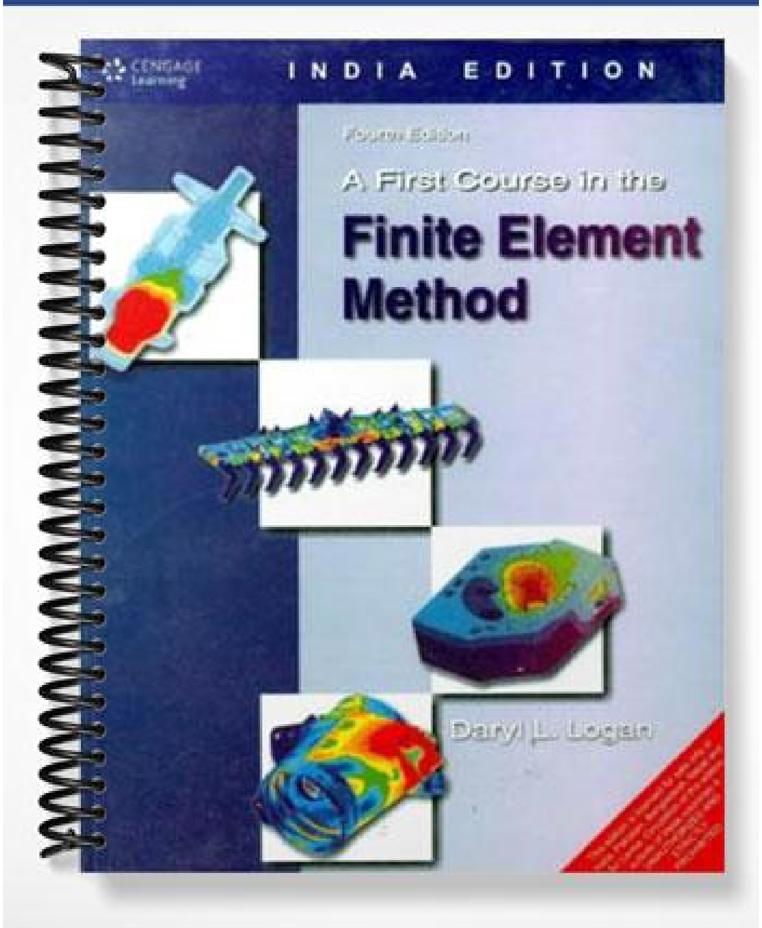
SOLUTIONS MANUAL



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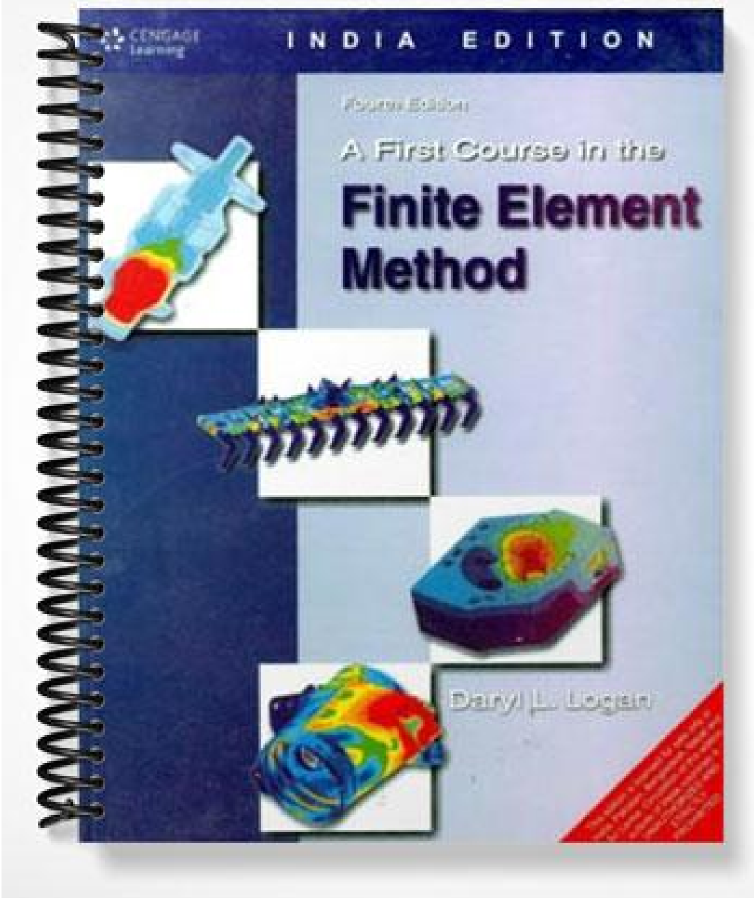
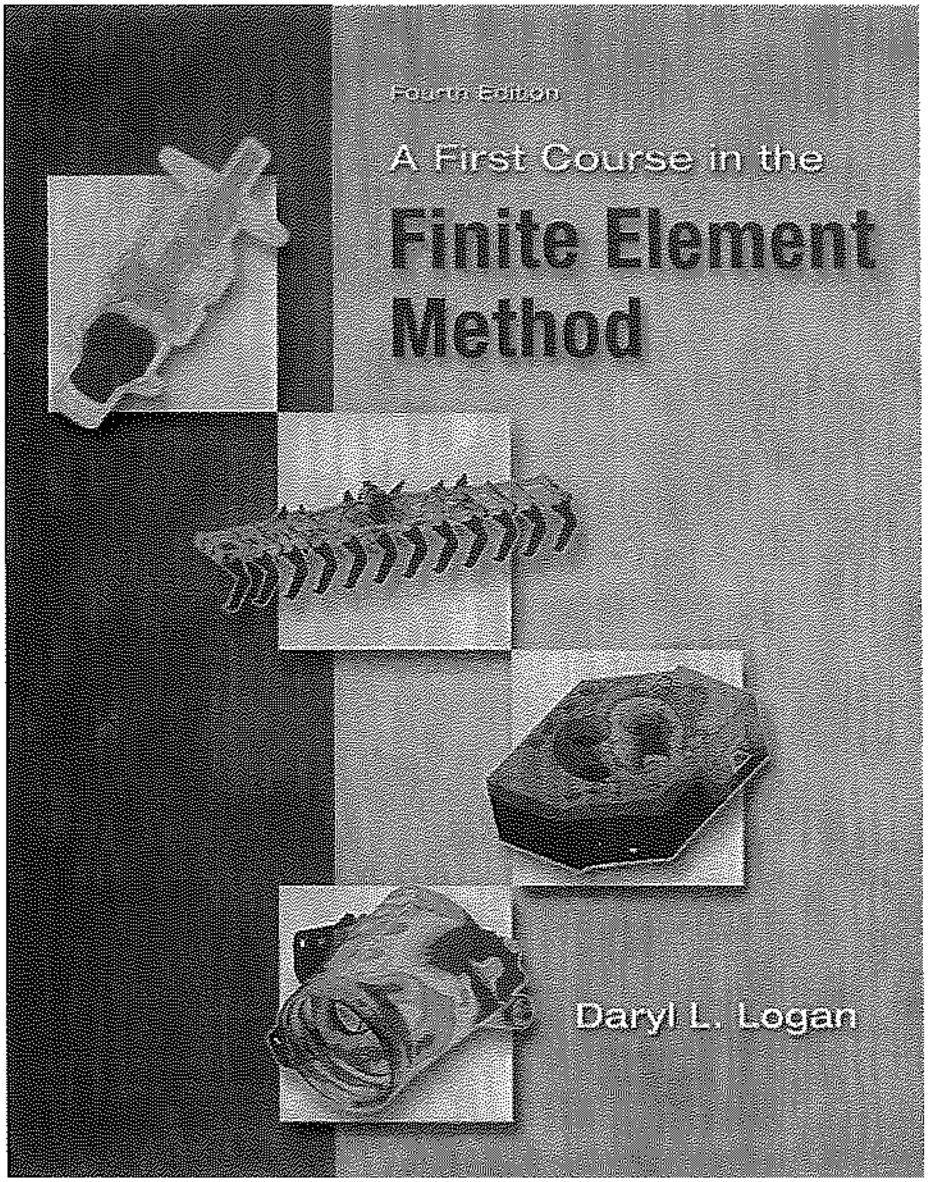


INSTRUCTOR'S SOLUTIONS MANUAL

to accompany



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INSTRUCTOR'S SOLUTIONS MANUAL FOR

A First Course in the
Finite Element Method

Fourth Edition

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Contents

Chapter 1

1. A finite element is a small body or unit interconnected to other units to model a larger structure or system.

2. Discretization means dividing the body (system) into an equivalent system of finite elements with associated nodes and elements.

3. The modern development of the finite element method began in 1941 with the work of Hrennikoff in the field of structural engineering.

4. The direct stiffness method was introduced in 1941 by Hrennikoff. However, it was not commonly known as the direct stiffness method until 1956.

5. A matrix is a rectangular array of quantities arranged in rows and columns that is often used to aid in expressing and solving a system of algebraic equations.

6. As computer developed it made possible to solve thousands of equations in a matter of minutes.

7. The following are the general steps of the finite element method:

   Step 1:
   Divide the body into an equivalent system of finite elements with associated nodes and choose the most appropriate element type.
   Step 2:
   Choose a displacement function within each element.
   Step 3:
   Relate the stresses to the strains through the stress/strain law-generally called the constitutive law.
   Step 4:
   Derive the element stiffness matrix and equations. Use the direct equilibrium method, a work or energy method, or a method of weighted residuals to relate the nodal forces to nodal displacements.
   Step 5:
   Assemble the element equations to obtain the global or total equations and introduce boundary conditions.
   Step 6:
   Solve for the unknown degrees of freedom (or generalized displacements).

   Step 7:
   Solve for the element strains and stresses.
   Step 8:
   Interpret and analyze the results for use in the design/analysis process.

8. The displacement method assumes displacements of the nodes as the unknowns of the problem. The problem is formulated such that a set of simultaneous equations is solved for nodal displacements.

9. Four common types of elements are: simple line elements, simple two-dimensional elements, simple three-dimensional elements, and simple axisymmetric elements.

10. Three common methods used to derive the element stiffness matrix and equations are
    1) direct equilibrium method
    2) work or energy methods
    3) methods of weighted residuals

11. The term "degrees of freedom" refers to rotations and displacements that are associated with each node.

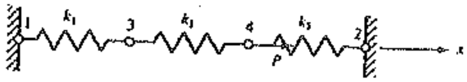
12. Five typical areas where the finite element is applied are:
    1) Structural/stress analysis
    2) Heat transfer analysis
    3) Fluid flow analysis
    4) Electric or magnetic potential distribution analysis
    5) Biomechanical engineering

13. Five advantages of the finite element method are the ability to:
    1) Model irregularly shaped bodies quite easily
    2) Handle general load conditions without difficulty
    3) Model bodies composed of several different materials because element equations are evaluated individually
    4) Handle unlimited numbers and kinds of boundary conditions
    5) Vary the size of the elements to make it possible to use small elements where necessary

Chapter. 2

2.1

a)



$$\underset{\sim}{k}^{(1)} = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & 0 & 0 & 0 \\ -k_1 & 0 & k_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad ; \quad \underset{\sim}{k}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k_2 & -k_2 \\ 0 & 0 & -k_2 & k_2 \end{bmatrix}$$

$$\underset{\sim}{k}_3^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_3 & 0 & -k_3 \\ 0 & 0 & 0 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix}$$

$$\underset{\sim}{K} = \underset{\sim}{k}^{(1)} + \underset{\sim}{k}^{(2)} + \underset{\sim}{k}^{(3)}$$

$$\underset{\sim}{k} = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_3 & 0 & -k_3 \\ -k_1 & 0 & k_1+k_2 & -k_2 \\ 0 & -k_3 & -k_2 & k_2+k_3 \end{bmatrix}$$

b) Nodes 1 & 2 are fixed so $d_{1x} = 0$ & $d_{2x} = 0$ and $\underset{\sim}{k}$ becomes

$$\underset{\sim}{k} = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}$$

$$\underset{\sim}{F} = \underset{\sim}{k} \cdot \underset{\sim}{d}$$

$$\begin{Bmatrix} F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} d_{3x} \\ d_{4x} \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} 0 \\ P \end{Bmatrix} = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} d_{3x} \\ d_{4x} \end{Bmatrix}$$

$$\underset{\sim}{F} = \underset{\sim}{k} \cdot \underset{\sim}{d} \Rightarrow \underset{\sim}{k}^{-1} \cdot \underset{\sim}{F} = \underset{\sim}{k}^{-1} \cdot \underset{\sim}{k} \cdot \underset{\sim}{d}$$

$$\Rightarrow \underset{\sim}{k}^{-1} \cdot \underset{\sim}{F} = \underset{\sim}{d}$$

Using the adjoint method to find $\underset{\sim}{k}^{-1} \Rightarrow$

$c_{11} = k_2 + k_3 \qquad c_{21} = (-1)^3 \cdot (-k_2)$

$c_{12} = (-1)^{1+2}(-k_2) = k_2 \qquad c_{22} = k_1 + k_2$

$$\underset{\sim}{c} = \begin{bmatrix} k_2+k_3 & k_2 \\ k_2 & k_1+k_2 \end{bmatrix} \text{ and } c^T = \begin{bmatrix} k_2+k_3 & k_2 \\ k_2 & k_1+k_2 \end{bmatrix}$$

$$\det \underset{\sim}{k} = |\underset{\sim}{k}| = (k_1+k_2)(k_2+k_3) - (-k_2)(-k_2)$$

$$\Rightarrow |k| = (k_1+k_2)(k_2+k_3) - k_2^2$$

CONT.

2.1 CONT.

$$\underset{\sim}{k}^{-1} = \frac{\underset{\sim}{c}^T}{\det k}$$

$$\underset{\sim}{k}^{-1} = \frac{\begin{bmatrix} k_2+k_3 & k_2 \\ k_2 & k_1+k_2 \end{bmatrix}}{(k_1+k_2)(k_2+k_3) - k_2^2} = \frac{\begin{bmatrix} k_2+k_3 & k_2 \\ k_2 & k_1+k_2 \end{bmatrix}}{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3}$$

$$\begin{Bmatrix} d_{3x} \\ d_{4x} \end{Bmatrix} = \frac{\begin{bmatrix} k_2+k_3 & k_2 \\ k_2 & k_1+k_2 \end{bmatrix} \begin{Bmatrix} 0 \\ P \end{Bmatrix}}{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3}$$

$$\Rightarrow d_{3x} = \frac{k_2 \cdot P}{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3}$$

$$d_{4x} = \frac{(k_1 + k_2) \cdot P}{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3}$$

c) In order to find the reaction forces we go back to the global matrix $\underset{\sim}{F} = \underset{\sim}{k} \cdot \underset{\sim}{d}$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_3 & 0 & -k_3 \\ -k_1 & 0 & k_1+k_2 & -k_2 \\ 0 & -k_3 & -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix}$$

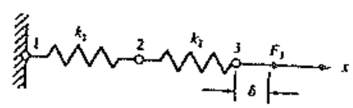
$$F_{1x} = -k_1 \cdot d_{3x} = -k_1 \cdot \frac{k_2 \cdot P}{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3}$$

$$\Rightarrow F_{1x} = \frac{-k_1 \cdot k_2 \cdot P}{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3}$$

$$F_{2x} = -k_3 \cdot d_{4x} = -k_3 \cdot \frac{(k_1 + k_2) \cdot P}{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3}$$

$$\Rightarrow F_{2x} = \frac{-k_3 \cdot (k_1 + k_2) \cdot P}{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3}$$

2.2



$k_1 = k_2 = k_3 = 1000$ lb/in

$$\underset{\sim}{k}^{(1)} = \begin{array}{cc} \begin{array}{cc} ① & ② \end{array} & \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} & \begin{array}{c} ① \\ ② \end{array} \end{array} \quad ; \quad \underset{\sim}{k}^{(2)} = \begin{array}{cc} \begin{array}{cc} ② & ③ \end{array} & \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} & \begin{array}{c} ② \\ ③ \end{array} \end{array}$$

By the method of superposition the global stiffness matrix is constructed

CONT.

2.2 CONT.

$$\underline{K} = \begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{bmatrix} k & -k & 0 \\ -k & k+k & -k \\ 0 & -k & k \end{bmatrix} \end{array} \begin{array}{c} ① \\ ② \\ ③ \end{array} \Rightarrow \underline{K} = \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}$$

Node 1 is fixed $\Rightarrow d_{1x} = 0$ and $d_{3x} = \delta$

$\underline{F} = \underline{k} \cdot \underline{d}$

$$\begin{Bmatrix} \cancel{F_{1x} = ?} \\ F_{2x} = 0 \\ F_{3x} = ? \end{Bmatrix} = \begin{bmatrix} \cancel{k \quad -k \quad 0} \\ \cancel{-k} \quad 2k \quad -k \\ \cancel{0} \quad -k \quad k \end{bmatrix} \begin{Bmatrix} \cancel{d_{1x} = 0} \\ d_{2x} = ? \\ d_{3x} = \delta \end{Bmatrix} \Rightarrow$$

$$\begin{Bmatrix} 0 \\ F_{3x} \end{Bmatrix} = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} d_{2x} \\ \delta \end{Bmatrix} \Rightarrow \begin{Bmatrix} 0 = 2k\, d_{2x} - k\delta \\ F_{3x} = -k\, d_{2x} + k\delta \end{Bmatrix}$$

$$\Rightarrow d_{2x} = \frac{k\delta}{2k} = \frac{\delta}{2} = \frac{1 \text{ inch}}{2} \Rightarrow d_{2x} = 0.5''$$

$$F_{3x} = -k\,(0.5'') + k\,(1'')$$

$$F_{3x} = (-1000 \text{ lb/in})(0.5'') + (1000 \text{ lb/in})(1'')$$

$$\Rightarrow F_{3x} = 500 \text{ lbs}$$

INTERNAL FORCES

Element ①

$$\begin{Bmatrix} \hat{f}_{1x}^{(1)} \\ \hat{f}_{2x}^{(1)} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} \hat{d}_{1x} = 0 \\ \hat{d}_{2x} = 0.5'' \end{Bmatrix}$$

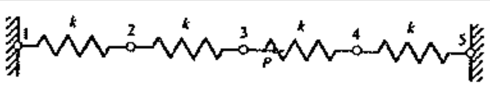
$$\Rightarrow \hat{f}_{1x}^{(1)} = (-1000 \text{ lb/in})(0.5'') \Rightarrow \hat{f}_{1x}^{(1)} = -500 \text{ lb}$$

$$\hat{f}_{2x}^{(1)} = (1000 \text{ lb/in})(0.5'') \Rightarrow \hat{f}_{2x}^{(1)} = 500 \text{ lb}$$

Element ②

$$\begin{Bmatrix} \hat{f}_{2x}^{(2)} \\ \hat{f}_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} d_{2x} = 0.5'' \\ d_{3x} = 1'' \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{2x}^{(2)} = -500 \text{ lb} \\ \hat{f}_{3x}^{(2)} = 500 \text{ lb} \end{matrix}$$

2.3 ⇒



a) $\underline{k}^{(1)} = \underline{k}^{(2)} = \underline{k}^{(3)} = \underline{k}^{(4)} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$

By the method of superposition we construct the global $\underline{K}$ and knowing $\underline{F} = \underline{k} \cdot \underline{d}$ we have :

CONT.

2.3 CONT

$$\begin{Bmatrix} \cancel{F_{1x} = ?} \\ F_{2x} = 0 \\ F_{3x} = P \\ F_{4x} = 0 \\ \cancel{F_{5x} = ?} \end{Bmatrix} = \begin{bmatrix} \cancel{k \quad -k \quad 0 \quad 0 \quad 0} \\ \cancel{-k} \quad 2k \quad -k \quad 0 \quad \cancel{0} \\ \cancel{0} \quad -k \quad 2k \quad -k \quad \cancel{0} \\ \cancel{0} \quad 0 \quad -k \quad 2k \quad \cancel{-k} \\ \cancel{0 \quad 0 \quad 0 \quad -k \quad k} \end{bmatrix} \begin{Bmatrix} \cancel{d_{1x} = 0} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ \cancel{d_{5x} = 0} \end{Bmatrix}$$

b)
$$\begin{Bmatrix} 0 \\ P \\ 0 \end{Bmatrix} = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix} \Rightarrow \begin{matrix} 0 = 2k d_{2x} - k d_{3x} \\ P = -k d_{2x} + 2k d_{3x} - k d_{4x} \\ 0 = -k d_{3x} + 2k d_{4x} \end{matrix}$$

$$\Rightarrow d_{2x} = \frac{d_{3x}}{2} \quad ; \quad d_{4x} = \frac{d_{3x}}{2}$$

Substituting in the equation in the middle

$$P = -k\, d_{2x} + 2k\, d_{3x} - k\, d_{4x}$$

$$\Rightarrow P = -k\left(\frac{d_{3x}}{2}\right) + 2k\, d_{3x} - k\left(\frac{d_{3x}}{2}\right)$$

$$\Rightarrow P = k \cdot d_{3x}$$

$$\Rightarrow d_{3x} = \frac{P}{k}$$

$$d_{2x} = \frac{P}{2k} \quad ; \quad d_{4x} = \frac{P}{2k}$$

c) In order to find the reactions at the fixed nodes 1 & 5 we go back to the global equation $\underline{F} = \underline{k} \cdot \underline{d}$

$$F_{1x} = -k \cdot d_{2x} = -k \cdot \frac{P}{2k} \Rightarrow F_{1x} = -\frac{P}{2}$$

$$F_{5x} = -k \cdot d_{4x} = -k \cdot \frac{P}{2k} \Rightarrow F_{5x} = -\frac{P}{2}$$

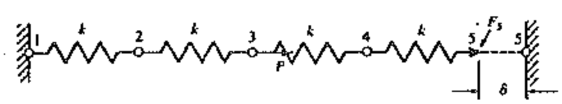
CHECK

$$\sum F_x = 0 \Rightarrow F_{1x} + F_{5x} + P = 0$$

$$\Rightarrow -\frac{P}{2} + \left(-\frac{P}{2}\right) + P = 0$$

$$\Rightarrow 0 = 0 \quad \underline{\underline{OK}}$$

2.4



a) $\underline{k}^{(1)} = \underline{k}^{(2)} = \underline{k}^{(3)} = \underline{k}^{(4)} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$

By the method of superposition the global $\underline{K}$ is constructed

CONT.

2.4 CONT

Also $\underline{F} = \underline{k} \cdot \underline{d}$ and $d_{1x} = 0$ & $d_{5x} = \delta$.

$$\begin{Bmatrix} \cancel{F_{1x} = ?} \\ F_{2x} = 0 \\ F_{3x} = 0 \\ F_{4x} = 0 \\ \cancel{F_{5x} = ?} \end{Bmatrix} = \begin{bmatrix} \cancel{k} & \cancel{-k} & \cancel{0} & \cancel{0} & \cancel{0} \\ -k & 2k & -k & 0 & 0 \\ 0 & -k & 2k & -k & 0 \\ 0 & 0 & -k & 2k & -k \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-k} & \cancel{k} \end{bmatrix} \begin{Bmatrix} \cancel{d_{1x} = 0} \\ d_{2x} = ? \\ d_{3x} = ? \\ d_{4x} = ? \\ \cancel{d_{5x} = \delta} \end{Bmatrix}$$

b) $0 = 2k\, d_{2x} - k\, d_{3x} \cdots (1)$

$0 = -k\, d_{2x} + 2k\, d_{3x} - k\, d_{4x} \cdots (2)$

$0 = -k\, d_{3x} + 2k\, d_{4x} - k \cdot \delta \cdots (3)$

From (1) $d_{3x} = 2 \cdot d_{2x}$

From (3) $d_{4x} = \dfrac{\delta + 2 \cdot d_{2x}}{2}$

Substituting in equation (2) $\Rightarrow$

$-k\,(d_{2x}) + 2k\,(2 \cdot d_{2x}) - k\left(\dfrac{\delta + 2 \cdot d_{2x}}{2}\right)$

$\Rightarrow -d_{2x} + 4\, d_{2x} - d_{2x} - \dfrac{\delta}{2} = 0 \Rightarrow d_{2x} = \dfrac{\delta}{4}$

$\Rightarrow d_{3x} = 2 \cdot \dfrac{\delta}{4} \Rightarrow d_{3x} = \dfrac{\delta}{2}$

$\Rightarrow d_{4x} = \dfrac{\delta + 2 \cdot \frac{\delta}{4}}{2} \Rightarrow d_{4x} = \dfrac{3 \cdot \delta}{4}$

c) Going back to the global equation

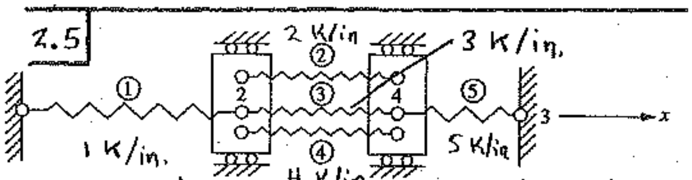
$\underline{F} = \underline{K} \cdot \underline{d}$

$F_{1x} = -k \cdot d_{2x} = -k \cdot \dfrac{\delta}{4} \Rightarrow F_{1x} = -\dfrac{k \cdot \delta}{4}$

$F_{5x} = -k\, d_{4x} + k \cdot \delta = -k\,\dfrac{(3 \cdot \delta)}{4} + k \cdot \delta$

$\Rightarrow F_{5x} = \dfrac{k \cdot \delta}{4}$

2.5



$$\underline{k}^{(1)} = \begin{array}{cc} d_1 & d_2 \\ \left[\begin{matrix} 1 & -1 \\ -1 & 1 \end{matrix}\right. & \left.\right] \end{array}, \quad \underline{k}^{(2)} = \begin{array}{cc} d_2 & d_4 \\ \left[\begin{matrix} 2 & -2 \\ -2 & 2 \end{matrix}\right. & \left.\right] \end{array}$$

$$\underline{k}^{(3)} = \begin{array}{cc} d_2 & d_4 \\ \left[\begin{matrix} 3 & -3 \\ -3 & 3 \end{matrix}\right. & \left.\right] \end{array}, \quad \underline{k}^{(4)} = \begin{array}{cc} d_2 & d_4 \\ \left[\begin{matrix} 4 & -4 \\ -4 & 4 \end{matrix}\right. & \left.\right] \end{array}$$

$$\underline{k}^{(5)} = \begin{array}{cc} d_4 & d_3 \\ \left[\begin{matrix} 5 & -5 \\ -5 & 5 \end{matrix}\right. & \left.\right] \end{array}$$

cont.

2.5 cont.

Assembling Global $\underline{K}$ using direct stiffness method:

$$\underline{K} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1+2+3+4 & 0 & -2-3-4 \\ 0 & 0 & 5 & -5 \\ 0 & -2-3-4 & -5 & 2+3+4+5 \end{bmatrix}$$

Simplifying

$$\underline{K} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 10 & 0 & -9 \\ 0 & 0 & 5 & -5 \\ 0 & -9 & -5 & 14 \end{bmatrix} \frac{\text{Kip}}{\text{in.}}$$

2.6 Now apply +2 kip at node 2 in spring assemblage of P 2.5.

$\therefore F_{2x} = 2$ kip

$[K]\{d\} = \{F\}$

$[K]$ from P 2.5

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 10 & 0 & -9 \\ 0 & 0 & 5 & -5 \\ 0 & -9 & -5 & 14 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} \\ d_{3x} = 0 \\ d_{4x} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 2 \\ F_3 \\ 0 \end{Bmatrix} \quad (A)$$

where $d_{1x} = 0$, $d_{3x} = 0$ as nodes 1 and 3 are fixed.

cont.

2.6 cont.

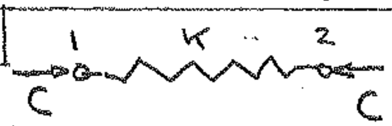
using eqs (1) & (3) of (A)

$$\begin{bmatrix} 10 & -9 \\ -9 & 14 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{4x} \end{Bmatrix} = \begin{Bmatrix} 2 \\ 0 \end{Bmatrix}$$

Solving

$d_{2x} = 0.475$ in., $d_{4x} = 0.305$ in. ◄

2.7



$+f_{1x}$  $+f_{2x}$

+ nodal force conv.

$f_{1x} = C \;,\; f_{2x} = -C$

$f = -k\,\delta = -k(d_{2x} - d_{1x})$

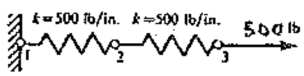
$\therefore f_{1x} = -k(d_{2x} - d_{1x})$

$f_{2x} = -(-k)(d_{2x} - d_{1x})$

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix}$$

$\therefore \underline{k} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$ ◄ Same as for tensile element

2.8



$\underline{k}_1 = 500 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$; $\underline{k}_2 = 500 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

So $\underline{K} = 500 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

$\underline{F} = \underline{K} \cdot \underline{d} \Rightarrow$

CONT.

2.8 CONT

$$\begin{Bmatrix} F_1 = ? \\ F_2 = 0 \\ F_3 = 1000 \end{Bmatrix} = 500 \begin{bmatrix} \cancel{1} & \cancel{-1} & \cancel{0} \\ \cancel{-1} & 2 & -1 \\ \cancel{0} & -1 & 1 \end{bmatrix} \begin{Bmatrix} \cancel{d_{1x} = 0} \\ d_{2x} = ? \\ d_{3x} = ? \end{Bmatrix}$$

$\Rightarrow 0 = 1000\, d_{2x} - 500\, d_{3x} \quad \cdots (1)$

$500 = -500\, d_{2x} + 500\, d_{3x} \quad \cdots (2)$

From (1) $d_{2x} = \frac{500}{1000} d_{3x} \Rightarrow d_{2x} = 0.5\, d_{3x}$ ↳ (3)

Substituting (3) into (2)

$\Rightarrow 500 = -500(0.5\, d_{3x}) + 500\, d_{3x}$

$\Rightarrow 500 = 250\, d_{3x}$

$\Rightarrow d_{3x} = 2$ in. ◄

$\Rightarrow d_{2x} = (0.5)(2 \text{ in}) \Rightarrow d_{2x} = 1$ in. ◄

Element 1-2

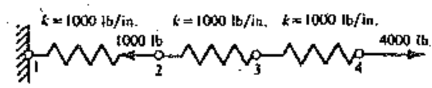
$$\begin{Bmatrix} \hat{f}_{1x}^{(1)} \\ \hat{f}_{2x}^{(1)} \end{Bmatrix} = 500 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \text{ in} \\ 1 \text{ in} \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{1x}^{(1)} = -500 \text{ lb} \\ \hat{f}_{2x}^{(1)} = 500 \text{ lb} \end{matrix}$$

Element 2-3

$$\begin{Bmatrix} \hat{f}_{2x}^{(2)} \\ \hat{f}_{3x}^{(2)} \end{Bmatrix} = 500 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 1 \text{ in} \\ 2 \text{ in} \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{2x}^{(2)} = -500 \text{ lb} \\ \hat{f}_{3x}^{(2)} = 500 \text{ lb} \end{matrix}$$

$$F_{1x} = 500 \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \text{ in} \\ 2 \text{ in} \end{Bmatrix} \Rightarrow F_{1x} = -500 \text{ lb}$$

2.9



$$\underline{k}^{(1)} = \begin{array}{cc} ① & ② \\ \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \end{array} \quad \underline{k}^{(2)} = \begin{array}{cc} ② & ③ \\ \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \end{array}$$

$$\underline{k}^{(3)} = \begin{array}{cc} ③ & ④ \\ \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \end{array}$$

$$\underline{K} = \begin{array}{cccc} ① & ② & ③ & ④ \\ \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -1000 & 0 \\ 0 & -1000 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \end{array}$$

CONT.

2.9 CONT

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = -1000 \\ F_{3x} = 0 \\ F_{4x} = 4000 \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -1000 & 0 \\ 0 & -1000 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix}$$

$$\Rightarrow \quad d_{1x} = 0 \text{ in}$$
$$d_{2x} = 3 \text{ in}$$
$$d_{3x} = 7 \text{ in}$$
$$d_{4x} = 11 \text{ in}$$

Reactions

$$F_{1x} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} = 3 \\ d_{3x} = 7 \\ d_{4x} = 11 \end{Bmatrix} \Rightarrow F_{1x} = -3000 \text{ lb}$$

Element Forces

Element ①

$$\begin{Bmatrix} \hat{f}_{1x}^{(1)} \\ \hat{f}_{2x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ 3 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{1x}^{(1)} = -3000 \text{ lb} \\ \hat{f}_{2x}^{(1)} = 3000 \text{ lb} \end{matrix}$$

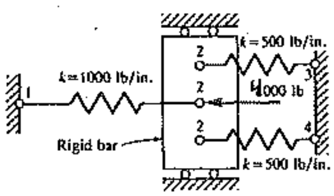
Element ②

$$\begin{Bmatrix} \hat{f}_{2x}^{(2)} \\ \hat{f}_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 3 \\ 7 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{2x}^{(2)} = -4000 \text{ lb} \\ \hat{f}_{3x}^{(2)} = 4000 \text{ lb} \end{matrix}$$

Element ③

$$\begin{Bmatrix} \hat{f}_{3x}^{(3)} \\ \hat{f}_{4x}^{(3)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 7 \\ 11 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{3x}^{(3)} = -4000 \text{ lb} \\ \hat{f}_{4x}^{(3)} = 4000 \text{ lb} \end{matrix}$$

2.10



$$\underset{\sim}{k}^{(1)} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \quad \underset{\sim}{k}^{(2)} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}$$

$$\underset{\sim}{k}^{(3)} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}$$

$$\underset{\sim}{F} = \underset{\sim}{K} \cdot \underset{\sim}{d}$$

CONT.

2.10 CONT

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = -4000 \\ F_{3x} = ? \\ F_{4x} = ? \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} = ? \\ d_{3x} = 0 \\ d_{4x} = 0 \end{Bmatrix}$$

$$\Rightarrow \quad d_{2x} = -\frac{4000}{2000} = -2 \text{ in}$$

Reactions

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ -2 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \quad \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{Bmatrix} 2000 \\ -4000 \\ 1000 \\ 1000 \end{Bmatrix} \text{ lb}$$

Element ①

$$\begin{Bmatrix} \hat{f}_{1x}^{(1)} \\ \hat{f}_{2x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ -2 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \hat{f}_{1x}^{(1)} \\ \hat{f}_{2x}^{(1)} \end{Bmatrix} = \begin{Bmatrix} 2000 \\ -2000 \end{Bmatrix} \text{ lb}$$

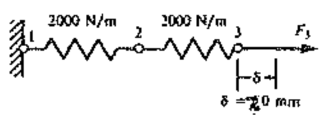
Element ②

$$\begin{Bmatrix} \hat{f}_{2x}^{(2)} \\ \hat{f}_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -2 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \hat{f}_{2x}^{(2)} \\ \hat{f}_{3x}^{(2)} \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 1000 \end{Bmatrix} \text{ lb}$$

Element ③

$$\begin{Bmatrix} \hat{f}_{2x}^{(3)} \\ \hat{f}_{4x}^{(3)} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -2 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \hat{f}_{2x}^{(3)} \\ \hat{f}_{4x}^{(3)} \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 1000 \end{Bmatrix} \text{ lb}$$

2.11



$$\underset{\sim}{k}^{(1)} = \begin{bmatrix} 2000 & -2000 \\ -2000 & 2000 \end{bmatrix} \quad ; \quad \underset{\sim}{k}^{(2)} = \begin{bmatrix} 2000 & -2000 \\ -2000 & 2000 \end{bmatrix}$$

$$\underset{\sim}{F} = \underset{\sim}{K} \cdot \underset{\sim}{d}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 0 \\ F_{3x} = ? \end{Bmatrix} = \begin{bmatrix} 2000 & -2000 & 0 \\ -2000 & 4000 & -2000 \\ 0 & -2000 & 2000 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} = ? \\ d_{3x} = 0.02 \text{ m} \end{Bmatrix}$$

$$\Rightarrow \quad d_{2x} = 0.01 \text{ m}$$

CONT.

2.11 CONT.

Reactions

$F_{1x} = (-2000)(0.01) \Rightarrow F_{1x} = -20\ N$

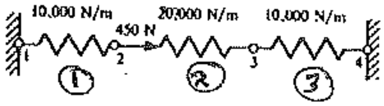
Element ①

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = \begin{bmatrix} 2000 & -2000 \\ -2000 & 2000 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.005 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = \begin{Bmatrix} -20 \\ 20 \end{Bmatrix} N$$

Element ②

$$\begin{Bmatrix} \hat{f}_{2x} \\ \hat{f}_{3x} \end{Bmatrix} = \begin{bmatrix} 2000 & -2000 \\ -2000 & 2000 \end{bmatrix} \begin{Bmatrix} 0.005 \\ 0.010 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \hat{f}_{2x} \\ \hat{f}_{3x} \end{Bmatrix} = \begin{Bmatrix} -20 \\ 20 \end{Bmatrix} N$$

2.12



$$\underline{k}^{(1)} = \underline{k}^{(3)} = 10000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad ; \quad \underline{k}^{(2)} = 10000 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$\underline{F} = \underline{k} \cdot \underline{d}$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 4500\ N \\ F_{3x} = 0 \\ F_{4x} = ? \end{Bmatrix} = 10000 \begin{bmatrix} 3 & -2 \\ -2 & 5 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} = ? \\ d_{3x} = ? \\ d_{4x} = 0 \end{Bmatrix}$$

$0 = -2d_{2x} + 3d_{3x} \Rightarrow d_{2x} = \frac{3}{2} d_{3x} \Rightarrow d_{2x} = 1.5 d_{3x}$

$450\ N = 30000(1.5 d_{3x}) - 20000\ d_{3x}$

$\Rightarrow 450\ N = (25000\ N/m)\ d_{3x} \Rightarrow d_{3x} = 1.8 \times 10^{-2}\ m$

$\Rightarrow d_{2x} = 1.5(1.8 \times 10^{-2}) \Rightarrow d_{2x} = 2.7 \times 10^{-2}\ m$

Element ①

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = 10000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 2.7 \times 10^{-2} \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{1x}^{(1)} = -270\ N \\ \hat{f}_{2x}^{(1)} = 270\ N \end{matrix}$$

Element ②

$$\begin{Bmatrix} \hat{f}_{2x} \\ \hat{f}_{3x} \end{Bmatrix} = 20000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 2.7 \times 10^{-2} \\ 1.8 \times 10^{-2} \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{2x}^{(2)} = 180\ N \\ \hat{f}_{3x}^{(2)} = -180\ N \end{matrix}$$

Element ③

$$\begin{Bmatrix} \hat{f}_{3x} \\ \hat{f}_{4x} \end{Bmatrix} = 10000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 1.8 \times 10^{-2} \\ 0 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{3x}^{(3)} = 180\ N \\ \hat{f}_{4x}^{(3)} = -180\ N \end{matrix}$$

Reactions

$$\{F_{1x}\} = (10000\ N/m)[1 \quad -1] \begin{Bmatrix} 0 \\ 2.7 \times 10^{-2} \end{Bmatrix} \Rightarrow F_{1x} = -270\ N$$

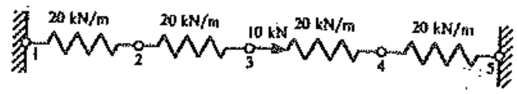
CONT.

2.12 CONT

$$\{F_{4x}\} = (10000\ N/m)[-1 \quad 1] \begin{Bmatrix} 1.8 \times 10^{-2} \\ 0 \end{Bmatrix}$$

$\Rightarrow F_{4x} = -180\ N$

2.13



$$\underline{k}^{(1)} = \underline{k}^{(2)} = \underline{k}^{(3)} = \underline{k}^{(4)} = 20 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$\underline{F} = \underline{k} \cdot \underline{d}$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 0 \\ F_{3x} = 10\ kN \\ F_{4x} = 0 \\ F_{5x} = ? \end{Bmatrix} = 20 \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} = ? \\ d_{3x} = ? \\ d_{4x} = ? \\ d_{5x} = 0 \end{Bmatrix}$$

$$\left. \begin{matrix} 0 = 2d_{2x} - d_{3x} \Rightarrow d_{2x} = 0.5\ d_{3x} \\ 0 = -d_{3x} + 2\ d_{4x} \Rightarrow d_{4x} = 0.5\ d_{3x} \end{matrix} \right\} \Rightarrow d_{2x} = d_{4x}$$

$\Rightarrow 10\ kN = -20\ d_{2x} + 40\ (2\ d_{2x}) - 20\ d_{2x}$

$\Rightarrow 10 = 40\ d_{2x} \Rightarrow d_{2x} = 0.25\ m$

$\Rightarrow d_{4x} = 0.25\ m$

$\Rightarrow d_{3x} = 2(0.25) \Rightarrow d_{3x} = 0.5\ m$

Element ①

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = 20 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.25 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{1x}^{(1)} = -5\ kN \\ \hat{f}_{2x}^{(1)} = 5\ kN \end{matrix}$$

Element ②

$$\begin{Bmatrix} \hat{f}_{2x} \\ \hat{f}_{3x} \end{Bmatrix} = 20 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.25 \\ 0.5 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{2x}^{(2)} = -5\ kN \\ \hat{f}_{3x}^{(2)} = 5\ kN \end{matrix}$$

Element ③

$$\begin{Bmatrix} \hat{f}_{3x} \\ \hat{f}_{4x} \end{Bmatrix} = 20 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0.25 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{3x}^{(3)} = 5\ kN \\ \hat{f}_{4x}^{(3)} = -5\ kN \end{matrix}$$

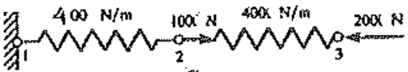
Element ④

$$\begin{Bmatrix} \hat{f}_{4x} \\ \hat{f}_{5x} \end{Bmatrix} = 20 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.25 \\ 0 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{4x}^{(4)} = 5\ kN \\ \hat{f}_{5x}^{(4)} = -5\ kN \end{matrix}$$

$F_{1x} = 20\,[1 \quad -1] \begin{Bmatrix} 0 \\ 0.25 \end{Bmatrix} \Rightarrow F_{1x} = -5\ kN$

$F_{5x} = 20\,[-1 \quad 1] \begin{Bmatrix} 0.25 \\ 0 \end{Bmatrix} \Rightarrow F_{5x} = -5\ kN$

2.14



$k^{(1)} = k^{(2)} = 400 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\underline{F} = \underline{k} \cdot \underline{d}$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 100 \\ F_{3x} = -200 \end{Bmatrix} = 400 \begin{bmatrix} \cancel{1} & \cancel{-1} & \cancel{0} \\ \cancel{-1} & 2 & -1 \\ \cancel{0} & -1 & 1 \end{bmatrix} \begin{Bmatrix} \cancel{d_{1x} = 0} \\ d_{2x} = ? \\ d_{3x} = ? \end{Bmatrix}$$

$$100 = 800\, d_{2x} - 400\, d_{3x}$$
$$-200 = -400\, d_{2x} + 400\, d_{3x}$$
$$-100 = 400\, d_{2x} \Rightarrow d_{2x} = -0.25 \text{ m}$$

$$100 = 800(-0.25) - 400\, d_{3x} \Rightarrow d_{3x} = -0.75 \text{ m}$$

Element ①

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = 400 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ -0.25 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{1x}^{(1)} = 100 \text{ N} \\ \hat{f}_{2x}^{(1)} = -100 \text{ N} \end{matrix}$$

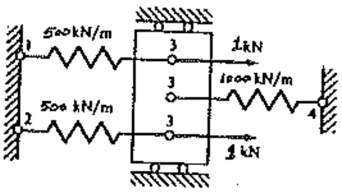
Element ②

$$\begin{Bmatrix} \hat{f}_{2x} \\ \hat{f}_{3x} \end{Bmatrix} = 400 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} -0.25 \\ -0.75 \end{Bmatrix} \Rightarrow \begin{matrix} \hat{f}_{2x}^{(2)} = 200 \text{ N} \\ \hat{f}_{3x}^{(2)} = -200 \text{ N} \end{matrix}$$

Reaction

$$\{F_{1x}\} = 400 \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{Bmatrix} 0 \\ -0.25 \end{Bmatrix} \Rightarrow F_{1x} = 100 \text{ N}$$

2.15



$k^{(1)} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \quad k^{(2)} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \quad k^{(3)} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix}$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = ? \\ F_{3x} = 2 \text{ kN} \\ F_{4x} = ? \end{Bmatrix} = \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} = 0 \\ d_{3x} = ? \\ d_{4x} = 0 \end{Bmatrix}$$

$$\Rightarrow d_{3x} = 0.001 \text{ m}$$

Reactions

$F_{1x} = (-500)(0.001) \Rightarrow F_{1x} = -0.5 \text{ kN}$

$F_{2x} = (-500)(0.001) \Rightarrow F_{2x} = -0.5 \text{ kN}$

$F_{4x} = (-1000)(0.001) \Rightarrow F_{4x} = -1.0 \text{ kN}$

CONT.

2.15 CONT

Element ①

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.001 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{3x} \end{Bmatrix} = \begin{Bmatrix} -0.5 \text{ kN} \\ 0.5 \text{ kN} \end{Bmatrix}$$

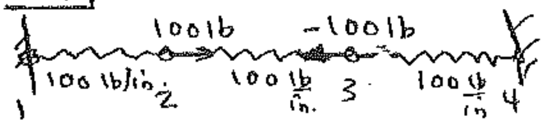
Element ②

$$\begin{Bmatrix} \hat{f}_{2x} \\ \hat{f}_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.001 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \hat{f}_{2x} \\ \hat{f}_{3x} \end{Bmatrix} = \begin{Bmatrix} -0.5 \text{ kN} \\ 0.5 \text{ kN} \end{Bmatrix}$$

Element ③

$$\begin{Bmatrix} \hat{f}_{3x} \\ \hat{f}_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0.001 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \hat{f}_{3x} \\ \hat{f}_{4x} \end{Bmatrix} = \begin{Bmatrix} 1 \text{ kN} \\ -1 \text{ kN} \end{Bmatrix}$$

2.16

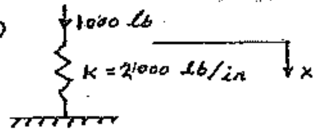


$$\begin{Bmatrix} F_{1x} \\ 100 \\ -100 \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 100+100 & -100 & 0 \\ 0 & -100 & 100+100 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \begin{Bmatrix} 0 \\ d_{2x} \\ d_{3x} \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 100 \\ -100 \end{Bmatrix} = \begin{bmatrix} 200 & -100 \\ -100 & 200 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix}$$

$$d_{2x} = 1/3 \text{ in.}, \quad d_{3x} = -1/3 \text{ in.}$$

2.17 (a)



As in Example 2.4

$\pi_p = U + \Omega$

$U = \frac{1}{2} k x^2 \,, \quad \Omega = -Fx$

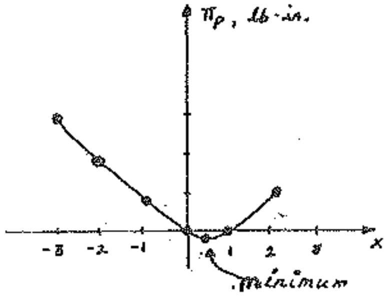
Set up Table

$\pi_p = \frac{1}{2}(2000)x^2 - 1000x = 1000x^2 - 1000x$

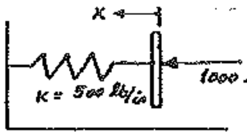
Deformation x, in.	π_p, lb-in.
-3.0	6000
-2.0	3000
-1.0	1000
0.0	0
0.5	-125
1.0	0
2.0	1000

2.17 cont

$\frac{\partial \Pi_p}{\partial x} = 2000x - 1000 = 0 \Rightarrow x = 0.5$ in, yields minimum Π_p as table verifies



(b)



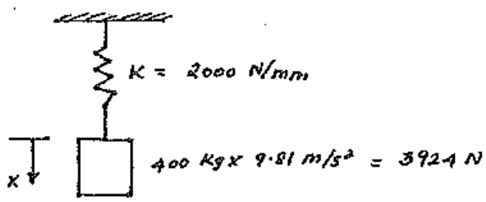
$\Pi_p = \frac{1}{2}kx^2 - Fx = 250x^2 - 1000x$

x, in	Π_p, lb-in
-3.0	11250
-2.0	3000
-1.0	1250
0	0
1.0	-750
2.0	-1000
3.0	-750

$\frac{\partial \Pi_p}{\partial x} = 500x - 1000 = 0 \Rightarrow$

$x = 2.0$ in yields Π_p minimum

(c)



$\Pi_p = \frac{1}{2}(2000)x^2 - 3924x = 1000x^2 - 3924x$

$\frac{\partial \Pi_p}{\partial x} = 2000x - 3924 = 0$

$\Rightarrow x = 1.962$ mm yields Π_p minimum

$\Pi_{p\,min} = \frac{1}{2}(2000)(1.962)^2 - 3924(1.962)$

$\Rightarrow \Pi_{p\,min} = -3849.45$ N·mm

(d) $\Pi_p = \frac{1}{2}(400)x^2 - 981x$

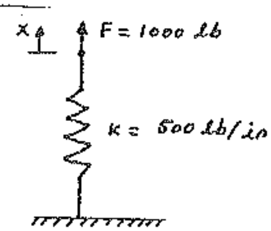
$\frac{\partial \Pi_p}{\partial x} = 400x - 981 = 0$

$\Rightarrow x = 2.4525$ mm yields Π_p minimum

$\Pi_{p\,min} = \frac{1}{2}(400)(2.4525)^2 - 981(2.4525)$

$\Rightarrow \Pi_{p\,min} = -1202.95$ N·mm

2.18



Now let positive x be upward

$\Pi_p = \frac{1}{2}kx^2 - Fx$

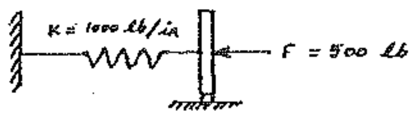
$\Pi_p = \frac{1}{2}(500)x^2 - 1000x$

$\Pi_p = 250x^2 - 1000x$

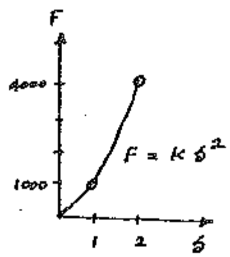
$\frac{\partial \Pi_p}{\partial x} = 500x - 1000 = 0$

$\Rightarrow x = 2.0$ in up

2.19



$F = k\delta^2 \qquad (x = \delta)$



$dU = F\,dx$

$U = \int_0^x (kx^2)\,dx$

$U = \frac{kx^3}{3}$

$\Omega = -Fx$

$\Pi_p = \frac{1}{3}kx^3 - 500x$

$\frac{\partial \Pi_p}{\partial x} = 0 = kx^2 - 500$

$0 = 1000x^2 - 500$

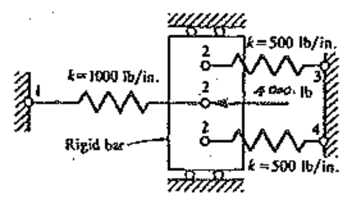
$\Rightarrow x = 0.707$ in (equilibrium value of displacement)

$\Pi_{p\,min} = \frac{1}{3}(1000)(0.707)^3 - 500(0.707)$

$\Pi_{p\,min} = -235.7$ lb·in

2.20

Solve Prob 2.10 using P.E approach



$$\Pi_p = \sum_{e=1}^{3} \Pi_p^{(e)} = \frac{1}{2}k_1(d_{2x} - d_{1x})^2 + \frac{1}{2}k_2(d_{3x} - d_{2x})^2$$
$$+ \frac{1}{2}k_3(d_{4x} - d_{2x})^2$$
$$- f_{1x}^{(1)} d_{1x} - f_{2x}^{(1)} d_{2x} - f_{2x}^{(2)} d_{2x}$$
$$- f_{3x}^{(2)} d_{3x} - f_{2x}^{(3)} d_{2x} - f_{4x}^{(3)} d_{4x}$$

CONT.

2.20 CONT.

$$\frac{\partial \Pi_p}{\partial d_{1x}} = -k_1 d_{2x} + k_1 d_{1x} - f_{1x}^{(1)} = 0 \qquad (1)$$

$$\frac{\partial \Pi_p}{\partial d_{2x}} = k_1 d_{2x} - k_1 d_{1x} - k_2 d_{3x} + k_2 d_{2x} - k_3 d_{4x} + k_3 d_{2x} - f_{2x}^{(1)} - f_{2x}^{(2)} - f_{2x}^{(3)} = 0 \qquad (2)$$

$$\frac{\partial \Pi_p}{\partial d_{3x}} = k_2 d_{3x} - k_2 d_{2x} - f_{3x}^{(2)} = 0 \qquad (3)$$

$$\frac{\partial \Pi_p}{\partial d_{4x}} = k_3 d_{4x} - k_3 d_{2x} - f_{4x}^{(3)} = 0 \qquad (4)$$

In matrix form (1) through (4) become

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2+k_3 & -k_2 & -k_3 \\ 0 & -k_2 & k_2 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix} = \begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} + f_{2x}^{(2)} + f_{2x}^{(3)} \\ f_{3x}^{(2)} \\ f_{4x}^{(3)} \end{Bmatrix} \qquad (5)$$

or using numerical values

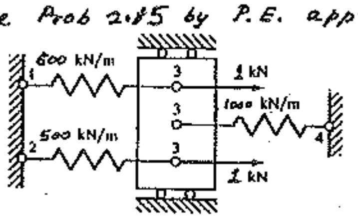
$$\begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} d_{1x} = 0 \\ d_{2x} \\ d_{3x} = 0 \\ d_{4x} = 0 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ -4000 \\ F_{3x} \\ F_{4x} \end{Bmatrix} \qquad (6)$$

Solution now follows as in Prob 2.10

Solve 2nd of Eqns (6) for $d_{2x} = -2$ in.

For reactions and element forces, see solution to Problem 2.10

2.21 Solve Prob 2.15 by P.E. approach



$$\Pi_p = \sum_{e=1}^{3} \pi_p^{(e)} = \tfrac{1}{2} k_1 (d_{3x} - d_{1x})^2 + \tfrac{1}{2} k_2 (d_{3x} - d_{2x})^2 + \tfrac{1}{2} k_3 (d_{4x} - d_{3x})^2 - f_{1x}^{(1)} d_{1x} - f_{3x}^{(1)} d_{3x} - f_{2x}^{(2)} d_{2x} - f_{3x}^{(2)} d_{3x} - f_{3x}^{(3)} d_{3x} - f_{4x}^{(3)} d_{4x}$$

$$\frac{\partial \Pi_p}{\partial d_{1x}} = 0 = -k_1 d_{3x} + k_1 d_{1x} - f_{1x}^{(1)}$$

CONT.

2.21 CONT.

$$\frac{\partial \Pi_p}{\partial d_{2x}} = 0 = -k_2 d_{3x} + k_2 d_{2x} - f_{2x}^{(2)}$$

$$\frac{\partial \Pi_p}{\partial d_{3x}} = 0 = k_1 d_{3x} + k_2 d_{3x} - k_2 d_{2x} - k_3 d_{4x} + k_3 d_{3x} - f_{3x}^{(3)} - f_{3x}^{(2)} - f_{3x}^{(1)} - k_1 d_{1x}$$

$$\frac{\partial \Pi_p}{\partial d_{4x}} = 0 = k_3 d_{4x} - k_3 d_{3x} - f_{4x}^{(3)}$$

In matrix form

$$\begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & -k_2 & 0 \\ -k_1 & -k_2 & k_1+k_2+k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} = 2\text{ kN} \\ F_{4x} \end{Bmatrix}$$

For rest of solution, see solution of Prob 2.15