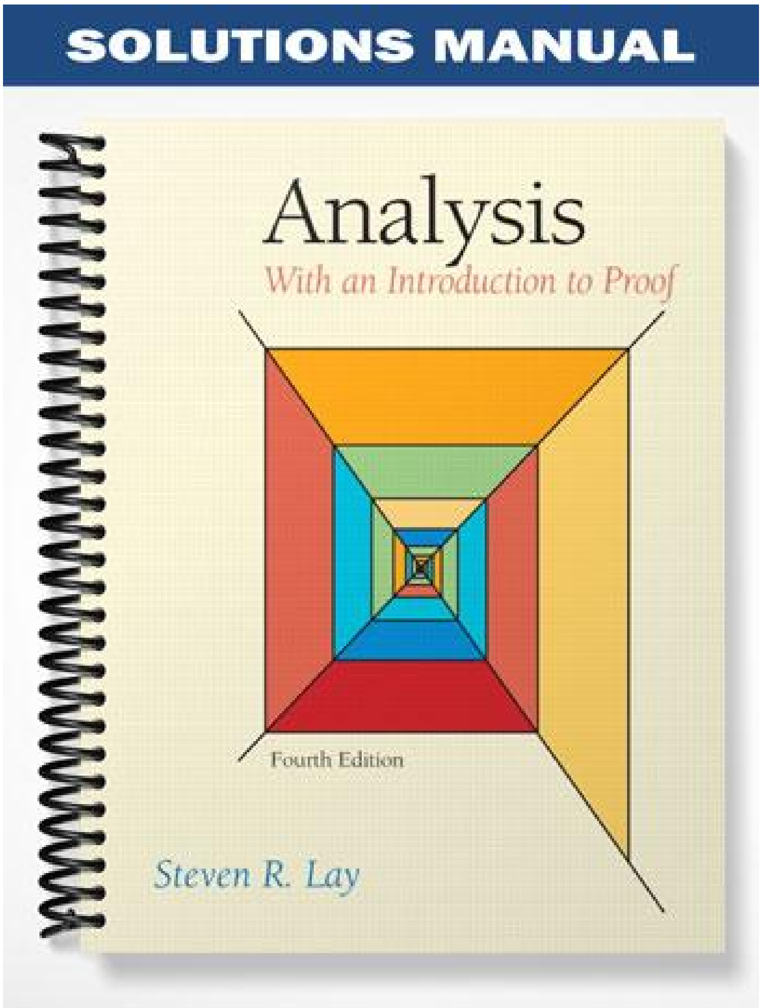
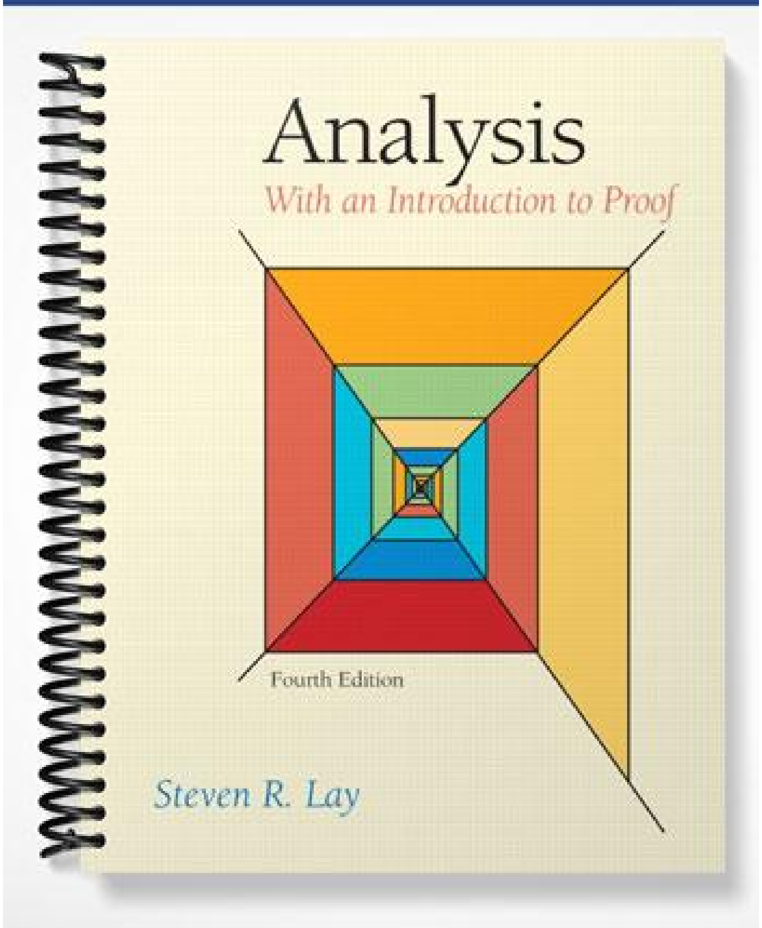

SOLUTIONS MANUAL
Analysis
With an Introduction to Proof
Fourth Edition
Steven R. Lay

SOLUTIONS MANUAL



Instructor's Solutions Manual
to accompany

ANALYSIS
with an Introduction to Proof

4th Edition

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This manual is intended to accompany Analysis with an Introduction to Proof (4^{th} edition) by Steven R. Lay [Prentice-Hall, 2005]. It contains solutions to nearly every exercise in the text. Those exercises that have hints (or answers) in the back of the book are numbered in **bold** print, and the hints are included here for reference. While many of the proofs have been given in full detail, some of the more routine proofs are only outlines. For some of the problems, other approaches may be equally acceptable. This is particularly true for those problems requesting a counterexample. I have not tried to be exhaustive in discussing each exercise, but rather to be suggestive.

Let me remind you that the starred exercises are not necessarily the more difficult ones. They are the exercises that are used in some way in subsequent sections. There is a table on page 2 that indicates where starred exercises are used later. The following notations are used throughout this manual:

$\mathbb{N}$ = the set of natural numbers $\{1, 2, 3, 4, \ldots\}$
$\mathbb{R}$ = the set of real numbers
$\forall$ = "for every"
$\exists$ = "there exists"
$\ni$ = "such that"

I have tried to be accurate in the preparation of this manual. Undoubtedly, however, some mistakes will inadvertently slip by. I would appreciate having any errors in this manual or the text brought to my attention.

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Table of Starred Exercises

Note: The prefix P indicates a practice problem, the prefix E indicates an example, the prefix T refers to a theorem or corollary, and the absence of a prefix before a number indicates an exercise.

Starred Exercise	Later Use	Starred Exercise	Later use
5.26	T13.11	18.14	19.5
6.10	8.26	19.10	33.14
7.32	9.3	19.16	34.9
10.3	E29.7	19.17	T34.3
10.4	29.7	20.14	26.8
10.6	E32.1	20.16	T26.9
10.7	18.10, 18.15, E32.7, T36.9	20.18	21.14, 22.15
10.8	P32.3	21.10	T30.8
10.22	16.7f, E22.7	21.11	30.9b
10.25	12.14	21.13	T22.5, T25.7, 29.13
10.28b	12.11, E16.11, 18.14	21.16	36.15
11.6a	16.9a, T17.1, 26.23, 30.16, T36.9	22.13b	T26.8, T26.10
11.6b	T27.8	25.6	T26.9, 26.14, 26.19
11.6c	T16.14	25.8	31.13
11.7	T33.5	25.17b	28.9
12.7	T30.4, 30.3	26.8	T30.1
12.12	29.14, T30.4	27.13d	37.16
13.15	14.12, T18.12	29.12	P30.5
13.19	14.7	29.13	30.5
14.8	36.15	29.16	30.17
15.12	24.9	30.9a	P31.7
16.6b	E17.2	30.11	T33.13
16.7f	T17.7, 18.10, E32.7	30.15	31.20
16.9a	21.10, 36.17	30.20	E31.9
16.11	E18.4	32.7	E33.6
16.12	20.15	32.8	33.13
16.13	20.13	32.13a	37.8
16.15b	19.11, 19.18, 22.,12	33.12	36.7, 36.8
16.16	20.15	33.14	T34.4
17.17	E28.3	35.15a	36.9
17.18	20.14, T35.10		

Solutions to Exercises

Section 1

1.1 (a) False: a statement may be false.
(b) False: a statement cannot be both true and false.
(c) True: comment after Practice 1.4.
(d) True: comment before Example 1.3.
(e) False: if the statement is false, then its negation is true.

1.2 (a) False: p is the antecedent
(b) True: Practice 1.6(a).
(c) True: first paragraph on page 5.
(d) False: "p whenever q" is "if q, then p".
(e) False: the negation of $p \Rightarrow q$ is $p \wedge \sim q$.

1.3 (a) M is not a cyclic subgroup.
(b) The interval [0,3] is not finite.
(c) Either the relation R is not reflexive or it is not symmetric.
(d) The set S is not finite and it is not denumerable.
(e) $x > 3$ and $f(x) \leq 7$.
(f) f is continuous and A is connected, but $f(x)$ is not connected.
(g) K is compact and either K is not closed or K is not bounded.

1.4 (a) The relation R is not transitive.
(b) The set of rational numbers is not bounded.
(c) Either the function f is not injective or it is not surjective.
(d) $x \geq 5$ and $x \leq 7$
(e) x is in A and $f(x)$ is in B.
(f) f is continuous, but either $f(x)$ is not closed or $f(x)$ is not bounded.
(g) K is closed and bounded, but K is not compact.

1.5 (a) Antecedent: M has a zero eigenvalue; consequent: M is singular.
(b) Antecedent: regularity; consequent: normality.
(c) Antecedent: it is Cauchy; consequent: a sequence is bounded.
(d) Antecedent: $x = 5$; consequent: $f(x) = 14$.

1.6 (a) Antecedent: $5n$ is odd; consequent: n is odd.
(b) Antecedent: it is monotone and bounded; consequent: a sequence is convergent.
(c) Antecedent: it is convergent; consequent: a real sequence is Cauchy.
(d) Antecedent: convergence; consequent: boundedness.

1.7 and 1.8 are routine.

1.9 (a) True (b) True (c) False (d) True (e) True (f) False (g) False (h) True
(i) True (j) True

1.10 (a) False (b) True (c) False (d) False (e) True (f) True (g) True (h) False
(i) False (j) True

1.11 (a) $\sim q \wedge p$ (b) $(p \vee q) \wedge \sim (p \wedge q)$ (c) $(p \vee q) \wedge \sim q$ (d) $\sim p \Rightarrow q$ (e) $p \Leftrightarrow \sim q$

1.12 (a) $\sim p \wedge q$ (b) $\sim(p \vee q)$ or $\sim p \wedge \sim q$ (c) $q \Rightarrow \sim p$ (d) $q \Rightarrow p$ (e) $q \Rightarrow p$

1.13 (a) and (b) are routine. (c) $p \wedge q$

1.14 These truth tables are all straightforward. Note that the tables for (c) through (f) have 8 rows because there are 3 letters, and therefore $2^3 = 8$ possible combinations of T and F.

Section 2

2.1 (a) True: comment before Example 2.1.
(b) False: the negation of a universal statement is an existential statement.
(c) True: comment before Example 2.1.

2.2 (a) False: it means there exists at least one.
(b) True: Example 2.1.
(c) True: comment after Practice 2.4.

2.3 (a) Some road in Yellowstone is not open.
(b) All fish are not green.
(c) There exists an even integer that is prime.
(d) $\forall\, x < 3, x^2 < 10$.
(e) $\exists\, x$ in $A \ni \forall\, y < k$, $f(y) \le 0$ or $f(y) \ge f(x)$.
(f) $n > N$ and $\exists\, x$ in $S \ni |f_n(x) - f(x)| \ge \varepsilon$.

2.4 (a) Some light is not on.
(b) All basketball players are Central High are not short.
(c) There exists a bounded interval that contains infinitely many integers.
(d) $\forall\, x$ in S, $x < 5$.
(e) $\exists\, x \ni 0 < x < 1$ and $3 \le f(x) \le 5$.
(f) $x > 5$ and $\forall\, y > 0$, $x^2 \le 25 + y$.

2.5 The True/False part of the answers is given as a hint.
(a) True. Let $x = 0$. Then given any y, let $z = y$. (A similar argument works for any x.)
(b) False. Given any x, let $y = -x$ and $z = 1$.
(c) True. Let $z = y - x$.
(d) False. Let $x = 0$ and $y = 1$. (It is a true statement for $x \neq 0$.)
(e) True. Let $x \le 0$.
(f) True. Take $z \le y$. This makes "$z > y$" false so that the implication is true. Or, choose $z > x + y$.

2.6 (a) True. Given x and y, let $z = x + y$.
(b) False. Let $x = 0$. Then given any y, let $z = y + 1$.
(c) True. Let $x = 1$. Then given any y, let $z = y$.
(d) False. Let $x = 1$ and $y = 0$.
(e) False. Let $x = 2$. Given any y, let $z = y + 1$. Then "$z > y$" is true, but "$z > x + y$" is false.
(f) True. Given any x and y, either choose $z > x + y$ or $z \le y$.

2.7 **(a)** You can use (*ii*) to prove (a) is true.
(b) You can use (*i*) to prove (b) is true.
(c) You can use (*ii*) to prove (c) is false.
(d) You can use (*i*) to prove (d) is false.

2.8 The best answer is (c).

2.9 (a) True (b) True (c) True (d) False (e) True (f) True (g) True (h) False

2.10 (a) True (b) False (c) True (d) False (e) False (f) False (g) True (h) True

2.11 (a) $\forall\, x,\ f(-x) = f(x)$; (b) $\exists\, x \ni f(-x) \neq f(x)$.

2.12 (a) $\exists\ k > 0 \ni \forall\, x,\ f(x+k) = f(x)$.
(b) $\forall\ k > 0, \exists\, x \ni f(x+k) \neq f(x)$.

2.13 (a) $\forall\, x$ and $\forall\, y,\ x \leq y \Rightarrow f(x) \leq f(y)$.
(b) $\exists\, x$ and $\exists\, y \ni x \leq y$ and $f(x) > f(y)$.

2.14 (a) $\forall\, x$ and $\forall\, y,\ x < y \Rightarrow f(x) > f(y)$.
(b) $\exists\, x$ and $\exists\, y \ni x < y$ and $f(x) \leq f(y)$.

2.15 (a) $\forall\, x$ and $\forall\, y,\ f(x) = f(y) \Rightarrow x = y$.
(b) $\exists$ x and $\exists\, y \ni f(x) = f(y)$ and $x \neq y$.

2.16 (a) $\forall\, y$ in B $\exists\, x$ in $A \ni f(x) = y$.
(b) $\exists\, y$ in $B \ni \forall\, x$ in A, $f(x) \neq y$.

2.17 (a) $\forall\ \varepsilon > 0, \exists\ \delta > 0 \ni \forall\, x \in D, |x - c| < \delta \Rightarrow |f(x) - f(c)| < \varepsilon$.
(b) $\exists\ \varepsilon > 0 \ni \forall\ \delta > 0, \exists\, x \in D \ni |x - c| < \delta$ and $|f(x) - f(c)| \geq \varepsilon$.

2.18 (a) $\forall\ \varepsilon > 0\ \exists\ \delta > 0 \ni \forall\, x$ in S and $\forall\, y$ in S, $|x - y| < \delta \Rightarrow |f(x) - f(y)| < \varepsilon$.
(b) $\exists\ \varepsilon > 0 \ni \forall\ \delta > 0, \exists\, x$ in S and $\exists\, y$ in $S \ni |x - y| < \delta$ and $|f(x) - f(y)| \geq \varepsilon$.

2.19 (a) $\forall\ \varepsilon > 0, \exists\ \delta > 0 \ni \forall\, x \in D,\ 0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$.
(b) $\exists\ \varepsilon > 0 \ni \forall\ \delta > 0, \exists\, x \in D \ni 0 < |x - c| < \delta$ and $|f(x) - L| \geq \varepsilon$.

2.20 Answers will vary.

Section 3

3.1 (a) False: p is the hypothesis.
(b) False: the contrapositive is $\sim q \Rightarrow \sim p$.
(c) False: the inverse is $\sim p \Rightarrow \sim q$.
(d) False: $p(n)$ must be true for <u>all</u> n.
(e) True: Example 3.1.

3.2 (a) True: comment in first paragraph on page 18.
(b) False: it's called a contradiction.
(c) True: comment after Practice 3.8.
(d) True: end of Example 3.1.
(e) False: must show $p(n)$ is true for all n.

3.3 (a) If some violets are not blue, then some roses are not red.
(b) If H is not normal, then H is regular.
(c) If K is not compact, then either K is not closed or K is not bounded.

3.4 (a) If all violets are blue, then all roses are red.
(b) If H is normal, then H is not regular.
(c) If K is compact, then K is closed and bounded.

3.5 (a) If some roses are not red, then some violets are not blue.
(b) If H is regular, then H is not normal.
(c) If K is not closed or K is not bounded, then K is not compact.

3.6 (a) $x = -3$ (b) $n = 40$ or $n = 41$ (c) an equilateral triangle (d) 101, 103, etc.
(e) 2 (f) $n = 1$ or any odd n (g) Let $x = 2$ and $y = 18$. (h) 0 has no reciprocal.
(i) If $0 < x < 1$, then $x^3 < x^2$. In particular, $(1/2)^3 = 1/8 < 1/4 = (1/2)^2$.
(j) The reciprocal of 1 is not less than 1.
(k) $3^5 + 2 = 245$ is not prime. (l) $x = 0$ (m) $x = -1$

3.7 (a) Suppose $p = 2k + 1$ and $q = 2r + 1$ for integers k and r. Then
$p + q = (2k + 1) + (2r + 1) = 2(k + r + 1)$, so $p + q$ is even.
(b) Suppose $p = 2k + 1$ and $q = 2r + 1$ for integers k and r. Then
$pq = (2k + 1)(2r + 1) = 4kr + 2k + 2r + 1 = 2(2kr + k + r) + 1$, so pq is odd.
(c) Suppose $p = 2k + 1$ and $q = 2r$ for integers k and r. Then
$p + q = (2k + 1) + 2r = 2(k + r) + 1$, so $p + q$ is odd.
(d) Suppose $p = 2k$ and $q = 2r$ for integers k and r. Then
$p + q = 2k + 2r = 2(k + r)$, so $p + q$ is even.
(e) Suppose $p = 2k$, then $pq = 2(kq)$, so pq is even. A similar argument applies when q is even.
(f) This is the contrapositive of part (e).
(g) Hint in book: look at the contrapositive.
Proof: To prove the contrapositive, suppose $p = 2k + 1$. Then
$p^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, so p^2 is odd.
(h) To prove the contrapositive, suppose $p = 2k$. Then
$p^2 = (2k)^2 = 4k^2 = 2(2k^2)$, so p^2 is even.

3.8 Suppose $f(x_1) = f(x_2)$. That is, $3x_1 - 5 = 3x_2 - 5$. Then $3x_1 = 3x_2$, so $x_1 = x_2$.

3.9 (a)

$r \Rightarrow \sim s$	hypothesis
$\sim s \Rightarrow \sim t$	contrapositive of hypothesis: 3.12(c)
$r \Rightarrow \sim t$	by 3.12(l)

(b)

$\sim t \Rightarrow (\sim r \vee \sim s)$	contrapositive of hypothesis: 3.12(c)
$\sim r \vee \sim s$	by 3.12(h)
$\sim s$	by 3.12(j)

(c)

$r \vee \sim r$	by 3.12(d)
$\sim s \Rightarrow \sim v$	contrapositive of hypothesis 4 [3.12(c)]
$r \Rightarrow \sim v$	hypothesis 1 and 3.12(l)
$\sim r \Rightarrow u$	hypotheses 2 and 3 and 3.12(l)
$\sim v \vee u$	by 3.12(o)

3.10 (a)

$\sim r$	hypothesis
$\sim r \Rightarrow (r \vee \sim s)$	contrapositive of hypothesis: 3.12(c)
$r \vee \sim s$	by 3.12(h)
$\sim s$	by lines 1 and 3, and 3.12(j)

(b)

$\sim t$	hypothesis
$\sim t \Rightarrow (\sim r \wedge \sim s)$	contrapositive of hypothesis: 3.12(c)
$\sim r \wedge \sim s$	by lines 1 and 2, and 3.12(h)
$\sim s$	by line 3 and 3.12(k)

(c)

$s \Rightarrow r$	contrapositive of hypothesis: 3.12(c)
$t \Rightarrow u$	hypothesis
$s \vee t$	hypothesis
$r \vee u$	by 3.12(o)

3.11 Let p: The basketball center is healthy. q: The point guard is hot.
r: The team will win. s: The fans are happy.
t: The coach is a millionaire. u: The college will balance the budget.
The hypotheses are $(p \vee q) \Rightarrow (r \wedge s)$ and $(s \vee t) \Rightarrow u$. The conclusion is $p \Rightarrow u$.