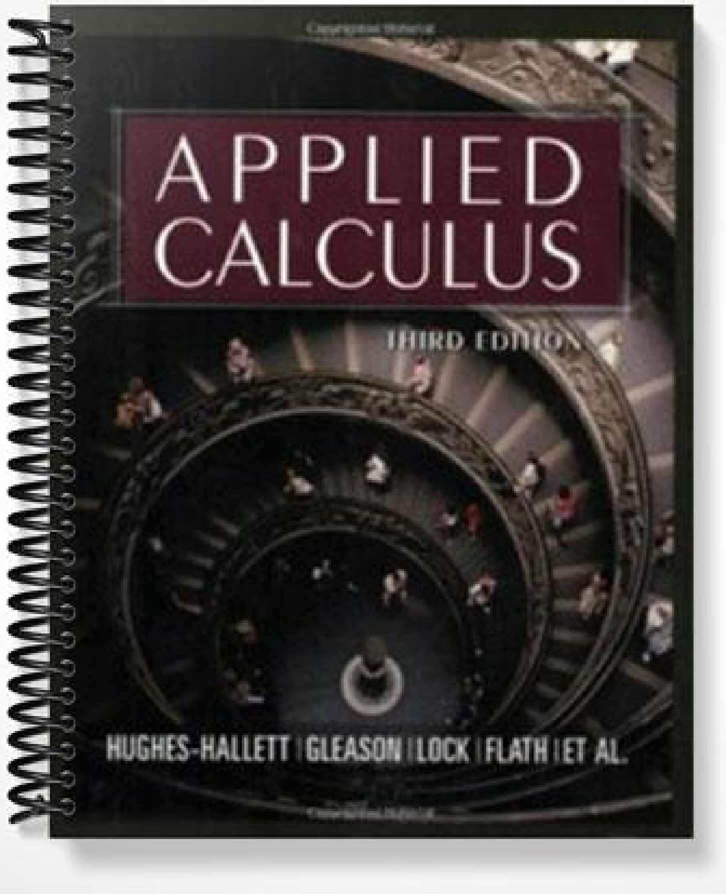
SOLUTIONS MANUAL



CHAPTER TWO

Solutions for Section 2.1

1. **(a)** The average velocity between $t = 3$ and $t = 5$ is

$$\frac{\text{Distance}}{\text{Time}} = \frac{s(5) - s(3)}{5 - 3} = \frac{25 - 9}{2} = \frac{16}{2} = 8 \text{ ft/sec.}$$

(b) Using an interval of size 0.1, we have

$$\begin{pmatrix} \text{Instantaneous velocity} \\ \text{at } t = 3 \end{pmatrix} \approx \frac{s(3.1) - s(3)}{3.1 - 3} = \frac{9.61 - 9}{0.1} = 6.1.$$

Using an interval of size 0.01, we have

$$\begin{pmatrix} \text{Instantaneous velocity} \\ \text{at } t = 3 \end{pmatrix} \approx \frac{s(3.01) - s(3)}{3.01 - 3} = \frac{9.0601 - 9}{0.01} = 6.01.$$

From this we guess that the instantaneous velocity at $t = 3$ is about 6 ft/sec.

2. **(a)** Let $s = f(t)$.

(i) We wish to find the average velocity between $t = 1$ and $t = 1.1$. We have

$$\text{Average velocity} = \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{7.84 - 7}{0.1} = 8.4 \text{ m/sec.}$$

(ii) We have

$$\text{Average velocity} = \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{7.0804 - 7}{0.01} = 8.04 \text{ m/sec.}$$

(iii) We have

$$\text{Average velocity} = \frac{f(1.001) - f(1)}{1.001 - 1} = \frac{7.008004 - 7}{0.001} = 8.004 \text{ m/sec.}$$

(b) We see in part (a) that as we choose a smaller and smaller interval around $t = 1$ the average velocity appears to be getting closer and closer to 8, so we estimate the instantaneous velocity at $t = 1$ to be 8 m/sec.

3. **(a)** The size of the tumor when $t = 0$ months is $S(0) = 2^0 = 1$ cubic millimeter. The size of the tumor when $t = 6$ months is $S(6) = 2^6 = 64$ cubic millimeters. The total change in the size of the tumor is $S(6) - S(0) = 64 - 1 = 63$ mm^3.

(b) The average rate of change in the size of the tumor during the first six months is:

$$\text{Average rate of change} = \frac{S(6) - S(0)}{6 - 0} = \frac{64 - 1}{6} = \frac{63}{6} = 10.5 \text{ cubic millimeters/month.}$$

(c) We will consider intervals to the right of $t = 6$:

t (months)	6	6.001	6.01	6.1
S (cubic millimeters)	64	64.0444	64.4452	68.5935

$$\text{Average rate of change} = \frac{68.5935 - 64}{6.1 - 6} = \frac{4.5935}{0.1} = 45.935$$
$$\text{Average rate of change} = \frac{64.4452 - 64}{6.01 - 6} = \frac{0.4452}{0.01} = 44.52$$
$$\text{Average rate of change} = \frac{64.0444 - 64}{6.001 - 6} = \frac{0.0444}{0.001} = 44.4$$

We can continue taking smaller intervals but the value of the average rate will not change much. Therefore, we can say that a good estimate of the growing rate of the tumor at $t = 6$ months is 44.4 cubic millimeters/month.

4.

Slope	-3	-1	0	$1/2$	1	2
Point	F	C	E	A	B	D

5. Since 2006 is 6 years after 2000, the rate of growth in 2006 is the derivative of $P(t)$ at $t = 6$. To estimate $P'(t)$ at $t = 6$, we take the interval between $t = 6$ and $t = 6.001$.

$$P'(6) \approx \frac{P(6.001) - P(6)}{6.001 - 6} = \frac{570(1.037)^{6.001} - 570(1.037)^6}{0.001}$$
$$= 25.754 \text{ thousand people/year.}$$

$$P'(6) \approx 25{,}754 \text{ people per year.}$$

6. **(a)** The average rate of change of a function over an interval is represented graphically as the slope of the secant line to its graph over the interval. See Figure 2.1. Segment AB is the secant line to the graph in the interval from $x = 0$ to $x = 3$ and segment BC is the secant line to the graph in the interval from $x = 3$ to $x = 5$.

We can easily see that slope of $AB >$ slope of BC. Therefore, the average rate of change between $x = 0$ and $x = 3$ is greater than the average rate of change between $x = 3$ and $x = 5$.

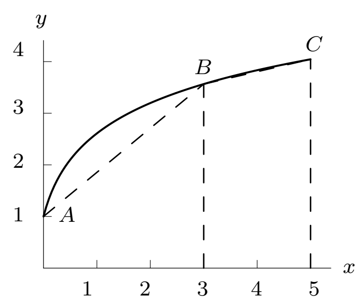


Figure 2.1

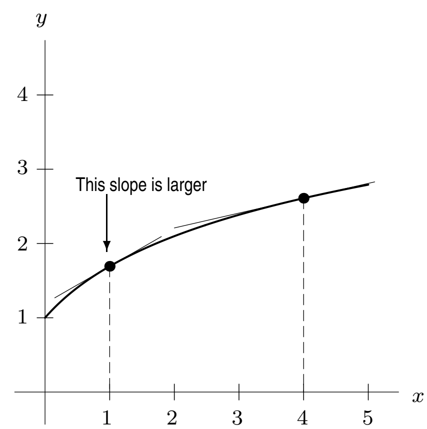


Figure 2.2

(b) We can see from the graph in Figure 2.2 that the function is increasing faster at $x = 1$ than at $x = 4$. Therefore, the instantaneous rate of change at $x = 1$ is greater than the instantaneous rate of change at $x = 4$.

(c) The units of rate of change are obtained by dividing units of cost by units of product: thousands of dollars/kilogram.

7. For the interval $0 \le t \le 0.8$, we have

$$\begin{pmatrix}\text{Average velocity}\\ 0 \le t \le 0.8\end{pmatrix} = \frac{s(0.8) - s(0)}{0.8 - 0} = \frac{6.5}{0.8} = 8.125 \text{ ft/sec.}$$
$$\begin{pmatrix}\text{Average velocity}\\ 0 \le t \le 0.2\end{pmatrix} = \frac{s(0.2) - s(0)}{0.2 - 0} = \frac{0.5}{0.2} = 2.5 \text{ ft/sec.}$$
$$\begin{pmatrix}\text{Average velocity}\\ 0.2 \le t \le 0.4\end{pmatrix} = \frac{s(0.4) - s(0.2)}{0.4 - 0.2} = \frac{1.3}{0.2} = 6.5 \text{ ft/sec.}$$

To find the velocity at $t = 0.2$, we find the average velocity to the right of $t = 0.2$ and to the left of $t = 0.2$ and average them. So a reasonable estimate of the velocity at $t = 0.2$ is the average of $\frac{1}{2}(6.5 + 2.5) = 4.5$ ft/sec.

8. **(a)** Since 75.2 percent live in the city in 1990 and 35.1 percent in 1890, we have

$$\text{Average rate of change} = \frac{75.2 - 35.1}{1990 - 1890} = \frac{40.1}{100} = 0.401.$$

Thus the average rate of change is 0.401 percent/year.

(b) By looking at the population in 1990 and 2000 we see that

$$\text{Average rate of change} = \frac{79.0 - 75.2}{2000 - 1990} = \frac{3.8}{10} = 0.38$$

which gives an average rate of change of 0.38 percent/year between 1990 and 2000. Alternatively, we look at the population in 1980 and 1990 and see that

$$\text{Average rate of change} = \frac{75.2 - 73.7}{1990 - 1980} = \frac{1.5}{10} = 0.15$$

giving an average rate of change of 0.15 percent/year between 1980 and 1990. We see that the rate of change in the year 1990 is somewhere between 0.38 and 0.15 percent/year. A good estimate is $(0.38 + 0.15)/2 = 0.265$ percent per year. In fact, the definition of urban area was changed for the 2000 data, so this estimate should be used with care.

(c) By looking at the population in 1830 and 1860 we see that

$$\text{Average rate of change} = \frac{19.8 - 9.0}{1860 - 1830} = \frac{10.8}{30} = 0.36,$$

giving an average rate of change of 0.36 percent/year between 1830 and 1860. Alternatively we can look at the population in 1800 and 1830 and see that

$$\text{Average rate of change} = \frac{9.0 - 6.0}{1830 - 1800} = \frac{3.0}{30} = 0.10.$$

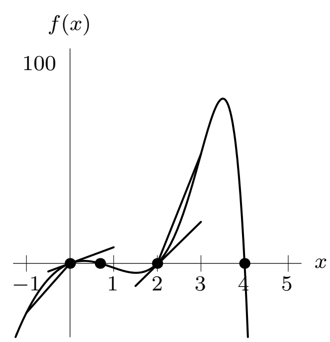
giving an average rate of change of 0.10 percent/year between 1800 and 1830 . We see that the rate of change at the year 1830 is somewhere between 0.10 and 0.36. This tells us that in the year 1830 the percent of the population in urban areas is changing by a rate somewhere between 0.10 percent/year and 0.36 percent/year.

(d) Looking at the data we see that the percent gets larger as time goes by. Thus, the function appears to be always increasing.

9. **(a)** $f'(x)$ is negative when the function is decreasing and positive when the function is increasing. Therefore, $f'(x)$ is positive at C and G. $f'(x)$ is negative at A and E. $f'(x)$ is zero at B, D, and F.

(b) $f'(x)$ is the largest when the graph of the function is increasing the fastest (i.e. the point with the steepest positive slope). This occurs at point G. $f'(x)$ is the most negative when the graph of the function is decreasing the fastest (i.e. the point with the steepest negative slope). This occurs at point A.

10.



(a) The graph shows four different zeros in the interval, at $x = 0$, $x = 2$, $x = 4$ and $x \approx 0.7$.

(b) At $x = 0$ and $x = 2$, we see that the tangent has a positive slope so f is increasing.
At $x = 4$, we notice that the tangent to the curve has negative slope, so f is decreasing.

(c) Comparing the slopes of the secant lines at these values, we can see that the average rate of change of f is greater on the interval $2 \leq x \leq 3$.

(d) Looking at the tangents of the function at $x = 0$ and $x = 2$, we see that the slope of the tangent at $x = 2$ is greater. Thus, the instantaneous rate of change of f is greater at $x = 2$.

11. We use the interval $x = 2$ to $x = 2.01$:

$$f'(2) \approx \frac{f(2.01) - f(2)}{2.01 - 2} = \frac{5^{2.01} - 5^2}{0.01} = \frac{25.4056 - 25}{0.01} = 40.56$$

For greater accuracy, we can use the smaller interval $x = 2$ to $x = 2.001$:

$$f'(2) \approx \frac{f(2.001) - f(2)}{2.001 - 2} = \frac{5^{2.001} - 5^2}{0.001} = \frac{25.040268 - 25}{0.001} = 40.268$$

12. (a) Figure 2.3 shows that for $t = 2$ the function $g(t) = (0.8)^t$ is decreasing. Therefore, $g'(2)$ is negative.

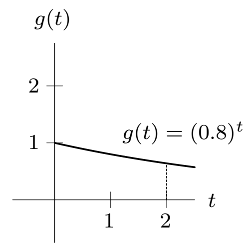


Figure 2.3

(b) To estimate $g'(2)$ we take the small interval between $t = 2$ and $t = 2.001$ to the right of $t = 2$.

$$g'(2) \approx \frac{g(2.001) - g(2)}{2.001 - 2} = \frac{0.8^{2.001} - 0.8^2}{0.001} = \frac{0.6399 - 0.64}{0.001} = \frac{-0.0001}{0.001} = -0.1$$

13. (a) From Figure 2.4 we can see that for $x = 1$ the value of the function is decreasing. Therefore, the derivative of $f(x)$ at $x = 1$ is negative.

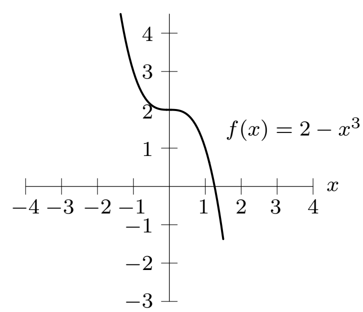


Figure 2.4

(b) $f'(1)$ is the derivative of the function at $x = 1$. This is the rate of change of $f(x) = 2 - x^3$ at $x = 1$. We estimate this by computing the average rate of change of $f(x)$ over intervals near $x = 1$.

Using the intervals $0.999 \le x \le 1$ and $1 \le x \le 1.001$, we see that

$$\begin{pmatrix}\text{Average rate of change}\\ \text{on } 0.999 \le x \le 1\end{pmatrix} = \frac{[2 - 1^3] - [2 - 0.999^3]}{1 - 0.999} = \frac{1 - 1.002997}{0.001} = -2.997,$$

$$\begin{pmatrix}\text{Average rate of change}\\ \text{on } 1 \le x \le 1.001\end{pmatrix} = \frac{[2 - 1.001^3] - [2 - 1^3]}{1.001 - 1} = \frac{0.996997 - 1}{0.001} = -3.003.$$

It appears that the rate of change of $f(x)$ at $x = 1$ is approximately -3, so we estimate $f'(1) = -3$.

14. Since $f'(x) = 0$ where the graph is horizontal, $f'(x) = 0$ at $x = d$. The derivative is positive at points b and c, but the graph is steeper at $x = c$. Thus $f'(x) = 0.5$ at $x = b$ and $f'(x) = 2$ at $x = c$. Finally, the derivative is negative at points a and e but the graph is steeper at $x = e$. Thus, $f'(x) = -0.5$ at $x = a$ and $f'(x) = -2$ at $x = e$. See Table 2.1.

Thus, we have $f'(d) = 0$, $f'(b) = 0.5$, $f'(c) = 2$, $f'(a) = -0.5$, $f'(e) = -2$.

Table 2.1

x	$f'(x)$
d	0
b	0.5
c	2
a	-0.5
e	-2

15. $P'(0)$ is the derivative of the function $P(t) = 200(1.05)^t$ at $t = 0$. This is the same as the rate of change of $P(t)$ at $t = 0$. We estimate this by computing the average rate of change over intervals near $t = 0$.

If we use the intervals $-0.001 \le t \le 0$ and $0 \le t \le 0.001$, we see that:

$$\begin{pmatrix}\text{Average rate of change} \\ \text{on } -0.001 \le t \le 0\end{pmatrix} = \frac{200(1.05)^0 - 200(1.05)^{-0.001}}{0-(-0.001)} = \frac{200-199.990}{0.001} = \frac{0.010}{0.001} = 10,$$

$$\begin{pmatrix}\text{Average rate of change} \\ \text{on } 0 \le t \le 0.001\end{pmatrix} = \frac{200(1.05)^{0.001} - 200(1.05)^{0}}{0.001-0} = \frac{200.010-200}{0.001} = \frac{0.010}{0.001} = 10.$$

It appears that the rate of change of $P(t)$ at $t = 0$ is 10, so we estimate $P'(0) = 10$.

16. Using the interval $1 \le x \le 1.001$, we estimate

$$f'(1) \approx \frac{f(1.001)-f(1)}{0.001} = \frac{3.0033-3.0000}{0.001} = 3.3$$

The graph of $f(x) = 3^x$ is concave up so we expect our estimate to be greater than $f'(1)$.

17. **(a)** Since the values of P go up as t goes from 4 to 6 to 8, we see that $f'(6)$ appears to be positive. The percent of households with cable television is increasing at $t = 6$.

(b) We estimate $f'(2)$ using the difference quotient for the interval to the right of $t = 2$, as follows:

$$f'(2) \approx \frac{\Delta P}{\Delta t} = \frac{63.4-61.5}{4-2} = \frac{1.9}{2} = 0.95.$$

The fact that $f'(2) = 0.95$ tells us that the percent of households with cable television in the United States was increasing at a rate of 0.95 percentage points per year when $t = 2$ (that means 1992).

Similarly:

$$f'(10) \approx \frac{\Delta P}{\Delta t} = \frac{68.9-67.8}{12-10} = \frac{1.1}{2} = 0.55.$$

The fact that $f'(10) = 0.55$ tells us that the percent of households in the United States with cable television was increasing at a rate of 0.55 percentage points per year when $t = 10$ (that means 2000).

18. **(a)** The function $N = f(t)$ is decreasing when $t = 1950$. Therefore, $f'(1950)$ is negative. That means that the number of farms in the US was decreasing in 1950.

(b) The function $N = f(t)$ is decreasing in 1960 as well as in 1980 but it is decreasing faster in 1960 than in 1980. Therefore, $f'(1960)$ is more negative than $f'(1980)$.

19. Using a difference quotient with $h = 0.001$, say, we find

$$f'(1) \approx \frac{1.001\ln(1.001) - 1\ln(1)}{1.001-1} = 1.0005$$

$$f'(2) \approx \frac{2.001\ln(2.001) - 2\ln(2)}{2.001-2} = 1.6934$$

The fact that f' is larger at $x = 2$ than at $x = 1$ suggests that f is concave up on the interval $[1, 2]$.

20. **(a)** The average rate of change is the slope of the secant line in Figure 2.5, which shows that this slope is positive.

(b) The instantaneous rate of change is the slope of the graph at $x = 3$, which we see from Figure 2.6 is negative.

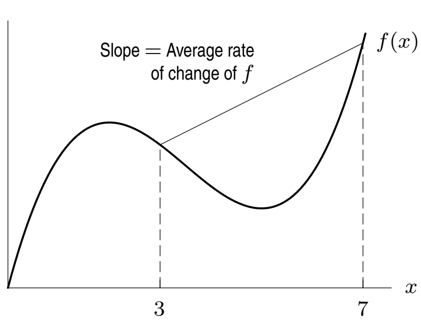


Figure 2.5

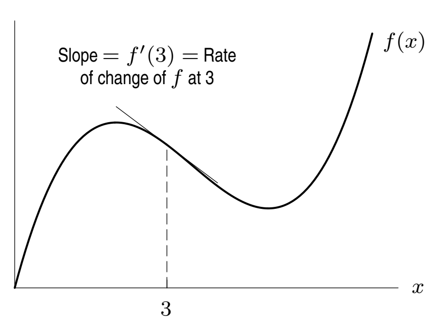


Figure 2.6

21. **(a)** Since the point $B = (2, 5)$ is on the graph of g, we have $g(2) = 5$.
(b) The slope of the tangent line touching the graph at $x = 2$ is given by

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{5 - 5.02}{2 - 1.95} = \frac{-0.02}{0.05} = -0.4.$$

Thus, $g'(2) = -0.4$.

22. **(a)** Since the point $A = (7, 3)$ is on the graph of f, we have $f(7) = 3$.
(b) The slope of the tangent line touching the curve at $x = 7$ is given by

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{3.8 - 3}{7.2 - 7} = \frac{0.8}{0.2} = 4.$$

Thus, $f'(7) = 4$.

23. The answers to parts (a)–(d) are shown in Figure 2.7.

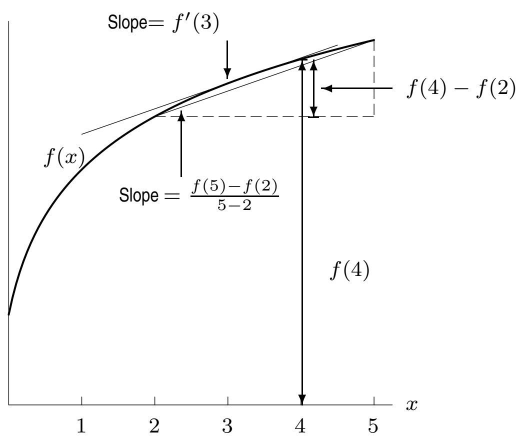


Figure 2.7

24. **(a)** Since f is increasing, $f(4) > f(3)$.
(b) From Figure 2.8, it appears that $f(2) - f(1) > f(3) - f(2)$.
(c) The quantity $\dfrac{f(2) - f(1)}{2 - 1}$ represents the slope of the secant line connecting the points on the graph at $x = 1$ and $x = 2$. This is greater than the slope of the secant line connecting the points at $x = 1$ and $x = 3$ which is $\dfrac{f(3) - f(1)}{3 - 1}$.
(d) The function is steeper at $x = 1$ than at $x = 4$ so $f'(1) > f'(4)$.

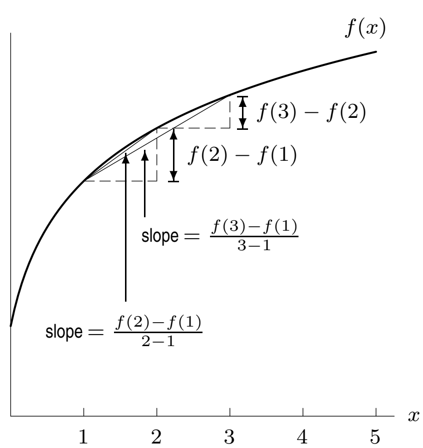


Figure 2.8

25.

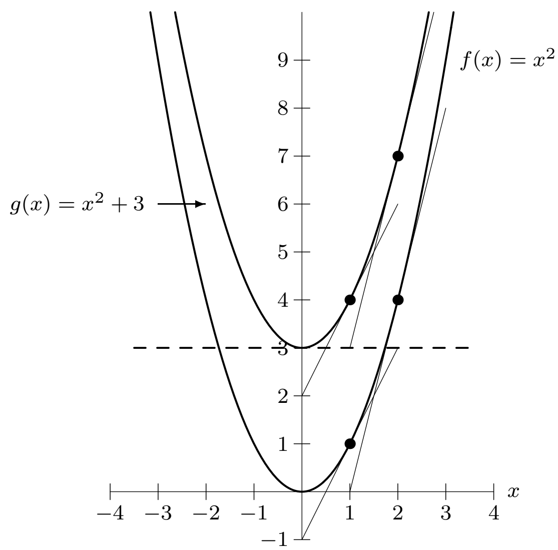


Figure 2.9

(a) The tangent line to the graph of $f(x) = x^2$ at $x = 0$ coincides with the x-axis and therefore is horizontal (slope $= 0$). The tangent line to the graph of $g(x) = x^2 + 3$ at $x = 0$ is the dashed line indicated in the figure and it also has a slope equal to zero. Therefore both tangent lines at $x = 0$ are parallel.

We see in Figure 2.9 that the tangent lines at $x = 1$ appear parallel, and the tangent lines at $x = 2$ appear parallel. The slopes of the tangent lines at any value $x = a$ will be equal.

(b) Adding a constant shifts the graph vertically, but does not change the slope of the curve.

26. **(a)** $f'(t)$ is negative or zero, because the average number of hours worked a week has been decreasing or constant over time.

$g'(t)$ is positive, because hourly wage has been increasing.

$h'(t)$ is positive, because average weekly earnings has been increasing.

(b) We use a difference quotient to the right for our estimates.

(i)

$$f'(1970) \approx \frac{36.0 - 37.0}{1975 - 1970} = -0.2 \quad \text{hours/year}$$
$$f'(1995) \approx \frac{34.3 - 34.3}{2000 - 1995} = 0 \quad \text{hours/year.}$$

In 1970, the average number of hours worked by a production worker in a week was decreasing at the rate of 0.2 hours per year. In 1995, the number of hours was not changing.

(ii)

$$g'(1970) \approx \frac{4.73 - 3.40}{1975 - 1970} = \$0.27 \quad \text{per year}$$
$$g'(1995) \approx \frac{14.00 - 11.64}{2000 - 1995} = \$0.47 \quad \text{per year.}$$

In 1970, the hourly wage was increasing at a rate of \$0.27 per year. In 1995, the hourly wage was increasing at a rate of \$0.47 per year.

(iii)

$$h'(1970) \approx \frac{170.28 - 125.80}{1975 - 1970} = \$8.90 \quad \text{per year}$$
$$h'(1995) \approx \frac{480.41 - 399.53}{2000 - 1995} = \$16.18 \quad \text{per year.}$$

In 1970, average weekly earnings were increasing at a rate of \$8.90 a year. In 1995, weekly earnings were increasing at a rate of \$16.18 a year.

Solutions for Section 2.2

1. Estimating the slope of the lines in Figure 2.10, we find that $f'(-2) \approx 1.0$, $f'(-1) \approx 0.3$, $f'(0) \approx -0.5$, and $f'(2) \approx -1$.

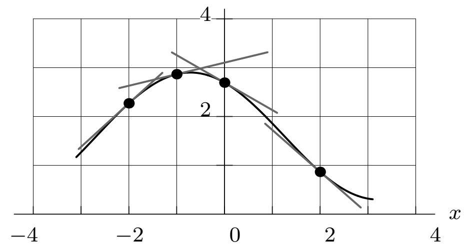


Figure 2.10

2. The graph is that of the line $y = -2x + 2$. The slope, and hence the derivative, is -2. See Figure 2.11.

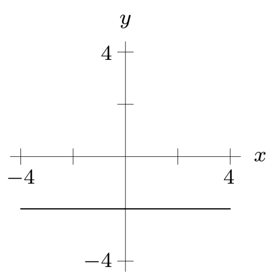


Figure 2.11

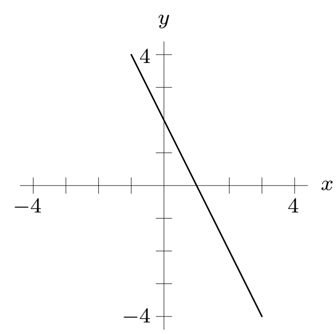


Figure 2.12

3. See Figure 2.12.

4. See Figure 2.13.

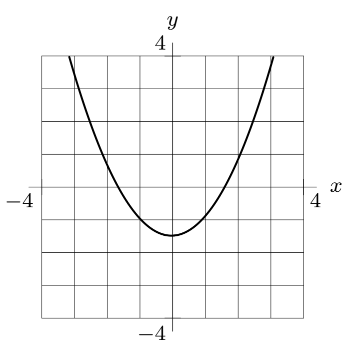


Figure 2.13

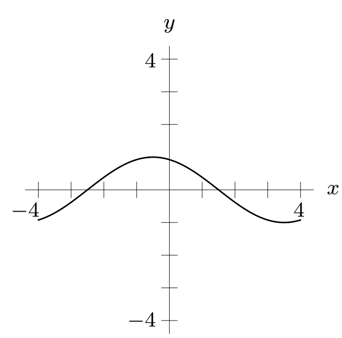


Figure 2.14

5. The slope of this curve is approximately -1 at $x = -4$ and at $x = 4$, approximately 0 at $x = -2.5$ and $x = 1.5$, and approximately 1 at $x = 0$. See Figure 2.14.

6. See Figure 2.15.

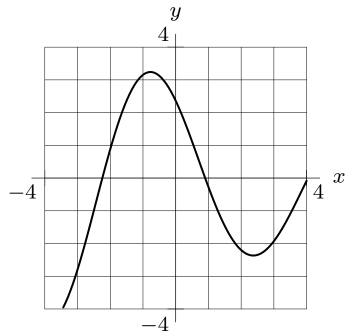


Figure 2.15

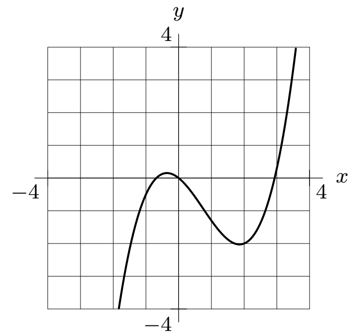


Figure 2.16

7. See Figure 2.16.

8. The graph is increasing for $0 < t < 10$ and is decreasing for $10 < t < 20$. One possible graph is shown in Figure 2.17. The units on the horizontal axis are years and the units on the vertical axis are people.

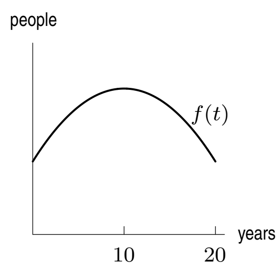


Figure 2.17

The derivative is positive for $0 < t < 10$ and negative for $10 < t < 20$. Two possible graphs are shown in Figure 2.18. The units on the horizontal axes are years and the units on the vertical axes are people per year.

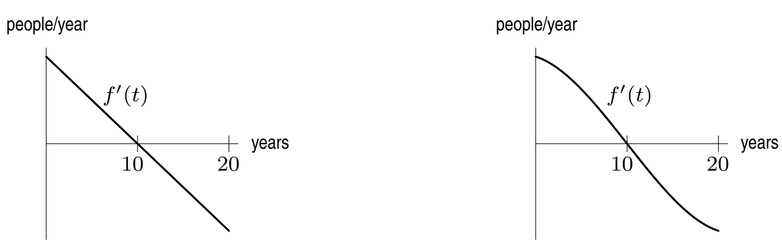


Figure 2.18

9. The function is decreasing for $x < -2$ and $x > 2$, and increasing for $-2 < x < 2$. The matching derivative must be negative (below the x-axis) for $x < -2$ and $x > 2$, positive (above the x-axis) for $-2 < x < 2$, and zero (on the x-axis) for $x = -2$ and $x = 2$. The matching derivative is in graph VIII.

10. The function is a line with negative slope, so $f'(x)$ is a negative constant, and the graph of $f'(x)$ is a horizontal line below the x-axis. The matching derivative is in graph IV.

11. The function is increasing for $x < 2$ and decreasing for $x > 2$. The corresponding derivative is positive (above the x-axis) for $x < 2$, negative (below the x-axis) for $x > 2$, and zero at $x = 2$. The matching derivative is in graph II.

12. The function is increasing for $x < -2$ and decreasing for $x > -2$. The corresponding derivative is positive (above the x-axis) for $x < -2$, negative (below the x-axis) for $x > -2$, and zero at $x = -2$. The derivatives in graphs VI and VII both satisfy these requirements. To decide which is correct, consider what happens as x gets large. The graph of $f(x)$ approaches an asymptote, gets more and more horizontal, and the slope gets closer and closer to zero. The derivative in graph VI meets this requirement and is the correct answer.

13. (a) x_3 (b) x_4 (c) x_5 (d) x_3

14. **(a)** We use the interval to the right of $x = 2$ to estimate the derivative. (Alternately, we could use the interval to the left of 2, or we could use both and average the results.) We have

$$f'(2) \approx \frac{f(4) - f(2)}{4 - 2} = \frac{24 - 18}{4 - 2} = \frac{6}{2} = 3.$$

We estimate $f'(2) \approx 3$.

(b) We know that $f'(x)$ is positive when $f(x)$ is increasing and negative when $f(x)$ is decreasing, so it appears that $f'(x)$ is positive for $0 < x < 4$ and is negative for $4 < x < 12$.

15. For $x = 0$, 5, 10, and 15, we use the interval to the right to estimate the derivative. For $x = 20$, we use the interval to the left. For $x = 0$, we have

$$f'(0) \approx \frac{f(5) - f(0)}{5 - 0} = \frac{70 - 100}{5 - 0} = \frac{-30}{5} = -6.$$

Similarly, we find the other estimates in Table 2.2.

Table 2.2

x	0	5	10	15	20
$f'(x)$	-6	-3	-1.8	-1.2	-1.2

16. Since $f'(x) > 0$ for $x < -1$, $f(x)$ is increasing on this interval.
Since $f'(x) < 0$ for $x > -1$, $f(x)$ is decreasing on this interval.
Since $f'(x) = 0$ at $x = -1$, the tangent to $f(x)$ is horizontal at $x = -1$.
One possible shape for $y = f(x)$ is shown in Figure 2.19.

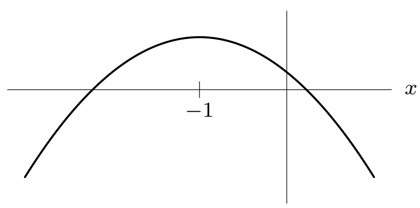


Figure 2.19

17. Since $f'(x) > 0$ for $1 < x < 3$, $f(x)$ is increasing on this interval.
Since $f'(x) < 0$ for $x < 1$ or $x > 3$, $f(x)$ is decreasing on these intervals.
Since $f'(x) = 0$ for $x = 1$ and $x = 3$, the tangent to $f(x)$ will be horizontal at these x's.
One of many possible shapes of $y = f(x)$ is shown in Figure 2.20.

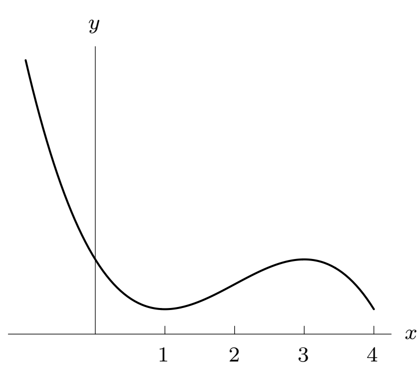


Figure 2.20

18. The value of $g(x)$ is increasing at a decreasing rate for $2.7 < x < 4.2$ and increasing at an increasing rate for $x > 4.2$.

$$\frac{\Delta y}{\Delta x} = \frac{7.4 - 6.0}{5.2 - 4.7} = 2.8 \qquad \text{between } x = 4.7 \text{ and } x = 5.2$$
$$\frac{\Delta y}{\Delta x} = \frac{9.0 - 7.4}{5.7 - 5.2} = 3.2 \qquad \text{between } x = 5.2 \text{ and } x = 5.7$$

Thus $g'(x)$ should be close to 3 near $x = 5.2$.

19. See Figure 2.21.

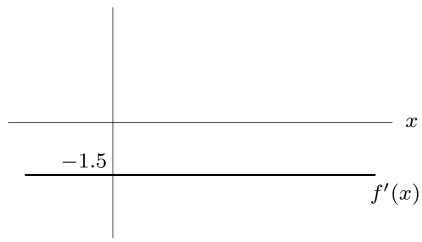


Figure 2.21

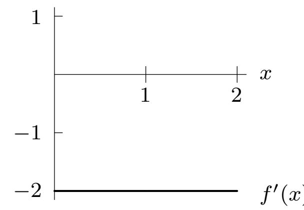


Figure 2.22

20. This is a line with slope -2, so the derivative is the constant function $f'(x) = -2$. The graph is a horizontal line at $y = -2$. See Figure 2.22.

21. See Figure 2.23.

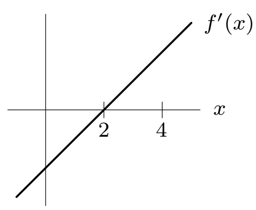


Figure 2.23

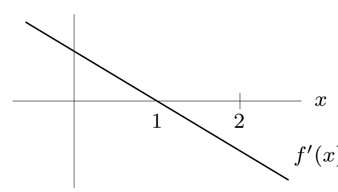


Figure 2.24

22. See Figure 2.24.

23. See Figure 2.25.

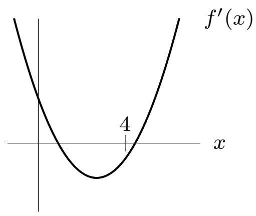


Figure 2.25

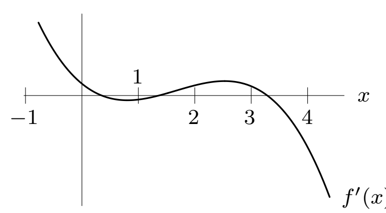


Figure 2.26

24. See Figure 2.26.

25. See Figure 2.27.

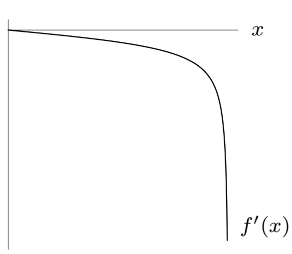


Figure 2.27

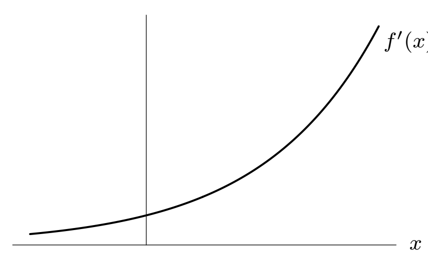


Figure 2.28

26. See Figure 2.28.

27. **(a)** $f'(1) \approx \dfrac{f(1.1) - f(1)}{0.1} = \dfrac{\ln(1.1) - \ln(1)}{0.1} \approx 0.95.$

$f'(2) \approx \dfrac{f(2.1) - f(2)}{0.1} = \dfrac{\ln(2.1) - \ln(2)}{0.1} \approx 0.49.$

$f'(3) \approx \dfrac{f(3.1) - f(3)}{0.1} = \dfrac{\ln(3.1) - \ln(3)}{0.1} \approx 0.33.$

$f'(4) \approx \dfrac{f(4.1) - f(4)}{0.1} = \dfrac{\ln(4.1) - \ln(4)}{0.1} \approx 0.25.$

$f'(5) \approx \dfrac{f(5.1) - f(5)}{0.1} = \dfrac{\ln(5.1) - \ln(5)}{0.1} \approx 0.20.$

(b) It looks like the derivative of $\ln(x)$ is $1/x$.

28.

Table 2.3

x	$f(x)$	x	$f(x)$	x	$f(x)$
1.998	2.6587	2.998	8.9820	3.998	21.3013
1.999	2.6627	2.999	8.9910	3.999	21.3173
2.000	2.6667	3.000	9.0000	4.000	21.3333
2.001	2.6707	3.001	9.0090	4.001	21.3493
2.002	2.6747	3.002	9.0180	4.002	21.3653

Near 2, the values of $f(x)$ seem to be increasing by 0.004 for each increase of 0.001 in x, so the derivative appears to be $\frac{0.004}{0.001} = 4$. Near 3, the values of $f(x)$ are increasing by 0.009 for each step of 0.001, so the derivative appears to be 9. Near 4, $f(x)$ increases by 0.016 for each step of 0.001, so the derivative appears to be 16. The pattern seems to be, then, that at a point x, the derivative of $f(x) = \frac{1}{3}x^3$ is $f'(x) = x^2$.

29. **(a)** The function f is increasing where f' is positive, so for $x_1 < x < x_3$.

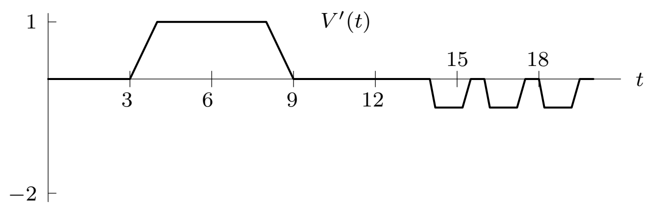
(b) The function f is decreasing where f' is negative, so for $0 < x < x_1$ or $x_3 < x < x_5$.

30. **(a)** $t = 3$

(b) $t = 9$

(c) $t = 14$

(d)



Solutions for Section 2.3

1. kilograms/meter

2. The derivative $f'(6)$ is the slope of the tangent line at $t = 6$. See Figure 2.29. We estimate that the slope of the tangent line is about $(250 - 50)/8 = 25$. We have $f'(6) \approx 25$ megawatts per year. In 1996, the world solar energy output was increasing at a rate of about 25 megawatts per year.

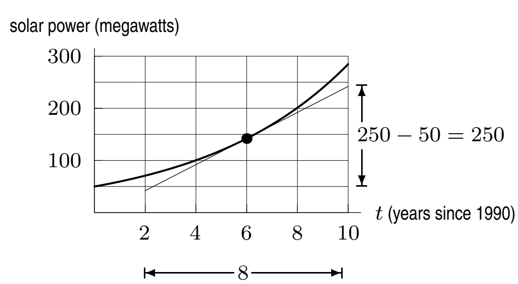


Figure 2.29

3. **(a)** The 12 represents the weight of the chemical; therefore, its units are pounds. The 5 represents the cost of the chemical; therefore, its units are dollars. The statement $f(12) = 5$ means that when the weight of the chemical is 12 pounds, the cost is 5 dollars.

(b) We expect the derivative to be positive since we expect the cost of the chemical to increase when the weight bought increases.

(c) Again, 12 is the weight of the chemical in pounds. The units of the 0.4 are dollars/pound since it is the rate of change of the cost as a function of the weight of the chemical bought. The statement $f'(12) = 0.4$ means that the cost is increasing at a rate of 0.4 dollars per pound when the weight is 12 pounds, or that an additional pound will cost an extra 40 cents.

4. **(a)** The statement $f(200) = 350$ means that it costs \$350 to produce 200 gallons of ice cream.

(b) The statement $f'(200) = 1.4$ means that when the number of gallons produced is 200, costs are increasing by about \$1.40 per gallon. In other words, it costs about \$1.40 to produce the next (the 201^{st}) gallon of ice cream.

5. **(a)** The statement $f(5) = 18$ means that when 5 milliliters of catalyst are present, the reaction will take 18 minutes. Thus, the units for 5 are ml while the units for 18 are minutes.

(b) As in part (a), 5 is measured in ml. Since f' tells how fast T changes per unit a, we have f' measured in minutes/ml. If the amount of catalyst increases by 1 ml (from 5 to 6 ml), the reaction time decreases by about 3 minutes.

6. The statement $f(20) = 57$ means that when $t = 20$, we have $P = 57$. This tells us that in 2002, 57% of households had a personal computer. The statement $f'(20) = 3$ tells us that in 2002, the percent of households with a personal computer is increasing at a rate of 3% a year.

7. **(a)** The yam is cooling off so T is decreasing and $f'(t)$ is negative.

(b) Since $f(t)$ is measured in degrees Fahrenheit and t is measured in minutes, df/dt must be measured in units of $^\circ\text{F}/\text{min}$.

8. **(a)** The statement $f(15) = 200$ tells us that when the price is \$15, we sell about 200 units of the product.

(b) The statement $f'(15) = -25$ tells us that if we increase the price by \$1 (from 15), we will sell about 25 fewer units of the product.

9. **(a)** Positive, since weight increases as the child gets older.

(b) $f(8) = 45$ tells us that when the child is 8 years old, the child weighs 45 pounds.

(c) The units of $f'(a)$ are lbs/year. $f'(a)$ tells the rate of growth in lbs/years at age a.

(d) $f'(8) = 4$ tells us that the 8-year-old child is growing at about 4 lbs/year.

(e) As a increases, $f'(a)$ will decrease since the rate of growth slows down as the child grows up.

10. **(a)** Since $f'(c)$ is negative, the function $P = f(c)$ is decreasing: pelican eggshells are getting thinner as the concentration, c, of PCBs in the environment is increasing.

(b) The statement $f(200) = 0.28$ means that the thickness of pelican eggshells is 0.28 mm when the concentration of PCBs in the environment is 200 parts per million (ppm).

The statement $f'(200) = -0.0005$ means that the thickness of pelican eggshells is decreasing (eggshells are becoming thinner) at a rate of 0.0005 mm per ppm of concentration of PCBs in the environment when the concentration of PCBs in the environment is 200 ppm.

11. **(a)** kilograms per week

(b) At week 24 the fetus is growing at a rate of 0.096 kg/week, or 96 grams per week.

12. **(a)** The tangent line to the weight graph is steeper at 36 weeks then at 20 weeks, so $g'(36)$ is greater than $g'(20)$.

(b) The fetus increases its weight more rapidly at week 36 than at week 20.

13. Compare the secant line to the graph from week 0 to week 40 to the tangent lines at week 20 and week 36.

(a) At week 20 the secant line is steeper than the tangent line. The instantaneous weight growth rate is less than the average.

(b) At week 36 the tangent line is steeper than the secant line. The instantaneous weight growth rate is greater than the average.

14. We estimate the derivatives at 20 and 36 weeks by drawing tangent lines to the weight graph, shown in Figure 2.30, and calculating their slopes.

(a) Two points on the tangent line at 20 weeks are $(16, 0)$ and $(40, 1.5)$. Thus,

$$g'(20) = \frac{1.5 - 0}{40 - 16} = 0.0625 \text{ kg/week}.$$

(b) Two points on the tangent line at 36 weeks are $(24, 0)$ and $(40, 3.0)$. Thus,

$$g'(36) = \frac{3-0}{40-24} = 0.19 \text{ kg/week.}$$

(c) The average rate of growth is the slope of the secant line from $(0, 0)$ to $(40, 3.1)$. Thus,

$$\text{Average rate of change} = \frac{3.1-0}{40-0} = 0.078\text{kg/week.}$$

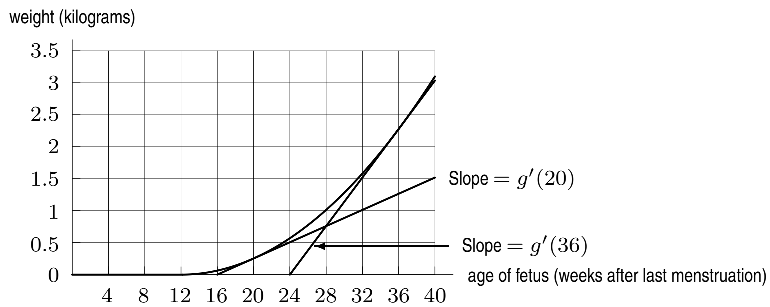


Figure 2.30

15. Moving away slightly from the center of the hurricane from a point 15 kilometers from the center moves you to a point with stronger winds. For example, the wind is stronger at 15.1 kilometers from the center of the hurricane than it is at 15 kilometers from the center.

16. After falling 20 meters the speed of the rock is increasing at a rate of 0.5 meters/second per meter.

17. Since $f(t) = 1.291(1.006)^t$, we have

$$f(6) = 1.291(1.006)^6 = 1.338.$$

To estimate $f'(6)$, we use a small interval around 6:

$$f'(6) \approx \frac{f(6.001) - f(6)}{6.001 - 6} = \frac{1.291(1.006)^{6.001} - 1.291(1.006)^6}{0.001} = 0.008.$$

We see that $f(6) = 1.338$ billion people and $f'(6) = 0.008$ billion (that is, 8 million) people per year. This model predicts that the population of China will be about 1,338,000,000 people in 2009 and growing at a rate of about 8,000,000 people per year at that time.

18. The derivative $f'(10)$ is the slope of the tangent line to the curve at $t = 10$. See Figure 2.31. Taking two points on the tangent line, we calculate its slope:

$$\text{Slope} \approx \frac{100-70}{5} = 6.$$

Since the slope is about 6, we have $f'(10) \approx 6$ cm/yr. At $t = 10$, the sturgeon was growing in length at a rate of about 6 centimeters a year.

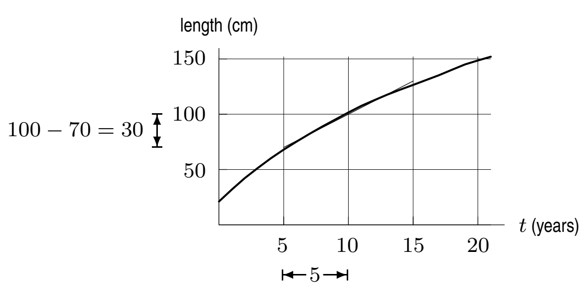


Figure 2.31

19. Since B is measured in dollars and t is measured in years, dB/dt is measured in dollars per year. We can interpret dB as the extra money added to your balance in dt years. Therefore dB/dt represents how fast your balance is growing, in units of dollars/year.

20. **(a)** The statement $f(140) = 120$ means that a patient weighing 140 pounds should receive a dose of 120 mg of the painkiller. The statement $f'(140) = 3$ tells us that if the weight of a patient increases by about one pound (from 140 pounds), the dose should be increased by about 3 mg.

(b) Since the dose for a weight of 140 lbs is 120 mg and at this weight the dose goes up by 3 mg for each pound, a 145 lb patient should get an additional $3(5) = 15$ mg. Thus, for a 145 lb patient, the correct dose is approximately

$$f(145) \approx 120 + 3(5) = 135 \text{ mg.}$$

21. Using $\Delta x = 1$, we can say that

$$\begin{aligned} f(21) &= f(20) + \text{change in } f(x) \\ &\approx f(20) + f'(20)\Delta x \\ &= 68 + (-3)(1) \\ &= 65. \end{aligned}$$

Similarly, using $\Delta x = -1$,

$$\begin{aligned} f(19) &= f(20) + \text{change in } f(x) \\ &\approx f(20) + f'(20)\Delta x \\ &= 68 + (-3)(-1) \\ &= 71. \end{aligned}$$

Using $\Delta x = 5$, we can write

$$\begin{aligned} f(25) &= f(20) + \text{change in } f(x) \\ &\approx f(20) + (-3)(5) \\ &= 68 - 15 \\ &= 53. \end{aligned}$$

22. Using the approximation $\Delta y \approx f'(x)\Delta x$ with $\Delta x = 2$, we have $\Delta y \approx f'(20) \cdot 2 = 6 \cdot 2$, so

$$f(22) \approx f(20) + f'(20) \cdot 2 = 345 + 6 \cdot 2 = 357.$$

23. **(a)** The statement $f(20) = 0.36$ means that 20 minutes after smoking a cigarette, there will be 0.36 mg of nicotine in the body. The statement $f'(20) = -0.002$ means that 20 minutes after smoking a cigarette, nicotine is leaving the body at a rate 0.002 mg per minute. The units are 20 minutes, 0.36 mg, and -0.002 mg/minute.

(b)

$$\begin{aligned} f(21) &\approx f(20) + \text{ change in } f \text{ in one minute} \\ &= 0.36 + (-0.002) \\ &= 0.358 \\ f(30) &\approx f(20) + \text{change in } f \text{ in 10 minutes} \\ &= 0.36 + (-0.002)(10) \\ &= 0.36 - 0.02 \\ &= 0.34 \end{aligned}$$

24. **(a)** The derivative $f'(t)$ appears to be positive, because according to the table, gold production is increasing.

(b) The derivative (or rate of change) appears to be greatest between 1996 and 1999.

(c) We have

$$f'(2002) \approx \frac{82.9 - 82.6}{2002 - 1999} = 0.1 \text{ million ounces/year.}$$

In 2002, gold production was increasing at a rate of approximately 0.1 million ounces per year.

(d) In 2002, gold production was 82.9 million ounces and was increasing at a rate of 0.1 million ounces each year. Therefore, in 2003 (one year later), we have

$$f(2003) \approx 82.9 + 0.1 = 83.0 \text{ million ounces,}$$

and in 2010 (8 years later), we have

$$f(2010) \approx 82.9 + 0.1(8) = 83.7 \text{ million ounces.}$$

We estimate that gold production in 2003 is 83.0 million troy ounces and gold production in 2010 is 83.7 million troy ounces.

25. (a) The tangent line is shown in Figure 2.32. Two points on the line are $(0, 16)$ and $(3.2, 0)$. Thus

$$\text{Slope} = \frac{0 - 16}{3.2 - 0} = -5 \text{ (cm/sec)/kg.}$$

(b) Since 50 grams $= 0.050$ kg, the contraction velocity changes by about $-5(\text{cm/sec})/\text{kg} \cdot 0.050\text{kg} = -0.25$ cm/sec. The velocity is reduced by 0.25 cm/sec or 2.5 mm/sec.

(c) Since $v(x)$ is the contraction velocity in cm/sec with a load of x kg, we have $v'(2) = -5$.

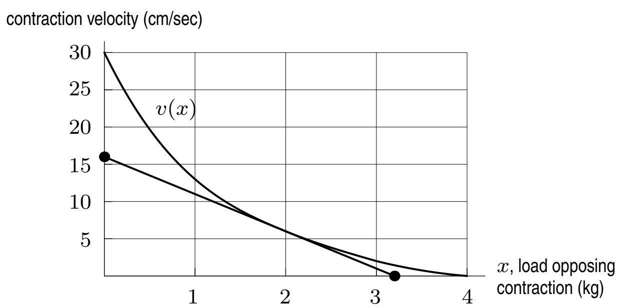


Figure 2.32

26. Units of $C'(r)$ are dollars/percent. Approximately, $C'(r)$ means the additional amount needed to pay off the loan when the interest rate is increased by 1%. The sign of $C'(r)$ is positive, because increasing the interest rate will increase the amount it costs to pay off a loan.

27. We have $\Delta h = h(6003) - h(6000) \approx h'(6000)\Delta x = (0.5)(3) = 1.5$ meters. The elevation increases approximately 1.5 meters as the climber moves from a position 6000 meters from the start of the trail to a position 6003 meters from the start. Thus his elevation increases from 8000 meters to 8001.5 meters. His new elevation is 8001.5 meters above sea level.

28. The fact that $f(80) = 0.05$ means that when the car is moving at 80 km/hr is it using 0.05 liter of gasoline for each kilometer traveled.

The derivative $f'(v)$ is the rate of change of gasoline consumption with respect to speed. That is, $f'(v)$ tells us how the consumption of gasoline changes as speeds vary. We are told that $f'(80) = 0.0005$. This means that a 1-kilometer increase in speed results in an increase in consumption of 0.0005 liter per km. At higher speeds, the vehicle burns more gasoline per km traveled than at lower speeds.

29. (a) The tangent line is shown in Figure 2.33. Two points on the line are $(0, 0.75)$ and $(2.5, 5)$. The

$$\text{Slope} = \frac{5 - 0.75}{2.5 - 0} = 1.7 \text{ (liters/minute)/hour.}$$

(b) The rate of change of the pumping rate is the slope of the tangent line. One minute $= 1/60$ hour, so in one minute the

$$\text{Pumping rate increases by about } 1.7 \frac{\text{(liter/minute)}}{\text{hour}} \cdot \frac{1}{60} \text{ hour } = 0.028 \text{ liter/minute.}$$

(c) Since $g(t)$ is the pumping rate in liters/minute at time t hours, we have $g'(2) = 1.7$.

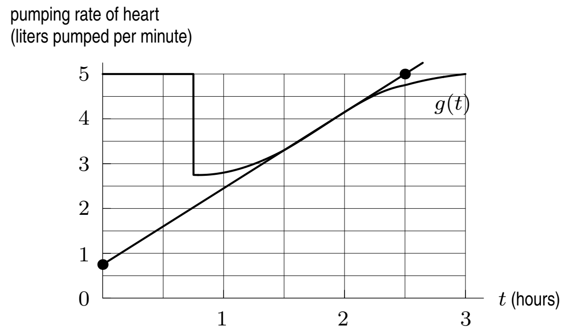


Figure 2.33

30. (a) Since the traffic flow is the number of cars per hour, it is the slope of the graph of $C(t)$. It is greatest where the graph of the function $C(t)$ is the steepest and increasing. This happens at approximately $t = 3$ hours, or 7am.

(b) By reading the values of $C(t)$ from the graph we see:

$$\begin{array}{c}\text{Average rate of change} \\ \text{on } 1 \leq t \leq 2\end{array} = \frac{C(2) - C(1)}{2 - 1} = \frac{1000 - 400}{1} = \frac{600}{1} = 600 \text{ cars/hour},$$

$$\begin{array}{c}\text{Average rate of change} \\ \text{on } 2 \leq t \leq 3\end{array} = \frac{C(3) - C(2)}{3 - 2} = \frac{2000 - 1000}{1} = \frac{1000}{1} = 1000 \text{ cars/hour}.$$

A good estimate of $C'(2)$ is the average of the last two results. Therefore:

$$C'(2) \approx \frac{(600 + 1000)}{2} = \frac{1600}{2} = 800 \text{ cars/hour}.$$

(c) Since $t = 2$ is 6 am, the fact that $C'(2) \approx 800$ cars/hour means that the traffic flow at 6 am is about 800 cars/hour.

31. The consumption rates (kg/week) are the rates at which the quantities are decreasing, that is, -1 times the derivatives of the storage functions. To compare rates at a given time, compare the steepness of the tangent lines to the graphs at that time.

(a) At 3 weeks, the tangent line to the fat storage graph is steeper than the tangent line to the protein storage graph. During the third week, fat is consumed at a greater rate than protein.

(b) At 7 weeks, the protein storage graph is steeper than the fat storage graph. During the seventh week, protein is consumed at a greater rate than fat.

32. Where the graph is linear, the derivative of the fat storage function is constant. The derivative gives the rate of fat consumption (kg/week). Thus, for the first four weeks the body burns fat at a constant rate.

33. The fat consumption rate (kg/week) is the rate at which the quantity of fat is decreasing, that is, -1 times the derivative of the fat storage function. We estimate the derivatives at 3, 6, and 8 weeks by drawing tangent lines to the storage graph, shown in Figure 2.34, and calculating their slopes.

(a) The tangent line at 3 weeks is the storage graph itself since that part of the graph is straight. Two points on the tangent line are $(0, 12)$ and $(4, 4)$. Then

$$\text{Slope} = \frac{4 - 12}{4 - 0} = -2.0 \text{ kg/week}.$$

The consumption rate is 2.0 kg/week.

(b) Two points on the tangent line are $(0, 4.7)$ and $(8, 0.1)$. Then,

$$\text{Slope} = \frac{0.1 - 4.7}{8 - 0} = -0.6 \text{ kg/week}.$$

The consumption rate is 0.6 kg/week.

(c) Two points on the tangent line are $(0, 2.7)$ and $(8, 0.6)$. Then

$$\text{Slope} = \frac{0.6 - 2.7}{8 - 0} = -0.3 \text{ kg/week}.$$

The consumption rate is 0.3 kg/week.

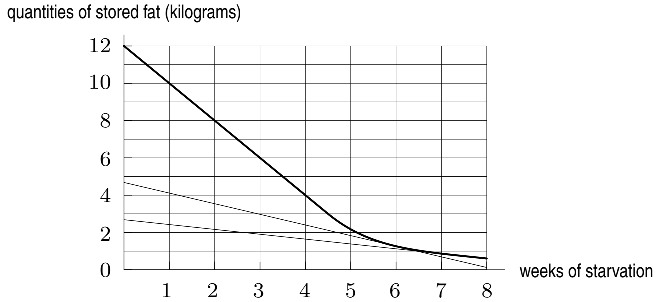


Figure 2.34: Rates of consumption are -1 times slopes of tangent lines

34. The body changes from burning more fat than protein to burning more protein. This is done by reducing the rate at which it burns fat and simultaneously increasing the rate at which it burns protein. The physiological reason is that the body has begun to run out of fat.

35. The graph of fat storage is linear for four weeks, then becomes concave up. Thus, the derivative of fat storage is constant for four weeks, then increases. This matches graph I.

The graph of protein storage is concave up for three weeks, then becomes concave down. Thus, the derivative of protein storage is increasing for three weeks and then becomes decreasing. This matches graph II.

36. The derivative is the instantaneous rate of change. To estimate the instantaneous rate of change in 2002 using the information given, we estimate the average rate of change on the interval from 2000 to 2002.

(a) We have

$$f'(2002) \approx \frac{803.3 - 942.5}{2002 - 2000} = -69.60 \text{ million CDs per year,}$$

and

$$g'(2002) \approx \frac{31.1 - 76.0}{2002 - 2000} = -22.45 \text{ million cassettes per year.}$$

In 2002, sales of music CDs were decreasing at a rate of about 69.60 million CDs per year and sales of music cassettes were decreasing at a rate of about 22.45 million cassettes per year.

(b) In 2002, sales of music CDs were 803.3 million and were decreasing at a rate of 69.60 million per year. Therefore, if we assume this rate of decrease stays constant, we have

$$f(2003) \approx 803.3 - 69.60 = 733.7 \text{ million CDs,}$$

and (since 2010 is 8 years after 2002)

$$f(2010) \approx 803.3 - 69.60(8) = 246.5 \text{ million CDs.}$$

We estimate sales of music CDS to be about 733.7 million in 2003 and about 246.5 million in 2010. We should view this estimate very cautiously, given the rapid expansion of competing technology.

37. (a) The units of compliance are units of volume per units of pressure, or liters per centimeter of water.

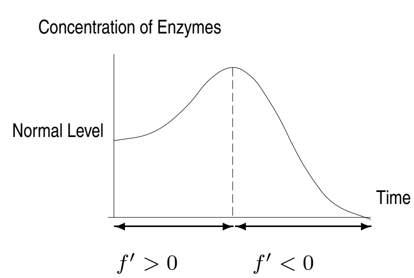
(b) The increase in volume for a 5 cm reduction in pressure is largest between 10 and 15 cm. Thus, the compliance appears maximum between 10 and 15 cm of pressure reduction. The derivative is given by the slope, so

$$\text{Compliance} \approx \frac{0.70 - 0.49}{15 - 10} = 0.042 \text{ liters per centimeter.}$$

(c) When the lung is nearly full, it cannot expand much more to accommodate more air.

38. (a) See (b).

(b)



(c) f' is the rate at which the concentration is increasing or decreasing. f' is positive at the start of the disease and negative toward the end. In practice, of course, one cannot measure f' directly. Checking the value of C in blood samples taken on consecutive days would tell us

$$f(t+1) - f(t) = \frac{f(t+1) - f(t)}{(t+1) - t},$$

which is our estimate of $f'(t)$.

Solutions for Section 2.4

1. **(a)** Since the graph is below the x-axis at $x = 2$, the value of $f(2)$ is negative.
(b) Since $f(x)$ is decreasing at $x = 2$, the value of $f'(2)$ is negative.
(c) Since $f(x)$ is concave up at $x = 2$, the value of $f''(2)$ is positive.

2. $f'(x) > 0$
$f''(x) > 0$

3. $f'(x) = 0$
$f''(x) = 0$

4. $f'(x) < 0$
$f''(x) = 0$

5. $f'(x) < 0$
$f''(x) > 0$

6. $f'(x) > 0$
$f''(x) < 0$

7. $f'(x) < 0$
$f''(x) < 0$

8. The derivative of $w(t)$ appears to be negative since the function is decreasing over the interval given. The second derivative, however, appears to be positive since the function is concave up, i.e., it is decreasing at a decreasing rate.

9. The derivative, $s'(t)$, appears to be positive since $s(t)$ is increasing over the interval given. The second derivative also appears to be positive or zero since the function is concave up or possibly linear between $t = 1$ and $t = 3$, i.e., it is increasing at a non-decreasing rate.

10. The derivative is positive on those intervals where the function is increasing and negative on those intervals where the function is decreasing. Therefore, the derivative is positive on the intervals $0 < t < 0.4$ and $1.7 < t < 3.4$, and negative on the intervals $0.4 < t < 1.7$ and $3.4 < t < 4$.

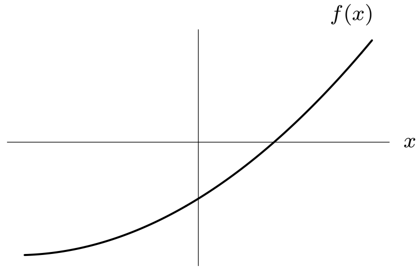
The second derivative is positive on those intervals where the graph of the function is concave up and negative on those intervals where the graph of the function is concave down. Therefore, the second derivative is positive on the interval $1 < t < 2.6$ and negative on the intervals $0 < t < 1$ and $2.6 < t < 4$.

11. The derivative is positive on those intervals where the function is increasing and negative on those intervals where the function is decreasing. Therefore, the derivative is positive on the interval $-2.3 < t < -0.5$ and negative on the interval $-0.5 < t < 4$.

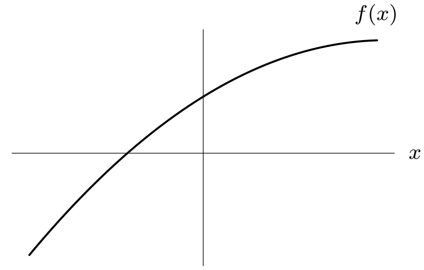
The second derivative is positive on those intervals where the graph of the function is concave up and negative on those intervals where the graph of the function is concave down. Therefore, the second derivative is positive on the interval $0.5 < t < 4$ and negative on the interval $-2.3 < t < 0.5$.

12.

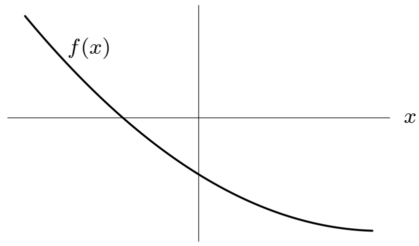
(a)



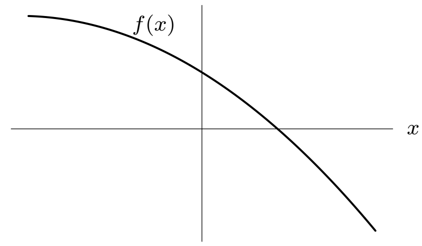
(b)



(c)



(d)



13. The two points at which $f' = 0$ are A and B. Since f' is nonzero at C and D and f'' is nonzero at all four points, we get the completed Table 2.4:

Table 2.4

Point	f	f'	f''
A	$-$	0	$+$
B	$+$	0	$-$
C	$+$	$-$	$-$
D	$-$	$+$	$+$

14. **(a)** minutes/kilometer.
(b) minutes/kilometer^2.

15. (b). The positive first derivative tells us that the temperature is increasing; the negative second derivative tells us that the rate of increase of the temperature is slowing.

16. (e). Since the smallest value of $f'(t)$ was $2°\text{C/hour}$, we know that $f'(t)$ was always positive. Thus, the temperature rose all day.

17. **(a)** The function appears to be decreasing and concave down, and so we conjecture that f' is negative and that f'' is negative.
(b) We use difference quotients to the right:
$f'(2) \approx \frac{137-145}{4-2} = -4$
$f'(8) \approx \frac{56-98}{10-8} = -21.$

18. Since $f(2) = 5$, the graph goes through the point $(2, 5)$. Since $f'(2) = 1/2$, the slope of the curve is $1/2$ when it passes through this point. Since $f''(2) > 0$, the graph is concave up at this point. One possible graph is shown in Figure 2.35. Many other answers are also possible.

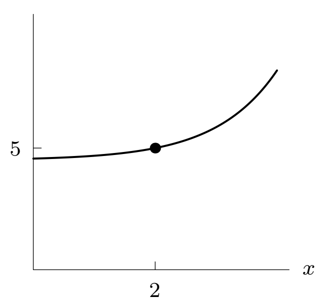


Figure 2.35

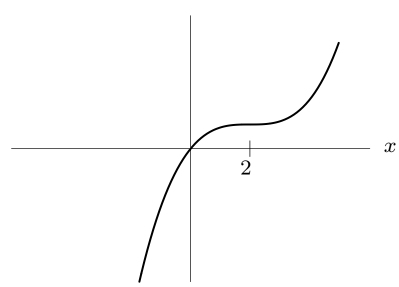


Figure 2.36

19. This graph is increasing for all x, and is concave down to the left of 2 and concave up to the right of 2. One possible answer is shown in Figure 2.36:

20. **(a)** At x_4 and x_5, because the graph is below the x-axis there.
(b) At x_3 and x_4, because the graph is sloping down there.
(c) At x_3 and x_4, because the graph is sloping down there. This is the same condition as part (b).
(d) At x_2 and x_3, because the graph is bending downward there.
(e) At x_1, x_2, and x_5, because the graph is sloping upward there.
(f) At x_1, x_4, and x_5, because the graph is bending upward there.

21. To the right of $x = 5$, the function starts by increasing, since $f'(5) = 2 > 0$ (though f may subsequently decrease) and is concave down, so its graph looks like the graph shown in Figure 2.37. Also, the tangent line to the curve at $x = 5$ has slope 2 and lies above the curve for $x > 5$. If we follow the tangent line until $x = 7$, we reach a height of 24. Therefore, $f(7)$ must be smaller than 24, meaning 22 is the only possible value for $f(7)$ from among the choices given.

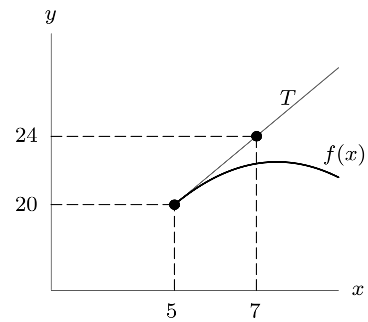


Figure 2.37

22. **(a)** The derivative, $f'(t)$, appears to be positive since the number of cars is increasing. The second derivative, $f''(t)$, appears to be positive during the period 1940–1980 because the rate of change is increasing. For example, between 1940 and 1950, the rate of change is $(40.3 - 27.5)/10 = 1.28$ million cars per year, while between 1950 and 1960, the rate of change is 2.14 million cars per year.

(b) We use the average rate of change formula on the interval 1970 to 1980 to estimate $f'(1975)$:

$$f'(1975) \approx \frac{121.6 - 89.2}{1980 - 1970} = \frac{32.2}{10} = 3.22.$$

We estimate that $f'(1975) \approx 3.22$ million cars per year. The number of passenger cars in the US was increasing at a rate of about 3.22 million cars per year in 1975.

23. **(a)** Let $N(t)$ be the number of people below the poverty line. See Figure 2.38.

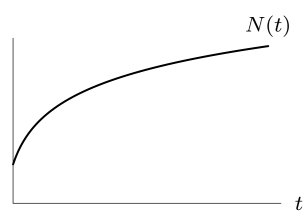


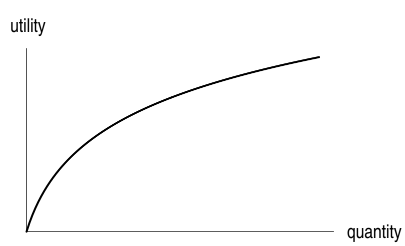
Figure 2.38

(b) dN/dt is positive, since people are still slipping below the poverty line. d^2N/dt^2 is negative, since the rate at which people are slipping below the poverty line, dN/dt, is decreasing.

24. **(a)** $dP/dt > 0$ and $d^2P/dt^2 > 0$.

(b) $dP/dt < 0$ and $d^2P/dt^2 > 0$ (but dP/dt is close to zero).

25. **(a)**



(b) As a function of quantity, utility is increasing but at a decreasing rate; the graph is increasing but concave down. So the derivative of utility is positive, but the second derivative of utility is negative.

26. **(a)** The EPA will say that the rate of discharge is still rising. The industry will say that the rate of discharge is increasing less quickly, and may soon level off or even start to fall.
(b) The EPA will say that the rate at which pollutants are being discharged is leveling off, but not to zero—so pollutants will continue to be dumped in the lake. The industry will say that the rate of discharge has decreased significantly.

27. Since velocity is positive and acceleration is negative, we have $f' > 0$ and $f'' < 0$, and so the graph is increasing and concave down. See Figure 2.39.

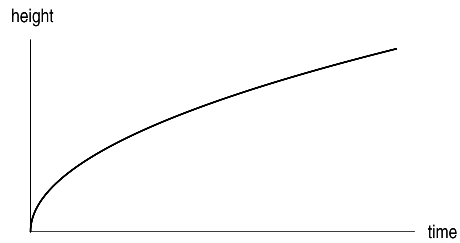


Figure 2.39

28. **(a)** At t_3, t_4, and t_5, because the graph is above the t-axis there.
(b) At t_2 and t_3, because the graph is sloping up there.
(c) At t_1, t_2, and t_5, because the graph is concave up there
(d) At t_1, t_4, and t_5, because the graph is sloping down there.
(e) At t_3 and t_4, because the graph is concave down there.

29. (a) IV, (b) III, (c) II, (d) I, (e) IV, (f) II

Solutions for Section 2.5

1. Drawing in the tangent line at the point $(10000, C(10000))$ we get Figure 2.40. We see that each vertical increase of 2500 in the tangent line gives a corresponding horizontal increase of roughly 6000. Thus the marginal cost at the production level of 10,000 units is

$$C'(10{,}000) = \begin{array}{c}\text{Slope of tangent line}\\ \text{to } C(q) \text{ at } q = 10{,}000\end{array} = \frac{2500}{6000} = 0.42.$$

This tells us that after producing 10,000 units, it will cost roughly \$0.42 to produce one more unit.

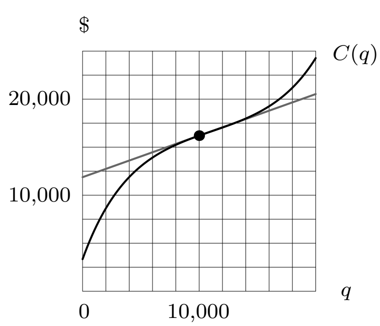


Figure 2.40

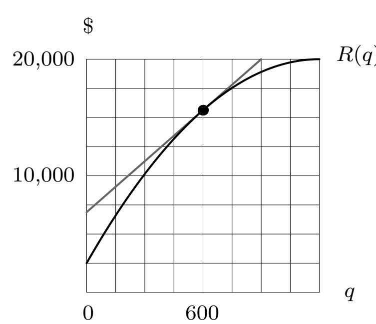


Figure 2.41

2. Drawing in the tangent line at the point $(600, R(600))$, we get Figure 2.41. We see that each vertical increase of 2500 in the tangent line gives a corresponding horizontal increase of roughly 150. The marginal revenue at the production level of 600 units is

$$R'(600) = \begin{array}{c}\text{Slope of tangent line}\\ \text{to } R(q) \text{ at } q = 600\end{array} = \frac{2500}{150} = 16.67.$$

This tells us that after producing 600 units, the revenue for producing the 601^{st} product will be roughly \$16.67.

3. **(a)** Marginal cost is the derivative $C'(q)$, so its units are dollars/barrel.
(b) It costs about $3 more to produce 101 barrels of olive oil than to produce 100 barrels.

4. The marginal cost is approximated by the difference quotient

$$MC \approx \frac{\Delta C}{\Delta q} = \frac{4830 - 4800}{1305 - 1295} = 3.$$

The marginal cost is approximately $3 per item.

5. Marginal cost $= C'(q)$. Therefore, marginal cost at q is the slope of the graph of $C(q)$ at q. We can see that the slope at $q = 5$ is greater than the slope at $q = 30$. Therefore, marginal cost is greater at $q = 5$. At $q = 20$, the slope is small, whereas at $q = 40$ the slope is larger. Therefore, marginal cost at $q = 40$ is greater than marginal cost at $q = 20$.

6. The slope of the revenue curve is greater than the slope of the cost curve at both q_1 and q_2, so the marginal revenue is greater at both production levels.

7. **(a)** We can approximate $C(16)$ by adding $C'(15)$ to $C(15)$, since $C'(15)$ is an estimate of the cost of the 16^{th} item.

$$C(16) \approx C(15) + C'(15) = \$2300 + \$108 = \$2408.$$

(b) We approximate $C(14)$ by subtracting $C'(15)$ from $C(15)$, where $C'(15)$ is an approximation of the cost of producing the 15^{th} item.

$$C(14) \approx C(15) - C'(15) = \$2300 - \$108 = \$2192.$$

8. We know $MC \approx C(1{,}001) - C(1{,}000)$. Therefore, $C(1{,}001) \approx C(1{,}000) + MC$ or $C(1{,}001) \approx 5000 + 25 = 5025$ dollars.

Since we do not know $MC(999)$, we will assume that $MC(999) = MC(1{,}000)$. Therefore:

$$MC(999) = C(1{,}000) - C(999).$$

Then:

$$C(999) \approx C(1{,}000) - MC(999) = 5{,}000 - 25 = 4{,}975 \text{ dollars.}$$

Alternatively, we can reason that

$$MC(1{,}000) \approx C(1{,}000) - C(999),$$

so

$$C(999) \approx C(1{,}000) - MC(1{,}000) = 4{,}975 \text{ dollars.}$$

Now for $C(1{,}000)$, we have

$$C(1{,}100) \approx C(1{,}000) + MC \cdot 100.$$

Since $1{,}100 - 1{,}000 = 100$,

$$C(1{,}100) \approx 5{,}000 + 25 \times 100 = 5{,}000 + 2{,}500 = 7{,}500 \text{ dollars.}$$

9. **(a)** The cost to produce 50 units is $4300 and the marginal cost to produce additional items is about $24 per unit. Producing two more units (from 50 to 52) increases cost by $48. We have

$$C(52) \approx 4300 + 24(2) = \$4348.$$

(b) When $q = 50$, the marginal cost is $24 per item and the marginal revenue is $35 per item. The profit on the 51^{st} item is $35 - 24 = \$11$.
(c) When $q = 100$, the marginal cost is $38 per item and the marginal revenue is $35 per item, so the company will lose $3 by producing the 101^{st} item. Since the company will lose money, it should not produce the 101^{st} item.

10. At $q = 50$, the slope of the revenue is larger than the slope of the cost. Thus, at $q = 50$, marginal revenue is greater than marginal cost and the 50^{th} bus should be added. At $q = 90$ the slope of revenue is less than the slope of cost. Thus, at $q = 90$ the marginal revenue is less than marginal cost and the 90^{th} bus should not be added.

11. **(a)** For $q = 500$

$$\text{Profit } = \pi(500) = R(500) - C(500) = 9400 - 7200 = 2200 \text{ dollars.}$$

(b) As production increases from $q = 500$ to $q = 501$,

$$\Delta R \approx R'(500)\Delta q = 20 \cdot 1 = 20 \text{ dollars,}$$
$$\Delta C \approx C'(500)\Delta q = 15 \cdot 1 = 15 \text{ dollars,}$$

Thus

$$\text{Change in profit } = \Delta\pi = \Delta R - \Delta C = 20 - 15 = 5 \text{ dollars.}$$

12. **(a)** At $q = 2000$, we have

$$\text{Profit } = R(2000) - C(2000) = 7780 - 5930 = 1850 \text{ dollars.}$$

(b) If q increases from 2000 to 2001,

$$\Delta R \approx R'(2000) \cdot \Delta q = 2.5 \cdot 1 = 2.5 \text{ dollars,}$$
$$\Delta C \approx C'(2000) \cdot \Delta q = 2.1 \cdot 1 = 2.1 \text{ dollars,}$$

Thus,

$$\text{Change in profit } = \Delta R - \Delta C = 2.5 - 2.1 = 0.4 \text{ dollars.}$$

Since increasing production increases profit, the company should increase production.

(c) By a calculation similar to that in part (b), as q increases from 2000 to 2001,

$$\text{Change in profit } = 4.32 - 4.77 = -0.45 \text{ dollars.}$$

Since increasing production reduces the profit, the company should decrease production.

13. **(a)** At $q = 2.1$ million,

$$\text{Profit } = \pi(2.1) = R(2.1) - C(2.1) = 6.9 - 5.1 = 1.8 \text{ million dollars.}$$

(b) If $\Delta q = 0.04$,

$$\text{Change in revenue, } \Delta R \approx R'(2.1)\Delta q = 0.7(0.04) = 0.028 \text{ million dollars } = \$28{,}000.$$

Thus, revenues increase by $28,000.

(c) If $\Delta q = -0.05$,

$$\text{Change in revenue, } \Delta R \approx R'(2.1)\Delta q = 0.7(-0.05) = -0.035 \text{ million dollars } = -\$35{,}000.$$

Thus, revenues decrease by $35,000.

(d) We find the change in cost by a similar calculation. For $\Delta q = 0.04$,

$$\text{Change in cost, } \Delta C \approx C'(2.1)\Delta q = 0.6(0.04) = 0.024 \text{ million dollars } = \$24{,}000$$
$$\text{Change in profit, } \Delta\pi = \$28{,}000 - \$24{,}000 = \$4000.$$

Thus, increasing production 0.04 million units increases profits by $4000.

For $\Delta q = -0.05$,

$$\text{Change in cost, } \Delta C \approx C'(2.1)\Delta q = 0.6(-0.05) = -0.03 \text{ million dollars } = -\$30{,}000$$
$$\text{Change in profit, } \Delta\pi = -\$35{,}000 - (-\$30{,}000) = -\$5000.$$

Thus, decreasing production 0.05 million units decreases profits by $5000.

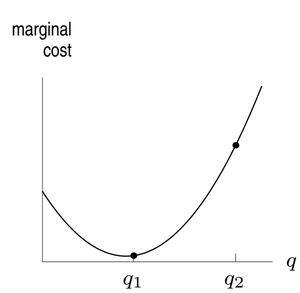
14. We have

$$C'(2000) \approx \frac{C(2500) - C(2000)}{2500 - 2000} = \frac{3825 - 3640}{500} = \$0.37\text{/ton.}$$

This means that recycling the 2001st ton of paper will cost around $0.37. The marginal cost is smallest at the point where the derivative of the function is smallest. Thus the marginal cost appears to be smallest on the interval $2500 \le q \le 3000$.

15. **(a)** The value of $C(0)$ represents the fixed costs before production, that is, the cost of producing zero units, incurred for initial investments in equipment, and so on.

(b) The marginal cost decreases slowly, and then increases as quantity produced increases.



(c) Concave down implies decreasing marginal cost, while concave up implies increasing marginal cost.
(d) An inflection point of the cost function is (locally) the point of maximum or minimum marginal cost.
(e) One would think that the more of an item you produce, the less it would cost to produce extra items. In economic terms, one would expect the marginal cost of production to decrease, so we would expect the cost curve to be concave down. In practice, though, it eventually becomes more expensive to produce more items, because workers and resources may become scarce as you increase production. Hence after a certain point, the marginal cost may rise again. This happens in oil production, for example.

Solutions for Chapter 2 Review

1. The slope is positive at A and D; negative at C and F. The slope is most positive at A; most negative at F.

2. The coordinates of A are $(4, 25)$. See Figure 2.42. The coordinates of B and C are obtained using the slope of the tangent line. Since $f'(4) = 1.5$, the slope is 1.5

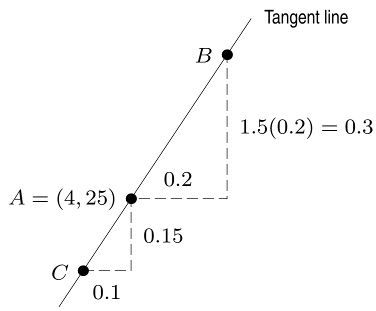


Figure 2.42

From A to B, $\Delta x = 0.2$, so $\Delta y = 1.5(0.2) = 0.3$. Thus, at C we have $y = 25 + 0.3 = 25.3$. The coordinates of B are $(4.2, 25.3)$.

From A to C, $\Delta x = -0.1$, so $\Delta y = 1.5(-0.1) = -0.15$. Thus, at C we have $y = 25 - 0.15 = 24.85$. The coordinates of C are $(3.9, 24.85)$.

3. We estimate $f'(2)$ using the average rate of change formula on a small interval around 2. We use the interval $x = 2$ to $x = 2.001$. (Any small interval around 2 gives a reasonable answer.) We have

$$f'(2) \approx \frac{f(2.001) - f(2)}{2.001 - 2} = \frac{3^{2.001} - 3^2}{2.001 - 2} = \frac{9.00989 - 9}{0.001} = 9.89.$$

4. **(a)** Let $s = f(t)$.

(i) We wish to find the average velocity between $t = 1$ and $t = 1.1$. We have

$$\text{Average velocity} = \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{3.63 - 3}{0.1} = 6.3 \text{ m/sec.}$$

(ii) We have

$$\text{Average velocity} = \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{3.0603 - 3}{0.01} = 6.03 \text{ m/sec.}$$

(iii) We have

$$\text{Average velocity} = \frac{f(1.001) - f(1)}{1.001 - 1} = \frac{3.006003 - 3}{0.001} = 6.003 \text{ m/sec.}$$

(b) We see in part (a) that as we choose a smaller and smaller interval around $t = 1$ the average velocity appears to be getting closer and closer to 6, so we estimate the instantaneous velocity at $t = 1$ to be 6 m/sec.

5. Between 1804 and 1927, the world's population increased 1 billion people in 123 years, for an average rate of change of $1/123$ billion people per year. We convert this to people per minute:

$$\frac{1,000,000,000}{123} \text{ people/year} \cdot \frac{1}{60 \cdot 24 \cdot 365} \text{ years/minute} = 15.47 \text{ people/minute}.$$

Between 1804 and 1927, the population of the world increased at an average rate of 15.47 people per minute. Similarly, we find the following:
Between 1927 and 1960, the increase was 57.65 people per minute.
Between 1960 and 1974, the increase was 135.90 people per minute.
Between 1974 and 1987, the increase was 146.35 people per minute.
Between 1987 and 1999, the increase was 158.55 people per minute.

6. **(a)** Let $s = f(t)$.

(i) We wish to find the average velocity between $t = 1$ and $t = 1.1$. We have

$$\text{Average velocity} = \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{0.808496 - 0.909297}{0.1} = -1.00801 \text{ m/sec}.$$

(ii) We have

$$\text{Average velocity} = \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{0.900793 - 0.909297}{0.01} = -0.8504 \text{ m/sec}.$$

(iii) We have

$$\text{Average velocity} = \frac{f(1.001) - f(1)}{1.001 - 1} = \frac{0.908463 - 0.909297}{0.001} = -0.834 \text{ m/sec}.$$

(b) We see in part (a) that as we choose a smaller and smaller interval around $t = 1$ the average velocity appears to be getting closer and closer to -0.83, so we estimate the instantaneous velocity at $t = 1$ to be -0.83 m/sec. In this case, more estimates with smaller values of h would be very helpful in making a better estimate.

7. We know that $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$. For this problem, we'll take the average of the values obtained for $h = 1$ and $h = -1$; that's the average of $f(x+1) - f(x)$ and $f(x) - f(x-1)$ which equals $\dfrac{f(x+1) - f(x-1)}{2}$. Thus,
$f'(0) \approx f(1) - f(0) = 13 - 18 = -5$.
$f'(1) \approx [f(2) - f(0)]/2 = [10 - 18]/2 = -4$.
$f'(2) \approx [f(3) - f(1)]/2 = [9 - 13]/2 = -2$.
$f'(3) \approx [f(4) - f(2)]/2 = [9 - 10]/2 = -0.5$.
$f'(4) \approx [f(5) - f(3)]/2 = [11 - 9]/2 = 1$.
$f'(5) \approx [f(6) - f(4)]/2 = [15 - 9]/2 = 3$.
$f'(6) \approx [f(7) - f(5)]/2 = [21 - 11]/2 = 5$.
$f'(7) \approx [f(8) - f(6)]/2 = [30 - 15]/2 = 7.5$.
$f'(8) \approx f(8) - f(7) = 30 - 21 = 9$.
The rate of change of $f(x)$ is positive for $4 \le x \le 8$, negative for $0 \le x \le 3$. The rate of change is greatest at about $x = 8$.

8. See Figure 2.43.

Figure 2.43

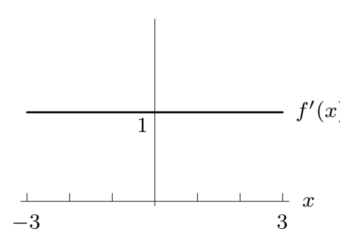


Figure 2.44

9. This is a line with slope 1, so the derivative is the constant function $f'(x) = 1$. The graph is the horizontal line $y = 1$. See Figure 2.44.

10. See Figure 2.45.

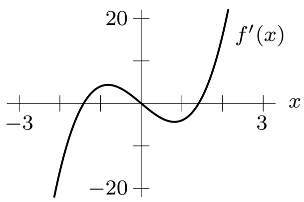


Figure 2.45

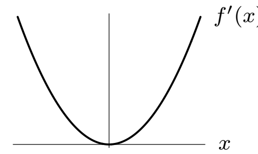


Figure 2.46

11. See Figure 2.46.

12. See Figure 2.47.

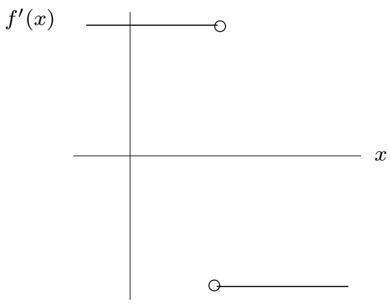


Figure 2.47

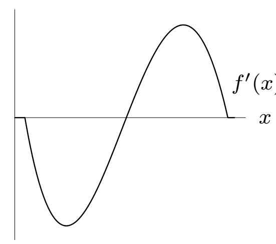


Figure 2.48

13. See Figure 2.48.

14. **(a)** Graph II
(b) Graph I
(c) Graph III

15. **(a)** As the cup of coffee cools, the temperature decreases, so $f'(t)$ is negative.
(b) Since $f'(t) = dH/dt$, the units are degrees Celsius per minute. The quantity $f'(20)$ represents the rate at which the coffee is cooling, in degrees per minute, 20 minutes after the cup is put on the counter.

16. (Note that we are considering the average temperature of the yam, since its temperature is different at different points inside it.)

(a) It is positive, because the temperature of the yam increases the longer it sits in the oven.
(b) The units of $f'(20)$ are °F/min. $f'(20) = 2$ means that at time $t = 20$ minutes, the temperature T increases by approximately 2°F for each additional minute in the oven.

17. The statements $f(100) = 35$ and $f'(100) = 3$ tell us that at $x = 100$, the value of the function is 35 and the function is increasing at a rate of 3 units for a unit increase in x. Since we increase x by 2 units in going from 100 to 102, the value of the function goes up by approximately $2 \cdot 3 = 6$ units, so

$$f(102) \approx 35 + 2 \cdot 3 = 35 + 6 = 41.$$

18. Since we do not have information beyond $t = 25$, we will assume that the function will continue to change at the same rate. Therefore,

$$\begin{aligned} f(26) &\approx f(25) + f'(25) \\ &= 3.6 + (-0.2) = 3.4. \end{aligned}$$

Since $30 = 25 + 5$, then

$$\begin{aligned} f(30) &\approx f(25) + f'(25)(5) \\ &= 3.6 + (-0.2)(5) = 2.6. \end{aligned}$$

19. **(a)** Since t represents the number of days from now, we are told $f(0) = 80$ and $f'(0) = 0.50$.

(b)

$$\begin{aligned} f(10) &\approx \text{value now} + \text{change in value in 10 days} \\ &= 80 + 0.50(10) \\ &= 80 + 5 \\ &= 85. \end{aligned}$$

In 10 days, we expect that the mutual fund will be worth about \$85 a share.

20. **(a)** Since $W = f(c)$ where W is weight in pounds and c is the number of Calories consumed per day:

$f(1800) = 155$	means that	consuming 1800 Calories per day results in a weight of 155 pounds.
$f'(2000) = 0$	means that	consuming 2000 Calories per day causes neither weight gain nor loss.

(b) The units of dW/dc are pounds/(Calories/day).

21. **(a)** This means that investing the \$1000 at 5% would yield \$1649 after 10 years.

(b) Writing $g'(r)$ as dB/dr, we see that the units of dB/dr are dollars per percent (interest). We can interpret dB as the extra money earned if interest rate is increased by dr percent. Therefore $g'(5) = \frac{dB}{dr}|_{r=5} \approx 165$ means that the balance, at 5% interest, would increase by about \$165 if the interest rate were increased by 1%. In other words, $g(6) \approx g(5) + 165 = 1649 + 165 = 1814$.

22. Units of $P'(t)$ are dollars/year. The practical meaning of $P'(t)$ is the rate at which the monthly payments change as the duration of the mortgage increases. Approximately, $P'(t)$ represents the change in the monthly payment if the duration is increased by one year. $P'(t)$ is negative because increasing the duration of a mortgage decreases the monthly payments.

23. The units of $f'(x)$ are feet/mile. The derivative, $f'(x)$, represents the rate of change of elevation with distance from the source, so if the river is flowing downhill everywhere, the elevation is always decreasing and $f'(x)$ is always negative. (In fact, there may be some stretches where the elevation is more or less constant, so $f'(x) = 0$.)

24. **(a)** If the price is \$150, then 2000 items will be sold.

(b) If the price goes up from \$150 by \$1 per item, about 25 fewer items will be sold. Equivalently, if the price is decreased from \$150 by \$1 per item, about 25 more items will be sold.

25. **(a)** The company hopes that increased advertising always brings in more customers instead of turning them away. Therefore, it hopes $f'(a)$ is always positive.

(b) If $f'(100) = 2$, it means that if the advertising budget is \$100,000, each extra dollar spent on advertising will bring in \$2 worth of sales. If $f'(100) = 0.5$, each dollar above \$100 thousand spent on advertising will bring in \$0.50 worth of sales.

(c) If $f'(100) = 2$, then as we saw in part (b), spending slightly more than \$100,000 will increase revenue by an amount greater than the additional expense, and thus more should be spent on advertising. If $f'(100) = 0.5$, then the increase in revenue is less than the additional expense, hence too much is being spent on advertising. The optimum amount to spend, a, is an amount that makes $f'(a) = 1$. At this point, the increases in advertising expenditures just pay for themselves. If $f'(a) < 1$, too much is being spent; if $f'(a) > 1$, more should be spent.

26. At B both dy/dx and d^2y/dx^2 could be positive because y is increasing and the graph is concave up there. At all the other points one or both of the derivatives could not be positive.

27. The function is everywhere increasing and concave up. One possible graph is shown in Figure 2.49.

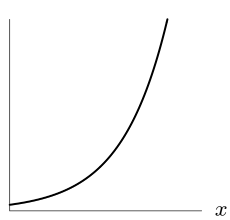


Figure 2.49

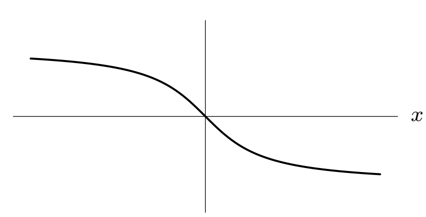


Figure 2.50

28. The graph must be everywhere decreasing and concave up on some intervals and concave down on other intervals. One possibility is shown in Figure 2.50.

29. Since all advertising campaigns are assumed to produce an increase in sales, a graph of sales against time would be expected to have a positive slope.

A positive second derivative means the rate at which sales are increasing is increasing. If a positive second derivative is observed during a new campaign, it is reasonable to conclude that this increase in the rate sales are increasing is caused by the new campaign–which is therefore judged a success. A negative second derivative means a decrease in the rate at which sales are increasing, and therefore suggests the new campaign is a failure.

30. **(a)** For the three years ending in 1995 we have

$$\frac{\Delta P}{\Delta t} = \frac{54.1 - 62.4}{1995 - 1992} = \frac{-8.3}{3} \approx -2.77 \text{ \%/year.}$$

For the three years ending in 1998 we have

$$\frac{\Delta P}{\Delta t} = \frac{48.0 - 54.1}{1998 - 1995} = \frac{-6.1}{3} \approx -2.03 \text{ \%/year.}$$

For the three years ending in 2001 we have

$$\frac{\Delta P}{\Delta t} = \frac{43.5 - 48.0}{2001 - 1998} = \frac{-4.5}{3} \approx -1.50 \text{ \%/year.}$$

For the three years ending in 2004 we have

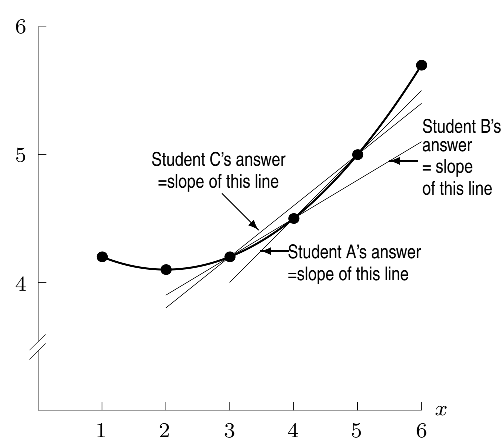
$$\frac{\Delta P}{\Delta t} = \frac{41.8 - 43.5}{2004 - 2001} = \frac{-1.7}{3} \approx -0.57 \text{ \%/year.}$$

(b) The fact that $\dfrac{\Delta P}{\Delta t}$ is increasing from 1992 to 2004 suggests that $\dfrac{d^2P}{dt^2}$ is positive.

(c) The values of P and $\dfrac{\Delta P}{\Delta t}$ are troublesome because they indicate that the percent of students graduating is low, and that the number is getting smaller each year.

(d) Since $\dfrac{d^2P}{dt^2}$ is positive, the percent of students graduating is not decreasing as fast as it once was. Also, in 2004 the magnitude of $\Delta P/\Delta t$ is less than 1% a year, so the level of drop-outs does in fact seem to be hitting its minimum at around 40%.

31. **(a)**



(b) The slope of f appears to be somewhere between student A's answer and student B's, so student C's answer, halfway in between, is probably the most accurate.

32. A possible graph of $y = f(x)$ is shown in Figure 2.51.

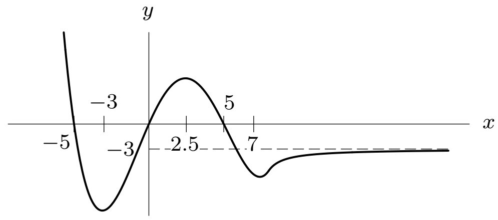


Figure 2.51

33. **(a)** The rate of energy consumption required when $v = 0$ is the vertical intercept, about 1.8 joules/sec.

(b) The graph shows $f(v)$ first decreases and then increases as v increases. This tells us that the bird expends more energy per second to remain still than to travel at slow speeds (say 0.5 to 1 meter/sec), but that the rate of energy consumption required increases again at speeds beyond 1 meter/sec. The upward concavity of the graph tells us that as the bird speeds up, it uses energy at a faster and faster rate.

(c) Figure 2.52 shows a possible graph of the derivative $f'(v)$. Other answers are possible.

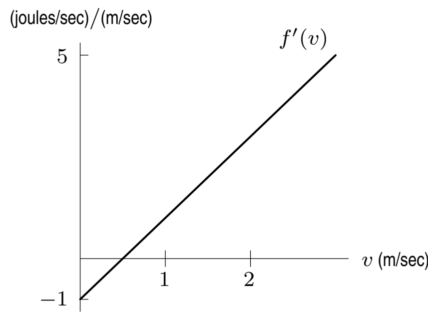


Figure 2.52

PROJECTS FOR CHAPTER TWO

1. **(a)** A possible graph is shown in Figure 2.53. At first, the yam heats up very quickly, since the difference in temperature between it and its surroundings is so large. As time goes by, the yam gets hotter and hotter, its rate of temperature increase slows down, and its temperature approaches the temperature of the oven as an asymptote. The graph is thus concave down. (We are considering the average temperature of the yam, since the temperature in its center and on its surface will vary in different ways.)

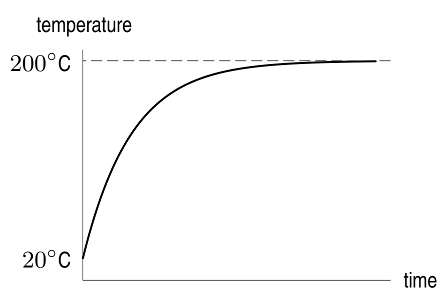


Figure 2.53

(b) If the rate of temperature increase were to remain 2°/min, in ten minutes the yam's temperature would increase 20°, from 120° to 140°. Since we know the graph is not linear, but concave down, the actual temperature is between 120° and 140°.

(c) In 30 minutes, we know the yam increases in temperature by $45°$ at an average rate of $45/30 = 1.5°$/min. Since the graph is concave down, the temperature at $t = 40$ is therefore between $120 + 1.5(10) = 135°$ and $140°$.

(d) If the temperature increases at $2°$/minute, it reaches $150°$ after 15 minutes, at $t = 45$. If the temperature increases at $1.5°$/minute, it reaches $150°$ after 20 minutes, at $t = 50$. So t is between 45 and 50 mins.

2. (a) Since illumination is concave up and temperature is concave down, the graph on the left side corresponds to the graph of illumination as a function of distance and the graph on the right side corresponds to the graph of temperature as a function of distance.

(b) The illumination drops from 75% at $d = 2$ to 56% at $d = 5$. Since $T = 47$ when $d = 5$ and $T = 53.5$ when $d = 2$,

$$\text{Average rate of change of temperature} = \frac{47 - 53.5}{5 - 2} = \frac{-6.5}{3} = -2.17° \text{ per foot.}$$

(c) A good estimate of the illumination when the distance is 3.5 feet is the average of the values of the illumination at 3 feet and at 4 feet. Therefore:

$$\text{Illumination at 3.5 feet} = \frac{67 + 60}{2} = 63.5\% < 65\%.$$

Since illumination is concave up, the 63.5% is likely to be an overestimate, so you are not likely to be able to read the watch.

(d) Let's represent the illumination as a function of the distance by $I(d)$ and the temperature as a function of the distance by $T(d)$. Therefore:

$$\begin{aligned} I(7) &= I(6) + \text{change in } I(d) \\ &\approx I(6) + \text{Average rate of change of } I(d) \\ &= 53\% + (-3\%) = 50\%. \end{aligned}$$

And

$$\begin{aligned} T(7) &= T(6) + \text{ change in } T(d) \\ &\approx T(6) + \text{Average rate of change of } T(d) \\ &= 43.5° + (-4.5°) = 39°F. \end{aligned}$$

(e) Let's calculate the distance when $T(d) = 40$ (when we are cold):

$$\begin{aligned} T(d) &= T(6) + T'(6) \cdot (d - 6) \\ 40 &= 43.5 + (-4.5)(d - 6) \\ 40 &= 43.5 - 4.5d + 27 \\ 40 &= 70.5 - 4.5d \\ -30.5 &= -4.5d \\ d &= \frac{-30.5}{-4.5} = 6.78 \text{ feet.} \end{aligned}$$

From part (d) we know that the illumination is 50% (darkness) when the distance is 7 feet. Therefore, as we walk away from the candle, we first get cold and then we are in darkness.

Solutions to Problems on Limits and the Definition of the Derivative

1. The answers to parts (a)–(f) are marked in Figure 2.54.

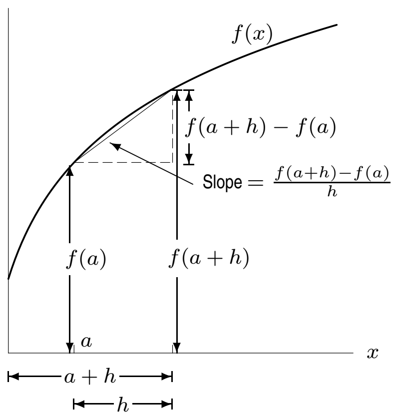


Figure 2.54

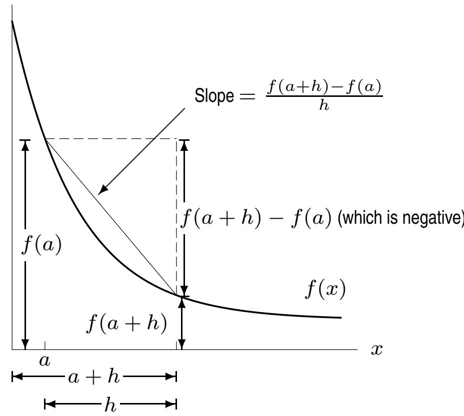


Figure 2.55

2. The answers to parts (a)–(f) are marked in Figure 2.55.

3. Figure 2.56 shows that as x approaches 0 from either side, the value of $\dfrac{\sin x}{x}$ appears to approach 1, suggesting that $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$. Zooming in on the graph near $x = 0$ provides further support for this conclusion. Notice that $\dfrac{\sin x}{x}$ is undefined at $x = 0$.

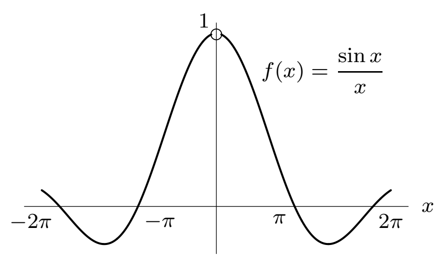


Figure 2.56

4. Figure 2.57 shows that as x approaches 0 from either side, the values of $\dfrac{5^x-1}{x}$ appear to approach 1.6, suggesting that

$$\lim_{x\to 0}\frac{5^x-1}{x} \approx 1.6.$$

Zooming in on the graph near $x = 0$ provides further support for this conclusion. Notice that $\dfrac{5^x-1}{x}$ is undefined at $x = 0$.

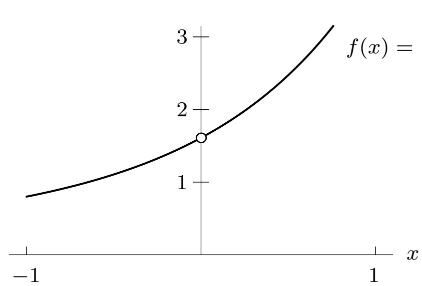


Figure 2.57

5. Using $h = 0.1, 0.01, 0.001$, we see

$$\frac{(3+0.1)^3 - 27}{0.1} = 27.91$$
$$\frac{(3+0.01)^3 - 27}{0.01} = 27.09$$
$$\frac{(3+0.001)^3 - 27}{0.001} = 27.009.$$

These calculations suggest that $\lim_{h \to 0} \frac{(3+h)^3 - 27}{h} = 27$.

6. Using $h = 0.1, 0.01, 0.001$, we see

$$\frac{7^{0.1} - 1}{0.1} = 2.148$$
$$\frac{7^{0.01} - 1}{0.01} = 1.965$$
$$\frac{7^{0.001} - 1}{0.001} = 1.948$$
$$\frac{7^{0.0001} - 1}{0.0001} = 1.946.$$

This suggests that $\lim_{h \to 0} \frac{7^h - 1}{h} \approx 1.9$.

7. Using $h = 0.1, 0.01, 0.001$, we see

h	$(e^{1+h} - e)/h$
0.01	2.7319
0.001	2.7196
0.0001	2.7184

These values suggest that $\lim_{h \to 0} \frac{e^{1+h} - e}{h} = 2.7$. In fact, this limit is e.

8. Using radians,

h	$(\cos h - 1)/h$
0.01	-0.005
0.001	-0.0005
0.0001	-0.00005

These values suggest that $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$.

9. Yes, $f(x)$ is continuous on $0 \le x \le 2$.

10. No, $f(x)$ is not continuous on $0 \le x \le 2$, but it is continuous on the interval $0 \le x \le 0.5$.

11. No, $f(x)$ is not continuous on $0 \le x \le 2$, but it is continuous on $0 \le x \le 0.5$.

12. No, $f(x)$ is not continuous on $0 \le x \le 2$, but it is continuous on $0 \le x \le 0.5$.

13. Yes: $f(x) = x + 2$ is continuous for all values of x.

14. Yes: $f(x) = 2^x$ is continuous function for all values of x.

15. Yes: $f(x) = x^2 + 2$ is a continuous function for all values of x.

16. Yes: $f(x) = 1/(x - 1)$ is a continuous function on any interval that does not contain $x = 1$.

17. No: $f(x) = 1/(x - 1)$ is not continuous on any interval containing $x = 1$.

18. Yes: $f(x) = 1/(x^2 + 1)$ is continuous for all values of x because the denominator is never 0.

19. This function is not continuous. Each time someone is born or dies, the number jumps by one.

20. Continuous

21. Since we can't make a fraction of a pair of pants, the number increases in jumps, so the function is not continuous.

22. Even though the car is stopping and starting, the distance traveled is a continuous function of time.

23. The time is not a continuous function of position as distance from your starting point, because every time you cross from one time zone into the next, the time jumps by 1 hour.

24. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{5(x+h)-5x}{h} \\ &= \lim_{h\to 0} \frac{5x+5h-5x}{h} \\ &= \lim_{h\to 0} \frac{5h}{h}. \end{aligned}$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $f'(x) = 5$.

25. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(3(x+h)-2)-(3x-2)}{h} \\ &= \lim_{h\to 0} \frac{3x+3h-2-3x+2}{h} \\ &= \lim_{h\to 0} \frac{3h}{h}. \end{aligned}$$

As h gets very close to zero without actually equaling zero, we can cancel the h in the numerator and denominator to obtain

$$f'(x) = \lim_{h\to 0}(3) = 3.$$

26. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{((x+h)^2+4)-(x^2+4)}{h} \\ &= \lim_{h\to 0} \frac{x^2+2xh+h^2+4-x^2-4}{h} \\ &= \lim_{h\to 0} \frac{2xh+h^2}{h} \\ &= \lim_{h\to 0} \frac{h(2x+h)}{h}. \end{aligned}$$

As h gets very close to zero without actually equaling zero, we can cancel the h in the numerator and denominator to obtain

$$f'(x) = \lim_{h\to 0}(2x+h) = 2x.$$

27. Using the definition of the derivative, we have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2-3x^2}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2)-3x^2}{h}$$
$$= \lim_{h\to 0} \frac{3x^2+6xh+3h^2-3x^2}{h}$$
$$= \lim_{h\to 0} \frac{6xh+3h^2}{h} = \lim_{h\to 0} \frac{h(6x+3h)}{h}.$$

As h gets very close to zero (but not equal to zero), we can cancel the h in the numerator and denominator to leave the following:

$$f'(x) = \lim_{h\to 0}(6x+3h).$$

As $h \to 0$, we have $f'(x) = 6x$.

28. Using the definition of the derivative, we have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{-2(x+h)^3-(-2x^3)}{h}$$
$$= \lim_{h\to 0} \frac{-2(x^3+3x^2h+3xh^2+h^3)+2x^3}{h}$$
$$= \lim_{h\to 0} \frac{-2x^3-6x^2h-6xh^2-2h^3+2x^3}{h}$$
$$= \lim_{h\to 0} \frac{-6x^2h-6xh^2-2h^3}{h}.$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $-6x^2 - 6xh - 2h^2$. Taking the limit as h goes to zero, we get $f'(x) = -6x^2$ since the other two terms go to zero.

29. Using the definition of the derivative, we have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{((x+h)-(x+h)^2)-(x-x^2)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h-(x^2+2xh+h^2))-x+x^2}{h}$$
$$= \lim_{h\to 0} \frac{x+h-x^2-2xh-h^2-x+x^2}{h}$$
$$= \lim_{h\to 0} \frac{h-2xh-h^2}{h}.$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $1 - 2x - h$. Taking the limit as h goes to zero, we get $f'(x) = 1 - 2x$ since the other term goes to zero.

30. Using the definition of the derivative, we have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(1-(x+h)^3)-(1-x^3)}{h}$$
$$= \lim_{h\to 0} \frac{(1-(x^3+3x^2h+3xh^2+h^3))-1+x^3}{h}$$
$$= \lim_{h\to 0} \frac{1-x^3-3x^2h-3xh^2-h^3-1+x^3}{h}$$
$$= \lim_{h\to 0} \frac{-3x^2h-3xh^2-h^3}{h}.$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $-3x^2 - 3xh - h^2$. Taking the limit as h goes to zero, we get $f'(x) = -3x^2$ since the other two terms go to zero.

31. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(5(x+h)^2+1)-(5x^2+1)}{h} \\ &= \lim_{h\to 0} \frac{5(x^2+2xh+h^2)+1-5x^2-1}{h} \\ &= \lim_{h\to 0} \frac{5x^2+10xh+5h^2+1-5x^2-1}{h} \\ &= \lim_{h\to 0} \frac{10xh+5h^2}{h} \\ &= \lim_{h\to 0} \frac{h(10x+5h)}{h}. \end{aligned}$$

As h gets very close to zero without actually equaling zero, we can cancel the h in the numerator and denominator to obtain

$$f'(x) = \lim_{h\to 0}(10x+5h) = 10x.$$

32. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(2(x+h)^2+(x+h))-(2x^2+x)}{h} \\ &= \lim_{h\to 0} \frac{2(x^2+2xh+h^2)+(x+h)-2x^2-x}{h} \\ &= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+x+h-2x^2-x}{h} \\ &= \lim_{h\to 0} \frac{4xh+2h^2+h}{h} \\ &= \lim_{h\to 0} \frac{h(4x+2h+1)}{h}. \end{aligned}$$

As h gets very close to zero without actually equaling zero, we can cancel the h in the numerator and denominator to obtain

$$f'(x) = \lim_{h\to 0}(4x+2h+1) = 4x+1.$$

33. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{1/(x+h)-1/x}{h}. \end{aligned}$$

Writing the numerator over a common denominator and simplifying, we get

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{(x-(x+h))/(x(x+h))}{h} \\ &= \lim_{h\to 0} \frac{-h/(x(x+h))}{h} \\ &= \lim_{h\to 0} \frac{-h}{hx(x+h)}. \end{aligned}$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $-1/(x(x+h))$. Taking the limit as h goes to zero, we get $f'(x) = -1/x^2$ since h goes to zero.