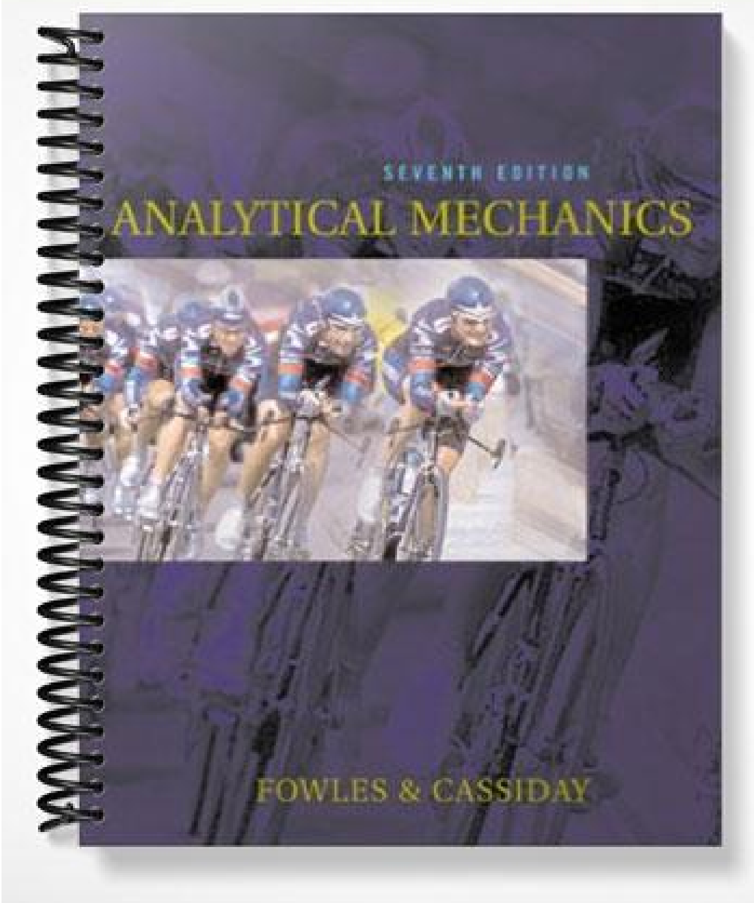
SOLUTIONS MANUAL



CHAPTER 2
NEWTONIAN MECHANICS:
RECTILINEAR MOTION OF A PARTICLE

2.1 (a) $\ddot{x} = \frac{1}{m}\left(F_\circ + ct\right)$

$$\dot{x} = \int_0^t \frac{1}{m}\left(F_\circ + ct\right)dt = \frac{F_\circ}{m}t + \frac{c}{2m}t^2$$

$$x = \int_0^t \left(\frac{F_\circ}{m}t + \frac{c}{2m}t^2\right)dt = \frac{F_\circ}{m}t^2 + \frac{c}{6m}t^3$$

(b) $\ddot{x} = \frac{F_\circ}{m}\sin ct$

$$\dot{x} = \int_0^t \frac{F_\circ}{m}\sin ct\, dt = -\frac{F_\circ}{cm}\cos ct\Big|_0^t = \frac{F_\circ}{cm}\left(1-\cos ct\right)$$

$$x = \int_0^t \frac{F_\circ}{cm}\left(1-\cos ct\right)dt = \frac{F_\circ}{cm}\left(1-\frac{1}{c}\sin ct\right)$$

(c) $\ddot{x} = \frac{F_\circ}{m}e^{ct}$

$$\dot{x} = \frac{F_\circ}{cm}e^{ct}\Big|_0^t = \frac{F_\circ}{cm}\left(e^{ct}-1\right)$$

$$x = \frac{F_\circ}{cm}\left(\frac{1}{c}e^{ct} - \frac{1}{c} - t\right) = \frac{F_\circ}{c^2 m}\left(e^{ct} - 1 - ct\right)$$

2.2 (a) $\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx}\cdot\frac{dx}{dt} = \dot{x}\frac{d\dot{x}}{dx}$

$$\dot{x}\frac{d\dot{x}}{dx} = \frac{1}{m}\left(F_\circ + cx\right)$$

$$\dot{x}d\dot{x} = \frac{1}{m}\left(F_\circ + cx\right)dx$$

$$\frac{1}{2}\dot{x}^2 = \frac{1}{m}\left(F_\circ x + \frac{cx^2}{2}\right)$$

$$\dot{x} = \left[\frac{x}{m}\left(2F_\circ + cx\right)\right]^{\frac{1}{2}}$$

(b) $\ddot{x} = \dot{x}\frac{d\dot{x}}{dx} = \frac{1}{m}F_\circ e^{-cx}$

$$\dot{x}d\dot{x} = \frac{1}{m}F_\circ e^{-cx}dx$$

$$\frac{1}{2}\dot{x}^2 = -\frac{F_\circ}{cm}\left(e^{-cx} - 1\right) = \frac{F_\circ}{cm}\left(1 - e^{-cx}\right)$$

$$\dot{x} = \left[\frac{2F_\circ}{cm}\left(1 - e^{-cx}\right)\right]^{\frac{1}{2}}$$

(c) $\ddot{x} = \dot{x}\dfrac{d\dot{x}}{dx} = \dfrac{1}{m}\left(F_\circ \cos cx\right)$

$$\dot{x}d\dot{x} = \frac{F_\circ}{m}\cos cx\, dx$$

$$\frac{1}{2}\dot{x}^2 = \frac{F_\circ}{cm}\sin cx$$

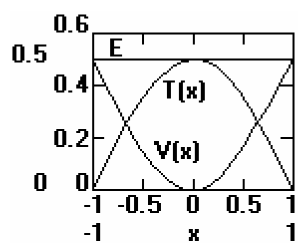
$$\dot{x} = \left(\frac{2F_\circ}{cm}\sin cx\right)^{\frac{1}{2}}$$

2.3 (a) $V(x) = -\int_{x_\circ}^{x}\left(F_\circ + cx\right)dx = -F_\circ x - \dfrac{cx^2}{2} + C$

(b) $V(x) = -\int_{x_\circ}^{x} F_\circ e^{-cx}dx = \dfrac{F_\circ}{c}e^{-cx} + C$

(c) $V(x) = -\int_{x_\circ}^{x} F_\circ \cos cx\, dx = -\dfrac{F_\circ}{c}\sin cx + C$

2.4 (a) $F(x) = -\dfrac{dV(x)}{dx} = -kx$



$$V(x) = \int_0^x kx\, dx = \frac{1}{2}kx^2$$

(b) $T_\circ = T(x) + V(x)$

$$T(x) = T_\circ - V(x) = \frac{1}{2}k\left(A - x^2\right)$$

(c) $E = T_\circ = \dfrac{1}{2}kA^2$

(d) *turning points @* $T(x_1) \to 0 \quad \therefore x_1 = \pm A$

2.5 (a) $F(x) - kx + \dfrac{kx^3}{A^2}$ so $V(x) = \int_0^x\left(kx - \dfrac{kx^3}{A^2}\right)dx = \dfrac{1}{2}kx^2 - \dfrac{1}{4}\dfrac{kx^4}{A^2}$

(b) $T(x) = T_\circ - V(x) = T_\circ - \dfrac{1}{2}kx^2 + \dfrac{1}{4}\dfrac{kx^4}{A^2}$

(c) $E = T_\circ$

(d) $V(x)$ has maximum at $\left|F(x_m)\right| \to 0$

$$kx_m - \frac{kx_m^{\ 3}}{A^2} = 0 \qquad x_m = \pm A$$

$$V(x_m) = \frac{1}{2}kA^2 - \frac{1}{4}\frac{kA^4}{A^2} = \frac{1}{4}kA^2$$

If $E < V(x_m)$ turning points exist.

Turning points @ $T(x_1) \to 0$ let $u = x_1^{\ 2}$

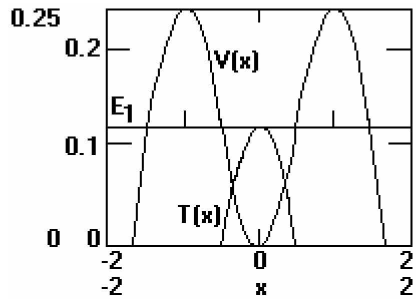
$$E - \frac{1}{2}ku + \frac{1}{4}\frac{ku^2}{A^2} = 0$$

solving for u, we obtain

$$u = A^2\left[1 \pm \left(1 - \frac{4E}{kA^2}\right)^{\frac{1}{2}}\right]$$

or

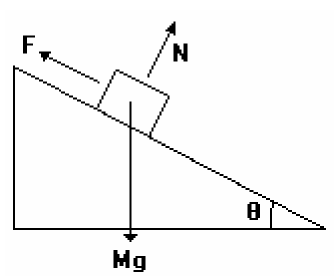
$$x_1 = \pm A\left[1 - \sqrt{\left(1 - \frac{4E}{kA^2}\right)}\right]^{\frac{1}{2}}$$



2.6 $\dot{x} = v(x) = \frac{\alpha}{x}$ $\qquad \ddot{x} = -\frac{\alpha}{x^2}\dot{x} = -\frac{\alpha^2}{x^3}$

$$F(x) = m\ddot{x} = -\frac{m\alpha^2}{x^3}$$

2.7



$$F \geq Mg\sin\theta$$

2.8 $F = m\ddot{x} = m\dot{x}\frac{d\dot{x}}{dx}$

$$\dot{x} = bx^{-3}$$

$$\frac{d\dot{x}}{dx} = -3bx^{-4}$$

$$F = m\left(bx^{-3}\right)\left(-3bx^{-4}\right)$$

$$F = -3mb^2x^{-7}$$

2.9 (a) $V = mgx = (.145kg)\left(9.8\frac{m}{s^2}\right)(1250\,ft)\left(.3048\frac{m}{ft}\right) = 541J$

(b) $T = \frac{1}{2}mv^2 = \frac{1}{2}mv_t^2 = \frac{1}{2}m\left(\frac{mg}{c_2}\right) = \frac{1}{2}\frac{m^2g}{.22D^2}$

$$T = \frac{(.145kg)^2\left(9.8\frac{m}{s^2}\right)}{(2)(.22)\left[(2)(.0366)\right]^2\frac{kg}{m}} = 87J$$

(c) $\int Fdx = \int -cv^2dx = -c\int v^3dt = -c\int\left(-v_t\tanh\left(\frac{t}{\tau}\right)\right)^3dt$

$$= cv_t^3\tau\left[-\frac{1}{2}\tanh^2\left(\frac{t}{\tau}\right) + \int\tanh\left(\frac{t}{\tau}\right)d\left(\frac{t}{\tau}\right)\right]$$

$$= cv_t^3\tau\left[-\frac{1}{2}\tanh^2\left(\frac{t}{\tau}\right) + \ln\cosh\left(\frac{t}{\tau}\right)\right]$$

Now $\tanh^2\left(\frac{t}{\tau}\right) \cong 1$ for $t >> \tau$

Meanwhile $x = \int vdt = \int\left(-v_t\tanh\left(\frac{t}{\tau}\right)\right)dt = v_t\tau\ln\cosh\left(\frac{t}{\tau}\right)$

$$\ln\cosh\left(\frac{t}{\tau}\right) = \frac{x}{v_t\tau}$$

$x = (1250\,ft)\left(.3048\frac{m}{ft}\right) = 381m$

$$v_t = \left(\frac{mg}{c_2}\right)^{\frac{1}{2}} = \left[\frac{(.145kg)\left(9.8\frac{m}{s^2}\right)}{(.22)(.0732)^2\frac{kg}{m}}\right]^{\frac{1}{2}} = 34.72\frac{m}{s}$$

$$\tau = \left(\frac{m}{c_2g}\right)^{\frac{1}{2}} = \left[\frac{(.145kg)}{(.22)(.0732)^2\frac{kg}{m}\left(9.8\frac{m}{s^2}\right)}\right]^{\frac{1}{2}} = 3.543s$$

$$\int Fdx = (.22)(.0732)^2(34.72)^3(3.543)\left[-.5 + \frac{3.81}{(34.72)(3.54)}\right] = 454J$$

$V - T = 541J - 87J = 454J$

2.10 Going up: $F_x = -mg\sin 30^\circ - \mu mg\cos 30^\circ$

$$\ddot{x} = -g\left(\sin 30^\circ + 0.1\cos 30^\circ\right) = -5.749\,\frac{m}{s^2}$$

$$v = v_\circ + at$$

at the highest point $v = 0$ so $t_{up} = -\dfrac{v_\circ}{a} = 0.174 v_\circ\ s$

$$x_{up} = v_\circ t_{up} + \frac{1}{2} a t_{up}^2 = 0.174 {v_\circ}^2 - .087 {v_\circ}^2 = 0.087 {v_\circ}^2 m$$

Going down: ${x_\circ}' = 0.087 {v_\circ}^2$, ${v_\circ}' = 0$, $a' = -9.8\left(0.5 - 0.0866\right)$

$$x_{down} = 0 = 0.087 {v_\circ}^2 - \frac{1}{2} 4.0513 t_{down}^2$$

$$t_{down} = 0.207 v_\circ\ s$$

$$t_{total} = t_{up} + t_{down} = 0.381 v_\circ\ s$$

2.11 $a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \cdot \dfrac{dx}{dt} = v \cdot \dfrac{dv}{dx} = -\dfrac{c}{m} v^{\frac{3}{2}}$

$$v^{-\frac{1}{2}} dv = -\frac{c}{m} dx$$

$$\int_{v_\circ}^{v} v^{-\frac{1}{2}} dv = \int_0^{x_{\max}} -\frac{c}{m} dx$$

$$-2 v_\circ^{\frac{1}{2}} = -\frac{c}{m} x_{\max}$$

$$x_{\max} = \frac{2 m v_\circ^{\frac{1}{2}}}{c}$$

2.12 Going up: $F_x = -mg - c_2 v^2$

$$a = v\frac{dv}{dx} = -g - kv^2, \quad k = \frac{{c_2}^2}{m}$$

$$\int_{v_\circ}^{v} \frac{v\,dv}{-g - kv^2} = \int_0^x dx$$ (Letting $x_0 = 0$ … starting from ground)

$$-\frac{1}{2k} \ln\left(-g - kv^2\right)\Big|_{v_\circ}^{v} = x$$

$$\frac{g + kv^2}{g + k{v_\circ}^2} = e^{-2kx}$$

$$v^2 = \left(\frac{g}{k} + {v_\circ}^2\right) e^{-2kx} - \frac{g}{k}$$

Going down: $F_x = -mg + c_2 v^2$

$$v\frac{dv}{dx} = -g + kv^2$$

$$\int_0^v \frac{vdv}{-g + kv^2} = \int_0^x dx$$

$$\frac{1}{2k}\ln\left(-g + kv^2\right)\Big|_0^v = x - x_\circ$$

$$1 - \frac{k}{g}v^2 = e^{2kx}e^{-2kx_\circ}$$

$$v^2 = \frac{g}{k} - \left(\frac{g}{k}e^{-2kx_\circ}\right)e^{2kx}$$

2.13 At the top $v = 0$ so $e^{-2kx_{\max}} = \dfrac{\frac{g}{k}}{\frac{g}{k} + v_\circ{}^2}$

Coming down $x_\circ = x_{\max}$ and at the bottom $x = 0$

$$v^2 = \frac{g}{k} - \left(\frac{g}{k}\right)^2 \frac{1}{\left(\frac{g}{k} + v_\circ{}^2\right)}(1) = \frac{\left(\frac{g}{k}\right)v_\circ{}^2}{\frac{g}{k} + v_\circ{}^2}$$

$$v = \frac{v_t v_\circ}{\left(v_t{}^2 + v_\circ{}^2\right)^{\frac{1}{2}}}, \quad v_t = \sqrt{\frac{g}{k}} = \sqrt{\frac{mg}{c_2}}$$

2.14 $a = v\dfrac{dv}{dx} = -\dfrac{k}{m}x^{-2}$

$$\int_0^v vdv = \int_b^x -\frac{kdx}{mx^2}$$

$$\frac{1}{2}v^2 = \frac{k}{m}\left(\frac{1}{x} - \frac{1}{b}\right)$$

$$v = \frac{dx}{dt} = \left[\frac{2k}{m}\left(\frac{1}{x} - \frac{1}{b}\right)\right]^{\frac{1}{2}} = \left[\frac{2k}{mb}\left(\frac{b-x}{x}\right)\right]^{\frac{1}{2}}$$

$$\int_0^t dt = \int_b^0 \left[\frac{mb}{2k}\left(\frac{x}{b-x}\right)\right]^{\frac{1}{2}} dx = \left(\frac{mb^3}{2k}\right)^{\frac{1}{2}} \int_1^0 \left(\frac{\frac{x}{b}}{1 - \frac{x}{b}}\right)^{\frac{1}{2}} d\left(\frac{x}{b}\right)$$

Since $x \le b$, say $\frac{x}{b} = \sin^2\theta$

$$t = \left(\frac{mb^3}{2k}\right)^{\frac{1}{2}} \int_{-\frac{\pi}{2}}^{0} \frac{\sin\theta\left(2\sin\theta\cos\theta d\theta\right)}{\cos\theta} = \left(\frac{2mb^3}{k}\right)^{\frac{1}{2}} \int_{-\frac{\pi}{2}}^{0} \sin^2\theta d\theta$$

$$t = \left(\frac{mb^3}{8k}\right)^{\frac{1}{2}} \pi$$

2.15 $$m\frac{dv}{dt} = mg - c_1 v - c_2 v^2$$

$$\int_0^t \frac{dt}{m} = \int_0^v \frac{dv}{mg - c_1 v - c_2 v^2}$$

Using $\int \frac{dx}{a + bx + cx^2} = \frac{1}{\sqrt{b^2 - 4ac}} \ln \frac{2cx + b - \sqrt{b^2 - 4ac}}{2cx + b + \sqrt{b^2 - 4ac}}$,

$$\frac{t}{m} = \frac{1}{\sqrt{c_1{}^2 + 4mgc_2}} \ln \left. \frac{-2c_2 v - c_1 - \sqrt{c_1{}^2 + 4mgc_2}}{-2c_2 v - c_1 + \sqrt{c_1{}^2 + 4mgc_2}} \right|_0^v$$

$$\frac{t}{m}\left(c_1{}^2 + 4mgc_2\right)^{\frac{1}{2}} = \ln \frac{\left(2c_2 v + c_1 + \sqrt{c_1{}^2 + 4mgc_2}\right)\left(c_1 - \sqrt{c_1{}^2 + 4mgc_2}\right)}{\left(2c_2 v + c_1 - \sqrt{c_1{}^2 + 4mgc_2}\right)\left(c_1 + \sqrt{c_1{}^2 + 4mgc_2}\right)}$$

as $t \to \infty$, $2c_2 v_t + c_1 - \sqrt{c_1{}^2 + 4mgc_2} = 0$

$$v_t = -\frac{c_1}{2c_2} + \left[\left(\frac{c_1}{2c_2}\right)^2 + \frac{mg}{c_2}\right]^{\frac{1}{2}}$$

Alternatively, when $v = v_t$,

$$m\frac{dv}{dt} = 0 = mg - c_1 v_t - c_2 v_t{}^2$$

$$v_t = -\frac{c_1}{2c_2} + \left[\left(\frac{c_1}{2c_2}\right)^2 + \frac{mg}{c_2}\right]^{\frac{1}{2}}$$

2.16 $$c_1 = \left(1.55\times 10^{-4}\right)\left(10^{-2}\right) = 1.55\times 10^{-6}\,\frac{kg}{s}$$

$$c_2 = \left(0.22\right)\left(10^{-2}\right)^2 = 2.2\times 10^{-5}\,\frac{kg}{s}$$

$$v_t = -\frac{1.55\times10^{-6}}{2\times2.2\times10^{-5}} + \left[\left(\frac{1.55\times10^{-6}}{2\times2.2\times10^{-5}}\right)^2 + \frac{(10^{-7})(9.8)}{2.2\times10^{-5}}\right]^{\frac{1}{2}}$$

$$v_t = 0.179\frac{m}{s}$$

Using Equation 2.4.10 … $v_t = \left(\frac{mg}{c_2}\right)^{\frac{1}{2}} = \sqrt{\frac{(10^{-7})(9.8)}{2.2\times10^{-5}}} = 0.211\frac{m}{s}$

2.17 $m\frac{dv}{dt} = mv\frac{dv}{dx} = f(x)g(v)$

$$\frac{mvdv}{g(v)} = f(x)dx$$

By integration, get $v = v(x) = \frac{dx}{dt}$

If $F(x,t) = f(x)g(t)$:

$$m\frac{d^2x}{dt^2} = m\frac{d}{dt}\left(\frac{dx}{dt}\right) = f(x)g(t)$$

This cannot, in general, be solved by integration.

If $F(v,t) = f(v)g(t)$:

$$m\frac{dv}{dt} = f(v)g(t)$$

$$\frac{mdv}{f(v)} = g(t)dt$$

Integration gives $v = v(t)$

$$\frac{dx}{dt} = v(t)$$

$$dx = v(t)dt$$

A second integration gives $x = x(t)$

2.18 $F = kxv = m\frac{dv}{dt}$

$$kx = \frac{m}{v}\frac{dv}{dt} = m\frac{dv}{dx}$$

$$\int kxdx = \int mdv$$

$$k\frac{x^2}{2} = mv + C$$

At x = 0, t = 0 $\quad v = v_0 \quad C = -mv_0$

$$k\frac{x^2}{2}+mv_0=m\frac{dx}{dt}$$

$$\int dt=\int\frac{dx}{kx^2/2m+v_0}=\int\frac{dx}{A^2+B^2x^2}\quad \text{where } A^2=v_0 \qquad B^2=\frac{k}{2m}$$

$$t=\frac{1}{AB}\tan^{-1}\frac{B}{A}x$$

Solving …

$$x=\left(\frac{2mv_0}{k}\right)^{\frac{1}{2}}\tan\left(\frac{kv_0}{2m}\right)^{\frac{1}{2}}t$$

2.19 $F(x)=-Ae^{\alpha\dot{x}}=m\ddot{x}$ or $F(v)=-Ae^{\alpha v}=m\dot{v}$ $\quad \frac{dv}{e^{\alpha v}}=-\frac{A}{m}dt$

Let $u=e^{\alpha v}$ $\quad du=\alpha e^{\alpha v}dv$ $\quad dv=\frac{du}{\alpha e^{\alpha v}}=\frac{du}{\alpha u}$ $\quad \therefore\frac{du}{u^2}=-\frac{\alpha A}{m}dt$

Integrating

$\frac{1}{u}-\frac{1}{u_\circ}=\frac{A}{m}\alpha t$ and substituting $e^{\alpha v}=u$

(a) $v=v_\circ-\frac{1}{\alpha}\ln\left[1+\frac{A}{m}e^{\alpha v_\circ}\alpha t\right]$

(b) $t=T\ @\ v=0$

$$\alpha v_\circ=\ln\left[1+\frac{A}{m}e^{\alpha v_\circ}\alpha T\right]$$

$$e^{\alpha v_\circ}=1+\frac{A}{m}e^{\alpha v_\circ}\alpha T \qquad T=\frac{m}{\alpha A}\left[1-e^{-\alpha v_\circ}\right]$$

(c) $v\frac{dv}{dx}=v=-\frac{A}{m}e^{\alpha v}$ $\quad \frac{vdv}{e^{\alpha v}}=-\frac{A}{m}dx$

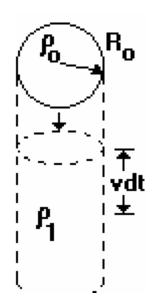
again, let $u=e^{\alpha v}$ $\quad du=\alpha udv$ or $dv=\frac{du}{\alpha u}$ $\quad v=\frac{1}{\alpha}\ln u$

$\frac{\left[\frac{1}{\alpha}\ln u\right]\frac{du}{\alpha u}}{u}=-\frac{A}{m}dx$ Integrating and solving

$$x=\frac{m}{\alpha^2A}\left[1-\left(1+\alpha v_\circ\right)e^{-\alpha v_\circ}\right]$$

2.20

$$F=\frac{d(mv)}{dt}=mv+vm=mg$$



but $m = \rho_{\circ} \frac{4}{3} \pi r^3 \qquad m = \rho_1 \pi r^2 v$

so (1) $\frac{4}{3} \pi \rho_{\circ} r^3 v + \pi \rho_1 r^2 v^2 = \frac{4}{3} \pi^2 = \frac{4}{3} \pi \rho_{\circ} r^3 g$

Now $\frac{\rho_1}{\rho_{\circ}} \approx 10^{-3}$ so, second term is negligible-small

hence $v \approx g$ and $\boxed{v \approx gt}$ speed $\propto t$ but

$\dot{m} = \rho_{\circ} 4\pi r^2 \dot{r} = \rho_1 \pi r^2 v$ or $\dot{r} \cong \frac{1}{4} \frac{\rho_1}{\rho_{\circ}} v$ Hence $\boxed{r \approx \frac{1}{4} \frac{\rho_1}{\rho_{\circ}} gt}$ and rate of growth $\propto t$

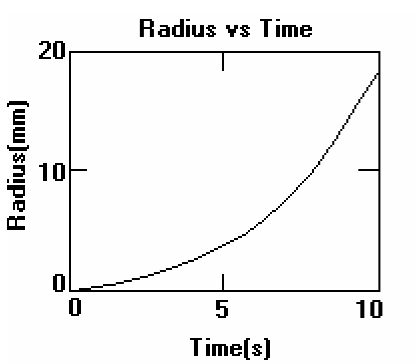
The exact differential equation from (1) above is:

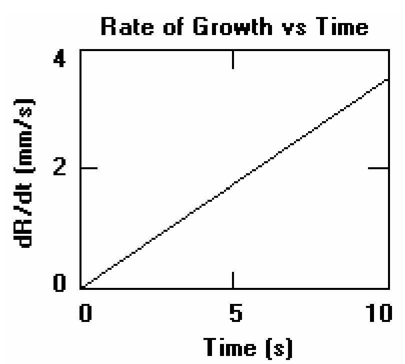
$$\frac{4}{3} \pi \rho_{\circ} r \left| \frac{4\rho_{\circ}}{\rho_1} \ddot{r} \right| + \pi \rho_1 \left| \frac{4\rho_{\circ} \dot{r}}{\rho_1} \right|^2 = \frac{4}{3} \pi \rho_{\circ} r g$$

which reduces to: $\ddot{r} + \frac{3\dot{r}^2}{r} = \frac{\rho_1}{4\rho_{\circ}} g$

Using Mathcad, solve the above non-linear d.e. letting $\frac{\rho_1}{\rho_{\circ}} \approx 10^{-3}$ and $R_{\circ} \approx 0.01mm$ (small raindrop). Graphs show that

$$v \propto \dot{r} \propto t \quad \text{and} \quad r \propto t^2$$





CHAPTER 3
OSCILLATIONS

3.1 $x = 0.002\sin\left[2\pi\left(512\,s^{-1}\right)t\right]\left[m\right]$

$$\dot{x}_{\max} = \left(0.002\right)\left(2\pi\right)\left(512\right)\left[\frac{m}{s}\right] = 6.43\left[\frac{m}{s}\right]$$

$$\ddot{x}_{\max} = \left(0.002\right)\left(2\pi\right)^2\left(512\right)^2\left[\frac{m}{s^2}\right] = 2.07\times10^4\left[\frac{m}{s^2}\right]$$

3.2 $x = 0.1\sin\omega_\circ t\ \ [m]$ $\qquad \dot{x} = 0.1\omega_\circ\cos\omega_\circ t\left[\frac{m}{s}\right]$

When $t = 0$, $x = 0$ and $\qquad \dot{x} = 0.5\left[\frac{m}{s}\right] = 0.1\omega_\circ$

$\omega_\circ = 5\,s^{-1}$ $\qquad T = \frac{2\pi}{\omega_\circ} = 1.26\,s$

3.3 $x\left(t\right) = x_\circ\cos\omega_\circ t + \frac{\dot{x}_\circ}{\omega_\circ}\sin\omega_\circ t$ and $\omega_\circ = 2\pi f$

$$x = 0.25\cos\left(20\pi t\right) + 0.00159\sin\left(20\pi t\right)\left[m\right]$$

3.4 $\cos\left(\alpha-\beta\right) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$x = A\cos\left(\omega_\circ t - \phi\right) = A\cos\phi\cos\omega_\circ t + A\sin\phi\sin\omega_\circ t$$

$$x = \mathrm{A}\cos\omega_\circ t + \mathrm{B}\sin\omega_\circ t\,,\ \ \mathrm{A} = A\cos\phi\,,\ \ \mathrm{B} = A\sin\phi$$

3.5 $\frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}kx_2^2$

$$k\left(x_1^2 - x_2^2\right) = m\left(\dot{x}_2^2 - \dot{x}_1^2\right)$$

$$\omega_\circ = \sqrt{\frac{k}{m}} = \left(\frac{\dot{x}_2^2 - \dot{x}_1^2}{x_1^2 - x_2^2}\right)^{\frac{1}{2}}$$

$$\frac{1}{2}kA^2 = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}kx_1^2$$

$$A^2 = \frac{m}{k}\dot{x}_1^2 + x_1^2 = \frac{x_1^2\dot{x}_1^2 - x_2^2\dot{x}_1^2}{\dot{x}_2^2 - \dot{x}_1^2} + x_1^2$$

$$A = \left(\frac{x_1^2\dot{x}_2^2 - x_2^2\dot{x}_1^2}{\dot{x}_2^2 - \dot{x}_1^2}\right)^{\frac{1}{2}}$$