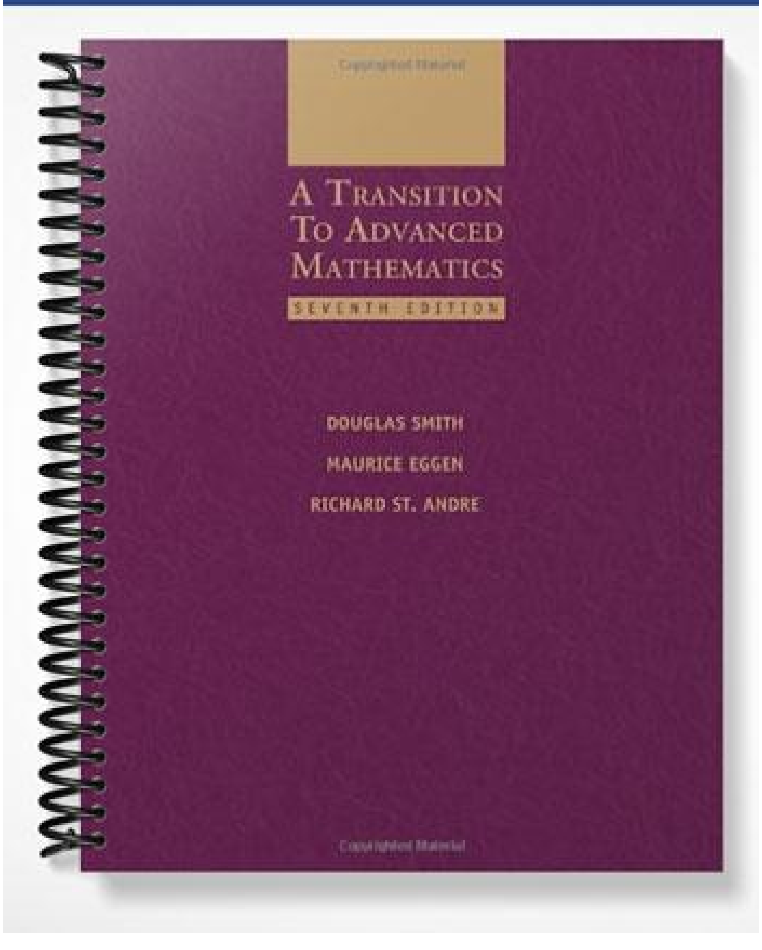
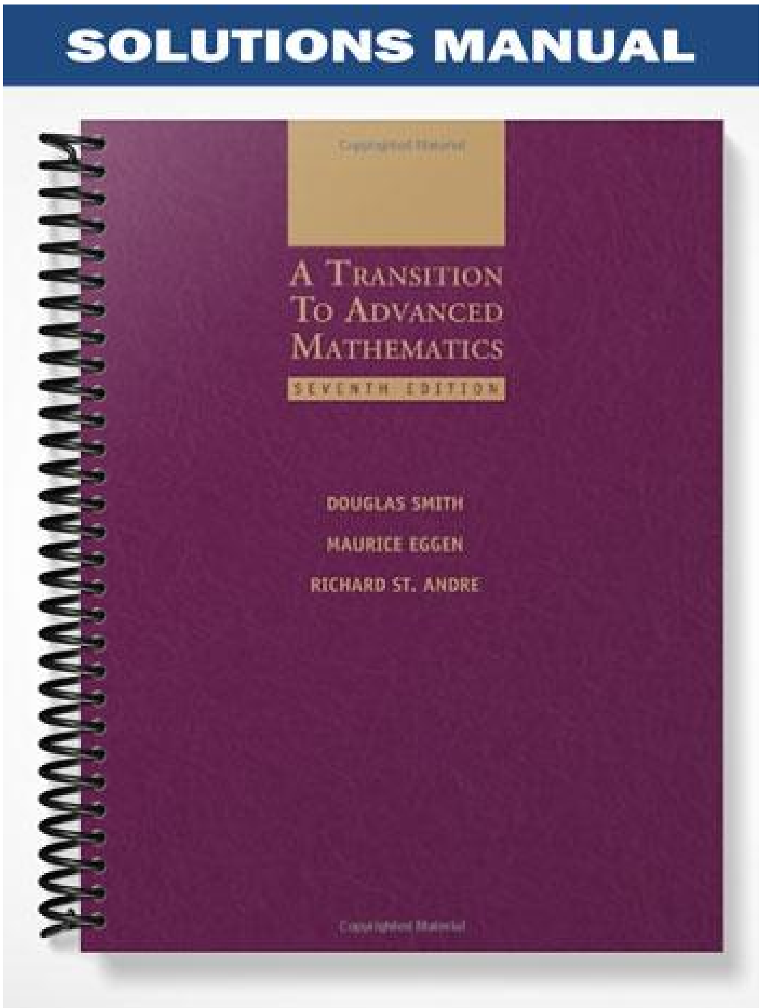
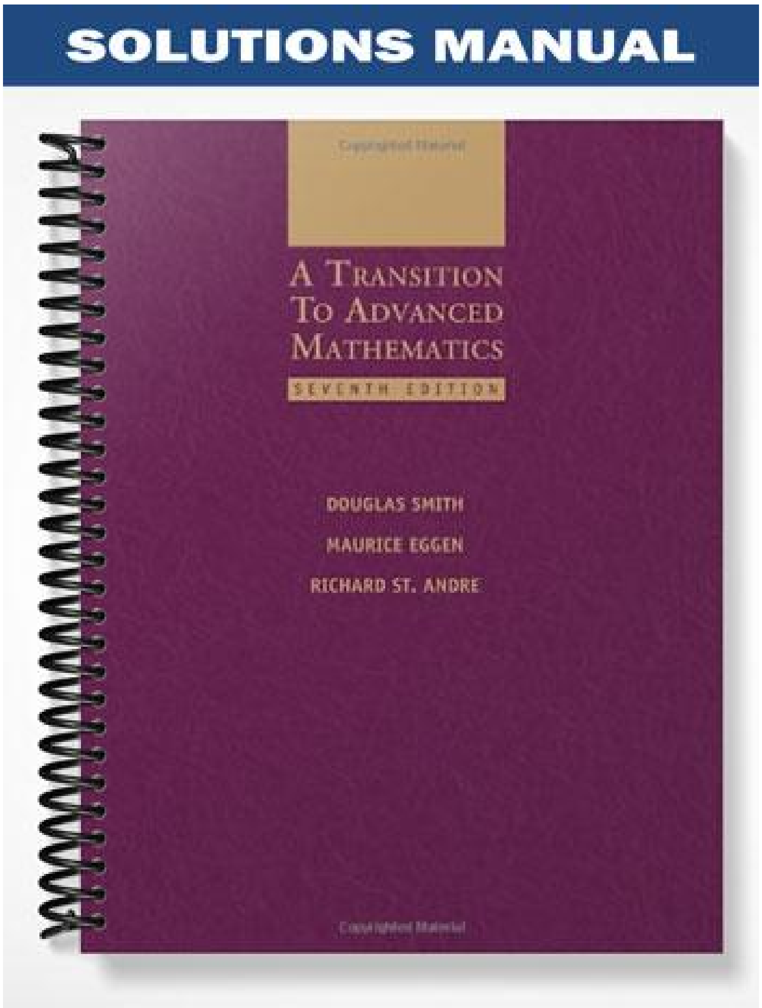
SOLUTIONS MANUAL




SOLUTIONS MANUAL
A TRANSITION TO ADVANCED MATHEMATICS
SEVENTH EDITION
DOUGLAS SMITH
MAURICE EGGEN
RICHARD ST. ANDRE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1 Logic and Proofs

1.1 Propositions and Connectives

1. (a) true (b) false (c) true (d) false (e) false (f) false (g) false (h) false

2. (a) Not a proposition
   (b) False proposition
   (c) Not a proposition. It would be a proposition if a value for x had been assigned.
   (d) Not a proposition. It would be a proposition if values for x and y had been assigned.
   (e) False proposition
   (f) True proposition
   (g) False proposition
   (h) True proposition
   (i) False proposition
   (j) Not a proposition. It is neither true nor false.

3. (a)

P	$\sim P$	$P \wedge \sim P$
T	F	T
F	T	F

(b)

P	$\sim P$	$P \vee \sim P$
T	F	T
F	T	T

(c)

P	Q	$\sim Q$	$P \wedge \sim Q$
T	T	F	F
F	T	F	F
T	F	T	T
F	F	T	F

(d)

P	Q	$\sim Q$	$Q \vee \sim Q$	$P \wedge (Q \vee \sim Q)$
T	T	F	T	T
F	T	F	T	F
T	F	T	T	T
F	F	T	T	F

(e)

P	Q	$\sim Q$	$P \wedge Q$	$(P \wedge Q) \vee \sim Q$
T	T	F	T	T
F	T	F	F	F
T	F	T	F	T
F	F	T	F	T

(f)

P	Q	$P \wedge Q$	$\sim (P \wedge Q)$
T	T	T	F
F	T	F	T
T	F	F	T
F	F	F	T

(g)

P	Q	R	$\sim Q$	$P \vee \sim Q$	$(P \vee \sim Q) \wedge R$
T	T	T	F	T	T
F	T	T	F	F	F
T	F	T	T	T	T
F	F	T	T	T	T
T	T	F	F	T	F
F	T	F	F	F	F
T	F	F	T	T	F
F	F	F	T	T	F

(h)

P	Q	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$
T	T	F	F	F
F	T	T	F	F
T	F	F	T	F
F	F	T	T	T

(i)

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$
T	T	T	T	T
F	T	T	T	F
T	F	T	T	T
F	F	T	T	F
T	T	F	T	T
F	T	F	T	F
T	F	F	F	F
F	F	F	F	F

(j)

P	Q	R	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T
F	T	T	F	F	F
T	F	T	F	T	T
F	F	T	F	F	F
T	T	F	T	F	T
F	T	F	F	F	F
T	F	F	F	F	F
F	F	F	F	F	F

4. (a) false (b) true (c) true (d) true
   (e) false (f) false (g) false (h) false
   (i) true (j) true (k) false (1) false

5. (a) No solution.

   (b)

P	Q	$P \vee Q$	$Q \vee P$
T	T	T	T
F	T	T	T
T	F	T	T
F	F	F	F

Since the third and fourth columns are the same, the propositions are equivalent.

(c)

P	Q	$P \wedge Q$	$Q \wedge P$
T	T	T	T
F	T	F	F
T	F	F	F
F	F	F	F

Since the third and fourth columns are the same, the propositions are equivalent.

(d)

P	Q	R	$Q \vee R$	$P \vee (Q \vee R)$	$P \vee Q$	$(P \vee Q) \vee R$
T	T	T	T	T	T	T
F	T	T	T	T	T	T
T	F	T	T	T	T	T
F	F	T	T	T	F	T
T	T	F	T	T	T	T
F	T	F	T	T	T	T
T	F	F	F	T	T	T
F	F	F	F	F	F	F

Since the fifth and seventh columns are the same, the propositions are equivalent.

(e)

P	Q	R	$Q \wedge R$	$P \wedge (Q \wedge R)$	$P \wedge Q$	$(P \wedge Q) \wedge R$
T	T	T	T	T	T	T
F	T	T	T	F	F	F
T	F	T	F	F	F	F
F	F	T	F	F	F	F
T	T	F	F	F	T	F
F	T	F	F	F	F	F
T	F	F	F	F	F	F
F	F	F	F	F	F	F

Since the fifth and seventh columns are the same, the propositions are equivalent.

(f)

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
F	T	T	T	F	F	F	F
T	F	T	T	T	F	T	T
F	F	T	T	F	F	F	F
T	T	F	T	T	T	F	T
F	T	F	T	F	F	F	F
T	F	F	F	F	F	F	F
F	F	F	F	F	F	F	F

Since the fifth and eighth columns are the same, the propositions are equivalent.

(g)

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T
F	F	T	F	F	F	T	F
T	T	F	F	T	T	T	T
F	T	F	F	F	T	F	F
T	F	F	F	T	T	T	T
F	F	F	F	F	F	F	F

Since the fifth and eighth columns are the same, the propositions are equivalent.

(h) No solution.

(i)

P	Q	$P \vee Q$	$\sim (P \vee Q)$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$
T	T	T	F	F	F	F
F	T	T	F	T	F	F
T	F	T	F	F	T	F
F	F	F	T	T	T	T

Since the fourth and eighth columns are the same, the propositions are equivalent.

6. (a) equivalent (b) equivalent
(c) equivalent (d) equivalent
(e) equivalent (f) not equivalent
(g) not equivalent (h) not equivalent

7. (a) $\sim P$, true (b) $P \wedge Q$, true
(c) P Q, true (d) $P \vee Q \vee R$, true

8. (a) Since P is equivalent to Q, P has the same truth table as Q. Therefore, Q has the same truth table as P, so Q is equivalent to P.

(b) Since P is equivalent to Q, P and Q have the same truth table. Since Q is equivalent to R, Q and R have the same truth table. Thus, P and R have the same truth table so P is equivalent to R.

(c) Since P is equivalent to Q, P and Q have the same truth table. That is, the truth table for P has value true on exactly the same lines that the truth table for Q has value true. Therefore the truth table for $\sim Q$ has value false on exactly the same lines that the truth table for $\sim P$ has the value false. Thus $\sim Q$ and $\sim P$ have the same truth table.

9. (a) $(P \wedge Q) \vee (\sim P \wedge \sim Q)$ is neither.

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$\sim P \wedge \sim Q$	$(P \wedge Q) \vee (\sim P \wedge \sim Q)$
T	T	F	F	T	F	T
F	T	T	F	F	F	F
T	F	F	T	F	F	F
F	F	T	T	F	T	T

(b) $\sim (P \wedge \sim P)$ is a tautology.

P	$\sim P$	$P \wedge \sim P$	$\sim (P \wedge \sim P)$
T	F	F	T
F	T	F	T

(c) $(P \wedge Q) \vee (\sim P \vee \sim Q)$ is a tautology.

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$\sim P \vee \sim Q$	$(P \wedge Q) \vee (\sim P \vee \sim Q)$
T	T	F	F	T	F	T
F	T	T	F	F	T	T
T	F	F	T	F	T	T
F	F	T	T	F	T	T

(d) $(A \wedge B) \vee (A \wedge \sim B) \vee (\sim A \wedge B) \vee (\sim A \wedge \sim B)$ is a tautology.

A	B	$\sim A$	$\sim B$	$A \wedge B$	$A \wedge \sim B$	$\sim A \wedge B$	$\sim A \wedge \sim B$	$(A \wedge B) \vee (A \wedge \sim B) \vee$ $(\sim A \wedge B) \vee (\sim A \wedge \sim B)$
T	T	F	F	T	F	F	F	T
F	T	T	F	F	F	T	F	T
T	F	F	T	F	T	F	F	T
F	F	T	T	F	F	F	T	T

(e) $(Q \wedge \sim P) \wedge \sim (P \wedge R)$ is neither.

P	Q	R	$\sim P$	$Q \wedge \sim P$	$P \wedge R$	$\sim (P \wedge R)$	$(Q \wedge \sim P) \wedge \sim (P \wedge R)$
T	T	T	F	F	T	F	F
F	T	T	T	T	F	T	T
T	F	T	F	F	T	F	F
F	F	T	T	F	F	T	F
T	T	F	F	F	F	T	F
F	T	F	T	T	F	T	T
T	F	F	F	F	F	T	F
F	F	F	T	F	F	T	F

(f) $P \vee [(\sim Q \wedge P) \wedge (R \vee Q)]$ is neither.

P	Q	R	$\sim Q$	$\sim Q \wedge P$	$R \vee Q$	$[(\sim Q \wedge P) \wedge (R \vee Q)]$	$P \vee [(\sim Q \wedge P) \wedge (R \vee Q)]$
T	T	T	F	F	T	F	T
F	T	T	F	F	T	F	F
T	F	T	T	T	T	T	T
F	F	T	T	F	T	F	F
T	T	F	F	F	T	F	T
F	T	F	F	F	T	F	F
T	F	F	T	T	F	F	T
F	F	F	T	F	F	F	F

10. (a) contradiction (b) tautology
(c) tautology (d) tautology

11. (a) x is not a positive integer.

(b) Cleveland will lose the first game and the second game. Or, Cleveland will lose both games.

(c) $5 < 3$

(d) 641,371 is not composite. Or 641,371 is prime.

(e) Roses are not red or violets are not blue.

(f) T is bounded and T is not compact.

(g) M is not odd or M is not one-to-one.

(h) The function.f does not have a positive first derivative at x or does not have a positive second derivative at x.

(i) The function g does not have a relative maximum at $x = 2$ (deleted comma) and does not have a relative maximum at $x = 4$, or else g does not have a relative minimum at $x = 3$.

(j) $z < s$ or $z \leq t$.

(k) R is not transitive or R is reflexive.

(l) If the function g has a relative minimum at $x = 2$ or $x = 4$, then g does not have a relative minimum at $x = 3$.

12. (a) $[\sim (\sim P)] \vee [(\sim Q) \wedge (\sim S)]$

(b) $[Q \wedge (\sim S)] \vee \sim (P \wedge [Q \wedge (\sim S)] \vee \sim ((\sim P \wedge Q)$.

(c) $[[P \wedge (\sim Q)] \vee [(\sim P) \wedge (\sim R)]] \vee [(\sim P) \wedge S]$

(d) $[(\sim P) \vee ([Q \wedge (\sim (\sim P))] \wedge Q)] \vee R$.

13. (a) i.

A	B	$A \veebar B$
T	T	F
F	T	T
T	F	T
F	F	F

ii.

A	B	$A \vee B$	$A \wedge B$	$\sim (A \wedge B)$	$(A \vee B) \wedge \sim (A \wedge B)$
T	T	T	T	F	F
F	T	T	F	T	T
T	F	T	F	T	T
F	F	F	F	T	F

Since the final columns of the two tables are identical, the two propositions have the same truth table, thus they are equivalent.

(b) i.

A	B	A NAND B	A NOR B
T	T	F	F
F	T	T	F
T	F	T	F
F	F	T	T

ii.

A	B	A NANDB	A NOR B	$(A$ NAND $B) \vee (A$ NOR $B)$
T	T	F	F	F
F	T	T	F	T
T	F	T	F	T
F	F	T	T	T

Since the third and last columns are equal, the propositions are equivalent.

iii.

A	B	A NAND B	A NOR B	$(A$ NAND $B) \wedge (A$ NOR $B)$
T	T	F	F	F
F	T	T	F	F
T	F	T	F	F
F	F	T	T	T

Since the fourth and last columns are equal, the propositions are equivalent.

1.2 Conditionals and Biconditionals

1. (a) Antecedent: squares have three.
   Consequent: triangles have four sides.

   (b) Antecedent: The moon is made of cheese.
   Consequent: 8 is an irrational number.

   (c) Antecedent: b divides 3.
   Consequent: b divides 9.

   (d) Antecedent: f is differentiable.
   Consequent: f is continuous.

   (e) Antecedent: a is convergent.
   Consequent: a is bounded.

   (f) Antecedent: f if integrable.
   Consequent: f is bounded.

   (g) Antecedent: $1 + 1 = 2$.
   Consequent: $1 + 2 = 3$.

(h) Antecedent: the fish bite.
Consequent: the moon is full.

(i) Antecedent: An athlete qualifies for the Olympic team.
Consequent: The athlete has a time of 3 minutes, 48 seconds or less (in the event).

2. (a) Converse: If triangles have four sides, then squares have three sides.
Contrapositive: If triangles do not have four sides, then squares do not have three sides.

(b) Converse: If 8 is irrational, then the moon is made of cheese.
Contrapositive: If 8 is rational, then the moon is not made of cheese.

(c) Converse: If b divides 9, then b divides 3.
Contrapositive: If b does not divide 9, then b does not divide 3.

(d) Converse: If f is continuous, then f is differentiable.
Contrapositive: If f is not continuous, then f is not differentiable.

(e) Converse: If a is bounded, then a is convergent.
Contrapositive: If a is not bounded, then a is not convergent.

(f) Converse: If f is bounded, then f is integrable.
Contrapositive: If f is not bounded, then f is not integrable.

(g) Converse: If $1 + 2 = 3$, then $1 + 1 = 2$.
Contrapositive: If $1 + 1 \neq 2$, then $1 + 2 \neq 3$.

(h) Converse: If the moon is full, then fish will bite.
Contrapositive: If the moon is not full, then fish will not bite.

(i) Converse: A time of 3 minutes, 48 seconds or less is sufficient to qualify for the Olympic team.
Contrapositive: If an athlete records a time that is not 3 minutes and 48 seconds or less, then that athlete does not qualify for the Olympic team.

3. (a) Q may be either true or false.

(b) Q must be true.

(c) Q must be false.

(d) Q must be false.

(e) Q must be false.

4. (a) Antecedent: $A(x)$ is an open sentence with variable x.
Consequent: $\sim (\forall x)A(x)$ is equivalent to $(\exists x) \sim A(x)$.

(b) Antecedent: Every even natural number greater than 2 is the sum of two primes.
Consequent: Every odd natural number greater than 5 is the sum of three primes.

(c) Antecedent: A is a set with n elements.
Consequent: $\mathcal{P}(A)$ is a set with 2^n elements.

(d) Antecedent: S is a subset of $\mathbb{N}$ such that $1 \in S$ and, for all $n \in \mathbb{N}$, if $n \in S$, then $n + 1 \in S$.
Consequent: $S = \mathbb{N}$.

(e) Antecedent: A is a finite set with m elements and B is a finite set with n elements.
Consequent: $\overline{\overline{A \times B}} = mn$.

(f) Antecedent: R is a partial order for A and $B \subseteq A$.
Consequent: If $\sup(B)$ exists, it is unique.

(g) Antecedent: A, B, C, and D are sets, f is a function from A to B, g is a function from B to C, and h is a function from C to D.
Consequent: $(h \circ g) \circ f = h \circ (g \circ f)$.

(h) Antecedent: A and B are disjoint finite sets.
Consequent: $A \cup B$ is finite and $\overline{\overline{A \cup B}} = \overline{\overline{A}} + \overline{\overline{B}}$.

5. (a) true (b) false (c) true (d) true
(e) true (f) true (g) true (h) false

6. (a) true (b) true (c) true (d) true
(e) false (f) true (g) false (The symbol for helium is He.)
(h) true (i) false (j) true (k) false

7. (a)

P	Q	$Q \wedge P$	$P \Rightarrow (Q \wedge P)$
T	T	T	T
F	T	F	T
T	F	F	F
F	F	F	T

(b)

P	Q	$\sim P$	$\sim P \Rightarrow Q$	$Q \Leftrightarrow P$	$(\sim P \Rightarrow Q) \vee (Q \Leftrightarrow P)$
T	T	F	T	T	T
F	T	T	T	F	T
T	F	F	T	F	T
F	F	T	F	T	T

(c)

P	Q	$\sim Q$	$Q \Leftrightarrow P$	$\sim Q \Rightarrow (Q \Leftrightarrow P)$
T	T	F	T	T
F	T	F	F	T
T	F	T	F	F
F	F	T	T	T

(d)

P	Q	$P \vee Q$	$P \wedge Q$	$(P \vee Q) \Rightarrow (P \wedge Q)$
T	T	T	T	T
F	T	T	F	F
T	F	T	F	F
F	F	F	F	T

(e)

P	Q	R	$P \wedge Q$	$Q \wedge R$	$P \vee R$	$(P \wedge Q) \vee (Q \wedge R)$	$(P \wedge Q) \vee (Q \wedge R) \Rightarrow P \vee R$
T	T	T	T	T	T	T	T
F	T	T	F	T	T	T	T
T	F	T	F	F	T	F	T
F	F	T	F	F	T	F	T
T	T	F	T	F	T	T	T
F	T	F	F	F	F	F	T
T	F	F	F	F	T	F	T
F	F	F	F	F	F	F	T

(f)

P	Q	R	S	$Q \Rightarrow S$	$Q \Rightarrow R$	$P \vee Q$	$S \vee R$	$(Q \Rightarrow S) \wedge (Q \Rightarrow R)$
T	T	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T	T
F	F	T	T	T	T	F	T	T
T	T	F	T	T	F	T	T	F
F	T	F	T	T	F	T	T	F
T	F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T	T
T	T	T	F	F	T	T	T	F
F	T	T	F	F	T	T	T	F
T	F	T	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
T	T	F	F	F	F	T	F	F
F	T	F	F	F	F	T	F	F
T	F	F	F	T	T	T	F	T
F	F	F	F	T	T	F	F	T

P	Q	R	S	$(P \vee Q) \Rightarrow (S \vee R)$	$[(Q \Rightarrow S) \wedge (Q \Rightarrow R)] \Rightarrow [(P \vee Q) \Rightarrow (S \vee R)]$
T	T	T	T	T	T
F	T	T	T	T	T
T	F	T	T	T	T
F	F	T	T	T	T
T	T	F	T	T	T
F	T	F	T	T	T
T	F	F	T	T	T
F	F	F	T	T	T
T	T	T	F	T	T
F	T	T	F	T	T
T	F	T	F	T	T
F	F	T	F	T	T
T	T	F	F	F	T
F	T	F	F	F	T
T	F	F	F	F	F
F	F	F	F	T	T

8. (a)

P	Q	$\sim P$	$P \Rightarrow Q$	$(\sim P) \vee Q$
T	T	F	T	T
F	T	T	T	T
T	F	F	F	F
F	F	T	T	T

Since the fourth and fifth columns are the same, the propositions $P \Rightarrow Q$ and $(\sim P) \vee Q$ are equivalent.

(b)

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	T	T	T	T	T
F	T	T	F	F	F
T	F	F	T	F	F
F	F	T	T	T	T

Since the fifth and sixth columns are the same, the propositions $P \Leftrightarrow Q$ and $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ are equivalent.

(c)

P	Q	$\sim Q$	$P \Rightarrow Q$	$\sim (P \Rightarrow Q)$	$P \wedge \sim Q$
T	T	F	T	F	F
F	T	F	T	F	F
T	F	T	F	T	T
F	F	T	T	F	F

Since the fifth and sixth columns are the same, the propositions $\sim (P \Rightarrow Q)$ and $P \wedge \sim Q$ are equivalent.

(d)

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$P \Rightarrow \sim Q$	$P \Rightarrow \sim Q$
T	T	F	F	T	F	F	F
F	T	T	F	F	T	T	T
T	F	F	T	F	T	T	T
F	F	T	T	F	T	T	T

Since the sixth, seventh and eighth columns are the same, all three propositions are equivalent.

(e)

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$
T	T	T	T	T	T	T
F	T	T	T	T	F	T
T	F	T	T	T	F	T
F	F	T	T	T	F	T
T	T	F	F	F	T	F
F	T	F	F	T	F	T
T	F	F	T	T	F	T
F	F	F	T	T	F	T

Since the fifth and seventh columns are the same, the propositions are equivalent.

(f)

P	Q	R	$Q \wedge R$	$P \Rightarrow (Q \wedge R)$	$P \Rightarrow Q$	$P \Rightarrow R$	$(P \Rightarrow Q) \wedge (P \Rightarrow R)$
T	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	T	F	F	F	T	F
F	F	T	F	T	T	T	T
T	T	F	F	F	T	F	F
F	T	F	F	T	T	T	T
T	F	F	F	F	F	F	F
F	F	F	F	T	T	T	T

Since the fifth and eighth columns are the same, the propositions are equivalent.

(g)

P	Q	R	$P \vee Q$	$P \vee Q) \Rightarrow R$	$P \Rightarrow R$	$Q \Rightarrow R$	$(P \Rightarrow R) \wedge (Q \Rightarrow R)$
T	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T
F	F	T	F	T	T	T	T
T	T	F	T	F	F	F	F
F	T	F	T	F	T	F	F
T	F	F	T	F	F	T	F
F	F	F	F	T	T	T	T

Since the fifth and seventh columns are the same, the propositions are equivalent.

9. (a) yes (b) no (c) yes
(d) yes (e) no (f) no

10. (a) $[(f$ has a relative minimum at $x_0) \wedge (f$ is differentiable at $x_0)] \Rightarrow (f'(x_0) = 0)$

(b) (n is prime) $\Rightarrow [(n = 2) \vee (n$ is odd$)]$

(c) (x is irrational) $\Rightarrow [(x$ is real$) \wedge \sim (x$ is rational$)]$

(d) $[(x = 1) \vee (x = -1)] \Rightarrow (|x| = 1)$

(e) (x_0 is a critical point for f) $\Leftrightarrow [(f'(x_0) = 0) \vee (f'(x_0)$ does not exist$)]$

(f) (S is compact) $\Leftrightarrow [(S$ is closed$) \wedge (S$ is bounded$)]$

(g) (B is invertible) $\Leftrightarrow$ (det B$\neq$0)

(h) $(6 \geq n - 3) \Rightarrow (n > 4) \vee (n > 10)$

(i) (x is Cauchy) $\Rightarrow$ (x is convergent)

(j) $(\lim_{x \to x_0} f(x) = f(x_0)) \Rightarrow$ (f is continuous at x_0)

(k) [(f is differentiable at x_0) $\wedge$ (f is strictly increasing at x_0)] $\Rightarrow (f'(x_0))$

11. (a) Let S be "I go to the store" and R be "It rains." The preferred translation: is $\sim S \Rightarrow R$ (or, equivalently, $\sim R \Rightarrow S$). This could be read as "If it doesn't rain, then I go to the store."

The speaker might mean "I go to the store if and only if it doesn't rain ($S \Rightarrow \sim R$) or possibly "If it rains, then I don't go to the store" ($R \Rightarrow \sim S$).

(b) There are three nonequivalent ways to translate the sentence, using the symbols D: "The Dolphins make the playoffs" and B: "The Bears win all the rest of their games." The first translation is preferred, but the speaker may have intended any of the three.

$\sim B \Rightarrow \sim D$ or, equivalently, $D \Rightarrow B$
$\sim D \Rightarrow \sim B$ or, equivalently, $B \Rightarrow D$
$\sim B \Leftrightarrow \sim D$ or, equivalently, $B \Leftrightarrow D$

(c) Let G be "You can go to the game" and H be "You do your homework first."

It is most likely that a student and parent both interpret this statement as a biconditional, $G \Leftrightarrow H$.

(d) Let W be "You win the lottery" and T be "You buy a ticket." Of the three common interpretations for the word "unless," only the form $\sim T \Rightarrow \sim W$ (or, equivalently, $W \Rightarrow T$) makes sense here.

12. (a)

P	Q	R	$P \vee Q$	$(P \vee Q) \Rightarrow R$	$\sim P \wedge \sim Q$	$\sim R \Rightarrow (\sim P \wedge \sim Q)$
T	T	T	T	T	F	T
F	T	T	T	T	F	T
T	F	T	T	T	F	T
F	F	T	F	T	T	T
T	T	F	T	F	F	F
F	T	F	T	F	F	F
T	F	F	T	F	F	F
F	F	F	F	T	T	T

Since the fifth and seventh columns are the same, $(P \vee Q) \Rightarrow R$ and $\sim R \Rightarrow (\sim P \wedge \sim Q)$ are equivalent.

(b)

P	Q	R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$	$\sim Q$	$\sim R$	$P \wedge \sim R$	$(P \wedge \sim R) \Rightarrow \sim Q$
T	T	T	T	T	F	F	F	T
F	T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	F	T
T	T	F	T	F	F	T	T	F
F	T	F	F	T	F	T	F	T
T	F	F	F	T	T	T	T	T
F	F	F	F	T	T	T	F	T

Since the fifth and ninth columns are the same, the propositions $(P \wedge Q) \Rightarrow R$ and $(P \wedge \sim R) \Rightarrow \sim Q$ are equivalent.

(c)

P	Q	R	$Q \wedge R$	$P \Rightarrow (Q \wedge R)$	$\sim Q \vee \sim R$	$(\sim Q \vee \sim R) \Rightarrow \sim P$
T	T	T	T	T	F	T
F	T	T	T	T	F	T
T	F	T	F	F	T	F
F	F	T	F	T	T	T
T	T	F	F	F	T	F
F	T	F	F	T	T	T
T	F	F	F	F	T	F
F	F	F	F	T	T	T

Since the fifth and seventh columns are the same, the propositions $P \Rightarrow (Q \wedge R)$ and $(\sim Q \vee \sim R) \Rightarrow \sim P$ are equivalent.

(d)

P	Q	R	$Q \vee R$	$P \Rightarrow (Q \vee R)$	$P \wedge \sim R$	$(P \wedge \sim R) \Rightarrow Q$
T	T	T	T	T	F	T
F	T	T	T	T	F	T
T	F	T	T	T	F	T
F	F	T	T	T	F	T
T	T	F	T	T	T	T
F	T	F	T	T	F	T
T	F	F	F	F	T	F
F	F	F	F	T	F	T

Since the fifth and seventh columns are the same, the propositions $P \Rightarrow (Q \vee R)$ and $(P \wedge \sim R) \Rightarrow Q$ are equivalent.

(e)

P	Q	R	$P \Rightarrow Q$	$(P \Rightarrow Q) \Rightarrow R$	$P \wedge \sim Q$	$(P \wedge \sim Q) \vee R$
T	T	T	T	T	F	T
F	T	T	T	T	F	T
T	F	T	F	T	T	T
F	F	T	T	T	F	T
T	T	F	T	F	F	F
F	T	F	T	F	F	F
T	F	F	F	T	T	T
F	F	F	T	F	F	F

Since the fifth and seventh columns are the same, the propositions $(P \Rightarrow Q) \Rightarrow R$ and $(P \wedge \sim Q) \vee R$ are equivalent.

(f)

P	Q	$P \Leftrightarrow Q$	$\sim P \vee Q$	$\sim Q \vee P$	$(\sim P \vee Q) \wedge (\sim Q \vee P)$
T	T	T	T	T	T
F	T	F	T	F	F
T	F	F	F	T	F
F	F	T	T	T	T

Since the third and sixth columns are the same, the propositions $P \Leftrightarrow Q$ and $(\sim P \vee Q) \wedge (\sim Q \vee P)$ are equivalent.

13. (a) If 6 is an even integer, then 7 is an odd integer.
    (b) If 6 is an odd integer, then 7 is an odd integer.
    (c) This is not possible.
    (d) If 6 is an even integer, then 7 is an even integer. (Any true conditional statement will work here.)

14. (a) If 7 is an odd integer, then 6 is an odd integer.
    (b) This is not possible.

(c) This is not possible.

(d) If 7 is an odd integer, then 6 is an odd integer. (Any false conditional statement will work here.)

15. (a) Converse: If $f'(x_0) = 0$, then f has a relative minimum at x_0 and is differentiable at x_0. False: $f(x) = x^3$ has first derivative 0 but no minimum at $x_0 = 0$.

Contrapositive: If $f'(x_0) \neq 0$, then f either has no relative minimum at x_0 or is not differentiable at x_0. True.

(b) Converse: If $n = 2$ or n is odd, then n is prime. False: 9 is odd but not prime.

Contrapositive: If n is even and not equal to 2, then n is not prime. True.

(c) Converse: If x is irrational, then x is real and not rational. True

Contrapositive: If x is not irrational, then x is not real or x is rational. True

(d) Converse: If $|x| = 1$, then $x = 1$ or $x = -1$. True.

Contrapositive: If $|x| \neq 1$, then $x \neq 1$ and $x \neq -1$. True.

16. (a) tautology (b) tautology (c) contradiction
(d) neither (e) tautology (f) neither
(g) contradiction (h) tautology (i) contradiction
(j) neither (k) tautology (l) neither

17. (a)

P	Q	$P \Rightarrow Q$	$\sim P$	$\sim Q$	$\sim P \Rightarrow \sim Q$
T	T	T	F	F	T
F	T	T	T	F	F
T	F	F	F	T	T
F	F	T	T	T	T

Comparison of the third and sixth columns of the truth table shows that $P \Rightarrow Q$ and $\sim P \Rightarrow \sim Q$ are not equivalent.

(b) We see from the truth table in part (a) that both propositions $P \Rightarrow Q$ and $\sim P \Rightarrow \sim Q$ are true only when P and Q have the same truth value.

(c) The converse of $P \Rightarrow Q$ is $Q \Rightarrow P$. The contrapositive of the inverse of $P \Rightarrow Q$ is $\sim\sim Q \Rightarrow \sim\sim P$, so the converse and the contrapositive of the inverse are equivalent.

The inverse of the contrapositive of $P \Rightarrow Q$ is also $\sim\sim Q \Rightarrow \sim\sim P$, so it too is equivalent to the converse.

1.3 Quantifiers

1. (a) $\sim (\forall x)(x$ is precious$\Rightarrow x$ is beautiful) or $(\exists\, x)(x$ is precious and x is not beautiful)

(b) $(\forall x)(x$ is precious$\Rightarrow x$ is not beautiful)

(c) $(\exists\, x)(x$ is isosceles$\wedge x$ is a right triangle)

(d) $(\forall x)(x$ is a right triangle$\Rightarrow x$ is not isosceles) or $\sim (\exists\, x)(x$ is a right triangle$\wedge x$ is isosceles)

(e) $(\forall x)(x$ is honest$)\vee \sim (\exists\, x)(x$ is honest)

(f) $(\exists\, x)(x$ is honest$) \wedge (\exists\, x)(x$ is not honest)

(g) $(\forall x)(x \neq 0 \Rightarrow (x > 0 \vee x < 0))$

(h) $(\forall x)(x$ is an integer $\Rightarrow (x > -4 \vee x < 6))$ or $(\forall x \in \mathbf{Z})(x > -4 \vee x < 6)$

(i) $(\forall x)(\exists y)(x > y)$

(j) $(\forall x)(\exists y)(x < y)$

(k) $(\forall x)(\forall y)[(x$ is an integer $\wedge y$ is an integer $\wedge y > x) \Rightarrow (\exists z)(y > z > x)]$ or $(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})[y > x \Rightarrow (\exists z)(y > z > x)]$

(l) $(\exists\, x)(x$ is a positive integer and x is smaller than all other positive integers) or $(\exists\, x)(x$ is a positive integer and $(\forall y)(y$ is a positive integer $\Rightarrow x \leq y))$ or $(\exists x \in \mathbb{Z})[x > 0 \wedge (\forall y \in \mathbb{Z})(y > 0 \Rightarrow y > x)]$

(m) $(\forall x)(\sim (\forall y)(x$ loves $y))$ or $\sim (\exists\, x)(\forall y)(x$ loves $y)$ or $(\forall x)(\exists\, y)(x$ does not love $y)$

(n) $(\forall x)(\exists\, y)(x$ loves $y)$

(o) $(\forall x)(x > 0 \Rightarrow (\exists! y)(2^y = x)$

2. (a) $(\forall x)(x$ is precious$\Rightarrow x$ is beautiful)

All precious stones are beautiful.

(b) $(\exists\, x)(x$ is precious$\wedge x$ is beautiful)

There is a beautiful precious stone, or Some precious stones are beautiful.

(c) $\sim (\exists\, x)(x$ is isosceles and x is a right triangle) or $(\forall x)(x$ is not isosceles or x is not a right triangle) or $(\forall x)(x$ is right triangle$\Rightarrow x$ is not isosceles) or $(\forall x)(x$ is isosceles$\Rightarrow x$ is not a right triangle).

There is no isosceles right triangle.

(d) $(\exists\, x)(x$ is isosceles$\wedge x$ is a right triangle)

There is an isosceles right triangle.

(e) $(\exists\, x)(x$ is dishonest$) \wedge (\exists\, x)(x$ is dishonest)

Some people are honest and some people are dishonest.

(f) $(\forall x)(x$ is honest$) \vee (\forall x)(x$ is dishonest)

All people are honest or no one is honest.

(g) $(\exists x)(x \neq 0 \wedge x$ is not positive $\wedge x$ is not negative)

There is a nonzero real number that is neither positive nor negative.

(h) $(\exists x)(x$ is an integer $\wedge x \leq -4 \wedge x \geq 6))$ or $(\exists x \in \mathbb{Z})(x \leq -4 \wedge x \geq 6)$

There is an integer that is less than or equal to -4 and greater than or equal to 6.

(i) $(\exists x)(\forall y)(x \leq y)$

Some integer is less than or equal to every integer, or There is a smallest integer.

(j) $(\exists x)(\forall y)(x \geq y)$

Some integer is greater than every other integer, or There is a largest integer.

(k) $(\exists x)(\exists y)[x$ is an integer $\wedge y$ is an integer $y > x \wedge (\forall z)(z \leq y \vee x \leq z)]$ or $(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})[y > x \wedge (\forall z)(z \leq y \vee x \leq z)]$

There is an integer x and a larger integer y such that there is no real number between them.

(l) $(\forall x)(x$ is a positive integer$\Rightarrow$ $(\exists y)(y$ is a positive integer$) \wedge x > y)$ or $(\forall x \in \mathbb{Z})[x \leq 0 \vee (\exists y \in \mathbb{Z})(y > 0 \wedge x > y)]$. For every positive integer there is a smaller positive integer.

Or, $\sim (\exists x)(x$ is a positive integer$\wedge(\forall y)(y$ is a positive integer$\Rightarrow x \leq y))$ or $\sim (\exists x \in \mathbb{Z})[x > 0 \wedge (\forall y \in \mathbb{Z}(y > 0 \Rightarrow y > x)]$

There is no smallest positive integer.

(m) $(\exists\ x)(\forall y)(x \text{ loves } y)$
There is someone who loves everyone.

(n) $(\exists\ x)(\forall y)(x \text{ does not loves } y)$ or $\sim (\forall x)(\exists\ y)(x \text{ loves } y)$.
Somebody doesn't love anyone.

(o) $(\exists\ x)(x > 0 \wedge \sim (\exists y)(2^y = x) \vee (\exists y)(\exists z)[y \neq z \wedge 2^y = x \wedge 2^z = x])$
There is a positive real number x for which there is no unique real number y such that $2^y = x$.
There is a nonzero complex number such that either every product of that number with any complex number is different from π, or there are at least two different complex numbers whose products with the given number are equal to π.

3. (a) $(\exists k)(k \text{ is an integer } \wedge x = 2k)$ or $(\exists k \in \mathbb{Z})(x = 2k)$
   (b) $(\exists j)(j \text{ is an integer } \wedge x = 2j + 1)$ or $(\exists j \in \mathbb{Z})(x = 2j + 1)$
   (c) $(\exists k)(k \text{ is an integer } \wedge b = ak)$ or $(\exists k \in \mathbb{Z})(b = ak)$
   (d) $n \neq 1 \wedge (\forall m \in \mathbb{Z})(m \text{ divides } n \Rightarrow (m = 1 \vee m = n)$
   (e) $n \neq 1 \wedge (\exists m \in \mathbb{Z})(m \text{ divides } n \wedge (m \neq 1 \vee m \neq n)$

4. (a) $(\forall x, y \in A)(xRy \Rightarrow yRx)$
   (b) $(\forall x, y, z \in A)(xRy \wedge yRz \Rightarrow xRz)$
   (c) $(\forall x, y \in A)(f(x) = f(y) \Rightarrow x = y)$
   (d) $(\forall x, y \in A)(x \cdot y = y \cdot x)$

5. The first interpretation may be translated as
$(\forall x)[x \text{ is a person} \Rightarrow (\forall y)(y \text{ is a tax} \Rightarrow x \text{ dislikes } y)]$.
The other sentences may be translated as
$(\forall x)[x \text{ is a person} \Rightarrow (\exists y)(y \text{ is a tax} \Rightarrow x \text{ dislikes } y)]$.
$(\exists x)[x \text{ is a person} \Rightarrow (\forall y)(y \text{ is a tax} \Rightarrow x \text{ dislikes } y)]$.
$(\exists x)[x \text{ is a person} \Rightarrow (\exists y)(y \text{ is a tax} \Rightarrow x \text{ dislikes } y)]$.

6. (a) T, U, V and W (b) T (c) T, U, V (d) T

7. (a) ***Proof.*** Let U be any universe. The sentences $\sim (\exists x)A(x)$ is true in U
iff $(\exists x)A(x)$ is false in U
iff the truth set for $A(x)$ is empty
iff the truth set for $\sim A(x)$ is U iff $(\forall x) \sim A(x)$ is true in U.

   (b) Let $A(x)$ be an open sentence with variable x. Then $\sim A(x)$ is an open sentence with variable x, so we may apply part (a) of Theorem 1.3.1(b). Thus $\sim (\forall x) \sim A(x)$ is equivalent to $(\exists\ x) \sim\sim A(x)$, which is equivalent to $(\exists\ x)A(x)$. Therefore $\sim (\exists\ x)A(x)$ is equivalent to $\sim\sim (\forall x) \sim A(x)$, which is equivalent to $(\forall x) \sim A(x)$.

8. (a) false (b) true (c) false (d) true
   (e) false (f) true (g) false (h) true
   (i) true (j) false (k) false (l) true

9. (a) Every natural number is greater than or equal to 1.
   (b) Exactly one real number is both nonnegative and nonpositive.
   (c) Every natural number that is prime and different from 2 is odd.
   (d) There is exactly one real number whose natural logarithm is 1.