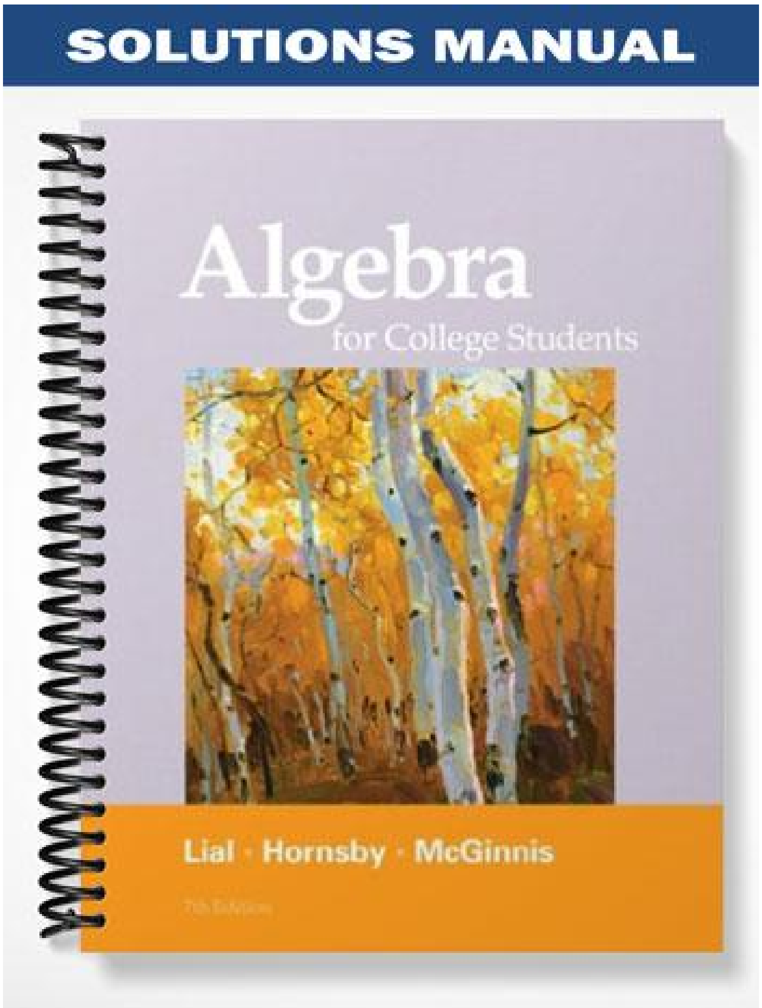
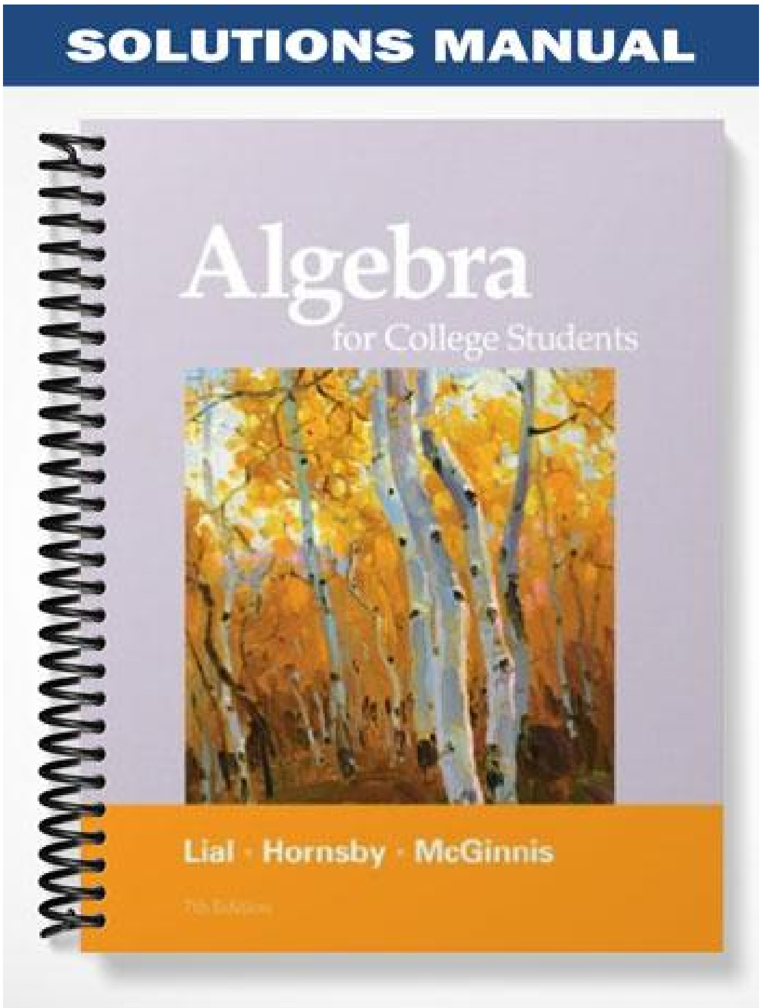

SOLUTIONS MANUAL
Algebra
for College Students
Lial · Hornsby · McGinnis


SOLUTIONS MANUAL
Algebra
for College Students
Lial · Hornsby · McGinnis

# CHAPTER 2 LINEAR EQUATIONS, INEQUALITIES, AND APPLICATIONS

## 2.1 Linear Equations in One Variable

### 2.1 Classroom Examples, Now Try Exercises

**1.** **(a)** $9x + 10 = 0$ is an *equation* because it contains an equals symbol.

**(b)** $9x + 10$ is an *expression* because it does not contain an equals symbol.

**N1.** **(a)** $2x + 17 - 3x$ is an *expression* because it does not contain an equals symbol.

**(b)** $2x + 17 = 3x$ is an *equation* because it contains an equals symbol.

**2.**

$$4x + 8x = -9 + 17x - 1 \quad \textit{Original equation}$$
$$12x = 17x - 10 \quad \textit{Combine terms.}$$
$$12x - 17x = 17x - 10 - 17x \quad \textit{Subtract 17x.}$$
$$-5x = -10 \quad \textit{Combine terms.}$$
$$\frac{-5x}{-5} = \frac{-10}{-5} \quad \textit{Divide by } -5.$$
$$x = 2 \quad \textit{Proposed solution}$$

**Check** by substituting 2 for $x$ in the *original* equation.

$$4x + 8x = -9 + 17x - 1 \quad \textit{Original equation}$$
$$4(2) + 8(2) \stackrel{?}{=} -9 + 17(2) - 1 \quad \textit{Let } x = 2.$$
$$8 + 16 \stackrel{?}{=} -9 + 34 - 1$$
$$24 \stackrel{?}{=} 25 - 1$$
$$24 = 24 \quad \textit{True}$$

The solution set is $\{2\}$.

**N2.**

$$5x + 11 = 2x - 13 - 3x \quad \textit{Original equation}$$
$$5x + 11 = -x - 13 \quad \textit{Combine terms.}$$
$$5x + 11 + x = -x - 13 + x \quad \textit{Add x.}$$
$$6x + 11 = -13 \quad \textit{Combine terms.}$$
$$6x + 11 - 11 = -13 - 11 \quad \textit{Subtract 11.}$$
$$6x = -24 \quad \textit{Combine terms.}$$
$$\frac{6x}{6} = \frac{-24}{6} \quad \textit{Divide by 6.}$$
$$x = -4 \quad \textit{Proposed solution}$$

**Check** by substituting $-4$ for $x$ in the *original* equation.

$$5x + 11 = 2x - 13 - 3x \quad \textit{Original equation}$$
$$5(-4) + 11 \stackrel{?}{=} 2(-4) - 13 - 3(-4) \quad \textit{Let } x = -4.$$
$$-20 + 11 \stackrel{?}{=} -8 - 13 + 12$$
$$-9 \stackrel{?}{=} -21 + 12$$
$$-9 = -9 \quad \textit{True}$$

The solution set is $\{-4\}$.

**3.**

$$6 - (4 + x) = 8x - 2(3x + 5)$$
$$6 - 4 - x = 8x - 6x - 10 \quad \textit{Distributive property}$$
$$2 - x = 2x - 10 \quad \textit{Combine terms.}$$
$$2 - x + x + 10 = 2x - 10 + x + 10 \quad \textit{Add x; add 10.}$$
$$12 = 3x \quad \textit{Combine terms.}$$
$$\frac{12}{3} = \frac{3x}{3} \quad \textit{Divide by 3.}$$
$$4 = x \quad \textit{Proposed solution}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

**Check** $x = 4$: $\quad -2 = 32 - 34 \quad$ *True*

The solution set is $\{4\}$.

**N3.**

$$5(x - 4) - 9 = 3 - 2(x + 16)$$
$$5x - 20 - 9 = 3 - 2x - 32 \quad \textit{Distributive property}$$
$$5x - 29 = -2x - 29 \quad \textit{Combine terms.}$$
$$5x - 29 + 2x + 29 = -2x - 29 + 2x + 29 \quad \textit{Add 2x; add 29.}$$
$$7x = 0 \quad \textit{Combine terms.}$$
$$\frac{7x}{7} = \frac{0}{7} \quad \textit{Divide by 7.}$$
$$x = 0 \quad \textit{Proposed solution}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

**Check** $x = 0$: $-20 - 9 = 3 - 32$ *True*

The solution set is $\{0\}$.

**4.** $\frac{x+1}{2} + \frac{x+3}{4} = \frac{1}{2}$

Multiply each side by the LCD, 4, and use the distributive property.

$$\begin{aligned} 4\left(\frac{x+1}{2}\right) + 4\left(\frac{x+3}{4}\right) &= 4\left(\frac{1}{2}\right) \\ 2(x+1) + 1(x+3) &= 2 \\ 2x + 2 + x + 3 &= 2 \\ 3x + 5 &= 2 \\ 3x &= -3 \quad \textit{Subtract 5.} \\ x &= -1 \quad \textit{Divide by 3.} \end{aligned}$$

**Check** $x = -1$: $0 + \frac{2}{4} = \frac{1}{2}$ *True*

The solution set is $\{-1\}$.

**N4.** $\frac{x-4}{4} + \frac{2x+4}{8} = 5$

Multiply each side by the LCD, 8, and use the distributive property.

$$\begin{aligned} 8\left(\frac{x-4}{4}\right) + 8\left(\frac{2x+4}{8}\right) &= 8(5) \\ 2(x-4) + 1(2x+4) &= 40 \\ 2x - 8 + 2x + 4 &= 40 \\ 4x - 4 &= 40 \\ 4x &= 44 \quad \textit{Add 4.} \\ x &= 11 \quad \textit{Divide by 4.} \end{aligned}$$

**Check** $x = 11$: $\frac{7}{4} + \frac{13}{4} = 5$ *True*

The solution set is $\{11\}$.

**5.** $0.02(60) + 0.04x = 0.03(50 + x)$

*Multiply each term by 100.*

$$\begin{aligned} 2(60) + 4x &= 3(50 + x) \\ 120 + 4x &= 150 + 3x \\ 4x &= 30 + 3x \\ x &= 30 \end{aligned}$$

**Check** $x = 30$: $1.2 + 1.2 = 2.4$ *True*

The solution set is $\{30\}$.

**N5.** $0.08x - 0.12(x - 4) = 0.03(x - 5)$

*Multiply each term by 100.*

$$\begin{aligned} 8x - 12(x-4) &= 3(x-5) \\ 8x - 12x + 48 &= 3x - 15 \\ -4x + 48 &= 3x - 15 \\ -4x + 63 &= 3x \\ 63 &= 7x \\ 9 &= x \end{aligned}$$

**Check** $x = 9$: $0.72 - 0.60 = 0.12$ *True*

The solution set is $\{9\}$.

**6. (a)** $5(x+2) - 2(x+1) = 3x + 1$

$$\begin{aligned} 5x + 10 - 2x - 2 &= 3x + 1 \\ 3x + 8 &= 3x + 1 \\ 3x + 8 - 3x &= 3x + 1 - 3x \quad \textit{Subtract 3x.} \\ 8 &= 1 \quad \textit{False} \end{aligned}$$

Since the result, $8 = 1$, is *false*, the equation has no solution and is called a *contradiction.*

The solution set is $\emptyset$.

**(b)** $\frac{x+1}{3} + \frac{2x}{3} = x + \frac{1}{3}$

Multiply each side by the LCD, 3, and use the distributive property.

$$\begin{aligned} 3\left(\frac{x+1}{3}\right) + 3\left(\frac{2x}{3}\right) &= 3\left(x + \frac{1}{3}\right) \\ x + 1 + 2x &= 3x + 1 \\ 3x + 1 &= 3x + 1 \end{aligned}$$

This is an *identity.* Any real number will make the equation true.

The solution set is {all real numbers}.

**(c)** $5(3x + 1) = x + 5$

$$\begin{aligned} 15x + 5 &= x + 5 \\ 14x + 5 &= 5 \quad \textit{Subtract x.} \\ 14x &= 0 \quad \textit{Subtract 5.} \\ x &= 0 \quad \textit{Divide by 14.} \end{aligned}$$

This is a *conditional equation.*

**Check** $x = 0$: $5(1) = 0 + 5$ *True*

The solution set is $\{0\}$.

**N6. (a)** $9x - 3(x + 4) = 6(x - 2)$

$$\begin{aligned} 9x - 3x - 12 &= 6x - 12 \\ 6x - 12 &= 6x - 12 \end{aligned}$$

This is an *identity.* Any real number will make the equation true.

The solution set is {all real numbers}.

**(b)** $-3(2x - 1) - 2x = 3 + x$

$$\begin{aligned} -6x + 3 - 2x &= 3 + x \\ -8x + 3 &= 3 + x \\ -9x + 3 &= 3 \quad \textit{Subtract x.} \\ -9x &= 0 \quad \textit{Subtract 3.} \\ x &= 0 \quad \textit{Divide by } -9. \end{aligned}$$

This is a *conditional equation.*

**Check** $x = 0$: $-3(-1) = 3$ *True*

The solution set is $\{0\}$.

**(c)**

$$\begin{aligned} 10x - 21 &= 2(x-5) + 8x \\ 10x - 21 &= 2x - 10 + 8x \\ 10x - 21 &= 10x - 10 \\ 10x - 21 - 10x &= 10x - 10 - 10x \quad \textit{Subtract 10x.} \\ -21 &= -10 \quad \textit{False} \end{aligned}$$

Since the result, $-21 = -10$, is *false*, the equation has no solution and is called a *contradiction.*

The solution set is $\emptyset$.

## 2.1 Section Exercises

**1.** **A.** $3x + x - 1 = 0$ can be written as $4x = 1$, so it is linear.

**C.** $6x + 2 = 9$ is in linear form.

**2.** **B.** $8 = x^2$ is not a linear equation because the variable is squared.

**D.** $\frac{1}{2}x - \frac{1}{x} = 0$ is not a linear equation because there is a variable in the denominator of the second term.

**3.**

$$\begin{aligned} 3(x+4) &= 5x \quad \textit{Original equation} \\ 3(6+4) &\stackrel{?}{=} 5 \cdot 6 \quad \textit{Let } x = 6. \\ 3(10) &\stackrel{?}{=} 30 \quad \textit{Add.} \\ 30 &= 30 \quad \textit{True} \end{aligned}$$

Since a true statement is obtained, 6 is a solution.

**4.**

$$\begin{aligned} 5(x+4) - 3(x+6) &= 9(x+1) \quad \textit{Original equation} \\ 5(-2+4) - 3(-2+6) &\stackrel{?}{=} 9(-2+1) \quad \textit{Let } x = -2. \\ 5(2) - 3(4) &\stackrel{?}{=} 9(-1) \quad \textit{Add.} \\ 10 - 12 &\stackrel{?}{=} -9 \quad \textit{Multiply.} \\ -2 &= -9 \quad \textit{False} \end{aligned}$$

Since a false statement is obtained, $-2$ is not a solution.

**5.** $-3x + 2 - 4 = x$ is an *equation* because it contains an equals symbol.

**6.** $-3x + 2 - 4 - x = 4$ is an *equation* because it contains an equals symbol.

**7.** $4(x+3) - 2(x+1) - 10$ is an *expression* because it does not contain an equals symbol.

**8.** $4(x+3) - 2(x+1) + 10$ is an *expression* because it does not contain an equals symbol.

**9.** $-10x + 12 - 4x = -3$ is an *equation* because it contains an equals symbol.

**10.** $-10x + 12 - 4x + 3 = 0$ is an *equation* because it contains an equals symbol.

In the following exercises, we show brief checks of the solutions. To be sure that your solution is correct, check it by substituting into the original equation.

**11.**

$$\begin{aligned} 7x + 8 &= 1 \\ 7x + 8 - 8 &= 1 - 8 \quad \textit{Subtract 8.} \\ 7x &= -7 \\ \frac{7x}{7} &= \frac{-7}{7} \quad \textit{Divide by 7.} \\ x &= -1 \end{aligned}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

**Check** $x = -1$: $\quad -7 + 8 = 1 \quad$ *True*

The solution set is $\{-1\}$.

**12.**

$$\begin{aligned} 5x - 4 &= 21 \\ 5x - 4 + 4 &= 21 + 4 \quad \textit{Add 4.} \\ 5x &= 25 \\ \frac{5x}{5} &= \frac{25}{5} \quad \textit{Divide by 5.} \\ x &= 5 \end{aligned}$$

**Check** $x = 5$: $\quad 25 - 4 = 21 \quad$ *True*

The solution set is $\{5\}$.

**13.**

$$\begin{aligned} 5x + 2 &= 3x - 6 \\ 5x + 2 - 3x &= 3x - 6 - 3x \quad \textit{Subtract 3x.} \\ 2x + 2 &= -6 \\ 2x + 2 - 2 &= -6 - 2 \quad \textit{Subtract 2.} \\ 2x &= -8 \\ \frac{2x}{2} &= \frac{-8}{2} \quad \textit{Divide by 2.} \\ x &= -4 \end{aligned}$$

**Check** $x = -4$: $\quad -20 + 2 = -12 - 6 \quad$ *True*

The solution set is $\{-4\}$.

**14.**

$$\begin{aligned} 9x + 1 &= 7x - 9 \\ 9x + 1 - 7x &= 7x - 9 - 7x \quad \textit{Subtract 7x.} \\ 2x + 1 &= -9 \\ 2x + 1 - 1 &= -9 - 1 \quad \textit{Subtract 1.} \\ 2x &= -10 \\ \frac{2x}{2} &= \frac{-10}{2} \quad \textit{Divide by 2.} \\ x &= -5 \end{aligned}$$

**Check** $x = -5$: $\quad -45 + 1 = -35 - 9 \quad$ *True*

The solution set is $\{-5\}$.

**15.**

$$\begin{aligned} 7x - 5x + 15 &= x + 8 \\ 2x + 15 &= x + 8 \quad \textit{Combine terms.} \\ 2x &= x - 7 \quad \textit{Subtract 15.} \\ x &= -7 \quad \textit{Subtract x.} \end{aligned}$$

**Check** $x = -7$: $\quad -49 + 35 + 15 = -7 + 8 \quad$ *True*

The solution set is $\{-7\}$.

**16.** $2x + 4 - x = 4x - 5$

$$\begin{aligned} x + 4 &= 4x - 5 && \textit{Combine terms.} \\ -3x + 4 &= -5 && \textit{Subtract 4x.} \\ -3x &= -9 && \textit{Subtract 4.} \\ x &= 3 && \textit{Divide by –3.} \end{aligned}$$

**Check** $x = 3$: $6 + 4 - 3 = 12 - 5$ *True*

The solution set is $\{3\}$.

**17.** $12w + 15w - 9 + 5 = -3w + 5 - 9$

$$\begin{aligned} 27w - 4 &= -3w - 4 && \textit{Combine terms.} \\ 30w - 4 &= -4 && \textit{Add 3w.} \\ 30w &= 0 && \textit{Add 4.} \\ w &= 0 && \textit{Divide by 30.} \end{aligned}$$

**Check** $w = 0$: $-9 + 5 = 5 - 9$ *True*

The solution set is $\{0\}$.

**18.** $-4x + 5x - 8 + 4 = 6x - 4$

$$\begin{aligned} x - 4 &= 6x - 4 && \textit{Combine terms.} \\ -5x - 4 &= -4 && \textit{Subtract 6x.} \\ -5x &= 0 && \textit{Add 4.} \\ x &= 0 && \textit{Divide by –5.} \end{aligned}$$

**Check** $x = 0$: $-8 + 4 = -4$ *True*

The solution set is $\{0\}$.

**19.** $3(2t - 4) = 20 - 2t$

$$\begin{aligned} 6t - 12 &= 20 - 2t && \textit{Distributive property} \\ 8t - 12 &= 20 && \textit{Add 2t.} \\ 8t &= 32 && \textit{Add 12.} \\ t &= 4 && \textit{Divide by 8.} \end{aligned}$$

**Check** $t = 4$: $3(4) = 20 - 8$ *True*

The solution set is $\{4\}$.

**20.** $2(3 - 2x) = x - 4$

$$\begin{aligned} 6 - 4x &= x - 4 && \textit{Distributive property} \\ 6 - 5x &= -4 && \textit{Subtract x.} \\ -5x &= -10 && \textit{Subtract 6.} \\ x &= 2 && \textit{Divide by } -5. \end{aligned}$$

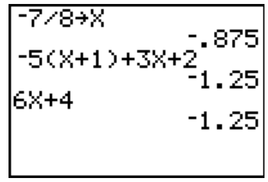
**Check** $x = 2$: $2(-1) = 2 - 4$ *True*

The solution set is $\{2\}$.

**21.** $-5(x + 1) + 3x + 2 = 6x + 4$

$$\begin{aligned} -5x - 5 + 3x + 2 &= 6x + 4 && \textit{Distributive property} \\ -2x - 3 &= 6x + 4 && \textit{Combine terms.} \\ -3 &= 8x + 4 && \textit{Add 2x.} \\ -7 &= 8x && \textit{Subtract 4.} \\ -\tfrac{7}{8} &= x && \textit{Divide by 8.} \end{aligned}$$

**Check:** Substitute $-\frac{7}{8}$ for $x$ and show that both sides equal $-1.25$. The screen shows a typical check on a calculator.




The solution set is $\left\{-\frac{7}{8}\right\}$.

**22.** $5(x + 3) + 4x - 5 = 4 - 2x$

$$\begin{aligned} 5x + 15 + 4x - 5 &= 4 - 2x && \textit{Distributive property} \\ 9x + 10 &= 4 - 2x && \textit{Combine terms.} \\ 11x + 10 &= 4 && \textit{Add 2x.} \\ 11x &= -6 && \textit{Subtract 10.} \\ x &= -\tfrac{6}{11} && \textit{Divide by 11.} \end{aligned}$$

**Check** $x = -\frac{6}{11}$:

$$\tfrac{135}{11} - \tfrac{24}{11} - \tfrac{55}{11} = \tfrac{44}{11} + \tfrac{12}{11} \quad \textit{True}$$

The solution set is $\left\{-\frac{6}{11}\right\}$.

**23.** $-2x + 5x - 9 = 3(x - 4) - 5$

$$\begin{aligned} 3x - 9 &= 3x - 12 - 5 \\ 3x - 9 &= 3x - 17 \\ -9 &= -17 && \textit{False} \end{aligned}$$

The equation is a *contradiction*.

The solution set is $\emptyset$.

**24.** $-6x + 2x - 11 = -2(2x - 3) + 4$

$$\begin{aligned} -4x - 11 &= -4x + 6 + 4 \\ -4x - 11 &= -4x + 10 \\ -11 &= 10 && \textit{False} \end{aligned}$$

The equation is a *contradiction*.

The solution set is $\emptyset$.

**25.** $2(x + 3) = -4(x + 1)$

$$\begin{aligned} 2x + 6 &= -4x - 4 && \textit{Remove parentheses.} \\ 6x + 6 &= -4 && \textit{Add 4x.} \\ 6x &= -10 && \textit{Subtract 6.} \\ x &= \tfrac{-10}{6} = -\tfrac{5}{3} && \textit{Divide by 6.} \end{aligned}$$

**Check** $x = -\frac{5}{3}$: $2\left(\frac{4}{3}\right) = -4\left(-\frac{2}{3}\right)$ *True*

The solution set is $\left\{-\frac{5}{3}\right\}$.

**26.** $4(x - 9) = 8(x + 3)$

$$\begin{aligned} 4x - 36 &= 8x + 24 && \textit{Remove parentheses.} \\ -4x - 36 &= 24 && \textit{Subtract 8x.} \\ -4x &= 60 && \textit{Add 36.} \\ x &= -15 && \textit{Divide by –4.} \end{aligned}$$

**Check** $x = -15$: $4(-24) = 8(-12)$ *True*

The solution set is $\{-15\}$.

**27.** $3(2x+1)-2(x-2)=5$

$$\begin{aligned} 6x+3-2x+4&=5 && \textit{Remove parentheses.} \\ 4x+7&=5 && \textit{Combine terms.} \\ 4x&=-2 && \textit{Subtract 7.} \\ x&=\tfrac{-2}{4}=-\tfrac{1}{2} && \textit{Divide by 4.} \end{aligned}$$

**Check** $x=-\frac{1}{2}$: $3(0)-2(-\frac{5}{2})=5$ *True*

The solution set is $\{-\frac{1}{2}\}$.

**28.** $4(x-2)+2(x+3)=6$

$$\begin{aligned} 4x-8+2x+6&=6 \\ 6x-2&=6 \\ 6x&=8 \\ x&=\tfrac{8}{6}=\tfrac{4}{3} \end{aligned}$$

**Check** $x=\frac{4}{3}$: $4(-\frac{2}{3})+2(\frac{13}{3})=6$ *True*

The solution set is $\{\frac{4}{3}\}$.

**29.** $2x+3(x-4)=2(x-3)$

$$\begin{aligned} 2x+3x-12&=2x-6 \\ 5x-12&=2x-6 \\ 3x&=6 \\ x&=\tfrac{6}{3}=2 \end{aligned}$$

**Check** $x=2$: $4+3(-2)=2(-1)$ *True*

The solution set is $\{2\}$.

**30.** $6x-3(5x+2)=4(1-x)$

$$\begin{aligned} 6x-15x-6&=4-4x \\ -9x-6&=4-4x \\ -5x&=10 \\ x&=\tfrac{10}{-5}=-2 \end{aligned}$$

**Check** $x=-2$: $-12-3(-8)=4(3)$ *True*

The solution set is $\{-2\}$.

**31.** $6x-4(3-2x)=5(x-4)-10$

$$\begin{aligned} 6x-12+8x&=5x-20-10 \\ 14x-12&=5x-30 \\ 9x&=-18 \\ x&=-2 \end{aligned}$$

**Check** $x=-2$: $-12-4(7)=5(-6)-10$ *True*

The solution set is $\{-2\}$.

**32.** $-2x-3(4-2x)=2(x-3)+2$

$$\begin{aligned} -2x-12+6x&=2x-6+2 \\ 4x-12&=2x-4 \\ 2x&=8 \\ x&=4 \end{aligned}$$

**Check** $x=4$: $-8-3(-4)=2(1)+2$ *True*

The solution set is $\{4\}$.

**33.** $-2(x+3)-x-4=-3(x+4)+2$

$$\begin{aligned} -2x-6-x-4&=-3x-12+2 \\ -3x-10&=-3x-10 \end{aligned}$$

The equation is an *identity*.

The solution set is {all real numbers}.

**34.** $4(2x+7)=2x+25+3(2x+1)$

$$\begin{aligned} 8x+28&=2x+25+6x+3 \\ 8x+28&=8x+28 \end{aligned}$$

The equation is an *identity*.

The solution set is {all real numbers}.

**35.** $2[x-(2x+4)+3]=2(x+1)$

$$\begin{aligned} 2[x-2x-4+3]&=2(x+1) \\ 2[-x-1]&=2(x+1) \\ -x-1&=x+1 && \textit{Divide by 2.} \\ -1&=2x+1 && \textit{Add x.} \\ -2&=2x && \textit{Subtract 1.} \\ -1&=x && \textit{Divide by 2.} \end{aligned}$$

**Check** $x=-1$: $2[-1-2+3]=0$ *True*

The solution set is $\{-1\}$.

**36.** $4[2x-(3-x)+5]=-(2+7x)$

$$\begin{aligned} 4[2x-3+x+5]&=-(2+7x) \\ 4[3x+2]&=-(2+7x) \\ 12x+8&=-2-7x \\ 19x+8&=-2 && \textit{Add 7x.} \\ 19x&=-10 && \textit{Subtract 8.} \\ x&=-\tfrac{10}{19} && \textit{Divide by 19.} \end{aligned}$$

**Check** $x=-\frac{10}{19}$:
$4[-\frac{20}{19}-\frac{67}{19}+\frac{95}{19}]=-(-\frac{32}{19})$ *True*

The solution set is $\{-\frac{10}{19}\}$.

**37.** $-[2x-(5x+2)]=2+(2x+7)$

$$\begin{aligned} -[2x-5x-2]&=2+2x+7 \\ -[-3x-2]&=2+2x+7 \\ 3x+2&=2x+9 \\ x&=7 \end{aligned}$$

**Check** $x=7$: $-[14-37]=2+21$ *True*

The solution set is $\{7\}$.

**38.** $-[6x-(4x+8)]=9+(6x+3)$

$$\begin{aligned} -[6x-4x-8]&=9+6x+3 \\ -(2x-8)&=6x+12 \\ -2x+8&=6x+12 \\ -8x&=4 \\ x&=\tfrac{4}{-8}=-\tfrac{1}{2} \end{aligned}$$

**Check** $x=-\frac{1}{2}$: $-[-3-6]=9+0$ *True*

The solution set is $\{-\frac{1}{2}\}$.

**39.**
$$\begin{aligned} -3x+6-5(x-1) &= -5x-(2x-4)+5 \\ -3x+6-5x+5 &= -5x-2x+4+5 \\ -8x+11 &= -7x+9 \\ -x &= -2 \\ x &= 2 \end{aligned}$$

**Check** $x=2$:
$$-6+6-5=-10-0+5 \quad \textit{True}$$

The solution set is $\{2\}$.

**40.**
$$\begin{aligned} 4(x+2)-8x-5 &= -3x+9-2(x+6) \\ 4x+8-8x-5 &= -3x+9-2x-12 \\ -4x+3 &= -5x-3 \\ x &= -6 \end{aligned}$$

**Check** $x=-6$:
$$-16+48-5=18+9-0 \quad \textit{True}$$

The solution set is $\{-6\}$.

**41.**
$$\begin{aligned} 7[2-(3+4x)]-2x &= -9+2(1-15x) \\ 7[2-3-4x]-2x &= -9+2-30x \\ 7[-1-4x]-2x &= -7-30x \\ -7-28x-2x &= -7-30x \\ -7-30x &= -7-30x \end{aligned}$$

The equation is an *identity*.

The solution set is $\{$all real numbers$\}$.

**42.**
$$\begin{aligned} 4[6-(1+2x)]+10x &= 2(10-3x)+8x \\ 4[6-1-2x]+10x &= 20-6x+8x \\ 4(5-2x)+10x &= 20+2x \\ 20-8x+10x &= 20+2x \\ 20+2x &= 20+2x \end{aligned}$$

The equation is an *identity*.

The solution set is $\{$all real numbers$\}$.

**43.**
$$\begin{aligned} -[3x-(2x+5)] &= -4-[3(2x-4)-3x] \\ -[3x-2x-5] &= -4-[6x-12-3x] \\ -[x-5] &= -4-[3x-12] \\ -x+5 &= -4-3x+12 \\ -x+5 &= -3x+8 \\ 2x &= 3 \\ x &= \tfrac{3}{2} \end{aligned}$$

**Check** $x=\frac{3}{2}$:
$$-\left[\tfrac{9}{2}-8\right]=-4-\left[-3-\tfrac{9}{2}\right] \quad \textit{True}$$

The solution set is $\left\{\frac{3}{2}\right\}$.

**44.**
$$\begin{aligned} 2[-(x-1)+4] &= 5+[-(6x-7)+9x] \\ 2[-x+1+4] &= 5+[-6x+7+9x] \\ 2[-x+5] &= 5+[3x+7] \\ -2x+10 &= 3x+12 \\ -5x &= 2 \\ x &= -\tfrac{2}{5} \end{aligned}$$

**Check** $x=-\frac{2}{5}$:
$$2\left[\tfrac{7}{5}+4\right]=5+\left[\tfrac{47}{5}-\tfrac{18}{5}\right] \quad \textit{True}$$

The solution set is $\left\{-\frac{2}{5}\right\}$.

**45.** The denominators of the fractions are 3, 4, and 1. The least common denominator is $(3)(4)(1)=12$, since it is the smallest number into which each denominator can divide without a remainder.

**46.** Yes, the coefficients will be larger, but you will get the correct solution. As long as you multiply both sides of the equation by the *same* nonzero number, the resulting equation is equivalent and the solution does not change.

**47.** **(a)** We need to make the coefficient of the first term on the left an integer. Since $0.05=\frac{5}{100}$, we multiply by $10^2$ or 100. This will also take care of the second term.

**(b)** We need to make 0.006, 0.007, and 0.009 integers. These numbers can be written as $\frac{6}{1000}$, $\frac{7}{1000}$, and $\frac{9}{1000}$. Multiplying by $10^3$ or 1000 will eliminate the decimal points (the denominators) so that all the coefficients are integers.

**48.**
$$\begin{aligned} &0.06(10-x)(100) \\ &= 0.06(100)(10-x) \\ &= 6(10-x) \\ &= 60-6x \quad \text{Choice } \mathbf{B} \text{ is correct.} \end{aligned}$$

**49.**
$$\begin{aligned} -\tfrac{5}{9}x &= 2 \\ -5x &= 18 && \textit{Multiply by 9.} \\ x &= \tfrac{18}{-5} = -\tfrac{18}{5} && \textit{Divide by } -5. \end{aligned}$$

**Check** $x=-\frac{18}{5}$: $\left(-\frac{5}{9}\right)\left(-\frac{18}{5}\right)=2$ *True*

The solution set is $\left\{-\frac{18}{5}\right\}$.

**50.**
$$\begin{aligned} \tfrac{3}{11}x &= -5 \\ 3x &= -55 && \textit{Multiply by 11.} \\ x &= \tfrac{-55}{3} = -\tfrac{55}{3} && \textit{Divide by 3.} \end{aligned}$$

**Check** $x=-\frac{55}{3}$: $\left(\frac{3}{11}\right)\left(-\frac{55}{3}\right)=-5$ *True*

The solution set is $\left\{-\frac{55}{3}\right\}$.

**51.**
$$\begin{aligned} \tfrac{6}{5}x &= -1 \\ 6x &= -5 && \textit{Multiply by 5.} \\ x &= \tfrac{-5}{6} = -\tfrac{5}{6} && \textit{Divide by 6.} \end{aligned}$$

**Check** $x=-\frac{5}{6}$: $\left(\frac{6}{5}\right)\left(-\frac{5}{6}\right)=-1$ *True*

The solution set is $\left\{-\frac{5}{6}\right\}$.

**52.**
$$\begin{aligned} -\tfrac{7}{8}x &= 6 \\ -7x &= 48 && \textit{Multiply by 8.} \\ x &= \tfrac{48}{-7} = -\tfrac{48}{7} && \textit{Divide by } -7. \end{aligned}$$

**Check** $x=-\frac{48}{7}$: $\left(-\frac{7}{8}\right)\left(-\frac{48}{7}\right)=6$ *True*
The solution set is $\left\{-\frac{48}{7}\right\}$.

**53.** $$\frac{x}{2}+\frac{x}{3}=5$$
*Multiply both sides by the LCD, 6.*
$$6\left(\frac{x}{2}+\frac{x}{3}\right)=6(5)$$
$$6\left(\frac{x}{2}\right)+6\left(\frac{x}{3}\right)=30 \quad \textit{Distributive property}$$
$$3x+2x=30$$
$$5x=30 \quad \textit{Add.}$$
$$x=6 \quad \textit{Divide by 5.}$$

**Check** $x=6$: $3+2=5$ *True*

The solution set is $\{6\}$.

**54.** $$\frac{x}{5}-\frac{x}{4}=1$$
*Multiply both sides by the LCD, 20.*
$$20\left(\frac{x}{5}-\frac{x}{4}\right)=20(1)$$
$$20\left(\frac{x}{5}\right)-20\left(\frac{x}{4}\right)=20 \quad \textit{Distributive property}$$
$$4x-5x=20$$
$$-x=20 \quad \textit{Subtract.}$$
$$x=-20 \quad \textit{Multiply by } -1.$$

**Check** $x=-20$: $-4+5=1$ *True*

The solution set is $\{-20\}$.

**55.** $$\frac{3x}{4}+\frac{5x}{2}=13$$
*Multiply both sides by the LCD, 4.*
$$4\left(\frac{3x}{4}+\frac{5x}{2}\right)=4(13)$$
$$4\left(\frac{3x}{4}\right)+4\left(\frac{5x}{2}\right)=4(13) \quad \textit{Distributive property}$$
$$3x+10x=52$$
$$13x=52 \quad \textit{Combine terms.}$$
$$x=4 \quad \textit{Divide by 13.}$$

**Check** $x=4$: $3+10=13$ *True*

The solution set is $\{4\}$.

**56.** $$\frac{8x}{3}-\frac{x}{2}=-13$$
*Multiply both sides by the LCD, 6.*
$$6\left(\frac{8x}{3}-\frac{x}{2}\right)=6(-13)$$
$$6\left(\frac{8x}{3}\right)-6\left(\frac{x}{2}\right)=6(-13) \quad \textit{Distributive property}$$
$$16x-3x=-78$$
$$13x=-78$$
$$x=-6 \quad \textit{Divide by 26.}$$

**Check** $x=-6$: $-16+3=-13$ *True*

The solution set is $\{-6\}$.

**57.** $$\frac{x-10}{5}+\frac{2}{5}=-\frac{x}{3}$$
*Multiply both sides by the LCD, 15.*
$$15\left(\frac{x-10}{5}+\frac{2}{5}\right)=15\left(-\frac{x}{3}\right)$$
$$3(x-10)+3(2)=-5x$$
$$3x-30+6=-5x$$
$$8x=24$$
$$x=\tfrac{24}{8}=3$$

**Check** $x=3$: $-\frac{7}{5}+\frac{2}{5}=-1$ *True*

The solution set is $\{3\}$.

**58.** $$\frac{2x-3}{7}+\frac{3}{7}=-\frac{x}{3}$$
*Multiply both sides by the LCD, 21.*
$$21\left(\frac{2x-3}{7}+\frac{3}{7}\right)=21\left(-\frac{x}{3}\right)$$
$$3(2x-3)+3(3)=7(-x)$$
$$6x-9+9=-7x$$
$$6x=-7x$$
$$13x=0$$
$$x=\tfrac{0}{13}=0$$

**Check** $x=0$: $-\frac{3}{7}+\frac{3}{7}=0$ *True*

The solution set is $\{0\}$.

**59.** $$\frac{3x-1}{4}+\frac{x+3}{6}=3$$
*Multiply both sides by the LCD, 12.*
$$12\left(\frac{3x-1}{4}+\frac{x+3}{6}\right)=12(3)$$
$$3(3x-1)+2(x+3)=36$$
$$9x-3+2x+6=36$$
$$11x+3=36$$
$$11x=33$$
$$x=3$$

**Check** $x=3$: $2+1=3$ *True*

The solution set is $\{3\}$.

**60.** $$\frac{3x+2}{7}-\frac{x+4}{5}=2$$
*Multiply both sides by the LCD, 35.*
$$35\left(\frac{3x+2}{7}-\frac{x+4}{5}\right)=35(2)$$
$$5(3x+2)-7(x+4)=70$$
$$15x+10-7x-28=70$$
$$8x-18=70$$
$$8x=88$$
$$x=11$$

**Check** $x=11$: $5-3=2$ *True*

The solution set is $\{11\}$.

**61.** $\frac{4x+1}{3}=\frac{x+5}{6}+\frac{x-3}{6}$

*Multiply both sides by the LCD, 6.*

$$\begin{aligned}6\left(\frac{4x+1}{3}\right)&=6\left(\frac{x+5}{6}+\frac{x-3}{6}\right)\\ 2(4x+1)&=(x+5)+(x-3)\\ 8x+2&=2x+2\\ 6x&=0\\ x&=0\end{aligned}$$

**Check** $x=0$: $\frac{1}{3}=\frac{5}{6}-\frac{3}{6}$ *True*

The solution set is $\{0\}$.

**62.** $\frac{2x+5}{5}=\frac{3x+1}{2}+\frac{-x+7}{2}$

*Multiply both sides by the LCD, 10.*

$$\begin{aligned}10\left(\frac{2x+5}{5}\right)&=10\left(\frac{3x+1}{2}+\frac{-x+7}{2}\right)\\ 2(2x+5)&=5(3x+1)+5(-x+7)\\ 4x+10&=15x+5-5x+35\\ 4x+10&=10x+40\\ -6x&=30\\ x&=\tfrac{30}{-6}=-5\end{aligned}$$

**Check** $x=-5$: $-1=-7+6$ *True*

The solution set is $\{-5\}$.

**63.** $0.05x+0.12(x+5000)=940$

*Multiply each term by 100.*

$$\begin{aligned}5x+12(x+5000)&=100(940)\\ 5x+12x+60{,}000&=94{,}000\\ 17x&=34{,}000\\ x&=2000\end{aligned}$$

**Check** $x=2000$: $100+840=940$ *True*

The solution set is $\{2000\}$.

**64.** $0.09x+0.13(x+300)=61$

*Multiply each term by* 100.

$$\begin{aligned}100[0.09x+0.13(x+300)]&=100(61)\\ 100(0.09x)+100(0.13)(x+300)&=6100\\ 9x+13(x+300)&=6100\\ 9x+13x+3900&=6100\\ 22x&=2200\\ x&=\tfrac{2200}{22}=100\end{aligned}$$

**Check** $x=100$: $9+52=61$ *True*

The solution set is $\{100\}$.

**65.** $0.02(50)+0.08x=0.04(50+x)$

*Multiply each term by 100.*

$$\begin{aligned}2(50)+8x&=4(50+x)\\ 100+8x&=200+4x\\ 4x&=100\\ x&=25\end{aligned}$$

**Check** $x=25$: $1+2=3$ *True*

The solution set is $\{25\}$.

**66.** $0.20(14{,}000)+0.14x=0.18(14{,}000+x)$

*Multiply each term by 100.*

$$\begin{aligned}100[0.20(14{,}000)+0.14x]&=100[0.18(14{,}000+x)]\\ 20(14{,}000)+14x&=18(14{,}000+x)\\ 280{,}000+14x&=252{,}000+18x\\ 28{,}000&=4x\\ x&=7000\end{aligned}$$

**Check** $x=7000$: $2800+980=3780$ *True*

The solution set is $\{7000\}$.

**67.** $0.05x+0.10(200-x)=0.45x$

*Multiply each term by 100.*

$$\begin{aligned}5x+10(200-x)&=45x\\ 5x+2000-10x&=45x\\ 2000-5x&=45x\\ 2000&=50x\\ 40&=x\end{aligned}$$

**Check** $x=40$: $2+16=18$ *True*

The solution set is $\{40\}$.

**68.** $0.08x+0.12(260-x)=0.48x$

*Multiply each term by 100.*

$$\begin{aligned}8x+12(260-x)&=48x\\ 8x+3120-12x&=48x\\ -4x+3120&=48x\\ 3120&=52x\\ x&=\tfrac{3120}{52}=60\end{aligned}$$

**Check** $x=60$: $4.8+24=28.8$ *True*

The solution set is $\{60\}$.

**69.** $0.006(x+2)=0.007x+0.009$

*Multiply each term by 1000.*

$$\begin{aligned}6(x+2)&=7x+9\\ 6x+12&=7x+9\\ 3&=x\end{aligned}$$

**Check** $x=3$: $0.03=0.021+0.009$ *True*

The solution set is $\{3\}$.

**70.** $0.004x+0.006(50-x)=0.004(68)$

*Multiply each term by 1000.*

$$\begin{aligned}4x+6(50-x)&=4(68)\\ 4x+300-6x&=272\\ -2x+300&=272\\ -2x&=-28\\ x&=14\end{aligned}$$

**Check** $x=14$: $0.056+0.216=0.272$ *True*

The solution set is $\{14\}$.

**71.** $2L + 2W; \quad L = 10, W = 8$

$$\begin{aligned} 2L + 2W &= 2(10) + 2(8) \\ &= 20 + 16 = 36 \end{aligned}$$

**72.** $rt; \quad r = 0.15, t = 3$

$$rt = 0.15(3) = 0.45$$

**73.** $\frac{1}{3}Bh; \quad B = 27, h = 8$

$$\begin{aligned} \tfrac{1}{3}Bh &= \tfrac{1}{3}(27)(8) \\ &= 9(8) = 72 \end{aligned}$$

**74.** $prt; \quad p = 8000, r = 0.06, t = 2$

$$\begin{aligned} prt &= 8000(0.06)(2) \\ &= 480(2) = 960 \end{aligned}$$

**75.** $\frac{5}{9}(F - 32); \quad F = 122$

$$\begin{aligned} \tfrac{5}{9}(F - 32) &= \tfrac{5}{9}(122 - 32) \\ &= \tfrac{5}{9}(90) = 50 \end{aligned}$$

**76.** $\frac{9}{5}C + 32; \quad C = 60$

$$\begin{aligned} \tfrac{9}{5}C + 32 &= \tfrac{9}{5}(60) + 32 \\ &= 108 + 32 = 140 \end{aligned}$$

## 2.2 Formulas and Percent

### 2.2 Classroom Examples, Now Try Exercises

**1.** To solve $d = rt$ for $r$, treat $r$ as the only variable.

$$\begin{aligned} d &= rt \\ \frac{d}{t} &= \frac{rt}{t} && \textit{Divide by t.} \\ \frac{d}{t} &= r, \text{ or } r = \frac{d}{t} \end{aligned}$$

**N1.** To solve $I = prt$ for $p$, treat $p$ as the only variable.

$$\begin{aligned} I &= prt \\ I &= p(rt) && \textit{Associative property} \\ \frac{I}{rt} &= \frac{p(rt)}{rt} && \textit{Divide by rt.} \\ \frac{I}{rt} &= p, \text{ or } p = \frac{I}{rt} \end{aligned}$$

**2.** Solve $P = 2L + 2W$ for $L$.

$$\begin{aligned} P &= 2L + 2W \\ P - 2W &= 2L && \textit{Subtract 2W.} \\ \frac{P - 2W}{2} &= \frac{2L}{2} && \textit{Divide by 2.} \\ \frac{P - 2W}{2} &= L, \text{ or } L = \frac{P}{2} - W \end{aligned}$$

**N2.** Solve $P = a + 2b + c$ for $b$.

$$\begin{aligned} P - a &= 2b + c && \textit{Subtract a.} \\ P - a - c &= 2b && \textit{Subtract c.} \\ \frac{P - a - c}{2} &= \frac{2b}{2} && \textit{Divide by 2.} \\ \frac{P - a - c}{2} &= b, \text{ or } b = \frac{P - a - c}{2} \end{aligned}$$

**3.** Solve $y = \frac{1}{2}(x + 3)$ for $x$.

$$\begin{aligned} 2y &= x + 3 && \textit{Multiply by 2.} \\ 2y - 3 &= x, \text{ or } x = 2y - 3 && \textit{Subtract 3.} \end{aligned}$$

**N3.** Solve $P = 2(L + W)$ for $L$.

$$\begin{aligned} \frac{P}{2} &= \frac{2(L + W)}{2} && \textit{Divide by 2.} \\ \frac{P}{2} &= L + W \\ \frac{P}{2} - W &= L && \textit{Subtract W.} \\ L &= \frac{P}{2} - W, \text{ or } L = \frac{P - 2W}{2} \end{aligned}$$

**4.** Solve $2x + 7y = 5$ for $y$.

$$\begin{aligned} 2x + 7y - 2x &= 5 - 2x && \textit{Subtract 2x.} \\ 7y &= 5 - 2x \\ \frac{7y}{7} &= \frac{5 - 2x}{7} && \textit{Divide by 7.} \\ y &= \frac{5 - 2x}{7}, \text{ or } y = \frac{2x - 5}{-7} \end{aligned}$$

**N4.** Solve $5x - 6y = 12$ for $y$.

$$\begin{aligned} -6y &= 12 - 5x && \textit{Subtract 5x.} \\ y &= \frac{12 - 5x}{-6}, && \textit{Divide by −6.} \\ \text{or } y &= \frac{5x - 12}{6} \end{aligned}$$

**5.** Use $d = rt$. Solve for $r$.

$$\begin{aligned} \frac{d}{t} &= \frac{rt}{t} && \textit{Divide by t.} \\ \frac{d}{t} &= r, \text{ or } r = \frac{d}{t} \end{aligned}$$

Now substitute $d = 15$ and $t = \frac{1}{3}$.

$$r = \frac{15}{\frac{1}{3}} = 15 \times \tfrac{3}{1} = 45$$

His average rate is 45 miles per hour.

**N5.** Use $d = rt$. Solve for $r$.

$$\begin{aligned} \frac{d}{t} &= \frac{rt}{t} && \textit{Divide by t.} \\ \frac{d}{t} &= r, \text{ or } r = \frac{d}{t} \end{aligned}$$

Now substitute $d = 21$ and $t = \frac{1}{2}$.

$$r = \frac{21}{\frac{1}{2}} = 21 \times \tfrac{2}{1} = 42$$

Her average rate is 42 miles per hour.

**6.** **(a)** The given amount of mixture is 20 oz. The part that is oil is 1 oz. Thus, the percent of oil is

$$\frac{\text{partial amount}}{\text{whole amount}} = \frac{1}{20} = 0.05 = 5\%.$$

**(b)** Let $x$ represent the amount of commission earned.

$$\frac{x}{12{,}000} = 0.08 \qquad \frac{\textit{partial}}{\textit{whole}} = \textit{percent}$$
$$x = 0.08(12{,}000) \qquad \textit{Multiply by 12,000.}$$
$$x = 960$$

The salesman earns \$960.

**N6.** **(a)** The given amount of mixture is 5 L. The part that is antifreeze is 2 L. Thus, the percent of antifreeze is

$$\frac{\text{partial amount}}{\text{whole amount}} = \frac{2}{5} = 0.40 = 40\%.$$

**(b)** Let $x$ represent the amount of interest earned.

$$\frac{x}{7500} = 0.025 \qquad \frac{\textit{partial}}{\textit{whole}} = \textit{percent}$$
$$x = 0.025(7500) \qquad \textit{Multiply by 7500.}$$
$$x = 187.50$$

The interest earned is \$187.50.

**7.** Let $x$ represent the amount spent on pet supplies/medicine.

$$\frac{x}{41.2} = 0.238 \qquad \textit{23.8\% = 0.238}$$
$$x = 0.238(41.2) \qquad \textit{Multiply by 41.2.}$$
$$x = 9.8056$$

Therefore, about \$9.8 billion was spent on pet supplies/medicine.

**N7.** Let $x$ represent the amount spent on vet care.

$$\frac{x}{41.2} = 0.245 \qquad \textit{24.5\% = 0.245}$$
$$x = 0.245(41.2) \qquad \textit{Multiply by 41.2.}$$
$$x = 10.094$$

Therefore, about \$10.1 billion was spent on vet care.

**8.** **(a)** Let $x =$ the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{base}}$$
$$x = \frac{1352 - 1300}{1300}$$
$$x = \frac{52}{1300}$$
$$x = 0.04$$

The percent increase was 4%.

**(b)** Let $x =$ the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{base}}$$
$$x = \frac{54.00 - 51.30}{54.00}$$
$$x = \frac{2.70}{54.00}$$
$$x = 0.05$$

The percent decrease was 5%.

**N8.** **(a)** Let $x =$ the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{base}}$$
$$x = \frac{80 - 56}{80}$$
$$x = \frac{24}{80}$$
$$x = 0.3$$

The percent markdown was 30%.

**(b)** Let $x =$ the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{base}}$$
$$x = \frac{689 - 650}{650}$$
$$x = \frac{39}{650}$$
$$x = 0.06$$

The percent increase was 6%.

## 2.2 Section Exercises

**1.** Solve $I = prt$ for $r$.

$$I = prt$$
$$\frac{I}{pt} = \frac{prt}{pt}$$
$$\frac{I}{pt} = r, \quad \text{or} \quad r = \frac{I}{pt}$$

**2.** Solve $d = rt$ for $t$.

$$\frac{d}{r} = \frac{rt}{r} \qquad \textit{Divide by r.}$$
$$\frac{d}{r} = t, \quad \text{or} \quad t = \frac{d}{r}$$

**3.** Solve $P = 2L + 2W$ for $L$.

$$P = 2L + 2W$$
$$P - 2W = 2L$$
$$\frac{P - 2W}{2} = \frac{2L}{2}$$
$$\frac{P - 2W}{2} = L, \quad \text{or} \quad L = \frac{P}{2} - W$$

**4.** Solve $\mathcal{A} = bh$ for $b$.

$$\frac{\mathcal{A}}{h} = b \qquad \textit{Divide by h.}$$

**5.** **(a)** Solve for $V = LWH$ for $W$.

$$V = LWH$$
$$\frac{V}{LH} = \frac{LWH}{LH}$$
$$\frac{V}{LH} = W, \text{ or } W = \frac{V}{LH}$$

**(b)** Solve for $V = LWH$ for $H$.

$$V = LWH$$
$$\frac{V}{LW} = \frac{LWH}{LW}$$
$$\frac{V}{LW} = H, \text{ or } H = \frac{V}{LW}$$

**6.** **(a)** Solve $P = a + b + c$ for $b$.

$$P - (a + c) = a + b + c - (a + c)$$ *Subtract (a + c).*

$$P - a - c = b$$

**(b)** Solve $P = a + b + c$ for $c$.

$$P - (a + b) = a + b + c - (a + b)$$ *Subtract (a + b).*

$$P - a - b = c$$

**7.** Solve $C = 2\pi r$ for $r$.

$$C = 2\pi r$$
$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \textit{Divide by } 2\pi.$$
$$\frac{C}{2\pi} = r$$

**8.** Solve $\mathcal{A} = \frac{1}{2}bh$ for $h$.

$$\frac{2\mathcal{A}}{b} = \frac{2}{b}\left(\frac{1}{2}bh\right) \quad \textit{Multiply by 2; divide by } b.$$
$$\frac{2\mathcal{A}}{b} = h$$

**9.** **(a)** Solve $\mathcal{A} = \frac{1}{2}h(b + B)$ for $h$.

$$2\mathcal{A} = h(b + B) \quad \textit{Multiply by 2.}$$
$$\frac{2\mathcal{A}}{b + B} = h \quad \textit{Divide by } b + B.$$

**(b)** Solve $\mathcal{A} = \frac{1}{2}h(b + B)$ for $B$.

$$2\mathcal{A} = h(b + B) \quad \textit{Multiply by 2.}$$
$$\frac{2\mathcal{A}}{h} = b + B \quad \textit{Divide by } h.$$
$$\frac{2\mathcal{A}}{h} - b = B \quad \textit{Subtract } b.$$

*Another method:*

$$2\mathcal{A} = h(b + B) \quad \textit{Multiply by 2.}$$
$$2\mathcal{A} = hb + hB \quad \textit{Distributive prop.}$$
$$2\mathcal{A} - hb = hB \quad \textit{Subtract } hb.$$
$$\frac{2\mathcal{A} - hb}{h} = B \quad \textit{Divide by } h.$$

**10.** Solve $S = 2\pi rh + 2\pi r^2$ for $h$.

$$S - 2\pi r^2 = 2\pi rh \quad \textit{Subtract } 2\pi r^2.$$
$$\frac{S - 2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r} \quad \textit{Divide by } 2\pi r.$$
$$\frac{S - 2\pi r^2}{2\pi r} = h, \text{ or } \frac{S}{2\pi r} - r = h$$

**11.** Solve $F = \frac{9}{5}C + 32$ for $C$.

$$F - 32 = \tfrac{9}{5}C$$
$$\tfrac{5}{9}(F - 32) = \tfrac{5}{9}\left(\tfrac{9}{5}C\right)$$
$$\tfrac{5}{9}(F - 32) = C$$

**12.** Solve $C = \frac{5}{9}(F - 32)$ for $F$.

$$\tfrac{9}{5}C = \tfrac{9}{5} \cdot \tfrac{5}{9}(F - 32) \quad \textit{Multiply by } \tfrac{9}{5}.$$
$$\tfrac{9}{5}C = F - 32$$
$$\tfrac{9}{5}C + 32 = F \quad \textit{Add 32.}$$

**13.**

$$\mathcal{A} = \tfrac{1}{2}bh$$
$$2\mathcal{A} = 2\left(\tfrac{1}{2}bh\right) \quad \textit{Multiply by 2.}$$
$$2\mathcal{A} = bh$$
$$\frac{2\mathcal{A}}{b} = \frac{bh}{b} \quad \textit{Divide by } b.$$
$$\frac{2\mathcal{A}}{b} = h$$
$$\frac{2\mathcal{A}}{b} = \frac{2}{1} \cdot \frac{\mathcal{A}}{b}$$
$$= 2\left(\frac{\mathcal{A}}{b}\right) \quad \textit{This choice is } \textbf{A}.$$
$$= 2\mathcal{A}\left(\frac{1}{b}\right) \quad \textit{This is choice } \textbf{B}.$$

To get choice **C**, divide $\mathcal{A} = \frac{1}{2}bh$ by $\frac{1}{2}b$.

$$\frac{\mathcal{A}}{\frac{1}{2}b} = \frac{\frac{1}{2}bh}{\frac{1}{2}b} \text{ gives us } h = \frac{\mathcal{A}}{\frac{1}{2}b}.$$

Choice **D**, $h = \frac{\frac{1}{2}\mathcal{A}}{b}$, can be multiplied by $\frac{2}{2}$ on the right side to get $h = \frac{\mathcal{A}}{2b}$, so it is *not* equivalent to $h = \frac{2\mathcal{A}}{b}$. Therefore, the correct answer is **D**.

**14.**

$$C = \tfrac{5}{9}(F - 32)$$
$$C = \tfrac{5}{9}F - \tfrac{160}{9} \quad \textit{This is choice } \textbf{A}.$$
$$C = \frac{5}{9} \cdot \frac{F}{1} - \frac{160}{9}$$
$$C = \frac{5F}{9} - \frac{160}{9} \quad \textit{This is choice } \textbf{B}.$$
$$C = \frac{5F - 160}{9} \quad \textit{This is choice } \textbf{C}.$$

Therefore, the correct answer is **D**.

**15.**
$$\begin{aligned} 4x + 9y &= 11 \\ 4x + 9y - 4x &= 11 - 4x && \textit{Subtract 4x.} \\ 9y &= 11 - 4x \\ \frac{9y}{9} &= \frac{11 - 4x}{9} && \textit{Divide by 9.} \\ y &= \frac{11 - 4x}{9} \end{aligned}$$

**16.**
$$\begin{aligned} 7x + 8y &= 11 \\ 8y &= 11 - 7x && \textit{Subtract 7x.} \\ y &= \frac{11 - 7x}{8} && \textit{Divide by 8.} \end{aligned}$$

**17.**
$$\begin{aligned} -3x + 2y &= 5 \\ 2y &= 5 + 3x && \textit{Add 3x.} \\ y &= \frac{5 + 3x}{2} && \textit{Divide by 2.} \end{aligned}$$

**18.**
$$\begin{aligned} -5x + 3y &= 12 \\ 3y &= 12 + 5x && \textit{Add 5x.} \\ y &= \frac{12 + 5x}{3} && \textit{Divide by 3.} \end{aligned}$$

**19.**
$$\begin{aligned} 6x - 5y &= 7 \\ -5y &= 7 - 6x && \textit{Subtract 6x.} \\ y &= \frac{7 - 6x}{-5}, && \textit{Divide by } -5. \\ \text{or } y &= \frac{6x - 7}{5} \end{aligned}$$

**20.**
$$\begin{aligned} 8x - 3y &= 4 \\ -3y &= 4 - 8x && \textit{Subtract 8x.} \\ y &= \frac{4 - 8x}{-3}, && \textit{Divide by } -3. \\ \text{or } y &= \frac{8x - 4}{3} \end{aligned}$$

**21.** Solve $d = rt$ for $t$.

$$t = \tfrac{d}{r}$$

To find $t$, substitute $d = 500$ and $r = 152.672$.

$$t = \tfrac{500}{152.672} \approx 3.275$$

His time was about 3.275 hours.

**22.** Solve $d = rt$ for $t$.

$$t = \tfrac{d}{r}$$

Replace $d$ by 415 and $r$ by 151.774.

$$t = \tfrac{415}{151.774} \approx 2.734$$

His time was about 2.734 hours.

**23.** Solve $d = rt$ for $r$.

$$\begin{aligned} r &= \frac{d}{t} \\ r &= \tfrac{520}{10} = 52 && \textit{Let } d = 520,\ t = 10. \end{aligned}$$

Her rate was 52 mph.

**24.** Solve $d = rt$ for $t$.

$$\begin{aligned} t &= \frac{d}{r} \\ t &= \frac{10{,}500}{500} = 21 && \textit{Let } d = 10{,}500,\ r = 500. \end{aligned}$$

The travel time is 21 hr.

**25.** Use the formula $F = \frac{9}{5}C + 32$.

$$\begin{aligned} F &= \tfrac{9}{5}(45) + 32 && \textit{Let } C = 45. \\ &= 81 + 32 \\ &= 113 \end{aligned}$$

The corresponding temperature is 113°F.

**26.** Use the formula $C = \frac{5}{9}(F - 32)$.

$$\begin{aligned} C &= \tfrac{5}{9}(-58 - 32) && \textit{Let } F = -58. \\ &= \tfrac{5}{9}(-90) \\ &= -50 \end{aligned}$$

The corresponding temperature is about $-50$°C.

**27.** Solve $P = 4s$ for $s$.

$$s = \tfrac{P}{4}$$

To find $s$, substitute 920 for $P$.

$$s = \tfrac{920}{4} = 230$$

The length of each side is 230 m.

**28.** Use $V = \pi r^2 h$.
Replace $r$ by $\frac{35}{2} = 17.5$ and $h$ by 588.

$$\begin{aligned} V &= \pi(17.5)^2(588) \\ &\approx 565{,}722.3 \end{aligned}$$

To the nearest whole number, the volume is 565,722 $\text{ft}^3$.

**29.** Use the formula $C = 2\pi r$.

$$\begin{aligned} 480\pi &= 2\pi r && \textit{Let } C = 480\pi. \\ \frac{480\pi}{2\pi} &= r && \textit{Divide by } 2\pi. \end{aligned}$$

So the radius of the circle is 240 inches and the diameter is twice that length, that is, 480 inches.

**30.** $d = 2r = 2(2.5) = 5$
The diameter is 5 inches.
$C = 2\pi r = 2\pi(2.5) = 5\pi$
The circumference is $5\pi$ inches.

**31.** Use $V = LWH$.
Let $V = 187$, $L = 11$, and $W = 8.5$.

$$\begin{aligned} 187 &= 11(8.5)H \\ 187 &= 93.5H \\ 2 &= H && \textit{Divide by 93.5.} \end{aligned}$$

The ream is 2 inches thick.

**32.** Use $V = LWH$.
Let $V = 238$, $W = 8.5$, and $H = 2$.

$$\begin{aligned} 238 &= L(8.5)(2) \\ 238 &= L(17) \\ 14 &= L \qquad \textit{Divide by 17.} \end{aligned}$$

The length of a legal sheet of paper is 14 inches.

**33.** The mixture is 36 oz and that part which is alcohol is 9 oz. Thus, the percent of alcohol is

$$\tfrac{9}{36} = \tfrac{1}{4} = \tfrac{25}{100} = 25\%.$$

The percent of water is

$$100\% - 25\% = 75\%.$$

**34.** Let $x =$ the amount of pure acid in the mixture. Then $x$ can be found by multiplying the total amount of the mixture by the percent of acid given as a decimal $(0.35)$.

$$x = 40(0.35) = 14$$

There are 14 L of pure acid. Since there are 40 L altogether, there are $40 - 14$, or 26 L of pure water in the mixture.

**35.** Find what percent \$6300 is of \$210,000.

$$\tfrac{6300}{210{,}000} = 0.03 = 3\%$$

The agent received a 3% rate of commission.

**36.** Solve $I = prt$ for $r$.

$$\begin{aligned} r &= \frac{I}{pt} \\ r &= \frac{221}{3400(1)} \\ &= 0.065 = 6.5\% \end{aligned}$$

The interest rate on this deposit is 6.5%.

In Exercises 37–40, use the rule of 78.

$$u = f \cdot \frac{k(k+1)}{n(n+1)}$$

**37.** Substitute 700 for $f$, 4 for $k$, and 36 for $n$.

$$\begin{aligned} u &= 700 \cdot \frac{4(4+1)}{36(36+1)} \\ &= 700 \cdot \frac{4(5)}{36(37)} \approx 10.51 \end{aligned}$$

The unearned interest is \$10.51.

**38.** Substitute 600 for $f$, 12 for $k$, and 36 for $n$.

$$\begin{aligned} u &= 600 \cdot \frac{12(12+1)}{36(36+1)} \\ &= 600 \cdot \frac{12(13)}{36(37)} \approx 70.27 \end{aligned}$$

The unearned interest is \$70.27.

**39.** Substitute 380.50 for $f$, 8 for $k$, and 24 for $n$.

$$\begin{aligned} u &= (380.50) \cdot \frac{8(8+1)}{24(24+1)} \\ &= (380.50) \cdot \frac{8(9)}{24(25)} \approx 45.66 \end{aligned}$$

The unearned interest is \$45.66.

**40.** Substitute 450 for $f$, 9 for $k$, and 24 for $n$.

$$\begin{aligned} u &= 450 \cdot \frac{9(9+1)}{24(24+1)} \\ &= 450 \cdot \frac{9(10)}{24(25)} \approx 67.50 \end{aligned}$$

The unearned interest is \$67.50.

**41.** **(a)** Boston:

$$\text{Pct.} = \frac{W}{W+L} = \frac{95}{95+67} = \frac{95}{162} \approx .586$$

**(b)** Tampa Bay:

$$\text{Pct.} = \frac{W}{W+L} = \frac{84}{84+78} = \frac{84}{162} \approx .519$$

**(c)** Toronto:

$$\text{Pct.} = \frac{W}{W+L} = \frac{75}{75+87} = \frac{75}{162} \approx .463$$

**(d)** Baltimore:

$$\text{Pct.} = \frac{W}{W+L} = \frac{64}{64+98} = \frac{64}{162} = .395$$

**42.** **(a)** Philadelphia:

$$\text{Pct.} = \frac{W}{W+L} = \frac{93}{93+69} = \frac{93}{162} \approx .574$$

**(b)** Atlanta:

$$\text{Pct.} = \frac{W}{W+L} = \frac{86}{86+76} = \frac{86}{162} \approx .531$$

**(c)** New York Mets:

$$\text{Pct.} = \frac{W}{W+L} = \frac{70}{70+92} = \frac{70}{162} \approx .432$$

**(d)** Washington:

$$\text{Pct.} = \frac{W}{W+L} = \frac{59}{59+103} = \frac{59}{162} \approx .364$$

**43.** $\dfrac{62.0 \text{ million}}{114.9 \text{ million}} \approx 0.54$

In 2009, about 54% of the U.S. households that owned at least one TV set owned at least 3 TV sets.

**44.** $\dfrac{102.2 \text{ million}}{114.9 \text{ million}} \approx 0.89$

In 2009, about 89% of the U.S. households that owned at least one TV set had a DVD player.

**45.** $0.88(114.9) = 101.112$

In 2009, about 101.1% of the U.S. households that owned at least one TV set received basic cable.

**46.** $0.35(114.9) = 40.215$

In 2009, about 40.2% of the U.S. households that owned at least one TV set received premium cable.

**47.** $0.32(221{,}190) = 70{,}780.80$

To the nearest dollar, \$70,781 will be spent to provide housing.

**48.** $0.08(221{,}190) = 17{,}695.20$

To the nearest dollar, \$17,695 will be spent for health care.

**49.** Since 16% is twice as much as 8%, the cost for child care and education will be $2(\$17{,}695) = \$35{,}390$.

**50.** $\dfrac{\$35{,}000}{\$221{,}190} \approx 0.1582$

So the food cost is about 16%, which agrees with the percent shown in the graph.

**51.** Let $x =$ the percent increase (as a decimal).

$$\begin{aligned} \text{percent increase} &= \frac{\text{amount of increase}}{\text{base}} \\ x &= \frac{11.34 - 10.50}{10.50} \\ x &= \frac{0.84}{10.50} \\ x &= 0.08 \end{aligned}$$

The percent increase was 8%.

**52.** Let $x =$ the percent decrease (as a decimal).

$$\begin{aligned} \text{percent decrease} &= \frac{\text{amount of decrease}}{\text{base}} \\ x &= \frac{70.00 - 59.50}{70.00} \\ x &= \frac{10.50}{70.00} \\ x &= 0.15 \end{aligned}$$

The percent discount was 15%.

**53.** Let $x =$ the percent decrease (as a decimal).

$$\begin{aligned} \text{percent decrease} &= \frac{\text{amount of decrease}}{\text{base}} \\ x &= \frac{134{,}953 - 129{,}798}{134{,}953} \\ x &= \frac{5155}{134{,}953} \\ x &\approx 0.038 \end{aligned}$$

The percent decrease was 3.8%.

**54.** Let $x =$ the percent increase (as a decimal).

$$\begin{aligned} \text{percent increase} &= \frac{\text{amount of increase}}{\text{base}} \\ x &= \frac{362{,}340 - 320{,}391}{320{,}391} \\ x &= \frac{41{,}949}{320{,}391} \\ x &\approx 0.131 \end{aligned}$$

The percent increase was 13.1%.

**55.**
$$\begin{aligned} \text{percent decrease} &= \frac{\text{amount of decrease}}{\text{base}} \\ &= \frac{18.98 - 9.97}{18.98} \\ &= \frac{9.01}{18.98} \approx 0.475 \end{aligned}$$

The percent discount was 47.5%.

**56.**
$$\begin{aligned} \text{percent decrease} &= \frac{\text{amount of decrease}}{\text{base}} \\ &= \frac{29.99 - 15.99}{29.99} \\ &= \frac{14.00}{29.99} \approx 0.467 \end{aligned}$$

The percent discount was 46.7%.

**57.**
$$\begin{aligned} 4x + 4(x + 7) &= 124 \\ 4x + 4x + 28 &= 124 \\ 8x + 28 &= 124 \\ 8x &= 96 \\ x &= \tfrac{96}{8} = 12 \end{aligned}$$

The solution set is $\{12\}$.

**58.**
$$\begin{aligned} x + 0.20x &= 66 \\ 1.2x &= 66 \\ x &= \tfrac{66}{1.2} = 55 \end{aligned}$$

The solution set is $\{55\}$.

**59.**
$$\begin{aligned} 2.4 + 0.4x &= 0.25(6 + x) \\ 240 + 40x &= 25(6 + x) \quad \textit{Multiply by 100.} \\ 240 + 40x &= 150 + 25x \\ 15x &= -90 \\ x &= -6 \end{aligned}$$

The solution set is $\{-6\}$.

**60.** $0.07x + 0.05(9000 - x) = 510$

$$
\begin{aligned}
7x + 5(9000 - x) &= 51{,}000 \quad \textit{Multiply by 100.}\\
7x + 45{,}000 - 5x &= 51{,}000\\
2x + 45{,}000 &= 51{,}000\\
2x &= 6000\\
x &= 3000
\end{aligned}
$$

The solution set is $\{3000\}$.

**61.** "The product of $-3$ and $5$, divided by $1$ less than $6$" is translated and evaluated as follows:

$$\frac{-3(5)}{6-1} = \frac{-15}{5} = -3$$

**62.** "Half of $-18$, added to the reciprocal of $\frac{1}{5}$" is translated and evaluated as follows:

$$\frac{1}{2}(-18) + \frac{1}{\frac{1}{5}} = -9 + 5 = -4$$

**63.** "The sum of 6 and $-9$, multiplied by the additive inverse of 2" is translated and evaluated as follows:

$$[6 + (-9)](-2) = -3(-2) = 6$$

**64.** "The product of $-2$ and $4$, added to the product of $-9$ and $-3$" is translated and evaluated as follows:

$$-2(4) + (-9)(-3) = -8 + 27 = 19$$

# 2.3 Applications of Linear Equations

## 2.3 Classroom Examples, Now Try Exercises

**1.** **(a)**

| The sum of a number and 6 | is | 28. |
|---|---|---|
| ↓ | ↓ | ↓ |
| $x + 6$ | $=$ | $28$ |

An equation is $x + 6 = 28$.

**(b)**

| The product of a number and 7 | is | twice the number | plus | 12. |
|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ |
| $7x$ | $=$ | $2x$ | $+$ | $12$ |

An equation is $7x = 2x + 12$.

**(c)**

| The quotient of a number and 6, | added to | twice the number, | is | 7. |
|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ |
| $\frac{x}{6}$ | $+$ | $2x$ | $=$ | $7$ |

An equation is $\frac{x}{6} + 2x = 7$, or equivalently, $2x + \frac{x}{6} = 7$.

**N1.** **(a)**

| The quotient of a number and 10 | is | twice the number. |
|---|---|---|
| ↓ | ↓ | ↓ |
| $\frac{x}{10}$ | $=$ | $2x$ |

An equation is $\frac{x}{10} = 2x$.

**(b)**

| The product of a number and 5, | decreased by | 7, | is | zero. |
|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ |
| $5x$ | $-$ | $7$ | $=$ | $0$ |

An equation is $5x - 7 = 0$.

**2.** **(a)** $5x - 3(x + 2) = 7$ is an *equation* because it has an equals symbol.

$$
\begin{aligned}
5x - 3(x + 2) &= 7\\
5x - 3x - 6 &= 7 \quad \textit{Distributive property}\\
2x - 6 &= 7 \quad \textit{Combine like terms.}\\
2x &= 13 \quad \textit{Add 6.}\\
x &= \tfrac{13}{2} \quad \textit{Divide by 2.}
\end{aligned}
$$

The solution set is $\{\frac{13}{2}\}$.

**(b)** $5x - 3(x + 2)$ is an *expression* because there is no equals symbol.

$$
\begin{aligned}
&5x - 3(x + 2)\\
&= 5x - 3x - 6 \quad \textit{Distributive property}\\
&= 2x - 6 \quad \textit{Combine like terms.}
\end{aligned}
$$

**N2.** **(a)** $3(x - 5) + 2x - 1$ is an *expression* because there is no equals symbol.

$$
\begin{aligned}
&3(x - 5) + 2x - 1\\
&= 3x - 15 + 2x - 1 \quad \textit{Distributive property}\\
&= 5x - 16 \quad \textit{Combine like terms.}
\end{aligned}
$$

**(b)** $3(x - 5) + 2x = 1$ is an *equation* because it has an equals symbol.

$$
\begin{aligned}
3(x - 5) + 2x &= 1\\
3x - 15 + 2x &= 1 \quad \textit{Distributive property}\\
5x - 15 &= 1 \quad \textit{Combine like terms.}\\
5x &= 16 \quad \textit{Add 15.}\\
x &= \tfrac{16}{5} \quad \textit{Divide by 5.}
\end{aligned}
$$

The solution set is $\{\frac{16}{5}\}$.

**3.** *Step 2*
The length and perimeter are given in terms of the width $W$. The length $L$ is 5 cm more than the width, so

$$L = W + 5.$$

The perimeter $P$ is 5 times the width, so

$$P = 5W.$$

*Step 3*
Use the formula for perimeter of a rectangle.

$$P = 2L + 2W$$
$$5W = 2(W + 5) + 2W \quad \textit{P = 5W; L = W + 5}$$

*Step 4*
Solve the equation.

| | |
|---|---|
| $5W = 2W + 10 + 2W$ | *Distributive property* |
| $5W = 4W + 10$ | *Combine terms.* |
| $W = 10$ | *Subtract 4W.* |

*Step 5*
The width is 10 and the length is

$$L = W + 5 = 10 + 5 = 15.$$

The rectangle is 10 cm by 15 cm.

*Step 6*
15 is 5 more than 10 and $P = 2(10) + 2(15) = 50$ is five times 10.

**N3.** *Step 2*
The length and perimeter are given in terms of the width $W$. The length $L$ is 2 ft more than twice the width, so

$$L = 2W + 2.$$

The perimeter $P$ is 34, so

$$P = 34.$$

*Step 3*
Use the formula for perimeter of a rectangle.

$$P = 2L + 2W$$
$$34 = 2(2W + 2) + 2W \quad \textit{P = 34; L = 2W + 2}$$

*Step 4*
Solve the equation.

| | |
|---|---|
| $34 = 4W + 4 + 2W$ | *Distributive property* |
| $34 = 6W + 4$ | *Combine terms.* |
| $30 = 6W$ | *Subtract 4.* |
| $5 = W$ | *Divide by 6.* |

*Step 5*
The width is 5 and the length is

$$L = 2W + 2 = 10 + 2 = 12.$$

The rectangle is 5 ft by 12 ft.

*Step 6*
12 is 2 more than twice 5 and
$P = 2(12) + 2(5) = 34$.

**4.** *Step 2*
Let $x =$ the number of hits for Jeter.
Then $x + 13 =$ the number of hits for Suzuki.

*Step 3*
The sum of their hits is 437, so an equation is

$$x + (x + 13) = 437.$$

*Step 4*
Solve the equation.

| | |
|---|---|
| $2x + 13 = 437$ | |
| $2x = 424$ | *Subtract 13.* |
| $x = 212$ | *Divide by 2.* |

*Step 5*
Jeter had 212 hits and Suzuki had
$212 + 13 = 225$ hits.

*Step 6*
225 is 13 more than 212, and the sum of 212 and 225 is 437.

**N4.** *Step 2*
Let $x =$ the number of TDs for Warner.
Then $x + 4 =$ the number of TDs for Brees.

*Step 3*
The sum of their TDs is 64, so an equation is

$$x + (x + 4) = 64.$$

*Step 4*
Solve the equation.

| | |
|---|---|
| $2x + 4 = 64$ | |
| $2x = 60$ | *Subtract 4.* |
| $x = 30$ | *Divide by 2.* |

*Step 5*
Warner had 30 TDs and Brees had
$30 + 4 = 34$ TDs.

*Step 6*
34 is 4 more than 30, and the sum of 30 and 34 is 64.

**5.** *Step 2*
Let $x =$ the number of area codes in 1947. Since $250\% = 250(0.01) = 2.5$, $2.5x =$ the number of additional area codes in 2002.

*Step 3*

| The number in 1947 | plus | the increase | is | 301. |
|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ |
| $x$ | $+$ | $2.5x$ | $=$ | 301 |

*Step 4*
Solve the equation.

| | |
|---|---|
| $1x + 2.5x = 301$ | *Identity property* |
| $3.5x = 301$ | *Combine like terms.* |
| $x = 86$ | *Divide by 3.5.* |

*Step 5*
There were 86 area codes in 1947.

*Step 6*
**Check** that the increase, $301 - 86 = 215$, is 250% of 86. $250\% \cdot 86 = 250(0.01)(86) = 215$, as required.

**N5.** *Step 2*
Let $x =$ the number of Introductory Statistics students in the fall of 1992. Since $700\% = 700(0.01) = 7$, $7x =$ the number of additional students in 2009.

*Step 3*

| The number in 1992 | plus | the increase | is | 96. |
|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ |
| $x$ | $+$ | $7x$ | $=$ | $96$ |

*Step 4*
Solve the equation.

$1x + 7x = 96$ *Identity property*
$8x = 96$ *Combine like terms.*
$x = 12$ *Divide by 8.*

*Step 5*
There were 12 students in the fall of 1992.

*Step 6*
**Check** that the increase, $96 - 12 = 84$, is 700% of 12. $700\% \cdot 12 = 700(0.01)(12) = 84$, as required.

**6.** Let $x =$ the amount invested at 5%.
Then $34{,}000 - x =$ the amount invested at 4%.

Use $I = prt$ with $t = 1$.

Make a table to organize the information.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.05 | $0.05x$ |
| $34{,}000 - x$ | 0.04 | $0.04(34{,}000 - x)$ |
| 34,000 | ← Totals → | 1545 |

The last column gives the equation.

$0.05x + 0.04(34{,}000 - x) = 1545$
$0.05x + 1360 - 0.04x = 1545$ *Distributive property*
$0.01x + 1360 = 1545$ *Combine terms.*
$0.01x = 185$ *Subtract 1360.*
$x = 18{,}500$ *Divide by 0.01.*

The man invested \$18,500 at 5% and \$34,000 − \$18,500 = \$15,500 at 4%.

**N6.** Let $x =$ the amount invested at 3%.
Then $20{,}000 - x =$ the amount invested at 2.5%.

Use $I = prt$ with $t = 1$.

Make a table to organize the information.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.03 | $0.03x$ |
| $20{,}000 - x$ | 0.025 | $0.025(20{,}000 - x)$ |
| 20,000 | ← Totals → | 575 |

The last column gives the equation.

$0.03x + 0.025(20{,}000 - x) = 575$
$0.03x + 500 - 0.025x = 575$ *Distributive property*
$0.005x + 500 = 575$ *Combine terms.*
$0.005x = 75$ *Subtract 500.*
$x = 15{,}000$ *Divide by 0.005.*

The man invested \$15,000 at 3% and \$20,000 − \$15,000 = \$5000 at 2.5%.

**7.** Let $x =$ the amount of \$8 per lb candy. Then $x + 100 =$ the amount of \$7 per lb candy.

Make a table to organize the information.

| Number of Pounds | Price per Pound | Value |
|---|---|---|
| $x$ | \$8 | $8x$ |
| 100 | \$4 | 400 |
| $x + 100$ | \$7 | $7(x + 100)$ |

The last column gives the equation.

$8x + 400 = 7(x + 100)$
$8x + 400 = 7x + 700$ *Distributive property*
$x + 400 = 700$ *Subtract 7x.*
$x = 300$ *Subtract 400.*

300 lb of candy worth \$8 per lb should be used.

**N7.** Let $x =$ the number of liters of 20% acid solution needed. Make a table.

| Liters of Solution | Percent (as a decimal) | Liters of Pure Acid |
|---|---|---|
| 5 | 0.30 | $0.30(5) = 1.5$ |
| $x$ | 0.20 | $0.20x$ |
| $x + 5$ | 0.24 | $0.24(x + 5)$ |

| Acid in 30% solution | + | acid in 20% solution | = | acid in 24% solution. |
|---|---|---|---|---|
| $1.5$ | $+$ | $0.20x$ | $=$ | $0.24(x + 5)$ |

$1.5 + 0.20x = 0.24x + 1.2$ *Distributive property*
$0.3 = 0.04x$ *Subtract 0.20x and 1.2.*
$x = 7.5$ *Divide by 0.04.*

$7\frac{1}{2}$ liters of the 20% solution are needed.

**Check** 30% of 5 is 1.5 and 20% of 7.5 is 1.5; $1.5 + 1.5 = 3.0$, which is the same as 24% of $(5 + 7.5)$.

**8.** Let $x =$ the number of liters of water.

| Number of Liters | Percent (as a decimal) | Liters of Pure Antifreeze |
|---|---|---|
| $x$ | $0\% = 0$ | $0$ |
| $20$ | $50\% = 0.50$ | $0.50(20)$ |
| $x + 20$ | $40\% = 0.40$ | $0.40(x + 20)$ |

The last column gives the equation.

$$\begin{aligned} 0 + 0.50(20) &= 0.40(x + 20) \\ 10 &= 0.4x + 8 \\ 2 &= 0.4x && \textit{Subtract 8.} \\ 5 &= x && \textit{Divide by 0.4.} \end{aligned}$$

5 L of water are needed.

**N8.** Let $x =$ the number of gallons of pure antifreeze.

| Number of Liters | Percent (as a decimal) | Liters of Pure Antifreeze |
|---|---|---|
| $x$ | $100\% = 1$ | $1x$ |
| $3$ | $30\% = 0.30$ | $0.30(3)$ |
| $x + 3$ | $40\% = 0.40$ | $0.40(x + 3)$ |

The last column gives the equation.

$$\begin{aligned} 1x + 0.30(3) &= 0.40(x + 3) \\ 1.0x + 0.9 &= 0.4x + 1.2 \\ 0.6x + 0.9 &= 1.2 && \textit{Subtract 0.4x.} \\ 0.6x &= 0.3 && \textit{Subtract 0.9.} \\ x &= 0.5 && \textit{Divide by 0.6.} \end{aligned}$$

$\frac{1}{2}$ gallon of pure antifreeze is needed.

## 2.3 Section Exercises

**1.** **(a)** 15 more than a number $\underline{x + 15}$

**(b)** 15 is more than a number. $\underline{15 > x}$

**2.** **(a)** 5 greater than a number $\underline{x + 5}$

**(b)** 5 is greater than a number. $\underline{5 > x}$

**3.** **(a)** 8 less than a number $\underline{x - 8}$

**(b)** 8 is less than a number. $\underline{8 < x}$

**4.** **(a)** 6 less than a number $\underline{x - 6}$

**(b)** 6 is less than a number. $\underline{6 < x}$

**5.** 40% can be written as $0.40 = 0.4 = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}$, so "40% of a number" can be written as $0.40x$, $0.4x$, or $\frac{2x}{5}$. We see that "40% of a number" cannot be written as $40x$, choice **D**.

**6.** $13 - x$ is the translation of "$x$ less than 13." The phrase "13 less than a number" is translated $x - 13$.

**7.** Twice a number, decreased by 13 $\underline{2x - 13}$

**8.** The product of 6 and a number, decreased by 14 $\underline{6x - 14}$

**9.** 12 increased by four times a number $\underline{12 + 4x}$

**10.** 15 more than one-half of a number $\underline{\frac{1}{2}x + 15}$

**11.** The product of 8 and 16 less than a number $\underline{8(x - 16)}$

**12.** The product of 8 more than a number and 5 less than the number $\underline{(x + 8)(x - 5)}$

**13.** The quotient of three times a number and 10 $\underline{\frac{3x}{10}}$

**14.** The quotient of 9 and five times a nonzero number $\underline{\frac{9}{5x}\ (x \neq 0)}$

**15.** The sentence "the sum of a number and 6 is $-31$" can be translated as

$$\begin{aligned} x + 6 &= -31. \\ x &= -37 && \textit{Subtract 6.} \end{aligned}$$

The number is $-37$.

**16.** The sentence "the sum of a number and $-4$ is 18" can be translated as

$$\begin{aligned} x + (-4) &= 18. \\ x &= 22 && \textit{Add 4.} \end{aligned}$$

The number is 22.

**17.** The sentence "if the product of a number and $-4$ is subtracted from the number, the result is 9 more than the number" can be translated as

$$\begin{aligned} x - (-4x) &= x + 9. \\ x + 4x &= x + 9 \\ 4x &= 9 \\ x &= \tfrac{9}{4} \end{aligned}$$

The number is $\frac{9}{4}$.

**18.** The sentence "if the quotient of a number and 6 is added to twice the number, the result is 8 less than the number" can be translated as

$$\begin{aligned} 2x + \frac{x}{6} &= x - 8. \\ 12x + x &= 6x - 48 && \textit{Multiply by 6.} \\ 13x &= 6x - 48 \\ 7x &= -48 && \textit{Subtract 6x.} \\ x &= -\tfrac{48}{7} && \textit{Divide by 7.} \end{aligned}$$

The number is $-\frac{48}{7}$.

**19.** The sentence "when $\frac{2}{3}$ of a number is subtracted from 14, the result is 10" can be translated as

$$\begin{aligned} 14 - \tfrac{2}{3}x &= 10. & \\ 42 - 2x &= 30 && \textit{Multiply by 3.} \\ -2x &= -12 && \textit{Subtract 42.} \\ x &= 6 && \textit{Divide by } -2. \end{aligned}$$

The number is 6.

**20.** The sentence "when 75% of a number is added to 6, the result is 3 more than the number" can be translated as

$$\begin{aligned} 6 + 0.75x &= x + 3. & \\ 600 + 75x &= 100x + 300 && \textit{Multiply by 100.} \\ 600 - 25x &= 300 && \textit{Subtract 100x.} \\ -25x &= -300 && \textit{Subtract 600.} \\ x &= 12 && \textit{Divide by } -25. \end{aligned}$$

The number is 12.

**21.** $5(x+3) - 8(2x-6)$ is an *expression* because there is no equals symbol.

$$\begin{aligned} &5(x+3) - 8(2x-6) \\ &= 5x + 15 - 16x + 48 && \textit{Distributive property} \\ &= -11x + 63 && \textit{Combine like terms.} \end{aligned}$$

**22.** $-7(x+4) + 13(x-6)$ has no equals symbol, so it is an *expression.*

$$\begin{aligned} &-7(x+4) + 13(x-6) \\ &= -7x - 28 + 13x - 78 && \textit{Distributive prop.} \\ &= 6x - 106 && \textit{Combine like terms.} \end{aligned}$$

**23.** $5(x+3) - 8(2x-6) = 12$ has an equals symbol, so this represents an *equation.*

$$\begin{aligned} 5(x+3) - 8(2x-6) &= 12 & \\ 5x + 15 - 16x + 48 &= 12 && \textit{Dist. prop.} \\ -11x + 63 &= 12 && \textit{Combine terms.} \\ -11x &= -51 && \textit{Subtract 63.} \\ x &= \tfrac{51}{11} && \textit{Divide by } -11. \end{aligned}$$

The solution set is $\{\frac{51}{11}\}$.

**24.** $-7(x+4) + 13(x-6) = 18$ has an equals symbol, so it is an *equation.*

$$\begin{aligned} -7(x+4) + 13(x-6) &= 18 & \\ -7x - 28 + 13x - 78 &= 18 && \textit{Dist. prop.} \\ 6x - 106 &= 18 && \textit{Combine terms.} \\ 6x &= 124 && \textit{Add 106.} \\ x = \tfrac{124}{6} &= \tfrac{62}{3} && \textit{Divide by 6.} \end{aligned}$$

The solution set is $\{\frac{62}{3}\}$.

**25.** $\frac{1}{2}x - \frac{1}{6}x + \frac{3}{2} - 8$ is an *expression* because there is no equals symbol.

$$\begin{aligned} &\tfrac{1}{2}x - \tfrac{1}{6}x + \tfrac{3}{2} - 8 \\ &= \tfrac{3}{6}x - \tfrac{1}{6}x + \tfrac{3}{2} - \tfrac{16}{2} && \textit{Common denominators} \\ &= \tfrac{2}{6}x - \tfrac{13}{2} && \textit{Combine like terms.} \\ &= \tfrac{1}{3}x - \tfrac{13}{2} && \textit{Reduce.} \end{aligned}$$

**26.** $\frac{1}{3}x + \frac{1}{5}x - \frac{1}{2} + 7$ is an *expression* because there is no equals symbol.

$$\begin{aligned} &\tfrac{1}{3}x + \tfrac{1}{5}x - \tfrac{1}{2} + 7 \\ &= \tfrac{5}{15}x + \tfrac{3}{15}x - \tfrac{1}{2} + \tfrac{14}{2} && \textit{Common denom.} \\ &= \tfrac{8}{15}x + \tfrac{13}{2} && \textit{Combine like terms.} \end{aligned}$$

**27.** *Step 1*
We are asked to find <u>the number of patents each corporation secured</u>.

*Step 2*
Let $x =$ the number of patents IBM secured.
Then $x - 667 =$ the number of <u>patents Samsung secured</u>.

*Step 3*
A total of 7671 patents were secured, so

$$\underline{x} + \underline{x - 667} = 7671.$$

*Step 4*

$$\begin{aligned} 2x - 667 &= 7671 \\ 2x &= 8338 \\ x &= \underline{4169} \end{aligned}$$

*Step 5*
IBM secured <u>4169</u> patents and Samsung secured $4169 - 667 =$ <u>3502</u> patents.

*Step 6*
The number of Samsung patents was <u>667</u> fewer than the number of <u>IBM patents</u> and the total number of patents was $4169 +$ <u>3502</u> $=$ <u>7671</u>.

**28.** *Step 1*
We are asked to find <u>the number of travelers to each country</u>.

*Step 2*
Let $x =$ the number of travelers to Mexico (in millions). Then
$x - 7.8 =$ the number of <u>travelers to Canada</u>.

*Step 3*
A total of 32.8 million U.S. residents traveled to Mexico and Canada, so

$$\underline{x} + \underline{(x - 7.8)} = 32.8.$$

*Step 4*

$$\begin{aligned} 2x - 7.8 &= 32.8 \\ 2x &= 40.6 \\ x &= \underline{20.3} \end{aligned}$$

*Step 5*
There were <u>20.3</u> million travelers to Mexico and $20.3 - 7.8 =$ <u>12.5</u> million travelers to Canada.

*Step 6*
The number of <u>travelers to Mexico</u> was <u>7.8</u> million more than the number of <u>travelers to Canada</u>, and the total number of these travelers was $20.3 +$ <u>12.5</u> $=$ <u>32.8</u> million.

**29.** *Step 2*
Let $W =$ the width of the base. Then $2W - 65$ is the length of the base.

*Step 3*
The perimeter of the base is 860 feet. Using $P = 2L + 2W$ gives us

$$2(2W - 65) + 2W = 860.$$

*Step 4*

$$\begin{aligned} 4W - 130 + 2W &= 860 \\ 6W - 130 &= 860 \\ 6W &= 990 \\ W &= \tfrac{990}{6} = 165 \end{aligned}$$

*Step 5*
The width of the base is 165 feet and the length of the base is $2(165) - 65 = 265$ feet.

*Step 6*
$2L + 2W = 2(265) + 2(165) = 530 + 330 = 860$, which is the perimeter of the base.

**30.** *Step 2*
Let $L =$ the length of the top floor. Then $\frac{1}{2}L + 20$ is the width of the top floor.

*Step 3*
The perimeter of the top floor is 520 feet. Using $P = 2L + 2W$ gives us

$$2L + 2(\tfrac{1}{2}L + 20) = 520.$$

*Step 4*

$$\begin{aligned} 2L + L + 40 &= 520 \\ 3L + 40 &= 520 \\ 3L &= 480 \\ L &= 160 \end{aligned}$$

*Step 5*
The length of the top floor is 160 feet and the width of the top floor is $\frac{1}{2}(160) + 20 = 100$ feet.

*Step 6*
$2L + 2W = 2(160) + 2(100) = 320 + 200 = 520$, which is the perimeter of the top floor.

**31.** *Step 2*
Let $x =$ the width of the painting. Then $x + 5.54 =$ the height of the painting.

*Step 3*
The perimeter of the painting is 108.44 inches. Using $P = 2L + 2W$ gives us

$$2(x + 5.54) + 2x = 108.44.$$

*Step 4*

$$\begin{aligned} 2x + 11.08 + 2x &= 108.44 \\ 4x + 11.08 &= 108.44 \\ 4x &= 97.36 \\ x &= 24.34 \end{aligned}$$

*Step 5*
The width of the painting is 24.34 inches and the height is $24.34 + 5.54 = 29.88$ inches.

*Step 6*
29.88 is 5.54 more than 24.34 and $2(29.88) + 2(24.34) = 108.44$, as required.

**32.** *Step 2*
Let $W =$ the width of the rectangle. Then $W + 12 =$ the length of the rectangle.

*Step 3*
The perimeter, $2L + 2W$, is 16 times the width, so

$$2(W + 12) + 2W = 16(W).$$

*Step 4*

$$\begin{aligned} 2W + 24 + 2W &= 16W \\ 4W + 24 &= 16W \\ 24 &= 12W \\ 2 &= W \end{aligned}$$

*Step 5*
The width of the rectangle is 2 centimeters. The length is $2 + 12 = 14$ centimeters.

*Step 6*
The length is 12 more than the width. The perimeter is $2(14) + 2(2) = 32$, which is 16 times the width, as required.

**33.** *Step 2*
Let $x =$ the length of the middle side. Then the shortest side is $x - 75$ and the longest side is $x + 375$.

*Step 3*
The perimeter of the Bermuda Triangle is 3075 miles. Using $P = a + b + c$ gives us

$$x + (x - 75) + (x + 375) = 3075.$$

*Step 4*

$$\begin{aligned} 3x + 300 &= 3075 \\ 3x &= 2775 && \textit{Subtract 300.} \\ x &= 925 && \textit{Divide by 3.} \end{aligned}$$

*Step 5*
The length of the middle side is 925 miles. The length of the shortest side is $x - 75 = 925 - 75 = 850$ miles. The length of the longest side is $x + 375 = 925 + 375 = 1300$ miles.

*Step 6*
The answer checks since $925 + 850 + 1300 = 3075$ miles, which is the correct perimeter.

**34.** *Step 2*
Let $x =$ the length of one of the sides of equal length.

*Step 3*
The perimeter of the triangle is 931.5 feet. Using $P = a + b + c$ gives us

$$x + x + 438 = 931.5$$

*Step 4*

$$\begin{aligned} 2x + 438 &= 931.5 \\ 2x &= 493.5 \quad \textit{Subtract 438.} \\ x &= 246.75 \quad \textit{Divide by 2.} \end{aligned}$$

*Step 5*
The two walls are each 246.75 feet long.

*Step 6*
The answer checks since $246.75 + 246.75 + 438 = 931.5$, which is the correct perimeter.

**35.** *Step 2*
Let $x =$ the amount of revenue for Exxon Mobil. Then $x - 37.3$ is the amount of revenue for Wal-Mart (in billions).

*Step 3*
The total revenue was \$848.5 billion, so

$$x + (x - 37.3) = 848.5.$$

*Step 4*

$$\begin{aligned} 2x - 37.3 &= 848.5 \\ 2x &= 885.8 \quad \textit{Add 37.3.} \\ x &= 442.9 \quad \textit{Divide by 2.} \end{aligned}$$

*Step 5*
The amount of revenue for Exxon Mobil was \$442.9 billion. The amount of revenue for Wal-Mart was $x - 37.3 = 442.9 - 37.3 = \$405.6$ billion.

*Step 6*
The answer checks since $442.9 + 405.6 = \$848.5$ billion, which is the correct total revenue.

**36.** *Step 2*
Let $x =$ the number of performances of *Cats*. Then $x - 805 =$ the number of performances of *Les Misérables*.

*Step 3*
There were 14,165 total performances, so

$$x + (x - 805) = 14{,}165.$$

*Step 4*

$$\begin{aligned} 2x - 805 &= 14{,}165 \\ 2x &= 14{,}970 \\ x &= 7485 \end{aligned}$$

*Step 5*
There were 7485 performances of *Cats* and $7485 - 805 = 6680$ performances of *Les Misérables*.

*Step 6*
The total number of performances is 14,165, as required.

**37.** *Step 2*
Let $x =$ the height of the Eiffel Tower. Then $x - 880 =$ the height of the Leaning Tower of Pisa.

*Step 3*
Together these heights are 1246 ft, so

$$x + (x - 880) = 1246.$$

*Step 4*

$$\begin{aligned} 2x - 880 &= 1246 \\ 2x &= 2126 \\ x &= 1063 \end{aligned}$$

*Step 5*
The height of the Eiffel Tower is 1063 feet and the height of the Leaning Tower of Pisa is $1063 - 880 = 183$ feet.

*Step 6*
183 feet is 880 feet shorter than 1063 feet and the sum of 183 feet and 1063 feet is 1246 feet.

**38.** *Step 2*
Let $x =$ the Yankees' payroll (in millions). Then $x - 65.6 =$ the Mets' payroll (in millions).

*Step 3*
The two payrolls totaled \$337.2 million, so

$$x + (x - 65.6) = 337.2$$

*Step 4*

$$\begin{aligned} 2x - 65.6 &= 337.2 \\ 2x &= 402.8 \\ x &= 201.4 \end{aligned}$$

*Step 5*
In 2009, the Yankees' payroll was \$201.4 million and the Mets' payroll was $201.4 - 65.6 = \$135.8$ million.

*Step 6*
\$135.8 million is \$65.6 million less than \$201.4 million and the sum of \$135.8 million and \$201.4 million is \$337.2 million.

**39.** *Step 2*
Let $x =$ number of electoral votes for McCain. Then $x + 192 =$ number of electoral votes for Obama.

*Step 3*
There were 538 total electoral votes, so

$$x + (x + 192) = 538.$$

*Step 4*

$$\begin{aligned}2x+192&=538\\2x&=346\\x&=173\end{aligned}$$

*Step 5*
McCain received 173 electoral votes, so Obama received $173+192=365$ electoral votes.

*Step 6*
365 is 192 more than 173 and the total is $173+365=538$.

**40.** *Step 2*
Let $x=$ the number of hits Williams got. Then $x+276=$ the number of hits Hornsby got.

*Step 3*
Their base hits totaled 5584, so

$$x+(x+276)=5584.$$

*Step 4*

$$\begin{aligned}2x+276&=5584\\2x&=5308\\x&=2654\end{aligned}$$

*Step 5*
Williams got 2654 base hits, and Hornsby got $2654+276=2930$ base hits.

*Step 6*
2930 is 276 more than 2654 and the total is $2654+2930=5584$.

**41.** Let $x=$ the percent increase.

$$\begin{aligned}x&=\frac{\text{amount of increase}}{\text{base number}}\\&=\frac{1{,}480{,}469-1{,}065{,}138}{1{,}065{,}138}=\frac{415{,}331}{1{,}065{,}138}\\&\approx 0.390=39.0\%\end{aligned}$$

The percent increase was approximately 39.0%.

**42.** Let $x=$ the percent increase.

$$\begin{aligned}x&=\frac{\text{amount of increase}}{\text{base score}}\\&=\frac{21.1-20.8}{20.8}=\frac{0.3}{20.8}\\&\approx 0.014=1.4\%\end{aligned}$$

The percent increase was approximately 1.4%.

**43.** Let $x=$ the approximate cost in 2009. Since $x$ is 150% more than the 1995 cost,

$$\begin{aligned}x&=2811+1.5(2811)\\&=2811+4216.5\\&=7027.5.\end{aligned}$$

To the nearest dollar, the cost was \$7028.

**44.** Let $x=$ the approximate cost in 2009. Since $x$ is 115.1% more than the 1995 cost,

$$\begin{aligned}x&=12{,}216+1.151(12{,}216)\\&\approx 12{,}216+14{,}060.62\\&=26{,}276.62.\end{aligned}$$

To the nearest dollar, the cost was \$26,277.

**45.** Let $x=$ the 2008 cost. Then

$$\begin{aligned}x-3.8\%(x)&=42.91.\\x-3.8(0.01)(x)&=42.91\\1x-0.038x&=42.91\\0.962x&=42.91\\x&=\tfrac{42.91}{0.962}\approx 44.60\end{aligned}$$

The 2008 cost was \$44.60.

**46.** Let $x=$ the 1987 cost. Then

$$\begin{aligned}x+60.4\%(x)&=42.91.\\x+60.4(0.01)(x)&=42.91\\1x+0.604x&=42.91\\1.604x&=42.91\\x&=\tfrac{42.91}{1.604}\approx 26.75\end{aligned}$$

The 1987 cost was \$26.75.

**47.** Let $x=$ the amount of the receipts excluding tax. Since the sales tax is 9% of $x$, the total amount is

$$\begin{aligned}x+0.09x&=2725\\1x+0.09x&=2725\\1.09x&=2725\\x&=\tfrac{2725}{1.09}=2500\end{aligned}$$

Thus, the tax was $0.09(2500)=\$225$.

**48.** Let $x=$ the amount of commission. Since $x$ is 6% of the selling price,

$$x=0.06(159{,}000)=9540.$$

So after the agent was paid, he had $159{,}000-9540=\$149{,}460$.

**49.** Let $x=$ the amount invested at 3%. Then $12{,}000-x=$ the amount invested at 4%. Complete the table. Use $I=prt$ with $t=1$.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.03 | $0.03x$ |
| $12{,}000-x$ | 0.04 | $0.04(12{,}000-x)$ |
| 12,000 | ← Totals → | 440 |

The last column gives the equation.

| Interest at 3% | + | interest at 4% | = | total interest. |
|---|---|---|---|---|
| $0.03x$ | + | $0.04(12{,}000-x)$ | = | 440 |

$$\begin{aligned}3x+4(12{,}000-x)&=44{,}000 \quad \textit{Multiply by 100.}\\3x+48{,}000-4x&=44{,}000\\-x&=-4000\\x&=4000\end{aligned}$$

He should invest \$4000 at 3% and $12{,}000 - 4000 = \$8000$ at 4%.

**Check** \$4000 @ 3% = \$120 and \$8000 @ 4% = \$320; \$120 + \$320 = \$440.

**50.** Let $x$ = the amount invested at 2%. Then $60{,}000 - x$ = the amount invested at 3%. Complete the table. Use $I = prt$ with $t = 1$.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.02 | $0.02x$ |
| $60{,}000 - x$ | 0.03 | $0.03(60{,}000 - x)$ |
| 60,000 | ← Totals → | 1600 |

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest at 2\%} & + & \text{interest at 3\%} & = & \text{total interest.} \\ 0.02x & + & 0.03(60{,}000 - x) & = & 1600 \end{array}$$

$$\begin{aligned} 2x + 3(60{,}000 - x) &= 160{,}000 \quad \textit{Multiply by 100.} \\ 2x + 180{,}000 - 3x &= 160{,}000 \\ -x &= -20{,}000 \\ x &= 20{,}000 \end{aligned}$$

She invested \$20,000 at 2% and $60{,}000 - x = 60{,}000 - 20{,}000 = \$40{,}000$ at 3%.

**Check** \$20,000 @ 2% = \$400 and \$40,000 @ 3% = \$1200; \$400 + \$1200 = \$1600.

**51.** Let $x$ = the amount invested at 4.5%. Then $2x - 1000$ = the amount invested at 3%. Use $I = prt$ with $t = 1$. Make a table.

| Principal | Rate (as a Decimal) | Interest |
|---|---|---|
| $x$ | 0.045 | $0.045x$ |
| $2x - 1000$ | 0.03 | $0.03(2x - 1000)$ |
| | Total → | 1020 |

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest at 4.5\%} & + & \text{interest at 3\%} & = & \text{total interest.} \\ 0.045x & + & 0.03(2x - 1000) & = & 1020 \end{array}$$

$$\begin{aligned} 45x + 30(2x - 1000) &= 1{,}020{,}000 \quad \textit{Multiply by 1000.} \\ 45x + 60x - 30{,}000 &= 1{,}020{,}000 \\ 105x &= 1{,}050{,}000 \\ x &= \frac{1{,}050{,}000}{105} = 10{,}000 \end{aligned}$$

She invested \$10,000 at 4.5% and $2x - 1000 = 2(10{,}000) - 1000 = \$19{,}000$ at 3%.

***Check*** \$19,000 is \$1000 less than two times \$10,000. \$10,000 @ 4.5% = \$450 and \$19,000 @ 3% = \$570; \$450 + \$570 = \$1020.

**52.** Let $x$ = the amount invested at 3.5%. Then $3x + 5000$ = the amount invested at 4%. Use $I = prt$ with $t = 1$. Make a table.

| Principal | Rate (as a Decimal) | Interest |
|---|---|---|
| $x$ | 0.035 | $0.035x$ |
| $3x + 5000$ | 0.04 | $0.04(3x + 5000)$ |
| | Total → | 1440 |

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest at 3.5\%} & + & \text{interest at 4\%} & = & 1440. \\ 0.035x & + & 0.04(3x + 5000) & = & 1440 \end{array}$$

$$\begin{aligned} 35x + 40(3x + 5000) &= 1{,}440{,}000 \quad \textit{Multiply by 1000.} \\ 35x + 120x + 200{,}000 &= 1{,}440{,}000 \\ 155x &= 1{,}240{,}000 \\ x &= \frac{1{,}240{,}000}{155} = 8000 \end{aligned}$$

He invested \$8000 at 3.5% and $3x + 5000 = 3(8000) + 5000 = \$29{,}000$ at 4%.

***Check*** \$29,000 is \$5000 more than three times \$8000. \$8000 @ 3.5% = \$280 and \$29,000 @ 4% = \$1160; \$280 + \$1160 = \$1440.

**53.** Let $x$ = the amount of additional money to be invested at 3%. Use $I = prt$ with $t = 1$. Make a table. Use the fact that the total return on the two investments is 4%.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| 12,000 | 0.06 | $0.06(12{,}000)$ |
| $x$ | 0.03 | $0.03x$ |
| $12{,}000 + x$ | 0.04 | $0.04(12{,}000 + x)$ |

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest at 6\%} & + & \text{interest at 3\%} & = & \text{interest at 4\%}. \\ 0.06(12{,}000) & + & 0.03x & = & 0.04(12{,}000 + x) \end{array}$$

$$\begin{aligned} 6(12{,}000) + 3x &= 4(12{,}000 + x) \quad \textit{Multiply by 100.} \\ 72{,}000 + 3x &= 48{,}000 + 4x \\ 24{,}000 &= x \end{aligned}$$

He should invest \$24,000 at 3%.

**Check** \$12,000 @ 6% = \$720 and \$24,000 @ 3% = \$720; \$720 + \$720 = \$1440, which is the same as (\$12,000 + \$24,000) @ 4%.

**54.** Let $x$ = the amount of additional money to be invested at 5%. Use $I = prt$ with $t = 1$. Make a table. Use the fact that the total return on the two investments is 6%.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| 17,000 | 0.065 | 0.065(17,000) |
| $x$ | 0.05 | $0.05x$ |
| $17{,}000 + x$ | 0.06 | $0.06(17{,}000 + x)$ |

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest at 6.5\%} & + & \text{interest at 5\%} & = & \text{interest at 6\%.} \\ 0.065(17{,}000) & + & 0.05x & = & 0.06(17{,}000 + x) \end{array}$$

$$65(17{,}000) + 50x = 60(17{,}000 + x)$$ *Multiply by 1000.*

$$1{,}105{,}000 + 50x = 1{,}020{,}000 + 60x$$
$$85{,}000 = 10x$$
$$8500 = x$$

She should invest \$8500 at 6%.

**Check** \$17,000 @ 6.5% = \$1105 and \$8500 @ 5% = \$425; \$1105 + \$425 = \$1530, which is the same as (\$17,000 + \$8500) @ 6%.

**55.** Let $x =$ the number of liters of 10% acid solution needed. Make a table.

| Liters of Solution | Percent (as a decimal) | Liters of Pure Acid |
|---|---|---|
| 10 | 0.04 | $0.04(10) = 0.4$ |
| $x$ | 0.10 | $0.10x$ |
| $x + 10$ | 0.06 | $0.06(x + 10)$ |

$$\begin{array}{ccccc} \text{Acid in 4\% solution} & + & \text{acid in 10\% solution} & = & \text{acid in 6\% solution.} \\ 0.4 & + & 0.10x & = & 0.06(x + 10) \end{array}$$

$$0.4 + 0.10x = 0.06x + 0.6$$ *Distributive property*

$$0.04x = 0.2$$ *Subtract 0.06x and 0.4.*

$$x = 5$$ *Divide by 0.04.*

Five liters of the 10% solution are needed.

**Check** 4% of 10 is 0.4 and 10% of 5 is 0.5; $0.4 + 0.5 = 0.9$, which is the same as 6% of $(10 + 5)$.

**56.** Let $x =$ the number of liters of 14% alcohol solution needed. Make a chart.

| Liters of Solution | Percent (as a decimal) | Liters of Pure Alcohol |
|---|---|---|
| $x$ | 0.14 | $0.14x$ |
| 20 | 0.50 | $0.50(20) = 10$ |
| $x + 20$ | 0.30 | $0.30(x + 20)$ |

$$\begin{array}{ccccc} \text{Alcohol in 14\% solution} & + & \text{alcohol in 50\% solution} & = & \text{alcohol in 30\% solution.} \\ 0.14x & + & 10 & = & 0.30(x + 20) \end{array}$$

$$14x + 1000 = 30(x + 20)$$ *Multiply by 100.*

$$14x + 1000 = 30x + 600$$ *Distributive property*

$$-16x = -400$$ *Subtract 30x and 1000.*

$$x = 25$$ *Divide by −16.*

25 L of 14% solution must be added.

**Check** 14% of 25 is 3.5 and 50% of 20 is 10; $3.5 + 10 = 13.5$, which is the same as 30% of $(25 + 20)$.

**57.** Let $x =$ the number of liters of the 20% alcohol solution. Make a table.

| Liters of Solution | Percent (as a decimal) | Liters of Pure Alcohol |
|---|---|---|
| 12 | 0.12 | $0.12(12) = 1.44$ |
| $x$ | 0.20 | $0.20x$ |
| $x + 12$ | 0.14 | $0.14(x + 12)$ |

$$\begin{array}{ccccc} \text{Alcohol in 12\% solution} & + & \text{alcohol in 20\% solution} & = & \text{alcohol in 14\% solution.} \\ 1.44 & + & 0.20x & = & 0.14(x + 12) \end{array}$$

$$144 + 20x = 14(x + 12)$$ *Multiply by 100.*

$$144 + 20x = 14x + 168$$ *Distributive property*

$$6x = 24$$ *Subtract 14x and 144.*

$$x = 4$$ *Divide by 6.*

4L of 20% alcohol solution are needed.

**Check** 12% of 12 is 1.44 and 20% of 4 is 0.8; $1.44 + 0.8 = 2.24$, which is the same as 14% of $(12 + 4)$.

**58.** Let $x =$ the number of liters of 10% alcohol solution. Make a chart.

| Liters of Solution | Percent (as a decimal) | Liters of Pure Alcohol |
|---|---|---|
| $x$ | 0.10 | $0.10x$ |
| 40 | 0.50 | $0.50(40) = 20$ |
| $x + 40$ | 0.40 | $0.40(x + 40)$ |

$$\begin{array}{ccccc} \text{Alcohol in 10\% solution} & + & \text{alcohol in 50\% solution} & = & \text{alcohol in 40\% solution.} \\ 0.10x & + & 20 & = & 0.40(x + 40) \end{array}$$

$$1x + 200 = 4(x + 40)$$ *Multiply by 10.*

$$x + 200 = 4x + 160$$ *Distributive property*

$$-3x = -40$$ *Subtract 4x and 200.*

$$x = \tfrac{40}{3} \text{ or } 13\tfrac{1}{3}$$ *Divide by −3.*

$13\frac{1}{3}$ L of 10% solution should be added.

**Check** 50% of 40 is 20 and 10% of $\frac{40}{3}$ is $\frac{4}{3}$; $20 + \frac{4}{3} = 21\frac{1}{3}$, which is the same as 40% of $(\frac{40}{3} + 40)$.

**59.** Let $x =$ the amount of pure dye used (pure dye is 100% dye). Make a table.

| Gallons of Solution | Percent (as a decimal) | Gallons of Pure Dye |
|---|---|---|
| $x$ | 1 | $1x = x$ |
| 4 | 0.25 | $0.25(4) = 1$ |
| $x + 4$ | 0.40 | $0.40(x + 4)$ |

Write the equation from the last column in the table.

$$x + 1 = 0.4(x + 4)$$
$$x + 1 = 0.4x + 1.6 \quad \textit{Distributive property}$$
$$0.6x = 0.6 \quad \textit{Subtract 0.4x and 1.}$$
$$x = 1 \quad \textit{Divide by 0.6.}$$

One gallon of pure (100%) dye is needed.

**Check** 100% of 1 is 1 and 25% of 4 is 1; $1 + 1 = 2$, which is the same as 40% of $(1 + 4)$.

**60.** Let $x =$ the number of gallons of water. Make a chart.

| Gallons of Solution | Percent (as a decimal) | Gallons of Pure Insecticide |
|---|---|---|
| $x$ | 0 | $0(x) = 0$ |
| 6 | 0.04 | $0.04(6) = 0.24$ |
| $x + 6$ | 0.03 | $0.03(x + 6)$ |

Insecticide in water + insecticide in 4% solution = insecticide in 3% solution.

$$0 + 0.24 = 0.03(x + 6)$$

$$0 + 24 = 3(x + 6) \quad \textit{Multiply by 100.}$$
$$24 = 3x + 18 \quad \textit{Distributive property}$$
$$6 = 3x \quad \textit{Subtract 18.}$$
$$2 = x \quad \textit{Divide by 3.}$$

2 gallons of water should be added.

**Check** 4% of 6 is 0.24, which is the same as 3% of $(2 + 6)$.

**61.** Let $x =$ the amount of \$6 per lb nuts. Make a table.

| Pounds of nuts | Cost per lb | Total Cost |
|---|---|---|
| 50 | \$2 | $2(50) = 100$ |
| $x$ | \$6 | $6x$ |
| $x + 50$ | \$5 | $5(x + 50)$ |

The total value of the \$2 per lb nuts and the \$6 per lb nuts must equal the value of the \$5 per lb nuts.

$$100 + 6x = 5(x + 50)$$
$$100 + 6x = 5x + 250$$
$$x = 150$$

He should use 150 lb of \$6 nuts.

**Check** 50 pounds of the \$2 per lb nuts are worth \$100 and 150 pounds of the \$6 per lb nuts are worth \$900; $\$100 + \$900 = \$1000$, which is the same as $(50 + 150)$ pounds worth \$5 per lb.

**62.** Let $x =$ the number of ounces of 2¢ per oz tea. Make a table.

| Ounces of Tea | Cost per oz | Total Cost |
|---|---|---|
| $x$ | 2¢ or 0.02 | $0.02x$ |
| 100 | 5¢ or 0.05 | $0.05(100) = 5$ |
| $x + 100$ | 3¢ or 0.03 | $0.03(x + 100)$ |

Cost of 2¢ tea + cost of 5¢ tea = cost of 3¢ tea.

$$0.02x + 5 = 0.03(x + 100)$$

$$2x + 500 = 3(x + 100) \quad \textit{Multiply by 100.}$$
$$2x + 500 = 3x + 300 \quad \textit{Distributive property}$$
$$200 = x \quad \textit{Subtract 2x and 300.}$$

200 oz of 2¢ per oz tea should be used.

**Check** 200 oz of 2¢ per oz tea is worth \$4 and 100 oz of 5¢ per oz tea is worth \$5; $\$4 + \$5 = \$9$, which is the same value as $(200 + 100)$ oz of 3¢ per oz tea.

**63.** We cannot expect the final mixture to be worth more than the more expensive of the two ingredients. Answers will vary.

**64.** Let $x =$ the number of liters of 30% acid solution. Make a chart.

| Liters of Solution | Percent (as a decimal) | Liters of Pure Acid |
|---|---|---|
| $x$ | 0.30 | $0.30x$ |
| 15 | 0.50 | $0.50(15) = 7.5$ |
| $x + 15$ | 0.60 | $0.60(x + 15)$ |

Acid in 30% solution + acid in 50% solution = acid in 60% solution.

$$0.30x + 7.5 = 0.60(x + 15)$$

$$3x + 75 = 6x + 90 \quad \textit{Multiply by 10.}$$
$$-3x = 15 \quad \textit{Subtract 6x and 75.}$$
$$x = -5 \quad \textit{Divide by } -3.$$

The solution, $-5$, is impossible since the number of liters of 30% acid solution cannot be negative. Therefore, this problem has no solution.

**65.** **(a)** Let $x =$ the amount invested at 5%.
$800 - x =$ the amount invested at 10%.

**(b)** Let $y =$ the amount of 5% acid used.
$800 - y =$ the amount of 10% acid used.

**66.** Organize the information in a table.

**(a)**

| Principal | Percent (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.05 | $0.05x$ |
| $800 - x$ | 0.10 | $0.10(800 - x)$ |
| 800 | 0.0875 | $0.0875(800)$ |

The amount of interest earned at 5% and 10% is found in the last column of the table, $0.05x$ and $0.10(800 - x)$.

**(b)**

| Liters of Solution | Percent (as a decimal) | Liters of Pure Acid |
|---|---|---|
| $y$ | 0.05 | $0.05y$ |
| $800 - y$ | 0.10 | $0.10(800 - y)$ |
| 800 | 0.0875 | $0.0875(800)$ |

The amount of pure acid in the 5% and 10% mixtures is found in the last column of the table, $0.05y$ and $0.10(800 - y)$.

**67.** Refer to the tables for Exercise 66. In each case, the last column gives the equation.

**(a)** $0.05x + 0.10(800 - x) = 0.0875(800)$

**(b)** $0.05y + 0.10(800 - y) = 0.0875(800)$

**68.** In both cases, multiply by 10,000 to clear the decimals.

**(a)**
$$\begin{aligned} 0.05x + 0.10(800 - x) &= 0.0875(800) \\ 500x + 1000(800 - x) &= 875(800) \\ 500x + 800{,}000 - 1000x &= 700{,}000 \\ -500x &= -100{,}000 \\ x &= 200 \end{aligned}$$

Jack invested \$200 at 5% and $800 - x = 800 - 200 = \$600$ at 10%.

**(b)**
$$\begin{aligned} 0.05y + 0.10(800 - y) &= 0.0875(800) \\ 500y + 1000(800 - y) &= 875(800) \\ 500y + 800{,}000 - 1000y &= 700{,}000 \\ -500y &= -100{,}000 \\ y &= 200 \end{aligned}$$

Jill used 200 L of 5% acid solution and $800 - y = 800 - 200 = 600$ L of 10% acid solution.

**(c)** The processes used to solve Problems A and B were virtually the same. Aside from the variables chosen, the problem information was organized in similar tables and the equations solved were the same.

**69.** $d = rt; \quad r = 50, t = 4$

$$d = 50(4) = 200$$

**70.** $P = 2L + 2W; \quad L = 10, W = 6$

$$P = 2(10) + 2(6) = 20 + 12 = 32$$

**71.** $P = a + b + c; \quad b = 13, c = 14, P = 46$

$$\begin{aligned} 46 &= a + 13 + 14 \\ 46 &= a + 27 \\ 19 &= a \end{aligned}$$

**72.** $\mathcal{A} = \frac{1}{2}h(b + B); \quad \mathcal{A} = 156, b = 12, B = 14$

$$\begin{aligned} 156 &= \tfrac{1}{2}h(12 + 14) \\ 312 &= h(26) \\ h &= \tfrac{312}{26} = 12 \end{aligned}$$

## 2.4 Further Applications of Linear Equations

### 2.4 Classroom Examples, Now Try Exercises

**1.** Let $x =$ the number of dimes.
Then $26 - x =$ the number of half-dollars.

| Number of Coins | Denomination | Value |
|---|---|---|
| $x$ | 0.10 | $0.10x$ |
| $26 - x$ | 0.50 | $0.50(26 - x)$ |
| | Total → | 8.60 |

Multiply the number of coins by the denominations, and add the results to get 8.60.

$$\begin{aligned} 0.10x + 0.50(26 - x) &= 8.60 \\ 1x + 5(26 - x) &= 86 \quad \textit{Multiply by 10.} \\ 1x + 130 - 5x &= 86 \\ -4x &= -44 \\ x &= 11 \end{aligned}$$

He has 11 dimes and $26 - 11 = 15$ half-dollars.

**Check** The number of coins is $11 + 15 = 26$ and the value of the coins is $\$0.10(11) + \$0.50(15) = \$8.60$, as required.

**N1.** Let $x =$ the number of dimes.
Then $52 - x =$ the number of nickels.

| Number of Coins | Denomination | Value |
|---|---|---|
| $x$ | 0.10 | $0.10x$ |
| $52 - x$ | 0.05 | $0.05(52 - x)$ |
| | Total → | 3.70 |

Multiply the number of coins by the denominations, and add the results to get 3.70.

$$\begin{aligned} 0.10x + 0.05(52 - x) &= 3.70 \\ 10x + 5(52 - x) &= 370 \quad \textit{Multiply by 100.} \\ 10x + 260 - 5x &= 370 \\ 5x &= 110 \\ x &= 22 \end{aligned}$$

He has 22 dimes and $52 - 22 = 30$ nickels.

**Check** The number of coins is $22 + 30 = 52$ and the value of the coins is $\$0.10(22) + \$0.05(30) = \$3.70$, as required.

**2.** Let $x =$ the amount of time needed for the cars to be 420 mi apart.

Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

| | Rate | Time | Distance |
|---|---|---|---|
| **Northbound Car** | 60 | $x$ | $60x$ |
| **Southbound Car** | 45 | $x$ | $45x$ |
| **Total** | | | 420 |

The total distance traveled is the sum of the distances traveled by each car, since they are traveling in opposite directions. This total is 420 mi.

$$60x + 45x = 420$$
$$105x = 420$$
$$x = \tfrac{420}{105} = 4$$

The cars will be 420 mi apart in 4 hr.

**Check** The northbound car traveled $60(4) = 240$ miles. The southbound car traveled $45(4) = 180$ miles, for a total of $240 + 180 = 420$, as required.

**N2.** Let $x =$ the amount of time needed for the trains to be 387.5 km apart.

Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

| | Rate | Time | Distance |
|---|---|---|---|
| **First Train** | 80 | $x$ | $80x$ |
| **Second Train** | 75 | $x$ | $75x$ |
| **Total** | | | 387.5 |

The total distance traveled is the sum of the distances traveled by each train, since they are traveling in opposite directions. This total is 387.5 km.

$$80x + 75x = 387.5$$
$$155x = 387.5$$
$$x = \tfrac{387.5}{155} = 2.5$$

The trains will be 387.5 km apart in 2.5 hr.

**Check** The first train traveled $80(2.5) = 200$ km. The second train traveled $75(2.5) = 187.5$ km, for a total of $200 + 187.5 = 387.5$, as required.

**3.** Let $x =$ the driving rate.
Then $x - 12 =$ the bus rate.

Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

| | Rate | Time | Distance |
|---|---|---|---|
| **Car** | $x$ | $\frac{1}{2}$ | $\frac{1}{2}x$ |
| **Bus** | $x - 12$ | $\frac{3}{4}$ | $\frac{3}{4}(x - 12)$ |

The distances are equal.

$$\tfrac{1}{2}x = \tfrac{3}{4}(x - 12)$$
$$2x = 3(x - 12) \quad \textit{Multiply by 4.}$$
$$2x = 3x - 36$$
$$36 = x$$

The distance he travels to work is

$$\tfrac{1}{2}x = \tfrac{1}{2}(36) = 18 \text{ miles.}$$

**Check** The distance he travels to work by bus is $\frac{3}{4}(x - 12) = \frac{3}{4}(36 - 12) = \frac{3}{4}(24) = 18$ miles, which is the same as we found above (by car).

**N3.** Let $x =$ the driving rate.
Then $x - 30 =$ the bicycling rate.

Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

| | Rate | Time | Distance |
|---|---|---|---|
| **Car** | $x$ | $\frac{1}{2}$ | $\frac{1}{2}x$ |
| **Bike** | $x - 30$ | $1\frac{1}{2} = \frac{3}{2}$ | $\frac{3}{2}(x - 30)$ |

The distances are equal.

$$\tfrac{1}{2}x = \tfrac{3}{2}(x - 30)$$
$$1x = 3(x - 30) \quad \textit{Multiply by 2.}$$
$$x = 3x - 90$$
$$90 = 2x$$
$$45 = x$$

The distance he travels to work is

$$\tfrac{1}{2}x = \tfrac{1}{2}(45) = 22.5 \text{ miles.}$$

**Check** The distance he travels to work by bike is $\frac{3}{2}(x - 30) = \frac{3}{2}(45 - 30) = \frac{3}{2}(15) = 22.5$ miles, which is the same as we found above (by car).

**4.** The sum of the three measures must equal 180°.

$$x + (x + 61) + (2x + 7) = 180$$
$$4x + 68 = 180$$
$$4x = 112$$
$$x = 28$$

The angles measure $28°$, $28 + 61 = 89°$, and $2(28) + 7 = 63°$.

**Check** Since $28° + 89° + 63° = 180°$, the answers are correct.

**N4.** The sum of the three measures must equal 180°.

$$x + (x + 11) + (3x - 36) = 180$$
$$5x - 25 = 180$$
$$5x = 205$$
$$x = 41$$

The angles measure $41°$, $41 + 11 = 52°$, and $3(41) - 36 = 87°$.

**Check** Since $41° + 52° + 87° = 180°$, the answers are correct.

## 2.4 Section Exercises

**1.** The total amount is

$$14(0.10) + 16(0.25) = 1.40 + 4.00 = \$5.40.$$

**2.** Use $d = rt$, or $t = \frac{d}{r}$.
Substitute 7700 for $d$ and 550 for $r$.

$$t = \frac{7700}{550} = 14$$

Its travel time is 14 hours.

**3.** Use $d = rt$, or $r = \frac{d}{t}$. Substitute 300 for $d$ and 10 for $t$.

$$r = \frac{300}{10} = 30$$

His rate was 30 mph.

**4.** Use $P = 4s$ or $s = \frac{P}{4}$.
Substitute 80 for $P$.

$$s = \frac{80}{4} = 20$$

The length of each side of the square is 20 in. This is also the length of each side of the equilateral triangle. To find the perimeter of the equilateral triangle, use $P = 3s$.
Substitute 20 for $s$.

$$P = 3(20) = 60$$

The perimeter would be 60 inches.

**5.** The problem asks for the distance Jeff travels to the workplace, so we must multiply the rate, 10 mph, by the time, $\frac{3}{4}$ hr, to get the distance, 7.5 mi.

**6.** Begin by subtracting 36° from 180°. Then find half of this difference to get the measure of each of the equal angles.

**7.** No, the answers must be whole numbers because they represent the number of coins.

**8.** Since the distance $d$ is in miles and the rate $r$ is $x$ miles per hour, we must have the time $t$ in hours. 10 minutes $= \frac{10}{60} = \frac{1}{6}$ hour, so

$$d = rt = x(\tfrac{1}{6}) \quad \text{or} \quad \tfrac{1}{6}x.$$

**9.** Let $x$ = the number of pennies. Then $x$ is also the number of dimes, and $44 - 2x$ is the number of quarters.

| Number of Coins | Denomination | Value |
|---|---|---|
| $x$ | 0.01 | $0.01x$ |
| $x$ | 0.10 | $0.10x$ |
| $44 - 2x$ | 0.25 | $0.25(44 - 2x)$ |
| 44 | ← Totals → | 4.37 |

The sum of the values must equal the total value.

$$0.01x + 0.10x + 0.25(44 - 2x) = 4.37$$
$$x + 10x + 25(44 - 2x) = 437 \quad \textit{Multiply both sides by 100.}$$
$$x + 10x + 1100 - 50x = 437$$
$$-39x + 1100 = 437$$
$$-39x = -663$$
$$x = 17$$

There are 17 pennies, 17 dimes, and $44 - 2(17) = 10$ quarters.

**Check** The number of coins is $17 + 17 + 10 = 44$ and the value of the coins is $\$0.01(17) + \$0.10(17) + \$0.25(10) = \$4.37$, as required.

**10.** Let $x$ = the number of nickels and the number of quarters. Then $2x$ is the number of half-dollars.

| Number of Coins | Denomination | Value |
|---|---|---|
| $x$ | 0.05 | $0.05x$ |
| $x$ | 0.25 | $0.25x$ |
| $2x$ | 0.50 | $0.50(2x)$ |
| | Total → | 2.60 |

The sum of the values must equal the total value.

$$0.05x + 0.25x + 0.50(2x) = 2.60$$
$$5x + 25x + 50(2x) = 260 \quad \textit{Multiply both sides by 100.}$$
$$5x + 25x + 100x = 260$$
$$130x = 260$$
$$x = 2$$

She found 2 nickels, 2 quarters, and $2(2) = 4$ half-dollars.

**Check** The number of coins is $2 + 2 + 4 = 8$ and the value of the coins is $\$0.05(2) + \$0.25(2) + \$0.50(4) = \$2.60$, as required.

**11.** Let $x$ = the number of loonies. Then $37 - x$ is the number of toonies.

| Number of Coins | Denomination | Value |
|---|---|---|
| $x$ | 1 | $1x$ |
| $37 - x$ | 2 | $2(37 - x)$ |
| 37 | ← Totals → | 51 |

The sum of the values must equal the total value.

$$1x + 2(37 - x) = 51$$
$$x + 74 - 2x = 51$$
$$-x + 74 = 51$$
$$23 = x$$

She has 23 loonies and $37 - 23 = 14$ toonies.

**Check** The number of coins is $23 + 14 = 37$ and the value of the coins is $\$1(23) + \$2(14) = \$51$, as required.

**12.** Let $x =$ the number of \$1 bills. Then $119 - x$ is the number of \$5 bills.

| Number of Bills | Denomination | Value |
|---|---|---|
| $x$ | 1 | $1x$ |
| $119 - x$ | 5 | $5(119 - x)$ |
| 119 | ← Totals → | 347 |

The sum of the values must equal the total value.

$$\begin{aligned} 1x + 5(119 - x) &= 347 \\ x + 595 - 5x &= 347 \\ -4x &= -248 \\ x &= 62 \end{aligned}$$

He has 62 \$1 bills and $119 - 62 = 57$ \$5 bills.

**Check** The number of bills is $62 + 57 = 119$ and the value of the bills is $\$1(62) + \$5(57) = \$62 + \$285 = \$347$, as required.

**13.** Let $x =$ the number of \$10 coins. Then $41 - x$ is the number of \$20 coins.

| Number of Coins | Denomination | Value |
|---|---|---|
| $x$ | 10 | $10x$ |
| $41 - x$ | 20 | $20(41 - x)$ |
| 41 | ← Totals → | 540 |

The sum of the values must equal the total value.

$$\begin{aligned} 10x + 20(41 - x) &= 540 \\ 10x + 820 - 20x &= 540 \\ -10x &= -280 \\ x &= 28 \end{aligned}$$

He has 28 \$10 coins and $41 - 28 = 13$ \$20 coins.

**Check** The number of coins is $28 + 13 = 41$ and the value of the coins is $\$10(28) + \$20(13) = \$540$, as required.

**14.** Let $x =$ the number of two-cent pieces. Then $3x$ is the number of three-cent pieces.

| Number of Coins | Denomination | Value |
|---|---|---|
| $x$ | 0.02 | $0.02x$ |
| $3x$ | 0.03 | $0.03(3x)$ |
| | Total → | 2.42 |

The sum of the values must equal the total value.

$$\begin{aligned} 0.02x + 0.03(3x) &= 2.42 \\ 2x + 3(3x) &= 242 \end{aligned}$$

*Multiply both sides by 100.*

$$\begin{aligned} 2x + 9x &= 242 \\ 11x &= 242 \\ x &= 22 \end{aligned}$$

She has 22 two-cent pieces and $3(22) = 66$ three-cent pieces.

**Check** 66 is three times 22 and the value of the coins is $\$0.02(22) + \$0.03(66) = \$2.42$, as required.

**15.** Let $x =$ the number of adult tickets sold. Then $1460 - x =$ the number of children and senior tickets sold.

| Cost of Ticket | Number Sold | Amount Collected |
|---|---|---|
| \$18 | $x$ | $18x$ |
| \$12 | $1460 - x$ | $12(1460 - x)$ |
| Totals | 1460 | \$22,752 |

Write the equation from the last column of the table.

$$\begin{aligned} 18x + 12(1460 - x) &= 22{,}752 \\ 18x + 17{,}520 - 12x &= 22{,}752 \\ 6x &= 5232 \\ x &= 872 \end{aligned}$$

872 adult tickets were sold; $1460 - 872 = 588$ children and senior tickets were sold.

**Check** The number of tickets sold was $872 + 588 = 1460$ and the amount collected was $\$18(872) + \$12(588) = \$15{,}696 + \$7056 = \$22{,}752$, as required.

**16.** Let $x =$ the number of student tickets sold. Then $480 - x =$ the number of nonstudent tickets sold.

| Cost of Ticket | Number Sold | Amount Collected |
|---|---|---|
| \$5 | $x$ | $5x$ |
| \$8 | $480 - x$ | $8(480 - x)$ |
| Totals | 480 | \$2895 |

Write the equation from the last column of the table.

$$\begin{aligned} 5x + 8(480 - x) &= 2895 \\ 5x + 3840 - 8x &= 2895 \\ -3x &= -945 \\ x &= 315 \end{aligned}$$

315 student tickets were sold; $480 - 315 = 165$ nonstudent tickets were sold.

**Check** The number of tickets sold was $315 + 165 = 480$ and the amount collected was $\$5(315) + \$8(165) = \$1575 + \$1320 = \$2895$, as required.

**17.** $d = rt$, so

$$r = \frac{d}{t} = \frac{100}{12.54} \approx 7.97$$

Her rate was about 7.97 m/sec.

**18.** $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{52.64} \approx 7.60$$

Her rate was about 7.60 m/sec.

**19.** $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{47.25} \approx 8.47$$

His rate was about 8.47 m/sec.

**20.** $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{43.75} \approx 9.14$$

His rate was about 9.14 m/sec.

**21.** Let $t =$ the time until they are 110 mi apart. Use the formula $d = rt$. Complete the table.

| | Rate | Time | Distance |
|---|---|---|---|
| **First Steamer** | 22 | $t$ | $22t$ |
| **Second Steamer** | 22 | $t$ | $22t$ |
| | | | 110 |

The total distance traveled is the sum of the distances traveled by each steamer, since they are traveling in opposite directions. This total is 110 mi.

$$\begin{aligned} 22t + 22t &= 110 \\ 44t &= 110 \\ t &= \tfrac{110}{44} = \tfrac{5}{2} \quad \text{or} \quad 2\tfrac{1}{2} \end{aligned}$$

It will take them $2\frac{1}{2}$ hr.

**Check** Each steamer traveled $22(2.5) = 55$ miles for a total of $2(55) = 110$ miles, as required.

**22.** Let $t =$ the time it takes for the trains to be 315 km apart. Use the formula $d = rt$. Complete the table.

| | Rate | Time | Distance |
|---|---|---|---|
| **First Train** | 85 | $t$ | $85t$ |
| **Second Train** | 95 | $t$ | $95t$ |
| | | | 315 |

The total distance traveled is the sum of the distances traveled by each train, since they are traveling in opposite directions. This total is 315 km.

$$\begin{aligned} 85t + 95t &= 315 \\ 180t &= 315 \\ t &= \tfrac{315}{180} = \tfrac{7}{4} \quad \text{or} \quad 1\tfrac{3}{4} \end{aligned}$$

It will take the trains $1\frac{3}{4}$ hr before they are 315 km apart.

**Check** The first train traveled $85(1.75) = 148.75$ km and the second train traveled $95(1.75) = 166.25$ km. The sum is 315 km, as required.

**23.** Let $t =$ Mulder's time. Then $t - \frac{1}{2} =$ Scully's time.

| | Rate | Time | Distance |
|---|---|---|---|
| **Mulder** | 65 | $t$ | $65t$ |
| **Scully** | 68 | $t - \frac{1}{2}$ | $68(t - \frac{1}{2})$ |

The distances are equal.

$$\begin{aligned} 65t &= 68(t - \tfrac{1}{2}) \\ 65t &= 68t - 34 \\ -3t &= -34 \\ t &= \tfrac{34}{3} \quad \text{or} \quad 11\tfrac{1}{3} \end{aligned}$$

Mulder's time will be $11\frac{1}{3}$ hr. Since he left at 8:30 A.M., $11\frac{1}{3}$ hr or 11 hr 20 min later is 7:50 P.M.

**Check** Mulder's distance was $65(\frac{34}{3}) = 736\frac{2}{3}$ miles. Scully's distance was $68(\frac{34}{3} - \frac{1}{2}) = 68(\frac{65}{6}) = 736\frac{2}{3}$, as required.

**24.** Let $x =$ Lois' travel time. Since Clark leaves 15 minutes after Lois, and $\frac{15}{60} = \frac{1}{4}$ hr, $x - \frac{1}{4} =$ time for Clark. Complete the table using the formula $rt = d$.

| | Rate | Time | Distance |
|---|---|---|---|
| **Lois** | 35 | $x$ | $35x$ |
| **Clark** | 40 | $x - \frac{1}{4}$ | $40(x - \frac{1}{4})$ |

Since Lois and Clark are going in opposite directions, we add their distances to get 140 mi.

$$\begin{aligned} 35x + 40(x - \tfrac{1}{4}) &= 140 \\ 35x + 40x - 10 &= 140 \\ 75x &= 150 \\ x &= 2 \end{aligned}$$

They will be 140 mi apart at 8 A.M. + 2 hr = 10 A.M.

**Check** Lois' distance was $35(2) = 70$. Clark's distance was $40(2 - \frac{1}{4}) = 40(\frac{7}{4}) = 70$. The sum is 140 miles, as required.

**25.** Let $x =$ her average rate on Sunday. Then $x + 5 =$ her average rate on Saturday.

| | Rate | Time | Distance |
|---|---|---|---|
| **Saturday** | $x + 5$ | 3.6 | $3.6(x + 5)$ |
| **Sunday** | $x$ | 4 | $4x$ |

The distances are equal.

$$\begin{aligned} 3.6(x+5) &= 4x \\ 3.6x + 18 &= 4x \\ 18 &= 0.4x \quad \textit{Subtract 3.6x.} \\ x &= \tfrac{18}{0.4} = 45 \end{aligned}$$

Her average rate on Sunday was 45 mph.
**Check** On Sunday, 4 hours @ 45 mph = 180 miles. On Saturday, 3.6 hours @ 50 mph = 180 miles. The distances are equal.

**26.** Let $x =$ her biking rate. Then $x - 7 =$ her walking rate.

| | Rate | Time | Distance |
|---|---|---|---|
| **Walking** | $x-7$ | $\frac{40}{60} = \frac{2}{3}$ hr | $\frac{2}{3}(x-7)$ |
| **Biking** | $x$ | $\frac{12}{60} = \frac{1}{5}$ hr | $\frac{1}{5}x$ |

The distances are equal.

$$\begin{aligned} \tfrac{2}{3}(x-7) &= \tfrac{1}{5}x \\ 10(x-7) &= 3x \quad \textit{Multiply by 15.} \\ 10x - 70 &= 3x \\ 7x &= 70 \\ x &= 10 \end{aligned}$$

The distance from her house to the train station is $\frac{1}{5}x = \frac{1}{5}(10) = 2$ miles.
**Check** The distance walking is $(3 \text{ mph})(\frac{2}{3} \text{ hr}) = 2$ mi. The distance biking is $(10 \text{ mph})(\frac{1}{5} \text{ hr}) = 2$ mi. The distances are equal.

**27.** Let $x =$ Anne's time. Then $x + \frac{1}{2} =$ Johnny's time.

| | Rate | Time | Distance |
|---|---|---|---|
| **Anne** | 60 | $x$ | $60x$ |
| **Johnny** | 50 | $x+\frac{1}{2}$ | $50(x+\frac{1}{2})$ |

The total distance is 80.

$$\begin{aligned} 60x + 50(x + \tfrac{1}{2}) &= 80 \\ 60x + 50x + 25 &= 80 \\ 110x &= 55 \\ x &= \tfrac{55}{110} = \tfrac{1}{2} \end{aligned}$$

They will meet $\frac{1}{2}$ hr after Anne leaves.

**Check** Anne travels $60(\frac{1}{2}) = 30$ miles. Johnny travels $50(\frac{1}{2} + \frac{1}{2}) = 50$ miles. The sum of the distances is 80 miles, as required.

**28.** Let $x =$ her rate (speed) during the first part of the trip. Then $x - 25 =$ her rate during rush-hour traffic. Make a table using the formula $rt = d$.

| | Rate | Time | Distance |
|---|---|---|---|
| **First Part** | $x$ | 2 | $2x$ |
| **Rush-Hour** | $x-25$ | $\frac{1}{2}$ | $\frac{1}{2}(x-25)$ |

The total distance was 125 miles.

$$\begin{aligned} 2x + \tfrac{1}{2}(x-25) &= 125 \\ 4x + x - 25 &= 250 \quad \textit{Multiply by 2.} \\ 5x &= 275 \\ x &= 55 \end{aligned}$$

The rate during the first part of the trip was 55 mph.

**Check** The distance traveled during the first part of the trip was $55(2) = 110$ miles. The distance traveled during the second part of the trip was $(55-25)(0.5) = 15$ miles. The sum of the distances is 125 miles, as required.

**29.** The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} (x-30) + (2x-120) + (\tfrac{1}{2}x + 15) &= 180 \\ \tfrac{7}{2}x - 135 &= 180 \\ 7x - 270 &= 360 \\ &\textit{Multiply by 2.} \\ 7x &= 630 \\ x &= 90 \end{aligned}$$

With $x = 90$, the three angle measures become

$$\begin{aligned} &(90-30)^\circ = 60^\circ, \\ &[2(90)-120]^\circ = 60^\circ, \\ \text{and} \quad &[\tfrac{1}{2}(90)+15]^\circ = 60^\circ. \end{aligned}$$

**30.** The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} (x+15) + (10x-20) + (x+5) &= 180 \\ 12x &= 180 \\ x &= 15 \end{aligned}$$

With $x = 15$, the three angle measures become

$$\begin{aligned} &(15+15)^\circ = 30^\circ, \\ &(10 \cdot 15 - 20)^\circ = 130^\circ, \\ \text{and} \quad &(15+5)^\circ = 20^\circ. \end{aligned}$$

**31.** The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} (3x+7) + (9x-4) + (4x+1) &= 180 \\ 16x + 4 &= 180 \\ 16x &= 176 \\ x &= 11 \end{aligned}$$

With $x = 11$, the three angle measures become

$$\begin{aligned} &(3 \cdot 11 + 7)^\circ = 40^\circ, \\ &(9 \cdot 11 - 4)^\circ = 95^\circ, \\ \text{and} \quad &(4 \cdot 11 + 1)^\circ = 45^\circ. \end{aligned}$$

**32.** The sum of the measures of the three angles of a triangle is $180°$.

$$\begin{aligned}(2x+7)+(x+61)+x&=180\\4x+68&=180\\4x&=112\\x&=28\end{aligned}$$

With $x=28$, the three angle measures become

$$(2\cdot 28+7)°=63°,\quad (28+61)°=89°,\quad \text{and}\quad 28°.$$

**33.** The sum of the measures of the angles of a triangle is $180°$.

$$\begin{aligned}x+2x+60&=180\\3x+60&=180\\3x&=120\\x&=40\end{aligned}$$

The measures of the unknown angles are $40°$ and $2x=80°$.

**34.** Two angles which form a straight line add to $180°$, so $180°-60°=120°$. The measure of the unknown angle is $120°$.

**35.** The sum of the measures of the unknown angles in Exercise 33 is $40°+80°=120°$. This is equal to the measure of the angle in Exercise 34.

**36.** The sum of the measures of angles (1) and (2) is equal to the measure of angle (3).

**37.** Vertical angles have equal measure.

$$\begin{aligned}8x+2&=7x+17\\x&=15\end{aligned}$$

$$8\cdot 15+2=122\quad\text{and}\quad 7\cdot 15+17=122.$$

The angles are both $122°$.

**38.** Vertical angles have equal measure.

$$\begin{aligned}9-5x&=25-3x\\9&=25+2x\\-16&=2x\\-8&=x\end{aligned}$$

$$9-5(-8)=49\quad\text{and}\quad 25-3(-8)=49.$$

The angles are both $49°$.

**39.** The sum of the two angles is $90°$.

$$\begin{aligned}(5x-1)+2x&=90\\7x-1&=90\\7x&=91\\x&=13\end{aligned}$$

The measures of the two angles are $[5(13)-1]°=64°$ and $[2(13)]°=26°$.

**40.** Supplementary angles have an angle measure sum of $180°$.

$$\begin{aligned}(3x+5)+(5x+15)&=180\\8x+20&=180\\8x&=160\\x&=20\end{aligned}$$

With $x=20$, the two angle measures become

$$(3\cdot 20+5)°=65°\quad\text{and}\quad (5\cdot 20+15)°=115°.$$

**41.** Let $x=$ the first consecutive integer. Then $x+1$ will be the second consecutive integer, and $x+2$ will be the third consecutive integer.

The sum of the first and twice the second is 17 more than twice the third.

$$\begin{aligned}x+2(x+1)&=2(x+2)+17\\x+2x+2&=2x+4+17\\3x+2&=2x+21\\x&=19\end{aligned}$$

Since $x=19$, $x+1=20$, and $x+2=21$. The three consecutive integers are 19, 20, and 21.

**42.** Let $x=$ the first integer. Then $x+1$, $x+2$, and $x+3$ are the next three consecutive integers. The sum of the first three integers is 54 more than the fourth.

$$\begin{aligned}x+(x+1)+(x+2)&=(x+3)+54\\3x+3&=x+57\\2x&=54\\x&=27\end{aligned}$$

The four consecutive integers are 27, 28, 29, and 30.

**43.** Let $x=$ my current age. Then $x+1$ will be my age next year.
The sum of these ages will be 103 years.

$$\begin{aligned}x+(x+1)&=103\\2x+1&=103\\2x&=102\\x&=51\end{aligned}$$

If my current age is 51, in 10 years I will be

$$51+10=61 \text{ years old.}$$

**44.** Let $x=$ the page number on one page. Then $x+1$ is the page number on the next page. The sum of the page numbers is 193.

$$\begin{aligned}x+(x+1)&=193\\2x+1&=193\\2x&=192\\x&=96\end{aligned}$$

The page numbers are 96 and 97.

**45.** Let $x =$ the least even integer. Then $x + 2$ and $x + 4$ are the next two even consecutive integers. The sum of the least integer and middle integer is 26 more than the greatest integer.

$$\begin{aligned} x + (x+2) &= (x+4) + 26 \\ 2x + 2 &= x + 30 \\ x &= 28 \end{aligned}$$

The three consecutive even integers are 28, 30, and 32.

**46.** Let $x =$ the least even integer. Then $x + 2$ and $x + 4$ are the next two even consecutive integers. The sum of the least integer and greatest integer is 12 more than the middle integer.

$$\begin{aligned} x + (x+4) &= (x+2) + 12 \\ 2x + 4 &= x + 14 \\ x &= 10 \end{aligned}$$

The three consecutive even integers are 10, 12, and 14.

**47.** Let $x =$ the least odd integer. Then $x + 2$ and $x + 4$ are the next two odd consecutive integers. The sum of the least integer and middle integer is 19 more than the greatest integer.

$$\begin{aligned} x + (x+2) &= (x+4) + 19 \\ 2x + 2 &= x + 23 \\ x &= 21 \end{aligned}$$

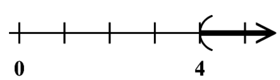
The three consecutive odd integers are 21, 23, and 25.

**48.** Let $x =$ the least odd integer. Then $x + 2$ and $x + 4$ are the next two odd consecutive integers. The sum of the least integer and greatest integer is 13 more than the middle integer.

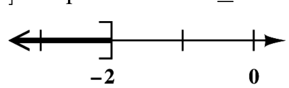
$$\begin{aligned} x + (x+4) &= (x+2) + 13 \\ 2x + 4 &= x + 15 \\ x &= 11 \end{aligned}$$

The three consecutive odd integers are 11, 13, and 15.

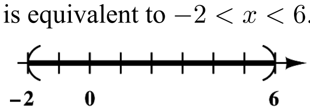
**49.** $(4, \infty)$ is equivalent to $x > 4$.



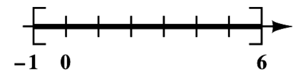

**50.** $(-\infty, -2]$ is equivalent to $x \le -2$.




**51.** $(-2, 6)$ is equivalent to $-2 < x < 6$.




**52.** $[-1, 6]$ is equivalent to $-1 \le x \le 6$.




## Summary Exercises on Solving Applied Problems

**1.** Let $x =$ the width of the rectangle. Then $x + 3$ is the length of the rectangle.

If the length were decreased by 2 inches and the width were increased by 1 inch, the perimeter would be 24 inches. Use the formula $P = 2L + 2W$, and substitute 24 for $P$, $(x+3) - 2$ or $x + 1$ for $L$, and $x + 1$ for $W$.

$$\begin{aligned} P &= 2L + 2W \\ 24 &= 2(x+1) + 2(x+1) \\ 24 &= 2x + 2 + 2x + 2 \\ 24 &= 4x + 4 \\ 20 &= 4x \\ 5 &= x \end{aligned}$$

The width of the rectangle is 5 inches, and the length is $5 + 3 = 8$ inches.

**2.** Let $x =$ the width. Then $2x$ is the length (twice the width).
Use $P = 2L + 2W$ and add one more width to cut the area into two parts. Thus, an equation is

$$\begin{aligned} 2L + 2W + W &= 210. \\ 2(2x) + 2x + x &= 210 \\ 7x &= 210 \\ x &= 30 \end{aligned}$$

The width is 30 meters, and the length is $2(30) = 60$ meters.

**3.** Let $x =$ the regular price of the item.
The sale price after a 46% (or 0.46) discount was \$46.97, so an equation is

$$\begin{aligned} x - 0.46x &= 46.97. \\ 0.54x &= 46.97 \\ x &= \tfrac{46.97}{0.54} \approx 86.98 \end{aligned}$$

To the nearest cent, the regular price was \$86.98.

**4.** Let $x =$ the regular price of the Blu-ray player.
The sale price after a discount of 40% (or 0.40) was \$255, so an equation is

$$\begin{aligned} x - 0.40x &= 255. \\ 0.60x &= 255 \\ x &= 425 \end{aligned}$$

The regular price of the Blu-ray player was \$425.

**5.** Let $x =$ the amount invested at 4%.
Then $2x$ is the amount invested at 5%.
Use $I = prt$ with $t = 1$ yr. Make a table.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.04 | $0.04x$ |
| $2x$ | 0.05 | $0.05(2x) = 0.10x$ |
| | Total → | 112 |

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest at 4\%} & + & \text{interest at 5\%} & = & \text{total interest.} \\ 0.04x & + & 0.10x & = & 112 \end{array}$$

$$\begin{aligned} 4x + 10x &= 11{,}200 \quad \textit{Multiply by 100.} \\ 14x &= 11{,}200 \\ x &= 800 \end{aligned}$$

\$800 is invested at 4% and $2(\$800) = \$1600$ at 5%.

**Check** \$800 @ 4% = \$32 and \$1600 @ 5% = \$80; \$32 + \$80 = \$112

**6.** Let $x$ = the amount invested at 3%. Then $x + 2000$ is the amount invested at 4%. Use $I = prt$ with $t = 1$ yr. Make a table.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.03 | $0.03x$ |
| $x + 2000$ | 0.04 | $0.04(x + 2000)$ |
| | Total → | 920 |

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest at 3\%} & + & \text{interest at 4\%} & = & \text{total interest.} \\ 0.03x & + & 0.04(x+2000) & = & 920 \end{array}$$

$$\begin{aligned} 3x + 4(x + 2000) &= 92{,}000 \quad \textit{Multiply by 100.} \\ 3x + 4x + 8000 &= 92{,}000 \\ 7x &= 84{,}000 \\ x &= 12{,}000 \end{aligned}$$

\$12,000 is invested at 3% and \$14,000 is invested at 4%.

**Check** \$12,000 @ 3% = \$360 and \$14,000 @ 4% = \$560; \$360 + \$560 = \$920

**7.** Let $x$ = the number of points scored by Wade in the 2008–2009 season. Then $x - 136$ = the number of points scored by James in the 2007–2008 season. The total number of points scored by both was 4636.

$$\begin{aligned} x + (x - 136) &= 4636 \\ 2x - 136 &= 4636 \\ 2x &= 4772 \\ x &= 2386 \end{aligned}$$

Wade scored 2386 points and James scored $2386 - 136 = 2250$ points.

**8.** Let $x$ = the amount grossed by *The Dark Knight.* Then $x + 67.5$ = the amount grossed by *Titanic* (in millions). Together they grossed \$1134.1 million.

$$\begin{aligned} x + (x + 67.5) &= 1134.1 \\ 2x + 67.5 &= 1134.1 \\ 2x &= 1066.6 \\ x &= 533.3 \end{aligned}$$

*The Dark Knight* grossed \$533.3 million and *Titanic* grossed $533.3 + 67.5 =$ \$600.8 million.

**9.** Let $t$ = the time it will take until John and Pat meet. Use $d = rt$ and make a table.

| | Rate | Time | Distance |
|---|---|---|---|
| **John** | 60 | $t$ | $60t$ |
| **Pat** | 28 | $t$ | $28t$ |

The total distance is 440 miles.

$$\begin{aligned} 60t + 28t &= 440 \\ 88t &= 440 \\ t &= 5 \end{aligned}$$

It will take 5 hours for John and Pat to meet.

**Check** John traveled $60(5) = 300$ miles and Pat traveled $28(5) = 140$ miles; $300 + 140 = 440$, as required.

**10.** Let $x$ = Merga's rate. Then $x - 1.9$ = Kosgei's rate.

| | Rate | Time | Distance |
|---|---|---|---|
| **Merga** | $x$ | 2.145 | $2.145x$ |
| **Kosgei** | $x - 1.9$ | 2.538 | $2.538(x - 1.9)$ |

The distances are equal.

$$\begin{aligned} 2.145x &= 2.538(x - 1.9) \\ 2145x &= 2538(x - 1.9) \quad \textit{Multiply by 1000.} \\ 2145x &= 2538x - 4822.2 \\ 4822.2 &= 393x \\ x &= \tfrac{4822.2}{393} \approx 12.27 \end{aligned}$$

Merga's rate was 12.27 mph and Kosgei's rate was $12.27 - 1.9 = 10.37$ mph.

**Check** Merga's distance was (12.27 mph)(2.145 hr) ≈ 26.32 miles. Kosgei's distance was (10.37 mph)(2.538 hr) ≈ 26.32 miles. (The values are slightly different due to rounding.)

**11.** Let $x$ = the number of liters of the 5% drug solution.

| Liters of Solution | Percent (as a decimal) | Liters of Pure Drug |
|---|---|---|
| 20 | 0.10 | $20(0.10) = 2$ |
| $x$ | 0.05 | $0.05x$ |
| $20 + x$ | 0.08 | $0.08(20 + x)$ |

$$\begin{array}{ccccc} \text{Drug in 10\%} & + & \text{drug in 5\%} & = & \text{drug in 8\%.} \\ 2 & + & 0.05x & = & 0.08(20+x) \end{array}$$

$$\begin{aligned} 200 + 5x &= 8(20 + x) \quad \textit{Multiply by 100.} \\ 200 + 5x &= 160 + 8x \\ 40 &= 3x \\ x &= \tfrac{40}{3} \text{ or } 13\tfrac{1}{3} \end{aligned}$$

The pharmacist should add $13\frac{1}{3}$ L.

**Check** 10% of 20 is 2 and 5% of $\frac{40}{3}$ is $\frac{2}{3}$; $2 + \frac{2}{3} = \frac{8}{3}$, which is the same as 8% of $(20 + \frac{40}{3})$.

**12.** Let $x =$ the number of kilograms of the metal that is 20% tin.

| Kilograms of Metal | Percent Tin (as a decimal) | Kilograms of Pure Tin |
|---|---|---|
| 80 | 0.70 | $80(0.70) = 56$ |
| $x$ | 0.20 | $0.20x$ |
| $80 + x$ | 0.50 | $0.50(80 + x)$ |

$$\begin{array}{ccccc} \text{Tin in 70\%} & + & \text{tin in 20\%} & = & \text{tin in 50\%.} \\ 56 & + & 0.20x & = & 0.50(80+x) \end{array}$$

$$\begin{aligned} 560 + 2x &= 5(80 + x) \quad \textit{Multiply by 10.} \\ 560 + 2x &= 400 + 5x \\ 160 &= 3x \\ x &= \tfrac{160}{3} \text{ or } 53\tfrac{1}{3} \end{aligned}$$

$53\frac{1}{3}$ kilograms should be added.

**Check** 70% of 80 is 56 and 20% of $\frac{160}{3}$ is $\frac{32}{3}$; $56 + \frac{32}{3} = 66\frac{2}{3}$, which is the same as 50% of $(80 + \frac{160}{3})$.

**13.** Let $x =$ the number of \$5 bills. Then $126 - x$ is the number of \$10 bills.

| Number of Bills | Denomination | Value |
|---|---|---|
| $x$ | 5 | $5x$ |
| $126 - x$ | 10 | $10(126 - x)$ |
| 126 | ← Totals → | 840 |

The sum of the values must equal the total value.

$$\begin{aligned} 5x + 10(126 - x) &= 840 \\ 5x + 1260 - 10x &= 840 \\ -5x &= -420 \\ x &= 84 \end{aligned}$$

There are 84 \$5 bills and $126 - 84 = 42$ \$10 bills.

**Check** The number of bills is $84 + 42 = 126$ and the value of the bills is $\$5(84) + \$10(42) = \$840$, as required.

**14.** Let $x =$ the number of \$7 tickets sold. Then $2460 - x =$ the number of \$9 tickets sold.

| Number Sold | Cost of Ticket | Amount Collected |
|---|---|---|
| $x$ | \$7 | $7x$ |
| $2460 - x$ | \$9 | $9(2460 - x)$ |
| 2460 | ← Totals → | \$20,520 |

Write the equation from the last column of the table.

$$\begin{aligned} 7x + 9(2460 - x) &= 20{,}520 \\ 7x + 22{,}140 - 9x &= 20{,}520 \\ -2x &= -1620 \\ x &= 810 \end{aligned}$$

810 \$7 tickets were sold and $2460 - 810 = 1650$ \$9 tickets were sold.

**Check** The number of tickets sold was $810 + 1650 = 2460$ and the amount collected was $\$7(810) + \$9(1650) = \$5670 + \$14{,}850 = \$20{,}520$, as required.

**15.** The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} x + (6x - 50) + (x - 10) &= 180 \\ 8x - 60 &= 180 \\ 8x &= 240 \\ x &= 30 \end{aligned}$$

With $x = 30$, the three angle measures become

$$(6 \cdot 30 - 50)° = 130°, \quad (30 - 10)° = 20°, \text{ and } 30°.$$

**16.** In the figure, the two angles are supplementary, so their sum is 180°.

$$\begin{aligned} (10x + 7) + (7x + 3) &= 180 \\ 17x + 10 &= 180 \\ 17x &= 170 \\ x &= 10 \end{aligned}$$

The two angle measures are $10(10) + 7 = 107°$ and $7(10) + 3 = 73°$.

**17.** Let $x =$ the least integer. Then $x + 1$ is the middle integer and $x + 2$ is the greatest integer.

"The sum of the least and greatest of three consecutive integers is 32 more than the middle integer" translates to

$$\begin{aligned} x + (x + 2) &= 32 + (x + 1). \\ 2x + 2 &= x + 33 \\ x &= 31 \end{aligned}$$

The three consecutive integers are 31, 32, and 33.

**Check** The sum of the least and greatest integers is $31 + 33 = 64$, which is the same as 32 more than the middle integer.

**18.** Let $x =$ the first odd integer. Then $x + 2$ is the next odd integer.

"If the lesser of two consecutive odd integers is doubled, the result is 7 more than the greater of the two integers" translates to

$$\begin{aligned} 2(x) &= 7 + (x + 2). \\ 2x &= x + 9 \\ x &= 9 \end{aligned}$$

The two consecutive odd integers are 9 and 11.

**Check** Doubling the lesser gives us $2(9) = 18$, which is equal to 7 more than 11.

**19.** Let $x =$ the length of the shortest side. Then $2x$ is the length of the middle side and $3x - 2$ is the length of the longest side.

The perimeter is 34 inches. Using $P = a + b + c$ gives us

$$\begin{aligned} x + 2x + (3x - 2) &= 34. \\ 6x - 2 &= 34 \\ 6x &= 36 \\ x &= 6 \end{aligned}$$

The lengths of the three sides are 6 inches, $2(6) = 12$ inches, and $3(6) - 2 = 16$ inches.

**Check** The sum of the lengths of the three sides is $6 + 12 + 16 = 34$ inches, as required.

**20.** Let $x =$ the length of the rectangle. Then the perimeter is $43 + x$.

$$\begin{aligned} P &= 2L + 2W \\ 43 + x &= 2x + 2(10) \quad \textit{Let } W = 10. \\ 43 + x &= 2x + 20 \\ 23 &= x \end{aligned}$$

The length of the rectangle is 23 inches.

**Check** $P = 2L + 2W = 2(23) + 2(10) = 66$ inches, which is 43 inches more than the length.

# 2.5 Linear Inequalities in One Variable

## 2.5 Classroom Examples, Now Try Exercises

**1.** $x - 5 > 1$

$x > 6$ *Add 5.*

**Check** Substitute 6 for $x$ in $x - 5 = 1$.

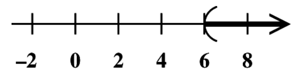
$$\begin{aligned} 6 - 5 &\stackrel{?}{=} 1 \\ 1 &= 1 \quad \textit{True} \end{aligned}$$

This shows that 6 is the boundary point. Choose 0 and 7 as test points.

$$x - 5 > 1$$

| | |
|---|---|
| *Let x = 0.* | *Let x = 7.* |
| $0 - 5 \stackrel{?}{>} 1$ | $7 - 5 \stackrel{?}{>} 1$ |
| $-5 > 1$ *False* | $2 > 1$ *True* |
| 0 is not in the solution set. | 7 is in the solution set. |

The check confirms that $(6, \infty)$ is the solution set.




**N1.** $x - 10 > -7$

$x > 3$ *Add 10.*

**Check** Substitute 3 for $x$ in $x - 10 = -7$.

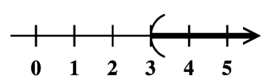
$$\begin{aligned} 3 - 10 &\stackrel{?}{=} -7 \\ -7 &= -7 \quad \textit{True} \end{aligned}$$

This shows that 3 is the boundary point. Choose 0 and 7 as test points.

$$x - 10 > -7$$

| | |
|---|---|
| *Let x = 0.* | *Let x = 7.* |
| $0 - 10 \stackrel{?}{>} -7$ | $7 - 10 \stackrel{?}{>} -7$ |
| $-10 > -7$ *False* | $-3 > -7$ *True* |
| 0 is not in the solution set. | 7 is in the solution set. |

The check confirms that $(3, \infty)$ is the solution set.




**2.** $5x + 3 \geq 4x - 1$

$x + 3 \geq -1$ *Subtract 4x.*

$x \geq -4$ *Subtract −3.*

**Check** Substitute $-4$ for $x$ in $5x + 3 = 4x - 1$.

$$\begin{aligned} 5(-4) + 3 &\stackrel{?}{=} 4(-4) - 1 \\ -17 &= -17 \quad \textit{True} \end{aligned}$$

So $-4$ satisfies the equality part of $\geq$. Choose $-5$ and 0 as test points.

$$5x + 3 \geq 4x - 1$$

*Let x = −5.*

$$\begin{aligned} 5(-5) + 3 &\stackrel{?}{\geq} 4(-5) - 1 \\ -22 &\geq -21 \quad \textit{False} \end{aligned}$$

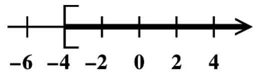
$-5$ is not in the solution set.

*Let x = 0.*

$$\begin{aligned} 5(0) + 3 &\stackrel{?}{\geq} 4(0) - 1 \\ 3 &\geq -1 \quad \textit{True} \end{aligned}$$

0 is in the solution set.

The check confirms that $[-4, \infty)$ is the solution set.




**N2.** $4x + 1 \geq 5x$

$1 \geq x$ *Subtract 4x.*

$x \leq 1$ *Equivalent inequality.*

**Check** Substitute 1 for $x$ in $4x + 1 = 5x$.

$4(1) + 1 \stackrel{?}{=} 5(1)$

$5 = 5$ *True*

So 1 satisfies the equality part of $\geq$. Choose 0 and 2 as test points.

$$4x + 1 \geq 5x$$

*Let x = 0.*

$4(0) + 1 \stackrel{?}{\geq} 5(0)$

$1 \geq 0$ *True*

0 is in the solution set.

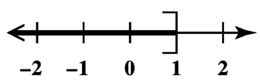
*Let x = 2.*

$4(2) + 1 \stackrel{?}{\geq} 5(2)$

$9 \geq 10$ *False*

2 is not in the solution set.

The check confirms that $(-\infty, 1]$ is the solution set.

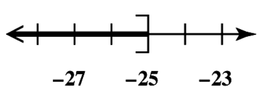



**3.** **(a)** $4x \leq -100$

$\frac{4x}{4} \leq \frac{-100}{4}$ *Divide by 4 > 0; do not reverse the symbol.*

$x \leq -25$

Check that the solution set is the interval $(-\infty, -25]$.

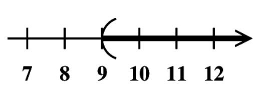



**(b)** $-9x < -81$

$\frac{-9x}{-9} > \frac{-81}{-9}$ *Divide by −9 < 0; reverse the symbol.*

$x > 9$

Check that the solution set is the interval $(9, \infty)$.

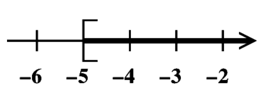



**N3.** **(a)** $8x \geq -40$

$\frac{8x}{8} \geq \frac{-40}{8}$ *Divide by 8 > 0; do not reverse the symbol.*

$x \geq -5$

Check that the solution set is the interval $[-5, \infty)$.

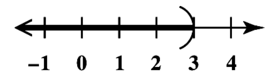



**(b)** $-20x > -60$

$\frac{-20x}{-20} < \frac{-60}{-20}$ *Divide by −20 < 0; reverse the symbol.*

$x < 3$

Check that the solution set is the interval $(-\infty, 3)$.




**4.** $6(x - 1) + 3x \geq -x - 3(x + 2)$

$6x - 6 + 3x \geq -x - 3x - 6$

$9x - 6 \geq -4x - 6$

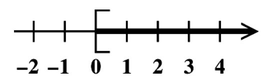
$13x - 6 \geq -6$

$13x \geq 0$

$\frac{13x}{13} \geq \frac{0}{13}$

$x \geq 0$

Check that the solution set is the interval $[0, \infty)$.




**N4.** $5 - 2(x - 4) \leq 11 - 4x$

$5 - 2x + 8 \leq 11 - 4x$

$-2x + 13 \leq 11 - 4x$

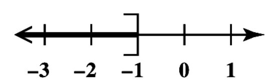
$2x + 13 \leq 11$

$2x \leq -2$

$\frac{2x}{2} \leq \frac{-2}{2}$

$x \leq -1$

Check that the solution set is the interval $(-\infty, -1]$.




**5.** $\frac{1}{4}(x + 3) + 2 \leq \frac{3}{4}(x + 8)$

$4\left[\frac{1}{4}(x + 3) + 2\right] \leq 4\left[\frac{3}{4}(x + 8)\right]$ *Multiply by 4.*

$(x + 3) + 8 \leq 3(x + 8)$

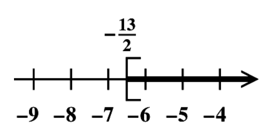
$x + 11 \leq 3x + 24$

$-2x + 11 \leq 24$ *Subtract 3x.*

$-2x \leq 13$ *Subtract 11.*

$x \geq -\frac{13}{2}$ *Divide by −2; reverse symbol.*

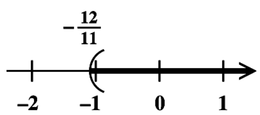
Check that the solution set is the interval $\left[-\frac{13}{2}, \infty\right)$.



**N5.**

$$\begin{aligned}
\tfrac{3}{4}(x-2)+\tfrac{1}{2} &> \tfrac{1}{5}(x-8) \\
20\left[\tfrac{3}{4}(x-2)+\tfrac{1}{2}\right] &> 20\left[\tfrac{1}{5}(x-8)\right] && \textit{Mult. by 20.} \\
15(x-2)+10 &> 4(x-8) \\
15x-30+10 &> 4x-32 \\
15x-20 &> 4x-32 \\
11x-20 &> -32 && \textit{Subtract 4x.} \\
11x &> -12 && \textit{Add 20.} \\
x &> -\tfrac{12}{11} && \textit{Divide by 11.}
\end{aligned}$$

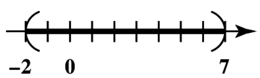
Check that the solution set is the interval $\left(-\frac{12}{11}, \infty\right)$.




**6.**

$$\begin{aligned}
-4 < x-2 &< 5 \\
-2 < \quad x \quad &< 7 && \textit{Add 2 to each part.}
\end{aligned}$$

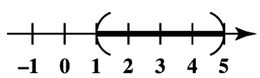
Check that the solution set is the interval $(-2, 7)$.




**N6.**

$$\begin{aligned}
-1 < x-2 &< 3 \\
1 < \quad x \quad &< 5 && \textit{Add 2 to each part.}
\end{aligned}$$

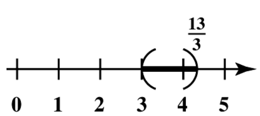
Check that the solution set is the interval $(1, 5)$.




**7.**

$$\begin{aligned}
5 < 3x-4 &< 9 \\
9 < \quad 3x \quad &< 13 && \textit{Add 4 to each part.} \\
\frac{9}{3} < \frac{3x}{3} &< \frac{13}{3} && \textit{Divide each part by 3.} \\
3 < \quad x \quad &< \tfrac{13}{3}
\end{aligned}$$

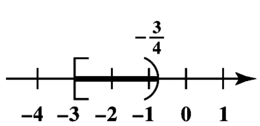
Check that the solution set is the interval $\left(3, \frac{13}{3}\right)$.




**N7.**

$$\begin{aligned}
-2 < -4x-5 &\le 7 \\
3 < \quad -4x \quad &\le 12 && \textit{Add 5 to each part.} \\
\frac{3}{-4} > \frac{-4x}{-4} &\ge \frac{12}{-4} && \textit{Divide by } -4. \textit{ Reverse inequalities.} \\
-\tfrac{3}{4} > \quad x \quad &\ge -3 && \textit{Reduce.} \\
-3 \le \quad x \quad &< -\tfrac{3}{4} && \textit{Equivalent inequality.}
\end{aligned}$$

Check that the solution set is the interval $\left[-3, -\frac{3}{4}\right)$.




**8.** *Step 2*

Let $h =$ the number of hours she can rent the leaf blower.

*Step 3*

She must pay \$5, plus \$$1.75h$, to rent the leaf blower for $h$ hours, and this amount must be *no more than* \$26.

| Cost of renting | is no more than | 26 dollars. |
|---|---|---|
| ↓ | ↓ | ↓ |
| $5 + 1.75h$ | $\le$ | $26$ |

*Step 4*

$$\begin{aligned}
1.75h &\le 21 && \textit{Subtract 5.} \\
h &\le 12 && \textit{Divide by 1.75.}
\end{aligned}$$

*Step 5*

She can use the leaf blower for a maximum of 12 hr. (She may use it for less time, as indicated by the inequality $h \le 12$.)

*Step 6*

If Marge uses the leaf blower for 12 hr, she will spend $5 + 1.75(12) = 26$, the maximum amount.

**N8.** *Step 2*

Let $x =$ the number of months she belongs to the health club.

*Step 3*

She must pay \$40, plus \$$35x$, to belong to the health club for $x$ months, and this amount must be *no more than* \$355.

| Cost of belonging | is no more than | 355 dollars. |
|---|---|---|
| ↓ | ↓ | ↓ |
| $40 + 35x$ | $\le$ | $355$ |

*Step 4*

$$\begin{aligned}
35x &\le 315 && \textit{Subtract 40.} \\
x &\le 9 && \textit{Divide by 35.}
\end{aligned}$$

*Step 5*

She can belong to the health club for a maximum of 9 months. (She may belong for less time, as indicated by the inequality $x \le 9$.)

*Step 6*

If Sara belongs for 9 months, she will spend $40 + 35(9) = 355$, the maximum amount.

**9.** Let $x =$ the grade she must make on the fourth test.

To find the average of the four tests, add them and divide by 4. This average must be at least 90, that is, greater than or equal to 90.

$$\begin{aligned} \frac{92+90+84+x}{4} &\geq 90 \\ \frac{266+x}{4} &\geq 90 \\ 266+x &\geq 360 \quad \textit{Multiply by 4.} \\ x &\geq 94 \quad \textit{Subtract 266.} \end{aligned}$$

Abby must score at least 94 on the fourth test.

**Check** $\frac{92+90+84+94}{4} = \frac{360}{4} = 90$

A score of 94 or more will give an average of at least 90, as required.

**N9.** Let $x =$ the grade he must make on the fourth test.

To find the average of the four tests, add them and divide by 4. This average must be at least 90, that is, greater than or equal to 90.

$$\begin{aligned} \frac{82+97+93+x}{4} &\geq 90 \\ \frac{272+x}{4} &\geq 90 \\ 272+x &\geq 360 \quad \textit{Multiply by 4.} \\ x &\geq 88 \quad \textit{Subtract 272.} \end{aligned}$$

Joel must score at least 88 on the fourth test.

**Check** $\frac{82+97+93+88}{4} = \frac{360}{4} = 90$

A score of 88 or more will give an average of at least 90, as required.

## 2.5 Section Exercises

**1.** $x \leq 3$

In interval notation, this inequality is written $(-\infty, 3]$. The bracket indicates that 3 is included. The answer is choice **D**.

**2.** $x > 3$

In interval notation, this inequality is written $(3, \infty)$. The parenthesis indicates that 3 is not included. The answer is choice **C**.

**3.** $x < 3$

In interval notation, this inequality is written $(-\infty, 3)$. The parenthesis indicates that 3 is not included. The graph of this inequality is shown in choice **B**.

**4.** $x \geq 3$

In interval notation, this inequality is written $[3, \infty)$. The bracket indicates that 3 is included. The graph of this inequality is shown in choice **A**.

**5.** $-3 \leq x \leq 3$

In interval notation, this inequality is written $[-3, 3]$. The brackets indicates that $-3$ and 3 are included. The answer is choice **F**.

**6.** $-3 < x < 3$

In interval notation, this inequality is written $(-3, 3)$. The parentheses indicate that neither $-3$ nor 3 is included. The answer is choice **E**.

**7.** Since $4 > 0$, the student should not have reversed the direction of the inequality symbol when dividing by 4. We reverse the inequality symbol only when multiplying or dividing by a *negative* number. The solution set is $[-16, \infty)$.

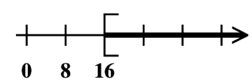
**8.** **(a)** A *small frame* corresponds to a wrist circumference $x$ that is less than 6.75 inches, which can be described by the inequality $x < 6.75$.

**(b)** A *medium frame* corresponds to a wrist circumference $x$ that is between 6.75 and 7.25 inches [inclusive], which can be described by the three-part inequality $6.75 \leq x \leq 7.25$.

**(c)** A *large frame* corresponds to a wrist circumference $x$ that is greater than 7.25 inches, which can be described by the inequality $x > 7.25$.

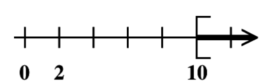
**9.**
$$\begin{aligned} x - 4 &\geq 12 \\ x &\geq 16 \quad \textit{Add 4.} \end{aligned}$$

Check that the solution set is the interval $[16, \infty)$.



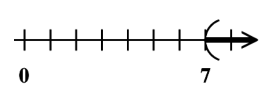

**10.**
$$\begin{aligned} x - 3 &\geq 7 \\ x &\geq 10 \quad \textit{Add 3.} \end{aligned}$$

Check that the solution set is the interval $[10, \infty)$.



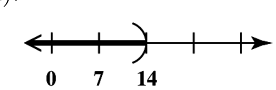

**11.**
$$\begin{aligned} 3k + 1 &> 22 \\ 3k &> 21 \quad \textit{Subtract 1.} \\ k &> 7 \quad \textit{Divide by 3.} \end{aligned}$$

Check that the solution set is the interval $(7, \infty)$.




**12.**
$$\begin{aligned} 5x + 6 &< 76 \\ 5x &< 70 \quad \textit{Subtract 6.} \\ x &< 14 \quad \textit{Divide by 5.} \end{aligned}$$

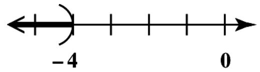
Check that the solution set is the interval $(-\infty, 14)$.



**13.** $4x < -16$

$x < -4$ *Divide by 4.*

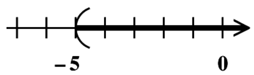
Check that the solution set is the interval $(-\infty, -4)$.




**14.** $2x > -10$

$x > -5$ *Divide by 2.*

Check that the solution set is the interval $(-5, \infty)$.



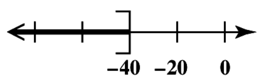

**15.** $-\frac{3}{4}x \geq 30$

Multiply both sides by $-\frac{4}{3}$ and reverse the inequality symbol.

$-\frac{4}{3}(-\frac{3}{4}x) \leq -\frac{4}{3}(30)$

$x \leq -40$

Check that the solution set is the interval $(-\infty, -40]$.



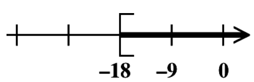

**16.** $-\frac{2}{3}x \leq 12$

Multiply both sides by $-\frac{3}{2}$ and reverse the inequality symbol.

$-\frac{3}{2}(-\frac{2}{3}x) \geq -\frac{3}{2}(12)$

$x \geq -18$

Check that the solution set is the interval $[-18, \infty)$.



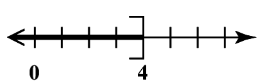

**17.** $-1.3x \geq -5.2$

Divide both sides by $-1.3$, and reverse the inequality symbol.

$$\frac{-1.3x}{-1.3} \leq \frac{-5.2}{-1.3}$$

$x \leq 4$

Check that the solution set is the interval $(-\infty, 4]$.



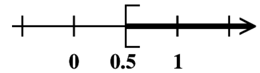

**18.** $-2.5x \leq -1.25$

Divide both sides by $-2.5$, and reverse the inequality symbol.

$$\frac{-2.5x}{-2.5} \geq \frac{-1.25}{-2.5}$$

$x \geq 0.5$

Check that the solution set is the interval $[0.5, \infty)$.

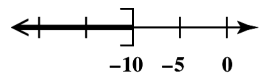



**19.** $5x + 2 \leq -48$

$5x \leq -50$ *Subtract 2.*

$x \leq -10$ *Divide by 5.*

Check that the solution set is the interval $(-\infty, -10]$.

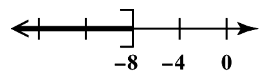



**20.** $4x + 1 \leq -31$

$4x \leq -32$ *Subtract 1.*

$x \leq -8$ *Divide by 4.*

Check that the solution set is the interval $(-\infty, -8]$.




**21.** $$\frac{5x-6}{8} < 8$$

$$8\left(\frac{5x-6}{8}\right) < 8 \cdot 8 \quad \textit{Multiply by 8.}$$

$5x - 6 < 64$

$5x < 70$ *Add 6.*

$x < 14$ *Divide by 5.*

**Check** Let $x = 14$ in the *equation* $\frac{5x-6}{8} = 8$.

$$\frac{5(14)-6}{8} \stackrel{?}{=} 8$$

$$\frac{64}{8} \stackrel{?}{=} 8$$

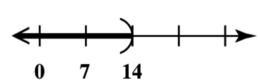
$8 = 8$ *True*

This shows that 14 is the boundary point. Now test a number on each side of 14. We choose 0 and 20.

$$\frac{5x-6}{8} < 8$$

| *Let $x = 0$.* | *Let $x = 20$.* |
|---|---|
| $\frac{5(0)-6}{8} \stackrel{?}{<} 8$ | $\frac{5(20)-6}{8} \stackrel{?}{<} 8$ |
| $-\frac{6}{8} < 8$ *True* | $\frac{94}{8}$ (or $11\frac{6}{8}$) $< 8$ *False* |
| 0 is in the solution set. | 20 is not in the solution set. |

The check confirms that $(-\infty, 14)$ is the solution set.



**22.** $\dfrac{3x-1}{4} > 5$

$4\left(\dfrac{3x-1}{4}\right) > 4(5)$ *Multiply by 4.*

$3x - 1 > 20$

$3x > 21$ *Add 1.*

$x > 7$ *Divide by 3.*

**Check** Let $x = 7$ in the *equation* $\dfrac{3x-1}{4} = 5$.

$\dfrac{3(7)-1}{4} \stackrel{?}{=} 5$

$\frac{20}{4} \stackrel{?}{=} 5$

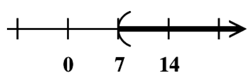
$5 = 5$ *True*

This shows that 7 is the boundary point. Now test a number on each side of 7. We choose 0 and 10.

$\dfrac{3x-1}{4} > 5$

| *Let $x = 0$.* | *Let $x = 10$.* |
|---|---|
| $\dfrac{3(0)-1}{4} \stackrel{?}{>} 5$ | $\dfrac{3(10)-1}{4} \stackrel{?}{>} 5$ |
| $\frac{-1}{4} > 5$ *False* | $\frac{29}{4} > 5$ *True* |
| 0 is not in the solution set. | 10 is in the solution set. |

The check confirms that $(7, \infty)$ is the solution set.




**23.** $\dfrac{2x-5}{-4} > 5$

Multiply both sides by $-4$, and reverse the inequality symbol.

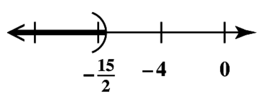
$-4\left(\dfrac{2x-5}{-4}\right) < -4(5)$

$2x - 5 < -20$

$2x < -15$ *Add 5.*

$x < -\frac{15}{2}$ *Divide by 2.*

Check that the solution set is the interval $(-\infty, -\frac{15}{2})$.




**24.** $\dfrac{3x-2}{-5} < 6$

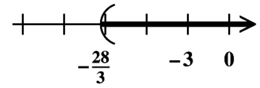
Multiply both sides by $-5$, and reverse the inequality symbol.

$3x - 2 > -30$

$3x > -28$ *Add 2.*

$x > -\frac{28}{3}$ *Divide by 3.*

Check that the solution set is the interval $(-\frac{28}{3}, \infty)$.



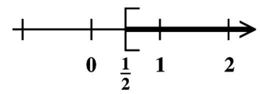

**25.** $6x - 4 \geq -2x$

$8x - 4 \geq 0$ *Add 2x.*

$8x \geq 4$ *Add 4.*

$x \geq \frac{4}{8} = \frac{1}{2}$ *Divide by 8.*

Check that the solution set is the interval $[\frac{1}{2}, \infty)$.



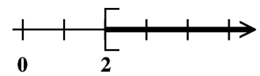

**26.** $2x - 8 \geq -2x$

$4x - 8 \geq 0$ *Add 2x.*

$4x \geq 8$ *Add 8.*

$x \geq \frac{8}{4} = 2$ *Divide by 4.*

Check that the solution set is the interval $[2, \infty)$.




**27.** $x - 2(x - 4) \leq 3x$

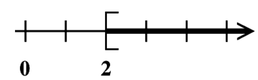
$x - 2x + 8 \leq 3x$

$-x + 8 \leq 3x$

$8 \leq 4x$ *Add x.*

$2 \leq x$, or $x \geq 2$

Check that the solution set is the interval $[2, \infty)$.




**28.** $x - 3(x + 1) \leq 4x$

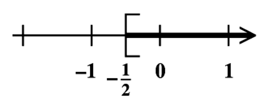
$x - 3x - 3 \leq 4x$

$-2x - 3 \leq 4x$

$-3 \leq 6x$ *Add 2x.*

$-\frac{1}{2} \leq x$, or $x \geq -\frac{1}{2}$

Check that the solution set is the interval $[-\frac{1}{2}, \infty)$.




**29.** $-(4 + r) + 2 - 3r < -14$

$-4 - r + 2 - 3r < -14$ *Distributive property*

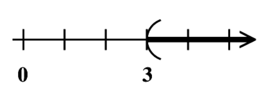
$-4r - 2 < -14$ *Combine terms.*

$-4r < -12$ *Add 2.*

Divide by $-4$, and reverse the inequality symbol.

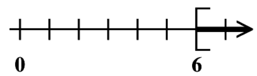
$r > 3$

Check that the solution set is the interval $(3, \infty)$.



**30.**
$$\begin{aligned} -(9+x)-5+4x &\ge 4 \\ -9-x-5+4x &\ge 4 \\ -14+3x &\ge 4 \\ 3x &\ge 18 \\ x &\ge 6 \end{aligned}$$

Check that the solution set is the interval $[6,\infty)$.




**31.** $-3(x-6) > 2x-2$

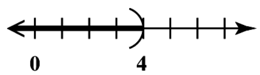
$-3x+18 > 2x-2$ *Distributive property*

$-5x > -20$ *Subtract 2x and 18.*

Divide by $-5$, and reverse the inequality symbol.

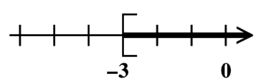
$x < 4$

Check that the solution set is the interval $(-\infty,4)$.




**32.**
$$\begin{aligned} -2(x+4) &\le 6x+16 \\ -2x-8 &\le 6x+16 \\ -8x &\le 24 \\ x &\ge -3 \end{aligned}$$
*Reverse symbol.*

Check that the solution set is the interval $[-3,\infty)$.




**33.** $\frac{2}{3}(3x-1) \ge \frac{3}{2}(2x-3)$

Multiply both sides by 6 to clear the fractions.

$6 \cdot \frac{2}{3}(3x-1) \ge 6 \cdot \frac{3}{2}(2x-3)$

$4(3x-1) \ge 9(2x-3)$

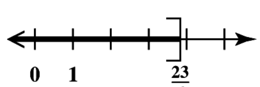
$12x-4 \ge 18x-27$ *Distributive property*

$-6x \ge -23$ *Subtract 18x; add 4.*

Divide by $-6$, and reverse the inequality symbol.

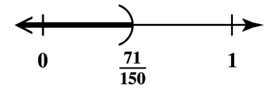
$$x \le \frac{23}{6}$$

Check that the solution set is the interval $(-\infty,\frac{23}{6}]$.




**34.**
$$\begin{aligned} \tfrac{7}{5}(10x-1) &< \tfrac{2}{3}(6x+5) \\ 15 \cdot \tfrac{7}{5}(10x-1) &< 15 \cdot \tfrac{2}{3}(6x+5) \\ 21(10x-1) &< 10(6x+5) \\ 210x-21 &< 60x+50 \\ 150x &< 71 \\ x &< \tfrac{71}{150} \end{aligned}$$

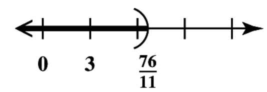
Check that the solution set is the interval $(-\infty,\frac{71}{150})$.




**35.** $-\frac{1}{4}(p+6)+\frac{3}{2}(2p-5) < 10$

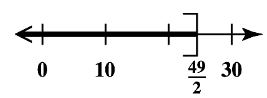
Multiply each term by 4 to clear the fractions.
$$\begin{aligned} -1(p+6)+6(2p-5) &< 40 \\ -p-6+12p-30 &< 40 \\ 11p-36 &< 40 \\ 11p &< 76 \\ p &< \tfrac{76}{11} \end{aligned}$$

Check that the solution set is the interval $(-\infty,\frac{76}{11})$.




**36.**
$$\begin{aligned} \tfrac{3}{5}(t-2)-\tfrac{1}{4}(2t-7) &\le 3 \\ 20 \cdot \tfrac{3}{5}(t-2) - 20 \cdot \tfrac{1}{4}(2t-7) &\le 20 \cdot 3 \\ 12(t-2)-5(2t-7) &\le 60 \\ 12t-24-10t+35 &\le 60 \\ 2t &\le 49 \\ t &\le \tfrac{49}{2} \end{aligned}$$

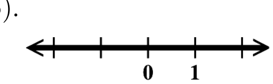
Check that the solution set is the interval $(-\infty,\frac{49}{2}]$.




**37.**
$$\begin{aligned} 3(2x-4)-4x &< 2x+3 \\ 6x-12-4x &< 2x+3 \\ 2x-12 &< 2x+3 \\ -12 &< 3 \end{aligned}$$
*True*

The statement is true for all values of $x$. Therefore, the original inequality is true for any real number.

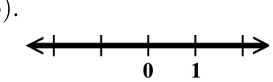
Check that the solution set is the interval $(-\infty,\infty)$.




**38.**
$$\begin{aligned} 7(4-x)+5x &< 2(16-x) \\ 28-7x+5x &< 32-2x \\ 28-2x &< 32-2x \\ 28 &< 32 \end{aligned}$$
*True*

This statement is true for all values of $x$.

Check that the solution set is the interval $(-\infty,\infty)$.



**39.** $8(\frac{1}{2}x+3) < 8(\frac{1}{2}x-1)$
$4x+24 < 4x-8$
$24 < -8$ *False*

This is a false statement, so the inequality is a contradiction.

Check that the solution set is $\emptyset$.

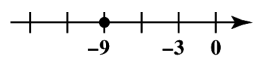
**40.** $10(\frac{1}{5}x+2) < 10(\frac{1}{5}x+1)$
$2x+20 < 2x+10$
$20 < 10$ *False*

This is a false statement, so the inequality is a contradiction.

Check that the solution set is $\emptyset$.

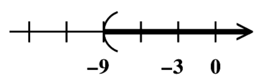
**41.** $5(x+3)-2(x-4) = 2(x+7)$
$5x+15-2x+8 = 2x+14$
$3x+23 = 2x+14$
$x = -9$

Check that the solution set is $\{-9\}$.
The graph is the point $-9$ on a number line.



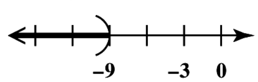

**42.** $5(x+3)-2(x-4) > 2(x+7)$
$5x+15-2x+8 > 2x+14$
$3x+23 > 2x+14$
$x > -9$

Check that the solution set is the interval $(-9,\infty)$. The graph extends from $-9$ to the right on a number line; $-9$ is not included in the graph.

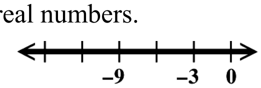



**43.** $5(x+3)-2(x-4) < 2(x+7)$
$5x+15-2x+8 < 2x+14$
$3x+23 < 2x+14$
$x < -9$

Check that the solution set is the interval $(-\infty,-9)$. The graph extends from $-9$ to the left on a number line; $-9$ is not included in the graph.




**44.** If we graph all the solution sets from Exercises 41–43; that is, $\{-9\}$, $(-9,\infty)$, and $(-\infty,-9)$, on the same number line, we will have graphed the set of all real numbers.



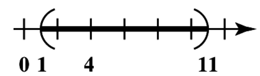

**45.** The solution set of the given equation is the point $-3$ on a number line. The solution set of the first inequality extends from $-3$ to the right (toward $\infty$) on the same number line. Based on Exercises 41–43, the solution set of the second inequality should then extend from $-3$ to the left (toward $-\infty$) on the number line. Complete the statement with $\underline{(-\infty,-3)}$.

**46.** $-2 < x$ is the same as $x > -2$. This inequality represents all real numbers greater than $-2$. Its graph is shown in choice **A**.

**47.** The goal is to isolate the variable $x$.

$$-4 < x-5 < 6$$
$$-4+5 < x-5+5 < 6+5 \quad \textit{Add 5.}$$
$$1 < x < 11$$

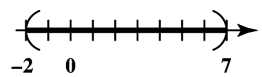
Check that the solution set is the interval $(1,11)$.




**48.** The goal is to isolate the variable $x$.

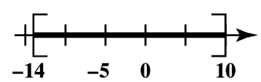
$$-1 < x+1 < 8$$
$$-1-1 < x+1-1 < 8-1 \quad \textit{Subtract 1.}$$
$$-2 < x < 7$$

Check that the solution set is the interval $(-2,7)$.



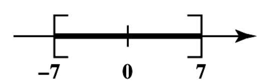

**49.**
$$-9 \le x+5 \le 15$$
$$-9-5 \le x+5-5 \le 15-5 \quad \textit{Subtract 5.}$$
$$-14 \le x \le 10$$

Check that the solution set is the interval $[-14,10]$.



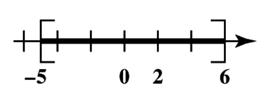

**50.**
$$-4 \le x+3 \le 10$$
$$-4-3 \le x+3-3 \le 10-3 \quad \textit{Subtract 3.}$$
$$-7 \le x \le 7$$

Check that the solution set is the interval $[-7,7]$.



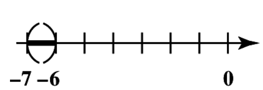

**51.**
$$-6 \le 2x+4 \le 16$$
$$-10 \le 2x \le 12 \quad \textit{Subtract 4.}$$
$$-5 \le x \le 6 \quad \textit{Divide by 2.}$$

Check that the solution set is the interval $[-5,6]$.



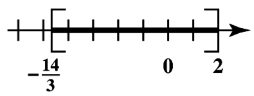

**52.**
$$-15 < 3x+6 < -12$$
$$-21 < 3x < -18 \quad \textit{Subtract 6.}$$
$$-7 < x < -6 \quad \textit{Divide by 3.}$$

Check that the solution set is the interval $(-7,-6)$.



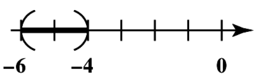
**53.**
$$\begin{aligned} -19 &\le 3x - 5 \le 1 \\ -14 &\le 3x \quad\ \ \le 6 \quad \textit{Add 5.} \\ -\tfrac{14}{3} &\le x \quad\ \ \ \le 2 \quad \textit{Divide by 3.} \end{aligned}$$

Check that the solution set is the interval $[-\frac{14}{3}, 2]$.



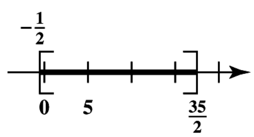

**54.**
$$\begin{aligned} -16 &< 3x + 2 < -10 \\ -18 &< 3x \quad\ \ < -12 \quad \textit{Subtract 2.} \\ -6 &< x \quad\ \ \ < -4 \quad \textit{Divide by 3.} \end{aligned}$$

Check that the solution set is the interval $(-6, -4)$.



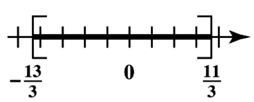

**55.**
$$\begin{aligned} -1 &\le \frac{2x-5}{6} \le 5 \\ -6 &\le 2x - 5 \le 30 \quad \textit{Multiply by 6.} \\ -1 &\le 2x \quad\ \ \le 35 \quad \textit{Add 5.} \\ -\tfrac{1}{2} &\le x \quad\ \ \ \le \tfrac{35}{2} \quad \textit{Divide by 2.} \end{aligned}$$

Check that the solution set is the interval $[-\frac{1}{2}, \frac{35}{2}]$.




**56.**
$$\begin{aligned} -3 &\le \frac{3x+1}{4} \le 3 \\ -12 &\le 3x + 1 \le 12 \quad \textit{Multiply by 4.} \\ -13 &\le 3x \quad\ \ \le 11 \quad \textit{Subtract 1.} \\ -\tfrac{13}{3} &\le x \quad\ \ \ \le \tfrac{11}{3} \quad \textit{Divide by 3.} \end{aligned}$$

Check that the solution set is the interval $[-\frac{13}{3}, \frac{11}{3}]$.

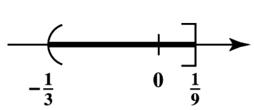



**57.**
$$\begin{aligned} 4 &\le -9x + 5 < 8 \\ -1 &\le -9x \quad\ \ < 3 \quad \textit{Subtract 5.} \\ \frac{1}{9} &\ge x \quad\ \ \ > -\frac{1}{3} \quad \textit{Divide by } -9. \textit{ Reverse inequalities.} \end{aligned}$$

The last inequality may be written as

$$-\tfrac{1}{3} < x \le \tfrac{1}{9}.$$

Check that the solution set is the interval $(-\frac{1}{3}, \frac{1}{9}]$.

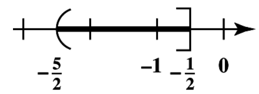



**58.**
$$\begin{aligned} 4 &\le -2x + 3 < 8 \\ 1 &\le -2x \quad\ \ < 5 \quad \textit{Subtract 3.} \\ -\frac{1}{2} &\ge x \quad\ \ \ > -\frac{5}{2} \quad \textit{Divide by } -2. \textit{ Reverse inequalities.} \end{aligned}$$

The last inequality may be written as

$$-\tfrac{5}{2} < x \le -\tfrac{1}{2}.$$

Check that the solution set is the interval $(-\frac{5}{2}, -\frac{1}{2}]$.




**59.** Six times a number is between $-12$ and 12.

$$\begin{aligned} -12 &< 6x < 12 \\ -2 &< \ x \ < 2 \quad \textit{Divide by 6.} \end{aligned}$$

This is the set of all numbers between $-2$ and 2—that is, $(-2, 2)$.

**60.** Half a number is between $-3$ and 2.

$$\begin{aligned} -3 &< \tfrac{1}{2}x < 2 \\ -6 &< \ x \ < 4 \quad \textit{Multiply by 2.} \end{aligned}$$

This is the set of all numbers between $-6$ and 4—that is, $(-6, 4)$.

**61.** When 1 is added to twice a number, the result is greater than or equal to 7.

$$\begin{aligned} 2x + 1 &\ge 7 \\ 2x &\ge 6 \quad \textit{Subtract 1.} \\ x &\ge 3 \quad \textit{Divide by 2.} \end{aligned}$$

This is the set of all numbers greater than or equal to 3—that is, $[3, \infty)$.

**62.** If 8 is subtracted from a number, then the result is at least 5.

$$\begin{aligned} x - 8 &\ge 5 \\ x &\ge 13 \quad \textit{Add 8.} \end{aligned}$$

This is the set of all numbers greater than or equal to 13—that is, $[13, \infty)$.

**63.** One third of a number is added to 6, giving a result of at least 3.

$$\begin{aligned} 6 + \tfrac{1}{3}x &\ge 3 \\ \tfrac{1}{3}x &\ge -3 \quad \textit{Subtract 6.} \\ x &\ge -9 \quad \textit{Multiply by 3.} \end{aligned}$$

This is the set of all numbers greater than or equal to $-9$—that is, $[-9, \infty)$.

**64.** Three times a number, minus 5, is no more than 7.

$$\begin{aligned} 3x-5 &\le 7 \\ 3x &\le 12 && \textit{Add 5.} \\ x &\le 4 && \textit{Divide by 3.} \end{aligned}$$

This is the set of all numbers less than or equal to 4—that is, $(-\infty, 4]$.

**65.** Let $x =$ her score on the third test.
Her average must be at least 84 ( $\ge 84$). To find the average of three numbers, add them and divide by 3.

$$\begin{aligned} \frac{90+82+x}{3} &\ge 84 \\ \frac{172+x}{3} &\ge 84 && \textit{Add.} \\ 172+x &\ge 252 && \textit{Multiply by 3.} \\ x &\ge 80 && \textit{Subtract 172.} \end{aligned}$$

She must score at least 80 on her third test.

**66.** Let $x =$ his score on the third test.
His average must be at least 90.

$$\begin{aligned} \frac{92+96+x}{3} &\ge 90 \\ 92+96+x &\ge 270 && \textit{Multiply by 3.} \\ 188+x &\ge 270 && \textit{Add.} \\ x &\ge 82 && \textit{Subtract 188.} \end{aligned}$$

He must score at least 82 on his third test.

**67.** Let $x =$ the number of months. The cost of Plan A is $54.99x$ and the cost of Plan B is $49.99x + 129$. To determine the number of months that would be needed to make Plan B less expensive, solve the following inequality.

$$\begin{aligned} \text{Plan B (cost)} &< \text{Plan A (cost)} \\ 49.99x+129 &< 54.99x \\ 129 &< 5x && \textit{Subtract 49.99x.} \\ 5x &> 129 && \textit{Equivalent} \\ x &> \tfrac{129}{5} \quad [=25.8] && \textit{Divide by 5.} \end{aligned}$$

It will take 26 months for Plan B to be the better deal.

**68.** Let $x =$ the number of miles driven. The cost of renting from Budget is \$34.95 plus the mileage cost of $\$0.25x$, while the cost of renting from U-Haul is \$29.95 plus the mileage cost of $\$0.28x$.

$$\begin{aligned} \text{Budget (cost)} &< \text{U-Haul (cost)} \\ 34.95+0.25x &< 29.95+0.28x \\ 5+0.25x &< 0.28x \\ 5 &< 0.03x \\ 0.03x &> 5 \\ x &> \tfrac{5}{0.03} \quad \left[=\tfrac{500}{3} \text{ or } 166\tfrac{2}{3}\right] \end{aligned}$$

The Budget rental will be a better deal than the U-Haul rental after 167 miles.

**69.** Cost $C = 20x + 100$; Revenue $R = 24x$
The business will show a profit only when $R > C$. Substitute the given expressions for $R$ and $C$.

$$\begin{aligned} R &> C \\ 24x &> 20x+100 \\ 4x &> 100 \\ x &> 25 \end{aligned}$$

The company will show a profit upon selling 26 DVDs.

**70.** $C = 3x + 2300$, $R = 5.50x$
To make a profit, $R > C$.

$$\begin{aligned} 5.50x &> 3x+2300 \\ 2.5x &> 2300 \\ x &> 920 \end{aligned}$$

The company will show a profit after making 921 deliveries.

**71.** $\text{BMI} = \dfrac{704 \times \text{(weight in pounds)}}{\text{(height in inches)}^2}$

**(a)** Let the height equal 72.

$$\begin{aligned} 19 \le\ &\text{BMI} \le 25 \\ 19 \le\ &\frac{704w}{72^2} \le 25 \\ 19(72^2) \le\ &704w \le 25(72^2) \\ \frac{19(72^2)}{704} \le\ &w \le \frac{25(72^2)}{704} \\ (\approx 139.91) \le\ &w \le (\approx 184.09) \end{aligned}$$

According to the BMI formula, the healthy weight range (rounded to the nearest pound) for a person who is 72 inches tall is 140 to 184 pounds.

**(b)** Let the height equal 63.

$$\begin{aligned} 19 \le\ &\text{BMI} \le 25 \\ 19 \le\ &\frac{704w}{63^2} \le 25 \\ 19(63^2) \le\ &704w \le 25(63^2) \\ \frac{19(63^2)}{704} \le\ &w \le \frac{25(63^2)}{704} \\ (\approx 107.12) \le\ &w \le (\approx 140.94) \end{aligned}$$

According to the BMI formula, the healthy weight range (rounded to the nearest pound) for a person who is 63 inches tall is 107 to 141 pounds.

**(c)** Answers will vary.

**72.** **(a)** Let $A = 35$.

$$\begin{array}{rcccl} 0.7(220-A) & \leq & \text{THR} & \leq & 0.85(220-A) \\ 0.7(220-35) & \leq & \text{THR} & \leq & 0.85(220-35) \\ 0.7(185) & \leq & \text{THR} & \leq & 0.85(185) \\ 129.5 & \leq & \text{THR} & \leq & 157.25 \end{array}$$

The range is about 130 to 157 beats per minute.

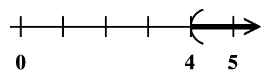
**(b)** Let $A = 55$.

$$\begin{array}{rcccl} 0.7(220-A) & \leq & \text{THR} & \leq & 0.85(220-A) \\ 0.7(220-55) & \leq & \text{THR} & \leq & 0.85(220-55) \\ 0.7(165) & \leq & \text{THR} & \leq & 0.85(165) \\ 115.5 & \leq & \text{THR} & \leq & 140.25 \end{array}$$

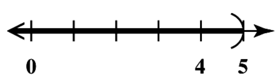
The range is about 116 to 140 beats per minute.

**(c)** Answers will vary.

**73.** **(a)** $x > 4$ is equivalent to $(4, \infty)$.

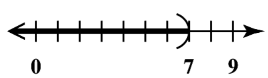



**(b)** $x < 5$ is equivalent to $(-\infty, 5)$.

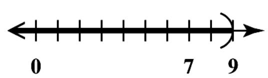



**(c)** All numbers greater than 4 and less than 5 (that is, those satisfying $4 < x < 5$) belong to *both* sets.

**74.** **(a)** $x < 7$ is equivalent to $(-\infty, 7)$.




**(b)** $x < 9$ is equivalent to $(-\infty, 9)$.




**(c)** All numbers less than 9 (that is, those satisfying $x < 9$) belong to *either* set.

## 2.6 Set Operations and Compound Inequalities

### 2.6 Classroom Examples, Now Try Exercises

**1.** Let $A = \{3, 4, 5, 6\}$ and $B = \{5, 6, 7\}$.

The set $A \cap B$, the intersection of $A$ and $B$, contains those elements that belong to both $A$ *and* $B$; that is, the numbers 5 and 6. Therefore,

$$A \cap B = \{5, 6\}.$$

**N1.** Let $A = \{2, 4, 6, 8\}$ and $B = \{0, 2, 6, 8\}$.

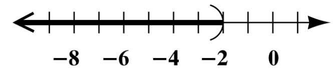
The set $A \cap B$, the intersection of $A$ and $B$, contains those elements that belong to both $A$ *and* $B$; that is, the numbers 2, 6, and 8. Therefore,

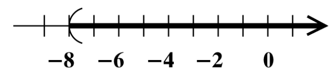
$$A \cap B = \{2, 6, 8\}.$$

**2.** $x + 3 < 1$ and $x - 4 > -12$

Solve each inequality.

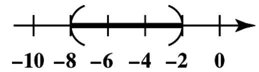
$$\begin{array}{rcl} x+3<1 & \text{and} & x-4>-12 \\ x+3-3<1-3 & \text{and} & x-4+4>-12+4 \\ x<-2 & \text{and} & x>-8 \end{array}$$




The values that satisfy both inequalities are the numbers between $-8$ and $-2$, excluding $-8$ and $-2$.

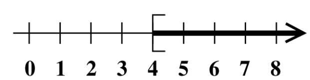
The solution set is $(-8, -2)$.




**N2.** $x - 2 \leq 5$ and $x + 5 \geq 9$

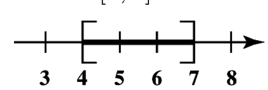
Solve each inequality.

$$\begin{array}{rcl} x-2\leq 5 & \text{and} & x+5\geq 9 \\ x-2+2\leq 5+2 & \text{and} & x+5-5\geq 9-5 \\ x\leq 7 & \text{and} & x\geq 4 \end{array}$$

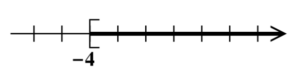



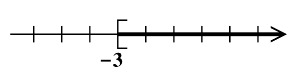

The values that satisfy both inequalities are the numbers between 4 and 7, including 4 and 7.

The solution set is $[4, 7]$.



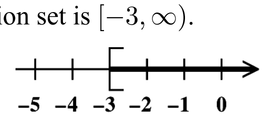

**3.**
$$\begin{array}{rcl} 2x \leq 4x+8 & \text{and} & 3x \geq -9 \\ -2x \leq 8 & & \\ x \geq -4 & \text{and} & x \geq -3 \end{array}$$




The overlap of the two graphs consists of the numbers that are greater than or equal to $-4$ and are also greater than or equal to $-3$; that is, the numbers greater than or equal to $-3$.

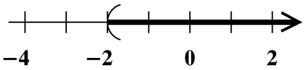
The solution set is $[-3, \infty)$.

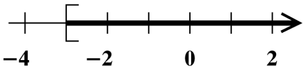


**N3.** $-4x - 1 < 7$ and $3x + 4 \geq -5$

$-4x < 8$ and $3x \geq -9$

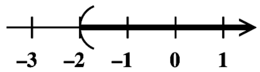
$x > -2$ and $x \geq -3$




The overlap of the two graphs consists of the numbers that are greater than $-2$ and are also greater than or equal to $-3$; that is, the numbers greater than $-2$.

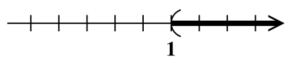
The solution set is $(-2, \infty)$.

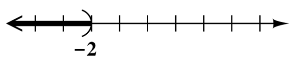



**4.** $x + 2 > 3$ and $2x + 1 < -3$

$2x < -4$

$x > 1$ and $x < -2$



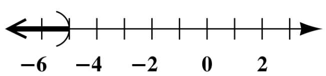



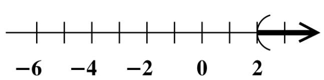

The two graphs do not overlap. Therefore, there is no number that is both greater than 1 *and* less than $-2$, so the given compound inequality has no solution. The solution set is $\emptyset$.

**N4.** $x - 7 < -12$ and $2x + 1 > 5$

$2x > 4$

$x < -5$ and $x > 2$




The two graphs do not overlap. Therefore, there is no number that is both greater than 2 *and* less than $-5$, so the given compound inequality has no solution. The solution set is $\emptyset$.

**5.** Let $A = \{3, 4, 5, 6\}$ and $B = \{5, 6, 7\}$.

The set $A \cup B$, the union of $A$ and $B$, consists of all elements in either $A$ *or* $B$ (or both). Start by listing the elements of set $A$: 3, 4, 5, 6. Then list any additional elements from set $B$. In this case, the elements 5 and 6 are already listed, so the only additional element is 7. Therefore,

$$A \cup B = \{3, 4, 5, 6, 7\}.$$

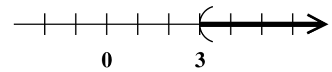
**N5.** Let $A = \{5, 10, 15, 20\}$ and $B = \{5, 15, 25\}$.

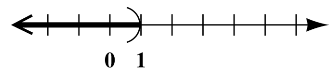
The set $A \cup B$, the union of $A$ and $B$, consists of all elements in either $A$ *or* $B$ (or both). Start by listing the elements of set $A$: 5, 10, 15, 20. Then list any additional elements from set $B$. In this case, the elements 5 and 15 are already listed, so the only additional element is 25. Therefore,

$$A \cup B = \{5, 10, 15, 20, 25\}.$$

**6.** $x - 1 > 2$ or $3x + 5 < 2x + 6$

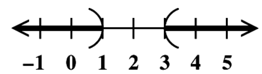
$x > 3$ or $x < 1$



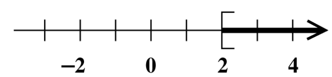

The graph of the solution set consists of all numbers greater than 3 *or* less than 1.

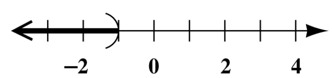
The solution set is $(-\infty, 1) \cup (3, \infty)$.




**N6.** $-12x \leq -24$ or $x + 9 < 8$

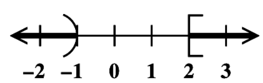
$x \geq 2$ or $x < -1$




The graph of the solution set consists of all numbers greater than or equal to 2 *or* less than $-1$.

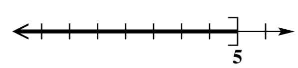
The solution set is $(-\infty, -1) \cup [2, \infty)$.

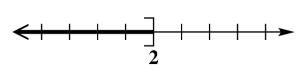



**7.** $3x - 2 \leq 13$ or $x + 5 \leq 7$

$3x \leq 15$

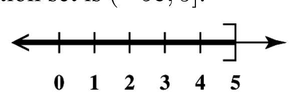
$x \leq 5$ or $x \leq 2$



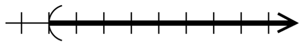



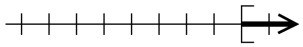

The solution set is all numbers that are either less than or equal to 5 *or* less than or equal to 2. All real numbers less than or equal to 5 are included.

The solution set is $(-\infty, 5]$.



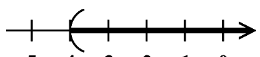
**N7.** $-x + 2 < 6$ or $6x - 8 \geq 10$
$-x < 4$ or $6x \geq 18$
$x > -4$ or $x \geq 3$



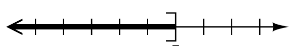



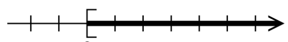

The solution set is all numbers that are either greater than $-4$ *or* greater than or equal to 3. All real numbers greater than $-4$ are included.

The solution set is $(-4, \infty)$.



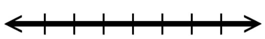

**8.** $3x - 2 \leq 13$ or $x + 5 \geq 7$
$3x \leq 15$
$x \leq 5$ or $x \geq 2$



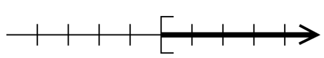



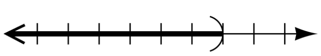

The solution set is all numbers that are either less than or equal to 5 *or* greater than or equal to 2. All real numbers are included.

The solution set is $(-\infty, \infty)$.



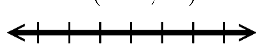

**N8.** $8x - 4 \geq 20$ or $-2x + 1 > -9$
$8x \geq 24$ or $-2x > -10$
$x \geq 3$ or $x < 5$




The solution set is all numbers that are either greater than or equal to 3 *or* less than 5. All real numbers are included.

The solution set is $(-\infty, \infty)$.




**9.** **(a)** All films had admissions greater than 130,000,000, but only *The Ten Commandments* had a gross income of less than \$950,000,000. Thus, the set of elements (films) that satisfy both sets is $\{$*The Ten Commandments*$\}$.

**(b)** Since all the films had admissions greater than 130,000,000 and we have an *or* statement, the second condition doesn't have an effect on the solution set. The required set is the set of all films; that is, $\{$*Gone with the Wind, Star Wars, The Sound of Music, E.T., The Ten Commandments*$\}$.

**N9.** **(a)** All films except *The Ten Commandments* had admissions greater than 140,000,000. *The Sound of Music, E.T.,* and *The Ten Commandments* had a gross income of less than \$1,200,000,000. Thus, the set of elements (films) that satisfy both sets is $\{$*The Sound of Music, E.T.*$\}$.

**(b)** *Star Wars, The Sound of Music, E.T.,* and *The Ten Commandments* had admissions less than 200,000,000. *The Sound of Music, E.T.,* and *The Ten Commandments* had a gross income of less than \$1,200,000,000. Thus, the set of elements (films) that satisfy either set is $\{$*Star Wars, The Sound of Music, E.T., The Ten Commandments*$\}$.

## 2.6 Section Exercises

**1.** This statement is *true*. The solution set of $x + 1 = 6$ is $\{5\}$. The solution set of $x + 1 > 6$ is $(5, \infty)$. The solution set of $x + 1 < 6$ is $(-\infty, 5)$. Taken together we have the set of real numbers. (See Section 2.5, Exercises 41–45, for a discussion of this concept.)

**2.** This statement is *false*. The intersection is $\{9\}$ since 9 is the element common to both sets.

**3.** This statement is *false*. The union is $(-\infty, 7) \cup (7, \infty)$. The only real number that is *not* in the union is 7.

**4.** This statement is *true* since 7 is the only element common to both sets.

**5.** This statement is *false* since 0 is a rational number but not an irrational number. The sets of rational numbers and irrational numbers have no common elements so their intersection is $\emptyset$.

**6.** This statement is *true*. The set of rational numbers together with the set of irrational numbers makes up the set of real numbers.

In Exercises 7–14, let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{1, 3, 5\}$, $C = \{1, 6\}$, and $D = \{4\}$.

**7.** The intersection of sets $B$ and $A$ contains only those elements in both sets $B$ and $A$.

$$B \cap A = \{1, 3, 5\} \text{ or set } B$$

**8.** The intersection of sets $A$ and $B$ contains only those elements in both sets $A$ and $B$.

$$A \cap B = \{1, 3, 5\} \text{ or set } B$$

Note that $A \cap B = B \cap A$.

**9.** The intersection of sets $A$ and $D$ is the set of all elements in both set $A$ and $D$. Therefore,

$$A \cap D = \{4\} \text{ or set } D.$$

**10.** 1 is the only element in both sets $B$ and $C$, so $B \cap C = \{1\}$.

**11.** The intersection of set $B$ and the set of no elements (empty set), $B \cap \emptyset$, is the set of no elements or $\emptyset$.

**12.** No element is common to both sets, so $A \cap \emptyset = \emptyset$, the empty set.

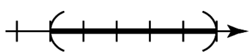
**13.** The union of sets $A$ and $B$ is the set of all elements that are in either set $A$ or set $B$ or both sets $A$ and $B$. Since all numbers in set $B$ are also in set $A$, the set $A \cup B$ will be the same as set $A$.

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \text{ or set } A$$

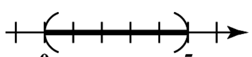
**14.** A union of sets contains all elements that belong to either set.

$$B \cup D = \{1, 3, 4, 5\}$$

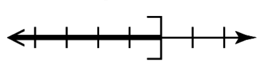
**15.** The first graph represents the set $(-\infty, 2)$. The second graph represents the set $(-3, \infty)$. The intersection includes the elements common to both sets, that is, $(-3, 2)$.



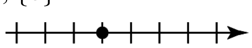

**16.** The first graph represents the set $(-\infty, 5)$. The second graph represents the set $(0, \infty)$. The intersection includes the elements common to both sets, that is, $(0, 5)$.




**17.** The first graph represents the set $(-\infty, 5]$. The second graph represents the set $(-\infty, 2]$. The intersection includes the elements common to both sets, that is, $(-\infty, 2]$.

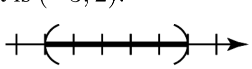



**18.** The first graph represents the set $[3, \infty)$. The second graph represents the set $(-\infty, 3]$. The intersection includes the elements common to both sets, that is, $\{3\}$.



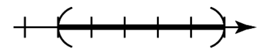

**19.** $x < 2$ and $x > -3$

The graph of the solution set will be all numbers that are both less than 2 and greater than $-3$. The solution set is $(-3, 2)$.



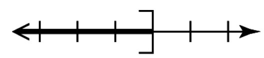

**20.** $x < 5$ and $x > 0$

*And* means intersection. The graph of the solution set will be all numbers that are both less than 5 and greater than 0. The overlap is the numbers between 0 and 5, not including 0 and 5. The solution set is $(0, 5)$.



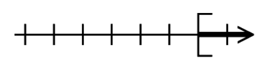

**21.** $x \leq 2$ and $x \leq 5$

The graph of the solution set will be all numbers that are both less than or equal to 2 and less than or equal to 5. The overlap is the numbers less than or equal to 2. The solution set is $(-\infty, 2]$.




**22.** $x \geq 3$ and $x \geq 6$

The graph of the solution set will be all numbers that are both greater than or equal to 3 and greater than or equal to 6. This will be all numbers greater than or equal to 6. The solution set is $[6, \infty)$.




**23.** $x \leq 3$ and $x \geq 6$

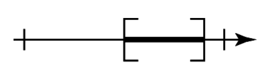
The graph of the solution set will be all numbers that are both less than or equal to 3 and greater than or equal to 6. There are no such numbers. The solution set is $\emptyset$.

**24.** $x \leq -1$ and $x \geq 3$

The graph of the solution set will be all numbers that are both less than or equal to $-1$ and greater than or equal to 3. There are no such numbers. The solution set is $\emptyset$.

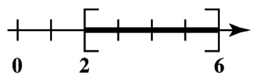
**25.**
$$\begin{aligned} x - 3 \leq 6 \quad &\text{and} \quad x + 2 \geq 7 \\ x \leq 9 \quad &\text{and} \quad x \geq 5 \end{aligned}$$

The graph of the solution set is all numbers that are both less than or equal to 9 and greater than or equal to 5. This is the intersection. The elements common to both sets are the numbers between 5 and 9, including the endpoints. The solution set is $[5, 9]$.



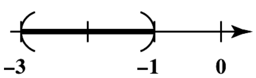

**26.**
$$\begin{aligned} x + 5 \leq 11 \quad &\text{and} \quad x - 3 \geq -1 \\ x \leq 6 \quad &\text{and} \quad x \geq 2 \end{aligned}$$

The graph of the solution set is all numbers that are both less than or equal to 6 and greater than or equal to 2. This is the intersection. The elements common to both sets are the numbers between $2$ and $6$, including the endpoints. The solution set is $[2, 6]$.



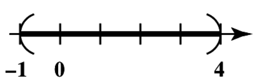
**27.** $-3x > 3$ and $x + 3 > 0$
$x < -1$ and $x > -3$

The graph of the solution set is all numbers that are both less than $-1$ and greater than $-3$. This is the intersection. The elements common to both sets are the numbers between $-3$ and $-1$, not including the endpoints. The solution set is $(-3, -1)$.



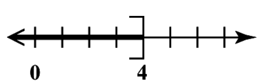
**28.** $-3x < 3$ and $x + 2 < 6$
$x > -1$ and $x < 4$

The graph of the solution set is all numbers that are both less than $4$ and greater than $-1$. This is the intersection. The elements common to both sets are the numbers between $-1$ and $4$, not including the endpoints. The solution set is $(-1, 4)$.



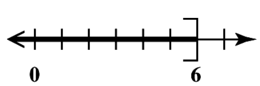
**29.** $3x - 4 \leq 8$ and $-4x + 1 \geq -15$
$3x \leq 12$ and $-4x \geq -16$
$x \leq 4$ and $x \leq 4$

Since both inequalities are identical, the graph of the solution set is the same as the graph of one of the inequalities. The solution set is $(-\infty, 4]$.

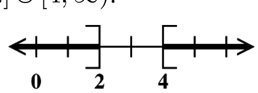


**30.** $7x + 6 \leq 48$ and $-4x \geq -24$
$7x \leq 42$
$x \leq 6$ and $x \leq 6$

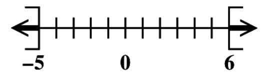
Since both inequalities are identical, the graph of the solution set is the same as the graph of one of the inequalities. The solution set is $(-\infty, 6]$.



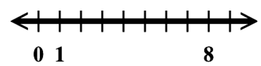
**31.** The first graph represents the set $(-\infty, 2]$. The second graph represents the set $[4, \infty)$. The union includes all elements in either set, or in both, that is, $(-\infty, 2] \cup [4, \infty)$.



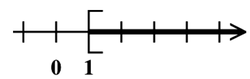
**32.** The first graph represents the set $(-\infty, -5]$. The second graph represents the set $[6, \infty)$. The union includes all elements in either set, or in both, that is, $(-\infty, -5] \cup [6, \infty)$.



**33.** The first graph represents the set $[1, \infty)$. The second graph represents the set $(-\infty, 8]$. The union includes all elements in either set, or in both, that is, $(-\infty, \infty)$.

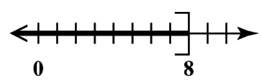


**34.** The first graph represents the set $[1, \infty)$. The second graph represents the set $[8, \infty)$. The union includes all elements in either set, or in both, that is, $[1, \infty)$.



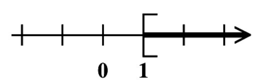
**35.** $x \leq 1$ or $x \leq 8$

The word "or" means to take the union of both sets. The graph of the solution set is all numbers that are either less than or equal to $1$ *or* less than or equal to $8$, or both. This is all numbers less than or equal to $8$. The solution set is $(-\infty, 8]$.



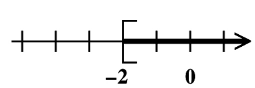
**36.** $x \geq 1$ or $x \geq 8$

The graph of the solution set will be all numbers that are either greater than or equal to 1 or greater than or equal to 8. The solution set is $[1, \infty)$.



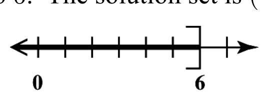
**37.** $x \geq -2$ or $x \geq 5$

The graph of the solution set will be all numbers that are either greater than or equal to $-2$ or greater than or equal to $5$. The solution set is $[-2, \infty)$.



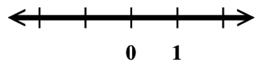
**38.** $x \leq -2$ or $x \leq 6$

The graph of the solution set will be all numbers that are either less than or equal to $-2$ or less than or equal to $6$. The solution set is $(-\infty, 6]$.



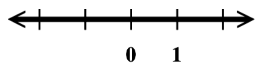
**39.** $x \geq -2$ or $x \leq 4$

The graph of the solution set will be all numbers that are either greater than or equal to $-2$ or less than or equal to $4$. This is the set of all real numbers. The solution set is $(-\infty, \infty)$.




**40.** $x \geq 5$ or $x \leq 7$

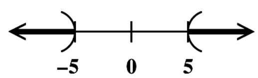
The graph of the solution set will be all numbers that are either greater than or equal to $5$ or less than or equal to $7$. The solution set is $(-\infty, \infty)$.




**41.**

$$\begin{aligned} x + 2 > 7 \quad &\text{or} \quad 1 - x > 6 \\ &\qquad\quad -x > 5 \\ x > 5 \quad &\text{or} \quad x < -5 \end{aligned}$$

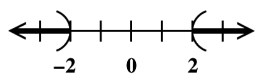
The graph of the solution set is all numbers either greater than $5$ or less than $-5$. This is the union. The solution set is $(-\infty, -5) \cup (5, \infty)$.




**42.**

$$\begin{aligned} x + 1 > 3 \quad &\text{or} \quad x + 4 < 2 \\ x > 2 \quad &\text{or} \quad x < -2 \end{aligned}$$

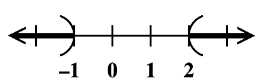
The graph of the solution set is all numbers either greater than $2$ or less than $-2$. This is the union. The solution set is $(-\infty, -2) \cup (2, \infty)$.




**43.**

$$\begin{aligned} x + 1 > 3 \quad &\text{or} \quad -4x + 1 > 5 \\ &\qquad\quad -4x > 4 \\ x > 2 \quad &\text{or} \quad x < -1 \end{aligned}$$

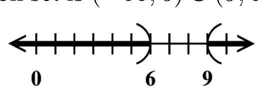
The graph of the solution set is all numbers either less than $-1$ or greater than $2$. This is the union. The solution set is $(-\infty, -1) \cup (2, \infty)$.




**44.**

$$\begin{aligned} 3x < x + 12 \quad &\text{or} \quad x + 1 > 10 \\ 2x < 12 \quad & \\ x < 6 \quad &\text{or} \quad x > 9 \end{aligned}$$

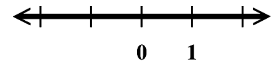
The graph of the solution set is all numbers either less than $6$ or greater than $9$. This is the union. The solution set is $(-\infty, 6) \cup (9, \infty)$.




**45.**

$$\begin{aligned} 4x + 1 \geq -7 \quad &\text{or} \quad -2x + 3 \geq 5 \\ 4x \geq -8 \quad &\text{or} \quad -2x \geq 2 \\ x \geq -2 \quad &\text{or} \quad x \leq -1 \end{aligned}$$

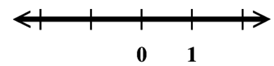
The graph of the solution set is all numbers either greater than or equal to $-2$ or less than or equal to $-1$. This is the set of all real numbers. The solution set is $(-\infty, \infty)$.




**46.**

$$\begin{aligned} 3x + 2 \leq -7 \quad &\text{or} \quad -2x + 1 \leq 9 \\ 3x \leq -9 \quad &\text{or} \quad -2x \leq 8 \\ x \leq -3 \quad &\text{or} \quad x \geq -4 \end{aligned}$$

The graph of the solution set is all numbers either less than or equal to $-3$ or greater than or equal to $-4$. This is the set of all real numbers. The solution set is $(-\infty, \infty)$.




**47.** $(-\infty, -1] \cap [-4, \infty)$

The intersection is the set of numbers less than or equal to $-1$ and greater than or equal to $-4$. The numbers common to both original sets are between, and including, $-4$ and $-1$. The simplest interval form is $[-4, -1]$.

**48.** $[-1, \infty) \cap (-\infty, 9] = (-\infty, 9] \cap [-1, \infty)$

The intersection is the set of numbers less than or equal to $9$ and greater than or equal to $-1$. The numbers common to both original sets are between, and including, $-1$ and $9$. The simplest interval form is $[-1, 9]$.

**49.** $(-\infty, -6] \cap [-9, \infty)$

The intersection is the set of numbers less than or equal to $-6$ and greater than or equal to $-9$. The numbers common to both original sets are between, and including, $-9$ and $-6$. The simplest interval form is $[-9, -6]$.

**50.** $(5, 11] \cap [6, \infty)$

The intersection is the set of numbers between $5$ and $11$, including $11$ but not $5$, and greater than or equal to $6$. The numbers common to both original sets are between, and including, $6$ and $11$. The simplest interval form is $[6, 11]$.

**51.** $(-\infty, 3) \cup (-\infty, -2)$

The union is the set of numbers that are either less than $3$ or less than $-2$, or both. This is all numbers less than $3$. The simplest interval form is $(-\infty, 3)$.

**52.** $[-9, 1] \cup (-\infty, -3)$

The union is the set of numbers between $-9$ and $1$, including both, or less than $-3$. This is all numbers less than, and including $1$. The simplest interval form is $(-\infty, 1]$.

**53.** $[3, 6] \cup (4, 9)$

The union is the set of numbers between, and including, 3 and 6, or between, but not including, 4 and 9. This is the set of numbers greater than or equal to 3 and less than 9. The simplest interval form is $[3, 9)$.

**54.** $[-1, 2] \cup (0, 5)$

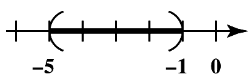
The union is the set of numbers between, and including, $-1$ and 2, or between, but not including, 0 and 5. This is the set of numbers greater than or equal to $-1$ and less than 5. The simplest interval form is $[-1, 5)$.

**55.** $x < -1$ and $x > -5$

The word "and" means to take the intersection of both sets. $x < -1$ and $x > -5$ is true only when

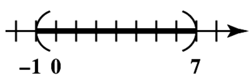
$$-5 < x < -1.$$

The graph of the solution set is all numbers greater than $-5$ *and* less than $-1$. This is all numbers between $-5$ and $-1$, not including $-5$ or $-1$. The solution set is $(-5, -1)$.



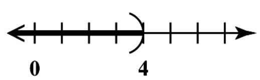

**56.** $x > -1$ and $x < 7$

This is an intersection. The solution set is $(-1, 7)$.



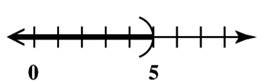

**57.** $x < 4$ or $x < -2$

The word "or" means to take the union of both sets. The graph of the solution set is all numbers that are either less than 4 *or* less than $-2$, or both. This is all numbers less than 4. The solution set is $(-\infty, 4)$.



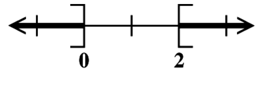

**58.** $x < 5$ or $x < -3$

This is a union. The solution set is $(-\infty, 5)$.



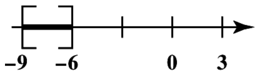

**59.**
$$\begin{aligned} -3x &\le -6 \quad \text{or} \quad -3x \ge 0 \\ x &\ge 2 \quad\;\; \text{or} \quad\;\; x \le 0 \end{aligned}$$

The word "or" means to take the union of both sets. The graph of the solution set is all numbers that are either greater than or equal to 2 *or* less than or equal to 0. The solution set is $(-\infty, 0] \cup [2, \infty)$.



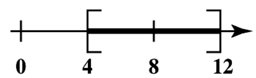

**60.**
$$\begin{aligned} 2x - 6 &\le -18 \quad \text{and} \quad 2x \ge -18 \\ 2x &\le -12 \\ x &\le -6 \quad\;\; \text{and} \quad\;\; x \ge -9 \end{aligned}$$

This is an intersection.
The solution set is $[-9, -6]$.



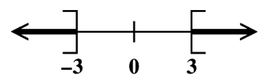

**61.**
$$\begin{aligned} x + 1 &\ge 5 \quad \text{and} \quad x - 2 \le 10 \\ x &\ge 4 \quad \text{and} \quad\quad\; x \le 12 \end{aligned}$$

The word "and" means to take the intersection of both sets. The graph of the solution set is all numbers that are both greater than or equal to 4 *and* less than or equal to 12. This is all numbers between, and including, 4 and 12. The solution set is $[4, 12]$.




**62.**
$$\begin{aligned} -8x &\le -24 \quad \text{or} \quad -5x \ge 15 \\ x &\ge 3 \quad\quad\; \text{or} \quad\quad\; x \le -3 \end{aligned}$$

This is a union.
The solution set is $(-\infty, -3] \cup [3, \infty)$.




**63.** The set of expenses that are less than \$6500 for public schools *and* are greater than \$10,000 for private schools is {Tuition and fees}.

**64.** The set of expenses that are greater than \$3000 for public schools *and* are less than \$4000 for private schools is {Board rates}.

**65.** The set of expenses that are less than \$6500 for public schools *or* are greater than \$10,000 for private schools is {Tuition and fees, Board rates, Dormitory charges}.

**66.** The set of expenses that are greater than \$12,000 *or* are between \$5000 and \$6000 is {Tuition and fees}.

**67.** Find "the yard can be fenced *and* the yard can be sodded."

A yard that can be fenced has $P \le 150$. Maria and Joe qualify.

A yard that can be sodded has $A \le 1400$. Again, Maria and Joe qualify.

Find the intersection. Maria's and Joe's yards are common to both sets, so Maria and Joe can have their yards both fenced and sodded.

**68.** Find "the yard can be fenced *and* the yard cannot be sodded."

A yard that can be fenced has $P \leq 150$. Maria and Joe qualify.

A yard that cannot be sodded has $A > 1400$. Luigi and Than qualify.

Find the intersection. There are no yards common to both sets, so none of them qualify.

**69.** Find "the yard cannot be fenced *and* the yard can be sodded."

A yard that cannot be fenced has $P > 150$. Luigi and Than qualify.

A yard that can be sodded has $A \leq 1400$. Maria and Joe qualify.

Find the intersection. There are no yards common to both sets, so none of them qualify.

**70.** Find "the yard cannot be fenced *and* the yard cannot be sodded."

A yard that cannot be fenced has $P > 150$. Luigi and Than qualify.

A yard that cannot be sodded has $A > 1400$. Again, Luigi and Than qualify.

Find the intersection. Luigi's and Than's yards are common to both sets, so Luigi and Than qualify.

**71.** Find "the yard can be fenced *or* the yard can be sodded." From Exercise 67, Maria's and Joe's yards qualify for both conditions, so the union is Maria and Joe.

**72.** Find "the yard cannot be fenced *or* the yard can be sodded." From Exercise 69, Luigi's and Than's yards cannot be fenced, and Maria's and Joe's yards can be sodded. The union includes all of them.

**73.**
$$\begin{aligned} 2x - 4 &\leq 3x + 2 \\ -x - 4 &\leq 2 \\ -x &\leq 6 \\ x &\geq -6 \end{aligned}$$

The solution set is $[-6, \infty)$.

**74.**
$$\begin{aligned} 5x - 8 &< 6x - 7 \\ -x - 8 &< -7 \\ -x &< 1 \\ x &> -1 \end{aligned}$$

The solution set is $(-1, \infty)$.

**75.**
$$\begin{aligned} -5 < 2x &+ 1 < 5 \\ -6 < 2x &< 4 \\ -3 < x &< 2 \end{aligned}$$

The solution set is $(-3, 2)$.

**76.**
$$\begin{aligned} -7 \leq 3x &- 2 < 7 \\ -5 \leq 3x &< 9 \\ -\tfrac{5}{3} \leq x &< 3 \end{aligned}$$

The solution set is $[-\frac{5}{3}, 3)$.

**77.**
$$\begin{aligned} -|6| - |-11| + (-4) &= -6 - (11) + (-4) \\ &= -17 + (-4) \\ &= -21 \end{aligned}$$

**78.**
$$\begin{aligned} (-5) - |-9| + |5 - 4| &= -5 - (9) + 1 \\ &= -14 + 1 \\ &= -13 \end{aligned}$$

**79.** The absolute value of 0 is 0, which is not a positive number, so the statement is *false*.

**80.** The absolute value of a negative number is positive, namely, its opposite; so $|a| = -a$ is a true statement for $a < 0$.

## 2.7 Absolute Value Equations and Inequalities

### 2.7 Classroom Examples, Now Try Exercises

**1.** $|3x - 4| = 11$

$$\begin{aligned} 3x - 4 &= 11 &\text{or}\quad 3x - 4 &= -11 \\ 3x &= 15 &\text{or}\quad 3x &= -7 \\ x &= 5 &\text{or}\quad x &= -\tfrac{7}{3} \end{aligned}$$

**Check** $x = 5$: $|11| = 11$ *True*

**Check** $x = -\frac{7}{3}$: $|-11| = 11$ *True*

The solution set is $\left\{-\frac{7}{3}, 5\right\}$.

**N1.** $|4x - 1| = 11$

$$\begin{aligned} 4x - 1 &= 11 &\text{or}\quad 4x - 1 &= -11 \\ 4x &= 12 &\text{or}\quad 4x &= -10 \\ x &= 3 &\text{or}\quad x &= -\tfrac{5}{2} \end{aligned}$$

**Check** $x = 3$: $|11| = 11$ *True*

**Check** $x = -\frac{5}{2}$: $|-11| = 11$ *True*

The solution set is $\left\{-\frac{5}{2}, 3\right\}$.

**2.** $|3x - 4| \geq 11$

$$\begin{aligned} 3x - 4 &\geq 11 &\text{or}\quad 3x - 4 &\leq -11 \\ 3x &\geq 15 &\text{or}\quad 3x &\leq -7 \\ x &\geq 5 &\text{or}\quad x &\leq -\tfrac{7}{3} \end{aligned}$$

Check $x = -3$, 0, and 6 in $|3x - 4| \geq 11$.

**Check** $x = -3$: $|-13| \geq 11$ *True*
**Check** $x = 0$: $|-4| \geq 11$ *False*
**Check** $x = 6$: $|14| \geq 11$ *True*

The solution set is $(-\infty, -\frac{7}{3}] \cup [5, \infty)$.

**N2.** $|4x-1|>11$

$$4x-1>11 \quad \text{or} \quad 4x-1<-11$$
$$4x>12 \quad \text{or} \quad 4x<-10$$
$$x>3 \quad \text{or} \quad x<-\tfrac{5}{2}$$

Check $x=-3$, 0, and 6 in $|4x-1|>11$.

**Check** $x=-3$: $|-13|>11$ *True*
**Check** $x=0$: $|-1|>11$ *False*
**Check** $x=6$: $|23|>11$ *True*

The solution set is $(-\infty,-\frac{5}{2})\cup(3,\infty)$.

**3.** $|3x-4|\le 11$

$$-11\le 3x-4\le 11$$
$$-7\le 3x \le 15$$
$$-\tfrac{7}{3}\le x \le 5$$

Check $x=-3$, 0, and 6 in $|3x-4|\le 11$.

**Check** $x=-3$: $|-13|\le 11$ *False*
**Check** $x=0$: $|-4|\le 11$ *True*
**Check** $x=6$: $|14|\le 11$ *False*

The solution set is $[-\frac{7}{3},5]$.

**N3.** $|4x-1|<11$

$$-11<4x-1<11$$
$$-10<4x<12$$
$$-\tfrac{5}{2}<x<3$$

Check $x=-5$, 0, and 5 in $|4x-1|<11$.

**Check** $x=-5$: $|-21|<11$ *False*
**Check** $x=0$: $|-1|<11$ *True*
**Check** $x=5$: $|19|<11$ *False*

The solution set is $(-\frac{5}{2},3)$.

**4.** $|3x+2|+4=15$

We first *isolate* the absolute value expression, that is, rewrite the equation so that the absolute value expression is alone on one side of the equals symbol.

$$|3x+2|=11$$
$$3x+2=11 \quad \text{or} \quad 3x+2=-11$$
$$3x=9 \quad \text{or} \quad 3x=-13$$
$$x=3 \quad \text{or} \quad x=-\tfrac{13}{3}$$

**Check** $x=3$: $|11|+4=15$ *True*
**Check** $x=-\frac{13}{3}$: $|-11|+4=15$ *True*

The solution set is $\{-\frac{13}{3},3\}$.

**N4.** $|10x-2|-2=12$

We first *isolate* the absolute value expression, that is, rewrite the equation so that the absolute value expression is alone on one side of the equals symbol.

$$|10x-2|=14$$
$$10x-2=14 \quad \text{or} \quad 10x-2=-14$$
$$10x=16 \quad \text{or} \quad 10x=-12$$
$$x=\tfrac{8}{5} \quad \text{or} \quad x=-\tfrac{6}{5}$$

**Check** $x=\frac{8}{5}$: $|14|-2=12$ *True*
**Check** $x=-\frac{6}{5}$: $|-14|-2=12$ *True*

The solution set is $\{-\frac{6}{5},\frac{8}{5}\}$.

**5.** **(a)** $|x+2|-3>2$

$$|x+2|>5 \quad \textit{Isolate.}$$
$$x+2>5 \quad \text{or} \quad x+2<-5$$
$$x>3 \quad \text{or} \quad x<-7$$

The solution set is $(-\infty,-7)\cup(3,\infty)$.

**(b)** $|x+2|-3<2$

$$|x+2|<5 \quad \textit{Isolate.}$$
$$-5<x+2<5$$
$$-7<x<3$$

The solution set is $(-7,3)$.

**N5.** **(a)** $|x-1|-4\le 2$

$$|x-1|\le 6 \quad \textit{Isolate.}$$
$$-6\le x-1\le 6$$
$$-5\le x\le 7$$

The solution set is $[-5,7]$.

**(b)** $|x-1|-4\ge 2$

$$|x-1|\ge 6 \quad \textit{Isolate.}$$
$$x-1\ge 6 \quad \text{or} \quad x-1\le -6$$
$$x\ge 7 \quad \text{or} \quad x\le -5$$

The solution set is $(-\infty,-5]\cup[7,\infty)$.

**6.** $|4x-1|=|3x+5|$

$$4x-1=3x+5 \quad \text{or} \quad 4x-1=-(3x+5)$$
$$x=6 \quad \text{or} \quad 4x-1=-3x-5$$
$$7x=-4$$
$$x=-\tfrac{4}{7}$$

**Check** $x=6$: $|23|=|23|$ *True*
**Check** $x=-\frac{4}{7}$: $\left|-\frac{23}{7}\right|=\left|\frac{23}{7}\right|$ *True*

The solution set is $\{-\frac{4}{7},6\}$.

**N6.** $|3x-4|=|5x+12|$

$$3x-4=5x+12 \quad \text{or} \quad 3x-4=-(5x+12)$$
$$-2x=16 \quad \text{or} \quad 3x-4=-5x-12$$
$$x=-8 \qquad 8x=-8$$
$$x=-1$$

**Check** $x=-8$: $|-28|=|-28|$ *True*
**Check** $x=-1$: $|-7|=|7|$ *True*

The solution set is $\{-8,-1\}$.

**7.** **(a)** $|6x+7| = -5$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

**(b)** $\left|\frac{1}{4}x - 3\right| = 0$

The expression $\frac{1}{4}x - 3$ will equal 0 *only* if

$\frac{1}{4}x - 3 = 0.$

$x - 12 = 0$ *Multiply by 4.*

$x = 12$

The solution set is $\{12\}$.

**N7.** **(a)** $|3x-8| = -2$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

**(b)** $|7x+12| = 0$

The expression $7x + 12$ will equal 0 *only* if

$7x + 12 = 0.$

$7x = -12$ *Subtract 12.*

$x = -\frac{12}{7}$ *Divide by 7.*

The solution set is $\left\{-\frac{12}{7}\right\}$.

**8.** **(a)** $|x| > -1$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.

The solution set is $(-\infty, \infty)$.

**(b)** $|x-10| - 2 \leq -3$

$|x-10| \leq -1$

There is no number whose absolute value is less than a negative number, so this inequality has no solution.

The solution set is $\emptyset$.

**(c)** $|x+2| \leq 0$

The value of $|x+2|$ will never be less than 0. However, $|x+2|$ will equal 0 when $x = -2$.

The solution set is $\{-2\}$.

**N8.** **(a)** $|x| > -10$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.

The solution set is $(-\infty, \infty)$.

**(b)** $|4x+1| + 5 < 4$

$|4x+1| < -1$

There is no number whose absolute value is less than a negative number, so this inequality has no solution.

The solution set is $\emptyset$.

**(c)** $|x-2| - 3 \leq -3$

$|x-2| \leq 0$ *Isolate abs. value.*

The value of $|x-2|$ will never be less than 0. However, $|x-2|$ will equal 0 when $x = 2$.

The solution set is $\{2\}$.

## 2.7 Section Exercises

**1.** $|x| = 5$ has two solutions, $x = 5$ or $x = -5$. The graph is Choice **E**.

$|x| < 5$ is written $-5 < x < 5$. Notice that $-5$ and 5 are not included. The graph is Choice **C**, which uses parentheses.

$|x| > 5$ is written $x < -5$ or $x > 5$. The graph is Choice **D**, which uses parentheses.

$|x| \leq 5$ is written $-5 \leq x \leq 5$. This time $-5$ and 5 are included. The graph is Choice **B**, which uses brackets.

$|x| \geq 5$ is written $x \leq -5$ or $x \geq 5$. The graph is Choice **A**, which uses brackets.

**2.** $|x| = 9$ has two solutions, $x = 9$ and $x = -9$. The graph is Choice **E**.

$|x| > 9$ is written $x < -9$ or $x > 9$. Notice that $-9$ and 9 are not included. The graph is Choice **D**, which uses parentheses.

$|x| \geq 9$ is written $x \leq -9$ or $x \geq 9$. This time $-9$ and 9 are included. The graph is Choice **A**, which uses brackets.

$|x| < 9$ is written $-9 < x < 9$. The graph is Choice **C**, which uses parentheses.

$|x| \leq 9$ is written $-9 \leq x \leq 9$. The graph is Choice **B**, which uses brackets.

**3.** **(a)** $|ax+b| = k$, $k = 0$

This means the distance from $ax + b$ to 0 is 0, so $ax + b = 0$, which has one solution.

**(b)** $|ax+b| = k$, $k > 0$

This means the distance from $ax + b$ to 0 is a positive number, so $ax + b = k$ or $ax + b = -k$. There are two solutions.

**(c)** $|ax+b| = k$, $k < 0$

This means the distance from $ax + b$ to 0 is a negative number, which is impossible because distance is always positive. There are no solutions.

**4.** When solving an absolute value equation or inequality of the form

$|ax+b|=k$, $|ax+b|<k$, or $|ax+b|>k$,

where $k$ is a positive number, use *or* for the equality statement and the $>$ statement. Use *and* for the $<$ statement.

**5.** $|x|=12$
$x=12$ or $x=-12$
The solution set is $\{-12, 12\}$.

**6.** $|x|=14$
$x=14$ or $x=-14$
The solution set is $\{-14, 14\}$.

**7.** $|4x|=20$
$4x=20$ or $4x=-20$
$x=5$ or $x=-5$
The solution set is $\{-5, 5\}$.

**8.** $|5x|=30$
$5x=30$ or $5x=-30$
$x=6$ or $x=-6$
The solution set is $\{-6, 6\}$.

**9.** $|x-3|=9$
$x-3=9$ or $x-3=-9$
$x=12$ or $x=-6$
The solution set is $\{-6, 12\}$.

**10.** $|x-5|=13$
$x-5=13$ or $x-5=-13$
$x=18$ or $x=-8$
The solution set is $\{-8, 18\}$.

**11.** $|2x-1|=11$
$2x-1=11$ or $2x-1=-11$
$2x=12$ $\quad 2x=-10$
$x=6$ or $x=-5$
The solution set is $\{-5, 6\}$.

**12.** $|2x+3|=19$
$2x+3=19$ or $2x+3=-19$
$2x=16$ $\quad 2x=-22$
$x=8$ or $x=-11$
The solution set is $\{-11, 8\}$.

**13.** $|4x-5|=17$
$4x-5=17$ or $4x-5=-17$
$4x=22$ $\quad 4x=-12$
$x=\frac{22}{4}=\frac{11}{2}$ or $x=-3$
The solution set is $\left\{-3, \frac{11}{2}\right\}$.

**14.** $|5x-1|=21$
$5x-1=21$ or $5x-1=-21$
$5x=22$ $\quad 5x=-20$
$x=\frac{22}{5}$ or $x=-4$
The solution set is $\left\{-4, \frac{22}{5}\right\}$.

**15.** $|2x+5|=14$
$2x+5=14$ or $2x+5=-14$
$2x=9$ $\quad 2x=-19$
$x=\frac{9}{2}$ or $x=-\frac{19}{2}$
The solution set is $\left\{-\frac{19}{2}, \frac{9}{2}\right\}$.

**16.** $|2x-9|=18$
$2x-9=18$ or $2x-9=-18$
$2x=27$ $\quad 2x=-9$
$x=\frac{27}{2}$ or $x=-\frac{9}{2}$
The solution set is $\left\{-\frac{9}{2}, \frac{27}{2}\right\}$.

**17.** $\left|\frac{1}{2}x+3\right|=2$
$\frac{1}{2}x+3=2$ or $\frac{1}{2}x+3=-2$
$\frac{1}{2}x=-1$ $\quad \frac{1}{2}x=-5$
$x=-2$ or $x=-10$
The solution set is $\{-10, -2\}$.

**18.** $\left|\frac{2}{3}x-1\right|=5$
$\frac{2}{3}x-1=5$ or $\frac{2}{3}x-1=-5$
$\frac{2}{3}x=6$ $\quad \frac{2}{3}x=-4$
$x=\frac{3}{2}(6)$ $\quad x=\frac{3}{2}(-4)$
$x=9$ or $x=-6$
The solution set is $\{-6, 9\}$.

**19.** $\left|1+\frac{3}{4}x\right|=7$
$1+\frac{3}{4}x=7$ or $1+\frac{3}{4}x=-7$
Multiply each side by 4.
$4+3x=28$ or $4+3x=-28$
$3x=24$ $\quad 3x=-32$
$x=8$ or $x=\frac{-32}{3}$
The solution set is $\left\{-\frac{32}{3}, 8\right\}$.

**20.** $\left|2-\frac{5}{2}x\right|=14$
$2-\frac{5}{2}x=14$ or $2-\frac{5}{2}x=-14$
$-\frac{5}{2}x=12$ $\quad -\frac{5}{2}x=-16$
$x=(-\frac{2}{5})(12)$ $\quad x=(-\frac{2}{5})(-16)$
$x=-\frac{24}{5}$ or $x=\frac{32}{5}$
The solution set is $\left\{-\frac{24}{5}, \frac{32}{5}\right\}$.

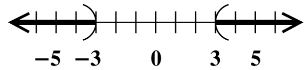
**21.** $|0.02x-1|=2.50$
$0.02x-1=2.50$ or $0.02x-1=-2.50$
$0.02x=3.50$ $\quad 0.02x=-1.50$
$x=50(3.5)$ $\quad x=50(-1.5)$
$x=175$ or $x=-75$
The solution set is $\{-75, 175\}$.

**22.** $|0.04x-3|=5.96$
$0.04x-3=5.96$ or $0.04x-3=-5.96$
$0.04x=8.96$ $\quad 0.04x=-2.96$

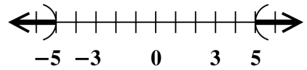
$$x = 25(8.96) \qquad x = 25(-2.96)$$
$$x = 224 \quad \text{or} \quad x = -74$$

The solution set is $\{-74, 224\}$.

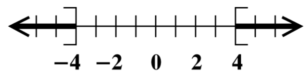
**23.** $|x| > 3 \quad \Leftrightarrow \quad x > 3$ or $x < -3$
The solution set is $(-\infty, -3) \cup (3, \infty)$.



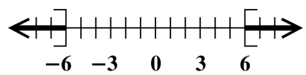

**24.** $|x| > 5 \quad \Leftrightarrow \quad x > 5$ or $x < -5$
The solution set is $(-\infty, -5) \cup (5, \infty)$.




**25.** $|x| \geq 4 \quad \Leftrightarrow \quad x \geq 4$ or $x \leq -4$
The solution set is $(-\infty, -4] \cup [4, \infty)$.



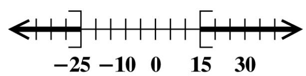

**26.** $|x| \geq 6 \quad \Leftrightarrow \quad x \geq 6$ or $x \leq -6$
The solution set is $(-\infty, -6] \cup [6, \infty)$.




**27.** $|r + 5| \geq 20$

$$r + 5 \leq -20 \quad \text{or} \quad r + 5 \geq 20$$
$$r \leq -25 \quad \text{or} \quad r \geq 15$$

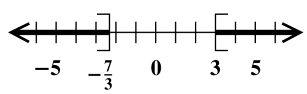
The solution set is $(-\infty, -25] \cup [15, \infty)$.




**28.** $|3r - 1| \geq 8$

$$3r - 1 \geq 8 \quad \text{or} \quad 3r - 1 \leq -8$$
$$3r \geq 9 \qquad 3r \leq -7$$
$$r \geq 3 \quad \text{or} \quad r \leq -\tfrac{7}{3}$$

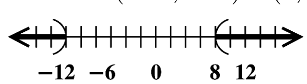
The solution set is $(-\infty, -\frac{7}{3}] \cup [3, \infty)$.




**29.** $|x + 2| > 10$

$$x + 2 > 10 \quad \text{or} \quad x + 2 < -10$$
$$x > 8 \quad \text{or} \quad x < -12$$

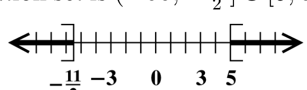
The solution set is $(-\infty, -12) \cup (8, \infty)$.




**30.** $|4x + 1| \geq 21$

$$4x + 1 \geq 21 \quad \text{or} \quad 4x + 1 \leq -21$$
$$4x \geq 20 \qquad 4x \leq -22$$
$$x \geq 5 \quad \text{or} \quad x \leq -\tfrac{11}{2}$$

The solution set is $(-\infty, -\frac{11}{2}] \cup [5, \infty)$.



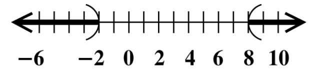

**31.** $|3 - x| > 5$

$$3 - x > 5 \quad \text{or} \quad 3 - x < -5$$
$$-x > 2 \quad \text{or} \quad -x < -8$$

Multiply by $-1$, and reverse the inequality symbols.

$$x < -2 \quad \text{or} \quad x > 8$$

The solution set is $(-\infty, -2) \cup (8, \infty)$.



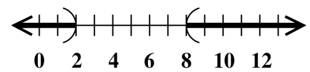

**32.** $|5 - x| > 3$

$$5 - x > 3 \quad \text{or} \quad 5 - x < -3$$
$$-x > -2 \qquad -x < -8$$

Multiply by $-1$, and reverse the inequality symbols.

$$x < 2 \quad \text{or} \quad x > 8$$

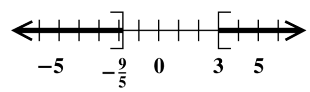
The solution set is $(-\infty, 2) \cup (8, \infty)$.




**33.** $|-5x + 3| \geq 12$

$$-5x + 3 \geq 12 \quad \text{or} \quad -5x + 3 \leq -12$$
$$-5x \geq 9 \qquad -5x \leq -15$$
$$x \leq -\tfrac{9}{5} \quad \text{or} \quad x \geq 3$$

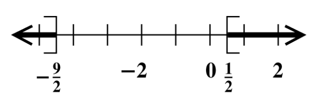
The solution set is $(-\infty, -\frac{9}{5}] \cup [3, \infty)$.




**34.** $|-2x - 4| \geq 5$

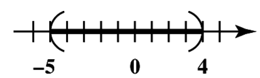
$$-2x - 4 \geq 5 \quad \text{or} \quad -2x - 4 \leq -5$$
$$-2x \geq 9 \qquad -2x \leq -1$$
$$x \leq -\tfrac{9}{2} \quad \text{or} \quad x \geq \tfrac{1}{2}$$

The solution set is $(-\infty, -\frac{9}{2}] \cup [\frac{1}{2}, \infty)$.



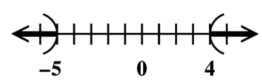

**35.** **(a)** $|2x + 1| < 9$

The graph of the solution set will be all numbers between $-5$ and $4$, since the absolute value is less than $9$.



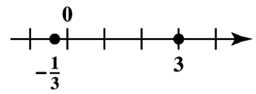

**(b)** $|2x + 1| > 9$

The graph of the solution set will be all numbers less than $-5$ or greater than $4$, since the absolute value is greater than $9$.



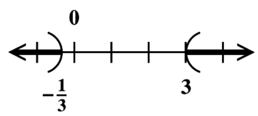
**36.** **(a)** $|3x - 4| = 5$

The solutions are the numbers at the endpoints in the given graph, $-\frac{1}{3}$ and 3.



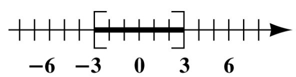

**(b)** $|3x - 4| > 5$

The solution set is composed of the numbers not in the given graph, not including the endpoints.



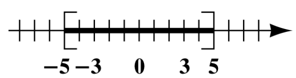

**37.** $|x| \le 3 \Leftrightarrow -3 \le x \le 3$

The solution set is $[-3, 3]$.



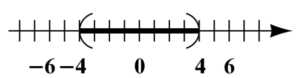

**38.** $|x| \le 5 \Leftrightarrow -5 \le x \le 5$

The solution set is $[-5, 5]$.



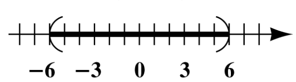

**39.** $|x| < 4 \Leftrightarrow -4 < x < 4$

The solution set is $(-4, 4)$.




**40.** $|x| < 6 \Leftrightarrow -6 < x < 6$

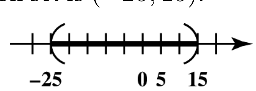
The solution set is $(-6, 6)$.




**41.** $|r + 5| < 20$

$-20 < r + 5 < 20$
$-25 < r \quad < 15$ *Subtract 5.*

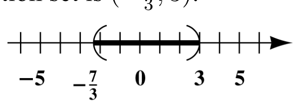
The solution set is $(-25, 15)$.




**42.** $|3r - 1| < 8$

$-8 < 3r - 1 < 8$
$-7 < \quad 3r \quad < 9$ *Add 1.*
$-\frac{7}{3} < \quad r \quad < 3$ *Divide by 3.*

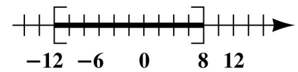
The solution set is $(-\frac{7}{3}, 3)$.




**43.** $|x + 2| \le 10$

$-10 \le x + 2 \le 10$
$-12 \le x \quad \le 8$

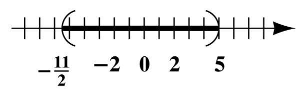
The solution set is $[-12, 8]$.




**44.** $|4x + 1| < 21$

$-21 < 4x + 1 < 21$
$-22 < 4x \quad < 20$
$-\frac{11}{2} < x \quad < 5$

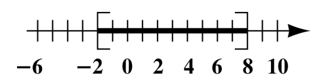
The solution set is $(-\frac{11}{2}, 5)$.




**45.** $|3 - x| \le 5$

$-5 \le 3 - x \le 5$
$-8 \le \quad -x \quad \le 2$
$8 \ge \quad x \quad \ge -2$ *Multiply by −1, reverse inequalities*
$-2 \le \quad x \quad \le 8$ *Equivalent inequality*

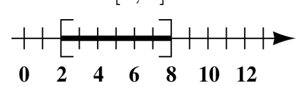
The solution set is $[-2, 8]$.




**46.** $|5 - x| \le 3$

$-3 \le 5 - x \le 3$
$-8 \le \quad -x \quad \le -2$
$8 \ge \quad x \quad \ge 2$ *Multiply by −1, reverse inequalities*
$2 \le \quad x \quad \le 8$ *Equivalent inequality*

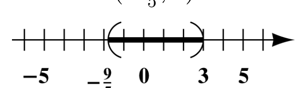
The solution set is $[2, 8]$.




**47.** $|-5x + 3| < 12$

$-12 < -5x + 3 < 12$
$-15 < \quad -5x \quad < 9$
$3 > \quad x \quad > -\frac{9}{5}$ *Divide by −5, reverse inequalities*
$-\frac{9}{5} < \quad x \quad < 3$ *Equivalent inequality*

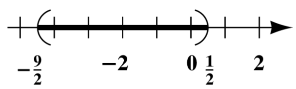
The solution set is $(-\frac{9}{5}, 3)$.



**48.** $|-2x - 4| < 5$

$$\begin{aligned} -5 < -2x - 4 &< 5 \\ -1 < \quad -2x \quad &< 9 \\ \frac{1}{2} > \quad x \quad &> -\frac{9}{2} \quad \textit{Divide by } -2, \textit{ reverse inequalities} \\ -\tfrac{9}{2} < \quad x \quad &< \tfrac{1}{2} \quad \textit{Equivalent inequality} \end{aligned}$$

The solution set is $(-\frac{9}{2}, \frac{1}{2})$.

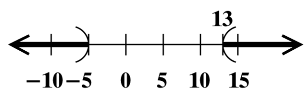



**49.** $|-4 + x| > 9$

$-4 + x > 9$ or $-4 + x < -9$

$x > 13$ or $x < -5$

The solution set is $(-\infty, -5) \cup (13, \infty)$.

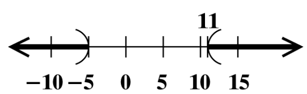



**50.** $|-3 + x| > 8$

$-3 + x > 8$ or $-3 + x < -8$

$x > 11$ or $x < -5$

The solution set is $(-\infty, -5) \cup (11, \infty)$.

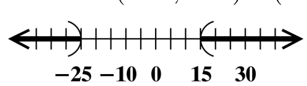



**51.** $|x + 5| > 20$

$x + 5 > 20$ or $x + 5 < -20$

$x > 15$ or $x < -25$

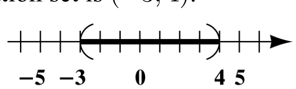
The solution set is $(-\infty, -25) \cup (15, \infty)$.




**52.** $|2x - 1| < 7$

$$\begin{aligned} -7 < 2x - 1 &< 7 \\ -6 < 2x \quad &< 8 \\ -3 < x \quad &< 4 \end{aligned}$$

The solution set is $(-3, 4)$.



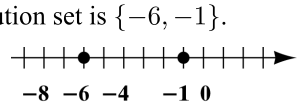

**53.** $|7 + 2x| = 5$

$7 + 2x = 5$ or $7 + 2x = -5$

$2x = -2$ $\quad$ $2x = -12$

$x = -1$ or $x = -6$

The solution set is $\{-6, -1\}$.



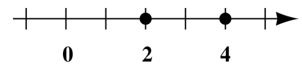

**54.** $|9 - 3x| = 3$

$9 - 3x = 3$ or $9 - 3x = -3$

$-3x = -6$ $\quad$ $-3x = -12$

$x = 2$ or $x = 4$

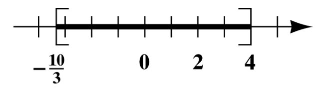
The solution set is $\{2, 4\}$.




**55.** $|3x - 1| \leq 11$

$$\begin{aligned} -11 \leq 3x - 1 &\leq 11 \\ -10 \leq 3x \quad &\leq 12 \\ -\tfrac{10}{3} \leq x \quad &\leq 4 \end{aligned}$$

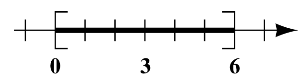
The solution set is $[-\frac{10}{3}, 4]$.




**56.** $|2x - 6| \leq 6$

$$\begin{aligned} -6 \leq 2x - 6 &\leq 6 \\ 0 \leq 2x \quad &\leq 12 \\ 0 \leq x \quad &\leq 6 \end{aligned}$$

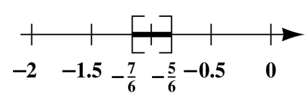
The solution set is $[0, 6]$.




**57.** $|-6x - 6| \leq 1$

$$\begin{aligned} -1 \leq -6x - 6 &\leq 1 \\ 5 \leq \quad -6x \quad &\leq 7 \\ -\frac{5}{6} \geq \quad x \quad &\geq -\frac{7}{6} \quad \textit{Divide by } -6. \textit{ Reverse inequalities.} \\ -\tfrac{7}{6} \leq \quad x \quad &\leq -\tfrac{5}{6} \quad \textit{Equivalent inequality} \end{aligned}$$

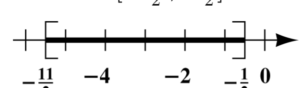
The solution set is $[-\frac{7}{6}, -\frac{5}{6}]$.




**58.** $|-2x - 6| \leq 5$

$$\begin{aligned} -5 \leq -2x - 6 &\leq 5 \\ 1 \leq \quad -2x \quad &\leq 11 \\ -\frac{1}{2} \geq \quad x \quad &\geq -\frac{11}{2} \quad \textit{Divide by } -2. \textit{ Reverse inequalities.} \\ -\tfrac{11}{2} \leq \quad x \quad &\leq -\tfrac{1}{2} \quad \textit{Equivalent inequality} \end{aligned}$$

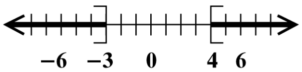
The solution set is $[-\frac{11}{2}, -\frac{1}{2}]$.



**59.** $|2x-1| \geq 7$

$$\begin{aligned} 2x-1 &\geq 7 \quad \text{or} \quad & 2x-1 &\leq -7 \\ 2x &\geq 8 \quad \text{or} \quad & 2x &\leq -6 \\ x &\geq 4 \quad \text{or} \quad & x &\leq -3 \end{aligned}$$

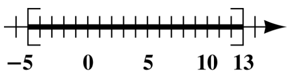
The solution set is $(-\infty, -3] \cup [4, \infty)$.




**60.** $|-4+x| \leq 9$

$$\begin{aligned} -9 \leq -4+x &\leq 9 \\ -5 \leq \quad x \quad &\leq 13 \end{aligned}$$

The solution set is $[-5, 13]$.




**61.** $|x+2| = 3$

$$\begin{aligned} x+2 &= 3 \quad \text{or} \quad & x+2 &= -3 \\ x &= 1 \quad \text{or} \quad & x &= -5 \end{aligned}$$

The solution set is $\{-5, 1\}$.

**62.** $|x+3| = 10$

$$\begin{aligned} x+3 &= 10 \quad \text{or} \quad & x+3 &= -10 \\ x &= 7 \quad \text{or} \quad & x &= -13 \end{aligned}$$

The solution set is $\{-13, 7\}$.

**63.** $|x-6| = 3$

$$\begin{aligned} x-6 &= 3 \quad \text{or} \quad & x-6 &= -3 \\ x &= 9 \quad \text{or} \quad & x &= 3 \end{aligned}$$

The solution set is $\{3, 9\}$.

**64.** $|x-4| = 1$

$$\begin{aligned} x-4 &= 1 \quad \text{or} \quad & x-4 &= -1 \\ x &= 5 \quad \text{or} \quad & x &= 3 \end{aligned}$$

The solution set is $\{3, 5\}$.

**65.** $|2-0.2x| = 2$

$$\begin{aligned} 2-0.2x &= 2 \quad \text{or} \quad & 2-0.2x &= -2 \\ -0.2x &= 0 & -0.2x &= -4 \\ x &= 0 \quad \text{or} \quad & x &= 20 \end{aligned}$$

The solution set is $\{0, 20\}$.

**66.** $|5-0.5x| = 4$

$$\begin{aligned} 5-0.5x &= 4 \quad \text{or} \quad & 5-0.5x &= -4 \\ -0.5x &= -1 & -0.5x &= -9 \\ x &= 2 \quad \text{or} \quad & x &= 18 \end{aligned}$$

The solution set is $\{2, 18\}$.

**67.** $|x| - 1 = 4$

$$|x| = 5$$

$$x = 5 \quad \text{or} \quad x = -5$$

The solution set is $\{-5, 5\}$.

**68.** $|x| + 3 = 10$

$$|x| = 7$$

$$x = 7 \quad \text{or} \quad x = -7$$

The solution set is $\{-7, 7\}$.

**69.** $|x+4| + 1 = 2$

$$|x+4| = 1$$

$$\begin{aligned} x+4 &= 1 \quad \text{or} \quad & x+4 &= -1 \\ x &= -3 \quad \text{or} \quad & x &= -5 \end{aligned}$$

The solution set is $\{-5, -3\}$.

**70.** $|x+5| - 2 = 12$

$$|x+5| = 14$$

$$\begin{aligned} x+5 &= 14 \quad \text{or} \quad & x+5 &= -14 \\ x &= 9 \quad \text{or} \quad & x &= -19 \end{aligned}$$

The solution set is $\{-19, 9\}$.

**71.** $|2x+1| + 3 > 8$

$$|2x+1| > 5$$

$$\begin{aligned} 2x+1 &> 5 \quad \text{or} \quad & 2x+1 &< -5 \\ 2x &> 4 & 2x &< -6 \\ x &> 2 \quad \text{or} \quad & x &< -3 \end{aligned}$$

The solution set is $(-\infty, -3) \cup (2, \infty)$.

**72.** $|6x-1| - 2 > 6$

$$|6x-1| > 8$$

$$\begin{aligned} 6x-1 &> 8 \quad \text{or} \quad & 6x-1 &< -8 \\ 6x &> 9 & 6x &< -7 \\ x &> \tfrac{3}{2} \quad \text{or} \quad & x &< -\tfrac{7}{6} \end{aligned}$$

The solution set is $(-\infty, -\frac{7}{6}) \cup (\frac{3}{2}, \infty)$.

**73.** $|x+5| - 6 \leq -1$

$$\begin{aligned} |x+5| &\leq 5 \\ -5 \leq x+5 &\leq 5 \\ -10 \leq x \quad &\leq 0 \end{aligned}$$

The solution set is $[-10, 0]$.

**74.** $|x-2| - 3 \leq 4$

$$\begin{aligned} |x-2| &\leq 7 \\ -7 \leq x-2 &\leq 7 \\ -5 \leq x \quad &\leq 9 \end{aligned}$$

The solution set is $[-5, 9]$.

**75.** $\left|\frac{1}{2}x + \frac{1}{3}\right| + \frac{1}{4} = \frac{3}{4}$

$$\left|\tfrac{1}{2}x + \tfrac{1}{3}\right| = \tfrac{1}{2}$$

$\frac{1}{2}x+\frac{1}{3}=\frac{1}{2}$ or $\frac{1}{2}x+\frac{1}{3}=-\frac{1}{2}$

$6(\frac{1}{2}x+\frac{1}{3})=6(\frac{1}{2})$ $\quad$ $6(\frac{1}{2}x+\frac{1}{3})=6(-\frac{1}{2})$

$3x+2=3$ $\quad$ $3x+2=-3$

$3x=1$ $\quad$ $3x=-5$

$x=\frac{1}{3}$ or $x=-\frac{5}{3}$

The solution set is $\left\{-\frac{5}{3},\frac{1}{3}\right\}$.

**76.** $\left|\frac{2}{3}x+\frac{1}{6}\right|+\frac{1}{2}=\frac{5}{2}$

$\left|\frac{2}{3}x+\frac{1}{6}\right|=2$

$\frac{2}{3}x+\frac{1}{6}=2$ or $\frac{2}{3}x+\frac{1}{6}=-2$

$6(\frac{2}{3}x+\frac{1}{6})=6(2)$ $\quad$ $6(\frac{2}{3}x+\frac{1}{6})=6(-2)$

$4x+1=12$ $\quad$ $4x+1=-12$

$4x=11$ $\quad$ $4x=-13$

$x=\frac{11}{4}$ or $x=-\frac{13}{4}$

The solution set is $\left\{-\frac{13}{4},\frac{11}{4}\right\}$.

**77.** $|0.1x-2.5|+0.3\geq 0.8$

$|0.1x-2.5|\geq 0.5$

$0.1x-2.5\geq 0.5$ or $0.1x-2.5\leq -0.5$

$0.1x\geq 3$ or $0.1x\leq 2$

$x\geq 30$ or $x\leq 20$

The solution set is $(-\infty,20]\cup[30,\infty)$.

**78.** $|0.5x-3.5|+0.2\geq 0.6$

$|0.5x-3.5|\geq 0.4$

$0.5x-3.5\geq 0.4$ or $0.5x-3.5\leq -0.4$

$0.5x\geq 3.9$ or $0.5x\leq 3.1$

$x\geq 7.8$ or $x\leq 6.2$

The solution set is $(-\infty,6.2]\cup[7.8,\infty)$.

**79.** $|3x+1|=|2x+4|$

$3x+1=2x+4$ or $3x+1=-(2x+4)$

$3x+1=-2x-4$

$5x=-5$

$x=3$ or $x=-1$

The solution set is $\{-1,3\}$.

**80.** $|7x+12|=|x-8|$

$7x+12=x-8$ or $7x+12=-(x-8)$

$6x=-20$ $\quad$ $7x+12=-x+8$

$8x=-4$

$x=-\frac{10}{3}$ or $x=-\frac{1}{2}$

The solution set is $\left\{-\frac{10}{3},-\frac{1}{2}\right\}$.

**81.** $\left|x-\frac{1}{2}\right|=\left|\frac{1}{2}x-2\right|$

$x-\frac{1}{2}=\frac{1}{2}x-2$ or $x-\frac{1}{2}=-(\frac{1}{2}x-2)$

*Multiply by 2.* $\quad$ $x-\frac{1}{2}=-\frac{1}{2}x+2$

*Multiply by 2.*

$2x-1=x-4$ $\quad$ $2x-1=-x+4$

$3x=5$

$x=-3$ or $x=\frac{5}{3}$

The solution set is $\left\{-3,\frac{5}{3}\right\}$.

**82.** $\left|\frac{2}{3}x-2\right|=\left|\frac{1}{3}x+3\right|$

$\frac{2}{3}x-2=\frac{1}{3}x+3$ or $\frac{2}{3}x-2=-(\frac{1}{3}x+3)$

$\frac{1}{3}x=5$ $\quad$ $\frac{2}{3}x-2=-\frac{1}{3}x-3$

$x=15$ or $x=-1$

The solution set is $\{-1,15\}$.

**83.** $|6x|=|9x+1|$

$6x=9x+1$ or $6x=-(9x+1)$

$-3x=1$ $\quad$ $6x=-9x-1$

$15x=-1$

$x=-\frac{1}{3}$ or $x=-\frac{1}{15}$

The solution set is $\left\{-\frac{1}{3},-\frac{1}{15}\right\}$.

**84.** $|13x|=|2x+1|$

$13x=2x+1$ or $13x=-(2x+1)$

$11x=1$ $\quad$ $13x=-2x-1$

$15x=-1$

$x=\frac{1}{11}$ or $x=-\frac{1}{15}$

The solution set is $\left\{-\frac{1}{15},\frac{1}{11}\right\}$.

**85.** $|2x-6|=|2x+11|$

$2x-6=2x+11$ or $2x-6=-(2x+11)$

$-6=11$ *False* $\quad$ $2x-6=-2x-11$

$4x=-5$

*No solution* or $x=-\frac{5}{4}$

The solution set is $\left\{-\frac{5}{4}\right\}$.

**86.** $|3x-1|=|3x+9|$

$3x-1=3x+9$ or $3x-1=-(3x+9)$

$-1=9$ *False* $\quad$ $3x-1=-3x-9$

$6x=-8$

*No solution* or $x=-\frac{4}{3}$

The solution set is $\left\{-\frac{4}{3}\right\}$.

**87.** $|x|\geq -10$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.

The solution set is $(-\infty,\infty)$.

**88.** $|x|\geq -15$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.

The solution set is $(-\infty,\infty)$.

**89.** $|12t - 3| = -8$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

**90.** $|13x + 1| = -3$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

**91.** $|4x + 1| = 0$

The expression $4x + 1$ will equal $0$ *only* for the solution of the equation

$$\begin{aligned} 4x + 1 &= 0. \\ 4x &= -1 \\ x &= \tfrac{-1}{4} \text{ or } -\tfrac{1}{4} \end{aligned}$$

The solution set is $\left\{-\frac{1}{4}\right\}$.

**92.** $|6x - 2| = 0$

The expression $6x - 2$ will equal $0$ *only* for the solution of the equation

$$\begin{aligned} 6x - 2 &= 0 \\ 6x &= 2 \\ x &= \tfrac{2}{6} = \tfrac{1}{3} \end{aligned}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

**93.** $|2x - 1| = -6$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

**94.** $|8x + 4| = -4$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

**95.** $|x + 5| > -9$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number $x$.

The solution set is $(-\infty, \infty)$.

**96.** $|x + 9| > -3$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number $x$.

The solution set is $(-\infty, \infty)$.

**97.** $|7x + 3| \leq 0$

The absolute value of an expression is always nonnegative (positive or zero), so this inequality is true only when

$$\begin{aligned} 7x + 3 &= 0 \\ 7x &= -3 \\ x &= -\tfrac{3}{7}. \end{aligned}$$

The solution set is $\left\{-\frac{3}{7}\right\}$.

**98.** $|4x - 1| \leq 0$

The absolute value of an expression is always nonnegative (positive or zero), so this inequality is true only when

$$\begin{aligned} 4x - 1 &= 0 \\ 4x &= 1 \\ x &= \tfrac{1}{4}. \end{aligned}$$

The solution set is $\left\{\frac{1}{4}\right\}$.

**99.** $|5x - 2| = 0$

The expression $5x - 2$ will equal $0$ *only* for the solution of the equation

$$\begin{aligned} 5x - 2 &= 0. \\ 5x &= 2 \\ x &= \tfrac{2}{5} \end{aligned}$$

The solution set is $\left\{\frac{2}{5}\right\}$.

**100.** $|7x + 4| = 0$

The expression $7x + 4$ will equal $0$ *only* for the solution of the equation

$$\begin{aligned} 7x + 4 &= 0 \\ 7x &= -4 \\ x &= -\tfrac{4}{7} \end{aligned}$$

The solution set is $\left\{-\frac{4}{7}\right\}$.

**101.** $|x - 2| + 3 \geq 2$

$|x - 2| \geq -1$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number $x$.

The solution set is $(-\infty, \infty)$.

**102.** $|x - 4| + 5 \geq 4$

$|x - 4| \geq -1$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number $x$.

The solution set is $(-\infty, \infty)$.

**103.** $|10x + 7| + 3 < 1$
$|10x + 7| < -2$

There is no number whose absolute value is less than $-2$, so this inequality has no solution.

The solution set is $\emptyset$.

**104.** $|4x + 1| - 2 < -5$
$|4x + 1| < -3$

There is no number whose absolute value is less than $-3$, so this inequality has no solution.

The solution set is $\emptyset$.

**105.** Let $x$ represent the calcium intake for a specific female. For $x$ to be within 100 mg of 1000 mg, we must have

$$|x - 1000| \leq 100.$$

$$\begin{aligned} -100 \leq x - 1000 &\leq 100 \\ 900 \leq x &\leq 1100 \end{aligned}$$

**106.** Let $x$ represent the clotting time for an individual. For $x$ to be within 3.6 seconds of 7.45 seconds, we must have

$$|x - 7.45| \leq 3.6$$

$$\begin{aligned} -3.6 \leq x - 7.45 &\leq 3.6 \\ 3.85 \leq x &\leq 11.05 \end{aligned}$$

**107.** Add the given heights with a calculator to get 8105. There are 10 numbers, so divide the sum by 10.

$$\frac{8105}{10} = 810.5$$

The average height is 810.5 ft.

**108.** $|x - k| < 50$

Substitute 810.5 for $k$ and solve the inequality.

$$|x - 810.5| < 50$$

$$\begin{aligned} -50 < x - 810.5 &< 50 \\ 760.5 < x &< 860.5 \end{aligned}$$

The buildings with heights between 760.5 ft and 860.5 ft are Bank of America Center and Texaco Heritage Plaza.

**109.** $|x - k| < 95$

Substitute 810.5 for $k$ and solve the inequality.

$$|x - 810.5| < 95$$

$$\begin{aligned} -95 < x - 810.5 &< 95 \\ 715.5 < x &< 905.5 \end{aligned}$$

The buildings with heights between 715.5 ft and 905.5 ft are Williams Tower, Bank of America Center, Texaco Heritage Plaza, Enterprise Plaza, Centerpoint Energy Plaza, Continental Center I, and Fulbright Tower.

**110.** **(a)** This would be the opposite of the inequality in Exercise 109, that is,

$$|x - 810.5| \geq 95.$$

**(b)** $|x - 810.5| \geq 95$

$$\begin{aligned} x - 810.5 \geq 95 \quad &\text{or} \quad x - 810.5 \leq -95 \\ x \geq 905.5 \quad &\text{or} \quad x \leq 715.5 \end{aligned}$$

**(c)** The buildings that are not within 95 ft of the average have height less than or equal to 715.5 or greater than or equal to 905.5. They are JPMorgan Chase Tower, Wells Fargo Plaza, and One Shell Plaza.

**(d)** The answer makes sense because it includes all the buildings *not* listed earlier which had heights within 95 ft of the average.

**111.** **(a)**
$$\begin{aligned} 3x + 2y &= 24 \\ 3(0) + 2y &= 24 \quad \textit{Let } x = 0. \\ 0 + 2y &= 24 \\ 2y &= 24 \\ y &= 12 \end{aligned}$$

**(b)**
$$\begin{aligned} -2x + 5y &= 20 \\ -2(0) + 5y &= 20 \quad \textit{Let } x = 0. \\ 0 + 5y &= 20 \\ 5y &= 20 \\ y &= 4 \end{aligned}$$

**112.** **(a)**
$$\begin{aligned} 3x + 2y &= 24 \\ 3(-2) + 2y &= 24 \quad \textit{Let } x = -2. \\ -6 + 2y &= 24 \\ 2y &= 30 \\ y &= 15 \end{aligned}$$

**(b)**
$$\begin{aligned} -2x + 5y &= 20 \\ -2(-2) + 5y &= 20 \quad \textit{Let } x = -2. \\ 4 + 5y &= 20 \\ 5y &= 16 \\ y &= \tfrac{16}{5} \end{aligned}$$

**113.** **(a)**
$$\begin{aligned} 3x + 2y &= 24 \\ 3(8) + 2y &= 24 \quad \textit{Let } x = 8. \\ 24 + 2y &= 24 \\ 2y &= 0 \\ y &= 0 \end{aligned}$$

**(b)**
$$\begin{aligned} -2x + 5y &= 20 \\ -2(8) + 5y &= 20 \quad \textit{Let } x = 8. \\ -16 + 5y &= 20 \\ 5y &= 36 \\ y &= \tfrac{36}{5} \end{aligned}$$

**114. (a)**
$$\begin{aligned} 3x+2y&=24 \\ 3(1.5)+2y&=24 \quad \textit{Let } x=1.5. \\ 4.5+2y&=24 \\ 2y&=19.5 \\ y&=9.75 \end{aligned}$$

**(b)**
$$\begin{aligned} -2x+5y&=20 \\ -2(1.5)+5y&=20 \quad \textit{Let } x=1.5. \\ -3+5y&=20 \\ 5y&=23 \\ y&=\tfrac{23}{5}, \quad \text{or} \quad 4.6 \end{aligned}$$

## Summary Exercises on Solving Linear and Absolute Value Equations and Inequalities

**1.**
$$\begin{aligned} 4x+1&=49 \\ 4x&=48 \\ x&=12 \end{aligned}$$

The solution set is $\{12\}$.

**2.** $|x-1|=6$

$$\begin{aligned} x-1&=6 \quad \text{or} \quad x-1=-6 \\ x&=7 \quad \text{or} \quad x=-5 \end{aligned}$$

The solution set is $\{-5,7\}$.

**3.**
$$\begin{aligned} 6x-9&=12+3x \\ 3x&=21 \\ x&=7 \end{aligned}$$

The solution set is $\{7\}$.

**4.**
$$\begin{aligned} 3x+7&=9+8x \\ -5x&=2 \\ x&=-\tfrac{2}{5} \end{aligned}$$

The solution set is $\left\{-\frac{2}{5}\right\}$.

**5.** $|x+3|=-4$

Since the absolute value of an expression is always nonnegative, there is no number that makes this statement true. Therefore, the solution set is $\emptyset$.

**6.**
$$\begin{aligned} 2x+1&\le x \\ x&\le -1 \end{aligned}$$

The solution set is $(-\infty,-1]$.

**7.**
$$\begin{aligned} 8x+2&\ge 5x \\ 3x&\ge -2 \\ x&\ge -\tfrac{2}{3} \end{aligned}$$

The solution set is $[-\frac{2}{3},\infty)$.

**8.**
$$\begin{aligned} 4(x-11)+3x&=20x-31 \\ 4x-44+3x&=20x-31 \\ 7x-44&=20x-31 \\ -13x&=13 \\ x&=-1 \end{aligned}$$

The solution set is $\{-1\}$.

**9.**
$$\begin{aligned} 2x-1&=-7 \\ 2x&=-6 \\ x&=-3 \end{aligned}$$

The solution set is $\{-3\}$.

**10.**
$$\begin{aligned} |3x-7|-4&=0 \\ |3x-7|&=4 \end{aligned}$$

$$\begin{aligned} 3x-7&=4 \quad \text{or} \quad 3x-7=-4 \\ 3x&=11 \qquad\qquad 3x=3 \\ x&=\tfrac{11}{3} \quad \text{or} \quad x=1 \end{aligned}$$

The solution set is $\left\{1,\frac{11}{3}\right\}$.

**11.**
$$\begin{aligned} 6x-5&\le 3x+10 \\ 3x&\le 15 \\ x&\le 5 \end{aligned}$$

The solution set is $(-\infty,5]$.

**12.**
$$\begin{aligned} |5x-8|+9&\ge 7 \\ |5x-8|&\ge -2 \end{aligned}$$

The absolute value of an expression is always nonnegative, so the inequality is true for any real number $x$.

The solution set is $(-\infty,\infty)$.

**13.**
$$\begin{aligned} 9x-3(x+1)&=8x-7 \\ 9x-3x-3&=8x-7 \\ 6x-3&=8x-7 \\ 4&=2x \\ 2&=x \end{aligned}$$

The solution set is $\{2\}$.

**14.** $|x|\ge 8 \Leftrightarrow \quad x\ge 8 \quad \text{or} \quad x\le -8$

The solution set is $(-\infty,-8]\cup[8,\infty)$.

**15.**
$$\begin{aligned} 9x-5&\ge 9x+3 \\ -5&\ge 3 \quad \textit{False} \end{aligned}$$

This is a false statement, so the inequality is a contradiction.

The solution set is $\emptyset$.

**16.**
$$\begin{aligned} 13x-5&>13x-8 \\ -5&>-8 \end{aligned}$$

This inequality is true for every value of $x$.

The solution set is $(-\infty,\infty)$.

**17.** $|x| < 5.5$
$-5.5 < x < 5.5$

The solution set is $(-5.5, 5.5)$.

**18.** $4x - 1 = 12 + x$
$3x = 13$
$x = \frac{13}{3}$

The solution set is $\left\{\frac{13}{3}\right\}$.

**19.** $\frac{2}{3}x + 8 = \frac{1}{4}x$
$8x + 96 = 3x$ *Multiply by 12.*
$5x = -96$
$x = -\frac{96}{5}$

The solution set is $\left\{-\frac{96}{5}\right\}$.

**20.** $-\frac{5}{8}x \geq -20$
$-\frac{8}{5}(-\frac{5}{8}x) \leq -\frac{8}{5}(-20)$
$x \leq 32$

The solution set is $(-\infty, 32]$.

**21.** $\frac{1}{4}x < -6$
$4(\frac{1}{4}x) < 4(-6)$
$x < -24$

The solution set is $(-\infty, -24)$.

**22.** $7x - 3 + 2x = 9x - 8x$
$9x - 3 = x$
$8x = 3$
$x = \frac{3}{8}$

The solution set is $\left\{\frac{3}{8}\right\}$.

**23.** $\frac{3}{5}x - \frac{1}{10} = 2$
$6x - 1 = 20$ *Multiply by 10.*
$6x = 21$
$x = \frac{21}{6} = \frac{7}{2}$

The solution set is $\left\{\frac{7}{2}\right\}$.

**24.** $|x - 1| < 7$

$-7 < x - 1 < 7$
$-6 < x < 8$

The solution set is $(-6, 8)$.

**25.** $x + 9 + 7x = 4(3 + 2x) - 3$
$8x + 9 = 12 + 8x - 3$
$8x + 9 = 8x + 9$
$0 = 0$ *True*

The last statement is true for any real number $x$.

The solution set is {all real numbers}.

**26.** $6 - 3(2 - x) < 2(1 + x) + 3$
$6 - 6 + 3x < 2 + 2x + 3$
$3x < 5 + 2x$
$x < 5$

The solution set is $(-\infty, 5)$.

**27.** $|2x - 3| > 11$

$2x - 3 > 11$ or $2x - 3 < -11$
$2x > 14$    $2x < -8$
$x > 7$ or $x < -4$

The solution set is $(-\infty, -4) \cup (7, \infty)$.

**28.** $\dfrac{x}{4} - \dfrac{2x}{3} = -10$
$3x - 8x = -120$ *Multiply by 12.*
$-5x = -120$
$x = 24$

The solution set is $\{24\}$.

**29.** $|5x + 1| \leq 0$

The expression $|5x + 1|$ is never less than 0 since an absolute value expression must be nonnegative. However, $|5x + 1| = 0$ if

$5x + 1 = 0$
$5x = -1$
$x = \frac{-1}{5} = -\frac{1}{5}$

The solution set is $\left\{-\frac{1}{5}\right\}$.

**30.** $5x - (3 + x) \geq 2(3x + 1)$
$5x - 3 - x \geq 6x + 2$
$4x - 3 \geq 6x + 2$
$-2x \geq 5$
$x \leq -\frac{5}{2}$

The solution set is $(-\infty, -\frac{5}{2}]$.

**31.** $-2 \leq 3x - 1 \leq 8$
$-1 \leq 3x \leq 9$
$-\frac{1}{3} \leq x \leq 3$

The solution set is $[-\frac{1}{3}, 3]$.

**32.** $-1 \leq 6 - x \leq 5$
$-7 \leq -x \leq -1$
$7 \geq x \geq 1$ *Multiply by −1. Reverse inequalities.*
$1 \leq x \leq 7$ *Equivalent inequality*
The solution set is $[1, 7]$.

**33.** $|7x - 1| = |5x + 3|$
$7x - 1 = 5x + 3$ or $7x - 1 = -(5x + 3)$
$2x = 4$    $7x - 1 = -5x - 3$
$12x = -2$
$x = 2$ or $x = \frac{-2}{12} = -\frac{1}{6}$
The solution set is $\left\{-\frac{1}{6}, 2\right\}$.

**34.** $|x+2| = |x+4|$

$x+2 = x+4$ or $x+2 = -(x+4)$

$2 = 4$ *False* $\quad x+2 = -x-4$

$2x = -6$

*No solution* or $x = -3$

The solution set is $\{-3\}$.

**35.** $|1-3x| \geq 4$

$1-3x \geq 4$ or $1-3x \leq -4$

$-3x \geq 3$ $\quad -3x \leq -5$

$x \leq -1$ or $x \geq \frac{5}{3}$

The solution set is $(-\infty, -1] \cup [\frac{5}{3}, \infty)$.

**36.** $\frac{1}{2} \leq \frac{2}{3}x \leq \frac{5}{4}$

$6 \leq 8x \leq 15$ *Multiply by 12.*

$\frac{6}{8} \leq x \leq \frac{15}{8}$

$\frac{3}{4} \leq x \leq \frac{15}{8}$

The solution set is $[\frac{3}{4}, \frac{15}{8}]$.

**37.** $-(x+4)+2 = 3x+8$

$-x-4+2 = 3x+8$

$-x-2 = 3x+8$

$-10 = 4x$

$x = \frac{-10}{4} = -\frac{5}{2}$

The solution set is $\{-\frac{5}{2}\}$.

**38.** $\dfrac{x}{6} - \dfrac{3x}{5} = x - 86$

$5x - 18x = 30x - 2580$ *Multiply by 30.*

$-43x = -2580$

$x = \frac{-2580}{-43} = 60$

The solution set is $\{60\}$.

**39.** $-6 \leq \frac{3}{2} - x \leq 6$

$-\frac{15}{2} \leq -x \leq \frac{9}{2}$ *Subtract $\frac{3}{2}$.*

$\dfrac{15}{2} \geq x \geq -\dfrac{9}{2}$ *Multiply by −1. Reverse inequalities.*

$-\frac{9}{2} \leq x \leq \frac{15}{2}$ *Equivalent inequality*

The solution set is $[-\frac{9}{2}, \frac{15}{2}]$.

**40.** $|5-x| < 4$

$-4 < 5-x < 4$

$-9 < -x < -1$ *Subtract 5.*

$9 > x > 1$ *Multiply by −1. Reverse inequalities.*

$1 < x < 9$ *Equivalent inequality*

The solution set is $(1, 9)$.

**41.** $|x-1| \geq -6$

The absolute value of an expression is always nonnegative, so the inequality is true for any real number $x$.

The solution set is $(-\infty, \infty)$.

**42.** $|2x-5| = |x+4|$

$2x-5 = x+4$ or $2x-5 = -(x+4)$

$2x-5 = -x-4$

$3x = 1$

$x = 9$ or $x = \frac{1}{3}$

The solution set is $\{\frac{1}{3}, 9\}$.

**43.** $8x - (1-x) = 3(1+3x) - 4$

$8x - 1 + x = 3 + 9x - 4$

$9x - 1 = 9x - 1$ *True*

This is an identity.

The solution set is {all real numbers}.

**44.** $8x - (x+3) = -(2x+1) - 12$

$8x - x - 3 = -2x - 1 - 12$

$7x - 3 = -2x - 13$

$9x = -10$

$x = -\frac{10}{9}$

The solution set is $\{-\frac{10}{9}\}$.

**45.** $|x-5| = |x+9|$

$x-5 = x+9$ or $x-5 = -(x+9)$

$-5 = 9$ *False* $\quad x-5 = -x-9$

$2x = -4$

*No solution* or $x = -2$

The solution set is $\{-2\}$.

**46.** $|x+2| < -3$

There are no numbers whose absolute value is negative, so this inequality has no solution. The solution set is $\emptyset$.

**47.** $2x+1 > 5$ or $3x+4 < 1$

$2x > 4$ $\quad 3x < -3$

$x > 2$ or $x < -1$

The solution set is $(-\infty, -1) \cup (2, \infty)$.

**48.** $1-2x \geq 5$ and $7+3x \geq -2$

$-2x \geq 4$ $\quad 3x \geq -9$

$x \leq -2$ and $x \geq -3$

This is an intersection.

The solution set is $[-3, -2]$.

## Chapter 2 Review Exercises

**1.** $-(8+3x)+5 = 2x+6$

$-8-3x+5 = 2x+6$

$-3x-3 = 2x+6$

$-5x = 9$

$x = -\frac{9}{5}$

The solution set is $\{-\frac{9}{5}\}$.

**2.** $-\frac{3}{4}x = -12$
$-3x = -48$ *Multiply by 4.*
$x = 16$

The solution set is $\{16\}$.

**3.** $\frac{2x+1}{3} - \frac{x-1}{4} = 0$
$4(2x+1) - 3(x-1) = 0$ *Multiply by 12.*
$8x + 4 - 3x + 3 = 0$
$5x + 7 = 0$
$5x = -7$
$x = -\frac{7}{5}$

The solution set is $\left\{-\frac{7}{5}\right\}$.

**4.** $5(2x-3) = 6(x-1) + 4x$
$10x - 15 = 6x - 6 + 4x$
$10x - 15 = 10x - 6$
$-15 = -6$ *False*

This is a false statement, so the equation is a contradiction.

The solution set is $\emptyset$.

**5.** $7x - 3(2x-5) + 5 + 3x = 4x + 20$
$7x - 6x + 15 + 5 + 3x = 4x + 20$
$4x + 20 = 4x + 20$
$20 = 20$ *True*

This equation is an *identity*.

The solution set is {all real numbers}.

**6.** $8x - 4x - (x-7) + 9x + 6 = 12x - 7$
$8x - 4x - x + 7 + 9x + 6 = 12x - 7$
$12x + 13 = 12x - 7$
$13 = -7$ *False*

This equation is a *contradiction*.

The solution set is $\emptyset$.

**7.** $-2x + 6(x-1) + 3x - (4-x) = -(x+5) - 5$
$-2x + 6x - 6 + 3x - 4 + x = -x - 5 - 5$
$8x - 10 = -x - 10$
$9x = 0$
$x = 0$

This equation is a *conditional* equation.

The solution set is $\{0\}$.

**8.** Solve $V = LWH$ for $L$.

$$\frac{V}{WH} = \frac{LWH}{WH}$$
$$\frac{V}{WH} = L, \text{ or } L = \frac{V}{WH}$$

**9.** Solve $\mathcal{A} = \frac{1}{2}h(b+B)$ for $b$.

$2\mathcal{A} = h(b+B)$ *Multiply by 2.*
$2\mathcal{A} = hb + hB$ *Distributive prop.*
$2\mathcal{A} - hB = hb$ *Subtract hB.*
$\frac{2\mathcal{A} - hB}{h} = b$ *Divide by h.*

*Another method:*

$2\mathcal{A} = h(b+B)$ *Multiply by 2.*
$\frac{2\mathcal{A}}{h} = b + B$ *Divide by h.*
$\frac{2\mathcal{A}}{h} - B = b,$ *Subtract B.*

**10.** Solve $M = -\frac{1}{4}(x+3y)$ for $x$.

$-4M = x + 3y$ *Multiply by −4.*
$x = -4M - 3y$

**11.** Solve $P = \frac{3}{4}x - 12$ for $x$.

$P + 12 = \frac{3}{4}x$ *Add 12.*
$x = \frac{4}{3}(P+12),$ *Multiply by 4/3.*
or $x = \frac{4}{3}P + 16$

**12.** Solve $-2x + 5 = 7$.

Begin by subtracting 5 from each side. Then divide each side by $-2$.

**13.** Use the formula $V = LWH$ and substitute 180 for $V$, 6 for $L$, and 5 for $W$.

$180 = 6(5)H$
$180 = 30H$
$6 = H$

The height is 6 feet.

**14.** Divide the amount of increase by the original amount (amounts in millions).

$$\frac{18.2 - 15.3}{15.3} = \frac{2.9}{15.3} \approx 0.190$$

The percent increase was about 19.0%.

**15.** Use the formula $I = prt$. Substitute \$7800 for $I$, \$30,000 for $p$, and 4 for $t$. Solve for $r$.

$I = prt$
$\$7800 = (\$30{,}000)r(4)$
$7800 = 120{,}000r$
$r = \frac{7800}{120{,}000} = 0.065$

The rate is 6.5%.

**16.** Use the formula $C = \frac{5}{9}(F - 32)$ and substitute 77 for $F$.

$$C = \tfrac{5}{9}(77 - 32) = \tfrac{5}{9}(45) = 25$$

The Celsius temperature is 25°.

**17.** The amount of money spent on Social Security in 2008 was about

$$0.207(\$2980 \text{ billion}) \approx \$617 \text{ billion}.$$

**18.** The amount of money spent on education and social services in 2008 was about

$$0.031(\$2980 \text{ billion}) \approx \$92.4 \text{ billion}$$

**19.** "One-third of a number, subtracted from 9" is written

$$9 - \tfrac{1}{3}x.$$

**20.** "The product of 4 and a number, divided by 9 more than the number" is written

$$\frac{4x}{x+9}.$$

**21.** Let $x =$ the width of the rectangle.
Then $2x - 3 =$ the length of the rectangle.

Use the formula $P = 2L + 2W$ with $P = 42$.

$$\begin{aligned} 42 &= 2(2x-3) + 2x \\ 42 &= 4x - 6 + 2x \\ 48 &= 6x \\ 8 &= x \end{aligned}$$

The width is 8 meters and the length is $2(8) - 3 = 13$ meters.

**22.** Let $x =$ the length of each equal side. Then $2x - 15 =$ the length of the third side.

Use the formula $P = a + b + c$ with $P = 53$.

$$\begin{aligned} 53 &= x + x + (2x - 15) \\ 53 &= 4x - 15 \\ 68 &= 4x \\ 17 &= x \end{aligned}$$

The lengths of the three sides are 17 inches, 17 inches, and $2(17) - 15 = 19$ inches.

**23.** Let $x =$ the number of kilograms of peanut clusters. Then $3x$ is the number of kilograms of chocolate creams.
The clerk has a total of 48 kg.

$$\begin{aligned} x + 3x &= 48 \\ 4x &= 48 \\ x &= 12 \end{aligned}$$

The clerk has 12 kilograms of peanut clusters.

**24.** Let $x =$ the number of liters of the 20% solution. Make a table.

| **Liters of Solution** | **Percent (as a decimal)** | **Liters of Pure Chemical** |
|---|---|---|
| $x$ | 0.20 | $0.20x$ |
| 15 | 0.50 | $0.50(15) = 7.5$ |
| $x + 15$ | 0.30 | $0.30(x + 15)$ |

The last column gives the equation.

$$\begin{aligned} 0.20x + 7.5 &= 0.30(x + 15) \\ 0.20x + 7.5 &= 0.30x + 4.5 \\ 3 &= 0.10x \\ 30 &= x \end{aligned}$$

30 L of the 20% solution should be used.

**25.** Let $x =$ the number of liters of water.

| **Liters of Solution** | **Percent (as a decimal)** | **Liters of Pure Acid** |
|---|---|---|
| 30 | 0.40 | $0.40(30) = 12$ |
| $x$ | 0 | $0(x) = 0$ |
| $30 + x$ | 0.30 | $0.30(30 + x)$ |

The last column gives the equation.

$$\begin{aligned} 12 + 0 &= 0.30(30 + x) \\ 12 &= 9 + 0.3x \\ 3 &= 0.3x \\ 10 &= x \end{aligned}$$

10 L of water should be added.

**26.** Let $x =$ the amount invested at 6%. Then $x - 4000 =$ the amount invested at 4%.

| **Principal** | **Rate (as a decimal)** | **Interest** |
|---|---|---|
| $x$ | 0.06 | $0.06x$ |
| $x - 4000$ | 0.04 | $0.04(x - 4000)$ |
| | Total → | $840 |

The last column gives the equation.

$$\begin{aligned} 0.06x + 0.04(x - 4000) &= 840 \\ 6x + 4(x - 4000) &= 84{,}000 \quad \textit{Multiply by 100.} \\ 6x + 4x - 16{,}000 &= 84{,}000 \\ 10x &= 100{,}000 \\ x &= 10{,}000 \end{aligned}$$

Eric should invest $10,000 at 6% and $10,000 − $4000 = $6000 at 4%.

**27.** Let $x =$ the number of quarters. Then $2x - 1$ is the number of dimes.

| **Number of Coins** | **Denomination** | **Value** |
|---|---|---|
| $x$ | 0.25 | $0.25x$ |
| $2x - 1$ | 0.10 | $0.10(2x - 1)$ |
| | Total → | 3.50 |

The sum of the values equals the total value.

$$\begin{aligned} 0.25x + 0.10(2x - 1) &= 3.50 \\ &\quad \textit{Multiply by 100.} \\ 25x + 10(2x - 1) &= 350 \\ 25x + 20x - 10 &= 350 \\ 45x &= 360 \\ x &= 8 \end{aligned}$$

There are 8 quarters and $2(8) - 1 = 15$ dimes.

**Check** $8(0.25) + 15(0.10) = 3.50$

**28.** Let $x =$ the number of nickels.
Then $19 - x$ is the number of dimes.

| Number of Coins | Denomination | Value |
|---|---|---|
| $x$ | 0.05 | $0.05x$ |
| $19 - x$ | 0.10 | $0.10(19 - x)$ |
| | Total → | 1.55 |

The sum of the values equals the total value.

$$0.05x + 0.10(19 - x) = 1.55$$
*Multiply by 100.*
$$5x + 10(19 - x) = 155$$
$$5x + 190 - 10x = 155$$
$$-5x = -35$$
$$x = 7$$

He had 7 nickels and $19 - 7 = 12$ dimes.

**Check** $7(0.05) + 12(0.10) = 1.55$

**29.** Use the formula $d = rt$ or $r = \frac{d}{t}$. Here, $d$ is about 400 mi and $t$ is about 8 hr. Since $\frac{400}{8} = 50$, the best estimate is choice **A**.

**30.** Use the formula $d = rt$.

**(a)** Here, $r = 53$ mph and $t = 10$ hr.

$$d = 53(10) = 530$$

The distance is 530 miles.

**(b)** Here, $r = 164$ mph and $t = 2$ hr.

$$d = 164(2) = 328$$

The distance is 328 miles.

**31.** Let $x =$ the time it takes for the trains to be 297 mi apart.

Use the formula $d = rt$.

| | Rate | Time | Distance |
|---|---|---|---|
| **Passenger Train** | 60 | $x$ | $60x$ |
| **Freight Train** | 75 | $x$ | $75x$ |
| | | | 297 |

The total distance traveled is the sum of the distances traveled by each train.

$$60x + 75x = 297$$
$$135x = 297$$
$$x = 2.2$$

It will take the trains 2.2 hours before they are 297 miles apart.

**32.** Let $x =$ the rate of the faster car and $x - 15 =$ the rate of the slower car.
Make a table.

| | Rate | Time | Distance |
|---|---|---|---|
| **Faster Car** | $x$ | 2 | $2x$ |
| **Slower Car** | $x - 15$ | 2 | $2(x - 15)$ |
| | | | 230 |

The total distance traveled is the sum of the distances traveled by each car.

$$2x + 2(x - 15) = 230$$
$$2x + 2x - 30 = 230$$
$$4x = 260$$
$$x = 65$$

The faster car travels at 65 km/hr, while the slower car travels at $65 - 15 = 50$ km/hr.

**Check** $2(65) + 2(50) = 230$

**33.** Let $x =$ amount of time spent averaging 45 miles per hour. Then $4 - x =$ amount of time at 50 mph.

| | Rate | Time | Distance |
|---|---|---|---|
| **First Part** | 45 | $x$ | $45x$ |
| **Second Part** | 50 | $4 - x$ | $50(4 - x)$ |
| **Total** | | | 195 |

From the last column:

$$45x + 50(4 - x) = 195$$
$$45x + 200 - 50x = 195$$
$$-5x = -5$$
$$x = 1$$

The automobile averaged 45 mph for 1 hour.

**Check** 45 mph for 1 hour $= 45$ miles and 50 mph for 3 hours $= 150$ miles; $45 + 150 = 195$.

**34.** Let $x =$ the average rate for the first hour. Then $x - 7 =$ the average rate for the second hour. Using $d = rt$, the distance traveled for the first hour is $x(1)$, for the second hour is $(x - 7)(1)$, and for the whole trip, 85.

$$x + (x - 7) = 85$$
$$2x - 7 = 85$$
$$2x = 92$$
$$x = 46$$

The average rate for the first hour was 46 mph.

**Check** 46 mph for 1 hour $= 46$ miles and $46 - 7 = 39$ mph for 1 hour $= 39$ miles; $46 + 39 = 85$.

**35.** The sum of the angles in a triangle is $180°$.

$$\begin{aligned}(3x+7)+(4x+1)+(9x-4)&=180\\16x+4&=180\\16x&=176\\x&=11\end{aligned}$$

The first angle is $3(11)+7=40°$.
The second angle is $4(11)+1=45°$.
The third angle is $9(11)-4=95°$.

**36.** The marked angles are supplements which have a sum of $180°$.

$$\begin{aligned}(15x+15)+(3x+3)&=180\\18x+18&=180\\18x&=162\\x&=9\end{aligned}$$

The angle measures are
$15(9)+15=150°$ and $3(9)+3=30°$.

**37.** $-\dfrac{2}{3}x<6$
$-2x<18$ *Multiply by 3.*
Divide by $-2$; reverse the inequality symbol.
$x>-9$

The solution set is $(-9,\infty)$.

**38.** $-5x-4\geq 11$
$-5x\geq 15$
Divide by $-5$; reverse the inequality symbol.
$x\leq -3$

The solution set is $(-\infty,-3]$.

**39.** $\dfrac{6x+3}{-4}<-3$
Multiply by $-4$; reverse the inequality symbol.
$6x+3>12$
$6x>9$
$x>\frac{9}{6}=\frac{3}{2}$

The solution set is $(\frac{3}{2},\infty)$.

**40.**
$$\begin{aligned}5-(6-4x)&\geq 2x-7\\5-6+4x&\geq 2x-7\\4x-1&\geq 2x-7\\2x&\geq -6\\x&\geq -3\end{aligned}$$

The solution set is $[-3,\infty)$.

**41.** $8\leq 3x-1<14$
$9\leq 3x<15$
$3\leq x<5$

The solution set is $[3,5)$.

**42.** $\frac{5}{3}(x-2)+\frac{2}{5}(x+1)>1$
$25(x-2)+6(x+1)>15$
*Multiply by 15.*
$$\begin{aligned}25x-50+6x+6&>15\\31x-44&>15\\31x&>59\\x&>\tfrac{59}{31}\end{aligned}$$

The solution set is $(\frac{59}{31},\infty)$.

**43.** Let $x=$ the other dimension of the rectangle. One dimension of the rectangle is 22 and the perimeter can be no greater than 120.

$$\begin{aligned}P&\leq 120\\2L+2W&\leq 120\\2(x)+2(22)&\leq 120\\2x+44&\leq 120\\2x&\leq 76\\x&\leq 38\end{aligned}$$

The other dimension must be 38 meters or less.

**44.** Let $x=$ the number of tickets that can be purchased. The total cost of the tickets is $\$48x$. Including the \$50 discount and staying within the available \$1600, we have

$$\begin{aligned}48x-50&\leq 1600.\\48x&\leq 1650\\x&\lesssim 34.4\end{aligned}$$

The group can purchase 34 tickets or fewer (but at least 15).

**45.** Let $x=$ the student's score on the fifth test. The average of the five test scores must be at least 70. The inequality is

$$\begin{aligned}\frac{75+79+64+71+x}{5}&\geq 70.\\75+79+64+71+x&\geq 350\\289+x&\geq 350\\x&\geq 61\end{aligned}$$

The student will pass algebra if any score greater than or equal to 61% on the fifth test is achieved.

**46.** The result, $-8<-13$, is a false statement. There are no real numbers that make this inequality true. The solution set is $\emptyset$.

For Exercises 47–50, let $A=\{$a, b, c, d$\}$, $B=\{$a, c, e, f$\}$, and $C=\{$a, e, f, g$\}$.

**47.** $A\cap B=\{\text{a, b, c, d}\}\cap\{\text{a, c, e, f}\}$
$=\{\text{a, c}\}$

**48.** $A\cap C=\{\text{a, b, c, d}\}\cap\{\text{a, e, f, g}\}$
$=\{\text{a}\}$

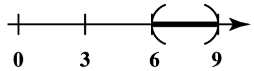
**49.** $B\cup C=\{\text{a, c, e, f}\}\cup\{\text{a, e, f, g}\}$
$=\{\text{a, c, e, f, g}\}$

**50.** $A \cup C = \{\text{a, b, c, d}\} \cup \{\text{a, e, f, g}\}$
$= \{\text{a, b, c, d, e, f, g}\}$

**51.** $x > 6$ and $x < 9$

The graph of the solution set will be all numbers which are both greater than 6 and less than 9. The overlap is the numbers between 6 and 9, not including the endpoints.

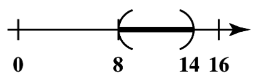
The solution set is $(6, 9)$.




**52.** $x + 4 > 12$ and $x - 2 < 12$
$x > 8$ and $x < 14$

The graph of the solution set will be all numbers between 8 and 14, not including the endpoints.

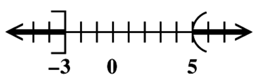
The solution set is $(8, 14)$.




**53.** $x > 5$ or $x \le -3$

The graph of the solution set will be all numbers that are either greater than 5 or less than or equal to $-3$.

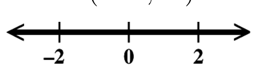
The solution set is $(-\infty, -3] \cup (5, \infty)$.




**54.** $x \ge -2$ or $x < 2$

The graph of the solution set will be all numbers that are either greater than or equal to $-2$ or less than 2. All real numbers satisfy these criteria.

The solution set is $(-\infty, \infty)$.




**55.** $x - 4 > 6$ and $x + 3 \le 10$
$x > 10$ and $x \le 7$

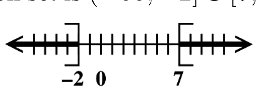
The graph of the solution set will be all numbers that are both greater than 10 and less than or equal to 7. There are no real numbers satisfying these criteria.

The solution set is $\emptyset$.

**56.** $-5x + 1 \ge 11$ or $3x + 5 \ge 26$
$-5x \ge 10$ $\quad 3x \ge 21$
$x \le -2$ or $x \ge 7$

The graph of the solution set will be all numbers that are either less than or equal to $-2$ or greater than or equal to 7.

The solution set is $(-\infty, -2] \cup [7, \infty)$.




**57.** $(-3, \infty) \cap (-\infty, 4)$
$(-3, \infty)$ includes all real numbers greater than $-3$.
$(-\infty, 4)$ includes all real numbers less than 4.
Find the intersection. The numbers common to both sets are greater than $-3$ and less than 4.

$$-3 < x < 4$$

The solution set is $(-3, 4)$.

**58.** $(-\infty, 6) \cap (-\infty, 2)$
$(-\infty, 6)$ includes all real numbers less than 6.
$(-\infty, 2)$ includes all real numbers less than 2.
Find the intersection. The numbers common to both sets are less than 2.

The solution set is $(-\infty, 2)$.

**59.** $(4, \infty) \cup (9, \infty)$
$(4, \infty)$ includes all real numbers greater than 4.
$(9, \infty)$ includes all real numbers greater than 9.
Find the union. The numbers in the first set, the second set, or in both sets are all the real numbers that are greater than 4.

The solution set is $(4, \infty)$.

**60.** $(1, 2) \cup (1, \infty)$
$(1, 2)$ includes the real numbers between 1 and 2, not including 1 and 2.
$(1, \infty)$ includes all real numbers greater than 1.
Find the union. The numbers in the first set, the second set, or in both sets are all real numbers greater than 1.

The solution set is $(1, \infty)$.

**61.** $|x| = 7 \Leftrightarrow x = 7$ or $x = -7$

The solution set is $\{-7, 7\}$.

**62.** $|x + 2| = 9$

$x + 2 = 9$ or $x + 2 = -9$
$x = 7$ or $x = -11$

The solution set is $\{-11, 7\}$.

**63.** $|3x - 7| = 8$

$3x - 7 = 8$ or $3x - 7 = -8$
$3x = 15$ $\quad 3x = -1$
$x = 5$ or $x = -\frac{1}{3}$

The solution set is $\left\{-\frac{1}{3}, 5\right\}$.

**64.** $|x - 4| = -12$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

**65.** $|2x-7|+4=11$
$|2x-7|=7$

$2x-7=7$ or $2x-7=-7$
$2x=14$ $\quad 2x=0$
$x=7$ or $x=0$

The solution set is $\{0,7\}$.

**66.** $|4x+2|-7=-3$
$|4x+2|=4$

$4x+2=4$ or $4x+2=-4$
$4x=2$ $\quad 4x=-6$
$x=\frac{2}{4}$ $\quad x=-\frac{6}{4}$
$x=\frac{1}{2}$ or $x=-\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2},\frac{1}{2}\right\}$.

**67.** $|3x+1|=|x+2|$

$3x+1=x+2$ or $3x+1=-(x+2)$
$2x=1$ $\quad 3x+1=-x-2$
$4x=-3$
$x=\frac{1}{2}$ or $x=-\frac{3}{4}$

The solution set is $\left\{-\frac{3}{4},\frac{1}{2}\right\}$.

**68.** $|2x-1|=|2x+3|$

$2x-1=2x+3$ or $2x-1=-(2x+3)$
$-1=3$ *False* $\quad 2x-1=-2x-3$
$4x=-2$
*No solution* or $x=-\frac{2}{4}=-\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}\right\}$.

**69.** $|x|<14 \Leftrightarrow -14<x<14$

The solution set is $(-14,14)$.

**70.** $|-x+6|\le 7$

$-7\le -x+6\le 7$
$-13\le -x \le 1$ *Subtract 6.*
$13\ge x \ge -1$ *Multiply by −1. Reverse inequalities.*
$-1\le x \le 13$ *Equivalent inequality*

The solution set is $[-1,13]$.

**71.** $|2x+5|\le 1$

$-1\le 2x+5\le 1$
$-6\le 2x \le -4$
$-3\le x \le -2$

The solution set is $[-3,-2]$.

**72.** $|x+1|\ge -3$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is *true* for any real number $x$.

The solution set is $(-\infty,\infty)$.

**73.** **[2.5]** $5-(6-4x)>2x-5$
$5-6+4x>2x-5$
$-1+4x>2x-5$
$2x>-4$
$x>-2$

The solution set is $(-2,\infty)$.

**74.** **[2.2]** Solve $ak+bt=6r$ for $k$.

$$ak=6r-bt$$
$$k=\frac{6r-bt}{a}$$

**75.** **[2.6]** $x<3$ and $x\ge -2$

The real numbers that are common to both sets are the numbers greater than or equal to $-2$ and less than 3.

$$-2\le x<3$$

The solution set is $[-2,3)$.

**76.** **[2.1]** $\dfrac{4x+2}{4}+\dfrac{3x-1}{8}=\dfrac{x+6}{16}$

Clear fractions by multiplying by the LCD, 16.
$4(4x+2)+2(3x-1)=x+6$
$16x+8+6x-2=x+6$
$22x+6=x+6$
$21x=0$
$x=0$

The solution set is $\{0\}$.

**77.** **[2.7]** $|3x+6|\ge 0$

The absolute value of an expression is always nonnegative, so the inequality is true for any real number $k$.

The solution set is $(-\infty,\infty)$.

**78.** **[2.5]** $-5x\ge -10$
$x\le 2$ *Divide by −5 < 0; reverse the symbol.*

The solution set is $(-\infty,2]$.

**79.** **[2.2]** Use the formula $V=LWH$, and solve for $H$.

$$\frac{V}{LW}=\frac{LWH}{LW}$$
$$\frac{V}{LW}=H, \text{ or } H=\frac{V}{LW}$$

Substitute 1.5 for $W$, 5 for $L$, and 75 for $V$.

$$H=\frac{75}{5(1.5)}=\frac{75}{7.5}=10$$

The height of the box is 10 ft.

**80.** **[2.4]** Let $x=$ the first consecutive integer. Then $x+1=$ the second consecutive integer and $x+2=$ the third consecutive integer.

The sum of the first and third integers is 47 more than the second integer, so an equation is

$$\begin{aligned} x+(x+2) &= 47+(x+1). \\ 2x+2 &= 48+x \\ x &= 46 \end{aligned}$$

Then $x+1=47$, and $x+2=48$.
The integers are 46, 47, and 48.

**81.** **[2.7]** $|3x+2|+4=9$

$$|3x+2|=5$$

$$\begin{aligned} 3x+2=5 \quad &\text{or} \quad 3x+2=-5 \\ 3x=3 \quad & \quad\quad 3x=-7 \\ x=1 \quad &\text{or} \quad x=-\tfrac{7}{3} \end{aligned}$$

The solution set is $\left\{-\frac{7}{3}, 1\right\}$.

**82.** **[2.1]** $0.05x+0.03(1200-x)=42$

$$\begin{aligned} 5x+3(1200-x) &= 4200 \quad \textit{Multiply by 100.} \\ 5x+3600-3x &= 4200 \\ 2x &= 600 \\ x &= 300 \end{aligned}$$

The solution set is $\{300\}$.

**83.** **[2.7]** $|x+3| \le 13$

$$\begin{aligned} -13 \le x+3 \le 13 \\ -16 \le \quad x \quad \le 10 \end{aligned}$$

The solution set is $[-16, 10]$.

**84.** **[2.5]** $\frac{3}{4}(x-2)-\frac{1}{3}(5-2x)<-2$

$$\begin{aligned} 9(x-2)-4(5-2x) &< -24 \quad \textit{Multiply by 12.} \\ 9x-18-20+8x &< -24 \\ 17x-38 &< -24 \\ 17x &< 14 \\ x &< \tfrac{14}{17} \end{aligned}$$

The solution set is $(-\infty, \frac{14}{17})$.

**85.** **[2.5]** $-4<3-2x<9$

$$\begin{aligned} -7<-2x \quad &<6 \quad \textit{Subtract 3.} \\ \frac{7}{2}>x \quad &>-3 \quad \textit{Divide by } -2. \textit{ Reverse inequalities.} \\ -3<x \quad &< \tfrac{7}{2} \quad \textit{Equivalent inequality} \end{aligned}$$

The solution set is $(-3, \frac{7}{2})$.

**86.** **[2.5]** $-0.3x+2.1(x-4) \le -6.6$

$$\begin{aligned} -3x+21(x-4) &\le -66 \quad \textit{Multiply by 10.} \\ -3x+21x-84 &\le -66 \\ 18x-84 &\le -66 \\ 18x &\le 18 \\ x &\le 1 \end{aligned}$$

The solution set is $(-\infty, 1]$.

**87.** **[2.4]** Let $x=$ the angle. Then $90-x$ is its complement and $180-x$ is its supplement. The complement of an angle measures 10° less than one-fifth of its supplement.

$$\begin{aligned} 90-x &= \tfrac{1}{5}(180-x)-10 \\ 450-5x &= 180-x-50 \quad \textit{Multiply by 5.} \\ 450-5x &= 130-x \\ 320 &= 4x \\ 80 &= x \end{aligned}$$

The measure of the angle is 80°.

**88.** **[2.5]** Let $x=$ the employee's earnings during the fifth month. The average of the five months must be at least \$1000.

$$\begin{aligned} \frac{900+1200+1040+760+x}{5} &\ge 1000 \\ 900+1200+1040+760+x &\ge 5000 \\ 3900+x &\ge 5000 \\ x &\ge 1100 \end{aligned}$$

Any amount greater than or equal to \$1100 will qualify the employee for the pension plan.

**89.** **[2.7]** $|5x-1|>14$

$$\begin{aligned} 5x-1>14 \quad &\text{or} \quad 5x-1<-14 \\ 5x>15 \quad & \quad\quad 5x<-13 \\ x>3 \quad &\text{or} \quad x<-\tfrac{13}{5} \end{aligned}$$

The solution set is $(-\infty, -\frac{13}{5}) \cup (3, \infty)$.

**90.** **[2.6]** $x \ge -2$ or $x<4$

The solution set includes all numbers either greater than or equal to $-2$ or all numbers less than 4. This is the union and is the set of all real numbers.

The solution set is $(-\infty, \infty)$.

**91.** **[2.3]** Let $x=$ the number of liters of the 20% solution. Then $x+10$ is the number of liters of the resulting 40% solution.

| Liters of Solution | Percent (as a decimal) | Liters of Mixture |
|---|---|---|
| $x$ | 0.20 | $0.20x$ |
| 10 | 0.50 | $0.50(10)=5$ |
| $x+10$ | 0.40 | $0.40(x+10)$ |

From the last column:

$$\begin{aligned} 0.20x+5 &= 0.40(x+10) \\ 0.20x+5 &= 0.40x+4 \\ 1 &= 0.20x \\ 5 &= x \end{aligned}$$

5 L of the 20% solution should be used.

**92.** **[2.7]** $|x-1| = |2x+3|$

$$\begin{aligned} x-1 &= 2x+3 \quad \text{or} \quad & x-1 &= -(2x+3) \\ & & x-1 &= -2x-3 \\ & & 3x &= -2 \\ -4 &= x \quad \text{or} & x &= -\tfrac{2}{3} \end{aligned}$$

The solution set is $\left\{-4, -\frac{2}{3}\right\}$.

**93.** **[2.1]** $\dfrac{3x}{5} - \dfrac{x}{2} = 3$

$$\begin{aligned} 6x - 5x &= 30 \quad \textit{Multiply by 10.} \\ x &= 30 \end{aligned}$$

The solution set is $\{30\}$.

**94.** **[2.7]** $|x+3| \le 1$

$$\begin{aligned} -1 \le x+3 &\le 1 \\ -4 \le x \quad &\le -2 \end{aligned}$$

The solution set is $[-4, -2]$.

**95.** **[2.7]** $|3x-7| = 4$

$$\begin{aligned} 3x-7 &= 4 \quad \text{or} \quad & 3x-7 &= -4 \\ 3x &= 11 & 3x &= 3 \\ x &= \tfrac{11}{3} \quad \text{or} & x &= 1 \end{aligned}$$

The solution set is $\left\{1, \frac{11}{3}\right\}$.

**96.** **[2.1]** $5(2x-7) = 2(5x+3)$

$$\begin{aligned} 10x - 35 &= 10x + 6 \\ -35 &= 6 \quad \textit{False} \end{aligned}$$

This equation is a *contradiction*.

The solution set is $\emptyset$.

**97.** **[2.7]** **(a)** $|5x+3| < k$

If $k < 0$, then $|5x+3|$ would be less than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\emptyset$.

**(b)** $|5x+3| > k$

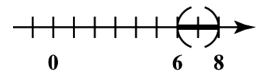
If $k < 0$, then $|5x+3|$ would be greater than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is the set of all real numbers, $(-\infty, \infty)$.

**(c)** $|5x+3| = k$

If $k < 0$, then $|5x+3|$ would be equal to a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\emptyset$.

**98.** **[2.6]** $x > 6$ and $x < 8$

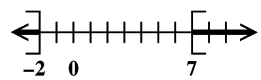
The graph of the solution set is all numbers both greater than 6 *and* less than 8. This is the intersection. The elements common to both sets are the numbers between 6 and 8, not including the endpoints. The solution set is $(6, 8)$.




**99.** **[2.6]** $-5x+1 \ge 11$ or $3x+5 \ge 26$

$$\begin{aligned} -5x &\ge 10 & 3x &\ge 21 \\ x &\le -2 \quad \text{or} & x &\ge 7 \end{aligned}$$

The graph of the solution set is all numbers either less than or equal to $-2$ *or* greater than or equal to 7. This is the union. The solution set is $(-\infty, -2] \cup [7, \infty)$.




**100.** **[2.6]** **(a)** All states have less than 3 million female workers. The set of states with more than 3 million male workers is {Illinois}. Illinois is the only state in both sets, so the set of states with less than 3 million female workers *and* more than 3 million male workers is {Illinois}.

**(b)** The set of states with less than 1 million female workers *or* more than 2 million male workers is {Illinois, Maine, North Carolina, Oregon, Utah}.

**(c)** It is easy to see that the sum of the female and male workers for each state doesn't exceed 7 million, so the set of states with a total of more than 7 million civilian workers is {}, or $\emptyset$.

## Chapter 2 Test

**1.**

$$\begin{aligned} 3(2x-2) - 4(x+6) &= 3x + 8 + x \\ 6x - 6 - 4x - 24 &= 4x + 8 \\ 2x - 30 &= 4x + 8 \\ -2x &= 38 \\ x &= -19 \end{aligned}$$

The solution set is $\{-19\}$.

**2.**

$$\begin{aligned} 0.08x + 0.06(x+9) &= 1.24 \\ 8x + 6(x+9) &= 124 \\ & \quad \textit{Multiply by 100.} \\ 8x + 6x + 54 &= 124 \\ 14x + 54 &= 124 \\ 14x &= 70 \\ x &= 5 \end{aligned}$$

The solution set is $\{5\}$.

**3.** $$\frac{x+6}{10}+\frac{x-4}{15}=\frac{x+2}{6}$$
$$3(x+6)+2(x-4)=5(x+2) \quad \textit{Multiply by 30.}$$
$$3x+18+2x-8=5x+10$$
$$5x+10=5x+10 \quad \textit{True}$$

This is an *identity*.

The solution set is $\{$all real numbers$\}$.

**4.** **(a)** $$3x-(2-x)+4x+2=8x+3$$
$$3x-2+x+4x+2=8x+3$$
$$8x=8x+3$$
$$0=3 \quad \textit{False}$$

The false statement indicates that the equation is a *contradiction*.

The solution set is $\emptyset$.

**(b)** $$\frac{x}{3}+7=\frac{5x}{6}-2-\frac{x}{2}+9$$
Multiply each side by the LCD, 6.
$$2x+42=5x-12-3x+54$$
$$2x+42=2x+42$$
$$0=0 \quad \textit{True}$$

This equation is an *identity.*

The solution set is $\{$all real numbers$\}$.

**(c)** $$-4(2x-6)=5x+24-7x$$
$$-8x+24=-2x+24$$
$$24=6x+24$$
$$0=6x$$
$$0=x$$

This is a *conditional equation.*

The solution set is $\{0\}$.

**5.** Solve $V=\frac{1}{3}bh$ for $h$.
$$V=\tfrac{1}{3}bh$$
$$3V=bh \quad \textit{Multiply by 3.}$$
$$\frac{3V}{b}=h \quad \textit{Divide by b.}$$

**6.** Solve $-16t^2+vt-S=0$ for $v$.
$$vt=S+16t^2 \quad \textit{Add S, 16t}^2.$$
$$v=\frac{S+16t^2}{t} \quad \textit{Divide by t.}$$

**7.** Solve $d=rt$ for $t$ and substitute 500 for $d$ and 150.318 for $r$.
$$t=\frac{d}{r}=\frac{500}{150.318}\approx 3.326$$

His time was about 3.326 hr.

**8.** Use $I=Prt$ and substitute \$2281.25 for $I$, \$36,500 for $P$, and 1 for $t$.
$$2281.25=36{,}500r(1)$$
$$r=\frac{2281.25}{36{,}500}=0.0625$$

The rate of interest is 6.25%.

**9.** $$\frac{27{,}232}{36{,}723}\approx 0.742$$

74.2% were classified as post offices.

**10.** Let $x=$ the amount invested at 3%. Then $28{,}000-x=$ the amount invested at 5%.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.03 | $0.03x$ |
| $28{,}000-x$ | 0.05 | $0.05(28{,}000-x)$ |
| \$28,000 | ← Totals → | \$1240 |

From the last column:
$$0.03x+0.05(28{,}000-x)=1240$$
$$3x+5(28{,}000-x)=124{,}000 \quad \textit{Multiply by 100.}$$
$$3x+140{,}000-5x=124{,}000$$
$$-2x=-16{,}000$$
$$x=8000$$

He invested \$8000 at 3% and \$28,000 − \$8000 = \$20,000 at 5%.

**11.** Let $x=$ the rate of the slower car. Then $x+15=$ the rate of the faster car.

Use the formula $d=rt$.

| | Rate | Time | Distance |
|---|---|---|---|
| **Slower Car** | $x$ | 6 | $6x$ |
| **Faster Car** | $x+15$ | 6 | $6(x+15)$ |
| | | | 630 |

The total distance traveled is the sum of the distances traveled by each car.
$$6x+6(x+15)=630$$
$$6x+6x+90=630$$
$$12x=540$$
$$x=45$$

The slower car traveled at 45 mph, while the faster car traveled at $45+15=60$ mph.

**12.** The sum of the three angle measures is $180°$.

$$\begin{aligned}(2x+20)+x+x&=180\\4x+20&=180\\4x&=160\\x&=40\end{aligned}$$

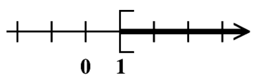
The three angle measures are $40°$, $40°$, and $(2 \cdot 40+20)° = 100°$.

**13.**
$$\begin{aligned}4-6(x+3)&\le -2-3(x+6)+3x\\4-6x-18&\le -2-3x-18+3x\\-6x-14&\le -20\\-6x&\le -6\end{aligned}$$

Divide by $-6$, and reverse the inequality symbol.

$$x \ge 1$$

The solution set is $[1,\infty)$.

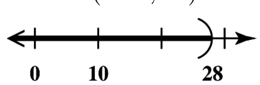


**14.**
$$-\frac{4}{7}x > -16$$
$$-4x > -112 \qquad \textit{Multiply by 7.}$$

Divide by $-4$, and reverse the inequality symbol.

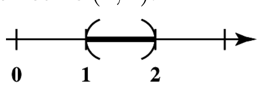
$$x < 28$$

The solution set is $(-\infty, 28)$.



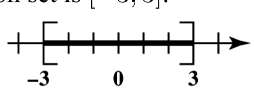
**15.**
$$\begin{aligned}-1 < 3x-4 &< 2\\3 < 3x &< 6 \quad \textit{Add 4.}\\1 < x &< 2 \quad \textit{Divide by 3.}\end{aligned}$$

The solution set is $(1,2)$.



**16.**
$$\begin{aligned}-6 \le \tfrac{4}{3}x-2 &\le 2\\-18 \le 4x-6 &\le 6 \quad \textit{Multiply by 3.}\\-12 \le 4x &\le 12 \quad \textit{Add 6.}\\-3 \le x &\le 3 \quad \textit{Divide by 4.}\end{aligned}$$

The solution set is $[-3,3]$.



**17.** For each inequality, divide both sides by $-3$ and reverse the direction of the inequality symbol.

**A.** $-3x < 9$, $x > -3$ **B.** $-3x > -9$, $x < 3$

**C.** $-3x > 9$, $x < -3$ **D.** $-3x < -9$, $x > 3$

Thus, inequality **C** is equivalent to $x < -3$.

**18.** Let $x =$ the score on the fourth test.

$$\begin{aligned}\frac{83+76+79+x}{4}&\ge 80\\\frac{238+x}{4}&\ge 80\\238+x&\ge 320\\x&\ge 82\end{aligned}$$

The minimum score must be 82 to guarantee a B.

**19.** **(a)** $A \cap B = \{1,2,5,7\} \cap \{1,5,9,12\} = \{1,5\}$

**(b)** $A \cup B = \{1,2,5,7\} \cup \{1,5,9,12\} = \{1,2,5,7,9,12\}$

**20.** $3x \ge 6$ and $x-4 < 5$
$x \ge 2$ and $x < 9$

The solution set is all numbers both greater than or equal to 2 *and* less than 9. This is the intersection. The numbers common to both sets are between 2 and 9, including 2 but not 9. The solution set is $[2,9)$.

**21.** $-4x \le -24$ or $4x-2 < 10$
$4x < 12$
$x \ge 6$ or $x < 3$

The solution set is all numbers less than 3 or greater than or equal to 6. This is the union. The solution set is $(-\infty,3) \cup [6,\infty)$.

**22.** $|4x+3| \le 7$

$$\begin{aligned}-7 \le 4x+3 &\le 7\\-10 \le 4x &\le 4\\-\tfrac{10}{4} \le x &\le \tfrac{4}{4}\\-\tfrac{5}{2} \le x &\le 1\end{aligned}$$

The solution set is $[-\frac{5}{2}, 1]$.

**23.** $|5-6x| > 12$

$5-6x > 12$ or $5-6x < -12$
$-6x > 7$ $\quad -6x < -17$
$x < -\frac{7}{6}$ or $x > \frac{17}{6}$

The solution set is $(-\infty, -\frac{7}{6}) \cup (\frac{17}{6}, \infty)$.

**24.** $|7-x| \le -1$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is *false* for any real number $x$.

The solution set is $\emptyset$.

**25.** $|-3x+4|-4<-1$

$|-3x+4|<3$

$-3<-3x+4<3$

$-7<-3x<-1$

$\frac{7}{3}>x>\frac{1}{3}$ *Reverse inequalities.*

$\frac{1}{3}<x<\frac{7}{3}$ *Equivalent inequality*

The solution set is $\left(\frac{1}{3},\frac{7}{3}\right)$.

**26.** $|3x-2|+1=8$

$|3x-2|=7$

$3x-2=7$ or $3x-2=-7$

$3x=9$ or $3x=-5$

$x=\frac{9}{3}=3$ or $x=-\frac{5}{3}$

The solution set is $\left\{-\frac{5}{3},3\right\}$.

**27.** $|3-5x|=|2x+8|$

$3-5x=2x+8$ or $3-5x=-(2x+8)$

$-7x=5$ $\quad$ $3-5x=-2x-8$

$-3x=-11$

$x=-\frac{5}{7}$ or $x=\frac{11}{3}$

The solution set is $\left\{-\frac{5}{7},\frac{11}{3}\right\}$.

**28.** **(a)** $|8x-5|<k$

If $k<0$, then $|8x-5|$ would be less than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\emptyset$.

**(b)** $|8x-5|>k$

If $k<0$, then $|8x-5|$ would be greater than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is the set of all real numbers, $(-\infty,\infty)$.

**(c)** $|8x-5|=k$

If $k<0$, then $|8x-5|$ would be equal to a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\emptyset$.

## Cumulative Review Exercises (Chapters 1–2)

Exercises 1–6 refer to set $A$.
Let $A=\left\{-8,-\frac{2}{3},-\sqrt{6},0,\frac{4}{5},9,\sqrt{36}\right\}$.

Simplify $\sqrt{36}$ to 6.

**1.** The elements 9 and 6 are natural numbers.

**2.** The elements 0, 9, and 6 are whole numbers.

**3.** The elements $-8$, 0, 9, and 6 are integers.

**4.** The elements $-8,-\frac{2}{3},0,\frac{4}{5},9,$ and 6 are rational numbers.

**5.** The element $-\sqrt{6}$ is an irrational number.

**6.** All the elements in set $A$ are real numbers.

**7.** $-\frac{4}{3}-\left(-\frac{2}{7}\right)=-\frac{4}{3}+\frac{2}{7}$

$=-\frac{28}{21}+\frac{6}{21}$

$=-\frac{22}{21}$

**8.** $|-4|-|2|+|-6|=4-2+6$

$=2+6$

$=8$

**9.** $(-2)^4+(-2)^3=16+(-8)=8$

**10.** $\sqrt{25}-5(-1)^0=5-5(1)=5-5=0$

**11.** $(-3)^5=(-3)(-3)(-3)(-3)(-3)=-243$

**12.** $\left(\frac{6}{7}\right)^3=\frac{6}{7}\cdot\frac{6}{7}\cdot\frac{6}{7}=\frac{216}{343}$

**13.** $\left(-\frac{2}{3}\right)^3=\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)=-\frac{8}{27}$

**14.** $-4^6=-(4\cdot4\cdot4\cdot4\cdot4\cdot4)=-4096$

For Exercises 15–17, let $a=2$, $b=-3$, and $c=4$.

**15.** $-3a+2b-c=-3(2)+2(-3)-4$

$=-6-6-4$

$=-16$

**16.** $-8\left(a^2+b^3\right)=-8\left[2^2+(-3)^3\right]$

$=-8[4+(-27)]$

$=-8(-23)$

$=184$

**17.** $\dfrac{3a^3-b}{4+3c}=\dfrac{3(2)^3-(-3)}{4+3(4)}$

$=\dfrac{3(8)-(-3)}{4+3(4)}$

$=\dfrac{24+3}{4+12}$

$=\frac{27}{16}$

**18.** $-7r+5-13r+12$

$=-7r-13r+5+12$

$=(-7-13)r+(5+12)$

$=-20r+17$

**19.** $-(3k+8)-2(4k-7)+3(8k+12)$

$=-3k-8-8k+14+24k+36$

$=-3k-8k+24k-8+14+36$

$=13k+42$

**20.** $(a+b)+4=4+(a+b)$

The order of the terms $(a+b)$ and 4 have been reversed. This is an illustration of the commutative property.

**21.** $4x + 12x = (4 + 12)x$

The common variable, $x$, has been removed from each term. This is an illustration of the distributive property.

**22.**
$$\begin{aligned} -4x + 7(2x + 3) &= 7x + 36 \\ -4x + 14x + 21 &= 7x + 36 \\ 10x + 21 &= 7x + 36 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

The solution set is $\{5\}$.

**23.**
$$\begin{aligned} -\tfrac{3}{5}x + \tfrac{2}{3}x &= 2 \\ 3(-3x) + 5(2x) &= 15(2) \quad \textit{Multiply by 15.} \\ -9x + 10x &= 30 \\ x &= 30 \end{aligned}$$

The solution set is $\{30\}$.

**24.**
$$\begin{aligned} 0.06x + 0.03(100 + x) &= 4.35 \\ 6x + 3(100 + x) &= 435 \quad \textit{Multiply by 100.} \\ 6x + 300 + 3x &= 435 \\ 9x &= 135 \\ x &= 15 \end{aligned}$$

The solution set is $\{15\}$.

**25.** Solve $P = a + b + c$ for $b$.

$P - a - c = b$, or $b = P - a - c$

**26.**
$$\begin{aligned} 4(2x - 6) + 3(x - 2) &= 11x + 1 \\ 8x - 24 + 3x - 6 &= 11x + 1 \\ 11x - 30 &= 11x + 1 \\ -30 &= 1 \quad \textit{False} \end{aligned}$$

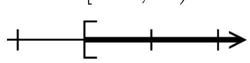
The solution set is $\emptyset$.

**27.**
$$\tfrac{2}{3}x + \tfrac{5}{8}x = \tfrac{31}{24}x$$
Multiply by the LCD, 24.
$$\begin{aligned} 8(2x) + 3(5x) &= 31x \\ 16x + 15x &= 31x \\ 31x &= 31x \quad \textit{True} \end{aligned}$$

The solution set is {all real numbers}.

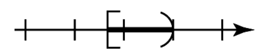
**28.**
$$\begin{aligned} 3 - 2(x + 7) &\le -x + 3 \\ 3 - 2x - 14 &\le -x + 3 \\ -2x - 11 &\le -x + 3 \\ -x &\le 14 \\ x &\ge -14 \quad \textit{Multiply by } -1. \textit{ Reverse inequality.} \end{aligned}$$

The solution set is $[-14, \infty)$.



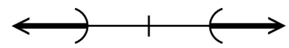

**29.**
$$\begin{aligned} -4 < 5 - 3x &\le 0 \\ -9 < -3x &\le -5 \quad \textit{Subtract 5.} \\ 3 > x &\ge \frac{5}{3} \quad \textit{Divide by } -3. \textit{ Reverse inequalities.} \\ \tfrac{5}{3} \le x &< 3 \quad \textit{Equivalent inequality} \end{aligned}$$

The solution set is $[\frac{5}{3}, 3)$.




**30.**
$$\begin{aligned} 2x + 1 > 5 \quad &\text{or} \quad 2 - x > 2 \\ 2x > 4 \quad & \quad\quad -x > 0 \\ x > 2 \quad &\text{or} \quad x < 0 \end{aligned}$$

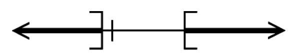
The solution set is $(-\infty, 0) \cup (2, \infty)$.




**31.** $|-7k + 3| \ge 4$

$$\begin{aligned} -7k + 3 \ge 4 \quad &\text{or} \quad -7k + 3 \le -4 \\ -7k \ge 1 \quad & \quad\quad -7k \le -7 \\ k \le -\tfrac{1}{7} \quad &\text{or} \quad k \ge 1 \end{aligned}$$

The solution set is $(-\infty, -\frac{1}{7}] \cup [1, \infty)$.




**32.** Let $x =$ the amount invested at 5%. Then $x + 2000 =$ the amount invested at 6%.

Use $I = prt$ and create a table.

| Principal | Rate (as a decimal) | Interest |
|---|---|---|
| $x$ | 0.05 | $0.05x$ |
| $x + 2000$ | 0.06 | $0.06(x + 2000)$ |
| | Total → | \$670 |

From the last column:

$$\begin{aligned} 0.05x + 0.06(x + 2000) &= 670 \\ 5x + 6(x + 2000) &= 67{,}000 \\ &\quad \textit{Multiply by 100.} \\ 5x + 6x + 12{,}000 &= 67{,}000 \\ 11x &= 55{,}000 \\ x &= 5000 \end{aligned}$$

\$5000 was invested at 5% and \$7000 at 6%.

**33.** Let $x =$ the amount of pure alcohol that should be added.

| Liters of Solution | Percent (as a decimal) | Liters of Pure Alcohol |
|---|---|---|
| $x$ | 1.00 | $1.00x$ |
| 7 | 0.10 | $0.10(7) = 0.7$ |
| $x + 7$ | 0.30 | $0.30(x + 7)$ |

From the last column:

$$\begin{aligned} 1.00x + 0.7 &= 0.30(x + 7) \\ 10x + 7 &= 3(x + 7) \quad \textit{Multiply by 10.} \\ 10x + 7 &= 3x + 21 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

2 L of pure alcohol should be added to the solution.

**34.** Clark's rule:

$$\frac{\text{Weight of child in pounds}}{150} \times \begin{matrix}\text{adult}\\ \text{dose}\end{matrix} = \begin{matrix}\text{child's}\\ \text{dose}\end{matrix}$$

If the child weighs 55 lb and the adult dosage is 120 mg, then

$$\frac{55}{150} \times 120 = 44.$$

The child's dosage is 44 mg.

**35.** **(a)** In 1975, there were 1756 daily newspapers. In 2008, there were 1408 daily newspapers. The difference is $1756 - 1408 = 348$, so the number of daily newspapers decreased by 348 from 1975 to 2008.

**(b)** $\frac{348}{1756} \approx 0.198$, or 19.8%.

The number of daily newspapers decreased by approximately 19.8% from 1975 to 2008.