SOLUTIONS MANUAL

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Solution Manual for Adaptive Control

Second Edition

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Preface

This Solution Manual contains solutions to selected problems in the second edition of Adaptive Control published by Addison-Wesley 1995, ISBN 0-201-55866-1.

PROBLEM SOLUTIONS

SOLUTIONS TO CHAPTER 1

1.5 Linearization of the valve shows that

$$\Delta v = 4v_0^3 \Delta u$$

The loop transfer function is then

$$G_0(s)G_{PI}(s)4v_0^3$$

where G_{PI} is the transfer function of a PI controller i.e.

$$G_{PI}(s) = K\left(1 + \frac{1}{sT_i}\right)$$

The characteristic equation for the closed loop system is

$$sT_i(s+1)^3 + K \cdot 4v_0^3(sT_i + 1) = 0$$

with $K = 0.15$ and $T_i = 1$ we get

$$(s+1)\left(s(s+1)^2 + 0.6v_0^3\right) = 0$$

$$(s+1)(s^3 + 2s^2 + s + 0.6v_0^3) = 0$$

The root locus of this equation with respect to v_o is sketched in Fig. 1. According to the Routh Hurwitz criterion the critical case is

$$0.6v_0^3 = 2 \qquad \Rightarrow v_0 = \sqrt[3]{\frac{10}{3}} = 1.49$$

Since the plant G_0 has unit static gain and the controller has integral action the steady-state output is equal to v_0 and the set point y_r. The closed-loop system is stable for $y_r = u_c = 0.3$ and 1.1 but unstable for $y_r = u_c = 5.1$. Compare with Fig. 1.9.

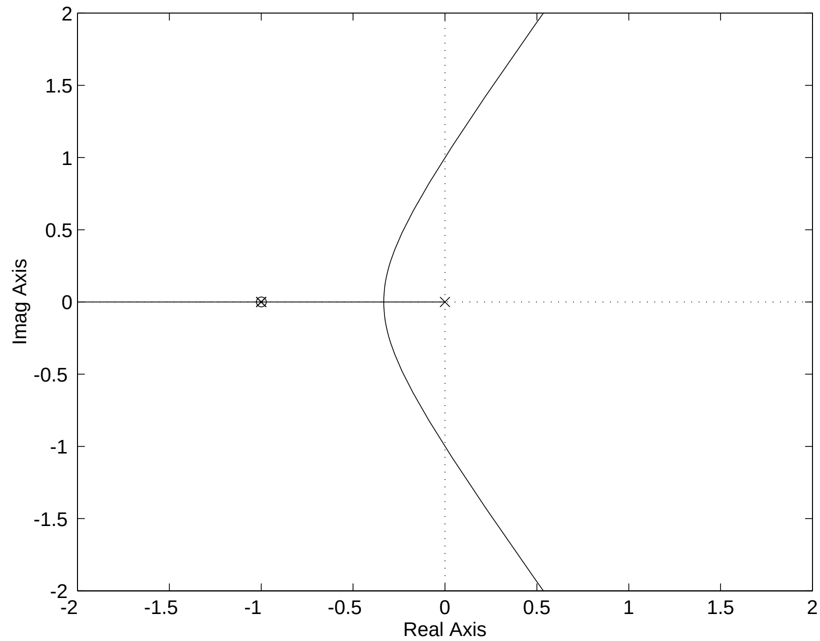

Figure 1. Root locus in Problem 1.5.

1.6 Tune the controller using the Ziegler-Nichols closed-loop method. The frequency ω_u, where the process has 180° phase lag is first determined. The controller parameters are then given by Table 8.2 on page 382 where

$$K_u = \frac{1}{|G_0(i\omega_u)}|$$

we have

$$G_0(s) = \frac{e^{-s/q}}{1+s/q}$$

$$\arg G_0(i\omega) = -\frac{\omega}{q} - \arctan\frac{\omega}{q} = -\pi$$

q	ω	$G_0(i\omega)$	K	T_i
0.5	1.0	0.45	1	5.24
	2.0	0.45	1	2.62
	4.1	0.45	1	1.3

A simulation of the system obtained when the controller is tuned for the smallest flow $q = 0.5$ is shown Fig. 2. The Ziegler-Nichols method is not the best tuning method in this case. In the Fig. 3 we show results for

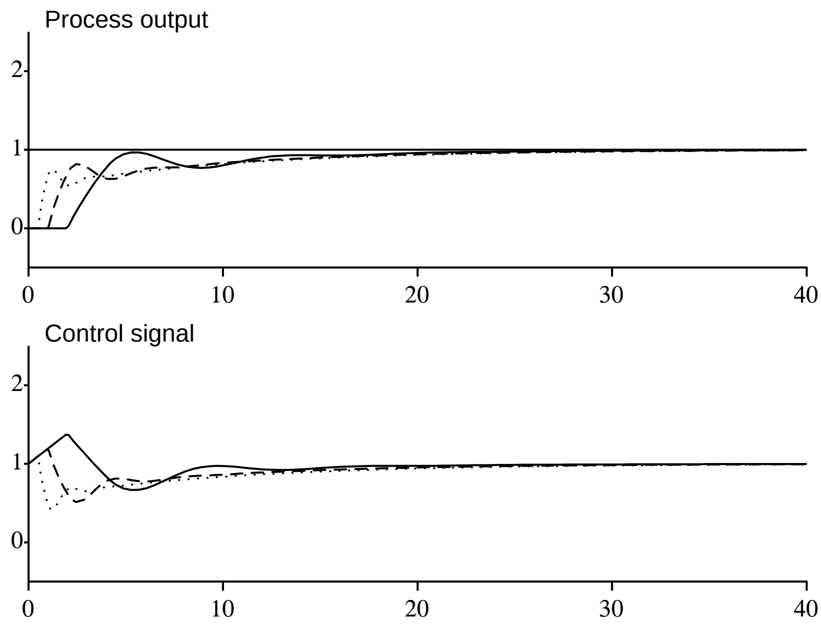

Figure 2. Simulation in Problem 1.6. Process output and control signal are shown for $q = 0.5$ (full), $q = 1$ (dashed), and $q = 2$ (dotted). The controller is designed for $q = 0.5$.

controller designed for $q = 1$ and in Fig. 4when the controller is designed for $q = 2$.

1.7 Introducing the feedback

$$u = -k_2 y_2$$

the system becomes

$$\frac{dx}{dt} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{pmatrix} x - k_2 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u_1$$

$$y_1 = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} x$$

The transfer function from u_1 to y_1 is

$$G(s) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} s+1 & 0 & 0 \\ 2k_2 & s+3 & 2k_2 \\ k_2 & 0 & s+1+k_2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{s^2 + (4 - k_2)s + 3 + k_2}{(s+1)(s+3)(s+1+k_2)}$$

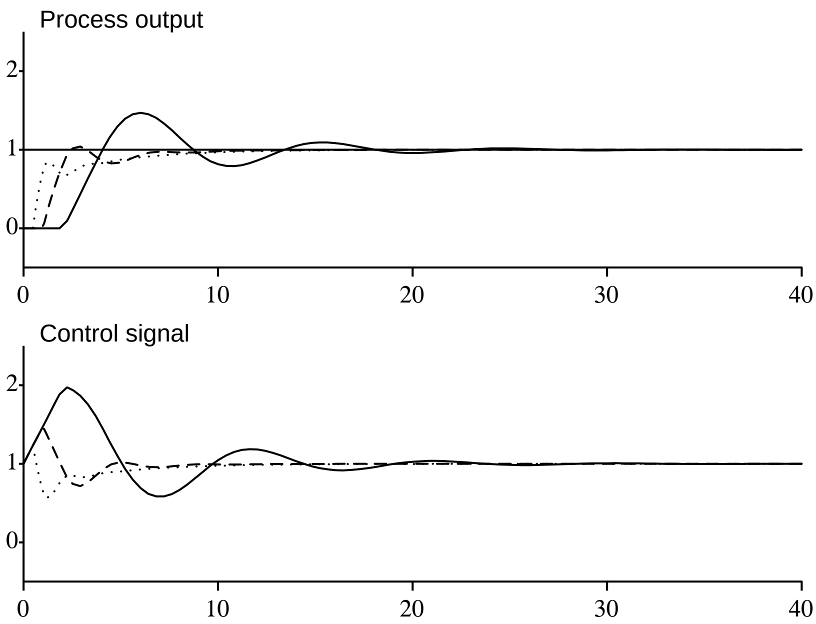

Figure 3. Simulation in Problem 1.6. Process output and control signal are shown for $q = 0.5$ (full), $q = 1$ (dashed), and $q = 2$ (dotted). The controller is designed for $q = 1$.

The static gain is

$$G(0) = \frac{3 + k_2}{3(1 + k_2)}$$

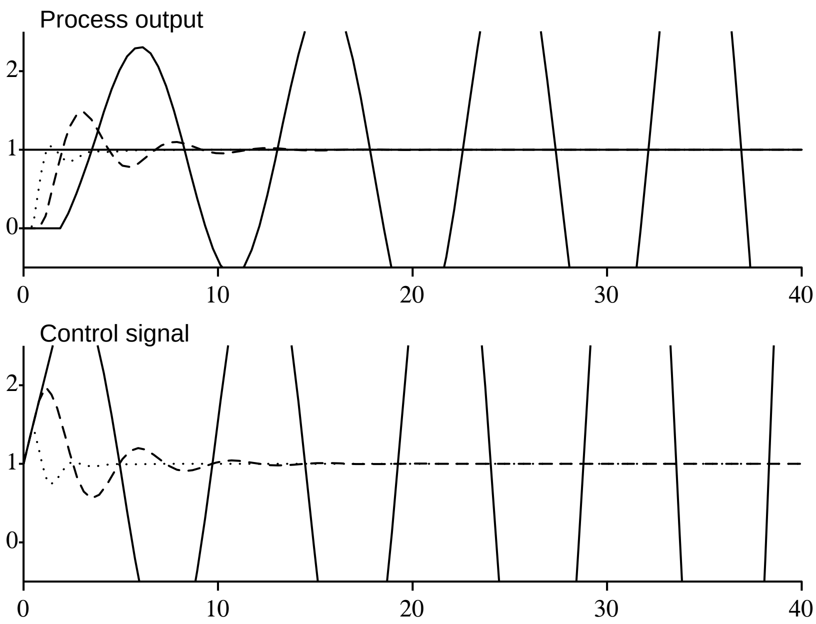

Figure 4. Simulation in Problem 1.6. Process output and control signal are shown for $q = 0.5$ (full), $q = 1$ (dashed), and $q = 2$ (dotted). The controller is designed for $q = 2$.