned} V &= LWH \\ &= (12)(8)(4) \\ V &= 384 \end{aligned}$$

35. $V = \frac{1}{3}Bh;\ B = 12,\ h = 13$

$$\begin{aligned} V &= \tfrac{1}{3}Bh \\ &= \tfrac{1}{3}(12)(13) \\ V &= 52 \end{aligned}$$

36. $V = \frac{1}{3}Bh;\ B = 36,\ h = 4$

$$\begin{aligned} V &= \tfrac{1}{3}Bh \\ &= \tfrac{1}{3}(36)(4) \\ V &= 48 \end{aligned}$$

37. $V = \frac{4}{3}\pi r^3;\ r = 12,\ \pi = 3.14$

$$\begin{aligned} V &= \tfrac{4}{3}\pi r^3 \\ &= \tfrac{4}{3}(3.14)(12)^3 \\ &= \tfrac{4}{3}(3.14)(1728) \\ V &= 7234.56 \end{aligned}$$

38. $V = \frac{4}{3}\pi r^3;\ r = 6,\ \pi = 3.14$

$$\begin{aligned} V &= \tfrac{4}{3}\pi r^3 \\ &= \tfrac{4}{3}(3.14)(6)^3 \\ &= \tfrac{4}{3}(3.14)(216) \\ V &= 904.32 \end{aligned}$$

39.

$$\begin{aligned} P &= 2L + 2W \\ 54 &= 2(W+9) + 2W \qquad \textit{Let } L = W + 9. \\ 54 &= 2W + 18 + 2W \\ 54 &= 4W + 18 \\ 36 &= 4W \\ 9 &= W \end{aligned}$$

The width is 9 inches and the length is $9 + 9 = 18$ inches.

40.

$$\begin{aligned} P &= 2L + 2W \\ 62 &= 2L + 2(L-3) \qquad \textit{Let } W = L - 3. \\ 62 &= 2L + 2L - 6 \\ 62 &= 4L - 6 \\ 68 &= 4L \\ 17 &= L \end{aligned}$$

The length is 17 feet and the width is $17 - 3 = 14$ feet.

41.
$$\begin{aligned} P &= 2l + 2w \\ 36 &= 2(3w+2) + 2w \quad \textit{Let } l = 3w + 2. \\ 36 &= 6w + 4 + 2w \\ 36 &= 8w + 4 \\ 32 &= 8w \\ 4 &= w \end{aligned}$$

The width is 4 meters and the length is $3(4) + 2 = 14$ meters.

42.
$$\begin{aligned} P &= 2l + 2w \\ 36 &= 2l + 2(2l - 18) \quad \textit{Let } w = 2l - 18. \\ 36 &= 2l + 4l - 36 \\ 36 &= 6l - 36 \\ 72 &= 6l \\ 12 &= l \end{aligned}$$

The length is 12 yards and the width is $2(12) - 18 = 6$ yards.

43. Let $s =$ the length of the shortest side, in inches;
$s + 2 =$ the length of the medium side, and,
$s + 3 =$ the length of the longest side.

The perimeter is 20 inches, so

$$\begin{aligned} s + (s+2) + (s+3) &= 20. \\ 3s + 5 &= 20 \\ 3s &= 15 \\ s &= 5 \end{aligned}$$

Since $s = 5$, $s + 2 = 7$, and $s + 3 = 8$. The lengths of the sides are 5, 7, and 8 inches. The perimeter is $5 + 7 + 8 = 20$, as required.

44. Let $s =$ the length of the shortest side, in feet.
$s + 4 =$ the length of the medium side, and,
$2s =$ the length of the longest side.

The perimeter is 28 feet, so

$$\begin{aligned} s + (s+4) + (2s) &= 28. \\ 4s + 4 &= 28 \\ 4s &= 24 \\ s &= 6 \end{aligned}$$

Since $s = 6$, $s + 4 = 10$, and $2s = 12$. The lengths of the sides are 6, 10, and 12 feet. The perimeter is $6 + 10 + 12 = 28$, as required.

45. Let $s =$ the length of the two sides that have equal length, in meters; and $2s - 4 =$ the length of the third side.

The perimeter is 24 meters, so

$$\begin{aligned} s + s + (2s - 4) &= 24. \\ 4s - 4 &= 24 \\ 4s &= 28 \\ s &= 7 \end{aligned}$$

Since $s = 7$, $2s - 4 = 10$. The lengths of the sides are 7, 7, and 10 meters. The perimeter is $7 + 7 + 10 = 24$, as required.

46. Let $s =$ the length of the shortest side, in yards;
$2s =$ the length of the medium side, and,
$3s - 7 =$ the length of the longest side.

The perimeter is 47 yards, so

$$\begin{aligned} s + 2s + (3s - 7) &= 47. \\ 6s - 7 &= 47 \\ 6s &= 54 \\ s &= 9 \end{aligned}$$

Since $s = 9$, $2s = 18$, and $3s - 7 = 20$. The lengths of the sides are 9, 18, and 20 yards. The perimeter is $9 + 18 + 20 = 47$, as required.

47. The diameter of the circle is 443 feet, so its radius is $\frac{443}{2} = 221.5$ ft. Use the area of a circle formula to find the enclosed area.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(221.5)^2 \\ &\approx 154{,}133.6 \text{ ft}^2, \end{aligned}$$

or about 154,000 ft^2. (If 3.14 is used for π, the value is 154,055.465.)

48. The diameter of the circular dome is 630 feet, so its radius is $\frac{630}{2} = 315$ ft. Use the circumference of a circle formula.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(315) \\ &\approx 1979.2 \text{ ft} \end{aligned}$$

If we use 3.14 for π, the answer is 1978.2 ft, or about 1978 ft.

49. The page is a rectangle with length 1.5 m and width 1.2 m, so use the formulas for the perimeter and area of a rectangle.

$$\begin{aligned} P &= 2L + 2W \\ &= 2(1.5) + 2(1.2) \\ &= 3 + 2.4 \\ P &= 5.4 \text{ meters} \end{aligned}$$

$$\begin{aligned} A &= LW \\ &= (1.5)(1.2) \\ A &= 1.8 \text{ square meters} \end{aligned}$$

50. Since the sand painting is a square with side 12.24 m, use the formulas for the perimeter and area of a square.

$$\begin{aligned} P &= 4s \\ &= 4(12.24) \\ P &= 48.96 \text{ meters} \end{aligned}$$

$$\begin{aligned} A &= s^2 \\ &= (12.24)^2 \\ A &= 149.8176 \text{ square meters} \end{aligned}$$

To the nearest hundredth of a square meter, the area is 149.82 square meters.

51. Use the formula for the area of a triangle with $A = 70$ and $b = 14$.

$$\begin{aligned} A &= \tfrac{1}{2}bh \\ 70 &= \tfrac{1}{2}(14)h \\ 70 &= 7h \\ 10 &= h \end{aligned}$$

The height of the sign is 10 feet.

52. To find the area of the drum face, use the formula for the area of a circle, $A = \pi r^2$. Since the diameter of the circle is 13 feet, the radius is $\left(\frac{1}{2}\right)(13) = 6.5$ feet.

$$\begin{aligned} A &= \pi r^2 \\ &\approx (3.14)(6.5)^2 \\ &= (3.14)(42.25) \\ A &= 132.665 \end{aligned}$$

The area of the drum face is about 132.665 square feet.

53. Use the formula for the area of a trapezoid with $B = 115.80,\ b = 171.00,$ and $h = 165.97.$

$$\begin{aligned} A &= \tfrac{1}{2}(B + b)h \\ &= \tfrac{1}{2}(115.80 + 171.00)(165.97) \\ &= \tfrac{1}{2}(286.80)(165.97) \\ &= 23{,}800.098 \end{aligned}$$

To the nearest hundredth of a square foot, the combined area of the two lots is 23,800.10 square feet.

54. Let $A =$ the area of Lot A. Use the formula for the area of a trapezoid with $B = 82.05,\ b = 26.84,$ and $h = 165.97.$

$$\begin{aligned} A &= \tfrac{1}{2}(B + b)h \\ &= \tfrac{1}{2}(82.05 + 26.84)(165.97) \\ &= 9036.23665 \end{aligned}$$

To the nearest hundredth of a square foot, the area is 9036.24 square feet.

55. The girth is $4 \cdot 18 = 72$ inches. Since the length plus the girth is 108, we have

$$\begin{aligned} L + G &= 108 \\ L + 72 &= 108 \\ L &= 36 \text{ in.} \end{aligned}$$

The volume of the box is

$$\begin{aligned} V &= LWH \\ &= (36)(18)(18) \\ &= 11{,}664 \text{ in.}^3 \end{aligned}$$

56. To find the volume of the sandwich, use the formula for the volume of a rectangular solid.

$$\begin{aligned} V &= LWH \\ &= (12)(12)\left(1\tfrac{11}{24}\right) \\ &= \frac{\overset{6}{\cancel{12}}}{1} \cdot \frac{\overset{1}{\cancel{12}}}{1} \cdot \frac{35}{\underset{\underset{1}{\cancel{2}}}{\cancel{24}}} \\ &= 210 \end{aligned}$$

The volume of the sandwich was 210 cubic feet.

57. The two angles are supplementary, so the sum of their measures is 180°.

$$\begin{aligned} (x + 1) + (4x - 56) &= 180 \\ 5x - 55 &= 180 \\ 5x &= 235 \\ x &= 47 \end{aligned}$$

Since $x = 47$, $x + 1 = 47 + 1 = 48$, and $4x - 56 = 4(47) - 56 = 132$.

The measures of the angles are 48° and 132°.

58. In the figure, the two angles are supplementary, so their sum is 180°.

$$\begin{aligned} (10x + 7) + (7x + 3) &= 180 \\ 17x + 10 &= 180 \\ 17x &= 170 \\ x &= 10 \end{aligned}$$

Since $x = 10$, $10x + 7 = 10(10) + 7 = 107$, and $7x + 3 = 7(10) + 3 = 73$.

The two angle measures are 107° and 73°.

59. The two angles are vertical angles, which have equal measures. Set their measures equal to each other and solve for x.

$$\begin{aligned} 5x - 129 &= 2x - 21 \\ 3x - 129 &= -21 \\ 3x &= 108 \\ x &= 36 \end{aligned}$$

Since $x = 36$, $5x - 129 = 5(36) - 129 = 51$, and $2x - 21 = 2(36) - 21 = 51$.

The measure of each angle is 51°.

60. The two angles are vertical angles, which have equal measures. Set their measures equal to each other and solve for x.

$$\begin{aligned} 7x+5 &= 3x+45 \\ 4x+5 &= 45 \\ 4x &= 40 \\ x &= 10 \end{aligned}$$

The measure of the first angle is $7(10)+5=75°$; the measure of the second angle is $3(10)+45$, which is also $75°$.

61. The angles are vertical angles, so their measures are equal.

$$\begin{aligned} 12x-3 &= 10x+15 \\ 2x-3 &= 15 \\ 2x &= 18 \\ x &= 9 \end{aligned}$$

Since $x=9$, $12x-3=12(9)-3=105$, and $10x+15=10(9)+15=105$.

The measure of each angle is $105°$.

62. The angles are vertical angles, which have equal measures. Set $11x-37$ equal to $7x+27$ and solve.

$$\begin{aligned} 11x-37 &= 7x+27 \\ 4x-37 &= 27 \\ 4x &= 64 \\ x &= 16 \end{aligned}$$

Since $x=16$, $11x-37=11(16)-37=139$, and $7x+27=7(16)+27=139$.

The angles both measure $139°$.

63. $d=rt$ for t

$$\begin{aligned} d &= rt \\ \frac{d}{r} &= \frac{rt}{r} \quad \textit{Divide by r.} \\ \frac{d}{r} &= t \quad \text{or} \quad t=\frac{d}{r} \end{aligned}$$

64. $d=rt$ for r

$$\begin{aligned} d &= rt \\ \frac{d}{t} &= \frac{rt}{t} \quad \textit{Divide by t.} \\ \frac{d}{t} &= r \quad \text{or} \quad r=\frac{d}{t} \end{aligned}$$

65. $A=bh$ for b

$$\begin{aligned} A &= bh \\ \frac{A}{h} &= \frac{bh}{h} \quad \textit{Divide by h.} \\ \frac{A}{h} &= b \quad \text{or} \quad b=\frac{A}{h} \end{aligned}$$

66. $A=LW$ for L

$$\begin{aligned} A &= LW \\ \frac{A}{W} &= \frac{LW}{W} \quad \textit{Divide by W.} \\ \frac{A}{W} &= L \quad \text{or} \quad L=\frac{A}{W} \end{aligned}$$

67. $C=\pi d$ for d

$$\begin{aligned} C &= \pi d \\ \frac{C}{\pi} &= \frac{\pi d}{\pi} \quad \textit{Divide by } \pi. \\ \frac{C}{\pi} &= d \quad \text{or} \quad d=\frac{C}{\pi} \end{aligned}$$

68. $P=4s$ for s

$$\begin{aligned} P &= 4s \\ \frac{P}{4} &= \frac{4s}{4} \quad \textit{Divide by 4.} \\ \frac{P}{4} &= s \quad \text{or} \quad s=\frac{P}{4} \end{aligned}$$

69. $V=LWH$ for H

$$\begin{aligned} V &= LWH \\ \frac{V}{LW} &= \frac{LWH}{LW} \quad \textit{Divide by LW.} \\ \frac{V}{LW} &= H \quad \text{or} \quad H=\frac{V}{LW} \end{aligned}$$

70. $V=LWH$ for W

$$\begin{aligned} V &= LWH \\ \frac{V}{LH} &= \frac{LWH}{LH} \quad \textit{Divide by LH.} \\ \frac{V}{LH} &= W \quad \text{or} \quad W=\frac{V}{LH} \end{aligned}$$

71. $I=prt$ for r

$$\begin{aligned} I &= prt \\ \frac{I}{pt} &= \frac{prt}{pt} \quad \textit{Divide by pt.} \\ \frac{I}{pt} &= r \quad \text{or} \quad r=\frac{I}{pt} \end{aligned}$$

72. $I=prt$ for p

$$\begin{aligned} I &= prt \\ \frac{I}{rt} &= \frac{prt}{rt} \quad \textit{Divide by rt.} \\ \frac{I}{rt} &= p \quad \text{or} \quad p=\frac{I}{rt} \end{aligned}$$

73. $A=\frac{1}{2}bh$ for h

$$\begin{aligned} 2A &= 2\left(\tfrac{1}{2}bh\right) \quad \textit{Multiply by 2.} \\ 2A &= bh \\ \frac{2A}{b} &= \frac{bh}{b} \quad \textit{Divide by b.} \\ \frac{2A}{b} &= h \quad \text{or} \quad h=\frac{2A}{b} \end{aligned}$$

74. $A = \frac{1}{2}bh$ for b

$$2A = 2\left(\tfrac{1}{2}bh\right) \quad \textit{Multiply by 2.}$$
$$2A = bh$$
$$\frac{2A}{h} = \frac{bh}{h} \quad \textit{Divide by h.}$$
$$\frac{2A}{h} = b \quad \text{or} \quad b = \frac{2A}{h}$$

75. $V = \frac{1}{3}\pi r^2 h$ for h

$$3V = 3\left(\tfrac{1}{3}\right)\pi r^2 h \quad \textit{Multiply by 3.}$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \quad \textit{Divide by } \pi r^2.$$
$$\frac{3V}{\pi r^2} = h \quad \text{or} \quad h = \frac{3V}{\pi r^2}$$

76. $V = \pi r^2 h$ for h

$$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \quad \textit{Divide by } \pi r^2.$$
$$\frac{V}{\pi r^2} = h \quad \text{or} \quad h = \frac{V}{\pi r^2}$$

77. $P = a + b + c$ for b

$$P - a - c = a + b + c - a - c$$
Subtract a and c.
$$P - a - c = b \quad \text{or} \quad b = P - a - c$$

78. $P = a + b + c$ for a

$$P - b - c = a + b + c - b - c$$
Subtract b and c.
$$P - b - c = a \quad \text{or} \quad a = P - b - c$$

79. $P = 2L + 2W$ for W

$$P - 2L = 2L + 2W - 2L \quad \textit{Subtract 2L.}$$
$$P - 2L = 2W$$
$$\frac{P - 2L}{2} = \frac{2W}{2} \quad \textit{Divide by 2.}$$
$$\frac{P - 2L}{2} = W, \quad \text{or} \quad W = \frac{P - 2L}{2}$$

80. $A = p + prt$ for r

$$A - p = p + prt - p \quad \textit{Subtract p.}$$
$$A - p = prt$$
$$\frac{A - p}{pt} = \frac{prt}{pt} \quad \textit{Divide by pt.}$$
$$\frac{A - p}{pt} = r, \quad \text{or} \quad r = \frac{A - p}{pt}$$

81. $y = mx + b$ for m

$$y - b = mx + b - b \quad \textit{Subtract b.}$$
$$y - b = mx$$
$$\frac{y - b}{x} = \frac{mx}{x} \quad \textit{Divide by x.}$$
$$\frac{y - b}{x} = m \quad \text{or} \quad m = \frac{y - b}{x}$$

82. $y = mx + b$ for x

$$y - b = mx + b - b \quad \textit{Subtract b.}$$
$$y - b = mx$$
$$\frac{y - b}{m} = \frac{mx}{m} \quad \textit{Divide by m.}$$
$$\frac{y - b}{m} = x \quad \text{or} \quad x = \frac{y - b}{m}$$

83. $Ax + By = C$ for y

$$By = C - Ax \quad \textit{Subtract Ax.}$$
$$\frac{By}{B} = \frac{C - Ax}{B} \quad \textit{Divide by B.}$$
$$y = \frac{C - Ax}{B}$$

84. $Ax + By = C$ for x

$$Ax = C - By \quad \textit{Subtract By.}$$
$$\frac{Ax}{A} = \frac{C - By}{A} \quad \textit{Divide by A.}$$
$$x = \frac{C - By}{A}$$

85. $M = C(1 + r)$ for r

$$M = C + Cr \quad \textit{Distributive Property}$$
$$M - C = Cr \quad \textit{Subtract C.}$$
$$\frac{M - C}{C} = \frac{Cr}{C} \quad \textit{Divide by C.}$$
$$\frac{M - C}{C} = r, \quad \text{or} \quad r = \frac{M - C}{C}$$

Alternative solution:

$$M = C(1 + r)$$
$$\frac{M}{C} = 1 + r \quad \textit{Divide by C.}$$
$$\frac{M}{C} - 1 = r \quad \textit{Subtract 1.}$$

86. $C = \frac{5}{9}(F - 32)$ for F

$$C = \tfrac{5}{9}(F - 32)$$
$$\tfrac{9}{5}C = \tfrac{9}{5} \cdot \tfrac{5}{9}(F - 32)$$
$$\tfrac{9}{5}C = F - 32$$
$$\tfrac{9}{5}C + 32 = F - 32 + 32$$
$$\tfrac{9}{5}C + 32 = F$$

87.
$$\frac{x}{12} = \frac{12}{72}$$
$$12\left(\tfrac{1}{12}x\right) = 12\left(\tfrac{1}{6}\right) \quad \textit{Multiply by 12.}$$
$$x = 2$$

The solution set is $\{2\}$.

88.
$$\frac{x}{15} = \frac{144}{60}$$
$$15\left(\tfrac{1}{15}x\right) = 15\left(\tfrac{12}{5}\right) \quad \textit{Multiply by 15.}$$
$$x = 36$$

The solution set is $\{36\}$.

89.
$$0.06x = 300$$
$$x = \tfrac{300}{0.06} = 5000$$

The solution set is $\{5000\}$.

90.
$$0.4x = 80$$
$$x = \tfrac{80}{0.4} = 200$$

The solution set is $\{200\}$.

91.
$$\tfrac{3}{4}x = 21$$
$$\tfrac{4}{3}\left(\tfrac{3}{4}x\right) = \tfrac{4}{3}(21)$$
$$x = 28$$

The solution set is $\{28\}$.

92.
$$-\tfrac{5}{6}x = 30$$
$$-\tfrac{6}{5}\left(-\tfrac{5}{6}x\right) = -\tfrac{6}{5}(30)$$
$$x = -36$$

The solution set is $\{-36\}$.

93.
$$-3x = \tfrac{1}{4}$$
$$\frac{-3x}{-3} = \frac{\frac{1}{4}}{-3}$$
$$x = \tfrac{1}{4}\left(-\tfrac{1}{3}\right) = -\tfrac{1}{12}$$

The solution set is $\left\{-\tfrac{1}{12}\right\}$.

94.
$$4x = \tfrac{1}{3}$$
$$\frac{4x}{4} = \frac{\frac{1}{3}}{4}$$
$$x = \tfrac{1}{3} \cdot \tfrac{1}{4} = \tfrac{1}{12}$$

The solution set is $\left\{\tfrac{1}{12}\right\}$.

2.6 Ratios and Proportions

2.6 Classroom Examples

1. **(a)** To find the ratio of 3 days to 2 weeks, first convert 2 weeks to days.

$$2 \text{ weeks} = 2 \cdot 7 = 14 \text{ days}$$

The ratio of 3 days to 2 weeks is then

$$\frac{3 \text{ days}}{2 \text{ weeks}} = \frac{3 \text{ days}}{14 \text{ days}} = \frac{3}{14}.$$

(b) 4 days $= 4 \cdot 24 = 96$ hours
The ratio of 12 hr to 4 days is then

$$\frac{12 \text{ hr}}{4 \text{ days}} = \frac{12 \text{ hr}}{96 \text{ hr}} = \frac{12}{96} = \frac{1}{8}.$$

2. The results in the following table are rounded to the nearest thousandth.

Size	**Unit Cost** (dollars per oz)
36 oz	$\frac{\$3.89}{36} = \$0.108$ $(*)$
24 oz	$\frac{\$2.79}{24} = \$0.116$
12 oz	$\frac{\$1.89}{12} = \$0.158$

Because the 36 oz size produces the lowest unit cost, it is the best buy. The unit cost, to the nearest thousandth, is \$0.108 per oz.

3. **(a)** $\dfrac{21}{15} = \dfrac{62}{45}$

Compare the cross products.

$$21 \cdot 45 = 945$$
$$15 \cdot 62 = 930$$

The cross products are *different*, so the proportion is *false*.

(b) $\dfrac{13}{17} = \dfrac{91}{119}$

Check to see whether the cross products are equal.

$$13 \cdot 119 = 1547$$
$$17 \cdot 91 = 1547$$

The cross products are *equal*, so the proportion is *true*.

4.
$$\frac{x}{6} = \frac{35}{42}$$
$$42x = 6 \cdot 35 \quad \textit{Cross products}$$
$$x = \frac{6 \cdot 35}{42} \quad \textit{Divide by 42.}$$
$$= \frac{6 \cdot 5 \cdot 7}{6 \cdot 7} \quad \textit{Factor.}$$
$$= 5 \quad \textit{Cancel.}$$

The solution set is $\{5\}$.

Note: We could have multiplied $6 \cdot 35$ to get 210 and then divided 210 by 42 to get 5. This may be the best approach if you are doing these calculations on a calculator. The factor and cancel method is preferable if you're not using a calculator.

5.
$$\frac{a+6}{2} = \frac{2}{5}$$
$$5(a+6) = 2(2) \quad \textit{Cross products}$$
$$5a + 30 = 4 \quad \textit{Distributive property}$$
$$5a = -26 \quad \textit{Subtract 30.}$$
$$a = -\tfrac{26}{5} \quad \textit{Divide by 5.}$$

The solution set is $\left\{-\tfrac{26}{5}\right\}$.

6. Let $x =$ the cost for 16.5 gallons.

$$\frac{\$37.68}{12} = \frac{x}{16.5}$$

$12x = 16.5(37.68)$ *Cross products*

$12x = 621.72$ *Multiply.*

$x = 51.81$ *Divide by 12.*

It would cost \$51.81.

2.6 Section Exercises

1. **(a)** 75 to 100 is $\frac{75}{100} = \frac{3}{4}$ or 3 to 4.
The answer is **C**.

(b) 5 to 4 or $\frac{5}{4} = \frac{5 \cdot 3}{4 \cdot 3} = \frac{15}{12}$ or 15 to 12.
The answer is **D**.

(c) $\frac{1}{2} = \frac{1 \cdot 50}{2 \cdot 50} = \frac{50}{100}$ or 50 to 100
The answer is **B**.

(d) 4 to 5 or $\frac{4}{5} = \frac{4 \cdot 20}{5 \cdot 20} = \frac{80}{100}$ or 80 to 100.
The answer is **A**.

2. The ratio 4 to 3 can be expressed as an equivalent ratio by multiplying 4 and 3 by the same nonzero number. For example, multiplying both numbers by 2, 3, and 10 gives the ratios $\frac{8}{6}$, $\frac{12}{9}$, and $\frac{40}{30}$.

3. The ratio of 40 miles to 30 miles is

$$\frac{40 \text{ miles}}{30 \text{ miles}} = \frac{40}{30} = \frac{4}{3}.$$

4. The ratio of 60 feet to 70 feet is

$$\frac{60 \text{ feet}}{70 \text{ feet}} = \frac{60}{70} = \frac{6}{7}.$$

5. The ratio of 120 people to 90 people is

$$\frac{120 \text{ people}}{90 \text{ people}} = \frac{4 \cdot 30}{3 \cdot 30} = \frac{4}{3}.$$

6. The ratio of 72 dollars to 220 dollars is

$$\frac{72 \text{ dollars}}{220 \text{ dollars}} = \frac{72}{220} = \frac{18 \cdot 4}{55 \cdot 4} = \frac{18}{55}.$$

7. To find the ratio of 20 yards to 8 feet, first convert 20 yards to feet.

$$20 \text{ yards} = 20 \text{ yards} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = 60 \text{ feet}$$

The ratio of 20 yards to 8 feet is then

$$\frac{60 \text{ feet}}{8 \text{ feet}} = \frac{60}{8} = \frac{15 \cdot 4}{2 \cdot 4} = \frac{15}{2}.$$

8. First convert 8 feet to inches.

$$8 \text{ feet} = 8 \cdot 12 = 96 \text{ inches}$$

The ratio of 30 inches to 8 feet is then

$$\frac{30 \text{ inches}}{96 \text{ inches}} = \frac{30}{96} = \frac{5 \cdot 6}{16 \cdot 6} = \frac{5}{16}.$$

9. Convert 2 hours to minutes.

$$2 \text{ hours} = 2 \text{ hours} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = 120 \text{ minutes}$$

The ratio of 24 minutes to 2 hours is then

$$\frac{24 \text{ minutes}}{120 \text{ minutes}} = \frac{24}{120} = \frac{1 \cdot 24}{5 \cdot 24} = \frac{1}{5}.$$

10. To find the ratio of 16 minutes to 1 hour, first convert 1 hour to minutes.

$$1 \text{ hour} = 60 \text{ minutes}$$

The ratio of 16 minutes to 1 hour is then

$$\frac{16 \text{ minutes}}{60 \text{ minutes}} = \frac{16}{60} = \frac{4 \cdot 4}{15 \cdot 4} = \frac{4}{15}.$$

11. 2 yards $= 2 \cdot 3 = 6$ feet
6 feet $= 6 \cdot 12 = 72$ inches
The ratio of 60 inches to 2 yards is then

$$\frac{60 \text{ inches}}{72 \text{ inches}} = \frac{5 \cdot 12}{6 \cdot 12} = \frac{5}{6}.$$

12. 5 days $= 5 \cdot 24 = 120$ hours
The ratio of 5 days to 40 hours is then

$$\frac{120 \text{ hours}}{40 \text{ hours}} = \frac{3 \cdot 40}{1 \cdot 40} = \frac{3}{1}, \text{ or } 3.$$

In Exercises 13–21, to find the best buy, divide the price by the number of units to get the unit cost. Each result was found by using a calculator and rounding the answer to three decimal places. The *best buy* (based on price per unit) is the smallest unit cost.

13.

Size	**Unit Cost** (dollars per lb)
4 lb	$\frac{\$1.78}{4} = \$0.445$
10 lb	$\frac{\$4.39}{10} = \$0.439$ (*)

The 10 lb size is the best buy.

14.

Size	**Unit Cost** (dollars per oz)
13 oz	$\frac{\$2.58}{13} = \$0.198$
39 oz	$\frac{\$4.44}{39} = \$0.114$ (*)

The 39 oz size is the best buy.

15.

Size	**Unit Cost** (dollars per oz)
16 oz	$\frac{\$2.44}{16} = \$0.153$
32 oz	$\frac{\$2.98}{32} = \$0.093$ (*)
48 oz	$\frac{\$4.95}{48} = \$0.103$

The 32 oz size is the best buy.

16.

Size	**Unit Cost** (dollars per oz)
2 oz	$\frac{\$1.79}{2} = \$0.895$
4 oz	$\frac{\$2.59}{4} = \$0.648$ (*)
8 oz	$\frac{\$5.59}{8} = \$0.699$

The 4 oz size is the best buy.

17.

Size	**Unit Cost** (dollars per oz)
16 oz	$\frac{\$1.54}{16} = \$0.096$
24 oz	$\frac{\$2.08}{24} = \$0.087$
64 oz	$\frac{\$3.63}{64} = \$0.057$
128 oz	$\frac{\$5.65}{128} = \$0.044$ (*)

The 128 oz size is the best buy.

18.

Size	**Unit Cost** (dollars per oz)
8.5 oz	$\frac{\$0.99}{8.5} = \$0.116$
16.9 oz	$\frac{\$1.87}{16.9} = \$0.111$
33.8 oz	$\frac{\$2.49}{33.8} = \$0.074$
50.7 oz	$\frac{\$2.99}{50.7} = \$0.059$ (*)

The 50.7 oz size is the best buy.

19.

Size	**Unit Cost** (dollars per oz)
14 oz	$\frac{\$1.39}{14} = \$0.099$
24 oz	$\frac{\$1.55}{24} = \$0.065$
36 oz	$\frac{\$1.78}{36} = \$0.049$ (*)
64 oz	$\frac{\$3.99}{64} = \$0.062$

The 36 oz size is the best buy.

20.

Size	**Unit Cost** (dollars per oz)
12 oz	$\frac{\$1.05}{12} = \$0.088$
18 oz	$\frac{\$1.73}{18} = \$0.096$
32 oz	$\frac{\$1.84}{32} = \$0.058$ (*)
48 oz	$\frac{\$2.88}{48} = \$0.060$

The 32 oz size is the best buy.

21.

Size	**Unit Cost** (dollars per oz)
87 oz	$\frac{\$7.88}{87} = \$0.091$
131 oz	$\frac{\$10.98}{131} = \$0.084$
263 oz	$\frac{\$19.96}{263} = \$0.076$ (*)

The 263 oz size is the best buy.

22. A ratio is a comparison, while a proportion is a statement that two ratios are equal. For example, $\frac{2}{3}$ is a ratio and $\frac{2}{3} = \frac{8}{12}$ is a proportion.

23. $\frac{5}{35} = \frac{8}{56}$

Check to see whether the cross products are equal.

$$5 \cdot 56 = 280$$
$$35 \cdot 8 = 280$$

The cross products are *equal*, so the proportion is *true*.

24. $\frac{4}{12} = \frac{7}{21}$

Check to see whether the cross products are equal.

$$4 \cdot 21 = 84$$
$$12 \cdot 7 = 84$$

The cross products are *equal*, so the proportion is *true*.

25. $\frac{120}{82} = \frac{7}{10}$

Compare the cross products.

$$120 \cdot 10 = 1200$$
$$82 \cdot 7 = 574$$

The cross products are *different*, so the proportion is *false*.

26. $\frac{27}{160} = \frac{18}{110}$

Compare the cross products.

$$27 \cdot 110 = 2970$$
$$160 \cdot 18 = 2880$$

The cross products are *different*, so the proportion is *false*.

27. $\frac{\frac{1}{2}}{5} = \frac{1}{10}$

Compare the cross products.

$$\tfrac{1}{2} \cdot 10 = 5$$
$$5 \cdot 1 = 5$$

The cross products are *equal*, so the proportion is *true*.

28. $\frac{\frac{1}{3}}{6} = \frac{1}{18}$

Compare the cross products.

$$\tfrac{1}{3} \cdot 18 = 6$$
$$6 \cdot 1 = 6$$

The cross products are *equal*, so the proportion is *true*.

29. $\frac{k}{4} = \frac{175}{20}$

$20k = 4(175)$ *Cross products are equal.*

$20k = 700$

$\frac{20k}{20} = \frac{700}{20}$ *Divide by 20.*

$k = 35$

The solution set is $\{35\}$.

30. $\frac{x}{6} = \frac{18}{4}$

$x \cdot 4 = 6 \cdot 18$ *Cross products are equal.*

$4x = 108$

$\frac{4x}{4} = \frac{108}{4}$ *Divide by 4.*

$x = 27$

The solution set is $\{27\}$.

31. $\frac{49}{56} = \frac{z}{8}$

$56z = 49(8)$ *Cross products are equal.*

$56z = 392$

$\frac{56z}{56} = \frac{392}{56}$ *Divide by 56.*

$z = 7$

The solution set is $\{7\}$.

32. $\frac{20}{100} = \frac{z}{80}$

$100 \cdot z = 20 \cdot 80$ *Cross products are equal.*

$100z = 1600$

$\frac{100z}{100} = \frac{1600}{100}$ *Divide by 100.*

$z = 16$

The solution set is $\{16\}$.

33. $\frac{a}{24} = \frac{15}{16}$

$16a = 24(15)$ *Cross products are equal.*

$16a = 360$

$\frac{16a}{16} = \frac{360}{16}$ *Divide by 16.*

$a = \frac{45 \cdot 8}{2 \cdot 8} = \frac{45}{2}$

The solution set is $\left\{\frac{45}{2}\right\}$.

34. $\frac{x}{4} = \frac{12}{30}$

$30x = 4(12)$ *Cross products are equal.*

$30x = 48$

$\frac{30x}{30} = \frac{48}{30}$ *Divide by 30.*

$x = \frac{8 \cdot 6}{5 \cdot 6} = \frac{8}{5}$

The solution set is $\left\{\frac{8}{5}\right\}$.

35. $\frac{z}{2} = \frac{z+1}{3}$

$3z = 2(z+1)$ *Cross products are equal.*

$3z = 2z + 2$ *Distributive property*

$z = 2$ *Subtract 2z.*

The solution set is $\{2\}$.

36. $\frac{m}{5} = \frac{m-2}{2}$

$2m = 5(m-2)$ *Cross products are equal.*

$2m = 5m - 10$ *Distributive property*

$-3m = -10$ *Subtract 5m.*

$m = \frac{10}{3}$ *Divide by −3.*

The solution set is $\left\{\frac{10}{3}\right\}$.

37. $\frac{3y-2}{5} = \frac{6y-5}{11}$

$11(3y-2) = 5(6y-5)$ *Cross products are equal.*

$33y - 22 = 30y - 25$ *Dist. prop.*

$3y - 22 = -25$ *Subtract 30y.*

$3y = -3$ *Add 22.*

$y = -1$ *Divide by 3.*

The solution set is $\{-1\}$.

38. $\frac{2r+8}{4} = \frac{3r-9}{3}$

$3(2r+8) = 4(3r-9)$ *Cross products are equal.*

$6r + 24 = 12r - 36$ *Dist. prop.*

$-6r + 24 = -36$ *Subtract 12r.*

$-6r = -60$ *Subtract 24.*

$r = 10$ *Divide by −6.*

The solution set is $\{10\}$.

39. $\frac{5k+1}{6} = \frac{3k-2}{3}$

$3(5k+1) = 6(3k-2)$ *Cross products*

$15k + 3 = 18k - 12$ *Dist. prop.*

$-3k + 3 = -12$ *Subtract 18k.*

$-3k = -15$ *Subtract 3.*

$k = 5$ *Divide by −3.*

The solution set is $\{5\}$.

40. $\frac{x+4}{6}=\frac{x+10}{8}$

$$\begin{aligned} 8(x+4)&=6(x+10) && \textit{Cross products}\\ 8x+32&=6x+60 && \textit{Dist. prop.}\\ 2x+32&=60 && \textit{Subtract 6x.}\\ 2x&=28 && \textit{Subtract 32.}\\ x&=14 && \textit{Divide by 2.} \end{aligned}$$

The solution set is $\{14\}$.

41. $\frac{2p+7}{3}=\frac{p-1}{4}$

$$\begin{aligned} 4(2p+7)&=3(p-1) && \textit{Cross products}\\ 8p+28&=3p-3 && \textit{Dist. prop.}\\ 5p+28&=-3 && \textit{Subtract 3p.}\\ 5p&=-31 && \textit{Subtract 28.}\\ p&=-\tfrac{31}{5} && \textit{Divide by 5.} \end{aligned}$$

The solution set is $\left\{-\frac{31}{5}\right\}$.

42. $\frac{3m-2}{5}=\frac{4-m}{3}$

$$\begin{aligned} 3(3m-2)&=5(4-m) && \textit{Cross products}\\ 9m-6&=20-5m && \textit{Dist. prop.}\\ 14m-6&=20 && \textit{Add 5m.}\\ 14m&=26 && \textit{Add 6.}\\ m&=\tfrac{26}{14}=\tfrac{13}{7} && \textit{Divide by 14.} \end{aligned}$$

The solution set is $\left\{\frac{13}{7}\right\}$.

43. $\frac{0.5x+2}{3}=\frac{2.25x+27}{9}$

$$\begin{aligned} 9(0.5x+2)&=3(2.25x+27) && \textit{Cross products}\\ 4.5x+18&=6.75x+81 && \textit{Dist. prop.}\\ 18&=2.25x+81 && \textit{Subtract 4.5x.}\\ -63&=2.25x && \textit{Subtract 81.}\\ -28&=x && \textit{Divide by 2.25.} \end{aligned}$$

The solution set is $\{-28\}$.

44. $\frac{x+4}{3}=\frac{x+5}{3}$

For the two expressions to be equal, their numerators must be equal. But there is no real number x such that $x+4=x+5$.

45. Let $x=$ the cost of 24 candy bars. Set up a proportion.

$$\begin{aligned} \frac{x}{24}&=\frac{\$20.00}{16}\\ 16x&=24(20)\\ 16x&=480\\ x&=30 \end{aligned}$$

The cost of 24 candy bars is \$30.00.

46. Let $x=$ the cost of 8 ring tones. Set up a proportion.

$$\begin{aligned} \frac{x}{8}&=\frac{\$30.00}{12}\\ 12x&=8(30)\\ 12x&=240\\ x&=20 \end{aligned}$$

The cost of 8 ring tones is \$20.00.

47. Let $x=$ the cost of 5 quarts of oil. Set up a proportion.

$$\begin{aligned} \frac{x}{5}&=\frac{\$14.00}{8}\\ 8x&=5(14)\\ 8x&=70\\ x&=8.75 \end{aligned}$$

The cost of five quarts of oil is \$8.75.

48. Let $x=$ the cost of 7 tires. Set up a proportion.

$$\begin{aligned} \frac{x}{7}&=\frac{\$398.00}{4}\\ 4x&=7(398)\\ 4x&=2786\\ x&=696.50 \end{aligned}$$

The cost of seven tires is \$696.50.

49. Let $x=$ the cost of five pairs of jeans.

$$\begin{aligned} \frac{9\text{ pairs}}{\$121.50}&=\frac{5\text{ pairs}}{x}\\ 9x&=5(121.50)\\ 9x&=607.5\\ \frac{9x}{9}&=\frac{607.5}{9}\\ x&=67.5 \end{aligned}$$

The cost of five pairs is \$67.50.

50. Let $x=$ the cost of 11 shirts.

$$\begin{aligned} \frac{7\text{ shirts}}{\$87.50}&=\frac{11\text{ shirts}}{x}\\ 7x&=11(87.50)\\ 7x&=962.5\\ \frac{7x}{7}&=\frac{962.5}{7}\\ x&=137.5 \end{aligned}$$

The cost of 11 shirts is \$137.50.

51. Let $x =$ the cost for filling a 15-gallon tank.
Set up a proportion.

$$\frac{x \text{ dollars}}{\$19.56} = \frac{15 \text{ gallons}}{6 \text{ gallons}}$$
$$6x = 15(19.56)$$
$$6x = 293.4$$
$$x = 48.90$$

It would cost \$48.90 to completely fill a 15-gallon tank.

52. Let $x =$ the sales tax on a \$120.00 compact disc player.

$$\frac{\$1.32}{\$x} = \frac{\$16}{\$120}$$
$$\frac{1.32}{x} = \frac{16}{120}$$
$$16x = 120(1.32)$$
$$16x = 158.4$$
$$x = 9.90$$

The sales tax on a \$120 compact disc player would be \$9.90.

53. Let $x =$ the distance between Memphis and Philadelphia on the map (in feet).

Set up a proportion with one ratio involving map distances and the other involving actual distances.

$$\frac{x \text{ feet}}{2.4 \text{ feet}} = \frac{1000 \text{ miles}}{600 \text{ miles}}$$
$$\frac{x}{2.4} = \frac{1000}{600}$$
$$600x = (2.4)(1000)$$
$$600x = 2400$$
$$x = 4$$

The distance on the map between Memphis and Philadelphia would be 4 feet.

54. Let $x =$ the number of inches between Mexico City and Cairo on the map.
Set up a proportion.

$$\frac{11 \text{ inches}}{x \text{ inches}} = \frac{3300 \text{ miles}}{7700 \text{ miles}}$$
$$\frac{11}{x} = \frac{3300}{7700}$$
$$3300x = 11(7700)$$
$$3300x = 84{,}700$$
$$x = \tfrac{84{,}700}{3300}$$
$$= \tfrac{847}{33} = \tfrac{77}{3}, \quad \text{or} \quad 25\tfrac{2}{3}$$

Mexico City and Cairo are $25\frac{2}{3}$ inches apart on the map.

55. Let $x =$ the number of inches between St. Louis and Des Moines on the map.
Set up a proportion.

$$\frac{8.5 \text{ inches}}{x \text{ inches}} = \frac{1040 \text{ miles}}{333 \text{ miles}}$$
$$1040x = 8.5(333)$$
$$1040x = 2830.5$$
$$x \approx 2.72$$

St. Louis and Des Moines are about 2.7 inches apart on the map.

56. Let $x =$ the number of inches between Milwaukee and Seattle on the map.
Set up a proportion.

$$\frac{8.0 \text{ inches}}{x \text{ inches}} = \frac{912 \text{ miles}}{1940 \text{ miles}}$$
$$912x = 8.0(1940)$$
$$912x = 15{,}520$$
$$x \approx 17.0$$

Milwaukee and Seattle are about 17.0 inches apart on the map.

57. Let $x =$ the number of inches between Moscow and Berlin on the globe.
Set up a proportion.

$$\frac{12.4 \text{ inches}}{x \text{ inches}} = \frac{10{,}080 \text{ km}}{1610 \text{ km}}$$
$$10{,}080x = 12.4(1610)$$
$$10{,}080x = 19{,}964$$
$$x \approx 1.98$$

Moscow and Berlin are about 2.0 inches apart on the globe.

58. Let $x =$ the number of inches between Paris and Stockholm on the globe.
Set up a proportion.

$$\frac{21.5 \text{ inches}}{x \text{ inches}} = \frac{17{,}615 \text{ km}}{1605 \text{ km}}$$
$$17{,}615x = 21.5(1605)$$
$$17{,}615x = 34{,}507.5$$
$$x \approx 1.96$$

Paris and Stockholm are about 2.0 inches apart on the globe.

59. Let $x =$ the number of cups of cleaner.
Set up a proportion with one ratio involving the number of cups of cleaner and the other involving the number of gallons of water.

$$\frac{x \text{ cups}}{\frac{1}{4} \text{ cup}} = \frac{10\frac{1}{2} \text{ gallons}}{1 \text{ gallon}}$$
$$x \cdot 1 = \tfrac{1}{4}\left(10\tfrac{1}{2}\right)$$
$$x = \tfrac{1}{4}\left(\tfrac{21}{2}\right) = \tfrac{21}{8}$$

The amount of cleaner needed is $2\frac{5}{8}$ cups.

60. Let $x =$ the number of cups of cleaner.
Set up a proportion with one ratio involving the number of cups of cleaner and the other involving the number of gallons of water.

$$\frac{x \text{ cups}}{\frac{1}{2}\text{ cup}} = \frac{15\frac{1}{2}\text{ gallons}}{1\text{ gallon}}$$
$$x \cdot 1 = \tfrac{1}{2}\left(15\tfrac{1}{2}\right)$$
$$x = \tfrac{1}{2}\left(\tfrac{31}{2}\right) = \tfrac{31}{4}$$

The amount of cleaner needed is $7\frac{3}{4}$ cups.

61. Let $x =$ the number of U.S. dollars Ashley exchanged.

Set up a proportion.

$$\frac{\$1.2128}{x\text{ dollars}} = \frac{1\text{ euro}}{300\text{ euros}}$$
$$x \cdot 1 = 1.2128(300)$$
$$x = 363.84$$

She exchanged $\$363.84$.

62. Let $x =$ the number of pesos that can be obtained for \$65.

Set up a proportion.

$$\frac{84.3\text{ pesos}}{x\text{ pesos}} = \frac{8\text{ dollars}}{65\text{ dollars}}$$
$$x \cdot 8 = 84.3(65)$$
$$8x = 5479.5$$
$$x = 684.9375$$

To the nearest tenth, one can obtain 684.9 pesos for \$65.

63. Let $x =$ the number of fish in Grand Bay.

Set up a proportion with one ratio involving the sample and the other involving the total number of fish.

$$\frac{7\text{ fish}}{700\text{ fish}} = \frac{500\text{ fish}}{x\text{ fish}}$$
$$7x = (700)(500)$$
$$7x = 350{,}000$$
$$x = 50{,}000$$

We estimate that there are $50{,}000$ fish in Grand Bay.

64. Let $x =$ the number of fish in Old River.

Set up a proportion with one ratio involving the sample and the other involving the total number of fish.

$$\frac{18\text{ fish}}{1000\text{ fish}} = \frac{840\text{ fish}}{x\text{ fish}}$$
$$18x = (1000)(840)$$
$$18x = 840{,}000$$
$$x = 46{,}666.\overline{6}$$

The approximate fish population of Old River, to the nearest hundred, is $46{,}700$ fish.

65. $\frac{x}{12} = \frac{3}{9}$
$9x = 12 \cdot 3 = 36$
$x = 4$

Other possibilities for the proportion are:

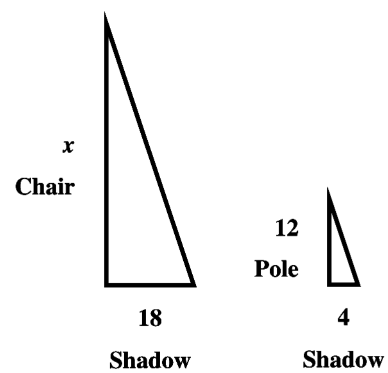
$$\frac{12}{x} = \frac{9}{3},\quad \frac{x}{12} = \frac{5}{15},\quad \frac{12}{x} = \frac{15}{5}$$

66. $\frac{x}{2} = \frac{12}{3}$
$3x = 2 \cdot 12 = 24$
$x = 8$

67. $\frac{x}{3} = \frac{2}{6}$ $\qquad \frac{y}{\frac{4}{3}} = \frac{6}{2}$
$6x = 3 \cdot 2 = 6$ $\qquad 2y = 6\left(\frac{4}{3}\right) = 8$
$x = 1$ $\qquad y = 4$

68. $\frac{x}{15} = \frac{12}{8}$ $\qquad \frac{y}{17} = \frac{12}{8}$
$8x = 15(12) = 180$ $\qquad 8y = 17(12) = 204$
$x = 22.5$ $\qquad y = 25.5$

69. **(a)**

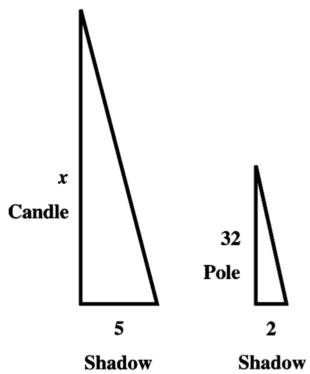


(b) These two triangles are similar, so their sides are proportional.

$$\frac{x}{12} = \frac{18}{4}$$
$$4x = 18(12)$$
$$4x = 216$$
$$x = 54$$

The chair is 54 feet tall.

70. **(a)**



(b) These two triangles are similar, so their sides are proportional.

$$\frac{x}{32} = \frac{5}{2}$$
$$2x = 5(32)$$
$$2x = 160$$
$$x = 80$$

The candle was 80 feet tall.

71. Let $x =$ the 1996 price of groceries.

$$\frac{\text{1990 price}}{\text{1990 index}} = \frac{\text{1996 price}}{\text{1996 index}}$$
$$\frac{120}{130.7} = \frac{x}{156.9}$$
$$130.7x = 120(156.9)$$
$$x = \frac{120(156.9)}{130.7} \approx 144.06$$

The 1996 price would be about \$144.

72. Let $x =$ the 2000 price of groceries.

$$\frac{\text{1990 price}}{\text{1990 index}} = \frac{\text{2000 price}}{\text{2000 index}}$$
$$\frac{120}{130.7} = \frac{x}{172.2}$$
$$130.7x = 120(172.2)$$
$$x = \frac{120(172.2)}{130.7} \approx 158.10$$

The 2000 price would be about \$158.

73. Let $x =$ the 2002 price of groceries.

$$\frac{\text{1990 price}}{\text{1990 index}} = \frac{\text{2002 price}}{\text{2002 index}}$$
$$\frac{120}{130.7} = \frac{x}{179.9}$$
$$130.7x = 120(179.9)$$
$$x = \frac{120(179.9)}{130.7} \approx 165.17$$

The 2002 price would be about \$165.

74. Let $x =$ the 2004 price of groceries.

$$\frac{\text{1990 price}}{\text{1990 index}} = \frac{\text{2004 price}}{\text{2004 index}}$$
$$\frac{120}{130.7} = \frac{x}{188.9}$$
$$130.7x = 120(188.9)$$
$$x = \frac{120(188.9)}{130.7} \approx 173.44$$

The 2004 price would be about \$173.

75.
$$I = prt$$
$$\$1280 = (\$8000)(r)(4)$$
$$r = \frac{1280}{8000(4)} = 0.04$$

The rate of interest is 4%.

76.
$$I = prt$$
$$\$75 = (\$200)(0.05)(t)$$
$$t = \frac{75}{200(0.05)} = 7.5$$

The money earned interest for 7.5 years.

77.
$$I = prt$$
$$\$5700 = (\$19{,}000)(0.03)(t)$$
$$t = \frac{5700}{19{,}000(0.03)} = 10$$

The money earned interest for 10 years.

78.
$$\text{Monetary value} = \text{number of coins} \times \text{denomination}$$
$$= 34 \times (0.25)$$
$$= 8.5$$

The monetary value of 34 quarters is \$8.50.

79.
$$0.15x + 0.30(3) = 0.20(3 + x)$$
$$100[0.15x + 0.30(3)] = 100[0.20(3 + x)]$$
$$15x + 30(3) = 20(3 + x)$$
$$15x + 90 = 60 + 20x$$
$$90 = 60 + 5x$$
$$30 = 5x$$
$$6 = x$$

The solution set is $\{6\}$.

80.
$$0.20(60) + 0.05x = 0.10(60 + x)$$
$$100[0.20(60) + 0.05x] = 100[0.10(60 + x)]$$
$$20(60) + 5x = 10(60 + x)$$
$$1200 + 5x = 600 + 10x$$
$$1200 = 600 + 5x$$
$$600 = 5x$$
$$120 = x$$

The solution set is $\{120\}$.

81.
$$\begin{aligned} 0.92x + 0.98(12 - x) &= 0.96(12) \\ 100[0.92x + 0.98(12 - x)] &= 100[0.96(12)] \\ 92x + 98(12 - x) &= 96(12) \\ 92x + 1176 - 98x &= 1152 \\ 1176 - 6x &= 1152 \\ -6x &= -24 \\ x &= 4 \end{aligned}$$

The solution set is $\{4\}$.

82.
$$\begin{aligned} 0.10(7) + 1.00x &= 0.30(7 + x) \\ 10[0.10(7) + 1.00x] &= 10[0.30(7 + x)] \\ 1(7) + 10x &= 3(7 + x) \\ 7 + 10x &= 21 + 3x \\ 7 + 7x &= 21 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

The solution set is $\{2\}$.

2.7 Further Applications of Linear Equations

2.7 Classroom Examples

1. **(a)** The amount of pure acid in 40 L of a 16% acid solution is

40 L	$\times$	0.16	$= 6.4$ L.
↑		↑	↑
Amount of solution		Rate of concentration	Amount of pure acid

(b) If \$5000 is invested for one year at 4% simple interest, the amount of interest earned is

\$5000	$\times$	0.04	$=$ \$200.
↑		↑	↑
Principal		Interest Rate	Interest earned

2. Let $x =$ the number of kilograms of metal that is 40% copper.
Then $x + 80 =$ the number of kilograms of metal that is 50% copper.

Number of kilograms of metal	x		80		$x + 80$
Percent of copper in metal	0.40	+	0.70	=	0.50

Copper in 40% metal plus copper in 70% metal
↓ ↓ ↓
$0.40x \quad + \quad 0.70(80)$

is copper in 50% metal.
↓ ↓
$= \quad 0.50(x + 80)$

Solve the equation.

$$0.40x + 0.70(80) = 0.50x + 40$$

Multiply by 10 to clear decimals.

$$\begin{aligned} 4x + 7(80) &= 5x + 400 \\ 4x + 560 &= 5x + 400 \\ 560 &= x + 400 \\ 160 &= x \end{aligned}$$

160 kilograms of metal that is 40% copper are needed.

Check $x = 160$:

LS and RS refer to the left side and right side of the original equation.

LS: $0.40(160) + 0.70(80) = 64 + 56 = 120$
RS: $0.50(160 + 80) = 0.50(240) = 120$

3. Let $x =$ the amount invested at 5%.
Then $2x + 3000 =$ the amount invested at 8%.

Amount invested (in dollars)	Rate of interest	Interest for one year
x	0.05	$0.05x$
$2x + 3000$	0.08	$0.08(2x + 3000)$

Since the total annual interest was \$1710, the equation is

$$0.05x + 0.08(2x + 3000) = 1710.$$

Multiply both sides by 100 to eliminate the decimals.

$$\begin{aligned} 5x + 8(2x + 3000) &= 100(1710) \\ 5x + 16x + 24{,}000 &= 171{,}000 \\ 21x + 24{,}000 &= 171{,}000 \\ 21x &= 147{,}000 \\ x &= 7000 \end{aligned}$$

The engineer invested \$7000 at 5%.

4. Let $x =$ the number of quarters.
Then $x + 9 =$ the number of nickels.

The value of quarters	plus	the value of nickels	is	\$2.55.
↓	↓	↓	↓	↓
$0.25x$	$+$	$0.05(x + 9)$	$=$	2.55

Multiply by 100.

$$\begin{aligned} 25x + 5(x + 9) &= 255 \\ 25x + 5x + 45 &= 255 \\ 30x + 45 &= 255 \\ 30x &= 210 \\ x &= 7 \end{aligned}$$

Since $x = 7$, $x + 9 = 16$.
The man has 16 nickels and 7 quarters.

5. $r = \dfrac{d}{t} = \dfrac{100 \text{ meters}}{9.77 \text{ seconds}} \approx 10.2354$

His rate was about 10.24 meters per second.

6. Let $t =$ the time it takes for the planes to be 3290 miles apart.
The slower plane flies at 410 mph and the faster plane flies at $410 + 120 = 530$ mph. Use the formula $d = rt$.

$$\begin{aligned} d_{\text{faster}} + d_{\text{slower}} &= d_{\text{total}} \\ 530t + 410t &= 3290 \\ 940t &= 3290 \\ t = \tfrac{3290}{940} &= 3.5 \end{aligned}$$

It will take 3.5 hours for the planes to be 3290 miles apart.

7. Let $x =$ the rate of the slower bus.
Then $x + 10 =$ the rate of the faster bus.

Use the formula $d = rt$ and the fact that each bus travels for $\frac{1}{5}$ hour.

$$\begin{aligned} d_{\text{slower}} + d_{\text{faster}} &= d_{\text{total}} \\ (x+10)\left(\tfrac{1}{5}\right) + x\left(\tfrac{1}{5}\right) + &= 12 \\ \tfrac{1}{5}x + 2 + \tfrac{1}{5}x &= 12 \\ \tfrac{2}{5}x + 2 &= 12 \\ \tfrac{2}{5}x &= 10 \\ \tfrac{5}{2}\left(\tfrac{2}{5}x\right) &= \tfrac{5}{2}(10) \\ x &= 25 \end{aligned}$$

Since $x = 25$, $x + 10 = 35$. The slower bus had a speed of 25 mph and the faster bus had a speed of 35 mph.

Check: The slower bus traveled $25\left(\frac{1}{5}\right) = 5$ miles and the faster bus traveled $35\left(\frac{1}{5}\right) = 7$ miles. The total miles traveled is $5 + 7 = 12$, as required.

2.7 Section Exercises

1. The amount of pure alcohol in 150 liters of a 30% alcohol solution is

150	×	0.30	= 45 liters.
↑		↑	↑
Amount of solution		Rate of concentration	Amount of pure alcohol

2. The amount of pure acid in 250 milliliters of a 14% acid solution is

250	×	0.14	= 35 milliliters.
↑		↑	↑
Amount of solution		Rate of concentration	Amount of pure acid

3. If $25,000 is invested at 3% simple interest for one year, the amount of interest earned is

$25,000	×	0.03	×	1	= $750.
↑		↑		↑	↑
Principal		Interest Rate		Time	Interest earned

4. If $10,000 is invested for one year at 3.5% simple interest, the amount of interest earned is

$10,000	×	0.035	= $350.
↑		↑	↑
Principal		Interest Rate	Interest earned

5. The monetary value of 35 half-dollars is

35	×	$0.50	= $17.50.
↑		↑	↑
Number of coins		Denomination	Monetary value

6. The monetary value of 283 nickels is

283	×	$0.05	= $14.15.
↑		↑	↑
Number of coins		Denomination	Monetary value

7. 42.1% is about 40%. The population of 1,819,000 is about 1.8 million, and 40% of 1.8 million is 720,000. So choice **A** is the correct answer.

8. 26.0% is about 25% or $\frac{1}{4}$. The population of 4,447,000 is about 4.4 million, and $\frac{1}{4}$ of 4.4 million is 1.1 million. So choice **C** is the correct answer.

9. **(a)** 13% of 4 million $= 0.13(4{,}000{,}000)$ $= 520{,}000$ white cars.

(b) 24% of 4 million $= 0.24(4{,}000{,}000)$ $= 960{,}000$ silver cars.

(c) 6% of 4 million $= 0.06(4{,}000{,}000)$ $= 240{,}000$ blue cars.

10. **(a)** 33% of \$184,320 $= 0.33(\$184{,}320)$ $= \$60{,}825.60 \approx \$60{,}826$ for housing.

(b) 17% of \$184,320 $= 0.17(\$184{,}320)$ $= \$31{,}334.40 \approx \$31{,}334$ for food.

(c) 8% of \$184,320 $= 0.08(\$184{,}320)$ $= \$14{,}745.60 \approx \$14{,}746$ for health care.

11. The problem can be stated as follows:
"8,149,000 is what percent of 147,401,000?"

Substitute $a = 8{,}149{,}000$ and $b = 147{,}401{,}000$ into the percent proportion; then find p.

$$\frac{p}{100} = \frac{a}{b}$$
$$\frac{p}{100} = \frac{8{,}149{,}000}{147{,}401{,}000}$$
$$p = \frac{8{,}149{,}000(100)}{147{,}401{,}000} \approx 5.53$$

The percent of unemployment, to the nearest tenth, is 5.5%.

12. The problem can be stated as follows:
"15,472,000 is what percent of 123,554,000?"

Substitute $a = 15{,}472{,}000$ and $b = 123{,}554{,}000$ into the percent proportion; then find p.

$$\frac{p}{100} = \frac{a}{b}$$
$$\frac{p}{100} = \frac{15{,}472{,}000}{123{,}554{,}000}$$
$$p = \frac{15{,}472{,}000(100)}{123{,}554{,}000} \approx 12.52$$

The percent of this labor force that belonged to unions, to the nearest tenth, is 12.5%.

13. The concentration of the new solution could not be more than the strength of the stronger of the original solutions, so the correct answer is **D**, since 32% is stronger than both 20% and 30%.

14. Because pure alcohol (100% concentration) is to be added, the new solution must be stronger than the original one. Therefore, the concentration of the new solution must be greater than 24%, so the correct answer is **A**, 22%.

15. *Step 2*
Let $x =$ the number of liters of 25% solution to be used.

Step 3
Use the box diagram in the textbook to write the equation.

25% acid solution → $0.25x$ +   40% acid solution → $0.40(80)$ =   30% acid solution → $0.30(x+80)$

$$0.25x + 0.40(80) = 0.30(x+80)$$

Step 4
Multiply by 100 to clear decimals.

$$25x + 40(80) = 30(x+80)$$
$$25x + 3200 = 30x + 2400$$
$$25x + 800 = 30x$$
$$800 = 5x$$
$$160 = x$$

Step 5
160 liters of 25% acid solution must be added.

Step 6
25% of 160 liters plus 40% of 80 liters is 40 liters plus 32 liters, or 72 liters, of pure acid; which is equal to 30% of (160 + 80) liters.
$[0.30(240) = 72]$

16. *Step 2*
Let $x =$ the number of gallons of 50% solution needed. Then
$x + 80 =$ the number of gallons of 40% solution.

Step 3
Use the box diagram in the textbook to write the equation.

Pure antifreeze in 50% solution → $0.50x$ plus → $+$ pure antifreeze in 20% solution → $0.20(80)$
is → $=$ pure antifreeze in 40% solution. → $0.40(x+80)$

Step 4
Solve the equation.

$$0.50x + 0.20(80) = 0.40(x+80)$$

Multiply by 10 to clear decimals.

$$5x + 2(80) = 4(x+80)$$
$$5x + 160 = 4x + 320$$
$$x + 160 = 320$$
$$x = 160$$

Step 5
160 gallons of 50% antifreeze are needed.

Step 6
50% of 160 gallons plus 20% of 80 gallons is 80 gallons plus 16 gallons, or 96 gallons, of pure antifreeze; which is equal to 40% of (160 + 80) gallons. $[0.40(240) = 96]$

17. Let $x =$ the number of liters of 5% drug solution.

Number of liters of solution	20		x		$x + 20$
		+		=	
Strength of solution	0.10		0.05		0.08

Pure drug solution in 10% solution → $0.10(20)$ plus → $+$ pure drug solution in 5% solution → $0.05x$

is pure drug solution in 8% solution.

↓ ↓

$= \quad 0.08(x+20)$

Solve the equation.

$$\begin{aligned} 0.10(20)+0.05x &= 0.08(x+20) \\ 10(20)+5x &= 8(x+20) \\ 200+5x &= 8x+160 \\ 200 &= 3x+160 \\ 40 &= 3x \\ x &= \tfrac{40}{3} = 13\tfrac{1}{3} \end{aligned}$$

The pharmacist needs $13\frac{1}{3}$ liters of 5% drug solution.

Check $x = 13\frac{1}{3}$:

LS and RS refer to the left side and right side of the original equation.

LS: $0.10(20)+0.05\left(13\frac{1}{3}\right) = 2\frac{2}{3}$

RS: $0.08\left(13\frac{1}{3}+20\right) = 2\frac{2}{3}$

18. Let $x =$ the number of kilograms of metal that is 20% tin.
Then $x+80 =$ the number of kilograms of metal that is 50% tin.

Number of kilograms of metal	x	+	80	=	$x+80$
Percent of tin in metal	0.20		0.70		0.50

Tin in 20% metal plus tin in 70% metal

↓ ↓ ↓

$0.20x \quad + \quad 0.70(80)$

is tin in 50% metal.

↓ ↓

$= \quad 0.50(x+80)$

Solve the equation.

$$0.20x+0.70(80) = 0.50x+40$$

Multiply by 10 to clear decimals.

$$\begin{aligned} 2x+7(80) &= 5x+400 \\ 2x+560 &= 5x+400 \\ 560 &= 3x+400 \\ 160 &= 3x \\ x &= \tfrac{160}{3} = 53\tfrac{1}{3} \end{aligned}$$

$53\frac{1}{3}$ kilograms of metal that is 20% tin are needed.

Check $x = 53\frac{1}{3}$:

LS and RS refer to the left side and right side of the original equation.

LS: $0.20\left(53\frac{1}{3}\right)+0.70(80) = 66\frac{2}{3}$

RS: $0.50\left(53\frac{1}{3}+80\right) = 66\frac{2}{3}$

19. Let $x =$ the number of liters of 60% acid solution.
Then $20-x =$ the number of liters of 75% acid solution.

Number of liters of solution	x	+	$20-x$	=	20
Concentration of acid	0.60		0.75		0.72

Pure acid in 60% solution plus pure acid in 75% solution

↓ ↓ ↓

$0.60x \quad + \quad 0.75(20-x)$

is pure acid in 72% solution.

↓ ↓

$= \quad 0.72(20)$

Solve the equation.

$$\begin{aligned} 0.60x+0.75(20-x) &= 0.72(20) \\ 60x+75(20-x) &= 72(20) \\ 60x+1500-75x &= 1440 \\ 1500-15x &= 1440 \\ -15x &= -60 \\ x &= 4 \end{aligned}$$

4 liters of 60% acid solution must used.

Check $x = 4$:

LS: $0.60(4)+0.75(20-4) = 14.4$

RS: $0.72(20) = 14.4$

20. Let $x =$ the number of liters of the 20% alcohol solution.
Complete the table.

Strength	Liters of solution	Liters of pure alcohol
12%	12	$0.12(12) = 1.44$
20%	x	$0.20x$
14%	$x+12$	$0.14(x+12)$

From the last column, we can formulate an equation that compares the number of liters of pure alcohol.

Alcohol in 12% solution + alcohol in 20% solution = alcohol in 14% solution.

continued

$$1.44 + 0.20x = 0.14(x + 12)$$
$$1.44 + 0.20x = 0.14x + 1.68$$
$$0.06x = 0.24$$
$$x = 4$$

4 L of the 20% alcohol solution are needed.

21. Let $x =$ the number of gallons of 12% indicator solution.
Then $10 - x =$ the number of gallons of 20% indicator solution.

Gallons of solution	x	+	$10 - x$	=	10
Strength of solution	0.12		0.20		0.14

Indicator in 12% solution + indicator in 20% solution = indicator in 14% solution

$$0.12x + 0.20(10 - x) = 0.14(10)$$

$$12x + 20(10 - x) = 14(10)$$
$$12x + 200 - 20x = 140$$
$$200 - 8x = 140$$
$$-8x = -60$$
$$x = \frac{-60}{-8}$$
$$x = \frac{15}{2} \text{ or } 7\tfrac{1}{2}$$

$7\frac{1}{2}$ gallons of 12% indicator solution must be used.

Check $x = 7\frac{1}{2}$:
LS: $0.12(7\frac{1}{2}) + 0.20(10 - 7\frac{1}{2}) = 1.4$
RS: $0.14(10) = 1.4$

22. Let $x =$ the number of liters of the 10% solution.

Strength	Liters of solution	Liters of pure alcohol
10%	x	$0.10x$
50%	40	$0.50(40) = 20$
40%	$x + 40$	$0.40(x + 40)$

Alcohol in 10% solution + alcohol in 50% solution = alcohol in 40% solution.

$$0.10x + 20 = 0.40(x + 40)$$

$$1x + 200 = 4(x + 40) \quad \textit{Multiply by 10.}$$
$$x + 200 = 4x + 160$$
$$-3x = -40$$
$$x = \frac{-40}{-3} = \frac{40}{3} \text{ or } 13\tfrac{1}{3}$$

$13\frac{1}{3}$ L of 10% alcohol solution should be added.

23. Let $x =$ the amount of water to be added.
Then $20 + x =$ the amount of 2% solution.

There is no minoxidil in water.

Number of milliliters of solution	x	+	20	=	$20 + x$
Concentration of solution	0		0.04		0.02

Pure minoxidil in x milliliters of water plus pure minoxidil in 4% solution is pure minoxidil in 2% solution

$$0(x) + 0.04(20) = 0.02(20 + x)$$

Solve the equation.

$$0x + 0.04(20) = 0.02(20 + x)$$
$$4(20) = 2(20 + x)$$
$$80 = 40 + 2x$$
$$40 = 2x$$
$$20 = x$$

20 milliliters of water should be used.

Check $x = 20$:
LS: $0(20) + 0.04(20) = 0.8$
RS: $0.02(20 + 20) = 0.8$

This answer should make common sense; that is, equal amounts of 0% and 4% solutions should produce a 2% solution.

24. Let $x =$ the number of milliliters of 4% solution.

Milliliters of solution	50	+	x	=	$50 + x$
Concentration of minoxidil	0.01		0.04		0.02

Pure minoxidil in 1% solution + pure minoxidil in 4% solution = pure minoxidil in 2% solution

$$0.01(50) + 0.04x = 0.02(50 + x)$$

$$1(50) + 4x = 2(50 + x)$$
$$50 + 4x = 100 + 2x$$
$$50 + 2x = 100$$
$$2x = 50$$
$$x = 25$$

The pharmacist must add 25 milliliters of 4% solution.

Check $x = 25$:
LS: $0.01(50) + 0.04(25) = 1.5$
RS: $0.02(50 + 25) = 1.5$

25. Let $x =$ the amount invested at 5% (in dollars).
Then $x - 1200 =$ the amount invested at 4% (in dollars).

Amount invested (in dollars)	Rate of interest	Interest for one year
x	0.05	$0.05x$
$x - 1200$	0.04	$0.04(x - 1200)$

Since the total annual interest was $141, the equation is

$$\begin{aligned} 0.05x + 0.04(x - 1200) &= 141. \\ 5x + 4(x - 1200) &= 100(141) \\ 5x + 4x - 4800 &= 14{,}100 \\ 9x - 4800 &= 14{,}100 \\ 9x &= 18{,}900 \\ x &= 2100 \end{aligned}$$

Since $x = 2100$, $x - 1200 = 900$.
Eduardo invested $2100 at 5% and $900 at 4%.

26. Let $x =$ the amount invested at 4%.
Then $x + 3000 =$ the amount invested at 6%.

Amount invested (in dollars)	Rate of interest	Interest for one year
x	0.04	$0.04x$
$x + 3000$	0.06	$0.06(x + 3000)$

Since the total annual interest was $780, the equation is

$$\begin{aligned} 0.04x + 0.06(x + 3000) &= 780. \\ 4x + 6(x + 3000) &= 100(780) \\ 4x + 6x + 18{,}000 &= 78{,}000 \\ 10x + 18{,}000 &= 78{,}000 \\ 10x &= 60{,}000 \\ x &= 6000 \end{aligned}$$

Since $x = 6000$, $x + 3000 = 9000$.
Roopa deposited $6000 at 4% and $9000 at 6%.

27. Let $x =$ the amount invested at 6%.
Then $3x + 6000 =$ the amount invested at 5%.

$$\begin{aligned} 0.06x + 0.05(3x + 6000) &= 825 \\ 6x + 5(3x + 6000) &= 100(825) \\ 6x + 15x + 30{,}000 &= 82{,}500 \\ 21x + 30{,}000 &= 82{,}500 \\ 21x &= 52{,}500 \\ x &= 2500 \end{aligned}$$

Since $x = 2500$, $3x + 6000 = 13{,}500$.
The artist invested $2500 at 6% and $13,500 at 5%.

28. Let $x =$ the amount invested at 3%.
Then $2x + 30{,}000 =$ the amount invested at 4%.

$$\begin{aligned} 0.03x + 0.04(2x + 30{,}000) &= 5600 \\ 3x + 4(2x + 30{,}000) &= 100(5600) \\ 3x + 8x + 120{,}000 &= 560{,}000 \\ 11x + 120{,}000 &= 560{,}000 \\ 11x &= 440{,}000 \\ x &= 40{,}000 \end{aligned}$$

Since $x = 40{,}000$, $2x + 30{,}000 = 110{,}000$.
The actor invested $40,000 at 3% and $110,000 at 4%.

29. Let $x =$ the number of nickels.
Then $x + 2 =$ the number of dimes.

The value of nickels	plus	the value of dimes	is	$1.70
↓	↓	↓	↓	↓
$0.05x$	$+$	$0.10(x + 2)$	$=$	1.70

$$\begin{aligned} 5x + 10(x + 2) &= 100(1.70) \\ 5x + 10x + 20 &= 170 \\ 15x + 20 &= 170 \\ 15x &= 150 \\ x &= 10 \end{aligned}$$

The collector has 10 nickels.

30. Let $x =$ the number of five-dollar bills.
Then $x + 5 =$ the number of twenty-dollar bills.

The value of fives	plus	the value of twenties	is	$725
↓	↓	↓	↓	↓
$5x$	$+$	$20(x + 5)$	$=$	725

$$\begin{aligned} 5x + 20x + 100 &= 725 \\ 25x + 100 &= 725 \\ 25x &= 625 \\ x &= 25 \end{aligned}$$

The teller has 25 five-dollar bills.

31. Let $x =$ the number of 39-cent stamps.
Then $45 - x =$ the number of 24-cent stamps.

The value of the 39-cent stamps is $0.39x$ and the value of the 24-cent stamps is $0.24(45 - x)$. The total value is $15.00, so

$$\begin{aligned} 0.39x + 0.24(45 - x) &= 15.00. \\ 39x + 24(45 - x) &= 100(15) \\ 39x + 1080 - 24x &= 1500 \\ 15x + 1080 &= 1500 \\ 15x &= 420 \\ x &= 28 \end{aligned}$$

Since $x = 28$, $45 - x = 17$. She bought 28 39-cent stamps valued at $10.92 and 17 24-cent stamps valued at $4.08, for a total value of $15.00.

32. Let $x =$ the number of adult tickets sold.
Then $600 - x =$ the number of children's tickets sold.

The value of the adult tickets is $8x$ and the value of the children's tickets is $5(600 - x)$. The total value was \$4116, so

$$\begin{aligned} 8x + 5(600 - x) &= 4116. \\ 8x + 3000 - 5x &= 4116 \\ 3x + 3000 &= 4116 \\ 3x &= 1116 \\ x &= 372 \end{aligned}$$

Since $x = 372$, $600 - x = 228$. There were 372 adult tickets valued at \$2976 and 228 children's tickets valued at \$1140, for a total value of \$4116.

33. Let $x =$ the number of pounds of Colombian Decaf beans.
Then $2x =$ the number of pounds of Arabian Mocha beans.

Number of Pounds	Cost per Pound	Total Value (in \$)
x	\$8.00	$8x$
$2x$	\$8.50	$8.5(2x)$

The total value is \$87.50, so

$$\begin{aligned} 8x + 8.5(2x) &= 87.50. \\ 8x + 17x &= 87.50 \\ 25x &= 87.50 \\ x &= 3.5 \end{aligned}$$

Since $x = 3.5$, $2x = 7$. She can buy 3.5 pounds of Colombian Decaf and 7 pounds of Arabian Mocha.

34. Let $x =$ the number of pounds of Kona Deluxe beans.

	Number of Pounds	Cost per Pound	Total Value (in \$)
Kona Deluxe	x	\$11.50	$11.50x$
French Roast	12	\$7.50	$7.50(12)$
Mixture	$x + 12$	\$10	$10(x + 12)$

The sum of the values of the Kona Deluxe beans and the French Roast beans must equal the value of the mixture, so

$$\begin{aligned} 11.50x + 7.50(12) &= 10(x + 12). \\ 115x + 75(12) &= 10[10(x + 12)] \\ 115x + 900 &= 100x + 1200 \\ 15x + 900 &= 1200 \\ 15x &= 300 \\ x &= 20 \end{aligned}$$

Mixing 20 pounds of Kona Deluxe (value \$230) with 12 pounds of French Roast (value \$90) yields a mixture of 32 pounds that sells for \$10 per pound (value \$320).

35. To estimate the average speed of the trip, round 405 to 400 and 8.2 to 8.

Use $r = \frac{d}{t}$ with $d = 400$ and $t = 8$.

$$r = \frac{d}{t} = \frac{400}{8} = 50$$

The best estimate is **A**, 50 miles per hour.

36. The distance traveled cannot be found by multiplying 45 and 30 because the rate is given in miles per hour, while the time is given in minutes. To find the correct distance, start by converting the time to hours.

$$\begin{aligned} 30 \text{ minutes} &= \tfrac{1}{2} \text{ hour} \\ d &= rt \\ &= 45\left(\tfrac{1}{2}\right) \\ &= 22.5 \end{aligned}$$

The car traveled 22.5 $\left(\text{or } 22\frac{1}{2}\right)$ miles.

37. Use the formula $d = rt$ with $r = 53$ and $t = 10$.

$$\begin{aligned} d &= rt \\ &= (53)(10) \\ &= 530 \end{aligned}$$

The distance between Memphis and Chicago is 530 miles.

38. Use $d = rt$ with $r = 164$ and $t = 2$.

$$\begin{aligned} d &= rt \\ &= 164(2) \\ &= 328 \end{aligned}$$

The distance from Warsaw to Rome is 328 miles.

39. Use $d = rt$ with $d = 500$ and $r = 157.603$.

$$\begin{aligned} d &= rt \\ 500 &= 157.603t \\ t &= \tfrac{500}{157.603} \approx 3.173 \end{aligned}$$

His time was about 3.173 hours.

40. Use $d = rt$ with $d = 400$ and $r = 118.782$.

$$\begin{aligned} d &= rt \\ 400 &= 118.782t \\ t &= \tfrac{400}{118.782} \approx 3.368 \end{aligned}$$

His time was about 3.368 hours.

41. $$r = \frac{d}{t} = \frac{100 \text{ meters}}{12.37 \text{ seconds}} \approx 8.084$$

Her rate was about 8.08 meters per second.

42. $r = \frac{d}{t} = \frac{400 \text{ meters}}{52.82 \text{ seconds}} \approx 7.573$

Her rate was about 7.57 meters per second.

43. $r = \frac{d}{t} = \frac{400 \text{ meters}}{47.63 \text{ seconds}} \approx 8.398$

His rate was about 8.40 meters per second.

44. $r = \frac{d}{t} = \frac{400 \text{ meters}}{44.00 \text{ seconds}} \approx 9.091$

His rate was about 9.09 meters per second.

45. Let $t =$ the number of hours until John and Pat meet.
The distance John travels and the distance Pat travels total 440 miles.

John's distance	and	Pat's distance	equal	total distance.
↓	↓	↓	↓	↓
$60t$	$+$	$28t$	$=$	440

$$\begin{aligned} 88t &= 440 \\ t &= 5 \end{aligned}$$

It will take 5 hours for them to meet.

46. Let $t =$ the time each plane travels.
Use the chart in the text to help write the equation.

Distance of plane leaving Portland	plus	distance of plane leaving St. Louis	is	distance between Portland and St. Louis.
↓	↓	↓	↓	↓
$90t$	$+$	$116t$	$=$	2060

$$\begin{aligned} 206t &= 2060 \\ t &= \tfrac{2060}{206} = 10 \end{aligned}$$

It will take the planes 10 hours to meet.

47. Let $t =$ the number of hours until the trains are 315 kilometers apart.

Distance of northbound train	plus	distance of southbound train	is	total distance
↓	↓	↓	↓	↓
$85t$	$+$	$95t$	$=$	315

$$\begin{aligned} 180t &= 315 \\ t &= \tfrac{315}{180} = \tfrac{7}{4} \end{aligned}$$

It will take $1\frac{3}{4}$ hours for the trains to be 315 kilometers apart.

48. Let $t =$ the number of hours until the steamers are 110 miles apart.
Each steamer will travel $22t$ miles and the total distance traveled will be 110 miles.

$$\begin{aligned} 22t + 22t &= 110 \\ 44t &= 110 \\ t &= \tfrac{110}{44} = \tfrac{5}{2} \end{aligned}$$

It will take $2\frac{1}{2}$ hours for the steamers to be 110 miles apart.

49. Let $t =$ the number of hours Marco and Celeste traveled.

Make a chart using the formula $d = rt$.

	r	t	d
Marco	10	t	$10t$
Celeste	12	t	$12t$

Marco's distance	minus	Celeste's distance	is	15.
↓	↓	↓	↓	↓
$12t$	$-$	$10t$	$=$	15

$$\begin{aligned} 12t - 10t &= 15 \\ 2t &= 15 \\ t &= \tfrac{15}{2} \text{ or } 7\tfrac{1}{2} \end{aligned}$$

They will be 15 miles apart in $7\frac{1}{2}$ hours.

50. Let $t =$ the number of hours until the steamboats will be 9 miles apart.

Make a chart using the formula $d = rt$.

	r	t	d
Slower boat	18	t	$18t$
Faster boat	24	t	$24t$

Distance traveled by faster boat	minus	Distance traveled by slower boat	is	9.
↓	↓	↓	↓	↓
$24t$	$-$	$18t$	$=$	9

$$\begin{aligned} 24t - 18t &= 9 \\ 6t &= 9 \\ t &= \tfrac{3}{2} \text{ or } 1\tfrac{1}{2} \end{aligned}$$

In $1\frac{1}{2}$ hours, the steamboats will be 9 miles apart.

51. Let $x =$ the rate of the westbound plane.
Then $x - 150 =$ the rate of the eastbound plane.

Using the formula $d = rt$ and the chart in the text, we see that

$$\begin{aligned} d_{\text{west}} + d_{\text{east}} &= d_{\text{total}} \\ x(3) + (x - 150)(3) &= 2250 \\ 3x + 3x - 450 &= 2250 \\ 6x &= 2700 \\ x &= 450 \end{aligned}$$

Since $x = 450$, $x - 150 = 300$.

The speed of the westbound plane is 450 mph and the speed of the eastbound plane is 300 mph.

52. Let $x =$ the rate of the northbound train.
Then $x + 20 =$ the rate of the southbound train.

Using the formula $d = rt$ and the chart in the text, we see that

$$\begin{aligned} d_{\text{north}} + d_{\text{south}} &= d_{\text{total}} \\ x(2) + (x+20)(2) &= 280 \\ 2x + 2x + 40 &= 280 \\ 4x &= 240 \\ x &= 60 \end{aligned}$$

Since $x = 60$, $x + 20 = 80$.

The speed of the northbound train is 60 mph and the speed of the southbound train is 80 mph.

53. Let $x =$ the rate of the slower car.
Then $x + 20 =$ the rate of the faster car.

Use the formula $d = rt$ and the fact that each car travels for 4 hours.

$$\begin{aligned} d_{\text{faster}} + d_{\text{slower}} &= d_{\text{total}} \\ (x+20)(4) + (x)(4) &= 400 \\ 4x + 80 + 4x &= 400 \\ 8x &= 320 \\ x &= 40 \end{aligned}$$

The speed of the slower car is 40 mph and the speed of the faster car is 60 mph.

54. Let $x =$ the rate of the faster car.
Then $x - 15 =$ the rate of the slower car.

Use the formula $d = rt$ and the fact that each car travels for 2 hours.

$$\begin{aligned} d_{\text{faster}} + d_{\text{slower}} &= d_{\text{total}} \\ x(2) + (x-15)(2) &= 230 \\ 2x + 2x - 30 &= 230 \\ 4x &= 260 \\ x &= 65 \end{aligned}$$

Since $x = 65$, $x - 15 = 50$.

The speed of the faster car is 65 km per hour and the speed of the slower car is 50 km per hour.

55. Let $x =$ Bob's current age.
Then $3x =$ Kevin's current age.

Three years ago, Bob's age was $x - 3$ and Kevin's age was $3x - 3$, and this sum was 22.

$$\begin{aligned} (x-3) + (3x-3) &= 22 \\ 4x - 6 &= 22 \\ 4x &= 28 \\ x &= 7 \end{aligned}$$

Bob is 7 years old and Kevin is $3(7) = 21$ years old.

56. Let $x =$ the number of pint cartons.
Then $6x =$ the number of quart cartons.

Since 1 quart $= 2$ pints, 1 pint $= \frac{1}{2}$ quart, and $\frac{1}{2}x$ is the number of quarts contained in pint cartons. The total number of quarts is 39, so

$$\begin{aligned} \tfrac{1}{2}x + 6x &= 39. \\ \tfrac{1}{2}x + \tfrac{12}{2}x &= 39 \\ \tfrac{13}{2}x &= 39 \\ \tfrac{2}{13}\left(\tfrac{13}{2}x\right) &= \tfrac{2}{13}(39) \\ x &= 6 \end{aligned}$$

There are 6 pint cartons and $6(6) = 36$ quart cartons.

57. Let $w =$ the width of the table.
Then $3w =$ the length of the table.

If we subtract 3 feet from the length $(3w - 3)$ and add 3 feet to the width $(w + 3)$, then the length and the width would be equal.

$$\begin{aligned} 3w - 3 &= w + 3 \\ 3w &= w + 6 \\ 2w &= 6 \\ w &= 3 \end{aligned}$$

The width is 3 feet and the length is $3(3) = 9$ feet.

58. Let $x =$ the number of hours worked.
Her gross pay (pay before deductions) is $6x$.

$$\begin{aligned} \text{gross pay} - \text{deductions} &= \text{take-home pay} \\ 6x - 0.25(6x) &= 450 \\ 6x - 1.5x &= 450 \\ 4.5x &= 450 \\ x &= \tfrac{450}{4.5} = 100 \end{aligned}$$

She must work 100 hours to take home \$450.

59. Let $x =$ her gross pay (pay before deductions).

$$\begin{aligned} \text{gross pay} - \text{deductions} &= \text{take-home pay} \\ x - 0.10(x) &= 585 \\ 0.90x &= 585 \\ x &= \tfrac{585}{0.90} = 650 \end{aligned}$$

She was paid \$650 before deductions.

60. Let $x =$ the amount of the sales.

$$\begin{aligned} \text{sales} + \text{tax} &= \text{total} \\ x + 0.05x &= 2394 \\ 1.05x &= 2394 \\ x &= \tfrac{2394}{1.05} = 2280 \end{aligned}$$

The amount of sales was \$2280.

61. $-2x + 6$
$= -2(3) + 6$ *Let x = 3.*
$= -6 + 6$
$= 0$

62. $-2x + 6$
$= -2(4) + 6$ *Let x = 4.*
$= -8 + 6$
$= -2$

63. $-2x + 6$
$= -2(0) + 6$ *Let x = 0.*
$= 0 + 6$
$= 6$

64. $-2x + 6$
$= -2(-5) + 6$ *Let x = −5.*
$= 10 + 6$
$= 16$

65. $x + 6 = 0$
$-6 + 6 = 0$ *Let x = −6.*
$0 = 0$ *True*

Since a true statement results, -6 is a solution of the given equation.

66. $-2x + 3 = 15$
$-2(-6) + 3 = 15$ *Let x = −6.*
$12 + 3 = 15$
$15 = 15$ *True*

Since a true statement results, -6 is a solution of the given equation.

67. $-6 + x = 0$
$-6 + (-6) = 0$ *Let x = −6.*
$-12 = 0$ *False*

Since a false statement results, -6 is not a solution of the given equation.

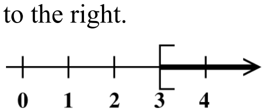
68. $2x = 12$
$2(-6) = 12$ *Let x = −6.*
$-12 = 12$ *False*

Since a false statement results, -6 is not a solution of the given equation.

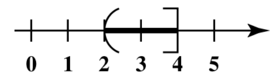
2.8 Solving Linear Inequalities

2.8 Classroom Examples

1. **(a)** The statement $x \geq 3$ says that x can represent any number greater than or equal to 3. The interval is written as $[3, \infty)$. To graph the inequality, place a bracket at 3 and a parenthesis at ∞ (to show that ∞ is *not* part of the graph) and draw an arrow extending to the right.



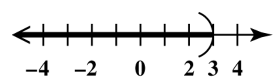
(b) The statement $2 < x \leq 4$ says that x can represent any number between 2 and 4, excluding 2 and including 4. To graph the inequality, place a parenthesis at 2 and a bracket at 4 and draw a line segment between them. The interval is written as $(2, 4]$.



2.
$-1 + 8x < 7x + 2$
$-1 + 8x + 1 < 7x + 2 + 1$ *Add 1.*
$8x < 7x + 3$
$8x - 7x < 7x + 3 - 7x$ *Subtract 7x.*
$x < 3$

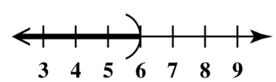
To graph this inequality, place a parenthesis at 3 on a number line and draw an arrow to the left.

Graph the solution set $(-\infty, 3)$.



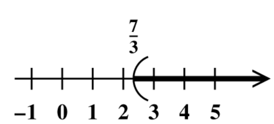
3.
$-2r > -12$
$\frac{-2r}{-2} < \frac{-12}{-2}$ *Divide by −2; reverse the symbol.*
$r < 6$

Graph the solution set $(-\infty, 6)$.



4.
$5x - x + 2 < 7x - 5$
$4x + 2 < 7x - 5$
$4x + 2 - 7x < 7x - 5 - 7x$ *Subtract 7x.*
$-3x + 2 < -5$
$-3x + 2 - 2 < -5 - 2$ *Subtract 2.*
$-3x < -7$
$\frac{-3x}{-3} > \frac{-7}{-3}$ *Divide by −3; reverse the symbol.*
$x > \frac{7}{3}$

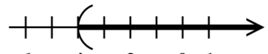
Graph the solution set $(\frac{7}{3}, \infty)$.



5.

$$\begin{aligned} 4(x-1)-3x &> -15-(2x+1) && \\ 4x-4-3x &> -15-2x-1 && \textit{Distributive property} \\ x-4 &> -16-2x && \\ x-4+2x &> -16-2x+2x && \textit{Add 2x.} \\ 3x-4 &> -16 && \\ 3x-4+4 &> -16+4 && \textit{Add 4.} \\ 3x &> -12 && \\ \frac{3x}{3} &> \frac{-12}{3} && \textit{Divide by 3.} \\ x &> -4 && \end{aligned}$$

Graph the solution set $(-4, \infty)$.



6. Let $x =$ Maggie's score on the fourth test.

The average	is at least	90.
↓	↓	↓
$\frac{98+86+88+x}{4}$	$\geq$	90

Solve the inequality.

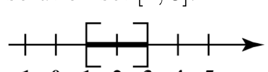
$$\begin{aligned} 4\left(\frac{272+x}{4}\right) &\geq 4(90) && \textit{Add in the numerator; multiply by 4.} \\ 272+x &\geq 360 && \\ 272+x-272 &\geq 360-272 && \textit{Subtract 272.} \\ x &\geq 88 && \textit{Combine terms.} \end{aligned}$$

She must score 88 or more on the fourth test to have an average of *at least* 90.

7.

$$\begin{aligned} 2 \leq 3x-1 &\leq 8 && \\ 2+1 \leq 3x-1+1 &\leq 8+1 && \textit{Add 1 to each part.} \\ 3 \leq 3x &\leq 9 && \\ \frac{3}{3} \leq \frac{3x}{3} &\leq \frac{9}{3} && \textit{Divide each part by 3.} \\ 1 \leq x &\leq 3 && \end{aligned}$$

Graph the solution set $[1, 3]$.



2.8 Section Exercises

1. Use a parenthesis if the inequality symbol is $>$ or $<$. Use a square bracket if the inequality symbol is $\geq$ or $\leq$. Examples:

A parenthesis would be used for the inequalities $x < 2$ and $x > 3$. A square bracket would be used for the inequalities $x \leq 2$ and $x \geq 3$. Note that a parenthesis is *always* used with the symbols $-\infty$ and ∞.

2. The graph of $x \geq -7$ includes the endpoint, -7, and therefore has a bracket at -7. The graph of $x > -7$ does not include -7 and therefore has a parenthesis at -7.

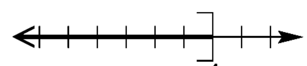
3. The set of numbers graphed corresponds to the inequality $x > -4$.

4. The set of numbers graphed corresponds to the inequality $x \geq -4$.

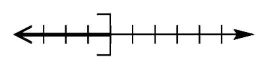
5. The set of numbers graphed corresponds to the inequality $x \leq 4$.

6. The set of numbers graphed corresponds to the inequality $x < 4$.

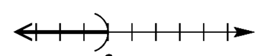
7. The statement $k \leq 4$ says that k can represent any number less than or equal to 4. The interval is written as $(-\infty, 4]$. To graph the inequality, place a square bracket at 4 (to show that 4 is part of the graph) and draw an arrow extending to the left.



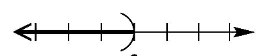
8. The statement $r \leq -11$ says that r can represent any number less than or equal to -11. The interval is written as $(-\infty, -11]$. To graph the inequality, place a square bracket at -11 (to show that -11 is part of the graph) and draw an arrow extending to the left.



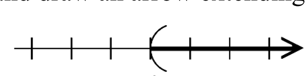
9. The statement $x < -3$ says that x can represent any number less than -3. The interval is written as $(-\infty, -3)$. To graph the inequality, place a parenthesis at -3 (to show that -3 is *not* part of the graph) and draw an arrow extending to the left.



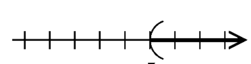
10. The statement $x < 3$ can be written as $(-\infty, 3)$.



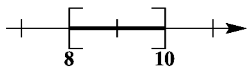
11. The statement $t > 4$ says that t can represent any number greater than 4. The interval is written $(4, \infty)$. To graph the inequality, place a parenthesis at 4 (to show that 4 is *not* part of the graph) and draw an arrow extending to the right.



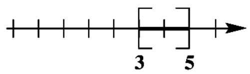
12. The statement $m > 5$ can be written as $(5, \infty)$.



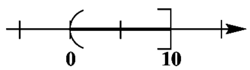
13. The statement $8 \leq x \leq 10$ says that x can represent any number between 8 and 10, including 8 and 10. To graph the inequality, place brackets at 8 and 10 (to show that 8 and 10 are part of the graph) and draw a line segment between the brackets. The interval is written as $[8, 10]$.



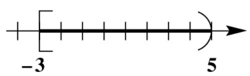
14. The statement $3 \leq x \leq 5$ can be written as $[3, 5]$.



15. The statement $0 < y \leq 10$ says that y can represent any number between 0 and 10, excluding 0 and including 10. To graph the inequality, place a parenthesis at 0 and a bracket at 10 and draw a line segment between them. The interval is written as $(0, 10]$.



16. The statement $-3 \leq x < 5$ can be written as $[-3, 5)$.



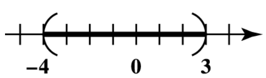
17.

$$4 > -x > -3$$

$$-1(4) < -1(-x) < -1(-3) \quad \textit{Multiply by } -1\textit{; reverse the symbols.}$$

$$-4 < x < 3$$

Graph the solution set $(-4, 3)$.



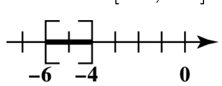
18.

$$6 \geq -x \geq 4$$

$$-1(6) \leq -1(-x) \leq -1(4) \quad \textit{Multiply by } -1\textit{; reverse the symbols.}$$

$$-6 \leq x \leq -4$$

Graph the solution set $[-6, -4]$.



19. It is wrong to write $3 < x < -2$ because it would imply that $3 < -2$, a false statement. Also, note that $3 < x < -2$ would require x to be a number which is both less than -2 and greater than 3 at the same time, which is impossible.

20. $p < q$ and $r < 0$

A. $pr < qr$

The second part of the multiplication property of inequality states that when both sides of an inequality are multiplied by a negative number, the inequality symbol is reversed. Since r is negative, this statement is *false*.

B. $pr > qr$

Both sides of the inequality $p < q$ have been multiplied by the negative number r and the inequality symbol has been reversed, so this statement is *true*.

C. $p + r < q + r$

By the addition property of inequality, this statement is *true*. When a negative number is added to both sides of an inequality, the inequality symbol is not reversed.

D. $p - r < q - r$

When the same number is subtracted from both sides of an inequality, the inequality symbol is not reversed. The statement is *true*. Therefore, only statement **A** is false.

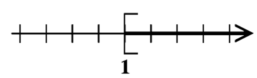
21.

$$z - 8 \geq -7$$

$$z - 8 + 8 \geq -7 + 8 \quad \textit{Add 8.}$$

$$z \geq 1$$

Graph the solution set $[1, \infty)$.



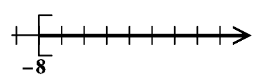
22.

$$p - 3 \geq -11$$

$$p - 3 + 3 \geq -11 + 3 \quad \textit{Add 3.}$$

$$p \geq -8$$

Graph the solution set $[-8, \infty)$.



23.

$$2k + 3 \geq k + 8$$

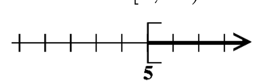
$$2k + 3 - k \geq k + 8 - k \quad \textit{Subtract k.}$$

$$k + 3 \geq 8$$

$$k + 3 - 3 \geq 8 - 3 \quad \textit{Subtract 3.}$$

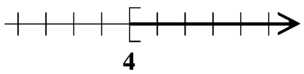
$$k \geq 5$$

Graph the solution set $[5, \infty)$.



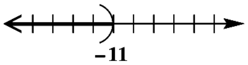
24.
$$\begin{aligned} 3x + 7 &\geq 2x + 11 \\ 3x + 7 - 2x &\geq 2x + 11 - 2x && \textit{Subtract } 2x. \\ x + 7 &\geq 11 \\ x + 7 - 7 &\geq 11 - 7 && \textit{Subtract 7.} \\ x &\geq 4 \end{aligned}$$

Graph the solution set $[4, \infty)$.



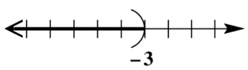
25.
$$\begin{aligned} 3n + 5 &< 2n - 6 \\ 3n - 2n + 5 &< 2n - 2n - 6 && \textit{Subtract } 2n. \\ n + 5 &< -6 \\ n + 5 - 5 &< -6 - 5 && \textit{Subtract 5.} \\ n &< -11 \end{aligned}$$

Graph the solution set $(-\infty, -11)$.



26.
$$\begin{aligned} 5x - 2 &< 4x - 5 \\ 5x - 2 - 4x &< 4x - 5 - 4x && \textit{Subtract } 4x. \\ x - 2 &< -5 \\ x - 2 + 2 &< -5 + 2 && \textit{Add 2.} \\ x &< -3 \end{aligned}$$

Graph the solution set $(-\infty, -3)$.



27. The inequality symbol must be reversed when one is multiplying or dividing by a negative number.

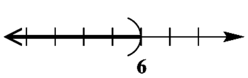
28. Dividing by 0.2 is the same as multiplying by the reciprocal of 0.2. Since $0.2 = \frac{1}{5}$, you must multiply both sides by the reciprocal of $\frac{1}{5}$, which is 5, to get just x on the left side.

29. To solve the inequality $-5x > 20$, divide both sides by -5 and reverse the inequality symbol to get $x < -4$.

30. To solve the inequality $6x < -42$, you would divide both sides by 6, a positive number, so you would not reverse the direction of the inequality. The direction of the inequality is reversed only when both sides are multiplied or divided by a negative number.

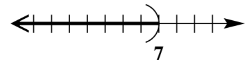
31.
$$\begin{aligned} 3x &< 18 \\ \frac{3x}{3} &< \frac{18}{3} && \textit{Divide by 3.} \\ x &< 6 \end{aligned}$$

Graph the solution set $(-\infty, 6)$.



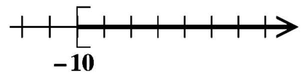
32.
$$\begin{aligned} 5x &< 35 \\ \frac{5x}{5} &< \frac{35}{5} && \textit{Divide by 5.} \\ x &< 7 \end{aligned}$$

Graph the solution set $(-\infty, 7)$.



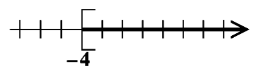
33.
$$\begin{aligned} 2y &\geq -20 \\ \frac{2y}{2} &\geq \frac{-20}{2} && \textit{Divide by 2.} \\ y &\geq -10 \end{aligned}$$

Graph the solution set $[-10, \infty)$.



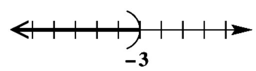
34.
$$\begin{aligned} 6m &\geq -24 \\ \frac{6m}{6} &\geq \frac{-24}{6} && \textit{Divide by 6.} \\ m &\geq -4 \end{aligned}$$

Graph the solution set $[-4, \infty)$.



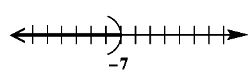
35.
$$\begin{aligned} -8t &> 24 \\ \frac{-8t}{-8} &< \frac{24}{-8} && \textit{Divide by } -8\textit{; reverse the symbol from } > \textit{ to } <. \\ t &< -3 \end{aligned}$$

Graph the solution set $(-\infty, -3)$.



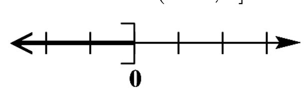
36.
$$\begin{aligned} -7x &> 49 \\ \frac{-7x}{-7} &< \frac{49}{-7} && \textit{Divide by } -7\textit{; reverse the symbol from } > \textit{ to } <. \\ x &< -7 \end{aligned}$$

Graph the solution set $(-\infty, -7)$.



37.
$$\begin{aligned} -x &\geq 0 \\ -1x &\geq 0 \\ \frac{-1x}{-1} &\leq \frac{0}{-1} && \textit{Divide by } -1\textit{; reverse the symbol from } \geq \textit{ to } \leq. \\ x &\leq 0 \end{aligned}$$

Graph the solution set $(-\infty, 0]$.



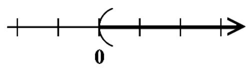
38. $-k < 0$

$-1k < 0$

$\dfrac{-1k}{-1} > \dfrac{0}{-1}$ *Divide by –1; reverse the symbol from $<$ to $>$.*

$k > 0$

Graph the solution set $(0, \infty)$.



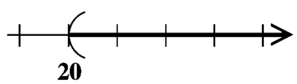
39. $-\frac{3}{4}r < -15$

$\left(-\frac{4}{3}\right)\left(-\frac{3}{4}r\right) > \left(-\frac{4}{3}\right)(-15)$

Multiply by $-\frac{4}{3}$ (the reciprocal of $-\frac{3}{4}$); reverse the symbol from $<$ to $>$.

$r > 20$

Graph the solution set $(20, \infty)$.



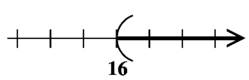
40. $-\frac{7}{8}t < -14$

$\left(-\frac{8}{7}\right)\left(-\frac{7}{8}t\right) > \left(-\frac{8}{7}\right)(-14)$

Multiply by $-\frac{8}{7}$ (the reciprocal of $-\frac{7}{8}$); reverse the symbol from $<$ to $>$.

$t > 16$

Graph the solution set $(16, \infty)$.

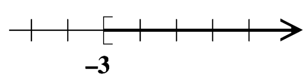


41. $-0.02x \le 0.06$

$\dfrac{-0.02x}{-0.02} \ge \dfrac{0.06}{-0.02}$ *Divide by –0.02; reverse the symbol from $\le$ to $\ge$.*

$x \ge -3$

Graph the solution set $[-3, \infty)$.

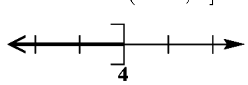


42. $-0.03v \ge -0.12$

$\dfrac{-0.03v}{-0.03} \le \dfrac{-0.12}{-0.03}$ *Divide by –0.03; reverse the symbol from $\ge$ to $\le$.*

$v \le 4$

Graph the solution set $(-\infty, 4]$.



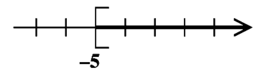
43. $5r + 1 \ge 3r - 9$

$2r + 1 \ge -9$ *Subtract 3r.*

$2r \ge -10$ *Subtract 1.*

$r \ge -5$ *Divide by 2.*

Graph the solution set $[-5, \infty)$.



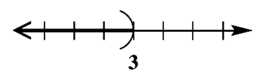
44. $6t + 3 < 3t + 12$

$3t + 3 < 12$ *Subtract 3t.*

$3t < 9$ *Subtract 3.*

$t < 3$ *Divide by 3.*

Graph the solution set $(-\infty, 3)$.



45. $6x + 3 + x < 2 + 4x + 4$

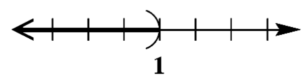
$7x + 3 < 4x + 6$ *Combine like terms.*

$3x + 3 < 6$ *Subtract 4x.*

$3x < 3$ *Subtract 3.*

$x < 1$ *Divide by 3.*

Graph the solution set $(-\infty, 1)$.



46. $-4w + 12 + 9w \ge w + 9 + w$

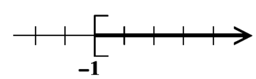
$5w + 12 \ge 2w + 9$ *Combine like terms.*

$3w + 12 \ge 9$ *Subtract 2w.*

$3w \ge -3$ *Subtract 12.*

$w \ge -1$ *Divide by 3.*

Graph the solution set $[-1, \infty)$.



47. $-x + 4 + 7x \le -2 + 3x + 6$

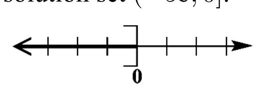
$6x + 4 \le 4 + 3x$

$3x + 4 \le 4$

$3x \le 0$

$x \le 0$

Graph the solution set $(-\infty, 0]$.



48. $14y - 6 + 7y > 4 + 10y - 10$

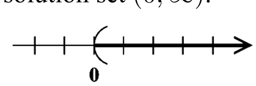
$21y - 6 > 10y - 6$

$11y - 6 > -6$

$11y > 0$

$y > 0$

Graph the solution set $(0, \infty)$.



49. $5(x+3)-6x \le 3(2x+1)-4x$

$5x+15-6x \le 6x+3-4x$

$-x+15 \le 2x+3$

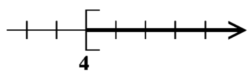
$-3x+15 \le 3$

$-3x \le -12$

$\frac{-3x}{-3} \ge \frac{-12}{-3}$ *Divide by –3; reverse the symbol.*

$x \ge 4$

Graph the solution set $[4, \infty)$.



50. $2(x-5)+3x < 4(x-6)+1$

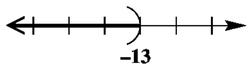
$2x-10+3x < 4x-24+1$

$5x-10 < 4x-23$

$x-10 < -23$

$x < -13$

Graph the solution set $(-\infty, -13)$.



51. $\frac{2}{3}(p+3) > \frac{5}{6}(p-4)$

$6\left(\frac{2}{3}\right)(p+3) > 6\left(\frac{5}{6}\right)(p-4)$

Multiply by 6, the LCD.

$4(p+3) > 5(p-4)$

$4p+12 > 5p-20$

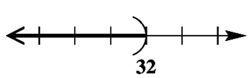
$-p+12 > -20$

$-p > -32$

$\frac{-p}{-1} < \frac{-32}{-1}$ *Divide by –1; reverse the symbol.*

$p < 32$

Graph the solution set $(-\infty, 32)$.



52. $\frac{7}{9}(y-4) \le \frac{4}{3}(y+5)$

$9\left(\frac{7}{9}\right)(y-4) \le 9\left(\frac{4}{3}\right)(y+5)$

Multiply by 9, the LCD.

$7(y-4) \le 12(y+5)$

$7y-28 \le 12y+60$

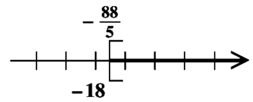
$-5y-28 \le 60$

$-5y \le 88$

$\frac{-5y}{-5} \ge \frac{88}{-5}$ *Divide by –5; reverse the symbol.*

$y \ge -\frac{88}{5}$

Graph the solution set $\left[-\frac{88}{5}, \infty\right)$.



53. $4x-(6x+1) \le 8x+2(x-3)$

$4x-6x-1 \le 8x+2x-6$

$-2x-1 \le 10x-6$

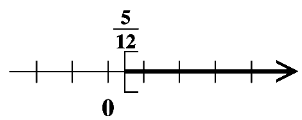
$-12x-1 \le -6$

$-12x \le -5$

$\frac{-12x}{-12} \ge \frac{-5}{-12}$ *Divide by –12; reverse the symbol.*

$x \ge \frac{5}{12}$

Graph the solution set $\left[\frac{5}{12}, \infty\right)$.



54. $2y-(4y+3) > 6y+3(y+4)$

$2y-4y-3 > 6y+3y+12$

$-2y-3 > 9y+12$

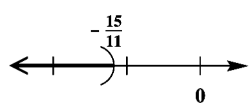
$-11y-3 > 12$

$-11y > 15$

$\frac{-11y}{-11} < \frac{15}{-11}$ *Divide by –11; reverse the symbol.*

$y < -\frac{15}{11}$

Graph the solution set $\left(-\infty, -\frac{15}{11}\right)$.



55. $5(2k+3)-2(k-8) > 3(2k+4)+k-2$

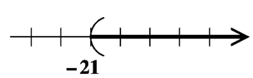
$10k+15-2k+16 > 6k+12+k-2$

$8k+31 > 7k+10$

$k+31 > 10$

$k > -21$

Graph the solution set $(-21, \infty)$.



56. $2(3z-5)+4(z+6) \ge 2(3z+2)+3z-15$

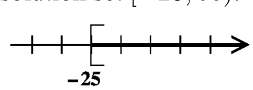
$6z-10+4z+24 \ge 6z+4+3z-15$

$10z+14 \ge 9z-11$

$z+14 \ge -11$

$z \ge -25$

Graph the solution set $[-25, \infty)$.



57. The graph corresponds to the inequality $-1 < x < 2$, excluding both -1 and 2.

58. The graph corresponds to the inequality $-1 \leq x < 2$.

59. The graph corresponds to the inequality $-1 < x \leq 2$, excluding -1 but including 2.

60. The graph corresponds to the inequality $-1 \leq x \leq 2$.

61.

$$-5 \leq 2x - 3 \leq 9$$

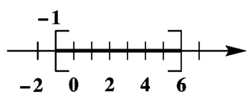
$$-5 + 3 \leq 2x - 3 + 3 \leq 9 + 3 \quad \textit{Add 3 to each part.}$$

$$-2 \leq 2x \leq 12$$

$$\frac{-2}{2} \leq \frac{2x}{2} \leq \frac{12}{2} \quad \textit{Divide each part by 2.}$$

$$-1 \leq x \leq 6$$

Graph the solution set $[-1, 6]$.



62.

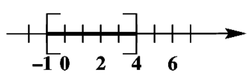
$$-7 \leq 3x - 4 \leq 8$$

$$-7 + 4 \leq 3x - 4 + 4 \leq 8 + 4 \quad \textit{Add 4 to each part.}$$

$$-3 \leq 3x \leq 12$$

$$-1 \leq x \leq 4 \quad \textit{Divide each part by 3.}$$

Graph the solution set $[-1, 4]$.



63.

$$5 < 1 - 6m < 12$$

$$5 - 1 < 1 - 6m - 1 < 12 - 1 \quad \textit{Subtract 1 from each part.}$$

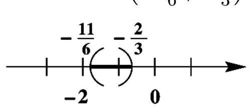
$$4 < -6m < 11$$

$$\frac{4}{-6} > \frac{-6m}{-6} > \frac{11}{-6} \quad \textit{Divide each part by –6; reverse both symbols.}$$

$$-\tfrac{2}{3} > m > -\tfrac{11}{6}$$

$$\text{or } -\tfrac{11}{6} < m < -\tfrac{2}{3} \quad \textit{Equivalent inequality}$$

Graph the solution set $\left(-\frac{11}{6}, -\frac{2}{3}\right)$.



64.

$$-1 \leq 1 - 5q \leq 16$$

$$-1 - 1 \leq 1 - 5q - 1 \leq 16 - 1 \quad \textit{Subtract 1 from each part.}$$

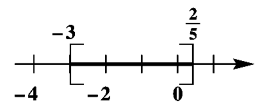
$$-2 \leq -5q \leq 15$$

$$\frac{-2}{-5} \geq \frac{-5q}{-5} \geq \frac{15}{-5} \quad \textit{Divide each part by –5; reverse both symbols.}$$

$$\tfrac{2}{5} \geq q \geq -3$$

$$\text{or } -3 \leq q \leq \tfrac{2}{5} \quad \textit{Equivalent inequality}$$

Graph the solution set $\left[-3, \frac{2}{5}\right]$.



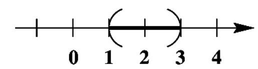
65.

$$10 < 7p + 3 < 24$$

$$7 < 7p < 21 \quad \textit{Subtract 3.}$$

$$1 < p < 3 \quad \textit{Divide by 7.}$$

Graph the solution set $(1, 3)$.



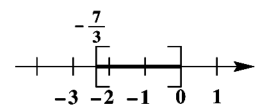
66.

$$-8 \leq 3r - 1 \leq -1$$

$$-7 \leq 3r \leq 0 \quad \textit{Add 1.}$$

$$-\tfrac{7}{3} \leq r \leq 0 \quad \textit{Divide by 3.}$$

Graph the solution set $\left[-\frac{7}{3}, 0\right]$.



67.

$$-12 \leq \tfrac{1}{2}z + 1 \leq 4$$

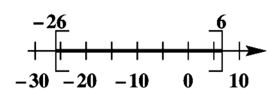
$$2(-12) \leq 2\left(\tfrac{1}{2}z + 1\right) \leq 2(4) \quad \textit{Multiply each part by 2.}$$

$$-24 \leq z + 2 \leq 8$$

$$-26 \leq z \leq 6 \quad \textit{Subtract 2.}$$

Note: We could have started this solution by subtracting 1 from each part.

Graph the solution set $[-26, 6]$.



68.

$$-6 \leq 3 + \tfrac{1}{3}a \leq 5$$

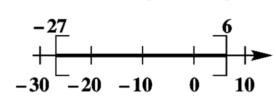
$$-9 \leq \tfrac{1}{3}a \leq 2 \quad \textit{Subtract 3.}$$

$$3(-9) \leq 3\left(\tfrac{1}{3}a\right) \leq 3(2) \quad \textit{Multiply each part by 3.}$$

$$-27 \leq a \leq 6$$

Note: We could have started this solution by multiplying each part by 3.

Graph the solution set $[-27, 6]$.



69. $1 \le 3 + \frac{2}{3}p \le 7$

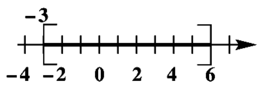
$3(1) \le 3\left(3 + \frac{2}{3}p\right) \le 3(7)$ *Multiply by 3.*

$3 \le 9 + 2p \le 21$

$-6 \le 2p \le 12$ *Subtract 9.*

$-3 \le p \le 6$ *Divide by 2.*

Graph the solution set $[-3, 6]$.



70. $2 < 6 + \frac{3}{4}x < 12$

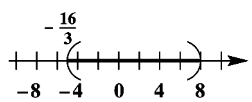
$4(2) < 4\left(6 + \frac{3}{4}x\right) < 4(12)$ *Multiply by 4.*

$8 < 24 + 3x < 48$

$-16 < 3x < 24$ *Subtract 24.*

$\frac{-16}{3} < x < 8$ *Divide by 3.*

Graph the solution set $\left(-\frac{16}{3}, 8\right)$.



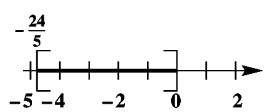
71. $-7 \le \frac{5}{4}r - 1 \le -1$

$-6 \le \frac{5}{4}r \le 0$ *Add 1.*

$\frac{4}{5}(-6) \le \frac{4}{5}\left(\frac{5}{4}r\right) \le \frac{4}{5}(0)$ *Multiply by $\frac{4}{5}$.*

$-\frac{24}{5} \le r \le 0$

Graph the solution set $\left[-\frac{24}{5}, 0\right]$.



72. $-12 \le \frac{3}{7}a + 2 \le -4$

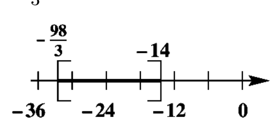
$7(-12) \le 7\left(\frac{3}{7}a + 2\right) \le 7(-4)$ *Multiply by 7.*

$-84 \le 3a + 14 \le -28$

$-98 \le 3a \le -42$ *Subtract 14.*

$-\frac{98}{3} \le a \le -14$ *Divide by 3.*

To graph the solution set $\left[-\frac{98}{3}, -14\right]$, note that $-\frac{98}{3} = -32\frac{2}{3}$.

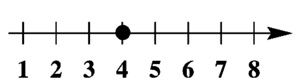


73. $3x + 2 = 14$

$3x = 12$

$x = 4$

Solution set: $\{4\}$

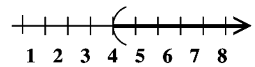


74. $3x + 2 > 14$

$3x > 12$

$x > 4$

Solution set: $(4, \infty)$

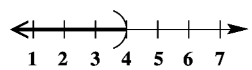


75. $3x + 2 < 14$

$3x < 12$

$x < 4$

Solution set: $(-\infty, 4)$



76. If you were to graph all the solutions from Exercises 73–75 on the same number line, the graph would be the complete number line, that is, all real numbers.

77. Let $x =$ the score on the third test.

The average of the three tests	is at least	80.
↓	↓	↓
$\frac{76 + 81 + x}{3}$	$\ge$	80

$$\frac{157 + x}{3} \ge 80$$

$$3\left(\frac{157 + x}{3}\right) \ge 3(80)$$

$$157 + x \ge 240$$

$$x \ge 83$$

In order to average at least 80, Inkie's score on her third test must be 83 or greater.

78. Let $x =$ the score on the third test.

The average of the three tests	is at least	90.
↓	↓	↓
$\frac{96 + 86 + x}{3}$	$\ge$	90

$$\frac{182 + x}{3} \ge 90$$

$$3\left(\frac{182 + x}{3}\right) \ge 3(90)$$

$$182 + x \ge 270$$

$$x \ge 88$$

In order to average at least 90, Mabimi's score on his third test must be 88 or greater.

79. Let $n =$ the number.
"When 2 is added to the difference between six times a number and 5, the result is greater than 13 added to five times the number" translates to

$$(6n - 5) + 2 > 5n + 13.$$

Solve the inequality.

$$\begin{aligned} 6n - 5 + 2 &> 5n + 13 \\ 6n - 3 &> 5n + 13 \\ n - 3 &> 13 && \textit{Subtract 5n.} \\ n &> 16 && \textit{Add 3.} \end{aligned}$$

All numbers greater than 16 satisfy the given condition.

80. Let $n =$ the number.
"When 8 is subtracted from the sum of three times a number and 6, the result is less than 4 more than the number" translates to

$$(3n + 6) - 8 < n + 4.$$

Solve the inequality.

$$\begin{aligned} 3n + 6 - 8 &< n + 4 \\ 3n - 2 &< n + 4 \\ 2n - 2 &< 4 && \textit{Subtract 1n.} \\ 2n &< 6 && \textit{Add 2.} \\ n &< 3 && \textit{Divide by 2.} \end{aligned}$$

All numbers less than 3 satisfy the given condition.

81. The Fahrenheit temperature must correspond to a Celsius temperature that is greater than or equal to -25 degrees.

$$\begin{aligned} C = \tfrac{5}{9}(F - 32) &\geq -25 \\ \tfrac{9}{5}\left[\tfrac{5}{9}(F - 32)\right] &\geq \tfrac{9}{5}(-25) \\ F - 32 &\geq -45 \\ F &\geq -13 \end{aligned}$$

The temperature in Minneapolis on a certain winter day is never less than $-13°$ Fahrenheit.

82. The Celsius temperature must give a Fahrenheit temperature that is less than or equal to 122 degrees.

$$\begin{aligned} F = \tfrac{9}{5}C + 32 &\leq 122 \\ \tfrac{9}{5}C &\leq 90 \\ \tfrac{5}{9}\left(\tfrac{9}{5}C\right) &\leq \tfrac{5}{9}(90) \\ C &\leq 50 \end{aligned}$$

The temperature of Phoenix has never exceeded 50° Celsius.

83. $P = 2L + 2W;\ P \geq 400$
From the figure, we have $L = 4x + 3$ and $W = x + 37$. Thus, we have the inequality

$$2(4x + 3) + 2(x + 37) \geq 400.$$

Solve this inequality.

$$\begin{aligned} 8x + 6 + 2x + 74 &\geq 400 \\ 10x + 80 &\geq 400 \\ 10x &\geq 320 \\ x &\geq 32 \end{aligned}$$

The rectangle will have a perimeter of at least 400 if the value of x is 32 or greater.

84. $P = a + b + c;\ P \geq 72$
From the figure, we have $a = x$, $b = x + 11$, and $c = 2x + 5$. Thus, we have the inequality

$$x + (x + 11) + (2x + 5) \geq 72.$$

Solve this inequality.

$$\begin{aligned} 4x + 16 &\geq 72 \\ 4x &\geq 56 \\ x &\geq \tfrac{56}{4} = 14 \end{aligned}$$

The triangle will have a perimeter of at least 72 if the value of x is 14 or greater.

85.

$$\begin{aligned} 2 + 0.30x &\leq 5.60 \\ 10(2 + 0.30x) &\leq 10(5.60) \\ 20 + 3x &\leq 56 \\ 3x &\leq 36 \\ x &\leq 12 \end{aligned}$$

Jorge can use the phone for a maximum of 12 minutes after the first three minutes. This means that the maximum *total* time he can use the phone is 15 minutes.

86. The cost of the call is represented by the expression $2 + 0.30x$, where x is the number of minutes after the first three minutes. If the cost of the call is between \$5.60 and \$6.50, we have

$$\begin{aligned} &5.60 < 2 + 0.30x < 6.50 \\ &3.60 < 0.30x < 4.50 \\ &\frac{3.60}{0.30} < \frac{0.30x}{0.30} < \frac{4.50}{0.30} \\ &12 < x < 15. \end{aligned}$$

This means that the call would last between 12 minutes and 15 minutes after the first three minutes. Therefore, the total length of the call would be between 15 minutes and 18 minutes.

87. Let $x =$ the number of gallons.
The amount she spends can be represented by $\$4.50 + \$3.20x$. This must be less than or equal to \$38.10.

$$\begin{aligned} 4.5 + 3.2x &\leq 38.10 \\ 3.2x &\leq 33.6 && \textit{Subtract 4.5.} \\ \frac{3.2x}{3.2} &\leq \frac{33.6}{3.2} && \textit{Divide by 3.2.} \\ x &\leq 10.5 \end{aligned}$$

She can purchase 10.5 gallons of gasoline.

88. Let $t =$ the number of hours he can rent the chain saw.
The amount he spends can be represented by $\$15 + \$2t$. This must be less than or equal to \$35.

$$15 + 2t \leq 35$$
$$2t \leq 20 \quad \textit{Subtract 15.}$$
$$t \leq 10 \quad \textit{Divide by 2.}$$

He can rent the chain saw for 10 hours.

89. "Revenue from the sales of the DVDs is \$5 per DVD less sales costs of \$100" translates to

$$R = 5x - 100,$$

where x represents the number of DVDs to be produced.

90. "Production costs are \$125 plus \$4 per DVD" translates to

$$C = 125 + 4x.$$

91. $P = R - C$
$= (5x - 100) - (125 + 4x)$
$= 5x - 100 - 125 - 4x$
$= x - 225$

We can use this expression for P to solve the inequality.

$$P > 0$$
$$x - 225 > 0$$
$$x > 225$$

92. To make a profit, more than 225 DVDs must be produced and sold.

In part (a) of Exercises 93–98, replace x with -2. In part (b), replace x with 4. Then use the order of operations.

93. **(a)** $y = 5x + 3$
$y = 5(-2) + 3$
$y = -10 + 3$
$y = -7$

(b) $y = 5x + 3$
$y = 5(4) + 3$
$y = 20 + 3$
$y = 23$

94. **(a)** $y = 4 - 3x$
$y = 4 - 3(-2)$
$y = 4 + 6$
$y = 10$

(b) $y = 4 - 3x$
$y = 4 - 3(4)$
$y = 4 - 12$
$y = -8$

95. **(a)** $6x - 2 = y$
$6(-2) - 2 = y$
$-12 - 2 = y$
$-14 = y$

(b) $6x - 2 = y$
$6(4) - 2 = y$
$24 - 2 = y$
$22 = y$

96. **(a)** $4x + 7y = 11$
$4(-2) + 7y = 11$
$-8 + 7y = 11$
$7y = 19$
$y = \frac{19}{7}$

(b) $4x + 7y = 11$
$4(4) + 7y = 11$
$16 + 7y = 11$
$7y = -5$
$y = -\frac{5}{7}$

97. **(a)** $2x - 5y = 10$
$2(-2) - 5y = 10$
$-4 - 5y = 10$
$-5y = 14$
$y = -\frac{14}{5}$

(b) $2x - 5y = 10$
$2(4) - 5y = 10$
$8 - 5y = 10$
$-5y = 2$
$y = -\frac{2}{5}$

98. **(a)** $y + 3x = 8$
$y + 3(-2) = 8$
$y - 6 = 8$
$y = 14$

(b) $y + 3x = 8$
$y + 3(4) = 8$
$y + 12 = 8$
$y = -4$

Chapter 2 Review Exercises

1. $x - 5 = 1$
$x = 6$ *Add 5.*

The solution set is $\{6\}$.

2. $x + 8 = -4$
$x = -12$ *Subtract 8.*

The solution set is $\{-12\}$.

3. $3k + 1 = 2k + 8$
$k + 1 = 8$ *Subtract 2k.*
$k = 7$ *Subtract 1.*

The solution set is $\{7\}$.

4. $5k = 4k + \frac{2}{3}$
$k = \frac{2}{3}$ *Subtract 4k.*

The solution set is $\left\{\frac{2}{3}\right\}$.

5. $(4r - 2) - (3r + 1) = 8$
$(4r - 2) - 1(3r + 1) = 8$ *Replace – with –1.*
$4r - 2 - 3r - 1 = 8$ *Dist. prop.*
$r - 3 = 8$
$r = 11$ *Add 3.*

The solution set is $\{11\}$.

6. $3(2x - 5) = 2 + 5x$
$6x - 15 = 2 + 5x$ *Dist. prop.*
$x - 15 = 2$ *Subtract 5y.*
$x = 17$ *Add 15.*

The solution set is $\{17\}$.

7. $7k = 35$
$k = 5$ *Divide by 7.*

The solution set is $\{5\}$.

8. $12r = -48$
$r = -4$ *Divide by 12.*

The solution set is $\{-4\}$.

9. $2p - 7p + 8p = 15$
$3p = 15$
$p = 5$ *Divide by 3.*

The solution set is $\{5\}$.

10. $\frac{m}{12} = -1$

$m = -12$ *Multiply by 12.*

The solution set is $\{-12\}$.

11. $\frac{5}{8}k = 8$

$\frac{8}{5}\left(\frac{5}{8}k\right) = \frac{8}{5}(8)$ *Multiply by $\frac{8}{5}$.*

$k = \frac{64}{5}$

The solution set is $\left\{\frac{64}{5}\right\}$.

12. $12m + 11 = 59$

$12m = 48$ *Subtract 11.*

$m = 4$ *Divide by 12.*

The solution set is $\{4\}$.

13. $3(2x+6) - 5(x+8) = x - 22$

$6x + 18 - 5x - 40 = x - 22$

$x - 22 = x - 22$

This is a true statement, so the solution set is {all real numbers}.

14. $5x + 9 - (2x - 3) = 2x - 7$

$5x + 9 - 2x + 3 = 2x - 7$

$3x + 12 = 2x - 7$

$x + 12 = -7$

$x = -19$

The solution set is $\{-19\}$.

15. $\frac{1}{2}r - \frac{r}{3} = \frac{r}{6}$

$6\left(\frac{1}{2}r\right) - 6\left(\frac{r}{3}\right) = 6\left(\frac{r}{6}\right)$ *Multiply by 6.*

$3r - 2r = r$

$r = r$

This is a true statement, so the solution set is {all real numbers}.

16. $0.1(x+80) + 0.2x = 14$

$10[0.1(x+80) + 0.2x] = 10(14)$ *Multiply by 10.*

$(x+80) + 2x = 140$ *Dist. prop.*

$3x + 80 = 140$

$3x = 60$

$x = 20$

The solution set is $\{20\}$.

17. $3x - (-2x + 6) = 4(x - 4) + x$

$3x + 2x - 6 = 4x - 16 + x$

$5x - 6 = 5x - 16$

$-6 = -16$

This statement is false, so there is no solution set, symbolized by $\emptyset$.

18. $2(y-3) - 4(y+12) = -2(y+27)$

$2y - 6 - 4y - 48 = -2y - 54$

$-2y - 54 = -2y - 54$

This is a true statement, so the solution set is {all real numbers}.

19. *Step 2*
Let $x =$ the number of Republicans.
Then $x - 13 =$ the number of Democrats.

Step 3 $x + (x - 13) = 101$

Step 4 $2x - 13 = 101$

$2x = 114$

$x = 57$

Step 5
Since $x = 57$, $x - 13 = 44$.
There were 44 Democrats and 57 Republicans.

Step 6
There are 13 fewer Democrats than Republicans and the total is 101.

20. *Step 2*
Let $x =$ the land area of Rhode Island.
Then $x + 5213 =$ land area of Hawaii.

Step 3
The areas total 7637 square miles, so

$$x + (x + 5213) = 7637.$$

Step 4 $2x + 5213 = 7637$

$2x = 2424$

$x = 1212$

Step 5
Since $x = 1212$, $x + 5213 = 6425$. The land area of Rhode Island is 1212 square miles and that of Hawaii is 6425 square miles.

Step 6
The land area of Hawaii is 5213 square miles greater than the land area of Rhode Island and the total is 7637 square miles.

21. *Step 2*
Let $x =$ the height of Rhaiadr Falls.
Then $\frac{11}{8}x =$ the height of Kegon Falls.

Step 3
The sum of the heights is 570 feet, so

$$x + \tfrac{11}{8}x = 570.$$

Step 4 $\frac{8}{8}x + \frac{11}{8}x = 570$

$\frac{19}{8}x = 570$

$\frac{8}{19}\left(\frac{19}{8}x\right) = \frac{8}{19}(570)$

$x = 240$

Step 5
Since $x = 240$, $\frac{11}{8}x = \frac{11}{8}(240) = 330$. The height of Rhaiadr Falls is 240 feet and that of Kegon Falls is 330 feet.

Step 6
The height of Kegon Falls is $\frac{11}{8}$ the height of Rhaiadr Falls and the sum is 570.

22. *Step 2*
Let $x =$ the height of Twin Falls.
Then $\frac{5}{2}x =$ the height of Seven Falls.

Step 3
The sum of the heights is 420 feet, so

$$x + \frac{5}{2}x = 420.$$

Step 4
$$\begin{aligned} 2\left(x + \tfrac{5}{2}x\right) &= 2(420) \\ 2x + 5x &= 840 \\ 7x &= 840 \\ x &= 120 \end{aligned}$$

Step 5
Since $x = 120$, $\frac{5}{2}x = \frac{5}{2}(120) = 300$. The height of Twin Falls is 120 feet and that of Seven Falls is 300 feet.

Step 6
The height of Seven Falls is $\frac{5}{2}$ the height of Twin Falls and the sum is 420.

23. *Step 2*
Let $x =$ the measure of the angle.
Then $90 - x =$ the measure of its complement and $180 - x =$ the measure of its supplement.

Step 3
$$180 - x = 10(90 - x)$$

Step 4
$$\begin{aligned} 180 - x &= 900 - 10x \\ 9x + 180 &= 900 \\ 9x &= 720 \\ x &= 80 \end{aligned}$$

Step 5
The measure of the angle is 80°.
Its complement measures $90° - 80° = 10°$, and its supplement measures $180° - 80° = 100°$.

Step 6
The measure of the supplement is 10 times the measure of the complement.

24. *Step 2*
Let $x =$ lesser odd integer.
Then $x + 2 =$ greater odd integer.

Step 3
$$x + 2(x + 2) = (x + 2) + 24$$

Step 4
$$\begin{aligned} x + 2x + 4 &= x + 26 \\ 3x + 4 &= x + 26 \\ 2x + 4 &= 26 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

Step 5
Since $x = 11$, $x + 2 = 13$. The consecutive odd numbers are 11 and 13.

Step 6
The lesser plus twice the greater is $11 + 2(13) = 37$, which is 24 more than the greater.

In Exercises 25–28, substitute the given values into the given formula and then solve for the remaining variable.

25. $A = \frac{1}{2}bh$; $A = 44$, $b = 8$

$$\begin{aligned} A &= \tfrac{1}{2}bh \\ 44 &= \tfrac{1}{2}(8)h \\ 44 &= 4h \\ 11 &= h \end{aligned}$$

26. $A = \frac{1}{2}h(b + B)$; $h = 8$, $b = 3$, $B = 4$

$$\begin{aligned} A &= \tfrac{1}{2}h(b + B) \\ A &= \tfrac{1}{2}(8)(3 + 4) \\ &= \tfrac{1}{2}(8)(7) \\ &= (4)(7) \\ A &= 28 \end{aligned}$$

27. $C = 2\pi r$; $C = 29.83$, $\pi = 3.14$

$$\begin{aligned} C &= 2\pi r \\ 29.83 &= 2(3.14)r \\ 29.83 &= 6.28r \\ \frac{29.83}{6.28} &= \frac{6.28r}{6.28} \\ 4.75 &= r \end{aligned}$$

28. $V = \frac{4}{3}\pi r^3$; $r = 6$, $\pi = 3.14$

$$\begin{aligned} V &= \tfrac{4}{3}\pi r^3 \\ &= \tfrac{4}{3}(3.14)(6)^3 \\ &= \tfrac{4}{3}(3.14)(216) \\ &= \tfrac{4}{3}(678.24) \\ V &= 904.32 \end{aligned}$$

29. $A = bh$ for h

$$\frac{A}{b} = \frac{bh}{b} \quad \textit{Divide by b.}$$

$$\frac{A}{b} = h \quad \text{or} \quad h = \frac{A}{b}$$

30. $A = \frac{1}{2}h(b+B)$ for h

$$2A = 2\left[\tfrac{1}{2}h(b+B)\right] \quad \textit{Multiply by 2.}$$
$$2A = h(b+B)$$
$$\frac{2A}{(b+B)} = \frac{h(b+B)}{(b+B)} \quad \textit{Divide by } b+B.$$
$$\frac{2A}{b+B} = h \quad \text{or} \quad h = \frac{2A}{b+B}$$

31. Because the two angles are supplementary,

$$(8x-1)+(3x-6)=180.$$
$$11x-7=180$$
$$11x=187$$
$$x=17$$

Since $x=17$, $8x-1=135$, and $3x-6=45$.

The measures of the two angles are 135° and 45°.

32. The angles are vertical angles, so their measures are equal.

$$3x+10=4x-20$$
$$10=x-20$$
$$30=x$$

Since $x=30$, $3x+10=100$, and $4x-20=100$.

Each angle has a measure of 100°.

33. Let W = the width of the rectangle.
Then $W+12$ = the length of the rectangle.

The perimeter of the rectangle is 16 times the width can be written as

$$2L+2W=16W$$

since the perimeter is $2L+2W$.

Because $L=W+12$, we have

$$2(W+12)+2W=16W.$$
$$2W+24+2W=16W$$
$$4W+24=16W$$
$$-12W+24=0$$
$$-12W=-24$$
$$W=2$$

The width is 2 cm and the length is $2+12=14$ cm.

34. First, use the formula for the circumference of a circle to find the value of r.

$$C=2\pi r$$
$$62.5=2(3.14)(r) \quad \textit{Let } C=62.5,\ \pi=3.14.$$
$$62.5=6.28r$$
$$\frac{62.5}{6.28}=\frac{6.28r}{6.28}$$
$$9.95\approx r$$

The radius of the turntable is approximately 9.95 feet. The diameter is twice the radius, so the diameter is approximately 19.9 feet.

Now use the formula for the area of a circle.

$$A=\pi r^2$$
$$=(3.14)(9.95)^2 \quad \textit{Let } \pi=3.14,\ r=9.95.$$
$$=(3.14)(99.0025)$$
$$A\approx 311$$

The area of the turntable is approximately 311 square feet.

35. The sum of the three marked angles in the triangle is 180°.

$$45^\circ+(x+12.2)^\circ+(3x+2.8)^\circ=180^\circ$$
$$4x+60=180$$
$$4x=120$$
$$x=30$$

Since $x=30$, $(x+12.2)^\circ=42.2^\circ$, and $(3x+2.8)^\circ=92.8^\circ$.

36. Knowing the values of h and b is not enough information to find the value of A. We would also need to know the value of B. Note that B and b are different variables. In general, to find the numerical value of one variable in a formula, we need to know the values of all the other variables.

37. The ratio of 60 centimeters to 40 centimeters is

$$\frac{60\text{ cm}}{40\text{ cm}}=\frac{3\cdot 20}{2\cdot 20}=\frac{3}{2}.$$

38. To find the ratio of 5 days to 2 weeks, first convert 2 weeks to days.

$$2\text{ weeks}=2\cdot 7=14\text{ days}$$

Thus, the ratio of 5 days to 2 weeks is $\frac{5}{14}$.

39. To find the ratio of 90 inches to 10 feet, first convert 10 feet to inches.

$$10\text{ feet}=10\cdot 12=120\text{ inches}$$

Thus, the ratio of 90 inches to 10 feet is

$$\frac{90}{120}=\frac{3\cdot 30}{4\cdot 30}=\frac{3}{4}.$$

40. To find the ratio of 3 months to 3 years, first convert 3 years to months.

$$3\text{ years}=3\cdot 12=36\text{ months}$$

Thus, the ratio of 3 months to 3 years is

$$\frac{3}{36}=\frac{1\cdot 3}{12\cdot 3}=\frac{1}{12}.$$

41. $\frac{p}{21} = \frac{5}{30}$

$30p = 105$ *Cross products are equal.*

$\frac{30p}{30} = \frac{105}{30}$ *Divide by 30.*

$p = \frac{105}{30} = \frac{7 \cdot 15}{2 \cdot 15} = \frac{7}{2}$

The solution set is $\left\{\frac{7}{2}\right\}$.

42. $\frac{5+x}{3} = \frac{2-x}{6}$

$6(5+x) = 3(2-x)$ *Cross products are equal.*

$30 + 6x = 6 - 3x$ *Distributive property*

$30 + 9x = 6$ *Add 3x.*

$9x = -24$ *Subtract 30.*

$x = \frac{-24}{9} = -\frac{8}{3}$

The solution set is $\left\{-\frac{8}{3}\right\}$.

43. Let $x =$ the number of pounds of fertilizer needed to cover 500 square feet.

$$\frac{x \text{ pounds}}{2 \text{ pounds}} = \frac{500 \text{ square feet}}{150 \text{ square feet}}$$
$$150x = 2(500)$$
$$x = \frac{1000}{150} = \frac{20 \cdot 50}{3 \cdot 50}$$
$$= \frac{20}{3} = 6\tfrac{2}{3}$$

$6\frac{2}{3}$ pounds of fertilizer will cover 500 square feet.

44. Let $x =$ the tax on a \$36.00 item.
Set up a proportion with one ratio involving sales tax and the other involving the costs of the items.

$$\frac{x \text{ dollars}}{\$2.04} = \frac{\$36}{\$24}$$
$$24x = (2.04)(36) = 73.44$$
$$x = \frac{73.44}{24} = 3.06$$

The sales tax on a \$36.00 item is \$3.06.

45. Let $x =$ the actual distance between the second pair of cities (in kilometers).

Set up a proportion with one ratio involving map distances and the other involving actual distances.

$$\frac{x \text{ kilometers}}{150 \text{ kilometers}} = \frac{80 \text{ centimeters}}{32 \text{ centimeters}}$$
$$32x = (150)(80) = 12{,}000$$
$$x = \frac{12{,}000}{32} = 375$$

The cities are 375 kilometers apart.

46. Let $x =$ the number of bronze medals earned by China.

$$\frac{x \text{ bronze medals}}{63 \text{ medals}} = \frac{2 \text{ bronze medals}}{9 \text{ medals}}$$
$$9x = 2(63) = 126$$
$$x = 14$$

At the 2004 Olympics, 14 bronze medals were earned by China.

In Exercises 47–48, to find the best buy, divide the price by the number of units to get the unit cost. Each result was found by using a calculator and rounding the answer to three decimal places. The *best buy* (based on price per unit) is the smallest unit cost.

47.

Size	**Unit Cost** (dollars per oz)
15 oz	$\frac{\$2.69}{15} = \$0.179$
20 oz	$\frac{\$3.29}{20} = \$0.165$
25.5 oz	$\frac{\$3.49}{25.5} = \$0.137$ $(*)$

The 25.5 oz size is the best buy.

48.

Size	**Unit Cost** (dollars per oz)
32 oz	$\frac{\$1.95}{32} = \$0.061$
48 oz	$\frac{\$2.89}{48} = \$0.060$
64 oz	$\frac{\$3.29}{64} = \$0.051$ $(*)$

The 64 oz size is the best buy.

49. \$160 million is what percent of \$290 million?

$$\frac{160}{290} \approx 0.5517$$

Approximately 55.2% of the cost of the new stadium was borrowed.

50. Let $x =$ the number of liters of the 60% solution to be used.
Then $x + 15 =$ the number of liters of the 20% solution.

Liters of solution	15	+	x	=	$x + 15$
Strength of solution	0.10		0.60		0.20

Drug amount in 10% solution plus Drug amount in 60% solution is Drug amount in 20% solution

$$0.10(15) + 0.60(x) = 0.20(x + 15)$$

Multiply by 10 to clear decimals.

$$1(15) + 6x = 2(x + 15)$$
$$15 + 6x = 2x + 30$$
$$15 + 4x = 30$$
$$4x = 15$$
$$x = \frac{15}{4} = 3.75$$

3.75 liters of 60% solution are needed.

51. Let $x =$ the amount invested at 5%.
Then $10{,}000 - x =$ the amount invested at 6%.

Interest at 5%	plus	Interest at 6%	equals	\$550.
↓	↓	↓	↓	↓
$0.05x$	$+$	$0.06(10{,}000 - x)$	$=$	550

$$5x + 6(10{,}000 - x) = 100(550)$$
$$5x + 60{,}000 - 6x = 55{,}000$$
$$-x = -5000$$
$$x = 5000$$

Todd invested \$5000 at 5% and $10{,}000 - 5000 =$ \$5000 at 6%.

52. Use the formula $d = rt$ or $r = \dfrac{d}{t}$.

$$r = \frac{d}{t} = \frac{3150}{384} \approx 8.203$$

Rounded to the nearest tenth, the *Yorkshire's* average speed was 8.2 mph.

53. Use the formula $d = rt$ or $t = \dfrac{d}{r}$.

$$t = \frac{d}{r} = \frac{819}{63} = 13$$

Honey drove for 13 hours.

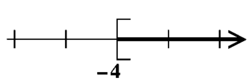
54. Let $t =$ the number of hours until the planes are 1925 miles apart.
Use $d = rt$.

The distance one plane flies north	plus	the distance the other plane flies south	is	the distance between the planes.
↓	↓	↓	↓	↓
$350t$	$+$	$420t$	$=$	1925

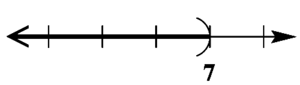
$$770t = 1925$$
$$t = \tfrac{1925}{770} = \tfrac{5}{2} = 2\tfrac{1}{2}$$

The planes will be 1925 miles apart in $2\frac{1}{2}$ hours.

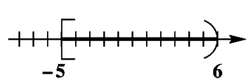
55. The statement $x \geq -4$ can be written as $[-4, \infty)$.



56. The statement $x < 7$ can be written as $(-\infty, 7)$.



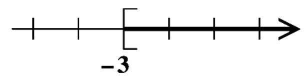
57. The statement $-5 \leq x < 6$ can be written as $[-5, 6)$.



58. By examining the choices, we see that $-4x \leq 36$ is the only inequality that has a negative coefficient of x. Thus, **B** is the only inequality that requires a reversal of the inequality symbol when it is solved.

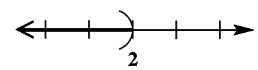
59.
$$x + 6 \geq 3$$
$$x \geq -3 \quad \textit{Subtract 6.}$$

Graph the solution set $[-3, \infty)$.



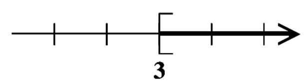
60.
$$5x < 4x + 2$$
$$x < 2 \qquad \textit{Subtract 4x.}$$

Graph the solution set $(-\infty, 2)$.



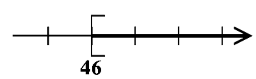
61.
$$-6x \leq -18$$
$$\frac{-6x}{-6} \geq \frac{-18}{-6} \quad \textit{Divide by –6; reverse the symbol.}$$
$$x \geq 3$$

Graph the solution set $[3, \infty)$.



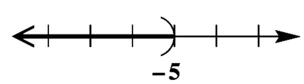
62.
$$8(x - 5) - (2 + 7x) \geq 4$$
$$8x - 40 - 2 - 7x \geq 4$$
$$x - 42 \geq 4$$
$$x \geq 46$$

Graph the solution set $[46, \infty)$.



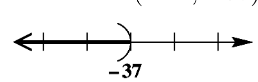
63.
$$4x - 3x > 10 - 4x + 7x$$
$$x > 10 + 3x$$
$$-2x > 10$$
$$\frac{-2x}{-2} < \frac{10}{-2} \quad \textit{Divide by –2; reverse the symbol.}$$
$$x < -5$$

Graph the solution set $(-\infty, -5)$.



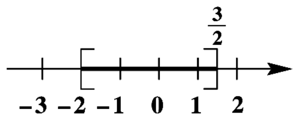
64.
$$3(2x + 5) + 4(8 + 3x) < 5(3x + 2) + 2x$$
$$6x + 15 + 32 + 12x < 15x + 10 + 2x$$
$$18x + 47 < 17x + 10$$
$$x + 47 < 10$$
$$x < -37$$

Graph the solution set $(-\infty, -37)$.



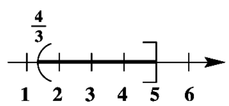
65. $-3 \le 2x + 1 \le 4$
$-4 \le 2x \le 3$ *Subtract 1.*
$-2 \le x \le \frac{3}{2}$ *Divide by 2.*

Graph the solution set $[-2, \frac{3}{2}]$.



66. $9 < 3x + 5 \le 20$
$4 < 3x \le 15$ *Subtract 5.*
$\frac{4}{3} < x \le 5$ *Divide by 3.*

Graph the solution set $\left(\frac{4}{3}, 5\right]$.



67. Let $x =$ the score on the third test.

The average of the three tests	is at least	90.
↓	↓	↓
$\frac{94 + 88 + x}{3}$	$\ge$	90

$$\frac{182 + x}{3} \ge 90$$
$$3\left(\frac{182 + x}{3}\right) \ge 3(90)$$
$$182 + x \ge 270$$
$$x \ge 88$$

In order to average at least 90, Carlotta's score on her third test must be 88 or more.

68. Let $n =$ the number.
"If nine times a number is added to 6, the result is at most 3" can be written as

$$9n + 6 \le 3.$$

Solve the inequality.

$9n \le -3$ *Subtract 6.*
$n \le \frac{-3}{9}$ *Divide by 9.*

All numbers less or equal to $-\frac{1}{3}$ satisfy the given condition.

69. **[2.6]** $\frac{x}{7} = \frac{x - 5}{2}$
$2x = 7(x - 5)$ *Cross products are equal.*
$2x = 7x - 35$
$-5x = -35$
$x = 7$

The solution set is $\{7\}$.

70. **[2.5]** $I = prt$ for r

$\frac{I}{pt} = \frac{prt}{pt}$ *Divide by pt.*

$\frac{I}{pt} = r$ or $r = \frac{I}{pt}$

71. **[2.8]** $-2x > -4$
$\frac{-2x}{-2} < \frac{-4}{-2}$ *Divide by –2; reverse the symbol.*
$x < 2$

The solution set is $(-\infty, 2)$.

72. **[2.3]** $2k - 5 = 4k + 13$
$-2k - 5 = 13$ *Subtract 4k.*
$-2k = 18$ *Add 5.*
$k = -9$ *Divide by –2.*

The solution set is $\{-9\}$.

73. **[2.2]** $0.05x + 0.02x = 4.9$
To clear decimals, multiply both sides by 100.

$$100(0.05x + 0.02x) = 100(4.9)$$
$$5x + 2x = 490$$
$$7x = 490$$
$$x = 70$$

The solution set is $\{70\}$.

74. **[2.3]** $2 - 3(x - 5) = 4 + x$
$2 - 3x + 15 = 4 + x$
$17 - 3x = 4 + x$
$17 - 4x = 4$
$-4x = -13$
$x = \frac{-13}{-4} = \frac{13}{4}$

The solution set is $\left\{\frac{13}{4}\right\}$.

75. **[2.3]** $9x - (7x + 2) = 3x + (2 - x)$
$9x - 7x - 2 = 3x + 2 - x$
$2x - 2 = 2x + 2$
$-2 = 2$

Because $-2 = 2$ is a false statement, the given equation has no solution, symbolized by $\emptyset$.

76. **[2.3]** $\frac{1}{3}s + \frac{1}{2}s + 7 = \frac{5}{6}s + 5 + 2$
$\frac{1}{3}s + \frac{1}{2}s = \frac{5}{6}s$ *Subtract 7.*
The least common denominator is 6.
$6\left(\frac{1}{3}s + \frac{1}{2}s\right) = 6\left(\frac{5}{6}s\right)$
$2s + 3s = 5s$
$5s = 5s$

Because $5s = 5s$ is a true statement, the solution set is $\{$all real numbers$\}$.

77. **[2.3]** Let $x = 6$ in the equation.

$$\begin{aligned} 3 - (8 + 4x) &= 2x + 7 \\ 3 - [8 + 4(6)] &= 2(6) + 7 \\ -29 &= 19 \end{aligned}$$

This is false, so $x = 6$ is not a solution of the equation.

Solve the equation.

$$\begin{aligned} 3 - (8 + 4x) &= 2x + 7 \\ 3 - 8 - 4x &= 2x + 7 \\ -5 - 4x &= 2x + 7 \\ -6x &= 12 \\ x &= -2 \end{aligned}$$

The solution set is $\{-2\}$. The student probably got the incorrect answer by writing

$$3 - (8 + 4x) = 3 - 8 + 4x$$

and then solving the equation, which *does* have solution set $\{6\}$.

78. **[2.6]** Let $x =$ the number of calories a 175-pound athlete can consume.
Set up a proportion with one ratio involving calories and the other involving pounds.

$$\begin{aligned} \frac{x \text{ calories}}{50 \text{ calories}} &= \frac{175 \text{ pounds}}{2.2 \text{ pounds}} \\ 2.2x &= 50(175) \\ x &= \tfrac{8750}{2.2} \approx 3977.3 \end{aligned}$$

To the nearest hundred calories, a 175-pound athlete in a vigorous training program can consume 4000 calories per day.

79. **[2.4]** Let $x =$ the length of the Brooklyn Bridge.
Then $x + 2604 =$ the length of the Golden Gate Bridge.

$$\begin{aligned} x + (x + 2604) &= 5796 \\ 2x + 2604 &= 5796 \\ 2x &= 3192 \\ x &= 1596 \end{aligned}$$

Since $x = 1596$, $x + 2604 = 4200$.

The length of the Brooklyn Bridge is 1596 feet and that of the Golden Gate Bridge is 4200 feet.

80. **[2.6]** The unit costs are rounded to four decimal places.

Size	**Unit Cost** (dollars per oz)
32 oz	$\frac{\$1.19}{32} = \$0.0372$
48 oz	$\frac{\$1.79}{48} = \$0.0373$
64 oz	$\frac{\$1.99}{64} = \$0.0311 \quad (*)$

The 64 ounce size is the best buy.

81. **[2.6]** Let $x =$ the number of quarts of oil needed for 192 quarts of gasoline.
Set up a proportion with one ratio involving oil and the other involving gasoline.

$$\begin{aligned} \frac{x \text{ quarts}}{1 \text{ quart}} &= \frac{192 \text{ quarts}}{24 \text{ quarts}} \\ x \cdot 24 &= 1 \cdot 192 && \textit{Cross products} \\ x &= 8 && \textit{Divide by 24.} \end{aligned}$$

The amount of oil needed is 8 quarts.

82. **[2.7]** Let $x =$ the speed of the slower train.
Then $x + 30 =$ the speed of the faster train.

	r	t	d
Slower train	x	3	$3x$
Faster train	$x + 30$	3	$3(x + 30)$

The sum of the distances traveled by the two trains is 390 miles, so

$$\begin{aligned} 3x + 3(x + 30) &= 390. \\ 3x + 3x + 90 &= 390 \\ 6x + 90 &= 390 \\ 6x &= 300 \\ x &= 50 \end{aligned}$$

Since $x = 50$, $x + 30 = 80$.

The speed of the slower train is 50 miles per hour and the speed of the faster train is 80 miles per hour.

83. **[2.5]** Let $x =$ the length of the first side.
Then $2x =$ the length of the second side.

Use the formula for the perimeter of a triangle, $P = a + b + c$, with perimeter 96 and third side 30.

$$\begin{aligned} x + 2x + 30 &= 96 \\ 3x + 30 &= 96 \\ 3x &= 66 \\ x &= 22 \end{aligned}$$

The sides have lengths 22 meters, 44 meters, and 30 meters. The length of the longest side is 44 meters.

84. **[2.8]** Let $s =$ the length of a side of the square.
The formula for the perimeter of a square is $P = 4s$.

The perimeter	cannot be greater than	200.
$\downarrow$	$\downarrow$	$\downarrow$
$4s$	$\leq$	200

$$\begin{aligned} 4s &\leq 200 \\ s &\leq 50 \end{aligned}$$

The length of a side is 50 meters or less.

Chapter 2 Test

1.
$$\begin{aligned} 5x + 9 &= 7x + 21 \\ -2x + 9 &= 21 \quad \textit{Subtract 7x} \\ -2x &= 12 \quad \textit{Subtract 9} \\ x &= -6 \quad \textit{Divide by –2} \end{aligned}$$

The solution set is $\{-6\}$.

2.
$$\begin{aligned} -\tfrac{4}{7}x &= -12 \\ \left(-\tfrac{7}{4}\right)\left(-\tfrac{4}{7}x\right) &= \left(-\tfrac{7}{4}\right)(-12) \\ x &= 21 \end{aligned}$$

The solution set is $\{21\}$.

3.
$$\begin{aligned} 7 - (x - 4) &= -3x + 2(x + 1) \\ 7 - x + 4 &= -3x + 2x + 2 \\ -x + 11 &= -x + 2 \end{aligned}$$

Because the last statement is false, the equation has no solution set, symbolized by $\emptyset$.

4. $0.6(x + 20) + 0.8(x - 10) = 46$
To clear decimals, multiply both sides by 10.

$$\begin{aligned} 10[0.6(x + 20) + 0.8(x - 10)] &= 10(46) \\ 6(x + 20) + 8(x - 10) &= 460 \\ 6x + 120 + 8x - 80 &= 460 \\ 14x + 40 &= 460 \\ 14x &= 420 \\ x &= 30 \end{aligned}$$

The solution set is $\{30\}$.

5.
$$\begin{aligned} -8(2x + 4) &= -4(4x + 8) \\ -16x - 32 &= -16x - 32 \end{aligned}$$

Because the last statement is true, the solution set is $\{\text{all real numbers}\}$.

6. Let $x =$ the number of games the Cardinals lost.
Then $2x - 24 =$ the number of games the Cardinals won.

The total number of games played was 162.

$$\begin{aligned} x + (2x - 24) &= 162 \\ 3x - 24 &= 162 \\ 3x &= 186 \\ x &= 62 \end{aligned}$$

Since $x = 62$, $2x - 24 = 100$.
The Cardinals won 100 games and lost 62 games.

7. Let $x =$ the area of Kauai (in square miles).
Then $x + 177 =$ the area of Maui (in square miles), and
$(x + 177) + 3293 = x + 3470 =$ the area of Hawaii.

$$\begin{aligned} x + (x + 177) + (x + 3470) &= 5300 \\ 3x + 3647 &= 5300 \\ 3x &= 1653 \\ x &= 551 \end{aligned}$$

Since $x = 551$, $x + 177 = 728$, and $x + 3470 = 4021$.

The area of Hawaii is 4021 square miles, the area of Maui is 728 square miles, and the area of Kauai is 551 square miles.

8. Let $x =$ the measure of the angle.
Then $90 - x =$ the measure of its complement, and $180 - x =$ the measure of its supplement.

$$\begin{aligned} 180 - x &= 3(90 - x) + 10 \\ 180 - x &= 270 - 3x + 10 \\ 180 - x &= 280 - 3x \\ 180 + 2x &= 280 \\ 2x &= 100 \\ x &= 50 \end{aligned}$$

The measure of the angle is 50°. The measure of its supplement, 130°, is 10° more than three times its complement, 40°.

9. **(a)** Solve $P = 2L + 2W$ for W.

$$\begin{aligned} P - 2L &= 2W \\ \frac{P - 2L}{2} &= W \quad \text{or} \quad W = \frac{P - 2L}{2} \end{aligned}$$

(b) Substitute 116 for P and 40 for L in the formula obtained in part (a).

$$\begin{aligned} W &= \frac{P - 2L}{2} \\ &= \frac{116 - 2(40)}{2} \\ &= \frac{116 - 80}{2} = \frac{36}{2} = 18 \end{aligned}$$

10. The angles are vertical angles, so their measures are equal.

$$\begin{aligned} 3x + 15 &= 4x - 5 \\ 15 &= x - 5 \\ 20 &= x \end{aligned}$$

Since $x = 20$, $3x + 15 = 75$ and $4x - 5 = 75$.

Both angles have measure 75°.

11.
$$\begin{aligned} \frac{z}{8} &= \frac{12}{16} \\ 16z &= 8(12) \quad \textit{Cross products are equal} \\ 16z &= 96 \\ \frac{16z}{16} &= \frac{96}{16} \quad \textit{Divide by 16} \\ z &= 6 \end{aligned}$$

The solution set is $\{6\}$.

12.
$$\begin{aligned} \frac{x + 5}{3} &= \frac{x - 3}{4} \\ 4(x + 5) &= 3(x - 3) \\ 4x + 20 &= 3x - 9 \\ x + 20 &= -9 \\ x &= -29 \end{aligned}$$

The solution set is $\{-29\}$.

13.

Size	**Unit Cost** (dollars per slice)
8 slices	$\frac{\$2.19}{8} = \$0.27375 \quad (*)$
12 slices	$\frac{\$3.30}{12} = \$0.275$

The better buy is 8 slices for $2.19.

14. Let $x =$ the actual distance between Seattle and Cincinnati.

$$\frac{x \text{ miles}}{1050 \text{ miles}} = \frac{92 \text{ inches}}{42 \text{ inches}}$$
$$42x = 92(1050) = 96{,}600$$
$$x = \tfrac{96{,}600}{42} = 2300$$

The actual distance between Seattle and Cincinnati is 2300 miles.

15. Let $x =$ the amount invested at 3%.
Then $x + 6000 =$ the amount invested at 4.5%.

Amount invested (in dollars)	**Rate of interest**	**Interest for one year**
x	0.03	$0.03x$
$x + 6000$	0.045	$0.045(x + 6000)$

$$0.03x + 0.045(x + 6000) = 870$$
$$1000(0.03x) + 1000[0.045(x + 6000)] = 1000(870)$$
$$30x + 45(x + 6000) = 870{,}000$$
$$30x + 45x + 270{,}000 = 870{,}000$$
$$75x + 270{,}000 = 870{,}000$$
$$75x = 600{,}000$$
$$x = 8000$$

Since $x = 8000$, $x + 6000 = 14{,}000$.

Keith invested $8000 at 3% and $14,000 at 4.5%.

16. Use the formula $d = rt$ and let t be the number of hours they traveled.

	r	***t***	***d***
First car	50	t	$50t$
Second car	65	t	$65t$

First car's distance	and	second car's distance
↓	↓	↓
$50t$	$+$	$65t$

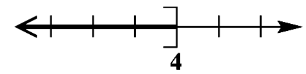
is	total distance.
↓	↓
$=$	460

$$50t + 65t = 460$$
$$115t = 460$$
$$t = 4$$

The two cars will be 460 miles apart in 4 hours.

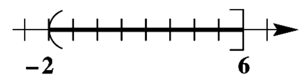
17.
$$-4x + 2(x - 3) \geq 4x - (3 + 5x) - 7$$
$$-4x + 2x - 6 \geq 4x - 3 - 5x - 7$$
$$-2x - 6 \geq -x - 10$$
$$-x - 6 \geq -10$$
$$-x \geq -4$$
$$\frac{-1x}{-1} \leq \frac{-4}{-1} \quad \textit{Divide by } -1; \textit{ reverse the symbol}$$
$$x \leq 4$$

Graph the solution set $(-\infty, 4]$.



18.
$$-10 < 3x - 4 \leq 14$$
$$-6 < 3x \leq 18 \qquad \textit{Add 4.}$$
$$-2 < x \leq 6 \qquad \textit{Divide by 3.}$$

Graph the solution set $(-2, 6]$.



19. Let $x =$ the score on the third test.

The average of the three tests	is at least	80.
↓	↓	↓
$\frac{76 + 81 + x}{3}$	$\geq$	80

$$\frac{157 + x}{3} \geq 80$$
$$3\left(\frac{157 + x}{3}\right) \geq 3(80)$$
$$157 + x \geq 240$$
$$x \geq 83$$

In order to average at least 80, Twylene's score on her third test must be 83 or more.

20. When an inequality is multiplied or divided by a negative number, the direction of the inequality symbol must be reversed.

Cumulative Review Exercises (Chapters 1–2)

1. $\frac{108}{144} = \frac{3 \cdot 36}{4 \cdot 36} = \frac{3}{4}$

2.
$$\frac{5}{6} + \frac{1}{4} - \frac{7}{15} = \frac{50}{60} + \frac{15}{60} - \frac{28}{60}$$
$$= \frac{65 - 28}{60}$$
$$= \tfrac{37}{60}$$

3. $\frac{9}{8} \cdot \frac{16}{3} \div \frac{5}{8} = \frac{9}{8} \cdot \frac{16}{3} \cdot \frac{8}{5}$
$= \frac{3 \cdot 3 \cdot 16 \cdot 8}{8 \cdot 3 \cdot 5}$
$= \frac{48}{5}$

4. "The difference between half a number and 18" is written

$$\tfrac{1}{2}x - 18.$$

5. "The quotient of 6 and 12 more than a number is 2" is written

$$\frac{6}{x+12} = 2.$$

6. $\frac{8(7) - 5(6+2)}{3 \cdot 5 + 1} \geq 1$
$\frac{8(7) - 5(8)}{3 \cdot 5 + 1} \geq 1$
$\frac{56 - 40}{15 + 1} \geq 1$
$\frac{16}{16} \geq 1$
$1 \geq 1$

The statement is *true*.

7. $9 - (-4) + (-2) = (9 + 4) + (-2)$
$= 13 - 2$
$= 11$

8. $\frac{-4(9)(-2)}{-3^2} = \frac{-36(-2)}{-1 \cdot 3^2}$
$= \frac{72}{-9}$
$= -8$

9. $(-7 - 1)(-4) + (-4) = (-8)(-4) + (-4)$
$= 32 + (-4)$
$= 28$

10. $\frac{3x^2 - y^3}{-4z} = \frac{3(-2)^2 - (-4)^3}{-4(3)}$ *Let x = −2, y = −4, z = 3.*
$= \frac{3(4) - (-64)}{-12}$
$= \frac{12 + 64}{-12}$
$= \frac{76}{-12}$
$= -\frac{19}{3}$

11. $7(k + m) = 7k + 7m$

The multiplication of 7 is distributed over the sum, which illustrates the distributive property.

12. $3 + (5 + 2) = 3 + (2 + 5)$

The order of the numbers added in the parentheses is changed, which illustrates the commutative property.

13. $-4(k + 2) + 3(2k - 1)$
$= (-4)(k) + (-4)(2) + (3)(2k) + (3)(-1)$
$= -4k - 8 + 6k - 3$
$= -4k + 6k - 8 - 3$
$= 2k - 11$

14. $2r - 6 = 8r$
$-6 = 6r$
$-1 = r$

Check $r = -1$: $-8 = -8$ *True*

The solution set is $\{-1\}$.

15. $4 - 5(a + 2) = 3(a + 1) - 1$
$4 - 5a - 10 = 3a + 3 - 1$
$-5a - 6 = 3a + 2$
$-8a - 6 = 2$
$-8a = 8$
$a = -1$

Check $a = -1$: $-1 = -1$ *True*

The solution set is $\{-1\}$.

16. $\frac{2}{3}x + \frac{3}{4}x = -17$
$12\left(\frac{2}{3}x + \frac{3}{4}x\right) = 12(-17)$ *LCD = 12*
$8x + 9x = -204$
$17x = -204$
$x = -12$

Check $x = -12$: $-17 = -17$ *True*

The solution set is $\{-12\}$.

17. $\frac{2x + 3}{5} = \frac{x - 4}{2}$
$(2x + 3)(2) = (5)(x - 4)$
$4x + 6 = 5x - 20$
$6 = x - 20$
$26 = x$

Check $x = 26$: $11 = 11$ *True*

The solution set is $\{26\}$.

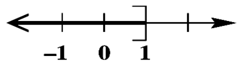
18. $3x + 4y = 24$ for y
$4y = 24 - 3x$
$y = \frac{24 - 3x}{4}$

19. $A = P(1 + ni)$ for n

$A = P + Pni$
$A - P = Pni$
$\frac{A - P}{Pi} = n$

20. $$\begin{aligned} 6(r-1)+2(3r-5) &\leq -4 \\ 6r-6+6r-10 &\leq -4 \\ 12r-16 &\leq -4 \\ 12r &\leq 12 \\ r &\leq 1 \end{aligned}$$

Graph the solution set $(-\infty, 1]$.

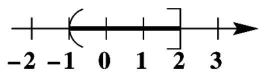


21. $$-18 \leq -9z < 9$$

$$2 \geq z > -1$$ *Divide by –9; reverse the symbols.*

or $-1 < z \leq 2$

Graph the solution set $(-1, 2]$.



22. Let $x =$ the length of the middle-sized piece.
Then $3x =$ the length of the longest piece, and
$x - 5 =$ the length of the shortest piece.

$$\begin{aligned} x+3x+(x-5) &= 40 \\ 5x-5 &= 40 \\ 5x &= 45 \\ x &= 9 \end{aligned}$$

The length of the middle-sized piece is 9 centimeters, of the longest piece is 27 centimeters, and of the shortest piece is 4 centimeters.

23. Let $r =$ the radius and use 3.14 for π.
Using the formula for circumference, $C = 2\pi r$, and $C = 78$, we have

$$\begin{aligned} 2\pi r &= 78. \\ r &= \frac{78}{2\pi} \approx 12.4204 \end{aligned}$$

To the nearest hundredth, the radius is 12.42 cm.

24. $$\frac{x \text{ cups}}{1\frac{1}{4} \text{ cups}} = \frac{20 \text{ people}}{6 \text{ people}}$$

$$\begin{aligned} 6x &= \left(1\tfrac{1}{4}\right)(20) \\ 6x &= 25 \\ x &= \tfrac{25}{6}, \quad \text{or} \quad 4\tfrac{1}{6} \text{ cups} \end{aligned}$$

$4\frac{1}{6}$ cups of cheese will be needed to serve 20 people.

25. Let $x =$ speed of slower car.
Then $x + 20 =$ speed of faster car.

Use the formula $d = rt$.

$$\begin{aligned} d_{\text{slower}} + d_{\text{faster}} &= d_{\text{total}} \\ (x)(4) + (x+20)(4) &= 400 \\ 4x + 4x + 80 &= 400 \\ 8x + 80 &= 400 \\ 8x &= 320 \\ x &= 40 \end{aligned}$$

The speeds are 40 mph and 60 mph.