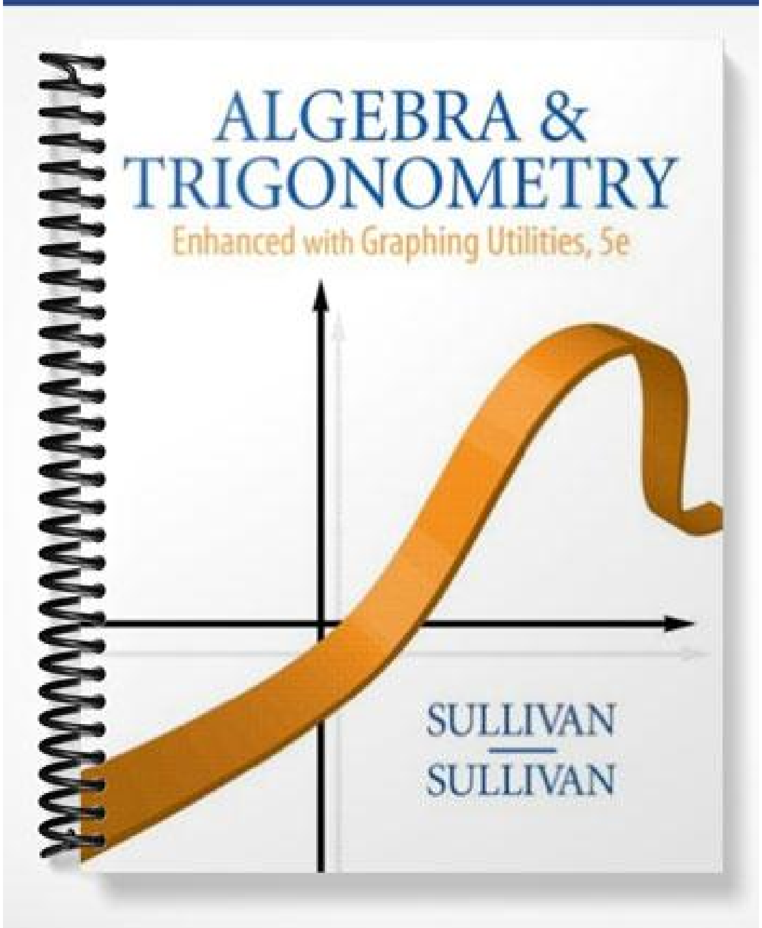
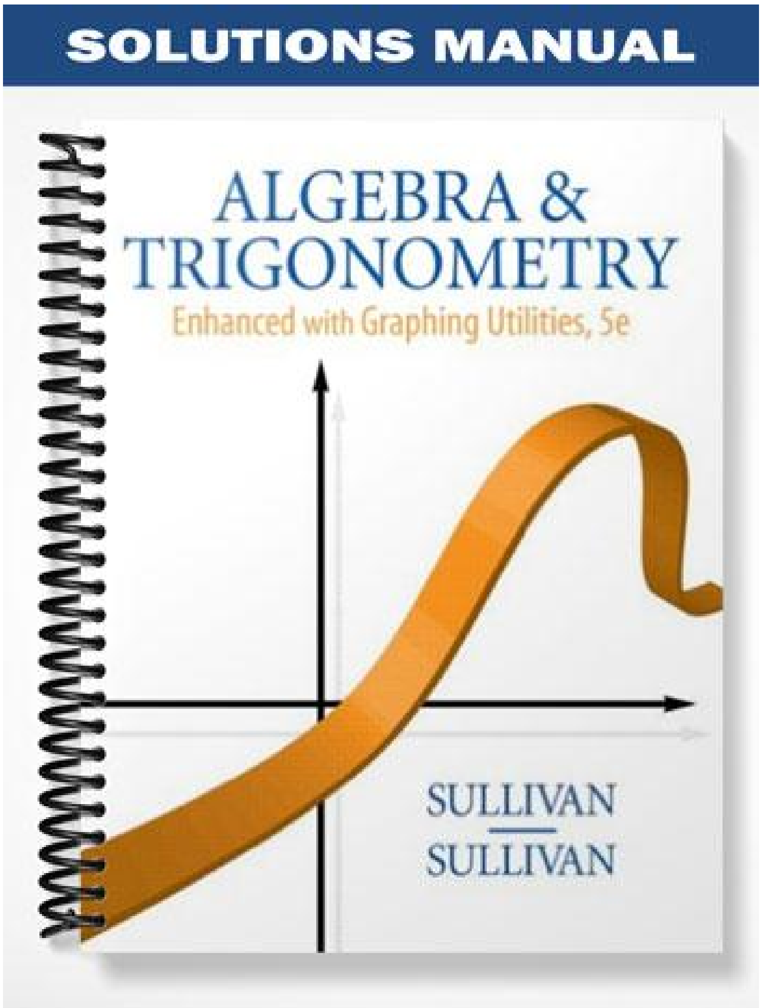
SOLUTIONS MANUAL



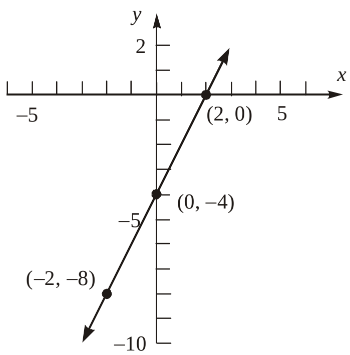

SOLUTIONS MANUAL
ALGEBRA &
TRIGONOMETRY
Enhanced with Graphing Utilities, 5e
SULLIVAN
SULLIVAN

Chapter 2
Graphs

Section 2.1

1. $y = 2x - 4$

x	$y = 2x - 4$	(x, y)
-2	$y = 2(-2) - 4 = -8$	$(-2, -8)$
0	$y = 2(0) - 4 = -4$	$(0, -4)$
2	$y = 2(2) - 4 = 0$	$(2, 0)$



Based on the graph, the intercepts are $(2, 0)$ and $(0, -4)$.

2. $x^2 - 4x - 12 = 0$

$(x - 6)(x + 2) = 0$

$x - 6 = 0$ or $x + 2 = 0$

$x = 6 \qquad x = -2$

The solution set is $\{-2, 6\}$.

3. $y = 0$

4. y-axis

5. 4

6. $(-3, 4)$

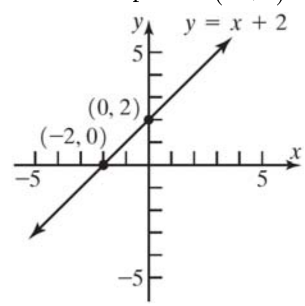
7. True

8. False; a graph can be symmetric with respect to both coordinate axes (in such cases it will also be symmetric with respect to the origin).
For example: $x^2 + y^2 = 1$

9. $y = x + 2$

x-intercept:	y-intercept:
$0 = x + 2$	$y = 0 + 2$
$-2 = x$	$y = 2$

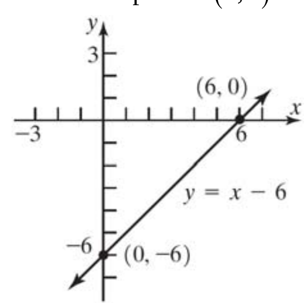
The intercepts are $(-2, 0)$ and $(0, 2)$.



10. $y = x - 6$

x-intercept:	y-intercept:
$0 = x - 6$	$y = 0 - 6$
$6 = x$	$y = -6$

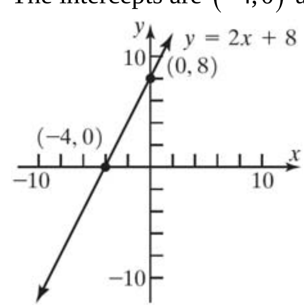
The intercepts are $(6, 0)$ and $(0, -6)$.



11. $y = 2x + 8$

x-intercept:	y-intercept:
$0 = 2x + 8$	$y = 2(0) + 8$
$2x = -8$	$y = 8$
$x = -4$	

The intercepts are $(-4, 0)$ and $(0, 8)$.

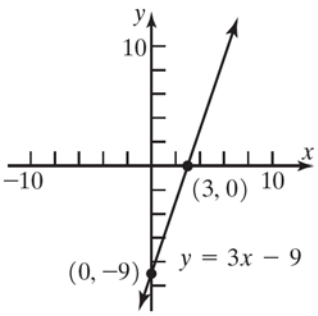


12. $y = 3x - 9$

x-intercept: $0 = 3x - 9$; $3x = 9$; $x = 3$

y-intercept: $y = 3(0) - 9$; $y = -9$

The intercepts are $(3,0)$ and $(0,-9)$.

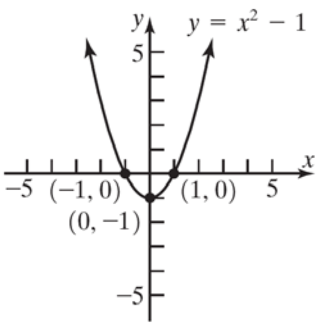


13. $y = x^2 - 1$

x-intercepts: $0 = x^2 - 1$; $x^2 = 1$; $x = \pm 1$

y-intercept: $y = 0^2 - 1$; $y = -1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

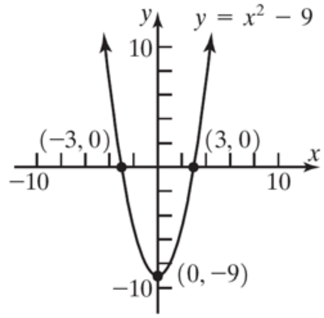


14. $y = x^2 - 9$

x-intercepts: $0 = x^2 - 9$; $x^2 = 9$; $x = \pm 3$

y-intercept: $y = 0^2 - 9$; $y = -9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

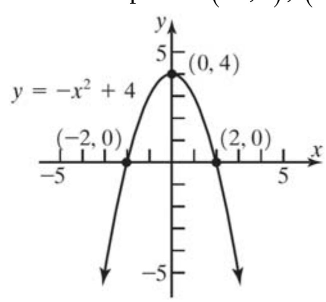


15. $y = -x^2 + 4$

x-intercepts: $0 = -x^2 + 4$; $x^2 = 4$; $x = \pm 2$

y-intercepts: $y = -(0)^2 + 4$; $y = 4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,4)$.

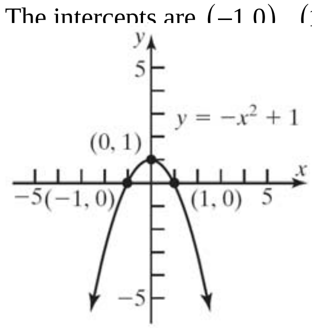


16. $y = -x^2 + 1$

x-intercepts: $0 = -x^2 + 1$; $x^2 = 1$; $x = \pm 1$

y-intercept: $y = -(0)^2 + 1$; $y = 1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,1)$.

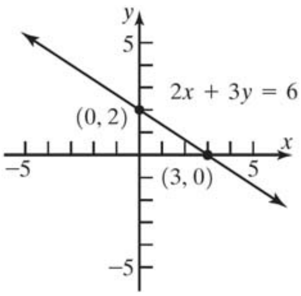


17. $2x+3y=6$

x-intercepts: $2x+3(0)=6$; $2x=6$; $x=3$

y-intercept: $2(0)+3y=6$; $3y=6$; $y=2$

The intercepts are $(3,0)$ and $(0,2)$.

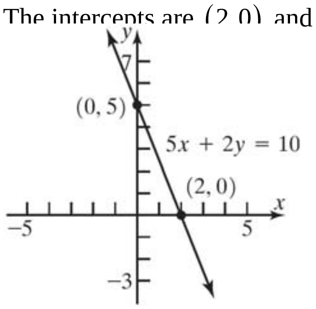


18. $5x+2y=10$

x-intercepts: $5x+2(0)=10$; $5x=10$; $x=2$

y-intercept: $5(0)+2y=10$; $2y=10$; $y=5$

The intercepts are $(2,0)$ and $(0,5)$.

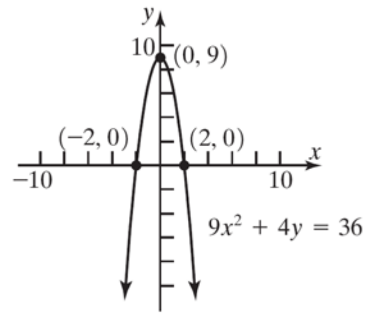


19. $9x^2+4y=36$

x-intercepts: $9x^2+4(0)=36$; $9x^2=36$; $x^2=4$; $x=\pm 2$

y-intercept: $9(0)^2+4y=36$; $4y=36$; $y=9$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,9)$.

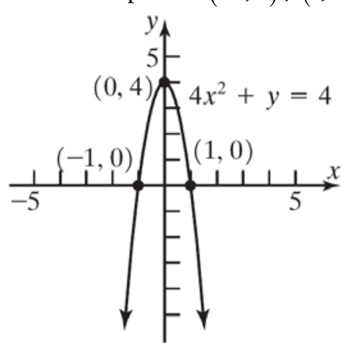


20. $4x^2+y=4$

x-intercepts: $4x^2+0=4$; $4x^2=4$; $x^2=1$; $x=\pm 1$

y-intercept: $4(0)^2+y=4$; $y=4$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,4)$.



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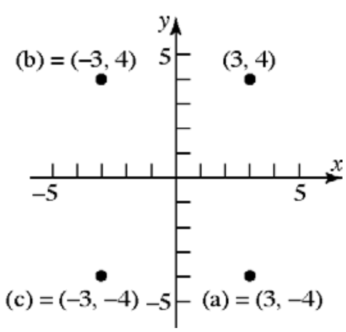

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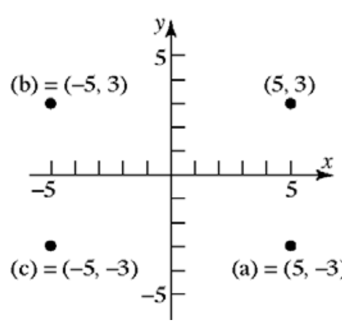

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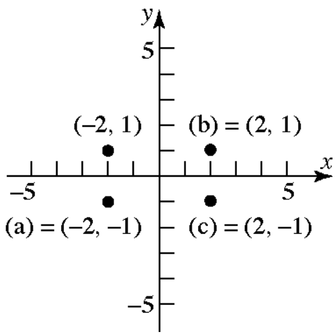

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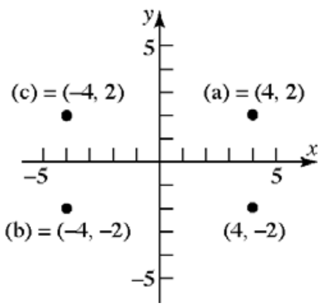

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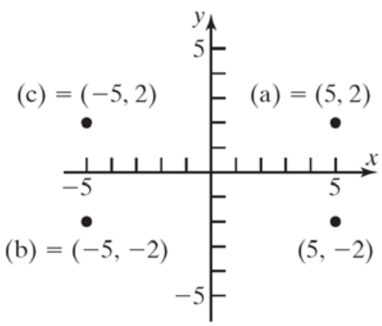

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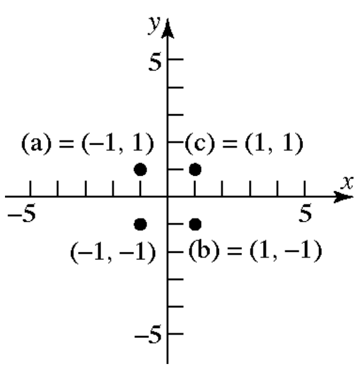

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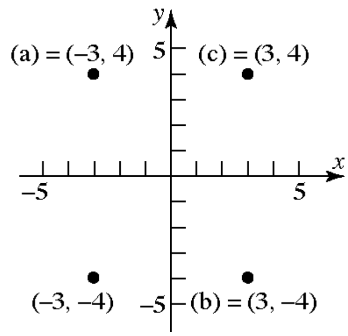

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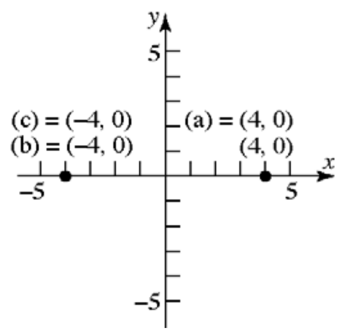

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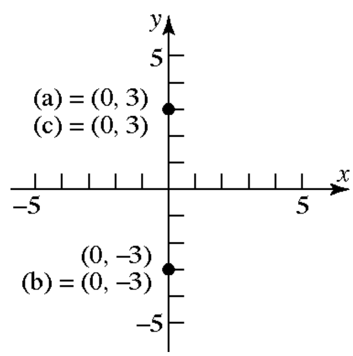

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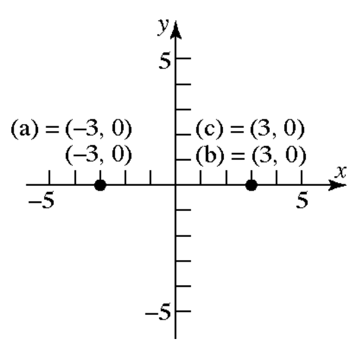


31. Symmetric with respect to the x-axis, y-axis, and the origin.

32. Not symmetric to x-axis, y-axis, or origin

33. Symmetric with respect to the y-axis.

34. Symmetric with respect to the y-axis.

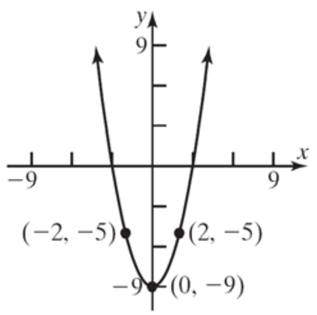
35. Symmetric with respect to the x-axis.

36. Symmetric with respect to the x-axis, y-axis, and the origin.

37. Symmetric with respect to the origin.

38. Symmetric with respect to the origin.

39.



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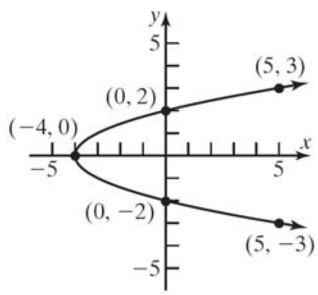

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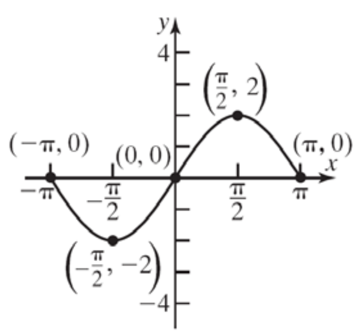

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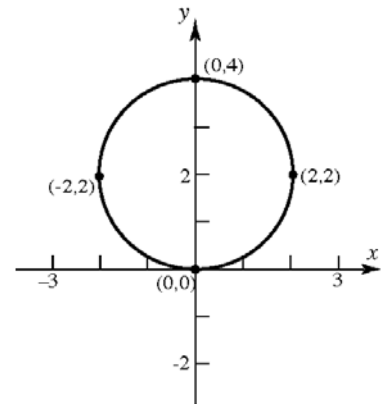
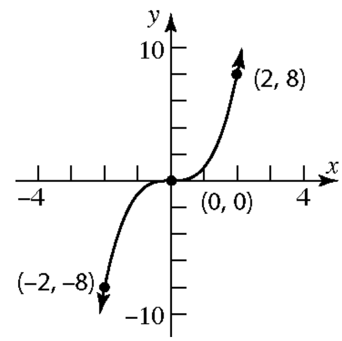
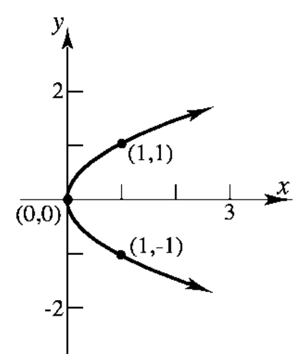


43. $y = x^3$



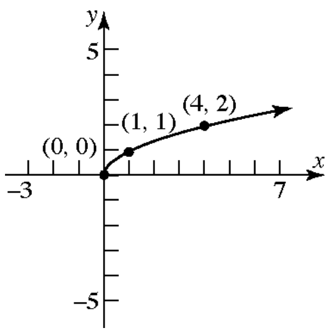
x-intercept : $(0,0)$; y-intercept : $(0,0)$

44. $x = y^2$



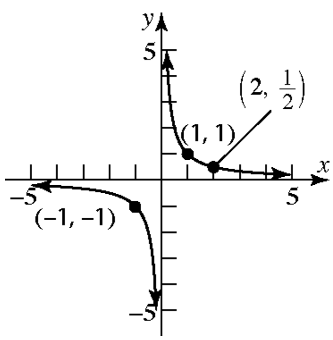
x-intercept : $(0,0)$; y-intercept : $(0,0)$

45. $y=\sqrt{x}$



x-intercept : $(0,0)$; y-intercept : $(0,0)$

46. $y=\dfrac{1}{x}$



no *x* or *y* intercepts

47. $y^2=x+4$

x-intercepts:	*y*-intercepts:
$0^2=x+4$	$y^2=0+4$
$-4=x$	$y^2=4$
	$y=\pm 2$

The intercepts are $(-4,0)$, $(0,-2)$ and $(0,2)$.

Test *x*-axis symmetry: Let $y=-y$

$(-y)^2=x+4$

$y^2=x+4$ same

Test *y*-axis symmetry: Let $x=-x$

$y^2=-x+4$ different

Test origin symmetry: Let $x=-x$ and $y=-y$.

$(-y)^2=-x+4$

$y^2=-x+4$ different

Therefore, the graph will have *x*-axis symmetry.

48. $y^2=x+9$

x-intercepts:	*y*-intercepts:
$(0)^2=-x+9$	$y^2=0+9$
$0=-x+9$	$y^2=9$
$x=9$	$y=\pm 3$

The intercepts are $(-9,0)$, $(0,-3)$ and $(0,3)$.

Test *x*-axis symmetry: Let $y=-y$

$(-y)^2=x+9$

$y^2=x+9$ same

Test *y*-axis symmetry: Let $x=-x$

$y^2=-x+9$ different

Test origin symmetry: Let $x=-x$ and $y=-y$.

$(-y)^2=-x+9$

$y^2=-x+9$ different

Therefore, the graph will have *x*-axis symmetry.

49. $y=\sqrt[3]{x}$

x-intercepts:	*y*-intercepts:
$0=\sqrt[3]{x}$	$y=\sqrt[3]{0}=0$
$0=x$	

The only intercept is $(0,0)$.

Test *x*-axis symmetry: Let $y=-y$

$-y=\sqrt[3]{x}$ different

Test *y*-axis symmetry: Let $x=-x$

$y=\sqrt[3]{-x}=-\sqrt[3]{x}$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=\sqrt[3]{-x}=-\sqrt[3]{x}$

$y=\sqrt[3]{x}$ same

Therefore, the graph will have origin symmetry.

50. $y = \sqrt[5]{x}$

x-intercepts: $0 = \sqrt[3]{x}$, $0 = x$

y-intercepts: $y = \sqrt[5]{0} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \sqrt[5]{x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \sqrt[5]{-x} = -\sqrt[5]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[5]{-x} = -\sqrt[5]{x}$

$y = \sqrt[5]{x}$ same

Therefore, the graph will have origin symmetry.

51. $y = x^4 - 8x^2 - 9$

x-intercepts: $x^4 - 8x^2 - 9 = 0$

$(x^2 - 9)(x^2 + 1) = 0$

$x^2 - 9 = 0$ or $x^2 + 1 = 0$

$x^2 = 9$ or $x^2 = -1$

$x = \pm 3$ no real solution

y-intercepts: $y = 0^4 - 8(0)^2 - 9$

$y = -9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^4 - 8x^2 - 9$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^4 - 8(-x)^2 - 9$

$y = x^4 - 8x^2 - 9$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^4 - 8(-x)^2 - 9$

$-y = x^4 - 8x^2 - 9$ different

Therefore, the graph will have y-axis symmetry.

52. $y = x^4 - 2x^2 - 8$

x-intercepts: $x^4 - 2x^2 - 8 = 0$

$(x^2 - 4)(x^2 + 2) = 0$

$x^2 - 4 = 0$ or $x^2 + 2 = 0$

$x^2 = 4$ or $x^2 = -2$

$x = \pm 2$ no real solution

y-intercepts: $y = 0^4 - 2(0)^2 - 8$

$y = -8$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-8)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^4 - 2x^2 - 8$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^4 - 2(-x)^2 - 8$

$y = x^4 - 2x^2 - 8$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^4 - 2(-x)^2 - 8$

$-y = x^4 - 2x^2 - 8$ different

Therefore, the graph will have y-axis symmetry.

53. $9x^2 + 4y^2 = 36$

x-intercepts: $9x^2 + 4(0)^2 = 36$, $9x^2 = 36$, $x^2 = 4$, $x = \pm 2$

y-intercepts: $9(0)^2 + 4y^2 = 36$, $4y^2 = 36$, $y^2 = 9$, $y = \pm 3$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-3)$, and $(0,3)$.

Test x-axis symmetry: Let $y = -y$

$9x^2 + 4(-y)^2 = 36$

$9x^2 + 4y^2 = 36$ same

Test y-axis symmetry: Let $x = -x$

$9(-x)^2 + 4y^2 = 36$

$9x^2 + 4y^2 = 36$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$9(-x)^2 + 4(-y)^2 = 36$

$9x^2 + 4y^2 = 36$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

54. $4x^2+y^2=4$

x-intercepts:

$$4x^2+0^2=4$$
$$4x^2=4$$
$$x^2=1$$
$$x=\pm 1$$

y-intercepts:

$$4(0)^2+y^2=4$$
$$y^2=4$$
$$y=\pm 2$$

The intercepts are $(-1,0)$, $(1,0)$, $(0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y=-y$

$$4x^2+(-y)^2=4$$
$$4x^2+y^2=4 \text{ same}$$

Test y-axis symmetry: Let $x=-x$

$$4(-x)^2+y^2=4$$
$$4x^2+y^2=4 \text{ same}$$

Test origin symmetry: Let $x=-x$ and $y=-y$

$$4(-x)^2+(-y)^2=4$$
$$4x^2+y^2=4 \text{ same}$$

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

55. $y=x^3+x^2-9x-9$

x-intercepts:
$$0=x^3+x^2-9x-9$$
$$0=x^2(x+1)-9(x+1)$$
$$0=(x^2-9)(x+1)$$
$$0=(x-3)(x+3)(x+1)$$
$$x=3,\quad x=-3,\quad x=-1$$

y-intercept:
$$y=0^3+0^2-9(0)-9$$
$$y=-9$$

The intercepts are $(-3,0)$, $(-1,0)$, $(3,0)$, and $(0,-9)$.

Test x-axis symmetry: Let $y=-y$

$$-y=x^3+x^2-9x-9 \text{ different}$$

Test y-axis symmetry: Let $x=-x$

$$y=(-x)^3+(-x)^2-9(-x)-9$$
$$y=-x^3+x^2+9x-9 \text{ different}$$

Test origin symmetry: Let $x=-x$ and $y=-y$

$$-y=(-x)^3+(-x)^2-9(-x)-9$$
$$-y=-x^3+x^2+9x-9$$
$$y=x^3-x^2-9x+9 \text{ different}$$

Therefore, the graph has none of the indicated symmetries.

56. $y=x^3+2x^2-4x-8$

x-intercepts:
$$0=x^3+2x^2-4x-8$$
$$0=x^2(x+2)-4(x+2)$$
$$0=(x^2-4)(x+2)$$
$$0=(x-2)(x+2)(x+2)$$
$$x=2,\quad x=-2$$

y-intercept:
$$y=0^3+2(0)^2-4(0)-8$$
$$y=-8$$

The intercepts are $(-2,0)$, $(-2,0)$, and $(0,-8)$.

Test x-axis symmetry: Let $y=-y$

$$-y=x^3+2x^2-4x-8 \text{ different}$$

Test y-axis symmetry: Let $x=-x$

$$y=(-x)^3+2(-x)^2-4(-x)-8$$
$$y=-x^3+2x^2+4x-8 \text{ different}$$

Test origin symmetry: Let $x=-x$ and $y=-y$

$$-y=(-x)^3+2(-x)^2-4(-x)-8$$
$$-y=-x^3+2x^2+4x-8$$
$$y=x^3-2x^2-4x+8 \text{ different}$$

Therefore, the graph has none of the indicated symmetries.

57. $y = |x| - 4$

x-intercepts:
$0 = |x| - 4$
$|x| = 4$
$x = -4$ or $x = 4$

y-intercepts:
$y = |0| - 4$
$y = -4$

The intercepts are $(-4, 0)$, $(4, 0)$, and $(0, -4)$.

Test *x*-axis symmetry: Let $y = -y$

$-y = |x| - 4$ different

Test *y*-axis symmetry: Let $x = -x$

$y = |-x| - 4 = |x| - 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = |-x| - 4$

$-y = |x| - 4$

$y = -|x| + 4$ different

Therefore, the graph will have *y*-axis symmetry.

58. $y = |x| - 2$

x-intercepts:
$0 = |x| - 2$
$|x| = 2$
$x = -2$ or $x = 2$

y-intercepts:
$y = |0| - 2$
$y = -2$

The intercepts are $(-2, 0)$, $(2, 0)$, and $(0, -2)$.

Test *x*-axis symmetry: Let $y = -y$

$-y = |x| - 2$ different

Test *y*-axis symmetry: Let $x = -x$

$y = |-x| - 2 = |x| - 2$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = |-x| - 2$

$-y = |x| - 2$

$y = -|x| + 2$ different

Therefore, the graph will have *y*-axis symmetry.

59. $y = \dfrac{3x}{x^2 + 9}$

x-intercepts:
$0 = \dfrac{3x}{x^2 + 9}$
$3x = 0$
$x = 0$

y-intercepts:
$y = \dfrac{3(0)}{0^2 + 9} = \dfrac{0}{9} = 0$

The only intercept is $(0, 0)$.

Test *x*-axis symmetry: Let $y = -y$

$-y = \dfrac{3x}{x^2 + 9}$ different

Test *y*-axis symmetry: Let $x = -x$

$y = \dfrac{3(-x)}{(-x)^2 + 9} = -\dfrac{3x}{x^2 + 9}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \dfrac{3(-x)}{(-x)^2 + 9}$

$-y = -\dfrac{3x}{x^2 + 9}$

$y = \dfrac{3x}{x^2 + 9}$ same

Therefore, the graph has origin symmetry.

60. $y = \dfrac{x^2 - 4}{2x}$

x-intercepts:
$0 = \dfrac{x^2 - 4}{2x}$
$x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$

y-intercepts:
$y = \dfrac{0^2 - 4}{2(0)} = \dfrac{-4}{0}$
undefined

The intercepts are $(-2, 0)$ and $(2, 0)$.

Test *x*-axis symmetry: Let $y = -y$

$-y = \dfrac{x^2 - 4}{2x}$ different

Test *y*-axis symmetry: Let $x = -x$

$y = \dfrac{(-x)^2 - 4}{2(-x)} = -\dfrac{x^2 - 4}{2x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \dfrac{(-x)^2 - 4}{2(-x)}$

$-y = \dfrac{x^2 - 4}{-2x}$

$y = \dfrac{x^2 - 4}{2x}$ same

Therefore, the graph has origin symmetry.

61. $y = \dfrac{-x^3}{x^2 - 9}$

x-intercepts:

$0 = \dfrac{-x^3}{x^2 - 9}$

$-x^3 = 0$

$x = 0$

y-intercepts:

$y = \dfrac{-0^3}{0^2 - 9} = \dfrac{0}{-9} = 0$

The only intercept is $(0, 0)$.

<u>Test *x*-axis symmetry:</u> Let $y = -y$

$-y = \dfrac{-x^3}{x^2 - 9}$

$y = \dfrac{x^3}{x^2 - 9}$ different

<u>Test *y*-axis symmetry:</u> Let $x = -x$

$y = \dfrac{-(-x)^3}{(-x)^2 - 9}$

$y = \dfrac{x^3}{x^2 - 9}$ different

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$

$-y = \dfrac{-(-x)^3}{(-x)^2 - 9}$

$-y = \dfrac{x^3}{x^2 - 9}$

$y = \dfrac{-x^3}{x^2 - 9}$ same

Therefore, the graph has origin symmetry.

62. $y = \dfrac{x^4 + 1}{2x^5}$

x-intercepts:

$0 = \dfrac{x^4 + 1}{2x^5}$

$x^4 = -1$

no real solution

y-intercepts:

$y = \dfrac{0^4 + 1}{2(0)^5} = \dfrac{1}{0}$

undefined

There are no intercepts for the graph of this equation.

<u>Test *x*-axis symmetry:</u> Let $y = -y$

$-y = \dfrac{x^4 + 1}{2x^5}$ different

<u>Test *y*-axis symmetry:</u> Let $x = -x$

$y = \dfrac{(-x)^4 + 1}{2(-x)^5}$

$y = \dfrac{x^4 + 1}{-2x^5}$ different

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$

$-y = \dfrac{(-x)^4 + 1}{2(-x)^5}$

$-y = \dfrac{x^4 + 1}{-2x^5}$

$y = \dfrac{x^4 + 1}{2x^5}$ same

Therefore, the graph has origin symmetry.

63. a. $0 = x^2 - 5$

$x^2 = 5$

$x = \pm\sqrt{5}$

The x-intercepts are $x = -\sqrt{5}$ and $x = \sqrt{5}$.

$y = (0)^2 - 5 = -5$

The y-intercept is $y = -5$.

The intercepts are $(-\sqrt{5}, 0)$, $(\sqrt{5}, 0)$, and $(0, -5)$.

b. x-axis (replace *y* by $-y$):

$-y = x^2 - 5$

$y = 5 - x^2$ different

y-axis (replace *x* by $-x$):

$y = (-x)^2 - 5 = x^2 - 5$ same

origin (replace *x* by $-x$ and *y* by $-y$):

$-y = (-x)^2 - 5$

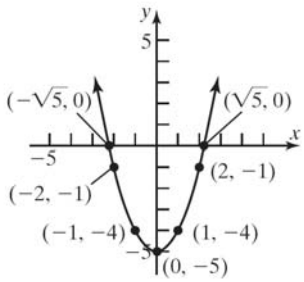
$y = 5 - x^2$ different

The equation has y-axis symmetry.

c. $y = x^2 - 5$

Additional points:

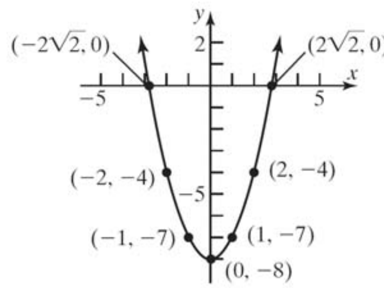
x	$y = x^2 - 5$	(x, y)
1	$y = 1^2 - 5 = -4$	$(1, -4)$
-1	from symmetry	$(-1, -4)$
2	$y = 2^2 - 5 = -1$	$(2, -1)$
-2	from symmetry	$(-2, -1)$



c. $y = x^2 - 8$

Additional points:

x	$y = x^2 - 5$	(x, y)
1	$y = 1^2 - 8 = -7$	$(1, -7)$
-1	from symmetry	$(-1, -7)$
2	$y = 2^2 - 8 = -4$	$(2, -4)$
-2	from symmetry	$(-2, -4)$



64. a. $0 = x^2 - 8$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

The x-intercepts are $x = -2\sqrt{2}$ and $x = 2\sqrt{2}$.

$y = (0)^2 - 8 = -8$

The y-intercept is $y = -8$.

The intercepts are $\left(-2\sqrt{2}, 0\right)$, $\left(2\sqrt{2}, 0\right)$, and $(0, -8)$.

b. x-axis (replace y by $-y$):

$-y = x^2 - 8$

$y = 8 - x^2$ different

y-axis (replace x by $-x$):

$y = (-x)^2 - 8 = x^2 - 8$ same

origin (replace x by $-x$ and y by $-y$):

$-y = (-x)^2 - 8$

$y = 8 - x^2$ different

The equation has y-axis symmetry.

65. a. $x - (0)^2 = -9$

$x = -9$

The x-intercept is $x = -9$.

$(0) - y^2 = -9$

$-y^2 = -9$

$y^2 = 9 \rightarrow y = \pm 3$

The y-intercepts are $y = -3$ and $y = 3$.

The intercepts are $(-9, 0)$, $(0, -3)$, and $(0, 3)$.

b. x-axis (replace y by $-y$):

$x - (-y)^2 = -9$

$x - y^2 = -9$ same

y-axis (replace x by $-x$):

$-x - y^2 = -9$

$x + y^2 = 9$ different

origin (replace x by $-x$ and y by $-y$):

$-x - (-y)^2 = -9$

$-x - y^2 = -9$

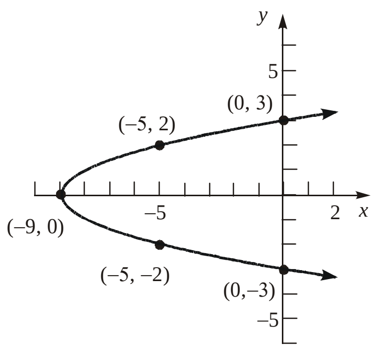
$x + y^2 = 9$ different

The equation has x-axis symmetry.

c. $x - y^2 = -9$ or $x = y^2 - 9$

Additional points:

y	$x = y^2 - 9$	(x, y)
2	$x = 2^2 - 9 = -5$	$(-5, 2)$
-2	from symmetry	$(-5, -2)$



66. a. $x + (0)^2 = 4$

$x = 4$

The x-intercept is $x = 4$.

$(0) + y^2 = 4$

$y^2 = 4 \rightarrow y = \pm 2$

The y-intercepts are $y = -2$ and $y = 2$.

The intercepts are $(4, 0)$, $(0, -2)$, and $(0, 2)$.

b. x-axis (replace y by $-y$):

$x + (-y)^2 = 4$

$x + y^2 = 4$ same

y-axis (replace x by $-x$):

$-x + y^2 = 4$

$x - y^2 = -4$ different

origin (replace x by $-x$ and y by $-y$):

$-x + (-y)^2 = 4$

$-x + y^2 = 4$

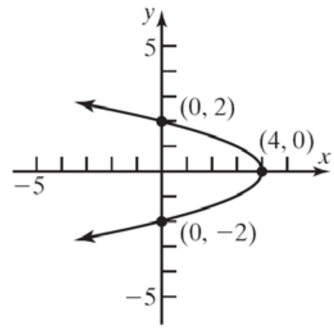
$x - y^2 = -4$ different

The equation has x-axis symmetry.

c. $x + y^2 = 4$ or $x = 4 - y^2$

Additional points:

y	$x = 4 - y^2$	(x, y)
1	$x = 4 - 1^2 = 3$	$(3, 1)$
-1	from symmetry	$(3, -1)$



67. a. $x^2 + (0)^2 = 9$

$x^2 = 9$

$x = \pm 3$

The x-intercepts are $x = -3$ and $x = 3$.

$(0)^2 + y^2 = 9$

$y^2 = 9$

$y = \pm 3$

The y-intercepts are $y = -3$ and $y = 3$.

The intercepts are $(-3, 0)$, $(3, 0)$, $(0, -3)$, and $(0, 3)$.

b. x-axis (replace y by $-y$):

$x^2 + (-y)^2 = 9$

$x^2 + y^2 = 9$ same

y-axis (replace x by $-x$):

$(-x)^2 + y^2 = 9$

$x^2 + y^2 = 9$ same

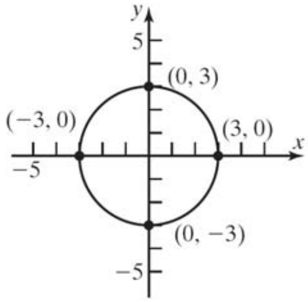
origin (replace x by $-x$ and y by $-y$):

$(-x)^2 + (-y)^2 = 9$

$x^2 + y^2 = 9$ same

The equation has x-axis symmetry, y-axis symmetry, and origin symmetry.

c. $x^2 + y^2 = 9$



68. a. $x^2 + (0)^2 = 16$

$x^2 = 16$

$x = \pm 4$

The x-intercepts are $x = -4$ and $x = 4$.

$(0)^2 + y^2 = 16$

$y^2 = 16$

$y = \pm 4$

The y-intercepts are $y = -4$ and $y = 4$.

The intercepts are $(-4,0)$, $(4,0)$, $(0,-4)$, and $(0,4)$.

b. x-axis (replace y by $-y$):

$x^2 + (-y)^2 = 16$

$x^2 + y^2 = 16$ same

y-axis (replace x by $-x$):

$(-x)^2 + y^2 = 16$

$x^2 + y^2 = 16$ same

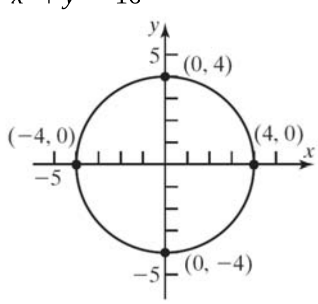
origin (replace x by $-x$ and y by $-y$):

$(-x)^2 + (-y)^2 = 16$

$x^2 + y^2 = 16$ same

The equation has x-axis symmetry, y-axis symmetry, and origin symmetry.

c. $x^2 + y^2 = 16$



69. a. $0 = x^3 - 4x$

$0 = x\left(x^2 - 4\right)$

$x = 0$ or $x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

The x-intercepts are $x = 0$, $x = -2$, and $x = 2$.

$y = 0^3 - 4(0) = 0$

The y-intercept is $y = 0$.

The intercepts are $(0,0)$, $(-2,0)$, and $(2,0)$.

b. x-axis (replace y by $-y$):

$-y = x^3 - 4x$

$y = 4x - x^3$ different

y-axis (replace x by $-x$):

$y = (-x)^3 - 4(-x)$

$y = -x^3 + 4x$ different

origin (replace x by $-x$ and y by $-y$):

$-y = (-x)^3 - 4(-x)$

$-y = -x^3 + 4x$

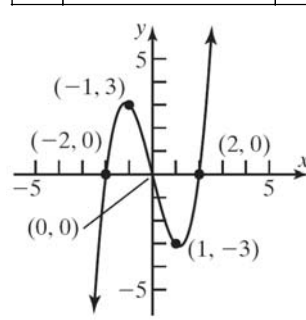
$y = x^3 - 4x$ same

The equation has origin symmetry.

c. $y = x^3 - 4x$

Additional points:

x	$y = x^3 - 4x$	(x, y)
1	$y = 1^3 - 4(1) = -3$	$(1,-3)$
−1	from symmetry	$(-1,3)$



70. **a.** $0 = x^3 - x$

$0 = x(x^2 - 1)$

$x = 0 \quad \text{or} \quad x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

The x-intercepts are $x = 0$, $x = -1$, and $x = 1$.

$y = 0^3 - (0) = 0$

The y-intercept is $y = 0$.

The intercepts are $(0,0)$, $(-1,0)$, and $(1,0)$.

b. x-axis (replace y by $-y$):

$-y = x^3 - x$

$y = x - x^3$ different

y-axis (replace x by $-x$):

$y = (-x)^3 - (-x)$

$y = -x^3 + x$ different

origin (replace x by $-x$ and y by $-y$):

$-y = (-x)^3 - (-x)$

$-y = -x^3 + x$

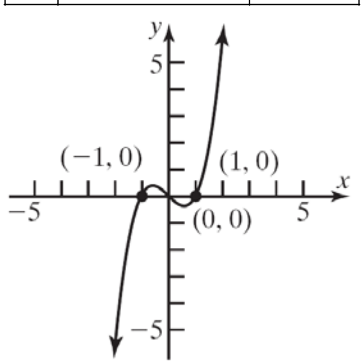
$y = x^3 - x$ same

The equation has origin symmetry.

c. $y = x^3 - x$

Additional points:

x	$y = x^3 - 4x$	(x, y)
2	$y = 2^3 - (2) = 6$	$(2, 6)$
–2	from symmetry	$(-2, -6)$



71. For a graph with origin symmetry, if the point (a, b) is on the graph, then so is the point $(-a, -b)$. Since the point $(1, 2)$ is on the graph of an equation with origin symmetry, the point $(-1, -2)$ must also be on the graph.

72. For a graph with *y*-axis symmetry, if the point (a, b) is on the graph, then so is the point $(-a, b)$. Since 6 is an *x*-intercept in this case, the point $(6, 0)$ is on the graph of the equation. Due to the *y*-axis symmetry, the point $(-6, 0)$ must also be on the graph. Therefore, -6 is another *x*-intercept.

73. For a graph with origin symmetry, if the point (a, b) is on the graph, then so is the point $(-a, -b)$. Since -4 is an *x*-intercept in this case, the point $(-4, 0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4, 0)$ must also be on the graph. Therefore, 4 is another *x*-intercept.

74. For a graph with *x*-axis symmetry, if the point (a, b) is on the graph, then so is the point $(a, -b)$. Since 2 is a *y*-intercept in this case, the point $(0, 2)$ is on the graph of the equation. Due to the *x*-axis symmetry, the point $(0, -2)$ must also be on the graph. Therefore, -2 is another *y*-intercept.

75. **a.** $(x^2 + y^2 - x)^2 = x^2 + y^2$

x-intercepts:

$(x^2 + (0)^2 - x)^2 = x^2 + (0)^2$

$(x^2 - x)^2 = x^2$

$x^4 - 2x^3 + x^2 = x^2$

$x^4 - 2x^3 = 0$

$x^3(x - 2) = 0$

$x^3 = 0 \quad \text{or} \quad x - 2 = 0$

$x = 0 \qquad\qquad x = 2$

y-intercepts:

$$\left((0)^2+y^2-0\right)^2=(0)^2+y^2$$
$$\left(y^2\right)^2=y^2$$
$$y^4=y^2$$
$$y^4-y^2=0$$
$$y^2\left(y^2-1\right)=0$$
$$y^2=0 \quad \text{or} \quad y^2-1=0$$
$$y=0 \qquad\qquad y^2=1$$
$$y=\pm 1$$

The intercepts are $(0,0)$, $(2,0)$, $(0,-1)$, and $(0,1)$.

b. Test x-axis symmetry: Let $y=-y$

$$\left(x^2+(-y)^2-x\right)^2=x^2+(-y)^2$$
$$\left(x^2+y^2-x\right)^2=x^2+y^2 \quad \text{same}$$

Test y-axis symmetry: Let $x=-x$

$$\left((-x)^2+y^2-(-x)\right)^2=(-x)^2+y^2$$
$$\left(x^2+y^2+x\right)^2=x^2+y^2 \quad \text{different}$$

Test origin symmetry: Let $x=-x$ and $y=-y$

$$\left((-x)^2+(-y)^2-(-x)\right)^2=(-x)^2+(-y)^2$$
$$\left(x^2+y^2+x\right)^2=x^2+y^2 \quad \text{different}$$

Thus, the graph will have x-axis symmetry.

76. a. $16y^2=120x-225$

x-intercepts:

$$16y^2=120(0)-225$$
$$16y^2=-225$$
$$y^2=-\frac{225}{16}$$

no real solution

y-intercepts:

$$16(0)^2=120x-225$$
$$0=120x-225$$
$$-120x=-225$$
$$x=\frac{-225}{-120}=\frac{15}{8}$$

The only intercept is $\left(\frac{15}{8},0\right)$.

b. Test x-axis symmetry: Let $y=-y$

$$16(-y)^2=120x-225$$
$$16y^2=120x-225 \quad \text{same}$$

Test y-axis symmetry: Let $x=-x$

$$16y^2=120(-x)-225$$
$$16y^2=-120x-225 \quad \text{different}$$

Test origin symmetry: Let $x=-x$ and $y=-y$

$$16(-y)^2=120(-x)-225$$
$$16y^2=-120x-225 \quad \text{different}$$

Thus, the graph will have x-axis symmetry.

77. Answers will vary.
Case 1: Graph has x-axis and y-axis symmetry, show origin symmetry.
(x,y) on graph $\rightarrow (x,-y)$ on graph
(from x-axis symmetry)
$(x,-y)$ on graph $\rightarrow (-x,-y)$ on graph
(from y-axis symmetry)
Since the point $(-x,-y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has x-axis and origin symmetry, show y-axis symmetry.
(x,y) on graph $\rightarrow (x,-y)$ on graph
(from x-axis symmetry)
$(x,-y)$ on graph $\rightarrow (-x,y)$ on graph
(from origin symmetry)
Since the point $(-x,y)$ is also on the graph, the graph has y-axis symmetry.

Case 3: Graph has y-axis and origin symmetry, show x-axis symmetry.
(x,y) on graph $\rightarrow (-x,y)$ on graph
(from y-axis symmetry)
$(-x,y)$ on graph $\rightarrow (x,-y)$ on graph
(from origin symmetry)
Since the point $(x,-y)$ is also on the graph, the graph has x-axis symmetry.

78. Answers may vary. The graph must contain the points $(-2,5)$, $(-1,3)$, and $(0,2)$. For the graph to be symmetric about the *y*-axis, the graph must also contain the points $(2,5)$ and $(1,3)$ (note that (0, 2) is on the *y*-axis).

For the graph to also be symmetric with respect to the *x*-axis, the graph must also contain the points $(-2,-5)$, $(-1,-3)$, $(0,-2)$, $(2,-5)$, and $(1,-3)$. Recall that a graph with two of the symmetries (x-axis, y-axis, origin) will necessarily have the third. Therefore, if the original graph with y-axis symmetry also has x-axis symmetry, then it will also have origin symmetry.

Section 2.2

1. undefined; 0

2. 3; 2

x-intercept: $2x+3(0)=6$

$$2x=6$$
$$x=3$$

y-intercept: $2(0)+3y=6$

$$3y=6$$
$$y=2$$

3. $y=b$; *y*-intercept

4. True

5. False; the slope is $\frac{3}{2}$.

$$2y=3x+5$$
$$y=\frac{3}{2}x+\frac{5}{2}$$

6. True; $2(1)+(2)\stackrel{?}{=}4$

$$2+2\stackrel{?}{=}4$$
$$4=4 \text{ True}$$

7. $m_1=m_2$; *y*-intercepts; $m_1\cdot m_2=-1$

8. 2

9. $-\frac{1}{2}$

10. False; perpendicular lines have slopes that are opposite-reciprocals of each other.

11. **a.** $\text{Slope}=\frac{1-0}{2-0}=\frac{1}{2}$

b. If x increases by 2 units, y will increase by 1 unit.

12. **a.** $\text{Slope}=\frac{1-0}{-2-0}=-\frac{1}{2}$

b. If x increases by 2 units, y will decrease by 1 unit.

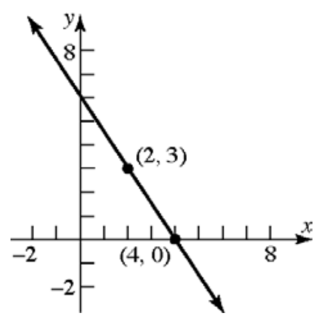
13. **a.** $\text{Slope}=\frac{1-2}{1-(-2)}=-\frac{1}{3}$

b. If x increases by 3 units, y will decrease by 1 unit.

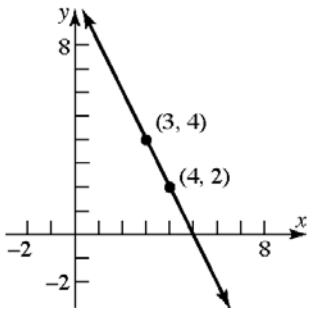
14. **a.** $\text{Slope}=\frac{2-1}{2-(-1)}=\frac{1}{3}$

b. If x increases by 3 units, y will increase by 1 unit.

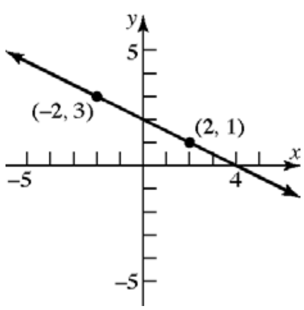
15. $\text{Slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{0-3}{4-2}=-\frac{3}{2}$



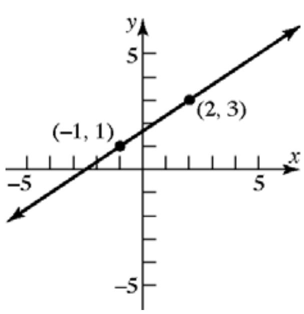
16. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-2}{3-4} = \dfrac{2}{-1} = -2$



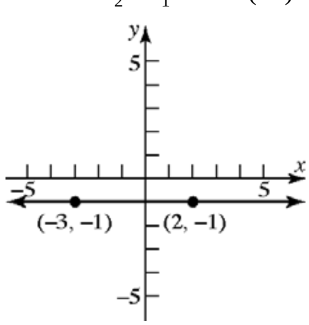
17. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-3}{2-(-2)} = \dfrac{-2}{4} = -\dfrac{1}{2}$



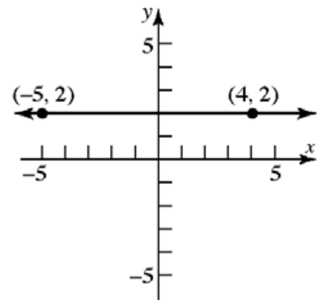
18. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-1}{2-(-1)} = \dfrac{2}{3}$



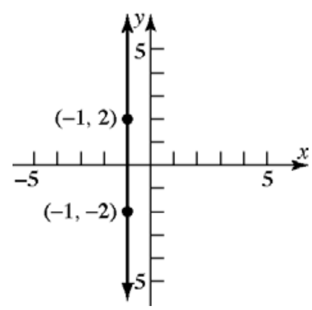
19. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-(-1)}{2-(-3)} = \dfrac{0}{5} = 0$



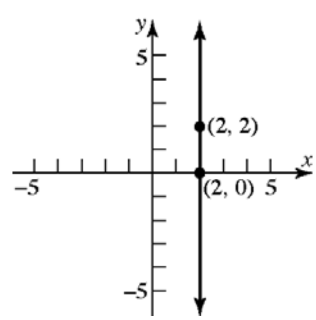
20. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-2}{-5-4} = \dfrac{0}{-9} = 0$



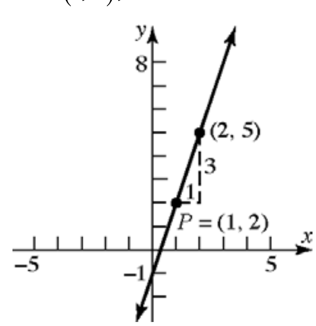
21. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-2}{-1-(-1)} = \dfrac{-4}{0}$ undefined.



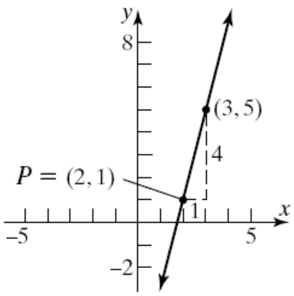
22. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{2-2} = \dfrac{2}{0}$ undefined.



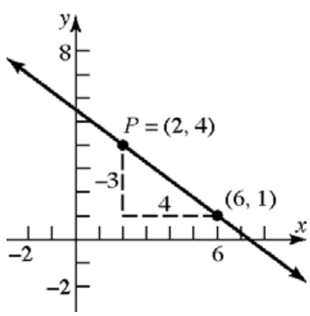
23. $P = (1, 2); m = 3$



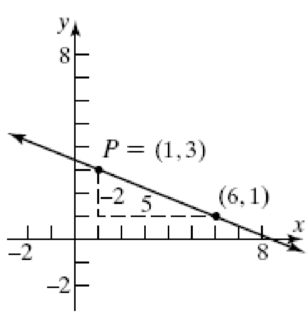
24. $P=(2,1); m=4$



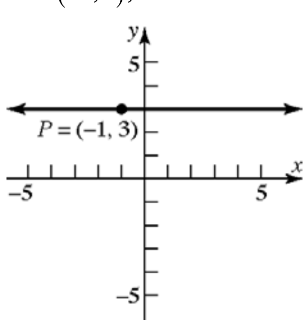
25. $P=(2,4); m=-\dfrac{3}{4}$



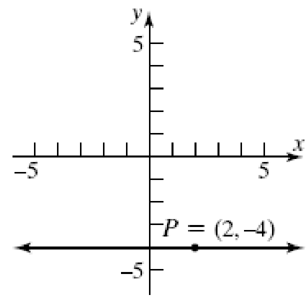
26. $P=(1,3); m=-\dfrac{2}{5}$



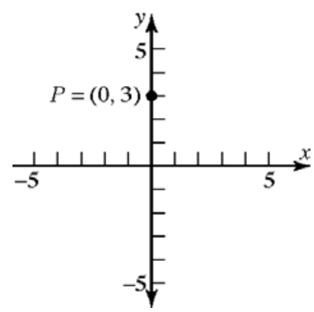
27. $P=(-1,3); m=0$



28. $P=(2,-4); m=0$

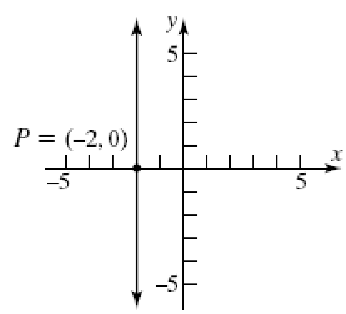


29. $P=(0,3);$ slope undefined



(note: the line is the *y*-axis)

30. $P=(-2,0);$ slope undefined



31. Slope $= 4 = \dfrac{4}{1}$; point: $(1,2)$

If x increases by 1 unit, then y increases by 4 units.

Answers will vary. Three possible points are:

$x=1+1=2$ and $y=2+4=6$

$(2,6)$

$x=2+1=3$ and $y=6+4=10$

$(3,10)$

$x=3+1=4$ and $y=10+4=14$

$(4,14)$

32. Slope $= 2 = \frac{2}{1}$; point: $(-2, 3)$

If x increases by 1 unit, then y increases by 2 units.

Answers will vary. Three possible points are:

$x = -2 + 1 = -1$ and $y = 3 + 2 = 5$

$(-1, 5)$

$x = -1 + 1 = 0$ and $y = 5 + 2 = 7$

$(0, 7)$

$x = 0 + 1 = 1$ and $y = 7 + 2 = 9$

$(1, 9)$

33. Slope $= -\frac{3}{2} = \frac{-3}{2}$; point: $(2, -4)$

If x increases by 2 units, then y decreases by 3 units.

Answers will vary. Three possible points are:

$x = 2 + 2 = 4$ and $y = -4 - 3 = -7$

$(4, -7)$

$x = 4 + 2 = 6$ and $y = -7 - 3 = -10$

$(6, -10)$

$x = 6 + 2 = 8$ and $y = -10 - 3 = -13$

$(8, -13)$

34. Slope $= \frac{4}{3}$; point: $(-3, 2)$

If x increases by 3 units, then y increases by 4 units.

Answers will vary. Three possible points are:

$x = -3 + 3 = 0$ and $y = 2 + 4 = 6$

$(0, 6)$

$x = 0 + 3 = 3$ and $y = 6 + 4 = 10$

$(3, 10)$

$x = 3 + 3 = 6$ and $y = 10 + 4 = 14$

$(6, 14)$

35. Slope $= -2 = \frac{-2}{1}$; point: $(-2, -3)$

If x increases by 1 unit, then y decreases by 2 units.

Answers will vary. Three possible points are:

$x = -2 + 1 = -1$ and $y = -3 - 2 = -5$

$(-1, -5)$

$x = -1 + 1 = 0$ and $y = -5 - 2 = -7$

$(0, -7)$

$x = 0 + 1 = 1$ and $y = -7 - 2 = -9$

$(1, -9)$

36. Slope $= -1 = \frac{-1}{1}$; point: $(4, 1)$

If x increases by 1 unit, then y decreases by 1 unit.

Answers will vary. Three possible points are:

$x = 4 + 1 = 5$ and $y = 1 - 1 = 0$

$(5, 0)$

$x = 5 + 1 = 6$ and $y = 0 - 1 = -1$

$(6, -1)$

$x = 6 + 1 = 7$ and $y = -1 - 1 = -2$

$(7, -2)$

37. (0, 0) and (2, 1) are points on the line.

Slope $= \frac{1 - 0}{2 - 0} = \frac{1}{2}$

y-intercept is 0; using $y = mx + b$:

$$y = \frac{1}{2}x + 0$$
$$2y = x$$
$$0 = x - 2y$$

$x - 2y = 0$ or $y = \frac{1}{2}x$

38. (0, 0) and (–2, 1) are points on the line.

Slope $= \frac{1 - 0}{-2 - 0} = \frac{1}{-2} = -\frac{1}{2}$

y-intercept is 0; using $y = mx + b$:

$$y = -\frac{1}{2}x + 0$$
$$2y = -x$$
$$x + 2y = 0$$

$x + 2y = 0$ or $y = -\frac{1}{2}x$

39. (–1, 3) and (1, 1) are points on the line.

$$\text{Slope} = \frac{1-3}{1-(-1)} = \frac{-2}{2} = -1$$

Using $y - y_1 = m(x - x_1)$

$$y-1 = -1(x-1)$$
$$y-1 = -x+1$$
$$y = -x+2$$
$$x+y=2 \text{ or } y=-x+2$$

40. (–1, 1) and (2, 2) are points on the line.

$$\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$$

Using $y - y_1 = m(x - x_1)$

$$y-1 = \frac{1}{3}\left(x-(-1)\right)$$
$$y-1 = \frac{1}{3}(x+1)$$
$$y-1 = \frac{1}{3}x+\frac{1}{3}$$
$$y = \frac{1}{3}x+\frac{4}{3}$$
$$x-3y=-4 \text{ or } y=\frac{1}{3}x+\frac{4}{3}$$

41. $y - y_1 = m(x - x_1),\ m = 2$

$$y-3 = 2(x-3)$$
$$y-3 = 2x-6$$
$$y = 2x-3$$
$$2x-y=3 \text{ or } y=2x-3$$

42. $y - y_1 = m(x - x_1),\ m = -1$

$$y-2 = -1(x-1)$$
$$y-2 = -x+1$$
$$y = -x+3$$
$$x+y=3 \text{ or } y=-x+3$$

43. $y - y_1 = m(x - x_1),\ m = -\frac{1}{2}$

$$y-2 = -\frac{1}{2}(x-1)$$
$$y-2 = -\frac{1}{2}x+\frac{1}{2}$$
$$y = -\frac{1}{2}x+\frac{5}{2}$$
$$x+2y=5 \text{ or } y=-\frac{1}{2}x+\frac{5}{2}$$

44. $y - y_1 = m(x - x_1),\ m = 1$

$$y-1 = 1(x-(-1))$$
$$y-1 = x+1$$
$$y = x+2$$
$$x-y=-2 \text{ or } y=x+2$$

45. Slope = 3; containing (–2, 3)

$$y - y_1 = m(x - x_1)$$
$$y-3 = 3(x-(-2))$$
$$y-3 = 3x+6$$
$$y = 3x+9$$
$$3x-y=-9 \text{ or } y=3x+9$$

46. Slope = 2; containing the point (4, –3)

$$y - y_1 = m(x - x_1)$$
$$y-(-3) = 2(x-4)$$
$$y+3 = 2x-8$$
$$y = 2x-11$$
$$2x-y=11 \text{ or } y=2x-11$$

47. Slope $= -\frac{2}{3}$; containing (1, –1)

$$y - y_1 = m(x - x_1)$$
$$y-(-1) = -\frac{2}{3}(x-1)$$
$$y+1 = -\frac{2}{3}x+\frac{2}{3}$$
$$y = -\frac{2}{3}x-\frac{1}{3}$$
$$2x+3y=-1 \text{ or } y=-\frac{2}{3}x-\frac{1}{3}$$

48. Slope $= \frac{1}{2}$; containing the point (3, 1)

$$y - y_1 = m(x - x_1)$$
$$y-1 = \frac{1}{2}(x-3)$$
$$y-1 = \frac{1}{2}x-\frac{3}{2}$$
$$y = \frac{1}{2}x-\frac{1}{2}$$
$$x-2y=1 \text{ or } y=\frac{1}{2}x-\frac{1}{2}$$

49. Containing (1, 3) and (–1, 2)

$$m=\frac{2-3}{-1-1}=\frac{-1}{-2}=\frac{1}{2}$$

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{1}{2}(x-1)$$

$$y-3=\frac{1}{2}x-\frac{1}{2}$$

$$y=\frac{1}{2}x+\frac{5}{2}$$

$$x-2y=-5 \text{ or } y=\frac{1}{2}x+\frac{5}{2}$$

50. Containing the points (–3, 4) and (2, 5)

$$m=\frac{5-4}{2-(-3)}=\frac{1}{5}$$

$$y-y_1=m(x-x_1)$$

$$y-5=\frac{1}{5}(x-2)$$

$$y-5=\frac{1}{5}x-\frac{2}{5}$$

$$y=\frac{1}{5}x+\frac{23}{5}$$

$$x-5y=-23 \text{ or } y=\frac{1}{5}x+\frac{23}{5}$$

51. Slope = –3; *y*-intercept =3

$$y=mx+b$$

$$y=-3x+3$$

$$3x+y=3 \text{ or } y=-3x+3$$

52. Slope = –2; *y*-intercept = –2

$$y=mx+b$$

$$y=-2x+(-2)$$

$$2x+y=-2 \text{ or } y=-2x-2$$

53. *x*-intercept = 2; *y*-intercept = –1
Points are (2,0) and (0,–1)

$$m=\frac{-1-0}{0-2}=\frac{-1}{-2}=\frac{1}{2}$$

$$y=mx+b$$

$$y=\frac{1}{2}x-1$$

$$x-2y=2 \text{ or } y=\frac{1}{2}x-1$$

54. *x*-intercept = –4; *y*-intercept = 4
Points are (–4, 0) and (0, 4)

$$m=\frac{4-0}{0-(-4)}=\frac{4}{4}=1$$

$$y=mx+b$$

$$y=1x+4$$

$$y=x+4$$

$$x-y=-4 \text{ or } y=x+4$$

55. Slope undefined; containing the point (2, 4)
This is a vertical line.
$x=2$ No slope-intercept form.

56. Slope undefined; containing the point (3, 8)
This is a vertical line.
$x=3$ No slope-intercept form.

57. Horizontal lines have slope $m=0$ and take the form $y=b$. Therefore, the horizontal line passing through the point $(-3,2)$ is $y=2$.

58. Vertical lines have an undefined slope and take the form $x=a$. Therefore, the vertical line passing through the point $(4,-5)$ is $x=4$.

59. Parallel to $y=2x$; Slope = 2
Containing (–1, 2)

$$y-y_1=m(x-x_1)$$

$$y-2=2(x-(-1))$$

$$y-2=2x+2 \rightarrow y=2x+4$$

$$2x-y=-4 \text{ or } y=2x+4$$

60. Parallel to $y=-3x$; Slope = –3; Containing the point (–1, 2)

$$y-y_1=m(x-x_1)$$

$$y-2=-3(x-(-1))$$

$$y-2=-3x-3 \rightarrow y=-3x-1$$

$$3x+y=-1 \text{ or } y=-3x-1$$

61. Parallel to $2x-y=-2$; Slope = 2
Containing the point (0, 0)

$$y-y_1=m(x-x_1)$$

$$y-0=2(x-0)$$

$$y=2x$$

$$2x-y=0 \text{ or } y=2x$$

62. Parallel to $x-2y=-5$;

Slope $=\frac{1}{2}$; Containing the point $(0,0)$

$y-y_1=m(x-x_1)$

$y-0=\frac{1}{2}(x-0)\rightarrow y=\frac{1}{2}x$

$x-2y=0$ or $y=\frac{1}{2}x$

63. Parallel to $x=5$; Containing (4,2)
This is a vertical line.
$x=4$ No slope-intercept form.

64. Parallel to $y=5$; Containing the point (4, 2)
This is a horizontal line. Slope = 0
$y=2$

65. Perpendicular to $y=\frac{1}{2}x+4$; Containing (1, –2)

Slope of perpendicular = –2

$y-y_1=m(x-x_1)$

$y-(-2)=-2(x-1)$

$y+2=-2x+2\rightarrow y=-2x$

$2x+y=0$ or $y=-2x$

66. Perpendicular to $y=2x-3$; Containing the point (1, –2)

Slope of perpendicular $=-\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y-(-2)=-\frac{1}{2}(x-1)$

$y+2=-\frac{1}{2}x+\frac{1}{2}\rightarrow y=-\frac{1}{2}x-\frac{3}{2}$

$x+2y=-3$ or $y=-\frac{1}{2}x-\frac{3}{2}$

67. Perpendicular to $2x+y=2$; Containing the point (–3, 0)

Slope of perpendicular $=\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y-0=\frac{1}{2}(x-(-3))\rightarrow y=\frac{1}{2}x+\frac{3}{2}$

$x-2y=-3$ or $y=\frac{1}{2}x+\frac{3}{2}$

68. Perpendicular to $x-2y=-5$; Containing the point (0, 4)
Slope of perpendicular = –2

$y=mx+b$

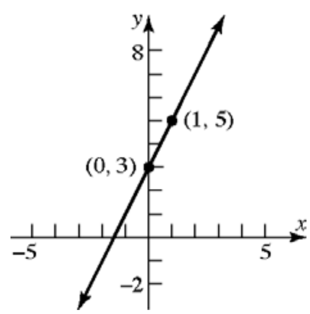
$y=-2x+4$

$2x+y=4$ or $y=-2x+4$

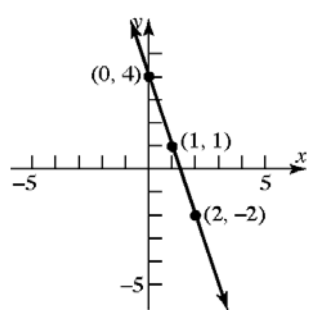
69. Perpendicular to $x=8$; Containing (3, 4)
Slope of perpendicular = 0 (horizontal line)
$y=4$

70. Perpendicular to $y=8$;
Containing the point (3, 4)
Slope of perpendicular is undefined (vertical line). $x=3$ No slope-intercept form.

71. $y=2x+3$; Slope = 2; y-intercept = 3

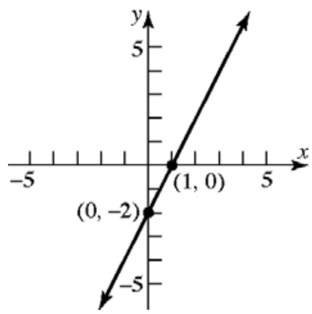


72. $y=-3x+4$; Slope = –3; y-intercept = 4



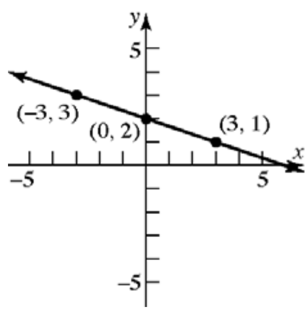
73. $\frac{1}{2}y = x - 1$; $y = 2x - 2$

Slope = 2; y-intercept = –2

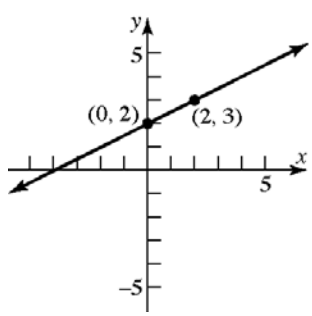


74. $\frac{1}{3}x + y = 2$; $y = -\frac{1}{3}x + 2$

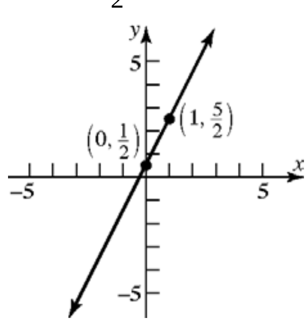
Slope = $-\frac{1}{3}$; y-intercept = 2



75. $y = \frac{1}{2}x + 2$; Slope = $\frac{1}{2}$; y-intercept = 2

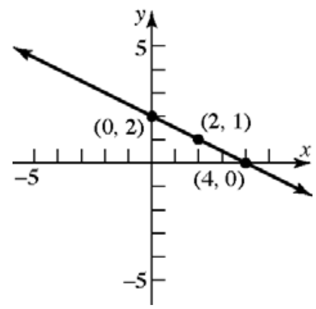


76. $y = 2x + \frac{1}{2}$; Slope = 2; y-intercept = $\frac{1}{2}$



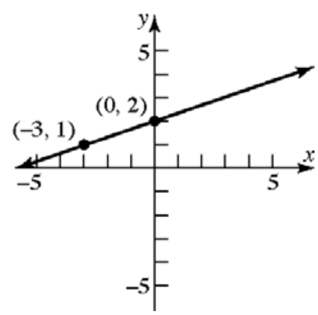
77. $x + 2y = 4$; $2y = -x + 4 \rightarrow y = -\frac{1}{2}x + 2$

Slope = $-\frac{1}{2}$; y-intercept = 2



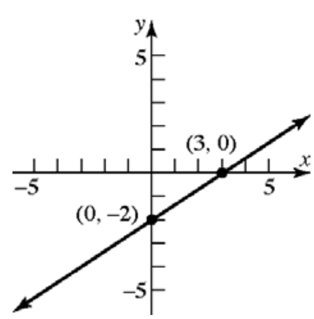
78. $-x + 3y = 6$; $3y = x + 6 \rightarrow y = \frac{1}{3}x + 2$

Slope = $\frac{1}{3}$; y-intercept = 2



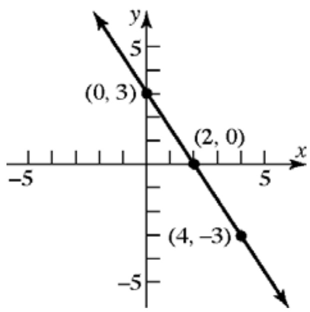
79. $2x - 3y = 6$; $-3y = -2x + 6 \rightarrow y = \frac{2}{3}x - 2$

Slope = $\frac{2}{3}$; y-intercept = –2



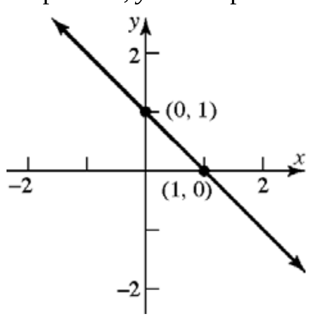
80. $3x+2y=6$; $2y=-3x+6 \rightarrow y=-\frac{3}{2}x+3$

Slope $=-\frac{3}{2}$; y-intercept = 3



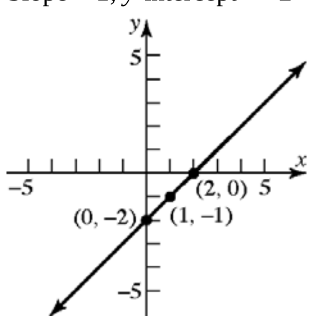
81. $x+y=1$; $y=-x+1$

Slope = –1; y-intercept = 1



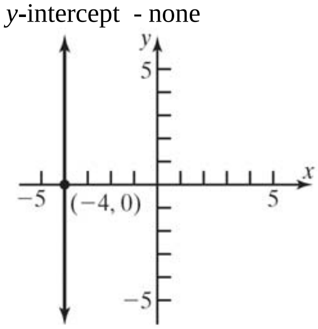
82. $x-y=2$; $y=x-2$

Slope = 1; y-intercept = –2

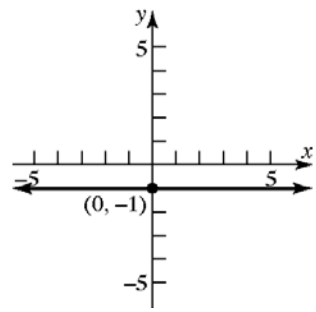


83. $x=-4$; Slope is undefined

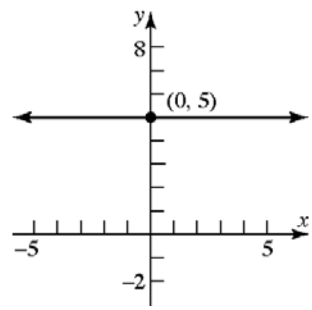
y-intercept - none



84. $y=-1$; Slope = 0; y-intercept = –1

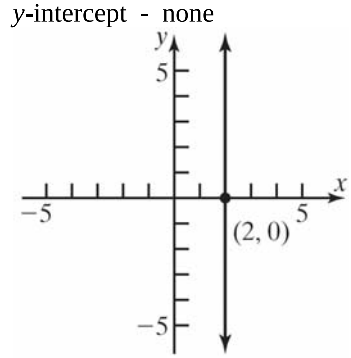


85. $y=5$; Slope = 0; y-intercept = 5



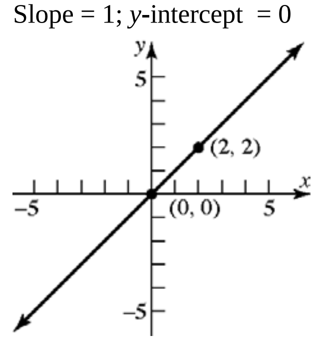
86. $x=2$; Slope is undefined

y-intercept - none



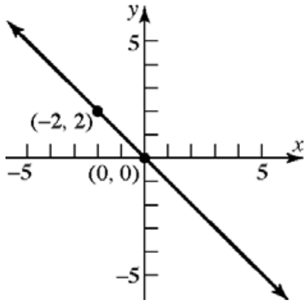
87. $y-x=0$; $y=x$

Slope = 1; y-intercept = 0



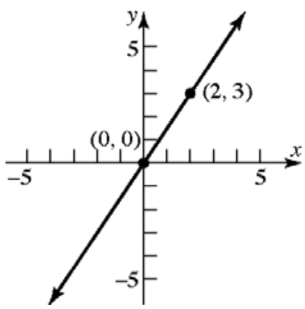
88. $x + y = 0$; $y = -x$

Slope = –1; y-intercept = 0



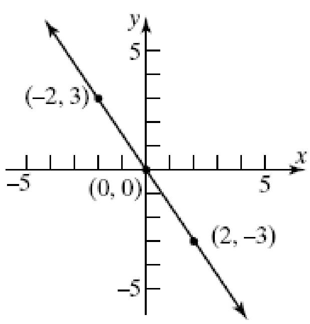
89. $2y - 3x = 0$; $2y = 3x \rightarrow y = \frac{3}{2}x$

Slope $= \frac{3}{2}$; y-intercept = 0



90. $3x + 2y = 0$; $2y = -3x \rightarrow y = -\frac{3}{2}x$

Slope $= -\frac{3}{2}$; y-intercept = 0



91. a. x-intercept: $2x + 3(0) = 6$

$$2x = 6$$
$$x = 3$$

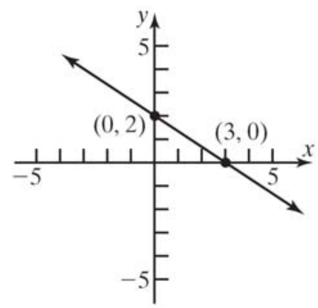
The point $(3, 0)$ is on the graph.

y-intercept: $2(0) + 3y = 6$

$$3y = 6$$
$$y = 2$$

The point $(0, 2)$ is on the graph.

b.



92. a. x-intercept: $3x - 2(0) = 6$

$$3x = 6$$
$$x = 2$$

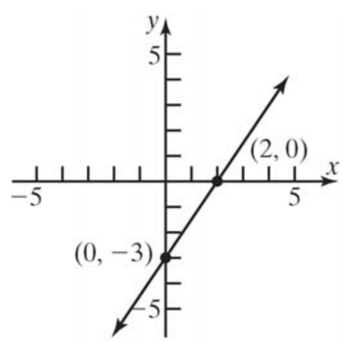
The point $(2, 0)$ is on the graph.

y-intercept: $3(0) - 2y = 6$

$$-2y = 6$$
$$y = -3$$

The point $(0, -3)$ is on the graph.

b.



93. a. x-intercept: $-4x+5(0)=40$

$$-4x=40$$
$$x=-10$$

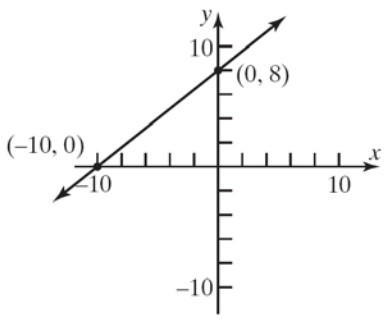
The point $(-10,0)$ is on the graph.

y-intercept: $-4(0)+5y=40$

$$5y=40$$
$$y=8$$

The point $(0,8)$ is on the graph.

b.



94. a. x-intercept: $6x-4(0)=24$

$$6x=24$$
$$x=4$$

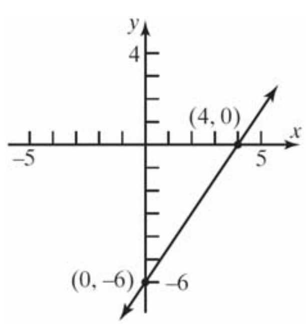
The point $(4,0)$ is on the graph.

y-intercept: $6(0)-4y=24$

$$-4y=24$$
$$y=-6$$

The point $(0,-6)$ is on the graph.

b.



95. a. x-intercept: $7x+2(0)=21$

$$7x=21$$
$$x=3$$

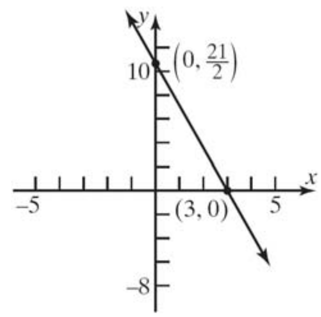
The point $(3,0)$ is on the graph.

y-intercept: $7(0)+2y=21$

$$2y=21$$
$$y=\frac{21}{2}$$

The point $\left(0,\frac{21}{2}\right)$ is on the graph.

b.



96. a. x-intercept: $5x+3(0)=18$

$$5x=18$$
$$x=\frac{18}{5}$$

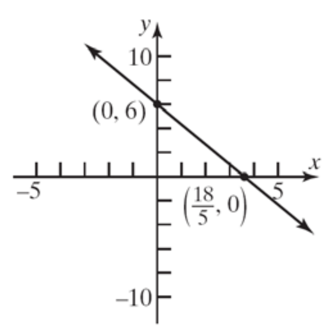
The point $\left(\frac{18}{5},0\right)$ is on the graph.

y-intercept: $5(0)+3y=18$

$$3y=18$$
$$y=6$$

The point $(0,6)$ is on the graph.

b.



97. a. x-intercept: $\frac{1}{2}x+\frac{1}{3}(0)=1$

$$\frac{1}{2}x=1$$
$$x=2$$

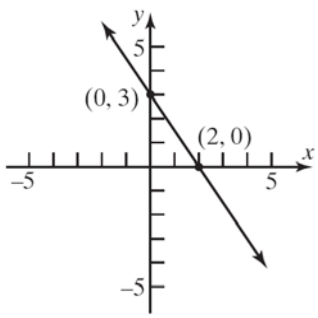
The point $(2,0)$ is on the graph.

y-intercept: $\frac{1}{2}(0)+\frac{1}{3}y=1$

$$\frac{1}{3}y=1$$
$$y=3$$

The point $(0,3)$ is on the graph.

b.



98. a. x-intercept: $x-\frac{2}{3}(0)=4$

$$x=4$$

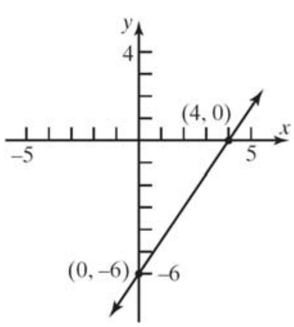
The point $(4,0)$ is on the graph.

y-intercept: $(0)-\frac{2}{3}y=4$

$$-\frac{2}{3}y=4$$
$$y=-6$$

The point $(0,-6)$ is on the graph.

b.



99. a. x-intercept: $0.2x-0.5(0)=1$

$$0.2x=1$$
$$x=5$$

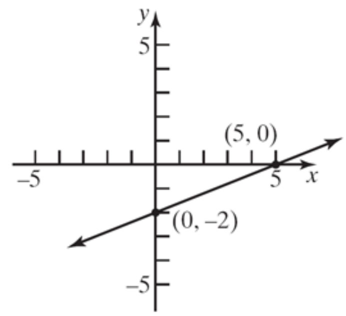
The point $(5,0)$ is on the graph.

y-intercept: $0.2(0)-0.5y=1$

$$-0.5y=1$$
$$y=-2$$

The point $(0,-2)$ is on the graph.

b.



100. a. x-intercept: $-0.3x+0.4(0)=1.2$

$$-0.3x=1.2$$
$$x=-4$$

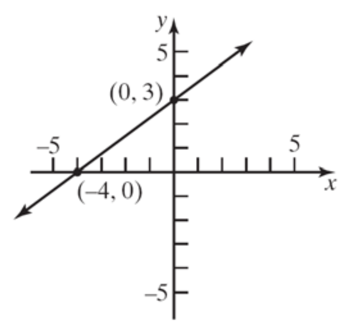
The point $(-4,0)$ is on the graph.

y-intercept: $-0.3(0)+0.4y=1.2$

$$0.4y=1.2$$
$$y=3$$

The point $(0,3)$ is on the graph.

b.



101. The equation of the x-axis is $y=0$. (The slope is 0 and the y-intercept is 0.)

102. The equation of the y-axis is $x=0$. (The slope is undefined.)

103. The slopes are the same but the y-intercepts are different. Therefore, the two lines are parallel.

104. The slopes are opposite-reciprocals. That is, their product is -1. Therefore, the lines are perpendicular.

105. The slopes are different and their product does not equal -1. Therefore, the lines are neither parallel nor perpendicular.

106. The slopes are different and their product does not equal -1 (in fact, the signs are the same so the product is positive). Therefore, the lines are neither parallel nor perpendicular.

107. (b)

108. (c)

109. (d)

110. (a)

111. $P_1=(-2,5)$, $P_2=(1,3)$: $m_1=\dfrac{5-3}{-2-1}=\dfrac{2}{-3}=-\dfrac{2}{3}$

$P_2=(1,3)$, $P_3=(-1,0)$: $m_2=\dfrac{3-0}{1-(-1)}=\dfrac{3}{2}$

Since $m_1\cdot m_2=-1$, the line segments $\overline{P_1P_2}$ and $\overline{P_2P_3}$ are perpendicular. Thus, the points P_1, P_2, and P_3 are vertices of a right triangle.

112. $P_1=(1,-1)$, $P_2=(4,1)$, $P_3=(2,2)$, $P_4=(5,4)$

$m_{12}=\dfrac{1-(-1)}{4-1}=\dfrac{2}{3}$; $m_{24}=\dfrac{4-1}{5-4}=3$;

$m_{34}=\dfrac{4-2}{5-2}=\dfrac{2}{3}$; $m_{13}=\dfrac{2-(-1)}{2-1}=3$

Each pair of opposite sides are parallel (same slope) and adjacent sides are not perpendicular. Therefore, the vertices are for a parallelogram.

113. $P_1=(-1,0)$, $P_2=(2,3)$, $P_3=(1,-2)$, $P_4=(4,1)$

$m_{12}=\dfrac{3-0}{2-(-1)}=\dfrac{3}{3}=1$; $m_{24}=\dfrac{1-3}{4-2}=-1$;

$m_{34}=\dfrac{1-(-2)}{4-1}=\dfrac{3}{3}=1$; $m_{13}=\dfrac{-2-0}{1-(-1)}=-1$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). Therefore, the vertices are for a rectangle.

114. $P_1=(0,0)$, $P_2=(1,3)$, $P_3=(4,2)$, $P_4=(3,-1)$

$m_{12}=\dfrac{3-0}{1-0}=3$; $m_{23}=\dfrac{2-3}{4-1}=-\dfrac{1}{3}$;

$m_{34}=\dfrac{-1-2}{3-4}=3$; $m_{14}=\dfrac{-1-0}{3-0}=-\dfrac{1}{3}$

$d_{12}=\sqrt{(1-0)^2+(3-0)^2}=\sqrt{1+9}=\sqrt{10}$

$d_{23}=\sqrt{(4-1)^2+(2-3)^2}=\sqrt{9+1}=\sqrt{10}$

$d_{34}=\sqrt{(3-4)^2+(-1-2)^2}=\sqrt{1+9}=\sqrt{10}$

$d_{14}=\sqrt{(3-0)^2+(-1-0)^2}=\sqrt{9+1}=\sqrt{10}$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). In addition, the length of all four sides is the same. Therefore, the vertices are for a square.

115. Let x = number of miles driven, and let C = cost in dollars.
Total cost = (cost per mile)(number of miles) + fixed cost
$C=0.20x+29$
When x = 110, $C=(0.20)(110)+29=\$51.00$.
When x = 230, $C=(0.20)(230)+29=\$75.00$.

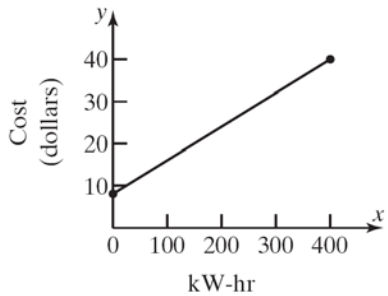
116. Let x = number of pairs of jeans manufactured, and let C = cost in dollars.
Total cost = (cost per pair)(number of pairs) + fixed cost
$C=8x+500$
When x = 400, $C=(8)(400)+500=\$3700$.
When x = 740, $C=(8)(740)+500=\$6420$.

117. Let x = number newspapers delivered, and let C = cost in dollars.
Total cost = (delivery cost per paper)(number of papers delivered) + fixed cost
$C=0.53x+1{,}070{,}000$

118. Let x = profit in dollars, and let S = salary in dollars.
Weekly salary = (% share of profit)(profit) + weekly pay
$S=0.05x+375$

119. a. $C = 0.08275x + 7.58$; $0 \le x \le 400$

b.



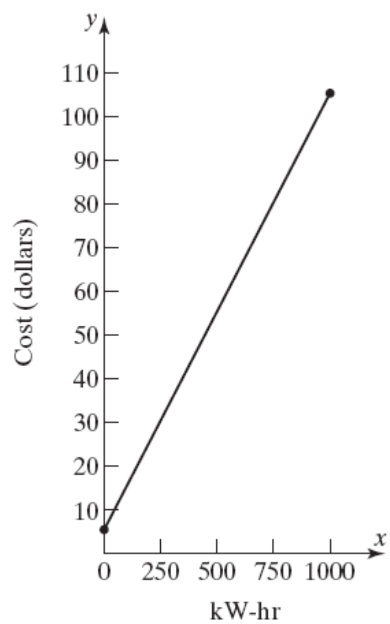
c. For 100 kWh,
$C = 0.08275(100) + 7.58 = \$15.86$

d. For 300 kWh,
$C = 0.08725(300) + 7.58 = \$32.41$

e. For each usage increase of 1 kWh, the monthly charge increases by \$0.08275 (that is, 8.275 cents).

120. a. $C = 0.1007x + 5.17$; $0 \le x \le 1000$

b.



c. For 200 kWh,
$C = 0.1007\left(200\right) + 5.17 = \$25.31$

d. For 500 kWh,
$C = 0.1007\left(500\right) + 5.17 = \$55.52$

e. For each usage increase of 1 kWh, the monthly charge increases by \$0.1007 (that is, 10.07 cents).

121. $(°C, °F) = (0, 32)$; $(°C, °F) = (100, 212)$

$$\text{slope} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

$$°F - 32 = \frac{9}{5}(°C - 0)$$

$$°F - 32 = \frac{9}{5}(°C)$$

$$°C = \frac{5}{9}(°F - 32)$$

If $°F = 70$, then

$$°C = \frac{5}{9}(70-32) = \frac{5}{9}(38)$$

$$°C \approx 21.1°$$

122. a. $K = °C + 273$

b.
$$°C = \frac{5}{9}(°F - 32)$$

$$K = \frac{5}{9}(°F - 32) + 273$$

$$K = \frac{5}{9}°F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}°F + \frac{2297}{9}$$

$$K = \frac{1}{9}\left(5°F + 2297\right)$$

123. a. The y-intercept is (0, 30), so $b = 30$. Since the ramp drops 2 inches for every 25 inches of run, the slope is $m = \frac{-2}{25} = -\frac{2}{25}$. Thus, the equation is $y = -\frac{2}{25}x + 30$.

b. Let $y = 0$.

$$0 = -\frac{2}{25}x + 30$$

$$\frac{2}{25}x = 30$$

$$\frac{25}{2}\left(\frac{2}{25}x\right) = \frac{25}{2}\left(30\right)$$

$$x = 375$$

The x-intercept is 375. This means that the ramp meets the floor 375 inches (or 31.25 feet) from the base of the platform.

c. No. From part (b), the run is 31.25 feet which exceeds the required maximum of 30 feet.

d. First, design requirements state that the maximum slope is a drop of 1 inch for each 12 inches of run. This means $|m| \le \frac{1}{12}$.

Second, the run is restricted to be no more than 30 feet = 360 inches. For a rise of 30 inches, this means the minimum slope is $\frac{30}{360} = \frac{1}{12}$. That is, $|m| \ge \frac{1}{12}$. Thus, the only possible slope is $|m| = \frac{1}{12}$. The diagram indicates that the slope is negative. Therefore, the only slope that can be used to obtain the 30-inch rise and still meet design requirements is $m = -\frac{1}{12}$. In words, for every 12 inches of run, the ramp must drop *exactly* 1 inch.

124. a. The year 1998 corresponds to $x = 0$, and the year 2005 corresponds to $x = 7$. Therefore, the points (0, 42) and (7, 22) are on the line. Thus, $m = \frac{22-42}{7-0} = -\frac{20}{7}$. The y-intercept is 42, so $b = 42$ and the equation is

$$y = -\frac{20}{7}x + 42$$

b. x-intercept:

$$0 = -\frac{20}{7}x + 42$$

$$\frac{20}{7}x = 42$$

$$\frac{7}{20}\left(\frac{20}{7}x\right) = \frac{7}{20}(42)$$

$$x = 14.7$$

y-intercept: $y = -\frac{20}{7}(0) + 42 = 42$

The intercepts are (14.7, 0) and (0, 42).

c. The y-intercept represents the percentage of teens in 1998 who had recently used cigarettes. The x-intercept represents the number of years after 1998 when 0% of teens will have recently used cigarettes.

d. The year 2019 corresponds to $x = 21$.

$$y = -\frac{20}{7}(21) + 42 = -60 + 42 = -18$$

This prediction is not reasonable because the percent cannot be negative.

125. a. Let x = number of boxes to be sold, and A = money, in dollars, spent on advertising. We have the points

$(x_1, A_1) = (100,000,\ 40,000)$;

$(x_2, A_2) = (200,000,\ 60,000)$

$$\text{slope} = \frac{60,000 - 40,000}{200,000 - 100,000} = \frac{20,000}{100,000} = \frac{1}{5}$$

$$A - 40,000 = \frac{1}{5}(x - 100,000)$$

$$A - 40,000 = \frac{1}{5}x - 20,000$$

$$A = \frac{1}{5}x + 20,000$$

b. If x = 300,000, then

$$A = \frac{1}{5}(300,000) + 20,000 = \$80,000$$

c. Each additional box sold requires an additional \$0.20 in advertising.

126. Find the slope of the line containing (a,b) and (b,a):

$$\text{slope} = \frac{a-b}{b-a} = -1$$

The slope of the line $y = x$ is 1.

Since $-1 \cdot 1 = -1$, the line containing the points (a,b) and (b,a) is perpendicular to the line $y = x$.

The midpoint of (a,b) and (b,a) is

$$M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right).$$

Since the coordinates are the same, the midpoint lies on the line $y = x$.

Note: $\frac{a+b}{2} = \frac{b+a}{2}$

127. $2x - y = C$

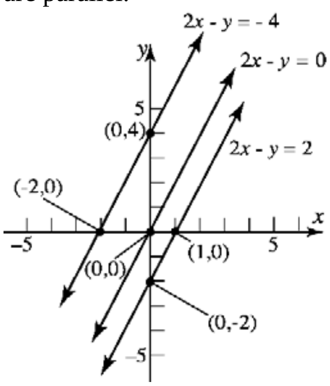
Graph the lines:

$2x - y = -4$

$2x - y = 0$

$2x - y = 2$

All the lines have the same slope, 2. The lines are parallel.



128. Refer to Figure 33.

length of $\overline{OA} = d(O, A) = \sqrt{1 + m_1^2}$

length of $\overline{OB} = d(O, B) = \sqrt{1 + m_2^2}$

length of $\overline{AB} = d(A, B) = m_1 - m_2$

Now consider the equation

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1 - m_2)^2$$

If this equation is valid, then ΔAOB is a right triangle with right angle at vertex *O*.

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1 - m_2)^2$$

$$1 + m_1^2 + 1 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

$$2 + m_1^2 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

But we are assuming that $m_1m_2 = -1$, so we have

$$2 + m_1^2 + m_2^2 = m_1^2 - 2(-1) + m_2^2$$

$$2 + m_1^2 + m_2^2 = m_1^2 + 2 + m_2^2$$

$$0 = 0$$

Therefore, by the converse of the Pythagorean Theorem, ΔAOB is a right triangle with right angle at vertex *O*. Thus Line 1 is perpendicular to Line 2.

129. (b), (c), (e) and (g)
The line has positive slope and positive *y*-intercept.

130. (a), (c), and (g)
The line has negative slope and positive *y*-intercept.

131. (c)
The equation $x - y = -2$ has slope 1 and *y*-intercept $(0, 2)$. The equation $x - y = 1$ has slope 1 and *y*-intercept $(0, -1)$. Thus, the lines are parallel with positive slopes. One line has a positive *y*-intercept and the other with a negative *y*-intercept.

132. (d)
The equation $y - 2x = 2$ has slope 2 and *y*-intercept $(0, 2)$. The equation $x + 2y = -1$ has slope $-\frac{1}{2}$ and *y*-intercept $\left(0, -\frac{1}{2}\right)$. The lines are perpendicular since $2\left(-\frac{1}{2}\right) = -1$. One line has a positive *y*-intercept and the other with a negative *y*-intercept.

133 – 135. Answers will vary.

136. No, the equation of a vertical line cannot be written in slope-intercept form because the slope is undefined.

137. No, a line does not need to have both an *x*-intercept and a *y*-intercept. Vertical and horizontal lines have only one intercept (unless they are a coordinate axis). Every line must have at least one intercept.

138. Two lines with equal slopes and equal *y*-intercepts are coinciding lines (i.e. the same).

139. Two lines that have the same *x*-intercept and *y*-intercept (assuming the *x*-intercept is not 0) are the same line since a line is uniquely defined by two distinct points.

140. No. Two lines with the same slope and different x-intercepts are distinct parallel lines and have no points in common.
Assume Line 1 has equation $y = mx + b_1$ and Line 2 has equation $y = mx + b_2$,
Line 1 has x-intercept $-\frac{b_1}{m}$ and y-intercept b_1.
Line 2 has x-intercept $-\frac{b_2}{m}$ and y-intercept b_2.
Assume also that Line 1 and Line 2 have unequal x-intercepts.
If the lines have the same y-intercept, then $b_1 = b_2$.

$$b_1 = b_2 \Rightarrow \frac{b_1}{m} = \frac{b_2}{m} \Rightarrow -\frac{b_1}{m} = -\frac{b_2}{m}$$

But $-\frac{b_1}{m} = -\frac{b_2}{m} \Rightarrow$ Line 1 and Line 2 have the same x-intercept, which contradicts the original assumption that the lines have unequal x-intercepts. Therefore, Line 1 and Line 2 cannot have the same y-intercept.

141. Yes. Two distinct lines with the same y-intercept, but different slopes, can have the same x-intercept if the x-intercept is $x = 0$.
Assume Line 1 has equation $y = m_1x + b$ and Line 2 has equation $y = m_2x + b$,
Line 1 has x-intercept $-\frac{b}{m_1}$ and y-intercept b.
Line 2 has x-intercept $-\frac{b}{m_2}$ and y-intercept b.
Assume also that Line 1 and Line 2 have unequal slopes, that is $m_1 \neq m_2$.
If the lines have the same x-intercept, then $-\frac{b}{m_1} = -\frac{b}{m_2}$.

$$-\frac{b}{m_1} = -\frac{b}{m_2}$$
$$-m_2b = -m_1b$$
$$-m_2b + m_1b = 0$$

But $-m_2b + m_1b = 0 \Rightarrow b(m_1 - m_2) = 0$
$\Rightarrow b = 0$
or $m_1 - m_2 = 0 \Rightarrow m_1 = m_2$

Since we are assuming that $m_1 \neq m_2$, the only way that the two lines can have the same x-intercept is if $b = 0$.

142. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{1-(-3)} = \frac{-6}{4} = -\frac{3}{2}$

It appears that the student incorrectly found the slope by dividing the change in x by the change in y.

Section 2.3

1. add; 25

2. $(x-2)^2 = 9$

$$x - 2 = \pm\sqrt{9}$$
$$x - 2 = \pm 3$$
$$x = 2 \pm 3$$
$$x = 5 \quad \text{or} \quad x = -1$$

The solution set is $\{-1, 5\}$.

3. False. For example, $x^2 + y^2 + 2x + 2y + 8 = 0$ is not a circle. It has no real solutions.

4. radius

5. True; $r^2 = 9 \rightarrow r = 3$

6. False; the center of the circle $(x+3)^2 + (y-2)^2 = 13$ is $(-3, 2)$.

7. Center = (2, 1)
Radius = distance from (0,1) to (2,1)
$= \sqrt{(2-0)^2 + (1-1)^2} = \sqrt{4} = 2$
Equation: $(x-2)^2 + (y-1)^2 = 4$

8. Center = (1, 2)
Radius = distance from (1,0) to (1,2)
$= \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$
Equation: $(x-1)^2 + (y-2)^2 = 4$

9. Center = midpoint of (1, 2) and (4, 2)
$= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$
Radius = distance from $\left(\frac{5}{2}, 2\right)$ to (4,2)
$= \sqrt{\left(4 - \frac{5}{2}\right)^2 + (2-2)^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$
Equation: $\left(x - \frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

10. Center = midpoint of (0, 1) and (2, 3)

$$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$$

Radius = distance from $(1,2)$ to (2,3)

$$= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$$

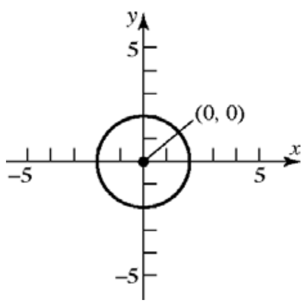
Equation: $(x-1)^2 + (y-2)^2 = 2$

11. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = 2^2$

$x^2 + y^2 = 4$

General form: $x^2 + y^2 - 4 = 0$

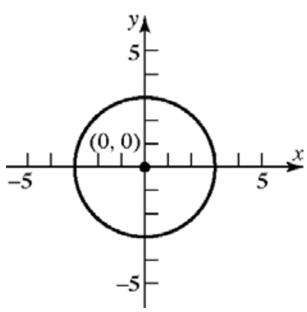


12. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = 3^2$

$x^2 + y^2 = 9$

General form: $x^2 + y^2 - 9 = 0$



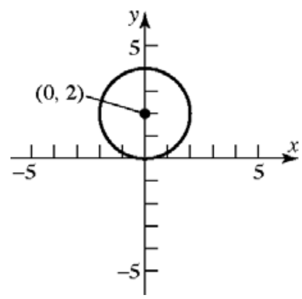
13. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-2)^2 = 2^2$

$x^2 + (y-2)^2 = 4$

General form: $x^2 + y^2 - 4y + 4 = 4$

$x^2 + y^2 - 4y = 0$



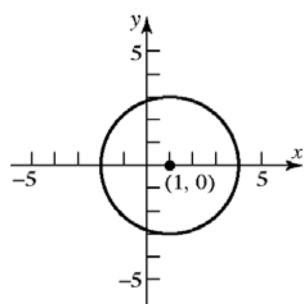
14. $(x-h)^2 + (y-k)^2 = r^2$

$(x-1)^2 + (y-0)^2 = 3^2$

$(x-1)^2 + y^2 = 9$

General form: $x^2 - 2x + 1 + y^2 = 9$

$x^2 + y^2 - 2x - 8 = 0$



15. $(x-h)^2 + (y-k)^2 = r^2$

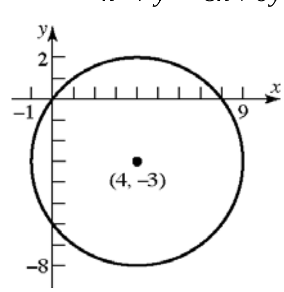
$(x-4)^2 + (y-(-3))^2 = 5^2$

$(x-4)^2 + (y+3)^2 = 25$

General form:

$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 8x + 6y = 0$



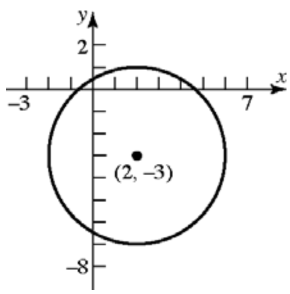
16. $(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-(-3))^2=4^2$

$(x-2)^2+(y+3)^2=16$

General form: $x^2-4x+4+y^2+6y+9=16$

$x^2+y^2-4x+6y-3=0$



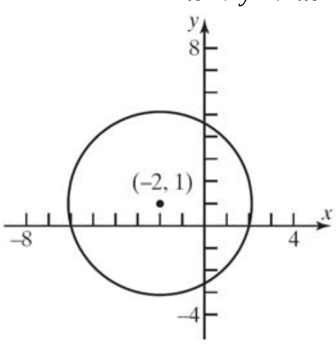
17. $(x-h)^2+(y-k)^2=r^2$

$(x-(-2))^2+(y-1)^2=4^2$

$(x+2)^2+(y-1)^2=16$

General form: $x^2+4x+4+y^2-2y+1=16$

$x^2+y^2+4x-2y-11=0$



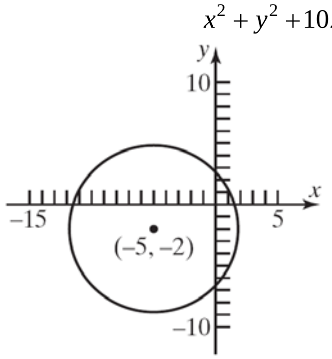
18. $(x-h)^2+(y-k)^2=r^2$

$(x-(-5))^2+(y-(-2))^2=7^2$

$(x+5)^2+(y+2)^2=49$

General form: $x^2+10x+25+y^2+4y+4=49$

$x^2+y^2+10x+4y-20=0$



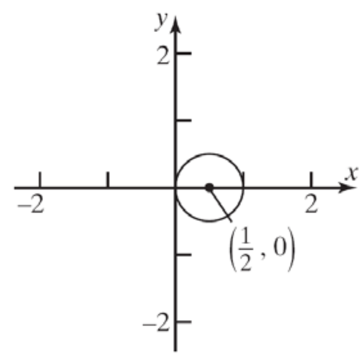
19. $(x-h)^2+(y-k)^2=r^2$

$\left(x-\frac{1}{2}\right)^2+(y-0)^2=\left(\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

General form: $x^2-x+\frac{1}{4}+y^2=\frac{1}{4}$

$x^2+y^2-x=0$



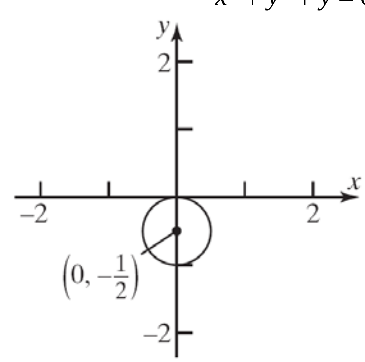
20. $(x-h)^2+(y-k)^2=r^2$

$(x-0)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2=\left(\frac{1}{2}\right)^2$

$x^2+\left(y+\frac{1}{2}\right)^2=\frac{1}{4}$

General form: $x^2+y^2+y+\frac{1}{4}=\frac{1}{4}$

$x^2+y^2+y=0$

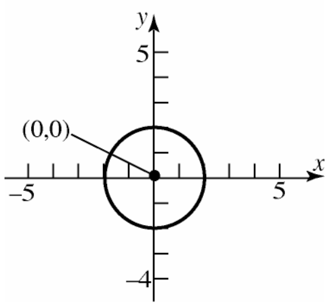


21. $x^2+y^2=4$

$x^2+y^2=2^2$

a. Center: $(0,0)$; Radius $=2$

b.



c. x-intercepts: $x^2+(0)^2=4$

$$x^2=4$$

$$x=\pm\sqrt{4}=\pm 2$$

y-intercepts: $(0)^2+y^2=4$

$$y^2=4$$

$$y=\pm\sqrt{4}=\pm 2$$

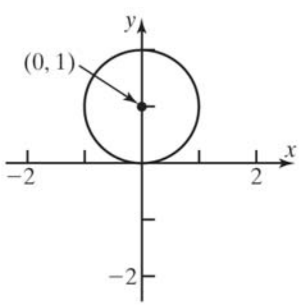
The intercepts are $(-2,0)$, $(2,0)$, $(0,-2)$, and $(0,2)$.

22. $x^2+(y-1)^2=1$

$$x^2+(y-1)^2=1^2$$

a. Center:$(0,1)$; Radius $=1$

b.



c. x-intercepts: $x^2+(0-1)^2=1$

$$x^2+1=1$$

$$x^2=0$$

$$x=\pm\sqrt{0}=0$$

y-intercepts: $(0)^2+(y-1)^2=1$

$$(y-1)^2=1$$

$$y-1=\pm\sqrt{1}$$

$$y-1=\pm 1$$

$$y=1\pm 1$$

$$y=2 \quad \text{or} \quad y=0$$

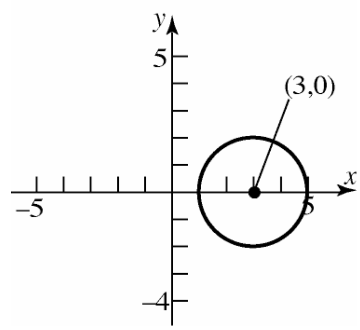
The intercepts are $(0,0)$ and $(0,2)$.

23. $2(x-3)^2+2y^2=8$

$$(x-3)^2+y^2=4$$

a. Center: (3, 0); Radius $=2$

b.



c. x-intercepts: $(x-3)^2+(0)^2=4$

$$(x-3)^2=4$$

$$x-3=\pm\sqrt{4}$$

$$x-3=\pm 2$$

$$x=3\pm 2$$

$$x=5 \quad \text{or} \quad x=1$$

y-intercepts: $(0-3)^2+y^2=4$

$$(-3)^2+y^2=4$$

$$9+y^2=4$$

$$y^2=-5$$

No real solution.

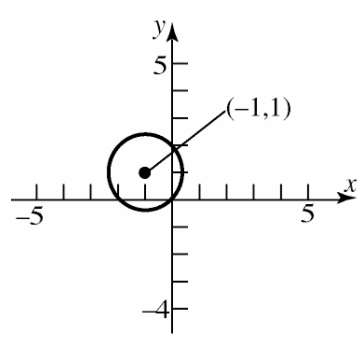
The intercepts are $(1,0)$ and $(5,0)$.

24. $3(x+1)^2+3(y-1)^2=6$

$$(x+1)^2+(y-1)^2=2$$

a. Center: (–1,1); Radius = $\sqrt{2}$

b.



c. x-intercepts: $(x+1)^2+(0-1)^2=2$

$$(x+1)^2+(-1)^2=2$$
$$(x+1)^2+1=2$$
$$(x+1)^2=1$$
$$x+1=\pm\sqrt{1}$$
$$x+1=\pm 1$$
$$x=-1\pm 1$$
$$x=0 \quad \text{or} \quad x=-2$$

y-intercepts: $(0+1)^2+(y-1)^2=2$

$$(1)^2+(y-1)^2=2$$
$$1+(y-1)^2=2$$
$$(y-1)^2=1$$
$$y-1=\pm\sqrt{1}$$
$$y-1=\pm 1$$
$$y=1\pm 1$$
$$y=2 \quad \text{or} \quad y=0$$

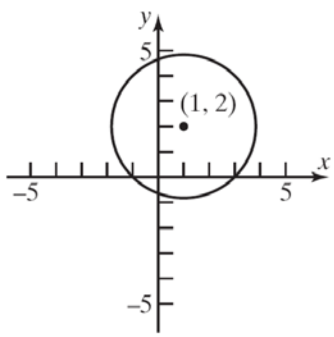
The intercepts are $(-2, 0)$, $(0, 0)$, and $(0, 2)$.

25. $x^2+y^2-2x-4y-4=0$

$$x^2-2x+y^2-4y=4$$
$$(x^2-2x+1)+(y^2-4y+4)=4+1+4$$
$$(x-1)^2+(y-2)^2=3^2$$

a. Center: (1, 2); Radius = 3

b.



c. x-intercepts: $(x-1)^2+(0-2)^2=3^2$

$$(x-1)^2+(-2)^2=3^2$$
$$(x-1)^2+4=9$$
$$(x-1)^2=5$$
$$x-1=\pm\sqrt{5}$$
$$x=1\pm\sqrt{5}$$

y-intercepts: $(0-1)^2+(y-2)^2=3^2$

$$(-1)^2+(y-2)^2=3^2$$
$$1+(y-2)^2=9$$
$$(y-2)^2=8$$
$$y-2=\pm\sqrt{8}$$
$$y-2=\pm 2\sqrt{2}$$
$$y=2\pm 2\sqrt{2}$$

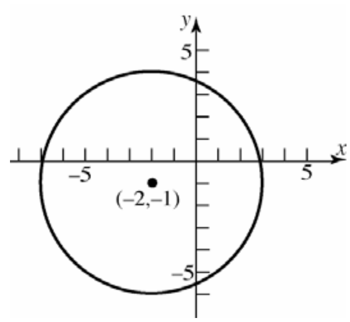
The intercepts are $(1-\sqrt{5}, 0)$, $(1+\sqrt{5}, 0)$, $(0, 2-2\sqrt{2})$, and $(0, 2+2\sqrt{2})$.

26. $x^2+y^2+4x+2y-20=0$

$$x^2+4x+y^2+2y=20$$
$$(x^2+4x+4)+(y^2+2y+1)=20+4+1$$
$$(x+2)^2+(y+1)^2=5^2$$

a. Center: (–2,–1); Radius = 5

b.



c. x-intercepts: $(x+2)^2+(0+1)^2=5^2$

$$(x+2)^2+1=25$$
$$(x+2)^2=24$$
$$x+2=\pm\sqrt{24}$$
$$x+2=\pm 2\sqrt{6}$$
$$x=-2\pm 2\sqrt{6}$$

y-intercepts: $(0+2)^2+(y+1)^2=5^2$

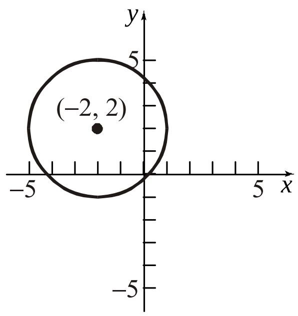
$$4+(y+1)^2=25$$
$$(y+1)^2=21$$
$$y+1=\pm\sqrt{21}$$
$$y=-1\pm\sqrt{21}$$

The intercepts are $(-2-2\sqrt{6}, 0)$, $(-2+2\sqrt{6}, 0)$, $(0, -1-\sqrt{21})$, and $(0, -1+\sqrt{21})$.

27.
$$x^2+y^2+4x-4y-1=0$$
$$x^2+4x+y^2-4y=1$$
$$(x^2+4x+4)+(y^2-4y+4)=1+4+4$$
$$(x+2)^2+(y-2)^2=3^2$$

a. Center: (–2, 2); Radius = 3

b.



c. x-intercepts: $(x+2)^2+(0-2)^2=3^2$
$$(x+2)^2+4=9$$
$$(x+2)^2=5$$
$$x+2=\pm\sqrt{5}$$
$$x=-2\pm\sqrt{5}$$

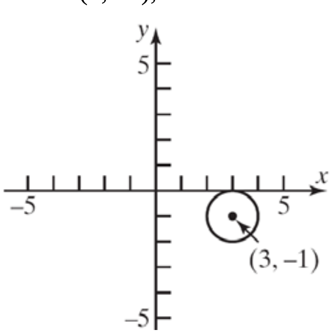
y-intercepts: $(0+2)^2+(y-2)^2=3^2$
$$4+(y-2)^2=9$$
$$(y-2)^2=5$$
$$y-2=\pm\sqrt{5}$$
$$y=2\pm\sqrt{5}$$

The intercepts are $\left(-2-\sqrt{5},\,0\right)$, $\left(-2+\sqrt{5},\,0\right)$, $\left(0,\,2-\sqrt{5}\right)$, and $\left(0,\,2+\sqrt{5}\right)$.

28.
$$x^2+y^2-6x+2y+9=0$$
$$x^2-6x+y^2+2y=-9$$
$$(x^2-6x+9)+(y^2+2y+1)=-9+9+1$$
$$(x-3)^2+(y+1)^2=1^2$$

a. Center: (3, –1); Radius = 1

b.



c. x-intercepts: $(x-3)^2+(0+1)^2=1^2$
$$(x-3)^2+1=1$$
$$(x-3)^2=0$$
$$x-3=0$$
$$x=3$$

y-intercepts: $(0-3)^2+(y+1)^2=1^2$
$$9+(y+1)^2=1$$
$$(y+1)^2=-8$$

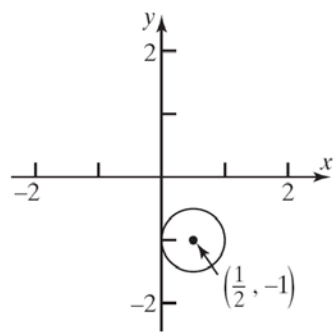
No real solution.

The intercept only intercept is $(3,0)$.

29.
$$x^2+y^2-x+2y+1=0$$
$$x^2-x+y^2+2y=-1$$
$$\left(x^2-x+\frac{1}{4}\right)+(y^2+2y+1)=-1+\frac{1}{4}+1$$
$$\left(x-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$$

a. Center: $\left(\frac{1}{2},-1\right)$; Radius = $\frac{1}{2}$

b.



c. x-intercepts: $\left(x-\frac{1}{2}\right)^2+(0+1)^2=\left(\frac{1}{2}\right)^2$
$$\left(x-\frac{1}{2}\right)^2+1=\frac{1}{4}$$
$$\left(x-\frac{1}{2}\right)^2=-\frac{3}{4}$$

No real solutions

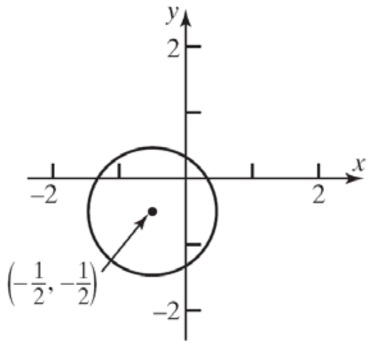
y-intercepts: $\left(0-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$
$$\frac{1}{4}+(y+1)^2=\frac{1}{4}$$
$$(y+1)^2=0$$
$$y+1=0$$
$$y=-1$$

The only intercept is $(0,-1)$.

30.
$$x^2+y^2+x+y-\frac{1}{2}=0$$
$$x^2+x+y^2+y=\frac{1}{2}$$
$$\left(x^2+x+\frac{1}{4}\right)+\left(y^2+y+\frac{1}{4}\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$$

a. Center: $\left(-\frac{1}{2},-\frac{1}{2}\right)$; Radius = 1

b.



c. x-intercepts: $\left(x+\frac{1}{2}\right)^2+\left(0+\frac{1}{2}\right)^2=1^2$
$$\left(x+\frac{1}{2}\right)^2+\frac{1}{4}=1$$
$$\left(x+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$x=\frac{-1\pm\sqrt{3}}{2}$$

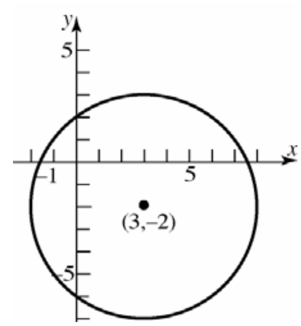
y-intercepts: $\left(0+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$
$$\frac{1}{4}+\left(y+\frac{1}{2}\right)^2=1$$
$$\left(y+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$y+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$y=\frac{-1\pm\sqrt{3}}{2}$$

The intercepts are $\left(\frac{-1-\sqrt{3}}{2}, 0\right)$, $\left(\frac{-1+\sqrt{3}}{2}, 0\right)$, $\left(0, \frac{-1-\sqrt{3}}{2}\right)$, and $\left(0, \frac{-1+\sqrt{3}}{2}\right)$.

31.
$$2x^2+2y^2-12x+8y-24=0$$
$$x^2+y^2-6x+4y=12$$
$$x^2-6x+y^2+4y=12$$
$$(x^2-6x+9)+(y^2+4y+4)=12+9+4$$
$$(x-3)^2+(y+2)^2=5^2$$

a. Center: (3,–2); Radius = 5

b.



c. x-intercepts: $(x-3)^2+(0+2)^2=5^2$
$$(x-3)^2+4=25$$
$$(x-3)^2=21$$
$$x-3=\pm\sqrt{21}$$
$$x=3\pm\sqrt{21}$$

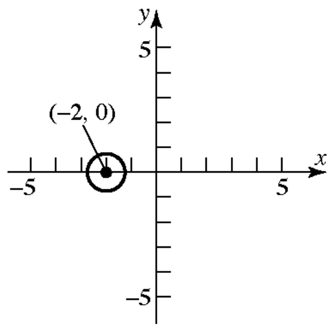
y-intercepts: $(0-3)^2+(y+2)^2=5^2$
$$9+(y+2)^2=25$$
$$(y+2)^2=16$$
$$y+2=\pm 4$$
$$y=-2\pm 4$$
$$y=2 \quad \text{or} \quad y=-6$$

The intercepts are $\left(3-\sqrt{21}, 0\right)$, $\left(3+\sqrt{21}, 0\right)$, $(0,-6)$, and $(0, 2)$.

32. a.
$$2x^2+2y^2+8x+7=0$$
$$2x^2+8x+2y^2=-7$$
$$x^2+4x+y^2=-\frac{7}{2}$$
$$(x^2+4x+4)+y^2=-\frac{7}{2}+4$$
$$(x+2)^2+y^2=\frac{1}{2}$$
$$(x+2)^2+y^2=\left(\frac{\sqrt{2}}{2}\right)^2$$

Center: (–2, 0); Radius = $\frac{\sqrt{2}}{2}$

b.



c. x-intercepts: $(x+2)^2+(0)^2=\frac{1}{2}$

$$(x+2)^2=\frac{1}{2}$$
$$x+2=\pm\sqrt{\frac{1}{2}}$$
$$x+2=\pm\frac{\sqrt{2}}{2}$$
$$x=-2\pm\frac{\sqrt{2}}{2}$$

y-intercepts: $(0+2)^2+y^2=\frac{1}{2}$

$$4+y^2=\frac{1}{2}$$
$$y^2=-\frac{7}{2}$$

No real solutions.

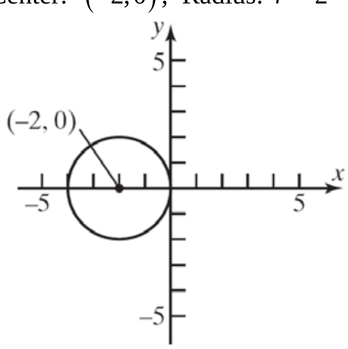
The intercepts are $\left(-2-\frac{\sqrt{2}}{2},0\right)$ and $\left(-2+\frac{\sqrt{2}}{2},0\right)$.

33. $2x^2+8x+2y^2=0$

$$x^2+4x+y^2=0$$
$$x^2+4x+4+y^2=0+4$$
$$(x+2)^2+y^2=2^2$$

a. Center: $(-2,0)$; Radius: $r=2$

b.



c. x-intercepts: $(x+2)^2+(0)^2=2^2$

$$(x+2)^2=4$$
$$(x+2)^2=\pm\sqrt{4}$$
$$x+2=\pm 2$$
$$x=-2\pm 2$$
$$x=0 \quad\text{or}\quad x=-4$$

y-intercepts: $(0+2)^2+y^2=2^2$

$$4+y^2=4$$
$$y^2=0$$
$$y=0$$

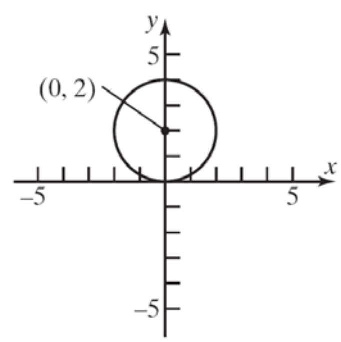
The intercepts are $(-4,0)$ and $(0,0)$.

34. $3x^2+3y^2-12y=0$

$$x^2+y^2-4y=0$$
$$x^2+y^2-4y+4=0+4$$
$$x^2+(y-2)^2=4$$

a. Center: $(0,2)$; Radius: $r=2$

b.



c. x-intercepts: $x^2+(0-2)^2=4$

$$x^2+4=4$$
$$x^2=0$$
$$x=0$$

y-intercepts: $0^2+(y-2)^2=4$

$$(y-2)^2=4$$
$$y-2=\pm\sqrt{4}$$
$$y-2=\pm 2$$
$$y=2\pm 2$$
$$y=4 \quad\text{or}\quad y=0$$

The intercepts are $(0,0)$ and $(0,4)$.

35. Center at (0, 0); containing point (–2, 3).
$r=\sqrt{(-2-0)^2+(3-0)^2}=\sqrt{4+9}=\sqrt{13}$

Equation: $(x-0)^2+(y-0)^2=\left(\sqrt{13}\right)^2$
$$x^2+y^2=13$$

36. Center at (1, 0); containing point (–3, 2).
$r=\sqrt{(-3-1)^2+(2-0)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$

Equation: $(x-1)^2+(y-0)^2=\left(\sqrt{20}\right)^2$
$$(x-1)^2+y^2=20$$

37. Center at (2, 3); tangent to the *x*-axis.
$r=3$

Equation: $(x-2)^2+(y-3)^2=3^2$
$$(x-2)^2+(y-3)^2=9$$

38. Center at (–3, 1); tangent to the *y*-axis.
$r=3$

Equation: $(x+3)^2+(y-1)^2=3^2$
$$(x+3)^2+(y-1)^2=9$$

39. Endpoints of a diameter are (1, 4) and (–3, 2).
The center is at the midpoint of that diameter:

Center: $\left(\frac{1+(-3)}{2},\frac{4+2}{2}\right)=(-1,3)$

Radius: $r=\sqrt{(1-(-1))^2+(4-3)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-(-1))^2+(y-3)^2=\left(\sqrt{5}\right)^2$
$$(x+1)^2+(y-3)^2=5$$

40. Endpoints of a diameter are (4, 3) and (0, 1).
The center is at the midpoint of that diameter:

Center: $\left(\frac{4+0}{2},\frac{3+1}{2}\right)=(2,2)$

Radius: $r=\sqrt{(4-2)^2+(3-2)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-2)^2+(y-2)^2=\left(\sqrt{5}\right)^2$
$$(x-2)^2+(y-2)^2=5$$

41. Center at (–1, 3); tangent to the line $y = 2$.
This means that the circle contains the point (–1, 2), so the radius is $r = 1$.

Equation: $(x+1)^2+(y-3)^2=(1)^2$
$$(x+1)^2+(y-3)^2=1$$

42. Center at (4, –2); tangent to the line $x = 1$.
This means that the circle contains the point (1, –2), so the radius is $r = 3$.

Equation: $(x-4)^2+(y+2)^2=(3)^2$
$$(x-4)^2+(y+2)^2=9$$

43. (c); Center: $(1,-2)$; Radius = 2

44. (d) ; Center: $(-3,3)$; Radius = 3

45. (b) ; Center: $(-1,2)$; Radius = 2

46. (a) ; Center: $(-3,3)$; Radius = 3

47. Let the upper-right corner of the square be the point (x,y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x=y$ at the upper-right corner of the square. Therefore, we get

$$x^2+y^2=9$$
$$x^2+x^2=9$$
$$2x^2=9$$
$$x^2=\frac{9}{2}$$
$$x=\sqrt{\frac{9}{2}}=\frac{3\sqrt{2}}{2}$$

The length of one side of the square is $2x$. Thus, the area is

$$A=s^2=\left(2\cdot\frac{3\sqrt{2}}{2}\right)^2=\left(3\sqrt{2}\right)^2=18 \text{ square units.}$$

48. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point (x, y).

The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square.

Therefore, we get

$$x^2 + y^2 = 36$$
$$x^2 + x^2 = 36$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = 3\sqrt{2}$$

The length of one side of the square is $2x$. Thus, the area of the square is $\left(2 \cdot 3\sqrt{2}\right)^2 = 72$ square units. From the equation of the circle, we have $r = 6$. The area of the circle is $\pi r^2 = \pi(6)^2 = 36\pi$ square units.

Therefore, the area of the shaded region is $A = 36\pi - 72$ square units.

49. The diameter of the Ferris wheel was 250 feet, so the radius was 125 feet. The maximum height was 264 feet, so the center was at a height of $264 - 125 = 139$ feet above the ground. Since the center of the wheel is on the *y*-axis, it is the point (0, 139). Thus, an equation for the wheel is:

$$(x-0)^2 + (y-139)^2 = 125^2$$
$$x^2 + (y-139)^2 = 15,625$$

50. The diameter of the wheel is 153 meters, so the radius is 76.5 meters. The maximum height is 160 meters, so the center of the wheel is at a height of $160 - 76.5 = 83.5$ meters above the ground. Since the center of the wheel is on the *y*-axis, it is the point (0, 83.5). Thus, an equation for the wheel is:

$$(x-0)^2 + (y-83.5)^2 = 76.5^2$$
$$x^2 + (y-83.5)^2 = 5852.25$$

51.
$$x^2 + y^2 + 2x + 4y - 4091 = 0$$
$$x^2 + 2x + y^2 + 4y - 4091 = 0$$
$$x^2 + 2x + 1 + y^2 + 4y + 4 = 4091 + 5$$
$$(x+1)^2 + (y+2)^2 = 4096$$

The circle representing Earth has center $(-1, -2)$ and radius $= \sqrt{4096} = 64$.
So the radius of the satellite's orbit is $64 + 0.6 = 64.6$ units.
The equation of the orbit is

$$(x+1)^2 + (y+2)^2 = (64.6)^2$$
$$x^2 + y^2 + 2x + 4y - 4168.16 = 0$$

52. a.
$$x^2 + (mx+b)^2 = r^2$$
$$x^2 + m^2x^2 + 2bmx + b^2 = r^2$$
$$(1+m^2)x^2 + 2bmx + b^2 - r^2 = 0$$

There is one solution if and only if the discriminant is zero.

$$(2bm)^2 - 4(1+m^2)(b^2 - r^2) = 0$$
$$4b^2m^2 - 4b^2 + 4r^2 - 4b^2m^2 + 4m^2r^2 = 0$$
$$-4b^2 + 4r^2 + 4m^2r^2 = 0$$
$$-b^2 + r^2 + m^2r^2 = 0$$
$$r^2(1+m^2) = b^2$$

b. Using the quadratic formula, the result from part (a), and knowing that the discriminant is zero, we get:

$$(1+m^2)x^2 + 2bmx + b^2 - r^2 = 0$$
$$x = \frac{-2bm}{2(1+m^2)} = \frac{-bm}{\left(\frac{b^2}{r^2}\right)} = \frac{-bmr^2}{b^2} = \frac{-mr^2}{b}$$
$$y = m\left(\frac{-mr^2}{b}\right) + b$$
$$= \frac{-m^2r^2}{b} + b = \frac{-m^2r^2 + b^2}{b} = \frac{r^2}{b}$$

c. The slope of the tangent line is m.
The slope of the line joining the point of tangency and the center is:

$$\frac{\left(\frac{r^2}{b} - 0\right)}{\left(\frac{-mr^2}{b} - 0\right)} = \frac{r^2}{b} \cdot \frac{b}{-mr^2} = -\frac{1}{m}$$

Therefore, the tangent line is perpendicular to the line containing the center of the circle and the point of tangency.

53. $x^2 + y^2 = 9$

Center: (0, 0)

Slope from center to $\left(1, 2\sqrt{2}\right)$ is

$$\frac{2\sqrt{2}-0}{1-0} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}.$$

Slope of the tangent line is $\frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$.

Equation of the tangent line is:

$$y - 2\sqrt{2} = -\frac{\sqrt{2}}{4}(x-1)$$
$$y - 2\sqrt{2} = -\frac{\sqrt{2}}{4}x + \frac{\sqrt{2}}{4}$$
$$4y - 8\sqrt{2} = -\sqrt{2}\,x + \sqrt{2}$$
$$\sqrt{2}\,x + 4y = 9\sqrt{2}$$

54. $x^2 + y^2 - 4x + 6y + 4 = 0$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$
$$(x-2)^2 + (y+3)^2 = 9$$

Center: (2, –3)

Slope from center to $\left(3, 2\sqrt{2}-3\right)$ is

$$\frac{2\sqrt{2}-3-(-3)}{3-2} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

Slope of the tangent line is: $\frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$

Equation of the tangent line:

$$y - \left(2\sqrt{2}-3\right) = -\frac{\sqrt{2}}{4}(x-3)$$
$$y - 2\sqrt{2} + 3 = -\frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4}$$
$$4y - 8\sqrt{2} + 12 = -\sqrt{2}x + 3\sqrt{2}$$
$$\sqrt{2}x + 4y = 11\sqrt{2} - 12$$

55. Let (h, k) be the center of the circle.

$$x - 2y + 4 = 0$$
$$2y = x + 4$$
$$y = \frac{1}{2}x + 2$$

The slope of the tangent line is $\frac{1}{2}$. The slope from (h, k) to (0, 2) is –2.

$$\frac{2-k}{0-h} = -2$$
$$2 - k = 2h$$

The other tangent line is $y = 2x - 7$, and it has slope 2.

The slope from (h, k) to (3, –1) is $-\frac{1}{2}$.

$$\frac{-1-k}{3-h} = -\frac{1}{2}$$
$$2 + 2k = 3 - h$$
$$2k = 1 - h$$
$$h = 1 - 2k$$

Solve the two equations in h and k:

$$2 - k = 2(1-2k)$$
$$2 - k = 2 - 4k$$
$$3k = 0$$
$$k = 0$$
$$h = 1 - 2(0) = 1$$

The center of the circle is (1, 0).

56. Find the centers of the two circles:

$$x^2 + y^2 - 4x + 6y + 4 = 0$$
$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$
$$(x-2)^2 + (y+3)^2 = 9$$

Center: $(2, -3)$

$$x^2 + y^2 + 6x + 4y + 9 = 0$$
$$(x^2 + 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$$
$$(x+3)^2 + (y+2)^2 = 4$$

Center: $(-3, -2)$

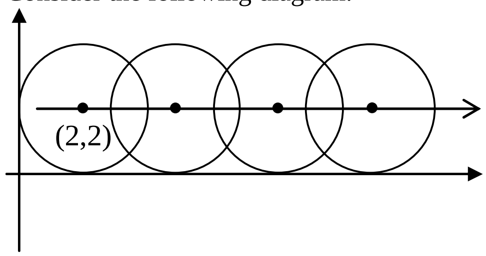
Find the slope of the line containing the centers:

$$m = \frac{-2-(-3)}{-3-2} = -\frac{1}{5}$$

Find the equation of the line containing the centers:

$$y + 3 = -\frac{1}{5}(x-2)$$
$$5y + 15 = -x + 2$$
$$x + 5y = -13$$

57. Consider the following diagram:



Therefore, the path of the center of the circle has the equation $y = 2$.

58.
$$C = 2\pi r$$
$$6\pi = 2\pi r$$
$$\frac{6\pi}{2\pi} = \frac{2\pi r}{2\pi}$$
$$3 = r$$
The radius is 3 units long.

59. (b), (c), (e) and (g)
We need $h, k > 0$ and $(0,0)$ on the graph.

60. (b), (e) and (g)
We need $h < 0$, $k = 0$, and $|h| > r$.

61. Answers will vary.

62. The student has the correct radius, but the signs of the coordinates of the center are incorrect. The student needs to write the equation in the standard form $(x-h)^2 + (y-k)^2 = r^2$.
$$(x+3)^2 + (y-2)^2 = 16$$
$$(x-(-3))^2 + (y-2)^2 = 4^2$$
Thus, $(h,k) = (-3,2)$ and $r = 4$.

Section 2.4

1. $y = kx$

2. False. If y varies directly with x, then $y = kx$, where k is a constant.

3.
$$y = kx$$
$$2 = 10k$$
$$k = \frac{2}{10} = \frac{1}{5}$$
$$y = \frac{1}{5}x$$

4.
$$v = kt$$
$$16 = 2k$$
$$8 = k$$
$$v = 8t$$

5.
$$A = kx^2$$
$$4\pi = k(2)^2$$
$$4\pi = 4k$$
$$\pi = k$$
$$A = \pi x^2$$

6.
$$V = kx^3$$
$$36\pi = k(3)^3$$
$$36\pi = 27k$$
$$k = \frac{36\pi}{27} = \frac{4}{3}\pi$$
$$V = \frac{4}{3}\pi x^3$$

7.
$$F = \frac{k}{d^2}$$
$$10 = \frac{k}{5^2}$$
$$10 = \frac{k}{25}$$
$$k = 250$$
$$F = \frac{250}{d^2}$$

8.
$$y = \frac{k}{\sqrt{x}}$$
$$4 = \frac{k}{\sqrt{9}}$$
$$4 = \frac{k}{3}$$
$$k = 12$$
$$y = \frac{12}{\sqrt{x}}$$

9. $z = k\left(x^2 + y^2\right)$

$5 = k\left(3^2 + 4^2\right)$

$5 = k(25)$

$k = \frac{5}{25} = \frac{1}{5}$

$z = \frac{1}{5}\left(x^2 + y^2\right)$

10. $T = k\left(\sqrt[3]{x}\right)\left(d^2\right)$

$18 = k\left(\sqrt[3]{8}\right)\left(3^2\right)$

$18 = k(18)$

$1 = k$

$T = d^2\sqrt[3]{x}$

11. $M = \frac{kd^2}{\sqrt{x}}$

$24 = \frac{k\left(4^2\right)}{\sqrt{9}}$

$24 = \frac{16k}{3}$

$k = 24\left(\frac{3}{16}\right) = \frac{9}{2}$

$M = \frac{9d^2}{2\sqrt{x}}$

12. $z = k\left(x^3 + y^2\right)$

$1 = k\left(2^3 + 3^2\right)$

$1 = k(17)$

$k = \frac{1}{17}$

$z = \frac{1}{17}\left(x^3 + y^2\right)$

13. $T^2 = \frac{ka^3}{d^2}$

$2^2 = \frac{k\left(2^3\right)}{4^2}$

$4 = \frac{k(8)}{16}$

$4 = \frac{k}{2}$

$k = 8$

$T^2 = \frac{8a^3}{d^2}$

14. $z^3 = k\left(x^2 + y^2\right)$

$2^3 = k\left(9^2 + 4^2\right)$

$8 = k(97)$

$k = \frac{8}{97}$

$z^3 = \frac{8}{97}\left(x^2 + y^2\right)$

15. $V = \frac{4\pi}{3}r^3$

16. $c^2 = a^2 + b^2$

17. $A = \frac{1}{2}bh$

18. $p = 2(l + w)$

19. $F = \left(6.67 \times 10^{-11}\right)\left(\frac{mM}{d^2}\right)$

20. $T = \frac{2\pi}{\sqrt{32}}\sqrt{l}$

21. $p = kB$

$6.49 = k(1000)$

$0.00649 = k$

Therefore we have the linear equation $p = 0.00649B$.

If $B = 145000$, then $p = 0.00649(145000) = \$941.05$.

22. $p = kB$

$8.99 = k(1000)$

$0.00899 = k$

Therefore we have the linear equation $p = 0.00899B$.

If $B = 175000$, then

$p = 0.00899(175000) = \$1573.25$.

23. $s = kt^2$

$16 = k(1)^2$

$k = 16$

Therefore, we have equation $s = 16t^2$.

If t = 3 seconds, then $s = 16(3)^2 = 144$ feet.

If s = 64 feet, then

$64 = 16t^2$

$t^2 = 4$

$t = \pm 2$

Time must be positive, so we disregard $t = -2$.

It takes 2 seconds to fall 64 feet.

24. $v = kt$

$64 = k(2)$

$k = 32$

Therefore, we have the linear equation $v = 32t$.

If t = 3 seconds, then $v = 32(3) = 96$ ft/sec.

25. $E = kW$

$3 = k(20)$

$k = \frac{3}{20}$

Therefore, we have the linear equation $E = \frac{3}{20}W$.

If W = 15, then $E = \frac{3}{20}(15) = 2.25$.

26. $R = \frac{k}{l}$

$256 = \frac{k}{48}$

$k = 12,288$

Therefore, we have the equation $R = \frac{12,288}{l}$.

If $R = 576$, then

$576 = \frac{12,288}{l}$

$576l = 12,288$

$l = \frac{12,288}{576} = \frac{64}{3}$ inches

27. $R = kg$

$47.40 = k(12)$

$3.95 = k$

Therefore, we have the linear equation $R = 3.95g$.

If $g = 10.5$, then $R = (3.95)(10.5) \approx \$41.48$.

28. $C = kA$

$23.75 = k(5)$

$4.75 = k$

Therefore, we have the linear equation $C = 4.75A$.

If $A = 3.5$, then $C = (4.75)(3.5) = \$16.63$.

29. $D = \frac{k}{p}$

a. $D = 156$, $p = 2.75$;

$156 = \frac{k}{2.75}$

$k = 429$

So, $D = \frac{429}{p}$.

b. $D = \frac{429}{3} = 143$ bags of candy

30. $t = \frac{k}{s}$

a. $t = 40$, $s = 30$;

$40 = \frac{k}{30}$

$k = 1200$

So, we have the equation $t = \frac{1200}{s}$.

b. $t = \frac{1200}{40} = 30$ minutes

31. $V = \frac{k}{P}$

$V = 600,\ P = 150$;

$600 = \frac{k}{150}$

$k = 90,000$

So, we have the equation $V = \frac{90,000}{P}$

If $P = 200$, then $V = \frac{90,000}{200} = 450$ cm^3.

32. $i = \frac{k}{R}$

If $i = 30,\ R = 8$, then $30 = \frac{k}{8}$ and $k = 240$.

So, we have the equation $i = \frac{240}{R}$.

If $R = 10$, then $i = \frac{240}{10} = 24$ amperes.

33. $W = \frac{k}{d^2}$

If $W = 125,\ d = 3960$ then

$125 = \frac{k}{3960^2}$ and $k = 1,960,200,000$

So, we have the equation $W = \frac{1,960,200,000}{d}$.

At the top of Mt. McKinley, we have $d = 3960 + 3.8 = 3963.8$, so

$W = \frac{1,960,200,000}{(3963.8)^2} \approx 124.76$ pounds.

34. $I = \frac{k}{d^2}$

If $I = 0.075,\ d = 2$, then

$0.075 = \frac{k}{2^2}$ and $k = 0.3$.

So, we have the equation $I = \frac{0.3}{d^2}$.

If $d = 5$, then $I = \frac{0.3}{5^2} = 0.012$ foot-candle.

35. $V = \pi r^2 h$

36. $V = \frac{\pi}{3} r^2 h$

37. $W = \frac{k}{d^2}$

$55 = \frac{k}{3960^2}$

$k = 862,488,000$

So, we have the equation $W = \frac{862,488,000}{d^2}$.

If d=3965, then

$W = \frac{862,488,000}{3965^2} \approx 54.86$ pounds.

38. $F = kAv^2$

$11 = k(20)(22)^2$

$11 = 9860k$

$k = \frac{11}{9680} = \frac{1}{880}$

So, we have the equation $F = \frac{1}{880} Av^2$.

If $A = 47.125$ and $v = 36.5$, then

$F = \frac{1}{880}(47.125)(36.5)^2 \approx 71.34$ pounds.

39. $h = ksd^3$

$36 = k(75)(2)^3$

$36 = 600k$

$0.06 = k$

So, we have the equation $h = 0.06sd^3$.

If $h = 45$ and $s = 125$, then

$45 = (0.06)(125)d^3$

$45 = 7.5d^3$

$6 = d^3$

$d = \sqrt[3]{6} \approx 1.82$ inches

40. $V = \dfrac{kT}{P}$

$100 = \dfrac{k(300)}{15}$

$100 = 20k$

$5 = k$

So, we have the equation $V = \dfrac{5T}{P}$.

If $V = 80$ and $T = 310$, then

$80 = \dfrac{5(310)}{P}$

$80P = 1550$

$P = \dfrac{1550}{80} = 19.375$ atmospheres

41. $K = kmv^2$

$1250 = k(25)(10)^2$

$1250 = 2500k$

$k = 0.5$

So, we have the equation $K = 0.5mv^2$.
If $m = 25$ and $v = 15$, then

$K = 0.5(25)(15)^2 = 2812.5$ Joules

42. $R = \dfrac{kl}{d^2}$

$1.24 = \dfrac{k(432)}{(4)^2}$

$1.24 = 27k$

$k = \dfrac{1.24}{27}$

So, we have the equation $R = \dfrac{1.24l}{27d^2}$.

If $R = 1.44$ and $d = 3$, then

$1.44 = \dfrac{1.24l}{27(3)^2}$

$1.44 = \dfrac{1.24l}{243}$

$349.92 = 1.24l$

$l = \dfrac{349.92}{1.24} \approx 282.2$ feet

43. $S = \dfrac{kpd}{t}$

$100 = \dfrac{k(25)(5)}{0.75}$

$75 = 125k$

$0.6 = k$

So, we have the equation $S = \dfrac{0.6pd}{t}$.

If $p = 40$, $d = 8$, and $t = 0.50$, then

$S = \dfrac{0.6(40)(8)}{0.50} = 384$ psi.

44. $S = \dfrac{kwt^2}{l}$

$750 = \dfrac{k(4)(2)^2}{8}$

$750 = 2k$

$375 = k$

So, we have the equation $S = \dfrac{375wt^2}{l}$.

If $l = 10$, $w = 6$, and $t = 2$, then

$S = \dfrac{375(6)(2)^2}{10} = 900$ pounds.

45 – 48. Answers will vary.

Chapter 2 Review Exercises

1. $P_1 = (0,0)$ and $P_2 = (4,2)$

a. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{2-0}{4-0} = \dfrac{2}{4} = \dfrac{1}{2}$

b. For every 2-unit change in *x*, *y* will change by 1 unit.

2. $P_1 = (0,0)$ and $P_2 = (-4,6)$

a. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{6-0}{-4-0} = \dfrac{6}{-4} = -\dfrac{3}{2}$

b. For every 2-unit change in *x*, *y* will change by -3. units.

3. $P_1 = (1,-1)$ and $P_2 = (-2,3)$

a. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{3-(-1)}{-2-1} = \dfrac{4}{-3} = -\dfrac{4}{3}$

b. For every 3-unit change in x, y will change by -4. units.

4. $P_1 = (-2,2)$ and $P_2 = (1,4)$

a. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{4-2}{1-(-2)} = \dfrac{2}{3}$

b. For every 3-unit change in x, y will change by 2 units.

5. $P_1 = (4,-4)$ and $P_2 = (4,8)$

a. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{8-(-4)}{4-4} = \dfrac{12}{0}$, undefined

b. An undefined slope means the points lie on a vertical line. For any change in y, there is no change in x.

6. $P_1 = (-3,4)$ and $P_2 = (2,4)$

a. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{4-4}{2-(-3)} = \dfrac{0}{5} = 0$

b. A slope of zero means the points lie on a horizontal line. For any change in x, there is no change in y.

7. $2x = 3y^2$

x-intercepts:	y-intercepts:
$2x = 3(0)^2$	$2(0) = 3y^2$
$2x = 0$	$0 = y^2$
$x = 0$	$y = 0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$2x = 3(-y)^2$

$2x = 3y^2$ same

Test y-axis symmetry: Let $x = -x$

$2(-x) = 3y^2$

$-2x = 3y^2$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$2(-x) = 3(-y)^2$

$-2x = 3y^2$ different

Therefore, the graph will have x-axis symmetry.

8. $y = 5x$

x-intercepts:	y-intercepts:
$0 = 5x$	$y = 5(0)$
$0 = x$	$y = 0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$-y = 5x$

$y = -5x$ different

Test y-axis symmetry: Let $x = -x$

$y = 5(-x)$

$y = -5x$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$-y = 5(-x)$

$y = 5x$ same

Therefore, the graph will have origin symmetry.

9. $x^2 + 4y^2 = 16$

x-intercepts:	y-intercepts:
$x^2 + 4(0)^2 = 16$	$(0)^2 + 4y^2 = 16$
$x^2 = 16$	$4y^2 = 16$
$x = \pm 4$	$y^2 = 4$
	$y = \pm 2$

The intercepts are $(-4,0)$, $(4,0)$, $(0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y = -y$

$x^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$ same

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + 4y^2 = 16$

$x^2 + 4y^2 = 16$ same

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-x)^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

10. $9x^2 - y^2 = 9$

x-intercepts:

$$9x^2 - (0)^2 = 9$$
$$9x^2 = 9$$
$$x^2 = 1$$
$$x = \pm 1$$

y-intercepts:

$$9(0)^2 - y^2 = 9$$
$$-y^2 = 9$$
$$y^2 = -9$$

no real solutions

The intercepts are $(-1, 0)$ and $(1, 0)$.

<u>Test x-axis symmetry</u>: Let $y = -y$

$$9x^2 - (-y)^2 = 9$$
$$9x^2 - y^2 = 9 \quad \text{same}$$

<u>Test y-axis symmetry</u>: Let $x = -x$

$$9(-x)^2 - y^2 = 9$$
$$9x^2 - y^2 = 9 \quad \text{same}$$

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.

$$9(-x)^2 - (-y)^2 = 9$$
$$9x^2 - y^2 = 9 \quad \text{same}$$

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

11. $y = x^4 + 2x^2 + 1$

x-intercepts:

$$0 = x^4 + 2x^2 + 1$$
$$0 = \left(x^2 + 1\right)\left(x^2 + 1\right)$$
$$x^2 + 1 = 0$$
$$x^2 = -1$$

no real solutions

y-intercepts:

$$y = (0)^4 + 2(0)^2 + 1$$
$$= 1$$

The only intercept is (0, 1).

<u>Test x-axis symmetry</u>: Let $y = -y$

$$-y = x^4 + 2x^2 + 1$$
$$y = -x^4 - 2x^2 - 1 \quad \text{different}$$

<u>Test y-axis symmetry</u>: Let $x = -x$

$$y = (-x)^4 + 2(-x)^2 + 1$$
$$y = x^4 + 2x^2 + 1 \quad \text{same}$$

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.

$$-y = (-x)^4 + 2(-x)^2 + 1$$
$$-y = x^4 + 2x^2 + 1$$
$$y = -x^4 - 2x^2 - 1 \quad \text{different}$$

Therefore, the graph will have y-axis symmetry.

12. $y = x^3 - x$

x-intercepts:

$$0 = x^3 - x$$
$$0 = x\left(x^2 - 1\right)$$
$$0 = x(x+1)(x-1)$$
$$x = 0,\ x = -1,\ x = 1$$

y-intercepts:

$$y = (0)^3 - 0$$
$$= 0$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(1, 0)$.

<u>Test x-axis symmetry</u>: Let $y = -y$

$$-y = x^3 - x$$
$$y = -x^3 + x \quad \text{different}$$

<u>Test y-axis symmetry</u>: Let $x = -x$

$$y = (-x)^3 - (-x)$$
$$y = -x^3 + x \quad \text{different}$$

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.

$$-y = (-x)^3 - (-x)$$
$$-y = -x^3 + x$$
$$y = x^3 - x \quad \text{same}$$

Therefore, the graph will have origin symmetry.

13. $x^2 + x + y^2 + 2y = 0$

x-intercepts:

$$x^2 + x + (0)^2 + 2(0) = 0$$
$$x^2 + x = 0$$
$$x(x+1) = 0$$
$$x = 0,\ x = -1$$

y-intercepts:

$$(0)^2 + 0 + y^2 + 2y = 0$$
$$y^2 + 2y = 0$$
$$y(y+2) = 0$$
$$y = 0,\ y = -2$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(0, -2)$.

<u>Test x-axis symmetry</u>: Let $y = -y$

$$x^2 + x + (-y)^2 + 2(-y) = 0$$
$$x^2 + x + y^2 - 2y = 0 \quad \text{different}$$

<u>Test y-axis symmetry</u>: Let $x = -x$

$$(-x)^2 + (-x) + y^2 + 2y = 0$$
$$x^2 - x + y^2 + 2y = 0 \quad \text{different}$$

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.

$$(-x)^2 + (-x) + (-y)^2 + 2(-y) = 0$$
$$x^2 - x + y^2 - 2y = 0 \quad \text{different}$$

The graph has none of the indicated symmetries.

14. $x^2+4x+y^2-2y=0$

x-intercepts: $x^2+4x+(0)^2-2(0)=0$

$$x^2+4x=0$$
$$x(x+4)=0$$
$$x=0,\ x=-4$$

y-intercepts: $(0)^2+4(0)+y^2-2y=0$

$$y^2-2y=0$$
$$y(y-2)=0$$
$$y=0,\ y=2$$

The intercepts are $(-4,0)$, $(0,0)$, and $(0,2)$.

Test x-axis symmetry: Let $y=-y$

$x^2+4x+(-y)^2-2(-y)=0$

$x^2+4x+y^2+2y=0$ different

Test y-axis symmetry: Let $x=-x$

$(-x)^2+4(-x)+y^2-2y=0$

$x^2-4x+y^2-2y=0$ different

Test origin symmetry: Let $x=-x$ and $y=-y$.

$(-x)^2+4(-x)+(-y)^2-2(-y)=0$

$x^2-4x+y^2+2y=0$ different

The graph has none of the indicated symmetries.

15. $(x-h)^2+(y-k)^2=r^2$

$\left(x-(-2)\right)^2+\left(y-3\right)^2=4^2$

$\left(x+2\right)^2+\left(y-3\right)^2=16$

16. $(x-h)^2+(y-k)^2=r^2$

$\left(x-3\right)^2+\left(y-4\right)^2=4^2$

$\left(x-3\right)^2+\left(y-4\right)^2=16$

17. $(x-h)^2+(y-k)^2=r^2$

$\left(x-(-1)\right)^2+\left(y-(-2)\right)^2=1^2$

$\left(x+1\right)^2+\left(y+2\right)^2=1$

18. $(x-h)^2+(y-k)^2=r^2$

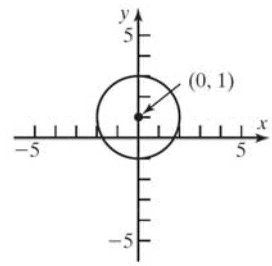
$\left(x-2\right)^2+\left(y-(-4)\right)^2=3^2$

$\left(x-2\right)^2+\left(y+4\right)^2=9$

19. $x^2+\left(y-1\right)^2=4$

$x^2+\left(y-1\right)^2=2^2$

Center: (0, 1); Radius = 2



x-intercepts: $x^2+\left(0-1\right)^2=4$

$$x^2+1=4$$
$$x^2=3$$
$$x=\pm\sqrt{3}$$

y-intercepts: $0^2+\left(y-1\right)^2=4$

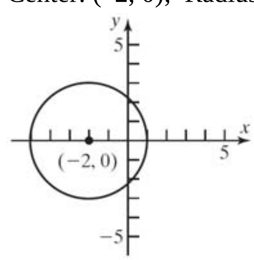
$$(y-1)^2=4$$
$$y-1=\pm 2$$
$$y=1\pm 2$$
$$y=3 \text{ or } y=-1$$

The intercepts are $\left(-\sqrt{3},0\right)$, $\left(\sqrt{3},0\right)$, $(0,-1)$, and $(0,3)$.

20. $\left(x+2\right)^2+y^2=9$

$\left(x+2\right)^2+y^2=3^2$

Center: (–2, 0); Radius = 3



x-intercepts:

$$\left(x+2\right)^2+0^2=9$$
$$\left(x+2\right)^2=9$$
$$x+2=\pm 3$$
$$x=-2\pm 3$$
$$x=1 \text{ or } x=-5$$

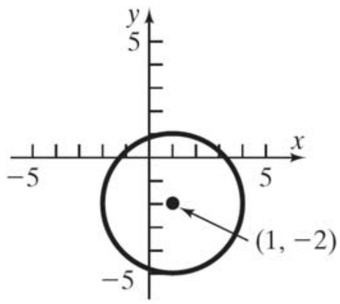
y-intercepts:

$$\left(0+2\right)^2+y^2=9$$
$$4+y^2=9$$
$$y^2=5$$
$$y=\pm\sqrt{5}$$

The intercepts are $(-5,0)$, $(1,0)$, $\left(0,-\sqrt{5}\right)$, and $\left(0,\sqrt{5}\right)$.

21. $$x^2+y^2-2x+4y-4=0$$
$$x^2-2x+y^2+4y=4$$
$$\left(x^2-2x+1\right)+\left(y^2+4y+4\right)=4+1+4$$
$$\left(x-1\right)^2+\left(y+2\right)^2=3^2$$

Center: (1, –2) Radius = 3



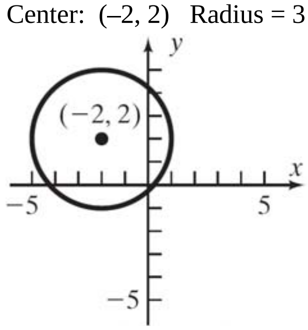
x-intercepts: $\left(x-1\right)^2+\left(0+2\right)^2=3^2$
$$\left(x-1\right)^2+4=9$$
$$\left(x-1\right)^2=5$$
$$x-1=\pm\sqrt{5}$$
$$x=1\pm\sqrt{5}$$

y-intercepts: $\left(0-1\right)^2+\left(y+2\right)^2=3^2$
$$1+\left(y+2\right)^2=9$$
$$\left(y+2\right)^2=8$$
$$y+2=\pm\sqrt{8}$$
$$y+2=\pm2\sqrt{2}$$
$$y=-2\pm2\sqrt{2}$$

The intercepts are $\left(1-\sqrt{5},0\right)$, $\left(1+\sqrt{5},0\right)$, $\left(0,-2-2\sqrt{2}\right)$, and $\left(0,-2+2\sqrt{2}\right)$.

22. $$x^2+y^2+4x-4y-1=0$$
$$x^2+4x+y^2-4y=1$$
$$\left(x^2+4x+4\right)+\left(y^2-4y+4\right)=1+4+4$$
$$\left(x+2\right)^2+\left(y-2\right)^2=3^2$$

Center: (–2, 2) Radius = 3



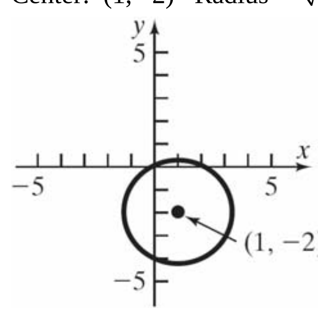
x-intercepts: $\left(x+2\right)^2+\left(0-2\right)^2=3^2$
$$\left(x+2\right)^2+4=9$$
$$\left(x+2\right)^2=5$$
$$x+2=\pm\sqrt{5}$$
$$x=-2\pm\sqrt{5}$$

y-intercepts: $\left(0+2\right)^2+\left(y-2\right)^2=3^2$
$$4+\left(y-2\right)^2=9$$
$$\left(y-2\right)^2=5$$
$$y-2=\pm\sqrt{5}$$
$$y=2\pm\sqrt{5}$$

The intercepts are $\left(-2-\sqrt{5},0\right)$, $\left(-2+\sqrt{5},0\right)$, $\left(0,2-\sqrt{5}\right)$, and $\left(0,2+\sqrt{5}\right)$.

23. $$3x^2+3y^2-6x+12y=0$$
$$x^2+y^2-2x+4y=0$$
$$x^2-2x+y^2+4y=0$$
$$\left(x^2-2x+1\right)+\left(y^2+4y+4\right)=1+4$$
$$\left(x-1\right)^2+\left(y+2\right)^2=\left(\sqrt{5}\right)^2$$

Center: (1, –2) Radius = $\sqrt{5}$



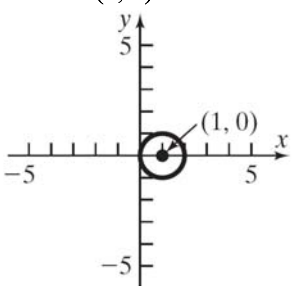
x-intercepts:
$$\left(x-1\right)^2+\left(0+2\right)^2=\left(\sqrt{5}\right)^2$$
$$\left(x-1\right)^2+4=5$$
$$\left(x-1\right)^2=1$$
$$x-1=\pm1$$
$$x=1\pm1$$
$$x=2 \quad\text{or}\quad x=0$$

y-intercepts:
$$\left(0-1\right)^2+\left(y+2\right)^2=\left(\sqrt{5}\right)^2$$
$$1+\left(y+2\right)^2=5$$
$$\left(y+2\right)^2=4$$
$$y+2=\pm2$$
$$y=-2\pm2$$
$$y=0 \quad\text{or}\quad y=-4$$

The intercepts are $\left(0,0\right)$, $\left(2,0\right)$, and $\left(0,-4\right)$.

24.
$$2x^2+2y^2-4x=0$$
$$x^2+y^2-2x=0$$
$$x^2-2x+y^2=0$$
$$\left(x^2-2x+1\right)+y^2=1$$
$$\left(x-1\right)^2+y^2=1^2$$

Center: (1, 0) Radius = 1



x-intercepts: $\left(x-1\right)^2+0^2=1^2$
$$\left(x-1\right)^2=1$$
$$x-1=\pm 1$$
$$x=1\pm 1$$
$$x=2 \quad \text{or} \quad x=0$$

y-intercepts: $\left(0-1\right)^2+y^2=y^2$
$$1+y^2=1$$
$$y^2=0$$
$$y=0$$

The intercepts are $\left(0,0\right)$ and $\left(2,0\right)$.

25. Slope = –2; containing (3,–1)
$$y-y_1=m\left(x-x_1\right)$$
$$y-(-1)=-2\left(x-3\right)$$
$$y+1=-2x+6$$
$$y=-2x+5 \quad \text{or} \quad 2x+y=5$$

26. Slope = 0; containing the point (–5, 4)
$$y-y_1=m\left(x-x_1\right)$$
$$y-4=0\left(x-(-5)\right)$$
$$y-4=0$$
$$y=4$$

27. vertical; containing (–3,4)
Vertical lines have equations of the form $x = a$, where a is the x-intercept. Now, a vertical line containing the point (–3, 4) must have an x-intercept of –3, so the equation of the line is $x=-3$. The equation does not have a slope-intercept form.

28. x-intercept = 2; containing the point (4, –5)
Points are (2, 0) and (4, –5).
$$m=\frac{-5-0}{4-2}=-\frac{5}{2}$$
$$y-y_1=m\left(x-x_1\right)$$
$$y-0=-\frac{5}{2}\left(x-2\right)$$
$$y=-\frac{5}{2}x+5 \quad \text{or} \quad 5x+2y=10$$

29. y-intercept = –2; containing (5,–3)
Points are (5,–3) and (0,–2)
$$m=\frac{-2-(-3)}{0-5}=\frac{1}{-5}=-\frac{1}{5}$$
$$y=mx+b$$
$$y=-\frac{1}{5}x-2 \quad \text{or} \quad x+5y=-10$$

30. Containing the points (3,–4) and (2, 1)
$$m=\frac{1-(-4)}{2-3}=\frac{5}{-1}=-5$$
$$y-y_1=m\left(x-x_1\right)$$
$$y-(-4)=-5\left(x-3\right)$$
$$y+4=-5x+15$$
$$y=-5x+11 \quad \text{or} \quad 5x+y=11$$

31. Parallel to $2x-3y=-4$
$$2x-3y=-4$$
$$-3y=-2x-4$$
$$\frac{-3y}{-3}=\frac{-2x-4}{-3}$$
$$y=\frac{2}{3}x+\frac{4}{3}$$

Slope $=\frac{2}{3}$; containing (–5,3)
$$y-y_1=m\left(x-x_1\right)$$
$$y-3=\frac{2}{3}\left(x-(-5)\right)$$
$$y-3=\frac{2}{3}\left(x+5\right)$$
$$y-3=\frac{2}{3}x+\frac{10}{3}$$
$$y=\frac{2}{3}x+\frac{19}{3} \quad \text{or} \quad 2x-3y=-19$$

32. Parallel to $x+y=2$

$$x+y=2$$
$$y=-x+2$$

Slope = –1; containing (1,–3)

$$y-y_1=m(x-x_1)$$
$$y-(-3)=-1(x-1)$$
$$y+3=-x+1$$
$$y=-x-2 \quad \text{or} \quad x+y=-2$$

33. Perpendicular to $x+y=2$

$$x+y=2$$
$$y=-x+2$$

The slope of this line is -1, so the slope of a line perpendicular to it is 1.
Slope = 1; containing (4,–3)

$$y-y_1=m(x-x_1)$$
$$y-(-3)=1(x-4)$$
$$y+3=x-4$$
$$y=x-7 \quad \text{or} \quad x-y=7$$

34. Perpendicular to $3x-y=-4$

$$3x-y=-4$$
$$y=3x+4$$

The slope of this line is 3, so the slope of a line perpendicular to it is $-\frac{1}{3}$.

Slope $=-\frac{1}{3}$; containing (–2, 4)

$$y-y_1=m(x-x_1)$$
$$y-4=-\frac{1}{3}\left(x-(-2)\right)$$
$$y-4=-\frac{1}{3}x-\frac{2}{3}$$
$$y=-\frac{1}{3}x+\frac{10}{3} \quad \text{or} \quad x+3y=10$$

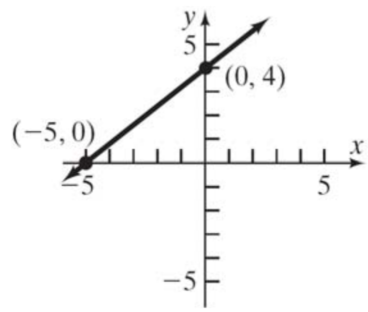
35. $4x-5y=-20$

$$-5y=-4x-20$$
$$y=\frac{4}{5}x+4$$

slope = $\frac{4}{5}$; y-intercept = 4

x-intercept: Let y = 0.

$$4x-5(0)=-20$$
$$4x=-20$$
$$x=-5$$



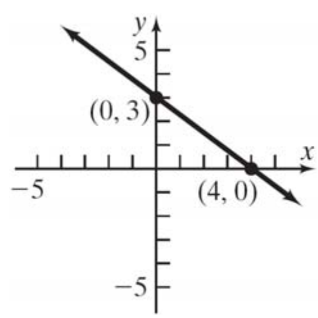
36. $3x+4y=12$

$$4y=-3x+12$$
$$y=-\frac{3}{4}x+3$$

slope = $-\frac{3}{4}$; y-intercept = 3

x-intercept: Let y = 0.

$$3x+4(0)=12$$
$$3x=12$$
$$x=4$$



37. $\frac{1}{2}x - \frac{1}{3}y = -\frac{1}{6}$

$-\frac{1}{3}y = -\frac{1}{2}x - \frac{1}{6}$

$y = \frac{3}{2}x + \frac{1}{2}$

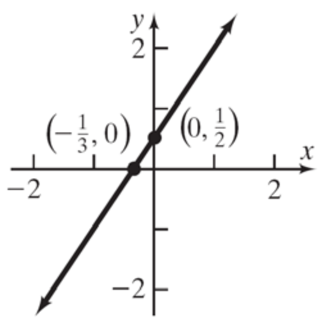
slope = $\frac{3}{2}$; y-intercept = $\frac{1}{2}$

x-intercept: Let $y = 0$.

$\frac{1}{2}x - \frac{1}{3}(0) = -\frac{1}{6}$

$\frac{1}{2}x = -\frac{1}{6}$

$x = -\frac{1}{3}$



38. $-\frac{3}{4}x + \frac{1}{2}y = 0$

$\frac{1}{2}y = \frac{3}{4}x$

$y = \frac{3}{2}x$

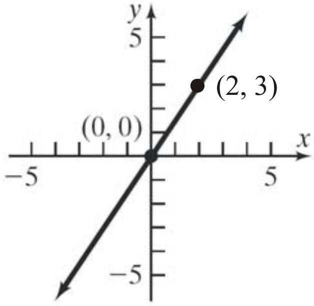
slope = $\frac{3}{2}$; y-intercept = 0

x-intercept: Let $y = 0$.

$-\frac{3}{4}x + \frac{1}{2}(0) = 0$

$-\frac{3}{4}x = 0$

$x = 0$

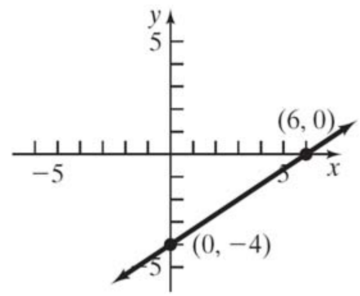


39. $2x - 3y = 12$

x-intercept:
$2x - 3(0) = 12$
$2x = 12$
$x = 6$

y-intercept:
$2(0) - 3y = 12$
$-3y = 12$
$y = -4$

The intercepts are $(6, 0)$ and $(0, -4)$.

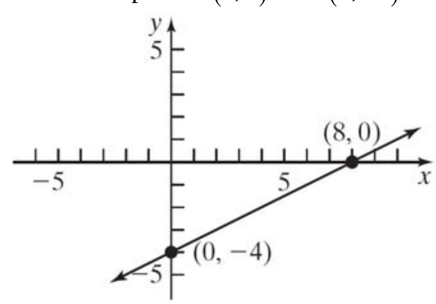


40. $x - 2y = 8$

x-intercept:
$x - 2(0) = 8$
$x = 8$

y-intercept:
$0 - 2y = 8$
$y = -4$

The intercepts are $(8, 0)$ and $(0, -4)$.

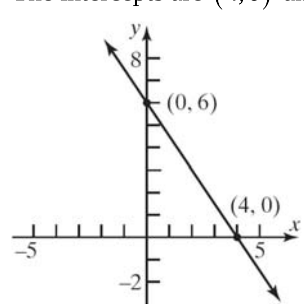


41. $\frac{1}{2}x + \frac{1}{3}y = 2$

x-intercept:
$\frac{1}{2}x + \frac{1}{3}(0) = 2$
$\frac{1}{2}x = 2$
$x = 4$

y-intercept:
$\frac{1}{2}(0) + \frac{1}{3}y = 2$
$\frac{1}{3}y = 2$
$y = 6$

The intercepts are $(4, 0)$ and $(0, 6)$.

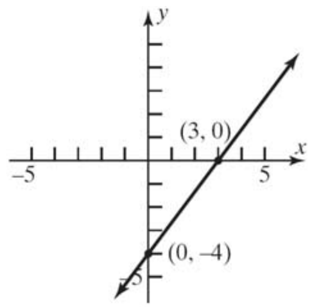


42. $\frac{1}{3}x - \frac{1}{4}y = 1$

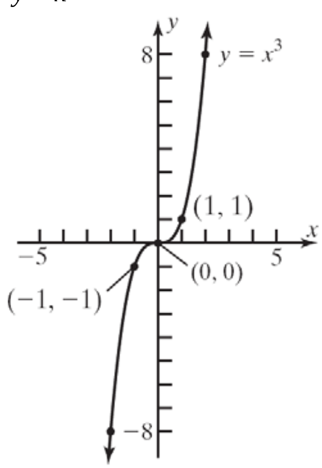
x-intercept:
$$\frac{1}{3}x - \frac{1}{4}(0) = 1$$
$$\frac{1}{3}x = 1$$
$$x = 3$$

y-intercept:
$$\frac{1}{3}(0) - \frac{1}{4}y = 1$$
$$-\frac{1}{4}y = 1$$
$$y = -4$$

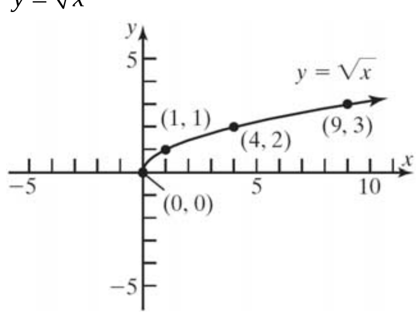
The intercepts are $(3,0)$ and $(0,-4)$.



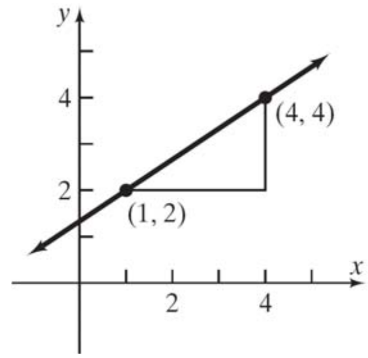
43. $y = x^3$



44. $y = \sqrt{x}$



45. slope = $\frac{2}{3}$, containing the point (1,2)



46. Endpoints of the diameter are (–3, 2) and (5,–6). The center is at the midpoint of the diameter:

Center: $\left(\frac{-3+5}{2}, \frac{2+(-6)}{2}\right) = (1,-2)$

Radius: $r = \sqrt{(1-(-3))^2 + (-2-2)^2}$
$$= \sqrt{16+16}$$
$$= \sqrt{32} = 4\sqrt{2}$$

Equation: $(x-1)^2 + (y+2)^2 = \left(4\sqrt{2}\right)^2$
$$(x-1)^2 + (y+2)^2 = 32$$

47. slope of $\overline{AB} = \frac{1-5}{6-2} = -1$

slope of $\overline{AC} = \frac{-1-5}{8-2} = -1$

slope of $\overline{BC} = \frac{-1-1}{8-6} = -1$

Therefore, the points lie on a line.

48. $p = kB$
$$854 = k(130,000)$$
$$k = \frac{854}{130,000} = \frac{427}{65,000}$$

Therefore, we have the equation $p = \frac{427}{65,000}B$.

If $B = 165,000$, then
$$p = \frac{427}{65,000}(165,000) = \$1083.92 .$$

49. $R = kg$

$46.67 = k(13)$

$k = \frac{46.67}{13} = 3.59$

Therefore, we have the equation $R = 3.59g$.

If $g = 11.2$, then $p = 3.59(11.2) \approx \$40.21$.

50. $w = \frac{k}{d^2}$

$200 = \frac{k}{3960^2}$

$k = (200)(3960^2) = 3{,}136{,}320{,}000$

Therefore, we have the equation

$w = \frac{3{,}136{,}320{,}000}{d^2}$.

If $d = 3960 + 1 = 3961$ miles, then

$w = \frac{3{,}136{,}320{,}000}{3961^2} \approx 199.9$ pounds.

51. $T^2 = ka^3$

$365^2 = (k)(93)^3$

$k = \frac{365^2}{93^3}$

Therefore, we have the equation

$T^2 = \frac{365^2}{93^3} a^3$.

If T = 88 days, then

$88^2 = \left(\frac{365^2}{93^3}\right)(a)^3$

$a^3 = (88^2)\left(\frac{93^3}{365^2}\right)$

$a = \sqrt[3]{(88^2)\left(\frac{93^3}{365^2}\right)} \approx 36$ million miles

The mean distance of the planet Mercury from the Sun is approximately 36 million miles.

52. Answers will vary.

53. **a.** The graph of $x = 0$ is a vertical line passing through the origin. That is, $x = 0$ is the equation of the *y*-axis.

b. The grapy of $y = 0$ is a horizontal line passing through the origin. That is, $y = 0$ is the equation of the *x*-axis.

c. $x + y = 0$

$y = -x$

The graph of $x + y = 0$ is line passing through the origin with slope = –1.

d. $xy = 0$

$y = 0$ or $x = 0$

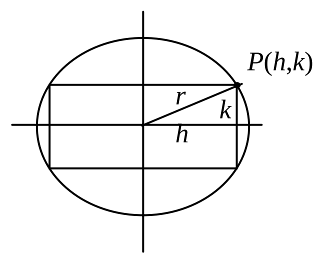
The graph of $xy = 0$ consists of the coordinate axes.

e. $x^2 + y^2 = 0$

$y = 0$ and $x = 0$

The graph of $x^2 + y^2 = 0$ is consists of the origin.

54. Set the axes so that the field's maximum dimension is along the *x*-axis.



Let $2h = width$, $2k = height$, therefore the point farthest from the origin has coordinates $P(h,k)$. So the distance from the origin to point *P* is $r = \sqrt{h^2 + k^2}$ = the radius of the circle .

Using 1 sprinkler arm:

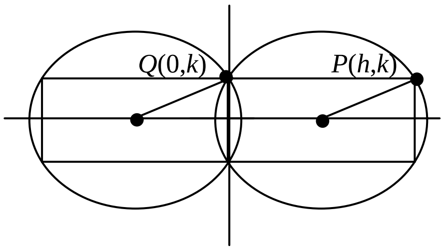
If we place the sprinkler at the origin, we get a circle with equation $x^2 + y^2 = r^2$, where $r = \sqrt{h^2 + k^2}$. So how much excess land is being watered ? The area of the field = $A_F = 4hk$.

The area of the circular water pattern

$A_C = \pi r^2 = \pi\left(\sqrt{h^2 + k^2}\right)^2 = \pi\left(h^2 + k^2\right)$.

Therefore the amount of excess land being watered = $A_C - A_F = \pi\left(h^2 + k^2\right) - 4hk$.

Using 2 sprinkler arms:



We want to place the sprinklers so that they overlap as little as possible while watering the entire field. The equation of the circle with center on the positive x-axis that passes through the point $P(h,k)$ and $Q(0,k)$ is

$\left(x-\frac{h}{2}\right)^2+y^2=\sqrt{\frac{1}{4}h^2+k^2}$, since the center is $\left(\frac{h}{2},0\right)$, and the radius is $\sqrt{\frac{1}{4}h^2+k^2}$.

The case for the other side is similar. Thus, the sprinklers should have their centers at $\left(-\frac{h}{2},0\right)$ and $\left(\frac{h}{2},0\right)$, with the arm lengths set at $\sqrt{\frac{1}{4}h^2+k^2}$.

Each sprinkler waters the same area, so the total area watered is

$$A_C=2\pi r^2=2\pi\left(\sqrt{\frac{1}{4}h^2+k^2}\right)^2=2\pi\left(\frac{1}{4}h^2+k^2\right).$$

The amount of excess land being watered is

$$A_C-A_F=2\pi\left(\frac{1}{4}h^2+k^2\right)-4hk .$$

Comparison:

In order to determine when to switch from 1 sprinkler to 2 sprinklers, we want to determine when 2 sprinklers water less excess land than 1 sprinkler waters. That is, we want to solve:

$$2\pi\left(\frac{1}{4}h^2+k^2\right)-4hk<\pi\left(h^2+k^2\right)-4hk .$$

$$2\pi\left(\frac{1}{4}h^2+k^2\right)-4hk<\pi\left(h^2+k^2\right)-4hk$$

$$\frac{\pi}{2}h^2+2\pi\ k^2-4hk<\pi\ h^2+\ \pi\ k^2-4hk$$

$$\frac{\pi}{2}h^2+2\pi\ k^2<\pi\ h^2+\ \pi\ k^2$$

$$\frac{1}{2}h^2+2\ k^2<h^2+\ k^2$$

$$k^2<\frac{1}{2}h^2$$

$$k<\sqrt{\frac{1}{2}}h$$

$$h>\sqrt{2}k$$

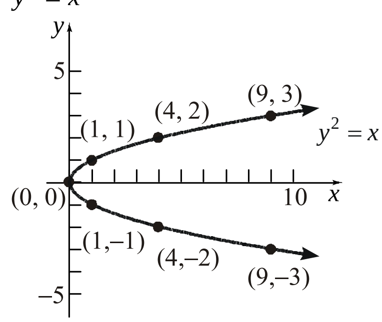
So 2 sprinklers is the better choice when the longer dimension of the rectangle exceeds the shorter dimension by a factor of more than $\sqrt{2}\approx 1.414$.

Chapter 2 Test

1. **a.** $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-1-3}{5-(-1)}=\frac{-4}{6}=-\frac{2}{3}$

   b. If x increases by 3 units, y will decrease by 2 units.

2. $y^2=x$



3. $x^2 + y = 9$

x-intercepts: y-intercept:

$x^2 + 0 = 9$ $\quad$ $(0)^2 + y = 9$

$x^2 = 9$ $\quad$ $y = 9$

$x = \pm 3$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y = -y$

$x^2 + (-y) = 9$

$x^2 - y = 9$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + y = 9$

$x^2 + y = 9$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

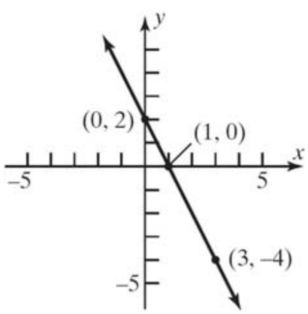
$(-x)^2 + (-y) = 9$

$x^2 - y = 9$ different

Therefore, the graph will have y-axis symmetry.

4. Slope = -2; containing $(3,-4)$

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -2(x - 3)$$
$$y + 4 = -2x + 6$$
$$y = -2x + 2$$



5. $(x-h)^2 + (y-k)^2 = r^2$

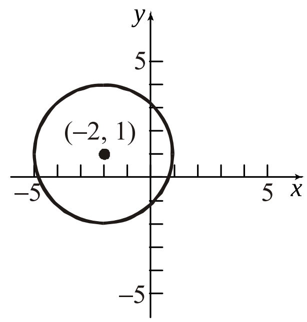
$$(x-4)^2 + (y-(-3))^2 = 5^2$$
$$(x-4)^2 + (y+3)^2 = 25$$

General form:

$$(x-4)^2 + (y+3)^2 = 25$$
$$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$$
$$x^2 + y^2 - 8x + 6y = 0$$

6.
$$x^2 + y^2 + 4x - 2y - 4 = 0$$
$$x^2 + 4x + y^2 - 2y = 4$$
$$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 4 + 1$$
$$(x+2)^2 + (y-1)^2 = 3^2$$

Center: (–2, 1); Radius = 3



7. $2x + 3y = 6$

$$3y = -2x + 6$$
$$y = -\frac{2}{3}x + 2$$

Parallel line

Any line parallel to $2x + 3y = 6$ has slope $m = -\frac{2}{3}$. The line contains $(1,-1)$:

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{2}{3}(x - 1)$$
$$y + 1 = -\frac{2}{3}x + \frac{2}{3}$$
$$y = -\frac{2}{3}x - \frac{1}{3}$$

Perpendicular line

Any line perpendicular to $2x + 3y = 6$ has slope $m = \frac{3}{2}$. The line contains $(0, 3)$:

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{3}{2}(x - 0)$$
$$y - 3 = \frac{3}{2}x$$
$$y = \frac{3}{2}x + 3$$

8. Let R = the resistance, l = length, and r = radius. Then $R = k \cdot \frac{l}{r^2}$. Now, R = 10 ohms, when l = 50 feet and $r = 6\times10^{-3}$ inch, so

$$10 = k \cdot \frac{50}{\left(6\times10^{-3}\right)^2}$$

$$k = 10 \cdot \frac{\left(6\times10^{-3}\right)^2}{50} = 7.2\times10^{-6}$$

Therefore, we have the equation

$$R = \left(7.2\times10^{-6}\right)\frac{l}{r^2}.$$

If $l = 100$ feet and $r = 7\times10^{-3}$ inch, then

$$R = \left(7.2\times10^{-6}\right)\frac{100}{\left(7\times10^{-3}\right)^2} \approx 14.69 \text{ ohms.}$$

Chapter 2 Cumulative Review

1.
$$\begin{aligned} 3x-5&=0\\ 3x&=5\\ x&=\frac{5}{3}\end{aligned}$$

The solution set is $\left\{\frac{5}{3}\right\}$.

2.
$$\begin{aligned} x^2-x-12&=0\\ (x-4)(x+3)&=0\\ x=4 &\text{ or } x=-3\end{aligned}$$

The solution set is $\{-3,4\}$.

3.
$$\begin{aligned} 2x^2-5x-3&=0\\ (2x+1)(x-3)&=0\\ x=-\frac{1}{2} &\text{ or } x=3\end{aligned}$$

The solution set is $\left\{-\frac{1}{2},3\right\}$.

4. $x^2-2x-2=0$

$$\begin{aligned} x&=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-2)}}{2(1)}\\ &=\frac{2\pm\sqrt{4+8}}{2}\\ &=\frac{2\pm\sqrt{12}}{2}\\ &=\frac{2\pm2\sqrt{3}}{2}\\ &=1\pm\sqrt{3}\end{aligned}$$

The solution set is $\left\{1-\sqrt{3}, 1+\sqrt{3}\right\}$.

5. $x^2+2x+5=0$

$$\begin{aligned} x&=\frac{-2\pm\sqrt{2^2-4(1)(5)}}{2(1)}\\ &=\frac{-2\pm\sqrt{4-20}}{2}\\ &=\frac{-2\pm\sqrt{-16}}{2}\end{aligned}$$

No real solutions

6.
$$\begin{aligned} \sqrt{2x+1}&=3\\ \left(\sqrt{2x+1}\right)^2&=3^2\\ 2x+1&=9\\ 2x&=8\\ x&=4\end{aligned}$$

Check: $\sqrt{2(4)+1}=3?$

$\sqrt{9}=3?$

$3=3$ True

The solution set is $\{4\}$.

7. $|x-2|=1$

$x-2=1$ or $x-2=-1$

$x=3$ $\quad$ $x=1$

The solution set is $\{1,3\}$.

8. $\sqrt{x^2+4x}=2$

$$\left(\sqrt{x^2+4x}\right)^2=2^2$$
$$x^2+4x=4$$
$$x^2+4x-4=0$$
$$x=\frac{-4\pm\sqrt{4^2-4(1)(-4)}}{2(1)}=\frac{-4\pm\sqrt{16+16}}{2}$$
$$=\frac{-4\pm\sqrt{32}}{2}=\frac{-4\pm4\sqrt{2}}{2}=-2\pm2\sqrt{2}$$

Check $x=-2+2\sqrt{2}$:

$$\sqrt{\left(-2+2\sqrt{2}\right)^2+4\left(-2+2\sqrt{2}\right)}=2?$$
$$\sqrt{4-8\sqrt{2}+8-8+8\sqrt{2}}=2?$$
$$\sqrt{4}=2 \text{ True}$$

Check $x=-2-2\sqrt{2}$:

$$\sqrt{\left(-2-2\sqrt{2}\right)^2+4\left(-2-2\sqrt{2}\right)}=2?$$
$$\sqrt{4+8\sqrt{2}+8-8-8\sqrt{2}}=2?$$
$$\sqrt{4}=2 \text{ True}$$

The solution set is $\left\{-2-2\sqrt{2},-2+2\sqrt{2}\right\}$.

9. $x^2=-9$

$$x=\pm\sqrt{-9}$$
$$x=\pm3i$$

The solution set is $\{-3i,3i\}$.

10. $x^2-2x+5=0$

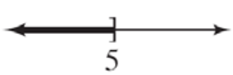
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)}=\frac{2\pm\sqrt{4-20}}{2}$$
$$=\frac{2\pm\sqrt{-16}}{2}=\frac{2\pm4i}{2}=1\pm2i$$

The solution set is $\{1-2i,1+2i\}$.

11. $2x-3\le7$

$$2x\le10$$
$$x\le5$$

$\{x\,|\,x\le5\}$ or $(-\infty,5]$



12. $-1<x+4<5$

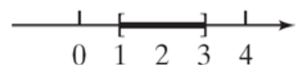
$$-5<x<1$$

$\{x\,|\,-5<x<1\}$ or $(-5,1)$

13. $|x-2|\le1$

$$-1\le x-2\le1$$
$$1\le x\le3$$

$\{x\,|\,1\le x\le3\}$ or $[1,3]$

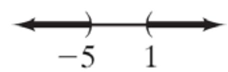


14. $|2+x|>3$

$2+x<-3$ or $2+x>3$

$x<-5$ or $x>1$

$\{x\,|\,x<-5 \text{ or } x>1\}$ or $(-\infty,-5)\cup(1,\infty)$



15. $d(P,Q)=\sqrt{(-1-4)^2+(3-(-2))^2}$

$$=\sqrt{(-5)^2+(5)^2}$$
$$=\sqrt{25+25}$$
$$=\sqrt{50}=5\sqrt{2}$$

$$\text{Midpoint}=\left(\frac{-1+4}{2},\frac{3+(-2)}{2}\right)=\left(\frac{3}{2},\frac{1}{2}\right)$$

16. $y=x^3-3x+1$

a. $(-2,-1)$:

$(-2)^3-(3)(-2)+1=-8+6+1=-1$

$(-2,-1)$ is on the graph.

b. $(2,3)$:

$(2)^3-(3)(2)+1=8-6+1=3$

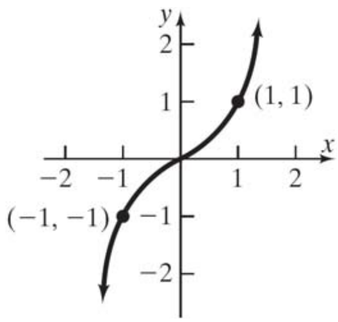
$(2,3)$ is on the graph.

c. $(3,1)$:

$(3)^3-(3)(3)+1=27-9+1=19\ne1$

$(3,1)$ is not on the graph.

17. $y = x^3$



18. The points (–1, 4) and (2,–2) are on the line.

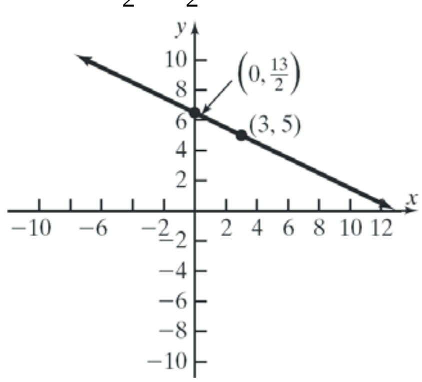
$$\text{Slope} = \frac{-2-4}{2-(-1)} = \frac{-6}{3} = -2$$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -2\left(x - (-1)\right)$$
$$y - 4 = -2\left(x + 1\right)$$
$$y = -2x - 2 + 4$$
$$y = -2x + 2$$

19. Perpendicular to $y = 2x + 1$; Contains $(3,5)$

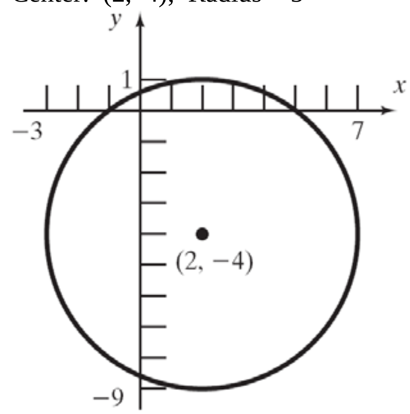
$$\text{Slope of perpendicular} = -\frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$
$$y - 5 = -\frac{1}{2}(x - 3)$$
$$y - 5 = -\frac{1}{2}x + \frac{3}{2}$$
$$y = -\frac{1}{2}x + \frac{13}{2}$$



20.
$$x^2 + y^2 - 4x + 8y - 5 = 0$$
$$x^2 - 4x + y^2 + 8y = 5$$
$$(x^2 - 4x + 4) + (y^2 + 8y + 16) = 5 + 4 + 16$$
$$(x-2)^2 + (y+4)^2 = 25$$
$$(x-2)^2 + (y+4)^2 = 5^2$$

Center: (2,–4); Radius = 5



Chapter 2 Projects

Project I

1. Men: Let $(x_1, y_1) = (1992, 2.22)$ and $(x_2, y_2) = (1996, 2.21)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.21 - 2.22}{1996 - 1992} = \frac{-0.01}{4} = -0.0025$$

$$y - y_1 = m\left(x - x_1\right)$$
$$y - 2.22 = -0.0025\left(x - 1992\right)$$
$$y - 2.22 = -0.0025x + 4.98$$
$$y = -0.0025x + 7.20$$

Women: Let $(x_1, y_1) = (1992, 2.54)$ and $(x_2, y_2) = (1996, 2.43)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.43 - 2.54}{1996 - 1992} = \frac{-0.11}{4} = -0.0275$$

$$y - y_1 = m\left(x - x_1\right)$$
$$y - 2.54 = -0.0275\left(x - 1992\right)$$
$$y - 2.54 = -0.0275x + 54.78$$
$$y = -0.0275x + 57.32$$

2. Men: The slope $m = -0.0025$ indicates that the winning times for men in the Olympic marathon are decreasing at an average rate of 0.0025 hour per year.

   Women: The slope $m = -0.0275$ indicates that the winning times for women in the Olympic marathon are decreasing at an average rate of 0.0275 hour per year.

   The *y*-intercepts do not have reasonable interpretations. In this case, the *y*-intercepts would indicate the winning times of the marathons in the year 0, which is not reasonable because it is too far away from the data on which our equations are based.

3. Men: If $x = 2004$, then $y = -0.0025(2004) + 7.20 = 2.19$ hours. This compares reasonably well to the actual result of 2.18 hours.

   Women: If $x = 2004$, then $y = -0.0275(2004) + 57.32 = 2.21$ hours. This does not compare well to the actual result of 2.44 hours.

4. **(1)** Men: Let $(x_1, y_1) = (1996, 2.21)$ and $(x_2, y_2) = (2000, 2.17)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.17 - 2.21}{2000 - 1996} = \frac{-0.04}{4} = -0.01$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2.21 &= -0.01(x - 1996) \\ y - 2.21 &= -0.01x + 19.96 \\ y &= -0.01x + 22.17 \end{aligned}$$

   Women: Let $(x_1, y_1) = (1996, 2.43)$ and $(x_2, y_2) = (2000, 2.39)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.39 - 2.43}{2000 - 1996} = \frac{-0.04}{4} = -0.01$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2.39 &= -0.01(x - 1996) \\ y - 2.39 &= -0.01x + 19.96 \\ y &= -0.01x + 22.35 \end{aligned}$$

   (2) Men: The slope $m = -0.01$ indicates that the winning times for men in the Olympic marathon are decreasing at a rate of 0.01 hour per year.

   Women: The slope $m = -0.01$ indicates that the winning times for women in the Olympic marathon are decreasing at a rate of 0.01 hour per year.

   The *y*-intercepts do not have reasonable interpretations. In this case, the *y*-intercepts would indicate the winning times of the marathons in the year 0, which is not reasonable because it is too far away from the data on which our equations are based.

   (3) Men: If $x = 2004$, then $y = -0.01(2004) + 22.17 = 2.13$ hours. This prediction is not extremely accurate, but it does compare reasonably well to the actual result of 2.18 hours.

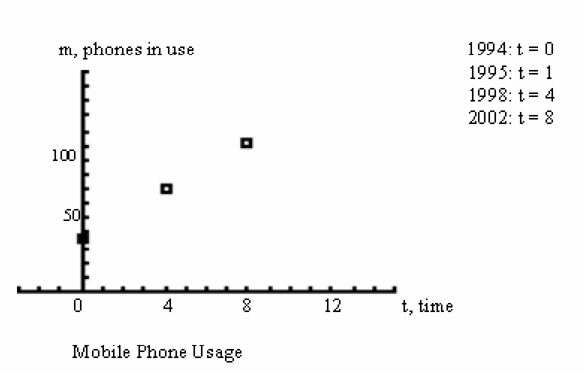
   Women: If $x = 2004$, then $y = -0.01(2004) + 22.35 = 2.31$ hours. This does not compare well to the actual result of 2.44 hours.

5. No. The year 2112 is too far away from the data on which our equations are based.

6. Answers will vary.

Project II

1.



2. The line of best fit: m = 8.125 t + 37.5

3. Slope indicates the rate at which mobile phone usage is growing per year; in this case, 8.125 million phones per year.

   The y-intercept (m-intercept) indicates how many phones were in use in the base year; in this case 37.5 million phones.

4. Using t = 12 for the year 2006,
m=8.125(12) + 37.5 = 135

Using this model, 135 million phones are predicted to be in use in 2006.

5. Answers will vary.

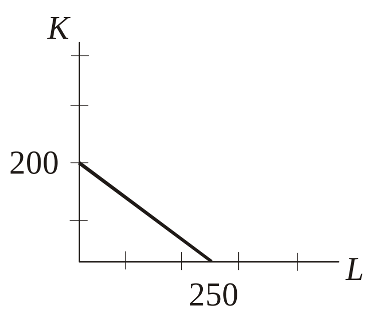
Project III

a. Both L and K should be positive because they are the number of hours needed of both labor and capital.

$$P_L \cdot L + P_K \cdot K = E$$
$$400(100) + 500K = 100000$$

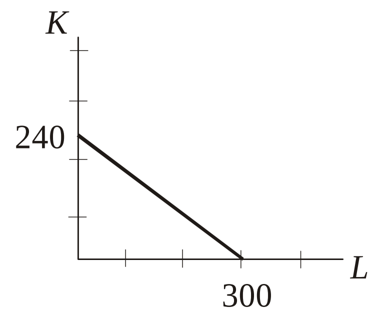
b. $40000 + 500K = 100000$
$$500K = 60000$$
$$K = 120$$
120 hours of capital are needed.

c. $400L + 500K = 100000$
$$K = -\frac{4}{5}L + 200$$



The line lies in the first quadrant because both L and K must be positive.

d. $400L + 500K = 120000$
$$K = -\frac{4}{5}L + 240$$



The slope is still the same but the x- and y-intercepts are larger. If the expenditures were to decrease, the slope would remain the same, but the x- and y-intercepts would decrease.

e. $P_L \cdot L + P_K \cdot K = E$
$$P_K \cdot K = -P_L \cdot L + E$$
$$K = -\frac{P_L}{P_K} \cdot L + \frac{E}{P_K}$$

The slope for the isocost line is $-\frac{P_L}{P_K}$. For the particular values in parts **a.-d.**, the slope is $-\frac{4}{5}$. This means that for every unit increase in the amount of labor, the amount of capital decreases by $\frac{4}{5}$ of a unit.

f. $500L + 500K = 100000$
$$K = -L + 200$$
The increase in the price of labor causes the graph to be steeper (decrease faster). The slope is -1.