

**SOLUTIONS MANUAL**

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**Analyzing  
Politics**

SECOND EDITION

**Rationality,  
Behavior, and  
Institutions**



# Analyzing Politics: Answer Key

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## Chapter 2: Problems and Discussion Questions

4. Either player can identify a top choice (or choices in the case of indifference) for any subset of size 2. This is always the case as long as a person's preferences satisfy *comparability*. There are 5 subsets with 3 or more outcomes: wxy, wxz, wyz, xyz and wxyz. Mr. i's and Ms. j's most-preferred outcomes over these subsets are shown in Table 1. Where either actor cannot state a most-preferred choice or choices, the table contains a '-'.  
a '-'

Subset	Mr. i		Ms. j	
	Top choice	Cycle?	Top choice	Cycle?
wxy	x	No	x,y	No
wxz	-	Yes	x	No
wyz	w	No	y	No
xyz	-	Yes	x,y	No
wxyz	-	Yes	x,y	No

Table 1: Mr. i's and Ms. j's preferences for within subsets of the outcomes.

Preference orderings over a subset of outcomes which contain a strict preference cycle through *all* of the elements in the subset have no articulable top choice. Preferences orderings over a subset of outcomes which contain a strict preference cycle through *less than all* of the outcomes in the subset may or may not have an articulable top choice. By way of comparison, note that i has no top choice among the subset wxyz, but an individual with the following preferences would strictly prefer w despite the presence of a cycle through x, y and z:  $wPx$ ,  $wPy$ ,  $wPz$ ,  $xPy$ ,  $yPz$ , and  $zPx$ .

Thus, transitive preferences are a sufficient condition for identifying top choices with respect to any possible set of outcomes. Technically, *acyclic* preferences over all triples of alternatives in a subset are necessary and sufficient for each subset to possess a maximal element. Acyclicity is defined, for any x, y, z, as: if  $xPy$ ,  $yPz$  then not  $zPx$ . Transitivity implies acyclicity and thus is sufficient for the existence of a maximal element. Inasmuch as maximizing behavior often relies on specifying clear ordinal rankings among outcomes, transitive preferences are effectively a prerequisite for a rational choice approach to individual decision-making.

5. A reasonable assumption is that Senator Clinton's preferences at that point were as follows:  $P > S > C$ . Because she chose the office of Secretary of State, it would not be reasonable to assume that  $P > C > S$ . The assumption that  $P > S$  is stronger, but corresponds with reports in the media at the time.

The fact that Senator Clinton chose the position within the administration implies the following:  $S > (1 - p)C + p(P)$ . The chance to serve as Secretary of State was preferred to a risky ‘lottery’ over remaining in the Senate and eventually becoming President. We can rearrange the parts of the inequality to derive the following statement:  $\frac{S-C}{P-C} > p$ . Thus, the lowest  $p$  which would induce Hillary Clinton to stay in the Senate is  $\frac{S-C}{P-C} = p$ , and any  $p$  greater than  $\frac{S-C}{P-C}$  would induce a strict preference for remaining in the Senate. This relation also suggests three interesting comparative statics when all other variables are held constant: 1. there is a threshold as  $S$  increases at which one cannot resist the offer of the Secretaryship; 2. there is a threshold as  $P$  increases at which one will reject the Secretaryship and hold out for a chance at the Presidency; and, 3. as  $C$  increases the threshold  $p$  at which one will accept the Secretaryship decreases. This latter effect occurs because as serving in Congress becomes more desirable, the lottery over  $C$  and  $P$  becomes more appealing relative to the  $S$ .

6. Recall that the theory of expected utility states that given two different lotteries,  $L$  and  $L'$ , over the same outcomes, then  $L \succ L'$  if and only if  $\sum_{x \in X} p(x)u(x) > \sum_{x \in X} p'(x)u(x)$ . Using this definition we can get the following two relations about  $L_1$  and  $L_2$ , and  $L_3$  and  $L_4$ :

$$L_1 \text{PL}_2 \text{ implies } u(y) > .10u(x) + .89u(y) + .01u(z)$$

and

$$L_4 \text{PL}_3 \text{ implies } .10u(x) + .90u(z) > .11u(y) + .89u(z)$$

The trick here is to manipulate these expressions to show that they imply a contradiction. Consider the following two steps: add  $.89u(z)$  to both sides of the first expression, and then subtract  $.89u(y)$  from both sides of the first expression. This yields:

$$.11u(y) + .89u(z) > .10u(x) + .90u(z) \text{ which implies } L_3 \text{PL}_4,$$

which contradicts our second expression.

Two steps in this process require further justification. First, is it okay to add and subtract constants to an expected utility expression? This is fine: expected utilities act like numbers and adding the same number to each side of an expression will not change the overall preference relation. If I prefer 2 apples to 1 orange, then I should also prefer 2 apples and 10 units of utility to 1 orange and 10 units of utility. Second, can we take these ‘manipulated’ expressions and treat them as identical to a genuine lottery? If we are smart about our manipulations (i.e. add and subtract things so that we still have a proper probability distribution where all of the probabilities are between 0 and 1, and also sum to 1) then we can treat the new objects as ordinary lotteries. This is what allows us to compare our manipulated version of expression 1 with expression 2.

## Chapter 3: Problems and Discussion Questions

- Using plurality rule and voting over all four alternatives, plan A wins with two votes. In a round-robin tournament, B is a *Condorcet winner*: it beats all of the other plans in a head-to-head competition. If plan B is removed from the competition, there is no Condorcet winner. Plan A defeats plan C in a head-to-head vote; plan C defeats plan D in a head-to-head vote; and, plan D defeats plan A in a head-to-head vote. Thus, there is a group preference cycle and the group has intransitive social preferences among the three outcomes, precluding identification of a top choice.
- The following table provides the outcomes of every head-to-head vote over  $q, r, s, t$  for the original and the revised preferences:

Matchup:	qr	qs	qt	sr	st	rt
Original	r	q	q	r	t	r
Revised	r	q	q	s	t	r

Under the original preferences, there are no group preference cycles. Under the revised preferences, there are preference cycles among  $(q, s, r)$ ,  $(r, t, s)$  and  $(q, r, s, t)$ . Under both sets of preferences, each individual's preferences are both complete and transitive, satisfying our very basic rationality requirement. However, for the revised preferences, these individually rational preferences give rise to collectively irrational social preferences which feature several preference cycles. This illustrates the fact that rational preferences at the individual level are no guarantee of rational preferences at the collective level.

- i cannot identify a sequential agenda to secure outcome  $q$  because outcome  $r$  is a Condorcet winner (it beats any other outcome so it survives every round of a sequential agenda and thus always beats  $q$  at some point in the voting). After k's preferences change,  $r$  is no longer a Condorcet winner and the following agendas will lead to  $q$  winning:  $rs|t|q$ ,  $rs|q|t$  and  $rt|s|q$ . This notation, for example with  $rs|t|q$ , means that in round 1,  $s$  and  $r$  face off; the winner of that round faces  $t$  in round 2; and, the winner of round 2 faces  $q$  in round 3.

Before k changes his mind, j will secure his most-preferred outcome with any agenda, because his top choice,  $r$ , is a Condorcet winner. However, similar to player i, k cannot secure his top choice,  $t$ , because  $r$  beats all challengers. After k changes his mind, j can secure his most-preferred outcome with several agendas:  $sq|r|t$ ,  $sq|t|r$ ,  $st|q|r$ ,  $st|r|q$  and  $qt|s|r$ . Likewise, k can secure his most-preferred outcome with  $rq|s|t$ . In answering this question, you may find it helpful to enumerate all 12 possible agendas and to determine the victor for each one.

It is not possible to fashion a sequential agenda which defeats a Condorcet winner, because by definition such an outcome will survive every head-to-head competition it faces in any agenda. Absent a Condorcet winner, all kinds of manipulation are possible although it is of course not possible to create an agenda which leads to the victory of an outcome which *loses* in a head-to-head competition with all other outcomes.

- If player i votes honestly under agenda  $st|r|q$  then  $r$  is the victor. However, if he misrepresents his preferences by voting strategically for option  $t$  in Round 2, then he secures his most-preferred outcome,  $q$

in the final round (assuming the others vote sincerely). Under agenda  $rq|s|t$ , if  $j$  (as well as  $i$  and  $k$ ) vote sincerely, then  $t$  will be the final outcome. However, if  $j$  strategically votes for  $q$  in round 1, then  $q$  will win (assuming the others vote sincerely) providing a superior outcome for  $j$ .