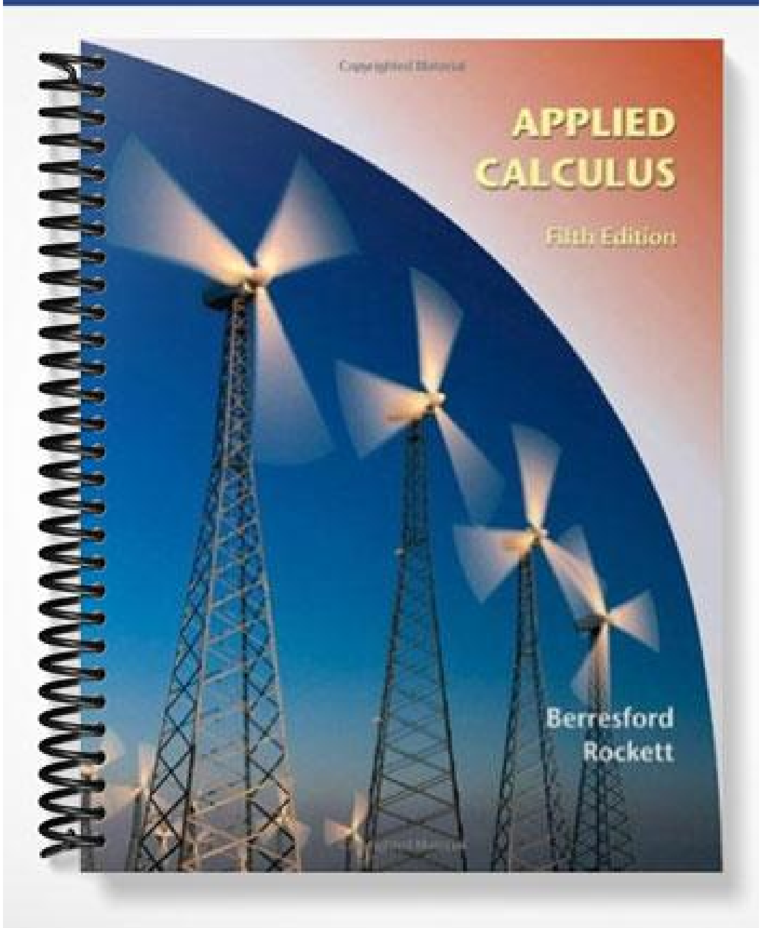
SOLUTIONS MANUAL



Chapter 2: Derivatives and Their Uses

EXERCISES 2.1

1.

x	$5x-7$
1.9	2.500
1.99	2.950
1.999	2.995

x	$5x-7$
2.1	3.500
2.01	3.050
2.001	3.005

a. $\lim_{x\to 2^-}(5x-7)=3$

b. $\lim_{x\to 2^+}(5x-7)=3$

c. $\lim_{x\to 2}(5x-7)=3$

2.

x	$2x+1$
3.9	8.800
3.99	8.980
3.999	8.998

x	$2x+1$
4.100	9.200
4.010	9.020
4.001	9.002

a. $\lim_{x\to 4^-}(2x+1)=9$

b. $\lim_{x\to 4^+}(2x+1)=9$

c. $\lim_{x\to 4}(2x+1)=9$

3.

x	$\frac{x^3-1}{x-1}$
0.900	2.710
0.990	2.970
0.999	2.997

x	$\frac{x^3-1}{x-1}$
1.1	3.310
1.01	3.030
1.001	3.003

a. $\lim_{x\to 1^-}\frac{x^3-1}{x-1}=3$

b. $\lim_{x\to 1^+}\frac{x^3-1}{x-1}=3$

c. $\lim_{x\to 1}\frac{x^3-1}{x-1}=3$

4.

x	$\frac{x^4-1}{x-1}$
0.9	3.439
0.99	3.940
0.999	3.994

x	$\frac{x^4-1}{x-1}$
1.1	4.641
1.01	4.060
1.001	4.006

a. $\lim_{x\to 1^-}\frac{x^4-1}{x-1}=4$

b. $\lim_{x\to 1^+}\frac{x^4-1}{x-1}=4$

c. $\lim_{x\to 1}\frac{x^4-1}{x-1}=4$

5.

x	$(1+2x)^{1/x}$
–0.1	9.313
–0.01	7.540
–0.001	7.404

x	$(1+2x)^{1/x}$
0.1	6.192
0.01	7.245
0.001	7.374

$\lim_{x\to 0}(1+2x)^{1/x}\approx 7.4$

6.

x	$(1-x)^{1/x}$
–0.1	0.386
–0.01	0.370
–0.001	0.368

x	$(1-x)^{1/x}$
0.1	0.349
0.01	0.366
0.001	0.368

$\lim_{x\to 0}(1-x)^{1/x}\approx 0.368$

7.

x	$\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$
1.9	–0.263
1.99	–0.251
1.999	–0.250

x	$\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$
2.1	-0.238
2.01	-0.249
2.001	-0.250

$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}=-0.25$

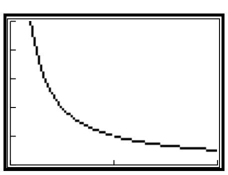
8.

x	$\frac{\sqrt{x}-1}{x-1}$
0.9	0.513
0.99	0.501
0.999	0.500

x	$\frac{\sqrt{x}-1}{x-1}$
1.1	0.488
1.01	0.499
1.001	0.500

$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=0.5$

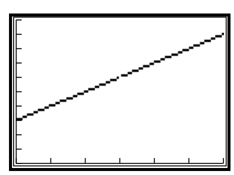
9.



on [0, 2] by [0, 5]

$\lim_{x\to 1}\frac{\frac{1}{x}-1}{1-x}=1$

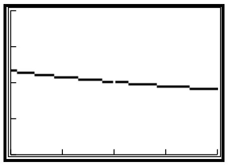
10.



on [0, 3] by [0, 10]

$\lim_{x\to 1.5}\frac{2x^2-4.5}{x-1.5}=6$

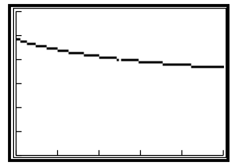
11.



on [3, 5] by [1, 3]

$$\lim_{x\to 4}\frac{x^{1.5}-4x^{0.5}}{x^{1.5}-2x}=2$$

12.



on [0.5, 1.5] by [0, 3]

$$\lim_{x\to 1}\frac{x-1}{x-\sqrt{x}}=2$$

13. $\lim_{x\to 3} 4x^2-10x+2=4(3)^2-10(3)+2=8$

14. $\lim_{x\to 7}\frac{x^2-x}{2x-7}=\frac{(7)^2-7}{2(7)-7}=6$

15. $\lim_{x\to 5}\frac{3x^2-5x}{7x-10}=\frac{3(5)^2-5(5)}{7(5)-10}=2$

16. $\lim_{t\to 3}\sqrt[3]{t^2+t-4}=\sqrt[3]{(3)^2+3-4}=2$

17. $\lim_{x\to 3}\sqrt{2}=\sqrt{2}$ because the limit of a constant is just the constant.

18. $\lim_{q\to 9}\frac{8+2\sqrt{q}}{8-2\sqrt{q}}=\frac{8+2\sqrt{9}}{8-2\sqrt{9}}=\frac{14}{2}=7$

19. $\lim_{t\to 25}\left[(t+5)t^{-1/2}\right]=(25+5)(25)^{-1/2}=6$

20. $\lim_{s\to 4}\left(s^{3/2}-3s^{1/2}\right)=\left[4^{3/2}-3(4)^{1/2}\right]=2$

21. $\lim_{h\to 0}(5x^3+2x^2h-xh^2)=5x^3+2x^2\cdot 0-x(0)^2=5x^3$

22. $\lim_{h\to 0}(2x^2+4xh+h^2)=2x^2+4x\cdot 0+(0)^2=2x^2$

23.
$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x+2)(x-2)}{x-2}=\lim_{x\to 2}(x+2)=2+2=4$$

24.
$$\lim_{x\to 1}\frac{x-1}{x^2+x-2}=\lim_{x\to 1}\frac{x-1}{(x+1)(x-1)}=\lim_{x\to 1}\frac{1}{x+2}=\frac{1}{1+2}=\frac{1}{3}$$

25.
$$\lim_{x\to -3}\frac{x+3}{x^2+8x+15}=\lim_{x\to -3}\frac{x+3}{(x+3)(x+5)}=\lim_{x\to -3}\frac{1}{x+5}=\frac{1}{-3+5}=\frac{1}{2}$$

26.
$$\lim_{x\to -4}\frac{x^2+9x+20}{x+4}=\lim_{x\to -4}\frac{(x+5)(x+4)}{x+4}=\lim_{x\to -4}x+5=-4+5=1$$

27.
$$\lim_{x\to -1}\frac{3x^3-3x^2-6x}{x^2+x}=\lim_{x\to -1}\frac{3x(x^2-x-2)}{x(x+1)}=\lim_{x\to -1}\frac{3x(x-2)(x+1)}{x(x+1)}=\lim_{x\to -1}3(x-2)=3(-1-2)=-9$$

28.
$$\lim_{x\to 0}\frac{x^2-x}{x^2+x}=\lim_{x\to 0}\frac{x(x-1)}{x(x+1)}=\lim_{x\to 0}\frac{x-1}{x+1}=-1$$

29.
$$\lim_{h\to 0}\frac{2xh-3h^2}{h}=\lim_{h\to 0}(2x-3h)=2x-3(0)=2x$$

30.
$$\lim_{h\to 0}\frac{5x^4h-9xh^2}{h}=\lim_{h\to 0}(5x^4-9xh)=5x^4-9x(0)=5x^4$$

31.
$$\lim_{h\to 0}\frac{4x^2h+xh^2-h^3}{h}=\lim_{h\to 0}(4x^2+xh-h^2)=4x^2+x(0)-(0)^2=4x^2$$

32.
$$\lim_{h\to 0}\frac{x^2h-xh^2+h^3}{h}=\lim_{h\to 0}(x^2-xh+h^2)=x^2-x(0)+(0)^2=x^2$$

33. **a.** $\lim_{x\to 2^-} f(x) = 1$

b. $\lim_{x\to 2^+} f(x) = 3$

c. $\lim_{x\to 2} f(x)$ does not exist.

34. **a.** $\lim_{x\to 2^-} f(x) = 1$

b. $\lim_{x\to 2^+} f(x) = 2$

c. $\lim_{x\to 2} f(x)$ does not exist.

35. **a.** $\lim_{x\to 2^-} f(x) = -1$

b. $\lim_{x\to 2^+} f(x) = -1$

c. $\lim_{x\to 2} f(x) = -1$

36. **a.** $\lim_{x\to 2^-} f(x) = 3$

b. $\lim_{x\to 2^+} f(x) = 3$

c. $\lim_{x\to 2} f(x) = 3$

37. **a.** $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}(3 - x) = 3 - (4) = -1$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}(10 - 2x) = 10 - 2(4) = 2$

c. $\lim_{x\to 4} f(x)$ does not exist.

38. **a.** $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}(5 - x) = 5 - 4 = 1$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}(2x - 5) = 2(4) - 5 = 3$

c. $\lim_{x\to 4} f(x)$ does not exist.

39. **a.** $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}(2 - x) = 2 - 4 = -2$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}(x - 6) = 4 - 6 = -2$

c. $\lim_{x\to 4} f(x) = -2$

40. **a.** $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}(2 - x) = 2 - 4 = -2$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}(2x - 10) = 2(4) - 10 = -2$

c. $\lim_{x\to 4} f(x) = -2$

41. **a.** $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(-x) = -0 = 0$

b. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(x) = 0$

c. $\lim_{x\to 0} f(x) = 0$

42. **a.** $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}[-(-x)] = -(-0) = 0$

b. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} -(x) = -0 = 0$

c. $\lim_{x\to 0} f(x) = 0$

43. **a.**

x	$\frac{\lvert x\rvert}{x}$
–0.1	–1
–0.01	–1
–0.001	–1

$\lim_{x\to 0} f(x) = -1$

b.

x	$\frac{\lvert x\rvert}{x}$
0.1	1
0.01	1
0.001	1

$\lim_{x\to 0} f(x) = 1$

c. $\lim_{x\to 0} f(x)$ does not exist.

44. **a.**

x	$-\frac{\lvert x\rvert}{x}$
–0.1	1
–0.01	1
–0.001	1

$\lim_{x\to 0} f(x) = 1$

b.

x	$-\frac{\lvert x\rvert}{x}$
0.1	–1
0.01	–1
0.001	–1

$\lim_{x\to 0} f(x) = -1$

c. $\lim_{x\to 0} f(x)$ does not exist.

45. $\lim_{x\to -\infty} f(x) = 0$; $\lim_{x\to 3^-} f(x) = \infty$ and $\lim_{x\to 3^+} f(x) = -\infty$, so $\lim_{x\to 3} f(x)$ does not exist and $\lim_{x\to\infty} f(x) = 0$.

46. $\lim_{x\to -\infty} f(x) = 0$; $\lim_{x\to -2^-} f(x) = \infty$ and $\lim_{x\to -2^+} f(x) = \infty$, so $\lim_{x\to -2} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = 0$.

47. $\lim\limits_{x\to-\infty} f(x)=0;\quad \lim\limits_{x\to 0^-} f(x)=\infty$ and $\lim\limits_{x\to 0^+} f(x)=\infty$,
so $\lim\limits_{x\to 0} f(x)=\infty$ and $\lim\limits_{x\to\infty} f(x)=0$.

48. $\lim\limits_{x\to-\infty} f(x)=0;\quad \lim\limits_{x\to 0^-} f(x)=-\infty$ and $\lim\limits_{x\to 0^+} f(x)=\infty$,
so $\lim\limits_{x\to 0} f(x)$ does not exist and $\lim\limits_{x\to\infty} f(x)=0$.

49. $\lim\limits_{x\to-\infty} f(x)=2;\quad \lim\limits_{x\to 1^-} f(x)=-\infty$ and $\lim\limits_{x\to 1^+} f(x)=\infty$,
so $\lim\limits_{x\to 1} f(x)$ does not exist and $\lim\limits_{x\to\infty} f(x)=2$.

50. $\lim\limits_{x\to-\infty} f(x)=1;\quad \lim\limits_{x\to -3^-} f(x)=\infty$ and $\lim\limits_{x\to -3^+} f(x)=-\infty$,
so $\lim\limits_{x\to -3} f(x)$ does not exist and $\lim\limits_{x\to\infty} f(x)=1$.

51. $\lim\limits_{x\to-\infty} f(x)=1;\quad \lim\limits_{x\to -2^-} f(x)=\infty$ and $\lim\limits_{x\to -2^+} f(x)=\infty$,
so $\lim\limits_{x\to -2} f(x)=\infty$ and $\lim\limits_{x\to\infty} f(x)=1$.

52. $\lim\limits_{x\to-\infty} f(x)=2;\quad \lim\limits_{x\to 1^-} f(x)=\infty$ and $\lim\limits_{x\to 1^+} f(x)=\infty$,
so $\lim\limits_{x\to 1} f(x)=\infty$ and $\lim\limits_{x\to\infty} f(x)=2$.

53. Continuous

54. Discontinuous at c because $f(c)$ is not defined.

55. Discontinuous at c because $\lim\limits_{x\to c} f(x)\neq f(c)$.

56. Discontinuous at c because $\lim\limits_{x\to c} f(x)$ does not exist.

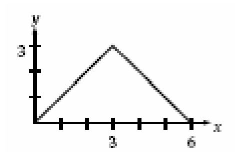
57. Discontinuous at c because $f(c)$ is not defined.

58. Continuous

59. Discontinuous at c because $\lim\limits_{x\to c} f(x)$ does not exist.

60. Discontinuous at c because $\lim\limits_{x\to c} f(x)\neq f(c)$.

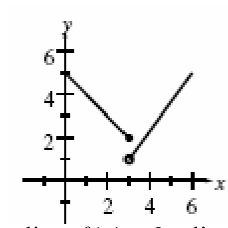
61. **a.**



b. $\lim\limits_{x\to 3^-} f(x)=3;\ \lim\limits_{x\to 3^+} f(x)=3$

c. Continuous

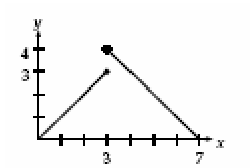
62. **a.**



b. $\lim\limits_{x\to 3^-} f(x)=2;\ \lim\limits_{x\to 3^+} f(x)=1$

c. Discontinuous because $\lim\limits_{x\to 3} f(x)$ does not exist.

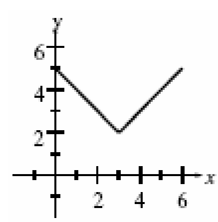
63. **a.**



b. $\lim\limits_{x\to 3^-} f(x)=3;\ \lim\limits_{x\to 3^+} f(x)=4$

c. Discontinuous because $\lim\limits_{x\to 3} f(x)$ does not exist.

64. **a.**



b. $\lim\limits_{x\to 3^-} f(x)=2;\ \lim\limits_{x\to 3^+} f(x)=2$

c. Continuous

65. Continuous

66. Continuous

67. Discontinuous at $x = 1$

68. Discontinuous at $x = -7$ and $x = 2$

69. $f(x)=\frac{12}{5x^3-5x}$ is discontinuous at values of x for which the denominator is zero. Thus, consider

$$5x^3-5x=0$$
$$5x(x^2-1)=0$$

$5x$ equals zero at $x=0$ and x^2-1 equals zero at $x=\pm1$.
Thus, the function is discontinuous at $x=0$, $x=-1$, and $x=1$.

70. $f(x)=\frac{x+2}{x^4-3x^3-4x^2}$ is discontinuous at values of x for which the denominator is zero. Thus, consider

$$x^4-3x^3-4x^2=0$$
$$x^2\left(x^2-3x-4\right)=0$$
$$x^2(x-4)(x+1)=0$$

x^2 equals zero at $x=0$, $x-4$ equals zero at $x=4$, and $x+1$ equals zero at $x=-1$.
Thus, the function is discontinuous at $x=0$, $x=4$, and $x=-1$.

71. From Exercise 37, we know $\lim\limits_{x\to4} f(x)$ does not exist. Therefore, the function is discontinuous at $x=4$.

72. From Exercise 38, we know $\lim\limits_{x\to4} f(x)$ does not exist. Therefore, the function is discontinuous at $x=4$.

73. From Exercise 39, we know $\lim\limits_{x\to4} f(x)=-2=f(4)$. Therefore, the function is continuous.

74. From Exercise 40, we know $\lim\limits_{x\to4} f(x)=-2=f(4)$. Therefore, the function is continuous.

75. From Exercise 41, we know $\lim\limits_{x\to0} f(x)=0=f(0)$. Therefore, the function is continuous.

76. From Exercise 43, we know $\lim\limits_{x\to0} f(x)$ does not exist. Therefore the function is discontinuous at $x=0$.

77. From the graph, we can see that $\lim\limits_{x\to6} f(x)$ does not exist because the left and right limits do not agree. $f(x)$ is discontinuous at $x=6$.

78. From the graph, we can see that $\lim\limits_{x\to7} f(x)$ does not exist because the left and right limits do not agree. $f(x)$ is discontinuous at $x=7$.

79. Since the function $\frac{(x-1)(x+2)}{x-1}$ is not defined at $x=1$ and the function $x+2$ equals 3 at $x=1$, the functions are not equal.

80. $\lim\limits_{v\to c}\sqrt{1-\left(\frac{v}{c}\right)^2}=\sqrt{1-\left(\frac{c}{c}\right)^2}=0$

81.

x	$\left(1+\frac{x}{10}\right)^{1/x}$
–0.1	1.11
–0.01	1.11
–0.001	1.11
–0.0001	1.11

x	$\left(1+\frac{x}{10}\right)^{1/x}$
0.1	1.10
0.01	1.11
0.001	1.11
0.0001	1.11

$\lim\limits_{x\to0}\left(1+\frac{x}{10}\right)^{1/x}\approx \$1.11$

82.

x	$\left(1+\frac{x}{20}\right)^{1/x}$
–0.1	1.05
–0.01	1.05
–0.001	1.05
–0.0001	1.05

x	$\left(1+\frac{x}{20}\right)^{1/x}$
0.1	1.05
0.01	1.05
0.001	1.05
0.0001	1.05

$\lim\limits_{x\to0}\left(1+\frac{x}{20}\right)^{1/x}\approx \$1.05$

83. $$\lim_{x\to0}\frac{100}{1+.001x^2}=\frac{100}{1+.001(0)^2}=\frac{100}{1}$$
$$=100$$

84. The left and right limits at 1 ounce and a 2 ounces do not agree. This function is discontinuous at 1 ounce and at 2 ounces.

85. As x approaches c, the function is approaching $\lim\limits_{x\to c} f(x)$ even if the value of the function at c is different, so the limit is where the function is "going".

86. In a continuous function, when x equals c, the function equals $\lim\limits_{x\to c} f(x)$. This is not true for a discontinuous function.

85. As x approaches c, the function is approaching $\lim_{x\to c} f(x)$ even if the value of the function at c is different, so the limit is where the function is "going".

86. In a continuous function, when x equals c, the function equals $\lim_{x\to c} f(x)$. This is not true for a discontinuous function.

87. False: The value of the function at 2 has nothing to do with the limit as x approaches 2.

88. False: There could be a "hole" or "jump" at $x = 2$.

89. False: Both one-sided limits would have to exist and agree to guarantee that the limit exists.

90. True: If $\lim_{x\to 2} f(x) = 7$, then $\lim_{x\to 2^+} f(x) = 7$ and $\lim_{x\to 2^-} f(x) = 7$.

91. False: On the left side of the limit exists and equals 2 (as we saw in Example 4), but on the right side of the denominator of the fraction is zero. Therefore one side of the equation is defined and the other is not.

92. True: A function must be defined at $x = c$ to be continuous at $x = c$.

93. True: The third requirement for continuity at $x = 2$ is that the limit and the value at 2 must agree, so if one is 7 the other must be 7.

94. True: If a function is continuous at every x-value, then its graph has no jumps or breaks. The jumps or breaks would make it discontinuous.

95. $\lim_{x\to 0^+} f(x)$ does not exist; $\lim_{x\to 0^-} f(x)$ does not exist; $\lim_{x\to 0} f(x)$ does not exist

EXERCISES 2.2

1. The slope is positive at P_1.
The slope is negative at P_2.
The slope is zero at P_3.

2. The slope is zero at P_1.
The slope is positive at P_2.
The slope is negative at P_3.

3. The slope is positive at P_1.
The slope is negative at P_2.
The slope is zero at P_3.

4. The slope is negative at P_1.
The slope is zero at P_2.
The slope is negative at P_3.

5. The tangent line at P_1 contains the points (0, 2) and (1, 5). The slope of this line is

$$m = \frac{5-2}{1-0} = 3.$$

The slope of the curve at P_1 is 3.
The tangent line at P_2 contains the points (3, 5) and (5, 4). The slope of this line is

$$m = \frac{4-5}{5-3} = -\frac{1}{2}$$

The slope of the curve at P_2 is $-\frac{1}{2}$.

6. The tangent line at P_1 contains the points (1, 4) and (4, 3). The slope of this line is

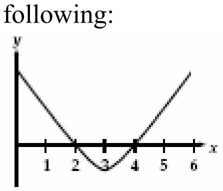
$$m = \frac{3-4}{4-1} = -\frac{1}{3}.$$

The slope of the curve at P_1 is $-\frac{1}{3}$.
The tangent line at P_2 contains the points (5, 3) and (6, 5). The slope of this line is

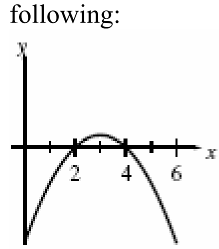
$$m = \frac{5-3}{6-5} = 2$$

The slope of the curve at P_2 is 2.

7. Your graph should look roughly like the following:



8. Your graph should look roughly like the following:



9.
a. $\frac{f(3)-f(1)}{2}=\frac{12-2}{2}=5$
b. $\frac{f(2)-f(1)}{1}=\frac{6-2}{1}=4$
c. $\frac{f(1.5)-f(1)}{.5}=\frac{3.75-2}{.5}=3.5$
d. $\frac{f(1.1)-f(1)}{.1}=\frac{2.31-2}{.1}=3.1$
e. $\frac{f(1.01)-f(1)}{.01}=\frac{2.0301-2}{.01}=3.01$
f. Answers seem to be approaching 3.

10.
a. $\frac{f(3)-f(1)}{2}=\frac{23-7}{2}=8$
b. $\frac{f(2)-f(1)}{1}=\frac{13-7}{1}=6$
c. $\frac{f(1.5)-f(1)}{.5}=\frac{9.5-7}{.5}=5$
d. $\frac{f(1.1)-f(1)}{.1}=\frac{7.42-7}{.1}=4.2$
e. $\frac{f(1.01)-f(1)}{.01}=\frac{7.0402-7}{.01}=4.02$
f. Answers seem to be approaching 4.

11.
a. $\frac{f(4)-f(2)}{2}=\frac{34-8}{2}=13$
b. $\frac{f(3)-f(2)}{1}=\frac{19-8}{1}=11$
c. $\frac{f(2.5)-f(2)}{.5}=\frac{13-8}{.5}=10$
d. $\frac{f(2.1)-f(2)}{.1}=\frac{8.92-8}{.1}=9.2$
e. $\frac{f(2.01)-f(1)}{.01}=\frac{8.0902-8}{.01}=9.02$
f. Answers seen to be approaching 9.

12.
a. $\frac{f(4)-f(2)}{2}=\frac{23-7}{2}=8$
b. $\frac{f(3)-f(2)}{1}=\frac{14-7}{1}=7$
c. $\frac{f(2.5)-f(2)}{.5}=\frac{10.25-7}{.5}=6.5$
d. $\frac{f(2.1)-f(2)}{.1}=\frac{7.61-7}{.1}=6.1$
e. $\frac{f(2.01)-f(1)}{.01}=\frac{7.0601-7}{.01}=6.01$
f. Answers seem to be approaching 6.

13.
a. $\frac{f(5)-f(3)}{2}=\frac{26-16}{2}=5$
b. $\frac{f(4)-f(3)}{1}=\frac{21-16}{1}=5$
c. $\frac{f(3.5)-f(3)}{.5}=\frac{18.5-16}{.5}=5$
d. $\frac{f(3.1)-f(3)}{.1}=\frac{16.5-16}{.1}=5$
e. $\frac{f(3.01)-f(3)}{.01}=\frac{16.05-16}{.01}=5$
f. Answers seem to be approaching 5.

14.
a. $\frac{f(5)-f(3)}{2}=\frac{33-19}{2}=7$
b. $\frac{f(4)-f(3)}{1}=\frac{26-19}{1}=7$
c. $\frac{f(3.5)-f(3)}{.5}=\frac{22.5-19}{.5}=7$
d. $\frac{f(3.1)-f(3)}{.1}=\frac{19.7-19}{.1}=7$
e. $\frac{f(3.01)-f(3)}{.01}=\frac{19.07-19}{.01}=7$
f. Answers seem to be approaching 7.

15.
a. $\frac{f(6)-f(4)}{2}=\frac{2.4495-2}{2}=0.2247$
b. $\frac{f(5)-f(4)}{1}=\frac{2.236-2}{1}=0.2361$
c. $\frac{f(4.5)-f(4)}{.5}=\frac{2.121-2}{.5}=0.2426$
d. $\frac{f(4.1)-f(4)}{.1}=\frac{2.025-2}{.1}=0.2485$
e. $\frac{f(4.01)-f(4)}{.01}=\frac{2.0025-2}{.01}=0.2498$
f. Answers seem to approach 0.25.

16.
a. $\frac{f(6)-f(4)}{2}=\frac{.6667-1}{2}=-0.1667$
b. $\frac{f(5)-f(4)}{1}=\frac{.8-1}{1}=-0.2000$
c. $\frac{f(4.5)-f(4)}{.5}=\frac{.889-1}{.5}=-0.2222$
d. $\frac{f(4.1)-f(4)}{.1}=\frac{.976-1}{.1}=-0.2439$
e. $\frac{f(4.01)-f(4)}{.01}=\frac{.9975-1}{.01}=-0.2494$
f. Answers seem to approach –0.25.

17. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} \frac{(x+h)^2+(x+h)-(x^2+x)}{h}$
$= \lim_{h\to 0} \frac{x^2+2xh+h^2+x+h-x^2-x}{h}$
$= \lim_{h\to 0} \frac{2xh+h^2+h}{h}$
$= \lim_{h\to 0} 2x+h+1 = 2x+1$
Evaluating at $x=1$ gives $2(1)+1=3$, which matches the answer from Exercise 9.

18. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} \frac{(x+h)^2+2(x+h)-1-(x^2+2x-1)}{h}$
$= \lim_{h\to 0} \frac{x^2+2xh+h^2+2x+2h-1-x^2-2x+1}{h}$
$= \lim_{h\to 0} \frac{2xh+h^2+2h}{h}$
$= \lim_{h\to 0} 2x+h+2 = 2x+2$
Evaluating at $x=2$ gives $2(2)+2=6$, which matches the answer from Exercise 12.

19. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} \frac{5(x+h)+1-(5x+1)}{h}$
$= \lim_{h\to 0} \frac{5x+5h+1-5x-1}{h}$
$= \lim_{h\to 0} \frac{5h}{h}$
$= \lim_{h\to 0} 5 = 5$
Evaluating at x=3 gives 5, which matches the answer from Exercise 13.

20. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} \frac{\frac{4}{(x+h)}-\frac{4}{x}}{h}$
$= \lim_{h\to 0} \frac{1}{h}\left(\frac{4}{x+h}-\frac{4}{x}\right)$
$= \lim_{h\to 0} \frac{1}{h}\left(\frac{4x-4(x+h)}{(x+h)x}\right)$
$= \lim_{h\to 0} \frac{4x-4x-4h}{h(x^2+xh)}$
$= \lim_{h\to 0} \frac{-4}{x^2+xh} = -\frac{4}{x^2}$
Evaluating at x=4 gives $-\frac{4}{4^2}$=-.25, which matches the answer from Exercise 16.

21. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} \frac{2(x+h)^2+(x+h)-2-(2x^2+x-2)}{h}$
$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+x+h-2-2x^2-x+2}{h}$
$= \lim_{h\to 0} \frac{4xh+2h^2+h}{h}$
$= \lim_{h\to 0} 4x+2h+1 = 4x+1$
The slope of the tangent line is at $x=2$ is $4(2)+1=9$, which matches the answer from Exercise 11.

22. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2+5-(2x^2+5)}{h}$
$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+5-2x^2-5}{h}$
$= \lim_{h\to 0} \frac{4xh+2h^2}{h}$
$= \lim_{h\to 0} 4x+2h = 4x$
The slope of the tangent line at $x=1$ is $4(1)=4$, which matches the answer from Exercise 10.

23. $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim\limits_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim\limits_{h \to 0} \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$

$= \lim\limits_{h \to 0} \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim\limits_{h \to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+0}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

The slope of the tangent line at $x = 4$ is $\frac{1}{2\sqrt{4}} = 0.25$, which matches the answer from Exercise 15.

24. $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h \to 0} \frac{7(x+h)-2-(7x-2)}{h}$

$= \lim\limits_{h \to 0} \frac{7x+7h-2-7x+2}{h}$

$= \lim\limits_{h \to 0} \frac{7h}{h}$

$= \lim\limits_{h \to 0} 7 = 7$

The slope of the tangent line at $x = 3$ is 7, which matches the answer from Exercise 14.

25. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-3(x+h)+5-(x^2-3x+5)}{h}$$
$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-3x-3h+5-x^2+3x-5}{h}$$
$$= \lim_{h\to 0}(2x+h-3)$$
$$= 2x-3$$

26. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2-5(x+h)+1-(2x^2-5x+1)}{h}$$
$$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2-5x-5h+1-2x^2+5x-1}{h}$$
$$= \lim_{h\to 0}(4x+2h-5)$$
$$= 4x-5$$

27. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{1-(x+h)^2-\left(1-x^2\right)}{h}$$
$$= \lim_{h\to 0} \frac{1-x^2-2xh-h^2-1+x^2}{h}$$
$$= \lim_{h\to 0}(-2x-h)$$
$$= -2x$$

28. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{1}{2}(x+h)^2+1-\left(\frac{1}{2}x^2+1\right)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2+1-\frac{1}{2}x^2-1}{h}$$
$$= \lim_{h\to 0} x+\tfrac{1}{2}h = x$$

29. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{9(x+h)-2-(9x-2)}{h}$$
$$= \lim_{h\to 0} \frac{9x+9h-2-9x+2}{h} = 9$$

30. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{-3(x+h)+5-(-3x+5)}{h}$$
$$= \lim_{h\to 0} \frac{-3x-3h+5+3x-5}{h} = -3$$

31. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{x+h}{2}-\frac{x}{2}}{h} = \lim_{h\to 0} \frac{\frac{h}{2}}{h}$$
$$= \lim_{h\to 0} \frac{h}{2h} = \frac{1}{2}$$

32. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{0.01(x+h)+0.05-(0.01x+0.05)}{h}$$
$$= \lim_{h\to 0} \frac{0.01x+0.01h+0.05-0.01x-0.05}{h}$$
$$= \lim_{h\to 0} \frac{0.01h}{h} = 0.01$$

33. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{4-4}{h}=0$

34. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\pi-\pi}{h}=0$

35.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{a(x+h)^2+b(x+h)+c-(ax^2+bx+c)}{h}\\&=\lim_{h\to 0}\frac{ax^2+2axh+ah^2+bx+bh+c-ax^2-bx-c}{h}\\&=\lim_{h\to 0}(2ax+ah+b)=2ax+b\end{aligned}$$

36. Use $(x+a)^2=x^2+2ax+a^2$.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+2a(x+h)+a^2-(x^2+2ax+a^2)}{h}\\&=\lim_{h\to 0}\frac{x^2+2xh+h^2+2ax+2ah+a^2-x^2-2ax-a^2}{h}\\&=\lim_{h\to 0}(2x+h+2a)\\&=2x+2a\end{aligned}$$

37.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{(x+h)^5-x^5}{h}\\&=\lim_{h\to 0}\frac{x^5+5x^4h+10x^3h^2+5xh^4+h^5-x^5}{h}\\&=\lim_{h\to 0}5x^4+10x^3h+5xh^3+h^4=5x^4\end{aligned}$$

38.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{(x+h)^4-x^4}{h}\\&=\lim_{h\to 0}\frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}\\&=\lim_{h\to 0}4x^3+6x^2h+4xh^2+h^3=4x^3\end{aligned}$$

39.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{\frac{2}{x+h}-\frac{2}{x}}{h}\\&=\lim_{h\to 0}\frac{\frac{2x}{x(x+h)}-\frac{2(x+h)}{x(x+h)}}{h}\\&=\lim_{h\to 0}\frac{2x-2x-2h}{x(x+h)}\cdot\frac{1}{h}\\&=\lim_{h\to 0}-\frac{2h}{x(x+h)}\cdot\frac{1}{h}\\&=\lim_{h\to 0}-\frac{2}{x(x+h)}\\&=-\frac{2}{x^2}\end{aligned}$$

40.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}\\&=\lim_{h\to 0}\frac{\frac{x^2}{x^2(x+h)^2}-\frac{(x+h)^2}{x^2(x+h)^2}}{h}\\&=\lim_{h\to 0}\frac{\frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}}{h}\\&=\lim_{h\to 0}-\frac{2xh+h^2}{x^2(x+h)^2}\cdot\frac{1}{h}\\&=\lim_{h\to 0}-\frac{2x+h}{x^2(x+h)^2}=-\frac{2x}{x^4}=-\frac{2}{x^3}\end{aligned}$$

41.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\\&=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\\&=\lim_{h\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\\&=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\\&=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}\end{aligned}$$

42. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}=\lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}=\lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{x-(x+h)}{\left(\sqrt{x}\sqrt{x+h}\right)\left(\sqrt{x}+\sqrt{x+h}\right)}}{h}=\lim_{h\to 0}\frac{-h}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=-\frac{1}{\sqrt{x}\sqrt{x}\left(\sqrt{x}+\sqrt{x}\right)}=-\frac{1}{x\left(2\sqrt{x}\right)}$$

$$=-\frac{1}{2x\sqrt{x}}$$

43. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^3+(x+h)^2-(x^3+x^2)}{h}$$

$$=\lim_{h\to 0}\frac{(x^3+3x^2h+3xh^2+h^3)+(x^2+2xh+h^2)-x^3-x^2}{h}$$

$$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3+2xh+h^2}{h}=\lim_{h\to 0}3x^2+3xh+h^2+2x+h$$

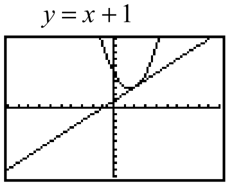
$$=3x^2+2x$$

44. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h}=\lim_{h\to 0}\frac{\frac{1}{2x+2h}-\frac{1}{2x}}{h}=\lim_{h\to 0}\frac{\frac{2x-(2x+2h)}{2x(2x+2h)}}{h}=\lim_{h\to 0}\frac{\frac{-2h}{2x(2x+2h)}}{h}$$

$$=\lim_{h\to 0}\frac{-h}{x(2x+2h)}\cdot\frac{1}{h}=\lim_{h\to 0}-\frac{1}{x(2x+2h)}=-\frac{1}{2x^2}$$

45. **a.** The slope of the tangent line at $x = 2$ is $f'(2)=2(2)-3=1$. To find the point of the curve at $x = 2$, we calculate $y=f(2)=2^2-3(2)+5=3$. Using the point-slope form with the point (2, 3), we have

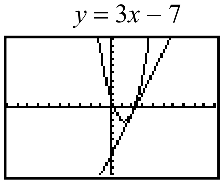
$$y-3=1(x-2)$$
$$y-3=x-2$$
$$y=x+1$$

b. 

on viewing window
[–10, 10] by [–10, 10]

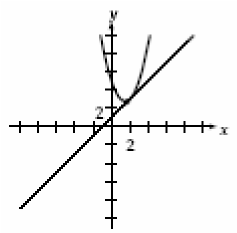
46. **a.** The slope of the tangent line at $x = 2$ is $f'(2)=4(2)-5=3$. To find the point of the curve at $x = 2$, we calculate $y=f(2)=2(2)^2-5(2)+1=-1$. Using the point-slope form with the point (2, –1), we have

$$y-(-1)=3(x-2)$$
$$y+1=3x-6$$
$$y=3x-7$$

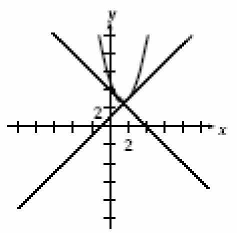
b. 

on viewing window
[–10, 10] by [–10, 10]

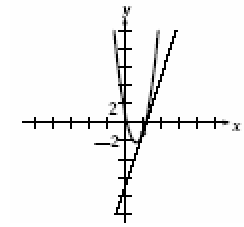
47. **a.**



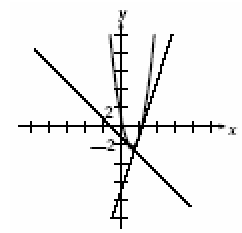
b.



48. **a.**



b.



49. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)-4-(3x-4)}{h} = \lim_{h\to 0} \frac{3x+3h-4-3x+4}{h} = \lim_{h\to 0} 3 = 3$

b. The graph of $f(x) = 3x - 4$ is a straight line with slope 3.

50. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)-9-(2x-9)}{h} = \lim_{h\to 0} \frac{2x+2h-9-2x+9}{h} = \lim_{h\to 0} 2 = 2$

b. The graph of $f(x) = 2x - 9$ is a straight line with slope 2.

51. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{5-5}{h} = \lim_{h\to 0} 0 = 0$

b. The graph of $f(x) = 5$ is a straight line with slope 0.

52. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{12-12}{h} = \lim_{h\to 0} 0 = 0$

b. The graph of $f(x) = 12$ is a straight line with slope 0.

53. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{m(x+h)+b-(mx+b)}{h} = \lim_{h\to 0} \frac{mx+mh+b-mx-b}{h} = \lim_{h\to 0} \frac{mh}{h}$

$= \lim_{h\to 0} m = m$

b. The graph of $f(x) = mx + b$ is a straight line with slope m.

54. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{b-b}{h} = \lim_{h\to 0} 0 = 0$

b. The graph of $f(x) = b$ is a straight line with slope 0.

55. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-8(x+h)+110-(x^2-8x+110)}{h}$

$= \lim_{h\to 0} \frac{x^2+2hx+h^2-8x-8h+110-x^2+8x-110}{h}$

$= \lim_{h\to 0} 2x+h-8 = 2x-8$

b. $f'(2) = 2(2) - 8 = -4$. The temperature is decreasing at a rate of 4 degrees per minute after 2 minutes.

c. $f'(5) = 2(5) - 8 = 2$. The temperature is increasing at a rate of 2 degrees per minute after 5 minutes.

56. **a.** $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{3(x+h)^2 - 12(x+h) + 200 - (3x^2 - 12x + 200)}{h}$

$= \lim\limits_{h\to 0} \dfrac{3x^2 + 6hx + 3h^2 - 12x - 12h + 200 - 3x^2 + 12x - 200}{h}$

$= \lim\limits_{h\to 0} 6x + 3h - 12 = 6x - 12$

b. $f'(1) = 6(1) - 12 = -6$. The population is decreasing by 6 people per week.

c. $f'(5) = 6(5) - 12 = 18$. The population is increasing by 18 people per week.

57. **a.** $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{2x^2 + 4hx + 2h^2 - x - h - 2x^2 + x}{h} = \lim\limits_{h\to 0} 4x + 2h - 1 = 4x - 1$

b. $f'(5) = 4(5) - 1 = 19$. When 5 words have been memorized, the memorization time is increasing at a rate of 19 seconds per word.

58. **a.** $S'(x) = \lim\limits_{h\to 0} \dfrac{S(x+h)-S(x)}{h} = \lim\limits_{h\to 0} \dfrac{-(x+h)^2 + 10(x+h) - (-x^2 + 10x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{-x^2 - 2hx - h^2 + 10x + 10h + x^2 - 10x}{h}$

$= \lim\limits_{h\to 0} -2x - h + 10 = -2x + 10$

b. $S'(3) = -2(3) + 10 = 4$. The number of cars sold on the third day of the advertising campaign is increasing at a rate of 4 cars per day.

c. $S'(6) = -2(6) + 10 = -2$. The number of cars sold on the sixth day of the advertising campaign is decreasing at a rate of 2 cars per day.

59. **a.** $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{2}(x+h)^2 - 3.7(x+h) + 12 - \left(\frac{1}{2}x^2 - 3.7x + 12\right)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - 3.7x - 3.7h + 12 - \left(\frac{1}{2}x^2 - 3.7x + 12\right)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - 3.7x - 3.7h + 12 - \frac{1}{2}x^2 + 3.7x - 12}{h}$

$= \lim\limits_{h\to 0} \left(x + \frac{1}{2}h - 3.7\right) = x - 3.7$

b. $f'(1) = 1 - 3.7 = -2.7$. In 1940, the percentage of immigrants was decreasing by 2.7 percentage points per decade (which is about 0.27) of a percentage point decrease per year).

c. $f'(7) = 7 - 3.7 = 3.3$. Increasing by 3.3 percentage points per decade (so about a third of a percentage point per year).

60. **a.** $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{2}(x+h)^2 - 2(x+h) + 25 - \left(\frac{1}{2}x^2 - 2x + 25\right)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - 2x - 2h + 25 - \left(\frac{1}{2}x^2 - 2x + 25\right)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - 2x - 2h + 25 - \frac{1}{2}x^2 + 2x - 25}{h}$

$= \lim\limits_{h\to 0} \left(x - \frac{1}{2}h - 2\right) = x - 2$

b. $f'(1) = 1 - 2 = -1$. In 2001, operating revenues were decreasing by \$1 billion per year.

c. $f'(6) = 8 - 2 = 4$. In 2006, operating revenues were increasing by \$4 billion per year.

61. The average rate of change requires two x-values and is the change in the function - values divided by the change in the x-values. The instantaneous rate of change is at a single x-value, and is found from the formula $f(x) = \lim_{x\to\infty} \frac{f(x+h)-f(x)}{h}$.

62. A secant line crosses the function twice while a tangent line crosses only once. To find the slope of a secant line, use the formula for average rate of change. For the slope of a tangent line, use the formula for instantaneous rate of change.

63. Substitution $h = 0$ would make the denominator zero, and we can't divide by zero. That's why we need to do some algebra on the difference quotient, to cancel out the terms that are zero so that afterwards we can evaluate by direct substitution.

64. The units of the derivative $f'(x)$ is in widgets per blivet because the derivative is equivalent to the slope $f(x)$ which is the change in f over the change in x.

65. The units of x are blargs and the units of f are prendles because the derivative $f'(x)$ is equivalent to the slope of $f(x)$, which is the change in f over the change in x.

66. The population is decreasing because the negative derivative implies a negative slope.

67. The patient's health is deteriorating during the first day (temperature is rising above normal). The patient's health is improving during the second day (temperature is falling).

68. The temperature at 6 a.m. is the lowest temperature throughout the first half of the day because the temperature falls until 6 a.m. and rises after 6 a.m.

EXERCISES 2.3

1. $f'(x) = \frac{d}{dx}(x^4) = 4x^{4-1} = 4x^3$

2. $f'(x) = \frac{d}{dx}(x^5) = 5x^{5-1} = 5x^4$

3. $f'(x) = \frac{d}{dx}(x^{500}) = 500x^{500-1} = 500x^{499}$

4. $f'(x) = \frac{d}{dx}(x^{1000}) = 1000x^{1000-1} = 1000x^{999}$

5. $f'(x) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2}$

6. $f'(x) = \frac{d}{dx}(x^{1/3}) = \frac{1}{3}x^{1/3-1} = \frac{1}{3}x^{-2/3}$

7. $g'(x) = \frac{d}{dx}\left(\frac{1}{4}x^4\right) = \frac{1}{2}\cdot 4x^{4-1} = 2x^3$

8. $g'(x) = \frac{d}{dx}\left(\frac{1}{3}x^9\right) = \frac{1}{3}\cdot 9x^{9-1} = 3x^8$

9. $g'(w) = \frac{d}{dw}(6w^{1/3}) = 6\cdot\frac{1}{3}w^{1/3-1} = 2w^{-2/3}$

10. $g'(w) = \frac{d}{dw}(12w^{1/2}) = 12\cdot\frac{1}{2}w^{1/2-1} = 6w^{-1/2}$

11. $h'(x) = \frac{d}{dx}(3x^{-2}) = 3(-2)x^{-2-1} = -6x^{-3}$

12. $h'(x) = \frac{d}{dx}(4x^{-3}) = 4(-3)x^{-3-1} = -12x^{-4}$

13.
$$\begin{aligned} f'(x) &= \frac{d}{dx}(4x^2 - 3x + 2) \\ &= 4\cdot 2x^{2-1} - 3\cdot 1x^{1-1} + 0 \\ &= 8x - 3 \end{aligned}$$

14.
$$\begin{aligned} f'(x) &= \frac{d}{dx}(3x^2 - 5x + 4) \\ &= 3\cdot 2x^{2-1} - 5\cdot 1x^{1-1} + 0 \\ &= 6x - 5 \end{aligned}$$

15.
$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{1}{x^{1/2}}\right) = \frac{d}{dx}\left(x^{-1/2}\right) \\ &= -\frac{1}{2}x^{-3/2} = -\frac{1}{2x^{3/2}} \end{aligned}$$

16.
$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{1}{x^{2/3}}\right) = \frac{d}{dx}\left(x^{-2/3}\right) \\ &= -\frac{2}{3}x^{-5/3} = -\frac{2}{3x^{5/3}} \end{aligned}$$

17. $f'(x)=\frac{d}{dx}\left(\frac{6}{\sqrt[3]{x}}\right)=\frac{d}{dx}\left(6x^{-1/3}\right)$
$=6\left(-\frac{1}{3}\right)x^{-4/3}=-\frac{2}{x^{4/3}}$

18. $f'(x)=\frac{d}{dx}\left(\frac{4}{\sqrt{x}}\right)=\frac{d}{dx}\left(4x^{-1/2}\right)$
$=4\left(-\frac{1}{2}\right)x^{-3/2}=-\frac{2}{x^{3/2}}$

19. $f'(r)=\frac{d}{dr}\left(\pi r^2\right)=\pi(2r)=2\pi r$

20. $f'(r)=\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi(3r^2)=4\pi r^2$

21. $f'(x)=\frac{d}{dx}\left(\frac{1}{6}x^3+\frac{1}{2}x^2+x+1\right)$
$=\frac{1}{6}(3x^2)+\frac{1}{2}(2x)+1$
$=\frac{1}{2}x^2+x+1$

22. $f'(x)=\frac{d}{dx}\left(\frac{1}{24}x^4+\frac{1}{6}x^3+\frac{1}{2}x^2+x+1\right)$
$=\frac{1}{24}(4x^3)+\frac{1}{6}(3x^2)+\frac{1}{2}(2x)+1$
$=\frac{1}{6}x^3+\frac{1}{2}x^2+x+1$

23. $g'(x)=\frac{d}{dx}(x^{1/2}-x^{-1})=\frac{1}{2}x^{1/2-1}-(-1)x^{-1-1}$
$=\frac{1}{2}x^{-1/2}+x^{-2}$

24. $g'(x)=\frac{d}{dx}(x^{1/3}-x^{-1})=\frac{1}{3}x^{1/3-1}-(-1)x^{-1-1}$
$=\frac{1}{3}x^{-2/3}+x^{-2}$

25. $h'(x)=\frac{d}{dx}\left(6x^{2/3}-12x^{-1/3}\right)=6\cdot\frac{3}{2}x^{2/3-1}-12\left(-\frac{1}{3}\right)x^{-1/3-1}$
$=4x^{-1/3}+4x^{-4/3}$

26. $h'(x)=\frac{d}{dx}\left(8x^{3/2}-8x^{-1/4}\right)=8\cdot\frac{3}{2}x^{3/2-1}-8\left(-\frac{1}{4}\right)x^{-1/4-1}$
$=12x^{1/2}+2x^{-5/4}$

27. $f'(x)=\frac{d}{dx}\left(10x^{-1/2}-\frac{9}{5}x^{5/3}+17\right)=10\left(-\frac{1}{2}\right)x^{-1/2-1}-\frac{9}{5}\left(\frac{5}{3}\right)x^{5/3-1}+0$
$=-5x^{-3/2}-3x^{2/3}$

28. $f'(x)=\frac{d}{dx}\left(\frac{9}{2}x^{-2/3}-16x^{5/2}-14\right)=\frac{9}{2}\left(-\frac{2}{3}\right)x^{-2/3-1}-16\left(\frac{5}{2}\right)x^{5/2-1}-0$
$=-3x^{-5/3}-40x^{3/2}$

29. $f'(x)=\frac{d}{dx}\left(\frac{x^2+x^3}{x}\right)=\frac{d}{dx}(x+x^2)=1+2x$

30. $f'(x)=\frac{d}{dx}\left(x^2(x+1)\right)=\frac{d}{dx}(x^3+x^2)=3x^2+2x$

31. $f'(x)=\frac{d}{dx}(x^5)=5x^4;\ f'(-2)=5(-2)^4=80$

32. $f'(x)=\frac{d}{dx}(x^4)=4x^3;\ f'(-3)=4(-3)^3=-108$

33. $f'(x)=\frac{d}{dx}(6x^{2/3}-48x^{-1/3})=6\left(\frac{2}{3}\right)x^{-1/3}-48\left(-\frac{1}{3}\right)x^{-4/3}$
$=4x^{-1/3}+16x^{-4/3}$
$f'(8)=4(8)^{-1/3}+16(8)^{-4/3}=\frac{4}{\sqrt[3]{8}}+\frac{16}{\sqrt[3]{8^4}}=\frac{4}{2}+\frac{16}{\sqrt[3]{8^3}\cdot\sqrt[3]{8}}=2+\frac{16}{8\cdot 2}=3$

34. $f'(x) = \frac{d}{dx}\left(12x^{2/3} + 48x^{-1/3}\right) = 12\left(\frac{2}{3}\right)x^{-1/3} + 48\left(-\frac{1}{3}\right)x^{-4/3}$

$= 4x^{-1/3} + 16x^{-4/3}$

$f'(8) = 4(8)^{-1/3} + 16(8)^{-4/3} = \frac{4}{\sqrt[3]{8}} + \frac{16}{\sqrt[3]{8^4}} = \frac{4}{2} + \frac{16}{\sqrt[3]{8^3}\cdot\sqrt[3]{8}} = 2 + \frac{16}{8\cdot 2} = 3$

35. $\frac{df}{dx} = \frac{d}{dx}(x^3) = 3x^2; \left.\frac{df}{dx}\right|_{x=-3} = 3(-3)^2 = 27$

36. $\frac{df}{dx} = \frac{d}{dx}(x^4) = 4x^3; \left.\frac{df}{dx}\right|_{x=-2} 4(-2)^3 = -32$

37. $\frac{df}{dx} = \frac{d}{dx}\left(16x^{-1/2} + 8x^{1/2}\right) = 16\left(-\frac{1}{2}\right)x^{-3/2} + 8\left(\frac{1}{2}\right)x^{-1/2}$

$= 8x^{-3/2} + 4x^{-1/2}$

$\left.\frac{df}{dx}\right|_{x=4} = -8(4)^{-3/2} + 4(4)^{-1/2} = -\frac{8}{\sqrt{4^3}} + \frac{4}{\sqrt{4}} = -\frac{8}{8} + \frac{4}{2} = 1$

38. $\frac{df}{dx} = \frac{d}{dx}\left(54x^{-1/2} + 12x^{1/2}\right) = 54\left(-\frac{1}{2}\right)x^{-3/2} + 12\left(\frac{1}{2}\right)x^{-1/2}$

$= -27x^{-3/2} + 6x^{-1/2}$

$\left.\frac{df}{dx}\right|_{x=9} = -27(9)^{-3/2} + 6(9)^{-1/2} = -\frac{27}{\sqrt{9^3}} + \frac{6}{\sqrt{9}} = -\frac{27}{27} + \frac{6}{3} = 1$

39. **a.** $f'(x) = 2\cdot x^1 - 2 + 0 = 2x + 2$

$f'(3) = 2(3) - 2 = 4$

The slope of the tangent line is 4.

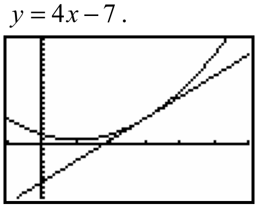
$y - 5 = 4(x - 3)$

$y - 5 = 4x - 12$

$y = 4x - 7$

The equation for the tangent line is $y = 4x - 7$.

b.



on [–1, 6] by [–10, 20]

40. **a.** $f'(x) = 2\cdot x^1 - 4 + 0 = 2x - 4$

$f'(1) = 2(1) - 4 = -2$

The slope of the tangent line is –2.

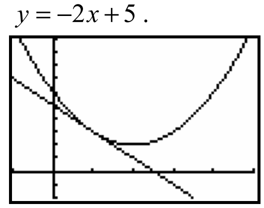
$y - 3 = -2(x - 1)$

$y - 3 = -2x + 2$

$y = -2x + 5$

The equation for the tangent line is $y = -2x + 5$.

b.



on [–1, 5] by [–2, 10]

41. **a.** $f'(x) = 3\cdot x^2 - 3(2)x^1 + 2 + 0 = 3x^2 - 6x + 2$

$f'(2) = 3(2)^2 - 6(2) + 2 = 2$

The slope of the tangent line is 2.

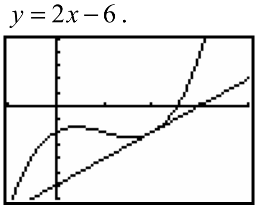
$y + 2 = 2(x - 2)$

$y + 2 = 2x - 4$

$y = 2x - 6$

The equation for the tangent line is $y = 2x - 6$.

b.



on [–1, 4] by [–7, 5]

42. **a.** $f'(x) = 3(2)x^1 - 3\cdot x^2 = 5x - 3x^2$

$f'(1) = 6(1) - 3(1)^2 = 3$

The slope of the tangent line is 3.

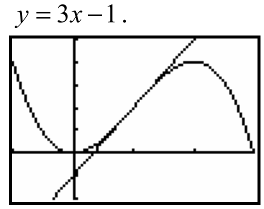
$y - 2 = 3(x - 1)$

$y - 2 = 3x - 3$

$y = 3x - 1$

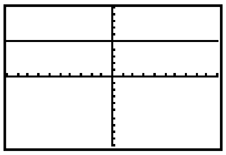
The equation for the tangent line is $y = 3x - 1$.

b.

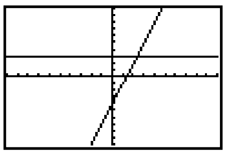


on [–1, 3] by [–2, 5]

43. For $y_1 = 5$ and viewing rectangle [–10, 10] by [–10, 10], your graph should look roughly like the following:



44. For $y_1 = 3x - 4$ and viewing rectangle [–10, 10] by [–10, 10], your graph should look roughly like the following:



45. **a.** $C'(x) = \frac{16}{\sqrt[3]{x}}$

$C'(8) = \frac{16}{\sqrt[3]{8}} = \frac{16}{2} = 8$

When 8 items are purchased, the cost of the last item is about \$8.

b. $C'(64) = \frac{16}{\sqrt[3]{64}} = \frac{16}{4} = 4$

When 8 items are purchased, the cost of the last item is about \$4.

46. **a.** $C'(x) = 140x^{-1/6}$

$C'(1) = \frac{140}{\sqrt[6]{1}} = 140$

When 1 license is purchased, the cost is about \$140.

b. $C'(64) = \frac{140}{\sqrt[6]{64}} = \frac{140}{2} = 70$

When 64 licenses are purchased, the cost is about \$70.

47. $C(64) - C(63) = 24(64)^{2/3} - 24(63)^{2/3} \approx 4.01$

The answer is close to \$4.

48. $C(64) - C(63) = 168(64)^{5/6} - 168(63)^{5/6} \approx 70.09$

The answer is close to \$70.

49. **a.** The rate of change of the population in x years is the derivative of the population function

$P'(x) = -x^2 + 5x - 3$

To find the rate of change of the population 20 years from now, evaluate $P'(x)$ for $x = 2$.

$P'(2) = -2^2 + 5(2) - 3 = 3$

In 2030, this population group will be increasing by 3 million per decade.

b. To find the rate of change of the population 0 years from now, evaluate $P'(x)$ for $x = 0$.

$P'(0) = -0^2 + 5(0) - 3 = -3$

In 2010, this population group will be decreasing by 3 million per decade.

50. **a.** The rate of change of the number of newly infected people is the derivative of the function

$f(t) = 13t^2 - t^3$

$f'(t) = 2 \cdot 13t - 3t^2 = 26t - 3t^2$

The rate of change on day 5 is found by evaluating $f'(t)$ for $t = 5$.

$f'(5) = 26(5) - 3(5)^2 = 55$

The number of newly infected people on day 5 is increasing by about 55 people per day.

b. The rate of change on day 10 is found by evaluating $f'(t)$ for $t = 10$.

$f'(10) = 26(10) - 3(10)^2 = -40$

The number of newly infected people on day 10 is decreasing by about 40 people per day.

51. The rate of change of the pool of potential customers is the derivative of the function

$N(x) = 400,000 - \dfrac{200,000}{x}$.

$N'(x) = \dfrac{d}{dx}(400,000 - 200,000x^{-1})$

$N'(x) = 0 - (-1)200,000x^{-2} = \dfrac{200,000}{x^2}$

To find the rate of change of the pool of potential customers when the ad has run for 5 days, evaluate $N'(x)$ for $x = 5$.

$N'(5) = \dfrac{200,000}{5^2} = 8000$

The pool of potential customers is increasing by about 8000 people per additional day.

52. $A(t) = 0.01t^2 \quad 1 \le t \le 5$

The instantaneous rate of change of the cross-sectional area t hours after administration of nitroglycerin is given by

$A'(t) = 2(0.01)t^{2-1} = 0.02t$

$A'(4) = 0.02(4) = 0.08$

After 4 hours the cross-sectional area is increasing by about 0.08 cm^2 per hour.

53. The rate of change of broadband access is the derivative of the function $f(x) = \frac{1}{4}x^2 + 5x + 6$.

$f'(x) = \frac{1}{2}x + 5$

The rate of change of broadband access in 10 years is found by evaluating $f'(x)$ at $x = 10$.

$f'(10) = \frac{1}{2}(10) + 5 = 10$

In 2010, the percentage of household with broadband internet access is increasing by 10% per year.

54. **a.** The rate of change of the function is the derivative of the function:

$f'(x) = -x^{-3/2} = -\dfrac{1}{\sqrt{x^3}}$.

Find the rate of change for $x = 1$.

$f'(1) = -\dfrac{1}{\sqrt{1^3}} = -1$

In 1980, the number of fatalities is decreasing by 1 per hundred million over the 5-year interval.

b. Find the rate of change for $x = 4$.

$f'(4) = -\dfrac{1}{\sqrt{4^3}} = -\dfrac{1}{8} = -0.125$

In 1995, the number of fatalities is decreasing by 0.125 per hundred million over the 5-year interval.

55. The instantaneous rate of change of the number of phrases students can memorize is the derivative of the function $p(t)=24\sqrt{t}$.

$$p'(t)=\frac{d}{dt}(24t^{1/2})$$
$$=\frac{1}{2}(24t^{-1/2})=12t^{-1/2}$$
$$p'(4)=12(4)^{-1/2}=\frac{12}{\sqrt{4}}=6$$

The number of phrases students can memorize after 4 hours is increasing by 6.

56. **a.** The instantaneous rate of change of the amount of dissolved oxygen x miles downstream is the derivative of the function $D(x)=0.2x^2-2x+10$.

$$D'(x)=2(0.2)x-2+0=0.4x-2$$
$$D'(1)=0.4(1)-2=-1.6$$

The amount of dissolved oxygen 1 mile downstream is decreasing by about 1.6 mpl per mile.

b. $D'(10)=0.4(10)-2=2$

The amount of dissolved oxygen 10 miles downstream is increasing by about 2 mpl per mile.

57. **a.** $U(x)=100\sqrt{x}=100x^{1/2}$

$$MU(x)=U'(x)=\frac{1}{2}(100)x^{1/2-1}=50x^{-1/2}$$

b. $MU(1)=U'(1)=50(1)^{-1/2}=50$

The marginal utility of the first dollar is 50.

c. $MU(1{,}000{,}000)=U'(1{,}000{,}000)=50(10^6)^{-1/2}=50(10)^{-3}=\frac{50}{1000}=0.05$

The marginal utility of the millionth dollar is 0.05.

58. **a.** $U(x)=12\sqrt[3]{x}=12x^{1/3}$

$$MU(x)=U'(x)=\frac{1}{3}(12x^{-2/3})=4x^{-2/3}$$

b. $MU(1)=U'(1)=4(1)^{-2/3}=4$

The marginal utility of the first dollar is 4.

c. $MU(1{,}000{,}000)=U'(1{,}000{,}000)=4(1{,}000{,}000)^{-2/3}=\frac{4}{(\sqrt[3]{1{,}000{,}000})^2}=\frac{4}{100^2}=0.0004$

The marginal utility of the millionth dollar is 0.0004.

59. **a.** $f(12)=0.831(12)^2-18.1(12)+137.3=39.764$

A smoker who is a high school graduate has a 39.8% chance of quitting.

$$f'(x)=0.831(2)x-18.1=1.662x-18.1$$
$$f'(12)=1.662(12)-18.1=1.844$$

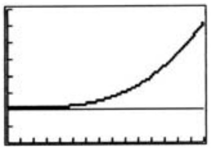
When a smoker has a high school diploma, the chance of quitting is increasing at the rate of 1.8% per year of education.

b. $f(16)=0.831(16)^2-18.1(16)+137.3=60.436$

A smoker who is a college graduate has a 60.4% chance of quitting.

$$f'(16)=1.662(16)-18.1=8.492$$

When a smoker has a college degree, the chance of quitting is increasing at the rate of 8.5% per year of education.

60. **a.** 

on [0, 15] by [–10, 30]

b. $N(10) = 0.00437(10)^{3.2} \approx 6.9$

A group with 10 years of exposure to asbestos will have 7 cases of lung cancer.

$N'(t) = 0.00437(3.2)t^{3.2-1} = 0.013984t^{2.2}$

$N'(10) = 0.013984(10)^{2.2} \approx 2.2$

When a group has 10 years of exposure to asbestos, the number of cases of lung cancer is increasing at the rate of 2.2 cases per year.

61. **a.** $f(6) = 400(6)^2 + 2500(6) + 7200$
$= \$36,600$

b. $f'(x) = 800x + 2500$

$f'(6) = 800(6) + 2500 = \$7300$

In 2017-2018, private college tuition will be increasing by \$7300 every 5 years.

c. \$1460

62. **a.** $f(6) = 200(6)^2 + 350(6) + 1600$
$= \$10,900$

b. $f'(x) = 400x + 350$

$f'(6) = 400(6) + 350 = \$2750$

In 2017-2018, public college tuition will be increasing by \$2750 every 5 years.

c. \$550

63. $f(x) = 2$ will have a graph that is a horizontal line at height 2, and the slope (the derivative) of a horizontal line is zero. A function that stays constant ill have a rate of change of zero, so its derivative (instantaneous rate of change) will be zero.

64. $f(x) = 3x - 5$ is a line with a slope (derivative) of 3. A function that has a slope of 3 will have a rate of change of 3, so its derivative (instantaneous rate of change) will be three.

65. If *f* has a particular rate of change, then the rate of change of $2 \cdot f(x)$ will be twice as large, and the rate of change of $c \cdot f(x)$ will be $c \cdot f'(x)$, which is just the constant multiple rule.

66. If *f* and *g* have particular rates of change, then the rate of change of $f(x) + g(x)$ would be the rate of change of $f(x)$ plus the rate of change of $g(x)$. For example, if $f(x) = 2x$ and $g(x) = 3x$, then the rate of change of $2x + 3x = 5x$ is the same as the rate of change of $2x$ plus the rate of change of $3x$. The derivative of $f(x) + g(x)$ will be $f'(x) + g'(x)$, which is just the sum rate.

67. Since –*f* slopes down by the same amount that *f* slopes up, the slope of –*f* should be the negative of the slope of *f*. The constant multiple rule with $c = -1$ also says that the slope of –*f* will be the negative of the slope of *f*.

68. The slopes of the *f* and $f + 10$ will be the same because $f + 10$ is just *f* raised 10 units. Using the sum rule with $g(x) = 10$, also says that the slope will just be *f* because $g'(x) = 0$.

69. Evaluating first would give a constant and the derivative of a constant is zero, so evaluating and the differentiating would always give zero, regardless of the function and number. This supports the idea that we should always differentiate and then evaluate to obtain anything meaningful.

70. To find a function that is positive but does not have a positive slope at a particular *x*-value, we need to find an equation with a negative slope. $y = -5x$ will work at $x = -1$.

71. Each additional year of education increases life expectancy by 1.7 years.

72. The probability of an accident increases by 13% as the speed exceeds the limit.

EXERCISES 2.4

1. **a.** Using the product rule:
$\frac{d}{dx}(x^4 \cdot x^6) = 4x^3 \cdot x^6 + x^4(6x^5) = 4x^9 + 6x^9 = 10x^9$

b. Using the power rule:
$\frac{d}{dx}(x^4 \cdot x^6) = \frac{d}{dx}(x^{10}) = 10x^9$

2. **a.** Using the product rule:
$\frac{d}{dx}(x^7 \cdot x^2) = 7x^6 \cdot x^2 + x^7 \cdot 2x = 7x^8 + 2x^8 = 9x^8$

b. Using the power rule:
$\frac{d}{dx}(x^7 \cdot x^2) = \frac{d}{dx}(x^9) = 9x^8$

3. **a.** Using the product rule:
$\frac{d}{dx}[x^4(x^5+1)] = 4x^3(x^5+1) + x^4(5x^4) = 4x^8 + 4x^3 + 5x^8 = 9x^8 + 4x^3$

b. Using the power rule:
$\frac{d}{dx}[x^4(x^5+1)] = \frac{d}{dx}(x^9 + x^4) = 9x^8 + 4x^3$

4. **a.** Using the product rule:
$\frac{d}{dx}[x^5(x^4+1)] = 5x^4(x^4+1) + x^5(4x^3) = 5x^8 + 5x^4 + 4x^8 = 9x^8 + 5x^4$

b. Using the power rule:
$\frac{d}{dx}[x^5(x^4+1)] = \frac{d}{dx}(x^9 + x^5) = 9x^8 + 5x^4$

5. $f'(x) = 2x(x^3+1) + x^2(3x^2) = 2x^4 + 2x + 3x^4 = 5x^4 + 2x$

6. $f'(x) = 3x^2(x^2+1) + x^3(2x) = 3x^4 + 3x^2 + 2x^4 = 5x^4 + 3x^2$

7. $f'(x) = 1(5x^2-1) + x(10x) = 5x^2 - 1 + 10x^2 = 15x^2 - 1$

8. $f'(x) = 2(x^4+1) + 2x(4x^3) = 2x^4 + 2 + 8x^4 = 10x^4 + 2$

9. $f'(x) = \frac{1}{2}x^{-1/2}(6x+2) + x^{1/2}(6) = 3x^{1/2} + x^{-1/2} + 6x^{1/2} = 9x^{1/2} + x^{-1/2} = 9\sqrt{x} + \frac{1}{\sqrt{x}}$

10. $f'(x) = 6\left(\frac{1}{3}x^{-2/3}\right)(2x+1) + 6x^{1/3}(2) = 4x^{1/3} + 2x^{-2/3} + 12x^{1/3} = 16x^{1/3} + 2x^{-2/3} = 16\sqrt[3]{x} + \frac{2}{\sqrt[3]{x^2}}$

11. $f'(x) = (2x)(x^2-1) + (x^2+1)(2x) = 2x^3 - 2x + 2x^3 + 2x = 4x^3$

12. $f'(x) = 3x^2(x^3+1) + (x^3-1)(3x^2) = 3x^5 + 3x^2 + 3x^5 - 3x^2 = 6x^5$

13. $f'(x) = (2x+1)(3x+1) + (x^2+x)(3) = 6x^2 + 5x + 1 + 3x^2 + 3x = 9x^2 + 8x + 1$

14. $f'(x) = (2x+2)(2x+1) + (x^2+2x)(2) = 4x^2 + 6x + 2 + 2x^2 + 4x = 6x^2 + 10x + 2$

15. $f'(x) = 2x(x^2+3x-1) + x^2(2x+3) = 2x^3 + 6x^2 - 2x + 2x^3 + 3x^2 = 4x^3 + 9x^2 - 2x$

16. $f'(x) = 3x^2(x^2-4x+3) + x^3(2x-4) = 3x^4 - 12x^3 + 9x^2 + 2x^4 - 4x^3 = 5x^4 - 16x^3 + 9x^2$

17. $f'(x)=(4x)(1-x)+(2x^2+1)(1)=4x-4x^2-2x^2-1=-6x^2+4x-1$

18. $f'(x)=(2x)(1-x^2)+(2x-1)(-2x)=2-2x^2-4x^2+2x=-6x^2+2x+2$

19. $f'(x)=\left(\frac{1}{2}x^{-1/2}\right)x^{1/2}+1+\left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-1/2}\right)=\frac{1}{2}+\frac{1}{2}x^{-1/2}+\frac{1}{2}-\frac{1}{2}x^{-1/2}=1$

20. $f'(x)=\left(\frac{1}{2}x^{-1/2}\right)\left(x^{1/2}-2\right)+\left(x^{1/2}+2\right)\left(\frac{1}{2}x^{-1/2}\right)=\frac{1}{2}-x^{-1/2}+\frac{1}{2}+x^{-1/2}=1$

21. $f'(t)=8t^{1/3}(3t^{2/3}+1)+6t^{4/3}(2t^{-1/3})=24t+8t^{1/3}+12t=36t+8t^{1/3}$

22. $f'(t)=6t^{1/2}(2t^{1/2}-1)+4t^{3/2}(t^{-1/2})=12t-6t^{1/2}+4t=16t-6t^{1/2}$

23. $$\begin{aligned}f'(z)&=(4z^3+2z)(z^3-z)+(z^4+z^2+1)(3z^2-1)\\&=4z^6-2z^4-2z^2+3z^6+3z^4+3z^2-z^4-z^2-1\\&=7z^6-1\end{aligned}$$

24. $$\begin{aligned}f'(z)&=\left(\frac{1}{4}z^{-3/4}+\frac{1}{2}z^{-1/2}\right)(z^{1/4}-z^{1/2})+(z^{1/4}+z^{1/2})\left(\frac{1}{4}z^{-3/4}-\tfrac{1}{2}z^{-1/2}\right)\\&=\frac{1}{4}z^{-1/2}-\frac{1}{4}z^{-1/4}+\frac{1}{2}z^{-1/4}-\frac{1}{2}+\frac{1}{4}z^{-1/2}-\frac{1}{2}z^{-1/4}+\frac{1}{4}z^{-1/4}-\frac{1}{2}\\&=\frac{1}{4}z^{-1/2}+\frac{1}{4}z^{-1/2}-\frac{1}{2}-\frac{1}{2}=\frac{1}{2}z^{-1/2}-1\end{aligned}$$

25. $$\begin{aligned}f'(x)&=\left(2+4\cdot\tfrac{1}{2}z^{-1/2}\right)\left(2z^{1/2}+1\right)+\left(2z+4z^{1/2}-1\right)\left(2\cdot\tfrac{1}{2}z^{-1/2}\right)\\&=4z^{1/2}+2+4+2z^{-1/2}+2z^{1/2}+4-z^{-1/2}\\&=6z^{1/2}+z^{-1/2}+10\\&=6\sqrt{z}+\frac{1}{\sqrt{z}}+10\end{aligned}$$

26. $$\begin{aligned}f'(x)&=\left(1+6\cdot\tfrac{1}{2}z^{-1/2}\right)\left(z-2z^{1/2}+1\right)+\left(z+6z^{1/2}\right)\left(1-2\cdot\tfrac{1}{2}z^{-1/2}\right)\\&=z+3z^{1/2}-2z^{1/2}-6+1+3z^{-1/2}+z-z^{1/2}+6z^{1/2}-6\\&=2z+6z^{1/2}+3z^{-1/2}-11\\&=2z+6\sqrt{z}+\frac{3}{\sqrt{z}}-11\end{aligned}$$

27. **a.** Using the quotient rule:

$$\frac{d}{dx}\left(\frac{x^8}{x^2}\right)=\frac{x^2(8x^7)-2x(x^8)}{(x^2)^2}=\frac{8x^9-2x^9}{x^4}=6x^5$$

b. Using the power rule:

$$\frac{d}{dx}\left(\frac{x^8}{x^2}\right)=\frac{d}{dx}(x^6)=6x^5$$

28. **a.** Using the quotient rule:

$$\frac{d}{dx}\left(\frac{x^9}{x^3}\right)=\frac{x^3(9x^8)-3x^2(x^9)}{(x^3)^2}=\frac{9x^{11}-3x^{11}}{x^6}=6x^5$$

b. Using the power rule:

$$\frac{d}{dx}\left(\frac{x^9}{x^3}\right)=\frac{d}{dx}(x^6)=6x^5$$

29. **a.** Using the quotient rule:

$$\frac{d}{dx}\left(\frac{1}{x^3}\right)=\frac{x^3(0)-3x^2(1)}{(x^3)^2}=-\frac{3x^2}{x^6}=-\frac{3}{x^4}$$

b. Using the power rule:

$$\frac{d}{dx}\left(\frac{1}{x^3}\right)=\frac{d}{dx}(x^{-3})=-3x^{-4}=-\frac{3}{x^4}$$

30. **a.** Using the quotient rule:

$$\frac{d}{dx}\left(\frac{1}{x^4}\right)=\frac{x^4(0)-4x^3(1)}{(x^4)^2}=-\frac{4x^3}{x^8}=-\frac{4}{x^5}$$

b. Using the power rule:

$$\frac{d}{dx}\left(\frac{1}{x^4}\right)=\frac{d}{dx}(x^{-4})=-4x^{-5}=-\frac{4}{x^5}$$

31. $$f'(x)=\frac{x^3(4x^3)-3x^2(x^4+1)}{(x^3)^2}=\frac{4x^6-3x^6-3x^2}{x^6}=\frac{x^6-3x^2}{x^6}=1-\frac{3}{x^4}$$

32. $$f'(x)=\frac{x^2(5x^4)-2x(x^5-1)}{(x^2)^2}=\frac{5x^6-2x^6+2x}{x^4}=\frac{3x^6+2x}{x^4}=3x^2+\frac{2}{x^3}$$

33. $$f'(x)=\frac{(x-1)(1)-(1)(x+1)}{(x-1)^2}=\frac{x-1-x-1}{(x-1)^2}=-\frac{2}{(x-1)^2}$$

34. $$f'(x)=\frac{(x+1)(1)-(1)(x-1)}{(x+1)^2}=\frac{x+1-x+1}{(x+1)^2}=\frac{2}{(x+1)^2}$$

35. $$f'(x)=\frac{(2+x)(3)-(1)(3x+1)}{(2+x)^2}=\frac{6+3x-3x-1}{(2+x)^2}=\frac{5}{(2+x)^2}$$

36. $$f'(x)=\frac{(2x^2+1)(1)-(4x)(x+1)}{\left(2x^2+1\right)^2}=\frac{2x^2+1-4x^2-4x}{\left(2x^2+1\right)^2}=\frac{-2x^2-4x+1}{\left(2x^2+1\right)^2}$$

37. $$f'(t)=\frac{(t^2+1)(2t)-(2t)(t^2-1)}{(t^2+1)^2}=\frac{2t^3+2t-2t^3+2t}{(t^2+1)^2}=\frac{4t}{(t^2+1)^2}$$

38. $$f'(t)=\frac{(t^2-1)(2t)-(2t)(t^2+1)}{(t^2-1)^2}=\frac{2t^3-2t-2t^3-2t}{(t^2-1)^2}=-\frac{4t}{(t^2-1)^2}$$

39. $$f'(s)=\frac{(s+1)(3s^2)-(1)(s^3-1)}{(s+1)^2}=\frac{3s^3+3s^2-s^3+1}{(s+1)^2}=\frac{2s^3+3s^2+1}{(s+1)^2}$$

40. $$f'(s)=\frac{(s-1)(3s^2)-(1)(s^3+1)}{(s-1)^2}=\frac{3s^3-3s^2-s^3-1}{(s-1)^2}=\frac{2s^3-3s^2-1}{(s-1)^2}$$

41. $$f'(x)=\frac{(x+1)(2x-2)-(1)(x^2-2x+3)}{(x+1)^2}=\frac{2x^2-2-x^2+2x-3}{(x+1)^2}=\frac{x^2+2x-5}{(x+1)^2}$$

42. $f'(x)=\dfrac{(x-1)(2x+3)-(1)(x^2+3x-1)}{(x-1)^2}=\dfrac{2x^2+x-3-x^2-3x+1}{(x-1)^2}=\dfrac{x^2-2x-2}{(x-1)^2}$

43. $f'(x)=\dfrac{(x^2+1)(4x^3+2x)-(2x)(x^4+x^2+1)}{(x^2+1)^2}=\dfrac{4x^3(x^2+1)+2x(x^2+1)-2x(x^4)-2x(x^2+1)}{(x^2+1)^2}$

$=\dfrac{4x^5+4x^3-2x^5}{(x^2+1)^2}=\dfrac{2x^5+4x^3}{(x^2+1)^2}$

44. $f(x)=\dfrac{x^5+x^3+x}{x^3+x}=\dfrac{x(x^4+x^2+1)}{x(x^2+1)}=\dfrac{x^4+x^2+1}{x^2+1}$

Thus, $f'(x)=\dfrac{2x^5+4x^3}{(x^2+1)^2}$.

45. $f'(t)=\dfrac{(t^2+t-3)(2t+2)-(t^2+2t-1)(2t+1)}{(t^2+t-3)^2}=\dfrac{2t^3+4t^2-4t-6-2t^3-5t^2+1}{(t^2+t-3)^2}=-\dfrac{t^2+4t+5}{(t^2+t-3)^2}$

46. $f'(t)=\dfrac{(t^2-t+2)(4t+1)-(2t^2+t-5)(2t-1)}{(t^2-t+2)^2}=\dfrac{4t^3-3t^2+7t+2-4t^3+11t-5}{(t^2-t+2)^2}=\dfrac{-3t^2+18t-3}{(t^2-t+2)^2}$

47.

Rewrite	Differentiate	Rewrite
$y=3x^{-1}$	$\dfrac{dy}{dx}=-3x^{-2}$	$\dfrac{dy}{dx}=-\dfrac{3}{x^2}$

48.

Rewrite	Differentiate	Rewrite
$y=\frac{1}{4}x^2$	$\dfrac{dy}{dx}=\frac{1}{2}x$	$\dfrac{dy}{dx}=\dfrac{x}{2}$

49.

Rewrite	Differentiate	Rewrite
$y=\frac{3}{8}x^4$	$\dfrac{dy}{dx}=\frac{3}{2}x^3$	$\dfrac{dy}{dx}=\dfrac{3x^3}{2}$

50.

Rewrite	Differentiate	Rewrite
$y=\frac{3}{2}x^{-2}$	$\dfrac{dy}{dx}=-3x^{-3}$	$\dfrac{dy}{dx}=-\dfrac{3}{x^3}$

51.

Rewrite	Differentiate	Rewrite
$y=\frac{1}{3}x^2-\frac{5}{3}x$	$\dfrac{dy}{dx}=\frac{2}{3}x-\frac{5}{3}$	$\dfrac{dy}{dx}=\dfrac{2x-5}{3}$

52.

Rewrite	Differentiate	Rewrite
$y=4x^{-1/2}$	$\dfrac{dy}{dx}=-2x^{-3/2}$	$\dfrac{dy}{dx}=\dfrac{-2}{\sqrt{x^3}}$

53. $\dfrac{d}{dx}\left[(x^3+2)\dfrac{x^2+1}{x+1}\right]=\dfrac{d}{dx}(x^3+2)\cdot\dfrac{x^2+1}{x+1}+(x^3+2)\cdot\dfrac{d}{dx}\left(\dfrac{x^2+1}{x+1}\right)$

$3x^2\left(\dfrac{x^2+1}{x+1}\right)+(x^3+2)\dfrac{(x+1)(2x)-(1)(x^2+1)}{(x+1)^2}$

$=3x^2\left(\dfrac{x^2+1}{x+1}\right)+(x^3+2)\dfrac{x^2+2x-1}{(x+1)^2}$

54. $\dfrac{d}{dx}\left[(x^5+1)\dfrac{x^3+2}{x+1}\right]=\dfrac{d}{dx}(x^5+1)\cdot\dfrac{x^3+2}{x+1}+(x^5+1)\cdot\dfrac{d}{dx}\left(\dfrac{x^3+2}{x+1}\right)$

$=5x^4\left(\dfrac{x^3+2}{x+1}\right)+(x^5+1)\dfrac{(x+1)(3x^2)-(1)(x^3+2)}{(x+1)^2}$

$=5x^4\left(\dfrac{x^3+2}{x+1}\right)+(x^5+1)\dfrac{3x^3+3x^2-x^3-2}{(x+1)^2}$

$=5x^4\left(\dfrac{x^3+2}{x+1}\right)+(x^5+1)\dfrac{2x^3+3x^2-2}{(x+1)^2}$

55. $$\frac{d}{dx}\frac{(x^2+3)(x^3+1)}{x^2+2} = \frac{(x^2+2)\frac{d}{dx}\left[(x^2+3)(x^3+1)\right] - \left[\frac{d}{dx}(x^2+2)\right]\left[(x^2+3)(x^3+1)\right]}{(x^2+2)^2}$$

$$= \frac{(x^2+2)\left\{\left[\frac{d}{dx}(x^2+3)\right](x^3+1)+(x^2+3)\frac{d}{dx}(x^3+1)\right\} - 2x\left[(x^2+3)(x^3+1)\right]}{(x^2+2)^2}$$

$$= \frac{(x^2+2)\left[(2x)(x^3+1)+(x^2+3)(3x^2)\right] - 2x(x^2+3)(x^3+1)}{(x^2+2)^2}$$

$$= \frac{(x^2+2)(2x^4+2x+3x^4+9x^2)-(2x)(x^5+3x^3+x^2+3)}{(x^2+2)^2}$$

$$= \frac{2x^6+2x^3+3x^6+9x^4+4x^4+4x+6x^4+18x^2-2x^6-6x^4-2x^3-6x}{(x^2+2)^2}$$

$$= \frac{3x^6+13x^4+18x^2-2x}{(x^2+2)^2}$$

56. $$\frac{d}{dx}\frac{(x^3+2)(x^2+2)}{x^3+1} = \frac{(x^3+1)\frac{d}{dx}\left[(x^3+2)(x^2+2)\right] - \left[\frac{d}{dx}(x^3+1)\right]\left[(x^3+2)(x^2+2)\right]}{(x^3+1)^2}$$

$$= \frac{(x^3+1)\left[(3x^2)(x^2+2)+(x^3+2)(2x)\right] - 3x^2(x^3+2)(x^2+2)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)(3x^2)(x^2+2)+(x^3+1)(x^3+2)(2x)-3x^2(x^3+2)(x^2+2)}{(x^3+1)^2}$$

$$= \frac{3x^2(x^2+2)\left[(x^3+1)-(x^3+2)\right]+(x^3+1)(x^3+2)(2x)}{(x^3+1)^2}$$

$$= \frac{-3x^4-6x^2+2x^7+6x^4+4x}{(x^3+1)^2}$$

$$= \frac{2x^7+3x^4-6x^2+4x}{(x^3+1)^2}$$

57. $$\frac{d}{dx}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) = \frac{d}{dx}\left(\frac{x^{1/2}-1}{x^{1/2}+1}\right) = \frac{(x^{1/2}+1)(\frac{1}{2}x^{-1/2})-\frac{1}{2}x^{-1/2}(x^{1/2}-1)}{(x^{1/2}+1)^2}$$

$$= \frac{\frac{1}{2}+\frac{1}{2}x^{-1/2}-\frac{1}{2}+\frac{1}{2}x^{-1/2}}{(x^{1/2}+1)^2} = \frac{x^{-1/2}}{(x^{1/2}+1)^2} = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$$

58. $$\frac{d}{dx}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) = \frac{d}{dx}\left(\frac{x^{1/2}+1}{x^{1/2}-1}\right) = \frac{(x^{1/2}-1)(\frac{1}{2}x^{-1/2})-\frac{1}{2}x^{-1/2}(x^{1/2}+1)}{(x^{1/2}-1)^2}$$

$$= \frac{\frac{1}{2}-\frac{1}{2}x^{-1/2}-\frac{1}{2}-\frac{1}{2}x^{-1/2}}{(x^{1/2}-1)^2} = -\frac{x^{-1/2}}{(x^{1/2}-1)^2} = -\frac{1}{\sqrt{x}(\sqrt{x}-1)^2}$$

59. $$\frac{d}{dx}\left[\frac{R(x)}{x}\right] = \frac{x \cdot R'(x) - 1 \cdot R(x)}{x^2} = \frac{xR'(x)-R(x)}{x^2}$$

60. $$\frac{d}{dx}\left[\frac{P(x)}{x}\right] = \frac{x \cdot P'(x) - 1 \cdot P(x)}{x^2} = \frac{xP'(x)-P(x)}{x^2}$$

61. **a.** The instantaneous rate of change of cost with respect to purity is the derivative of the cost function $C(x)=\frac{100}{100-x}$ on $50 \le x < 100$.

$$C'(x)=\frac{(100-x)\cdot 0-(-1)(100)}{(100-x)^2}=\frac{100}{(100-x)^2} \quad \text{on } 50 \le x < 100$$

b. To find the rate of change for a purity of 95%, evaluate $C'(x)$ at $x = 95$.

$$C'(95)=\frac{100}{(100-95)^2}=\frac{100}{5^2}=4$$

The cost is increasing by 4 cents per additional percent of purity.

c. To find the rate of change for a purity of 98%, evaluate $C'(x)$ at $x = 98$.

$$C'(98)=\frac{100}{(100-98)^2}=\frac{100}{2^2}=25$$

The cost is increasing by 25 cents per additional percent of purity.

62. **a.** $AC(x)=\frac{C(x)}{x}=\frac{6x+50}{x}$

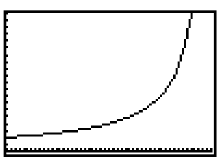
b. The marginal average cost function $MAC(x)$ is the derivative of the average cost function $AC(x)=\frac{C(x)}{x}=\frac{6x+50}{x}$.

$$MAC(x)=\frac{d}{dx}\left(\frac{6x+50}{x}\right)=\frac{x(6)-1(6x+50)}{x^2}$$
$$=\frac{6x-6x-50}{x^2}=\frac{-50}{x^2}$$

c. $MAC(25)=\frac{-50}{25^2}=-0.08$

The average cost is decreasing at the rate of 8 cents per additional truck.

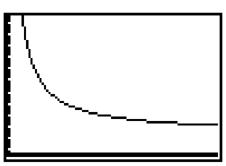
63. **a.**



on [50, 100] by [0, 20]

b. Rate of change of cost is 4 for $x = 95$; rate of change of cost is 25 for $x = 98$.

64. **a.**



on [0, 400] by [0,50]

b. Rate of change is –0.03 for $x = 200$.

65. **a.** $AC(x)=\frac{C(x)}{x}=\frac{6x+45}{x}$

b. The marginal average cost function $MAC(x)$ is the derivative of the average cost function $AC(x)=\frac{C(x)}{x}=\frac{6x+45}{x}$.

$$MAC(x)=\frac{d}{dx}\left(\frac{6x+45}{x}\right)=\frac{x(6)-1(6x+45)}{x^2}=\frac{6x-6x-45}{x^2}=\frac{-45}{x^2}$$

c. $MAC(30)=\frac{-45}{30^2}=-0.05$

The average cost is decreasing at the rate of 5 cents per alarm clock.

66. To find the rate of change of the number of bottles sold, find $N'(p)$.

$$N'(p) = \frac{(p+7)\cdot 0 - (1)(2250)}{(p+7)^2} = -\frac{2250}{(p+7)^2}$$

When $p = 8$, $N'(8) = -\frac{2250}{(8+7)^2} = -\frac{2250}{225} = -10$.

At \$8 per bottle, the number of bottles of whiskey sold will decrease by about 10 bottles for each \$1 increase in price.

67. To find the rate of change of temperature, find $T'(x)$.

$$T'(x) = 3x^2(4 - x^2) + x^3(-2x)$$
$$= 12x^2 - 3x^4 - 2x^4 = 12x^2 - 5x^4$$

For $x = 1$, $T'(1) = 12(1)^2 - 5(1)^4 = 12 - 5 = 7$.

After 1 hour, the person's temperature is increasing by 7 degrees per hour.

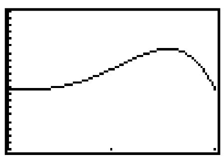
68. To find the rate of change of the sales, find $S'(x)$.

$$S'(x) = 2x(8 - x^3) + x^2(-3x^2)$$
$$= 16x - 2x^4 - 3x^4 = 16x - 5x^4$$

For $x = 1$, $S'(1) = 16(1) - 5(1)^4 = 16 - 5 = 11$.

After 1 month, sales are increasing by 11 thousand per month.

69. **a.**

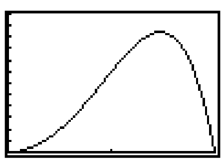


on [0, 2] by [90, 110]

b. The rate of change at $x = 1$ is 7.

c. The maximum temperature is about 104.5 degrees.

70. **a.**



on [0, 2] by [0, 12]

b. The rate of change at $x = 1$ is 11.

c. The maximum sales are about 10.4 thousand.

71. **a.** $y_3(10) = 33.928$

The per capita national debt in 2010 would be \$33,928.

$y_3(20) = 46.301$

The per capita national debt in 2020 would be \$46,301.

b. $y_3'(10) = 1.342$

In 2010 the per capita national debt will be growing by \$1342 per year.

$y_3'(20) = 1.141$

In 2020 the per capita national debt will be growing by \$1141 per year.

72. **a.** $$g'(x)=\frac{(x^2-110x+3500)(-30x+1125)-(2x-110)(-15x^2+1125x)}{(x^2-110x+3500)^2}$$
$$=\frac{525x^2-105,000x+3,937,500}{(x^2-110x+3500)^2}$$

b. $$g'(40)=\frac{525(40)^2-105,000(40)+3,937,500}{\left((40)^2-110(40)+3500\right)^2}$$
$$=1.1785714\ \frac{\text{mi/gallon}}{\text{mi/hour}}$$

At 40 mph, your gas mileage increases by 1.1785714 for each additional mile per hour.

$$g'(50)=\frac{525(50)^2-105,000(50)+3,937,500}{\left((50)^2-110(50)+3500\right)^2}$$
$$=0\ \frac{\text{mi/gallon}}{\text{mi/hour}}$$

At 50 mph, your gas mileage will not change for each additional mile per hour.

$$g'(60)=\frac{525(60)^2-105,000(60)+3,937,500}{\left((60)^2-110(60)+3500\right)^2}$$
$$=-1.89\ \frac{\text{mi/gallon}}{\text{mi/hour}}$$

At 60 mph, your gas mileage decreases by 1.89 for each additional mile per hour.

c. The positive sign of $g'(40)$ tells you that gas mileage increases with speed when driving at 40 mph. The negative sign of $g'(60)$ tells you that gas mileage decreases with speed when driving at 60 mph. The fact that $g'(50)$ is zero tells you that gas mileage neither increases nor decreases with speed when driving at 50 mph. This means it is the most economical speed.

73. **a.** $$\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{d}{dx}(x^{-2})=-2x^{-3}=-\frac{2}{x^3}$$
$$\left.\frac{d}{dx}\right|_{x=0}=-\frac{2}{0^3}\quad \text{Undefined}$$

b. Answers will vary.

74. **a.** $$\frac{d}{dx}\left(\frac{1}{x}\right)=\frac{d}{dx}(x^{-1})=-x^{-2}=-\frac{1}{x^2}$$
$$\left.\frac{d}{dx}\right|_{x=0}=-\frac{1}{0^2}\quad \text{Undefined}$$

b. Answers will vary.

75. False: the product rule gives the correct right-hand side.

76. False: the quotient rule gives the correct right-hand side.

77. True: $\frac{d}{dx}(x\cdot f)=\frac{d}{dx}x\cdot f+x\cdot f'=f+x\cdot f'$

78. True: $\frac{d}{dx}\left(\frac{f}{x}\right)=\frac{x\cdot f'-\frac{d}{dx}x\cdot f}{(x)^2}=\frac{x\cdot f'-f}{x^2}$

79. $\frac{d}{dx}(f\cdot g)=\frac{(f\cdot g)f'}{f}+\frac{(f\cdot g)g'}{g}=g\cdot f'+f\cdot g'$

The right-hand side multiplies out to $g\cdot f'+f\cdot g'$ which agrees with the product rule.

80. $$\frac{d}{dx}\left(\frac{f}{g}\right)=\left(\frac{f}{g}\right)\left(\frac{f'}{f}\right)-\left(\frac{f}{g}\right)\left(\frac{g'}{g}\right)$$
$$=\frac{f'}{g}-\frac{f\cdot g'}{g^2}$$
$$=\frac{g\cdot f'+f\cdot g'}{g^2}$$

The right-hand side multiplies out to $\frac{g\cdot f'+f\cdot g'}{g^2}$ which agrees with the quotient rule.

81. False: This would be the same as saying that the derivative (instantaneous rate of change) of a product is a product of the derivatives. The product rule gives the correct way of finding the derivative of a product.

82. False: This would be the same as saying that the derivative (instantaneous rate of change) of a quotient is a quotient of the derivatives. The quotient rule gives the correct way of finding the derivative of a quotient.

83. $$\frac{d}{dx}(f\cdot g\cdot h)=\frac{d}{dx}[f\cdot(g\cdot h)]=\frac{df}{dx}\cdot(g\cdot h)+f\cdot\frac{d}{dx}(g\cdot h)$$
$$=\frac{df}{dx}\cdot(g\cdot h)+f\left(\frac{dg}{dx}\cdot h+g\cdot\frac{dh}{dx}\right)$$
$$=\frac{df}{dx}\cdot g\cdot h+f\cdot\frac{dg}{dx}\cdot h+f\cdot g\cdot\frac{dh}{dx}$$

84.

a. $Q(x)=\frac{f(x)}{g(x)}$

b. $g(x)\cdot Q(x)=\frac{f(x)}{g(x)}\cdot g(x)$

$Q(x)\cdot g(x)=f(x)$

c. $Q'(x)\cdot g(x)+Q(x)\cdot g'(x)=f'(x)$

d. $Q'(x)\cdot g(x)=f'(x)-Q(x)\cdot g'(x)$

$$Q'(x)=\frac{f'(x)-Q(x)\cdot g'(x)}{g(x)}$$

e. $$Q'(x)=\frac{f'(x)-\frac{f(x)}{g(x)}\cdot g'(x)}{g(x)}=\frac{\frac{1}{g(x)}[g(x)\cdot f'(x)-f(x)\cdot g'(x)]}{g(x)}=\frac{g(x)\cdot f'(x)-f(x)\cdot g'(x)}{[g(x)]^2}$$

85. $$\frac{d}{dx}[f(x)]^2=\frac{d}{dx}[f(x)\cdot f(x)]$$
$$=\left[\frac{d}{dx}f(x)\right]f(x)+f(x)\left[\frac{d}{dx}f(x)\right]$$
$$=f'(x)\cdot f(x)+f(x)\cdot f'(x)=2f(x)\cdot f'(x)$$

86. Rewrite $[f(x)]^{-1}=\frac{1}{f(x)}$ and find $\frac{d}{dx}\left[\frac{1}{f(x)}\right]$.

$$\frac{d}{dx}\left[\frac{1}{f(x)}\right]=\frac{f(x)\cdot 0-f'(x)\cdot 1}{[f(x)]^2}=-\frac{f'(x)}{[f(x)]^2}$$

87. $$\frac{dy}{dx}\left(\frac{Rx}{1+\left(\frac{R-1}{K}\right)x}\right)=\frac{\left(1+\left(\frac{R-1}{K}\right)x\right)R-Rx\left(\frac{R-1}{K}\right)}{\left(1+\left(\frac{R-1}{K}\right)x\right)^2}=\frac{R+Rx\left(\frac{R-1}{K}\right)-Rx\left(\frac{R-1}{K}\right)}{\left(1+\left(\frac{R-1}{K}\right)x\right)^2}=\frac{R}{\left(1+\left(\frac{R-1}{K}\right)x\right)^2}$$

To show $y'>0$, show that both the numerator and denominator are greater than zero.

$R > 1 > 0 \Rightarrow$ numerator is greater than zero.

$R > 1 \Rightarrow R - 1 > 0.$

$R - 1 > 0; K > 0 \Rightarrow \frac{R-1}{K} > 0$

$\frac{R-1}{K} > 0,\ x > 0 \Rightarrow \left(1+\left(\frac{R-1}{K}\right)x\right) > 0 \Rightarrow$ denominator is greater than zero.

$\therefore y' > 0.$

This means the density of the offspring is always increasing faster than the density of the parents.

88. $w > 5$ and $3.5 > 0$, $(w-1.5)^2 > 0$ and $w-1.5>0 \Rightarrow R'(w)=\frac{w-5}{w-1.5}>0.$

A task that expends w kcal/min of work for $w > 5$ requires more than w minutes of rest.

EXERCISES 2.5

1.
a. $f'(x)=4x^3-3(2)x^2-2(3)x+5-0$
$=4x^3-6x^2-6x+5$
b. $f''(x)=3(4)x^2-2(6)x-6+0=12x^2-12x-6$
c. $f'''(x)=2(12)x-12-0=24x-12$
d. $f^{(4)}(x)=24$

2.
a. $f'(x)=4x^3-9x^2+4x-8$
b. $f''(x)=12x^2-18x+4$
c. $f'''(x)=24x-18$
d. $f^{(4)}(x)=24$

3.
a. $f'(x)=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4$
b. $f''(x)=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3$
c. $f'''(x)=1+x+\frac{1}{2}x^2$
d. $f^{(4)}(x)=1+x$

4.
a. $f'(x)=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3$
b. $f''(x)=1+x+\frac{1}{2}x^2$
c. $f'''(x)=1+x$
d. $f^{(4)}(x)=1$

5. $f(x)=\sqrt{x^5}=x^{5/2}$
a. $f'(x)=\frac{5}{2}x^{3/2}$
b. $f''(x)=\frac{3}{2}\left(\frac{5}{2}\right)x^{1/2}=\frac{15}{4}\sqrt{x}$
c. $f'''(x)=\frac{d}{dx}\left(\frac{15}{4}x^{1/2}\right)=\frac{1}{2}\left(\frac{15}{4}\right)x^{-1/2}$
$=\frac{15}{8}x^{-1/2}=\frac{15}{8\sqrt{x}}$
d. $f^{(4)}(x)=\frac{d}{dx}\left(\frac{15}{8}x^{-1/2}\right)=-\frac{1}{2}\left(\frac{15}{8}\right)x^{-3/2}$
$=-\frac{15}{16}x^{-3/2}$

6. $f(x)=\sqrt{x^3}=x^{3/2}$
a. $f'(x)=\frac{3}{2}x^{1/2}=\frac{3}{2}\sqrt{x}$
b. $f''(x)=\frac{3}{4}x^{-1/2}=\frac{3}{4\sqrt{x}}$
c. $f'''(x)=-\frac{3}{8}x^{-3/2}$
d. $f^{(4)}(x)=\frac{9}{16}x^{-5/2}$

7. $f(x)=\frac{x-1}{x}=1-\frac{1}{x}$

a. $f'(x)=\frac{1}{x^2}$

$f''(x)=\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{d}{dx}(x^{-2})=-2x^{-3}$

$=-\frac{2}{x^3}$

b. $f''(3)=-\frac{2}{3^3}=-\frac{2}{27}$

8. $f(x)=\frac{x+2}{x}=1+\frac{2}{x}$

a. $f'(x)=-\frac{2}{x^2}$

$f''(x)=\frac{d}{dx}\left(-\frac{2}{x^2}\right)=\frac{d}{dx}(-2x^{-2})=4x^{-3}$

$=\frac{4}{x^3}$

b. $f''(3)=\frac{4}{3^3}=\frac{4}{27}$

9. $f(x)=\frac{x+1}{2x}=\frac{1}{2}+\frac{1}{2x}$

a. $f'(x)=-\frac{1}{2x^2}$

$f''(x)=\frac{d}{dx}\left(-\frac{1}{2x^2}\right)=\frac{d}{dx}\left(-\frac{1}{2}x^{-2}\right)=\frac{1}{x^3}$

b. $f''(3)=\frac{1}{3^3}=\frac{1}{27}$

10. $f(x)=\frac{x-2}{4x}=\frac{1}{4}-\frac{1}{2x}$

a. $f'(x)=\frac{1}{2x^2}$

$f''(x)=\frac{d}{dx}\left(\frac{1}{2x^2}\right)=\frac{d}{dx}\left(\frac{1}{2}x^{-2}\right)=-\frac{1}{x^3}$

b. $f''(3)=-\frac{1}{3^3}=-\frac{1}{27}$

11. $f(x)=\frac{1}{6x^2}$

a. $f'(x)=-2\left(\frac{1}{6}\right)x^{-3}=-\frac{1}{3x^3}$

$f''(x)=\frac{d}{dx}\left(-\frac{1}{3}x^{-3}\right)=x^{-4}=\frac{1}{x^4}$

b. $f''(3)=\frac{1}{3^4}=\frac{1}{81}$

12. $f(x)=\frac{1}{12x^3}$

a. $f'(x)=-\frac{1}{4}x^{-4}$

$f''(x)=\frac{d}{dx}\left(-\frac{1}{4}x^{-4}\right)=x^{-5}=\frac{1}{x^5}$

b. $f''(3)=\frac{1}{3^5}=\frac{1}{243}$

13. $f(x)=(x^2-2)(x^2+3)=x^4+x^2-6$

$f'(x)=4x^3+2x$

$f''(x)=12x^2+2$

14. $f(x)=(x^2-1)(x^2+2)=x^4+x^2-2$

$f'(x)=4x^3+2x$

$f''(x)=12x^2+2$

15. $f(x)=\frac{27}{\sqrt[3]{x}}=27x^{-1/3}$

$f'(x)=\left(-\frac{1}{3}\right)(27x^{-4/3})=-9x^{-4/3}$

$f''(x)=\left(-\frac{4}{3}\right)(-9x^{-7/3})=12x^{-7/3}$

16. $f(x)=\frac{32}{\sqrt[4]{x}}=32x^{-1/4}$

$f'(x)=\left(-\frac{1}{4}\right)(32x^{-5/4})=-8x^{-5/4}$

$f''(x)=\left(-\frac{5}{4}\right)(-8x^{-9/4})=10x^{-9/4}$

17. $f(x)=\frac{x}{x-1}$

$$f'(x)=\frac{(x-1)\frac{dx}{dx}-\left[\frac{d}{dx}(x-1)\right](x)}{(x-1)^2}=\frac{(x-1)(1)-(1)(x)}{(x-1)^2}=-\frac{1}{(x-1)^2}=-\frac{1}{x^2-2x+1}$$

$$f''(x)=\frac{(x^2-2x+1)\frac{d}{dx}(-1)-\left[\frac{d}{dx}(x^2-2x+1)\right](-1)}{[(x-1)^2]^2}=\frac{0-(2x-2)(-1)}{(x-1)^4}=\frac{2x-2}{(x-1)^4}=\frac{2(x-1)}{(x-1)^4}=\frac{2}{(x-1)^3}$$

18. $f(x)=\dfrac{x}{x-2}$

$$f'(x)=\frac{(x-2)(1)-(1)(x)}{(x-2)^2}=\frac{x-2-x}{(x-2)^2}=-\frac{2}{(x-2)^2}=-\frac{2}{x^2-4x+4}$$

$$f''(x)=\frac{(x^2-4x+4)\cdot 0-(2x-4)(-2)}{(x^2-4x+4)^2}=\frac{4x-8}{[(x-2)^2]^2}=\frac{4(x-2)}{(x-2)^4}=\frac{4}{(x-2)^3}$$

19. $\dfrac{d}{dr}(\pi r^2)=2\pi r$

$\dfrac{d^2}{dr^2}(\pi r^2)=\dfrac{d}{dr}(2\pi r)=2\pi$

20. $\dfrac{d}{dr}(\frac{4}{3}\pi r^3)=4\pi r^2$

$\dfrac{d^2}{dr^2}(\frac{4}{3}\pi r^3)=\dfrac{d}{dr}(4\pi r^2)=8\pi r$

$\dfrac{d^3}{dr^3}(\frac{4}{3}\pi r^3)=\dfrac{d}{dr}(8\pi r)=8\pi$

21. $\dfrac{d}{dx}x^{10}=10x^9$

$\dfrac{d^2}{dx^2}x^{10}=\dfrac{d}{dx}(10x^9)=90x^8$

$\left.\dfrac{d^2}{dx^2}x^{10}\right|_{x=-1}=90(-1)^8=90$

22. $\dfrac{d}{dx}x^{11}=11x^{10}$

$\dfrac{d^2}{dx^2}x^{11}=\dfrac{d}{dx}(11x^{10})=110x^9$

$\left.\dfrac{d^2}{dx^2}x^{11}\right|_{x=-1}=110(-1)^9=-110$

23. From Exercise 21, we know

$\dfrac{d^2}{dx^2}x^{10}=90x^8$

$\dfrac{d^3}{dx^3}x^{10}=\dfrac{d}{dx}(90x^8)=720x^7$

Thus, $\left.\dfrac{d^3}{dx^3}x^{10}\right|_{x=-1}=720(-1)^7=-720$

24. From Exercise 22, we know

$\dfrac{d^2}{dx^2}x^{11}=110x^9$

$\dfrac{d^3}{dx^3}x^{11}=\dfrac{d}{dx}(110x^9)=990x^8$

Thus, $\left.\dfrac{d^3}{dx^3}x^{11}\right|_{x=-1}=990(-1)^8=990$

25. $\dfrac{d}{dx}\sqrt{x^3}=\dfrac{d}{dx}x^{3/2}=\dfrac{3}{2}x^{1/2}$

$\dfrac{d^2}{dx^2}\sqrt{x^3}=\dfrac{d}{dx}\left(\dfrac{3}{2}x^{1/2}\right)=\dfrac{3}{4}x^{-1/2}$

$\left.\dfrac{d^2}{dx^2}\sqrt{x^3}\right|_{x=1/16}=\dfrac{3}{4}\left(\dfrac{1}{16}\right)^{-1/2}=\dfrac{3}{4}\sqrt{16}=3$

26. $\dfrac{d}{dx}\sqrt[3]{x^4}=\dfrac{d}{dx}x^{4/3}=\dfrac{4}{3}x^{1/3}$

$\dfrac{d^2}{dx^2}\sqrt[3]{x^4}=\dfrac{d}{dx}\left(\dfrac{4}{3}x^{1/3}\right)=\dfrac{4}{9}x^{-2/3}$

$\left.\dfrac{d^2}{dx^2}\left(\sqrt[3]{x^4}\right)\right|_{x=1/27}=\dfrac{4}{9}\left(\dfrac{1}{27}\right)^{-2/3}=\dfrac{4}{9}(27)^{2/3}=4$

27.

$$\begin{aligned}\frac{d}{dx}[(x^2-x+1)(x^3-1)]&=(2x-1)(x^3-1)+(x^2-x+1)(3x^2)\\&=2x^4-x^3-2x+1+3x^4-3x^3+3x^2\\&=5x^4-4x^3+3x^2-2x+1\end{aligned}$$

$$\begin{aligned}\frac{d^2}{dx^2}[(x^2-x+1)(x^3-1)]&=\frac{d}{dx}(5x^4-4x^3+3x^2-2x+1)\\&=20x^3-12x^2+6x-2\end{aligned}$$

28.

$$\begin{aligned}\frac{d}{dx}[(x^3+x-1)(x^3+1)]&=(3x^2+1)(x^3+1)+(x^3+x-1)(3x^2)\\&=3x^5+3x^2+x^3+1+3x^5+3x^3-3x^2=6x^5+4x^3+1\end{aligned}$$

$$\frac{d^2}{dx^2}[(x^3+x-1)(x^3+1)]=\frac{d}{dx}(6x^5+4x^3+1)=30x^4+12x^2$$

29. $\frac{d}{dx}\left(\frac{x}{x^2+1}\right) = \frac{(x^2+1)\left(\frac{dx}{dx}\right) - x\frac{d}{dx}(x^2+1)}{(x^2+1)^2} = \frac{x^2+1-x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{1-x^2}{x^4+2x^2+1}$

$$\frac{d^2}{dx^2}\left(\frac{x}{x^2+1}\right) = \frac{d}{dx}\left(\frac{1-x^2}{x^4+2x^2+1}\right) = \frac{(x^4+2x^2+1)\frac{d}{dx}(1-x^2)-(1-x^2)\frac{d}{dx}(x^4+2x^2+1)}{[(x^2+1)^2]^2}$$

$$= \frac{(x^4+2x^2+1)(-2x)-(1-x^2)(4x^3+4x)}{(x^2+1)^4} = \frac{(x^2+1)[(x^2+1)(-2x)-(1-x^2)(4x)]}{(x^2+1)^4}$$

$$= \frac{-2x^3-2x-4x+4x^3}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}$$

30. $\frac{d}{dx}\left(\frac{x}{x^2-1}\right) = \frac{(x^2-1)\cdot 1-(2x)x}{(x^2-1)^2} = \frac{x^2-2x^2-1}{(x^2-1)^2} = \frac{-x^2-1}{(x^2-1)^2} = \frac{-x^2-1}{x^4-2x^2+1}$

$$\frac{d^2}{dx^2}\left(\frac{x}{x^2-1}\right) = \frac{(x^2-1)^2(-2x)-(4x^3-4x)(-x^2-1)}{(x^2-1)^4}$$

$$= \frac{-2x^5+4x^3-2x-(-4x^5-4x^3+4x^3+4x)}{(x^2-1)^4}$$

$$= \frac{-2x^5+4x^3-2x+4x^5-4x}{(x^2-1)^4} = \frac{2x^5+4x^3-6x}{(x^2-1)^4}$$

31. $\frac{d}{dx}\left(\frac{2x-1}{2x+1}\right) = \frac{(2x+1)\cdot 2-2(2x-1)}{(2x+1)^2} = \frac{4x+2-4x+2}{4x^2+4x+1} = \frac{4}{4x^2+4x+1}$

$$\frac{d^2}{dx^2}\left(\frac{2x-1}{2x+1}\right) = \frac{(4x^2+4x+1)\cdot 0-(8x+4)(4)}{(4x^2+4x+1)^2} = -\frac{32x+16}{[(2x+1)^2]^2} = -\frac{32x+16}{(2x+1)^4}$$

$$= -\frac{16(2x+1)}{(2x+1)^4} = -\frac{16}{(2x+1)^3}$$

32. $\frac{d}{dx}\left(\frac{3x+1}{3x-1}\right) = \frac{(3x-1)(3)-3(3x+1)}{(3x-1)^2} = \frac{9x-3-9x-3}{9x^2-6x+1} = -\frac{6}{9x^2-6x+1}$

$$\frac{d^2}{dx^2}\left(\frac{3x+1}{3x-1}\right) = -\frac{(9x^2-6x+1)\cdot 0-6(18x-6)}{(9x^2-6x+1)^2} = \frac{36(3x-1)}{[(3x-1)^2]^2} = \frac{36}{(3x-1)^3}$$

33. To find velocity, we differentiate the distance function $s(t) = 18t^2 - 2t^3$.
$v(t) = s'(t) = 36t - 6t^2$

a. For $t = 3$, $v(3) = s'(3) = 36(3) - 6(3)^2 = 108 - 54 = 54$ miles per hour

b. For $t = 7$, $v(7) = s'(7) = 36(7) - 6(7)^2 = 252 - 294 = -42$ miles per hour

c. To find the acceleration, we differentiate the velocity function $s'(t) = 36t - 6t^2$.
$a(t) = s''(t) = 36 - 12t$.
For $t = 1$, $a(1) = s''(1) = 36 - 12(1) = 24$ mi / hr^2

34. To find velocity, we differentiate the distance function $s(t) = 24t^2 - 2t^3$.
$v(t) = s'(t) = 48t - 6t^2$

a. For $t = 4$, $v(4) = s'(4) = 48(4) - 6(4)^2 = 96$ miles per hour

b. For $t = 10$, $v(10) = s'(10) = 48(10) - 6(10)^2 = -120$ miles per hour

c. To find the acceleration, we differentiate the velocity function $s'(t) = 48t - 6t^2$.
$a(t) = s''(t) = 48 - 12t$.
For $t = 1$, $a(1) = s''(1) = 48 - 12(1) = 36$ mi / hr^2

35. $v(t) = h'(t) = 3t^2 + 2(0.5)t = 3t^2 + t$

$v(10) = 3(10)^2 + 10 = 310$ feet per second

$a(t) = v'(t) = \frac{d}{dt}(3t^2 + t) = 6t + 1$

$a(10) = 6(10) + 1 = 61 \text{ ft/sec}^2$

36. $v(t) = s'(t) = 60 + \frac{(t+3)\cdot 0 - (1)(100)}{(t+3)^2}$

$= 60 - \frac{100}{(t+3)^2}$

$v(2) = 60 - \frac{100}{(2+3)^2} = 60 - \frac{100}{25} = 56$ miles per hour

37. **a.** To find when the steel ball will reach the ground, we need to determine what value of t produces $s(t) = 0$. Thus, set $s(t) = 0$ and solve the equation.

$0 = s(t) = 1667 - 16t^2$

$0 = 1667 - 16t^2$

$16t^2 = 1667$

$t^2 = 104.1875$

$t \approx 10.20722783$ sec

The steel ball will reach the ground after about 10.2 seconds.

b. $v(t) = s'(t) = -32t$

$v(10.2) = s'(10.2) = 326.4$ feet per second

38. **a.** To find how long it will take to reach the ground, we need to determine what value of t produces $s(t) = 0$. Thus, set $s(t) = 0$ and solve the equation.

$s(t) = 1454 - 16t^2 = 0$

$t^2 = \frac{1454}{16}$

$t \approx 9.53$

It will take about 9.53 seconds to reach the ground.

b. $v(t) = s'(t) = -32t$

$v(9.53) = -32(9.53) \approx -305$ feet per second

39. **a.** $v(t) = s'(t) = -32t + 1280$

b. When $s(t)$ is a maximum, $s'(t) = 0$.

$-32t + 1280 = 0$

$32t = 1280$

$t = 40$ seconds

c. At $t = 40$,

$s(40) = -16(40)^2 + 1280(40)$

$= -25{,}600 + 51{,}200$

$= 25{,}600$ feet

40. $T(2) = 98 + \frac{8}{\sqrt{2}} \approx 103.7$

The temperature of a patient 2 hours after taking the medicine is 103.7 degrees.

$T(t) = 98 + 8t^{-1/2}$

$T'(t) = -4t^{-3/2} = -\frac{4}{t^{3/2}}$

$T'(2) = -\frac{4}{2^{3/2}} \approx -1.4$

Two hours after taking the medicine, the patient's temperature is decreasing at the rate of 1.4 degrees per hour.

$T''(t) = 6t^{-5/2} = \frac{6}{t^{5/2}}$

$T''(2) = \frac{6}{2^{5/2}} \approx 1.1$

After 2 hours, the rate of decrease of the patient's temperature is increasing by about 1.1 degree per hour each hour.

41. $D'(t) = \frac{4}{3}\left(9t^{1/3}\right) = 12t^{1/3}$

$D'(8) = 12(8)^{1/3} = 12(2) = 24$

The national debt is increasing by 24 billion dollars per year after 8 years.

$D''(t) = \frac{d}{dt}\left(12t^{1/3}\right) = \frac{1}{3}\left(12t^{-2/3}\right) = 4t^{-2/3}$

$D''(8) = 4(8)^{-2/3} = 4\left(\frac{1}{8}\right)^{2/3} = 4\left(\frac{1}{4}\right) = 1$

The rate of growth of the national debt is increasing by 1 billion dollars per year each year after 8 years.

42. $T(10) \approx 6.3$

In 2100 global temperatures will have risen by about 6.3°F.

$T'(t) = 0.35t^{0.4}$

$T'(10) \approx 0.88$

In 2100 global temperatures will be rising by 0.88 degrees per decade, or about a tenth of a degree per year.

$T''(t) = 0.14t^{-0.6}$

$T''(10) \approx 0.035$

The temperature increases will be speeding up (by about 0.035 degrees per decade per decade).

43. $L(10) = 9.3$

By 2100, sea levels may have risen by 93 cm (about 3 feet).

$L'(t) = 0.06x^4 - 0.14x + 8$

$L'(10) = 12.6$

In 2100 sea levels will be rising by about 12.6 centimeters per decade, or about 1.26 cm (about half an inch) per year.

$L''(x) = 0.12x - 0.14$

$L''(10) = 1.06$

The rise in the sea level will be speeding up (by about 1 cm per decade per decade).

44. $P'(x) = 1.581x^{-0.7} - 0.70376x^{0.52}$

$P''(x) \approx -1.1067x^{-1.7} - 0.3660x^{-0.48}$

$P(3) \approx 4.87$

The profit 3 years from now will be \$4.87 million.

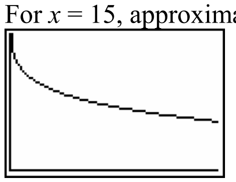
$P'(3) \approx -0.51$

The profit will be decreasing by about \$0.51 million per year 3 years from now.

$P''(3) \approx -0.39$

In 3 years, the rate of decline of profit will be accelerating by about \$0.39 million per year each year.

45. **a.** For $x = 15$, approximately 21.589°; for $x = 30$, approximately 17.597°.

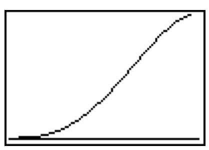


on [0,50] by [0,40]

b. Each 1-mph increase in wind speed lowers the wind-chill index. As wind speed increases, the rate with which the wind-chill index decreases slows.

c. For $x = 15$, $y' = -0.363$. For a wind speed of 15 mph, each additional mile per hour decreases the wind-chill index by about 0.363. For $x = 30$, $y' = -0.203$. For a wind speed of 30 mph, each additional mile per hour decreases the wind-chill index by about 0.203˚.

46. **a.**



on [1, 20] by [0, 800]

b. The second derivative is $f''(x) = -0.2184x^2 + 3.156x - 2.6$, which equals zero when x is approximately equal to 13.57. The AIDS epidemic began to slow in 1993.

47. **a.** The first derivative is negative because the temperature is dropping.

b. The second derivative is negative because the temperature is dropping increasingly rapidly making the change in temperature speed up in a negative direction.

48. **a.** The first derivative is positive because the economy is growing.

b. The second derivative is negative because the growth is slowing.

49. **a.** The first derivative is negative, because the stock market is declining.

b. The second derivative is positive because the change in the stock market is slowing down in a negative direction.

50. **a.** The first derivative is positive because the population is growing.

b. The second derivative is positive because the change in population is speeding up.

51. True: For example $\frac{d^4}{dx^4}x^3 = 0$.

52. **a.** $f'(1)$ will be positive because the altitude is increasing.

$f''(1)$ will be positive because acceleration is positive.

b. $f'(59)$ will be negative because the altitude is decreasing.

$f''(59)$ will be negative because acceleration is negative.

53. **a.** iii (showing a stop, then a slower velocity)
b. i (showing a stop, then a negative velocity)
c. ii (showing stops and starts and then a higher velocity)

54. **a.** ii (showing positive and increasing slope)
b. iii (showing negative but increasing slope)
c. i (showing positive but decreasing slope)

55. $\frac{d^{100}}{dx^{100}}(x^{99} - 4x^{98} + 3x^{50} + 6) = 0$

56. $\frac{d}{dx}(x^{-1}) = -x^{-2}$

$\frac{d^2}{dx^2}(x^{-1}) = 2x^{-3} = \frac{2!}{x^3}$

$\frac{d^3}{dx^3}(x^{-1}) = -6x^{-4} = -\frac{3!}{x^4}$

$\frac{d^4}{dx^4}(x^{-1}) = 24x^{-5} = \frac{4!}{x^5}$

$\vdots$

$\frac{d^n}{dx^n}(x^{-1}) = (-1)^n \frac{n!}{x^{n+1}}$

57. $\frac{d^2}{dx^2}(f \cdot g) = \frac{d}{dx}\left(\frac{df}{dx} \cdot g + f \cdot \frac{dg}{dx}\right) \quad \left[\text{or } \frac{d}{dx}(f' \cdot g + f \cdot g')\right]$

$= \frac{d}{dx}\left(\frac{df}{dx} \cdot g\right) + \frac{d}{dx}\left(f \cdot \frac{dg}{dx}\right) \quad \left[\text{or } \frac{d}{dx}(f' \cdot g + \frac{d}{dx}(f \cdot g')\right]$

$\frac{d^2 f}{dx^2} \cdot g + \frac{df}{dx} \cdot \frac{dg}{dx} + \frac{df}{dx} \cdot \frac{dg}{dx} + f \cdot \frac{d^2 g}{dx^2} \quad [\text{or } f'' \cdot g + f' \cdot g' + f' \cdot g' + f \cdot g'']$

$= f'' \cdot g + 2f' \cdot g' + f \cdot g''$

58. From Exercise 31, we know that $\frac{d^2}{dx^2}(f \cdot g) = f'' \cdot g + 2f' \cdot g' + f \cdot g''$.

$\frac{d^3}{dx^3}(f \cdot g) = \frac{d}{dx}\left[\frac{d^2}{dx^2}(f \cdot g)\right] = \frac{d}{dx}(f'' \cdot g + 2f' \cdot g' + f \cdot g'') = f''' \cdot g + f'' \cdot g' + 2f'' \cdot g' + 2f' \cdot g'' + f' \cdot g'' + f \cdot g'''$

$= f''' \cdot g + 3f'' \cdot g' + 3f' \cdot g'' + f \cdot g'''$

EXERCISES 2.6

1. $f(g(x)) = \sqrt{x^2 - 3x + 1}$

The outside function is $\sqrt{x}$ and the inside function is $x^2 - 3x + 1$.

Thus, we take $\begin{cases} f(x) = \sqrt{x} \\ g(x) = x^2 - 3x + 1 \end{cases}$

2. $f(g(x)) = (5x^2 - x + 2)^4$

The outside function is x^4 and the inside function is $5x^2 - x + 2$.

Thus, we take $\begin{cases} f(x) = x^4 \\ g(x) = 5x^2 - x + 2 \end{cases}$

3. $f(g(x)) = (x^2 - x)^{-3}$

The outside function is x^{-3} and the inside function is $x^2 - x$.

Thus, we take $\begin{cases} f(x) = x^{-3} \\ g(x) = x^2 - x \end{cases}$

4. $f(g(x)) = \frac{1}{x^2 + x}$

The outside function is $\frac{1}{x}$ and the inside function is $x^2 + x$.

Thus, we take $\begin{cases} f(x) = \frac{1}{x} \\ g(x) = x^2 + x \end{cases}$

5. $f(g(x)) = \frac{x^3+1}{x^3-1}$
$f(x) = \frac{x+1}{x-1}$
$g(x) = x^3$

6. $f(g(x)) = \frac{\sqrt{x}-1}{\sqrt{x}+1}$
$f(x) = \frac{x-1}{x+1}$
$g(x) = \sqrt{x}$

7. $f(g(x)) = \left(\frac{x+1}{x-1}\right)^4$
$f(x) = x^4$
$g(x) = \frac{x+1}{x-1}$

8. $f(g(x)) = \sqrt{\frac{x-1}{x+1}}$
$f(x) = \sqrt{x}$
$g(x) = \frac{x-1}{x+1}$

9. $f(g(x)) = \sqrt{x^2-9}+5$
$f(x) = \sqrt{x}+5$
$g(x) = x^2-9$

10. $f(g(x)) = \sqrt[3]{x^3+8}-5$
$f(x) = \sqrt[3]{x}-5$
$g(x) = x^3+8$

11. $f(x) = (x^2+1)^3$
$f'(x) = 3(x^2+1)^2(2x) = 6x(x^2+1)^2$

12. $f(x) = (x^3+1)^4$
$f'(x) = 4(x^3+1)^3(3x^2) = 12x^2(x^3+1)^3$

13. $g(x) = (2x^2-7x+3)^4$
$g'(x) = 4(2x^2-7x+3)^3(4x-7)$

14. $g(x) = (3x^3-x^2+1)^5$
$g'(x) = 5(3x^3-x^2+1)^4(9x^2-2x)$

15. $h(z) = (3z^2-5z+2)^4$
$h'(z) = 4(3z^2-5z+2)^3(6z-5)$

16. $h(z) = (5z^2+3z-1)^3$
$h'(z) = 3(5z^2+3z-1)^2(10z+3)$

17. $f(x) = \sqrt{x^4-5x+1} = (x^4-5x+1)^{1/2}$
$f'(x) = \frac{1}{2}(x^4-5x+1)^{-1/2}(4x^3-5)$

18. $f(x) = \sqrt{x^6+3x-1} = (x^6+3x-1)^{1/2}$
$f'(x) = \frac{1}{2}(x^6+3x-1)^{-1/2}(6x^5+3)$

19. $w(z) = \sqrt[3]{9z-1} = (9z-1)^{1/3}$
$w'(z) = \frac{1}{3}(9z-1)^{-2/3}(9) = 3(9z-1)^{-2/3}$

20. $w(z) = \sqrt[5]{10z-4} = (10z-4)^{1/5}$
$w'(z) = \frac{1}{5}(10z-4)^{-4/5}(10) = 2(10z-4)^{-4/5}$

21. $y = (4-x^2)^4$
$y' = 4(4-x^2)^3(-2x) = -8x(4-x^2)^3$

22. $y = (1-x)^{50}$
$y' = 50(1-x)^{49}(-1) = -50(1-x)^{49}$

23. $y = \left(\frac{1}{w^3-1}\right)^4 = [(w^3-1)^{-1}]^4 = (w^3-1)^{-4}$
$y' = -4(w^3-1)^{-5}(3w^2) = -12w^2(w^3-1)^{-5}$

24. $y = \left(\frac{1}{w^4+1}\right)^5 = [(w^4+1)^{-1}]^5 = (w^4+1)^{-5}$
$y' = -5(w^4+1)^{-6}(4w^3) = -20w^3(w^4+1)^{-6}$

25. $y = x^4+(1-x)^4$
$y' = 4x^3+4(1-x)^3(-1) = 4x^3-4(1-x)^3$

26. $f(x) = (x^2+4)^3-(x^2+4)^2$
$f'(x) = 3(x^2+4)^2(2x)-2(x^2+4)(2x)$
$= 6x(x^2+4)^2-4x(x^2+4)$

27. $f(x)=\dfrac{1}{\sqrt{3x^2-5x+1}}=\left(3x^2-5x+1\right)^{-1/2}$
$f'(x)=-\dfrac{1}{2}\left(3x^2-5x+1\right)^{-3/2}(6x-5)$

28. $f(x)=\dfrac{1}{\sqrt{2x^2-7x+1}}=\left(2x^2-7x+1\right)^{-1/2}$
$f'(x)=-\dfrac{1}{2}\left(2x^2-7x+1\right)^{-3/2}(4x-7)$

29. $f(x)=\dfrac{1}{\sqrt[3]{(9x+1)^2}}=\dfrac{1}{(9x+1)^{2/3}}=(9x+1)^{-2/3}$
$f'(x)=-\dfrac{2}{3}(9x+1)^{-5/3}(9)=-6(9x+1)^{-5/3}$

30. $f(x)=\dfrac{1}{\sqrt[3]{(3x-1)^2}}=(3x-1)^{-2/3}$
$f'(x)=-\dfrac{2}{3}(3x-1)^{-5/3}(3)=-2(3x-1)^{-5/3}$

31. $f(x)=\dfrac{1}{\sqrt[3]{\left(2x^2-3x+1\right)^2}}=\left(2x^2-3x+1\right)^{-2/3}$
$f'(x)=-\dfrac{2}{3}\left(2x^2-3x+1\right)^{-5/3}(4x-3)$

32. $f(x)=\dfrac{1}{\sqrt[3]{\left(x^2+x-9\right)^2}}=\left(x^2+x-9\right)^{-2/3}$
$f'(x)=-\dfrac{2}{3}\left(x^2+x-9\right)^{-5/3}(2x+1)$

33. $f(x)=[(x^2+1)^3+x]^3$
$f'(x)=3[(x^2+1)^3+x]^2[3(x^2+1)^2(2x)+1]$
$=3[(x^2+1)^3+x]^2[6x(x^2+1)^2+1]$

34. $f(x)=[(x^3+1)^2-x]^4$
$f'(x)=4[(x^3+1)^2-x]^3[2(x^3+1)(3x^2)-1]$
$=4[(x^3+1)^2-x]^3[6x^2(x^3+1)-1]$

35. $f(x)=3x^2(2x+1)^5$
$f'(x)=6x(2x+1)^5+3x^2[5(2x+1)^4(2)]$
$=6x(2x+1)^5+30x^2(2x+1)^4$

36. $f(x)=2x(x^3-1)^4$
$f'(x)=2(x^3-1)^4+2x[4(x^3-1)^3(3x^2)]$
$=2(x^3-1)^4+24x^3(x^3-1)^3$

37. $f(x)=(2x+1)^3(2x-1)^4$
$f'(x)=3(2x+1)^2(2)(2x-1)^4+(2x+1)^3[4(2x-1)^3(2)]$
$=6(2x+1)^2(2x-1)^4+8(2x+1)^3(2x-1)^3$

38. $f(x)=(2x-1)^3(2x+1)^4$
$f'(x)=3(2x-1)^2(2)(2x+1)^4+(2x-1)^3[4(2x+1)^3(2)]$
$=6(2x-1)^2(2x+1)^4+8(2x-1)^3(2x+1)^3$

39. $g(z)=2z\left(3z^2-z+1\right)^4$
$g'(z)=2\left(3z^2-z+1\right)^4+2z(4)\left(3z^2-z+1\right)^3(6z-1)$

40. $g(z)=z^2\left(2z^3-z+5\right)^4$
$g'(z)=2z\left(2z^2-z+5\right)^4+2^2(4)\left(2z^2-z+5\right)^3\left(6z^2-1\right)$
$=2z\left(2z^2-z+5\right)^3\left(14z^3-3z+5\right)$

41. $f(x)=\left(\dfrac{x+1}{x-1}\right)^3$

$$f'(x)=3\left(\frac{x+1}{x-1}\right)^2\left[\frac{(x-1)(1)-(1)(x+1)}{(x-1)^2}\right]$$
$$=3\left(\frac{x+1}{x-1}\right)^2\left[\frac{x-1-x-1}{(x-1)^2}\right]=3\left(\frac{x+1}{x-1}\right)^2\left[\frac{-2}{(x-1)^2}\right]$$
$$=-6\frac{(x+1)^2}{(x-1)^4}$$

42. $f(x)=\left(\dfrac{x-1}{x+1}\right)^5$

$$f'(x)=5\left(\frac{x-1}{x+1}\right)^4\left[\frac{(x+1)(1)-(1)(x-1)}{(x+1)^2}\right]$$
$$=5\left(\frac{x-1}{x+1}\right)^4\left[\frac{x+1-x+1}{(x+1)^2}\right]=10\frac{(x-1)^4}{(x+1)^6}$$

43. $f(x)=x^2\sqrt{1+x^2}=x^2\left(1+x^2\right)^{1/2}$

$$f'(x)=2x\left(1+x^2\right)^{1/2}+x^2\left[\frac{1}{2}\left(1+x^2\right)^{-1/2}(2x)\right]$$
$$=2x\left(1+x^2\right)^{1/2}+x^3\left(1+x^2\right)^{-1/2}$$

44. $f(x)=x^2\sqrt{x^2-1}=x^2\left(x^2-1\right)^{1/2}$

$$f'(x)=2x\left(x^2-1\right)^{1/2}+x^2\left[\frac{1}{2}\left(x^2-1\right)^{-1/2}(2x)\right]$$
$$=2x\left(x^2-1\right)^{1/2}+x^3\left(x^2-1\right)^{-1/2}$$

45. $f(x)=\sqrt{1+\sqrt{x}}=\left(1+x^{1/2}\right)^{1/2}$

$$f'(x)=\tfrac{1}{2}\left(1+x^{1/2}\right)^{-1/2}\left(\tfrac{1}{2}x^{-1/2}\right)$$
$$=\tfrac{1}{4}x^{-1/2}\left(1+x^{1/2}\right)^{-1/2}$$

46. $f(x)=\sqrt[3]{1+\sqrt[3]{x}}=\left(1+x^{1/3}\right)^{1/3}$

$$f'(x)=\tfrac{1}{3}\left(1+x^{1/3}\right)^{-2/3}\left(\tfrac{1}{3}x^{-2/3}\right)$$
$$=\tfrac{1}{9}x^{-2/3}\left(1+x^{1/3}\right)^{-2/3}$$

47. **a.** $\dfrac{d}{dx}[(x^2+1)^2]=2(x^2+1)(2x)=4x^3+4x$

b. $\dfrac{d}{dx}[(x^2+1)^2]=\dfrac{d}{dx}(x^4+2x^2+1)=4x^3+4x$

48. **a.** $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right)=\dfrac{x^2\cdot 0-(2x)(1)}{(x^2)^2}=\dfrac{-2x}{x^4}=-\dfrac{2}{x^3}$

b. $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right)=\dfrac{d}{dx}[(x^2)^{-1}]=-1(x^2)^{-2}(2x)$

$$=-\frac{2}{x^3}$$

c. $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right)=\dfrac{d}{dx}(x^{-2})=-2x^{-3}=-\dfrac{2}{x^3}$

49. **a.** $\dfrac{d}{dx}\left(\dfrac{1}{3x+1}\right)=\dfrac{(3x+1)0-3(1)}{(3x+1)^2}=-\dfrac{3}{(3x+1)^2}$

b. $\dfrac{d}{dx}[(3x+1)^{-1}]=-(3x+1)^{-2}(3)$

$$=-\frac{3}{(3x+1)^2}$$

50. $\dfrac{d}{dx}f(g(h(x)))=f'(g(h(x)))g'(h(x))h'(x)$

51. $f(x)=(x^2+1)^{10}$

$$f'(x)=10(x^2+1)^9(2x)=20x(x^2+1)^9$$
$$f''(x)=20(x^2+1)^9+20x[9(x^2+1)^8(2x)]$$
$$=20(x^2+1)^9+40x^2[9(x^2+1)^8]$$
$$=20(x^2+1)^9+360x^2(x^2+1)^8$$

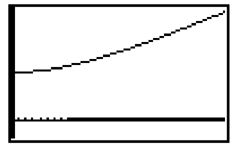
52. $f(x)=(x^3-1)^5$

$$f'(x)=5(x^3-1)^4(3x^2)=15x^2(x^3-1)^4$$
$$f''(x)=30x(x^3-1)^4+15x^2[4(x^3-1)^3(3x^2)]$$
$$=30x(x^3-1)^4+180x^4(x^3-1)^3$$

53. The marginal cost function is the derivative of the function $C(x) = \sqrt{4x^2 + 900} = (4x^2 + 900)^{1/2}$.

$$MC(x) = \frac{1}{2}(4x^2 + 900)^{-1/2}(8x)$$
$$= 4x(4x^2 + 900)^{-1/2}$$
$$MC(20) = 4(20)[4(20)^2 + 900]^{-1/2}$$
$$= 80[4(400) + 900]^{-1/2}$$
$$= 80(2500)^{-1/2} = \frac{80}{50} = 1.60$$

54.



on [0, 30] by [–10, 70]

55. $x = 27$

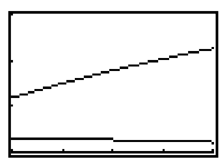
56.
$$S(e) = 0.22(e+4)^{2.1}$$
$$S'(e) = 0.462(e+4)^{1.1}$$
$$S'(12) = 0.462(12+4)^{1.1} \approx 9.75$$
At a level of 12 units, a person's social status increases by about 9.75 units per additional year of education.

57.
$$S(i) = 17.5(i-1)^{0.53}$$
$$S'(i) = 9.275(i-1)^{-0.47}$$
$$S'(25) = 9.275(25-1)^{-0.47} \approx 2.08$$
At an income of $25,000 social status increases by about 2.08 units per additional $1000 of income.

58.
$$V(r) = 1000(1 + 0.01r)^5$$
$$V'(r) = 5000(1 + 0.01r)^4(0.01)$$
$$= 50(1 + 0.01r)^4$$
$$V'(6) = 50[1 + 0.01(6)]^4 \approx 63.12$$
At a rate of 6%, the value increases by about $63.12 for each additional percentage point of interest.

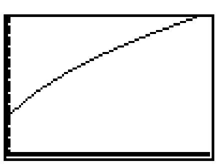
59.
$$R(x) = 4x\sqrt{11+0.5x} = 4x(11+0.5x)^{1/2}$$
$$R'(x) = 4x\left(\frac{1}{2}\right)(11+0.5x)^{-1/2}(0.5) + 4(11+0.5x)^{1/2}$$
$$= \frac{x}{\sqrt{11+0.5x}} + 4\sqrt{11+0.5x}$$
The sensitivity to a dose of 50 mg is
$$R'(50) = \frac{x}{\sqrt{11+0.5(50)}} + 4\sqrt{11+0.5(50)}$$
$$= \frac{50}{\sqrt{11+25}} + 4\sqrt{11+25} = \frac{50}{\sqrt{36}} + 4\sqrt{36}$$
$$= \frac{50}{6} + 24 = \frac{25+72}{3} = \frac{97}{3} = 32\frac{1}{3}$$

60.



on [1, 5] by [0, 3]
$x = 3.6$ years

61.



on [0, 140] by [0, 50]
$x \approx 26$ mg

62.
$$R(x) = 0.25(1+x)^4$$
$$R'(x) = 4(0.25)(1+x)^4 = (1+x)^3$$

a. $R'(0) = (1+0)^3 = 1$

b. $R'(1) = (1+1)^3 = 8$

63. $P(t) = 0.02(12+2t)^{3/2} + 1$
$P'(t) = 0.03(12+2t)^{1/2}(2)$
$= 0.06(12+2t)^{1/2}$
$P'(2) = 0.06[12+2(2)]^{1/2} = 0.24$

64. $T(p) = 36(p+1)^{-1/3}$
$T'(p) = -\frac{1}{3}(36)(p+1)^{-4/3} = -12(p+1)^{-4/3}$
$T'(7) = -12(7+1)^{-4/3} = -\frac{3}{4}$
The time is decreasing by about $\frac{3}{4}$ minute for each additional practice session.

65. **b.** 0.0456
c. $\frac{1.8}{x} = 0.0456$
$x \approx 39.47$, so about 40 years.

66. $h(40) = 8.2 - (0.01(40) - 2.8)^2 = 2.44$
$h'(x) = -2(0.01t - 2.8)(0.01) = -0.0002t + 0.056$
$h'(40) = 0.048$
At a temperature of 40 degrees, happiness approximately 2.4, and each additional degree of temperature would raise happiness by about 0.05.

67. False: There should not be a prime of the first g on the right-hand side.

68. False: There should not be a prime of the first g on the right-hand side.

69. The generalized power rule is a special case of the chain rule, when the outer function is just a power of the variable.

70. The power rule has only x as the inner function, but the generalized power rule can have any inner function.

71. True: $\frac{d}{dx} f(5x) = \frac{d}{dx} 5x \cdot f'(5x) = 5 \cdot f'(5x)$

72. True: $\frac{d}{dx} f\left(\frac{x}{2}\right) = \frac{d}{dx}\frac{x}{2} \cdot f'\left(\frac{x}{2}\right) = \frac{f'\left(\frac{x}{2}\right)}{2}$

73. False: The outer function, $\sqrt{x}$, was not differentiated. The correct right-hand side is $\frac{1}{2}[g(x)]^{-1/2} \cdot g'(x)$.

74. True: $\frac{d}{dx} f(x+5) = f'(x+5) \cdot (1) = f'(x+5)$

75. No: Since instantaneous rates of change are derivatives, this would be saying that $\frac{d}{dx}[f(x)]^2 = [f'(x)]^2$, where $f(x)$ is the length of a side. The chain rule gives the correct derivative of $[f(x)]^2$.

76. No: since instantaneous rates of change are derivatives, this would be saying that $\frac{d}{dx}[f(x)]^3 = [f'(x)]^3$, where $f(x)$ is the length of a side. The chain rule gives the correct derivative of $[f(x)]^3$.

77. $\frac{d}{dx} L(g(x)) = L'(g(x))g'(x)$
But since $L'(x) = \frac{1}{x}$, $L'(g(x)) = \frac{1}{g(x)}$.
Thus, $\frac{d}{dx} L(g(x)) = \frac{g'(x)}{g(x)}$.

78. $\frac{d}{dx} E(g(x)) = E'(g(x)) \cdot g'(x)$
Since $E'(x) = E(x)$, $E'(g(x)) = E(g(x))$.
Thus, $\frac{d}{dx} E(g(x)) = E(g(x)) \cdot g'(x)$.

EXERCISES 2.7

1. The derivative does not exist at the corner points $x = -2,\ 0,\ 2.$

2. The derivative does not exist at the discontinuous points $x = -3,\ 0,\ 3.$

3. The derivative does not exist at the discontinuous points $x = -3,\ 3.$

4. The derivative does not exist at the vertical tangents at $x = -4,\ -2,\ 2,\ 4.$

5. For positive h, $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{|2x+2h|-|2x|}{h}$. For $x = 0$, this becomes

$\lim\limits_{h\to 0}\dfrac{|0+2h|-|0|}{h} = \dfrac{|2h|}{h} = 2$ because h is positive. For h negative,

$\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{|2x+2h|-|2x|}{h}$. Since $x = 0$, we get

$\lim\limits_{h\to 0}\dfrac{|0+2h|-|0|}{h} = \dfrac{|2h|}{h} = -2$. Thus, the derivative does not exist.

6. For positive h and $x = 0$, $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{|0+3h|-|0|}{h} = \dfrac{3h}{h} = 3$.

For negative h and $x = 0$, $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{|0+3h|-|0|}{h} = \dfrac{-3h}{h} = -3$.

Thus, the derivative does not exist.

7. For $x = 0$, $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{(0+h)^{2/5}-0}{h} = \lim\limits_{h\to 0}\dfrac{h^{2/5}}{h} = \lim\limits_{h\to 0}\dfrac{1}{h^{3/5}}$ which does not exist. Thus, the derivative does not exist at $x = 0$.

8. For $x = 0$, $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{(0+h)^{4/5}-0}{h} = \lim\limits_{h\to 0}\dfrac{h^{4/5}}{h} = \lim\limits_{h\to 0}\dfrac{1}{h^{1/5}}$ which does not exist.

Thus, the derivative does not exist at $x = 0$.

9. If you get a numerical answer, it is wrong because the function is undefined at $x = 0$. Thus, the derivative at $x = 0$ does not exist.

10. If you get a numerical answer, it is wrong because the function is undefined at $x = 0$. Thus, the derivative at $x = 0$ does not exist.

11. **a.** For $x = 0$,

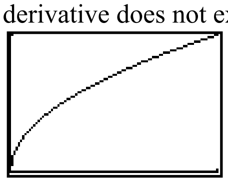
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{0+h}-\sqrt{0}}{h} = \lim_{h\to 0}\frac{\sqrt{h}}{h}$$

b.

h	$\frac{\sqrt{h}}{h}$
0.1	3.162
0.001	31.62
0.00001	316.2

c. No, the limit does not exist. No, the derivative does not exist at $x = 0$.

d.



On [0, 1] by [0, 1]

12. **a.** For $x = 0$,

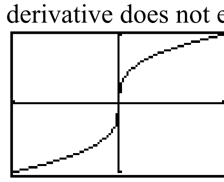
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt[3]{0+h}-\sqrt[3]{0}}{h} = \lim_{h\to 0}\frac{\sqrt[3]{h}}{h}$$

b.

h	$\frac{\sqrt[3]{h}}{h}$
0.1	4.64
0.0001	464.16
0.0000001	46,415.89

c. No, the limit does not exist. No, the derivative does not exist at $x = 0$.

d.



on [–1, 1] by [–1, 1]

13. At a corner point, a proposed tangent line can tip back and forth and so there is no well-defined slope.

14. A vertical tangent has an undefined slope, or derivative.

15. At a discontinuity, the values of the function take a sudden jump, and so a (steady) rate of change cannot be defined.

16. False: A function can have a corner or vertical tangent and still be continuous.

17. True: For a function to be differentiable, it cannot jump or break.

18. False: A function could have a vertical tangent.

REVIEW EXERCISES FOR CHAPTER 2

1.

x	$4x+2$
1.9	9.6
1.99	9.96
1.999	9.996

x	$4x+2$
2.1	10.4
2.01	10.04
2.001	10.004

a. $\lim\limits_{x\to 2^-} 4x+2=10$

b. $\lim\limits_{x\to 2^+} 4x+2=10$

c. $\lim\limits_{x\to 2} 4x+2=10$

2.

x	$\frac{\sqrt{x+1}-1}{x}$
–0.1	0.513
–0.01	0.501
–0.0001	0.500

x	$\frac{\sqrt{x+1}-1}{x}$
0.1	0.488
0.01	0.499
0.001	0.500

a. $\lim\limits_{x\to 0^-} \dfrac{\sqrt{x+1}-1}{x}=0.5$

b. $\lim\limits_{x\to 0^+} \dfrac{\sqrt{x+1}-1}{x}=0.5$

c. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}=0.5$

3. **a.** $\lim\limits_{x\to 5^-} f(x)=\lim\limits_{x\to 5^-}(2x-7)$
$=2(5)-7)=3$

b. $\lim\limits_{x\to 5^+} f(x)=\lim\limits_{x\to 5+}(3-x)$
$=3-5=-2$

c. $\lim\limits_{x\to 5} f(x)$ does not exist.

4. **a.** $\lim\limits_{x\to 5^-} f(x)=\lim\limits_{x\to 5^-}(4-x)$
$=4-5=-1$

b. $\lim\limits_{x\to 5^+} f(x)=\lim\limits_{x\to 5^+}(2x-11)$
$=2(5)-11=-1$

c. $\lim\limits_{x\to 5} f(x)=-1$

5. $\lim\limits_{x\to 4}\sqrt{x^2+x+5}=\sqrt{4^2+4+5}=\sqrt{16+9}$
$=\sqrt{25}=5$

6. $\lim\limits_{x\to 0}\pi=\pi$

7. $\lim\limits_{s\to 16}\left(\frac{1}{2}s-s^{1/2}\right)=\frac{1}{2}(16)-16^{1/2}=8-4=2$

8. $\lim\limits_{r\to 8}\dfrac{r}{r^2-30\sqrt[3]{r}}=\dfrac{8}{8^2-30\sqrt[3]{8}}=\dfrac{8}{64-30(2)}=2$

9. $\lim\limits_{x\to 1}\dfrac{x^2-x}{x^2-1}=\lim\limits_{x\to 1}\dfrac{x(x-1)}{(x+1)(x-1)}$
$=\lim\limits_{x\to 1}\dfrac{x}{x+1}=\dfrac{1}{2}$

10. $\lim\limits_{x\to -1}\dfrac{3x^3-3x}{2x^2+2x}=\lim\limits_{x\to -1}\dfrac{3x(x^2-1)}{2x(x+1)}$
$=\lim\limits_{x\to -1}\dfrac{3x(x-1)(x+1)}{2x(x+1)}$
$=\lim\limits_{x\to -1}\dfrac{3(x-1)}{2}=\dfrac{3(-1-1)}{2}=-3$

11. $\lim\limits_{h\to 0}\dfrac{2x^2h-xh^2}{h}=\lim\limits_{h\to 0}\dfrac{xh(2x-h)}{h}$
$=\lim\limits_{h\to 0}x(2x-h)$
$=x(2x-0)=2x^2$

12. $\lim\limits_{h\to 0}\dfrac{6xh^2-x^2h}{h}=\lim\limits_{h\to 0}\dfrac{xh(6h-x)}{h}$
$=\lim\limits_{h\to 0}x(6h-x)$
$=x(0-x)=-x^2$

13. $\lim_{x\to-\infty} f(x)=0;\quad \lim_{x\to-2^-} f(x)=\infty$ and $\lim_{x\to-2^+} f(x)=\infty$,
so $\lim_{x\to-2} f(x)=\infty$ and $\lim_{x\to\infty} f(x)=0$.

14. $\lim_{x\to-\infty} f(x)=3;\quad \lim_{x\to2^-} f(x)=-\infty$ and $\lim_{x\to2^+} f(x)=\infty$,
so $\lim_{x\to2} f(x)$ does not exist and $\lim_{x\to\infty} f(x)=3$.

15. Continuous

16. Continuous

17. Discontinuous at $x = -1$

18. Continuous

19. The function is discontinuous at values of x for which the denominator is zero. Thus, we consider $x^2+x=0$ and solve.

$$x^2+x=0$$
$$x(x+1)=0$$
$$x=0,-1$$

Discontinuous at $x = 0, -1$

20. Discontinuous at $x = -3, 3$

21. From Exercise 3, we know $\lim_{x\to5} f(x)$ does not exist. Therefore, the function is discontinuous at $x = 5$.

22. From Exercise 4, we know $\lim_{x\to5} f(x)=-1=f(5)$. Therefore, the function is continuous.

23. $$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{2(x+h)^2+3(x+h)-1-(2x^2+3x-1)}{h}=\lim_{h\to0}\frac{2(x^2+2xh+h^2)+3x+3h-1-2x^2-3x+1}{h}$$
$$=\lim_{h\to0}\frac{2x^2+4xh+2h^2+3x-3x+3h-1+1-2x^2}{h}=\lim_{h\to0}\frac{4xh+2h^2+3h}{h}=\lim_{h\to0}4x+2h+3=4x+3$$

24. $$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{3(x+h)^2+2(x+h)-3-(3x^2+2x-3)}{h}=\lim_{h\to0}\frac{3x^2+6xh+3h^2+2x+2h-3-3x^2-2x+3}{h}$$
$$=\lim_{h\to0}\frac{6xh+3h^2+2h}{h}=\lim_{h\to0}6x+3h+2=6x+2$$

25. $$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{\frac{3}{x+h}-\frac{3}{x}}{h}=\lim_{h\to0}\frac{\frac{3x-3(x+h)}{x(x+h)}}{h}=\lim_{h\to0}\frac{3x-3x-3h}{x(x+h)}\cdot\frac{1}{h}=\lim_{h\to0}\frac{-3h}{x(x+h)h}=\lim_{h\to0}-\frac{3}{x(x+h)}=-\frac{3}{x^2}$$

26. $$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{4\sqrt{x+h}-4\sqrt{x}}{h}=\lim_{h\to0}4\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\lim_{h\to0}4\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$=\lim_{h\to0}4\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim_{h\to0}\frac{4}{\sqrt{x+h}+\sqrt{x}}=\frac{4}{2\sqrt{x}}=\frac{2}{\sqrt{x}}$$

27. $$f(x)=6\sqrt[3]{x^5}-\frac{4}{\sqrt{x}}+1=6x^{5/3}-4x^{-1/2}+1$$
$$f'(x)=\frac{5}{3}\left(6x^{2/3}\right)-\left(-\frac{1}{2}\right)\left(4x^{-3/2}\right)+0$$
$$=10x^{2/3}+2x^{-3/2}$$

28. $$f(x)=4\sqrt{x^5}-\frac{6}{\sqrt[3]{x}}+1=4x^{5/2}-6x^{-1/3}+1$$
$$f'(x)=\frac{5}{2}\left(4x^{3/2}\right)-\left(-\frac{1}{3}\right)\left(6x^{-4/3}\right)+0$$
$$=10x^{3/2}+2x^{-4/3}$$

29. $f(x)=\frac{1}{x^2}=x^{-2}$

$f'(x)=-2x^{-3}$

$f'\left(\frac{1}{2}\right)=-2\left(\frac{1}{2}\right)^{-3}=-2(2)^3=-16$

30. $f(x)=\frac{1}{x}=x^{-1}$

$f'(x)=-x^{-2}$

$f'\left(\frac{1}{3}\right)=-\left(\frac{1}{3}\right)^{-2}=-(3)^2=-9$

31. $f(x)=12\sqrt[3]{x}=12x^{1/3}$

$f'(x)=\frac{1}{3}\left(12x^{-2/3}\right)=4x^{-2/3}$

$f'(8)=4(8)^{-2/3}=4\left(\frac{1}{8}\right)^{2/3}=4\left(\frac{1}{8^{2/3}}\right)$

$=4\left(\frac{1}{\sqrt[3]{8^2}}\right)=4\left(\frac{1}{\sqrt[3]{64}}\right)=1$

32. $f(x)=6\sqrt[3]{x}=6x^{1/3}$

$f'(x)=\frac{1}{3}\left(6x^{-2/3}\right)=2x^{-2/3}$

$f'(-8)=2(-8)^{-2/3}=2\left(-\frac{1}{8}\right)^{2/3}=2\left(\frac{1}{\sqrt[3]{(-8)^2}}\right)=\frac{1}{2}$

33. **a.** $C'(x)=20x^{-1/6}$

$C'(1)=\frac{20}{\sqrt[6]{1}}=20$

Costs are increasing by about \$20 per additional license.

b. $C'(64)=\frac{20}{\sqrt[6]{64}}=\frac{20}{2}=10$

Costs are increasing by about \$10 per additional license.

34. $f(x)=150x^{-0.322}$

$f'(x)=-0.322(150x^{-1.322})=-48.3x^{-1.322}$

$f'(10)=-48.3(10)^{-1.322}\approx-2.3$

After 10 planes, construction time is decreasing by about 2300 hours for each additional plane built.

35. **a.** $A=\pi r^2$

$A'=2\pi r$

b. As the radius increases, the area grows by "a circumference."

36. **a.** $V=\frac{4}{3}\pi r^3$

$A'=3\left(\frac{4}{3}\pi r^2\right)=4\pi r^2$

b. As the radius increases, the volume grows by "a surface area."

37. $f(x)=2x(5x^3+3)$

$f'(x)=2(5x^3+3)+2x(15x^2)$

$=2(5x^3+3)+30x^3=40x^3+6$

38. $f(x)=x^2(3x^3-1)$

$f'(x)=2x(3x^3-1)+x^2(9x^2)$

$=2x(3x^3-1)+9x^4=15x^4-2x$

39. $f(x)=(x^2+5)(x^2-5)$

$f'(x)=2x(x^2-5)+(x^2+5)(2x)$

$=2x^3-10x+2x^3+10x=4x^3$

40. $f(x)=(x^2+3)(x^2-3)$

$f'(x)=2x(x^2-3)+(x^2+3)(2x)$

$=2x^3-6x+2x^3+6x=4x^3$

41. $y=(x^4+x^2+1)(x^5-x^3+x)$

$y'=(4x^3+2x)(x^5-x^3+x)+(x^4+x^2+1)(5x^4-3x^2+1)$

$=4x^8-4x^6+4x^4+2x^6-2x^4+2x^2+5x^8-3x^6+x^4+5x^6-3x^4+x^2+5x^4-3x^2+1$

$=9x^8+5x^4+1$

42. $y=(x^5+x^3+x)(x^4-x^2+1)$

$y'=(5x^4+3x^2+1)(x^4-x^2+1)+(x^5+x^3+x)(4x^3-2x)$

$=5x^8-5x^6+5x^4+3x^6-3x^4+3x^2+x^4-x^2+1+4x^8+4x^6+4x^4-2x^6-2x^4-2x^2$

$=5x^8-2x^6+3x^4+2x^2+1+4x^8+2x^6+2x^4-2x^2$

$9x^8+5x^4+1$

43. $y=\dfrac{x-1}{x+1}$

$$y'=\frac{(x+1)(1)-(1)(x-1)}{(x+1)^2}=\frac{x+1-x+1}{(x+1)^2}=\frac{2}{(x+1)^2}$$

44. $y=\dfrac{x+1}{x-1}$

$$y'=\frac{(x-1)(1)-(1)(x+1)}{(x-1)^2}=\frac{x-1-x-1}{(x-1)^2}=-\frac{2}{(x-1)^2}$$

45. $y=\dfrac{x^5+1}{x^5-1}$

$$y'=\frac{(x^5-1)(5x^4)-(5x^4)(x^5+1)}{(x^5-1)^2}$$
$$=\frac{5x^9-5x^4-5x^9-5x^4}{(x^5-1)^2}=-\frac{10x^4}{(x^5-1)^2}$$

46. $y=\dfrac{x^6-1}{x^6+1}$

$$y'=\frac{(x^6+1)(6x^5)-(6x^5)(x^6-1)}{(x^6+1)^2}$$
$$=\frac{6x^{11}+6x^5-6x^{11}+6x^5}{(x^6+1)^2}=\frac{12x^5}{(x^6+1)^2}$$

47. **a.** $f(x)=\dfrac{2x+1}{x}$

$$f'(x)=\frac{x(2)-(1)(2x+1)}{x^2}$$
$$=\frac{2x-2x-1}{x^2}=-\frac{1}{x^2}$$

b. $f(x)=(2x+1)(x^{-1})$

$$f'(x)=2(x^{-1})+(2x+1)(-1)x^{-2}$$
$$=\frac{2}{x}-\frac{2x+1}{x^2}=\frac{2x}{x^2}-\frac{2x+1}{x^2}=-\frac{1}{x^2}$$

c. Dividing $2x+1$ by x, we get

$$f(x)=2+\frac{1}{x}=2+x^{-1}$$
$$f'(x)=0+(-1)x^{-2}=-\frac{1}{x^2}$$

48. $S(x)=\dfrac{2250}{x+9}=2250(x+9)^{-1}$

$$S'(x)=-2250(x+9)^{-2}=-\frac{2250}{(x+9)^2}$$
$$S'(6)=-\frac{2250}{(6+9)^2}=-10$$

At \$6 per tape, the number of tapes sold is decreasing by 10 per dollar increase in price.

49. **a.** To find the average profit function, divide $P(x)=6x-200$ by x.

$$AP(x)=\frac{6x-200}{x}$$

b. To find the marginal average profit, take the derivative of $AP(x)$.

$$MAP(x)=\frac{x(6)-(1)(6x-200)}{x^2}=\frac{200}{x^2}$$

c. $MAP(10)=\dfrac{200}{(10)^2}=\dfrac{200}{100}=2$

Average profit is increasing by about \$2 for each additional unit.

50. **a.** $AC(x)=\dfrac{C(x)}{x}=\dfrac{7.5x+50}{x}=7.5+\dfrac{50}{x}$

b. $MAC(x)=\dfrac{d}{dx}\left(7.5+\dfrac{50}{x}\right)=\dfrac{-50}{x^2}$

c. $MAC(50)=\dfrac{-50}{50^2}=-\dfrac{1}{50}=-0.02$

Average cost is decreasing by about \$0.02 per additional mouse.

51. $f(x)=12\sqrt{x^3}-9\sqrt[3]{x}=12x^{3/2}-9x^{1/3}$

$$f'(x)=\frac{3}{2}\left(12x^{1/2}\right)-\frac{1}{3}\left(9x^{-2/3}\right)=18x^{1/2}-3x^{-2/3}$$
$$f''(x)=\frac{1}{2}\left(18x^{-1/2}\right)-\frac{2}{3}\left(-3x^{-5/3}\right)$$
$$=9x^{-1/2}+2x^{-5/3}$$

52. $f(x)=18\sqrt[3]{x^2}-4\sqrt{x^3}=18x^{2/3}-4x^{3/2}$

$$f'(x)=12x^{-1/3}-6x^{1/2}$$
$$f''(x)=-4x^{-4/3}-3x^{-1/2}$$

53. $f(x) = \dfrac{1}{3x^2} = \dfrac{1}{3}x^{-2}$

$f'(x) = -2\left(\dfrac{1}{3}x^{-3}\right) = -\dfrac{2}{3}x^{-3}$

$f''(x) = -3\left(-\dfrac{2}{3}x^{-4}\right) = 2x^{-4}$

54. $f(x) = \dfrac{1}{2x^3} = \dfrac{1}{2}x^{-3}$

$f'(x) = -\dfrac{3}{2}x^{-4}$

$f''(x) = 6x^{-5}$

55. $f(x) = \dfrac{2}{x^3} = 2x^{-3}$

$f'(x) = -3\left(2x^{-4}\right) = -6x^{-4} = -\dfrac{6}{x^4}$

$f''(x) = -4\left(-6x^{-5}\right) = 24x^{-5} = \dfrac{24}{x^5}$

$f''(-1) = \dfrac{24}{(-1)^5} = -24$

56. $f(x) = \dfrac{3}{x^4} = 3x^{-4}$

$f'(x) = -12x^{-5}$

$f''(x) = 60x^{-6} = \dfrac{60}{x^6}$

$f''(-1) = \dfrac{60}{(-1)^6} = 60$

57. $\dfrac{d}{dx}x^6 = 6x^5$

$\dfrac{d^2}{dx^2}x^6 = \dfrac{d}{dx}6x^5 = 30x^4$

$\left.\dfrac{d^2}{dx^2}\right|_{x=-2} x^6 = 30(-2)^4 = 480$

58. $\dfrac{d}{dx}x^{-2} = -2x^{-3}$

$\dfrac{d^2}{dx^2}x^{-2} = \dfrac{d}{dx}(-2x^{-3}) = 6x^{-4} = \dfrac{6}{x^4}$

$\left.\dfrac{d^2}{dx^2}\right|_{x=-2} x^{-2} = \dfrac{6}{(-2)^4} = \dfrac{6}{16} = \dfrac{3}{8}$

59. $\dfrac{d}{dx}\sqrt{x^5} = \dfrac{d}{dx}x^{5/2} = \dfrac{5}{2}x^{3/2}$

$\dfrac{d^2}{dx^2}\sqrt{x^5} = \dfrac{d}{dx}\left(\dfrac{5}{2}x^{3/2}\right) = \dfrac{15}{4}x^{1/2}$

$\left.\dfrac{d^2}{dx^2}\right|_{x=16}\sqrt{x^5} = \dfrac{15}{4}(16)^{1/2} = 15$

60. $\dfrac{d}{dx}\sqrt{x^7} = \dfrac{d}{dx}x^{7/2} = \dfrac{7}{2}x^{5/2}$

$\dfrac{d^2}{dx^2}\sqrt{x^7} = \dfrac{d}{dx}\left(\dfrac{7}{2}x^{5/2}\right) = \dfrac{35}{4}x^{3/2}$

$\left.\dfrac{d^2}{dx^2}\right|_{x=4}\sqrt{x^7} = \dfrac{35}{4}(4)^{3/2} = 70$

61. $P(t) = 0.25t^3 - 3t^2 + 5t + 200$

$P'(t) = 3(0.25t^2) - 2(3t) + 1(5) + 0 = 0.75t^2 - 6t + 5$

$P''(t) = 2(0.75t) - 6(1) + 0 = 1.5t - 6$

$P(10) = 0.25(10)^3 - 3(10)^2 + 5(10) + 200 = 250 - 300 + 50 + 200 = 200$

In 10 years, the population will be 200,000.

$P'(10) = 0.75(10)^2 - 6(10) + 5 = 75 - 60 + 5 = 20$

In 10 years, the population will be increasing by about 20,000 per year.

$P''(10) = 1.5(10) - 6 = 9$

In 10 years, the rate of growth of the increase will be 9000 per year each year.

62. $s(t) = 8t^{5/2}$

$v(t) = s'(t) = \dfrac{5}{2}(8t^{3/2}) = 20t^{3/2}$

$a(t) = v'(t) = \dfrac{3}{2}(20t^{1/2}) = 30t^{1/2}$

$v(25) = 20(25)^{3/2} = 2500$ ft / sec

$a(25) = 30(25)^{1/2} = 150$ ft / sec^2

63. **a.** When the height is a maximum, the velocity is zero. Thus, to find the maximum height, set $v(t) = 0$ and solve. First, we find $v(t) = s'(t)$.

$s(t) = -16t^2 + 148t + 5$

$v(t) = s'(t) = -32t + 148$

Now set $v(t) = 0$ and solve.

$v(t) = -32t + 148 = 0$

$-32t = -148$

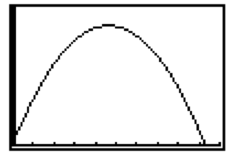
$t = 4.625$

To find the height when $t = 4.625$, evaluate $s(4.625)$.

$s(4.625) = -16(4.625)^2 + 148(4.625) + 5$

$= 347.25$ feet

b.



64. $h(z) = (4z^2 - 3z + 1)^3$

$h'(z) = 3(4z^2 - 3z + 1)^2(8z - 3)$

65. $h(z) = (3z^2 - 5z - 1)^4$

$h'(z) = 4(3z^2 - 5z - 1)^3(6z - 5)$

66. $g(x) = (100 - x)^5$

$g'(x) = 5(100 - x)^4(-1) = -5(100 - x)^4$

67. $g(x) = (1000 - x)^4$

$g'(x) = 4(1000 - x)^3(-1) = -4(1000 - x)^3$

68. $f(x) = \sqrt{x^2 - x + 2} = (x^2 - x + 2)^{1/2}$

$f'(x) = \frac{1}{2}(x^2 - x + 2)^{-1/2}(2x - 1)$

69. $f(x) = \sqrt{x^2 - 5x - 1} = (x^2 - 5x - 1)^{1/2}$

$f'(x) = \frac{1}{2}(x^2 - 5x - 1)^{-1/2}(2x - 5)$

70. $w(z) = \sqrt[3]{6z - 1} = (6z - 1)^{1/3}$

$w'(z) = \frac{1}{3}(6z - 1)^{-2/3}(6) = 2(6z - 1)^{-2/3}$

71. $w(z) = \sqrt[3]{3z + 1} = (3z + 1)^{1/3}$

$w'(z) = \frac{1}{3}(3z + 1)^{-2/3}(3) = (3z + 1)^{-2/3}$

72. $h(x) = \dfrac{1}{\sqrt[5]{(5x + 1)^2}} = (5x + 1)^{-2/5}$

$h'(x) = -\frac{2}{5}(5x + 1)^{-7/5}(5) = -2(5x + 1)^{-7/5}$

73. $h(x) = \dfrac{1}{\sqrt[5]{(10x + 1)^3}} = (10x + 1)^{-3/5}$

$h'(x) = -\frac{3}{5}(10x + 1)^{-8/5}(10) = -6(10x + 1)^{-8/5}$

74. $g(x) = x^2(2x - 1)^4$

$g'(x) = 2x(2x - 1)^4 + x^2[4(2x - 1)^3(2)]$

$= 2x(2x - 1)^4 + 8x^2(2x - 1)^3$

75. $g(x) = 5x(x^3 - 2)^4$

$g'(x) = 5(x^3 - 2)^4 + 5x[4(x^3 - 2)^3(3x^2)]$

$= 5(x^3 - 2)^4 + 60x^3(x^3 - 2)^3$

76. $y = x^3\sqrt[3]{x^3 + 1} = x^3(x^3 + 1)^{1/3}$

$y' = 3x^2(x^3 + 1)^{1/3} + x^3\left[\frac{1}{3}(x^3 + 1)^{-2/3}(3x^2)\right]$

$= 3x^2(x^3 + 1)^{1/3} + x^5(x^3 + 1)^{-2/3}$

77. $y = x^4\sqrt{x^2 + 1} = x^4(x^2 + 1)^{1/2}$

$y' = 4x^3(x^2 + 1)^{1/2} + x^4\left[\frac{1}{2}(x^2 + 1)^{-1/2}(2x)\right]$

$= 4x^3(x^2 + 1)^{1/2} + x^5(x^2 + 1)^{-1/2}$

78. $f(x) = [(2x^2 + 1)^4 + x^4]^3$

$f'(x) = 3[(2x^2 + 1)^4 + x^4]^2[4(2x^2 + 1)^3(4x) + 4x^3]$

$= 3[(2x^2 + 1)^4 + x^4]^2[16x(2x^2 + 1)^3 + 4x^3]$

79. $f(x) = [(3x^2 - 1)^3 + x^3]^2$

$f'(x) = 2[(3x^2 - 1)^3 + x^3][3(3x^2 - 1)^2(6x) + 3x^2]$

$= 2[(3x^2 - 1)^3 + x^3][18x(3x^2 - 1)^2 + 3x^2]$

80. $f(x)=\sqrt{(x^2+1)^4-x^4}=[(x^2+1)^4-x^4]^{1/2}$

$f'(x)=\frac{1}{2}[(x^2+1)^4-x^4]^{-1/2}[4(x^2+1)^3(2x)-4x^3]$

$=\frac{1}{2}[(x^2+1)^4-x^4]^{-1/2}[8x(x^2+1)^3-4x^3]$

81. $f(x)=\sqrt{(x^3+1)^2+x^2}=[(x^3+1)^2+x^2]^{1/2}$

$f'(x)=\frac{1}{2}[(x^3+1)^2+x^2]^{-1/2}[2(x^3+1)(3x^2)+2x]$

$=\frac{1}{2}[(x^3+1)^2+x^2]^{-1/2}[6x^2(x^3+1)+2x]$

$=[(x^3+1)^2+x^2]^{-1/2}[3x^2(x^3+1)+x]$

82. $f(x)=(3x+1)^4(4x+1)^3$

$f'(x)=4(3x+1)^3(3)(4x+1)^3+(3x+1)^4(3)(4x+1)^2(4)$

$=12(3x+1)^3(4x+1)^3+12(3x+1)^4(4x+1)^2$

$=12(3x+1)^3(4x+1)^2[(4x+1)+(3x+1)]$

$=12(3x+1)^3(4x+1)^2(7x+2)$

83. $f(x)=(x^2+1)^3(x^2-1)^4$

$f'(x)=3(x^2+1)^2(2x)(x^2-1)^4+(x^2+1)^3[4(x^2-1)^3(2x)]$

$=6x(x^2+1)^2(x^2-1)^4+8x(x^2+1)^3(x^2-1)^3$

84. $f(x)=\left(\frac{x+5}{x}\right)^4$

$f'(x)=4\left(\frac{x+5}{x}\right)^3\left[\frac{x(1)-(1)(x+5)}{x^2}\right]$

$=4\left(\frac{x+5}{x}\right)^3\left(\frac{x-x-5}{x^2}\right)=-\frac{20}{x^2}\frac{(x+5)^3}{x^3}$

$=-\frac{20(x+5)^3}{x^5}$

85. $f(x)=\left(\frac{x+4}{x}\right)^5$

$f'(x)=5\left(\frac{x+4}{x}\right)^4\left[\frac{x(1)-(1)(x+4)}{x^2}\right]$

$=5\left(\frac{x+4}{x}\right)^4\left(\frac{x-x-4}{x^2}\right)=-\frac{20}{x^2}\left(\frac{x+4}{x}\right)^4$

$=-\frac{20(x+4)^4}{x^6}$

86. $h(w)=(2w^2-4)^5$

$h'(w)=5(2w^2-4)^4(4w)$

$h''(w)=20(2w^2-4)^3(4w)(4w)+5(2w^2-4)^4(4)$

$=20(16w^2)(2w^2-4)^3+20(2w^2-4)^4$

$=320w^2(2w^2-4)^3+20(2w^2-4)^4$

87. $h(w)=(3w^2+1)^4$

$h'(w)=4(3w^2+1)^3(6w)$

$h''(w)=12(3w^2+1)^2(6w)(6w)+4(3w^2+1)^3(6)$

$=432w^2(3w^2+1)^2+24(3w^2+1)^3$

88. $g(z)=z^3(z+1)^3$

$g'(z)=3z^2(z+1)^3+z^3[3(z+1)^2]=3z^2(z+1)^3+3z^3(z+1)^2$

$g''(z)=6z(z+1)^3+3z^2[3(z+1)^2]+9z^2(z+1)^2+3z^3[2(z+1)]$

$=6z(z+1)^3+18z^2(z+1)^2+6z^3(z+1)$

89. $g(z)=z^4(z+1)^4$

$g'(z)=4z^3(z+1)^4+4z^4(z+1)^3$

$g''(z)=12z^2(z+1)^4+4z^3[4(z+1)^3]+16z^3(z+1)^3+4z^4[3(z+1)^2]$

$=12z^2(z+1)^4+32z^3(z+1)^3+12z^4(z+1)^2$

90. **a.** $\frac{d}{dx}(x^3-1)^2=2(x^3-1)(3x^2)=6x^2(x^3-1)=6x^5-6x^2$

b. $\frac{d}{dx}(x^3-1)^2=\frac{d}{dx}(x^6-2x^3+1)=6x^5-6x^2$

91. **a.** $\frac{d}{dx}\left(\frac{1}{x^3+1}\right) = \frac{(x^3+1)(0)-(3x^2)(1)}{(x^3+1)^2} = -\frac{3x^2}{(x^3+1)^2}$

b. $\frac{d}{dx}\left(\frac{1}{x^3+1}\right) = \frac{d}{dx}(x^3+1)^{-1} = -1(x^3+1)^{-2}(3x^2) = -\frac{3x^2}{(x^3+1)^2}$

92. $P(x) = \sqrt{x^3-3x+34} = (x^3-3x+34)^{1/2}$

$P'(x) = \frac{1}{2}(x^3-3x+34)^{-1/2}(3x^2-3) = \frac{3x^2-3}{2\sqrt{x^3-3x+34}}$

$P'(5) = \frac{3(5)^2-3}{2\sqrt{(5)^3-3(5)+34}} = \frac{72}{2\sqrt{144}} = \frac{72}{24} = 3$

When 5 tons is produced, profit is increasing at about $3000 for each additional ton.

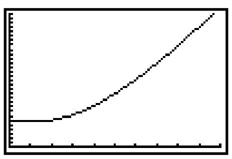
93. $V(r) = 500(1+0.01r)^3$

$V'(r) = 1500(1+0.01r)^2(0.01) = 15(1+0.01r)^2$

$V'(8) = 15[1+0.01(8)]^2 \approx 17.50$

For 8 percent interest, the value increases by about $17.50 for each additional percent interest.

94.



on [0, 10] by [0, 30]

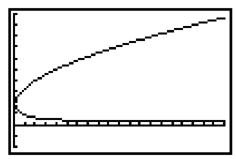
a. $P(5)-P(4) \approx 12-9.27 = 2.73$

$P(6)-P(5) \approx 15.23-12 = 3.23$

Both values are near 3.

b. $x \approx 7.6$

95.



on [0, 20] by [–2, 10]

$x \approx 16$

96. $R(x) = 0.25(0.01x+1)^4$

$R'(x) = 0.25[4(0.01x+1)^3(0.01)]$

$= 0.01(0.01x+1)^3$

$R'(100) = 0.01[0.01(100)+1)]^3 = 0.08$

97. $N(x) = 1000\sqrt{100-x} = 1000(100-x)^{1/2}$

$N'(x) = 500(100-x)^{-1/2}(-1) = -\frac{500}{\sqrt{100-x}}$

$N'(96) = -\frac{500}{\sqrt{100-96}} = -250$

At age 96, the number of survivors is decreasing by 250 people per year.

98. The derivative does not exist at corner points $x=-3$ and $x=3$ and at the discontinuous point $x=1$.

99. The derivative does not exist at the corner point $x=2$ and at the discontinuous point $x=-2$.

100. The derivative does not exist at the corner point $x=3.5$ and the discontinuous point $x=0$.

101. The derivative does not exist at the corner points $x=0$ and $x=3$.

102. For positive h,

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{|5x+5h|-|5x|}{h}=\lim_{h\to 0}\frac{|5(0)+5h|-|5\cdot 0|}{h}\quad\text{for}\quad x=0$$

$$=\lim_{h\to 0}\frac{5h}{h}=5$$

For negative h,

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{|5x+5h|-|5x|}{h}=\lim_{h\to 0}\frac{|5(0)+5h|-|5(0)|}{h}\quad\text{for}\quad x=0$$

$$=\lim_{h\to 0}-\frac{5h}{h}=-5$$

Thus, the limit does not exist, and so the derivative does not exist.

103. $$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^{3/5}-x^{3/5}}{h}=\lim_{h\to 0}\frac{h^{3/5}}{h}\quad\text{for}\quad x=0$$

$$=\lim_{h\to 0}\frac{1}{h^{2/5}}$$

which does not exist. Thus, the derivative does not exist.