

SOLUTIONS MANUAL



A BRIEF COURSE IN

Mathematical Statistics



ELLIOT A. TANIS • ROBERT V. HOGG

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**INSTRUCTOR'S
SOLUTIONS MANUAL**

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MATHEMATICAL STATISTICS**

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Contents

Preface	v
1 Probability	1
1.1 Basic Concepts	1
1.2 Methods of Enumeration	2
1.3 Conditional Probability	2
1.4 Independent Events	3
1.5 Bayes's Theorem	4
2 Discrete Distributions	7
2.1 Discrete Probability Distributions	7
2.2 Expectations	10
2.3 Special Discrete Distributions	11
2.4 Estimation	13
2.5 Linear Functions of Independent Random Variables	14
2.6 Multivariate Discrete Distributions	15
3 Continuous Distributions	17
3.1 Descriptive Statistics and EDA	17
3.2 Continuous Probability Distributions	20
3.3 Special Continuous Distributions	24
3.4 The Normal Distribution	25
3.5 Estimation in the Continuous Case	27
3.6 The Central Limit Theorem	28
3.7 Approximations for Discrete Distributions	30
4 Applications of Statistical Inference	33
4.1 Summary of Necessary Theoretical Results	33
4.2 Confidence Intervals using χ^2 , F , and T	33
4.3 Confidence Intervals and Tests of Hypotheses	35
4.4 Basic Tests Concerning One Parameter	35
4.5 Tests of the Equality of Two Parameters	37
4.6 Simple Linear Regression	39
4.7 More on Linear Regression	41
4.8 One-Factor Analysis of Variance	46
4.9 Distribution-Free Confidence and Tolerance Intervals	47
4.10 Chi-Square Goodness of Fit Tests	49
4.11 Contingency Tables	52

5	Computer Oriented Techniques	57
5.1	Computation of Statistics	57
5.2	Computer Algebra Systems	59
5.3	Simulation	62
5.4	Resampling	76
6	Sampling Distribution Theory	85
6.1	Moment-Generating Function Technique	85
6.2	M.G.F. of Linear Functions	86
6.3	Limiting Moment-Generating Functions	88
6.4	Use of Order Statistics in Non-regular Cases	89

Preface

This solutions manual provides answers for the even-numbered exercises in *A Brief Course in Mathematical Statistics* by Elliot A. Tanis and Robert V. Hogg. Complete solutions are given for most of these exercises. You, the instructor, may decide how many of these answers you want to make available to your students. Note that the answers for the odd-numbered exercises are given in the textbook.

All of the figures in this manual were generated using *Maple*, a computer algebra system. Most of the figures were generated and many of the solutions, especially those involving data, were solved using procedures that were written by Zaven Karian from Denison University. We thank him for providing these. These procedures are available free of charge for your use. They are available on a “CD” that can be downloaded from Prentice-Hall. Short descriptions of these procedures are provided on the “Maple Card” on the “CD”. Complete descriptions of these procedures are given in *Probability and Statistics: Explorations with MAPLE*, second edition, 1999, written by Zaven Karian and Elliot Tanis, published by Prentice Hall (ISBN 0-13-021536-8).

Our hope is that this solutions manual will be helpful to each of you in your teaching.

If you find an error or wish to make a suggestion, send these to Elliot Tanis at tanis@hope.edu and he will post corrections on his web page, <http://www.math.hope.edu/tanis/>.

E.A.T.
R.V.H.

Chapter 1

Probability

1.1 Basic Concepts

1.1-2 (a) $O = \{\text{HHHH}, \text{HHHT}, \text{HHTH}, \text{HTHH}, \text{THHH}, \text{HHTT}, \text{HTTH}, \text{TTHH}, \text{HTHT}, \text{THTH}, \text{THHT}, \text{HTTT}, \text{THTT}, \text{TTHT}, \text{T TTH}, \text{TTTT}\};$

(b) (i) $5/16$, (ii) 0 , (iii) $11/16$, (iv) $4/16$, (v) $4/16$, (vi) $9/16$, (vii) $4/16$.

1.1-4 (a) $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$;

$$\begin{aligned} \text{(b)} \quad A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.4 &= P(A \cap B') + 0.3 \\ P(A \cap B) &= 0.1; \end{aligned}$$

(c) $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.3 = 0.7$.

1.1-6 (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.7 = 0.4 + 0.5 - P(A \cap B)$$

$$P(A \cap B) = 0.2;$$

(b) $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B)$

$$= 1 - 0.2$$

$$= 0.8.$$

1.1-8 (a) $O = \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\};$

(b) (i) $1/10$; (ii) $5/10$.

$$1.1-10 \quad P(A) = \frac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \frac{\sqrt{3}}{2}.$$

$$1.1-12 \quad A \cup B \cup C = A \cup (B \cup C)$$

$$P(A \cup B \cup C) = P(A) + P(B \cup C) - P[A \cap (B \cup C)]$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)]$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C)$$

$$+ P(A \cap B \cap C).$$

1.2 Methods of Enumeration

1.2-2 (4)(3)(2) = 24.

1.2-4 (a) (4)(5)(2) = 40; (b) (2)(2)(2) = 8.

1.2-6 $O = \{ \text{FFF, FFRF, FRFF, RFFF, FFRRF, FRFRF, RFFRF, FRRFF, RFRFF, RRFFF, RRR, RRFR, RFRR, FRRR, RRFFR, RFRFR, FRRFR, RFFRR, FRFRR, FFRRR} \}$ so there are 20 possibilities.

1.2-8
$$\binom{n-1}{r} + \binom{n-1}{r-1} = \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!}$$

$$= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}.$$

1.2-10
$$0 = (1-1)^n = \sum_{r=0}^n \binom{n}{r} (-1)^r (1)^{n-r} = \sum_{r=0}^n (-1)^r \binom{n}{r}.$$

$$2^n = (1+1)^n = \sum_{r=0}^n \binom{n}{r} (1)^r (1)^{n-r} = \sum_{r=0}^n \binom{n}{r}.$$

1.3 Conditional Probability

1.3-2 (a) $\frac{1041}{1456}$;

(b) $\frac{392}{633}$;

(c) $\frac{649}{823}$.

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.3-4 (a) $P(\text{HH}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$;

(b) $P(\text{HC}) = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204}$;

(c) $P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$
 $= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{51}{52 \cdot 51} = \frac{1}{52}.$

1.3-6 (a) $\frac{8}{14} \cdot \frac{7}{13} = \frac{56}{182}$;

(b) $\frac{6}{14} \cdot \frac{5}{13} = \frac{30}{182}$;

(c) $2 \left(\frac{8}{14} \cdot \frac{6}{13} \right) = \frac{96}{182}$ or $1 - \left[\frac{56}{182} + \frac{30}{182} \right] = \frac{96}{182}.$

1.3-8
$$\frac{\binom{2}{0} \binom{8}{5}}{\binom{10}{5}} \cdot \frac{2}{5} + \frac{\binom{2}{1} \binom{8}{4}}{\binom{10}{5}} \cdot \frac{1}{5} = \frac{1}{5}.$$

1.3-10 (a) It doesn't matter because $P(B_1) = \frac{1}{18}$, $P(B_5) = \frac{1}{18}$, $P(B_{18}) = \frac{1}{18}$;