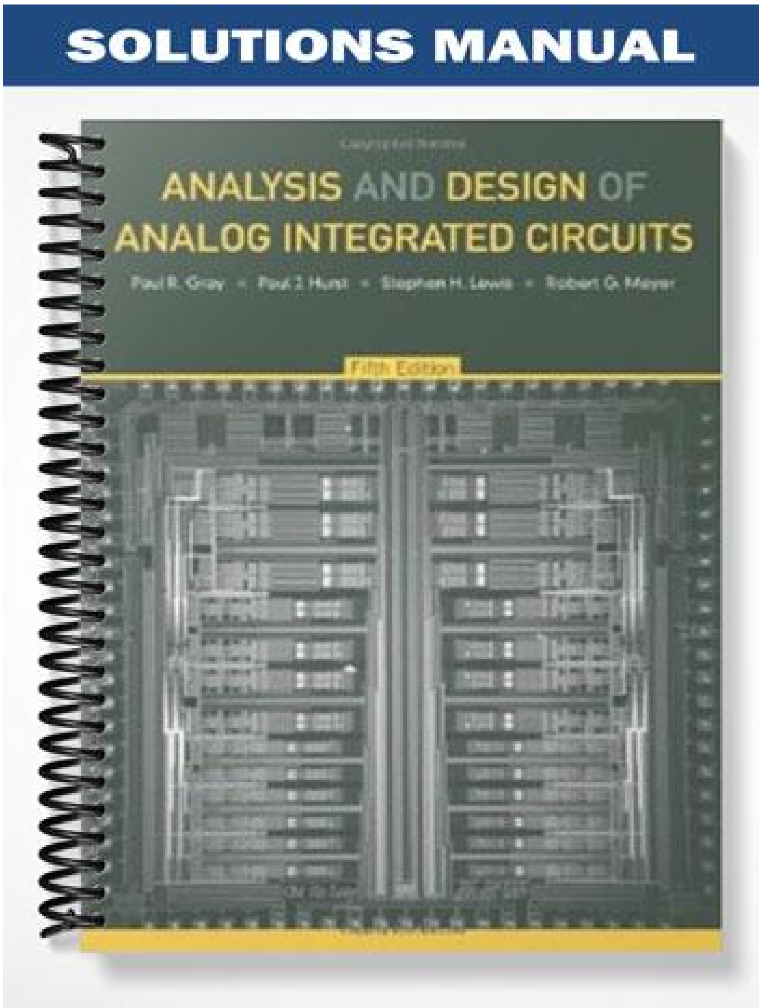
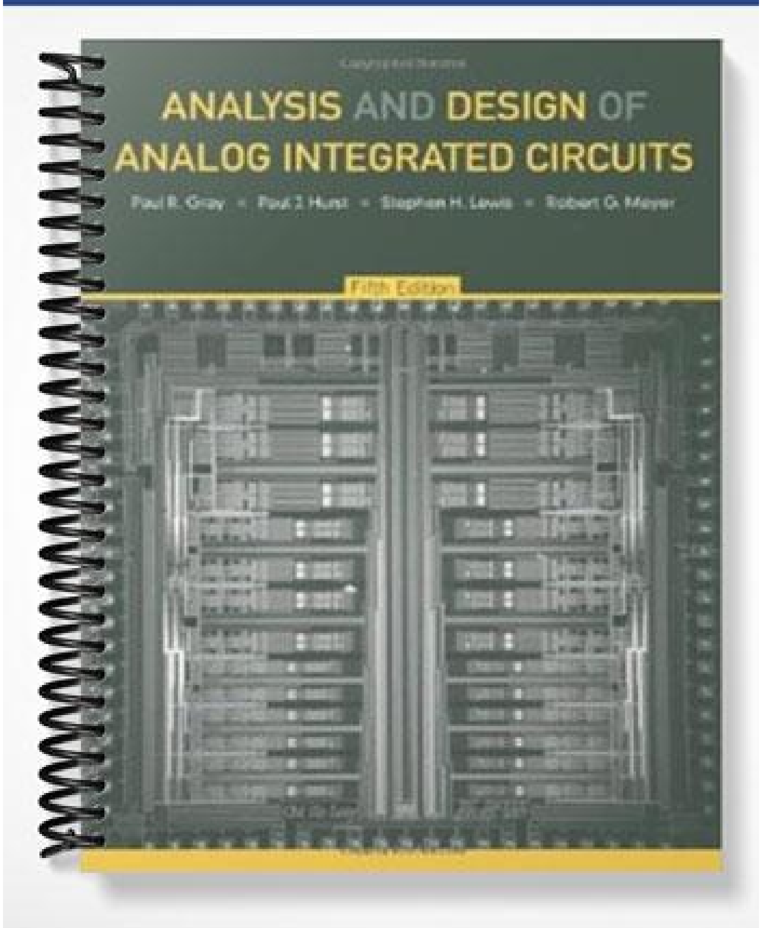

SOLUTIONS MANUAL
ANALYSIS AND DESIGN OF
ANALOG INTEGRATED CIRCUITS
Paul R. Gray • Paul J. Hurst • Stephen H. Lewis • Robert G. Meyer
Fifth Edition

SOLUTIONS MANUAL



CHAPTER 2

2.1

From Fig. (2.2)

For p-type Si,

$N_A = 3 \times 10^{17}$ atoms/cm^3

For n-type Si,

$N_D = 10^{17}$ atoms/cm^3

2.2

$$R_{\square} = \frac{\rho}{T} = 1\,\frac{\Omega\text{-cm}}{2\,\mu m} = \frac{1\,\Omega\text{-cm}}{2\times10^{-4}\,cm}$$

$$= 5000\ \Omega/\square$$

2.3

The doping profile has the form

$$N_D(x) = N_{DO} \exp\left(-\frac{x}{L}\right)$$

The first step is to determine N_{DO} and L. Since at $x=0$,

$N_D(x) = 10^{17}\ cm^{-3}$, then

$N_{DO} = 10^{17} cm^{-3}$

At $x = 0.5\ \mu m$, $N_D(x) = \frac{1}{e} N_D(0)$

$\therefore\ N_{DO} e^{-1} = N_{DO} \exp\left(-\frac{0.5\ \mu m}{L}\right)$

$\therefore\ L = 0.5\ \mu m$

Thus,
$$N_D(x) = 10^{17} \exp\left(-\frac{x}{0.5 \mu m}\right)$$

The junction will exist at the point where the net doping

$N_D(x) - N_A(x) = 0$

$\therefore\ 10^{17} \exp\left(-\frac{x_j}{0.5\ \mu m}\right) = 10^{15}$

$\therefore\ x_j = 0.5\ \mu m\ \ln \frac{10^{17}}{10^{15}} = 2.3\ \mu m$

The sheet resistance is given by:

$$R_\square = \left[q \bar{\mu}_n \int_0^{x_j} [N_D(x) - N_A(x)]\, dx \right]^{-1}$$

Since the <u>effective</u> doping is $N_D(x) - N_A(x)$

$$\therefore\ q\left[1000 \frac{cm^2}{V\text{-}sec}\right] \int_0^{2.3\mu m} [10^{17} e^{-\frac{x}{0.5\mu m}} - 10^{15}]\, dx$$

$$= q\left[1000 \frac{cm^2}{V\text{-}sec}\right] \left\{\left[-0.5\times 10^{11} - 2.3\times 10^{11}\right] + 0.5\times 10^{13}\right\}$$

$$= 1.6 \times 10^{-19} \times 1000 \times 4.72 \times 10^{12}$$

$$= 1.6 \times 10 \times 4.72 \times 10^{-5}$$

$$\therefore\ R_\square = [7.55 \times 10^{-4}]^{-1} = 1300\ \Omega/\square$$

2.4

For the resistor $\frac{L}{W} = \frac{100\ \mu m}{5\ \mu m} = 20$

Base-diffused

$R = 20 \times 100\ \Omega/\square = 2\ K\Omega$

Emitter-diffused

$R = 20 \times 5\ \Omega/\square = 100\ \Omega$

Pinch

$R = 20 \times 5\ K\Omega/\square = 100\ K\Omega$

2.5

From (2.17), for an npn-transistor,

$$Q_B = \frac{A \bar{D}_n q n_i^2}{I_c} \exp\left(\frac{V_{BE}}{V_T}\right)$$

For $V_{BE} = 520$ mV,

$$Q_B = \frac{(10^{-4}\,cm^2)(13\,\frac{cm^2}{sec})(1.6\times10^{-19})(2\times10^{20})}{2\times10^{-5}}\, e^{(520/26)}$$

$= 1.01 \times 10^{12}$ atoms/cm^2

For $V_{BE} = 580$ mV, same expression gives : $Q_B = 1\times10^{13}$ atoms/cm^2

For $Q_B = 1.01 \times 10^{12}$

$$R_\square = [\,q\,\bar{\mu}_p\,Q_B\,]^{-1}$$

$$= \left[(1.6\times10^{-19})(150\,\frac{cm^2}{V\text{-}sec})(1.01\times10^{12})\right]^{-1}$$

$= (2.42 \times 10^{-17})^{-1} = .42$ K$\Omega/\square$

2.6

(a) Series base R :

Emitter periphery adjacent to a base contact is :

$p = 4 \times 40\ \mu m = 160\ \mu m$

Distance from base contact to emitter is : $10\ \mu m$

$\therefore R_B = R_{\square}\left(\frac{10}{160}\right) = 100\ \Omega/_{\square}\left(\frac{1}{16}\right)$

$= 6.2\ \Omega$

(b) Series collector resistance :

For each of two emitters the effective buried layer dimensions are :

$W_{BL} = (W+2T) = 20\ \mu m + 2(10\ \mu m)$

$= 40\ \mu m$

$L_{BL} = (L+2T) = 40\ \mu m + 2(10\ \mu m)$

$= 60\ \mu m$

Using (2.18),

$a = \left(\frac{20}{40}\right)^{-1}, \quad b = \left(\frac{40}{60}\right)^{-1}$

$2r_{c1} = \frac{(5\ \Omega\text{-cm})(10\ \mu m)}{(20\ \mu m)(40\ \mu m)} \ln \frac{2/1.5}{0.5}$

$= 360\ \Omega$

$\therefore r_{c1} = 180\ \Omega$

$r_{c2} = R_{BL\square}\left(\frac{L}{W}\right) = 20\ \frac{\Omega}{\square}\left(\frac{40}{140}\right) = 5.7\ \Omega$

r_{c3} : for buried layer

$L_{eff} = 140\ \mu m$

$W_{eff} = 14\ \mu m + 2T = 34\ \mu m$

For top N^+: $L = 140\ \mu m$

$W = 14\ \mu m$

$\therefore a = \frac{34}{14} = 2.43 \quad ; b = 1$

$\therefore r_{c3} = \frac{(5\ \Omega\text{-cm})(10\ \mu m)}{(140\ \mu m)(14\ \mu m)} \ln \frac{2.43}{2.43-1}$

$= 158\ \Omega$

$\therefore R_c = 158 + 180 + 5.7 = 344\ \Omega$

(c) $C_{jc} = (A_{bottom} + A_{sidewall})\ C_{jc}/_{area}$

$= \left[(140\ \mu m)(60\ \mu m) + \frac{\pi}{2} \times 3\ \mu m(280\ \mu m + 120\ \mu m)\right] \times 10^{-4}\ pF/_{(\mu m)^2}$

$= (8400 + 1885) \times 10^{-4}$

$= 1.03\ pF$

(d) $C_{je} \approx (C_{jebottom} + C_{jesidewall})$

$= 2\left[A_{bottom} + A_{sidewall}\right] C_{je}/_{area}$

$= 2\left[(20\ \mu m)(40\ \mu m) + \frac{\pi}{2} \times 2\ \mu m(40\ \mu m + 80\ \mu m)\right] \times 10^{-4}\ pF/_{(\mu m)^2}$

$= 2.4\ pF$

(e) $C_{substrate} = C_{sidewall} + C_{epi\text{-}sub} + C_{BL\text{-}sub}$

$C_{sidewall} = \frac{\pi}{2} \times (17\ \mu m)\left[175\ \mu m \times 2 + 140\ \mu m \times 2\right] \times 10^{-4}\ pF/_{(\mu m)^2}$

$= 1.6\ pF$

$C_{epi\text{-}sub} = (175 \times 140 - 85 \times 126) \times 10^{-4}\ \frac{pF}{(\mu m)^2}$

$= 1.4\ pF$

$C_{BL\text{-}sub} = (85 \times 126) \times 3.3 \times 10^{-4} = 3.5\ pF$

$\therefore C_{substrate} = 6.5\ pF$

(f) $I_S = \dfrac{q n_i^2}{Q_B/\bar{D}_n} A_{EB}$

From text example,

$$\frac{Q_B}{\bar{D}_n} = 5.7 \times 10^{11} \text{ cm}^{-4} \text{ sec}$$

$$\therefore I_S = \frac{1.6 \times 10^{-19} \times 2 \times 10^{20} \times 1600 \ \mu m^2 \times \left(10^{-8} \frac{cm^2}{\mu m^2}\right)}{5.7 \times 10^{11}}$$

$$= 0.9 \times 10^{-15} \text{ A}$$

2.7

From (2.24), the β falloff begins at $I_C = qAN_D \frac{D_p}{W_B}$

For this structure, the area A is the product of the emitter diffusion periphery 120 μm, and the effective sidewall depth. Regarding the sidewall as a quarter-cylinder this effective depth is the emitter junction depth multiplied by $\pi/2$. Thus,

$$A_{eff} = (120\ \mu m)(3\ \mu m)(\pi/2) = 5.65 \times 10^{-6}\ cm^2$$

From Fig. (2.2), the donor density corresponding to a resistivity of $0.5\ \Omega\text{-cm}$ is $1.2 \times 10^{16}\ cm^{-3}$. Thus,

$$I_C = \frac{(1.6\times10^{-19})(5.65\times10^{-6})(1.2\times10^{16})(10)}{5\times10^{-4}}$$

$$= 21.7 \times 10^{-5} = 217\ \mu A$$

2.8

From (2.17), for a pnp transistor

$$Q_B = qA\bar{D}_p \frac{n_i^2}{I_c} \exp\left(\frac{V_{BE}}{V_T}\right)$$

For this device,

$$A = 2\times(30\mu m \times 75\mu m) + 2\times(10\mu m \times 30\mu m)$$

$$= 5100\,(\mu m)^2$$

For $V_{BE} = 525$ mV

$$Q_B = \frac{1.6\times10^{-19}\times5100\times10^{-8}\times10\times2\times10^{20}}{10^{-5}} \exp\left(\frac{525}{26}\right)$$

$$= 9.6\times10^{11}$$

Using Fig. (2.2), the donor density corresponding to $2\,\Omega$-cm is $2.5\times10^{15}\,cm^{-3}$

since, $Q_B = W_B N_D$, then

$$W_B = \frac{Q_B}{N_D} = \frac{9.6\times10^{11}}{2.5\times10^{15}} = 3.84\times10^{-4}\,cm$$

$$= 3.84\,\mu m$$

Since the p-diffusion depth is 3 μm, the total epi thickness is 6.84 μm, for $V_{BE} = 525$ mV. By a similar calculation, the total depth for $V_{BE} = 560$ mV is 17.45 μm.

The same resistances are:

For $V_{BE} = 525$ mV

$$R_\square = \left[q\bar{\mu}_n Q_B\right]^{-1}$$

$$= \left[(1.6\times10^{-19})(800)(9.6\times10^{11})\right]^{-1}$$

$$= \left[12\times10^{-5}\right]^{-1} = 8\ k\Omega/\square$$

For $V_{BE} = 560$ mV

$$R_\square = 2.12\ k\Omega/\square$$

2.9.

(1) $I_S = A_{EB}\, q \dfrac{n_i^2}{Q_B} \bar{D}_{Pb}$

$= \left[90\,\mu m \times 75\,\mu m - 30\,\mu m \times 55\,\mu m\right] \times 10^{-8}\dfrac{cm^2}{\mu^2} \times \dfrac{(1.6\times10^{-19})(2\times10^{20})\,(10\ cm^2/sec)}{\left(10^{15}\dfrac{atoms}{cm^3}\right)\times 14\,\mu m \times 10^{-4}\dfrac{cm}{\mu m}}$

$= 1.17 \times 10^{-14}\, A$

(2) $C_{je} = A_{bottom} \times 10^{-4}\ pF/(\mu m)^2 + A_{sidewall} \times 10^{-3}\ pF/(\mu m)^2$

$= (90\times75 - 30\times55)(\mu m)^2 \times 10^{-4}\dfrac{pF}{(\mu m)^2} + \dfrac{\pi}{2}(60+110)(3)(\mu m)^2 \times 10^{-3}\dfrac{pF}{(\mu m)^2}$

$= 0.51 + 0.8 = 1.31\ pF$

(3) $C_\mu = C_{epi\text{-}sidewall} + C_{bottom}$

$= \dfrac{\pi}{2}(40\times2 + 125\times2)\times(17)(\mu m)^2 \times 10^{-4}\dfrac{pF}{(\mu m)^2} + (140\times125)(\mu m)^2 \times 10^{-4}\dfrac{pF}{(\mu m)^2}$

$= 1.4 + 1.75 = 3.15\ pF$

(4) $\tau_F = \dfrac{W_B^2}{2D_p} = \dfrac{\left[(14\,\mu m)(10^{-4}\,cm/\mu m)\right]^2}{2\times10\ cm^2/sec}$

$= 98\ nsec$

2.10

First consider the 6 μm resistor, For 10 kΩ we need 100 squares or 600 μm of length. The total resistor area is that of the body plus clubheads, or

$$A_{bottom} = 600\ \mu m \times 6\ \mu m + 2\ (26\ \mu m)^2$$
$$= 4952\ (\mu m)^2$$

The sidewall area is equal to the total periphery multiplied by $(3\ \mu m)(\pi/2)$.

The periphery is,

$$P = 600\ \mu m + 600\ \mu m + 6 \times 26\ \mu m + 2(26-6)\ \mu m$$
$$= 1396\ \mu m$$

Thus,

$$A_{sidewall} = 1396\ \mu m \times 3\ \mu m \times \frac{\pi}{2}$$
$$= 6578\ (\mu m)^2$$

The total area is,

$$A_{total} = 11{,}530\ (\mu m)^2$$

For the $10^{15}\ cm^{-3}$ epi concentration the capacitance per unit area for zero bias is from Fig.(2.29) equal to $10^{-4}\ pF/(\mu m)^2$

Thus, $C_{total} = 11{,}530\ (\mu m)^2 \times 10^{-4} \dfrac{pF}{(\mu m)^2}$

$$= 1.15\ pF$$

For the 12 μm resistor, we need 1200 μm of length

Thus, $A_{bottom} = 12\ \mu m \times 1200\ \mu m + 2(26\mu)^2$

$$= 15{,}750\ (\mu m)^2$$

$$A_{sidewall} = \frac{\pi}{2}\left[1200 \times 2 + 26 \times 6 + 2(26-12)\right] \times 3$$
$$= 12{,}177\ (\mu m)^2$$

$\therefore\ A_{total} = 27{,}929\ (\mu m)^2$

$\therefore\ C_{total} = 27{,}929\ (\mu m)^2 \times 10^{-4} \dfrac{pF}{(\mu m)^2} = 2.8\ pF$

2.11

For an npn transistor

$$Q_B = \frac{qA\bar{D}_n n_i^2}{I_C}\exp\left(\frac{V_{BE}}{V_T}\right)$$

$$= \frac{(1.6\times10^{-19})(10^{-4}\,cm^2)(13)(2\times10^{20})}{10^{-5}}e^{\frac{480}{26}}$$

$$= 4.3\times10^{11}\ atoms/cm^2$$

$$R_\square = [q\mu_p Q_B]^{-1}$$

$$= \left[1.6\times10^{-19}\times150\,\frac{cm^2}{V\text{-}sec}\times4.3\times10^{11}\right]^{-1}$$

$$= 97\ k\Omega/\square$$

In order to contain $4.3\times10^{11}\,\frac{atoms}{cm^2}$ of the n-type epi-impurity, the width of the collector depletion layer at punch-thru is,

$$W_\square = \frac{4.3\times10^{11}\ atoms/cm^2}{10^{15}\ cm^{-3}}$$

$$= 4.3\times10^{-4}\ cm = 4.3\ \mu m$$

Using (1.15) and assuming that $N_A >> N_D$,

$$W_\square = \sqrt{\frac{2\epsilon(\psi_o+\psi_R)}{qN_D}} = 4.3\ \mu m$$

$$= \sqrt{\frac{2\times1.04\times10^{-12}\times(0.55+V_R)}{1.6\times10^{-19}\times10^{15}}}$$

$$\therefore V_R = 13.7\ V$$

For $V_{BE} = 560$ mV, same expressions give, $Q_B = 9.4\times10^{12}$ atoms/cm²

$$R_\square = 4.43\ k\Omega/\square$$

$$W_\square = 94\ \mu m$$

$$V_R = 6,796\ V$$

2.12 From (1.157),

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2$$

$$100\,\mu A = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (1.5 - V_t)^2 \rightarrow (1)$$

$$10\,\mu A = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (0.8 - V_t)^2 \rightarrow (2)$$

Divide (1) by (2) and solve for $V_t \rightarrow$ gives $V_t = 0.48\,V$.

Substituting for V_t in (1) gives,

$$\mu_n C_{ox} \frac{W}{L} = 191\ \mu A/V$$

<u>2.13</u> $V_t = \phi_{MS} + 2\phi_f + \frac{Q_b}{C_{ox}} - \frac{Q_{ss}}{C_{ox}}$, (2.27)

(i) For unimplanted transistors,

From Table 2.1, $\phi_{MS} = -0.1\,V$

From (2.28),

$$\phi_f = \frac{KT}{q} \ln \frac{N_A}{n_i} = 0.026 \ln \frac{10^{16}}{1.45\times10^{10}}$$

$$= 0.35.$$

From (2.30),

$$C_{ox} = 0.86 \ fF/(\mu m)^2$$

$$\frac{Q_{ss}}{C_{ox}} = \frac{1.6\times10^{-19}\times 10^{11}}{8.6\times10^{-8}} = 0.19\,V$$

From (1.137),

$$Q_{bo} = \sqrt{2q N_A \varepsilon . 2\phi_f}$$

$$= \left[2\times1.6\times10^{-19}\times10^{16}\times11.6\times8.86\times10^{-14} \times 2\times0.35\right]^{1/2} = 4.8\times10^{-8}\,\frac{\text{coulombs}}{cm^2}$$

$$\therefore V_t = -0.1 - 2\times0.35 - \frac{4.8\times10^{-8}}{8.6\times10^{-8}} - 0.19$$

$$= -1.55$$

(ii) For implanted transistors,

$$N_A = 1\times10^{16} - 0.9\times10^{16} = 10^{15}\,cm^{-3}$$

From Fig.(2.29), the depletion layer width corresponding to a doping level of $10^{15} cm^{-3}$ is $\sim 1\,\mu m$, while the implant depth is only $0.3\,\mu m$

$$\therefore V_{t(implant)} = -1.55 + \frac{0.9\times10^{16}\times0.3\times10^{-4}\times1.6\times10^{-19}}{8.6\times10^{-8}}$$

$$= -1.05$$

2.14

The metallurgical channel length is

$$L = L_{drawn} - 2L_d = 7\,\mu m - 2(0.3) = 6.4\,\mu m$$

The effective channel length is L minus the width of the depletion region at the drain. In the active region, the voltage at the drain end of the channel is $= V_{GS} - V_t = V_{ov}$

To calculate V_{ov}, assume at first that $L_{eff} \simeq L$. Then from (1.166)

$$V_{ov} = \sqrt{\frac{2I_D}{k' W/L}} = \sqrt{\frac{2(10\,\mu A)}{700\,\frac{cm^2}{V \cdot s} \cdot 0.86\,\frac{fF}{\mu^2}\left(\frac{100}{6.4}\right)}}$$

$$= 0.15\ V$$

Thus, the voltage across the drain depletion region $= 5 - 0.15 = 4.85\,V$

To estimate the depletion-region width, assume it is a one-sided step junction that mainly exists in the lightly doped side. Since the channel and the drain are both n-type regions, the built in potential of the junction is near zero. Using (1.14) and assuming $N_D >> N_A$,

$$X_d = \sqrt{\frac{2\varepsilon(V_{DS} - V_{OV})}{q N_A}} = \sqrt{\frac{2(1.04 \times 10^{-12})(5 - 0.15)}{1.6 \times 10^{-19}(2 \times 10^{16} + 10^{15})}}$$

$$= 0.55\,\mu m$$

So, $L_{eff} = 7\,\mu m - 2(0.3\,\mu m) - 0.55\,\mu m$

$$= 5.85\,\mu m$$

Therefore, since $r_o = \frac{1}{\lambda I_D} = \frac{L_{eff}}{I_D} \Big/ \frac{dX_d}{dV_{DS}}$

$$\Rightarrow r_o = 5\,M\Omega = \frac{5.85\,\mu m}{10\,\mu A} \Big/ \frac{dX_d}{dV_{DS}}$$

So, $\frac{dX_d}{dV_{DS}} = 0.12\,\frac{\mu m}{V}$. Since, the other device uses the same technology, $\frac{dX_d}{dV_{DS}}$ is unchanged

But, should calculate V_{ov} for 2^{nd} transistor,

$$V_{ov} = \sqrt{\frac{2I}{k'\frac{W}{L}}} = \sqrt{\frac{2(30)}{(60.2)\frac{50}{12 - 2(0.3)}}} = 0.48\,V$$

$$\text{So, } X_d = \sqrt{\frac{2(1.04 \times 10^{-12})(5 - 0.48)}{1.6 \times 10^{-19}(2.1 \times 10^{16})}}$$

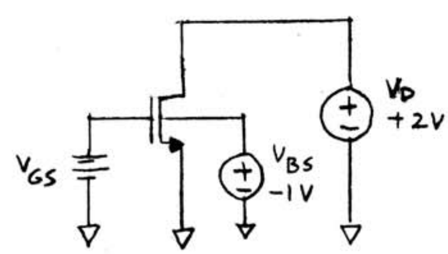
$$= 0.53\,\mu m$$

$$L_{eff_2} = 12 - 2(0.3) - 0.53) = 10.87\,\mu m$$

$$r_o = \frac{L_{eff}}{I_D} \Big/ \frac{dX_d}{dV_{DS}} = \frac{10.87\,\mu m}{30\,\mu A} \Big/ 0.12 = 3.02\,M\Omega$$

Note that using the same V_{ov} as for the 1^{st} transistor (0.15V) gives an answer of 3.1 MΩ. (The change hardly matters)

2.15



First to estimate X_d and L_{eff}

$$L_{eff} = L_{drawn} - 2L_d - X_d \longrightarrow (1)$$

$$X_d = \sqrt{\frac{2\epsilon_{si}(V_{DS} - V_{ov})}{q(N_A + N_i)}} \longrightarrow (2)$$

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L_{eff}} (V_{GS} - V_t)^2 \longrightarrow (3)$$

L_{eff}, X_d and $(V_{GS} - V_t)$ can be found by solving (1), (2), (3).

To avoid solving non-linear equations, the following procedure is used: find x_d by substituting $L_{drawn} - 2L_D$ for L_{eff} in (3) and use the result in (2).

From (3),

$$V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L_{eff}}}$$

$$= \sqrt{\frac{2 \times 20\,\mu A}{450 \frac{cm^2}{V.sec}\, 4.32 \frac{fF}{(\mu m)^2} \left(\frac{10\,\mu m}{1\,\mu m - 2(0.09\,\mu m)}\right)}}$$

$$= 0.130\ V$$

From (2),

$$x_d = \sqrt{\frac{2 \times 11.6 \times 8.86 \times 10^{-14}(2 - 0.13)}{1.6 \times 10^{-19}(5 \times 10^{15} + 4 \times 10^{16})}}$$

$$= 0.231\,\mu m$$

From (1),

$$L_{eff} \cong 1\,\mu m - 2(0.09)\,\mu m - 0.231\,\mu m$$

$$= 0.589\,\mu m$$

(a) $$g_m = \sqrt{2K' \frac{W}{L_{eff}} I_D}$$

$$= \sqrt{2 \times 194 \frac{\mu A}{V^2} \times \frac{10}{0.589} \times 20\,\mu A}$$

$$= 363\ \mu A/V$$

(b) From (1.200), $g_{mb} = \dfrac{\gamma g_m}{2\sqrt{2\phi_f + V_{SB}}}$

where, $\gamma = \dfrac{\sqrt{2q\varepsilon_{si} N_T}}{C_{ox}}$ and

N_T is the effective dopant density at the bottom of the channel-substrate depletion layer, it can be either N_A (substrate doping) or $N_A + N_i$. To determine which one applies, we have to find $x'_{d\,max}$, i.e., the depletion depth under the channel,

$$x'_{d\,max} = \sqrt{\frac{2\varepsilon_{si}(2|\phi_p| + V_{SB})}{q(N_A + N_i)}}$$

But,

$$\phi_p = \frac{KT}{q} \ln \frac{N_A + N_i}{n_i}$$

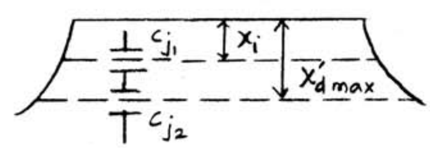
$$= 0.026 \ln \frac{4 \times 10^{16} + 5 \times 10^{15}}{1.45 \times 10^{10}} = 0.389$$

$$\therefore x'_{d\,max} = \sqrt{\frac{2 \times 11.6 \times 8.86 \times 10^{-14}(2 \times 0.389 + 1)}{1.6 \times 10^{-19}(4 \times 10^{16} + 5 \times 10^{15})}}$$

$$= 0.225\,\mu m$$

$> 0.16\,\mu m$ (effective implant depth)

since $x'_{d\,max} > x_i$, the depletion capacitor is the series combination of the capacitors as shown,



Now χ can be calculated from the following relationships,

$$\chi = \frac{C_{js}}{C_{ox}}, \quad C_{js} = \frac{C_{j1} C_{j2}}{C_{j1} + C_{j2}}$$

$$C_{j1} = \frac{\varepsilon_{si}}{x_i}, \quad C_{j2} = \frac{\varepsilon_{si}}{x_{d\,max} - x_i}$$

The actual depletion depth is

$$x_{d_{max.}} = \left[\frac{2\epsilon_{si}}{qN_A}\left(\phi_f + |\phi_p| + V_{SB} - x_i^2 \frac{N_i}{N_A}\right)\right]^{1/2}$$

(cf. Muller and kamins : Device Electronics for Integrated circuits, 2nd ed., 1986; equation (10.6.2))

$$\phi_f = \frac{KT}{q} \ln \frac{N_A}{N_i} = 0.026 \ln \frac{5\times10^5}{1.45\times10^{10}} = 0.33\,V$$

$$x_{d_{max.}} = \left[\frac{2\times11.6\times8.86\times10^{-4}(0.33+0.389+1)}{1.6\times10^{-19}\times5\times10^{15}} - \frac{(0.16\times10^{-4}cm)^2(4\times10^{16})}{5\times10^{15}} \right]^{1/2}$$

$$= 0.453\ \mu m$$

$$C_{j1} = \frac{11.6\times8.86\times10^{-14}}{0.16\times10^{-4}} = 6.42\times10^{-8}\ F/cm^2$$

$$C_{j2} = \frac{11.6\times8.86\times10^{-14}}{(0.453-0.16)\times10^{-4}} = 3.51\times10^{-8}\ F/cm^2$$

$$C_{js} = 2.27\times10^{-8}\ F/cm^2$$

$$\therefore\ \chi = \frac{2.27\times10^{-8}}{4.32\times10^{-7}} = 0.0526$$

$$g_{mb} = \chi g_m = 0.0526\times363\times10^{-6} = 19.1\ \frac{\mu A}{V}$$

(c) From (2.37),

$$V_{ov} = \sqrt{\frac{2I_D}{K'(W/L_{eff})}} = \sqrt{\frac{2\times20\ \mu A}{194\ \mu A/V^2\,(10\mu/0.589\mu)}}$$

$$= 0.110\ V$$

$$x_d = \sqrt{\frac{2\epsilon_{si}(V_{DS}-V_{ov})}{q(N_A+N_i)}}$$

$$\frac{dx_d}{dV_{DS}} = \frac{x_d}{2(V_{DS}-V_{ov})} = \frac{0.231\ \mu m}{2(2-0.11)} = 0.061\ \mu m/V$$

$$\therefore\ r_o = \left(\frac{I_D}{L_{eff}}\frac{dx_d}{dV_{DS}}\right)^{-1} = \left(\frac{20\ \mu A}{0.589\ \mu m}\times0.061\ \mu m/V\right)^{-1}$$

$$= 482.78\ k\Omega$$

(d) $C_{gs} = \frac{2}{3}WL_{eff}C_{ox} + C_{OL}$

$$= \frac{2}{3}\times10\times0.589\times4.32\times10^{-15} + 0.35\times10^{-15}\times10$$

$$= 20.5\ fF$$

(e) $C_{gd} = C_{OL} = 0.35\times10^{-15}\times10 = 3.5\ fF$

(f) $C_{db} = \dfrac{C_{jo}\times(\text{drain area}) + C_{jswo}\times\text{periphery}}{\sqrt{1+V_{DB}/\phi_o}}$

$$= \frac{0.2\times10\times1 + 1.2\,(10+1\times2)}{\sqrt{1+(3/0.7)}}\ fF$$

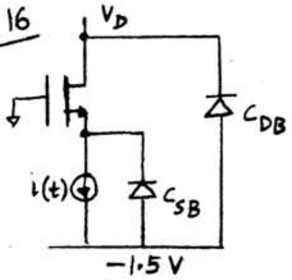
$$= 7.1\ fF$$

(g) $C_{sb} = \dfrac{C_{jo}\times(\text{source area}) + C_{jswo}\times\text{periphery}}{\sqrt{1+V_{SB}/\phi_o}}$

$$= \frac{0.2\times10\times1 + 1.2\,(10+1\times2)}{\sqrt{1+(1/0.7)}}\ fF$$

$$= 10.5\ fF$$

2.16



$V_D(t=0) = V_s(t=0) = 1.5\,V$

(i) Initially transistor is off, current source discharges $C_{OL} + C_{SB}$. The rate of voltage change is,

$$\frac{V}{t} = \frac{I}{C}$$

$$\simeq \frac{10\,\mu A}{0.35\,\frac{fF}{\mu m} \times 10\,\mu m + 0.2\,\frac{fF}{\mu m^2} \times 10\,\mu m \times (1\,\mu m + 0.09\,\mu m) + 1.2\,\frac{fF}{\mu m}(12\,\mu m)}$$

$$= 500\ V/\mu sec$$

(ii) Transistor enters saturation region ; drain current starts to flow, discharges $C_{OL} + C_{DB}$ and $\frac{2}{3} C_{ox} WL + C_{OL} + C_{SB}$. The discharge rate of drain voltage is

$$\frac{V}{t} \simeq \frac{10\,\mu A}{C_{OL} + C_{DB}}$$

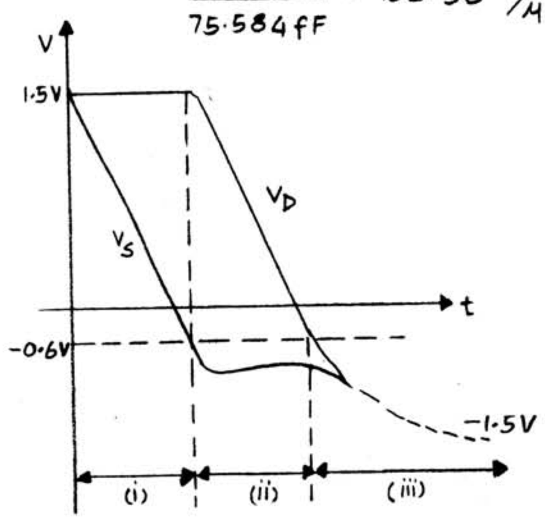
$$= \frac{10\,\mu A}{0.35\,\frac{fF}{\mu m} \times 10\,\mu m + 0.2\,\frac{fF}{\mu m^2} \times 10\,\mu m \times (1\,\mu m + 0.09\,\mu m) + 1.2\,\frac{fF}{\mu m}(12\,\mu m)}$$

$$= 500\ V/\mu s$$

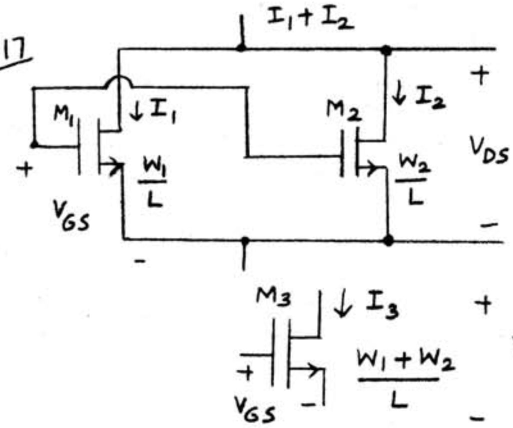
(iii) Transistor enters triode region, current discharges $C_{ox} WL + C_{DB} + C_{SB} + 2\,C_{OL}$ at a rate of,

$$\frac{V}{t} \simeq \frac{10\,\mu A}{\{4.32(10)(1-2(0.09)) + 0.2(10)(1+0.09)\,2 + 1.2(12)\,2 + \underbrace{0.35(2)(10)}_{\text{2 overlaps}}\}}$$

$$= \frac{10\,\mu A}{75.584\,fF} = 132.30\ V/\mu s$$



2.17



Note that all the transistors have equal terminal voltages. so,

$$V_{GS_1} = V_{GS_2} = V_{GS_3} = V_{GS}$$

$$V_{DS_1} = V_{DS_2} = V_{DS_3} = V_{DS}$$

$$V_{SB_1} = V_{SB_2} = V_{SB_3} = V_{SB}$$

If $V_{SB} \neq 0$, there is a body effect but $V_{t_1} = V_{t_2} = V_{t_3} = V_t$

Case 1 : All active

$$I_1 = \frac{k'}{2} \frac{W_1}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$I_2 = \frac{k'}{2} \frac{W_2}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$I_1 + I_2 = \frac{k'}{2} \left(\frac{W_1}{L} + \frac{W_2}{L} \right) (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$I_3 = \frac{k'}{2} \left(\frac{W_1 + W_2}{L} \right) (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$
$$= I_1 + I_2$$

Case 2 : All triode

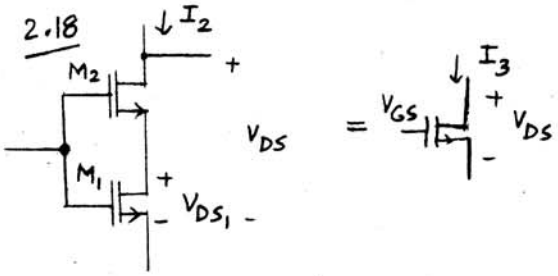
$$I_1 = \frac{k'}{2} \frac{W_1}{L} \left[2(V_{GS} - V_t) V_{DS} - V_{DS}^2 \right]$$

$$I_2 = \frac{k'}{2} \frac{W_2}{L} \left[2(V_{GS} - V_t) V_{DS} - V_{DS}^2 \right]$$

$$I_1 + I_2 = \frac{k'}{2} \left(\frac{W_1}{L} + \frac{W_2}{L} \right) \left[2(V_{GS} - V_t) V_{DS} - V_{DS}^2 \right]$$

$$I_3 = \frac{k'}{2} \left(\frac{W_1 + W_2}{L} \right) \left[2(V_{GS} - V_t) V_{DS} - V_{DS}^2 \right]$$
$$= I_1 + I_2$$

2.18



M_2 can operate in active or triode region. M_1 always operates in triode region

$$I_1 = \frac{k'}{2}\left(\frac{W}{L}\right)_1 \left[2(V_{GS}-V_t)V_{DS_1} - V_{DS_1}^2\right] = I_2$$

solve for V_{DS_1},

$$V_{DS_1}^2 - 2(V_{GS}-V_t)V_{DS_1} + \frac{2I_2}{k'\left(\frac{W}{L}\right)_1} = 0$$

$$V_{DS_1} = \frac{2(V_{GS}-V_t) \pm \sqrt{4(V_{GS}-V_t)^2 - 4\frac{2I_2}{k'\left(\frac{W}{L}\right)_1}}}{2}$$

V_{DS_1} must be $< V_{GS}-V_t$ or M_1 would be in active region.

$$V_{DS_1} = V_{GS}-V_t - \sqrt{(V_{GS}-V_t)^2 - \frac{2I_2}{k'\left(\frac{W}{L}\right)_1}}$$

Assume M_2 is active,

$$I_2 = \frac{k'}{2}\left(\frac{W}{L}\right)_2 (V_{GS}-V_{DS_1}-V_t)^2$$

$$= \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left[V_{GS}-V_t-(V_{GS}-V_t)+\sqrt{(V_{GS}-V_t)^2 - \frac{2I_2}{k'\left(\frac{W}{L}\right)_1}}\right]^2$$

$$= \frac{k'}{2}\left(\frac{W}{L}\right)_2 (V_{GS}-V_t)^2 - \left(\frac{W}{L}\right)_2 \frac{I_2}{\left(\frac{W}{L}\right)_1}$$

$$I_2\left(1+\left(\frac{W}{L}\right)_2 \Big/ \left(\frac{W}{L}\right)_1\right) = \frac{k'}{2}\left(\frac{W}{L}\right)_2 (V_{GS}-V_t)^2$$

$$I_2 = \left[\frac{k'}{2}\left\{\left(\frac{W}{L}\right)_2 \Big/ 1 + \left(\frac{W}{L}\right)_2 \Big/ \left(\frac{W}{L}\right)_1\right\}\right](V_{GS}-V_t)^2$$

$$= \frac{k'}{2}\left(\frac{W}{L_1+L_2}\right)(V_{GS}-V_t)^2$$

If M_2 and M_3 are active,

$$I_3 = \frac{k'}{2}\left(\frac{W}{L_1+L_2}\right)(V_{GS}-V_t)^2 = I_2$$

Assume M_2 is in triode region,

$$I_2 = \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left[2(V_{GS}-V_{DS_1}-V_t)(V_{DS}-V_{DS_1}) - (V_{DS}-V_{DS_1})^2\right]$$

$$V_{DS_1} = V_{GS}-V_t - \sqrt{(V_{GS}-V_t)^2 - \frac{2I_2}{k'\left(\frac{W}{L}\right)_1}}$$

$$I_2 = \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left\{2\sqrt{(V_{GS}-V_t)^2 - \frac{I_2}{k'\left(\frac{W}{L}\right)_1}}\left[V_{DS} - (V_{GS}-V_t) + \sqrt{(V_{GS}-V_t)^2 - \frac{2I_2}{k'\left(\frac{W}{L}\right)_1}}\right] - \left[V_{DS}-(V_{GS}-V_t) + \sqrt{(V_{GS}-V_t)^2 - \frac{2I_2}{k'\left(\frac{W}{L}\right)_1}}\right]^2\right\}$$

Let $X = V_{DS} - (V_{GS}-V_t)$ and

$$Y = \sqrt{(V_{GS}-V_t)^2 - \frac{I_2}{k'\left(\frac{W}{L}\right)_1}}$$

Then,

$$I_2 = \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left[2Y(X+Y) - (X+Y)^2\right]$$

$$I_2 = \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left[2XY + 2Y^2 - X^2 - Y^2 - 2XY\right]$$

$$I_2 = \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left[Y^2 - X^2\right]$$

$$= \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left[(V_{GS}-V_t)^2 - \frac{I_2}{k'\left(\frac{W}{L}\right)_1} - \{V_{DS}-(V_{GS}-V_t)\}^2\right]$$

$$= \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left[(V_{GS}-V_t)^2 - \frac{I_2}{k'\left(\frac{W}{L}\right)_1} - V_{DS}^2 - 2V_{DS}(V_{GS}-V_t) + (V_{GS}-V_t)^2\right]$$

$$I_2\left(1+\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}\right) = \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left\{-V_{DS}^2 + 2V_{DS}(V_{GS}-V_t)\right\}$$

$$I_2 = \frac{k'}{2}\frac{W}{L_1+L_2}\left[2(V_{GS}-V_t)V_{DS} - V_{DS}^2\right] = I_3$$

2.19

(a) Curve B ;

(i) The cost of a 40,000 mil^2 chip is

$$C = \frac{C_W}{(N Y_{ws}) Y_{df} Y_{ft}} = \frac{100}{(47)(10.9)(0.8)} + \frac{0.6}{0.8}$$

$$= 3.71$$

$N Y_{ws} = 47$ is obtained from fig.(2.70)

(ii) The cost of two 20,000 mil^2 chips is,

$$C_{Y_2} = \frac{100}{(200)(0.9)(10.8)} + \frac{0.6}{0.8} = 1.44$$

$$C_{total} = C_{1/2} \times 2 = 2.88$$

∴ Putting the system on two chips is more economical

(b) Curve A ;

(i) For a 40,000 mil^2 chip,

$$C = \frac{100}{(115)(0.9)(0.8)} + \frac{0.6}{0.8} = 1.1$$

$$C_{total} = 1.1 \times 2 = 2.2$$

∴ one chip is more economical.

(c) Curve C ;

(i) For a 40,000 mil^2 chip,

$$C = \frac{100}{12(0.9)(0.8)} + \frac{0.6}{0.8} = 12.3$$

(ii) For two 20,000 mil^2 chips,

$$C_{1/2} = \frac{100}{(80)(0.9)(0.8)} + \frac{0.6}{0.8} = 2.49$$

$$C_{total} = 2.49 \times 2 = 4.98$$

∴ using two chips is more economical.

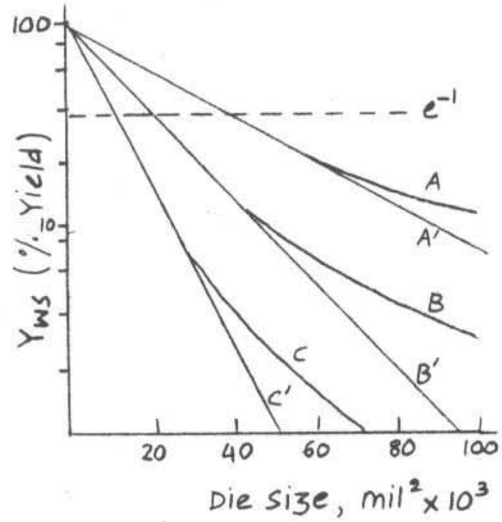
2.20

(a) When $Y_{ws} = e^{-1} = 0.37$

From curve A, $A_o = 38{,}000\ mil^2$

curve B, $A_o = 20{,}000\ mil^2$

curve C, $A_o = 11{,}000\ mil^2$



curves A', B' and C' are predicted by the equation.

(b) In fig (2.69), gross die/wafer is inversely proportional to the die size, i.e., $N = KA^{-1}$, K is the proportionality constant related to the wafer size (more specifically, K is the effective or usable area on the wafer). By (2.56), the cost per unit silicon area is,

$$\frac{C}{A} = \frac{C_W}{A N Y_{ws} Y_{df} Y_{ft}} + \frac{C_P}{A Y_{ft}}$$

$$= \frac{C_W}{K e^{-A/A_o} Y_{df} Y_{ft}} + \frac{C_P}{A Y_{ft}}$$

Each K can be obtained from fig (2.69). e.g. $K \doteq 1.15 \times 10^5\ mil^2$ for 4' wafer.

2.21

Area = (150 mils)^2 = 22,500 mil^2

(a) curve A,

$$C = \frac{C_W}{(NY_{WS})(Y_{df})(Y_{ft})} + \frac{C_P}{Y_{ft}}$$

$$= \frac{130}{(360)(0.8)(0.8)} + \frac{0.4}{0.8} = 1.06$$

$NY_{WS} = 360$ is obtained from Fig.(2.70)

(b) curve B,

$$C = \frac{130}{(140)(0.8)(0.8)} + \frac{0.4}{0.8} = 1.95$$

(c) curve C,

$$C = \frac{130}{(60)(0.8)(0.8)} + \frac{0.4}{0.8} = 3.89$$

2.22

First estimate x_d and L_{eff}

$$L_{eff} = L_{drawn} - 2L_d - x_d \quad (1)$$

$$x_d = \sqrt{\frac{2\epsilon_{si}(V_{DS} - V_{ov})}{q(N_A + N_i)}} \quad (2)$$

$$I_D = \frac{k'_n}{2}\frac{W}{L_{eff}}(V_{GS} - V_t)^2 \quad (3)$$

x_d, L_{eff} and $V_{ov} = V_{GS} - V_t$ can be found by solving equations (1), (2) and (3). However, solving these nonlinear equations is difficult. Instead, we approximately solve them. First, estimate x_d by substituting $L_{drawn} - 2L_d$ for L_{eff} in (3) and use that result in (2).

From (3) and using values in Table 2.6:

$$\begin{aligned} V_{ov} &= V_{GS} - V_t = \sqrt{\frac{2I_D}{k'_n W/L_{eff}}} \\ &\approx \sqrt{\frac{2(100\mu\text{A})}{(538\frac{\mu\text{A}}{\text{V}^2})\frac{0.5\mu\text{m}}{0.1\mu\text{m}-2(0.005\mu\text{m})}}} \\ &= 0.27\text{V} \end{aligned}$$

where $k'_n = \mu_n C_{ox} = 390\frac{\text{cm}^2}{\text{V-s}}(\frac{13.8\text{fF}}{\mu\text{m}^2})\frac{10^4\mu\text{m}}{1\text{cm}} = 538\mu\text{A/V}^2$ and $C_{ox} = 13.8\frac{\text{fF}}{\mu\text{m}^2}$ (based on $t_{ox} = 25$ Angstroms) have been used.

Using (2) and $V_{DS} = 0.6$ V:

$$\begin{aligned} x_d &= \sqrt{\frac{2(11.6)(8.86\times10^{-14})(0.6-0.27)}{1.6\times10^{-19}(1\times10^{17}+5\times10^{17})}}\ \text{cm} \\ &= 0.027\mu\text{m} \end{aligned}$$

From (1),

$$L_{eff} = 0.1\mu\text{m} - 2(0.005\mu\text{m}) - 0.027\mu\text{m} = 0.063\mu\text{m}$$

Now we can find g_m:

$$\begin{aligned} g_m &= \sqrt{2k'_n(W/L_{eff})I_D} \\ &= \sqrt{2(538\mu\text{A/V}^2)(0.5/0.063)100\mu\text{A}} \\ &= 924\mu\text{A/V} \end{aligned}$$

Next we calculate r_o:

$$\begin{aligned} r_o &= [(I_D/L_{eff})\partial x_d/\partial \text{V}_{\text{DS}}]^{-1} \\ &= [(100\mu\text{A}/0.063\mu\text{m})0.06\mu\text{m/V}]^{-1} \\ &= 10.5k\Omega \end{aligned}$$

Since the transitor is active (or saturated):

$$\begin{aligned} C_{gs} &= \frac{2}{3}WL_{eff}C_{ox} + C_{ol} \\ &= \frac{2}{3}(0.5\mu\text{m})(0.063\mu\text{m})13.8\text{fF}/\mu\text{m}^2 \\ &\quad + (0.5\mu\text{m})0.10\text{fF}/\mu\text{m} \\ &= 0.29\text{fF} + 0.05\text{fF} = 0.34\text{fF} \end{aligned}$$

and

$$\begin{aligned} C_{gd} &= C_{ol} \\ &= (0.5\mu\text{m})0.10\text{fF}/\mu\text{m} \\ &= 0.05\text{fF} \end{aligned}$$

The gate-leakage current can be estimated by

$$\begin{aligned} I_G &\approx J_G W L \\ &\approx (1.2\ \text{nA}/\mu\text{m}^2)(0.50\mu\text{m})[0.1\mu\text{m} - 2(0.005\mu\text{m})] \\ &= 54\text{pA} \end{aligned}$$

2.23

First estimate x_d and L_{eff}

$$L_{eff} = L_{drawn} - 2L_d - x_d \qquad (1)$$

$$x_d = \sqrt{\frac{2\epsilon_{si}(V_{DS} - V_{ov})}{q(N_A + N_i)}} \qquad (2)$$

$$I_D = \frac{k_n'}{2}\frac{W}{L_{eff}}(V_{GS} - V_t)^2 \qquad (3)$$

x_d, L_{eff} and $V_{ov} = V_{GS} - V_t$ can be found by solving equations (1), (2) and (3). However, solving these nonlinear equations is difficult. Instead, we approximately solve them. First, estimate x_d by substituting $L_{drawn} - 2L_d$ for L_{eff} in (3) and use that result in (2).

From (3) and using values in Table 2.5:

$$\begin{aligned} V_{ov} &= V_{GS} - V_t = \sqrt{\frac{2I_D}{k_n' W/L_{eff}}} \\ &\approx \sqrt{\frac{2(100\mu\text{A})}{(246\frac{\mu\text{A}}{\text{V}^2})\frac{0.9\mu\text{m}}{0.2\mu\text{m}-2(0.01\mu\text{m})}}} \\ &= 0.40\text{V} \end{aligned}$$

where $k_n' = \mu_n C_{ox} = 300\frac{\text{cm}^2}{\text{V-s}}(8.21\frac{\text{fF}}{\mu\text{m}^2})\frac{10^4\mu m}{1\text{cm}} = 246\mu\text{A/V}^2$ and $C_{ox} = 8.21\frac{\text{fF}}{\mu\text{m}^2}$ (based on $t_{ox} = 42$ Angstroms) have been used.

Using (2) and $V_{DS} = 1.0$ V:

$$\begin{aligned} x_d &= \sqrt{\frac{2(11.6)(8.86\times10^{-14})(1-0.40)}{1.6\times10^{-19}(8\times10^{16}+1\times10^{17})}}\ \text{cm} \\ &= 0.065\mu\text{m} \end{aligned}$$

From (1),

$$L_{eff} = 0.2\mu\text{m} - 2(0.01\mu\text{m}) - 0.065\mu\text{m} \approx 0.12\mu\text{m}$$

Now we can calculate g_m:

$$\begin{aligned} g_m &= \sqrt{2k_n'(W/L_{eff})I_D} \\ &= \sqrt{2(246\mu\text{A/V}^2)(0.9/0.12)100\mu\text{A}} \\ &= 607\mu\text{A/V} \end{aligned}$$

Next we find r_o:

$$\begin{aligned} r_o &= [(I_D/L_{eff})\partial x_d/\partial \text{V}_{\text{DS}}]^{-1} \\ &= [(100\mu\text{A}/0.12\mu\text{m})0.028\mu\text{m/V}]^{-1} \\ &= 43k\Omega \end{aligned}$$

Since the transitor is active (or saturated):

$$\begin{aligned} C_{gs} &= \frac{2}{3}WL_{eff}C_{ox} + C_{ol} \\ &= \frac{2}{3}(0.9\mu\text{m})(0.12\mu\text{m})8.21\text{fF}/\mu\text{m}^2 \\ &\quad + (0.9\mu\text{m})0.36\text{fF}/\mu\text{m} \\ &= 0.59\text{fF} + 0.32\text{fF} = 0.91\text{fF} \end{aligned}$$

and

$$\begin{aligned} C_{gd} &= C_{ol} \\ &= (0.9\mu\text{m})0.36\text{fF}/\mu\text{m} \\ &= 0.32\text{fF} \end{aligned}$$