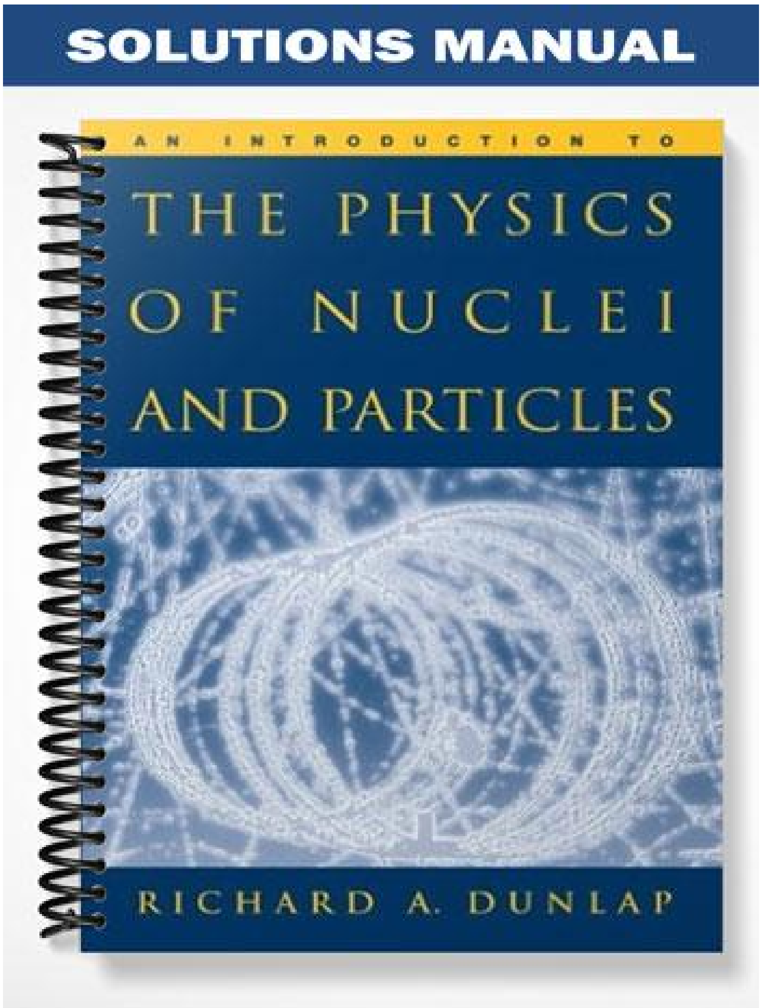
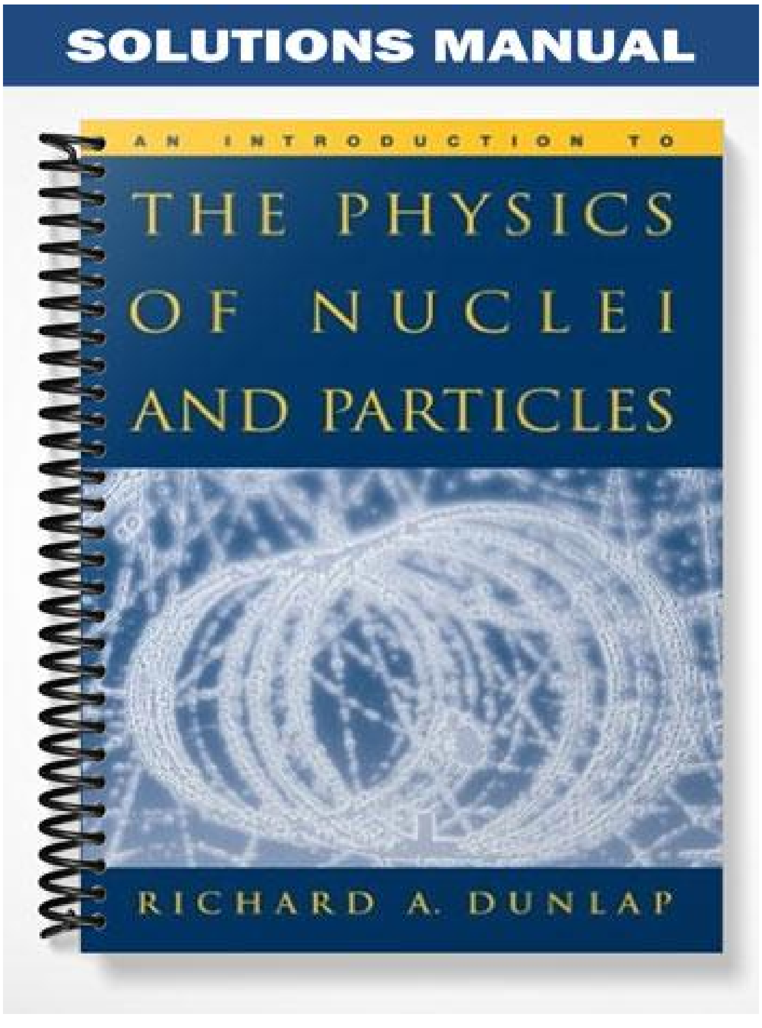

SOLUTIONS MANUAL
AN INTRODUCTION TO
THE PHYSICS OF NUCLEI AND PARTICLES
RICHARD A. DUNLAP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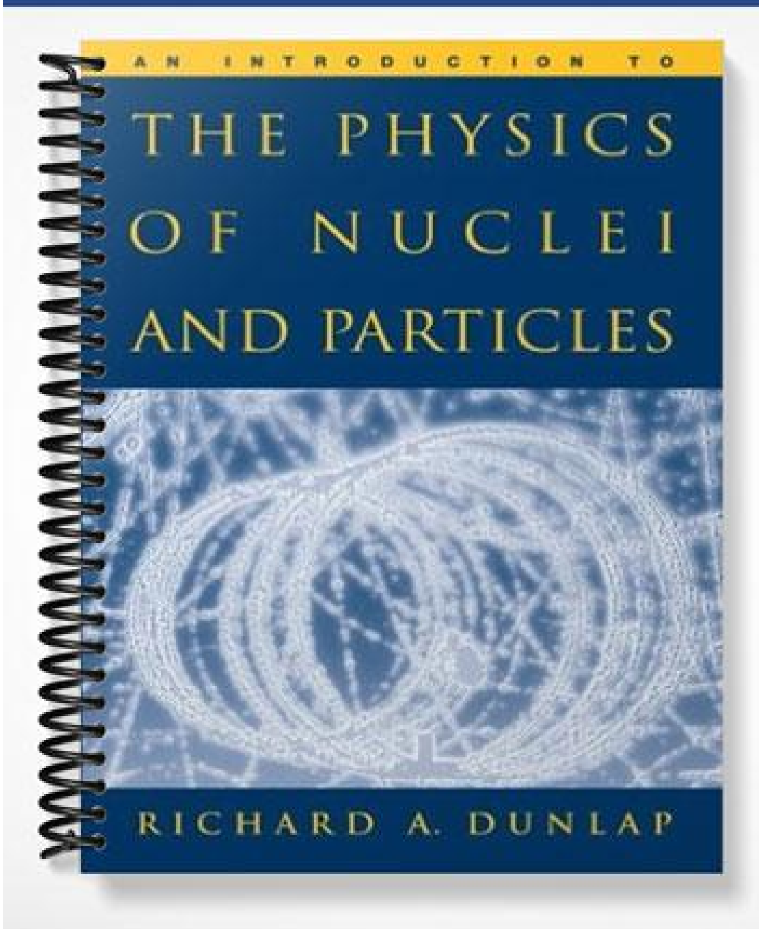

Solutions to Problems in

Chapter 3: Nuclear Composition and Size

3.1. (a) The initial kinetic energy, E_0, is equal to the Coulombic potential at the point of closest approach, so

$$E_0 = \frac{Zze^2}{4\pi\varepsilon_0 d}$$

Then

$$d = \frac{Zze^2}{4\pi\varepsilon_0 E_0}$$

(b) Using E_0 = 8 MeV, Z = 79, z = 2 and the value of the Coulomb constant

$$\frac{e^2}{4\pi\varepsilon_0} = 1.44 \quad \text{MeV·fm}$$

we find b = 28.4 fm.

3.2. Data may be analyzed on the basis of equation (3.12). This may be written in terms of the incident energy, E, as

$$\frac{d\sigma}{d\Omega} = \left(\frac{Zze^2}{4\pi\varepsilon_0}\right)^2 \left(\frac{1}{4E}\right)^2 \csc^4\left(\frac{\theta}{2}\right).$$

Measurements are made for a fixed angle, θ, as a function of energy. Since the detector will subtend a sold angle, Ω, the total cross section for scattering into the detector will be

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi \left(\frac{Zze^2}{4\pi\varepsilon_0}\right)^2 \left(\frac{1}{4E}\right)^2 \int \csc^4\left(\frac{\theta}{2}\right) \sin\theta d\theta\ .$$

The number of scattered particles observed will be proportional to σ and hence to E^{-2}.

3.3. The minimum energy will occur for an impact parameter b = 0. In this case the initial kinetic energy, E_0, is equal to the Coulomb potential when the distance between the nuclei is such that their surfaces are just in contact. This distance is

$$d = R_\alpha + R_{\text{Au}}$$

Using $R_0 = 1.2 * A^{1/3}$ fm then

$$d = 1.2 * \left[4^{1/3} + 197^{1/3}\right] = 8.89 \text{ fm.}$$

Equating the energies gives

$$E_0 = \frac{Zze^2}{4\pi\varepsilon_0 d}$$

Using $Z = 79$ and $z = 2$ gives the energy $E_0 = 25.6$ MeV.

3.4. (a) The scattering angle, θ, is related to the impact parameter, b, as

$$\tan\frac{\theta}{2} = \frac{Zze^2}{4\pi\varepsilon_0 m{v_0}^2 b} .$$

Solving for b and expressing the velocity in terms of the initial kinetic energy, E_0;

$$b = \frac{e^2}{4\pi\varepsilon_0}\frac{Zz}{2E_0 \tan\frac{\theta}{2}} .$$

Using $Z = 79$, $z = 2$, $E_0 = 8$ MeV and θ, = 90° gives

$$b = 14.2 \text{ fm.}$$

(b) Conservation of energy gives

$$E_0 = \frac{1}{2}m{v_c}^2 + \frac{e^2}{4\pi\varepsilon_0}\frac{Zz}{r_c}$$

where the subscript c denotes the point of closest approach. Conservation of angular momentum gives

$$mv_0 b = mv_c r_c .$$

Solving for v_c and substituting into the expression for E_0 gives

$$E_0 = E_0\frac{b^2}{{r_c}^2} + \frac{e^2}{4\pi\varepsilon_0}\frac{Zz}{r_c} .$$

Substituting $E_0 = 8$ MeV and $b = 14.2$ fm gives a quadratic in r_c;

$$8{r_c}^2 - 228r_c - 1618 = 0 .$$

Solving for r_c gives $r_c = 34.4$ fm.

(c) The kinetic energy will be

$$E_c = \frac{1}{2}m{v_c}^2 = E_0\frac{b^2}{{r_c}^2} .$$

Substituting numerical values gives $E_c = 1.36$ MeV.

3.5. The radius of a ^{208}Pb nucleus is $R_0 = 1.2{*}A^{1/3} = 7.11$ fm. The volume (assuming a well defined edge at R_0) is

$$V = \frac{4}{3}\pi R_0^{\ 3} = 1505 \text{ fm}^3$$

The number of nucleons is 208 so assuming a uniform density inside R_0 the number of nucleons per unit volume is

$$\rho(0) = \frac{A}{V} = 0.138 \text{ nucleons/fm}^3.$$

This is consistent with the more detailed picture, Figure 3.9. Since one nucleon has a mass of ≈1 u then

$$\rho(0) = 0.138 \text{ u/fm}^3.$$

In more conventional units this is

$$\rho(0) = 2.3\times 10^{-25} \text{ g/fm}^3$$

or

$$\rho(0) = 2.3\times 10^{14} \text{ g/cm}^3$$

3.6. (a) All three nuclei have $\rho(0) \approx 0.16$ fm^{-3}. The values of r_{90} and r_{10} are found so that

$$\rho(r_{90}) = 0.9\times 0.16 = 0.144 \text{ fm}^{-3}$$
$$\rho(r_{10}) = 0.1\times 0.16 = 0.016 \text{ fm}^{-3}.$$

The width of the surface region is then given as $r_{10} - r_{90}$. Reading values from the graph gives values in the table.

nucleus	r_{10} (fm)	r_{90} (fm)	($r_{10} - r_{90}$) (fm)
^{16}O	3.6	1.2	2.4
^{118}Sn	6.3	3.9	2.4
^{197}Au	7.7	5.3	2.4

(b) Using equation (2.4) and assuming

$$r_{90} = R_0 - 1.2\times 2.4 = R_0 - 1.2 \text{ (fm)}$$

we find that

$$r_{10} = R_0 + 1.2 \text{ (fm)}.$$

This gives

$$\rho_{90} = \frac{0.16}{1+\exp\left[-\frac{1.2}{a}\right]}$$

and

$$\rho_{10} = \frac{0.0144}{1+\exp\left[+\frac{1.2}{a}\right]}.$$

Either equation may be solved to give a = 0.55 fm.

3.7. (a) We write equation (3.9) as

$$\sigma = \pi\left(\frac{Zze^2}{4\pi\varepsilon_0}\right)^2\left(\frac{1}{4E}\right)^2\cot^2\left(\frac{\theta}{2}\right)$$

Using Z=79 and z=2. This gives

$$\sigma = 3.14\left(\frac{79\cdot 2\cdot 1.44}{2\cdot 10}\right)^2\cot^2\left(\frac{\theta}{2}\right) = 406\cdot\cot^2\left(\frac{\theta}{2}\right) \qquad \text{fm}^2$$

For the angles given we find

θ (°)	σ (fm^2)
1	5.3×10^6
5	2.1×10^5
20	1.3×10^4

(b) We write equation (3.12) as

$$\frac{d\sigma}{d\Omega} = \left(\frac{Zze^2}{4\pi\varepsilon_0}\right)^2\left(\frac{1}{4E}\right)^2\csc^4\left(\frac{\theta}{2}\right)$$

And substituting values as above

$$\frac{d\sigma}{d\Omega} = 32.4\cdot\csc^4\left(\frac{\theta}{2}\right) \qquad \text{fm}^2\text{sr}^{-1}$$

This gives the following results

θ (°)	σ ($\text{fm}^2\text{sr}^{-1}$)
1	5.6×10^9
5	8.9×10^6
20	3.6×10^4

3.8. The relativistic scattering cross section is given as

$$\left(\frac{d\sigma}{d\Omega}\right)_{rel} = \left(\frac{d\sigma}{d\Omega}\right)_{nonrel}\left[1-\frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right].$$

We define the relative size of the relativistic correction as

$$f = \frac{\left| \left(\frac{d\sigma}{d\Omega}\right)_{rel} - \left(\frac{d\sigma}{d\Omega}\right)_{nonrel} \right|}{\left(\frac{d\sigma}{d\Omega}\right)_{nonrel}} = \frac{v^2}{c^2}\sin^2\frac{\theta}{2} .$$

For $E = 0.1$ MeV then $mv^2/2 << mc^2$ and we calculate

$$\frac{v^2}{c^2} = \frac{2\left(\frac{1}{2}mv^2\right)}{c^2} = \frac{0.2}{0.511} = 0.39 .$$

For $E = 1$ MeV or 100 MeV the $E > mc^2$ so $v \approx c$ giving $v^2/c^2 = 1$. Using these values in the above expression gives the results in the table.

E (MeV)	v^2/c^2	θ (°)	f
0.1	0.39	20	0.011
0.1	0.39	90	0.195
1.0	1.0	20	0.12
1.0	1.0	90	0.5
100	1.0	20	0.12
100	1.0	90	0.5

3.9. We define the central part of the nucleus as $r < r_{90}$. The volume of the central region is therefore $\frac{4}{3}\pi r_{90}{}^3$ and the number of nucleons in this volume is

$$N_{core} = \rho_0 \frac{4}{3}\pi r_{90}{}^3$$

The fraction of surface nucleons will be

$$f = \frac{N_{surf}}{N_{total}} = \frac{N_{total} - N_{core}}{N_{total}} = \frac{A - N_{core}}{A}$$

Reading values from the appropriate graphs gives the values in the table.

nucleus	r_{90} (fm)	N_{core}	f
^{16}O	1.2	1.2	0.92
^{118}Sn	3.9	40	0.63
^{197}Au	5.3	100	0.52

3.10. (a) Conservation of energy gives

$$m_\alpha (v_{\alpha i}{}^2 - v_{\alpha f}{}^2) = m_A v_{Af}{}^2 .$$

This may be rearranged to give

$$m_\alpha \left(v_{\alpha i} - v_{\alpha f}\right)\left(v_{\alpha i} + v_{\alpha f}\right) = m_{\mathrm{A}} v_{\mathrm{A}f}{}^2 .$$

Conservation of momentum gives

$$m_\alpha \left(v_{\alpha i} - v_{\alpha f}\right) = m_{\mathrm{A}} v_{\mathrm{A}f} .$$

Combining these expressions yields

$$v_{\alpha i} + v_{\alpha f} = v_{\mathrm{A}f} .$$

Using this with the conservation of momentum equation gives

$$v_{\mathrm{A}f} = 2\frac{m_\alpha v_{\alpha i}}{m_\alpha + m_{\mathrm{A}}} .$$

The recoil energy (kinetic energy) is obtained from this as

$$R_{\mathrm{A}} = \frac{2 m_{\mathrm{A}} m_\alpha{}^2 v_{\alpha i}{}^2}{\left(m_\alpha + m_{\mathrm{A}}\right)^2} .$$

Using

$$E_{\alpha i} = \frac{1}{2} m_\alpha v_{\alpha i}{}^2$$

this may be rewritten as

$$R_{\mathrm{A}} = \frac{4 m_\alpha m_{\mathrm{A}} E_{\alpha i}}{\left(m_\alpha + m_{\mathrm{A}}\right)^2} .$$

(b) Using $m_\alpha = 4.0015$ u and $E_{\alpha i} = 10$ MeV we obtain the following results:

nucleus	m_{A} (u)	R_{A} (MeV)
^{16}O	15.995	6.4
^{118}Sn	117.902	1.27
^{197}Au	196.967	0.78