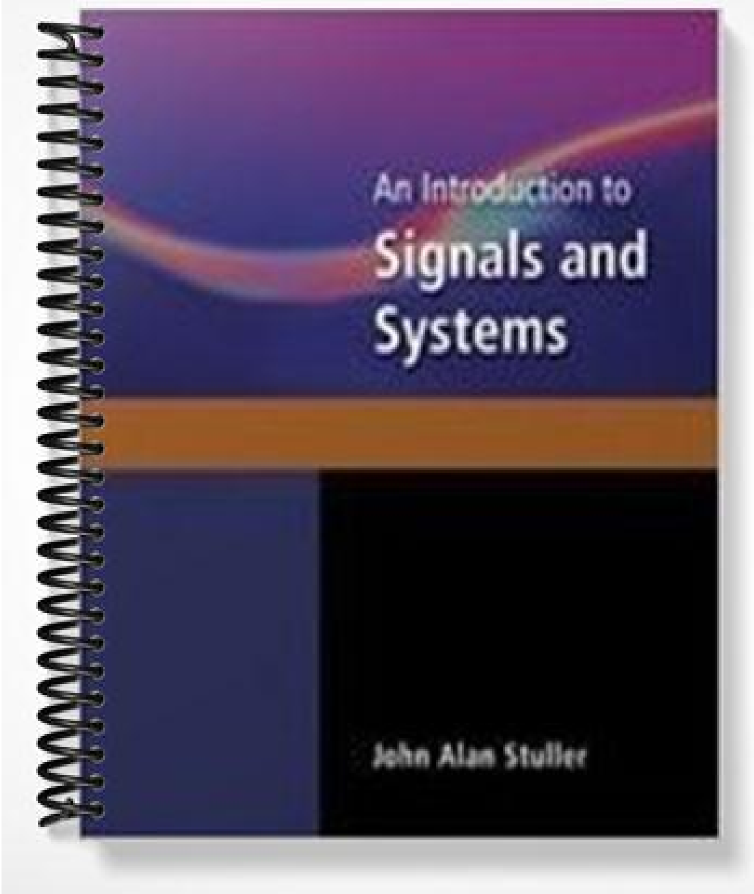
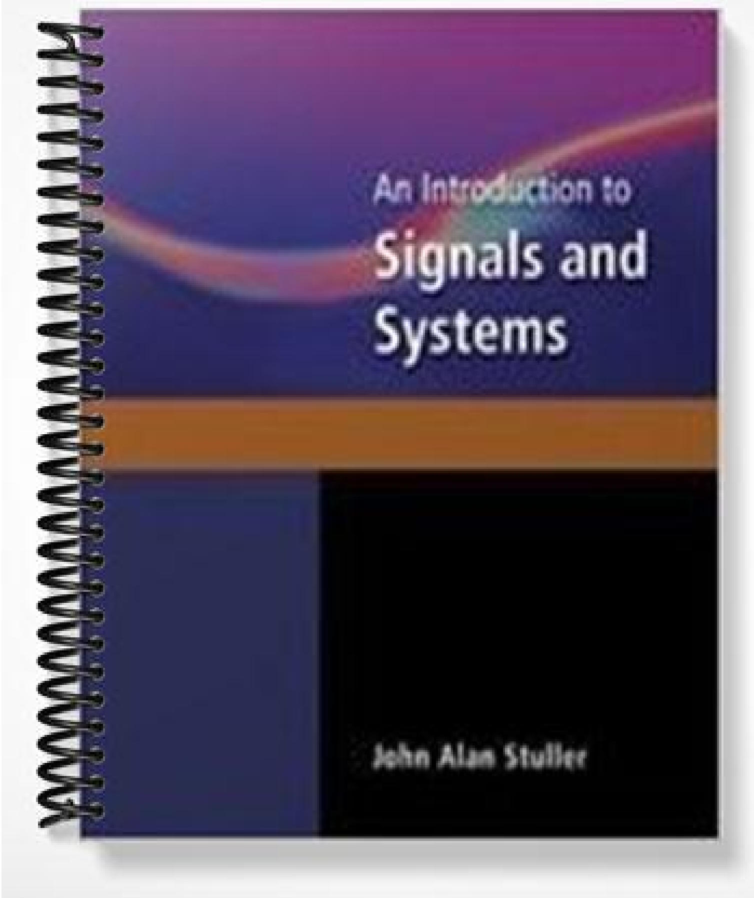
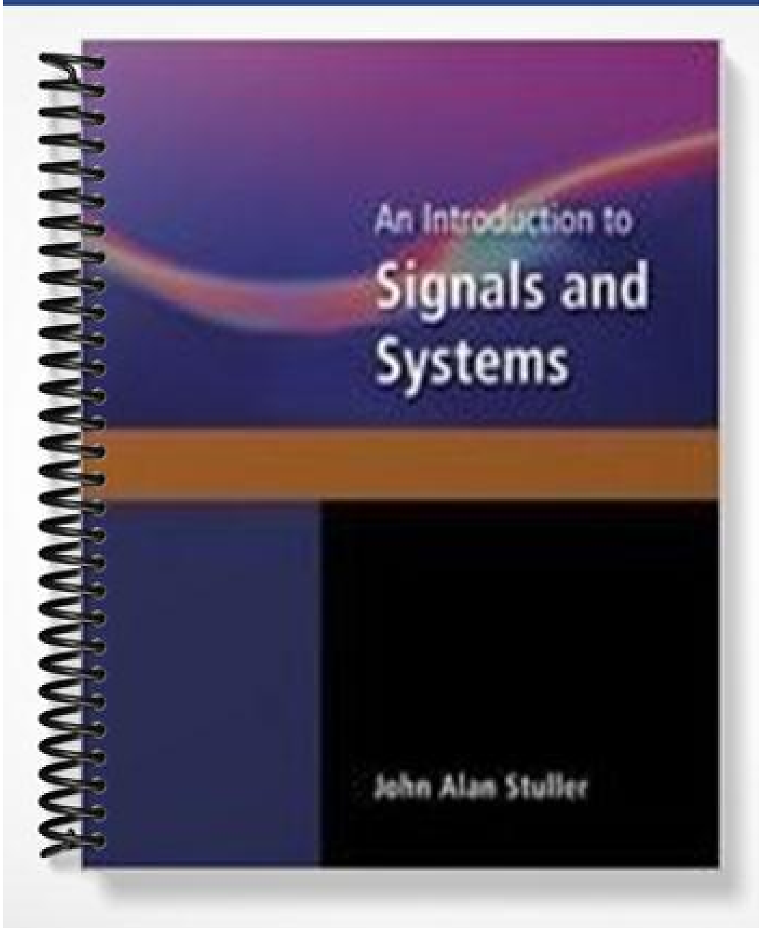
SOLUTIONS MANUAL



SOLUTIONS MANUAL



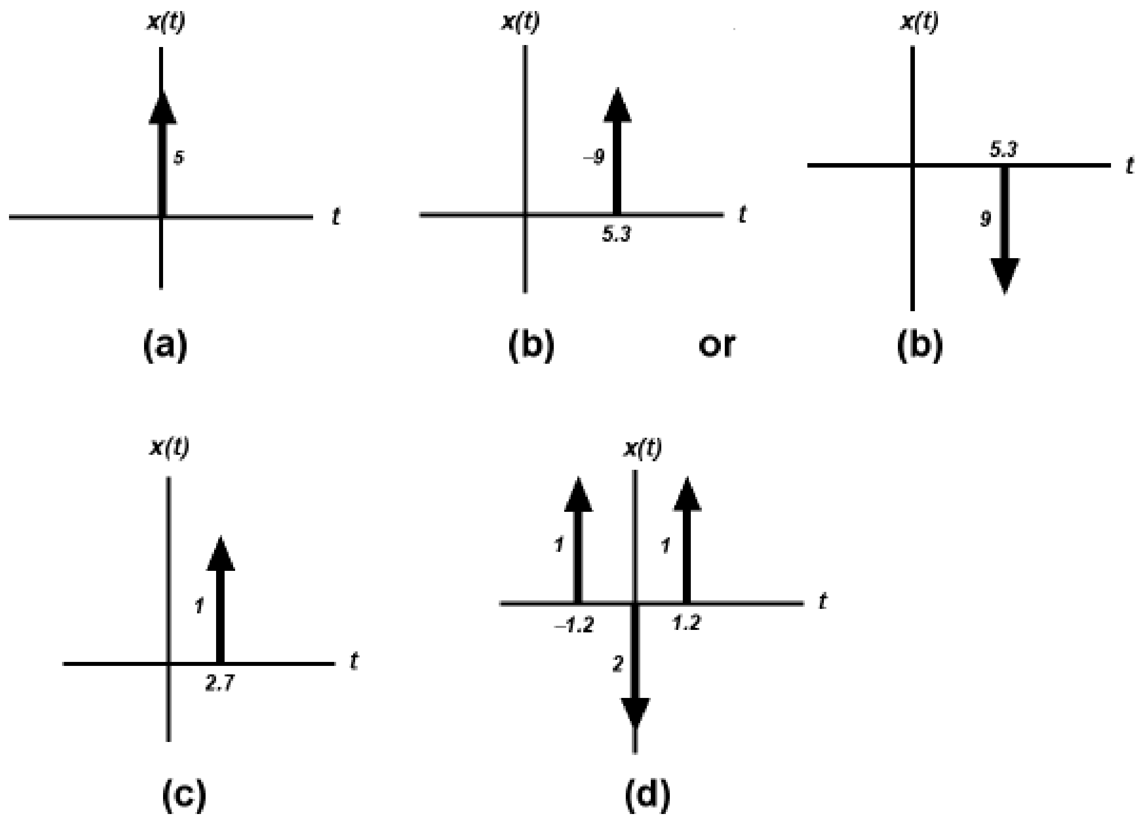
SOLUTIONS MANUAL



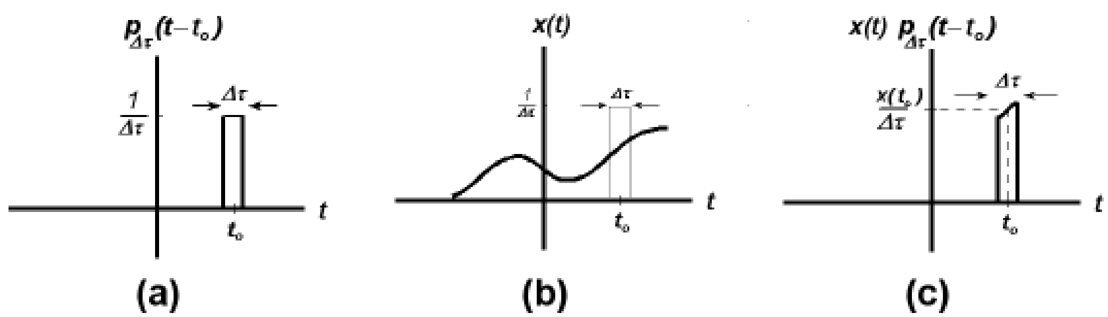
CHAPTER 2CT

2CT.2 - UNIT IMPULSE

2CT.2.1



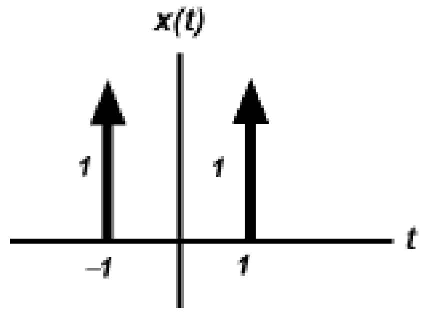
2CT.2.2



(d) We can see from plot (c) that the limit $\Delta\tau \to 0$, $x(t)p_{\Delta\tau}(t-t_o) = x(t_o)\delta(t-t_o)$. This result is consistent with the sampling property: $x(t)\delta(t-t_o) = x(t_o)\delta(t-t_o)$

2CT.2.3

$$t^2\delta(t-1) + \sin(t)\delta(t) + t^2\delta(t+1) = \delta(t-1) + \delta(t+1)$$



2CT.2.4

a) Let $x = 3t$

$$\int_{-\infty}^{\infty} \delta(3t)dt = \frac{1}{3}\int_{-\infty}^{\infty} \delta(x)dx = \frac{1}{3}$$

b) Let $x = at$
For $a > 0$

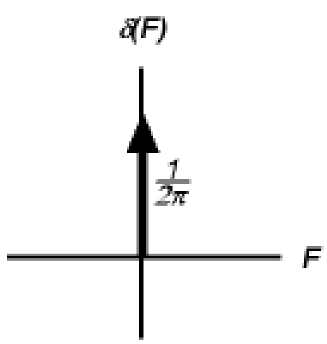
$$\int_{-\infty}^{\infty} \delta(at)dt = \frac{1}{a}\int_{-\infty}^{\infty} \delta(x)dx = \frac{1}{a}$$

For $a < 0$

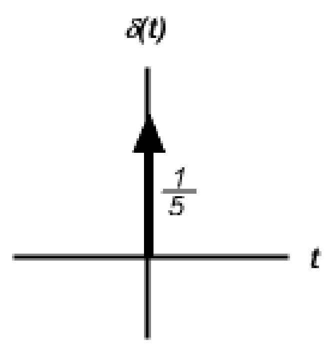
$$\int_{-\infty}^{\infty} \delta(at)dt = \frac{1}{a}\int_{\infty}^{-\infty} \delta(x)dx = -\frac{1}{a}\int_{-\infty}^{\infty} \delta(x)dx = -\frac{1}{a}$$

Therefore, for any $a \neq 0$,

$$\int_{-\infty}^{\infty} \delta(at)dt = \frac{1}{|a|}$$



(c)



(d)

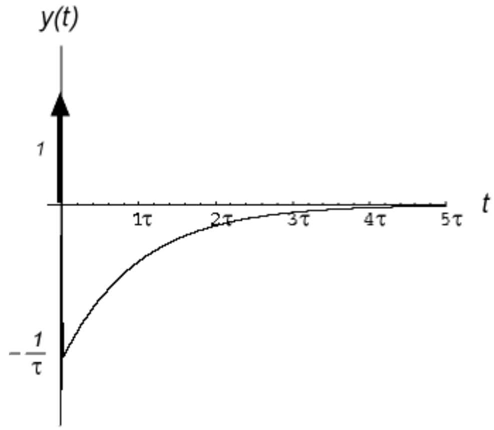
2CT.3 - IMPULSE RESPONSE

2CT.3.1

a)

$$y(t) = \frac{d}{dt}\left(e^{-\frac{t}{\tau}}u(t)\right) = e^{-\frac{t}{\tau}}\frac{d}{dt}u(t) + u(t)\frac{d}{dt}e^{-\frac{t}{\tau}} = \delta(t) - \frac{1}{\tau}e^{-\frac{t}{\tau}}u(t)$$

b)

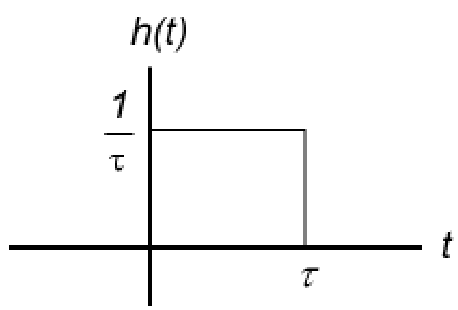


2CT.3.2

a)

$$h(t) = \frac{1}{\tau}\int_{t\text{-}\tau}^{t}\delta(\lambda)d\lambda = \frac{1}{\tau}\int_{-\infty}^{t}\delta(\lambda)d\lambda - \frac{1}{\tau}\int_{-\infty}^{t\text{-}\tau}\delta(\lambda)d\lambda = \frac{1}{\tau}[u(t) - u(t-\tau)]$$

b)

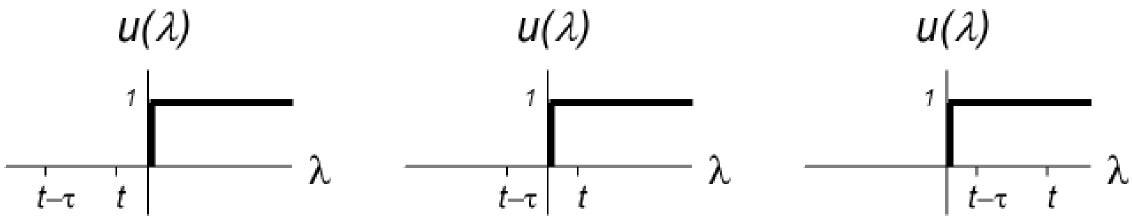


2CT.3.3

With $x(t) = u(t)$, (78) becomes

$$y(t) = \frac{1}{\tau}\int_{t\text{-}\tau}^{t}x(\lambda)d\lambda = \frac{1}{\tau}\int_{t\text{-}\tau}^{t}u(\lambda)d\lambda$$

Evaluation of the integral is facilitated by referring to the Figure below. The value of the integral is equal to the area under $u(\lambda)$ evaluated from $\lambda = t\text{-}\tau$ to $\lambda = t$. You can see from the figure that this area depends upon the value of t.



By inspection

$$\int_{t\text{-}\tau}^{t} u(\lambda)d\lambda = \begin{cases} 0; & t < 0 \\ t; & 0 \le t < \tau \\ \tau; & \tau \le t \end{cases}$$

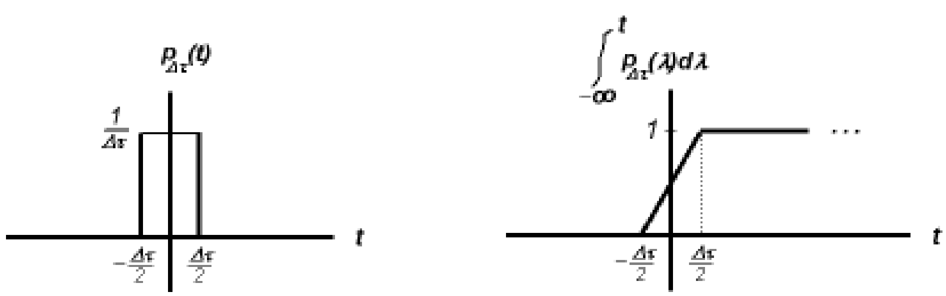
Therefore,

$$y(t) = \begin{cases} 0; & t < 0 \\ t/\tau; & 0 \le t < \tau \\ 1; & \tau \le t \end{cases}$$

2CT.3.4

$$h(t) = \frac{1}{\tau}[u(t) - u(t-\tau)]$$

2CT.3.5
a)



b) In the limit $\Delta\tau \to 0$, $p_{\Delta\tau}(t) = \delta(t)$ and $\int_{-\infty}^{t} p_{\Delta\tau}(\lambda)d\lambda = u(t)$.

2CT.4 - WAVEFORM REPRESENTED AS AN INTEGRAL OF SHIFTED IMPULSES

2CT.4.1

$$\int_{-\infty}^{\infty} g(t)\delta(t-t_o)dt = \int_{-\infty}^{\infty} g(t_o)\delta(t-t_o)dt = g(t_o)\int_{-\infty}^{\infty}\delta(t-t_o)dt = g(t_o)$$

2CT.4.2
a)

$$x(t) = \int_{-\infty}^{\infty} \sqrt{\tau}\,\delta(25-\tau)d\tau = 5$$

b)

$$x(t) = \int_{-\infty}^{\infty} e^{-\tau}\,u(\tau)\delta(t-\tau)d\tau = e^{-t}\,u(t)$$

2CT.5 - CONVOLUTION INTEGRAL

2CT.5.1
Linearity

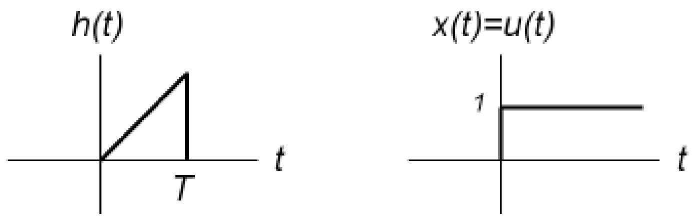
$$y(t) = \int_{-\infty}^{\infty} h(t-\lambda)(ax_1(\lambda)+bx_2(\lambda))d\lambda$$

$$= a\int_{-\infty}^{\infty} h(t-\lambda)x_1(\lambda)d\lambda + b\int_{-\infty}^{\infty} h(t-\lambda)x_2(\lambda)d\lambda$$

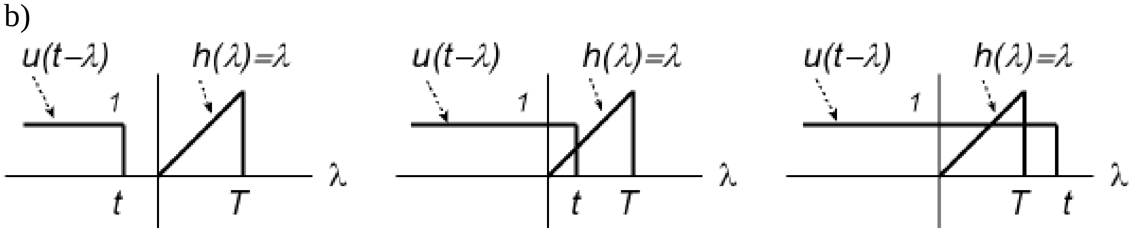
Time invariance. Let $\beta = \lambda - \tau$

$$z(t) = \int_{-\infty}^{\infty} h(t-\lambda)x(\lambda-\tau)d\lambda = \int_{-\infty}^{\infty} h(t-\tau-\beta)x(\beta)d\beta = y(t-\tau)$$

5.2
a)

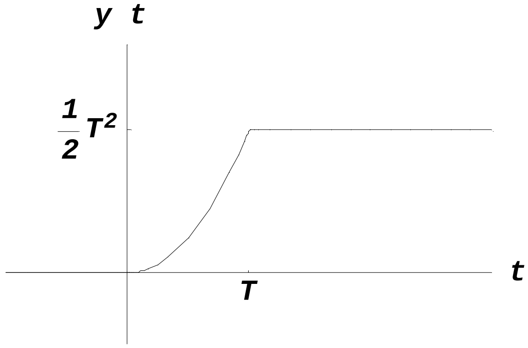


b)



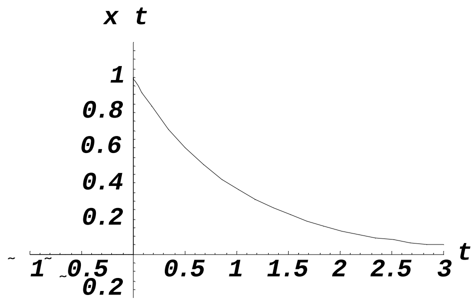
$$y(t) = \begin{cases} 0; & t<0 \\ \int_0^t \lambda d\lambda = \frac{1}{2}t^2; & 0<t<T \\ \frac{1}{2}T^2; & T \le t \end{cases}$$

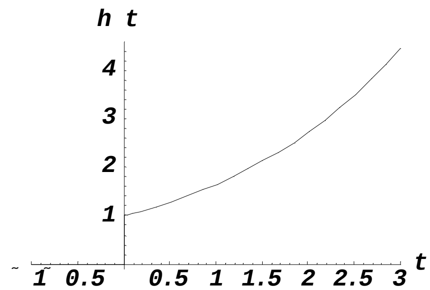
c)



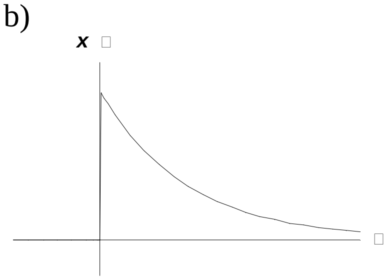
2CT.5.3

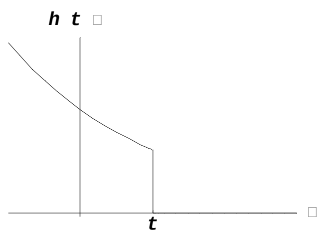
a)

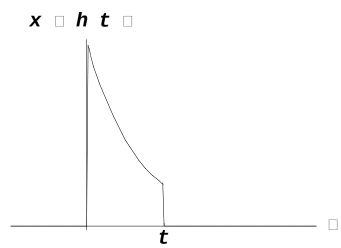




b)







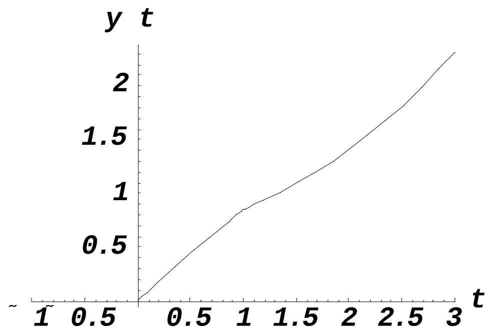
$t > 0$ in the above plots.

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda = \int_0^1 e^{-\lambda}e^{\frac{1}{2}(t-\lambda)}u((t-\lambda))d\lambda$$

$$= e^{\beta t}\int_0^1 e^{(-1-\frac{1}{2})\lambda}u((t-\lambda))d\lambda = \begin{cases} 0; & t < 0 \\ e^{\frac{1}{2}t}\int_0^t e^{-\frac{3}{2}\lambda}d\lambda; & 0 \le t < 1 \\ e^{\frac{1}{2}t}\int_0^1 e^{-\frac{3}{2}\lambda}d\lambda; & 1 \le t \end{cases}$$

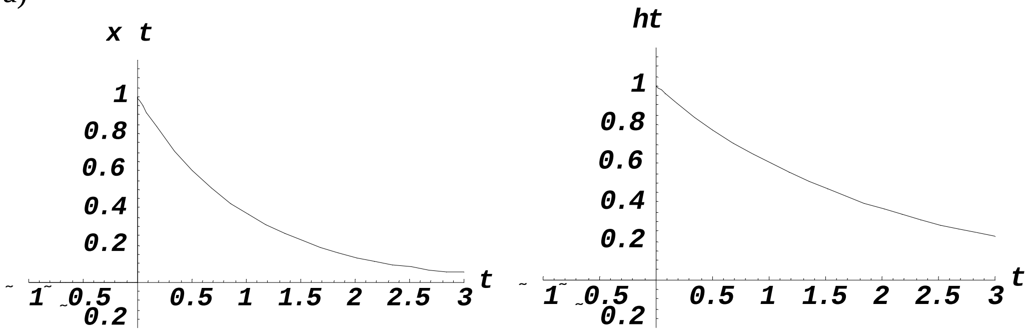
$$y(t) = \begin{cases} 0; & t < 0 \\ \frac{2}{3}e^{\frac{1}{2}t}\left(1 - e^{-\frac{3}{2}t}\right); & 0 \le t < 1 \\ \frac{2}{3}e^{\frac{1}{2}t}\left(1 - e^{-\frac{3}{2}}\right); & 1 \le t \end{cases}$$

c)



2CT.5.4

a)



b) The plots for the graphical method are analogous to those in problem 5.3.

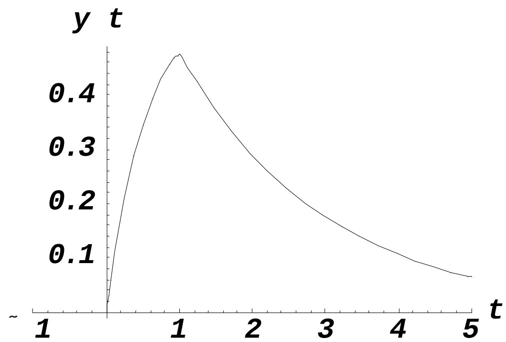
$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda = \int_0^1 e^{\alpha\lambda}e^{\beta(t-\lambda)}u((t-\lambda))d\lambda$$

$$= e^{\beta t}\int_0^1 e^{(\alpha-\beta)\lambda}u((t-\lambda))d\lambda = \begin{cases} 0; & t < 0 \\ e^{\beta t}\int_0^t e^{(\alpha-\beta)\lambda}d\lambda; & 0 \le t < 1 \\ e^{\beta t}\int_0^1 e^{(\alpha-\beta)\lambda}d\lambda; & 1 \le t \end{cases}$$

$$y(t) = \begin{cases} 0; & t < 0 \\ \frac{1}{\alpha-\beta}e^{\beta t}\left(e^{(\alpha-\beta)t} - 1\right); & 0 \le t < 1 \\ \frac{1}{\alpha-\beta}e^{\beta t}\left(e^{(\alpha-\beta)} - 1\right); & 1 \le t \end{cases}$$

For $\alpha = -1$ and $\beta = -\frac{1}{2}$ the above becomes

$$y(t) = \begin{cases} 0; & t < 0 \\ 2e^{-t/2}\left(1 - e^{-t/2}\right); & 0 \le t < 1 \\ 2e^{-t/2}\left(1 - e^{-1/2}\right); & 1 \le t \end{cases}$$



2CT.5.5

We can find $y(t)$ in a manner similar to that of problem 3. Alternatively, let's take the limit $\beta \to \alpha$ in the solution to problem 5.4

$$y(t) = \lim_{\beta \to \alpha} \begin{cases} 0; & t < 0 \\ \frac{1}{\alpha-\beta} e^{\beta t}\left(e^{(\alpha-\beta)t} - 1\right); & 0 \le t < 1 \\ \frac{1}{\alpha-\beta} e^{\beta t}\left(e^{(\alpha-\beta)} - 1\right); & 1 \le t \end{cases}$$

Set $\epsilon = \alpha - \beta$. L'Hôspital's rule gives us

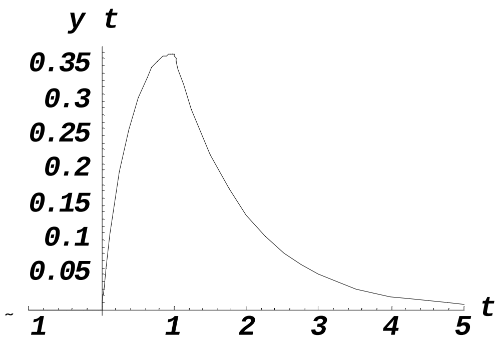
$$\lim_{\substack{\epsilon \to 0 \\ \beta \to \alpha}} \frac{1}{\epsilon} e^{\beta t}\left(e^{\epsilon t} - 1\right) = \lim_{\beta \to \alpha} \frac{\lim_{\epsilon \to 0} \frac{d}{d\epsilon} e^{\beta t}(e^{\epsilon t} - 1)}{\frac{d}{d\epsilon}\epsilon} = te^{\alpha t}$$

and

$$\lim_{\substack{\epsilon \to 0 \\ \beta \to \alpha}} \frac{1}{\epsilon} e^{\beta t}(e^{\epsilon} - 1) = \lim_{\beta \to \alpha} \frac{\lim_{\epsilon \to 0} \frac{d}{d\epsilon} e^{\beta t}(e^{\epsilon} - 1)}{\frac{d}{d\epsilon}\epsilon} = e^{\alpha t}$$

$$y(t) = \lim_{\beta \to \alpha} \begin{cases} 0; & t < 0 \\ te^{\alpha t}; & 0 \le t < 1 \\ e^{\alpha t}; & 1 \le t \end{cases}$$

A plot of $y(t)$ for $\alpha = -1$ follows.



2CT.5.6

$$y(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau = \int_{0}^{\infty} h(t-\tau)d\tau = \int_{-\infty}^{t} h(\lambda)d\lambda$$

where we let $\lambda = t - \tau$.

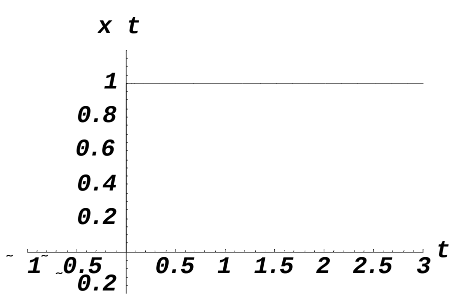
2CT.5.7

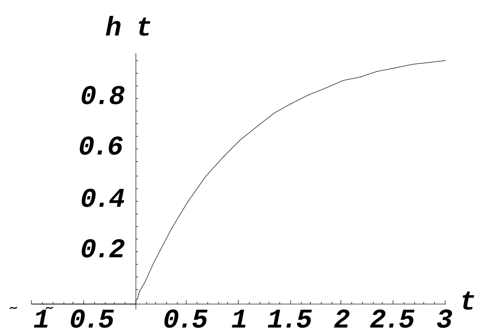
$$y(t) = x(t)*u(t) = \int_{-\infty}^{\infty} u(t-\lambda)x(\lambda)d\lambda = \int_{-\infty}^{\infty} u(t-\lambda)x(\lambda)d\lambda = \int_{-\infty}^{t} x(\lambda)d\lambda$$

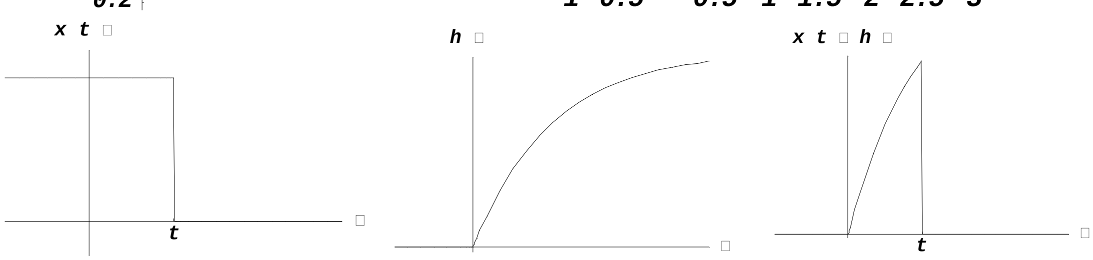
2CT.5.8

a) For this problem, we prefer to use the commutivity property to write the convolution as

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$







This plot assumes $t > 0$

We see that $y(t) = 0$ for $t < 0$. We see that for $t \geq 0$,

$$y(t) = \int_0^t h(\lambda)d\lambda = \int_0^t (1 - e^{-\lambda})d\lambda = t - \left(1 - e^{-t}\right)$$

Thus

$$y(t) = \left[t - \left(1 - e^{-t}\right)\right]u(t)$$

b) We can also use the result of problem 5.6 to write down

$$y(t) = \int_0^t h(\lambda)d\lambda$$

and continue as in Problem 5.8.

2CT.5.9

If we set $\lambda = t - \tau$ we obtain

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

Therefore,

$$x(t)*h(t) = h(t)*x(t)$$

2CT.5.10

a)

$$\int_{-\infty}^{\infty} x(t-\lambda)\delta(\lambda)d\lambda = x(t)$$

b)

$$\int_{-\infty}^{\infty} x(t-t_o-\lambda)\delta(\lambda)d\lambda = x(t-t_o)$$

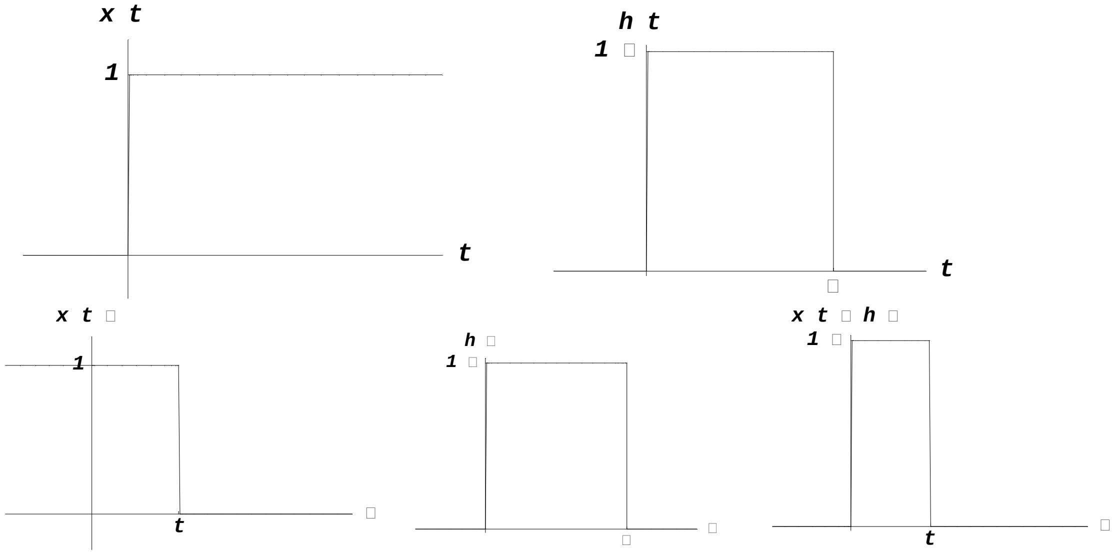
c)

$$\int_{-\infty}^{\infty} e^{j2\pi F(t-\lambda)}x(\lambda)d\lambda = e^{j2\pi Ft}\int_{-\infty}^{\infty} e^{-j2\pi F\lambda}x(\lambda)d\lambda = e^{j2\pi Ft}X(F)$$

where

$$X(F) = \int_{-\infty}^{\infty} e^{-j2\pi F\lambda}x(\lambda)d\lambda$$

2CT.5.11



Above plots assume $0 < t < \tau$

For $0 < t < \tau$ we see by inspection of the plot that x$(t-\lambda)h(\lambda)$ has area t/τ. Working similarly for $t \leq 0$ and for $t \geq \tau$ we obtain.

$$y(t) = \begin{cases} \frac{t}{\tau}u(t); & t < \tau \\ 1; & t \geq \tau \end{cases}$$

The plot of $y(t)$ is straightforward. Here is another method. From problem 3.2,

$$h(t) = \frac{1}{\tau}[u(t) - u(t-\tau)]$$

It follows from the superposition and time invariance properties of convolution that

$$y(t) = \frac{1}{\tau}u(t)*u(t) - \frac{1}{\tau}u(t)*u(t-\tau) = \frac{t}{\tau}u(t) - \frac{t-\tau}{\tau}u(t-\tau)$$

$$= \begin{cases} \frac{t}{\tau}u(t); & t < \tau \\ 1; & t \geq \tau \end{cases}$$

A third method is to use Eq (88).

2CT.5.12

We can interpret the expression $\delta(t-1.1)*\delta(t-2.2)*\delta(t-3.3)$ as the output of a system having impulse response $\delta(t-2.2)*\delta(t-3.3)$ driven by an input $\delta(t-1.1)$. The system is the cascade of two delay systems: one having delay 2.2 and the other having delay 3.3. Therefore, by inspection, the output is

$$\delta(t-1.1)*\delta(t-2.2)*\delta(t-3.3) = \delta(t-6.6)$$

2CT.5.13
No, the response of the same system to the input $x(2t)$ is, in general, not $y(2t)$. For example, assume that $x(t) = \delta(t)$. Then $y(t) = h(t)$ is the impulse response. The response to $x(2t) = \delta(2t)$ is

$$\int_{-\infty}^{\infty} h(t-\lambda)\delta(2\lambda)d\lambda = \int_{-\infty}^{\infty} h\left(t - \frac{1}{2}\beta\right)\delta(\beta)\frac{1}{2}d\beta = \frac{1}{2}h(t) \neq h(2t) = y(2t)$$

In general, the response to $x(2t)$, is given by $\int_{-\infty}^{\infty} h(t-\lambda)x(2\lambda)d\lambda$. This integral must be evaluated for the particular $x(\,\cdot\,)$ in question.

2CT.6 - BIBO STABILITY

2CT.6.1
a) Not BIBO stable because $\int_{-\infty}^{\infty} |u(t)|dt = \infty$.
b) BIBO stable because $\int_{-\infty}^{\infty} |\delta(t)|dt = 1 < \infty$.
c) BIBO stable because $\int_{-\infty}^{\infty} |\frac{1}{\tau}e^{-t/\tau}u(t)|dt = 1 < \infty$ for all $\tau > 0$.
d) BIBO stable because $\int_{-\infty}^{\infty} |u(t) - u(t-2)|dt = 2 < \infty$ for all $\tau > 0$.
e) Not BIBO stable because $\int_{-\infty}^{\infty} |\cos(100t)u(t)|dt = \infty$.

2CT.6.2
a) For a BIBO stable system,

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty$$

and it follows from the given inequality that

$$\int_{-\infty}^{\infty} |h(t)|^2 dt < \infty$$

Therefore, if a LTI system is BIBO stable, its impulse response has finite energy.
b) The converse is not true. Consider the system in the hint where $h(t) = \frac{1}{t}u(t-1)$. The energy in $h(t)$ is finite

$$\int_{-\infty}^{\infty} |h(t)|^2 dt = \int_{1}^{\infty} \frac{1}{t^2}dt = -\frac{1}{3t^3}\bigg|_{1}^{\infty} = \frac{1}{3}$$

However, the system is not BIBO stable because

$$\int_{-\infty}^{\infty} |h(t)|dt = \int_{1}^{\infty} \frac{1}{t}dt = \ln(t)\bigg|_{1}^{\infty} = \infty$$

2CT.7 - CAUSAL SYSTEMS

2CT.7.1

a) Causal because $h(t) = 0$ for $t < 0$.
b) Noncausal because $h(t) \neq 0$ for $t < 0$.
c) Noncausal because $h(t) \neq 0$ for $t < 0$.
d) Noncausal because output values for $t < 0$ depend on input values for $t > 0$. (For example $y(-3) = x(+3)$. Therefore, some output values depend on input values before these input values occurs.)

2CT.8 - BASIC CONNECTIONS OF CT LTI SYSTEMS

2CT.8.1

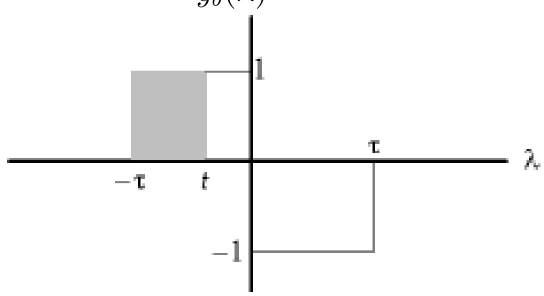
$$y_b(\lambda) = \sqcap\left(\frac{\lambda + \frac{\tau}{2}}{\tau}\right) - \sqcap\left(\frac{\lambda - \frac{\tau}{2}}{\tau}\right)$$

is sketched below on the left. The right-hand figure is the integral

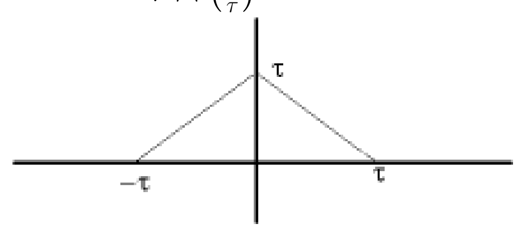
$$y(t) = \int_{-\infty}^{y} y_b(\lambda) d\lambda = \tau \wedge \left(\frac{t}{\tau}\right)$$

given by the shaded region. As t increases from $t = -\tau$, this shaded region increases linearly and equals τ when $t = 0$. As t grows beyond 0, the total area decreases linearly and equals 0 at $t = \tau$.

$y_b(\lambda)$



$\tau \wedge \left(\frac{t}{\tau}\right)$



2CT.8.2

a)

$$\frac{d}{dt}\left(1 - e^{-\frac{t}{\tau}}\right)u(t) = \frac{d}{dt}u(t) - \frac{d}{dt}e^{-\frac{t}{\tau}}u(t) = \delta(t) + \frac{1}{\tau}e^{-\frac{t}{\tau}}u(t) - \delta(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}u(t)$$

b)-c) The step response of an LTI system is given by

$$h_{-1}(t) = u(t)*h(t) = \int_{-\infty}^{\infty} h(\lambda)u(t-\lambda)d\lambda = \int_{-\infty}^{t} h(\lambda)d\lambda$$

If we differentiate the above we obtain

$$\frac{d}{dt}h_{-1}(t) = h(t)$$

2CT.8.3

Consider first the problem of evaluating $r(t)*r(t)$. A system having impulse response $h(t) = r(t)$ can be represented by 2 integrators in series.. Thus, we can find $r(t)*r(t)$ by integrating a ramp twice. The output of the first integration is

$$\int_{-\infty}^{t} r(\beta)d\beta = \frac{1}{2}t^2u(t)$$

and the output of the second integration is

$$r(t)*r(t) = \int_{-\infty}^{t} \frac{1}{2}\beta^2u(t)d\beta = \frac{1}{6}t^3u(t)$$

Since convolution is a LTI operation, we can use time invariance to find the output for an input $r(t-t_o)$. The result is

$$r(t)*r(t-t_o) = \frac{1}{6}(t-t_o)^3u(t-t_o)$$

2CT.8.4

a) If we substitute $h_1(t) = \delta(t-1)$ into (53) we obtain

$$h(t) = \delta(t) + \delta(t-1) + \delta(t-1)*\delta(t-1) + \delta(t-1)*\delta(t-1)*\delta(t-1) + \cdots$$

which simplifies to

$$h(t) = \delta(t) + \delta(t-1) + \delta(t-2) + \delta(t-3) + \cdots$$

b) The same result can be obtained by recognizing that the system $h_1(t) = \delta(t-1)$ is a delay element with delay 1. If we apply an impulse, $\delta(t)$, to the loop we see the impulse is delayed by 1 each time it goes around the loop.

2CT.8.5

It is possible to connect stable systems together in such a way that the overall system is unstable. Consider the system of Problem 8.4. The delay element is stable. However, the overall system is not stable. You can establish this result by noting that $h(t)$ from the preceding problem is not absolutely integrable. Alternatively, you can notice that if you apply the bounded input $x(t) = u(t)$, the output will be an unbounded:

$$u(t)*h(t) = u(t)*\delta(t) + u(t)*\delta(t-1) + u(t)*\delta(t-2) + u(t)*\delta(t-3) + \cdots$$

$$= u(t) + u(t-1) + u(t-2) + u(t-3) + \cdots$$

2CT.9 - SINGULARITY FUNCTIONS AND THEIR APPLICATIONS

2CT.9.1

Plots of $u(\beta)$, $u(t-\beta)$, and $u(\beta)u(t-\beta)$ versus β will show that

$$\int_{-\infty}^{\infty} u(\beta)u(t-\beta)d\beta = \left\{\begin{matrix} 0 & \text{for } t<0 \\ \int_0^t u(\beta)u(t-\beta)d\beta & \text{for } y \geq 0 \end{matrix}\right\} = \left\{\begin{matrix} 0 & \text{for } t<0 \\ t & \text{for } y \geq 0 \end{matrix}\right\} = r(t)$$

b) We can view $u(t)*u(t)$ as the response of an integrator to a step. The integral of a step is a ramp.

2CT.9.2

$$h(t) = \frac{d}{dt}\delta(t) = \delta_1(t) \text{ (a unit doublet)}$$

2CT.9.3

$$\sqcap\left(\frac{t}{\tau}\right) * \sqcap\left(\frac{t}{\tau}\right) = \int_{-\infty}^{\infty} \sqcap\left(\frac{t-\lambda}{\tau}\right)\sqcap\left(\frac{\lambda}{\tau}\right)d\lambda$$

The stated result follows by plotting $\sqcap\left(\frac{t-\lambda}{\tau}\right)$, $\sqcap\left(\frac{\lambda}{\tau}\right)$ and $\sqcap\left(\frac{t-\lambda}{\tau}\right)\sqcap\left(\frac{\lambda}{\tau}\right)$ versus λ for different values of t. For example, plot these functions for $0 \leq t < \tau$ to show that

$$\int_{-\infty}^{\infty} \sqcap\left(\frac{t-\lambda}{\tau}\right)\sqcap\left(\frac{\lambda}{\tau}\right)d\lambda = \tau - t$$

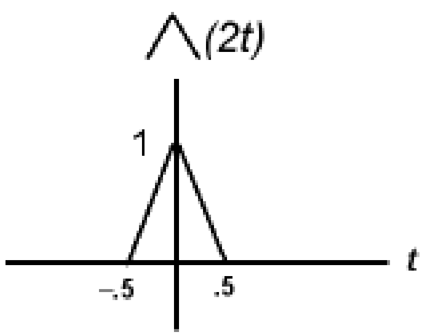
when $0 \leq t < \tau$. Plot the functions for $\tau < t$ to show that

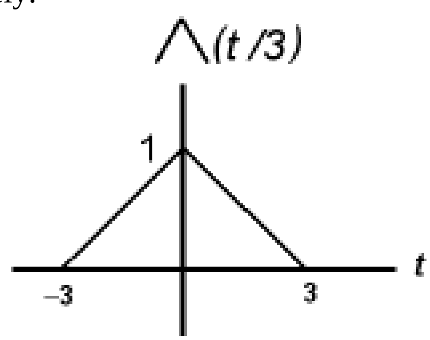
$$\int_{-\infty}^{\infty} \sqcap\left(\frac{t-\lambda}{\tau}\right)\sqcap\left(\frac{\lambda}{\tau}\right)d\lambda = 0$$

when $\tau < t$ and so on.

2CT.9.4

Notice that the base of $\wedge(t)$ touches 0 at $t = \pm 1$. The bases of $\wedge(2t)$ and $\wedge(t/3)$ therefore touch 0 at $t = \pm 0.5$ and $t = \pm 3$ respectively.





2CT.9.5

a) Step response:

$$h_{-1}(t) = u(t)*h(t) = \int_{-\infty}^{\infty} u(t-\lambda)h(\lambda)d\lambda = \int_{-\infty}^{t} h(\lambda)d\lambda$$

b) If we differentiate the above we have the result

$$h(t) = \frac{d}{dt}h_{-1}(t)$$

c) A unit ramp can be expressed as

$$r(t) = u(t)*u(t)$$

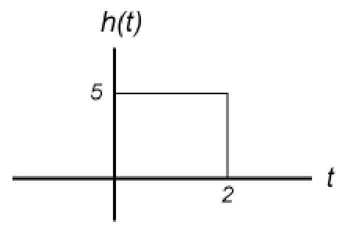
Therefore the ramp response of the LTI system is

$$h_{-2}(t) = r(t)*h(t) = [u(t)*u(t)]*h(t) = u(t)*[u(t)*h(t)]$$

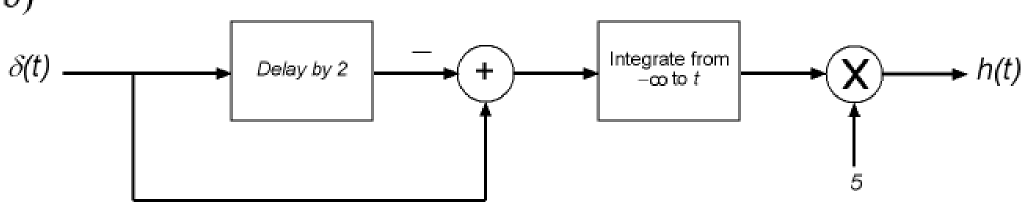
$$= u(t)*h_{-1}(t) = \int_{-\infty}^{\infty} u(t-\lambda)h_{-1}(t)d\lambda = \int_{-\infty}^{t} h_{-1}(t)d\lambda$$

2CT.9.6

a)



b)



c)

$$y(t) = x(t)*h(t) = \int_{-\infty}^{\infty} x(t-\lambda)5\sqcap\left(\frac{\lambda-1}{2}\right)d\lambda = 5\int_{0}^{2} x(t-\lambda)d\lambda = 5\int_{t-2}^{t} x(\beta)d\beta$$

2CT.9.7

$$x(t) = 2\wedge(t+4) + 2\wedge(t+3) + 4\wedge(t+2) + 4\wedge(t+1)$$

$$-\wedge(t-1) - 2\wedge(t-2) + 2\wedge(t-3)$$

2CT.9.8
We know from (72) that

$$\wedge\left(\frac{t}{\tau}\right)=\frac{r(t+\tau)-2r(t)+r(t-\tau)}{\tau}$$

The system's ramp response is give as $h_{-2}(t)$. Superposition and time-invariance then yield:

$$\wedge\left(\frac{t}{\tau}\right)\xrightarrow{h(t)}\frac{1}{\tau}h_{-2}(t+\tau)-\frac{2}{\tau}h_{-2}(t)+\frac{1}{\tau}h_{-2}(t-\tau)$$

Therefore, for $\tau=1$:

$$\wedge(t)\xrightarrow{h(t)}h_{-2}(t+1)-2h_{-2}(t)+h_{-2}(t-1)$$

2CT.9.9
We showed in the solution to 9.8 that:

$$\wedge\left(\frac{t}{\tau}\right)\xrightarrow{h(t)}\frac{1}{\tau}h_{-2}(t+\tau)-\frac{2}{\tau}h_{-2}(t)+\frac{1}{\tau}h_{-2}(t-\tau)$$

When we set $\tau=6$ the above becomes:

$$\wedge\left(\frac{t}{6}\right)\xrightarrow{h(t)}\frac{1}{6}h_{-2}(t+6)-\frac{1}{3}h_{-2}(t)+\frac{1}{6}h_{-2}(t-6)$$

MISCELLANEOUS PROBLEMS

2CT.1
a) By Newton's second law of motion, force equals mass times acceleration. Two forces act on the mass in the $+x$ direction: the applied force $F(t)$ and the friction $-kv(t)$ where $v(t)=\dot{x}(t)$ and the dot is Newton's notation for differentiation with respect to t. By Newton's third law, therefore,

$$F(t)-kv(t)=m\dot{v}(t)$$

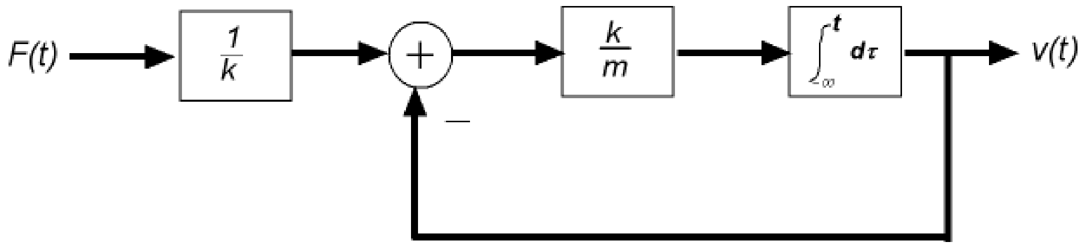
which rearranges to

$$m\dot{v}(t)+kv(t)=F(t)$$

b)
The above equation can be written as

$$\frac{m}{k}\dot{v}(t)=\frac{1}{k}F(t)-v(t)$$

The following system implements this equation:



c) Notice that the above system is identical to Figure 11 in the book with $\tau_c = m/k$, except that the input is multiplied by $1/k$. We can use the analysis of Example 3 to write

$$h(t) = \frac{1}{k\tau_c} e^{-\frac{t}{\tau_c}} u(t) = \frac{1}{m} e^{-\frac{kt}{m}} u(t)$$

The response of the system $F(t) = F_o u(t)$ is the scaled integral of the impulse response

$$v(t) = F_o \int_{-\infty}^{t} h(t)dt = \frac{F_o}{k}\left(1 - e^{-\frac{kt}{m}}\right) u(t)$$

The steady state velocity is therefore $v(\infty) = F_o/k$.

2CT.2
If we combine the equations in the hint we get

$$y(t) = \frac{1}{RC} \int_{-\infty}^{t} (y(t) - x(t))d\lambda$$

which is one form of the I-O equation for the circuit.

2CT.3
By Kirchoff's voltage law,

$$v_R(t) + v_C(t) = x(t)$$

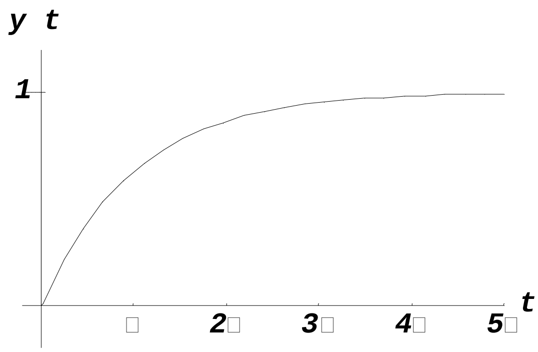
where $v_R(t)$ and $v_C(t)$ are, respectively, the voltage across the resistor and the capacitor. We have already shown that if $x(t) = \delta(t)$, then $v_C(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$ where $\tau = RC$. Therefore, when $x(t) = \delta(t)$,

$$v_R(t) = h(t) = \delta(t) - \frac{1}{\tau} e^{-t/\tau} u(t)$$

A plot is shown in the solution to Problem 3.1.

2CT.4 The step response is the integral of the impulse response given in the solution to 3 above.

$$y(t) = h_{-1}(t) = \int_{-\infty}^{t} \left(\delta(\lambda) - \frac{1}{\tau} e^{-\lambda/\tau} u(\lambda) \right) d\lambda = \left(1 - e^{-t/\tau}\right) u(t)$$



2CT.5

$$y(t) = \int_{-\infty}^{\infty} r(t-\lambda) x(\lambda) d\lambda = \int_{-\infty}^{\infty} (t-\lambda) u(t-\lambda) x(\lambda) d\lambda = \int_{-\infty}^{t} (t-\lambda) x(\lambda) d\lambda$$

This equation suffices. However, we recognize that if $h(t) = r(t)$ then the system is composed of two integrators in series. Therefore, an alternative expression is

$$y(t) = \int_{-\infty}^{t} \int_{-\infty}^{\beta} x(\lambda) d\lambda d\beta$$

We can also write the I-O equation as

$$\frac{d^2}{dt^2} y(t) = x(t)$$

2CT.6

a) An inspection of the system yields, for $\xi(t) = 0$,

$$v(t) = x(t) - \beta y(t)$$

and

$$y(t) = AGv(t)$$

If we eliminate $v(t)$ and solve for $y(t)$ we obtain

$$y(t) = \frac{AG}{1 + AG\beta} x(t)$$

b) If $AG\beta \gg 1$, then $1 + AG\beta \approx AG\beta$ and the answer to (a) becomes

$$y(t) \approx \frac{1}{\beta} x(t)$$

c) $AG\beta \gg 1$ if an only if $A \gg \frac{1}{G\beta}$.

2CT.7
a) By inspection of the system, we see that

$$v(t) = x(t) - \beta y(t)$$

and

$$y(t) = AGv(t) + \xi(t)$$

If we eliminate $v(t)$ and solve for $y(t)$ we obtain

$$y(t) = \frac{AG}{1 + AG\beta}x(t) + \frac{1}{1 + AG\beta}\xi(t)$$

c) For $AG\beta \gg 1$, the above becomes

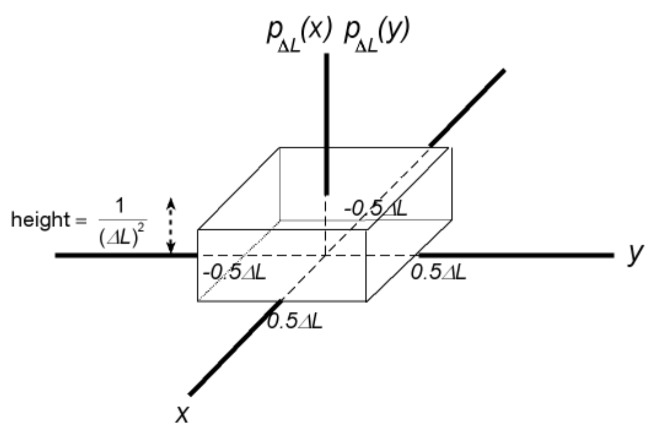
$$y(t) \approx \frac{1}{\beta}x(t) + \frac{1}{AG\beta}\xi(t)$$

which is

$$y(t) \approx \frac{1}{\beta}x(t)$$

for sufficiently large $AG\beta$.

2CT.8
a)



As $\Delta L \to 0$, the sides of the box-like function shrink to zero and the amplitude increases to infinity. The volume of the box equals unity. The result is a two-dimensional impulse.

b)

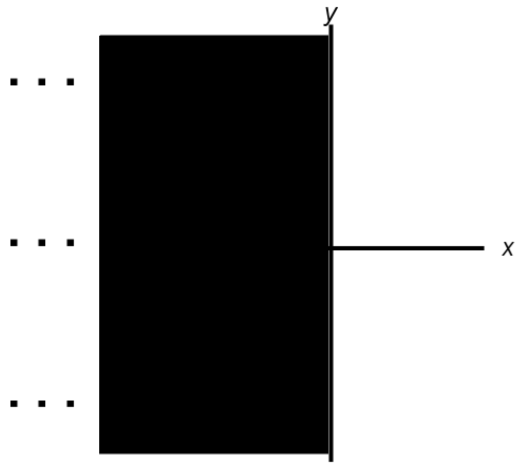
$$I_o(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(\alpha - x_o, \beta - y_o)h(x-\alpha, y-\beta)d\alpha d\beta$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(\alpha - x_o)\delta(\beta - y_o)h(x-\alpha, y-\beta)d\alpha d\beta$$

$$= \int_{-\infty}^{\infty}\delta(\beta - y_o)\left\{\int_{-\infty}^{\infty}\delta(\alpha - x_o)h(x-\alpha, y-\beta)d\alpha\right\}d\beta$$

$$= \int_{-\infty}^{\infty}\delta(\beta - y_o)h(x - x_o, y-\beta)d\beta = h(x-x_o, y-y_o)$$

c)



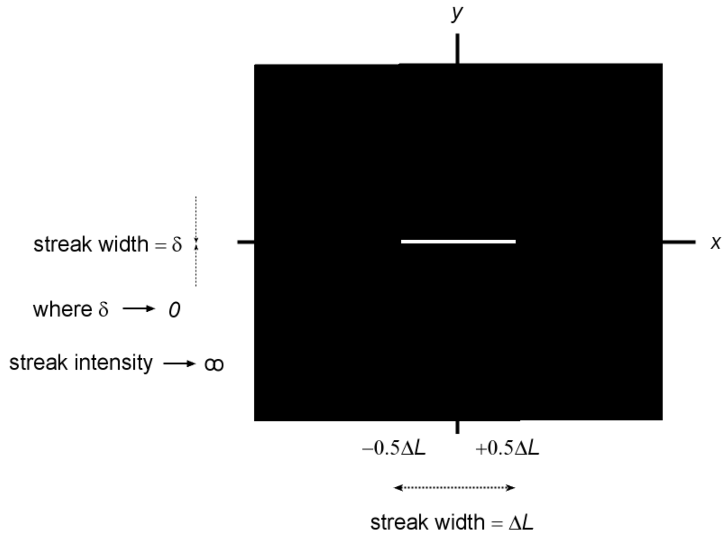
d)

$$I_o(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u(\alpha)\delta(x-\alpha, y-\beta)d\alpha d\beta = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u(\alpha)\delta(x-\alpha)\delta(y-\beta)d\alpha d\beta$$

$$= \int_{-\infty}^{\infty}u(\alpha)\delta(x-\alpha)d\alpha\int_{-\infty}^{\infty}\delta(y-\beta)d\beta = \int_{-\infty}^{\infty}u(x)\delta(y-\beta)d\beta = u(x)$$

2CT.9

a)



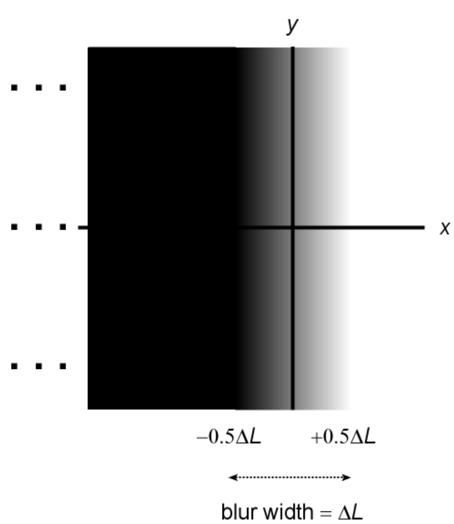
b)

$$I_o(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(\alpha)\sqcap\left(\frac{x-\alpha}{\Delta L}\right)\delta(y-\beta)d\alpha d\beta$$

$$= \int_{-\infty}^{\infty} u(\alpha)\sqcap\left(\frac{x-\alpha}{\Delta L}\right)d\alpha\int_{-\infty}^{\infty}\delta(y-\beta)d\beta\alpha = \int_{-\infty}^{\infty} u(\alpha)\sqcap\left(\frac{x-\alpha}{\Delta L}\right)d\alpha$$

$$= \int_{-\infty}^{\infty} u(\alpha)u(x-\alpha+0.5\Delta L) - u(x-\alpha-0.5\Delta L)d\alpha$$

$$= r(x+0.5\Delta L) - r(x-0.5\Delta L)$$



Notice that camera motion during the time of exposure has blurred the edge.