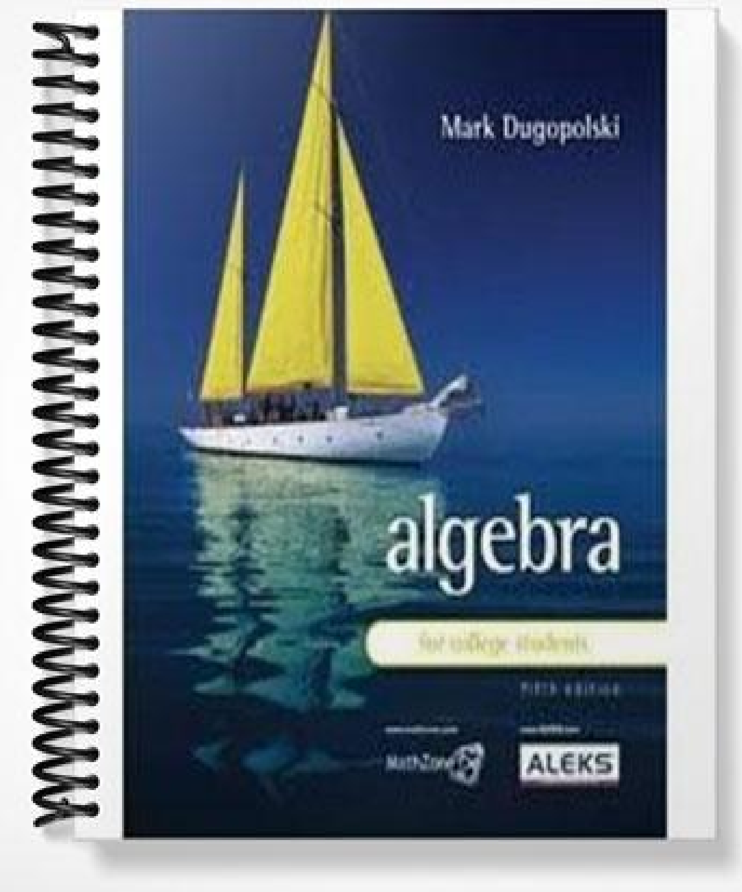
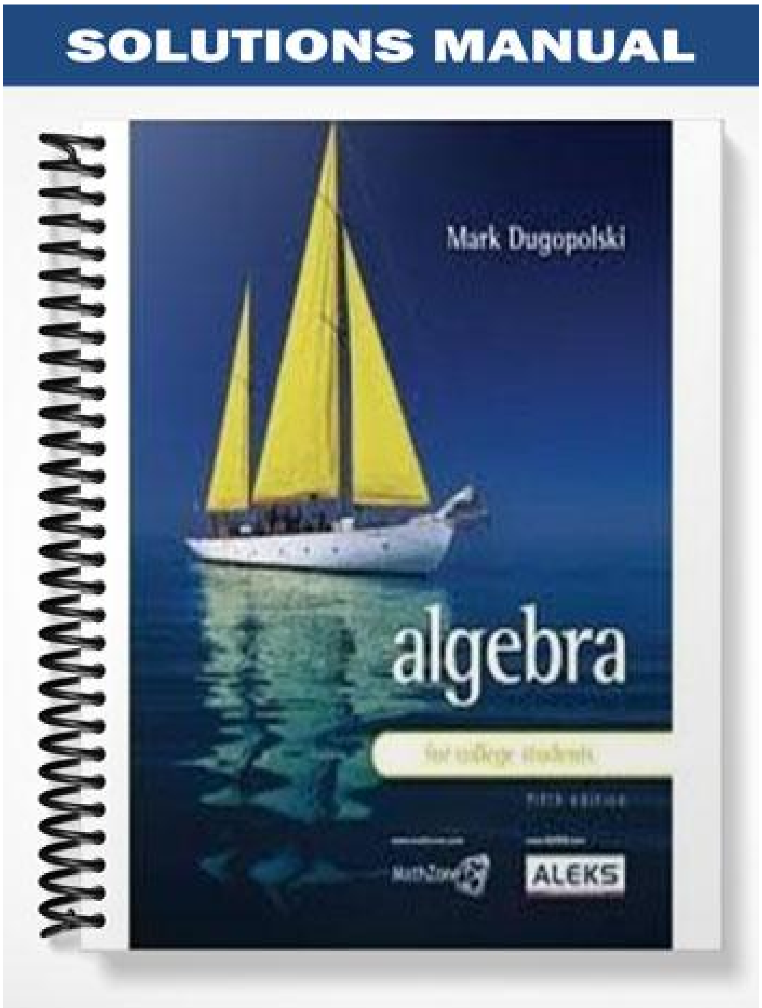
SOLUTIONS MANUAL




SOLUTIONS MANUAL
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algebra
for college students
ALEKS

2.1 WARM-UPS

1. False, it is equivalent to $-2x = 5$.
2. True, because
$x - (x - 3) = x - x + 3 = 3$.
3. False, multiply each side by $\frac{4}{3}$.
4. True, because we can multiply each side by -1.
5. True, because all of the denominators divide into the LCD.
6. True, subtract 5 from each side and then divide each side by 3.
7. True, because it simplifies to $8 = 12$.
8. True, it is equivalent to $x = -3$.
9. True, it is equivalent to $0.8x = 0.8x$.
10. True, since it is equivalent to $3x = 12$.

2.1 EXERCISES

1. An equation is a sentence that expresses the equality of two algebraic expressions.
2. A number satisfies the equation if the equation is true when the variable is replaced by the number.
3. Equivalent equation are equations that have the same solution set.
4. A linear equation in one variable is an equation of the form $ax = b$ with $a \neq 0$.
5. If the equation involves fractions then multiply each side by the LCD.
6. An identity is an equation that is satisfied by all values of the variable for which both sides are defined.
7. A conditional equation is an equation that has at least one solution but is not an identity.
8. An inconsistent equation is an equation that has no solution.
9. Yes, because $3 \cdot (-4) + 7 = -5$ is correct.
10. Yes, because $-3(-6) - 5 = 13$ is correct.
11. Yes, because $\frac{1}{2} \cdot 12 - 4 = \frac{1}{3} \cdot 12 - 2$.
12. Yes, because $\frac{9-7}{2} - \frac{1}{3} = \frac{9-7}{3}$.
13. No, because $0.2(200 - 50) = 30$ and $20 - 0.05 \cdot 200 = 10$.
14. No because $0.1 \cdot 80 - 30 = -22$ and $16 - 0.06 \cdot 80 = 11.2$.
15.
$$\begin{aligned} x + 3 &= 24 \\ x + 3 - 3 &= 24 - 3 \\ x &= 21 \end{aligned}$$
The solution set is $\{21\}$.
16.
$$\begin{aligned} x - 5 &= 12 \\ x - 5 + 5 &= 12 + 5 \\ x &= 17 \end{aligned}$$
The solution set is $\{17\}$.
17.
$$\begin{aligned} 5x &= 20 \\ \frac{1}{5} \cdot 5x &= \frac{1}{5} \cdot 20 \\ x &= 4 \end{aligned}$$
The solution set is $\{4\}$.
18.
$$\begin{aligned} 3x &= 51 \\ \frac{1}{3} \cdot 3x &= \frac{1}{3} \cdot 51 \\ x &= 17 \end{aligned}$$
The solution set is $\{17\}$.
19.
$$\begin{aligned} 2x - 3 &= 25 \\ 2x - 3 + 3 &= 25 + 3 \\ 2x &= 28 \\ \frac{1}{2} \cdot 2x &= \frac{1}{2} \cdot 28 \\ x &= 14 \end{aligned}$$
The solution set is $\{14\}$.
20.
$$\begin{aligned} 3x + 5 &= 26 \\ 3x + 5 - 5 &= 26 - 5 \\ 3x &= 21 \\ \frac{1}{3} \cdot 3x &= \frac{1}{3} \cdot 21 \\ x &= 7 \end{aligned}$$
The solution set is $\{7\}$.
21.
$$\begin{aligned} -72 - x &= 15 \\ -72 - x + 72 &= 15 + 72 \\ -x &= 87 \\ (-1)(-x) &= -1 \cdot 87 \\ x &= -87 \end{aligned}$$
The solution set is $\{-87\}$.
22.
$$\begin{aligned} 51 - x &= -9 \\ -x &= -9 - 51 \\ -x &= -60 \\ x &= 60 \end{aligned}$$
The solution set is $\{60\}$.
23.
$$\begin{aligned} -3x - 19 &= 5 - 2x \\ -3x &= 24 - 2x \\ -3x + 2x &= 24 \\ -x &= 24 \\ x &= -24 \end{aligned}$$
The solution set is $\{-24\}$.
24.
$$\begin{aligned} -5x + 4 &= -9 - 4x \\ -5x &= -13 - 4x \\ -x &= -13 \\ x &= 13 \end{aligned}$$
The solution set is $\{13\}$.

25.
$$\begin{aligned} 2x - 3 &= 0 \\ 2x - 3 + 3 &= 0 + 3 \\ 2x &= 3 \\ \frac{2x}{2} &= \frac{3}{2} \\ x &= \frac{3}{2} \end{aligned}$$
The solution set is $\left\{\frac{3}{2}\right\}$.

26.
$$\begin{aligned} 5x + 7 &= 0 \\ 5x + 7 - 7 &= 0 - 7 \\ 5x &= -7 \\ x &= -\frac{7}{5} \end{aligned}$$
The solution set is $\left\{-\frac{7}{5}\right\}$.

27.
$$\begin{aligned} -2x + 5 &= 7 \\ -2x + 5 - 5 &= 7 - 5 \\ -2x &= 2 \\ \frac{-2x}{-2} &= \frac{2}{-2} \\ x &= -1 \end{aligned}$$
The solution set is $\{-1\}$.

28.
$$\begin{aligned} -3x - 4 &= 11 \\ -3x &= 15 \\ x &= -5 \end{aligned}$$
The solution set is $\{-5\}$.

29.
$$\begin{aligned} -12x - 15 &= 21 \\ -12x - 15 + 15 &= 21 + 15 \\ -12x &= 36 \\ \frac{-12x}{-12} &= \frac{36}{-12} \\ x &= -3 \end{aligned}$$
The solution set is $\{-3\}$.

30.
$$\begin{aligned} -13x + 7 &= -19 \\ -13x &= -26 \\ x &= 2 \end{aligned}$$
The solution set is $\{2\}$.

31.
$$\begin{aligned} 26 &= 4x + 16 \\ 26 - 16 &= 4x + 16 - 16 \\ 10 &= 4x \\ \frac{10}{4} &= \frac{4x}{4} \\ \frac{5}{2} &= x \end{aligned}$$
The solution set is $\left\{\frac{5}{2}\right\}$.

32.
$$\begin{aligned} 14 &= -5x - 21 \\ 35 &= -5x \\ -7 &= x \end{aligned}$$
The solution set is $\{-7\}$.

33.
$$\begin{aligned} -3(x - 16) &= 12 - x \\ -3x + 48 &= 12 - x \\ -3x &= -36 - x \\ -2x &= -36 \\ x &= 18 \end{aligned}$$
The solution set is $\{18\}$.

34.
$$\begin{aligned} -2(x + 17) &= 13 - x \\ -2x - 34 &= 13 - x \\ -2x &= 47 - x \\ -x &= 47 \\ x &= -47 \end{aligned}$$
The solution set is $\{-47\}$.

35.
$$\begin{aligned} 2(x + 9) - x &= 36 \\ 2x + 18 - x &= 36 \\ x + 18 &= 36 \\ x + 18 - 18 &= 36 - 18 \\ x &= 18 \end{aligned}$$
The solution set is $\{18\}$.

36.
$$\begin{aligned} 3(x - 13) - x &= 9 \\ 3x - 39 - x &= 9 \\ 2x - 39 &= 9 \\ 2x &= 48 \\ x &= 24 \end{aligned}$$
The solution set is $\{24\}$.

37.
$$\begin{aligned} 2 + 3(x - 1) &= x - 1 \\ 2 + 3x - 3 &= x - 1 \\ 3x - 1 &= x - 1 \\ 3x - x &= -1 + 1 \\ 2x &= 0 \\ x &= 0 \end{aligned}$$
The solution set is $\{0\}$.

38.
$$\begin{aligned} x + 9 &= 1 - 4x + 8 \\ x + 9 &= 9 - 4x \\ 5x &= 0 \\ x &= 0 \end{aligned}$$
The solution set is $\{0\}$.

39.
$$\begin{aligned} -\frac{3}{7}x &= 4 \\ -\frac{7}{3}\left(-\frac{3}{7}x\right) &= -\frac{7}{3}(4) \\ x &= -\frac{28}{3} \end{aligned}$$
The solution set is $\left\{-\frac{28}{3}\right\}$.

40.
$$\begin{aligned} \frac{5}{6}x &= -2 \\ \frac{6}{5}\left(\frac{5}{6}x\right) &= \frac{6}{5}(-2) \\ x &= -\frac{12}{5} \end{aligned}$$
The solution set is $\left\{-\frac{12}{5}\right\}$.

41. $-\frac{5}{7}x - 1 = 3$

$$-\frac{5}{7}x = 4$$
$$-\frac{7}{5}\left(-\frac{5}{7}x\right) = -\frac{7}{5}(4)$$
$$x = -\frac{28}{5}$$

The solution set is $\left\{-\frac{28}{5}\right\}$.

42. $4 - \frac{3}{5}x = -6$

$$5\left(4 - \frac{3}{5}x\right) = 5(-6)$$
$$20 - 3x = -30$$
$$-3x = -50$$
$$x = \frac{50}{3}$$

The solution set is $\left\{\frac{50}{3}\right\}$.

43. $\frac{x}{3} + \frac{1}{2} = \frac{7}{6}$

$$6\left(\frac{x}{3} + \frac{1}{2}\right) = 6 \cdot \frac{7}{6}$$
$$2x + 3 = 7$$
$$2x = 4$$
$$x = 2$$

The solution set is $\{2\}$.

44. $\frac{1}{4} + \frac{1}{5} = \frac{x}{2}$

$$20\left(\frac{1}{4} + \frac{1}{5}\right) = 20 \cdot \frac{x}{2}$$
$$5 + 4 = 10x$$
$$9 = 10x$$
$$\frac{9}{10} = x$$

The solution set is $\left\{\frac{9}{10}\right\}$.

45. $\frac{2}{3}x + 5 = -\frac{1}{3}x + 17$

$$3\left(\frac{2}{3}x + 5\right) = 3\left(-\frac{1}{3}x + 17\right)$$
$$2x + 15 = -x + 51$$
$$3x + 15 = 51$$
$$3x = 36$$
$$x = 12$$

The solution set is $\{12\}$.

46. $\frac{1}{4}x - 6 = -\frac{3}{4}x + 14$

$$4\left(\frac{1}{4}x - 6\right) = 4\left(-\frac{3}{4}x + 14\right)$$
$$x - 24 = -3x + 56$$
$$4x - 24 = 56$$
$$4x = 80$$
$$x = 20$$

The solution set is $\{20\}$.

47. $\frac{1}{2}x + \frac{1}{4} = \frac{1}{4}\left(x - 6\right)$

$$4\left(\frac{1}{2}x + \frac{1}{4}\right) = 4 \cdot \frac{1}{4}\left(x - 6\right)$$
$$4\left(\frac{1}{2}x + \frac{1}{4}\right) = 4 \cdot \frac{1}{4}\left(x - 6\right)$$
$$2x + 1 = x - 6$$
$$x + 1 = -6$$
$$x = -7$$

The solution set is $\{-7\}$.

48. $\frac{1}{3}(x - 2) = \frac{2}{3}x - \frac{13}{3}$

$$3 \cdot \frac{1}{3}(x - 2) = 3 \cdot \frac{2}{3}x - 3 \cdot \frac{13}{3}$$
$$x - 2 = 2x - 13$$
$$x = 2x - 11$$
$$-x = -11$$
$$x = 11$$

The solution set is $\{11\}$.

49. $8 - \frac{x-2}{2} = \frac{x}{4}$

$$4\left(8 - \frac{x-2}{2}\right) = 4\left(\frac{x}{4}\right)$$
$$32 - 2(x - 2) = x$$
$$32 - 2x + 4 = x$$
$$-3x = -36$$
$$x = 12$$

The solution set is $\{12\}$.

50. $\frac{x}{3} - \frac{x-5}{5} = 3$

$$15 \cdot \frac{x}{3} - 15 \cdot \frac{x-5}{5} = 15 \cdot 3$$
$$5x - 3(x - 5) = 45$$
$$5x - 3x + 15 = 45$$
$$2x + 15 = 45$$
$$2x = 30$$
$$x = 15$$

The solution set is $\{15\}$.

51. $\frac{y-3}{3} - \frac{y-2}{2} = -1$

$$6\left(\frac{y-3}{3}\right) - 6\left(\frac{y-2}{2}\right) = 6(-1)$$
$$2y - 6 - 3y + 6 = -6$$
$$-y = -6$$
$$y = 6$$

The solution set is $\{6\}$.

52. $\frac{x-2}{2} - \frac{x-3}{4} = \frac{7}{4}$

$$4 \cdot \frac{x-2}{2} - 4 \cdot \frac{x-3}{4} = 4 \cdot \frac{7}{4}$$
$$2(x - 2) - (x - 3) = 7$$
$$2x - 4 - x + 3 = 7$$
$$x - 1 = 7$$
$$x = 8$$

The solution set is $\{8\}$.

53. $x - 0.2x = 72$
$10(x - 0.2x) = 10(72)$
$10x - 2x = 720$
$8x = 720$
$x = 90$
The solution set is $\{90\}$.
54. $x - 0.1x = 63$
$0.9x = 63$
$x = \frac{63}{0.9} = 70$
The solution set is $\{70\}$.
55. $0.03(x + 200) + 0.05x = 86$
$0.03x + 0.03(200) + 0.05x = 86$
$0.08x + 6 = 86$
$0.08x = 80$
$x = 1000$
The solution set is $\{1000\}$.
56. $0.02(x - 100) + 0.06x = 62$
$0.02x - 2 + 0.06x = 62$
$0.08x = 64$
$x = 800$
The solution set is $\{800\}$.
57. $0.1x + 0.05(x - 300) = 105$
$0.1x + 0.05x - 0.05(300) = 105$
$0.15x - 15 = 105$
$0.15x = 120$
$x = 800$
The solution set is $\{800\}$.
58. $0.2x - 0.05(x - 100) = 35$
$0.20x - 0.05x + 5 = 35$
$0.15x + 5 = 35$
$0.15x = 30$
$x = 200$
The solution set is $\{200\}$.
59. $2(x + 1) = 2(x + 3)$
$2x + 2 = 2x + 6$
$2 = 6$
The solution set is $\emptyset$. The equation is inconsistent.
60. $2x + 3x = 6x$
$5x = 6x$
$0 = x$
The solution set is $\{0\}$. The equation is conditional.
61. $x + x = 2x$
$2x = 2x$
The solution set is R. The equation is an identity.
62. $4x - 3x = x$
$x = x$
The solution set is the set of all real numbers, R. The equation is an identity.
63. $x + x = 2$
$2x = 2$
$x = 1$
The solution set is $\{1\}$ and the equation is conditional.
64. $4x - 3x = 5$
$x = 5$
The solution set is $\{5\}$. The equation is conditional.
65. $\frac{4x}{4} = x$
$x = x$
The solution set is R and the equation is an identity.
66. $5x \div 5 = x$
$x = x$
The solution set is R. The equation is an identity.
67. $x \cdot x = x^2$
$x^2 = x^2$
The solution set is R and the equation is an identity.
68. $\frac{2x}{2x} = 1$

$\frac{x}{x} = 1$
This equation is satisfied by any nonzero real number. The solution set is $\{x \mid x \neq 0\}$. The equation is an identity.
69. $2(x + 3) - 7 = 5(5 - x) + 7(x + 1)$
$2x + 6 - 7 = 25 - 5x + 7x + 7$
$2x - 1 = 32 + 2x$
$-1 = 32$
The solution set is $\emptyset$. The equation is inconsistent.
70. $2(x + 4) - 8 = 2x + 1$
$2x + 8 - 8 = 2x + 1$
$2x = 2x + 1$
$0 = 1$
The solution set is $\emptyset$. The equation is inconsistent.
71. $2\left(\frac{1}{2}x + \frac{3}{2}\right) - \frac{7}{2} = \frac{3}{2}(x + 1) - \left(\frac{1}{2}x + 2\right)$

$x + 3 - \frac{7}{2} = \frac{3}{2}x + \frac{3}{2} - \frac{1}{2}x - 2$
$2x + 6 - 7 = 3x + 3 - x - 4$
$2x - 1 = 2x - 1$
The solution set to this identity is R.

72. $2\left(\frac{1}{4}x+1\right)-2=\frac{1}{2}x$
$$\frac{1}{2}x+2-2=\frac{1}{2}x$$
$$\frac{1}{2}x=\frac{1}{2}x$$
The solution set to this identity is R.

73. $2(0.5x+1.5)-3.5=3(0.5x+0.5)$
$$x+3-3.5=1.5x+1.5$$
$$x-0.5=1.5x+1.5$$
$$10x-5=15x+15$$
$$-20=5x$$
$$-4=x$$
The solution set is to this conditional equation is $\{-4\}$.

74. $2(0.25x+1)-2=0.75x-1.75$
$$0.5x+2-2=0.75x-1.75$$
$$0.5x=0.75x-1.75$$
$$50x=75x-175$$
$$-25x=-175$$
$$x=7$$
The solution set to this conditional equation is $\{7\}$.

75. $4-6(2x-3)+1=3+2(5-x)$
$$4-12x+18+1=3+10-2x$$
$$-12x+23=13-2x$$
$$-10x=-10$$
$$x=1$$
The solution set is $\{1\}$.

76. $3x-5(6-2x)=4(x-8)+3$
$$3x-30+10x=4x-32+3$$
$$13x-30=4x-29$$
$$9x=1$$
$$x=\frac{1}{9}$$
The solution set is $\left\{\frac{1}{9}\right\}$.

77. $5x-2(3x+6)=4-(2+x)+7$
$$5x-6x-12=4-2-x+7$$
$$-x-12=-x+9$$
$$-12=9$$
The solution set is the empty set, $\emptyset$.

78. $-1+5(2x-3)=16x-2(3x+8)$
$$-1+10x-15=16x-6x-16$$
$$10x-16=10x-16$$
All real numbers satisfy this equation. The solution set is R or $(-\infty,\infty)$.

79. $\frac{2x-5}{4}-\frac{3x-1}{6}=-\frac{13}{12}$
$$12\cdot\frac{2x-5}{4}-12\cdot\frac{3x-1}{6}=-\frac{13}{12}\cdot 12$$
$$6x-15-6x+2=-13$$
$$-13=-13$$
All real numbers satisfy this equation. The solution set is R or $(-\infty,\infty)$.

80. $\frac{x-1}{2}-\frac{3x-4}{6}=\frac{1}{3}$
$$6\cdot\frac{x-1}{2}-6\cdot\frac{3x-4}{6}=\frac{1}{3}\cdot 6$$
$$3x-3-3x+4=2$$
$$1=2$$
No real numbers satisfy this equation. The solution set is the empty set, $\emptyset$.

81. $\frac{1}{2}\left(y-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3y\right)$
$$\frac{1}{2}y-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-y$$
$$6y-1+8=10+2-12y$$
$$6y+7=12-12y$$
$$18y=5$$
$$y=\frac{5}{18}$$
The solution set is $\left\{\frac{5}{18}\right\}$.

82. $\frac{3}{4}-\frac{1}{3}\left(\frac{1}{2}y-2\right)=3\left(y-\frac{1}{4}\right)$
$$\frac{3}{4}-\frac{1}{6}y+\frac{2}{3}=3y-\frac{3}{4}$$
$$9-2y+8=36y-9$$
$$-2y+17=36y-9$$
$$-38y=-26$$
$$y=\frac{13}{19}$$
The solution set is $\left\{\frac{13}{19}\right\}$.

83. $\frac{40x-5}{2}+\frac{5}{2}=\frac{33-2x}{3}-11$
$$20x-\frac{5}{2}+\frac{5}{2}=11-\frac{2}{3}x-11$$
$$20x=-\frac{2}{3}x$$
$$60x=-2x$$
$$62x=0$$
$$x=0$$
The solution set is $\{0\}$.

84. $\frac{a-3}{4}-\frac{2a-5}{2}=\frac{a+1}{3}-\frac{1}{6}$
$$12\cdot\frac{a-3}{4}-12\cdot\frac{2a-5}{2}$$
$$=12\cdot\frac{a+1}{3}-12\cdot\frac{1}{6}$$
$$3a-9-12a+30=4a+4-2$$
$$-9a+21=4a+2$$
$$-13a=-19$$
$$a=\frac{19}{13}$$
The solution set is $\left\{\frac{19}{13}\right\}$.

85. $1.3 - 0.2(6 - 3x) = 0.1(0.2x + 3)$
$$1.3 - 1.2 + 0.6x = 0.02x + 0.3$$
$$130 - 120 + 60x = 2x + 30$$
$$10 + 60x = 2x + 30$$
$$58x = 20$$
$$x = \frac{20}{58} = \frac{10}{29}$$
The solution set is $\left\{\frac{10}{29}\right\}$.

86. $0.01(500 - 30x) = 5.4x + 200$
$$5 - 0.3x = 5.4x + 200$$
$$50 - 3x = 54x + 2000$$
$$-57x = 1950$$
$$x = -\frac{1950}{57} = -\frac{650}{19}$$
The solution set is $\left\{-\frac{650}{19}\right\}$.

87. $3x - 9 = 0$
$$3x = 9$$
$$x = 3$$
The solution set is $\{3\}$.

88. $5x + 1 = 0$
$$5x = -1$$
$$x = -\frac{1}{5}$$
The solution set is $\left\{-\frac{1}{5}\right\}$.

89. $7 - z = -9$
$$16 = z$$
The solution set is $\{16\}$.

90. $-3 - z = 3$
$$-z = 6$$
$$z = -6$$
The solution set is $\{-6\}$.

91. $\frac{2}{3}x = \frac{1}{2}$
$$x = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$$
The solution set is $\left\{\frac{3}{4}\right\}$.

92. $\frac{3}{2}x = -\frac{9}{5}$
$$x = \frac{2}{3} \cdot \left(-\frac{9}{5}\right) = -\frac{18}{15} = -\frac{6}{5}$$
The solution set is $\left\{-\frac{6}{5}\right\}$.

93. $-\frac{3}{5}y = 9$
$$y = -\frac{5}{3} \cdot 9 = -15$$
The solution set is $\{-15\}$.

94. $-\frac{2}{7}w = 4$
$$w = -\frac{7}{2} \cdot 4 = -14$$
The solution set is $\{-14\}$.

95. $3y + 5 = 4y - 1$
$$6 = y$$
The solution set is $\{6\}$.

96. $2y - 7 = 3y + 1$
$$-8 = y$$
The solution set is $\{-8\}$.

97. $5x + 10(x + 2) = 110$
$$5x + 10x + 20 = 110$$
$$15x = 90$$
$$x = 6$$
The solution set is $\{6\}$.

98. $1 - 3(x - 2) = 4(x - 1) - 3$
$$1 - 3x + 6 = 4x - 4 - 3$$
$$7 - 3x = 4x - 7$$
$$-7x = -14$$
$$x = 2$$
The solution set is $\{2\}$.

99. $\frac{P+7}{3} - \frac{P-2}{5} = \frac{7}{3} - \frac{P}{15}$
$$15\left(\frac{P+7}{3}\right) - 15\left(\frac{P-2}{5}\right) = 15\left(\frac{7}{3}\right) - 15\left(\frac{P}{15}\right)$$
$$5P + 35 - 3P + 6 = 35 - P$$
$$2P + 41 = 35 - P$$
$$3P = -6$$
$$P = -2$$
The solution set is $\{-2\}$.

100. $\frac{w-3}{8} - \frac{5-w}{4} = \frac{4w-1}{8} - 1$
$$8 \cdot \frac{w-3}{8} - 8 \cdot \frac{5-w}{4} = 8 \cdot \frac{4w-1}{8} - 8 \cdot 1$$
$$w - 3 - 2(5 - w) = 4w - 1 - 8$$
$$w - 3 - 10 + 2w = 4w - 9$$
$$3w - 13 = 4w - 9$$
$$-w - 13 = -9$$
$$-w = 4$$
$$w = -4$$
The solution set is $\{-4\}$.

101. $x - 0.06x = 50{,}000$
$$0.94x = 50{,}000$$
$$x = \frac{50{,}000}{0.94} \approx 53{,}191.49$$
The solution set is $\{53{,}191.49\}$.

102. $x - 0.05x = 800$
$$0.95x = 800$$
$$x = \frac{800}{0.95} \approx 842.11$$
The solution set is $\{842.11\}$.

103.
$$2.365x + 3.694 = 14.8095$$
$$2.365x = 14.8095 - 3.694$$
$$2.365x = 11.1155$$
$$x = \frac{11.1155}{2.365} = 4.7$$
The solution set is $\{4.7\}$.

104.
$$-3.48x + 6.981 = 4.329x - 6.851$$
$$-7.809x = -13.832$$
$$x = \frac{-13.832}{-7.809} \approx 1.77$$
The solution set is $\{1.77\}$.

105. a) For 1992, $x = 7$.
$$0.45(7) + 39.05 = 42.2$$
Public school enrollment in 1992 was 42.2 million.

b)
$$0.45x + 39.05 = 50$$
$$0.45x = 10.95$$
$$x \approx 24.3$$
So in 24.3 years (or during the 25th year) public school enrollment will reach 50 million. $1985 + 25 = 2010$.

c) Judging from the graph, enrollment is increasing.

106. a) $553.7(8) + 27{,}966 \approx 32{,}396$
Average teacher salary in 1993 was $32,396.

b)
$$553.7x + 27{,}966 = 45{,}000$$
$$553.7x = 17{,}034$$
$$x \approx 31$$
Average teacher salary will reach $45,000 in the year $1985 + 31$, or 2016.

107. To eliminate decimals multiply each side by an appropriate power of 10.

108. Multiplying each side by 0 will produce the identity $0 = 0$.

2.2 WARM-UPS

1. False, P is on both sides of the equation.
2. False, because we use the distributive property to factor out P on the right side.
3. False, because we divide each side by Pr to get $t = \frac{I}{Pr}$.
4. True, because $\frac{5 \cdot 6}{2} = 15$.
5. True, because $P = 2L + 2W$.
6. True, because $V = LWH$.
7. False, because $A = \frac{1}{2}h(b_1 + b_2)$.
8. True, $x - y = 5$ is equivalent to $x = 5 + y$, or $y = x - 5$.
9. True, because $-2(-3) - 4 = 6 - 4 = 2$.
10. False, perimeter is the distance around the outside edge.

2.2 EXERCISES

1. A formula is an equation involving two or more variables.
2. A formula is used to express a relationship between variables.
3. Solving for a variable means to rewrite the formula with the indicated variable isolated.
4. If a variable occurs twice, then we usually use the distributive property to isolate it.
5. To find the value a variable, solve for that variable, then replace all other variables with the given numbers.
6. The value of A can be determined from the value of s.

7.
$$I = Prt$$
$$\frac{I}{Pr} = \frac{Prt}{Pr}$$
$$\frac{I}{Pr} = t$$
$$t = \frac{I}{Pr}$$

8.
$$d = rt$$
$$\frac{d}{t} = \frac{rt}{t}$$
$$r = \frac{d}{t}$$

9.
$$F = \frac{9}{5}C + 32$$
$$F - 32 = \frac{9}{5}C$$
$$\frac{5}{9}(F - 32) = \frac{5}{9} \cdot \frac{9}{5}C$$
$$C = \frac{5}{9}(F - 32)$$

10.
$$A = \frac{1}{2}bh$$
$$2A = bh$$
$$\frac{2A}{b} = \frac{bh}{b}$$
$$h = \frac{2A}{b}$$

11. $A = LW$
$\frac{A}{L} = \frac{LW}{L}$
$\frac{A}{L} = W$
$W = \frac{A}{L}$

12. $C = 2\pi r$
$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$
$r = \frac{C}{2\pi}$

13. $A = \frac{1}{2}(b_1 + b_2)$
$2A = 2 \cdot \frac{1}{2}(b_1 + b_2)$
$2A = b_1 + b_2$
$2A - b_2 = b_1$
$b_1 = 2A - b_2$

14. $A = \frac{1}{2}(b_1 + b_2)$
$2A = b_1 + b_2$
$2A - b_1 = b_2$
$b_2 = 2A - b_1$

15. $P = 2L + 2W$
$P - 2W = 2L$
$\frac{P - 2W}{2} = L$
$L = \frac{P - 2W}{2}$ or $L = \frac{1}{2}P - W$

16. $P = 2L + 2W$
$P - 2L = 2W$
$\frac{P - 2L}{2} = W$
$W = \frac{P - 2L}{2}$ or $W = \frac{1}{2}P - L$

17. $V = \pi r^2 h$
$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$
$h = \frac{V}{\pi r^2}$

18. $V = \frac{1}{3}\pi r^2 h$
$3V = \pi r^2 h$
$\frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$
$h = \frac{3V}{\pi r^2}$

19. $2x + 3y = 9$
$3y = -2x + 9$
$\frac{3y}{3} = \frac{-2x}{3} + \frac{9}{3}$
$y = -\frac{2}{3}x + 3$

20. $4y + 5x = 8$
$4y = -5x + 8$
$y = -\frac{5}{4}x + 2$

21. $x - y = 4$
$-y = -x + 4$
$\frac{-y}{-1} = \frac{-x}{-1} + \frac{4}{-1}$
$y = x - 4$

22. $y - x = 6$
$y = x + 6$

23. $\frac{1}{2}x - \frac{1}{3}y = 2$
$-\frac{1}{3}y = -\frac{1}{2}x + 2$
$-3\left(-\frac{1}{3}y\right) = -3\left(-\frac{1}{2}x + 2\right)$
$y = \frac{3}{2}x - 6$

24. $\frac{1}{3}x - \frac{1}{4}y = 1$
$-\frac{1}{4}y = -\frac{1}{3}x + 1$
$-4\left(-\frac{1}{4}y\right) = -4\left(-\frac{1}{3}x + 1\right)$
$y = \frac{4}{3}x - 4$

25. $y - 2 = \frac{1}{2}(x - 3)$
$y - 2 = \frac{1}{2}x - \frac{3}{2}$
$y = \frac{1}{2}x - \frac{3}{2} + 2$
$y = \frac{1}{2}x + \frac{1}{2}$

26. $y - 3 = \frac{1}{3}(x - 4)$
$y - 3 = \frac{1}{3}x - \frac{4}{3}$
$y = \frac{1}{3}x - \frac{4}{3} + \frac{9}{3}$
$y = \frac{1}{3}x + \frac{5}{3}$

27. $A = P + Prt$
$A - P = Prt$
$\frac{A - P}{Pr} = \frac{Prt}{Pr}$
$\frac{A - P}{Pr} = t$
$t = \frac{A - P}{Pr}$

28. $A = P + Prt$
$A - P = Prt$
$\frac{A - P}{Pt} = \frac{Prt}{Pt}$
$r = \frac{A - P}{Pt}$

29. $ab + a = 1$
$a(b + 1) = 1$
$\frac{a(b + 1)}{b + 1} = \frac{1}{b + 1}$
$a = \frac{1}{b + 1}$

30.
$$\begin{aligned} y - wy &= m \\ y(1-w) &= m \\ y &= \frac{m}{1-w} \end{aligned}$$

31.
$$\begin{aligned} xy + 5 &= y - 7 \\ xy - y &= -5 - 7 \\ y(x-1) &= -12 \\ y &= \frac{-12}{x-1} \\ y &= \frac{-12(-1)}{(x-1)(-1)} \\ y &= \frac{12}{1-x} \end{aligned}$$

32.
$$\begin{aligned} xy + 5 &= x + 7 \\ xy - x + 5 &= 7 \\ xy - x &= 2 \\ x(y-1) &= 2 \\ x &= \frac{2}{y-1} \end{aligned}$$

33.
$$\begin{aligned} xy^2 + xz^2 &= xw^2 - 6 \\ xy^2 + xz^2 - xw^2 &= -6 \\ x(y^2 + z^2 - w^2) &= -6 \\ x &= \frac{-6}{y^2 + z^2 - w^2} \\ x &= \frac{6}{w^2 - y^2 - z^2} \end{aligned}$$

34.
$$\begin{aligned} xz^2 + xw^2 &= xy^2 + 5 \\ xz^2 + xw^2 - xy^2 &= 5 \\ x(z^2 + w^2 - y^2) &= 5 \\ x &= \frac{5}{z^2 + w^2 - y^2} \end{aligned}$$

35.
$$\begin{aligned} 3.35x - 54.6 &= 44.3 - 4.58x \\ 3.35x + 4.58x &= 44.3 + 54.6 \\ x(3.35 + 4.58) &= 44.3 + 54.6 \\ x &= \frac{44.3 + 54.6}{3.35 + 4.58} \\ x &\approx 12.472 \end{aligned}$$

36.
$$\begin{aligned} -4.487x - 33.41 &= 55.83 - 22.49x \\ -4.487x + 22.49x &= 55.83 + 33.41 \\ x(-4.487 + 22.49) &= 55.83 + 33.41 \\ x &= \frac{55.83 + 33.41}{-4.487 + 22.49} \\ x &\approx 4.957 \end{aligned}$$

37.
$$\begin{aligned} 4.59x - 66.7 &= 3.2(x - 5.67) \\ 4.59x - 66.7 &= 3.2x - 3.2(5.67) \\ x(4.59 - 3.2) &= 66.7 - 3.2(5.67) \\ x &= \frac{66.7 - 3.2(5.67)}{4.59 - 3.2} \\ x &\approx 34.932 \end{aligned}$$

38.
$$\begin{aligned} 457(36x - 99) &= 34(28x - 239) \\ 457 \cdot 36x - 457 \cdot 99 &= 34 \cdot 28x - 34 \cdot 239 \\ 457 \cdot 36x - 34 \cdot 28x &= 457 \cdot 99 - 34 \cdot 239 \\ x(457 \cdot 36 - 34 \cdot 28) &= 457 \cdot 99 - 34 \cdot 239 \\ x &= \frac{457 \cdot 99 - 34 \cdot 239}{457 \cdot 36 - 34 \cdot 28} \\ x &\approx 2.395 \end{aligned}$$

39.
$$\begin{aligned} \frac{x}{19} - \frac{3}{23} &= \frac{4}{31} + \frac{3x}{7} \\ \frac{x}{19} + \frac{3x}{7} &= \frac{4}{31} + \frac{3}{23} \\ x\left(\frac{1}{19} + \frac{3}{7}\right) &= \frac{4}{31} + \frac{3}{23} \\ x &= \frac{\frac{4}{31} + \frac{3}{23}}{\frac{1}{19} + \frac{3}{7}} \approx 0.539 \end{aligned}$$

40. $\frac{1}{8} - \frac{5}{7}\left(x - \frac{5}{22}\right) = \frac{4x}{9} + \frac{1}{12}$

$$\begin{aligned} \frac{1}{8} - \frac{5x}{7} + \frac{25}{154} &= \frac{4x}{9} + \frac{1}{12} \\ -\frac{5x}{7} - \frac{4x}{9} &= \frac{1}{12} - \frac{1}{8} - \frac{25}{154} \\ x\left(-\frac{5}{7} - \frac{4}{9}\right) &= \frac{1}{12} - \frac{1}{8} - \frac{25}{154} \end{aligned}$$
$$x = \frac{\frac{1}{12} - \frac{1}{8} - \frac{25}{154}}{-\frac{5}{7} - \frac{4}{9}} = \frac{1131}{6424} \approx 0.176$$

41.
$$\begin{aligned} 2x - 3y &= 5 \\ 2(3) - 3y &= 5 \\ -3y &= -1 \\ y &= \frac{1}{3} \end{aligned}$$

42.
$$\begin{aligned} -3x - 4y &= 4 \\ -4y &= 3x + 4 \\ y &= -\frac{3}{4}x - 1 \end{aligned}$$

If $x = 3$, then
$$y = -\frac{3}{4}(3) - 1 = -\frac{9}{4} - \frac{4}{4} = -\frac{13}{4}.$$

43.
$$\begin{aligned} -4x + 2y &= 1 \\ -4(3) + 2y &= 1 \\ 2y &= 13 \\ y &= \frac{13}{2} \end{aligned}$$

44.
$$\begin{aligned} x - y &= 7 \\ -y &= -x + 7 \\ y &= x - 7 \end{aligned}$$

If $x = 3$, then $y = 3 - 7 = -4$.

45.
$$\begin{aligned} y &= -2x + 5 \\ y &= -2(3) + 5 \\ y &= -6 + 5 \\ y &= -1 \end{aligned}$$

46. If $x = 3$, then $y = -3(3) - 6 = -15$.

47.
$$\begin{aligned} -x+2y&=5\\ -3+2y&=5\\ 2y&=8\\ y&=4 \end{aligned}$$

48.
$$\begin{aligned} -x-3y&=6\\ -3y&=x+6\\ y&=\frac{x+6}{-3} \end{aligned}$$

If $x=3$, then $y=\frac{3+6}{-3}=\frac{9}{-3}=-3$.

49.
$$\begin{aligned} y-1.046&=2.63(x-5.09)\\ y-1.046&=2.63(3-5.09)\\ y-1.046&=-5.4967\\ y&=-4.4507 \end{aligned}$$

50. Use $x=3$ in the equation.
$$\begin{aligned} y-2.895&=-1.07(x-2.89)\\ y-2.895&=-1.07(3-2.89)\\ y-2.895&=-0.1177\\ y&=2.7773 \end{aligned}$$

51.
$$\begin{aligned} wxy&=5\\ 4x(2)&=5\\ 8x&=5\\ x&=\tfrac{5}{8} \end{aligned}$$

52. Use $w=4$ and $z=-3$ in $wxz=4$.
$$\begin{aligned} 4x(-3)&=4\\ -12x&=4\\ x&=-\tfrac{4}{12}=-\tfrac{1}{3} \end{aligned}$$

53.
$$\begin{aligned} x+xz&=7\\ x+x(-3)&=7\\ -2x&=7\\ x&=-\tfrac{7}{2} \end{aligned}$$

54. Use $w=4$ in $xw-x=3$.
$$\begin{aligned} x(4)-x&=3\\ 3x&=3\\ x&=1 \end{aligned}$$

55.
$$\begin{aligned} w(x-z)&=y(x-4)\\ 4[x-(-3)]&=2(x-4)\\ 4x+12&=2x-8\\ 2x&=-20\\ x&=-10 \end{aligned}$$

56. Use $z=-3$ and $y=2$ in $z(x-y)=y(x+5)$.
$$\begin{aligned} -3(x-2)&=2(x+5)\\ -3x+6&=2x+10\\ -5x&=4\\ x&=-\tfrac{4}{5} \end{aligned}$$

57.
$$\begin{aligned} w&=\tfrac{1}{2}xz\\ 4&=\tfrac{1}{2}x(-3)\\ 4&=-\tfrac{3}{2}x\\ -\tfrac{2}{3}(4)&=-\tfrac{2}{3}\left(-\tfrac{3}{2}x\right)\\ -\tfrac{8}{3}&=x \end{aligned}$$

58. Use $y=2$ and $w=4$ in $y=\frac{1}{2}wx$.
$$\begin{aligned} 2&=\tfrac{1}{2}\cdot 4x\\ 2&=2x\\ 1&=x \end{aligned}$$

59.
$$\begin{aligned} \frac{1}{w}+\frac{1}{x}&=\frac{1}{y}\\ \frac{1}{4}+\frac{1}{x}&=\frac{1}{2}\\ 4x\cdot\frac{1}{4}+4x\cdot\frac{1}{x}&=4x\cdot\frac{1}{2}\\ x+4&=2x\\ 4&=x \end{aligned}$$

60. Use $w=4$ and $y=2$ in $\frac{1}{w}+\frac{1}{y}=\frac{1}{x}$.
$$\begin{aligned} \frac{1}{4}+\frac{1}{2}&=\frac{1}{x}\\ \frac{3}{4}&=\frac{1}{x}\\ 4x\cdot\frac{3}{4}&=4x\cdot\frac{1}{x}\\ 3x&=4\\ x&=\frac{4}{3} \end{aligned}$$

61.
$$\begin{aligned} I&=Prt\\ 300&=1000\cdot 2\cdot \mathrm{r}\\ r&=\frac{300}{2000}=15\% \end{aligned}$$

62.
$$\begin{aligned} I&=Prt\\ 2000&=20{,}000\cdot 5\cdot \mathrm{r}\\ r&=\frac{2000}{100{,}000}=2\% \end{aligned}$$

63.
$$\begin{aligned} I&=Prt\\ 20&=500\cdot \mathrm{r}\cdot\frac{2}{52}\\ r&=\frac{20}{500\cdot\frac{2}{52}}=104\% \end{aligned}$$

64.
$$\begin{aligned} I&=Prt\\ 19&=200\cdot \mathrm{r}\cdot\frac{2}{52}\\ r&=\frac{19}{200\cdot\frac{2}{52}}=247\% \end{aligned}$$

65. Use $P = 2000$, $r = 0.18$, and $I = 180$ in the formula for simple interest $I = Prt$.

$180 = 2000(0.18)t$
$180 = 360t$
$0.5 = t$

The time is one-half year.

66. Use $P = 10{,}000$, $r = 0.06$, and $I = 3000$ in the formula for simple interest $I = Prt$.

$3000 = 10{,}000(0.06)t$
$3000 = 600t$
$5 = t$

The time is 5 years.

67. The formula for the area of a circle is $A = \pi r^2$.

68. The formula for the circumference of a circle is $C = \pi d$.

69. Since $C = 2\pi r$, we can divide each side by 2π to get $r = \dfrac{C}{2\pi}$.

70. Because $C = \pi d$, we can divide each side by π to get $d = \dfrac{C}{\pi}$.

71. Because $P = 2L + 2W$, we can subtract $2L$ from each side to get $2W = P - 2L$, and then divide each side by 2 to get $W = \dfrac{P - 2L}{2}$ or $W = \frac{1}{2}P - L$.

72. Because $A = LW$, we can divide each side by W to get $L = \dfrac{A}{W}$.

73. $A = LW$
$23 = L(4)$
$L = \dfrac{23}{4} = 5.75$ yards

74. Use $L = 7$ and $A = 55$ in the formula for the area of a rectangle, $A = LW$.

$55 = 7W$
$\dfrac{55}{7} = W$

The width is $\dfrac{55}{7}$ meters.

75. $V = LWH$
$36 = 2(2.5)H$
$36 = 5H$
$H = 7.2$ feet

76. Use $V = 2.5$, $H = 1$, and $W = 1.25$ in the formula for the volume of a rectangular solid $V = LWH$.

$2.5 = L(1.25)(1)$
$2.5 = 1.25L$
$2 = L$

The length is 2 meters.

77. $V = 900 \text{ gal.}\left(\dfrac{1 \text{ ft}^3}{7.5 \text{ gal}}\right) = 120 \text{ ft}^3$
$V = LWH$
$120 = 4 \cdot 6 \cdot H$
$H = \dfrac{120}{24} = 5$ feet

78. First convert 60,000 gallons to cubic feet.

60,000 gallons $\cdot \dfrac{1 \text{ cubic foot}}{7.5 \text{ gallons}} = 8000$ cubic feet

Use $V = 8000$, $W = 40$, and $L = 100$ in the formula for the volume of a rectangular solid $V = LWH$.

$8000 = 100 \cdot 40 \cdot H$
$8000 = 4000H$
$2 = \text{H}$

The pool is 2 feet deep.

79. $A = \frac{1}{2}bh$
$30 = \frac{1}{2}4h$
$30 = 2h$
$h = 15$ feet

80. Use $A = 40$ and $h = 10$ in the formula for the area of a triangle $A = \frac{1}{2}bh$.

$40 = \frac{1}{2}b(10)$
$40 = 5b$
$8 = b$

The base is 8 meters.

81. $A = \frac{1}{2}h(b_1 + b_2)$
$300 = \frac{1}{2}(20)(16 + b_2)$
$300 = 10(16 + b_2)$
$30 = 16 + b_2$
$b_2 = 14$ inches

82. Use $A = 200$, $b_1 = 16$, and $b_2 = 24$ in the formula for the area of a trapezoid $A = \frac{1}{2}h(b_1 + b_2)$.

$200 = \frac{1}{2}h(16 + 24)$
$200 = 20h$
$10 = h$

The height is 10 centimeters.

83. $P = 2L + 2W$
$600 = 2L + 2(132)$
$336 = 2L$
$L = 168$ feet

84. Use $P = 306\frac{2}{3} = \frac{920}{3}$ and $L = 100$ in the formula for the perimeter of a rectangle $P = 2L + 2W$.

$$\frac{920}{3} = 2(100) + 2W$$
$$\frac{920}{3} = 200 + 2W$$
$$920 = 600 + 6W$$
$$320 = 6W$$
$$\frac{320}{6} = W$$

The width is $\frac{160}{3}$ or $53\frac{1}{3}$ yards or 160 feet.

85. $C = 2\pi r$
$$3\pi = 2\pi r$$
$$r = \frac{3\pi}{2\pi}$$
$$r = 1.5 \text{ meters}$$

86. Use $C = 12\pi$ in the formula $C = 2\pi r$.
$$12\pi = 2\pi r$$
$$6 = r$$
Since the radius is 6 inches, the diameter is 12 inches.

87. $C = 2\pi r$
$$r = \frac{C}{2\pi}$$
$$r = \frac{25{,}000}{2\pi}$$
$$r \approx 3979 \text{ miles}$$

88. Use $C = 26{,}000$ in the formula $C = 2\pi r$.
$$26{,}000 = 2\pi r$$
$$r = \frac{26{,}000}{2\pi} \approx 4138$$
Since the radius of the earth is 3979 (from the previous exercise), the satellite is $4138 - 3979$, or approximately 159 miles above the earth.

89. If the diameter is 3 in., then the radius is 1.5 in.
$$V = \pi r^2 h$$
$$30 = \pi(1.5)^2 h$$
$$30 = 2.25\pi h$$
$$h = \frac{30}{2.25\pi} \approx 4.24 \text{ inches}$$

90. Use $V = 6.3$ and $r = 0.6$ in the formula for the volume of a cylinder $V = \pi r^2 h$.
$$6.3 = \pi(0.6)^2 h$$
$$6.3 = 0.36\pi h$$
$$h = \frac{6.3}{0.36\pi} \approx 5.57$$
The height is approximately 5.57 meters.

91. Let $W = 62$ lb/ft^3, $D = 32$ ft, and $A = 48$ ft^2 in $F = WDA$:
$$F = 62 \text{ lb/ft}^3 \cdot 32 \text{ ft} \cdot 48 \text{ ft}^2 = 95{,}232 \text{ lb}$$

92. Let $s = 38$ lb/in.^{2} and $r = 30$ in. in $F = s\pi r^2$:
$$F = 38 \text{ lb/in.}^2 \cdot \pi \cdot 30^2 \text{ in.}^2 \approx 107{,}442 \text{ lb}$$
It would take a force of $107,442$ pounds of larger to shear the concrete in the shaft.
Since the shear strength of the concrete shaft is larger than the force of the water, the concrete will hold back the water.

93. \$1000 ÷ \$2 = 500 feet
$$b_1 + b_2 = 500$$
$$A = \frac{1}{2}h(b_1 + b_2)$$
$$50{,}000 = \frac{1}{2}h \cdot 500$$
$$h = 200 \text{ feet}$$

94. Use $A = 25{,}000$ and $h = 200$ (from Exercise 95) in the formula for the area of a triangle $A = \frac{1}{2}bh$.
$$25{,}000 = \frac{1}{2}b(200)$$
$$25{,}000 = 100b$$
$$250 = b$$
Since she has 250 feet of frontage, her assessment will be \$500.

95. $A = bh$
$$60{,}000 = b \cdot 200$$
$$b = 300 \text{ feet}$$
There is 300 ft on each street. So 600 ft at \$2 each is a \$1200 assessment.

96. Let $L =$ the length of the driveway. The driveway is a rectangular solid with a volume of 12 cubic yards. The driveway is 12 ft or 4 yd wide. It is 4 in. or $\frac{4}{36} = \frac{1}{9}$ yard thick. Use the formula $V = LWH$.
$$12 = L(4)\frac{1}{9}$$
$$12 = \frac{4}{9}L$$
$$108 = 4L$$
$$27 = L$$
It is 27 yards or 81 feet from the street to her house.

97. Since August has 31 days, the total is $1 + 2 + 3 + 4 + \ldots + 31$. Use $n = 31$ in the formula $S = \frac{n(n+1)}{2}$:
$$S = \frac{31(32)}{2} = 31 \cdot 16 = 496$$
So her total jogging time is 496 minutes.

98. The total area in square inches is
$1^2 + 2^2 + 3^2 + 4^2 + \ldots + 40^2$.
Use $n = 40$ in the formula
$S = \frac{n(n+1)(2n+1)}{6}$:

$$S = \frac{40(41)(81)}{6} = 22{,}140 \text{ in.}^2$$
$$22{,}140 \text{ in.}^2 \cdot \frac{1 \text{ ft}^2}{144 \text{ in.}^2} = 153.75 \text{ ft}^2$$

An 8ft by 15 ft wall is 120 ft^2. So he cannot fit all 40 squares on the wall.

99. a) $N = B + S - 1$
$N = 2003 + 455 - 1 = 2457$
b) $1452 = 1033 + S - 1$
$420 = S$

100. a) $P = 47.9 - 0.94(32) = 17.82$
So 17.82% of persons in the 18 to 25 age group are smokers in 2006.
b) According to the graph it looks like smoking in this group will be eliminated in about 50 years after 1974 or in 2024.
c) $47.9 - 0.94n = 0$
$-0.94n = -47.9$
$n = \frac{-47.9}{-0.94} \approx 51$
So according to the formula smoking will be eliminated from this age group in the year $1974 + 51$, or 2025.

2.3 WARM-UPS

1. False, first identify what the variable stands for.
2. True, we must know what the letters represent.
3. False, you may have the wrong equation.
4. False, odd integers differ by 2.
5. True, because $x + 6 - x = 6$.
6. True, because $x + 7 - x = 7$.
7. False, $5x - 2 = 3(x + 20)$ since $5x$ is larger than $3(x + 20)$.
8. True, because 8% of x is $0.08x$.
9. False, because 10% of \$88,000 is \$8,800 and \$88,000 − \$8,800 is not \$80,000.
10. False, because the acid in the mixture must be between 10% and 14%.

2.3 EXERCISES

1. Three unknown consecutive integers are represented by x, $x + 1$, and $x + 2$.
2. In either case we use x, $x + 2$, and $x + 4$, but for odd integers x represents an odd integer and for even integers x represents an even integer.
3. The formula $P = 2L + 2W$ expresses the perimeter in terms of the length and width.
4. Addition can be indicated by the words, sum, more than, or plus.
5. The commission is a percentage of the selling price.
6. Uniform motion is motion at a constant rate.
7. Since two consecutive even integers differ by 2, we can use x and $x + 2$ to represent them.
8. Since consecutive odd integers differ by 2, we can use x and $x + 2$ to represent two consecutive odd integers.
9. The expressions x and $10 - x$ have a sum of 10, since $x + 10 - x = 10$.
10. The expressions x and $-6 - x$ have a sum of -6, since $x + (-6 - x) = -6$.
11. Two numbers with a difference of 2 can be represented as x and $x + 2$ or x and $x - 2$.
12. Two numbers with a difference of 3 can be represented as x and $x + 3$ or x and $x - 3$.
13. If x is the selling price, then eighty-five percent of the selling price is $0.85x$.
14. The product of an unknown number, x, and 3 can be represented as $3x$.

15. Since $D = RT$, the distance is $3x$ miles.

16. Since $D = RT$, the time it takes to travel 100 miles at $x + 5$ miles per hour is represented as $\frac{100}{x+5}$ hours.

17. Since the perimeter is twice the length $(x + 5)$ plus twice the width (x), we can represent the perimeter by $2(x + 5) + 2(x)$ or $4x + 10$.

18. The sum of the length and width is one-half the perimeter, or 10 meters. If the length is x meters, then the width is $10 - x$ meters.

19. Let $x =$ the first integer, $x + 1 =$ the second integer, and $x + 2 =$ the third integer. Their sum is 84:

$$\begin{aligned} x + x + 1 + x + 2 &= 84 \\ 3x + 3 &= 84 \\ 3x &= 81 \\ x &= 27 \end{aligned}$$

If $x = 27$, then $x + 1 = 28$, and $x + 2 = 29$. The integers are 27, 28, and 29.

20. Let $x =$ the first integer, $x + 1 =$ the second integer, and $x + 2 =$ the third integer. Since their sum is 171, we can write the following equation.

$$\begin{aligned} x + x + 1 + x + 2 &= 171 \\ 3x + 3 &= 171 \\ 3x &= 168 \\ x &= 56 \\ x + 1 &= 57 \\ x + 2 &= 58 \end{aligned}$$

The three consecutive integers are 56, 57, and 58.

21. Let $x =$ the first even integer, $x + 2 =$ the second, and $x + 4 =$ the third. Their sum is 252:

$$\begin{aligned} x + x + 2 + x + 4 &= 252 \\ 3x + 6 &= 252 \\ 3x &= 246 \\ x &= 82 \end{aligned}$$

If $x = 82$, then $x + 2 = 84$ and $x + 4 = 86$. The integers are 82, 84, and 86.

22. Let $x =$ the first even integer, $x + 2 =$ the second, and $x + 4 =$ the third. Since their sum is 84, we can write the following equation.

$$\begin{aligned} x + x + 2 + x + 4 &= 84 \\ 3x + 6 &= 84 \\ 3x &= 78 \\ x &= 26 \\ x + 2 &= 28 \\ x + 4 &= 30 \end{aligned}$$

The three consecutive even integers are 26, 28, and 30.

23. Let $x =$ the first odd integer and $x + 2 =$ the second. Their sum is 128:

$$\begin{aligned} x + x + 2 &= 128 \\ 2x &= 126 \\ x &= 63 \end{aligned}$$

If $x = 63$, then $x + 2 = 65$. The integers are 63 and 65.

24. Let $x =$ the first odd integer, $x + 2 =$ the second, $x + 4 =$ the third, and $x + 6 =$ the fourth. Since their sum is 56, we can write the following equation.

$$\begin{aligned} x + x + 2 + x + 4 + x + 6 &= 56 \\ 4x + 12 &= 56 \\ 4x &= 44 \\ x &= 11 \\ x + 2 &= 13 \\ x + 4 &= 15 \\ x + 6 &= 17 \end{aligned}$$

The four consecutive odd integers are 11, 13, 15, and 17.

25. Let $x =$ the number.

$$\begin{aligned} x + 5 &= -8 \\ x &= -13 \end{aligned}$$

The number is -13.

26. Let $x =$ the number.

$$\begin{aligned} x + (-12) &= 6 \\ x &= 18 \end{aligned}$$

The number is 18.

27. Let $x =$ the number.

$$\begin{aligned} 2x + 6 &= 52 \\ 2x &= 46 \\ x &= 23 \end{aligned}$$

The number is 23.

28. Let $x =$ the number.

$$\begin{aligned} 2x - 3 &= 31 \\ 2x &= 34 \\ x &= 17 \end{aligned}$$

The number is 17.

29. Let $x =$ the number.

$$\begin{aligned} \frac{1}{6}x - \frac{1}{7}x &= 1 \\ 42\left(\frac{1}{6}x - \frac{1}{7}x\right) &= 42 \\ 7x - 6x &= 42 \\ x &= 42 \end{aligned}$$

The number is 42.

30. Let $x =$ the number.

$$\begin{aligned} \frac{1}{5}x + \frac{1}{6}x &= 33 \\ 30\left(\frac{1}{5}x + \frac{1}{6}x\right) &= 30(33) \\ 6x + 5x &= 990 \\ 11x &= 990 \\ x &= 90 \end{aligned}$$

The number is 90.

31. Let $x =$ the length and $x - 2 =$ the width. Since $P = 2L + 2W$ we can write the following equation.

$$2x + 2(x - 2) = 16$$
$$4x - 4 = 16$$
$$4x = 20$$
$$x = 5$$
$$x - 2 = 3$$

So the length is 5 ft and the width is 3 ft.

32. Let $x =$ the width of the frame and $x + 2 =$ the length of the frame. Since the perimeter of the frame is 10 feet, we can write the following equation.

$$2x + 2(x + 2) = 10$$
$$2x + 2x + 4 = 10$$
$$4x = 6$$
$$x = 1.5$$
$$x + 2 = 3.5$$

The width of the frame is 1.5 feet and the length of the frame is 3.5 feet.

33. Let $x =$ the width and $2x + 4 =$ the length. Since the perimeter is 26 in., we can write the following equation.

$$2x + 2(2x + 4) = 26$$
$$2x + 4x + 8 = 26$$
$$6x = 18$$
$$x = 3$$
$$2x + 4 = 10$$

The width of the glass is 3 in. and the length of the glass is 10 in.

34. Let $x =$ the width and $2x + 2 =$ the length. Since the perimeter is 34 yd, we can write the following equation.

$$2x + 2(2x + 2) = 34$$
$$2x + 4x + 4 = 34$$
$$6x = 30$$
$$x = 5$$
$$2x + 2 = 12$$

The width of the courtyard is 5 yd and the length is 12 yd.

35. Let $x =$ the length and $\frac{1}{2}x - 2 =$ the width. Since the perimeter is 44 cm, we can write the following equation.

$$2x + 2(\tfrac{1}{2}x - 2) = 44$$
$$2x + x - 4 = 44$$
$$3x = 48$$
$$x = 16$$
$$\frac{1}{2}x - 2 = 6$$

The width of the sign is 6 cm and the length is 16 cm.

36. Let $x =$ the length and $\frac{1}{2}x - 1 =$ the width. Since the perimeter is 40 ft, we can write the following equation.

$$2x + 2(\tfrac{1}{2}x - 1) = 40$$
$$2x + x - 2 = 40$$
$$3x = 42$$
$$x = 14$$
$$\frac{1}{2}x - 1 = 6$$

The width of the closet is 6 ft and the length is 14 ft.

37. Let $x =$ the width and $x + 2 =$ the length. Since he is fencing two width and one length, we can write the following equation.

$$2x + 1(x + 2) = 14$$
$$2x + x + 2 = 14$$
$$3x = 12$$
$$x = 4$$
$$x + 2 = 6$$

The width of the region is 4 ft and the length is 6 ft.

38. Let $x =$ the height and $x + 9 =$ the length of the side parallel to the ground. Since he is trimming two heights and one length, we can write the following equation.

$$2x + 1(x + 9) = 30$$
$$2x + x + 9 = 30$$
$$3x = 21$$
$$x = 7$$
$$x + 9 = 16$$

The height of the doorway 7 ft and the width is 16 ft.

39. Let $x =$ the width, and $2x + 5 =$ the length. To fence the 3 sides, we use 2 widths and 1 length:

$$2(x) + (2x + 5) = 50$$
$$4x = 45$$
$$x = 11.25$$
$$2x + 5 = 27.5$$

Width is 11.25 feet and the length is 27.5 feet.

40. Let $x =$ the width and $2x + 1 =$ the height of the doorway. The equation expresses the fact that the total length of the 3 pieces is 17 feet.

$$x + 2(2x + 1) = 17$$
$$x + 4x + 2 = 17$$
$$5x = 15$$

$$x = 3$$
$$2x + 1 = 7$$

The doorway is 3 feet wide and 7 feet high.

41. Let $x =$ the length of the first side, $2x - 10 =$ the length of the second side, and $x + 50 =$ the length of the third side. Since the perimeter is 684, we have the following equation.

$$x + 2x - 10 + x + 50 = 684$$
$$4x + 40 = 684$$
$$4x = 644$$
$$x = 161$$

If $x = 161$ feet, then $2x - 10 = 312$ feet, and $x + 50 =$ 211 feet.

42. Let $x =$ the length of each of the equal sides and $x - 3.5 =$ the length of the base. Since the perimeter of the triangle is 49 inches, we can write the following equation.

$$x + x + x - 3.5 = 49$$
$$3x - 3.5 = 49$$
$$3x = 52.5$$
$$x = 17.5$$

The length of each of the equal sides is 17.5 inches.

43. Let $x =$ the amount invested at 5% and $2x =$ the amount invested at 9%. Since the interest on the investments is $0.05x$ and $0.09(2x)$ we can write the equation

$$0.05x + 0.09(2x) = 920$$
$$0.05x + 0.18x = 920$$
$$0.23x = 920$$
$$x = 4000$$
$$2x = 8000$$

He invested \$4000 at 5% and \$8000 at 9%.

44. Let $x =$ the amount invested at 3% and $x + 3000 =$ the amount invested at 7%. Since the interest on the investments is $0.03x$ and $0.07(x + 3000)$ we can write the equation

$$0.03x + 0.07(x + 3000) = 810$$
$$0.10x + 210 = 810$$
$$0.10x = 600$$
$$x = 6000$$
$$x + 3000 = 9000$$

He invested \$6000 at 3% and \$9000 at 7%.

45. Let $x =$ the amount invested at 6% and $x + 1000 =$ the amount invested at 10%. Since the interest on the investments is $0.06x$ and $0.10(x + 1000)$ we can write the equation

$$0.06x + 0.10(x + 1000) = 340$$
$$0.16x + 100 = 340$$
$$0.16x = 240$$
$$x = 1500$$
$$x + 1000 = 2500$$

He invested \$1500 at 6% and \$2500 at 10%.

46. Let $x =$ the amount lent to her brother at 9% and $\frac{1}{2}x =$ the amount lent to her sister at 16%. The equation expresses the fact that the total interest is 34 cents.

$$0.09x + 0.16 \cdot \frac{1}{2}x = 0.34$$
$$0.09x + 0.08x = 0.34$$
$$0.17x = 0.34$$
$$x = 2$$
$$\frac{1}{2}x = 1$$

She lent her brother \$2 and her sister \$1.

47. Let $x =$ the amount of his inheritance. He invests $\frac{1}{2}x$ at 10% and $\frac{1}{4}x$ at 12%. His total income of \$6400 can be expressed as

$$0.10\left(\frac{1}{2}x\right) + 0.12\left(\frac{1}{4}x\right) = 6400$$
$$0.05x + 0.03x = 6400$$
$$0.08x = 6400$$
$$x = 80{,}000$$

His inheritance was \$80,000.

48. Let $x =$ the amount of Gary's insurance settlement, $\frac{1}{3}x =$ the amount he invested at 12%, and $\frac{1}{3}x =$ the amount invested at 15%. The equation expresses the fact that the total income from these two investments was \$10,800.

$$0.12 \cdot \frac{1}{3}x + 0.15 \cdot \frac{1}{3}x = 10{,}800$$
$$0.04x + 0.05x = 10{,}800$$
$$0.09x = 10{,}800$$
$$x = 120{,}000$$

His insurance settlement was \$120,000.

49. Let $x =$ the amount of Claudette's inheritance, $\frac{1}{2}x =$ the amount he invested at 5%, and $\frac{1}{3}x =$ the amount invested at 6%.

$$\frac{1}{2}x \cdot 0.05 + \frac{1}{3}x \cdot 0.06 = 9000$$

Multiply by 6:

$$3x \cdot 0.05 + 2x \cdot 0.06 = 54000$$
$$0.15x + 0.12x = 54000$$
$$0.27x = 54000$$
$$x = 200,000$$

The inheritance was \$200,000.

50. Let $x =$ the amount of Wanda's winnings,
$\frac{1}{2}x =$ the amount invested at 8%, and
$\frac{1}{4}x =$ the amount invested at 3%, and
$\frac{1}{4}x =$ the amount invested at 4%.
$\frac{1}{2}x \cdot 0.08 + \frac{1}{4}x \cdot 0.03 + \frac{1}{4}x \cdot 0.04 = 5750$
Multiply by 4:
$$2x \cdot 0.08 + x \cdot 0.03 + x \cdot 0.04 = 23000$$
$$0.16x + 0.03x + 0.04x = 23000$$
$$0.23x = 23000$$
$$x = 100,000$$
The amount won in the lottery was $100,000.

51. Let $x =$ the number of gallons of 5% solution. In the 5% solution there are $0.05x$ gallons of acid, and in the 20 gallons of 10% solution there are $0.10(20)$ gallons of acid. The final mixture consists of $x + 20$ gallons of which 8% is acid. The total acid in the mixture is the sum of the acid from each solution mixed together:
$$0.05x + 0.10(20) = 0.08(x + 20)$$
$$0.05x + 2 = 0.08x + 1.6$$
$$0.4 = 0.03x$$
$$x = \frac{0.4}{0.03} = \frac{40}{3}$$
Use $\frac{40}{3}$ gallons of 5% solution.

52. Let $x =$ the number of liters of 10% solution. In the 10% solution there are $0.10x$ liters of alcohol. In the 12 liters of 20% solution there are $0.20(12)$ liters of alcohol. In the $x + 12$ liters in the final 14% solution there are $0.14(x + 12)$ liters of alcohol.
$$0.10x + 0.20(12) = 0.14(x + 12)$$
$$0.10x + 2.4 = 0.14x + 1.68$$
$$2.40 - 1.68 = 0.04x$$
$$0.72 = 0.04x$$
$$18 = x$$
Use 18 liters of 10% solution.

53. Let $x =$ the amount of $8 per pound cherries and $x + 12$ = the amount of the $7 per pound mixture.
$$12(5) + x(8) = (x + 12)7$$
$$60 + 8x = 7x + 84$$
$$x = 24$$
He should use 24 pounds of cherries.

54. Let $x =$ the amount of $12 per pound dried peaches and $x + 5 =$ the amount of the $10 per pound mixture.
$$5(4) + x(12) = (x + 5)10$$
$$20 + 12x = 10x + 50$$
$$2x = 30$$
$$x = 15$$
She should use 15 pounds of peaches.

55. Let $x =$ the amount of 5% solution and $6 - x =$ the amount of 13% solution.
$$0.05x + 0.13(6 - x) = 0.08(6)$$
$$0.05x + 0.78 - 0.13x = 0.48$$
$$-0.08x = -0.30$$
$$x = 3.75$$
$$6 - x = 2.25$$
He should us 3.75 gallons of 5% solution and 2.25 gallons of 13% solution.

56. Let $x =$ the amount of 6% solution and $10 - x =$ the amount of 14% solution.
$$0.06x + 0.14(10 - x) = 0.12(10)$$
$$0.06x + 1.4 - 0.14x = 1.2$$
$$-0.08x = -0.2$$
$$x = 2.5$$
$$10 - x = 7.5$$
She should us 2.5 gallons of 6% solution and 7.5 gallons of 14% solution.

57. Let $x =$ the number of gallons of pure acid. After mixing we will have $1 + x$ gallons of 6% solution. The original gallon has $0.05(1)$ gallons of acid in it. The equation totals up the acid:
$$0.05(1) + x = 0.06(1 + x)$$
$$0.05 + x = 0.06 + 0.06x$$
$$0.94x = 0.01$$
$$x = 0.010638 \text{ gallons}$$
Use 1 gallon $= 128$ ounces, to get approximately 1.36 ounces of pure acid.

58. Let $x =$ the number of ounces of sodium hypochlorite to be added. Multiply 8.3 pounds by 16 to get 132.8 ounces. In the original 5.25% gallon there are $0.0525(132.8)$ ounces of sodium hypochlorite. In the final mixture there are $0.06(132.8 + x)$ ounces of sodium hypochlorite. We can write the following equation.
$$0.0525(132.8) + x = 0.06(132.8 + x)$$
$$6.972 + x = 7.968 + 0.06x$$
$$1.00x - 0.06x = 7.968 - 6.972$$
$$0.94x = 0.996$$
$$x \approx 1.0596$$
Add approximately 1.0596 ounces of sodium hypochlorite.

59. Let $x =$ his speed in the fog and $x + 30 =$ his increased speed. Since $D = RT$, his distance in the fog was $3x$ and his distance later was $6(x + 30)$. The equation gives the total distance:

$$3x + 6(x + 30) = 540$$
$$9x + 180 = 540$$
$$9x = 360$$
$$x = 40$$

His speed in the fog was 40 mph.

60. Let $x =$ her walking speed and $2x =$ her running speed. The distance that she walked is $2x$ miles and the distance that she ran is $2x(1.5) = 3x$ miles. Since her total distance was 20 miles, we can write the following equation.

$$2x + 3x = 20$$
$$5x = 20$$
$$x = 4$$
$$2x = 8$$

She walks 4 miles per hour and runs 8 miles per hour.

61. Let $x =$ the speed of the commuter bus and $x + 25 =$ the speed of the express bus. Use $D = RT$ to get the distance traveled by each as $2x$ and $\frac{3}{4}(x + 25)$. The equation expresses the fact that they travel the same distance:

$$2x = \frac{3}{4}(x + 25)$$
$$4 \cdot 2x = 4 \cdot \frac{3}{4}(x + 25)$$
$$8x = 3x + 75$$
$$5x = 75$$
$$x = 15$$

The speed of the commuter bus was 15 mph.

62. Let $x =$ the speed of the freight train and $x + 40 =$ the speed of the passenger train in miles per hour. Since the time for the freight train is 1.25 hours, the distance for the freight is $1.25x$ miles. Since the time for the passenger train is 0.75 hours, the distance for the passenger train is $0.75(x + 40)$ miles. Since the distance traveled for each train is the same, we can write the following equation.

$$1.25x = 0.75(x + 40)$$
$$1.25x = 0.75x + 30$$
$$0.50x = 30$$
$$x = 60$$
$$x + 40 = 100$$

The speed of the passenger train is 100 miles per hour.

63. Let $x =$ the speed before lunch and $x + 15 =$ the speed after lunch. The distance before is $3x$ and the distance after is $4(x + 15)$. Write the equation about the total distance:

$$3x + 4(x + 15) = 410$$
$$3x + 4x + 60 = 410$$
$$7x = 350$$
$$x = 50$$
$$x + 15 = 65$$

His average speed before lunch was 50 mph.

64. Let $x =$ the speed before lunch and $x - 10 =$ the speed after lunch. The distance before is $4x$ and the distance after is $5(x - 10)$. Write the equation about the total distance:

$$4x + 5(x - 10) = 688$$
$$4x + 5x - 50 = 688$$
$$9x = 738$$
$$x = 82$$
$$x - 10 = 72$$

His average speed after lunch was 72 mph.

65. Let $x =$ Candy's speed and $x + 10 =$ Fran's speed. The distance for Candy is $\frac{1}{2}x$ and the distance for Fran is $\frac{1}{3}(x + 10)$. Write the equation about the fact that the distances are the same:

$$\frac{1}{2}x = \frac{1}{3}(x + 10)$$

Multiply each side by 6:

$$3x = 2(x + 10)$$
$$3x = 2x + 20$$
$$x = 20$$

Candy's speed is 20 mph.

66. Let $x =$ Violet's speed and $x - 10 =$ Veronica's speed. The distance for Violet is $\frac{3}{4}x$ and the distance for Fran is $\frac{54}{60}(x - 10)$ or $\frac{9}{10}(x - 10)$. Write the equation about the fact that the distances are the same:

$$\frac{3}{4}x = \frac{9}{10}(x - 10)$$

Multiply each side by 20:

$$15x = 18(x - 10)$$
$$15x = 18x - 180$$

$$-3x = -180$$
$$x = 60$$
$$x - 10 = 50$$

Veronica's average speed is 50 mph.

67. If $x =$ the selling price, then the commission is $0.08x$. The owner gets the selling price minus the commission:

$$x - 0.08x = 80{,}000$$
$$0.92x = 80{,}000$$
$$x \approx \$86{,}957$$

68. Let $x =$ the amount of her sales and $0.05x =$ the amount of sales tax. Since her total was 915.60, we can write the following equation.

$$x + 0.05x = 915.60$$
$$1.05x = 915.60$$
$$x = \frac{915.60}{1.05} = 872$$
$$0.05x = 43.60$$

The sales tax that she collected was \$43.60.

69. If $x =$ the selling price, then $0.07x =$ the amount of sales tax. The selling price plus the sales tax is the total amount paid:

$$x + 0.07x = 9041.50$$
$$1.07x = 9041.50$$
$$x = \$8450$$

70. Let $x =$ the selling price and $0.10x =$ the broker's commission. Since Roy wants to get \$3000, we can write the following equation.

$$x - 0.10x = 3000$$
$$0.90x = 3000$$
$$x = \frac{3000}{0.9} \approx 3{,}333.33$$

The selling price should be \$3,333.33 to the nearest cent.

71. Let $x - 3 =$ the distance from the base line to the service line and $x =$ the distance from the service line to the net. The total is 39 feet:

$$x + x - 3 = 39$$
$$2x = 42$$
$$x = 21$$

The distance from service line to the net is 21 ft.

72. Let $x =$ the width of the doubles court and $x + \frac{1}{3}x =$ the width of the singles court.

$$x + \frac{1}{3}x = 36$$
$$\frac{4}{3}x = 36$$
$$x = 27$$

The width of the singles court is 27 ft.

73. Let $x =$ the width, and $2x + 1 =$ the length. Since $P = 2L + 2W$ we can write the following equation.

$$2(x) + 2(2x + 1) = 38$$
$$6x + 2 = 38$$
$$6x = 36$$
$$x = 6$$

If $x = 6$, then $2x + 1 = 13$. The width is 6 m and the length is 13 m.

74. Let $x =$ the width, and $x + 6 =$ the length. Since $P = 2L + 2W$ we can write the following equation.

$$2(x) + 2(x + 6) = 116$$
$$4x + 12 = 116$$
$$4x = 104$$
$$x = 26$$
$$x + 6 = 32$$

The width is 26 m and the length is 32 m.

75. Let $x =$ the driving time for Suzie and $x + 3 =$ the driving time for Scott. The distance for Suzie is $54x$ and the distance for Scott is $58(x + 3)$. Write the equation about the total distance:

$$54x + 58(x + 3) = 734$$
$$54x + 58x + 174 = 734$$
$$112x = 560$$
$$x = 5$$
$$x + 3 = 8$$

So Scott drove for 8 hours.

76. Let $x =$ the driving time for Sam and $x + 3 =$ the driving time for Dave. The distance for Sam is $60x$ and the distance for Dave is $57(x + 3)$. Write the equation about the total distance:

$$60x + 57(x + 3) = 873$$
$$60x + 57x + 171 = 873$$
$$117x = 702$$
$$x = 6$$
$$x + 3 = 9$$

So Sam drove for 6 hours.

77. Let $x =$ the price per pound of the mixture, which contains 5 pounds of nuts. Write an equation about the total cost:

$$5(x) = 3(8) + 2(6)$$
$$5x = 36$$
$$x = 7.20$$

The mixture should cost \$7.20 per pound.

78. Let $x =$ the price per pound of the mixture, which contains 1.25 pounds of nuts. Write an equation about the total cost:

$$1.25x = 0.5(2.50) + 0.75(5.50)$$
$$1.25x = 5.375$$
$$x = 4.30$$

The mixture should cost \$4.30 per pound.

79. Let $x =$ the length and $\frac{1}{2}x - 3 =$ the width. Use $P = 2L + 2W$:

$$2(x) + 2\left(\frac{1}{2}x - 3\right) = 48$$
$$2x + x - 6 = 48$$
$$3x = 54$$
$$x = 18$$
$$\frac{1}{2}x - 3 = 6$$

So the length is 18 cm and the width is 6 cm.

80. Let $x =$ the width, and $2x + 1 =$ the length. Since $P = 2L + 2W$ we can write the following equation.

$$2(x) + 2(2x + 1) = 278$$
$$6x + 2 = 278$$
$$6x = 276$$
$$x = 46$$

If $x = 46$, then $2x + 1 = 93$. The width is 46 meters and the length is 93 meters.

81. Let $x =$ the number of points scored by the Packers and $x - 25 =$ the number of points scored by the Chiefs.

$$x + x - 25 = 45$$
$$2x = 70$$
$$x = 35$$

The score was Packers 35, Chiefs 10.

82. Let $x =$ the market share for Wal-Mart and $x - 4 =$ the market share for Toys R Us.

$$x + x - 4 = 38$$
$$2x = 42$$
$$x = 21$$
$$x - 4 = 17$$

Wal-Mart had 21% of the toy market and Toys R Us had 17%.

83. Let $x =$ the price per pound for the blended coffee. The price of 0.75 lb of Brazilian coffee at \$10 per lb is \$7.50, and the price of 1.5 lb of Colombian coffee at \$8 per lb is \$12. The total price for 2.25 lb of blended coffee at x dollars per kg is $2.25x$ dollars. We write an equation expressing the total cost:

$$7.50 + 12 = 2.25x$$
$$19.50 = 2.25x$$
$$x \approx 8.67$$

The blended coffee should sell for \$8.67 per lb to the nearest cent.

84. Let $x =$ the price per ounce of the mixture. The total cost of 200 ounces of cinnamon at \$1.80 per ounce is 200(1.80). The total cost of 100 ounces of nutmeg at \$1.60 per ounce is 100(1.60). The total cost of 100 ounces of cloves at \$1.40 per ounce is 100(1.40). Since there will be 400 ounces altogether, the total cost of the mix should be $400x$ dollars.

$$200(1.80) + 100(1.60) + 100(1.40) = 400x$$
$$660 = 400x$$
$$x = \frac{660}{400} = 1.65$$

The mixture should sell for \$1.65/ounce.

85. Let $x =$ the number of pounds of apricots. The total cost of 10 pounds of bananas at \$3.20 per pound is \$32, the total cost of x pounds of apricots at \$4 per pound is $4x$, and the total cost $x + 10$ pounds of mix at \$3.80 per pound is $3.80(x + 10)$. Write an equation expressing the total cost:

$$32 + 4x = 3.80(x + 10)$$
$$32 + 4x = 3.80x + 38$$
$$0.20x = 6$$
$$x = 30$$

The mix should contain 30 pounds of apricots.

86. Let $x =$ the number of pounds of cashews. Note that x pounds of cashews at \$4.80 per pound cost $4.80x$ and 20 pounds of Brazil nuts at \$6.00 per pound cost 6.00(20). Since the final mixture of $x + 20$ pounds should cost \$5.20 per pound, we can write the following equation.

$$4.80x + 6.00(20) = 5.20(x + 20)$$
$$4.80x + 120 = 5.20x + 104$$
$$16 = 0.40x$$
$$40 = x$$

Use 40 pounds of cashews.

87. Let $x =$ the number of quarts to be drained out. In the original 20 qt radiator there are 0.30(20) qts of antifreeze. In the x qts that are drained there are $0.30x$ qts of antifreeze, but in the x qts put back in there are x qts of antifreeze. The equation accounts for all of the antifreeze:

$$0.30(20) - 0.30x + x = 0.50(20)$$
$$6 + 0.70x = 10$$
$$0.7x = 4$$

$$x = \frac{4}{0.7} = \frac{40}{7}$$

The amount drained should be $\frac{40}{7}$ qts.

88. Let $x =$ the amount of the 40% solution that is to be replaced with a 70% solution. The amount of antifreeze in the original 16 quarts is $0.40(16)$. The amount of antifreeze in the x quarts that are removed is $0.40x$. The amount of antifreeze in the x quarts when they are put back in is $0.70x$. The amount of antifreeze in the radiator when the process is complete is $0.50(16)$.

$$\begin{aligned} 0.40(16) - 0.40x + 0.70x &= 0.50(16) \\ 6.4 + 0.30x &= 8 \\ 0.30x &= 1.6 \\ x &= \frac{1.6}{0.3} = \frac{16}{3} \end{aligned}$$

Replace $\frac{16}{3}$ quarts with 70% solution.

89. If $x =$ the profit in the third quarter of the previous year.

$$\begin{aligned} x + 0.158x &= 44 \\ 1.158x &= 44 \\ x &\approx 38 \end{aligned}$$

The profit per share in the previous year was 38 cents.

90. If $x =$ the market value of Exxon Mobil.

$$\begin{aligned} x - 0.07x &= 374 \\ 0.93x &= 374 \\ x &\approx 402 \end{aligned}$$

The market value of Exxon Mobil was \$402 billion.

91. Let $x =$ Brian's inheritance, $\frac{1}{2}x =$ Daniel's inheritance, and $\frac{1}{3}x - 1000 =$ Raymond's inheritance. The sum of the three amounts is \$25,400:

$$\begin{aligned} x + \frac{1}{2}x + \frac{1}{3}x - 1000 &= 25400 \\ \frac{11}{6}x &= 26400 \\ x &= \frac{6}{11} \cdot 26400 \\ &= 14{,}400 \\ \frac{1}{2}x &= 7200 \\ \frac{1}{3}x - 1000 &= 3800 \end{aligned}$$

Brian gets \$14,400, Daniel \$7200, and Raymond \$3800.

92. Let $x =$ Lauren's share, $\frac{1}{2}x =$ Lisa's share, and $\frac{1}{4}x =$ Lena's share. Since the lawyer gets $0.10x$, we can write the following equation.

$$\begin{aligned} x + 0.50x + 0.25x + 0.10x &= 164{,}428 \\ 1.85x &= 164{,}428 \\ x &= 88{,}880 \\ 0.5x &= 44{,}440 \\ 0.25x &= 22{,}220 \\ 0.10x &= 8{,}888 \end{aligned}$$

Lauren gets \$88,880, Lisa gets \$44,440, Lena gets \$22,220, and the lawyer gets \$8888.

93. Let $x =$ the first integer and $x + 1 =$ the second. Subtract the larger from twice the smaller to get 21:

$$\begin{aligned} 2x - (x + 1) &= 21 \\ x - 1 &= 21 \\ x &= 22 \\ x + 1 &= 23 \end{aligned}$$

The integers are 22 and 23.

94. Let $x =$ the first consecutive odd integer and $x + 2 =$ the second. If the smaller is subtracted from twice the larger, the result is 13.

$$\begin{aligned} 2(x + 2) - x &= 13 \\ 2x + 4 - x &= 13 \\ x + 4 &= 13 \\ x &= 9 \\ x + 2 &= 11 \end{aligned}$$

The integers are 9 and 11.

95. Let $x =$ Berenice's time and $x - 2$ $=$ Jarrett's time. Berenice's distance is $50x$ and Jarrett's distance is $56(x - 2)$.

$$\begin{aligned} 50x + 56(x - 2) &= 683 \\ 106x - 112 &= 683 \\ 106x &= 795 \\ x &= 7.5 \end{aligned}$$

Berenice drove for 7.5 hours.

96. Let $x =$ Fernell's time and $x - 3 =$ Dabney's time. Fernell's distance is $50x$ and Dabney's distance is $64(x - 3)$

$$\begin{aligned} 50x + 18 &= 64(x - 3) \\ 50x + 18 &= 64x - 192 \\ -14x &= -210 \\ x &= 15 \end{aligned}$$

Fernell drove for 15 hours.

97. Let $x =$ the length of a side of the square. She will use x meters of fencing for each of 3 sides, but only $\frac{1}{2}x$ meters for the side with the opening.

$$\begin{aligned} 3x + \frac{1}{2}x &= 70 \\ \frac{7}{2}x &= 70 \end{aligned}$$

$$x = \frac{2}{7} \cdot 70 = 20$$

The square will by 20 meters by 20 meters.

98. Let $x =$ the length of the side of the square. He will use x linear feet of siding on each of the 3 sides. On the side with the door he will use $x - 4$ linear feet of siding. Since the total number of linear feet should be 32, we can write the following equation.

$$3x + x - 4 = 32$$
$$4x = 36$$
$$x = 9$$

The foundation should be 9 feet by 9 feet.

99. Let $x =$ the amount invested at 8% and $3000 - x =$ the amount invested at 10%. Income on the first investment is $0.08x$ and income on the second is $0.10(3000 - x)$. The total income is \$290:

$$0.08x + 0.10(3000 - x) = 290$$
$$0.08x + 300 - 0.10x = 290$$
$$-0.02x = -10$$
$$x = \frac{-10}{-0.02} = 500$$
$$3000 - x = 2500$$

She invested \$500 at 8% and \$2500 at 10%.

100. Let $x =$ the amount invested at 6% and $8000 - x =$ the amount invested at 9%. Write an equation e$x$pressing the fact that the total income from the investments was \$690.

$$0.06x + 0.09(8000 - x) = 690$$
$$0.06x + 720 - 0.09x = 690$$
$$-0.03x = -30$$
$$x = 1000$$
$$8000 - x = 7000$$

She invested \$1000 at 6% and \$7000 at 9%.

101. Let $x =$ the number of gallons of 5% alcohol and $5 - x =$ the number of gallons of 10% alcohol. The alcohol in the final 5 gallons is the sum of the alcohol in the two separate quantities that are mixed together:

$$0.05x + 0.10(5 - x) = 0.08(5)$$
$$0.05x + 0.5 - 0.10x = 0.4$$
$$-0.05x = -0.1$$
$$x = \frac{-0.1}{-0.05} = 2$$
$$5 - x = 3$$

Use 2 gallons of 5% solution and 3 gallons of 10% solution.

102. Let $x =$ the number of liters of 12% alcohol and $6 - x =$ the number of liters of water. In the x liters there are $0.12x$ liters of alcohol. In the water there is no alcohol. In the 6 liters there are $0.10(6)$ liters of alcohol.

$$0.12x + 0 = 0.10(6)$$
$$0.12x = 0.6$$
$$x = 5$$

She should use 5 liters of 12% alcohol and 1 liter of water to obtain 6 liters of 10% alcohol.

103. Let $x =$ the number of gallons of ethanol.

$$x = 0.85(x + 90)$$
$$x = 0.85x + 76.5$$
$$0.15x = 76.5$$
$$x = 510$$

Use 510 gallons of ethanol.

104. Let $x =$ the number of gallons of gasoline.

$$765 = 0.85(x + 765)$$
$$765 = 0.85x + 650.25$$
$$0.85x = 114.75$$
$$x = 135$$

Use 135 gallons of gasoline.

105. Let $x =$ Darla's age now and $78 - x =$ Todd's age now. In 6 years Todd will be $78 - x + 6$ or $84 - x$, and 6 years ago Darla was $x - 6$. Todd's age in 6 years is twice what Darla's age was 6 years ago:

$$84 - x = 2(x - 6)$$
$$84 - x = 2x - 12$$
$$-3x = -96$$
$$x = 32$$
$$78 - x = 46$$

Todd is 46 now and Darla is 32 now.

106. Let $x =$ the number of years of experience for Al and $2x =$ the number of years of experience for Bart. Since altogether they have 100 years of experience, Carl has $100 - 3x$ years of experience. In 3 years Carl will have $100 - 3x + 3$ years of experience. A year ago Al had $x - 1$ years of experience. In 3 years Carl will have twice the experience that Al had a year ago.

$$100 - 3x + 3 = 2(x - 1)$$
$$103 - 3x = 2x - 2$$
$$-5x = -105$$
$$x = 21$$
$$2x = 42$$
$$100 - 3x = 37$$

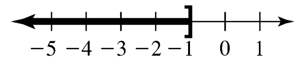
Al has 21 years of experience, Bart has 42 years, and Carl has 37 years of experience.

2.4 WARM-UPS

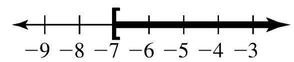
1. False, $0 = 0$.
2. False, $-300 < -2$.
3. True, because $-60 = -60$.
4. False, since $6 < x$ is equivalent to $x > 6$.
5. False, $-2x < 10$ is equivalent to $x > -5$.
6. False, $3x \geq -12$ is equivalent to $x \geq -4$.
7. True, multiply each side by -1.
8. True, because of the trichotomy property.
9. True, because of the trichotomy property.
10. True, because $3 - 4(-2) \leq 11$ is correct.

2.4 EXERCISES

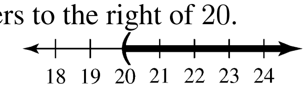
1. An inequality is a statement that expresses inequality between two algebraic expressions.
2. To express inequality we use the symbols $<$, $\leq$, $>$, and $\geq$.
3. If a is less than b, then a lies to the left of b on the number line.
4. A linear inequality is an inequality of the form $ax < b$ or with any of the other inequality symbols used in place of $<$.
5. When you multiply or divide by a negative number, the inequality symbol is reversed.
6. We can verbally indicate inequality with words like less than, at least, greater than, and at most.
7. False, because $-3 > -9$.
8. False, because -8 is to the left of -7 on the number line $(-8 < -7)$.
9. True, because $0 < 8$.
10. True, because -6 is to the right of -8 on the number line $(-6 > -8)$.
11. True, because $-60 > -120$.
12. False, because $3 < -5$ is incorrect.
13. True, because $9 - (-3) = 12$.
14. True, because the left side is 22 and $22 \geq 21$ is correct.
15. Yes, because $2(-3) - 4 < 8$ simplifies to $-10 < 8$.
16. No, because $5 - 3(6) > -1$ simplifies to $-13 > -1$.
17. No, because $2(5) - 3 \leq 3(5) - 9$ simplifies to $7 \leq 6$.
18. Yes, because $6 - 3(-4) \geq 10 - 2(-4)$ simplifies to $18 \geq 18$.
19. No, because $5 - (-1) < 4 - 2(-1)$ simplifies to $6 < 6$.
20. Yes, because $3(9) - 7 \geq 3(9) - 10$ simplifies to $20 \geq 17$.
21. The solution set is the interval $(-\infty, -1]$. To graph it, shade the numbers to the left of -1 and including -1.



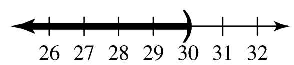
22. The solution set is the interval $[-7, \infty)$. To graph it, shade the numbers to the right of -7 and including -7.



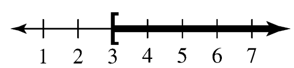
23. The solution set is the interval $(20, \infty)$. To graph it, shade numbers to the right of 20.



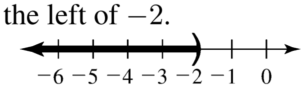
24. The solution set is the interval $(-\infty, 30)$. To graph it, shade the numbers to the left of 30.



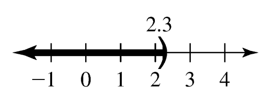
25. The solution set is the interval $[3, \infty)$ because $3 \leq x$ is equivalent to $x \geq 3$. We shade the numbers to the right of 3, including 3.



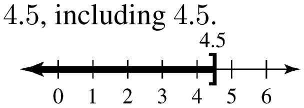
26. The solution set is the interval $(-\infty, -2)$ because $-2 > x$ is equivalent to $x < -2$. Shade the numbers to the left of -2.



27. The solution set is the interval $(-\infty, 2.3)$. Shade to the left of 2.3.

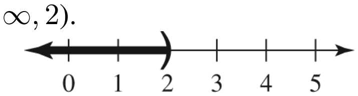


28. The solution set is $(-\infty, 4.5]$. Shade the numbers to the left of 4.5, including 4.5.

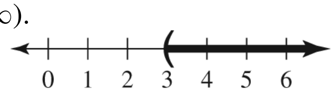


29. $x + 5 > 12$ is equivalent to $x > 7$.
30. $2x - 3 \leq -4$ is equivalent to $2x \leq -1$.
31. $-x < 6$ is equivalent to $x > -6$.
32. $-5 \geq -x$ is equivalent to $5 \leq x$.
33. $-2x \geq 8$ is equivalent to $x \leq -4$.
34. $-5x > -10$ is equivalent to $x < 2$.

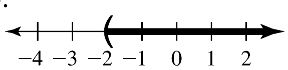
35. $4 < x$ is equivalent to $x > 4$.
36. $-3 \geq x$ is equivalent to $x \leq -3$.
37. $-9 \leq -x$ is equivalent to $x \leq 9$.
38. $6 > -x$ is equivalent to $x > -6$.
39. $x + 3 < 5$
$x < 2$
Solution set is $(-\infty, 2)$.



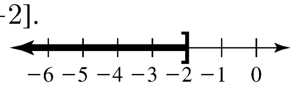
40. $x - 9 > -6$
$x > 3$
Solution set is $(3, \infty)$.



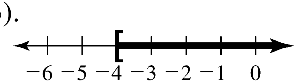
41. $7x > -14$
$x > -2$
Solution set is $(-2, \infty)$.



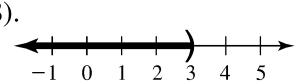
42. $4x \leq -8$
$x \leq -2$
Solution set is $(-\infty, -2]$.



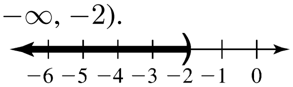
43. $-3x \leq 12$
$x \geq -4$
Solution set is $[-4, \infty)$.



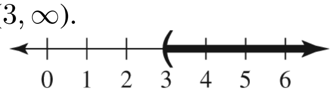
44. $-2x > -6$
$x < 3$
Solution set is $(-\infty, 3)$.



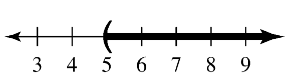
45. $-x > 2$
$x < -2$
Solution set is $(-\infty, -2)$.



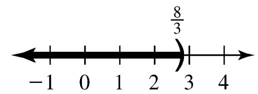
46. $-x < -3$
$x > 3$
Solution set is $(3, \infty)$.



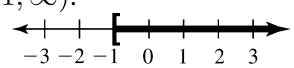
47. $2x - 3 > 7$
$2x > 10$
$x > 5$
Solution set is $(5, \infty)$.



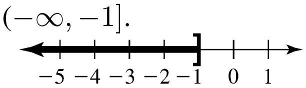
48. $3x - 2 < 6$
$3x < 8$
$x < \frac{8}{3}$
Solution set is $(-\infty, 8/3)$.



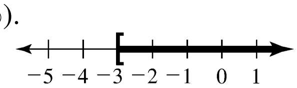
49. $4 - x \leq 3$
$-x \leq -1$
$x \geq 1$
Solution set is $[1, \infty)$.



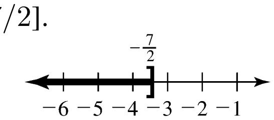
50. $-2 - x \geq -1$
$-x \geq 1$
$x \leq -1$
Solution set is $(-\infty, -1]$.



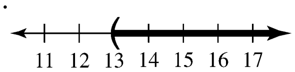
51. $18 \geq 3 - 5x$
$15 \geq -5x$
$-3 \leq x$
$x \geq -3$
Solution set is $[-3, \infty)$.



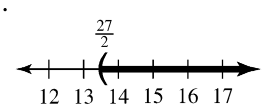
52. $19 \leq 5 - 4x$
$14 \leq -4x$
$-\frac{7}{2} \geq x$
$x \leq -\frac{7}{2}$
Solution set is $(-\infty, -7/2]$.



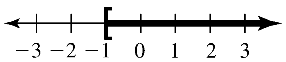
53. $\frac{x-3}{-5} < -2$
$x - 3 > 10$
$x > 13$
Solution set is $(13, \infty)$.



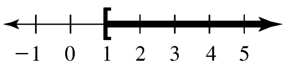
54. $\frac{2x-3}{4} > 6$
$2x - 3 > 24$
$2x > 27$
$x > \frac{27}{2}$
Solution set is $(27/2, \infty)$.



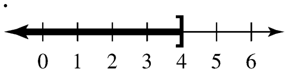
55.
$$\begin{aligned} 2 &\geq \frac{5-3x}{4} \\ 8 &\geq 5-3x \\ 3 &\geq -3x \\ -1 &\leq x \\ x &\geq -1 \end{aligned}$$
Solution set is $[-1, \infty)$.



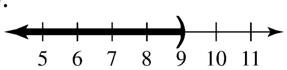
56.
$$\begin{aligned} -1 &\leq \frac{7-5x}{-2} \\ 2 &\geq 7-5x \\ -5 &\geq -5x \\ 1 &\leq x \\ x &\geq 1 \end{aligned}$$
Solution set is $[1, \infty)$.



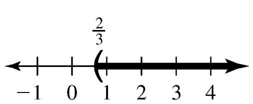
57.
$$\begin{aligned} 3 - \frac{1}{4}x &\geq 2 \\ 4\left(3 - \frac{1}{4}x\right) &\geq 4(2) \\ 12 - x &\geq 8 \\ -x &\geq -4 \\ x &\leq 4 \end{aligned}$$
Solution set is $(-\infty, 4]$.



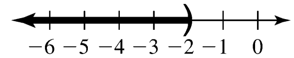
58.
$$\begin{aligned} 5 - \frac{1}{3}x &> 2 \\ 15 - x &> 6 \\ -x &> -9 \\ x &< 9 \end{aligned}$$
Solution set is $(-\infty, 9)$.



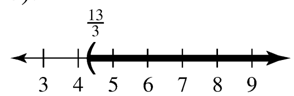
59.
$$\begin{aligned} \frac{1}{4}x - \frac{1}{2} &< \frac{1}{2}x - \frac{2}{3} \\ 12\left(\frac{1}{4}x - \frac{1}{2}\right) &< 12\left(\frac{1}{2}x - \frac{2}{3}\right) \\ 3x - 6 &< 6x - 8 \\ -3x &< -2 \\ x &> \frac{2}{3} \end{aligned}$$
Solution set is $(2/3, \infty)$.



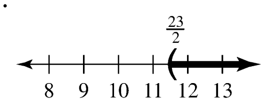
60.
$$\begin{aligned} \frac{1}{3}x - \frac{1}{6} &< \frac{1}{6}x - \frac{1}{2} \\ 6\left(\frac{1}{3}x - \frac{1}{6}\right) &< 6\left(\frac{1}{6}x - \frac{1}{2}\right) \\ 2x - 1 &< x - 3 \\ x &< -2 \end{aligned}$$
Solution set is $(-\infty, -2)$.



61.
$$\begin{aligned} \frac{y-3}{2} &> \frac{1}{2} - \frac{y-5}{4} \\ 4 \cdot \frac{y-3}{2} &> 4 \cdot \frac{1}{2} - 4 \cdot \frac{y-5}{4} \\ 2y - 6 &> 2 - y + 5 \\ 3y &> 13 \\ y &> \frac{13}{3} \end{aligned}$$
Solution set is $(13/3, \infty)$.



62.
$$\begin{aligned} \frac{y-1}{3} - \frac{y+1}{5} &> 1 \\ 15 \cdot \frac{y-1}{3} - 15 \cdot \frac{y+1}{5} &> 15 \cdot 1 \\ 5y - 5 - 3y - 3 &> 15 \\ 2y &> 23 \\ y &> \frac{23}{2} \end{aligned}$$
Solution set is $(23/2, \infty)$.

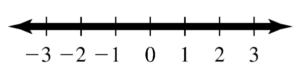


63.
$$\begin{aligned} x - 3 &> x \\ -3 &> 0 \end{aligned}$$
Since $-3 > 0$ is false, the inequality has no solutions. The solution set is the empty set, $\emptyset$.

64.
$$\begin{aligned} 5 - x &< 1 - x \\ 5 &< 1 \end{aligned}$$
Since $5 < 1$ is false, the inequality has no solutions. The solution set is the empty set, $\emptyset$.

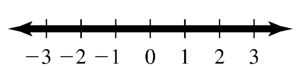
65. $x \geq x$

Subtract x from each side:
$$0 \geq 0$$
Since $0 \geq 0$ is true, all real numbers satisfy the original inequality. The solution set is R or $(-\infty, \infty)$.

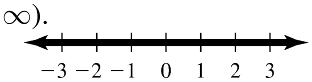


66. $x - 5 \leq x + 5$

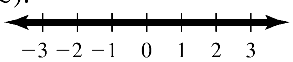
Subtract x from each side:
$$-5 \leq 5$$
Since $-5 \leq 5$ is true, all real numbers satisfy the original inequality. The solution set is R or $(-\infty, \infty)$.



67. $3(x+2) \le 9+3x$
$3x+6 \le 9+3x$
$6 \le 9$
Solution set is $(-\infty, \infty)$.



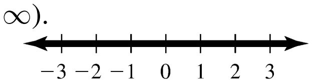
68. $2x+3 > 2(x-4)$
$2x+3 > 2x-8$
$3 > -8$
Solution set is $(-\infty, \infty)$.



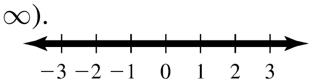
69. $-2(5x-1) \le -5(5+2x)$
$-10x+2 \le -25-10x$
$2 \le -25$
Solution set is $\emptyset$.

70. $-4(2x-5) \le 2(6-4x)$
$-8x+20 \le 12-8x$
$20 \le 12$
Solution set is $\emptyset$.

71. $3x-(4-2x) < 5-(2-5x)$
$3x-4+2x < 5-2+5x$
$5x-4 < 3+5x$
$-4 < 3$
Solution set is $(-\infty, \infty)$.



72. $6-(5-3x) > 7x-(3+4x)$
$6-5+3x > 7x-3-4x$
$1+3x > -3+3x$
$1 > -3$
Solution set is $(-\infty, \infty)$.



73. $\frac{1}{2}x+\frac{1}{4}x < \frac{1}{8}(6x-4)$
$\frac{3}{4}x < \frac{3}{4}x - \frac{1}{2}$
$0 < -\frac{1}{2}$
Solution set is $\emptyset$.

74. $\frac{3}{8}x - \frac{1}{4}x < \frac{1}{6}\left(\frac{3}{4}x-6\right)$
$\frac{1}{8}x < \frac{1}{8}x - 1$
$0 < -1$
Solution set is $\emptyset$.

75. If $x =$ Tony's height in feet, then $x > 6$ feet.

76. If $a =$ Glenda's age in years, then $a < 60$ years.

77. If $s =$ Wilma's salary in dollars, then $s <$ 80,000.

78. If $w =$ Bubba's weight in pounds, then $w > 80$ pounds.

79. If $v =$ speed of the Concorde in mph, then $v \le 1450$ mph.

80. If $s =$ speed on the freeway in mph, then $s \ge 45$.mph

81. If $a =$ amount in dollars that Julie can afford, then $a \le$ \$400.

82. If $a =$ Fred's grade point average, then $a \ge 3.2$.

83. If $b =$ Burt's height in feet, then $b \le 5$ feet.

84. If $r =$ Ernie's speed in mph, then $r \le 10$ mph.

85. If $t =$ Tina's hourly wage in dollars, then $t \le$ \$8.20.

86. If $s =$ selling price in dollars, then $s \ge$ \$12,000.

87. Let $x =$ the price of the car in dollars and $0.08x =$ the amount of tax. To spend less than \$10,000 we must satisfy the inequality
$x + 0.08x + 172 < 10{,}000$
$1.08x < 9828$
$x < 9100$
The price range for the car is $x <$ \$9100.

88. Let $x =$ the price of the sewing machine and $0.10x =$ the amount of sales tax. His total must be less than or equal to \$700.
$x + 0.10x \le 700$
$1.1x \le 700$
$x \le$ \$636.36
The price of the sewing machine must be less than or equal to \$636.36 to the nearest cent.

89. Let $x =$ the price of the truck in dollars and $0.09x =$ the amount of sales tax. The total cost of at least \$10,000 is expressed as
$x + 0.09x + 80 \ge 10{,}000$
$1.09x \ge 9920$
$x \ge 9100.9174$
The price range for the truck is $x \ge$ \$9100.92 to the nearest cent.

90. Let $x =$ the number of movies rented per month. The total cost of renting x DVDs locally is $3.98x$. The Internet is a better deal when $3.98x > 19.95$, or $x > 5.013$. So if you rent 6 or more movies per month it is cheaper to rent them through the Internet.

91. **a)** Decreasing
b) $-0.52n + 71.1 < 55$
$-0.52n < -16.1$
$n > 30.96$
Round to 31 years and add 31 to 1980 to get 2011 as the first year in which the number of births will be less than 55 per 1000 women.

92. $16.45n + 980.2 > 1500$
$16.45n > 519.8$
$n > 31.6$
Round to 32 years and add to 1985 to get 2017 as the first year in which the number of bachelor's degrees will exceed 1.5 million.

93. Let $x =$ the final exam score. One-third of the midterm plus two-thirds of the final must be at least 70:

$$\frac{1}{3}(56) + \frac{2}{3}x \geq 70$$
$$3\left(\frac{1}{3}(56) + \frac{2}{3}x\right) \geq 3(70)$$
$$56 + 2x \geq 210$$
$$2x \geq 154$$
$$x \geq 77$$

The final exam score must satisfy $x \geq 77$.

94. Let $x =$ his final exam score. We can write the following inequality.

$$\frac{2}{3}(56) + \frac{1}{3}x \geq 70$$
$$112 + x \geq 210$$
$$x \geq 98$$

Wilburt must score at least 98 to get an average at least 70.

95. Let $x =$ the price of a pair of A-Mart jeans and $x + 50 =$ the price of a pair of designer jeans. Four pairs of A-Mart jeans cost less than one pair of designer jeans is written as follows.
$4x < x + 50$
$3x < 50$
$x < 16.6666$
The price range for A-Mart jeans is $x < \$16.67$.

96. Let $x =$ Al's rate and $x + 20 =$ Rita's rate. In 5 hours Al drove $5x$ miles and in 3 hours Rita drove $3(x + 20)$ miles. Since his distance is less than Rita's, we can write the following inequality.
$5x < 3(x + 20)$
$5x < 3x + 60$
$2x < 60$
$x < 30$
Al's rate is less than 30 miles per hour.

97. **a)** Write the inequality $x \geq 2$ and subtract -6 from each side to get $x - (-6) \geq 2 - (-6)$, or $x + 6 \geq 8$. So the results are all greater than or equal to 8, which is the interval $[8, \infty)$.
b) Write the inequality $x < -3$ and multiply each side by 2 to get $2x < -6$. So the results are all less than -6, which is the interval $(-\infty, -6)$.
c) If every number in $(8, \infty)$ is divided by 4 the result is the interval $(2, \infty)$.
d) If every number in $(6, \infty)$ is multiplied by -2, the result is the interval $(-\infty, -12)$
e) If every number in $(-\infty, -10)$ is divided by -5, the result is the interval $(2, \infty)$.

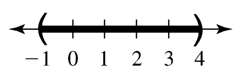
2.5 WARM-UPS

1. True, because both inequalities are true.
2. True, because both inequalities are correct.
3. False, because $3 > 5$ is incorrect.
4. True, because $3 \leq 10$ is correct.
5. True, because both inequalities are correct.
6. True, because both are correct.
7. False, because $0 < -2$ is incorrect.
8. True, because only numbers larger than 8 are larger than 3 and larger than 8.
9. False, because $(3, \infty) \cup [8, \infty) = (3, \infty)$.
10. True, because the numbers greater than -2 and less than 9 are between -2 and 9.

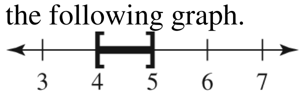
2.5 EXERCISES

1. A compound inequality consists of two inequalities joined with the words “and” or “or.”
2. A compound inequality using and is true only when both simple inequalities are true.
3. A compound inequality using or is true when either one or the other or both inequalities is true.

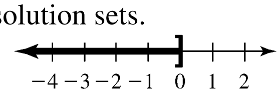
4. Solve each simple inequality and then find either the union or intersection of the solution sets.
5. The inequality $a < b < c$ means that $a < b$ and $b < c$.
6. The inequality $5 < x > 7$ has no meaning. All inequality symbols must point in the same direction in this notation.
7. No, because $-6 > -3$ is incorrect.
8. Yes, because both inequalities are correct.
9. Yes, because both inequalities are correct.
10. Yes, because $3 < 5$ is correct.
11. No, because both inequalities are incorrect.
12. Yes, because $0 \leq 0$ is correct.
13. No, because $-4 > -3$ is incorrect.
14. Yes, because both inequalities in $-4 > -5$ and $-4 < 0$ are correct.
15. Yes, because even though $-4 > -3$ is incorrect, $-4 < 5$ is correct.
16. No, because neither inequality is satisfied if x is replaced by -4.
17. Yes, because $-4 - 3 \geq -7$ is correct.
18. Yes, because $2(-4) \leq -8$ and $5(-4) \leq 0$ are both correct.
19. Yes, because $2(-4) - 1 < -7$ is correct.
20. Yes, because $-3(-4) > 0$ and $3(-4) - 4 < 11$ are both correct.
21. The solution set is the set of numbers between -1 and 4:



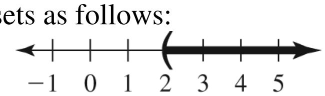
22. If x is less than or equal to 5 and greater than or equal to 4, then x is between 4 and 5 inclusive as shown on the following graph.



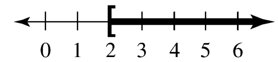
23. Numbers that satisfy both $x \leq 3$ and $x \leq 0$ must be less than or equal to 0. Graph the intersection of the two solution sets.



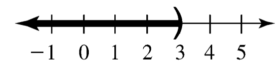
24. Numbers that satisfy both $x > 2$ and $x > 0$ must be greater than 2. Graph the intersection of the two solution sets as follows:



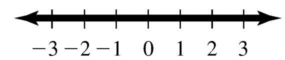
25. Numbers that satisfy $x \geq 2$ or $x \geq 5$ are greater than or equal to 2. Graph the union of the two solution sets:



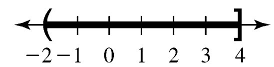
26. The union of $(-\infty, -1)$ with $(-\infty, 3)$ is the interval $(-\infty, 3)$. Graph the union of the two solution sets.



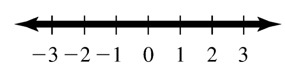
27. The union of $(-\infty, 6]$ with $(-2, \infty)$ is the interval $(-\infty, \infty)$. The union of the two solution sets consists of all real numbers:



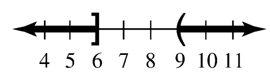
28. The intersection of the two solution sets consists of the numbers greater than -2 and less than or equal to 4.



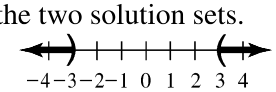
29. The solution set is $\emptyset$, because no number is greater than 9 and less than or equal to 6. There is no graph.
30. Graph the union of the two solution sets. Every real number is either less than 7 or greater than 0.



31. The union of the two solution sets is graphed as follows:



32. The intersection of the two solution sets is the empty set $\emptyset$. There is no real number that is greater than or equal to 4 and less than or equal to -4. There is no graph.
33. The solution set is $\emptyset$, because there is no intersection to the two solution sets. There is no graph.
34. Graph the union of the two solution sets.

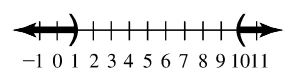


35. $x - 3 > 7$ or $3 - x > 2$

$x > 10$ or $-x > -1$

$x > 10$ or $x < 1$

$(-\infty, 1) \cup (10, \infty)$

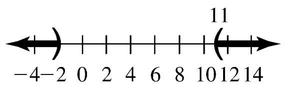


36. $x - 5 > 6$ or $2 - x > 4$

$x > 11$ or $-x > 2$

$x > 11$ or $x < -2$

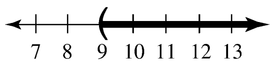
$(-\infty, -2) \cup (11, \infty)$



37. $3 < x$ and $1 + x > 10$

$x > 3$ and $x > 9$

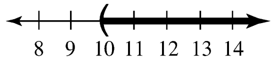
$(9, \infty)$



38. $-0.3x < 9$ and $0.2x > 2$

$x > -30$ and $x > 10$

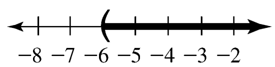
$(10, \infty)$



39. $\frac{1}{2}x > 5$ or $-\frac{1}{3}x < 2$

$x > 10$ or $x > -6$

$(-6, \infty)$

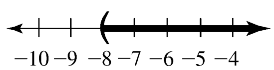


40. $5 < x$ or $3 - \frac{1}{2}x < 7$

$5 < x$ or $-\frac{1}{2}x < 4$

$x > 5$ or $x > -8$

$(-8, \infty)$

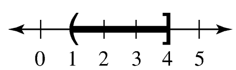


41. $2x - 3 \leq 5$ and $x - 1 > 0$

$2x \leq 8$ and $x > 1$

$x \leq 4$ and $x > 1$

$(1, 4]$



42. $\frac{3}{4}x < 9$ and $-\frac{1}{3}x \leq -15$

$x < 12$ and $x \geq 45$

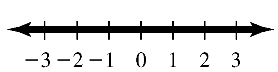
The solution set is $\emptyset$, because there are no real numbers less than 12 and greater than or equal to 45.

43. $\frac{1}{2}x - \frac{1}{3} \geq -\frac{1}{6}$ or $\frac{2}{7}x \leq \frac{1}{10}$

$3x - 2 \geq -1$ or $x \leq \frac{7}{2} \cdot \frac{1}{10}$

$x \geq \frac{1}{3}$ or $x \leq \frac{7}{20}$

$(-\infty, \infty)$



44. $\frac{1}{4}x - \frac{1}{3} > -\frac{1}{5}$ and $\frac{1}{2}x < 2$

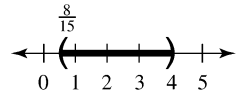
$60\left(\frac{1}{4}x - \frac{1}{3}\right) > 60\left(-\frac{1}{5}\right)$ and $2 \cdot \frac{1}{2}x < 2 \cdot 2$

$15x - 20 > -12$ and $x < 4$

$15x > 8$ and $x < 4$

$x > \frac{8}{15}$ and $x < 4$

$(8/15, 4)$



45. $0.5x < 2$ and $-0.6x < -3$

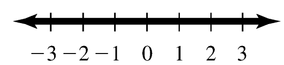
$x < 4$ and $x > 5$

The solution set is $\emptyset$, because there are no numbers that are less than 4 and greater than 5. There is no graph.

46. $0.3x < 0.6$ or $0.05x > -4$

$x < 2$ or $x > -80$

$(-\infty, \infty)$

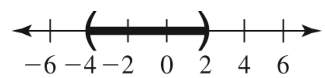


47. $-3 < x + 1 < 3$

$-3 - 1 < x + 1 - 1 < 3 - 1$

$-4 < x < 2$

$(-4, 2)$



48. $-4 \leq x - 4 \leq 1$

$-4 + 4 \leq x - 4 + 4 < 1 + 4$

$0 \leq x \leq 5$

$[0, 5]$

49. $5 < 2x - 3 < 11$

$5 + 3 < 2x - 3 + 3 < 11 + 3$

$8 < 2x < 14$

$4 < x < 7$

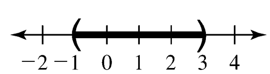
$(4, 7)$

50. $-2 < 3x + 1 < 10$

$-3 < 3x < 9$

$-1 < x < 3$

$(-1, 3)$



51. $-1 < 5 - 3x \leq 14$

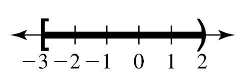
$-6 < -3x \leq 9$

$\frac{-6}{-3} > \frac{-3x}{-3} \geq \frac{9}{-3}$

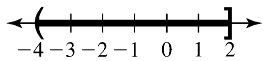
$2 > x \geq -3$

$-3 \leq x < 2$

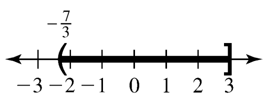
$[-3, 2)$



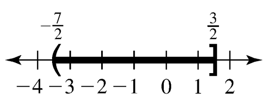
52.
$$\begin{aligned} -1 &\le 3 - 2x < 11 \\ -4 &\le -2x < 8 \\ 2 &\ge x > -4 \\ -4 &< x \le 2 \end{aligned}$$
$(-4, 2]$



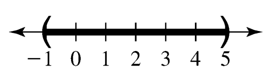
53.
$$\begin{aligned} -3 &< \frac{3m+1}{2} \le 5 \\ 2(-3) &< 2 \cdot \frac{3m+1}{2} \le 2 \cdot 5 \\ -6 &< 3m + 1 \le 10 \\ -7 &< 3m \le 9 \\ -\frac{7}{3} &< m \le 3 \end{aligned}$$
$(-7/3, 3]$



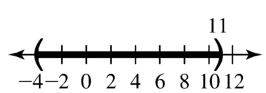
54.
$$\begin{aligned} 0 &\le \frac{3-2x}{2} < 5 \\ 0 &\le 3 - 2x < 10 \\ -3 &\le -2x < 7 \\ \frac{3}{2} &\ge x > -\frac{7}{2} \\ -\frac{7}{2} &< x \le \frac{3}{2} \end{aligned}$$
$(-7/2, 3/2]$



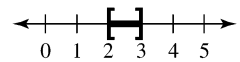
55.
$$\begin{aligned} -2 &< \frac{1-3x}{-2} < 7 \\ -2(-2) &> -2 \cdot \frac{1-3x}{-2} > -2(7) \\ 4 &> 1 - 3x > -14 \\ 3 &> -3x > -15 \\ -1 &< x < 5 \end{aligned}$$
$(-1, 5)$



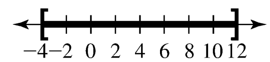
56.
$$\begin{aligned} -3 &< \frac{2x-1}{3} < 7 \\ -9 &< 2x - 1 < 21 \\ -8 &< 2x < 22 \\ -4 &< x < 11 \end{aligned}$$
$(-4, 11)$



57.
$$\begin{aligned} 3 &\le 3 - 5(x-3) \le 8 \\ 3 &\le 3 - 5x + 15 \le 8 \\ 3 &\le 18 - 5x \le 8 \\ -15 &\le -5x \le -10 \\ 3 &\ge x \ge 2 \\ 2 &\le x \le 3 \end{aligned}$$
$[2, 3]$



58.
$$\begin{aligned} 2 &\le 4 - \frac{1}{2}(x-8) \le 10 \\ 2 &\le 4 - \frac{1}{2}x + 4 \le 10 \\ 2 &\le 8 - \frac{1}{2}x \le 10 \\ -6 &\le -\frac{1}{2}x \le 2 \\ 12 &\ge x \ge -4 \\ -4 &\le x \le 12 \end{aligned}$$
$[-4, 12]$



59. $(2, \infty) \cup (4, \infty) = (2, \infty)$
60. $(-3, \infty) \cup (-6, \infty) = (-6, \infty)$
61. $(-\infty, 5) \cap (-\infty, 9) = (-\infty, 5)$
62. $(-\infty, -2) \cap (-\infty, 1) = (-\infty, -2)$
63. $(-\infty, 4] \cap [2, \infty) = [2, 4]$
64. $(-\infty, 8) \cap [3, \infty) = [3, 8)$
65. $(-\infty, 5) \cup [-3, \infty) = (-\infty, \infty)$
66. $(-\infty, -2] \cup (2, \infty)$
67. $(3, \infty) \cap (-\infty, 3] = \emptyset$
68. $[-4, \infty) \cap (-\infty, -6] = \emptyset$
69. $(3, 5) \cap [4, 8) = [4, 5)$
70. $[-2, 4] \cap (0, 9) = (0, 4]$
71. $[1, 4) \cup (2, 6] = [1, 6]$
72. $[1, 3) \cup (0, 5) = (0, 5)$
73. The graph shows real numbers to the right of 2: $x > 2$
74. The graph shows the real numbers to the left of and including 5. The inequality is $x \le 5$.
75. The graph shows the real numbers to the left of 3: $x < 3$
76. The graph shows the real numbers less than -4 together with the real numbers to the right of 3. The inequality is $x < -4$ or $x > 3$.
77. This graph is the union of the numbers greater than 2 with the numbers less than or equal to -1: $x > 2$ or $x \le -1$
78. The graph shows the real numbers greater than -1 and less than 2. The inequality is $-1 < x < 2$.
79. This graph shows real numbers between -2 and 3, including -2: $-2 \le x < 3$
80. The graph shows the real numbers less than 2. The inequality is $x < 2$.

81. The graph shows real numbers greater than or equal to -3: $x \geq -3$

82. The graph shows the real numbers less than or equal to 0 together with the real numbers greater than 1: $x \leq 0$ or $x > 1$.

83. Let $x =$ the final exam score. We write an inequality expressing the fact that $1/3$ of the midterm plus $2/3$ of the final must be between 70 and 79 inclusive.

$$70 \leq \frac{1}{3} \cdot 64 + \frac{2}{3}x \leq 79$$
$$210 \leq 64 + 2x \leq 237$$
$$146 \leq 2x \leq 173$$
$$73 \leq x \leq 86.5$$

84. Let $x =$ the final exam score. His average is calculated as $\frac{2}{3}(64) + \frac{1}{3}x$.

$$70 \leq \frac{2}{3}(64) + \frac{1}{3}x \leq 79$$
$$210 \leq 2(64) + x \leq 237$$
$$210 \leq 128 + x \leq 237$$
$$82 \leq x \leq 109$$

85. The car is replaced when
$0.0004x + 20 > 40$ and
$20{,}000 - 0.2x < 12{,}000$.
Solve these inequalities to get
$x > 50{,}000$ and $x > 40{,}000$,
which is equivalent to $x > 50{,}000$. So the car is replaced when x is in the interval $(50{,}000, \infty)$.

86. If the word in changed to "or" in the previous exercise, then $x > 50{,}000$ or $x > 40{,}000$, which is equivalent to $x > 40{,}000$. So the car is replaced when x is in the interval $(40{,}000, \infty)$.

87. The president worries if $20 + 0.1x < 22$ or $30 - 0.5x < 15$. Solve each inequality to get $x < 20$ or $x > 30$. So the president worries if x is in the union $(-\infty, 20) \cup (30, \infty)$.

88. The country is in recession if $20 + 0.1x > 23$ and $30 - 0.5x < 14$. Solve each inequality to get $x > 30$ and $x > 32$. So the government gets involved if x is in the intersection of the intervals, which is $(32, \infty)$.

89. Let $x =$ the price of the truck. The total spent will be $x + 0.08x + 84$.

$$12{,}000 \leq x + 0.08x + 84 \leq 15{,}000$$
$$12{,}000 \leq 1.08x + 84 \leq 15{,}000$$
$$11{,}916 \leq 1.08x \leq 14{,}916$$
$$\$11{,}033 \leq x \leq \$13{,}811$$

90. Let $x =$ the selling price and $0.10x =$ the broker's commission. Renee gets the selling price minus the commission. Renee must get at least \$13,104.

$$x - 0.10x \geq 13{,}104$$
$$0.90x \geq 13{,}104$$
$$x \geq 14{,}560$$

Renee needs at least \$14,560 to pay off the loan, but the car also will not sell for more than \$14,900. So we can write the inequality $\$14{,}560 \leq x \leq \$14{,}900$.

91. Let $x =$ the number of cigarettes smoked on the run, giving the equivalent of $\frac{1}{2}x$ cigarettes smoked. Thus, she smokes $3 + \frac{1}{2}x$ whole cigarettes per day and this number is between 5 and 12 inclusive:

$$5 \leq 3 + \frac{1}{2}x \leq 12$$
$$10 \leq 6 + x \leq 24$$
$$4 \leq x \leq 18$$

She smokes from 4 to 18 cigarettes on the run.

92. Let $w =$ the width and $w + 20 =$ the length. Since $P = 2L + 2W$ for a rectangle, we can write the following inequality.

$$80 < 2w + 2(w + 20) < 100$$
$$80 < 2w + 2w + 40 < 100$$
$$80 < 4w + 40 < 100$$
$$40 < 4w < 60$$
$$10 < w < 15$$

93. a) In 2000, we have $n = 10$.
$16.45(10) + 1062.45 = 1226.95$
In 2000, there were 1,226,950 bachelors degrees awarded.

b) $16.45n + 1062.45 = 1{,}400$

$$16.45n = 337.55$$
$$n \approx 21$$

$1990 + 21 = 2011$

c) $16.45n + 1062.45 > 1{,}400$

$$16.45n > 337.55$$
$$n > 20.52$$

$$7.79n + 326.82 > 550$$
$$7.79n > 223.18$$
$$n > 28.64$$

Both happen in the year $1990 + 29$, or 2019.

d) Either happens in the year $1990 + 21$, or 2011.

94. a) For 2000, $n = 10$.
$s = 0.38(20) + 31.2 = 35$
In 2000 there were 35 million seniors.
b) $-0.25n + 12.2 = 7$
$-0.25n = -5.2$
$n = 20.8$
$1990 + 21 = 2011$
c) $0.38n + 31.2 > 40$
$0.38n > 8.8$
$n > 23.2$
$1990 + 23.2 = 2013.2$
Both (b) and (c) will be achieved in around 2014.
95. If $a < b$ and $a < -x < b$, we can multiply each part of this inequality by -1 to get $-a > x > -b$ or $-b < x < -a$. In words, x is between $-b$ and $-a$.
96. Notation is used correctly in (a) and (e). The notation is incorrect in (c) and (b) because the inequality symbols point in different directions. The notation is incorrect in (d) because 6 is not less than -8.
97. a) If $3 < x < 8$, then $12 < 4x < 32$.
$(12,\ 32)$
b) If $-2 \le x < 4$, then
$(-5)(-2) \ge -5x > (-5)(4)$
$10 \ge -5x > -20$
$-20 < -5x \le 10.$
$(-20, 10]$
c) If $-3 < x < 6$, then $0 < x + 3 < 9$.
$(0, 9)$
d) If $3 \le x \le 9$, then
$\frac{3}{-3} \ge \frac{x}{-3} \ge \frac{9}{-3}$
$-1 \ge \frac{x}{-3} \ge -3$
$-3 \le \frac{x}{-3} \le -1.$
$[-3, -1]$
98. a) If $x > s$ and $x < t$, then the solution is (s, t) provided $s < t$. There is no solution if $t \le s$.
b) If $x > s$ and $x > t$, then the solution is (s, ∞) if $s \ge t$, or (t, ∞) if $t \ge s$. The solution set is never the empty set.

2.6 WARM-UPS

1. True, because both 2 and -2 have absolute value 2.
2. False, because $|x| = 0$ has only one solution and $|x| = -1$ has no solutions.
3. False, because it is equivalent to $2x - 3 = 7$ or $2x - 3 = -7$.
4. True, because $|x| > 5$ means that x is more than 5 units away from 0 and that is true to the right of 5 or to the left of -5.
5. False, because this equation has no solution.
6. True, because only 3 satisfies the equation.
7. False, because only inequalities that express x between two numbers are written this way.
8. False, because $|x| < 7$ is equivalent to $-7 < x < 7$.
9. True, because 2 is subtracted from each side.
10. False, because if $x = 0$, then $|x| = 0$ and 0 is not positive.

2.6 EXERCISES

1. Absolute value of a number is the number's distance from 0 on the number line.
2. Only 0 is 0 units from 0 on the number line.
3. Since both 4 and -4 are four units from 0, $|x| = 4$ has two solutions.
4. Since $|x| \ge 0$ for every real number x, $|x| = -3$ is impossible.
5. Since the distance from 0 for every number on the number line is greater than or equal to 0, $|x| \ge 0$.
6. Since $|x| \ge 0$ for all x, $|x| < -3$ is impossible.
7. $|a| = 5$
$a = 5$ or $a = -5$
Solution set: $\{-5, 5\}$
8. $|x| = 2$
$x = 2$ or $x = -2$
The solution set is $\{-2, 2\}$.
9. $|x - 3| = 1$
$x - 3 = 1$ or $x - 3 = -1$
$x = 4$ or $x = 2$
Solution set: $\{2, 4\}$
10. $|x - 5| = 2$
$x - 5 = 2$ or $x - 5 = -2$
$x = 7$ or $x = 3$
The solution set is $\{3, 7\}$.

11. $|3 - x| = 6$

$3 - x = 6$ or $3 - x = -6$

$-x = 3$ or $-x = -9$

$x = -3$ or $x = 9$

Solution set: $\{-3, 9\}$

12. $|7 - x| = 6$

$7 - x = 6$ or $7 - x = -6$

$-x = -1$ or $-x = -13$

$x = 1$ or $x = 13$

The solution set is $\{1, 13\}$.

13. $|3x - 4| = 12$

$3x - 4 = 12$ or $3x - 4 = -12$

$3x = 16$ or $3x = -8$

$x = \frac{16}{3}$ or $x = -\frac{8}{3}$

Solution set: $\left\{-\frac{8}{3}, \frac{16}{3}\right\}$

14. $\left|3 - \frac{3}{4}x\right| = \frac{1}{4}$

$3 - \frac{3}{4}x = \frac{1}{4}$ or $3 - \frac{3}{4}x = -\frac{1}{4}$

$12 - 3x = 1$ or $12 - 3x = -1$

$-3x = -11$ or $-3x = -13$

$x = \frac{11}{3}$ or $x = \frac{13}{3}$

The solution set is $\left\{\frac{11}{3}, \frac{13}{3}\right\}$.

15. $\left|\frac{2}{3}x - 8\right| = 0$

$\frac{2}{3}x - 8 = 0$

$\frac{2}{3}x = 8$

$x = 12$

Solution set: $\{12\}$

16. $|5 - 0.1x| = 0$

$5 - 0.1x = 0$

$-0.1x = -5$

$x = 50$

The solution set is $\{50\}$.

17. $|5x + 2| = -3$

The solution set is $\emptyset$, because the absolute value of any quantity is nonnegative.

18. Since absolute value is nonnegative, the solution set is $\emptyset$.

19. $6 - 0.2x = 10$ or $6 - 0.2x = -10$

$-0.2x = 4$ or $-0.2x = -16$

$x = -20$ or $x = 80$

Solution set: $\{-20, 80\}$

20. $|2(a + 3)| = 15$

$2a + 6 = 15$ or $2a + 6 = -15$

$2a = 9$ or $2a = -21$

$a = 9/2$ or $a = -21/2$

The solution set is $\{-21/2, 9/2\}$.

21. $|2(x - 4) + 3| = 5$

$2(x - 4) + 3 = 5$ or $2(x - 4) + 3 = -5$

$2x - 5 = 5$ or $2x - 5 = -5$

$2x = 10$ or $2x = 0$

$x = 5$ or $x = 0$

Solution set: $\{0, 5\}$

22. $|3(x - 2) + 7| = 6$

$|3x - 6 + 7| = 6$

$|3x + 1| = 6$

$3x + 1 = 6$ or $3x + 1 = -6$

$3x = 5$ or $3x = -7$

$x = \frac{5}{3}$ or $x = -\frac{7}{3}$

The solution set is $\left\{\frac{5}{3}, -\frac{7}{3}\right\}$.

23. $|7.3x - 5.26| = 4.215$

$7.3x - 5.26 = 4.215$

$7.3x = 9.475$

$x \approx 1.298$

or $7.3x - 5.26 = -4.215$

$7.3x = 1.045$

$x \approx 0.143$

Solution set: $\{0.143, 1.298\}$

24. $|5.74 - 2.17x| = 10.28$

$5.74 - 2.17x = 10.28$ or

$5.74 - 2.17x = -10.28$

$-2.17x = 4.54$ or $-2.17x = -16.02$

$x \approx -2.092$ or $x \approx 7.382$

The solution set is $\{-2.092, 7.382\}$.

25. $3 + |x| = 5$

$|x| = 2$

$x = 2$ or $x = -2$

Solution set: $\{-2, 2\}$

26. $|x| - 10 = -3$

$|x| = 7$

$x = 7$ or $x = -7$

The solution set is $\{-7, 7\}$.

27. $3|a| - 6 = 21$

$3|a| = 27$

$|a| = 9$

$a = 9$ or $a = -9$

The solution set is $\{-9, 9\}$.

28. $-2|b| + 3 = -9$

$-2|b| = -12$

$|b| = 6$

$b = 6$ or $b = -6$

The solution set is $\{-6, 6\}$.

29. $3\,|\,w+1\,|\,-2=7$
$3\,|w+1\,|\,=9$
$|\,w+1\,|\,=3$
$w+1=3$ or $w+1=-3$
$w=2$ or $w=-4$
The solution set is $\{-4, 2\}$.

30. $2\,|\,y-3\,|\,-11=-1$
$2\,|y-3\,|\,=10$
$|\,y-3\,|\,=5$
$y-3=5$ or $y-3=-5$
$y=8$ or $y=-2$
The solution set is $\{-2, 8\}$.

31. $2-|\,x+3|=-6$
$-|\,x+3|=-8$
$|x+3|=8$
$x+3=8$ or $x+3=-8$
$x=5$ or $x=-11$
Solution set: $\{-11, 5\}$

32. $4-3\,|\,x-2\,|\,=-8$
$-3\,|\,x-2\,|\,=-12$
$|\,x-2\,|\,=4$
$x-2=4$ or $x-2=-4$
$x=6$ or $x=-2$
The solution set is $\{-2, 6\}$.

33. $5-\dfrac{|3-2x|}{3}=4$
$15-\,|\,3-2x\,|\,=12$
$-\,|\,3-2x\,|\,=-3$
$|\,3-2x\,|\,=3$
$3-2x=3$ or $3-2x=-3$
$-2x=0$ or $-2x=-6$
$x=0$ or $x=3$
The solution set is $\{0, 3\}$.

34. $3-\frac{1}{2}\left|\frac{1}{2}x-4\right|=2$
$6-\left|\frac{1}{2}x-4\right|=4$
$-\left|\frac{1}{2}x-4\right|=-2$
$\left|\frac{1}{2}x-4\right|=2$
$\frac{1}{2}x-4=2$ or $\frac{1}{2}x-4=-2$
$\frac{1}{2}x=6$ or $\frac{1}{2}x=2$
$x=12$ or $x=4$
The solution set is $\{4, 12\}$.

35. $x-5=2x+1$ or $x-5=-(2x+1)$
$-6=x$ or $x-5=-2x-1$
$3x=4$
$x=4/3$
Solution set: $\left\{-6, \frac{4}{3}\right\}$

36. $|\,w-6\,|\,=\,|\,3-2w\,|$
$w-6=3-2w$ or $w-6=-(3-2w)$
$3w=9$ or $w-6=-3+2w$
$w=3$ or $-3=w$
The solution set is $\{-3, 3\}$.

37. $\left|\frac{5}{2}-x\right|=\left|2-\frac{x}{2}\right|$
$\frac{5}{2}-x=2-\frac{x}{2}$ or $\frac{5}{2}-x=-\left(2-\frac{x}{2}\right)$
$5-2x=4-x$ or $\frac{5}{2}-x=-2+\frac{x}{2}$
$1=x$ or $5-2x=-4+x$
$-3x=-9$
$x=3$
Solution set: $\{1, 3\}$

38. $\left|x-\frac{1}{4}\right|=\left|\frac{1}{2}x-\frac{3}{4}\right|$
$x-\frac{1}{4}=\frac{1}{2}x-\frac{3}{4}$ or $x-\frac{1}{4}=-\left(\frac{1}{2}x-\frac{3}{4}\right)$
$4x-1=2x-3$ or $4x-1=-2x+3$
$2x=-2$ or $6x=4$
$x=-1$ or $x=2/3$
The solution set is $\left\{-1, \frac{2}{3}\right\}$.

39. $|\,x-3|=|\,3-x\,|$
$x-3=3-x$ or $x-3=-(3-x)$
$2x=6$ or $x-3=-3+x$
$x=3$ or $x-3=x-3$
The second equation is an identity. So the solution set is the set of real numbers, which is written as R or $(-\infty, \infty)$.

40. $|\,a-6\,|\,=\,|\,6-a\,|$
$a-6=6-a$ or $a-6=-(6-a)$
$2a=12$ or $a-6=-6+a$
$a=6$ or $a-6=a-6$
Since the second equation is an identity, the solution set is R, or $(-\infty, \infty)$.

41. The graph shows the real numbers between -2 and 2: $|\,x\,|\,<2$

42. The graph shows the real numbers between -5 and 5. The inequality is $|\,x\,|\,\le 5$.

43. The graph shows numbers greater than 3 or less than -3: $|\,x\,|\,>3$

44. The graph shows the real numbers greater than or equal to 6 together with the real numbers less than or equal to -6. The inequality is $|\,x\,|\,\ge 6$.

45. The graph shows numbers between -1 and 1 inclusive: $|\,x\,|\,\le 1$

46. The graph shows the real numbers between -1 and 1. The inequality is $|\,x\,|\,<1$.

47. The graph shows numbers two or more units away from 0: $|x| \geq 2$

48. The graph shows the real numbers larger than 4 or smaller than -4. The inequality is $|x| > 4$.

49. No, because $|x| < 3$ is equivalent to $-3 < x < 3$.

50. No, because $|x| > 3$ is equivalent to $x > 3$ or $x < -3$.

51. Yes.

52. Yes.

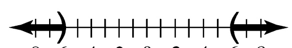
53. No, because $|x-3| \geq 1$ is equivalent to $x-3 \geq 1$ or $x-3 \leq -1$.

54. No, because $|x-3| > 0$ is equivalent to $x-3 > 0$ or $x-3 < 0$.

55. $|x| > 6$

$x > 6$ or $x < -6$

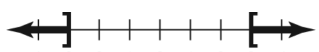
$(-\infty, -6) \cup (6, \infty)$



56. $|w| \geq 3$

$w \geq 3$ or $w \leq -3$

$(-\infty, -3] \cup [3, \infty)$



57. $|t| \leq 2$

$-2 \leq t \leq 2$

$[-2, 2]$

58. $|b| < 4$

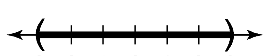
$-4 < b < 4$

$(-4, 4)$

59. $|2a| < 6$

$-6 < 2a < 6$

$-3 < a < 3$



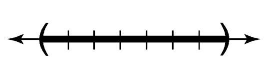
$(-3, 3)$

60. $|3x| < 21$

$-21 < 3x < 21$

$-7 < x < 7$

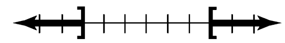
$(-7, 7)$



61. $x-2 \geq 3$ or $x-2 \leq -3$

$x \geq 5$ or $x \leq -1$

$(-\infty, -1] \cup [5, \infty)$

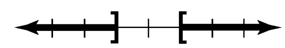


62. $|x-5| \geq 1$

$x-5 \geq 1$ or $x-5 \leq -1$

$x \geq 6$ or $x \leq 4$

$(-\infty, 4] \cup [6, \infty)$



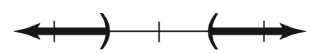
63. $3|a| - 3 > 21$

$3|a| > 24$

$|a| > 8$

$a > 8$ or $a < -8$

$(-\infty, -8) \cup (8, \infty)$



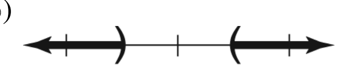
64. $-2|b| + 5 < -9$

$-2|b| < -14$

$|b| > 7$

$b > 7$ or $b < -7$

$(-\infty, -7) \cup (7, \infty)$



65. $3|w+1| - 5 \leq 7$

$3|w+1| \leq 12$

$|w+1| \leq 4$

$-4 \leq w+1 \leq 4$

$-5 \leq w \leq 3$

$[-5, 3]$

66. $2|y-3| - 7 \leq -1$

$|y-3| \leq 3$

$-3 \leq y-3 \leq 3$

$0 \leq y \leq 6$

$[0, 6]$

67. $\frac{1}{5}|2x-4| < 1$

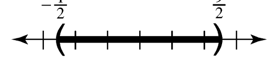
$|2x-4| < 5$

$-5 < 2x-4 < 5$

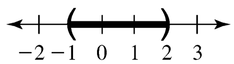
$-1 < 2x < 9$

$-\frac{1}{2} < x < \frac{9}{2}$

$\left(-\frac{1}{2}, \frac{9}{2}\right)$

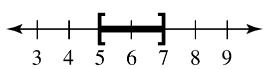


68. $\frac{1}{3}\mid 2x-1 \mid \;< 1$
$\mid 2x-1 \mid \;< 3$
$-3 < 2x-1 < 3$
$-2 < 2x < 4$
$-1 < x < 2$
$(-1, 2)$

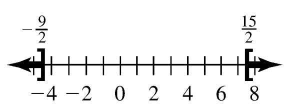


69. $-2\mid 5-x \mid \;\geq -14$
$\mid 5-x \mid \;\leq 7$
$-7 \leq 5-x \leq 7$
$-12 \leq -x \leq 2$
$12 \geq x \geq -2$
$[-2, 12]$

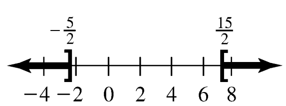
70. $-3\mid 6-x \mid \;\geq -3$
$\mid 6-x \mid \;\leq 1$
$-1 \leq 6-x \leq 1$
$-7 \leq -x \leq -5$
$7 \geq x \geq 5$
$5 \leq x \leq 7$
$[5, 7]$



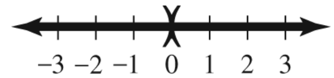
71. $2\mid 3-2x \mid \;-6 \geq 18$
$2\mid 3-2x \mid \;\geq 24$
$\mid 3-2x \mid \;\geq 12$
$3-2x \geq 12$ or $3-2x \leq -12$
$-2x \geq 9$ or $-2x \leq -15$
$x \leq -\frac{9}{2}$ or $x \geq \frac{15}{2}$
$(-\infty, -\frac{9}{2}] \cup [\frac{15}{2}, \infty)$



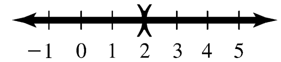
72. $2\mid 5-2x \mid \;-15 \geq 5$
$2\mid 5-2x \mid \;\geq 20$
$\mid 5-2x \mid \;\geq 10$
$5-2x \geq 10$ or $5-2x \leq -10$
$-2x \geq 5$ or $-2x \leq -15$
$x \leq -\frac{5}{2}$ or $x \geq \frac{15}{2}$
$(-\infty, -\frac{5}{2}] \cup [\frac{15}{2}, \infty)$



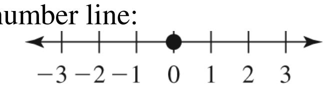
73. $|x| > 0$ is true except when $x = 0$ so the solution set is the set of all real numbers except 0, which is written in interval notation as $(-\infty, 0) \cup (0, \infty)$ and graphed as follows:



74. $\mid x-2 \mid \;> 0$, except when $x = 2$.
$(-\infty, 2) \cup (2, \infty)$

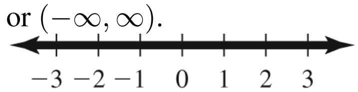


75. $|x| \leq 0$ is true only when $x = 0$ because the absolute value of any nonzero real number is positive. The solution set is $\{0\}$ and it is a single point on the number line:

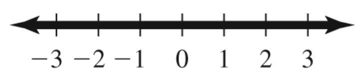


76. $\mid x \mid \;< 0$ is never true because the absolute value of any nonzero real number is positive and the absolute value of 0 is 0. The solution set is the empty set $\emptyset$ and there is no graph.

77. The inequality $\mid x-5 \mid \;\geq 0$ is satisfied by every real number because the absolute value of any real number is nonnegative. The solution set is R, or $(-\infty, \infty)$.



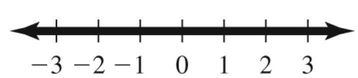
78. The inequality $\mid 3x-7 \mid \;\geq -3$ is satisfied by every real number because the absolute value of any real number is nonnegative. The solution set is R or $(-\infty, \infty)$.



79. $-2\mid 3x-7 \mid \;> 6$ is equivalent to
$\mid 3x-7 \mid \;< -3$.
Absolute value of an expression cannot be less than a negative number. Solution set is $\emptyset$.

80. $-3\mid 7x-42 \mid \;> 18$ is equivalent to
$\mid x-42 \mid \;< -6$.
Absolute value of an expression cannot be less than a negative number because absolute value of any real number is nonnegative. The solution set is $\emptyset$.

81. $\mid 2x+3 \mid \;+6 > 0$
$\mid 2x+3 \mid \;> -6$
Since the absolute value of any expression is greater than or equal to zero, it is greater than any negative number. The solution set is R or $(-\infty, \infty)$.



82. $|5-x|+5>5$
$|5-x|>0$
Since the absolute value of any real number is greater than or equal to 0, the only value of x that does not satisfy this inequality is one for which $|5-x|=0$. This inequality is satisfied by every real number except 5. The solution set is $(-\infty, 5) \cup (5, \infty)$.

83. $1 < |x+2|$
$|x+2| > 1$
$x+2>1$ or $x+2<-1$
$x>-1$ or $x<-3$
$(-\infty, -3) \cup (-1, \infty)$

84. $5 \geq |x-4|$
$|x-4| \leq 5$
$-5 \leq x-4 \leq 5$
$-1 \leq x \leq 9$
$[-1, 9]$

85. $5 > |x+1|$
$|x|+1<5$
$|x|<4$
$-4<x<4$
$(-4, 4)$

86. $4 \leq |x|-6$
$10 \leq |x|$
$|x| \geq 10$
$x \geq 10$ or $x \leq -10$
$(-\infty, -10] \cup [10, \infty)$

87. $3-5|x|>-2$
$-5|x|>-5$
$|x|<1$
$-1<x<1$
$(-1, 1)$

88. $1-2|x|<-7$
$-2|x|<-8$
$|x|>4$
$x>4$ or $x<-4$
$(-\infty, -4) \cup (4, \infty)$

89. $|5.67x-3.124|<1.68$
$-1.68<5.67x-3.124<1.68$
$1.444<5.67x<4.804$
$0.255<x<0.847$
$(0.255, 0.847)$

90. $|4.67-3.2x| \geq 1.43$
$4.67-3.2x \geq 1.43$ or $4.67-3.2x \leq -1.43$
$-3.2x \geq -3.24$ or $-3.2x \leq -6.1$
$x \leq 1.0125$ or $x \geq 1.90625$
$(-\infty, 1.0125] \cup [1.90625, \infty)$

91. Let x = the year of the battle of Orleans. Since the difference between x and 1415 is 14 years, we have $|x-1415|=14$.
$x-1415=14$ or $x-1415=-14$
$x=1429$ or $x=1401$
The battle Agincourt was either in 1401 or 1429.

92. Let x = the number of the day in July on which Cram set the record in the 1 mile race. We have $|x-16|=11$.
$x-16=11$ or $x-16=-11$
$x=27$ or $x=5$
So the 1 mile record was set July 5 or July 27.

93. Let x = the weight of Kathy. The difference between their weights is less than 6 pounds is expressed by the absolute value inequality
$|x-127|<6$
$-6<x-127<6$
$121<x<133$
Kathy weighs between 121 and 133 pounds.

94. Let x = Jude's IQ score. Since Jude's score is more than 15 points away from Sherry's, we can write the following inequality.
$|x-110|>15$
$x-110>15$ or $x-110<-15$
$x>125$ or $x<95$
Jude's IQ score is either greater than 125 or less than 95.

95. a) If x is the percentage that approve of the president then x is between $39\% + 5$ and $39\% - 5\%$. That is, the percentage is between 34% and 39%.
b) The difference between x and 39% or 0.39 is less than 0.05.
$|x-0.39|<0.05$

96. a) If x is the time of death, then x is between 3 AM $+$ 2 hours and 3 AM $-$ 2 hours. That is, the time of death is between 1 AM and 5 AM.
b) The difference between x and 3 AM is less than 2 hours. So $|x-3|<2$.

97. a) From the graph it appears that the balls are at the same height when $t=1$ second.

b) Height of first ball is $S = -16t^2 + 50t$ and height of second ball is
$S = -16t^2 + 40t + 10$.
When the balls are at the same height we have
$-16t^2 + 50t = -16t^2 + 40t + 10$
$50t = 40t + 10$
$10t = 10$
$t = 1$
The balls are at the same height when $t = 1$ sec.
c) The difference between the heights is less than 5 feet when
$| -16t^2 + 50t - (-16t^2 + 40t + 10) | < 5$
$| 10t - 10 | < 5$
$-5 < 10t - 10 < 5$
$5 < 10t < 15$
$0.5 < t < 1.5$
98. The height of the dropped ball is $S = -16t^2 + 60$ and the height of the ball tossed upward is $S = -16t^2 + 80t$. The difference between the heights is less than or equal to 10 when
$| -16t^2 + 60 - (-16t^2 + 80t) | \leq 10$
$| 60 - 80t | \leq 10$
$-10 \leq 60 - 80t \leq 10$
$-70 \leq -80t \leq -50$
$0.875 \geq t \geq 0.625$
Now $0.875 - 0.625 = 0.25$. So the balls are less than 10 feet apart for 0.25 sec.
99. a) The equation $|m - n| = |n - m|$ is satisfied for all real numbers, because $m - n$ and $n - m$ are opposites of each other and opposites always have the same absolute value. So both m and n can be in the interval $(-\infty, \infty)$.
b) $|mn| = |m| \cdot |n|$ is satisfied for all real numbers, because of the rules for multiplying real numbers. So both m and n can be in the interval $(-\infty, \infty)$.
c) Since you cannot have 0 in a denominator, the equation is satisfied by all real numbers except if $n = 0$.
100. a) $|m + n| = 8$ or 2 in each case, while $|m| + |n| = 8$ in each case.
b) So we can conclude that
$| m + n | \leq | m | + | n |$.

Chapter 2 Wrap-Up

Enriching Your Mathematical Word Power

1. c **2.** b **3.** d **4.** c
5. c **6.** d **7.** d **8.** a
9. b **10.** d **11.** d **12.** d
13. c **14.** d

CHAPTER 2 REVIEW

1. $2x - 7 = 9$
$2x = 16$
$x = 8$
Solution set: $\{8\}$
2. $5x - 7 = 38$
$5x = 45$
$x = 9$
The solution set is $\{9\}$.
3. $5 - 4x = 11$
$-4x = 6$
$x = \frac{6}{-4} = -\frac{3}{2}$
Solution set: $\left\{-\frac{3}{2}\right\}$
4. $7 - 3x = -8$
$-3x = -15$
$x = 5$
The solution set is $\{5\}$.
5. $x - 6 - (x - 6) = 0$
$x - 6 - x + 6 = 0$
$0 = 0$
Solution set: R or $(-\infty, \infty)$
6. $x - 6 - 2(x - 3) = 0$
$x - 6 - 2x + 6 = 0$
$-x = 0$
$x = 0$
The solution set is $\{0\}$.
7. $2(x - 3) - 5 = 5 - (3 - 2x)$
$2x - 6 - 5 = 5 - 3 + 2x$
$2x - 11 = 2 + 2x$
$-11 = 2$
Solution set: $\emptyset$
8. $2(x - 4) + 5 = -(3 - 2x)$
$2x - 8 + 5 = -3 + 2x$
$2x - 3 = 2x - 3$
The equation is an identity, the solution set is R.

9. $\frac{3}{17}x = 0$

$x = 0$

Solution set: $\{0\}$

10. $-\frac{3}{8}x = \frac{1}{2}$

$x = -\frac{8}{3} \cdot \frac{1}{2} = -\frac{4}{3}$

Solution set: $\{-4/3\}$

11. $\frac{1}{4}x - \frac{1}{5} = \frac{1}{5}x + \frac{4}{5}$

$20\left(\frac{1}{4}x - \frac{1}{5}\right) = 20\left(\frac{1}{5}x + \frac{4}{5}\right)$

$5x - 4 = 4x + 16$

$x = 20$

Solution set: $\{20\}$

12. $\frac{1}{2}x - 1 = \frac{1}{3}x$

$6\left(\frac{1}{2}x - 1\right) = 6 \cdot \frac{1}{3}x$

$3x - 6 = 2x$

$x = 6$

The solution set is $\{6\}$.

13. $\frac{t}{2} - \frac{t-2}{3} = \frac{3}{2}$

$6\left(\frac{t}{2}\right) - 6\left(\frac{t-2}{3}\right) = 6\left(\frac{3}{2}\right)$

$3t - 2t + 4 = 9$

$t + 4 = 9$

$t = 5$

Solution set: $\{5\}$

14. $\frac{y+1}{4} - \frac{y-1}{6} = y + 5$

$12 \cdot \frac{y+1}{4} - 12 \cdot \frac{y-1}{6} = 12(y+5)$

$3y + 3 - 2(y-1) = 12y + 60$

$3y + 3 - 2y + 2 = 12y + 60$

$y + 5 = 12y + 60$

$-11y = 55$

$y = -5$

The solution set is $\{-5\}$.

15. $1 - 0.4(x-4) + 0.6(x-7) = -0.6$

$1 - 0.4x + 1.6 + 0.6x - 4.2 = -0.6$

$0.2x - 1.6 = -0.6$

$0.2x = 1$

$x = 5$

The solution set is $\{5\}$.

16. $0.04x - 0.06(x-8) = 0.1x$

$0.04x - 0.06x + 0.48 = 0.1x$

$-0.12x = -0.48$

$x = 4$

The solution set is $\{4\}$.

17. $ax + b = 0$

$ax = -b$

$x = \frac{-b}{a}$

$x = -\frac{b}{a}$

18. $mx + c = d$

$mx = d - c$

$x = \frac{d-c}{m}$

19. $ax + 2 = cx$

$ax - cx = -2$

$x(a-c) = -2$

$\frac{x(a-c)}{a-c} = \frac{-2}{a-c}$

$x = \frac{2}{c-a}$

20. $mx = 3 - x$

$mx + x = 3$

$x(m+1) = 3$

$x = \frac{3}{m+1}$

21. $mwx = P$

$\frac{mwx}{mw} = \frac{P}{mw}$

$x = \frac{P}{mw}$

22. $xyz = 2$

$x = \frac{2}{yz}$

23. $\frac{x}{2} + \frac{a}{6} = \frac{x}{3}$

$6\left(\frac{x}{2} + \frac{a}{6}\right) = 6\left(\frac{x}{3}\right)$

$3x + a = 2x$

$x = -a$

24. $\frac{x}{4} - \frac{x}{3} = \frac{a}{2}$

$12\left(\frac{x}{4} - \frac{x}{3}\right) = 12\left(\frac{a}{2}\right)$

$3x - 4x = 6a$

$-x = 6a$

$x = -6a$

25. $3x - 2y = -6$

$-2y = -3x - 6$

$y = \frac{-3x-6}{-2}$

$y = \frac{3}{2}x + 3$

26. $4x - 3y + 9 = 0$

$-3y = -4x - 9$

$y = \frac{4}{3}x + 3$

27. $y - 2 = -\frac{1}{3}x + 2$

$y = -\frac{1}{3}x + 4$

28. $y+6=\frac{1}{2}(x-4)$
$y+6=\frac{1}{2}x-2$
$y=\frac{1}{2}x-8$

29. $\frac{1}{2}x-\frac{1}{4}y=5$
$4\left(\frac{1}{2}x-\frac{1}{4}y\right)=4(5)$
$2x-y=20$
$-y=-2x+20$
$y=2x-20$

30. $-\frac{x}{3}+\frac{y}{2}=\frac{5}{8}$

$\frac{y}{2}=\frac{x}{3}+\frac{5}{8}$

$2\cdot\frac{y}{2}=2\cdot\frac{x}{3}+2\cdot\frac{5}{8}$

$y=\frac{2}{3}x+\frac{5}{4}$

31. Let $W=$ the width and $W+5.5=$ the length. Since $2L+2W=P$, we have
$2(W+5.5)+2W=45$
$2W+11+2W=45$
$4W=34$
$W=8.5$
Length is 14 inches and width is 8.5 inches.

32. Let $x=$ the length of the lower base and $x-2=$ the length of the upper base. Use the formula for the area of a trapezoid to write the following equation.

$\frac{1}{2}\cdot 5(x+x-2)=45$
$5(2x-2)=90$
$10x-10=90$
$10x=100$
$x=10$
The length of the lower base is 10 feet.

33. Let $x=$ the wife's income and $x+8000=$ Roy's income. Roy saves $0.10(x+8000)$ and his wife saves $0.08x$. Since the total saved is \$5660, we can write
$0.10(x+8000)+0.08x=5660$
$0.10x+800+0.08x=5660$
$0.18x=4860$
$x=27{,}000$
Roy's wife earns \$27,000 and Roy earns \$35,000.

34. Let $x=$ Duane's income and $x+1000=$ his wife's income. Write an equation about the amount that each one gives to charity.
$0.05x+0.10(x+1000)=2500$
$0.05x+0.10x+100=2500$
$0.15x=2400$
$x=16{,}000$
$x+1000=17{,}000$
Duane makes \$16,000 per year and his wife makes \$17,000 per year.

35. Let $x=$ the list price and $0.20x=$ the discount. The list price minus the discount is equal to \$7,600.
$x-0.20x=7600$
$0.80x=7600$
$x=9500$
The list price was \$9500.

36. Let $x=$ the list price and $0.25x=$ the amount of the discount. Since the selling price is the list price minus the discount, we can write the following equation.
$x-0.25x=465$
$0.75x=465$
$x=620$
The list price was \$620.

37. Let $x=$ the number of nickels and $15-x=$ the number of dimes. The value of x nickels is $5x$ cents and the value of $15-x$ dimes is $10(15-x)$ cents. Since she has a total of 95 cents, we can write the equation
$5x+10(15-x)=95$
$5x+150-10x=95$
$-5x=-55$
$x=11$
$15-x=4$
She has 11 nickels and 4 dimes.

38. Let $x=$ the number of quarters, $6x=$ the number of nickels, and $19-x-6x=$ the number of dimes. Write an equation expressing the total value of the coins.
$0.25x+0.05(6x)+0.10(19-7x)=1.60$
$25x+5(6x)+10(19-7x)=160$
$25x+30x+190-70x=160$

$$-15x = -30$$
$$x = 2$$
$$6x = 12$$
$$19 - 7x = 5$$

She has 2 quarters, 12 nickels, and 5 dimes.

39. Let $x =$ her walking speed and $x + 9 =$ her riding speed. Since she walked for 3 hours, $3x$ is the number of miles that she walked. Since she rode for 5 hours, $5(x + 9)$ is the number of miles that she rode. Her total distance was 85 miles.

$$3x + 5(x + 9) = 85$$
$$3x + 5x + 45 = 85$$
$$8x = 40$$
$$x = 5$$

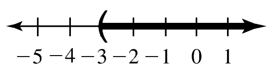
She walked 5 miles per hour for 3 hours, so she walked 15 miles.

40. Let $x =$ his flying speed and $x - 150 =$ his driving speed. His time flying was 90 minutes or 1.5 hours. His distance flying was $1.5x$ and his distance driving was $6(x - 150)$. Since the distance is the same for either driving or flying, we can write the following equation.

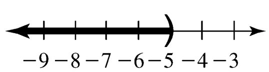
$$1.5x = 6(x - 150)$$
$$1.5x = 6x - 900$$
$$-4.5x = -900$$
$$x = 200$$

His flying speed was 200 miles per hour.

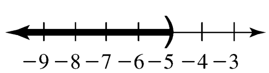
41. $3 - 4x < 15$
$$-4x < 12$$
$$x > -3$$
$(-3, \infty)$



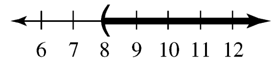
42. $5 - 6x > 35$
$$-6x > 30$$
$$x < -5$$
$(-\infty, -5)$



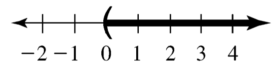
43. $-3 - x > 2$
$$-x > 5$$
$$x < -5$$
$(-\infty, -5)$



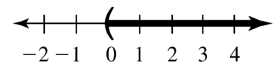
44. $5 - x < -3$
$$-x < -8$$
$$x > 8$$
$(8, \infty)$



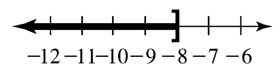
45. $2(x - 3) > -6$
$$2x - 6 > -6$$
$$2x > 0$$
$$x > 0$$
$(0, \infty)$



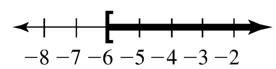
46. $4(5 - x) < 20$
$$5 - x < 5$$
$$-x < 0$$
$$x > 0$$
$(0, \infty)$



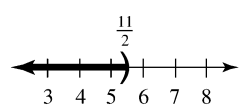
47. $-\frac{3}{4}x \geq 6$
$$-3x \geq 24$$
$$x \leq -8$$
$(-\infty, -8]$



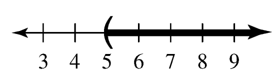
48. $-\frac{2}{3}x \leq 4$
$$-2x \leq 12$$
$$x \geq -6$$
$[-6, \infty)$



49. $3(x + 2) > 5(x - 1)$
$$3x + 6 > 5x - 5$$
$$-2x > -11$$
$$x < \frac{11}{2}$$
$(-\infty, 11/2)$



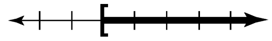
50. $4 - 2(x - 3) < 0$
$$4 - 2x + 6 < 0$$
$$-2x + 10 < 0$$
$$-2x < -10$$
$$x > 5$$
$(5, \infty)$



51. $\frac{1}{2}x + 7 \leq \frac{3}{4}x - 5$
$$4\left(\frac{1}{2}x + 7\right) \leq 4\left(\frac{3}{4}x - 5\right)$$

$$2x + 28 \le 3x - 20$$
$$48 \le x$$

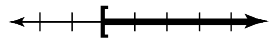
[48, ∞)



52. $\frac{5}{6}x - 3 \ge \frac{2}{3}x + 7$

$$6 \cdot \frac{5}{6}x - 6 \cdot 3 \ge 6 \cdot \frac{2}{3}x + 6 \cdot 7$$
$$5x - 18 \ge 4x + 42$$
$$x \ge 60$$

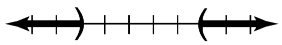
[60, ∞)



53. $x + 2 > 3$ or $x - 6 < -10$

$x > 1$ or $x < -4$

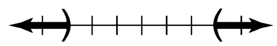
$(-\infty, -4) \cup (1, \infty)$



54. $x - 2 > 5$ or $x - 2 < -1$

$x > 7$ or $x < 1$

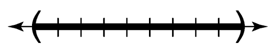
$(-\infty, 1) \cup (7, \infty)$



55. $x > 0$ and $x - 6 < 3$

$x > 0$ and $x < 9$

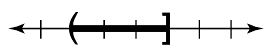
(0, 9)



56. $x \le 0$ and $x + 6 > 3$

$x \le 0$ and $x > -3$

$(-3, 0]$

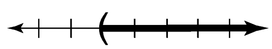


57. $6 - x < 3$ or $-x < 0$

$-x < -3$ or $-x < 0$

$x > 3$ or $x > 0$

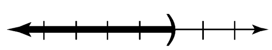
(0, ∞)



58. $-x > 0$ or $x + 2 < 7$

$x < 0$ or $x < 5$

$(-\infty, 5)$



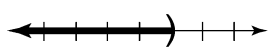
59. $2x < 8$ and $2(x - 3) < 6$

$2x < 8$ and $2x - 6 < 6$

$x < 4$ and $2x < 12$

$x < 4$ and $x < 6$

$(-\infty, 4)$



60. $\frac{1}{3}x > 2$ and $\frac{1}{4}x > 2$

$x > 6$ and $x > 8$

(8, ∞)

61. $x - 6 > 2$ and $6 - x > 0$

$x > 8$ and $-x > -6$

$x > 8$ and $x < 6$

No number is greater than 8 and less than 6.

$\emptyset$

62. $-\frac{1}{2}x < 6$ or $\frac{2}{3}x < 4$

$x > -12$ or $x < 6$

Every real number is either greater than -12 or less than 6. The solution set is R or $(-\infty, \infty)$.

63. $0.5x > 10$ or $0.1x < 3$

$x > 20$ or $x < 30$

Every number is either greater than 20 or less than 30. Solution set is R or $(-\infty, \infty)$

64. $0.02x > 4$ and $0.2x < 3$

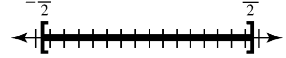
$x > 200$ and $x < 15$

No real number is greater than 200 and less than 15. The solution set is $\emptyset$.

65. $-2 \le \frac{2x - 3}{10} \le 1$

$$10(-2) \le 10 \cdot \frac{2x - 3}{10} \le 10(1)$$
$$-20 \le 2x - 3 \le 10$$
$$-17 \le 2x \le 13$$
$$-\frac{17}{2} \le x \le \frac{13}{2}$$

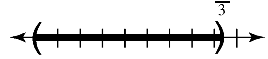
$\left[-\frac{17}{2}, \frac{13}{2}\right]$



66. $-3 < \frac{4 - 3x}{5} < 2$

$$-15 < 4 - 3x < 10$$
$$-19 < -3x < 6$$
$$\frac{19}{3} > x > -2$$

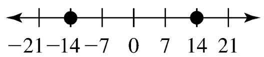
$\left(-2, \frac{19}{3}\right)$



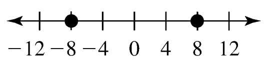
67. $[1, 4) \cup (2, \infty) = [1, \infty)$

68. $(2, 5) \cup (-1, \infty) = (-1, \infty)$

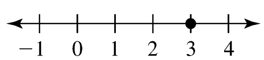
69. $(3, 6) \cap [2, 8] = (3, 6)$
70. $[-1, 3] \cap [0, 8] = [0, 3]$
71. $(-\infty, 5) \cup [5, \infty) = (-\infty, \infty)$
72. $(-\infty, 1) \cup (0, \infty) = (-\infty, \infty)$
73. $(-3, -1] \cap [-2, 5] = [-2, -1]$
74. $[-2, 4] \cap (4, 7] = \emptyset$
75. $|\,x\,| + 2 = 16$
$|\,x\,| = 14$
$x = 14$ or $x = -14$
$\{-14, 14\}$



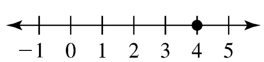
76. $\left|\frac{x}{2}\right| - 5 = -1$
$\left|\frac{x}{2}\right| = 4$
$\frac{x}{2} = 4$ or $\frac{x}{2} = -4$
$x = 8$ or $x = -8$
$\{-8, 8\}$



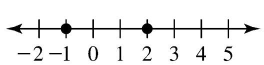
77. $|4x - 12| = 0$
$4x - 12 = 0$
$4x = 12$
$x = 3$
$\{3\}$



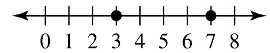
78. $|2x - 8| = 0$
$2x - 8 = 0$
$2x = 8$
$x = 4$
$\{4\}$



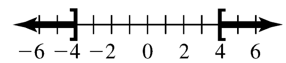
79. Since $|\,x\,| \geq 0$ for any real number, the solution set is $\emptyset$.
80. Since $|\,x\,| \geq 0$ for any real number, the solution set is $\emptyset$.
81. $|2x - 1| - 3 = 0$
$|2x - 1| = 3$
$2x - 1 = 3$ or $2x - 1 = -3$
$2x = 4$ or $2x = -2$
$x = 2$ or $x = -1$
$\{-1, 2\}$



82. $|5 - x| - 2 = 0$
$|5 - x| = 2$
$5 - x = 2$ or $5 - x = -2$
$-x = -3$ or $-x = -7$
$x = 3$ or $x = 7$
$\{3, 7\}$

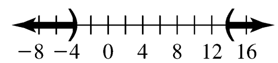


83. $|2x| \geq 8$
$2x \geq 8$ or $2x \leq -8$
$x \geq 4$ or $x \leq -4$
$(-\infty, -4] \cup [4, \infty)$

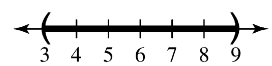


84. $|\,5x - 1\,| \leq 14$
$-14 \leq 5x - 1 \leq 14$
$-13 \leq 5x \leq 15$
$-\frac{13}{5} \leq x \leq 3$
$\left[-\frac{13}{5}, 3\right]$

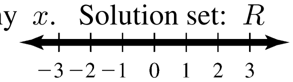
85. $\left|1 - \frac{x}{5}\right| > \frac{9}{5}$
$1 - \frac{x}{5} > \frac{9}{5}$ or $1 - \frac{x}{5} < -\frac{9}{5}$
$5 - x > 9$ or $5 - x < -9$
$-x > 4$ or $-x < -14$
$x < -4$ or $x > 14$
$(-\infty, -4) \cup (14, \infty)$



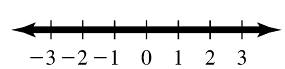
86. $\left|1 - \frac{1}{6}x\right| < \frac{1}{2}$
$-\frac{1}{2} < 1 - \frac{1}{6}x < \frac{1}{2}$
$-3 < 6 - x < 3$
$-9 < -x < -3$
$9 > x > 3$
$(3, 9)$



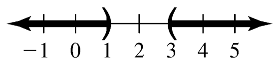
87. Since $|\,x - 3| \geq 0$ for any value of x, the solution set is $\emptyset$.
88. Since the absolute value of any real number is nonnegative, there is no value for x that will make $|\,x - 7\,| \leq -4$ true. The solution set is $\emptyset$.
89. Since $|\,x + 4\,| \geq 0$ for any value of x, $|\,x + 4\,| \geq -1$ for any x. Solution set: R



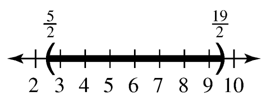
90. Since the absolute value of any real number is nonnegative, $|\,6x - 1\,| \geq 0$ is correct for any real number x.
The solution set is R.



91. $1 - \frac{3}{2}|x-2| < -\frac{1}{2}$
$2 - 3\,|\,x-2\,| < -1$
$-3\,|\,x-2\,| < -3$
$|\,x-2\,| > 1$
$x - 2 > 1$ or $x - 2 < -1$
$x > 3$ or $x < 1$
$(-\infty, 1) \cup (3, \infty)$



92. $1 > \frac{1}{2}|6-x| - \frac{3}{4}$
$4 > 2\,|\,6-x\,| - 3$
$7 > 2\,|\,6-x\,|$
$\frac{7}{2} > |\,6-x\,|$
$-\frac{7}{2} < 6 - x < \frac{7}{2}$
$-7 < 12 - 2x < 7$
$-19 < -2x < -5$
$\frac{19}{2} > x > \frac{5}{2}$
$\left(\frac{5}{2}, \frac{19}{2}\right)$



93. Let x = the rental price, $0.45x =$ the overhead per tape, and $x - 0.45x$ or $0.55x =$ the profit per video. The rental price must be less than or equal to $5 and satisfy the inequality
$0.55x \geq 1.65$
$x \geq 3$
The range of the rental price is $3 \leq x \leq$ $5.

94. Let $x =$ the number of hours that she works per week. Since her pay is $6.80 per hour, she makes $6.80x$ dollars for x hours of work. Since she may not make more than $51, we can write the following equation.
$0 \leq 6.80x \leq 51$
$0 \leq x \leq 7.5$
The number of hours must be in [0, 7.5].

95. Since $150 < h < 180$, we have
$150 < 60.089 + 2.238F < 180$
$89.911 < 2.238F < 119.911$
$40.2 < F < 53.6$
The length of the femur is in $(40.2, 53.6)$.

96. a) From the graph the approximate femur length is 43 cm.
b) If height is over 170 cm, then we have
$61.412 + 2.317F > 170$
$2.317F > 108.588$
$F > 46.9$
Femur is greater than 46.9 cm, or $F > 46.9$.

97. Let $x =$ the number on the mile marker where Dane was picked up. We can write the absolute value equation
$|\,x - 86\,| = 5$
$x - 86 = 5$ or $x - 86 = -5$
$x = 91$ or $x = 81$
He was either at 81 or 91.

98. Let $x =$ Katie's score. Since Katie's score was more than 16 points away from Scott's, we can write the following inequality.
$|\,x - 72\,| > 16$
$x - 72 > 16$ or $x - 72 < -16$
$x > 88$ or $x < 56$
Katie's score was either greater than 88 or less than 56.

99. $b = 0.20(300{,}000 - b)$
$b = 60{,}000 - 0.20b$
$1.2b = 60{,}000$
$b = 50{,}000$
Bonus is $50,000 according to the accountant, and $60,000 according the employees.

100. If the bonus is $60,000, then the profit is $240,000 and the bonus is 25% of the profit.

101. Let $x =$ the number of cows in Washington County and $3600 - x =$ the number of cows in Cade County. We have the equation
$0.30x + 0.60(3600 - x) = 0.50(3600)$
$-0.30x + 2160 = 1800$
$-0.30x = -360$
$x = 1200$
$3600 - x = 2400$
There are 1200 cows in Washington County and 2400 in Cade County.

102. If $x =$ the amount of sales of best vinyl siding, then $90{,}000 + x =$ total sales. Profit on the good and better siding is 0.2(40,000) + 0.3(50,000) or $23,000. Total profit is at least 50% of total sales:
$23{,}000 + 0.6x \geq 0.5(x + 90{,}000)$
$0.1x \geq 22{,}000$
$x \geq 220{,}000$
UHI should sell at least $220,000 in best siding.

103. The numbers to the right of 1 are described by the inequality $x > 1$.

104. The numbers to left of and including 2 are described by the inequality $x \le 2$.
105. The number 2 satisfies the equation $|x-2| = 0$.
106. The graph shows the numbers greater than or equal to 3 and less than 5. These numbers are the solution set to $3 \le x < 5$.
107. The numbers 3 and -3 both satisfy the equation $|x| = 3$.
108. The graph shows the number 1. So $|x-1| = 0$.
109. The numbers to the left of and including -1 satisfy $x \le -1$.
110. The numbers greater than 2 or less than -2 are described by $|x| > 2$.
111. The numbers between -2 and 2 including the endpoints satisfy $|x| \le 2$.
112. The numbers 5 and -5 form the solution set to the absolute value equation $|x| = 5$.
113. $x \le 2$ or $x \ge 7$
114. The numbers between -1 and 1 inclusive satisfy the absolute value inequality $|x| \le 1$.
115. The numbers greater than 3 or less than -3 satisfy $|x| > 3$.
116. The numbers greater than 3 or less than -1 satisfy $x > 3$ or $x < -1$.
117. $5 < x < 7$ or $|x-6| < 1$
118. The numbers greater than 4 or less than -4 satisfy $|x| > 4$.
119. Every number except 0 has a positive absolute value and satisfies $|x| > 0$.
120. The numbers greater than or equal to -6 and less than 6 satisfy $-6 \le x < 6$.

CHAPTER 2 TEST

1. $-10x - 5 + 4x = -4x + 3$
$$-6x - 5 = -4x + 3$$
$$-2x = 8$$
$$x = -4$$
Solution set: $\{-4\}$

2. $\frac{y}{2} - \frac{y-3}{3} = \frac{y+6}{6}$
$$6\left(\frac{y}{2}\right) - 6\left(\frac{y-3}{3}\right) = 6\left(\frac{y+6}{6}\right)$$
$$3y - 2y + 6 = y + 6$$
$$y + 6 = y + 6$$
The equation is an identity. Solution set: R

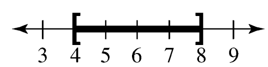
3. $|w| + 3 = 9$
$$|w| = 6$$
$$w = 6 \text{ or } w = -6$$
Solution set: $\{-6, 6\}$

4. $|3 - 2(5 - x)| = 3$
$$|-7 + 2x| = 3$$
$$-7 + 2x = 3 \text{ or } -7 + 2x = -3$$
$$2x = 10 \text{ or } 2x = 4$$
$$x = 5 \text{ or } x = 2$$
Solution set: $\{2, 5\}$

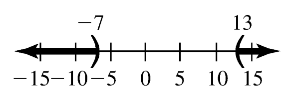
5. $2x - 5y = 20$
$$-5y = -2x + 20$$
$$y = \frac{2}{5}x - 4$$

6. $y = 3xy + 5$
$$y - 3xy = 5$$
$$y(1 - 3x) = 5$$
$$y = \frac{5}{1 - 3x}$$

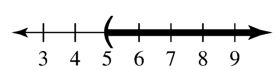
7. $|m - 6| \le 2$
$$-2 \le m - 6 \le 2$$
$$4 \le m \le 8$$
[4, 8]



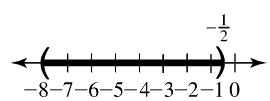
8. $2|x - 3| - 5 > 15$
$$2|x - 3| > 20$$
$$|x - 3| > 10$$
$$x - 3 > 10 \text{ or } x - 3 < -10$$
$$x > 13 \text{ or } x < -7$$
$(-\infty, -7) \cup (13, \infty)$



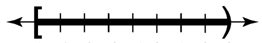
9. $2 - 3(w - 1) < -2w$
$$2 - 3w + 3 < -2w$$
$$-w < -5$$
$$w > 5$$
$(5, \infty)$



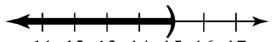
10. $2 < \frac{5 - 2x}{3} < 7$
$$3(2) < 3\left(\frac{5 - 2x}{3}\right) < 3(7)$$
$$6 < 5 - 2x < 21$$
$$1 < -2x < 16$$
$$-\frac{1}{2} > x > -8$$
$\left(-8, -\frac{1}{2}\right)$



11. $3x - 2 < 7$ and $-3x \le 15$
$3x < 9$ and $x \ge -5$
$x < 3$ and $x \ge -5$
$[-5, 3)$



12. $\frac{2}{3}y < 4$ or $y - 3 < 12$
$\frac{3}{2}\left(\frac{2}{3}y\right) < \frac{3}{2}(4)$ or $y - 3 < 12$
$y < 6$ or $y < 15$
$(-\infty, 15)$



13. The equation $|\ 2x - 7\ | = -3$ has no solution because the absolute value of any real number is greater than or equal to zero. The solution set is $\emptyset$.
14. $x - 4 > 1$ or $x < 12$
$x > 5$ or $x < 12$
$(-\infty, \infty)$
15. $3x < 0$ and $x - 5 > 2$
$x < 0$ and $x > 7$
No real number is both less than 0 and greater than 7. Solution set: $\emptyset$
16. Since no real number satisfies $|2x - 5| < 0$, we need only solve $|2x - 5| = 0$, which is equivalent to $2x - 5 = 0$, or $x = 2.5$.
Solution set: $\{2.5\}$
17. Since no real number satisfies $|x - 3| < 0$, the solution set is $\emptyset$.
18. Since $x + 3x = 4x$ is equivalent to $4x = 4x$, the solution set is R or $(-\infty, \infty)$.
19. $2(x + 7) = 2x + 9$
$2x + 14 = 2x + 9$
$14 = 9$
Solution set is $\emptyset$.
20. Since $|x - 6| \ge 0$ for any real number x, $|\ x - 6\ | \ge -6$ for any real number x. The solution set is R or $(-\infty, \infty)$.
21. $x - 0.04(x - 10) = 96.4$
$x - 0.04x + 0.4 = 96.4$
$0.96x = 96$
$x = 100$
$\{100\}$
22. Let W = the width and $W + 16$ = the length. Use $2W + 2L = P$ to write the equation
$2W + 2(W + 16) = 84$
$4W + 32 = 84$
$4W = 52$
$W = 13$
The width is 13 meters.

23. Let h = the height. Use the formula $A = \frac{1}{2}bh$ to write the following equation.
$\frac{1}{2} \cdot 3h = 21$
$3h = 42$
$h = 14$
The height is 14 inches.

24. Let x = the original price and $0.30x$ = the amount of discount. The original price minus the discount is equal to the price she paid.
$x - 0.30x = 210$
$0.70x = 210$
$x = 300$
The original price was \$300.
25. Let x = the number of liters of 11% alcohol. The mixture will be $x + 60$ liters of 7% alcohol. There are $0.11x$ liters of alcohol in the 11% solution, 0.05(60) liters of alcohol in the 5% solution, and $0.07(x + 60)$ liters of alcohol in the 7% solution.
$0.11x + 0.05(60) = 0.07(x + 60)$
$0.11x + 3 = 0.07x + 4.2$
$0.04x = 1.2$
$x = 30$
Use 30 liters of 11% alcohol solution.
26. Let b = Brenda's salary.
$|\ b - 28{,}000\ | > 3000$
$b - 28{,}000 > 3000$ or $b - 28{,}000 < -3000$
$b > 31{,}000$ or $b < 25{,}000$
Brenda's salary is either greater than \$31,000 or less than \$25,000.

Making Connections

Chapters 1-2
1. $5x + 6x = 11x$
2. $5x \cdot 6x = 30x^2$
3. $\frac{6x + 2}{2} = \frac{6x}{2} + \frac{2}{2} = 3x + 1$
4. $5 - 4(2 - x) = 5 - 8 + 4x = 4x - 3$
5. $(30 - 1)(30 + 1) = 29 \cdot 31 = 899$
6. $(30 + 1)^2 = 31^2 = 961$
7. $(30 - 1)^2 = 29^2 = 841$
8. $(2 + 3)^2 = 5^2 = 25$

9. $2^2 + 3^2 = 4 + 9 = 13$
10. $(8 - 3)(3 - 8) = 5(-5) = -25$
11. $(-1)(3 - 8) = -1(-5) = 5$
12. $-2^2 = -(2^2) = -4$
13. $3x + 8 - 5(x - 1) = 3x + 8 - 5x + 5$
$= -2x + 13$
14. $(-6)^2 - 4(-3)2 = 36 + 24 = 60$
15. $3^2 \cdot 2^3 = 9 \cdot 8 = 72$
16. $4(-6) - (-5)(3) = -24 + 15 = -9$
17. $-3x \cdot x \cdot x = -3x^3$
18. $(-1)^6 = 1$

19.
$$\begin{aligned} 5x + 6x &= 8x \\ 11x &= 8x \\ 3x &= 0 \\ x &= 0 \end{aligned}$$
Solution set: $\{0\}$

20.
$$\begin{aligned} 5x + 6x &= 11x \\ 11x &= 11x \end{aligned}$$
This equation is an identity. Solution set is R or $(-\infty, \infty)$.

21.
$$\begin{aligned} 5x + 6x &= 0 \\ 11x &= 0 \\ x &= 0 \end{aligned}$$
Solution set: $\{0\}$

22.
$$\begin{aligned} 5x + 6 &= 11x \\ -6x &= -6 \\ x &= 1 \end{aligned}$$
Solution set: $\{1\}$

23.
$$\begin{aligned} 3x + 1 &= 0 \\ 3x &= -1 \\ x &= -\frac{1}{3} \end{aligned}$$
Solution set: $\left\{-\frac{1}{3}\right\}$

24.
$$\begin{aligned} 5 - 4(2 - x) &= 1 \\ 5 - 8 + 4x &= 1 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$
Solution set: $\{1\}$

25.
$$\begin{aligned} 3x + 6 &= 3(x + 2) \\ 3x + 6 &= 3x + 6 \end{aligned}$$
This equation is an identity. Solution set is R or $(-\infty, \infty)$.

26.
$$\begin{aligned} x - 0.01x &= 990 \\ 0.99x &= 990 \\ x &= 1000 \end{aligned}$$
Solution set: $\{1000\}$

27. $|\,5x + 6\,| = 11$
$$\begin{aligned} 5x + 6 &= 11 &\quad\text{or}\quad 5x + 6 &= -11 \\ 5x &= 5 &\quad\text{or}\quad 5x &= -17 \\ x &= 1 &\quad\text{or}\quad x &= -17/5 \end{aligned}$$
Solution set: $\{-17/5, 1\}$

28. a) From the graph it appears that the cost of renting and buying are equal at around 87,500 copies.
b) If $x =$ the number of copies made in 5 years, then the cost for renting is
$R = 60(75) + 0.06x$ dollars
or $R = 4500 + 0.06x$ dollars.
The cost if the copier is purchased is
$P = 8000 + 0.02x$ dollars.
c)
$$\begin{aligned} 60(75) + 0.06x &= 8000 + 0.02x \\ 0.04x &= 3500 \\ x &= 87{,}500 \end{aligned}$$
Five-year cost is same for 87,500 copies.

d) If 120,000 copies are made, then renting cost is \$11,700 and buying cost is \$10,400. So buying is \$1300 cheaper.

e)
$$\begin{aligned} &|60(75) + 0.06x - (8000 + 0.02x)| < 500 \\ &|-3500 + 0.04x| < 500 \\ &-500 < -3500 + 0.04x < 500 \\ &3000 < 0.04x < 4000 \\ &75{,}000 < x < 100{,}000 \end{aligned}$$
If the number of copies is between 75,000 and 100,000, then the plans differ by less than \$500.

Critical Thinking

Chapter 2

1. The numbers in the table are the cubes of the counting numbers: 1^3, 2^3, 3^3, etc. Column C contains the cubes of the multiples of 3: 3^3, 6^3, 9^3, etc. So column C contains 999^3. One billion is 1000^3 so one billion is in column A.

2. Let x be the number of peanuts in the bowl originally. Larry passes $\frac{2}{3}(x-1)$ or $\frac{2}{3}x - \frac{2}{3}$ peanuts to Curly. Curly passes $\frac{2}{3}\left(\frac{2}{3}x - \frac{2}{3} - 1\right)$ or $\frac{4}{9}x - \frac{10}{9}$ peanuts to Moe. Moe passes $\frac{2}{3}\left(\frac{4}{9}x - \frac{10}{9} - 1\right)$ or $\frac{8}{27}x - \frac{38}{27}$ peanuts to the elephant. The elephant then gives $\frac{1}{3}\left(\frac{8}{27}x - \frac{38}{27}\right)$ or $\frac{8}{81}x - \frac{38}{81}$ peanuts to each of the three guys. So now we need to find a natural number x such that $\frac{8}{81}x - \frac{38}{81}$ is a natural number greater than or equal to 1. Make a table on a graphing calculator and scroll through it to find that the smallest value for x is 25. So the smallest number of peanuts that could have been in the bowl to start with is 25.

3. The numbers between 0 and 100 that are divisible by their units digit are

$1, 2, 3, 4, 5, 6, 7, 8, 9$
$11, 12, 15, 21, 22, 24, 25,$
$31, 32, 33, 35, 36, 41, 42, 44, 45, 48,$
$51, 52, 55, 61, 62, 63, 64, 65, 66,$
$71, 72, 75, 77, 81, 82, 84, 85, 88,$
$91, 92, 93, 95, 96, 99$

There are 50 numbers in this list.

4. Let x be the tens digit and y be the ones digit. The number is $10x + y$. So $10x + y = 2xy$. Solve for y to get

$$\begin{aligned} 2xy - y &= 10x \\ y(2x-1) &= 10x \\ y &= (10x)/(2x-1) \end{aligned}$$

Since x is the tens digit, it must be one of the natural numbers from 1 through 9 inclusive. The only value for x that gives a single digit natural number for y is $x = 3$. If $x = 3$, then $y = 6$ and the number is 36.

5. If the mean score is 14, then the sum of all of the scores is $9(14)$ or 126. The lowest scores that the lowest eight students could get are $1, 2, 3, 4, 5, 6, 7,$ and 8, which have a sum of 36. Subtract 36 from 126 to get the highest possible score of 90.

6. Let x be the income in globs and y be the take-home pay. A Slob who makes x globs pays $x\%$ and takes home $x - \frac{x}{100} \cdot x$. So $y = -0.01x^2 + x$. You could graph this or make a table to find the maximum value of y. The maximum value of y is 25 and it occurs when $x = 50$. So the maximum take home pay is 25 globs per week.

7. The volume of rain in the bottle is $\pi(1)^2(1.5)$ in.^{3} If the bottle was a cylinder with a diameter of 0.75 in. (radius 0.375 in.) then the height of the rain water would indicate how much rain had fallen. So solve the following equation:

$$\begin{aligned} \pi(0.375)^2 h &= \pi(1)^2(1.5) \\ h &= \frac{\pi(1)^2 1.5}{\pi(0.375)^2} \approx 10.666 \end{aligned}$$

So the amount of rain was $10\frac{2}{3}$ in.

8. If the cube is 3 units on each side there are 27 smaller cubes. If any side is painted then at least 9 of the cubes get paint and there could not be 24 unpainted cubes. If the cube is 4 units on each side and consists of 64 cubes, then there only 8 smaller cubes that are in the interior and don't get painted no matter which sides are painted. If a side is painted 16 cubes have paint on them. If a pair of opposite sides are painted, then 32 cubes have paint on them. If any one the remaining sides is painted then 8 more cubes get paint on them, for a total of 40 cubes with paint and 24 without.