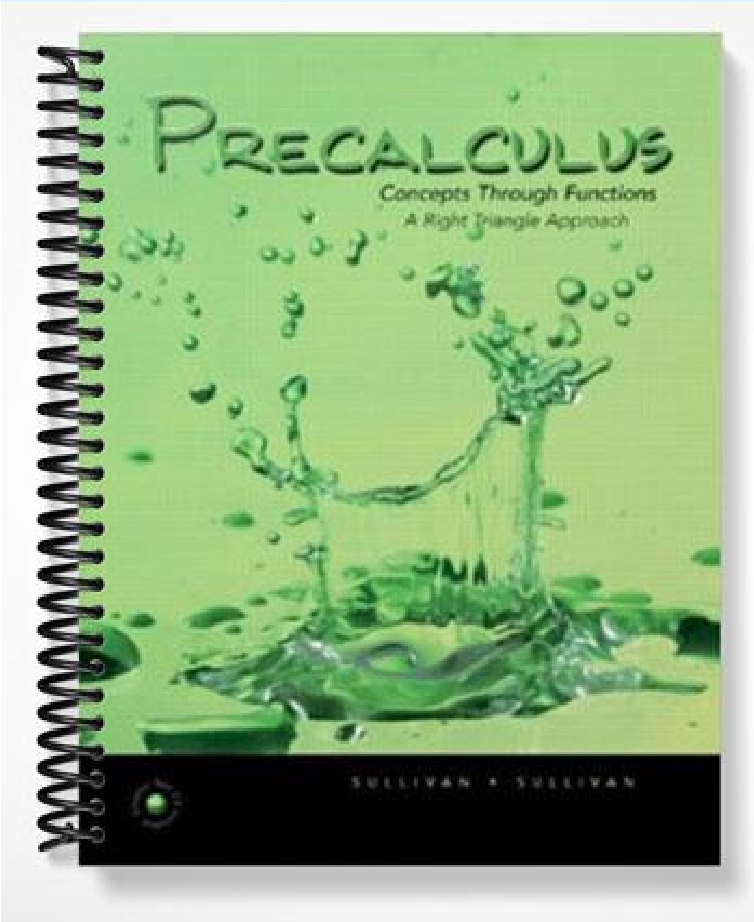
SOLUTIONS MANUAL

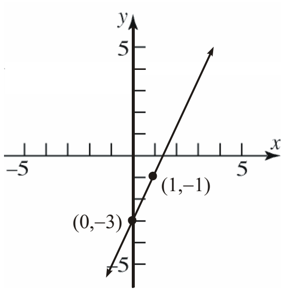


Chapter 2

Linear and Quadratic Functions

Section 2.1

1. From the equation $y = 2x - 3$, we see that the y-intercept is -3. Thus, the point $(0, -3)$ is on the graph. We can obtain a second point by choosing a value for x and finding the corresponding value for y. Let $x = 1$, then $y = 2(1) - 3 = -1$. Thus, the point $(1, -1)$ is also on the graph. Plotting the two points and connecting with a line yields the graph below.



2. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - 5}{-1 - 2} = \dfrac{-2}{-3} = \dfrac{2}{3}$

3. To find the y-intercept, let $x = 0$:
$0^2 + 2y = 4$
$y = 2$
So, the y-intercept is (0, 2).
To find the x-intercept(s), let $y = 0$:
$x^2 + 2(0) = 4$
$x^2 = 4$
$x = \pm 2$
So, the x-intercepts are $(-2, 0)$ and $(2, 0)$.

4. $60x - 900 = -15x + 2850$
$75x - 900 = 2850$
$75x = 3750$
$x = 50$
The solution set is $\{50\}$.

5. $f(-2) = (-2)^2 - 4 = 4 - 4 = 0$

6. True

7. slope; y-intercept

8. -4; 3

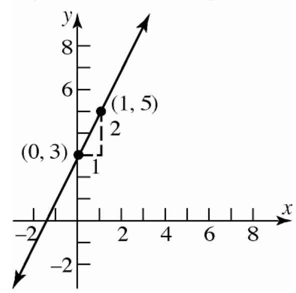
9. positive

10. True

11. False. If x increases by 3, then y increases by 2.

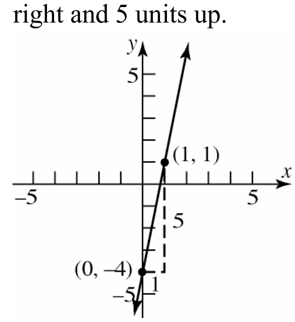
12. False. The y-intercept is 8. The zero is -4.

13. $f(x) = 2x + 3$
    - **a.** Slope = 2; y-intercept = 3
    - **b.** Plot the point (0, 3). Use the slope to find an additional point by moving 1 unit to the right and 2 units up.



    - **c.** average rate of change = 2
    - **d.** increasing

14. $g(x) = 5x - 4$
    - **a.** Slope = 5; y-intercept = -4
    - **b.** Plot the point $(0, -4)$. Use the slope to find an additional point by moving 1 unit to the right and 5 units up.

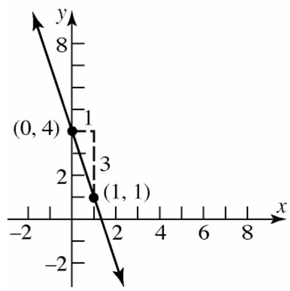


    - **c.** average rate of change = 5
    - **d.** increasing

15. $h(x) = -3x + 4$

a. Slope = -3 ; y-intercept = 4

b. Plot the point (0, 4). Use the slope to find an additional point by moving 1 unit to the right and 3 units down.



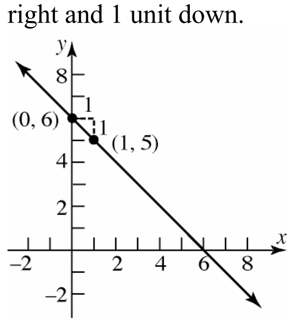
c. average rate of change = -3

d. decreasing

16. $p(x) = -x + 6$

a. Slope = -1 ; y-intercept = 6

b. Plot the point (0, 6). Use the slope to find an additional point by moving 1 unit to the right and 1 unit down.



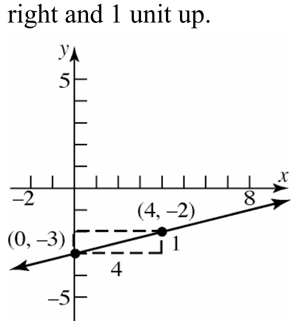
c. average rate of change = -1

d. decreasing

17. $f(x) = \frac{1}{4}x - 3$

a. Slope = $\frac{1}{4}$; y-intercept = -3

b. Plot the point $(0, -3)$. Use the slope to find an additional point by moving 4 units to the right and 1 unit up.



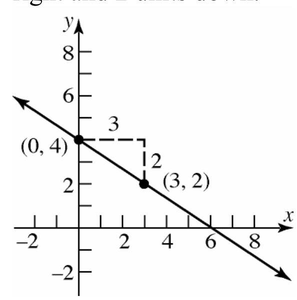
c. average rate of change = $\frac{1}{4}$

d. increasing

18. $h(x) = -\frac{2}{3}x + 4$

a. Slope = $-\frac{2}{3}$; y-intercept = 4

b. Plot the point (0, 4). Use the slope to find an additional point by moving 3 units to the right and 2 units down.



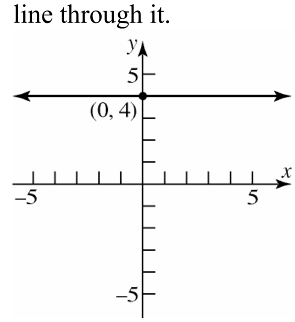
c. average rate of change = $-\frac{2}{3}$

d. decreasing

19. $F(x) = 4$

a. Slope = 0; y-intercept = 4

b. Plot the point (0, 4) and draw a horizontal line through it.



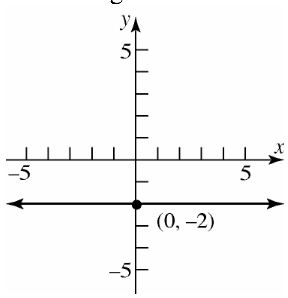
c. average rate of change = 0

d. constant

20. $G(x) = -2$

a. Slope = 0; y-intercept = -2

b. Plot the point $(0, -2)$ and draw a horizontal line through it.



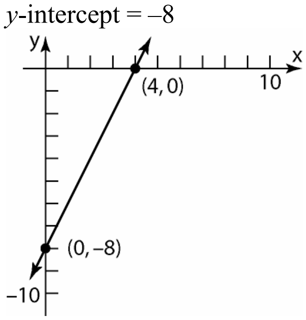
c. average rate of change = 0

d. constant

21. $g(x) = 2x - 8$

a.
$$\begin{aligned} 2x - 8 &= 0 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

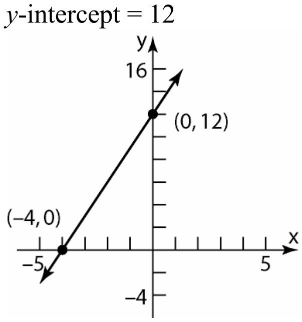
b. y-intercept = -8



22. $g(x) = 3x + 12$

a.
$$\begin{aligned} 3x + 12 &= 0 \\ 3x &= -12 \\ x &= -4 \end{aligned}$$

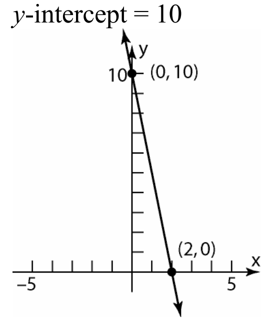
b. y-intercept = 12



23. $f(x) = -5x + 10$

a.
$$\begin{aligned} -5x + 10 &= 0 \\ -5x &= -10 \\ x &= 2 \end{aligned}$$

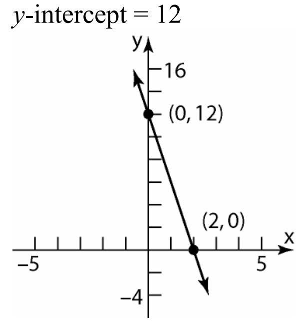
b. y-intercept = 10



24. $f(x) = -6x + 12$

a.
$$\begin{aligned} -6x + 12 &= 0 \\ -6x &= -12 \\ x &= 2 \end{aligned}$$

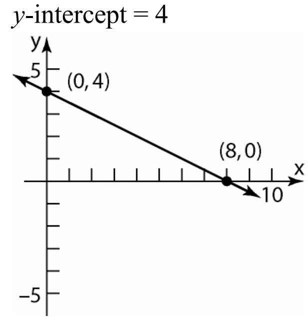
b. y-intercept = 12



25. $H(x) = -\dfrac{1}{2}x + 4$

a.
$$\begin{aligned} -\frac{1}{2}x + 4 &= 0 \\ -\frac{1}{2}x &= -4 \\ x &= 8 \end{aligned}$$

b. y-intercept = 4



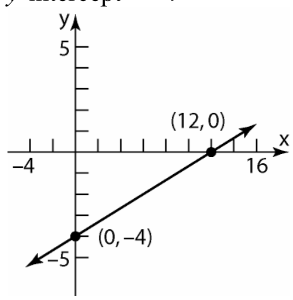
26. $G(x)=\frac{1}{3}x-4$

a. $\frac{1}{3}x-4=0$

$\frac{1}{3}x=4$

$x=12$

b. y-intercept = –4



27.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
–2	4	
–1	1	$\frac{1-4}{-1-(-2)}=\frac{-3}{1}=-3$
0	–2	$\frac{-2-1}{0-(-1)}=\frac{-3}{1}=-3$
1	–5	$\frac{-5-(-2)}{1-0}=\frac{-3}{1}=-3$
2	–8	$\frac{-8-(-5)}{2-1}=\frac{-3}{1}=-3$

This is a linear function with slope = –3, since the average rate of change is constant at –3.

28.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
–2	$\frac{1}{4}$	
–1	$\frac{1}{2}$	$\frac{\left(\frac{1}{2}-\frac{1}{4}\right)}{-1-(-2)}=\frac{\frac{1}{4}}{1}=\frac{1}{4}$
0	1	$\frac{\left(1-\frac{1}{2}\right)}{0-(-1)}=\frac{\frac{1}{2}}{1}=\frac{1}{2}$
1	2	
2	4	

This is not a linear function since the average rate of change is not constant.

29.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
–2	–8	
–1	–3	$\frac{-3-(-8)}{-1-(-2)}=\frac{5}{1}=5$
0	0	$\frac{0-(-3)}{0-(-1)}=\frac{3}{1}=3$
1	1	
2	0	

This is not a linear function, since the average rate of change is not constant.

30.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
–2	–4	
–1	0	$\frac{0-(-4)}{-1-(-2)}=\frac{4}{1}=4$
0	4	$\frac{4-0}{0-(-1)}=\frac{4}{1}=4$
1	8	$\frac{8-4}{1-0}=\frac{4}{1}=4$
2	12	$\frac{12-8}{2-1}=\frac{4}{1}=4$

This is a linear function with slope = 4, since the average rate of change is constant at 4.

31.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
–2	–26	
–1	–4	$\frac{-4-(-26)}{-1-(-2)}=\frac{22}{1}=22$
0	2	$\frac{2-(-4)}{0-(-1)}=\frac{6}{1}=6$
1	–2	
2	–10	

This is not a linear function, since the average rate of change is not constant.

32.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
–2	–4	
–1	–3.5	$\frac{-3.5-(-4)}{-1-(-2)}=\frac{0.5}{1}=0.5$
0	–3	$\frac{-3-(-3.5)}{0-(-1)}=\frac{0.5}{1}=0.5$
1	–2.5	$\frac{-2.5-(-3)}{1-0}=\frac{0.5}{1}=0.5$
2	–2	$\frac{-2-(-2.5)}{2-1}=\frac{0.5}{1}=0.5$

This is a linear function, since the average rate of change is constant at 0.5

33.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
–2	8	
–1	8	$\frac{8-8}{-1-(-2)}=\frac{0}{1}=0$
0	8	$\frac{8-8}{0-(-1)}=\frac{0}{1}=0$
1	8	$\frac{8-8}{1-0}=\frac{0}{1}=0$
2	8	$\frac{8-8}{2-1}=\frac{0}{1}=0$

This is a linear function with slope = 0, since the average rate of change is constant at 0.

34.

x	$y=f(x)$	Avg. rate of change $=\frac{\Delta y}{\Delta x}$
–2	0	
–1	1	$\frac{1-0}{-1-(-2)}=\frac{1}{1}=1$
0	4	$\frac{4-1}{0-(-1)}=\frac{3}{1}=3$
1	9	
2	16	

This is not a linear function, since the average rate of change is not constant.

35. $f(x)=4x-1;\quad g(x)=-2x+5$

a. $f(x)=0$

$4x-1=0$

$x=\frac{1}{4}$

b. $f(x)>0$

$4x-1>0$

$x>\frac{1}{4}$

The solution set is $\left\{x \middle| x>\frac{1}{4}\right\}$ or $\left(\frac{1}{4},\ \infty\right)$.

c. $f(x)=g(x)$

$4x-1=-2x+5$

$6x=6$

$x=1$

d. $f(x)\le g(x)$

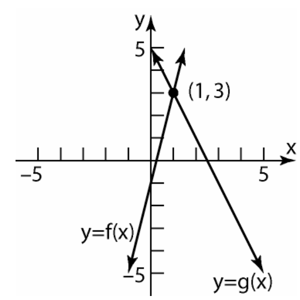
$4x-1\le -2x+5$

$6x\le 6$

$x\le 1$

The solution set is $\{x \mid x\le 1\}$ or $(-\infty,\ 1]$.

e.



36. $f(x)=3x+5;\quad g(x)=-2x+15$

a. $f(x)=0$

$3x+5=0$

$x=-\frac{5}{3}$

b. $f(x)<0$

$3x+5<0$

$x<-\frac{5}{3}$

The solution set is $\left\{x \middle| x<-\frac{5}{3}\right\}$ or $\left(-\infty,-\frac{5}{3}\right)$.

c. $f(x) = g(x)$

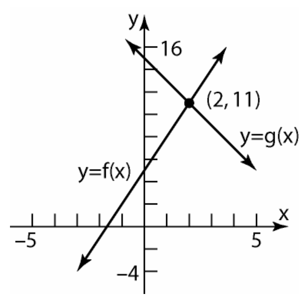
$$\begin{aligned} 3x + 5 &= -2x + 15 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

d. $f(x) \geq g(x)$

$$\begin{aligned} 3x + 5 &\geq -2x + 15 \\ 5x &\geq 10 \\ x &\geq 2 \end{aligned}$$

The solution set is $\{x \mid x \geq 2\}$ or $[2, \infty)$.

e.



37. a. The point (40, 50) is on the graph of $y = f(x)$, so the solution to $f(x) = 50$ is $x = 40$.

b. The point (88, 80) is on the graph of $y = f(x)$, so the solution to $f(x) = 80$ is $x = 88$.

c. The point $(-40, 0)$ is on the graph of $y = f(x)$, so the solution to $f(x) = 0$ is $x = -40$.

d. The *y*-coordinates of the graph of $y = f(x)$ are above 50 when the *x*-coordinates are larger than 40. Thus, the solution to $f(x) > 50$ is $\{x \mid x > 40\}$ or $(40, \infty)$.

e. The *y*-coordinates of the graph of $y = f(x)$ are below 80 when the *x*-coordinates are smaller than 88. Thus, the solution to $f(x) \leq 80$ is $\{x \mid x \leq 88\}$ or $(-\infty, 88]$.

f. The *y*-coordinates of the graph of $y = f(x)$ are between 0 and 80 when the *x*-coordinates are between -40 and 88. Thus, the solution to $0 < f(x) < 80$ is $\{x \mid -40 < x < 88\}$ or $(-40, 88)$.

38. a. The point (5, 20) is on the graph of $y = g(x)$, so the solution to $g(x) = 20$ is $x = 5$.

b. The point $(-15, 60)$ is on the graph of $y = g(x)$, so the solution to $g(x) = 60$ is $x = -15$.

c. The point (15, 0) is on the graph of $y = g(x)$, so the solution to $g(x) = 0$ is $x = 15$.

d. The *y*-coordinates of the graph of $y = g(x)$ are above 20 when the *x*-coordinates are smaller than 5. Thus, the solution to $g(x) > 20$ is $\{x \mid x < 5\}$ or $(-\infty, 5)$.

e. The *y*-coordinates of the graph of $y = f(x)$ are below 60 when the *x*-coordinates are larger than -15. Thus, the solution to $g(x) \leq 60$ is $\{x \mid x \geq -15\}$ or $[-15, \infty)$.

f. The *y*-coordinates of the graph of $y = f(x)$ are between 0 and 60 when the *x*-coordinates are between -15 and 15. Thus, the solution to $0 < f(x) < 60$ is $\{x \mid -15 < x < 15\}$ or $(-15, 15)$.

39. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = -4$.

b. $f(x) \leq g(x)$ when the graph of *f* is above the graph of *g*. Thus, the solution is $\{x \mid x < -4\}$ or $(-\infty, -4)$.

40. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = 2$.

b. $f(x) \leq g(x)$ when the graph of *f* is below or intersects the graph of *g*. Thus, the solution is $\{x \mid x \leq 2\}$ or $(-\infty, 2]$.

41. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = -6$.

b. $g(x) \leq f(x) < h(x)$ when the graph of *f* is above or intersects the graph of *g* and below the graph of *h*. Thus, the solution is $\{x \mid -6 \leq x < 5\}$ or $[-6, 5)$.

42. **a.** $f(x)=g(x)$ when their graphs intersect. Thus, $x=7$.

b. $g(x)\le f(x)<h(x)$ when the graph of *f* is above or intersects the graph of *g* and below the graph of *h*. Thus, the solution is $\{x|-4\le x<7\}$ or $[-4,7)$.

43. $C(x)=0.25x+35$

a. $C(40)=0.25(40)+35=\$45$.

b. Solve $C(x)=0.25x+35=80$

$$0.25x+35=80$$
$$0.25x=45$$
$$x=\frac{45}{0.25}=180 \text{ miles}$$

c. Solve $C(x)=0.25x+35<100$

$$0.25x+35<100$$
$$0.25x<65$$
$$x<\frac{65}{0.25}=260 \text{ miles}$$

44. $C(x)=0.38x+5$

a. $C(50)=0.38(50)+5=\$24$.

b. Solve $C(x)=0.38x+5=29.32$

$$0.38x+5=29.32$$
$$0.38x=24.32$$
$$x=\frac{24.32}{0.38}=64 \text{ minutes}$$

c. Solve $C(x)=0.38x+5\le 60$

$$0.38x+5\le 60$$
$$0.38x\le 55$$
$$x\le\frac{55}{0.38}\approx 144 \text{ minutes}$$

45. $B(t)=19.25t+585.72$

a. $B(10)=19.25(10)+585.72=\$778.22$

b. Solve $B(t)=19.25t+585.72=893.72$

$$19.25t+585.72=893.72$$
$$19.25t=308$$
$$t=\frac{308}{19.25}=16 \text{ years}$$

Therefore, the average monthly benefit will be \$893.72 in the year 2006.

c. Solve $B(t)=19.25t+585.72>1000$

$$19.25t+585.72>1000$$
$$19.25t>414.28$$
$$t>\frac{414.28}{19.25}\approx 21.52 \text{ years}$$

Therefore, the average monthly benefit will exceed \$1000 in the year 2012.

46. $E(t)=26t+411$

a. $E(10)=26(10)+411=\$671$ billion.

b. Solve $E(t)=26t+411=879$

$$26t+411=879$$
$$26t=468$$
$$t=\frac{468}{26}=18 \text{ years}$$

Therefore, the total private expenditure will be \$879 billion in the year 2008.

c. Solve $E(t)=26t+411>1000$

$$26t+411>1000$$
$$26t>589$$
$$t>\frac{589}{26}\approx 22.65 \text{ years}$$

Therefore, the total private expenditure will exceed \$1 trillion in the year 2013.

47. $S(p)=-200+50p$; $D(p)=1000-25p$

a. Solve $S(p)=D(p)$.

$$-200+50p=1000-25p$$
$$75p=1200$$
$$p=\frac{1200}{75}=16$$
$$S(16)=-200+50(16)=600$$

Thus, the equilibrium price is \$16, and the equilibrium quantity is 600 T-shirts.

b. Solve $D(p)>S(p)$.

$$1000-25p>-200+50p$$
$$1200>75p$$
$$\frac{1200}{75}>p$$
$$16>p$$

The demand will exceed supply when the price is less than \$16 (but still greater than \$0).

c. The price will eventually be increased.

48. $S(p) = -2000 + 3000p;\ D(p) = 10000 - 1000p$

a. Solve $S(p) = D(p)$.

$$-2000 + 3000p = 10000 - 1000p$$
$$4000p = 12000$$
$$p = \frac{12000}{4000} = 3$$
$$S(3) = -2000 + 3000(3) = 7000$$

Thus, the equilibrium price is \$3, and the equilibrium quantity is 7000 hot dogs.

b. Solve $D(p) < S(p) \Rightarrow 10000 - 1000p < -2000 + 3000p$

$$10000 - 1000p < -2000 + 3000p$$
$$12000 < 4000p$$
$$\frac{12000}{4000} < p$$
$$3 < p$$

The demand will be less than the supply when the price is greater than \$3.

c. The price will eventually be decreased.

49. a. We are told that the tax function T is for adjusted gross incomes x between \$7,300 and \$29,700, inclusive. Thus, the domain is $\{x \mid 7{,}300 \le x \le 29{,}700\}$ or $[7300,\ 29700]$.

b. $T(18000) = 0.15(18000 - 7300) + 730 = 2335$

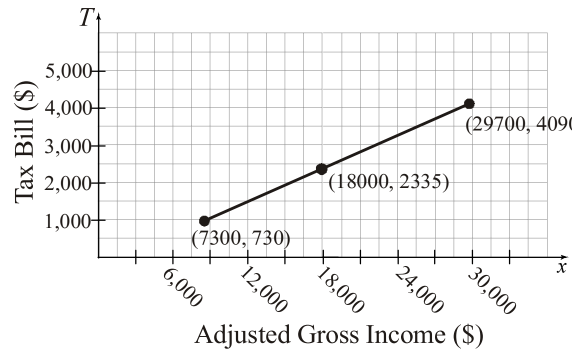
If a single filer's adjusted gross income is \$18,000, this his or her tax bill will be \$2,335 .

c. The independent variable is adjusted gross income, x. The dependent variable is the tax bill, T.

d. Evaluate T at $x = 7300,\ 18000,\ \text{and}\ 29700$.

$$T(7300) = 0.15(7300 - 7300) + 730 = 730$$
$$T(18000) = 0.15(18000 - 7300) + 730 = 2335$$
$$T(29700) = 0.15(29700 - 7300) + 730 = 4090$$

Thus, the points $(7300, 730)$, $(18000, 2335)$, and $(29700, 4090)$ are on the graph.



e. We must solve $T(x) = 2860$.

$$0.15(x - 7300) + 730 = 2860$$
$$0.15x - 1095 + 730 = 2860$$
$$0.15x - 365 = 2860$$
$$0.15x = 3225$$
$$x = 21{,}500$$

A single filer with an adjusted gross income of \$21,500 will have a tax bill of \$2860.

50. a. The independent variable is payroll, p. The payroll tax only applies if the payroll is \$128 million or more. Thus, the domain of T is $\{p \mid p \ge 128\}$ or $[128,\ \infty)$.

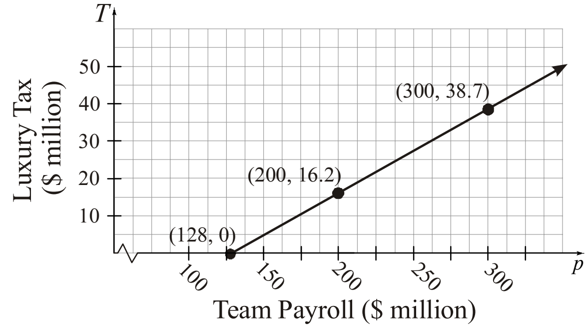
b. $T(160) = 0.225(160 - 128) = 7.2$

The luxury tax whose payroll is \$160 million would be \$7.2 million.

c. Evaluate T at $p = 128$, 200, and 300 million.

$$T(128) = 0.225(128 - 128) = 0$$
$$T(200) = 0.225(200 - 128) = 16.2 \text{ million}$$
$$T(300) = 0.225(300 - 128) = 38.7 \text{ million}$$

Thus, the points $(128 \text{ million},\ 0)$, $(200 \text{ million},\ 16.2 \text{ million})$, and $(300 \text{ million},\ 38.7 \text{ million})$ are on the graph.



d. We must solve $T(p) = 11.7$.

$$0.225(p-128) = 11.7$$
$$0.225p - 28.8 = 11.7$$
$$0.225p = 40.5$$
$$p = 180$$

If the luxury tax is $11.7 million, then the payroll of the team is $180 million.

51. $R(x) = 8x;\ \ C(x) = 4.5x + 17500$

a. Solve $R(x) = C(x)$.

$$8x = 4.5x + 17500$$
$$3.5x = 17500$$
$$x = \frac{17500}{3.5} = 5000$$

The break-even point occurs when the company sells 5000 units.

b. Solve $R(x) > C(x)$

$$8x > 4.5x + 17500$$
$$3.5x > 17500$$
$$x > \frac{17500}{3.5} = 5000$$

The company makes a profit if it sells more than 5000 units.

52. $R(x) = 12x;\ \ C(x) = 10x + 15000$

a. Solve $R(x) = C(x)$

$$12x = 10x + 15000$$
$$2x = 15000$$
$$x = \frac{15000}{2} = 7500$$

The break-even point occurs when the company sells 7500 units.

b. Solve $R(x) > C(x)$

$$12x > 10x + 15000$$
$$2x > 15000$$
$$x > \frac{15000}{2} = 7500$$

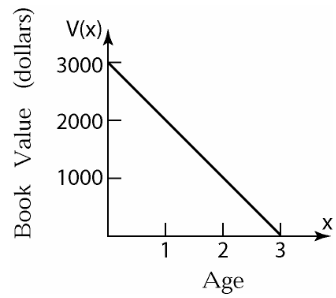
The company makes a profit if it sells more than 7500 units.

53. a. Consider the data points (x, y), where x = the age in years of the computer and y = the value in dollars of the computer. So we have the points $(0,3000)$ and $(3,0)$. The slope formula yields:

$$m = \frac{\Delta y}{\Delta x} = \frac{0-3000}{3-0} = \frac{-3000}{3} = -1000$$

The y-intercept is $(0,3000)$, so $b = 3000$.
Therefore, the linear function is
$V(x) = mx + b = -1000x + 3000$.

b. The graph of $V(x) = -1000x + 3000$



c. $V(2) = -1000(2) + 3000 = 1000$

The computer's value after 2 years will be $1000.

d. Solve $V(x) = 2000$

$$-1000x + 3000 = 2000$$
$$-1000x = -1000$$
$$x = 1$$

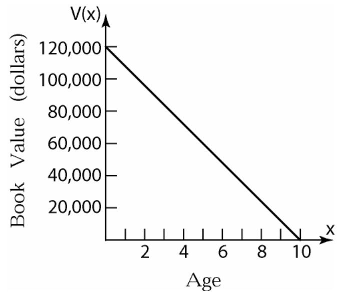
The computer will be worth $2000 after 1 year.

54. a. Consider the data points (x, y), where x = the age in years of the machine and y = the value in dollars of the machine. So we have the points $(0,120000)$ and $(10,0)$. The slope formula yields:

$$m = \frac{\Delta y}{\Delta x} = \frac{0-120000}{10-0} = \frac{-120000}{10} = -12000$$

The y-intercept is $(0,120000)$, so $b = 120000$.
Therefore, the linear function is
$V(x) = mx + b = -12{,}000x + 120{,}000$.

b. The graph of $V(x) = -12,000x + 120,000$



c. $V(4) = -12000(4) + 120000 = 72000$
The machine's value after 4 years is given by \$72,000.

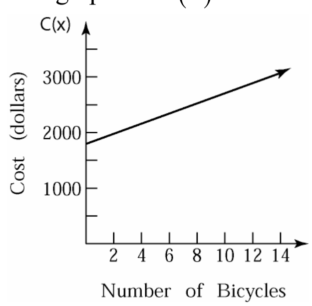
d. Solve $V(x) = 72000$.

$$-12000x + 120000 = 72000$$
$$-12000x = -48000$$
$$x = 4$$

The machine will be worth \$72,000 after 4 years.

55. a. Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with m = 90 and b = 1800. Therefore $C(x) = 90x + 1800$

b. The graph of $C(x) = 90x + 1800$



c. The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1800 = \$3060$.

d. Solve $C(x) = 90x + 1800 = 3780$

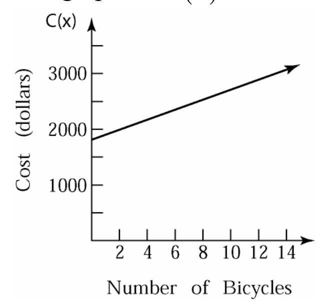
$$90x + 1800 = 3780$$
$$90x = 1980$$
$$x = 22$$

So 22 bicycles could be manufactured for \$3780.

56. a. The new daily fixed cost is
$$1800 + \frac{100}{20} = \$1805$$

b. Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with m = 90 and b = 1805.
Therefore $C(x) = 90x + 1805$

c. The graph of $C(x) = 90x + 1805$



d. The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1805 = \$3065$.

e. Solve $C(x) = 90x + 1805 = 3780$

$$90x + 1805 = 3780$$
$$90x = 1975$$
$$x \approx 21.94$$

So approximately 21 bicycles could be manufactured for \$3780.

57. a. Let x = number of miles driven, and let C = cost in dollars. Total cost = (cost per mile)(number of miles) + fixed cost
$C(x) = 0.07x + 29$

b. $C(110) = (0.07)(110) + 29 = \$36.70$
$C(230) = (0.07)(230) + 29 = \$45.10$

58. a. Let x = number of minutes used, and let C = cost in dollars. Total cost = (cost per minute)(number of minutes) + fixed cost
$C(x) = 0.05x + 5$

b. $C(105) = (0.05)(105) + 5 = \$10.25$
$C(180) = (0.05)(180) + 5 = \$14$

59. The graph shown has a positive slope and a positive y-intercept. Therefore, the function from (d) and (e) might have the graph shown.

60. The graph shown has a negaive slope and a positive y-intercept. Therefore, the function from (b) and (e) might have the graph shown.

61. A linear function $f(x)=mx+b$ will be odd provided $f(-x)=-f(x)$.

That is, provided $m(-x)+b=-(mx+b)$.

$$-mx+b=-mx-b$$
$$b=-b$$
$$2b=0$$
$$b=0$$

So a linear function $f(x)=mx+b$ will be odd provided $b=0$.

A linear function $f(x)=mx+b$ will be even provided $f(-x)=f(x)$.

That is, provided $m(-x)+b=mx+b$.

$$-mx+b=mx+b$$
$$-mxb=mx$$
$$0=2mx$$
$$m=0$$

So a linear function $f(x)=mx+b$ will be even provided $m=0$.

62. The French word *monter* means "to climb". Urban legend has it that the usage of *m* for slope resulted from *monter*. However, this is uncertain. Another theory is that the *m* comes from the term "modulus of the slope".

63. The grade of a road is related to the slope of a line in that both terms describe "steepness". A 4% grade means that for every 100 units of horizontal change in the road there is 4 units of vertical change. Answers will vary.

64. The pitch of a roof/staircase is related to the slope of a line in that both terms describe "steepness". Answers will vary.

Section 2.2

1. No, the relation is not a function because the *x*-coordinate 1 is paired with two different *y*-coordinates, 5 and 12.

2. Yes, the relation defined by $y = 2x + 1$ is a function because it is a non-vertical line, which means each *x*-coordinate will be paired with a distinct *y*-coordinate.

3. scatter diagram

4. $y=kx$

5. True

6. True

7. Linear relation, $m>0$

8. Nonlinear relation

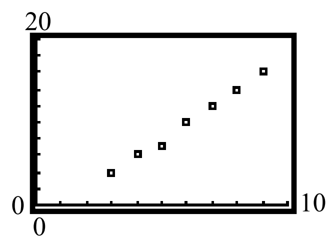
9. Linear relation, $m<0$

10. No relation

11. Nonlinear relation

12. Nonlinear relation

13. **a.**



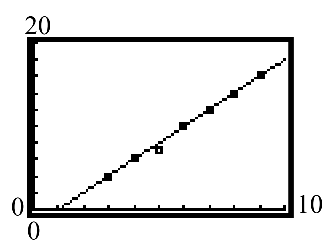
b. Answers will vary. We select (3, 4) and (9, 16). The slope of the line containing these points is:

$$m=\frac{16-4}{9-3}=\frac{12}{6}=2$$

The equation of the line is:

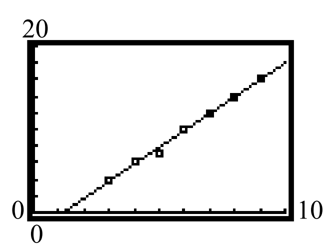
$$y-y_1=m(x-x_1)$$
$$y-4=2(x-3)$$
$$y-4=2x-6$$
$$y=2x-2$$

c.

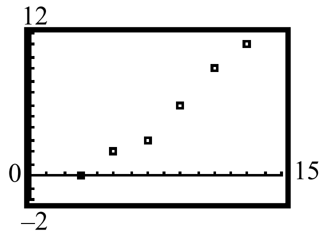


d. Using the LINear REGresssion program, the line of best fit is:
$y=2.0357x-2.3571$

e.



14. a.

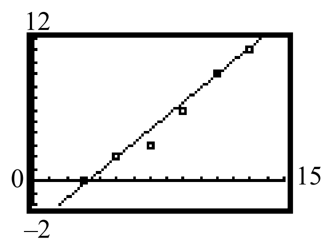


b. Answers will vary. We select (5, 2) and (11, 9). The slope of the line containing these points is: $m = \dfrac{9-2}{11-5} = \dfrac{7}{6}$

The equation of the line is:

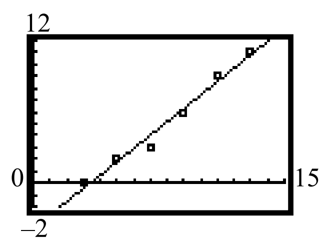
$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{7}{6}(x - 5)$$
$$y - 2 = \frac{7}{6}x - \frac{35}{6}$$
$$y = \frac{7}{6}x - \frac{23}{6}$$

c.

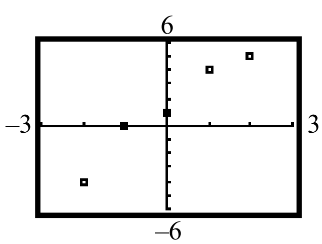


d. Using the LINear REGression program, the line of best fit is:
$y = 1.1286x - 3.8619$

e.



15. a.



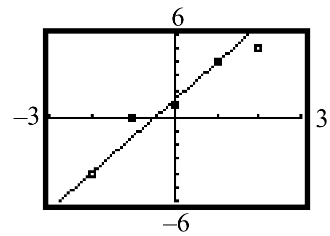
b. Answers will vary. We select (–2,–4) and (1, 4). The slope of the line containing these points is:

$$m = \frac{4-(-4)}{1-(-2)} = \frac{8}{3}.$$

The equation of the line is:

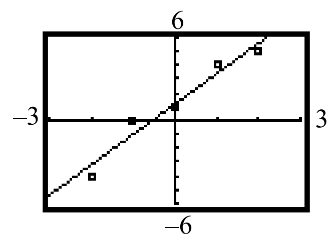
$$y - y_1 = m(x - x_1)$$
$$y - (-4) = \frac{8}{3}(x - (-2))$$
$$y + 4 = \frac{8}{3}x + \frac{16}{3}$$
$$y = \frac{8}{3}x + \frac{4}{3}$$

c.

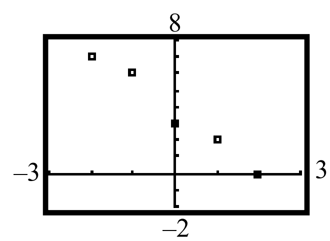


d. Using the LINear REGresssion program, the line of best fit is:
$y = 2.2x + 1.2$

e.



16. a.



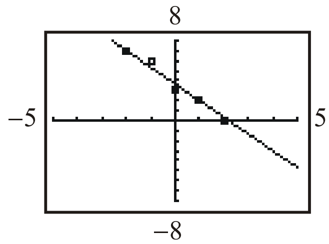
b. Answers will vary. We select (–2, 7) and (1, 2). The slope of the line containing these points is:

$$m = \frac{2-7}{1-(-2)} = \frac{-5}{3} = -\frac{5}{3}$$

The equation of the line is:

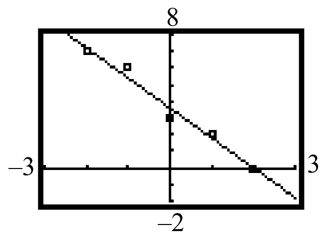
$$y - y_1 = m(x - x_1)$$
$$y - 7 = -\frac{5}{3}(x - (-2))$$
$$y - 7 = -\frac{5}{3}x - \frac{10}{3}$$
$$y = -\frac{5}{3}x + \frac{11}{3}$$

c.

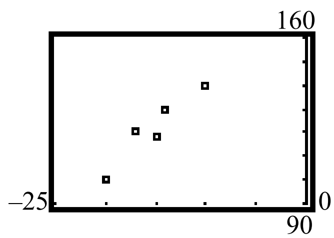


d. Using the LINear REGression program, the line of best fit is:
$y = -1.8x + 3.6$

e.



17. a.



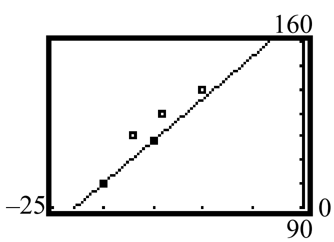
b. Answers will vary. We select (–20,100) and (–15,118). The slope of the line containing these points is:

$$m = \frac{118-100}{-15-(-20)} = \frac{18}{5} = 3.6$$

The equation of the line is:

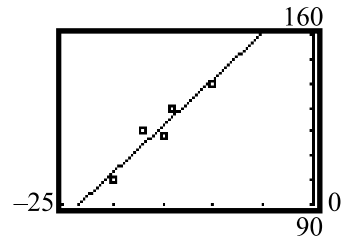
$$y - y_1 = m(x - x_1)$$
$$y - 100 = \frac{18}{5}(x - (-20))$$
$$y - 100 = \frac{18}{5}x + 72$$
$$y = \frac{18}{5}x + 172$$

c.

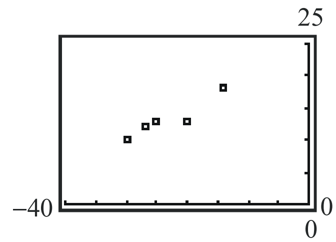


d. Using the LINear REGresssion program, the line of best fit is:
$y = 3.8613x + 180.2920$

e.



18. a.



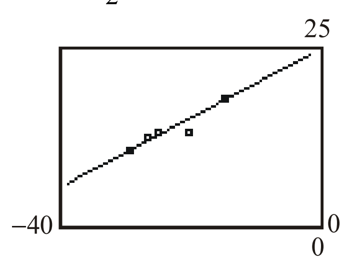
b. Selection of points will vary. We select (–30, 10) and (–14, 18). The slope of the line containing these points is:

$$m = \frac{18-10}{-14-(-30)} = \frac{8}{16} = \frac{1}{2}$$

The equation of the line is:

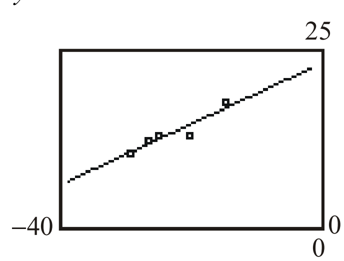
$$y - y_1 = m(x - x_1)$$
$$y - 10 = \frac{1}{2}(x - (-30))$$
$$y - 10 = \frac{1}{2}x + 15$$
$$y = \frac{1}{2}x + 25$$

c.



d. Using the LINear REGression program, the line of best fit is:
$y = 0.4421x + 23.4559$

e.



19. $p = kB$

$6.49 = k(1000)$

$0.00649 = k$

Therefore we have the linear function

$p(B) = 0.00649B$.

If $B = 145000$, then

$p = (0.00649)(145000) = \$941.05$.

20. $p = kB$

$8.99 = k(1000)$

$0.00899 = k$

Therefore we have the linear function

$p(B) = 0.00899B + 0 = 0.00899B$.

If $B = 175000$, then

$p = (0.00899)(175000) = \$1573.25$.

21. $R = kg$

$23.40 = k(12)$

$1.95 = k$

Therefore we have the linear function

$R(g) = 1.95g + 0 = 1.95g$.

If $g = 10.5$, then $R = (1.95)(10.5) = \$20.48$.

22. $C = kA$

$23.75 = k(5)$

$4.75 = k$

Therefore we have the linear function

$C(A) = 4.75A + 0 = 4.75A$.

If $A = 3.5$, then $C = (4.75)(3.5) = \$16.63$.

23. $W = kS$

$1.875 = k(15)$

$0.125 = k$

For 40 gallons of sand:

$W = 0.125(40) = 5$ gallons of water.

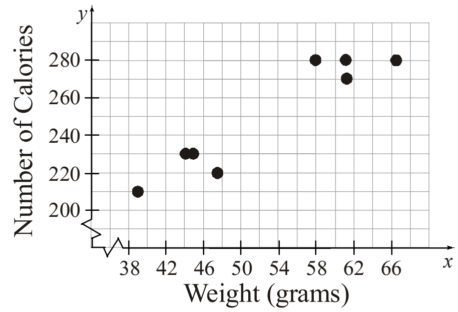
24. $v = kt$

$64 = k(2)$

$k = 32$

In 3 seconds, $v = (32)(3) = 96$ feet per second

25. a.



b. Linear.

c. Answers will vary. We will use the points $(39.52,\ 210)$ and $(66.45,\ 280)$.

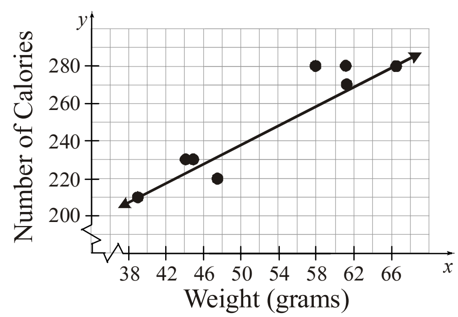
$$m = \frac{280 - 210}{66.45 - 39.52} = \frac{70}{26.93} \approx 2.599$$

$$y - 210 = 2.599(x - 39.52)$$

$$y - 210 = 2.599x - 102.712$$

$$y = 2.599x + 107.288$$

d.



e. $x = 62.3$: $y = 2.599(62.3) + 107.288$

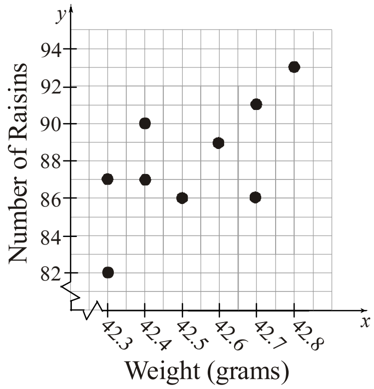
≈ 269

We predict that a candy bar weighing 62.3 grams will contain 269 calories.

f. The slope of the line found is 2.599 calories per gram. This means that if the weight of a candy bar is increased by 1 gram, then the number of calories will increase by 2.599.

26. a. No, the relation does not represent a function. Several *x*-coordinates are paired with multiple *y*-coordinates. For example, the *x*-coordinate 42.3 is paired with the two different *y*-coordinates 87 and 82.

b.

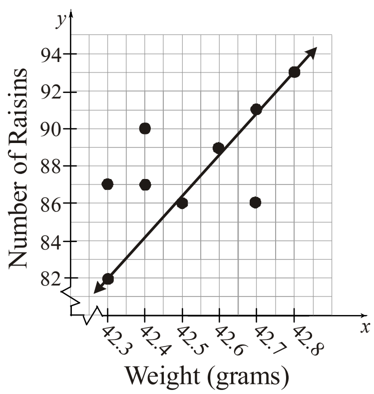


c. Answers will vary. We will use the points $(42.3,\ 82)$ and $(42.8,\ 93)$.

$$m=\frac{93-82}{42.8-42.3}=\frac{11}{0.5}=22$$

$$y-82=22(x-42.3)$$
$$y-82=22x-930.6$$
$$y=22x-848.6$$

d.



e. Let N represent the number of raisins in the box, and let w represent the weight (in grams) of the box of raisins.

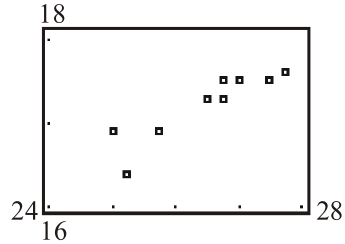
$N(w)=22w-848.6$

f. $N(42.5)=22(42.5)-848.6=86.4$

We predict that approximately 86 raisins will be in a box weighing 42.5 grams.

g. The slope of the line found is 22 raisins per gram. This means that if the weight is to be increased by one gram, then the number of raisins must be increased by 22 raisins.

27. a.



b. Using the LINear REGression program, the line of best fit is:

$C(H)=0.3734H+7.3268$

c. For each 1 inch increase in height, the circumference increases by 0.3734 inch.

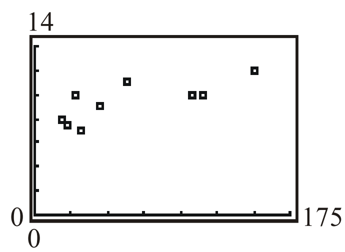
d. $C(26)=0.3734(26)+7.3268\ \approx 17.0$ inches

e. To find the height, we solve the following equation:

$$17.4=0.3734H+7.3268$$
$$10.0732=0.3734H$$
$$26.98\approx H$$

A child with a head circumference of 17.4 inches would have a height of about 26.98 inches.

28. a.



b. Using the LINear REGression program, the line of best fit is:

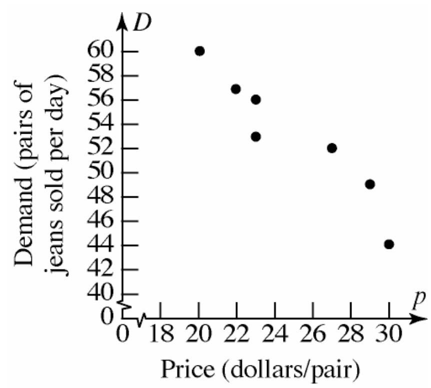
$L(G)=0.0261G+7.8738$

c. For each 1 day increase in Gestation period, the life expectancy increases by 0.0261 years (about 9.5 days).

d. $L(89)=0.0261(89)+7.8738\ \approx 10.2$ years

29. a. The relation is not a function because 23 is paired with both 56 and 53.

b.



c. Using the LINear REGression program, the line of best fit is:
$D = -1.3355p + 86.1974$

d. As the price of the jeans increases by \$1, the demand for the jeans decreases by about 1.34 pairs per day.

e. $D(p) = -1.3355p + 86.1974$

f. Domain: $\{p \mid 0 \le p \le 64\}$

Note that the p-intercept is roughly 64.54 and that the number of pairs cannot be negative.

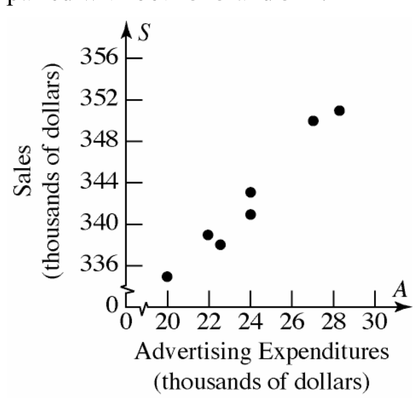
g. $D(28) = -1.3355(28) + 86.1974$

≈ 48.8034

Demand is about 49 pairs.

30. a. The relation is not a function because 24 is paired with both 343 and 341.

b.



c. Using the LINear REGression program, the line of best fit is: $S = 2.0667A + 292.8869$

d. As the advertising expenditure increases by \$1000, the sales increase by about \$2067.

e. $S(A) = 2.0667A + 292.8869$

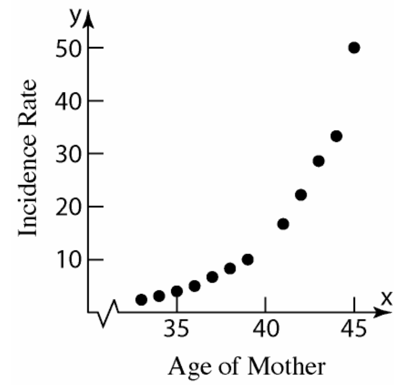
f. Domain: $\{A \mid A \ge 0\}$

g. $S(25) = 2.0667(25) + 292.8869$

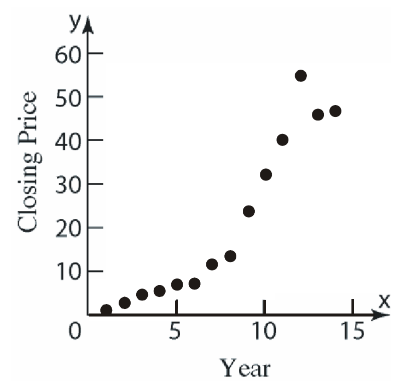
≈ 344.5544

Sales are about \$344,554.

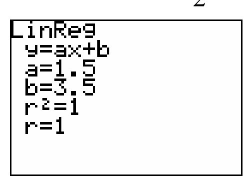
31. The data do not follow a linear pattern so it would not make sense to find the line of best fit.



32. The data do not follow a linear pattern so it would not make sense to find the line of best fit.



33. Using the ordered pairs $(1,5)$ and $(3,8)$, the line of best fit is $y = \frac{3}{2}x + \frac{7}{2}$ or $y = 1.5x + 3.5$.



The correlation coefficient is $r = 1$. This makes sense because two points completely determine a line.

34. A correlation coefficient of 0 implies that the data do not have a linear relationship.

35. If the student's average in the class is directly proportional to the amount of time the student studies, then the relationship between the average and time studying would be linear. That is, we could express the relationship as $A = kT$ where T is the time spent studying and A is the student's average.

Section 2.3

1. $x^2 - 5x - 6 = (x-6)(x+1)$

2. $2x^2 - x - 3 = (2x-3)(x+1)$

3. $2x - 3 = 11$
$2x = 14$
$x = 7$
The solution set is $\{7\}$.

4. $(x-3)(3x+5) = 0$
$x - 3 = 0 \quad \text{or} \quad 3x + 5 = 0$
$x = 3 \qquad\qquad 3x = -5$
$x = -\frac{5}{3}$
The solution set is $\left\{-\frac{5}{3}, 3\right\}$.

5. add; $\left(\frac{1}{2}\cdot 6\right)^2 = 9$

6. If $f(4) = 10$, then the point (4, 10) is on the graph of f.

7. repeated; multiplicity 2

8. discriminant; negative

9. False. If the discriminant of a quadratic equation is 0, then the equation has one repeated real solution. If the discriminant is negative, then the equation has no real solutions.

10. True.

11. $f(x) = 0$
$x^2 - 9x = 0$
$x(x-9) = 0$
$x = 0 \quad \text{or} \quad x - 9 = 0$
$x = 9$
The zeros of $f(x) = x^2 - 9x$ are 0 and 9. The x-intercepts of the graph of f are 0 and 9.

12. $f(x) = 0$
$x^2 + 4x = 0$
$x(x+4) = 0$
$x = 0 \quad \text{or} \quad x + 4 = 0$
$x = -4$
The zeros of $f(x) = x^2 + 4x$ are -4 and 0. The x-intercepts of the graph of f are -4 and 0.

13. $g(x) = 0$
$x^2 - 25 = 0$
$(x+5)(x-5) = 0$
$x + 5 = 0 \quad \text{or} \quad x - 5 = 0$
$x = -5 \qquad\qquad x = 5$
The zeros of $g(x) = x^2 - 25$ are -5 and 5. The x-intercepts of the graph of g are -5 and 5.

14. $G(x) = 0$
$x^2 - 9 = 0$
$(x+3)(x-3) = 0$
$x + 3 = 0 \quad \text{or} \quad x - 3 = 0$
$x = -3 \qquad\qquad x = 3$
The zeros of $G(x) = x^2 - 9$ are -3 and 3. The x-intercepts of the graph of G are -3 and 3.

15. $F(x) = 0$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$
$x + 3 = 0 \quad \text{or} \quad x - 2 = 0$
$x = -3 \qquad\qquad x = 2$
The zeros of $F(x) = x^2 + x - 6$ are -3 and 2. The x-intercepts of the graph of F are -3 and 2.

16. $H(x)=0$

$x^2+7x+6=0$

$(x+6)(x+1)=0$

$x+6=0$ or $x+1=0$

$x=-6$ $\quad x=-1$

The zeros of $H(x)=x^2+7x+6$ are -6 and -1.

The x-intercepts of the graph of H are -6 and -1.

17. $g(x)=0$

$2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$2x+1=0$ or $x-3=0$

$x=-\frac{1}{2}$ $\quad x=3$

The zeros of $g(x)=2x^2-5x-3$ are $-\frac{1}{2}$ and 3.

The x-intercepts of the graph of g are $-\frac{1}{2}$ and 3.

18. $f(x)=0$

$3x^2+5x+2=0$

$(3x+2)(x+1)=0$

$3x+2=0$ or $x+1=0$

$x=-\frac{2}{3}$ $\quad x=-1$

The zeros of $f(x)=3x^2+5x+2$ are -1 and $-\frac{2}{3}$. The x-intercepts of the graph of f are -1 and $-\frac{2}{3}$.

19. $P(x)=0$

$3x^2-48=0$

$3(x^2-16)=0$

$3(x+4)(x-4)=0$

$t+4=0$ or $t-4=0$

$t=-4$ $\quad t=4$

The zeros of $P(x)=3x^2-48$ are -4 and 4.

The x-intercepts of the graph of P are -4 and 4.

20. $H(x)=0$

$2x^2-50=0$

$2(x^2-25)=0$

$2(x+5)(x-5)=0$

$y+5=0$ or $y-5=0$

$y=-5$ $\quad y=5$

The zeros of $H(x)=2x^2-50$ are -5 and 5.

The x-intercepts of the graph of H are -5 and 5.

21. $g(x)=0$

$x(x+8)+12=0$

$x^2+8x+12=0$

$(x+6)(x+2)=0$

$x=-6$ or $x=-2$

The zeros of $g(x)=x(x+8)+12$ are -6 and -2.

The x-intercepts of the graph of g are -6 and -2.

22. $f(x)=0$

$x(x-4)-12=0$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=-2$ or $x=6$

The zeros of $f(x)=x(x-4)-12$ are -2 and 6.

The x-intercepts of the graph of f are -2 and 6.

23. $G(x)=0$

$4x^2+9-12x=0$

$4x^2-12x+9=0$

$(2x-3)(2x-3)=0$

$2x-3=0$ or $2x-3=0$

$x=\frac{3}{2}$ $\quad x=\frac{3}{2}$

The only zero of $G(x)=4x^2+9-12x$ is $\frac{3}{2}$.

The only x-intercept of the graph of G is $\frac{3}{2}$.

24.
$$F(x)=0$$
$$25x^2+16-40x=0$$
$$25x^2-40x+16=0$$
$$(5x-4)(5x-4)=0$$
$$5x-4=0 \quad \text{or} \quad 5x-4=0$$
$$x=\frac{4}{5} \qquad x=\frac{4}{5}$$
The only zero of $F(x)=25x^2+16-40x$ is $\frac{4}{5}$.

The only x-intercept of the graph of F is $\frac{4}{5}$.

25.
$$f(x)=0$$
$$x^2-25=0$$
$$x^2=25$$
$$x=\pm\sqrt{25}=\pm 5$$
The zeros of $f(x)=x^2-25$ are -5 and 5. The x-intercepts of the graph of f are -5 and 5.

26.
$$g(x)=0$$
$$x^2-36=0$$
$$x^2=36$$
$$x=\pm\sqrt{36}=\pm 6$$
The zeros of $g(x)=x^2-36$ are -6 and 6. The x-intercepts of the graph of g are -6 and 6.

27.
$$g(x)=0$$
$$(x-1)^2-4=0$$
$$(x-1)^2=4$$
$$x-1=\pm\sqrt{4}$$
$$x-1=\pm 2$$
$$x-1=2 \quad \text{or} \quad x-1=-2$$
$$x=3 \qquad x=-1$$
The zeros of $g(x)=(x-1)^2-4$ are -1 and 3.
The x-intercepts of the graph of g are -1 and 3.

28.
$$G(x)=0$$
$$(x+2)^2-1=0$$
$$(x+2)^2=1$$
$$x+2=\pm\sqrt{1}$$
$$x+2=\pm 1$$
$$x+2=1 \quad \text{or} \quad x+2=-1$$
$$x=-1 \qquad x=-3$$
The zeros of $G(x)=(x+2)^2-1$ are -3 and -1.
The x-intercepts of the graph of G are -3 and -1.

29.
$$F(x)=0$$
$$(2x+3)^2-9=0$$
$$(2x+3)^2=9$$
$$2x+3=\pm\sqrt{9}$$
$$2x+3=\pm 3$$
$$2x+3=3 \quad \text{or} \quad 2x+3=-3$$
$$2x=0 \qquad 2x=-6$$
$$x=0 \qquad x=-3$$
The zeros of $F(x)=(2x+3)^2-9$ are -3 and 0.
The x-intercepts of the graph of F are -3 and 0.

30.
$$G(x)=0$$
$$(3x-2)^2-4=0$$
$$(3x-2)^2=4$$
$$3x-2=\pm\sqrt{4}$$
$$3x-2=\pm 2$$
$$3x-2=2 \quad \text{or} \quad 3x-2=-2$$
$$3x=4 \qquad 3x=0$$
$$x=\frac{4}{3} \qquad x=0$$
The zeros of $G(x)=(3x-2)^2-4$ are 0 and $\frac{4}{3}$.

The x-intercepts of the graph of G are 0 and $\frac{4}{3}$.

31.
$$\begin{aligned} f(x) &= 0 \\ x^2+4x-21 &= 0 \\ x^2+4x &= 21 \\ x^2+4x+4 &= 21+4 \\ (x+2)^2 &= 25 \\ x+2 &= \pm\sqrt{25} \\ x+2 &= \pm 5 \\ x &= -2\pm 5 \\ x=3 \ \text{ or } \ x&=-7 \end{aligned}$$

The zeros of $f(x)=x^2+4x-21$ are -7 and 3. The x-intercepts of the graph of f are -7 and 3.

32.
$$\begin{aligned} f(x) &= 0 \\ x^2-6x-13 &= 0 \\ x^2-6x+9 &= 13+9 \\ (x-3)^2 &= 22 \\ x-3 &= \pm\sqrt{22} \\ x &= 3\pm\sqrt{22} \end{aligned}$$

The zeros of $f(x)=x^2-6x-13$ are $3-\sqrt{22}$ and $3+\sqrt{22}$. The x-intercepts of the graph of f are $3-\sqrt{22}$ and $3+\sqrt{22}$.

33.
$$\begin{aligned} g(x) &= 0 \\ x^2-\frac{1}{2}x-\frac{3}{16} &= 0 \\ x^2-\frac{1}{2}x &= \frac{3}{16} \\ x^2-\frac{1}{2}x+\frac{1}{16} &= \frac{3}{16}+\frac{1}{16} \\ \left(x-\frac{1}{4}\right)^2 &= \frac{1}{4} \\ x-\frac{1}{4} &= \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2} \\ x &= \frac{1}{4}\pm\frac{1}{2} \\ x=\frac{3}{4} \ \text{ or } \ x &= -\frac{1}{4} \end{aligned}$$

The zeros of $g(x)=x^2-\frac{1}{2}x-\frac{3}{16}$ are $-\frac{1}{4}$ and $\frac{3}{4}$.

The x-intercepts of the graph of g are $-\frac{1}{4}$ and $\frac{3}{4}$.

34.
$$\begin{aligned} g(x) &= 0 \\ x^2+\frac{2}{3}x-\frac{1}{3} &= 0 \\ x^2+\frac{2}{3}x &= \frac{1}{3} \\ x^2+\frac{2}{3}x+\frac{1}{9} &= \frac{1}{3}+\frac{1}{9} \\ \left(x+\frac{1}{3}\right)^2 &= \frac{4}{9} \\ x+\frac{1}{3} &= \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3} \\ x &= -\frac{1}{3}\pm\frac{2}{3} \\ x=\frac{1}{3} \ \text{ or } \ x &= -1 \end{aligned}$$

The zeros of $g(x)=x^2+\frac{2}{3}x-\frac{1}{3}$ are -1 and $\frac{1}{3}$.

The x-intercepts of the graph of g are -1 and $\frac{1}{3}$.

35.
$$\begin{aligned} F(x) &= 0 \\ 3x^2+x-\frac{1}{2} &= 0 \\ x^2+\frac{1}{3}x-\frac{1}{6} &= 0 \\ x^2+\frac{1}{3}x &= \frac{1}{6} \\ x^2+\frac{1}{3}x+\frac{1}{36} &= \frac{1}{6}+\frac{1}{36} \\ \left(x+\frac{1}{6}\right)^2 &= \frac{7}{36} \\ x+\frac{1}{6} &= \pm\sqrt{\frac{7}{36}} = \pm\frac{\sqrt{7}}{6} \\ x &= \frac{-1\pm\sqrt{7}}{6} \end{aligned}$$

The zeros of $F(x)=3x^2+x-\frac{1}{2}$ are $\frac{-1-\sqrt{7}}{6}$ and $\frac{-1+\sqrt{7}}{6}$. The x-intercepts of the graph of F are $\frac{-1-\sqrt{7}}{6}$ and $\frac{-1+\sqrt{7}}{6}$.

36.
$$G(x)=0$$
$$2x^2-3x-1=0$$
$$x^2-\frac{3}{2}x-\frac{1}{2}=0$$
$$x^2-\frac{3}{2}x=\frac{1}{2}$$
$$x^2-\frac{3}{2}x+\frac{9}{16}=\frac{1}{2}+\frac{9}{16}$$
$$\left(x-\frac{3}{4}\right)^2=\frac{17}{16}$$
$$x-\frac{3}{4}=\pm\sqrt{\frac{17}{16}}=\pm\frac{\sqrt{17}}{4}$$
$$x=\frac{3\pm\sqrt{17}}{4}$$

The zeros of $G(x)=2x^2-3x-1$ are $\frac{3-\sqrt{17}}{4}$ and $\frac{3+\sqrt{17}}{4}$. The x-intercepts of the graph of G are $\frac{3-\sqrt{17}}{4}$ and $\frac{3+\sqrt{17}}{4}$.

37.
$$f(x)=0$$
$$x^2-4x+2=0$$
$$a=1,\quad b=-4,\quad c=2$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}=\frac{4\pm\sqrt{16-8}}{2}$$
$$=\frac{4\pm\sqrt{8}}{2}=\frac{4\pm2\sqrt{2}}{2}=2\pm\sqrt{2}$$

The zeros of $f(x)=x^2-4x+2$ are $2-\sqrt{2}$ and $2+\sqrt{2}$. The x-intercepts of the graph of f are $2-\sqrt{2}$ and $2+\sqrt{2}$.

38.
$$f(x)=0$$
$$x^2+4x+2=0$$
$$a=1,\quad b=4,\quad c=2$$
$$x=\frac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)}=\frac{-4\pm\sqrt{16-8}}{2}$$
$$=\frac{-4\pm\sqrt{8}}{2}=\frac{-4\pm2\sqrt{2}}{2}=-2\pm\sqrt{2}$$

The zeros of $f(x)=x^2+4x+2$ are $-2-\sqrt{2}$ and $-2+\sqrt{2}$. The x-intercepts of the graph of f are $-2-\sqrt{2}$ and $-2+\sqrt{2}$.

39.
$$g(x)=0$$
$$x^2-4x-1=0$$
$$a=1,\quad b=-4,\quad c=-1$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-1)}}{2(1)}=\frac{4\pm\sqrt{16+4}}{2}$$
$$=\frac{4\pm\sqrt{20}}{2}=\frac{4\pm2\sqrt{5}}{2}=2\pm\sqrt{5}$$

The zeros of $g(x)=x^2-4x-1$ are $2-\sqrt{5}$ and $2+\sqrt{5}$. The x-intercepts of the graph of g are $2-\sqrt{5}$ and $2+\sqrt{5}$.

40.
$$g(x)=0$$
$$x^2+6x+1=0$$
$$a=1,\quad b=6,\quad c=1$$
$$x=\frac{-6\pm\sqrt{6^2-4(1)(1)}}{2(1)}=\frac{-6\pm\sqrt{36-4}}{2}$$
$$=\frac{-6\pm\sqrt{32}}{2}=\frac{-6\pm4\sqrt{2}}{2}=-3\pm2\sqrt{2}$$

The zeros of $g(x)=x^2+6x+1$ are $-3-2\sqrt{2}$ and $-3+2\sqrt{2}$. The x-intercepts of the graph of g are $-3-2\sqrt{2}$ and $-3+2\sqrt{2}$.

41.
$$F(x)=0$$
$$2x^2-5x+3=0$$
$$a=2,\quad b=-5,\quad c=3$$
$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(3)}}{2(2)}=\frac{5\pm\sqrt{25-24}}{4}$$
$$=\frac{5\pm1}{4}=\frac{3}{2}\text{ or }1$$

The zeros of $F(x)=2x^2-5x+3$ are 1 and $\frac{3}{2}$.

The x-intercepts of the graph of F are 1 and $\frac{3}{2}$.

42.
$$g(x)=0$$
$$2x^2+5x+3=0$$
$$a=2,\quad b=5,\quad c=3$$
$$x=\frac{-5\pm\sqrt{5^2-4(2)(3)}}{2(2)}=\frac{-5\pm\sqrt{25-24}}{4}$$
$$=\frac{-5\pm1}{4}=-1\text{ or }-\frac{3}{2}$$

The zeros of $g(x)=2x^2+5x+3$ are $-\frac{3}{2}$ and -1.

The x-intercepts of the graph of g are $-\frac{3}{2}$ and -1.

43. $P(x)=0$

$4x^2-x+2=0$
$a=4,\quad b=-1,\quad c=2$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(4)(2)}}{2(4)}=\frac{1\pm\sqrt{1-32}}{8}$$
$$=\frac{1\pm\sqrt{-31}}{8}=\text{not real}$$

The function $P(x)=4x^2-x+2$ has no real zeros, and the graph of P has no x-intercepts.

44. $H(x)=0$

$4x^2+x+1=0$
$a=4,\quad b=1,\quad c=1$

$$t=\frac{-1\pm\sqrt{1^2-4(4)(1)}}{2(4)}=\frac{-1\pm\sqrt{1-16}}{8}$$
$$=\frac{-1\pm\sqrt{-15}}{8}=\text{not real}$$

The function $H(x)=4x^2+x+1$ has no real zeros, and the graph of H has no x-intercepts.

45. $f(x)=0$

$4x^2-1+2x=0$
$4x^2+2x-1=0$
$a=4,\quad b=2,\quad c=-1$

$$x=\frac{-2\pm\sqrt{2^2-4(4)(-1)}}{2(4)}=\frac{-2\pm\sqrt{4+16}}{8}$$
$$=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm2\sqrt{5}}{8}=\frac{-1\pm\sqrt{5}}{4}$$

The zeros of $f(x)=4x^2-1+2x$ are $\frac{-1-\sqrt{5}}{4}$ and $\frac{-1+\sqrt{5}}{4}$. The x-intercepts of the graph of f are $\frac{-1-\sqrt{5}}{4}$ and $\frac{-1+\sqrt{5}}{4}$.

46. $f(x)=0$

$2x^2-1+2x=0$
$2x^2+2x-1=0$
$a=2,\quad b=2,\quad c=-1$

$$x=\frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}=\frac{-2\pm\sqrt{4+8}}{4}$$
$$=\frac{-2\pm\sqrt{12}}{4}=\frac{-2\pm2\sqrt{3}}{4}=\frac{-1\pm\sqrt{3}}{2}$$

The zeros of $f(x)=2x^2-1+2x$ are $\frac{-1-\sqrt{3}}{2}$ and $\frac{-1+\sqrt{3}}{2}$. The x-intercepts of the graph of f are $\frac{-1-\sqrt{3}}{2}$ and $\frac{-1+\sqrt{3}}{2}$.

47. $G(x)=0$

$4x^2-9x-2=0$
$a=4,\quad b=-9,\quad c=-2$

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(4)(-2)}}{2(4)}=\frac{9\pm\sqrt{81+32}}{8}$$
$$=\frac{9\pm\sqrt{113}}{8}$$

The zeros of $G(x)=4x^2-9x-2$ are $\frac{9-\sqrt{113}}{8}$ and $\frac{9+\sqrt{113}}{8}$. The x-intercepts of the graph of G are $\frac{9-\sqrt{113}}{8}$ and $\frac{9+\sqrt{113}}{8}$.

48. $F(x)=0$

$5x-4x^2-5=0$
$0=4x^2-5x+5$
$a=4,\quad b=-5,\quad c=5$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(4)(5)}}{2(4)}=\frac{5\pm\sqrt{25-80}}{8}$$
$$=\frac{5\pm\sqrt{-55}}{8}=\text{not real}$$

The function $F(x)=5x-4x^2-5$ has no real zeros, and the graph of F has no x-intercepts.

49. $f(x)=0$

$x^2-5=0$
$x^2=5\Rightarrow x=\pm\sqrt{5}$

The zeros of $f(x)=x^2-5$ are $-\sqrt{5}$ and $\sqrt{5}$. The x-intercepts of the graph of f are $-\sqrt{5}$ and $\sqrt{5}$.

50. $f(x)=0$

$x^2-6=0$
$x^2=6\Rightarrow x=\pm\sqrt{6}$

The zeros of $f(x)=x^2-6$ are $-\sqrt{6}$ and $\sqrt{6}$. The x-intercepts of the graph of f are $-\sqrt{6}$ and $\sqrt{6}$.

51.
$$g(x)=0$$
$$16x^2-8x+1=0$$
$$(4x-1)^2=0$$
$$4x-1=0 \Rightarrow x=\frac{1}{4}$$
The only real zero of $g(x)=16x^2-8x+1$ is $\frac{1}{4}$.

The only x-intercept of the graph of g is $\frac{1}{4}$.

52.
$$F(x)=0$$
$$4x^2-12x+9=0$$
$$(2x-3)^2=0$$
$$2x-3=0 \Rightarrow x=\frac{3}{2}$$
The only real zero of $F(x)=4x^2-12x+9$ is $\frac{3}{2}$.

The only x-intercept of the graph of F is $\frac{3}{2}$.

53.
$$G(x)=0$$
$$10x^2-19x-15=0$$
$$(5x+3)(2x-5)=0$$
$$5x+3=0 \quad \text{or} \quad 2x-5=0$$
$$x=-\frac{3}{5} \qquad x=\frac{5}{2}$$
The zeros of $G(x)=10x^2-19x-15$ are $-\frac{3}{5}$ and $\frac{5}{2}$. The x-intercepts of the graph of G are $-\frac{3}{5}$ and $\frac{5}{2}$.

54.
$$f(x)=0$$
$$6x^2+7x-20=0$$
$$(3x-4)(2x+5)=0$$
$$3x-4=0 \quad \text{or} \quad 2x+5=0$$
$$x=\frac{4}{3} \qquad x=-\frac{5}{2}$$
The zeros of $f(x)=6x^2+7x-20$ are $-\frac{5}{2}$ and $\frac{4}{3}$.

The x-intercepts of the graph of f are $-\frac{5}{2}$ and $\frac{4}{3}$.

55.
$$P(x)=0$$
$$6x^2-x-2=0$$
$$(3x-2)(2x+1)=0$$
$$3x-2=0 \quad \text{or} \quad 2x+1=0$$
$$x=\frac{2}{3} \qquad x=-\frac{1}{2}$$
The zeros of $P(x)=6x^2-x-2$ are $-\frac{1}{2}$ and $\frac{2}{3}$.

The x-intercepts of the graph of P are $-\frac{1}{2}$ and $\frac{2}{3}$.

56.
$$H(x)=0$$
$$6x^2+x-2=0$$
$$(3x+2)(2x-1)=0$$
$$3x+2=0 \quad \text{or} \quad 2x-1=0$$
$$x=-\frac{2}{3} \qquad x=\frac{1}{2}$$
The zeros of $H(x)=6x^2+x-2$ are $-\frac{2}{3}$ and $\frac{1}{2}$.

The x-intercepts of the graph of H are $-\frac{2}{3}$ and $\frac{1}{2}$.

57.
$$G(x)=0$$
$$x^2+\sqrt{2}x-\frac{1}{2}=0$$
$$2\left(x^2+\sqrt{2}x-\frac{1}{2}\right)=(0)(2)$$
$$2x^2+2\sqrt{2}x-1=0$$
$$a=2, \quad b=2\sqrt{2}, \quad c=-1$$
$$x=\frac{-(2\sqrt{2})\pm\sqrt{(2\sqrt{2})^2-4(2)(-1)}}{2(2)}$$
$$=\frac{-2\sqrt{2}\pm\sqrt{8+8}}{4}=\frac{-2\sqrt{2}\pm\sqrt{16}}{4}$$
$$=\frac{-2\sqrt{2}\pm 4}{4}=\frac{-\sqrt{2}\pm 2}{2}$$
The zeros of $G(x)=x^2+\sqrt{2}x-\frac{1}{2}$ are $\frac{-\sqrt{2}-2}{2}$ and $\frac{-\sqrt{2}+2}{2}$. The x-intercepts of the graph of G are $\frac{-\sqrt{2}-2}{2}$ and $\frac{-\sqrt{2}+2}{2}$.

58.
$$F(x) = 0$$
$$\frac{1}{2}x^2 - \sqrt{2}x - 1 = 0$$
$$2\left(\frac{1}{2}x^2 - \sqrt{2}x - 1\right) = (0)(2)$$
$$x^2 - 2\sqrt{2}x - 2 = 0$$
$$a = 1, \quad b = -2\sqrt{2}, \quad c = -2$$
$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{2\sqrt{2} \pm \sqrt{16}}{2} = \frac{2\sqrt{2} \pm 4}{2} = \frac{\sqrt{2} \pm 2}{1}$$

The zeros of $F(x) = \frac{1}{2}x^2 - \sqrt{2}x - 1$ are $\sqrt{2} - 2$ and $\sqrt{2} + 2$. The *x*-intercepts of the graph of *F* are $\sqrt{2} - 2$ and $\sqrt{2} + 2$.

59.
$$f(x) = 0$$
$$x^2 + x - 4 = 0$$
$$a = 1, \quad b = 1, \quad c = -4$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1 + 16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

The zeros of $f(x) = x^2 + x - 4$ are $\frac{-1 - \sqrt{17}}{2}$ and $\frac{-1 + \sqrt{17}}{2}$. The *x*-intercepts of the graph of *f* are $\frac{-1 - \sqrt{17}}{2}$ and $\frac{-1 + \sqrt{17}}{2}$.

60.
$$g(x) = 0$$
$$x^2 + x - 1 = 0$$
$$a = 1, \quad b = 1, \quad c = -1$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

The zeros of $g(x) = x^2 + x - 1$ are $\frac{-1 - \sqrt{5}}{2}$ and $\frac{-1 + \sqrt{5}}{2}$. The *x*-intercepts of the graph of *g* are $\frac{-1 - \sqrt{5}}{2}$ and $\frac{-1 + \sqrt{5}}{2}$.

61.
$$f(x) = g(x)$$
$$x^2 + 6x + 3 = 3$$
$$x^2 + 6x = 0$$
$$x(x + 6) = 0$$
$$x = 0 \quad \text{or} \quad x + 6 = 0$$
$$x = -6$$

The *x*-coordinates of the points of intersection are -6 and 0. The *y*-coordinates are $g(-6) = 3$ and $g(0) = 3$. The graphs of the *f* and *g* intersect at the points $(-6, 3)$ and $(0, 3)$.

62.
$$f(x) = g(x)$$
$$x^2 - 4x + 3 = 3$$
$$x^2 - 4x = 0$$
$$x(x - 4) = 0$$
$$x = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = 4$$

The *x*-coordinates of the points of intersection are 0 and 4. The *y*-coordinates are $g(0) = 3$ and $g(4) = 3$. The graphs of the *f* and *g* intersect at the points $(0, 3)$ and $(4, 3)$.

63.
$$f(x) = g(x)$$
$$-2x^2 + 1 = 3x + 2$$
$$0 = 2x^2 + 3x + 1$$
$$0 = (2x + 1)(x + 1)$$
$$2x + 1 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -\frac{1}{2} \qquad x = -1$$

The *x*-coordinates of the points of intersection are -1 and $-\frac{1}{2}$. The *y*-coordinates are $g(-1) = 3(-1) + 2 = -3 + 2 = -1$ and $g\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right) + 2 = -\frac{3}{2} + 2 = \frac{1}{2}$.
The graphs of the *f* and *g* intersect at the points $(-1, -1)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

64.
$$\begin{aligned} f(x) &= g(x) \\ 3x^2 - 7 &= 10x + 1 \\ 3x^2 - 10x - 8 &= 0 \\ (3x+2)(x-4) &= 0 \end{aligned}$$
$$3x + 2 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -\frac{2}{3} \qquad x = 4$$
The x-coordinates of the points of intersection are $-\frac{2}{3}$ and 4. The y-coordinates are $g\left(-\frac{2}{3}\right) = 10\left(-\frac{2}{3}\right) + 1 = -\frac{20}{3} + 1 = -\frac{17}{3}$ and $g(4) = 10(4) + 1 = 40 + 1 = 41$.
The graphs of the f and g intersect at the points $\left(-\frac{2}{3}, -\frac{17}{3}\right)$ and $(4, 41)$.

65.
$$\begin{aligned} f(x) &= g(x) \\ x^2 - x + 1 &= 2x^2 - 3x - 14 \\ 0 &= x^2 - 2x - 15 \\ 0 &= (x+3)(x-5) \end{aligned}$$
$$x + 3 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = -3 \qquad x = 5$$
The x-coordinates of the points of intersection are -3 and 5. The y-coordinates are $f(-3) = (-3)^2 - (-3) + 1 = 9 + 3 + 1 = 13$ and $f(5) = 5^2 - 5 + 1 = 25 - 5 + 1 = 21$.
The graphs of the f and g intersect at the points $(-3, 13)$ and $(5, 21)$.

66.
$$\begin{aligned} f(x) &= g(x) \\ x^2 + 5x - 3 &= 2x^2 + 7x - 27 \\ 0 &= x^2 + 2x - 24 \\ 0 &= (x+6)(x-4) \end{aligned}$$
$$x + 6 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -6 \qquad x = 4$$
The x-coordinates of the points of intersection are -6 and 4. The y-coordinates are $f(-6) = (-6)^2 + 5(-6) - 3 = 36 - 30 - 3 = 3$ and $f(4) = 4^2 + 5(4) - 3 = 16 + 20 - 3 = 33$.
The graphs of the f and g intersect at the points $(-6, 3)$ and $(4, 33)$.

67.
$$\begin{aligned} P(x) &= 0 \\ x^4 - 6x^2 - 16 &= 0 \\ (x^2+2)(x^2-8) &= 0 \end{aligned}$$
$$x^2 + 2 = 0 \quad \text{or} \quad x^2 - 8 = 0$$
$$x^2 = -2 \qquad x^2 = 8$$
$$x = \pm\sqrt{-2} \qquad x = \pm\sqrt{8}$$
$$= \text{not real} \qquad = \pm 2\sqrt{2}$$
The zeros of $P(x) = x^4 - 6x^2 - 16$ are $-2\sqrt{2}$ and $2\sqrt{2}$. The x-intercepts of the graph of P are $-2\sqrt{2}$ and $2\sqrt{2}$.

68.
$$\begin{aligned} H(x) &= 0 \\ x^4 - 3x^2 - 4 &= 0 \\ (x^2+1)(x^2-4) &= 0 \end{aligned}$$
$$x^2 + 1 = 0 \quad \text{or} \quad x^2 - 4 = 0$$
$$x^2 = -1 \qquad x^2 = 4$$
$$x = \pm\sqrt{-1} \qquad x = \pm\sqrt{4}$$
$$= \text{not real} \qquad = \pm 2$$
The zeros of $H(x) = x^4 - 3x^2 - 4$ are -2 and 2. The x-intercepts of the graph of H are -2 and 2.

69.
$$\begin{aligned} f(x) &= 0 \\ x^4 - 5x^2 + 4 &= 0 \\ (x^2-4)(x^2-1) &= 0 \end{aligned}$$
$$x^2 - 4 = 0 \quad \text{or} \quad x^2 - 1 = 0$$
$$x = \pm 2 \quad \text{or} \quad x = \pm 1$$
The zeros of $f(x) = x^4 - 5x^2 + 4$ are -2, -1, 1, and 2. The x-intercepts of the graph of f are -2, -1, 1, and 2.

70.
$$\begin{aligned} f(x) &= 0 \\ x^4 - 10x^2 + 25 &= 0 \\ (x^2-5)^2 &= 0 \\ x^2 - 5 &= 0 \\ x^2 &= 5 \\ x &= \pm\sqrt{5} \end{aligned}$$
The zeros of $f(x) = x^4 - 10x^2 + 25$ are $-\sqrt{5}$ and $\sqrt{5}$. The x-intercepts of the graph of f are $-\sqrt{5}$ and $\sqrt{5}$.

71.
$$G(x)=0$$
$$3x^4-2x^2-1=0$$
$$(3x^2+1)(x^2-1)=0$$
$$3x^2+1=0 \quad \text{or} \quad x^2-1=0$$
$$x^2=-\frac{1}{3} \qquad x^2=1$$
$$x=\pm\sqrt{-\frac{1}{3}} \qquad x=\pm\sqrt{1}=\pm 1$$
$$x=\text{not real}$$

The zeros of $G(x)=3x^4-2x^2-1$ are -1 and 1. The x-intercepts of the graph of G are -1 and 1.

72.
$$F(x)=0$$
$$2x^4-5x^2-12=0$$
$$(2x^2+3)(x^2-4)=0$$
$$2x^2+3=0 \quad \text{or} \quad x^2-4=0$$
$$x^2=-\frac{3}{2} \qquad x^2=4$$
$$x=\pm\sqrt{-\frac{3}{2}} \qquad x=\pm\sqrt{4}=\pm 2$$
$$=\text{not real}$$

The zeros of $F(x)=2x^4-5x^2-12$ are -2 and 2. The x-intercepts of the graph of F are -2 and 2.

73.
$$g(x)=0$$
$$x^6+7x^3-8=0$$
$$(x^3+8)(x^3-1)=0$$
$$x^3+8=0 \quad \text{or} \quad x^3-1=0$$
$$x^3=-8 \qquad x^3=1$$
$$x=-2 \qquad x=1$$

The zeros of $g(x)=x^6+7x^3-8$ are -2 and 1. The x-intercepts of the graph of g are -2 and 1.

74.
$$g(x)=0$$
$$x^6-7x^3-8=0$$
$$(x^3-8)(x^3+1)=0$$
$$x^3-8=0 \quad \text{or} \quad x^3+1=0$$
$$x^3=8 \qquad x^3=-1$$
$$x=2 \qquad x=-1$$

The zeros of $g(x)=x^6-7x^3-8$ are -1 and 2. The x-intercepts of the graph of g are -1 and 2.

75.
$$G(x)=0$$
$$(x+2)^2+7(x+2)+12=0$$
Let $u=x+2 \rightarrow u^2=(x+2)^2$
$$u^2+7u+12=0$$
$$(u+3)(u+4)=0$$
$$u+3=0 \quad \text{or} \quad u+4=0$$
$$u=-3 \qquad u=-4$$
$$x+2=-3 \qquad x+2=-4$$
$$x=-5 \qquad x=-6$$

The zeros of $G(x)=(x+2)^2+7(x+2)+12$ are -6 and -5. The x-intercepts of the graph of G are -6 and -5.

76.
$$f(x)=0$$
$$(2x+5)^2-(2x+5)-6=0$$
Let $u=2x+5 \rightarrow u^2=(2x+5)^2$
$$u^2-u-6=0$$
$$(u-3)(u+2)=0$$
$$u-3=0 \quad \text{or} \quad u+2=0$$
$$u=3 \qquad u=-2$$
$$2x+5=3 \qquad 2x+5=-2$$
$$x=-1 \qquad x=-\frac{7}{2}$$

The zeros of $f(x)=(2x+5)^2-(2x+5)-6$ are $-\frac{7}{2}$ and -1. The x-intercepts of the graph of f are $-\frac{7}{2}$ and -1.

77.
$$f(x)=0$$
$$(3x+4)^2-6(3x+4)+9=0$$
Let $u=3x+4 \rightarrow u^2=(3x+4)^2$
$$u^2-6u+9=0$$
$$(u-3)^2=0$$
$$u-3=0$$
$$u=3$$
$$3x+4=3$$
$$x=-\frac{1}{3}$$

The only zero of $f(x)=(3x+4)^2-6(3x+4)+9$ is $-\frac{1}{3}$. The x-intercept of the graph of f is $-\frac{1}{3}$.

78. $H(x) = 0$

$(2-x)^2 + (2-x) - 20 = 0$

Let $u = 2-x \to u^2 = (2-x)^2$

$u^2 + u - 20 = 0$

$(u+5)(u-4) = 0$

$u+5=0$ or $u-4=0$

$u=-5$ $\quad$ $u=4$

$2-x=-5$ $\quad$ $2-x=4$

$x=7$ $\quad$ $x=-2$

The zeros of $H(x) = (2-x)^2 + (2-x) - 20$ are -2 and 7. The *x*-intercepts of the graph of *H* are -2 and 7.

79. $P(x) = 0$

$2(x+1)^2 - 5(x+1) - 3 = 0$

Let $u = x+1 \to u^2 = (x+1)^2$

$2u^2 - 5u - 3 = 0$

$(2u+1)(u-3) = 0$

$2u+1=0$ or $u-3=0$

$u = -\frac{1}{2}$ $\quad$ $u=3$

$x+1 = -\frac{1}{2}$ $\quad$ $x+1=3$

$x = -\frac{3}{2}$ $\quad$ $x=2$

The zeros of $P(x) = 2(x+1)^2 - 5(x+1) - 3$ are $-\frac{3}{2}$ and 2. The *x*-intercepts of the graph of *P* are $-\frac{3}{2}$ and 2.

80. $H(x) = 0$

$3(1-x)^2 + 5(1-x) + 2 = 0$

Let $u = 1-x \to u^2 = (1-x)^2$

$3u^2 + 5u + 2 = 0$

$(3u+2)(u+1) = 0$

$3u+2=0$ or $u+1=0$

$u = -\frac{2}{3}$ $\quad$ $u=-1$

$1-x = -\frac{2}{3}$ $\quad$ $1-x=-1$

$x = \frac{5}{3}$ $\quad$ $x=2$

The zeros of $H(x) = 3(1-x)^2 + 5(1-x) + 2$ are $\frac{5}{3}$ and 2. The *x*-intercepts of the graph of *H* are $\frac{5}{3}$ and 2.

81. $G(x) = 0$

$x - 4\sqrt{x} = 0$

Let $u = \sqrt{x} \to u^2 = x$

$u^2 - 4u = 0$

$u(u-4) = 0$

$u=0$ or $u-4=0$

$u=4$

$\sqrt{x}=0$ $\quad$ $\sqrt{x}=4$

$x = 0^2 = 0$ $\quad$ $x = 4^2 = 16$

Check:

$G(0) = 0 - 4\sqrt{0} = 0$

$G(16) = 16 - 4\sqrt{16} = 16 - 16 = 0$

The zeros of $G(x) = x - 4\sqrt{x}$ are 0 and 16. The *x*-intercepts of the graph of *G* are 0 and 16.

82. $f(x) = 0$

$x + 8\sqrt{x} = 0$

Let $u = \sqrt{x} \to u^2 = x$

$u^2 + 8u = 0$

$u(u+8) = 0$

$u=0$ or $u+8=0$

$u=-8$

$\sqrt{x}=0$ $\quad$ $\sqrt{x}=-8$

$x = 0^2 = 0$ $\quad$ x = not real

Check: $f(0) = 0 + 8\sqrt{0} = 0$

The only zero of $f(x) = x + 8\sqrt{x}$ is 0. The only *x*-intercept of the graph of *f* is 0.

83. $g(x)=0$

$x+\sqrt{x}-20=0$

Let $u=\sqrt{x} \to u^2=x$

$u^2+u-20=0$

$(u+5)(u-4)=0$

$u+5=0$ or $u-4=0$

$u=-5$ $\quad u=4$

$\sqrt{x}=-5$ $\quad \sqrt{x}=4$

$x=$ not real $\quad x=4^2=16$

Check: $g(16)=16+\sqrt{16}-20=16+4-20=0$

The only zero of $g(x)=x+\sqrt{x}-20$ is 16. The only x-intercept of the graph of g is 16.

84. $f(x)=0$

$x+\sqrt{x}-2=0$

Let $u=\sqrt{x} \to u^2=x$

$u^2+u-2=0$

$(u-1)(u+2)=0$

$u-1=0$ or $u+2=0$

$u=1$ $\quad u=-2$

$\sqrt{x}=1$ $\quad \sqrt{x}=-2$

$x=1^2=1$ $\quad x=$ not real

Check: $f(1)=1+\sqrt{1}-2=1+1-2=0$

The only zero of $f(x)=x+\sqrt{x}-2$ is 1. The only x-intercept of the graph of f is 1.

85. $A(x)=143$

$x(x+2)=143$

$x^2+2x-143=0$

$(x+13)(x-11)=0$

$\cancel{x=-13}$ or $x=11$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 11 feet and the length is 13 feet.

86. $A(x)=306$

$x(x+1)=306$

$x^2+x-306=0$

$(x+18)(x-17)=0$

$\cancel{x=-18}$ or $x=17$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 17 cm and the length is 18 cm.

87. $V(x)=4$

$(x-2)^2=4$

$x-2=\pm\sqrt{4}$

$x-2=\pm 2$

$x=2\pm 2$

$x=4$ or $x=0$

Discard $x=0$ since that is not a feasible length for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

88. $V(x)=4$

$(x-2)^2=16$

$x-2=\pm\sqrt{16}$

$x-2=\pm 4$

$x=2\pm 4$

$x=6$ or $\cancel{x=-2}$

Discard $x=-2$ since width cannot be negative. Therefore, the original sheet should measure 6 feet on each side.

89. **a.** When the ball strikes the ground, the distance from the ground will be 0. Therefore, we solve

$s=0$

$96+80t-16t^2=0$

$-16t^2+80t+96=0$

$t^2-5t-6=0$

$(t-6)(t+1)=0$

$t=6$ or $\cancel{t=-1}$

Discard the negative solution since the time of flight must be positive. The ball will strike the ground after 6 seconds.

b. When the ball passes the top of the building, it will be 96 feet from the ground. Therefore, we solve

$s=96$

$96+80t-16t^2=96$

$-16t^2+80t=0$

$t^2-5t=0$

$t(t-5)=0$

$t=0$ or $t=5$

The ball is at the top of the building at time $t=0$ seconds when it is thrown. It will pass the top of the building on the way down after 5 seconds.

90. a. To find when the object will be 15 meters above the ground, we solve

$$s = 15$$
$$-4.9t^2 + 20t = 15$$
$$-4.9t^2 + 20t - 15 = 0$$
$$a = -4.9,\ b = 20,\ c = -15$$
$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-15)}}{2(-4.9)}$$
$$= \frac{-20 \pm \sqrt{106}}{-9.8}$$
$$= \frac{20 \pm \sqrt{106}}{9.8}$$
$$t \approx 0.99 \quad \text{or} \quad t \approx 3.09$$

The object will be 15 meters above the ground after about 0.99 seconds (on the way up) and about 3.09 seconds (on the way down).

b. The object will strike the ground when the distance from the ground is 0. Thus, we solve

$$s = 0$$
$$-4.9t^2 + 20t = 0$$
$$t(-4.9t + 20) = 0$$
$$t = 0 \quad \text{or} \quad -4.9t + 20 = 0$$
$$-4.9t = -20$$
$$t \approx 4.08$$

The object will strike the ground after about 4.08 seconds.

c.

$$s = 100$$
$$-4.9t^2 + 20t = 100$$
$$-4.9t^2 + 20t - 100 = 0$$
$$a = -4.9,\ b = 20,\ c = -100$$
$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-100)}}{2(-4.9)}$$
$$= \frac{-20 \pm \sqrt{-1560}}{-9.8}$$

There is no real solution. The object never reaches a height of 100 meters.

91. For the sum to be 210, we solve

$$S(n) = 210$$
$$\frac{1}{2}n(n+1) = 210$$
$$n(n+1) = 420$$
$$n^2 + n - 420 = 0$$
$$(n-20)(n+21) = 0$$
$$n - 20 = 0 \quad \text{or} \quad n + 21 = 0$$
$$n = 20 \qquad \cancel{n = -21}$$

Discard the negative solution since the number of consecutive integers must be positive. For a sum of 210, we must add the 20 consecutive integers, starting at 1.

92. To determine the number of sides when a polygon has 65 diagonals, we solve

$$D(n) = 65$$
$$\frac{1}{2}n(n-3) = 65$$
$$n(n-3) = 130$$
$$n^2 - 3n - 130 = 0$$
$$(n+10)(n-13) = 0$$
$$n + 10 = 0 \quad \text{or} \quad n - 13 = 0$$
$$\cancel{n = -10} \qquad n = 13$$

Discard the negative solution since the number of sides must be positive. A polygon with 65 diagonals will have 13 sides.

To determine the number of sides if a polygon has 80 diagonals, we solve

$$D(n) = 80$$
$$\frac{1}{2}n(n-3) = 80$$
$$n(n-3) = 160$$
$$n^2 - 3n - 160 = 0$$
$$a = 1,\ b = -3,\ c = -160$$
$$t = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-160)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{649}}{2}$$

Since the solutions are not integers, a polygon with 80 diagonals is not possible.

93. The roots of a quadratic equation are
$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a},$$
so the sum of the roots is
$$\begin{aligned} x_1 + x_2 &= \frac{-b-\sqrt{b^2-4ac}}{2a} + \frac{-b+\sqrt{b^2-4ac}}{2a} \\ &= \frac{-b-\sqrt{b^2-4ac}-b+\sqrt{b^2-4ac}}{2a} \\ &= \frac{-2b}{2a} = -\frac{b}{a} \end{aligned}$$

94. The roots of a quadratic equation are
$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a},$$
so the product of the roots is
$$\begin{aligned} x_1 \cdot x_2 &= \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) \\ &= \frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{(2a)^2} = \frac{b^2-(b^2-4ac)}{4a^2} \\ &= \frac{b^2-b^2+4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a} \end{aligned}$$

95. In order to have one repeated real zero, we need the discriminant to be 0.
$$\begin{aligned} b^2-4ac &= 0 \\ 1^2-4(k)(k) &= 0 \\ 1-4k^2 &= 0 \\ 4k^2 &= 1 \\ k^2 &= \frac{1}{4} \\ k &= \pm\sqrt{\frac{1}{4}} \\ k &= \frac{1}{2} \quad \text{or} \quad k = -\frac{1}{2} \end{aligned}$$

96. In order to have one repeated real zero, we need the discriminant to be 0.
$$\begin{aligned} b^2-4ac &= 0 \\ (-k)^2-4(1)(4) &= 0 \\ k^2-16 &= 0 \\ (k-4)(k+4) &= 0 \\ k = 4 \quad &\text{or} \quad k = -4 \end{aligned}$$

97. For $f(x) = ax^2+bx+c=0$:
$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$
For $f(x) = ax^2-bx+c=0$:
$$\begin{aligned} x_1^* &= \frac{-(-b)-\sqrt{(-b)^2-4ac}}{2a} \\ &= \frac{b-\sqrt{b^2-4ac}}{2a} = -\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) = -x_2 \end{aligned}$$
and
$$\begin{aligned} x_2^* &= \frac{-(-b)+\sqrt{(-b)^2-4ac}}{2a} \\ &= \frac{b+\sqrt{b^2-4ac}}{2a} = -\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = -x_1 \end{aligned}$$

98. For $f(x) = ax^2+bx+c=0$:
$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$
For $f(x) = cx^2+bx+a=0$:
$$\begin{aligned} x_1^* &= \frac{-b-\sqrt{b^2-4(c)(a)}}{2c} = \frac{-b-\sqrt{b^2-4ac}}{2c} \\ &= \frac{-b-\sqrt{b^2-4ac}}{2c} \cdot \frac{-b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}} \\ &= \frac{b^2-(b^2-4ac)}{2c\left(-b+\sqrt{b^2-4ac}\right)} = \frac{4ac}{2c\left(-b+\sqrt{b^2-4ac}\right)} \\ &= \frac{2a}{-b+\sqrt{b^2-4ac}} = \frac{1}{x_2} \end{aligned}$$
and
$$\begin{aligned} x_2^* &= \frac{-b+\sqrt{b^2-4(c)(a)}}{2c} = \frac{-b+\sqrt{b^2-4ac}}{2c} \\ &= \frac{-b+\sqrt{b^2-4ac}}{2c} \cdot \frac{-b-\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}} \\ &= \frac{b^2-(b^2-4ac)}{2c\left(-b-\sqrt{b^2-4ac}\right)} = \frac{4ac}{2c\left(-b-\sqrt{b^2-4ac}\right)} \\ &= \frac{2a}{-b-\sqrt{b^2-4ac}} = \frac{1}{x_1} \end{aligned}$$

99. **a.** $x^2 = 9$ and $x = 3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x = -3$.

b. $x = \sqrt{9}$ and $x = 3$ are equivalent because $\sqrt{9} = 3$.

c. $(x-1)(x-2) = (x-1)^2$ and $x - 2 = x - 1$ are not equivalent because they do not have the same solution set. The first equation has the solution set $\{1\}$ while the second equation has no solutions.

100. Answers may vary. Methods discussed in this section include factoring, the square root method, completing the square, and the quadratic formula.

101. Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.

102. Answers will vary. One possibility:
Two distinct: $f(x) = x^2 - 3x - 18$
One repeated: $f(x) = x^2 - 14x + 49$
No real: $f(x) = x^2 + x + 4$

103. Answers will vary.

104. Two quadratic functions can intersect 0, 1, or 2 times.

Section 2.4

1. To find the y-intercept, let $x = 0$:
$y = 0^2 - 9 = -9$.
To find the x-intercept(s), let $y = 0$:

$$x^2 - 9 = 0$$
$$x^2 = 9$$
$$x = \pm\sqrt{9} = \pm 3$$

The intercepts are $(0,-9), (-3, 0),$ and $(3, 0)$.

2. To find the y-intercept, let $x = 0$:
$y = 2(0)^2 + 7(0) - 4 = -4$.
To find the x-intercept(s), let $y = 0$:

$$2x^2 + 7x - 4 = 0$$
$$(2x-1)(x+4) = 0$$
$$2x - 1 = 0 \quad \text{or} \quad x + 4 = 0$$
$$2x = 1 \qquad x = -4$$
$$x = \frac{1}{2}$$

The intercepts are $(0,-4)$, $\left(\frac{1}{2},0\right)$, and $(-4,0)$.

3. $\left[\frac{1}{2}(-5)\right]^2 = \frac{25}{4}$

4. right; 4

5. parabola

6. axis (or axis of symmetry)

7. $-\frac{b}{2a}$

8. True; $a = 2 > 0$.

9. True; $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$

10. True

11. C

12. E

13. F

14. A

15. G

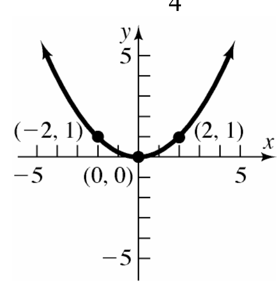
16. B

17. H

18. D

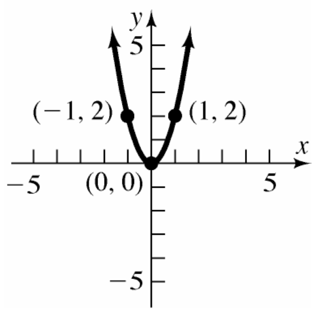
19. $f(x) = \frac{1}{4}x^2$

Using the graph of $y = x^2$, compress vertically by a factor of $\frac{1}{4}$.



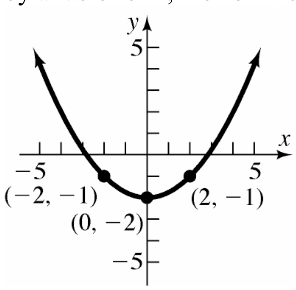
20. $f(x) = 2x^2$

Using the graph of $y = x^2$, stretch vertically by a factor of 2.



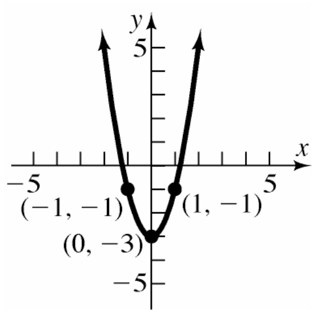
21. $f(x) = \frac{1}{4}x^2 - 2$

Using the graph of $y = x^2$, compress vertically by a factor of 2, then shift down 2 units.



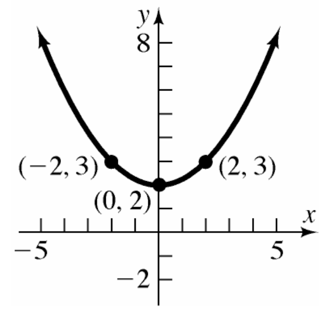
22. $f(x) = 2x^2 - 3$

Using the graph of $y = x^2$, stretch vertically by a factor of 2, then shift down 3 units.



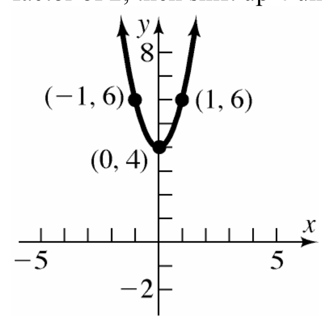
23. $f(x) = \frac{1}{4}x^2 + 2$

Using the graph of $y = x^2$, compress vertically by a factor of $\frac{1}{4}$, then shift up 2 units.



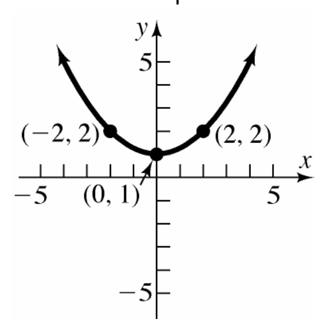
24. $f(x) = 2x^2 + 4$

Using the graph of $y = x^2$, stretch vertically by a factor of 2, then shift up 4 units.



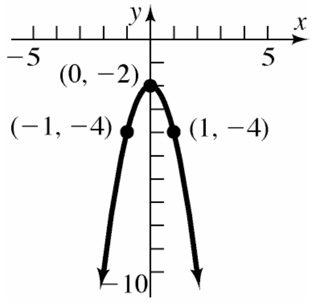
25. $f(x) = \frac{1}{4}x^2 + 1$

Using the graph of $y = x^2$, compress vertically by a factor of $\frac{1}{4}$, then shift up 1 unit.



26. $f(x) = -2x^2 - 2$

Using the graph of $y = x^2$, stretch vertically by a factor of 2, reflect across the *x*-axis, then shift down 2 units.

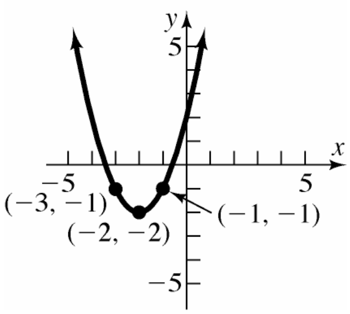


27. $f(x) = x^2 + 4x + 2$

$= (x^2 + 4x + 4) + 2 - 4$

$= (x + 2)^2 - 2$

Using the graph of $y = x^2$, shift left 2 units, then shift down 2 units.

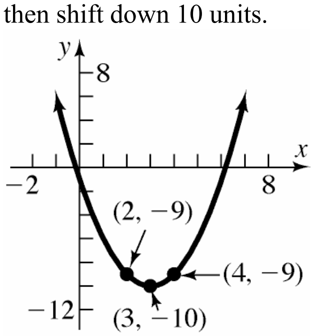


28. $f(x) = x^2 - 6x - 1$

$= (x^2 - 6x + 9) - 1 - 9$

$= (x - 3)^2 - 10$

Using the graph of $y = x^2$, shift right 3 units, then shift down 10 units.



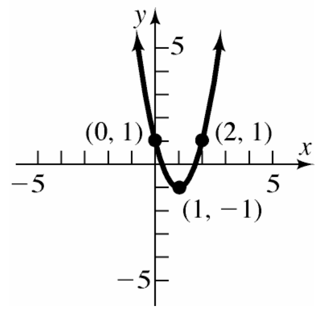
29. $f(x) = 2x^2 - 4x + 1$

$= 2\left(x^2 - 2x\right) + 1$

$= 2(x^2 - 2x + 1) + 1 - 2$

$= 2(x - 1)^2 - 1$

Using the graph of $y = x^2$, shift right 1 unit, stretch vertically by a factor of 2, then shift down 1 unit.



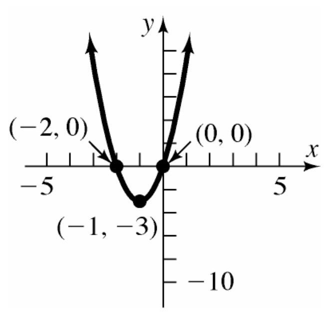
30. $f(x) = 3x^2 + 6x$

$= 3\left(x^2 + 2x\right)$

$= 3(x^2 + 2x + 1) - 3$

$= 3(x + 1)^2 - 3$

Using the graph of $y = x^2$, shift left 1 unit, stretch vertically by a factor of 3, then shift down 3 units.



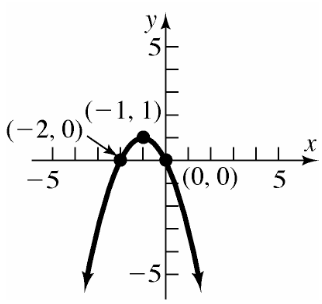
31. $f(x) = -x^2 - 2x$

$= -\left(x^2 + 2x\right)$

$= -(x^2 + 2x + 1) + 1$

$= -(x + 1)^2 + 1$

Using the graph of $y = x^2$, shift left 1 unit, reflect across the *x*-axis, then shift up 1 unit.



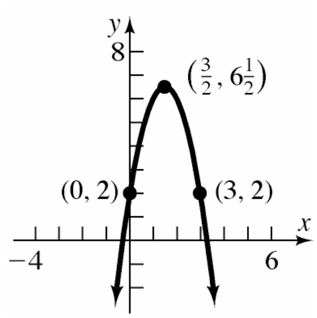
32. $f(x) = -2x^2 + 6x + 2$

$$= -2\left(x^2 - 3x\right) + 2$$

$$= -2\left(x^2 - 3x + \frac{9}{4}\right) + 2 + \frac{9}{2}$$

$$= -2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$$

Using the graph of $y = x^2$, shift right $\frac{3}{2}$ units, reflect about the *x*-axis, stretch vertically by a factor of 2, then shift up $\frac{13}{2}$ units.



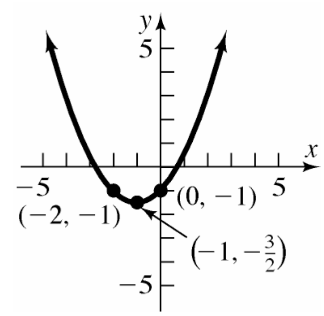
33. $f(x) = \frac{1}{2}x^2 + x - 1$

$$= \frac{1}{2}\left(x^2 + 2x\right) - 1$$

$$= \frac{1}{2}\left(x^2 + 2x + 1\right) - 1 - \frac{1}{2}$$

$$= \frac{1}{2}\left(x + 1\right)^2 - \frac{3}{2}$$

Using the graph of $y = x^2$, shift left 1 unit, compress vertically by a factor of $\frac{1}{2}$, then shift down $\frac{3}{2}$ units.



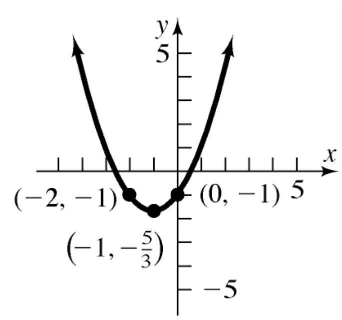
34. $f(x) = \frac{2}{3}x^2 + \frac{4}{3}x - 1$

$$= \frac{2}{3}\left(x^2 + 2x\right) - 1$$

$$= \frac{2}{3}\left(x^2 + 2x + 1\right) - 1 - \frac{2}{3}$$

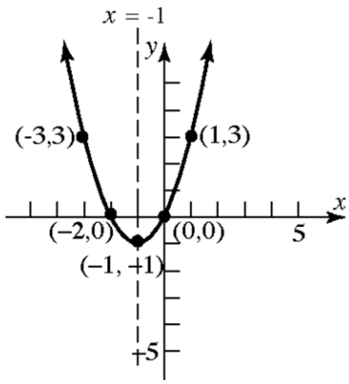
$$= \frac{2}{3}\left(x + 1\right)^2 - \frac{5}{3}$$

Using the graph of $y = x^2$, shift left 1 unit, compress vertically by a factor of $\frac{2}{3}$, then shift down $\frac{5}{3}$ unit.



35. a. For $f(x) = x^2 + 2x$, $a = 1$, $b = 2$, $c = 0$.
Since $a = 1 > 0$, the graph opens up.
The *x*-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-(2)}{2(1)} = \frac{-2}{2} = -1$.
The *y*-coordinate of the vertex is
$f\left(\frac{-b}{2a}\right) = f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1.$
Thus, the vertex is $(-1, -1)$.
The axis of symmetry is the line $x = -1$.
The discriminant is
$b^2 - 4ac = (2)^2 - 4(1)(0) = 4 > 0$, so the graph has two *x*-intercepts.
The *x*-intercepts are found by solving:

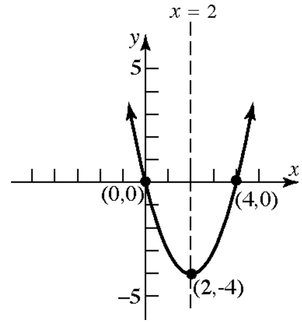
$x^2 + 2x = 0$
$x(x+2) = 0$
$x = 0$ or $x = -2$
The x-intercepts are –2 and 0 .
The y-intercept is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$. The range is $[-1, \infty)$.

c. Increasing on $(-1, \infty)$; decreasing on $(-\infty, -1)$.

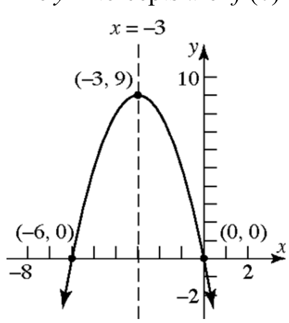
36. a. For $f(x) = x^2 - 4x$, $a = 1$, $b = -4$, $c = 0$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \dfrac{-b}{2a} = \dfrac{-(-4)}{2(1)} = \dfrac{4}{2} = 2$.
The y-coordinate of the vertex is
$f\left(\dfrac{-b}{2a}\right) = f(2) = (2)^2 - 4(2) = 4 - 8 = -4.$
Thus, the vertex is $(2, -4)$.
The axis of symmetry is the line $x = 2$.
The discriminant is:
$b^2 - 4ac = (-4)^2 - 4(1)(0) = 16 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2 - 4x = 0$
$x(x-4) = 0$
$x = 0$ or $x = 4$.
The x-intercepts are 0 and 4.
The y-intercept is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$. The range is $[-4, \infty)$.

c. Increasing on $(2, \infty)$; decreasing on $(-\infty, 2)$.

37. a. For $f(x) = -x^2 - 6x$, $a = -1$, $b = -6$, $c = 0$. Since $a = -1 < 0,$ the graph opens down. The x-coordinate of the vertex is
$x = \dfrac{-b}{2a} = \dfrac{-(-6)}{2(-1)} = \dfrac{6}{-2} = -3.$
The y-coordinate of the vertex is
$f\left(\dfrac{-b}{2a}\right) = f(-3) = -(-3)^2 - 6(-3)$
$= -9 + 18 = 9.$
Thus, the vertex is $(-3, 9)$.
The axis of symmetry is the line $x = -3$.
The discriminant is:
$b^2 - 4ac = (-6)^2 - 4(-1)(0) = 36 > 0$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$-x^2 - 6x = 0$
$-x(x+6) = 0$
$x = 0$ or $x = -6.$
The x-intercepts are -6 and 0 .
The y-intercepts are $f(0) = 0$.



b. The domain is $(-\infty, \infty)$. The range is $(-\infty, 9]$.

c. Increasing on $(-\infty, -3)$; decreasing on $(-3, \infty)$.

38. a. For $f(x) = -x^2 + 4x,\ \ a = -1,\ \ b = 4$, $c = 0$.

Since $a = -1 < 0$, the graph opens down.

The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(2)$$
$$= -(2)^2 + 4(2)$$
$$= 4.$$

Thus, the vertex is $(2, 4)$.

The axis of symmetry is the line $x = 2$.

The discriminant is:

$$b^2 - 4ac = 4^2 - 4(-1)(0) = 16 > 0,$$

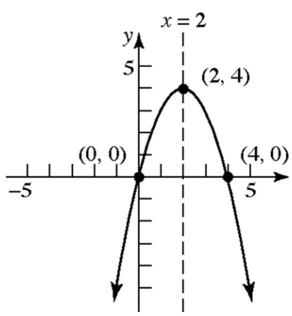
so the graph has two x-intercepts.

The x-intercepts are found by solving:

$$-x^2 + 4x = 0$$
$$-x(x-4) = 0$$
$$x = 0 \text{ or } x = 4.$$

The x-intercepts are 0 and 4.

The y-intercept is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$. The range is $(-\infty, 4]$.

c. Increasing on $(-\infty, 2)$; decreasing on $(2, \infty)$.

39. a. For $f(x) = 2x^2 - 8x,\ \ a = 2,\ b = -8,\ c = 0$.

Since $a = 2 > 0$, the graph opens up.

The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(2) = 2(2)^2 - 8(2) = 8 - 16 = -8.$$

Thus, the vertex is $(2, -8)$.

The axis of symmetry is the line $x = 2$.

The discriminant is:

$$b^2 - 4ac = (-8)^2 - 4(2)(0) = 64 > 0,$$

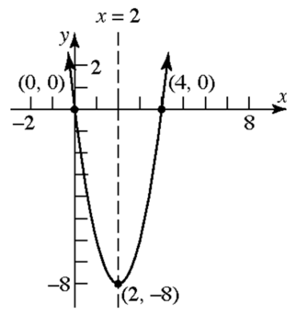
so the graph has two x-intercepts.

The x-intercepts are found by solving:

$$2x^2 - 8x = 0$$
$$2x(x-4) = 0$$
$$x = 0 \text{ or } x = 4.$$

The x-intercepts are 0 and 4.

The y-intercepts is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$. The range is $[-8, \infty)$.

c. Increasing on $(2, \infty)$; decreasing on $(-\infty, 2)$.

40. a. For $f(x) = 3x^2 + 18x,\ \ a = 3,\ \ b = 18,\ \ c = 0$.

Since $a = 3 > 0$, the graph opens up.

The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-18}{2(3)} = \frac{-18}{6} = -3.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(-3)$$
$$= 3(-3)^2 + 18(-3)$$
$$= 27 - 54$$
$$= -27.$$

Thus, the vertex is $(-3, -27)$.

The axis of symmetry is the line $x = -3$.

The discriminant is:

$$b^2 - 4ac = (18)^2 - 4(3)(0) = 324 > 0,$$

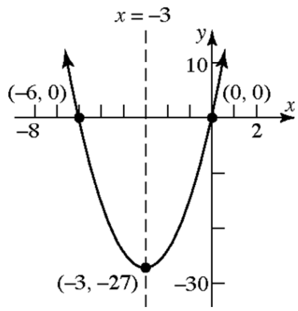
so the graph has two x-intercepts.

The x-intercepts are found by solving:

$$3x^2 + 18x = 0$$
$$3x(x+6) = 0$$
$$x = 0 \text{ or } x = -6.$$

The x-intercepts are 0 and -6.

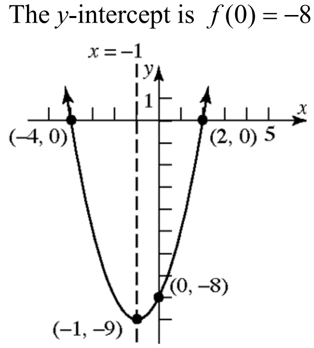
The y-intercept is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$. The range is $[-27, \infty)$.

c. Increasing on $(-3, \infty)$; decreasing on $(-\infty, -3)$.

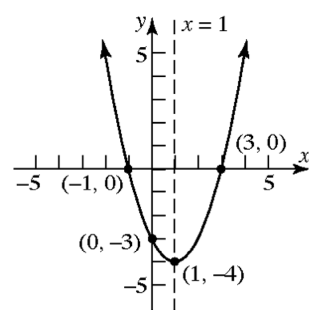
41. a. For $f(x) = x^2 + 2x - 8$, $a = 1$, $b = 2$, $c = -8$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \dfrac{-b}{2a} = \dfrac{-2}{2(1)} = \dfrac{-2}{2} = -1$.
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(-1)$$
$$= (-1)^2 + 2(-1) - 8 = 1 - 2 - 8 = -9.$$
Thus, the vertex is $(-1, -9)$.
The axis of symmetry is the line $x = -1$.
The discriminant is:
$b^2 - 4ac = 2^2 - 4(1)(-8) = 4 + 32 = 36 > 0$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2 + 2x - 8 = 0$
$(x+4)(x-2) = 0$
$x = -4$ or $x = 2$.
The x-intercepts are -4 and 2.
The y-intercept is $f(0) = -8$.



b. The domain is $(-\infty, \infty)$. The range is $[-9, \infty)$.

c. Increasing on $(-1, \infty)$; decreasing on $(-\infty, -1)$.

42. a. For $f(x) = x^2 - 2x - 3$, $a = 1$, $b = -2$, $c = -3$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \dfrac{-b}{2a} = \dfrac{-(-2)}{2(1)} = \dfrac{2}{2} = 1$.
The y-coordinate of the vertex is
$f\left(\dfrac{-b}{2a}\right) = f(1) = 1^2 - 2(1) - 3 = -4$.
Thus, the vertex is $(1, -4)$.
The axis of symmetry is the line $x = 1$.
The discriminant is:
$b^2 - 4ac = (-2)^2 - 4(1)(-3) = 4 + 12 = 16 > 0$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$x = -1$ or $x = 3$.
The x-intercepts are -1 and 3.
The y-intercept is $f(0) = -3$.



b. The domain is $(-\infty, \infty)$. The range is $[-4, \infty)$.

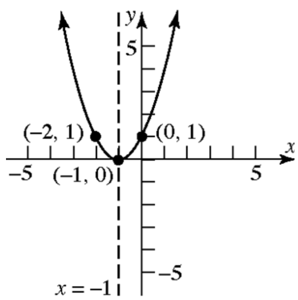
c. Increasing on $(1, \infty)$; decreasing on $(-\infty, 1)$.

43. a. For $f(x) = x^2 + 2x + 1$, $a = 1$, $b = 2$, $c = 1$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \dfrac{-b}{2a} = \dfrac{-2}{2(1)} = \dfrac{-2}{2} = -1$.
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(-1)$$
$$= (-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0.$$
Thus, the vertex is $(-1, 0)$.
The axis of symmetry is the line $x = -1$.
The discriminant is:
$b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0$,

so the graph has one x-intercept.
The x-intercept is found by solving:

$$x^2+2x+1=0$$
$$(x+1)^2=0$$
$$x=-1.$$

The x-intercept is -1.
The y-intercept is $f(0)=1$.



b. The domain is $(-\infty,\infty)$. The range is $[0,\infty)$.

c. Increasing on $(-1,\infty)$; decreasing on $(-\infty,-1)$.

44. a. For $f(x)=x^2+6x+9$, $a=1$, $b=6$, $c=9$.
Since $a=1>0$, the graph opens up.
The x-coordinate of the vertex is

$$x=\frac{-b}{2a}=\frac{-6}{2(1)}=\frac{-6}{2}=-3.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right)=f(-3)$$
$$=(-3)^2+6(-3)+9=9-18+9=0.$$

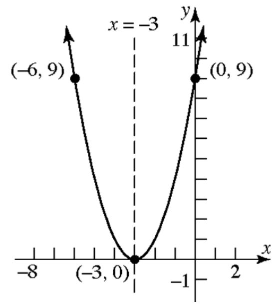
Thus, the vertex is $(-3,0)$.
The axis of symmetry is the line $x=-3$.
The discriminant is:

$$b^2-4ac=6^2-4(1)(9)=36-36=0,$$

so the graph has one x-intercept.
The x-intercept is found by solving:

$$x^2+6x+9=0$$
$$(x+3)^2=0$$
$$x=-3.$$

The x-intercept is -3.
The y-intercept is $f(0)=9$.



b. The domain is $(-\infty,\infty)$. The range is $[0,\infty)$.

c. Increasing on $(-3,\infty)$; decreasing on $(-\infty,-3)$.

45. a. For $f(x)=2x^2-x+2$, $a=2$, $b=-1$, $c=2$.
Since $a=2>0$, the graph opens up.
The x-coordinate of the vertex is

$$x=\frac{-b}{2a}=\frac{-(-1)}{2(2)}=\frac{1}{4}.$$

The y-coordinate of the vertex is

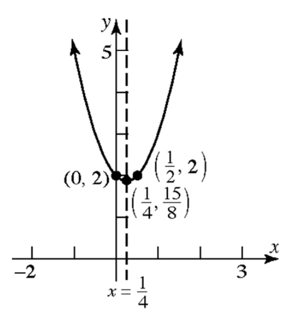
$$f\left(\frac{-b}{2a}\right)=f\left(\frac{1}{4}\right)=2\left(\frac{1}{4}\right)^2-\frac{1}{4}+2$$
$$=\frac{1}{8}-\frac{1}{4}+2=\frac{15}{8}.$$

Thus, the vertex is $\left(\frac{1}{4},\frac{15}{8}\right)$.

The axis of symmetry is the line $x=\frac{1}{4}$.
The discriminant is:

$$b^2-4ac=(-1)^2-4(2)(2)=1-16=-15,$$

so the graph has no x-intercepts.
The y-intercept is $f(0)=2$.



b. The domain is $(-\infty,\infty)$. The range is $\left[\frac{15}{8},\infty\right)$.

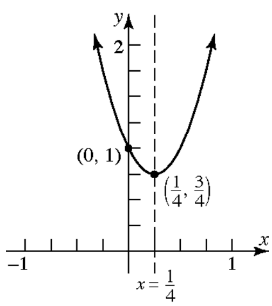
c. Increasing on $\left(\frac{1}{4},\infty\right)$; decreasing on $\left(-\infty,\frac{1}{4}\right)$.

46. a. For $f(x) = 4x^2 - 2x + 1$, $a = 4$, $b = -2$, $c = 1$.
Since $a = 4 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-(-2)}{2(4)} = \frac{2}{8} = \frac{1}{4}$.
The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right) + 1$$
$$= \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}.$$

Thus, the vertex is $\left(\frac{1}{4}, \frac{3}{4}\right)$.

The axis of symmetry is the line $x = \frac{1}{4}$.
The discriminant is:
$b^2 - 4ac = (-2)^2 - 4(4)(1) = 4 - 16 = -12$,
so the graph has no x-intercepts.
The y-intercept is $f(0) = 1$.



b. The domain is $(-\infty, \infty)$. The range is $\left[\frac{3}{4}, \infty\right)$.

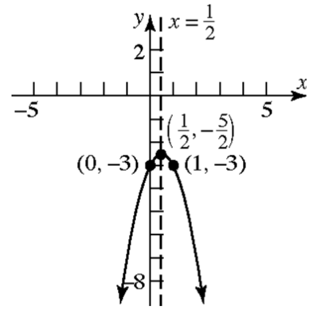
c. Increasing on $\left(\frac{1}{4}, \infty\right)$; decreasing on $\left(-\infty, \frac{1}{4}\right)$.

47. a. For $f(x) = -2x^2 + 2x - 3$, $a = -2$, $b = 2$, $c = -3$. Since $a = -2 < 0$, the graph opens down.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-(2)}{2(-2)} = \frac{-2}{-4} = \frac{1}{2}$.
The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 3$$
$$= -\frac{1}{2} + 1 - 3 = -\frac{5}{2}.$$

Thus, the vertex is $\left(\frac{1}{2}, -\frac{5}{2}\right)$.

The axis of symmetry is the line $x = \frac{1}{2}$.
The discriminant is:
$b^2 - 4ac = 2^2 - 4(-2)(-3) = 4 - 24 = -20$,
so the graph has no x-intercepts.
The y-intercept is $f(0) = -3$.



b. The domain is $(-\infty, \infty)$. The range is $\left(-\infty, -\frac{5}{2}\right]$.

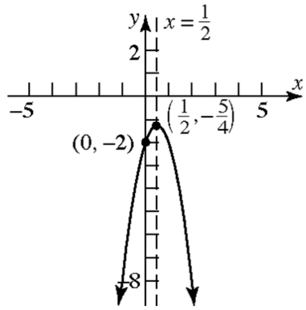
c. Increasing on $\left(-\infty, \frac{1}{2}\right)$; decreasing on $\left(\frac{1}{2}, \infty\right)$.

48. a. For $f(x) = -3x^2 + 3x - 2$, $a = -3$, $b = 3$, $c = -2$. Since $a = -3 < 0$, the graph opens down.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-3}{2(-3)} = \frac{-3}{-6} = \frac{1}{2}$.
The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = -3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2$$
$$= -\frac{3}{4} + \frac{3}{2} - 2 = -\frac{5}{4}.$$

Thus, the vertex is $\left(\frac{1}{2}, -\frac{5}{4}\right)$.

The axis of symmetry is the line $x = \frac{1}{2}$.
The discriminant is:
$b^2 - 4ac = 3^2 - 4(-3)(-2) = 9 - 24 = -15$,
so the graph has no x-intercepts.
The y-intercept is $f(0) = -2$.



b. The domain is $(-\infty, \infty)$. The range is $\left(-\infty, -\frac{5}{4}\right]$.

c. Increasing on $\left(-\infty, \frac{1}{2}\right)$; decreasing on $\left(\frac{1}{2}, \infty\right)$.

49. a. For $f(x) = 3x^2 + 6x + 2$, $a = 3$, $b = 6$, $c = 2$. Since $a = 3 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-6}{2(3)} = \frac{-6}{6} = -1.$$

The y-coordinate of the vertex is

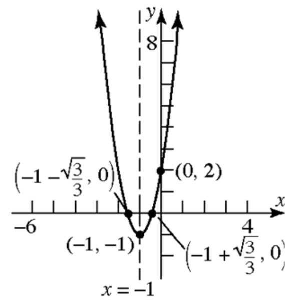
$$f\left(\frac{-b}{2a}\right) = f(-1) = 3(-1)^2 + 6(-1) + 2$$
$$= 3 - 6 + 2 = -1.$$

Thus, the vertex is $(-1, -1)$.
The axis of symmetry is the line $x = -1$.
The discriminant is:
$b^2 - 4ac = 6^2 - 4(3)(2) = 36 - 24 = 12$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$3x^2 + 6x + 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6 \pm \sqrt{12}}{6} = \frac{-6 \pm 2\sqrt{3}}{6} = \frac{-3 \pm \sqrt{3}}{3}$$

The x-intercepts are $-1 - \frac{\sqrt{3}}{3}$ and $-1 + \frac{\sqrt{3}}{3}$.

The y-intercept is $f(0) = 2$.



b. The domain is $(-\infty, \infty)$. The range is $[-1, \infty)$.

c. Increasing on $(-1, \infty)$; decreasing on $(-\infty, -1)$.

50. a. For $f(x) = 2x^2 + 5x + 3$, $a = 2$, $b = 5$, $c = 3$. Since $a = 2 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-5}{2(2)} = -\frac{5}{4}.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{5}{4}\right)$$
$$= 2\left(-\frac{5}{4}\right)^2 + 5\left(-\frac{5}{4}\right) + 3$$
$$= \frac{25}{8} - \frac{25}{4} + 3$$
$$= -\frac{1}{8}.$$

Thus, the vertex is $\left(-\frac{5}{4}, -\frac{1}{8}\right)$.

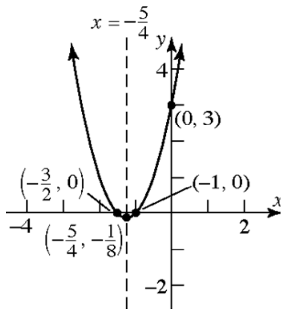
The axis of symmetry is the line $x = -\frac{5}{4}$.

The discriminant is:
$b^2 - 4ac = 5^2 - 4(2)(3) = 25 - 24 = 1$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$2x^2 + 5x + 3 = 0$$
$$(2x + 3)(x + 1) = 0$$
$$x = -\frac{3}{2} \text{ or } x = -1.$$

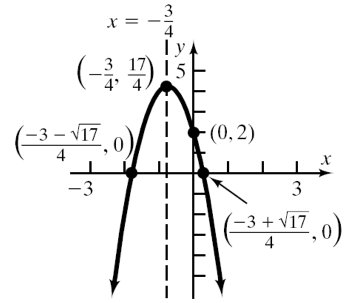
The x-intercepts are $-\frac{3}{2}$ and -1.

The y-intercept is $f(0) = 3$.



b. The domain is $(-\infty, \infty)$. The range is $\left[-\frac{1}{8}, \infty\right)$.

c. Increasing on $\left(-\frac{5}{4}, \infty\right)$; decreasing on $\left(-\infty, -\frac{5}{4}\right)$.



b. The domain is $(-\infty, \infty)$. The range is $\left(-\infty, \frac{17}{4}\right]$.

c. Increasing on $\left(-\infty, -\frac{3}{4}\right)$; decreasing on $\left(-\frac{3}{4}, \infty\right)$.

51. a. For $f(x) = -4x^2 - 6x + 2$, $a = -4$, $b = -6$, $c = 2$. Since $a = -4 < 0$, the graph opens down.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-6)}{2(-4)} = \frac{6}{-8} = -\frac{3}{4}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{3}{4}\right) = -4\left(-\frac{3}{4}\right)^2 - 6\left(-\frac{3}{4}\right) + 2$$
$$= -\frac{9}{4} + \frac{9}{2} + 2 = \frac{17}{4}.$$
Thus, the vertex is $\left(-\frac{3}{4}, \frac{17}{4}\right)$.

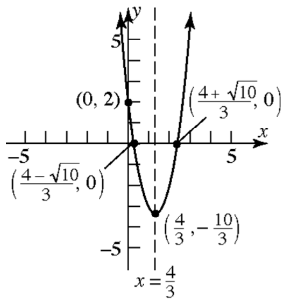
The axis of symmetry is the line $x = -\frac{3}{4}$.

The discriminant is:
$b^2 - 4ac = (-6)^2 - 4(-4)(2) = 36 + 32 = 68$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$-4x^2 - 6x + 2 = 0$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{68}}{2(-4)}$$
$$= \frac{6 \pm \sqrt{68}}{-8} = \frac{6 \pm 2\sqrt{17}}{-8} = \frac{3 \pm \sqrt{17}}{-4}$$
The x-intercepts are $\frac{-3+\sqrt{17}}{4}$ and $\frac{-3-\sqrt{17}}{4}$.
The y-intercept is $f(0) = 2$.

52. a. For $f(x) = 3x^2 - 8x + 2$, $a = 3$, $b = -8$, $c = 2$. Since $a = 3 > 0$, the graph opens up.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-8)}{2(3)} = \frac{8}{6} = \frac{4}{3}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 2$$
$$= \frac{16}{3} - \frac{32}{3} + 2 = -\frac{10}{3}.$$
Thus, the vertex is $\left(\frac{4}{3}, -\frac{10}{3}\right)$.

The axis of symmetry is the line $x = \frac{4}{3}$.

The discriminant is:
$b^2 - 4ac = (-8)^2 - 4(3)(2) = 64 - 24 = 40$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$3x^2 - 8x + 2 = 0$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{40}}{2(3)}$$
$$= \frac{8 \pm \sqrt{40}}{6} = \frac{8 \pm 2\sqrt{10}}{6} = \frac{4 \pm \sqrt{10}}{3}$$
The x-intercepts are $\frac{4+\sqrt{10}}{3}$ and $\frac{4-\sqrt{10}}{3}$.
The y-intercept is $f(0) = 2$.



b. The domain is $(-\infty, \infty)$. The range is $\left[-\frac{10}{3}, \infty\right)$.

c. Increasing on $\left(\frac{4}{3}, \infty\right)$; decreasing on $\left(-\infty, \frac{4}{3}\right)$.

53. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-1,-2)$ so we have $h = -1$ and $k = -2$. The graph also passes through the point $(x,y) = (0,-1)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$-1 = a(0-(-1))^2 + (-2)$

$-1 = a(1)^2 - 2$

$-1 = a - 2$

$1 = a$

The quadratic function is

$f(x) = (x+1)^2 - 2 = x^2 + 2x - 1$.

54. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(2,1)$ so we have $h = 2$ and $k = 1$. The graph also passes through the point $(x,y) = (0,5)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$5 = a(0-2)^2 + 1$

$5 = a(-2)^2 + 1$

$5 = 4a + 1$

$4 = 4a$

$1 = a$

The quadratic function is

$f(x) = (x-2)^2 + 1 = x^2 - 4x + 5$.

55. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-3,5)$ so we have $h = -3$ and $k = 5$. The graph also passes through the point $(x,y) = (0,-4)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$-4 = a(0-(-3))^2 + 5$

$-4 = a(3)^2 + 5$

$-4 = 9a + 5$

$-9 = 9a$

$-1 = a$

The quadratic function is

$f(x) = -(x+3)^2 + 5 = -x^2 - 6x - 4$.

56. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(2,3)$ so we have $h = 2$ and $k = 3$. The graph also passes through the point $(x,y) = (0,-1)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$-1 = a(0-2)^2 + 3$

$-1 = a(-2)^2 + 3$

$-1 = 4a + 3$

$-4 = 4a$

$-1 = a$

The quadratic function is

$f(x) = -(x-2)^2 + 3 = -x^2 + 4x - 1$.

57. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(1,-3)$ so we have $h = 1$ and $k = -3$. The graph also passes through the point $(x,y) = (3,5)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$5 = a(3-1)^2 + (-3)$

$5 = a(2)^2 - 3$

$5 = 4a - 3$

$8 = 4a$

$2 = a$

The quadratic function is

$f(x) = 2(x-1)^2 - 3 = 2x^2 - 4x - 1$.

58. Consider the form $y = a\left(x-h\right)^2 + k$. From the graph we know that the vertex is $\left(-2,6\right)$ so we have $h=-2$ and $k=6$. The graph also passes through the point $\left(x,y\right) = \left(-4,-2\right)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$$-2 = a\left(-4-(-2)\right)^2 + 6$$
$$-2 = a\left(-2\right)^2 + 6$$
$$-2 = 4a + 6$$
$$-8 = 4a$$
$$-2 = a$$

The quadratic function is
$f\left(x\right) = -2\left(x+2\right)^2 + 6 = -2x^2 - 8x - 2$.

59. For $f(x) = 2x^2 + 12x$, $a=2$, $b=12$, $c=0$. Since $a = 2 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \dfrac{-b}{2a} = \dfrac{-12}{2(2)} = \dfrac{-12}{4} = -3.$

The minimum value is
$f(-3) = 2(-3)^2 + 12(-3) = 18 - 36 = -18$.

60. For $f(x) = -2x^2 + 12x$, $a=-2$, $b=12$, $c=0$,. Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \dfrac{-b}{2a} = \dfrac{-12}{2(-2)} = \dfrac{-12}{-4} = 3.$

The maximum value is
$f(3) = -2(3)^2 + 12(3) = -18 + 36 = 18$.

61. For $f(x) = 2x^2 + 12x - 3$, $a=2$, $b=12$, $c=-3$. Since $a = 2 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \dfrac{-b}{2a} = \dfrac{-12}{2(2)} = \dfrac{-12}{4} = -3.$ The minimum value is
$f(-3) = 2(-3)^2 + 12(-3) - 3 = 18 - 36 - 3 = -21$.

62. For $f(x) = 4x^2 - 8x + 3$, $a=4$, $b=-8$, $c=3$. Since $a = 4 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \dfrac{-b}{2a} = \dfrac{-(-8)}{2(4)} = \dfrac{8}{8} = 1.$ The minimum value is
$f(1) = 4(1)^2 - 8(1) + 3 = 4 - 8 + 3 = -1$.

63. For $f(x) = -x^2 + 10x - 4$, $a=-1$, $b=10$, $c=-4$. Since $a = -1 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \dfrac{-b}{2a} = \dfrac{-10}{2(-1)} = \dfrac{-10}{-2} = 5$.

The maximum value is
$f\left(5\right) = -\left(5\right)^2 + 10\left(5\right) - 4 = -25 + 50 - 4 = 21$.

64. For $f(x) = -2x^2 + 8x + 3$, $a=-2$, $b=8$, $c=3$. Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \dfrac{-b}{2a} = \dfrac{-8}{2(-2)} = \dfrac{-8}{-4} = 2$. The maximum value is
$f\left(2\right) = -2\left(2\right)^2 + 8\left(2\right) + 3 = -8 + 16 + 3 = 11$.

65. For $f(x) = -3x^2 + 12x + 1$, $a=-3$, $b=12$, $c=1$. Since $a = -3 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \dfrac{-b}{2a} = \dfrac{-12}{2(-3)} = \dfrac{-12}{-6} = 2$.

The maximum value is
$f\left(2\right) = -3\left(2\right)^2 + 12\left(2\right) + 1 = -12 + 24 + 1 = 13$.

66. For $f(x) = 4x^2 - 4x$, $a=4$, $b=-4$, $c=0$. Since $a = 4 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \dfrac{-b}{2a} = \dfrac{-(-4)}{2(4)} = \dfrac{4}{8} = \dfrac{1}{2}$. The minimum value is

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) = 1 - 2 = -1.$$

67. Use the form $f\left(x\right) = a\left(x-h\right)^2 + k$.
The vertex is $(0,2)$, so $h = 0$ and $k = 2$.
$f\left(x\right) = a\left(x-0\right)^2 + 2 = ax^2 + 2$.
Since the graph passes through $(1,8)$, $f(1) = 8$.

$$f\left(x\right) = ax^2 + 2$$
$$8 = a\left(1\right)^2 + 2$$
$$8 = a + 2$$
$$6 = a$$

$f\left(x\right) = 6x^2 + 2$.
$a = 6, b = 0, c = 2$

68. Use the form $f(x) = a(x-h)^2 + k$.

The vertex is $(1, 4)$, so $h = 1$ and $k = 4$.

$f(x) = a(x-1)^2 + 4$.

Since the graph passes through $(-1, -8)$,

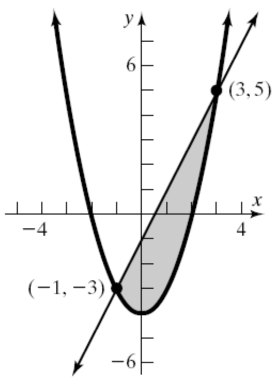
$f(-1) = -8$.

$$\begin{aligned} -8 &= a(-1-1)^2 + 4 \\ -8 &= a(-2)^2 + 4 \\ -8 &= 4a + 4 \\ -12 &= 4a \\ -3 &= a \end{aligned}$$

$$\begin{aligned} f(x) &= -3(x-1)^2 + 4 \\ &= -3(x^2 - 2x + 1) + 4 \\ &= -3x^2 + 6x - 3 + 4 \\ &= -3x^2 + 6x + 1 \end{aligned}$$

$a = -3,\ b = 6,\ c = 1$

69. **a** and **d.**



b. $f(x) = g(x)$

$$\begin{aligned} 2x - 1 &= x^2 - 4 \\ 0 &= x^2 - 2x - 3 \\ 0 &= (x+1)(x-3) \end{aligned}$$

$x + 1 = 0$ or $x - 3 = 0$

$x = -1 \qquad x = 3$

c. $f(-1) = 2(-1) - 1 = -2 - 1 = -3$

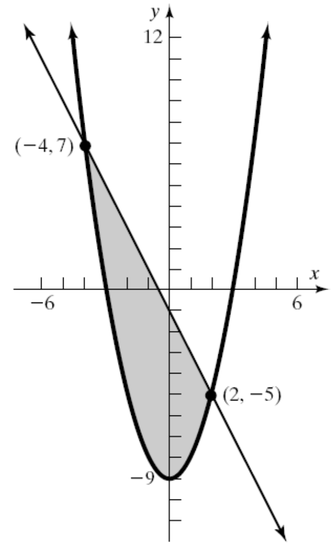
$g(-1) = (-1)^2 - 4 = 1 - 4 = -3$

$f(3) = 2(3) - 1 = 6 - 1 = 5$

$g(3) = 3^2 - 4 = 9 - 4 = 5$

Thus, the graphs of f and g intersect at the points $(-1, -3)$ and $(3, 5)$.

70. **a** and **d.**



b. $f(x) = g(x)$

$$\begin{aligned} -2x - 1 &= x^2 - 9 \\ 0 &= x^2 + 2x - 8 \\ 0 &= (x+4)(x-2) \end{aligned}$$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4 \qquad x = 2$

c. $f(-4) = -2(-4) - 1 = 8 - 1 = 7$

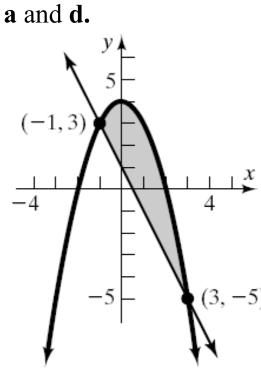
$g(-4) = (-4)^2 - 9 = 16 - 9 = 7$

$f(2) = -2(2) - 1 = -4 - 1 = -5$

$g(2) = 2^2 - 9 = 4 - 9 = -5$

Thus, the graphs of f and g intersect at the points $(-4, 7)$ and $(2, -5)$.

71. **a** and **d.**



b. $f(x)=g(x)$

$-x^2+4=-2x+1$

$0=x^2-2x-3$

$0=(x+1)(x-3)$

$x+1=0$ or $x-3=0$

$x=-1$ $\quad x=3$

c. $f(1)=-(-1)^2+4=-1+4=3$

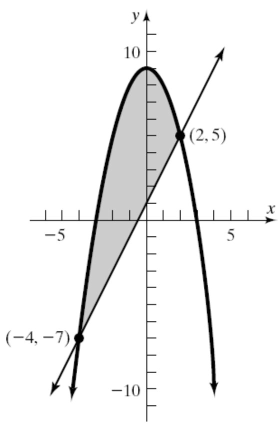
$g(1)=-2(-1)+1=2+1=3$

$f(3)=-(3)^2+4=-9+4=-5$

$g(3)=-2(3)+1=-6+1=-5$

Thus, the graphs of *f* and *g* intersect at the points $(-1,\ 3)$ and $(3,-5)$.

72. a and **d.**



b. $f(x)=g(x)$

$-x^2+9=2x+1$

$0=x^2+2x-8$

$0=(x+4)(x-2)$

$x+4=0$ or $x-2=0$

$x=-4$ $\quad x=2$

c. $f(-4)=-(-4)^2+9=-16+9=-7$

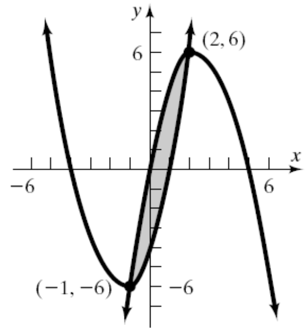
$g(-4)=2(-4)+1=-8+1=-7$

$f(2)=-(2)^2+9=-4+9=5$

$g(2)=2(2)+1=4+1=5$

Thus, the graphs of *f* and *g* intersect at the points $(-4,-7)$ and $(2,\ 5)$.

73. a and **d.**



b. $f(x)=g(x)$

$-x^2+5x=x^2+3x-4$

$0=2x^2-2x-4$

$0=x^2-x-2$

$0=(x+1)(x-2)$

$x+1=0$ or $x-2=0$

$x=-1$ $\quad x=2$

c. $f(-1)=-(-1)^2+5(-1)=-1-5=-6$

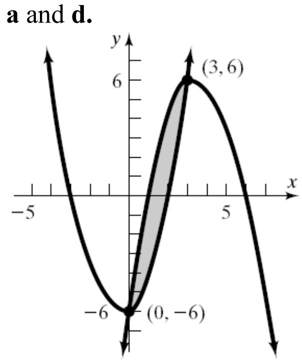
$g(-1)=(-1)^2+3(-1)-4=1-3-4=-6$

$f(2)=-(2)^2+5(2)=-4+10=6$

$g(2)=2^2+3(2)-4=4+6-4=6$

Thus, the graphs of *f* and *g* intersect at the points $(-1,-6)$ and $(2,\ 6)$.

74. a and **d.**



b. $f(x)=g(x)$

$-x^2+7x-6=x^2+x-6$

$0=2x^2-6x$

$0=2x(x-3)$

$2x=0$ or $x-3=0$

$x=0$ $\quad x=3$

c. $f(0) = -(0)^2 + 7(0) - 6 = -6$

$g(0) = 0^2 + 0 - 6 = -6$

$f(3) = -(3)^2 + 7(3) - 6 = -9 + 21 - 6 = 6$

$g(3) = 3^2 + 3 - 6 = 9 + 3 - 6 = 6$

Thus, the graphs of *f* and *g* intersect at the points $(0, -6)$ and $(3, 6)$.

75. a. For $a = 1$:

$$\begin{aligned} f(x) &= a(x - r_1)(x - r_2) \\ &= 1(x - (-3))(x - 1) \\ &= (x + 3)(x - 1) = x^2 + 2x - 3 \end{aligned}$$

For $a = 2$:

$$\begin{aligned} f(x) &= 2(x - (-3))(x - 1) \\ &= 2(x + 3)(x - 1) \\ &= 2(x^2 + 2x - 3) = 2x^2 + 4x - 6 \end{aligned}$$

For $a = -2$:

$$\begin{aligned} f(x) &= -2(x - (-3))(x - 1) \\ &= -2(x + 3)(x - 1) \\ &= -2(x^2 + 2x - 3) = -2x^2 - 4x + 6 \end{aligned}$$

For $a = 5$:

$$\begin{aligned} f(x) &= 5(x - (-3))(x - 1) \\ &= 5(x + 3)(x - 1) \\ &= 5(x^2 + 2x - 3) = 5x^2 + 10x - 15 \end{aligned}$$

b. The *x*-intercepts are not affected by the value of *a*. The *y*-intercept is multiplied by the value of a.

c. The axis of symmetry is unaffected by the value of a. For this problem, the axis of symmetry is $x = -1$ for all values of *a*.

d. The *x*-coordinate of the vertex is not affected by the value of *a*. The *y*-coordinate of the vertex is multiplied by the value of a.

e. The *x*-coordinate of the vertex is the midpoint of the *x*-intercepts.

76. a. For $a = 1$:

$$\begin{aligned} f(x) &= 1(x - (-5))(x - 3) \\ &= (x + 5)(x - 3) = x^2 + 2x - 15 \end{aligned}$$

For $a = 2$:

$$\begin{aligned} f(x) &= 2(x - (-5))(x - 3) \\ &= 2(x + 5)(x - 3) \\ &= 2(x^2 + 2x - 15) = 2x^2 + 4x - 30 \end{aligned}$$

For $a = -2$:

$$\begin{aligned} f(x) &= -2(x - (-5))(x - 3) \\ &= -2(x + 5)(x - 3) \\ &= -2(x^2 + 2x - 15) = -2x^2 - 4x + 30 \end{aligned}$$

For $a = 5$:

$$\begin{aligned} f(x) &= 5(x - (-5))(x - 3) \\ &= 5(x + 5)(x - 3) \\ &= 5(x^2 + 2x - 15) = 5x^2 + 10x - 75 \end{aligned}$$

b. The *x*-intercepts are not affected by the value of *a*. The *y*-intercept is multiplied by the value of a.

c. The axis of symmetry is unaffected by the value of a. For this problem, the axis of symmetry is $x = -1$ for all values of *a*.

d. The *x*-coordinate of the vertex is not affected by the value of *a*. The *y*-coordinate of the vertex is multiplied by the value of a.

e. The *x*-coordinate of the vertex is the midpoint of the *x*-intercepts.

77. a. $x = -\dfrac{b}{2a} = -\dfrac{4}{2(1)} = -2$

$y = f(-2) = (-2)^2 + 4(-2) - 21 = -25$

The vertex is $(-2, -25)$.

b.

$$\begin{aligned} f(x) &= 0 \\ x^2 + 4x - 21 &= 0 \\ (x + 7)(x - 3) &= 0 \end{aligned}$$

$x + 7 = 0$ or $x - 3 = 0$

$x = -7$ $\quad x = 3$

The zeros of *f* are -7 and 3. Likewise, the *x*-intercepts of *f* are -7 and 3.

c.

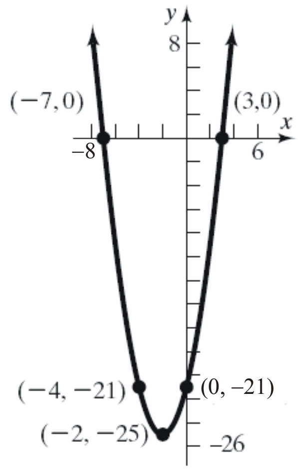
$$\begin{aligned} f(x) &= -21 \\ x^2 + 4x - 21 &= -21 \\ x^2 + 4x &= 0 \\ x(x + 4) &= 0 \end{aligned}$$

$x = 0$ or $x + 4 = 0$

$x = -4$

The solutions $f(x) = -21$ are -4 and 0. Thus, the points $(-4, -21)$ and $(0, -21)$ are on the graph of *f*.

d.



78. a. $x = -\dfrac{b}{2a} = -\dfrac{2}{2(1)} = -1$

$y = f(-1) = (-1)^2 + 2(-1) - 8 = -9$

The vertex is $(-1, -9)$.

b. $f(x) = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ $\quad$ $x = 2$

The zeros of *f* are -4 and 2. Likewise, the *x*-intercepts of *f* are -4 and 2.

c. $f(x) = -8$

$x^2 + 2x - 8 = -8$

$x^2 + 2x = 0$

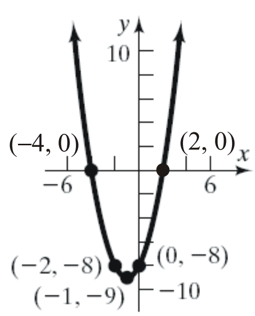
$x(x+2) = 0$

$x = 0$ or $x + 2 = 0$

$x = -2$

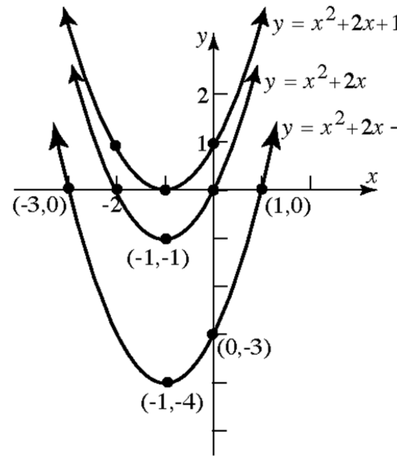
The solutions $f(x) = -8$ are -2 and 0. Thus, the points $(-2, -8)$ and $(0, -8)$ are on the graph of *f*.

d.



79. Answers will vary.

80. $y = x^2 + 2x - 3$; $y = x^2 + 2x + 1$; $y = x^2 + 2x$



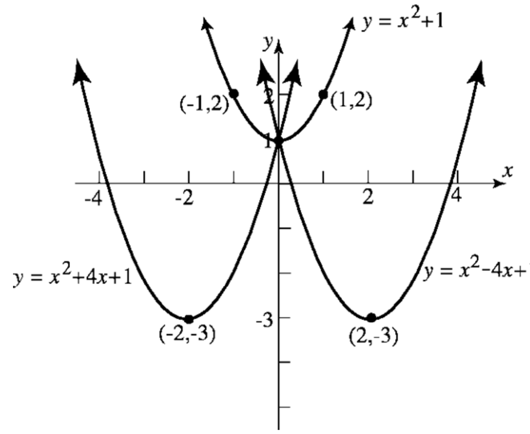
Each member of this family will be a parabola with the following characteristics:

(i) opens upwards since a > 0;

(ii) vertex occurs at $x = -\dfrac{b}{2a} = -\dfrac{2}{2(1)} = -1$;

(iii) There is at least one *x*-intercept since $b^2 - 4ac \geq 0$.

81. $y = x^2 - 4x + 1$; $y = x^2 + 1$; $y = x^2 + 4x + 1$



Each member of this family will be a parabola with the following characteristics:

(i) opens upwards since $a > 0$

(ii) *y*-intercept occurs at (0, 1).

82. The graph of the quadratic function $f(x) = ax^2 + bx + c$ will not have any *x*-intercepts whenever $b^2 - 4ac < 0$.

83. By completing the square on the quadratic function $f(x) = ax^2 + bx + c$ we obtain the equation $y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$. We can then draw the graph by applying transformations to the graph of the basic parabola $y = x^2$, which opens up. When $a > 0$, the basic parabola will either be stretched or compressed vertically. When $a < 0$, the basic parabola will either be stretched or compressed vertically as well as reflected across the x-axis. Therefore, when $a > 0$, the graph of $f(x) = ax^2 + bx + c$ will open up, and when $a < 0$, the graph of $f(x) = ax^2 + bx + c$ will open down.

84. No. We know that the graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola with vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right]$. If $a > 0$, then the vertex is a minimum point, so the range is $\left[f\left(-\frac{b}{2a}\right), \infty\right)$. If $a < 0$, then the vertex is a maximum point, so the range is $\left(-\infty, f\left(-\frac{b}{2a}\right)\right)$. Therefore, it is impossible for the range to be $(-\infty, \infty)$.

Section 2.5

1. $-3x - 2 < 7$
$$-3x < 9$$
$$x > -3$$
The solution set is $\{x \mid x > -3\}$ or $(-3, \infty)$.

2. $(-2, 7]$ represent the numbers between -2 and 7, including 7 but not including -2. Using inequality notation, this is equivalent to $-2 < x \le 7$.

3. a. $f(x) > 0$ when the graph of f is above the x-axis. Thus, $\{x \mid x < -2 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

b. $f(x) \le 0$ when the graph of f is below or intersects the x-axis. Thus, $\{x \mid -2 \le x \le 2\}$ or, using interval notation, $[-2, 2]$.

4. a. $g(x) < 0$ when the graph of g is below the x-axis. Thus, $\{x \mid x < -1 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, -1) \cup (4, \infty)$.

b. $g(x) \ge 0$ when the graph of f is above or intersects the x-axis. Thus, $\{x \mid -1 \le x \le 4\}$ or, using interval notation, $[-1, 4]$.

5. a. $g(x) \ge f(x)$ when the graph of g is above or intersects the graph of f. Thus $\{x \mid -2 \le x \le 1\}$, or, using interval notation, $[-2, 1]$.

b. $f(x) > g(x)$ when the graph of f is above the graph of g. Thus, $\{x \mid x < -2 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -2) \cup (1, \infty)$.

6. a. $f(x) < g(x)$ when the graph of f is below the graph of g. Thus, $\{x \mid x < -3 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -3) \cup (1, \infty)$.

b. $f(x) \ge g(x)$ when the graph of f is above or intersects the graph of g. Thus, $\{x \mid -3 \le x \le 1\}$ or, using interval notation, $[-3, 1]$.

7. $x^2 - 3x - 10 < 0$
$$f(x) = x^2 - 3x - 10$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$x = 5, x = -2$ are the zeros of f.

Interval	$(-\infty, -2)$	$(-2, 5)$	$(5, \infty)$
Test Number	-3	0	6
Value of f	8	-10	8
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -2 < x < 5\}$ or, using interval notation, $(-2, 5)$.

8. $x^2+3x-10>0$

$f(x)=x^2+3x-10$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x=-5, x=2$ are the zeros of f.

Interval	$(-\infty,-5)$	$(-5, 2)$	$(2,\infty)$
Test Number	-6	0	6
Value of f	8	-10	8
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-5 \text{ or } x>2\}$ or, using interval notation, $(-\infty,-5)\cup(2,\infty)$.

9. $x^2-4x>0$

$f(x)=x^2-4x$

$x^2-4x=0$

$x(x-4)=0$

$x=0, x=4$ are the zeros of f.

Interval	$(-\infty, 0)$	$(0, 4)$	$(4,\infty)$
Test Number	-1	1	5
Value of f	5	-3	5
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<0 \text{ or } x>4\}$ or, using interval notation, $(-\infty, 0)\cup(4,\infty)$.

10. $x^2+8x>0$

$f(x)=x^2+8x$

$x^2+8x=0$

$x(x+8)=0$

$x=-8, x=0$ are the zeros of f.

Interval	$(-\infty,-8)$	$(-8, 0)$	$(0,\infty)$
Test Number	-9	-1	1
Value of f	9	-7	9
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-8 \text{ or } x>0\}$ or, using interval notation, $(-\infty,-8)\cup(0,\infty)$.

11. $x^2-9<0$

$f(x)=x^2-9$

$x^2-9=0$

$(x-3)(x+3)=0$

$x=-3, x=3$ are the zeros of f.

Interval	$(-\infty,-3)$	$(-3, 3)$	$(3,\infty)$
Test Number	-4	0	4
Value of f	7	-9	7
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -3<x<3\}$ or, using interval notation, $(-3, 3)$.

12. $x^2-1<0$

$f(x)=x^2-1$

$x^2-1=0$

$(x-1)(x+1)=0$

$x=-1, x=1$ are the zeros of f.

Interval	$(-\infty,-1)$	$(-1, 1)$	$(1,\infty)$
Test Number	-2	0	2
Value of f	3	-1	3
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -1<x<1\}$ or, using interval notation, $(-1, 1)$.

13. $x^2+x>12$

$x^2+x-12>0$

$f(x)=x^2+x-12$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4, x=3$ are the zeros of f

Interval	$(-\infty,-4)$	$(-4, 3)$	$(3,\infty)$
Test Number	-5	0	4
Value of f	8	-12	8
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-4 \text{ or } x>3\}$ or, using interval notation, $(-\infty,-4)\cup(3,\infty)$.

14. $x^2+7x<-12$

$x^2+7x+12<0$

$f(x)=x^2+7x+12$

$x^2+7x+12=0$

$(x+4)(x+3)=0$

$x=-4,\ x=-3$ are the zeros of f.

Interval	$(-\infty,-4)$	$(-4,-3)$	$(-3,\infty)$
Test Number	–5	–3.5	0
Value of f	2	–0.25	12
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -4<x<-3\}$ or, using interval notation, $(-4,-3)$.

15. $2x^2<5x+3$

$2x^2-5x-3<0$

$f(x)=2x^2-5x-3$

$2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$x=-\frac{1}{2},\ x=3$ are the zeros of f

Interval	$\left(-\infty,-\frac{1}{2}\right)$	$\left(-\frac{1}{2},3\right)$	$(3,\infty)$
Test Number	–1	0	4
Value of f	4	–3	9
Conclusion	Positive	Negative	Positive

The solution set is $\left\{x \,\middle|\, -\frac{1}{2}<x<3\right\}$ or, using interval notation, $\left(-\frac{1}{2},\ 3\right)$.

16. $6x^2<6+5x$

$6x^2-5x-6<0$

$f(x)=6x^2-5x-6$

$6x^2-5x-6=0$

$(3x+2)(2x-3)=0$

$x=-\frac{2}{3},\ x=\frac{3}{2}$ are the zeros

Interval	$\left(-\infty,-\frac{2}{3}\right)$	$\left(-\frac{2}{3},\frac{3}{2}\right)$	$\left(\frac{3}{2},\infty\right)$
Test Number	–1	0	2
Value of f	5	–6	8
Conclusion	Positive	Negative	Positive

The solution set is $\left\{x \,\middle|\, -\frac{2}{3}<x<\frac{3}{2}\right\}$ or, using interval notation, $\left(-\frac{2}{3},\ \frac{3}{2}\right)$.

17. $x(x-7)>8$

$x^2-7x>8$

$x^2-7x-8>0$

$f(x)=x^2-7x-8$

$x^2-7x-8=0$

$(x+1)(x-8)=0$

$x=-1,\ x=8$ are the zeros of f.

Interval	$(-\infty,-1)$	$(-1,8)$	$(8,\infty)$
Test Number	–2	0	9
Value of f	10	–8	10
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-1 \text{ or } x>8\}$ or, using interval notation, $(-\infty,-1)\cup(8,\ \infty)$.

18. $x(x+1)>20$

$x^2+x>20$

$x^2+x-20>0$

$f(x)=x^2+x-20$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x=-5,\ x=4$ are the zeros of f.

Interval	$(-\infty,-5)$	$(-5,4)$	$(4,\infty)$
Test Number	–6	0	5
Value of f	10	–20	10
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-5 \text{ or } x>4\}$ or, using interval notation, $(-\infty,-5)\cup(4,\ \infty)$.

19. $4x^2 + 9 < 6x$

$4x^2 - 6x + 9 < 0$

$f(x) = 4x^2 - 6x + 9$

$b^2 - 4ac = (-6)^2 - 4(4)(9) = 36 - 144 = -108$

Since the discriminant is negative, f has no real zeros. Therefore, f is either always positive or always negative. Choose any value and test.

For $x = 0$, $f(0) = 4(0)^2 - 6(0) + 9 = 9 > 0$

Thus, there is no real solution.

20. $25x^2 + 16 < 40x$

$25x^2 - 40x + 16 < 0$

$(5x-4)^2 < 0$

Since the square of a real number is always non-negative, there are no values of x for which the expression is negative. Thus, there is no solution.

21. $6(x^2 - 1) > 5x$

$6x^2 - 6 > 5x$

$6x^2 - 5x - 6 > 0$

$(3x+2)(2x-3) > 0$

$f(x) = 6x^2 - 5x - 6$

$x = -\frac{2}{3}, x = \frac{3}{2}$ are the zeros of f.

Interval	$\left(-\infty, -\frac{2}{3}\right)$	$\left(-\frac{2}{3}, \frac{3}{2}\right)$	$\left(\frac{3}{2}, \infty\right)$
Test Number	-1	0	2
Value of f	5	-6	8
Conclusion	Positive	Negative	Positive

The solution set is $\left\{ x \,\middle|\, x < -\frac{2}{3} \text{ or } x > \frac{3}{2} \right\}$ or, using interval notation, $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{3}{2}, \infty\right)$.

22. $2(2x^2 - 3x) > -9$

$4x^2 - 6x + 9 > 0$

$f(x) = 4x^2 - 6x + 9$

$$\begin{aligned} b^2 - 4ac &= (-6)^2 - 4(4)(9) \\ &= 36 - 144 \\ &= -108 \end{aligned}$$

Since the discriminant is negative, f has no real zeros. Therefore, f is either always positive or always negative. Choose any value and test.

For $x = 0$, $f(0) = 4(0)^2 - 6(0) + 9 = 9 \geq 0$

Thus, the solution set is $\{x \mid x \text{ is any real number}\}$ or, using interval notation, $(-\infty, \infty)$.

23. The domain of the expression $f(x) = \sqrt{x^2 - 16}$ includes all values for which $x^2 - 16 \geq 0$.

$p(x) = x^2 - 16$

$(x+4)(x-4) = 0$

$x = -4, x = 4$ are the zeros are of p.

Interval	$(-\infty, -4)$	$(-4, 4)$	$(4, \infty)$
Test Number	-5	0	5
Value of p	9	-16	9
Conclusion	Positive	Negative	Positive

The domain of f is $\{x \mid x \leq -4 \text{ or } x \geq 4\}$ or, using interval notation, $(-\infty, -4] \cup [4, \infty)$.

24. The domain of the expression $f(x) = \sqrt{x - 3x^2}$ includes all values for which $x - 3x^2 \geq 0$.

$p(x) = x - 3x^2$

$x(1-3x) = 0$

$x = 0, x = \frac{1}{3}$ are the zeros are of p.

Interval	$(-\infty, 0)$	$\left(0, \frac{1}{3}\right)$	$\left(\frac{1}{3}, \infty\right)$
Test Number	-1	$\frac{1}{6}$	1
Value of p	-4	$-\frac{13}{12}$	-4
Conclusion	Negative	Positive	Negative

The domain of f is $\left\{ x \,\middle|\, 0 \leq x \leq \frac{1}{3} \right\}$ or, using interval notation, $\left[0, \frac{1}{3}\right]$.

25. $f(x)=x^2-1; \quad g(x)=3x+3$

a. $f(x)=0$

$x^2-1=0$
$(x-1)(x+1)=0$
$x=1; x=-1$

Solution set: $\{-1,1\}$.

b. $g(x)=0$

$3x+3=0$
$3x=-3$
$x=-1$

Solution set: $\{-1\}$.

c. $f(x)=g(x)$

$x^2-1=3x+3$
$x^2-3x-4=0$
$(x-4)(x+1)=0$
$x=4; x=-1$

Solution set: $\{-1,4\}$.

d. $f(x)>0$

$x^2-1>0$
$(x-1)(x+1)>0$

The zeros are $x=-1$ and $x=1$.

Interval	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
Test Number	-2	0	2
Value of f	3	-1	3
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-1 \text{ or } x>1\}$ or, using interval notation, $(-\infty,-1)\cup(1,\infty)$.

e. $g(x)\le 0$

$3x+3\le 0$
$3x\le -3$
$x\le -1$

The solution set is $\{x \mid x\le -1\}$ or, using interval notation, $(-\infty,-1]$.

f. $f(x)>g(x)$

$x^2-1>3x+3$
$x^2-3x-4>0$
$(x-4)(x+1)>0$

The zeros are $x=-1$ and $x=4$.
$p(x)=x^2-3x-4$

Interval	$(-\infty,-1)$	$(-1,4)$	$(4,\infty)$
Test Number	-2	0	5
Value of p	6	-4	6
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-1 \text{ or } x>4\}$ or, using interval notation, $(-\infty,-1)\cup(4,\infty)$.

g. $f(x)\ge 1$

$x^2-1>1$
$x^2-2>0$
$(x-\sqrt{2})(x+\sqrt{2})>0$

The zeros are $x=-\sqrt{2}$ and $x=\sqrt{2}$.
$p(x)=x^2-2$

Interval	$(-\infty,-\sqrt{2})$	$(-\sqrt{2},\sqrt{2})$	$(\sqrt{2},\infty)$
Test Number	-2	0	5
Value of p	2	-2	2
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-\sqrt{2} \text{ or } x>\sqrt{2}\}$ or, using interval notation, $(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$.

26. $f(x)=x^2-1; \quad g(x)=-3x+3$

a. $f(x)=0$

$x^2-1=0$
$(x-1)(x+1)=0$
$x=1; x=-1$

Solution set: $\{-1,1\}$.

b. $g(x)=0$

$-3x+3=0$
$-3x=-3$
$x=1$

Solution set: $\{1\}$.

c. $f(x)=g(x)$

$x^2-1=-3x+3$
$x^2+3x-4=0$
$(x+4)(x-1)=0$
$x=-4; x=1$

Solution set: $\{-4,1\}$.

d. $f(x) > 0$

$x^2 - 1 > 0$
$(x-1)(x+1) > 0$

The zeros are $x = -1$ and $x = 1$.

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Test Number	–2	0	2
Value of f	3	–1	3
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x < -1 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -1) \cup (1, \infty)$.

e. $g(x) \le 0$

$-3x + 3 \le 0$
$-3x \le -3$
$x \ge 1$

The solution set is $\{x \mid x \ge 1\}$ or, using interval notation, $[1, \infty)$.

f. $f(x) > g(x)$

$x^2 - 1 > -3x + 3$
$x^2 + 3x - 4 > 0$
$(x+4)(x-1) > 0$

The zeros are $x = -4$ and $x = 1$.
$p(x) = x^2 + 3x - 4$

Interval	$(-\infty, -4)$	$(-4, 1)$	$(1, \infty)$
Test Number	–5	0	2
Value of p	6	–4	6
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x < -4 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -4) \cup (1, \infty)$.

g. $f(x) \ge 1$

$x^2 - 1 > 1$
$x^2 - 2 > 0$
$(x - \sqrt{2})(x + \sqrt{2}) > 0$

The zeros are $x = -\sqrt{2}$ and $x = \sqrt{2}$.
$p(x) = x^2 - 2$

Interval	$(-\infty, -\sqrt{2})$	$(-\sqrt{2}, \sqrt{2})$	$(\sqrt{2}, \infty)$
Test Number	–2	0	5
Value of p	2	–2	2
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x < -\sqrt{2} \text{ or } x > \sqrt{2}\}$ or, using interval notation, $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

27. $f(x) = -x^2 + 1$; $g(x) = 4x + 1$

a. $f(x) = 0$

$-x^2 + 1 = 0$
$1 - x^2 = 0$
$(1-x)(1+x) = 0$
$x = 1;\ x = -1$

Solution set: $\{-1, 1\}$.

b. $g(x) = 0$

$4x + 1 = 0$
$4x = -1$
$x = -\frac{1}{4}$

Solution set: $\left\{-\frac{1}{4}\right\}$.

c. $f(x) = g(x)$

$-x^2 + 1 = 4x + 1$
$0 = x^2 + 4x$
$0 = x(x+4)$
$x = -4; x = 0$

Solution set: $\{-4, 0\}$.

d. $f(x) > 0$

$-x^2 + 1 > 0$
$1 - x^2 > 0$
$(1-x)(1+x) > 0$
$x = 1; x = -1$

The zeros are $x = -1$ and $x = 1$.

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Test Number	–2	0	2
Value of f	–3	1	–3
Conclusion	Negative	Positive	Negative

The solution set is $\{x \mid -1 < x < 1\}$ or, using interval notation, $(-1, 1)$.

e. $g(x) \le 0$

$4x + 1 \le 0$
$4x \le -1$
$x \le -\frac{1}{4}$

The solution set is $\left\{ x \middle| x \le -\frac{1}{4} \right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{4}\right]$.

f. $f(x) > g(x)$

$-x^2 + 1 > 4x + 1$
$0 > x^2 + 4x$
$0 > x(x+4)$

The zeros are $x = -4$ and $x = 0$.
$p(x) = x(x+4)$

Interval	$(-\infty, -4)$	$(-4, 0)$	$(0, \infty)$
Test Number	–5	–1	1
Value of p	5	–3	5
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -4 < x < 0\}$ or, using interval notation, $(-4, 0)$.

g. $f(x) \ge 1$

$-x^2 + 1 \ge 1$
$-x^2 \ge 0$
$x^2 \le 0$

The zero is $x = 0$. $p(x) = x^2$

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Number	–1	1
Value of p	1	1
Conclusion	Positive	Positive

The solution set is $\{0\}$.

28. $f(x) = -x^2 + 4;\quad g(x) = -x - 2$

a. $f(x) = 0$

$-x^2 + 4 = 0$
$4 - x^2 = 0$
$(2-x)(2+x) = 0$
$x = 2; x = -2$

Solution set: $\{-2, 2\}$.

b. $g(x) = 0$

$-x - 2 = 0$
$-x = 2$
$x = -2$

Solution set: $\{-2\}$.

c. $f(x) = g(x)$

$-x^2 + 4 = -x - 2$
$0 = x^2 - x - 6$
$0 = (x-3)(x+2)$
$x = 3; x = -2$

Solution set: $\{-2, 3\}$.

d. $f(x) > 0$

$-x^2 + 4 > 0$
$4 - x^2 > 0$
$(2-x)(2+x) > 0$

The zeros are $x = -2$ and $x = 2$.

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Test Number	–3	0	3
Value of f	–5	2	–5
Conclusion	Negative	Positive	Negative

The solution set is $\{x \mid -2 < x < 2\}$ or, using interval notation, $(-2, 2)$.

e. $g(x) \le 0$

$-x - 2 \le 0$
$-x \le 2$
$x \ge -2$

The solution set is $\{x \mid x \ge -2\}$ or, using interval notation, $[-2, \infty)$.

f. $f(x) > g(x)$

$-x^2 + 4 > -x - 2$
$0 > x^2 - x - 6$
$0 > (x-3)(x+2)$

The zeros are $x = -2$ and $x = 3$.
$p(x) = (x-3)(x+2)$

Interval	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
Test Number	–3	0	4
Value of f	6	–6	6
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -2 < x < 3\}$ or, using interval notation, $(-2, 3)$.

g. $f(x) \geq 1$

$$-x^2 + 4 > 1$$
$$0 \geq x^2 - 3$$
$$0 \geq \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$$

The zeros are $x = -\sqrt{3}$ and $x = \sqrt{3}$.

$$p(x) = \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$$

Interval	$\left(-\infty, -\sqrt{3}\right)$	$\left(-\sqrt{3}, \sqrt{3}\right)$	$\left(\sqrt{3}, \infty\right)$
Test Number	–2	0	2
Value of *f*	1	–3	1
Conclusion	Positive	Negative	Positive

The solution set is $\left\{ x \middle| -\sqrt{3} \leq x \leq \sqrt{3} \right\}$ or, using interval notation, $\left[-\sqrt{3}, \sqrt{3}\right]$.

29. $f(x) = x^2 - 4$; $g(x) = -x^2 + 4$

a. $f(x) = 0$

$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$$x = 2; x = -2$$

Solution set: $\{-2, 2\}$.

b. $g(x) = 0$

$$-x^2 + 4 = 0$$
$$4 - x^2 = 0$$
$$(2-x)(2+x) = 0$$
$$x = 2; x = -2$$

Solution set: $\{-2, 2\}$.

c. $f(x) = g(x)$

$$x^2 - 4 = -x^2 + 4$$
$$2x^2 - 8 = 0$$
$$2\left(x^2 - 4\right) = 0$$
$$2(x-2)(x+2) = 0$$
$$x = 2; x = -2$$

Solution set: $\{-2, 2\}$.

d. $f(x) > 0$

$$x^2 - 4 > 0$$
$$(x-2)(x+2) > 0$$

The zeros are $x = -2$ and $x = 2$.

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Test Number	–3	0	3
Value of *f*	5	–4	5
Conclusion	Positive	Negative	Positive

The solution set is $\{ x \mid x < -2 \text{ or } x > 2 \}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

e. $g(x) \leq 0$

$$-x^2 + 4 \leq 0$$
$$4 - x^2 \leq 0$$
$$(2-x)(2+x) \leq 0$$

The zeros are $x = -2$ and $x = 2$.

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Test Number	–3	0	3
Value of *g*	–5	4	–5
Conclusion	Negative	Positive	Negative

The solution set is $\{ x \mid x \leq -2 \text{ or } x \geq 2 \}$ or, using interval notation, $(-\infty, -2] \cup [2, \infty)$.

f. $f(x) > g(x)$

$$x^2 - 4 > -x^2 + 4$$
$$2x^2 - 8 > 0$$
$$2\left(x^2 - 4\right) > 0$$
$$2(x-2)(x+2) > 0$$

The zeros are $x = -2$ and $x = 2$.

$$p(x) = 2(x-2)(x+2)$$

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Test Number	–3	0	3
Value of *p*	10	–8	10
Conclusion	Positive	Negative	Positive

The solution set is $\{ x \mid x < -2 \text{ or } x > 2 \}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

g. $f(x) \ge 1$

$x^2 - 4 \ge 1$
$x^2 - 5 \ge 0$
$\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right) \ge 0$

The zeros are
$x = -\sqrt{5}$ and $x = \sqrt{5}$.
$p(x) = \left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)$

Interval	$\left(-\infty, -\sqrt{5}\right)$	$\left(-\sqrt{5}, \sqrt{5}\right)$	$\left(\sqrt{5}, \infty\right)$
Test Number	–3	0	3
Value of p	4	–5	4
Conclusion	Positive	Negative	Positive

The solution set is $\left\{x \middle| x \le -\sqrt{5} \text{ or } x \ge \sqrt{5}\right\}$ or, using interval notation, $\left(-\infty, -\sqrt{5}\right] \cup \left[\sqrt{5}, \infty\right)$.

30. $f(x) = x^2 - 2x + 1$; $g(x) = -x^2 + 1$

a. $f(x) = 0$

$x^2 - 2x + 1 = 0$
$(x-1)(x-1) = 0$
$x = 1$

Solution set: $\{1\}$.

b. $g(x) = 0$

$-x^2 + 1 = 0$
$1 - x^2 = 0$
$(1-x)(1+x) = 0$
$x = 1; x = -1$

Solution set: $\{-1, 1\}$.

c. $f(x) = g(x)$

$x^2 - 2x + 1 = -x^2 + 1$
$2x^2 - 2x = 0$
$2x(x-1) = 0$
$x = 0, x = 1$

Solution set: $\{0, 1\}$.

d. $f(x) > 0$

$x^2 - 2x + 1 = 0$
$(x-1)(x-1) = 0$

The zero is $x = 1$.

Interval	$(-\infty, 1)$	$(1, \infty)$
Test Number	0	2
Value of f	1	1
Conclusion	Positive	Positive

The solution set is $\{x \mid x < 1 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, 1) \cup (1, \infty)$.

e. $g(x) \le 0$

$-x^2 + 1 \le 0$
$1 - x^2 \le 0$
$(1-x)(1+x) \le 0$

The zeros are $x = -1$ and $x = 1$.

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Test Number	–2	0	2
Value of g	–3	1	–3
Conclusion	Negative	Positive	Negative

The solution set is $\{x \mid x \le -1 \text{ or } x \ge 1\}$ or, using interval notation, $(-\infty, -1] \cup [1, \infty)$.

f. $f(x) > g(x)$

$x^2 - 2x + 1 = -x^2 + 1$
$2x^2 - 2x = 0$
$2x(x-1) = 0$
$x = 0, x = 1$

The zeros are $x = 0$ and $x = 1$.
$p(x) = 2x(x-1)$

Interval	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
Test Number	–1	0.5	2
Value of p	4	–0.5	4
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x < 0 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, 0) \cup (1, \infty)$.

g. $f(x) \ge 1$

$x^2 - 2x + 1 \ge 1$
$x^2 - 2x \ge 0$
$x(x-2) \ge 0$

The zeros are $x = 0$ and $x = 2$.
$p(x) = x(x-2)$

Interval	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
Test Number	–1	1	3
Value of p	3	–1	3
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x \le 0 \text{ or } x \ge 2\}$ or, using interval notation, $(-\infty, 0] \cup [2, \infty)$.

31. $f(x)=x^2-x-2;\quad g(x)=x^2+x-2$

a. $f(x)=0$

$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=2, x=-1$$

Solution set: $\{-1, 2\}$.

b. $g(x)=0$

$$x^2+x-2=0$$
$$(x+2)(x-1)=0$$
$$x=-2; x=1$$

Solution set: $\{-2, 1\}$.

c. $f(x)=g(x)$

$$x^2-x-2=x^2+x-2$$
$$-2x=0$$
$$x=0$$

Solution set: $\{0\}$.

d. $f(x)>0$

$$x^2-x-2>0$$
$$(x-2)(x+1)>0$$

The zeros are $x=-1$ and $x=2$.

Interval	$(-\infty,-1)$	$(-1, 2)$	$(2, \infty)$
Test Number	–2	0	3
Value of f	4	–2	4
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x<-1 \text{ or } x>2\}$ or, using interval notation, $(-\infty,-1)\cup(2, \infty)$.

e. $g(x)\le 0$

$$x^2+x-2\le 0$$
$$(x+2)(x-1)\le 0$$

The zeros are $x=-2$ and $x=1$.

Interval	$(-\infty,-2)$	$(-2, 1)$	$(1, \infty)$
Test Number	–3	0	2
Value of g	4	–2	4
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -2\le x\le 1\}$ or, using interval notation, $[-2, 1]$.

f. $f(x)>g(x)$

$$x^2-x-2>x^2+x-2$$
$$-2x>0$$
$$x<0$$

The solution set is $\{x \mid x<0\}$ or, using interval notation, $(-\infty, 0)$.

g. $f(x)\ge 1$

$$x^2-x-2\ge 1$$
$$x^2-x-3\ge 0$$
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-3)}}{2(1)}$$
$$=\frac{1\pm\sqrt{1+12}}{2}=\frac{1\pm\sqrt{13}}{2}$$

The zeros are

$$x=\frac{1-\sqrt{13}}{2}\approx -1.30 \text{ and } x=\frac{1+\sqrt{13}}{2}\approx 2.30$$

$$p(x)=x^2-x-3$$

Interval	$\left(-\infty, \frac{1-\sqrt{13}}{2}\right)$	$\left(\frac{1-\sqrt{13}}{2}, \frac{1+\sqrt{13}}{2}\right)$	$\left(\frac{1+\sqrt{13}}{2}, \infty\right)$
Test Number	–2	0	3
Value of p	3	–3	3
Conclusion	Positive	Negative	Positive

The solution set is

$\left\{x \,\middle|\, x\le\frac{1-\sqrt{13}}{2} \text{ or } x\ge\frac{1+\sqrt{13}}{2}\right\}$ or, using interval notation,

$$\left(-\infty, \frac{1-\sqrt{13}}{2}\right]\cup\left[\frac{1+\sqrt{13}}{2}, \infty\right).$$

32. $f(x)=-x^2-x+1;\quad g(x)=-x^2+x+6$

a. $f(x)=0$

$$-x^2-x+1=0$$
$$x^2+x-1=0$$
$$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

Solution set: $\left\{\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right\}$.

b. $g(x)=0$

$$-x^2+x+6=0$$
$$x^2-x-6=0$$
$$(x-3)(x+2)=0$$
$$x=3; x=-2$$

Solution set: $\{-2, 3\}$.

c. $f(x)=g(x)$

$$-x^2-x+1=-x^2+x+6$$
$$-2x-5=0$$
$$-2x=5$$
$$x=-\frac{5}{2}$$

Solution set: $\left\{-\frac{5}{2}\right\}$.

d. $f(x)>0$

$$-x^2-x+1>0$$
$$x^2+x-1<0$$
$$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

The zeros are

$$x=\frac{-1-\sqrt{5}}{2}\approx-1.62 \text{ and } x=\frac{-1+\sqrt{5}}{2}\approx 0.62$$

$p(x)=x^2+x-1$

Interval	$\left(-\infty, \frac{-1-\sqrt{5}}{2}\right)$	$\left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$	$\left(\frac{-1+\sqrt{5}}{2}, \infty\right)$
Test Number	–1	0	2
Value of p	3	–3	3
Conclusion	Positive	Negative	Positive

The solution set is $\left\{x \,\middle|\, \frac{-1-\sqrt{5}}{2}<x<\frac{-1+\sqrt{5}}{2}\right\}$

or, using interval notation, $\left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$.

e. $g(x)\le 0$

$$-x^2+x+6\le 0$$
$$x^2-x-6\ge 0$$
$$(x-3)(x+2)\ge 0$$

The zeros are $x=-2$ and $x=3$.

$p(x)=x^2-x-6$

Interval	$(-\infty,-2)$	$(-2, 3)$	$(3, \infty)$
Test Number	–3	0	4
Value of p	6	–6	6
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x\le -2 \text{ or } x\ge 3\}$ or, using interval notation, $(-\infty, 2]\cup[3, \infty)$.

f. $f(x)>g(x)$

$$-x^2-x+1>-x^2+x+6$$
$$-2x>5$$
$$x<-\frac{5}{2}$$

The solution set is $\left\{x \,\middle|\, x<-\frac{5}{2}\right\}$ or, using interval notation, $\left(-\infty,-\frac{5}{2}\right)$.

g. $f(x)\ge 1$

$$-x^2-x+1\ge 1$$
$$-x^2-x\ge 0$$
$$-x(x+1)\ge 0$$

The zeros are $x=-1$ and $x=0$.

$p(x)=-x(x+1)$

Interval	$(-\infty,-1)$	$(-1, 0)$	$(0, \infty)$
Test Number	–2	–0.5	1
Value of p	–2	0.25	–2
Conclusion	Negative	Positive	Negative

The solution set is $\{x \mid -1\le x\le 0\}$ or, using interval notation, $[-1, 0]$.

33. a. The ball strikes the ground when $s(t)=80t-16t^2=0$.

$$80t-16t^2=0$$
$$16t(5-t)=0$$
$$t=0, t=5$$

The ball strikes the ground after 5 seconds.

b. Find the values of t for which

$$80t-16t^2>96$$
$$-16t^2+80t-96>0$$
$$16t^2-80t+96<0$$
$$16(t^2-5t+6)<0$$
$$16(t-2)(t-3)<0$$

The zeros are $t=2$ and $t=3$.

$s(t)=16t^2-80t+96$

Interval	$(-\infty, 2)$	$(2, 3)$	$(3, \infty)$
Test Number	1	2.5	4
$s(t)$	2.5	–4	32
Conclusion	Positive	Negative	Positive

The solution set is $\{t \mid 2 < t < 3\}$ or, using interval notation, $(2, 3)$. The ball is more than 96 feet above the ground for times between 2 and 3 seconds.

34. a. The ball strikes the ground when $s(t) = 96t - 16t^2 = 0$.

$96t - 16t^2 = 0$
$16t(6 - t) = 0$
$t = 0, t = 6$

The ball strikes the ground after 6 seconds.

b. Find the values of t for which

$96t - 16t^2 > 128$
$-16t^2 + 96t - 128 > 0$
$16t^2 - 96t + 128 < 0$
$16(t^2 - 6t + 8) < 0$
$16(t - 4)(t - 2) < 0$

The zeros are $t = 2$ and $t = 4$.

$s(t) = -16t^2 + 96t - 128$

Interval	$(-\infty, 2)$	$(2, 4)$	$(4, \infty)$
Test Number	1	3	5
$s(t)$	–32	32	–32
Conclusion	Negative	Positive	Negative

The solution set is $\{t \mid 2 < t < 4\}$ or, using interval notation, $(2, 4)$. The ball is more than 128 feet above the ground for times between 2 and 4 seconds.

35. a. $R(p) = -4p^2 + 4000p = 0$

$-4p(p - 1000) = 0$
$p = 0, p = 1000$

Thus, the revenue equals zero when p = \$0 and when p = \$1000.

b. Find the values of p for which

$R(p) = -4p^2 + 4000p > 800000$
$-4p^2 + 4000p > 800000$
$-4p^2 + 4000p - 800000 > 0$
$4p^2 - 4000p + 800000 < 0$
$4(p^2 - 1000p + 200000) < 0$
$p^2 - 1000p + 200000 = 0$

$$p = \frac{-(-1000) \pm \sqrt{(-1000)^2 - 4(1)(200000)}}{2(1)} = \frac{1000 \pm \sqrt{200000}}{2} = \frac{1000 \pm 200\sqrt{5}}{2} = 500 \pm 100\sqrt{5}$$

The zeros are $p \approx 276.39$ and $p \approx 723.61$.

$s(p) = -4p^2 + 4000p - 800000$

Int.	$(-\infty, 276.39)$	$(276.39, 723.61)$	$(723.61, \infty)$
Test No.	276	277	724
$s(p)$	–704	1084	–704
Concl.	Negative	Positive	Negative

The solution set is $\{p \mid 276.39 < p < 723.61\}$ or, using interval notation, $(276.39, 723.61)$. The revenue is more than \$800,000 for prices between \$276.39 and \$723.61.

36. a. $R(p) = -\frac{1}{2}p^2 + 1900p = 0$

$-\frac{1}{2}p(p - 3800) = 0$
$p = 0, p = 3800$

Thus, the revenue equals zero when p = \$0 and when p = \$3800.

b. Find the values of p for which

$R(p) = -\frac{1}{2}p^2 + 1900p > 1200000$
$-\frac{1}{2}p^2 + 1900p > 1200000$
$-\frac{1}{2}p^2 + 1900p - 1200000 > 0$
$\frac{1}{2}p^2 - 1900p + 1200000 < 0$
$\frac{1}{2}(p^2 - 3800p + 2400000) < 0$
$\frac{1}{2}(p - 800)(p - 3000) < 0$

The zeros are $p = 800$ and $p = 3000$.

$s(p) = -\frac{1}{2}p^2 + 1900p - 1200000$

Interval	$(-\infty, 800)$	$(800, 3000)$	$(3000, \infty)$
Test Number	276	277	724
$s(p)$	-704	1084	-704
Conclusion	Negative	Positive	Negative

The solution set is $\{p \mid 800 < p < 3000\}$ or, using interval notation, $(800, 3000)$. The revenue is more than \$1,200,000 for prices between \$800 and \$3000.

37. Solving $(x-4)^2 \le 0$

The only zero is $x = 4$.

$f(x) = (x-4)^2$

Interval	$(-\infty, 4)$	$(4, \infty)$
Test Number	3	5
Value of f	1	1
Conclusion	Positive	Positive

The solution is $\{x \mid x = 4\}$. Therefore, the given inequality has exactly one real solution.

38. Solving $(x-2)^2 > 0$

The only zero is $x = 2$.

$f(x) = (x-2)^2$

Interval	$(-\infty, 2)$	$(2, \infty)$
Test Number	1	3
Value of f	1	1
Conclusion	Positive	Positive

The solution is $\{x \mid x < 2 \text{ or } x > 2\}$. Therefore, the given inequality has exactly one real number that is not a solution, namely $x \neq 2$.

39. Solving $x^2 + x + 1 > 0$

The discriminant $b^2 - 4ac = 1^2 - 4(1)(1) = -3$, so $f(x) = x^2 + x + 1$ has no real zeros, which means it is either always positive or always negative. Now, $f(-5) = (-5)^2 + (-5) + 1 = 21$ is positive, so the solution is $\{x \mid x \text{ is any real number}\}$ or, using interval notation, $(-\infty, \infty)$.

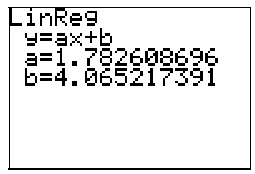
40. Solving $x^2 - x + 1 < 0$

The discriminant $b^2 - 4ac = (-1)^2 - 4(1)(1) = -3$, so $f(x) = x^2 - x + 1$ has no real zeros, which means it is either always positive or always negative. Now, $f(-5) = (-5)^2 - (-5) + 1 = 31$ is positive, so the solution is { } or $\varnothing$.

Section 2.6

1. $R = 3x$

2. $y = 1.7826x + 4.0652$



3. $R(p) = -4p^2 + 4000p$, $a = -4, b = 4000, c = 0$. Since $a = -4 < 0$, the graph is a parabola that opens down, so the vertex is a maximum point. The maximum occurs at

$p = \dfrac{-b}{2a} = \dfrac{-4000}{2(-4)} = 500$. Thus, the unit price should be \$500 for maximum revenue.

The maximum revenue is

$$\begin{aligned} R(500) &= -4(500)^2 + 4000(500) \\ &= -1000000 + 2000000 \\ &= \$1,000,000 \end{aligned}$$

4. $R(p) = -\dfrac{1}{2}p^2 + 1900p$,

$a = -\dfrac{1}{2}, b = 1900, c = 0$. Since $a = -\dfrac{1}{2} < 0$, the graph is a parabola that opens down, so the vertex is a maximum point. The maximum occurs at $p = \dfrac{-b}{2a} = \dfrac{-1900}{2(-1/2)} = \dfrac{-1900}{-1} = 1900$.

Thus, the unit price should be \$1900 for maximum revenue. The maximum revenue is

$$\begin{aligned} R(1900) &= -\frac{1}{2}(1900)^2 + 1900(1900) \\ &= -1805000 + 3610000 \\ &= \$1,805,000 \end{aligned}$$

5. $C(x) = x^2 - 80x + 2000$,
$a = 1, b = -80, c = 2000$. Since $a = 1 > 0$, the graph opens up, so the vertex is a minimum point. The minimum marginal cost occurs at
$x = \dfrac{-b}{2a} = \dfrac{-(-80)}{2(1)} = \dfrac{80}{2} = 40$ televisions produced. The minimum marginal cost is

$$f\left(\frac{-b}{2a}\right) = f(40) = (40)^2 - 80(40) + 2000$$
$$= 1600 - 3200 + 2000$$
$$= \$400$$

6. $C(x) = 5x^2 - 200x + 4000$,
$a = 5, b = -200, c = 4000$. Since $a = 5 > 0$, the graph opens up, so the vertex is a minimum point. The minimum marginal cost occurs at
$x = \dfrac{-b}{2a} = \dfrac{-(-200)}{2(5)} = \dfrac{200}{10} = 20$ cell phones manufactured. The minimum marginal cost is

$$f\left(\frac{-b}{2a}\right) = f(20) = 5(20)^2 - 200(20) + 4000$$
$$= 2000 - 4000 + 4000$$
$$= \$2000$$

7. a. $a = -\dfrac{32}{2500}, b = 1, c = 200$. The maximum height occurs when

$$x = \frac{-b}{2a} = \frac{-1}{2(-32/2500)} = \frac{2500}{64} \approx 39 \text{ feet}$$

from base of the cliff.

b. The maximum height is

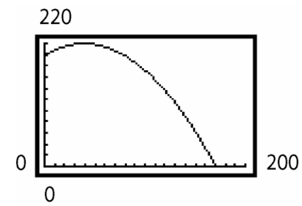
$$h(39.0625) = \frac{-32(39.0625)^2}{2500} + 39.0625 + 200$$
$$\approx 219.5 \text{ feet.}$$

c. Solving when $h(x) = 0$:

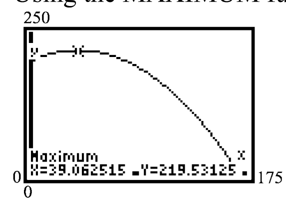
$$-\frac{32}{2500}x^2 + x + 200 = 0$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4(-32/2500)(200)}}{2(-32/2500)}$$
$$x \approx \frac{-1 \pm \sqrt{11.24}}{-0.0256}$$

$x \approx -91.90$ or $x \approx 170$
Since the distance cannot be negative, the projectile strikes the water approximately 170 feet from the base of the cliff.

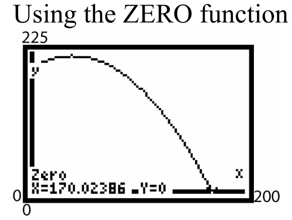
d.



e. Using the MAXIMUM function



Using the ZERO function



f.
$$-\frac{32}{2500}x^2 + x + 200 = 100$$
$$-\frac{32}{2500}x^2 + x + 100 = 0$$
$$x = \frac{\sqrt{1^2 - 4(-32/2500)(100)}}{2(-32/2500)} = \frac{-1 \pm \sqrt{6.12}}{-0.0256}$$

$x \approx -57.57$ or $x \approx 135.70$
Since the distance cannot be negative, the projectile is 100 feet above the water when it is approximately 135.7 feet from the base of the cliff.

8. a. $a = -\dfrac{32}{10000}, b = 1, c = 0.$
The maximum height occurs when

$$x = \frac{-b}{2a} = \frac{-1}{2\left(-\frac{32}{10000}\right)}$$
$$= \frac{10000}{64} = 156.25 \text{ feet}$$

b. The maximum height is

$$h(156.25) = \frac{-32(156.25)^2}{10000} + 156.25$$
$$= 78.125 \text{ feet}$$

c. Solving when $h(x) = 0$:

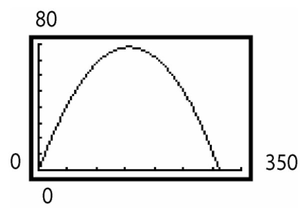
$$-\frac{32}{10000}x^2 + x = 0$$

$$x\left(-\frac{32}{10000}x + 1\right) = 0$$

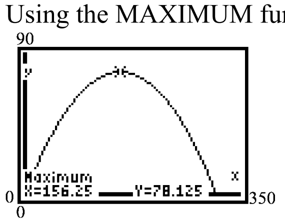
$x = 0$ or $x = 312.5$

Since the distance cannot be zero, the projectile lands 312.5 feet from where it was fired.

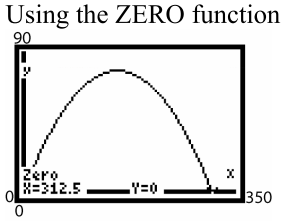
d.



e. Using the MAXIMUM function



Using the ZERO function



f. Solving when $h(x) = 50$:

$$-\frac{32}{10000}x^2 + x = 50$$

$$-\frac{32}{10000}x^2 + x - 50 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-32/10000)(-50)}}{2(-32/10000)}$$

$$= \frac{-1 \pm \sqrt{0.36}}{-0.0064} \approx \frac{-1 \pm 0.6}{-0.0064}$$

$x = 62.5$ or $x = 250$

The projectile is 50 feet above the ground 62.5 feet and 250 feet from where it was fired.

9. a. $a = -3.24, b = 242.1, c = -738.4$

The maximum number of hunters occurs when the income level is

$$x = \frac{-b}{2a} = \frac{-242.1}{2(-3.24)} = \frac{-242.1}{-6.48} \approx 37.4 \text{ years old}$$

The number of hunters this old is: $H(37.4)$

$$= -3.24(37.4)^2 + 242.1(37.4) - 738.4$$

$$\approx 3784 \text{ hunters}$$

b. The maximum occurs when $x = 37.4$, so the function increases on the interval $(0, 37.4)$ and decreases on the interval $(37.4, \infty)$

Therefore, the number of hunters is decreasing for individuals who are between ages 40 and 45 years of age.

10. a. $a = -0.008, b = 0.815, c = -9.983$

The maximum number of hunters occurs when the income level is

$$x = \frac{-b}{2a} = \frac{-0.815}{2(-0.008)} = \frac{-0.815}{-0.016} \approx 51 \text{ years old.}$$

The percentage for this age is:

$$P(51) = -0.008(51)^2 + 0.815(51) - 9.983$$

$$\approx 10.8\%$$

b. The maximum occurs when $x = 51$, so the function increases on the interval $(0, 51)$ and decreases on the interval $(51, \infty)$.

Therefore, the percentage of Americans who have earned advanced degrees if increasing for individuals who are between ages 40 and 50 years of age.

11. a. $M(23) = 1.00(23)^2 - 136.74(23) + 4764.89$

$$\approx 2149 \text{ male murder victims}$$

b. $1.00x^2 - 136.74x + 4764.89 = 1456$

$1.00x^2 - 136.74x + 3308.89 = 0$

$a = 1.00, b = -136.74, c = 3308.89$

$$x = \frac{-(-136.74) \pm \sqrt{(-136.74)^2 - 4(1.00)(3308.89)}}{2(1.00)}$$

$$= \frac{136.74 \pm \sqrt{5462.2676}}{2} \approx \frac{136.74 \pm 73.91}{2}$$

$x \approx 31$ or $x \approx 105$

Disregard 105 since it falls outside the domain for the function ($20 \le x < 90$).

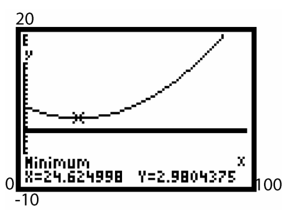
Thus, the number of male murder victims is 1456 for 31 year olds.

c. A minimum occurs when
$$x = \frac{-b}{2a} = \frac{-(-136.74)}{2(1.00)} = \frac{136.74}{2} = 68.37,$$
so the function decreases on the interval $(20, 68.37)$ and increases on the interval $(68.37, 90)$. As age increases between 20 and 65, the number of murder victims decreases.

12. a. $H(45) = 0.004(45)^2 - 0.197(45) + 54.06$
$\approx 4.6\%$

b. Solve for *x*:
$$H(x) = 0.004x^2 - 0.197x + 5.406 = 10$$
$$0.004x^2 - 0.197x + 5.406 = 10$$
$$0.004x^2 - 0.197x - 4.594 = 0$$
$a = 0.004,\ b = -0.197,\ c = -4.594$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-0.197) \pm \sqrt{(-0.197)^2 - 4(0.004)(-4.594)}}{2(0.004)}$$
$$= \frac{0.197 \pm \sqrt{0.112313}}{0.008} \approx \frac{0.197 \pm 0.335}{0.008}$$
≈ 66.5 or -17.25, which is not practical
The only practical solution is $x \approx 67$ years old.

c.



d. As age increases between the ages of 0 and 20.62 years, the percentage decreases. After age 20.62 years, percentage increases as age increases.

13. a. $R(x) = 75x - 0.2x^2$
$a = -0.2,\ b = 75,\ c = 0$
The maximum revenue occurs when
$$x = \frac{-b}{2a} = \frac{-75}{2(-0.2)} = \frac{-75}{-0.4} = 187.5$$
The maximum revenue occurs when $x = 187$ or $x = 188$.
The maximum revenue is:
$$R(187) = 75(187) - 0.2(187)^2 = \$7031.20$$
$$R(188) = 75(188) - 0.2(188)^2 = \$7031.20$$

b. $P(x) = R(x) - C(x)$
$= 75x - 0.2x^2 - (32x + 1750)$
$= -0.2x^2 + 43x - 1750$

c. $P(x) = -0.2x^2 + 43x - 1750$
$a = -0.2,\ b = 43,\ c = -1750$
$$x = \frac{-b}{2a} = \frac{-43}{2(-0.2)} = \frac{-43}{-0.4} = 107.5$$
The maximum profit occurs when $x = 107$ or $x = 108$.
The maximum profit is:
$$P(107) = -0.2(107)^2 + 43(107) - 1750 = \$561.20$$
$$P(108) = -0.2(108)^2 + 43(108) - 1750 = \$561.20$$

d. Answers will vary.

14. a. $R(x) = 9.5x - 0.04x^2$
$a = -0.04,\ b = 9.5,\ c = 0$
The maximum revenue occurs when
$$x = \frac{-b}{2a} = \frac{-9.5}{2(-0.04)} = \frac{-9.5}{-0.08}$$
$= 118.75 \approx 119$ boxes
The maximum revenue is:
$$R(119) = 9.5(119) - 0.04(119)^2 = \$564.06$$

b. $P(x) = R(x) - C(x)$
$= 9.5x - 0.04x^2 - (1.25x + 250)$
$= -0.04x^2 + 8.25x - 250$

c. $P(x) = -0.04x^2 + 8.25x - 250$
$a = -0.04,\ b = 8.25,\ c = -250$
The maximum profit occurs when
$$x = \frac{-b}{2a} = \frac{-8.25}{2(-0.04)} = \frac{-8.25}{-0.08}$$
$= 103.125 \approx 103$ boxes
The maximum profit is:
$$P(103) = -0.04(103)^2 + 8.25(103) - 250 = \$175.39$$

d. Answers will vary.

15. a. $R(x) = x\left(-\frac{1}{6}x + 100\right) = -\frac{1}{6}x^2 + 100x$

b. $$R(200) = -\frac{1}{6}(200)^2 + 100(200)$$
$$= \frac{-20000}{3} + 20000$$
$$= \frac{40000}{3} \approx \$13,333.33$$

c. $x = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{1}{6}\right)} = \frac{-100}{\left(-\frac{1}{3}\right)} = \frac{300}{1} = 300$

The maximum revenue is
$$R(300) = -\frac{1}{6}(300)^2 + 100(300)$$
$$= -15000 + 30000$$
$$= \$15,000$$

d. $p = -\frac{1}{6}(300) + 100 = -50 + 100 = \$50$

16. a. $R(x) = x\left(-\frac{1}{3}x + 100\right) = -\frac{1}{3}x^2 + 100x$

b. $$R(100) = -\frac{1}{3}(100)^2 + 100(100)$$
$$= \frac{-10000}{3} + 10000$$
$$= \frac{20000}{3} \approx \$6,666.67$$

c. $x = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{1}{3}\right)} = \frac{-100}{\left(-\frac{2}{3}\right)} = \frac{300}{2} = 150$

The maximum revenue is
$$R(150) = -\frac{1}{3}(150)^2 + 100(150)$$
$$= -7500 + 15000$$
$$= \$7,500$$

d. $p = -\frac{1}{3}(150) + 100 = -50 + 100 = \$50$

17. a. If $x = -5p + 100$, then $p = \frac{100 - x}{5}$.

$$R(x) = x\left(\frac{100 - x}{5}\right) = -\frac{1}{5}x^2 + 20x$$

b. $$R(15) = -\frac{1}{5}(15)^2 + 20(15)$$
$$= -45 + 300$$
$$= \$255$$

c. $x = \frac{-b}{2a} = \frac{-20}{2\left(-\frac{1}{5}\right)} = \frac{-20}{\left(-\frac{2}{5}\right)} = \frac{100}{2} = 50$

The maximum revenue is
$$R(50) = -\frac{1}{5}(50)^2 + 20(50)$$
$$= -500 + 1000 = \$500$$

d. $p = \frac{100 - 50}{5} = \frac{50}{5} = \$10$

18. a. If $x = -20p + 500$, then $p = \frac{500 - x}{20}$.

$$R(x) = x\left(\frac{500 - x}{20}\right) = -\frac{1}{20}x^2 + 25x$$

b. $$R(20) = -\frac{1}{20}(20)^2 + 25(20)$$
$$= -20 + 500$$
$$= \$480.$$

c. $x = \frac{-b}{2a} = \frac{-25}{2\left(-\frac{1}{20}\right)} = \frac{-25}{\left(-\frac{1}{10}\right)} = \frac{250}{1} = 250$.

The maximum revenue is
$$R(250) = -\frac{1}{20}(250)^2 + 25(250)$$
$$= -3125 + 6250$$
$$= \$3125.$$

d. $p = \frac{500 - 250}{20} = \frac{250}{20} = \$12.50$.

19. a. Let w = width and l = length of the rectangular area.
Solving $P = 2w + 2l = 400$ for l:
$$l = \frac{400 - 2w}{2} = 200 - w.$$
Then $A(w) = (200 - w)w = 200w - w^2$.
$$= -w^2 + 200w$$

b. $w = \frac{-b}{2a} = \frac{-200}{2(-1)} = \frac{-200}{-2} = 100$ yards

c. $$A(100) = -100^2 + 200(100)$$
$$= -10000 + 20000$$
$$= 10,000 \text{ sq yds.}$$

20. **a.** Let $x =$ length and $y =$ width of the rectangular field.
Solving $P = 2x + 2y = 3000$ for y:
$$y = \frac{3000 - 2x}{2} = 1500 - x.$$
Then $A(x) = (1500 - x)x = 1500x - x^2$
$$= -x^2 + 1500x.$$

b. $x = \frac{-b}{2a} = \frac{-1500}{2(-1)} = \frac{-1500}{-2} = 750$ feet

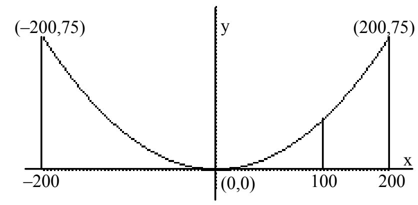
c. $A(750) = -750^2 + 1500(750)$
$$= -562500 + 1125000$$
$$= 562,500 \text{ ft}^2$$

21. Let $x =$ width and $y =$ length of the rectangular area. Solving $P = 2x + y = 4000$ for y:
$y = 4000 - 2x$. Then
$$A(x) = (4000 - 2x)x = 4000x - 2x^2 = -2x^2 + 4000x$$
$$x = \frac{-b}{2a} = \frac{-4000}{2(-2)} = \frac{-4000}{-4} = 1000 \text{ meters}$$
maximizes area.
$$A(1000) = -2(1000)^2 + 4000(1000).$$
$$= -2000000 + 4000000$$
$$= 2,000,000$$
The largest area that can be enclosed is 2,000,000 square meters.

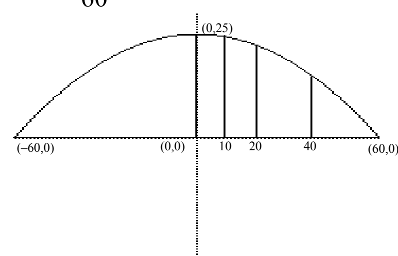
22. Let $x =$ width and $y =$ length of the rectangular area.
$2x + y = 2000 \quad \Rightarrow \quad y = 2000 - 2x$ Then
$$A(x) = (2000 - 2x)x$$
$$= 2000x - 2x^2 = -2x^2 + 2000x$$
$$x = -\frac{b}{2a} = -\frac{2000}{2(-2)} = -\frac{2000}{-4} = 500 \text{ meters}$$
maximizes area
$$A(500) = -2(500)^2 + 2000(500)$$
$$= -500,000 + 1,000,000 = 500,000$$
The largest area that can be enclosed is 500,000 square meters.

23. Locate the origin at the point where the cable touches the road. Then the equation of the parabola is of the form: $y = ax^2$, where $a > 0$. Since the point (200, 75) is on the parabola, we can find the constant a:

Since $75 = a(200)^2$, then
$$a = \frac{75}{200^2} = 0.001875.$$
When $x = 100$, we have:
$$y = 0.001875(100)^2 = 18.75 \text{ meters}.$$



24. Locate the origin at the point directly under the highest point of the arch. Then the equation of the parabola is of the form:
$y = -ax^2 + k$, where $a > 0$. Since the maximum height is 25 feet, when $x = 0$, $y = k = 25$. Since the point (60, 0) is on the parabola, we can find the constant a:
Since $0 = -a(60)^2 + 25$ then
$$a = \frac{25}{60^2} \approx 0.006944$$
The equation of the parabola is:
$$y = -\frac{25}{60^2}x^2 + 25.$$



At $x = 10$:
$$y = -\frac{25}{60^2}(10)^2 + 25 = -\frac{25}{36} + 25 \approx 24.3 \text{ ft.}$$
At $x = 20$:
$$y = -\frac{25}{60^2}(20)^2 + 25 = -\frac{25}{9} + 25 \approx 22.2 \text{ ft.}$$
At $x = 40$:
$$y = -\frac{25}{60^2}(40)^2 + 25 = -\frac{100}{9} + 25 \approx 13.9 \text{ ft.}$$

25. Let $x =$ the depth of the gutter and $y =$ the width of the gutter.
Then $A = xy$ is the cross-sectional area of the gutter. Since the aluminum sheets for the gutter are 12 inches wide, we have $2x + y = 12$. Solving for y: $y = 12 - 2x$. The area is to be maximized, so: $A = xy = x(12-2x) = -2x^2 + 12x$. This equation is a parabola opening down; thus, it has a maximum when $x = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$.
Thus, a depth of 3 inches produces a maximum cross-sectional area.

26. Let $x =$ width of the window and $y =$ height of the rectangular part of the window. The perimeter of the window is: $x + 2y + \frac{\pi x}{2} = 20.$

Solving for y: $y = \frac{40 - 2x - \pi x}{4}$

The area of the window is:

$$A(x) = x\left(\frac{40-2x-\pi x}{4}\right) + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$
$$= 10x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$$
$$= \left(-\frac{1}{2} - \frac{\pi}{8}\right)x^2 + 10x.$$

This equation is a parabola opening down; thus, it has a maximum when

$$x = \frac{-b}{2a} = \frac{-10}{2\left(-\frac{1}{2}-\frac{\pi}{8}\right)} = \frac{10}{\left(1+\frac{\pi}{4}\right)} \approx 5.6 \text{ feet}$$
$$y = \frac{40 - 2(5.60) - \pi(5.60)}{4} \approx 2.8 \text{ feet}$$

The width of the window is about 5.6 feet and the height of the rectangular part is approximately 2.8 feet. The radius of the semicircle is roughly 2.8 feet, so the total height is about 5.6 feet.

27. Let $x =$ the width of the rectangle or the diameter of the semicircle and let $y =$ the length of the rectangle.

The perimeter of each semicircle is $\frac{\pi x}{2}$.

The perimeter of the track is given by: $\frac{\pi x}{2} + \frac{\pi x}{2} + y + y = 1500$.

Solving for x:

$$\pi x + 2y = 1500$$
$$\pi x = 1500 - 2y$$
$$x = \frac{1500-2y}{\pi}$$

The area of the rectangle is:

$$A = xy = \left(\frac{1500-2y}{\pi}\right)y = \frac{-2}{\pi}y^2 + \frac{1500}{\pi}y.$$

This equation is a parabola opening down; thus, it has a maximum when

$$y = \frac{-b}{2a} = \frac{\frac{-1500}{\pi}}{2\left(\frac{-2}{\pi}\right)} = \frac{-1500}{-4} = 375..$$

Thus, $x = \frac{1500 - 2(375)}{\pi} = \frac{750}{\pi} \approx 238.73$

The dimensions for the rectangle with maximum area are $\frac{750}{\pi} \approx 238.73$ meters by 375 meters.

28. Let $x =$ width of the window and $y =$ height of the rectangular part of the window.
The perimeter of the window is:

$$3x + 2y = 16$$
$$y = \frac{16-3x}{2}\cdot$$

The area of the window is

$$A(x) = x\left(\frac{16-3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$$
$$= 8x - \frac{3}{2}x^2 + \frac{\sqrt{3}}{4}x^2$$
$$= \left(-\frac{3}{2} + \frac{\sqrt{3}}{4}\right)x^2 + 8x$$

This equation is a parabola opening down; thus, it has a maximum when

$$x = \frac{-b}{2a} = \frac{-8}{2\left(-\frac{3}{2}+\frac{\sqrt{3}}{4}\right)} = \frac{-8}{\left(-3+\frac{\sqrt{3}}{2}\right)} \approx 3.75 \text{ ft}, .$$
$$y = \frac{16 - 3(3.75)}{2} \approx 2.38$$

The window is approximately 3.75 feet wide and 2.38 feet high (rectangular part). The height of the equilateral triangle is $\frac{\sqrt{3}}{2}(3.75) \approx 3.25$ feet, so the total height is about 5.63 feet.

29. We are given: $V(x) = kx(a-x) = -kx^2 + akx$.
The reaction rate is a maximum when:
$x = \dfrac{-b}{2a} = \dfrac{-ak}{2(-k)} = \dfrac{ak}{2k} = \dfrac{a}{2}$.

30. We have:
$a(-h)^2 + b(-h) + c = ah^2 - bh + c = y_0$
$a(0)^2 + b(0) + c = c = y_1$
$a(h)^2 + b(h) + c = ah^2 + bh + c = y_2$
Equating the two equations for the area, we have:
$$y_0 + 4y_1 + y_2 = ah^2 - bh + c + 4c + ah^2 + bh + c$$
$$= 2ah^2 + 6c.$$
Therefore,
$\text{Area} = \dfrac{h}{3}\left(2ah^2 + 6c\right) = \dfrac{h}{3}\left(y_0 + 4y_1 + y_2\right)$ sq. units.

31. $f(x) = -5x^2 + 8,\ h = 1$
$$\text{Area} = \frac{h}{3}\left(2ah^2 + 6c\right) = \frac{1}{3}\left(2(-5)(1)^2 + 6(8)\right)$$
$$= \frac{1}{3}(-10 + 48) = \frac{38}{3} \approx 12.67 \text{ sq. units}$$

32. $f(x) = 2x^2 + 8, \quad h = 2$
$$\text{Area} = \frac{h}{3}(2ah^2 + 6c) = \frac{2}{3}\left(2(2)(2)^2 + 6(8)\right)$$
$$= \frac{2}{3}(16 + 48) = \frac{2}{3}(64)$$
$$= \frac{128}{3} \approx 42.67 \text{ sq. units.}$$

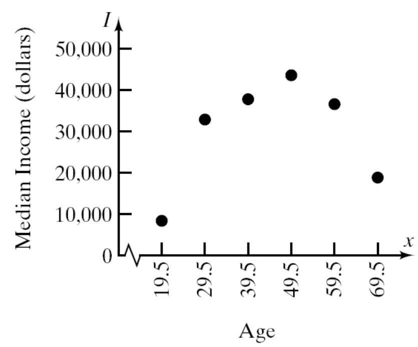
33. $f(x) = x^2 + 3x + 5, \ h = 4$
$$\text{Area} = \frac{h}{3}\left(2ah^2 + 6c\right) = \frac{4}{3}\left(2(1)(4)^2 + 6(5)\right)$$
$$= \frac{4}{3}(32 + 30) = \frac{248}{3} \approx 82.67 \text{ sq. units.}$$

34. $f(x) = -x^2 + x + 4, \quad h = 1$
$$\text{Area} = \frac{h}{3}(2ah^2 + 6c) = \frac{1}{3}\left(2(-1)(1)^2 + 6(4)\right)$$
$$= \frac{1}{3}(-2 + 24) = \frac{1}{3}(22)$$
$$= \frac{22}{3} \approx 7.33 \text{ sq. units}$$

35. The area function is:
$A(x) = x(10 - x) = -x^2 + 10x$.
The maximum value occurs at the vertex:
$x = \dfrac{-b}{2a} = \dfrac{-10}{2(-1)} = \dfrac{-10}{-2} = 5$
The maximum area is:
$A(5) = -(5)^2 + 10(5) = -25 + 50 = 25$ sq. units.

36. If x is even, then ax^2 and bx are even. When two even numbers are added to an odd number the result is odd. Thus, $f(x)$ is odd. If x is odd, then ax^2 and bx are odd. The sum of three odd numbers is an odd number. Thus, $f(x)$ is odd.

37. a. From the graph, the data appear to be quadratic with $a < 0$.



b. $x = \dfrac{-b}{2a} = \dfrac{-3998}{2(-42.7)} \approx 46.8$

An individual will earn the most income at an age of 46.8 years.

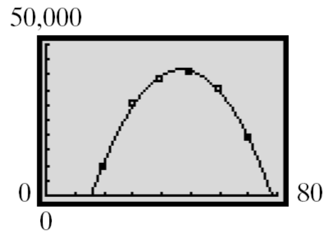
c. The maximum income will be:
$$I(46.8) = -42.7(46.8)^2 + 3998(46.8) - 51873$$
$$\approx \$41,710$$

d. Using the QUADratic REGression program

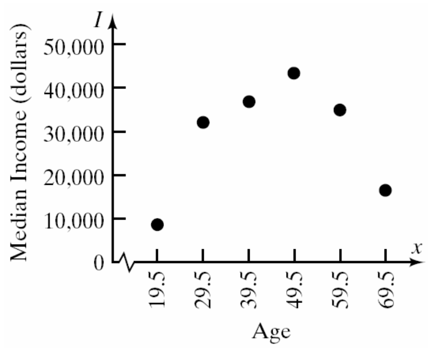
```
QuadReg
 y=ax²+bx+c
 a=-42.67464286
 b=3998.271786
 c=-51873.19545
```

$I(x) = -42.7x^2 + 3998x - 51873$

e.



38. a. From the graph, the data appear to be quadratic with $a < 0$.



b. $x = \dfrac{-b}{2a} = \dfrac{-3963}{2(-42.3)} \approx 46.8$

An individual will earn the most income at an age of 46.8 years.

c. The maximum income will be:

$I(46.8) = -42.3(46.8)^2 + 3963(46.8) - 51070$

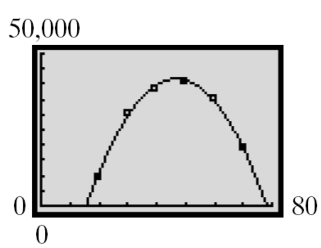
$\approx \$41,751$

d. Using the QUADratic REGression program

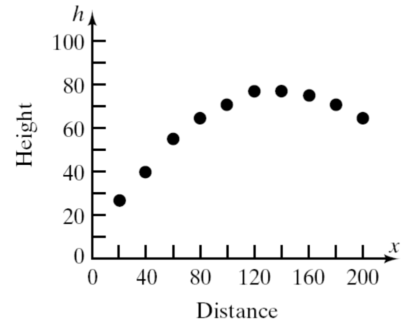
```
QuadReg
 y=ax²+bx+c
 a=-42.32285714
 b=3963.44
 c=-51070.24214
```

$I(x) = -42.3x^2 + 3963x - 51070$

e.



39. a. From the graph, the data appear to be quadratic with $a < 0$.



b. $x = \dfrac{-b}{2a} = \dfrac{-1.03}{2(-0.0037)} \approx 139.2$

The ball will travel 139.2 feet before it reaches its maximum height.

c. The maximum height will be:

$I(139.2)$

$= -0.0037(139.2)^2 + 1.03(139.2) + 5.7$

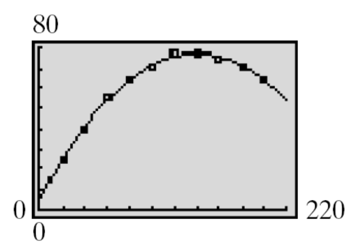
≈ 77.4 feet

d. Using the QUADratic REGression program

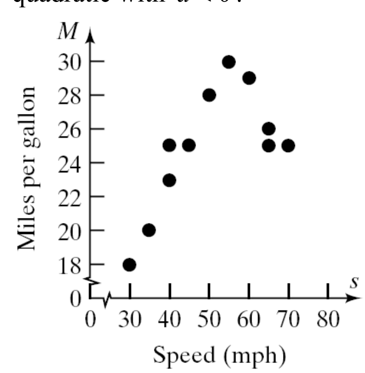
```
QuadReg
 y=ax²+bx+c
 a=-.0037121212
 b=1.031818182
 c=5.666666667
```

$I(x) = -0.0037x^2 + 1.03x + 5.7$

e.



40. a. From the graph, the data appear to be quadratic with $a < 0$.



b. $x = \dfrac{-b}{2a} = \dfrac{-1.93}{2(-0.018)} \approx 53.6$

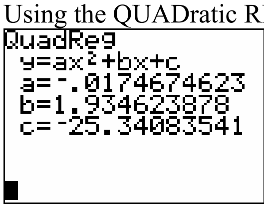
The speed that maximizes miles per gallon is 53.6 mile per hour.

c. The miles per gallon for a speed of 53.6 miles per hour is:

$M(63) = -0.018(53.6)^2 + 1.93(53.6) - 25.34$

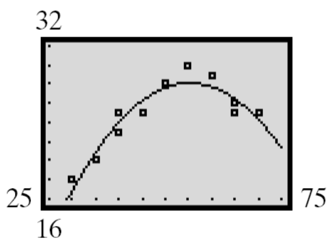
≈ 26.4 miles per gallon

d. Using the QUADratic REGression program



$M(s) = -0.0175s^2 + 1.93s - 25.34$

e.



41. Answers will vary. One possibility follows: If the price is \$140, no one will buy the calculators, thus making the revenue \$0.

Section 2.7

1. Integers: $\{-3, 0\}$

Rationals: $\left\{-3, 0, \dfrac{6}{5}\right\}$

2. True; the set of real numbers consists of all rational and irrational numbers.

3. $10 - 5i$

4. True

5. True

6. $9i$

7. False. If $2 - 3i$ is the zero of a quadratic function with real coefficients, then $2 + 3i$ is also a zero.

8. True

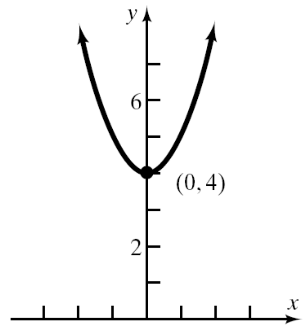
9. $f(x) = 0$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

The zero are $-2i$ and $2i$.



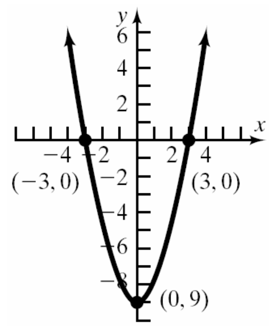
10. $f(x) = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm\sqrt{9} = \pm 3$

The zeros are -3 and 3.



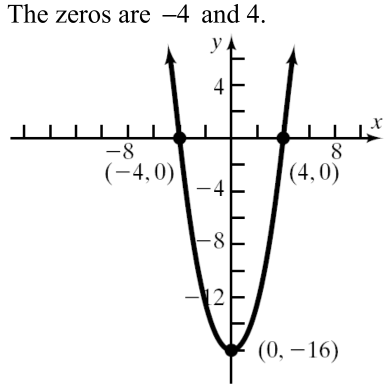
11. $f(x) = 0$

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm\sqrt{16} = \pm 4$

The zeros are -4 and 4.



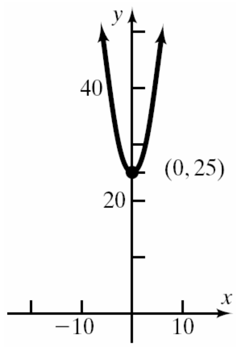
12. $f(x)=0$

$x^2+25=0$

$x^2=-25$

$x=\pm\sqrt{-25}=\pm 5i$

The zeros are $-5i$ and $5i$.



13. $f(x)=0$

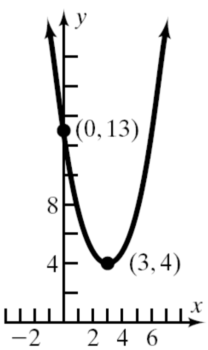
$x^2-6x+13=0$

$a=1, b=-6, c=13,$

$b^2-4ac=(-6)^2-4(1)(13)=36-52=-16$

$$x=\frac{-(-6)\pm\sqrt{-16}}{2(1)}=\frac{6\pm 4i}{2}=3\pm 2i$$

The zeros are $3-2i$ and $3+2i$.



14. $f(x)=0$

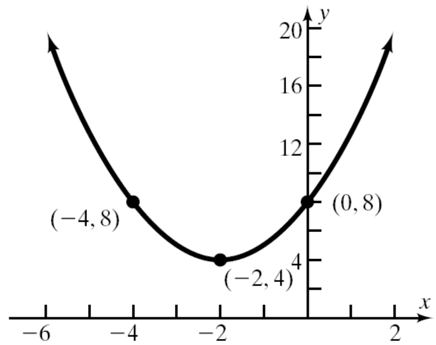
$x^2+4x+8=0$

$a=1, b=4, c=8$

$b^2-4ac=4^2-4(1)(8)=16-32=-16$

$$x=\frac{-4\pm\sqrt{-16}}{2(1)}=\frac{-4\pm 4i}{2}=-2\pm 2i$$

The zeros are $-2-2i$ and $-2+2i$.



15. $f(x)=0$

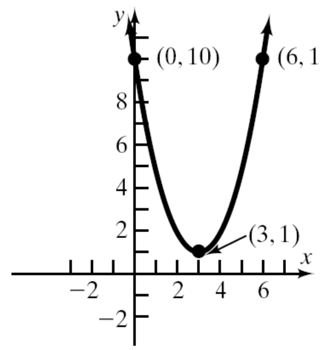
$x^2-6x+10=0$

$a=1, b=-6, c=10$

$b^2-4ac=(-6)^2-4(1)(10)=36-40=-4$

$$x=\frac{-(-6)\pm\sqrt{-4}}{2(1)}=\frac{6\pm 2i}{2}=3\pm i$$

The zeros are $3-i$ and $3+i$.



16. $f(x)=0$

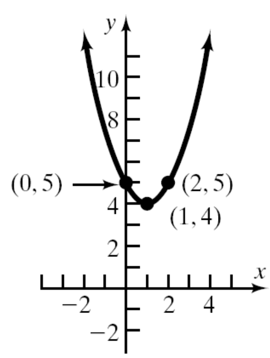
$x^2-2x+5=0$

$a=1, b=-2, c=5$

$b^2-4ac=(-2)^2-4(1)(5)=4-20=-16$

$$x=\frac{-(-2)\pm\sqrt{-16}}{2(1)}=\frac{2\pm 4i}{2}=1\pm 2i$$

The zeros are $1-2i$ and $1+2i$.



17. $f(x)=0$

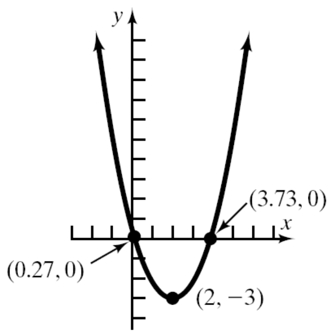
$x^2-4x+1=0$

$a=1, b=-4, c=1$

$b^2-4ac=(-4)^2-4(1)(1)=16-4=12$

$$x=\frac{-(-4)\pm\sqrt{12}}{2(1)}=\frac{4\pm 2\sqrt{3}}{2}=2\pm\sqrt{3}$$

The zeros are $2-\sqrt{3}$ and $2+\sqrt{3}$, or approximately 0.27 and 3.73.



18. $f(x)=0$

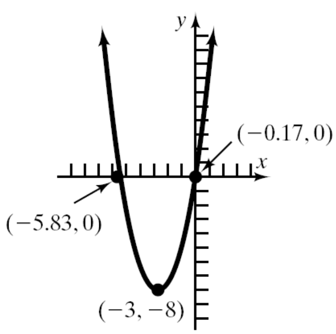
$x^2+6x+1=0$

$a=1, b=6, c=1$

$b^2-4ac=6^2-4(1)(1)=36-4=32$

$$x=\frac{-6\pm\sqrt{32}}{2(1)}=\frac{-6\pm 4\sqrt{2}}{2}=-3\pm 2\sqrt{2}$$

The zeros are $-3-2\sqrt{2}$ and $-3+2\sqrt{2}$, or approximately -5.83 and -0.17.



19. $f(x)=0$

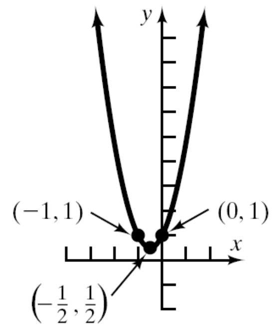
$2x^2+2x+1=0$

$a=2, b=2, c=1$

$b^2-4ac=(2)^2-4(2)(1)=4-8=-4$

$$x=\frac{-2\pm\sqrt{-4}}{2(2)}=\frac{-2\pm 2i}{4}=-\frac{1}{2}\pm\frac{1}{2}i$$

The zeros are $-\frac{1}{2}-\frac{1}{2}i$ and $-\frac{1}{2}+\frac{1}{2}i$.



20. $f(x)=0$

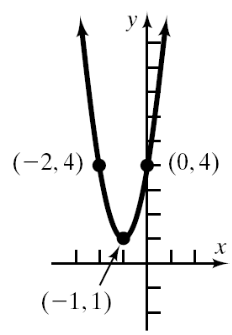
$3x^2+6x+4=0$

$a=3, b=6, c=4$

$b^2-4ac=(6)^2-4(3)(4)=36-48=-12$

$$x=\frac{-6\pm\sqrt{-12}}{2(3)}=\frac{-6\pm 2\sqrt{3}i}{6}=-1\pm\frac{\sqrt{3}}{3}i$$

The zeros are $-1-\frac{\sqrt{3}}{3}i$ and $-1+\frac{\sqrt{3}}{3}i$.



21. $f(x)=0$

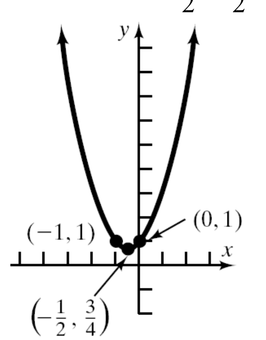
$x^2+x+1=0$

$a=1, b=1, c=1,$

$b^2-4ac=1^2-4(1)(1)=1-4=-3$

$$x=\frac{-1\pm\sqrt{-3}}{2(1)}=\frac{-1\pm\sqrt{3}\,i}{2}=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$$

The zeros are $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ and $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$.



22. $f(x)=0$

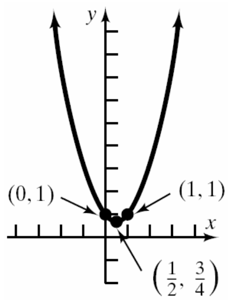
$x^2-x+1=0$

$a=1, b=-1, c=1$

$b^2-4ac=(-1)^2-4(1)(1)=1-4=-3$

$x=\dfrac{-(-1)\pm\sqrt{-3}}{2(1)}=\dfrac{1\pm\sqrt{3}\,i}{2}=\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$

The zeros are $\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$ and $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$.



23. $f(x)=0$

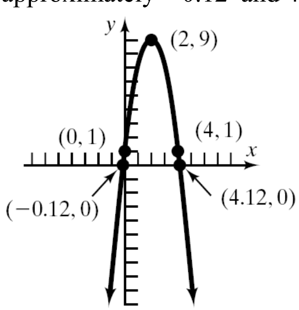
$-2x^2+8x+1=0$

$a=-2, b=8, c=1$

$b^2-4ac=8^2-4(-2)(1)=64+8=72$

$x=\dfrac{-8\pm\sqrt{72}}{2(-2)}=\dfrac{-8\pm6\sqrt{2}}{-4}=\dfrac{4\pm3\sqrt{2}}{2}=2\pm\dfrac{3\sqrt{2}}{2}$

The zeros are $\dfrac{4-3\sqrt{2}}{2}$ and $\dfrac{4+3\sqrt{2}}{2}$, or approximately -0.12 and 4.12.



24. $f(x)=0$

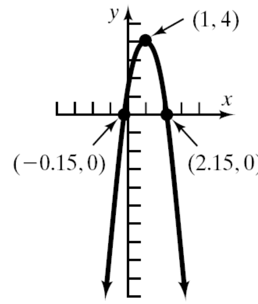
$-3x^2+6x+1=0$

$a=-3, b=6, c=1$

$b^2-4ac=6^2-4(-3)(1)=36+12=48$

$x=\dfrac{-6\pm\sqrt{48}}{2(-3)}=\dfrac{-6\pm4\sqrt{3}}{-6}=\dfrac{3\pm2\sqrt{3}}{3}=1\pm\dfrac{2\sqrt{3}}{3}$

The zeros are $\dfrac{3-2\sqrt{3}}{3}$ and $\dfrac{3+2\sqrt{3}}{3}$, or approximately -0.15 and 2.15.



25. $3x^2-3x+4=0$

$a=3, b=-3, c=4$

$b^2-4ac=(-3)^2-4(3)(4)=9-48=-39$

The equation has two complex solutions that are conjugates of each other.

26. $2x^2-4x+1=0$

$a=2, b=-4, c=1$

$b^2-4ac=(-4)^2-4(2)(1)=16-8=8$

The equation has two unequal real number solutions.

27. $2x^2+3x-4=0$

$a=2, b=3, c=-4$

$b^2-4ac=3^2-4(2)(-4)=9+32=41$

The equation has two unequal real solutions.

28. $x^2+2x+6=0$

$a=1, b=2, c=6$

$b^2-4ac=(2)^2-4(1)(6)=4-24=-20$

The equation has two complex solutions that are conjugates of each other.

29. $9x^2-12x+4=0$

$a=9, b=-12, c=4$

$b^2-4ac=(-12)^2-4(9)(4)=144-144=0$

The equation has a repeated real solution.

30. $4x^2+12x+9=0$

$a=4, b=12, c=9$

$b^2-4ac=12^2-4(4)(9)=144-144=0$

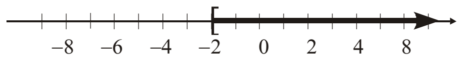
The equation has a repeated real solution.

31. The other solution is the conjugate of $2+3i$, which is $2-3i$.

32. The other solution is the conjugate of $4-i$, which is $4+i$.

Section 2.8

1. $x \geq -2$



2. False. $|0| = 0$ is not positive.

3. $4x - 3 = 9$

$4x = 12$

$x = 3$

The solution set is $\{3\}$.

4. $3x - 2 > 7$

$3x > 9$

$x > 3$

The solution set is $\{x \mid x > 3\}$ or, using interval notation, $(3, \infty)$.

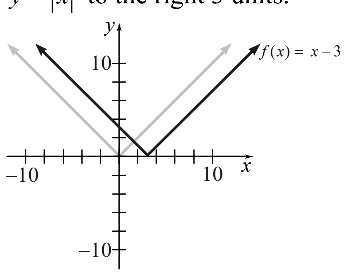
5. $-1 < 2x + 5 < 13$

$-6 < 2x < 8$

$-3 < x < 4$

The solution set is $\{x \mid -3 < x < 4\}$ or, using interval notation, $(-3, 4)$.

6. To graph $f(x) = |x - 3|$, shift the graph of $y = |x|$ to the right 3 units.



7. $-a$; a

8. $-a < u < a$

9. $<$

10. True

11. False. Any real number will be a solution of $|x| > -2$ since the absolute value of any real number is positive.

12. False. $|u| > a$ is equivalent to $u < -a$ or $u > a$.

13. **a.** Since the graphs of *f* and *g* intersect at the points $(-9, 6)$ and $(3, 6)$, the solution set of $f(x) = g(x)$ is $\{-9, 3\}$.

b. Since the graph of *f* is below the graph of *g* when *x* is between -9 and 3, the solution set of $f(x) \leq g(x)$ is $\{x \mid -9 \leq x \leq 3\}$ or, using interval notation, $[-9, 3]$.

c. Since the graph of *f* is above the graph of *g* to the left of $x = -9$ and to the right of $x = 3$, the solution set of $f(x) > g(x)$ is $\{x \mid x < -9 \text{ or } x > 3\}$ or, using interval notation, $(-\infty, -9) \cup (3, \infty)$.

14. **a.** Since the graphs of *f* and *g* intersect at the points $(0, 2)$ and $(4, 2)$, the solution set of $f(x) = g(x)$ is $\{0, 4\}$.

b. Since the graph of *f* is below the graph of *g* when *x* is between 0 and 4, the solution set of $f(x) \leq g(x)$ is $\{x \mid 0 \leq x \leq 4\}$ or, using interval notation, $[0, 4]$.

c. Since the graph of *f* is above the graph of *g* to the left of $x = 0$ and to the right of $x = 4$, the solution set of $f(x) > g(x)$ is $\{x \mid x < 0 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, 0) \cup (4, \infty)$.

15. **a.** Since the graphs of *f* and *g* intersect at the points $(-2, 5)$ and $(3, 5)$, the solution set of $f(x) = g(x)$ is $\{-2, 3\}$.

b. Since the graph of *f* is above the graph of *g* to the left of $x = -2$ and to the right of $x = 3$, the solution set of $f(x) \geq g(x)$ is $\{x \mid x \leq -2 \text{ or } x \geq 3\}$ or, using interval notation, $(-\infty, -2] \cup [3, \infty)$.

c. Since the graph of *f* is below the graph of *g* when *x* is between -2 and 3, the solution set of $f(x) < g(x)$ is $\{x \mid -2 < x < 3\}$ or, using interval notation, $(-2, 3)$.

16. **a.** Since the graphs of *f* and *g* intersect at the points $(-4, 7)$ and $(3, 7)$, the solution set of $f(x) = g(x)$ is $\{-4, 3\}$.

b. Since the graph of *f* is above the graph of *g* to the left of $x = -4$ and to the right of $x = 3$, the solution set of $f(x) \geq g(x)$ is $\{x \mid x \leq -4 \text{ or } x \geq 3\}$ or, using interval notation, $(-\infty, -4] \cup [3, \infty)$.

c. Since the graph of f is below the graph of g when x is between -4 and 3, the solution set of $f(x) < g(x)$ is $\{x \mid -4 < x < 3\}$ or, using interval notation, $(-4, 3)$.

17. $|x| = 6$

$x = 6$ or $x = -6$

The solution set is $\{-6, 6\}$.

18. $|x| = 12$

$x = 12$ or $x = -12$

The solution set is $\{-12, 12\}$.

19. $|2x+3| = 5$

$2x+3 = 5$ or $2x+3 = -5$

$2x = 2$ or $2x = -8$

$x = 1$ or $x = -4$

The solution set is $\{-4, 1\}$.

20. $|3x-1| = 2$

$3x-1 = 2$ or $3x-1 = -2$

$3x = 3$ or $3x = -1$

$x = 1$ or $x = -\frac{1}{3}$

The solution set is $\left\{-\frac{1}{3}, 1\right\}$.

21. $|1-4t| + 8 = 13$

$|1-4t| = 5$

$1-4t = 5$ or $1-4t = -5$

$-4t = 4$ or $-4t = -6$

$t = -1$ or $t = \frac{3}{2}$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

22. $|1-2z| + 6 = 9$

$|1-2z| = 3$

$1-2z = 3$ or $1-2z = -3$

$-2z = 2$ or $-2z = -4$

$z = -1$ or $z = 2$

The solution set is $\{-1, 2\}$.

23. $|-2x| = 8$

$-2x = 8$ or $-2x = -8$

$x = -4$ or $x = 4$

The solution set is $\{-4, 4\}$.

24. $|-x| = 1$

$-x = 1$ or $-x = -1$

The solution set is $\{-1, 1\}$.

25. $4 - |2x| = 3$

$-|2x| = -1$

$|2x| = 1$

$2x = 1$ or $2x = -1$

$x = \frac{1}{2}$ or $x = -\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

26. $5 - \left|\frac{1}{2}x\right| = 3$

$-\left|\frac{1}{2}x\right| = -2$

$\left|\frac{1}{2}x\right| = 2$

$\frac{1}{2}x = 2$ or $\frac{1}{2}x = -2$

$x = 4$ or $x = -4$

The solution set is $\{-4, 4\}$.

27. $\frac{2}{3}|x| = 9$

$|x| = \frac{27}{2}$

$x = \frac{27}{2}$ or $x = -\frac{27}{2}$

The solution set is $\left\{-\frac{27}{2}, \frac{27}{2}\right\}$.

28. $\frac{3}{4}|x| = 9$

$|x| = 12$

$x = 12$ or $x = -12$

The solution set is $\{-12, 12\}$.

29. $\left|\frac{x}{3}+\frac{2}{5}\right|=2$

$\frac{x}{3}+\frac{2}{5}=2 \quad \text{or} \quad \frac{x}{3}+\frac{2}{5}=-2$

$5x+6=30 \quad \text{or} \quad 5x+6=-30$

$5x=24 \quad \text{or} \quad 5x=-36$

$x=\frac{24}{5} \quad \text{or} \quad x=-\frac{36}{5}$

The solution set is $\left\{-\frac{36}{5}, \frac{24}{5}\right\}$.

30. $\left|\frac{x}{2}-\frac{1}{3}\right|=1$

$\frac{x}{2}-\frac{1}{3}=1 \quad \text{or} \quad \frac{x}{2}-\frac{1}{3}=-1$

$3x-2=6 \quad \text{or} \quad 3x-2=-6$

$3x=8 \quad \text{or} \quad 3x=-4$

$x=\frac{8}{3} \quad \text{or} \quad x=-\frac{4}{3}$

The solution set is $\left\{-\frac{4}{3}, \frac{8}{3}\right\}$.

31. $|u-2|=-\frac{1}{2}$

No solution, since absolute value always yields a non-negative number.

32. $|2-v|=-1$

No solution, since absolute value always yields a non-negative number.

33. $|x^2-9|=0$

$x^2-9=0$

$x^2=9$

$x=\pm 3$

The solution set is $\{-3, 3\}$.

34. $|x^2-16|=0$

$x^2-16=0$

$x^2=16$

$x=\pm 4$

The solution set is $\{-4, 4\}$.

35. $|x^2-2x|=3$

$x^2-2x=3 \quad \text{or} \quad x^2-2x=-3$

$x^2-2x-3=0 \quad \text{or} \quad x^2-2x+3=0$

$(x-3)(x+1)=0 \quad \text{or} \quad x=\frac{2\pm\sqrt{4-12}}{2}$

$=\frac{2\pm\sqrt{-8}}{2}$ no real sol.

$x=3 \quad \text{or} \quad x=-1$

The solution set is $\{-1, 3\}$.

36. $|x^2+x|=12$

$x^2+x=12 \quad \text{or} \quad x^2+x=-12$

$x^2+x-12=0 \quad \text{or} \quad x^2+x+12=0$

$(x-3)(x+4)=0 \quad \text{or} \quad x=\frac{-1\pm\sqrt{1-48}}{2}$

$=\frac{1\pm\sqrt{-47}}{2}$ no real sol.

$x=3 \quad \text{or} \quad x=-4$

The solution set is $\{-4, 3\}$.

37. $|x^2+x-1|=1$

$x^2+x-1=1 \quad \text{or} \quad x^2+x-1=-1$

$x^2+x-2=0 \quad \text{or} \quad x^2+x=0$

$(x-1)(x+2)=0 \quad \text{or} \quad x(x+1)=0$

$x=1, x=-2 \quad \text{or} \quad x=0, x=-1$

The solution set is $\{-2, -1, 0, 1\}$.

38. $|x^2+3x-2|=2$

$x^2+3x-2=2 \quad \text{or} \quad x^2+3x-2=-2$

$x^2+3x=4 \quad \text{or} \quad x^2+3x=0$

$x^2+3x-4=0 \quad \text{or} \quad x(x+3)=0$

$(x+4)(x-1)=0 \quad \text{or} \quad x=0, x=-3$

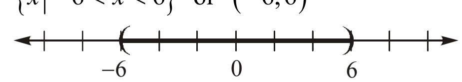
$x=-4, x=1$

The solution set is $\{-4, -3, 0, 1\}$.

39. $|x|<6$

$-6<x<6$

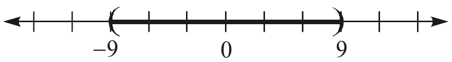
$\{x \mid -6<x<6\}$ or $(-6, 6)$



40. $|x| < 9$

$-9 < x < 9$

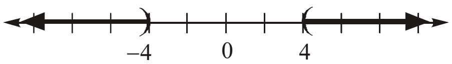
$\{x \mid -9 < x < 9\}$ or $(-9, 9)$



41. $|x| > 4$

$x < -4$ or $x > 4$

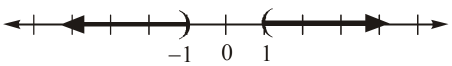
$\{x \mid x < -4 \text{ or } x > 4\}$ or $(-\infty, -4) \cup (4, \infty)$



42. $|x| > 1$

$x < -1$ or $x > 1$

$\{x \mid x < -1 \text{ or } x > 1\}$ or $(-\infty, -1) \cup (1, \infty)$

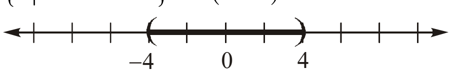


43. $|2x| < 8$

$-8 < 2x < 8$

$-4 < x < 4$

$\{x \mid -4 < x < 4\}$ or $(-4, 4)$

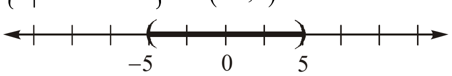


44. $|3x| < 15$

$-15 < 3x < 15$

$-5 < x < 5$

$\{x \mid -5 < x < 5\}$ or $(-5, 5)$

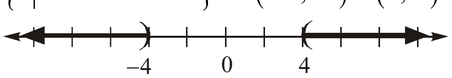


45. $|3x| > 12$

$3x < -12$ or $3x > 12$

$x < -4$ or $x > 4$

$\{x \mid x < -4 \text{ or } x > 4\}$ or $(-\infty, -4) \cup (4, \infty)$

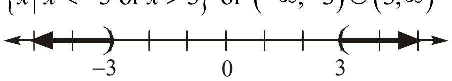


46. $|2x| > 6$

$2x < -6$ or $2x > 6$

$x < -3$ or $x > 3$

$\{x \mid x < -3 \text{ or } x > 3\}$ or $(-\infty, -3) \cup (3, \infty)$



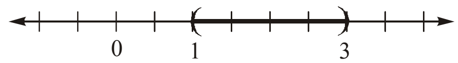
47. $|x - 2| + 2 < 3$

$|x - 2| < 1$

$-1 < x - 2 < 1$

$1 < x < 3$

$\{x \mid 1 < x < 3\}$ or $(1, 3)$



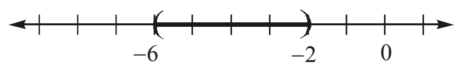
48. $|x + 4| + 3 < 5$

$|x + 4| < 2$

$-2 < x + 4 < 2$

$-6 < x < -2$

$\{x \mid -6 < x < -2\}$ or $(-6, -2)$



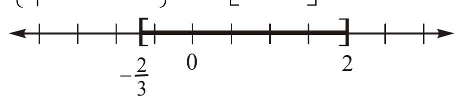
49. $|3t - 2| \le 4$

$-4 \le 3t - 2 \le 4$

$-2 \le 3t \le 6$

$-\frac{2}{3} \le t \le 2$

$\left\{t \,\middle|\, -\frac{2}{3} \le t \le 2\right\}$ or $\left[-\frac{2}{3}, 2\right]$



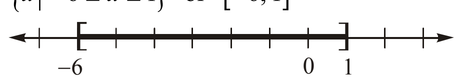
50. $|2u + 5| \le 7$

$-7 \le 2u + 5 \le 7$

$-12 \le 2u \le 2$

$-6 \le u \le 1$

$\{u \mid -6 \le u \le 1\}$ or $[-6, 1]$

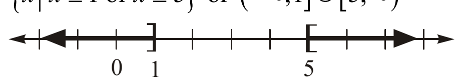


51. $|x - 3| \ge 2$

$x - 3 \le -2$ or $x - 3 \ge 2$

$x \le 1$ or $x \ge 5$

$\{x \mid x \le 1 \text{ or } x \ge 5\}$ or $(-\infty, 1] \cup [5, \infty)$

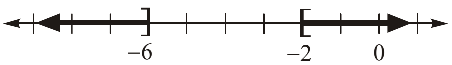


52. $|x+4| \ge 2$

$x+4 \le -2$ or $x+4 \ge 2$

$x \le -6$ or $x \ge -2$

$\{x \mid x \le -6 \text{ or } x \ge -2\}$ or $(-\infty,-6] \cup [-2,\infty)$



53. $|1-4x| - 7 < -2$

$|1-4x| < 5$

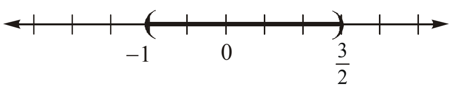
$-5 < 1-4x < 5$

$-6 < -4x < 4$

$\frac{-6}{-4} > x > \frac{4}{-4}$

$\frac{3}{2} > x > -1$ or $-1 < x < \frac{3}{2}$

$\left\{x \middle| -1 < x < \frac{3}{2}\right\}$ or $\left(-1, \frac{3}{2}\right)$



54. $|1-2x| - 4 < -1$

$|1-2x| < 3$

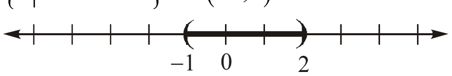
$-3 < 1-2x < 3$

$-4 < -2x < 2$

$\frac{-4}{-2} > x > \frac{2}{-2}$

$2 > x > -1$ or $-1 < x < 2$

$\{x \mid -1 < x < 2\}$ or $(-1,2)$



55. $|1-2x| > |-3|$

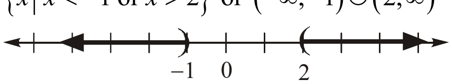
$|1-2x| > 3$

$1-2x < -3$ or $1-2x > 3$

$-2x < -4$ or $-2x > 2$

$x > 2$ or $x < -1$

$\{x \mid x < -1 \text{ or } x > 2\}$ or $(-\infty,-1) \cup (2,\infty)$



56. $|2-3x| > |-1|$

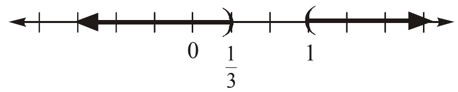
$|2-3x| > 1$

$2-3x < -1$ or $2-3x > 1$

$-3x < -3$ or $-3x > -1$

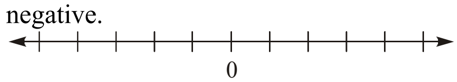
$x > 1$ or $x < \frac{1}{3}$

$\left\{x \middle| x < \frac{1}{3} \text{ or } x > 1\right\}$ or $\left(-\infty, \frac{1}{3}\right) \cup (1,\infty)$



57. $|2x+1| < -1$

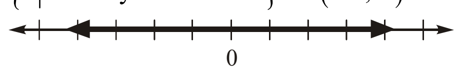
No solution since absolute value is always non-negative.



58. $|3x-4| \ge 0$

All real numbers since absolute value is always non-negative.

$\{x \mid x \text{ is any real number}\}$ or $(-\infty,\infty)$



59. $|x-10| < 2$

$-2 < x-10 < 2$

$8 < x < 12$

Solution set: $\{x \mid 8 < x < 12\}$ or $(8, 12)$

60. $|x-(-6)| < 3$

$|x+6| < 3$

$-3 < x+6 < 3$

$-9 < x < -3$

Solution set: $\{x \mid -9 < x < -3\}$ or $(-9, -3)$

61. $|2x-(-1)| > 5$

$|2x+1| > 5$

$2x+1 < -5$ or $2x+1 > 5$

$2x < -6$ or $2x > 4$

$x < -3$ or $x > 2$

Solution set: $\{x \mid x < -3 \text{ or } x > 2\}$ or $(-\infty,-3) \cup (2,\infty)$

62. $|2x-3|>1$

$2x-3<-1$ or $2x-3>1$

$2x<2$ or $2x>4$

$x<1$ or $x>2$

Solution set: $\{x \mid x<1 \text{ or } x>2\}$ or

$(-\infty,1)\cup(2,\infty)$

63. $|x-5.7|\le 0.0005$

$-0.0005 < x-5.7 < 0.0005$

$5.6995 < x < 5.7005$

The acceptable lengths of the rod is from 5.6995 inches to 5.7005 inches.

64. $|x-6.125|\le 0.0005$

$-0.0005 < x-6.125 < 0.0005$

$6.1245 < x < 6.1255$

The acceptable lengths of the rod is from 6.1245 inches to 6.1255 inches.

65. $\left|\dfrac{x-100}{15}\right|>1.96$

$\dfrac{x-100}{15}<-1.96$ or $\dfrac{x-100}{15}>1.96$

$x-100<-29.4$ or $x-100>29.4$

$x<70.6$ or $x>129.4$

Since IQ scores are whole numbers, any IQ less than 71 or greater than 129 would be considered unusual.

66. $\left|\dfrac{x-266}{16}\right|>1.96$

$\dfrac{x-266}{16}<-1.96$ or $\dfrac{x-266}{16}>1.96$

$x-266<-31.36$ or $x-266>31.36$

$x<234.64$ or $x>297.36$

Pregnancies less than 235 days long or greater than 297 days long would be considered unusual.

67. $|5x+1|+7=5$

$|5x+1|=-2$

No matter what real number is substituted for x, the absolute value expression on the left side of the equation must always be zero or larger. Thus, it can never equal -2.

68. $|x|>-1$

No matter what real number is substituted for x, the absolute value expression on the left side of the equation must always be zero or larger. Thus, it will always be larger than -1. Thus, the solution is the set of all real numbers.

69. $|2x-1|\le 0$

No matter what real number is substituted for x, the absolute value expression on the left side of the equation must always be zero or larger. Thus, the only solution to the inequality above will be when the absolute value expression equals 0:

$|2x-1|=0$

$2x-1=0$

$2x=1$

$x=\dfrac{1}{2}$

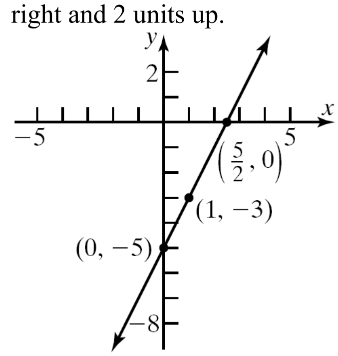
Thus, the solution set is $\left\{\dfrac{1}{2}\right\}$.

Chapter 2 Review Exercises

1. $f(x)=2x-5$

a. Slope = 2; y-intercept = -5

b. Plot the point $(0,-5)$. Use the slope to find an additional point by moving 1 unit to the right and 2 units up.

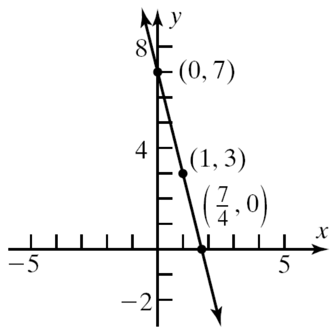


c. increasing

2. $g(x)=-4x+7$

a. Slope = -4; y-intercept = 7

b. Plot the point $(0, 7)$. Use the slope to find an additional point by moving 1 unit to the right and 4 units down.

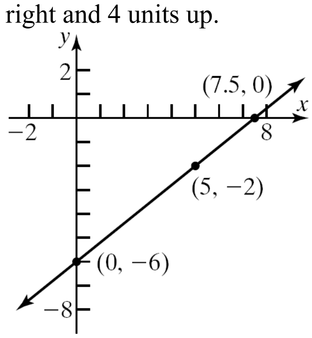


c. decreasing

3. $h(x) = \frac{4}{5}x - 6$

a. Slope = $\frac{4}{5}$; y-intercept = -6

b. Plot the point $(0, -6)$. Use the slope to find an additional point by moving 5 units to the right and 4 units up.

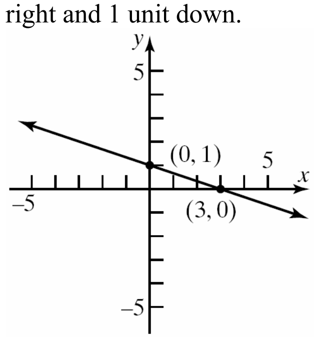


c. increasing

4. $F(x) = -\frac{1}{3}x + 1$

a. Slope = $-\frac{1}{3}$; y-intercept = 1

b. Plot the point $(0, 1)$. Use the slope to find an additional point by moving 3 units to the right and 1 unit down.

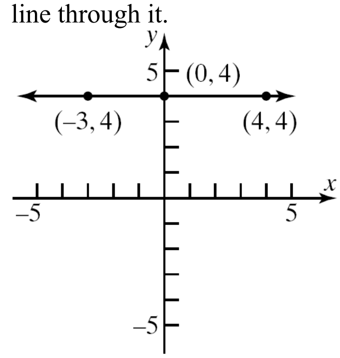


c. decreasing

5. $G(x) = 4$

a. Slope = 0; y-intercept = 4

b. Plot the point (0, 4) and draw a horizontal line through it.

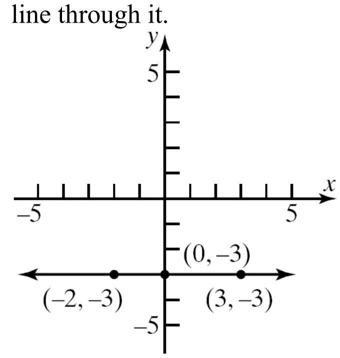


c. constant

6. $H(x) = -3$

a. Slope = 0; y-intercept = -3

b. Plot the point $(0, -3)$ and draw a horizontal line through it.



c. constant

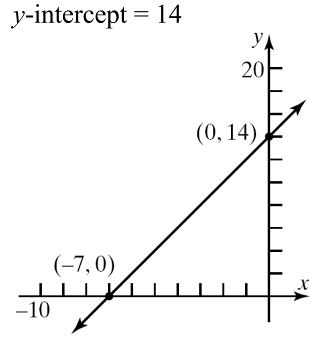
7. $f(x) = 2x + 14$

zero: $f(x) = 2x + 14 = 0$

$$2x = -14$$

$$x = -7$$

y-intercept = 14



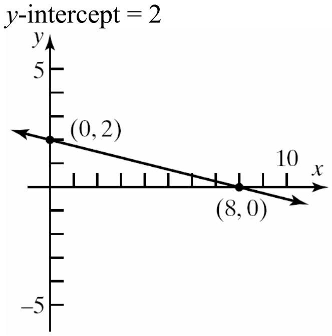
8. $f(x) = -\frac{1}{4}x + 2$

zero: $f(x) = -\frac{1}{4}x + 2 = 0$

$$-\frac{1}{4}x = -2$$

$$x = 8$$

y-intercept = 2



9.

x	$y = f(x)$	Avg. rate of change $= \frac{\Delta y}{\Delta x}$
–1	–2	
0	3	$\frac{3-(-2)}{0-(-1)} = \frac{5}{1} = 5$
1	8	$\frac{8-3}{1-0} = \frac{5}{1} = 5$
2	13	$\frac{13-8}{2-1} = \frac{5}{1} = 5$
3	18	$\frac{18-13}{3-2} = \frac{5}{1} = 5$

This is a linear function with slope = 5, since the average rate of change is constant at 5.

10.

x	$y = f(x)$	Avg. rate of change $= \frac{\Delta y}{\Delta x}$
–1	–3	
0	4	$\frac{4-(-3)}{0-(-1)} = \frac{7}{1} = 7$
1	7	$\frac{7-4}{1-0} = \frac{3}{1} = 3$
2	6	
3	1	

This is not a linear function, since the average rate of change is not constant.

11.
$$f(x) = 0$$
$$x^2 + x - 72 = 0$$
$$(x+9)(x-8) = 0$$
$$x + 9 = 0 \quad \text{or} \quad x - 8 = 0$$
$$x = -9 \qquad\qquad x = 8$$

The zeros of $f(x) = x^2 + x - 72$ are -9 and 8.
The x-intercepts of the graph of f are -9 and 8.

12.
$$G(x) = 0$$
$$x^2 - 3x - 40 = 0$$
$$(x+5)(x-8) = 0$$
$$x + 5 = 0 \quad \text{or} \quad x - 8 = 0$$
$$x = -5 \qquad\qquad x = 8$$

The zeros of $G(x) = x^2 - 3x - 40$ are -5 and 8.
The x-intercepts of the graph of G are -5 and 8.

13.
$$P(t) = 0$$
$$6t^2 - 13t - 5 = 0$$
$$(3t+1)(2t-5) = 0$$
$$3t + 1 = 0 \quad \text{or} \quad 2t - 5 = 0$$
$$t = -\frac{1}{3} \qquad\qquad t = \frac{5}{2}$$

The zeros of $P(t) = 6t^2 - 13t - 5$ are $-\frac{1}{3}$ and $\frac{5}{2}$.

The t-intercepts of the graph of P are $-\frac{1}{3}$ and $\frac{5}{2}$.

14.
$$H(z) = 0$$
$$3z^2 - 17z + 20 = 0$$
$$(3z-5)(z-4) = 0$$
$$3z - 5 = 0 \quad \text{or} \quad z - 4 = 0$$
$$z = \frac{5}{3} \qquad\qquad z = 4$$

The zeros of $H(z) = 3z^2 - 17z + 20$ are $\frac{5}{3}$ and 4.

The x-intercepts of the graph of H are $\frac{5}{3}$ and 4.

15.
$$g(x)=0$$
$$(x-3)^2-4=0$$
$$(x-3)^2=4$$
$$x-3=\pm\sqrt{4}$$
$$x-3=\pm 2$$
$$x=3\pm 2$$
$$x=3-2=1 \quad \text{or} \quad x=3+2=5$$

The zeros of $g(x)=(x-3)^2-4$ are 1 and 5. The x-intercepts of the graph of g are 1 and 5.

16.
$$F(x)=0$$
$$(x+5)^2-9=0$$
$$(x+5)^2=9$$
$$x+5=\pm\sqrt{9}$$
$$x+5=\pm 3$$
$$x=-5\pm 3$$
$$x=-5-3=-8 \quad \text{or} \quad x=-5+3=-2$$

The zeros of $F(x)=(x+5)^2-9$ are -8 and -2. The x-intercepts of the graph of F are -8 and -2.

17.
$$h(x)=0$$
$$9x^2+6x+1=0$$
$$(3x+1)(3x+1)=0$$
$$3x+1=0 \quad \text{or} \quad 3x+1=0$$
$$x=-\frac{1}{3} \qquad x=-\frac{1}{3}$$

The only zero of $h(x)=9x^2+6x+1$ is $-\frac{1}{3}$.

The only x-intercept of the graph of h is $-\frac{1}{3}$.

18.
$$f(x)=0$$
$$4x^2-4x+1=0$$
$$(2x-1)(2x-1)=0$$
$$2x-1=0 \quad \text{or} \quad 2x-1=0$$
$$x=\frac{1}{2} \qquad x=\frac{1}{2}$$

The only zero of $f(x)=4x^2-4x+1$ is $\frac{1}{2}$. The only x-intercept of the graph of f is $\frac{1}{2}$.

19.
$$G(x)=0$$
$$2x^2-4x-1=0$$
$$x^2-2x-\frac{1}{2}=0$$
$$x^2-2x=\frac{1}{2}$$
$$x^2-2x+1=\frac{1}{2}+1$$
$$(x-1)^2=\frac{3}{2}$$
$$x-1=\pm\sqrt{\frac{3}{2}}=\pm\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\pm\frac{\sqrt{6}}{2}$$
$$x=1\pm\frac{\sqrt{6}}{2}=\frac{2\pm\sqrt{6}}{2}$$

The zeros of $G(x)=2x^2-4x-1$ are $\frac{2-\sqrt{6}}{2}$ and $\frac{2+\sqrt{6}}{2}$. The x-intercepts of the graph of G are $\frac{2-\sqrt{6}}{2}$ and $\frac{2+\sqrt{6}}{2}$.

20.
$$P(x)=0$$
$$3x^2+5x+1=0$$
$$x^2+\frac{5}{3}x+\frac{1}{3}=0$$
$$x^2+\frac{5}{3}x=-\frac{1}{3}$$
$$x^2+\frac{5}{3}x+\frac{25}{36}=-\frac{1}{3}+\frac{25}{36}$$
$$\left(x+\frac{5}{6}\right)^2=\frac{13}{36}$$
$$x+\frac{5}{6}=\pm\sqrt{\frac{13}{36}}=\pm\frac{\sqrt{13}}{6}$$
$$x=-\frac{5}{6}\pm\frac{\sqrt{13}}{6}=\frac{-5\pm\sqrt{13}}{6}$$

The zeros of $P(x)=3x^2+5x+1$ are $\frac{-5-\sqrt{13}}{6}$ and $\frac{-5+\sqrt{13}}{6}$. The x-intercepts of the graph of P are $\frac{-5-\sqrt{13}}{6}$ and $\frac{-5+\sqrt{13}}{6}$.

21. $f(x)=0$

$-2x^2+x+1=0$

$2x^2-x-1=0$

$(2x+1)(x-1)=0$

$2x+1=0$ or $x-1=0$

$x=-\frac{1}{2}$ $\quad x=1$

The zeros of $f(x)=-2x^2+x+1$ are $-\frac{1}{2}$ and 1.

The x-intercepts of the graph of f are $-\frac{1}{2}$ and 1.

22. $H(x)=0$

$-x^2-2x+5=0$

$x^2+2x-5=0$

$x^2+2x=5$

$x^2+2x+1=5+1$

$(x+1)^2=6$

$x+1=\pm\sqrt{6}$

$x=-1\pm\sqrt{6}$

The zeros of $H(x)=-x^2-2x+5$ are $-1-\sqrt{6}$ and $-1+\sqrt{6}$. The x-intercepts of the graph of H are $-1-\sqrt{6}$ and $-1+\sqrt{6}$.

23. $f(x)=g(x)$

$(x-3)^2=16$

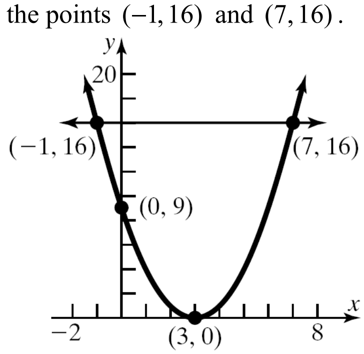
$x-3=\pm\sqrt{16}=\pm4$

$x=3\pm4$

$x=3-4=-1$ or $x=3+4=7$

The solution set is $\{-1, 7\}$.

The x-coordinates of the points of intersection are -1 and 7. The y-coordinates are $g(-1)=16$ and $g(7)=16$. The graphs of the f and g intersect at the points $(-1,16)$ and $(7,16)$.



24. $f(x)=g(x)$

$x^2+6x=16$

$x^2+6x-16=0$

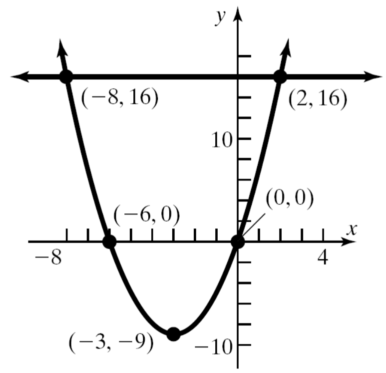
$(x+8)(x-2)=0$

$x+8=0$ or $x-2=0$

$x=-8$ $\quad x=2$

The solution set is $\{-8, 2\}$.

The x-coordinates of the points of intersection are -8 and 2. The y-coordinates are $g(-8)=16$ and $g(2)=16$. The graphs of the f and g intersect at the points $(-8,16)$ and $(2,16)$.



25. $f(x)=g(x)$

$x^2+4x-5=4x-1$

$x^2-4=0$

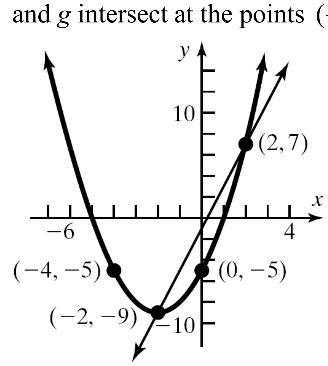
$(x+2)(x-2)=0$

$x+2=0$ or $x-2=0$

$x=-2$ $\quad x=2$

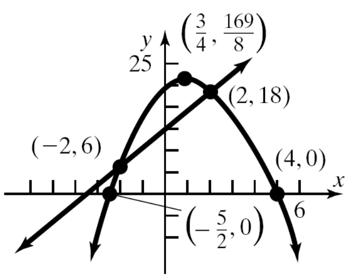
The solution set is $\{-2, 2\}$.

The x-coordinates of the points of intersection are -2 and 2. The y-coordinates are $g(-2)=4(-2)-1=-8-1=-9$ and $g(2)=4(2)-1=8-1=7$. The graphs of the f and g intersect at the points $(-2,-9)$ and $(2,7)$.



26.
$$f(x) = g(x)$$
$$-2x^2 + 3x + 20 = 3x + 12$$
$$-2x^2 + 8 = 0$$
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x + 2 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -2 \qquad x = 2$$
The solution set is $\{-2, 2\}$.

The x-coordinates of the points of intersection are -2 and 2. The y-coordinates are $g(-2) = 3(-2) + 12 = -6 + 12 = 6$ and $g(2) = 3(2) + 12 = 6 + 12 = 18$. The graphs of the f and g intersect at the points $(-2, 6)$ and $(2, 18)$.



27.
$$f(x) = 0$$
$$x^4 - 5x^2 + 4 = 0$$
$$(x^2 - 4)(x^2 - 1) = 0$$
$$x^2 - 4 = 0 \text{ or } x^2 - 1 = 0$$
$$x = \pm 2 \text{ or } x = \pm 1$$
The zeros of $f(x) = x^4 - 5x^2 + 4$ are -2, -1, 1, and 2. The x-intercepts of the graph of f are -2, -1, 1, and 2.

28.
$$g(x) = 0$$
$$3x^4 + 4x^2 + 1 = 0$$
$$(3x^2 + 1)(x^2 + 1) = 0$$
$$3x^2 + 1 = 0 \qquad \text{or} \qquad x^2 + 1 = 0$$
$$x^2 = -\frac{1}{3} \qquad x^2 = -1$$
$$x = \pm\sqrt{-\frac{1}{3}} \qquad x = \pm\sqrt{-1}$$
$$x = \text{not real} \qquad x = \text{not real}$$
The function $g(x) = 3x^4 + 4x^2 + 1$ has no real solution. The graph of g has no x-intercepts.

29.
$$F(x) = 0$$
$$(x-3)^2 - 2(x-3) - 48 = 0$$
Let $u = x - 3 \rightarrow u^2 = (x-3)^2$
$$u^2 - 2u - 48 = 0$$
$$(u+6)(u-8) = 0$$
$$u + 6 = 0 \quad \text{or} \quad u - 8 = 0$$
$$u = -6 \qquad u = 8$$
$$x - 3 = -6 \qquad x - 3 = 8$$
$$x = -3 \qquad x = 11$$
The zeros of $F(x) = (x-3)^2 - 2(x-3) - 48$ are -3 and 11. The x-intercepts of the graph of F are -3 and 11.

30.
$$G(x) = 0$$
$$2(x+4)^2 + 3(x+4) - 14 = 0$$
Let $u = x + 4 \rightarrow u^2 = (x+4)^2$
$$2u^2 + 3u - 14 = 0$$
$$(2u+7)(u-2) = 0$$
$$2u + 7 = 0 \quad \text{or} \quad u - 2 = 0$$
$$u = -\frac{7}{2} \qquad u = 2$$
$$x + 4 = -\frac{7}{2} \qquad x + 4 = 2$$
$$x = -\frac{15}{2} \qquad x = -2$$
The zeros of $G(x) = 2(x+4)^2 + 3(x+4) - 14$ are $-\frac{15}{2}$ and -2. The x-intercepts of the graph of G are $-\frac{15}{2}$ and -2.

31.
$$h(x) = 0$$
$$3x - 13\sqrt{x} - 10 = 0$$
Let $u = \sqrt{x} \rightarrow u^2 = x$
$$3u^2 - 13u - 10 = 0$$
$$(3u+2)(u-5) = 0$$
$$3u + 2 = 0 \quad \text{or} \quad u - 5 = 0$$
$$u = -\frac{2}{3} \qquad u = 5$$
$$\sqrt{x} = -\frac{2}{3} \qquad \sqrt{x} = 5$$
$$x = \text{not real} \qquad x = 5^2 = 25$$

Check: $h(25) = 3(25) - 13\sqrt{25} - 10$

$$= 3(25) - 13(5) - 10$$
$$= 75 - 65 - 10 = 0$$

The only zero of $h(x) = 3x - 13\sqrt{x} - 10$ is 25.

The only x-intercept of the graph of h is 25.

32. $$f(x) = 0$$

$$\left(\frac{1}{x}\right)^2 - 4\left(\frac{1}{x}\right) - 12 = 0$$

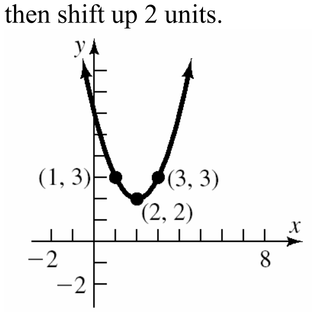
Let $u = \frac{1}{x} \rightarrow u^2 = \left(\frac{1}{x}\right)^2$

$$u^2 - 4u - 12 = 0$$
$$(u+2)(u-6) = 0$$
$$u + 2 = 0 \quad \text{or} \quad u - 6 = 0$$
$$u = -2 \qquad u = 6$$
$$\frac{1}{x} = -2 \qquad \frac{1}{x} = 6$$
$$x = -\frac{1}{2} \qquad x = \frac{1}{6}$$

The zeros of $f(x) = \left(\frac{1}{x}\right)^2 - 4\left(\frac{1}{x}\right) - 12$ are $-\frac{1}{2}$ and $\frac{1}{6}$. The x-intercepts of the graph of f are $-\frac{1}{2}$ and $\frac{1}{6}$.

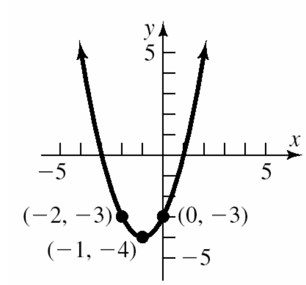
33. $f(x) = (x-2)^2 + 2$

Using the graph of $y = x^2$, shift right 2 units, then shift up 2 units.



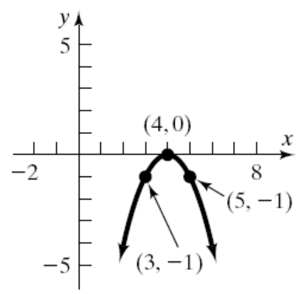
34. $f(x) = (x+1)^2 - 4$

Using the graph of $y = x^2$, shift left 1 unit, then shift down 4 units.



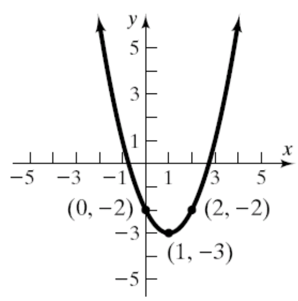
35. $f(x) = -(x-4)^2$

Using the graph of $y = x^2$, shift the graph 4 units right, then reflect about the x-axis.



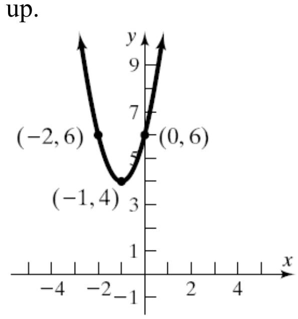
36. $f(x) = (x-1)^2 - 3$

Using the graph of $y = x^2$, shift the graph 1 unit right and shift 3 units down.



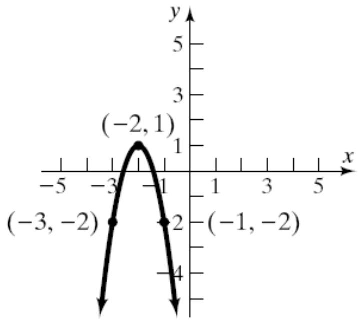
37. $f(x) = 2(x+1)^2 + 4$

Using the graph of $y = x^2$, stretch vertically by a factor of 2, then shift 1 unit left, then shift 4 units up.



38. $f(x) = -3(x+2)^2 + 1$

Using the graph of $y = x^2$, stretch vertically by a factor of 3, then shift 2 units left, then reflect about the x-axis, then shift 1 unit up.



39. a. $f(x) = (x-2)^2 + 2$

$= x^2 - 4x + 4 + 2$

$= x^2 - 4x + 6$

$a = 1, b = -4, c = 6.$ Since $a = 1 > 0,$ the graph opens up. The x-coordinate of the vertex is $x = -\dfrac{b}{2a} = -\dfrac{-4}{2(1)} = \dfrac{4}{2} = 2$.

The y-coordinate of the vertex is

$f\left(-\dfrac{b}{2a}\right) = f(2) = (2)^2 - 4(2) + 6 = 2$.

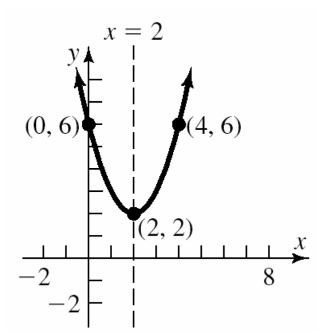
Thus, the vertex is (2, 2).

The axis of symmetry is the line $x = 2$.

The discriminant is:

$b^2 - 4ac = (-4)^2 - 4(1)(6) = -8 < 0$, so the graph has no x-intercepts.

The y-intercept is $f(0) = 6$.



b. Domain: $(-\infty, \infty)$. Range: $[2, \infty)$.

c. Decreasing on $(-\infty, 2)$; increasing on $(2, \infty)$.

40. a. $f(x) = (x+1)^2 - 4$

$= x^2 + 2x + 1 - 4$

$= x^2 + 2x - 3$

$a = 1, b = 2, c = 2.$ Since $a = 1 > 0,$ the graph opens up. The x-coordinate of the vertex is $x = -\dfrac{b}{2a} = -\dfrac{2}{2(1)} = -1$.

The y-coordinate of the vertex is

$f\left(-\dfrac{b}{2a}\right) = f(-1) = (-1)^2 + 2(-1) - 3 = -4$.

Thus, the vertex is (–1, –4).

The axis of symmetry is the line $x = -1$.

The discriminant is:

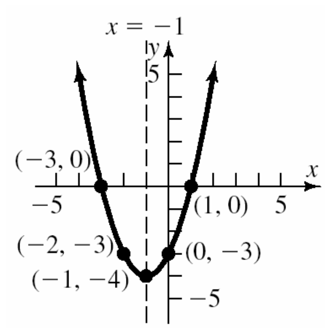
$b^2 - 4ac = (2)^2 - 4(1)(-3) = 16 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving:

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$



b. Domain: $(-\infty, \infty)$. Range: $[-4, \infty)$.

c. Decreasing on $(-\infty, -1)$; increasing on $(-1, \infty)$.

41. a. $f(x) = \dfrac{1}{4}x^2 - 16$

$a = \dfrac{1}{4}, b = 0, c = -16.$ Since $a = \dfrac{1}{4} > 0,$ the graph opens up. The x-coordinate of the vertex is $x = -\dfrac{b}{2a} = -\dfrac{-0}{2\left(\dfrac{1}{4}\right)} = -\dfrac{0}{\dfrac{1}{2}} = 0$.

The y-coordinate of the vertex is

$f\left(-\dfrac{b}{2a}\right) = f(0) = \dfrac{1}{4}(0)^2 - 16 = -16$.

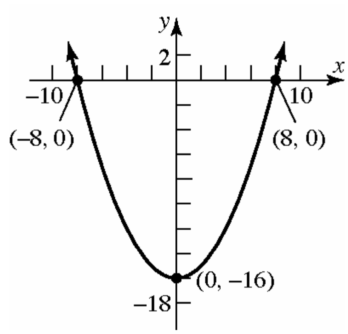
Thus, the vertex is (0, –16).

The axis of symmetry is the line $x = 0$.
The discriminant is:

$b^2 - 4ac = (0)^2 - 4\left(\frac{1}{4}\right)(-16) = 16 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$\frac{1}{4}x^2 - 16 = 0$$
$$x^2 - 64 = 0$$
$$x^2 = 64$$
$$x = 8 \text{ or } x = -8$$

The x-intercepts are –8 and 8.
The y-intercept is $f(0) = -16$.



b. Domain: $(-\infty, \infty)$. Range: $[-16, \infty)$.

c. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.

42. a. $f(x) = -\frac{1}{2}x^2 + 2$

$a = -\frac{1}{2}, b = 0, c = 2$. Since $a = -\frac{1}{2} < 0$, the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{0}{2\left(-\frac{1}{2}\right)} = -\frac{0}{-1} = 0.$

The y-coordinate of the vertex is

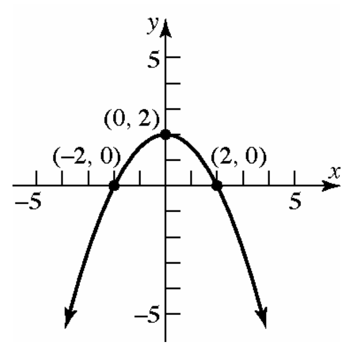
$f\left(-\frac{b}{2a}\right) = f(0) = -\frac{1}{2}(0)^2 + 2 = 2$.

The axis of symmetry is the line $x = 0$.
he discriminant is:

$b^2 - 4ac = (0)^2 - 4\left(-\frac{1}{2}\right)(2) = 4 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$-\frac{1}{2}x^2 + 2 = 0$$
$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = -2 \text{ or } x = 2$$

The x-intercepts are –2 and 2.
The y-intercept is $f(0) = 2$.



b. Domain: $(-\infty, \infty)$. Range: $(-\infty, 2]$.

c. Increasing on $(-\infty, 0)$; decreasing on $(0, \infty)$.

43. a. $f(x) = -4x^2 + 4x$

$a = -4, b = 4, c = 0$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{4}{2(-4)} = -\frac{4}{-8} = \frac{1}{2}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)$$
$$= -1 + 2 = 1$$

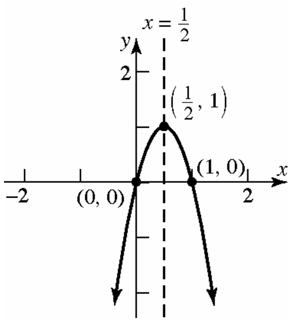
Thus, the vertex is $\left(\frac{1}{2}, 1\right)$.

The axis of symmetry is the line $x = \frac{1}{2}$.
The discriminant is:

$b^2 - 4ac = 4^2 - 4(-4)(0) = 16 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$-4x^2 + 4x = 0$$
$$-4x(x - 1) = 0$$
$$x = 0 \text{ or } x = 1$$

The x-intercepts are 0 and 1.
The y-intercept is $f(0) = -4(0)^2 + 4(0) = 0$.



b. Domain: $(-\infty, \infty)$. Range: $(-\infty, 1]$.

c. Increasing on $\left(-\infty, \dfrac{1}{2}\right)$; decreasing on $\left(\dfrac{1}{2}, \infty\right)$.

44. a. $f(x) = 9x^2 - 6x + 3$

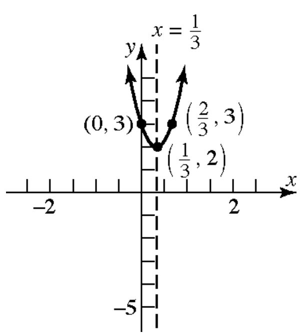
$a = 9, b = -6, c = 3$. Since $a = 9 > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\dfrac{b}{2a} = -\dfrac{-6}{2(9)} = \dfrac{6}{18} = \dfrac{1}{3}$. The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{3}\right) = 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 3$$
$$= 1 - 2 + 3 = 2$$

Thus, the vertex is $\left(\dfrac{1}{3}, 2\right)$.

The axis of symmetry is the line $x = \dfrac{1}{3}$.

The discriminant is:
$b^2 - 4ac = (-6)^2 - 4(9)(3) = -72 < 0$, so the graph has no x-intercepts.
The y-intercept is
$f(0) = 9(0)^2 - 6(0) + 3 = 3$.



b. Domain: $(-\infty, \infty)$. Range: $[2, \infty)$.

c. Decreasing on $\left(-\infty, \dfrac{1}{3}\right)$; increasing on $\left(\dfrac{1}{3}, \infty\right)$.

45. a. $f(x) = \dfrac{9}{2}x^2 + 3x + 1$

$a = \dfrac{9}{2}, b = 3, c = 1$. Since $a = \dfrac{9}{2} > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\dfrac{b}{2a} = -\dfrac{3}{2\left(\frac{9}{2}\right)} = -\dfrac{3}{9} = -\dfrac{1}{3}$.

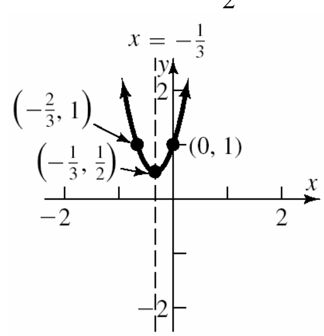
The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(-\frac{1}{3}\right) = \frac{9}{2}\left(-\frac{1}{3}\right)^2 + 3\left(-\frac{1}{3}\right) + 1$$
$$= \frac{1}{2} - 1 + 1 = \frac{1}{2}$$

Thus, the vertex is $\left(-\dfrac{1}{3}, \dfrac{1}{2}\right)$.

The axis of symmetry is the line $x = -\dfrac{1}{3}$.

The discriminant is:
$b^2 - 4ac = 3^2 - 4\left(\dfrac{9}{2}\right)(1) = 9 - 18 = -9 < 0$,
so the graph has no x-intercepts. The y-intercept is $f(0) = \dfrac{9}{2}(0)^2 + 3(0) + 1 = 1$.



b. Domain: $(-\infty, \infty)$. Range: $\left[\dfrac{1}{2}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\dfrac{1}{3}\right)$; increasing on $\left(-\dfrac{1}{3}, \infty\right)$.

46. a. $f(x) = -x^2 + x + \frac{1}{2}$

$a = -1, b = 1, c = \frac{1}{2}$. Since $a = -1 < 0$, the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{1}{2(-1)} = -\frac{1}{-2} = \frac{1}{2}$.

The y-coordinate of the vertex is

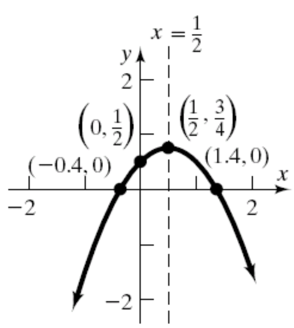
$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) + \frac{1}{2}$$
$$= -\frac{1}{4} + 1 = \frac{3}{4}$$

Thus, the vertex is $\left(\frac{1}{2}, \frac{3}{4}\right)$. The axis of symmetry is the line $x = \frac{1}{2}$. The discriminant is: $b^2 - 4ac = 1^2 - 4(-1)\left(\frac{1}{2}\right) = 3 > 0$, so the graph has two x-intercepts. The x-intercepts are found by solving: $-x^2 + x + \frac{1}{2} = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{3}}{2(-1)} = \frac{-1 \pm \sqrt{3}}{-2} = \frac{1 \pm \sqrt{3}}{2}$$

The x-intercepts are $\frac{1 - \sqrt{3}}{2} \approx -0.4$ and $\frac{1 + \sqrt{3}}{2} \approx 1.4$.

The y-intercept is $f(0) = -(0)^2 + (0) + \frac{1}{2} = \frac{1}{2}$.



b. Domain: $(-\infty, \infty)$. Range: $\left(-\infty, \frac{3}{4}\right]$.

c. Increasing on $\left(-\infty, \frac{1}{2}\right)$; decreasing on $\left(\frac{1}{2}, \infty\right)$.

47. a. $f(x) = 3x^2 + 4x - 1$

$a = 3, b = 4, c = -1$. Since $a = 3 > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{4}{2(3)} = -\frac{4}{6} = -\frac{2}{3}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 + 4\left(-\frac{2}{3}\right) - 1$$
$$= \frac{4}{3} - \frac{8}{3} - 1 = -\frac{7}{3}$$

Thus, the vertex is $\left(-\frac{2}{3}, -\frac{7}{3}\right)$.

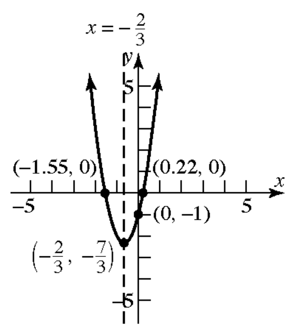
The axis of symmetry is the line $x = -\frac{2}{3}$.

The discriminant is:
$b^2 - 4ac = (4)^2 - 4(3)(-1) = 28 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$3x^2 + 4x - 1 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{28}}{2(3)}$$
$$= \frac{-4 \pm 2\sqrt{7}}{6} = \frac{-2 \pm \sqrt{7}}{3}$$

The x-intercepts are $\frac{-2 - \sqrt{7}}{3} \approx -1.55$ and $\frac{-2 + \sqrt{7}}{3} \approx 0.22$.

The y-intercept is $f(0) = 3(0)^2 + 4(0) - 1 = -1$.



b. Domain: $(-\infty, \infty)$. Range: $\left[-\frac{7}{3}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{2}{3}\right)$; increasing on $\left(-\frac{2}{3}, \infty\right)$.

48. **a.** $f(x) = -2x^2 - x + 4$

$a = -2, b = -1, c = 4$. Since $a = -2 < 0,$ the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{-1}{2(-2)} = \frac{1}{-4} = -\frac{1}{4}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(-\frac{1}{4}\right) = -2\left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right) + 4$$
$$= -\frac{1}{8} + \frac{1}{4} + 4 = \frac{33}{8}$$

Thus, the vertex is $\left(-\frac{1}{4}, \frac{33}{8}\right)$.

The axis of symmetry is the line $x = -\frac{1}{4}$.

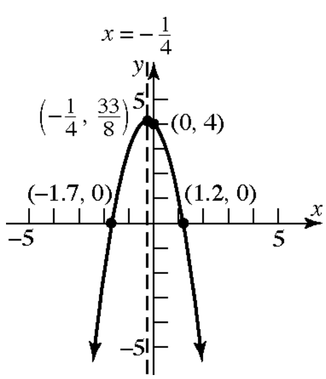
The discriminant is:

$b^2 - 4ac = (-1)^2 - 4(-2)(4) = 33 > 0$, so the graph has two x-intercepts. The x-intercepts are found by solving: $-2x^2 - x + 4 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{33}}{2(-2)}$$
$$= \frac{1 \pm \sqrt{33}}{-4} = \frac{-1 \pm \sqrt{33}}{4}$$

The x-intercepts are $\frac{-1 - \sqrt{33}}{4} \approx -1.7$ and $\frac{-1 + \sqrt{33}}{4} \approx 1.2$.

The y-intercept is $f(0) = -2(0)^2 - 0 + 4 = 4$.



b. Domain: $(-\infty, \infty)$. Range: $\left(-\infty, \frac{33}{8}\right]$.

c. Increasing on $\left(-\infty, -\frac{1}{4}\right)$; decreasing on $\left(-\frac{1}{4}, \infty\right)$.

49. $f(x) = 3x^2 - 6x + 4$

$a = 3, b = -6, c = 4$. Since $a = 3 > 0,$ the graph opens up, so the vertex is a minimum point. The minimum occurs at

$$x = -\frac{b}{2a} = -\frac{-6}{2(3)} = \frac{6}{6} = 1.$$

The minimum value is

$$f\left(-\frac{b}{2a}\right) = f(1) = 3(1)^2 - 6(1) + 4$$
$$= 3 - 6 + 4 = 1$$

50. $f(x) = 2x^2 + 8x + 5$

$a = 2, b = 8, c = 5$. Since $a = 2 > 0,$ the graph opens up, so the vertex is a minimum point. The minimum occurs at

$$x = -\frac{b}{2a} = -\frac{8}{2(2)} = -\frac{8}{4} = -2.$$

The minimum value is

$$f\left(-\frac{b}{2a}\right) = f(-2) = 2(-2)^2 + 8(-2) + 5$$
$$= 8 - 16 + 5 = -3$$

51. $f(x) = -x^2 + 8x - 4$

$a = -1, b = 8, c = -4$. Since $a = -1 < 0,$ the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{8}{2(-1)} = -\frac{8}{-2} = 4.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(4) = -(4)^2 + 8(4) - 4$$
$$= -16 + 32 - 4 = 12$$

52. $f(x) = -x^2 - 10x - 3$

$a = -1, b = -10, c = -3$. Since $a = -1 < 0,$ the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{-10}{2(-1)} = \frac{10}{-2} = -5.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(-5) = -(-5)^2 - 10(-5) - 3$$
$$= -25 + 50 - 3 = 22$$

53. $f(x) = -3x^2 + 12x + 4$

$a = -3, b = 12, c = 4.$ Since $a = -3 < 0,$ the graph opens down, so the vertex is a maximum point. The maximum occurs at

$x = -\frac{b}{2a} = -\frac{12}{2(-3)} = -\frac{12}{-6} = 2$.

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(2) = -3(2)^2 + 12(2) + 4$$
$$= -12 + 24 + 4 = 16$$

54. $f(x) = -2x^2 + 4$

$a = -2, b = 0, c = 4.$ Since $a = -2 < 0,$ the graph opens down, so the vertex is a maximum point. The maximum occurs at

$x = -\frac{b}{2a} = -\frac{0}{2(-2)} = 0$.

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(0) = -2(0)^2 + 4 = 4.$$

55. $x^2 + 6x - 16 < 0$

$f(x) = x^2 + 6x - 16$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8, x = 2$ are the zeros of f.

Interval	$(-\infty, -8)$	$(-8, 2)$	$(2, \infty)$
Test Number	–9	0	3
Value of *f*	11	–16	11
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -8 < x < 2\}$ or, using interval notation, $(-8, 2)$.

56. $3x^2 - 2x - 1 \geq 0$

$f(x) = 3x^2 - 2x - 1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3}, x = 1$ are the zeros of *f*.

Interval	$\left(-\infty, -\frac{1}{3}\right)$	$\left(-\frac{1}{3}, 1\right)$	$(1, \infty)$
Test Number	–1	0	2
Value of *f*	4	–1	7
Conclusion	Positive	Negative	Positive

The solution set is $\left\{x \middle| x \leq -\frac{1}{3} \text{ or } x \geq 1\right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{3}\right] \cup [1, \infty)$.

57. $3x^2 \geq 14x + 5$

$3x^2 - 14x - 5 \geq 0$

$f(x) = 3x^2 - 14x - 5$

$3x^2 - 14x - 5 = 0$

$(3x+1)(x-5) = 0$

$x = -\frac{1}{3}, x = 5$ are the zeros of f.

Interval	$\left(-\infty, -\frac{1}{3}\right)$	$\left(-\frac{1}{3}, 5\right)$	$(5, \infty)$
Test Number	–1	0	2
Value of *f*	12	–5	19
Conclusion	Positive	Negative	Positive

The solution set is $\left\{x \middle| x \leq -\frac{1}{3} \text{ or } x \geq 5\right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{3}\right] \cup [5, \infty)$.

58. $4x^2 < 13x - 3$

$4x^2 - 13x + 3 < 0$

$f(x) = 4x^2 - 13x + 3$

$4x^2 - 13x + 3 = 0$

$(4x-1)(x-3) = 0$

$x = \frac{1}{4}, x = 3$ are the zeros of f.

Interval	$\left(-\infty, \frac{1}{4}\right)$	$\left(\frac{1}{4}, 3\right)$	$(3, \infty)$
Test Number	0	1	4
Value of *f*	0	–6	15
Conclusion	Positive	Negative	Positive

The solution set is $\left\{x \middle| \frac{1}{4} < x < 3\right\}$ or, using interval notation, $\left(\frac{1}{4}, 3\right)$.

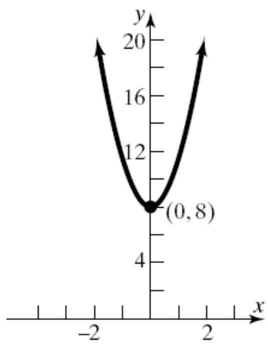
59. $f(x) = 0$

$x^2 + 8 = 0$

$x^2 = -8$

$x = \pm\sqrt{-8} = \pm 2\sqrt{2}\, i$

The zero are $-2\sqrt{2}\, i$ and $2\sqrt{2}\, i$.



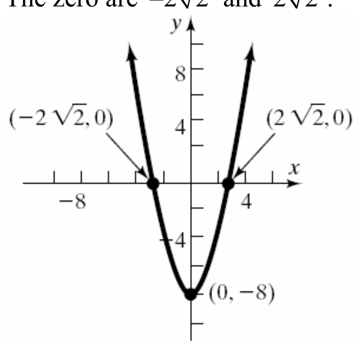
60. $f(x) = 0$

$x^2 - 8 = 0$

$x^2 = 8$

$x = \pm\sqrt{8} = \pm 2\sqrt{2}$

The zero are $-2\sqrt{2}$ and $2\sqrt{2}$.



61. $g(x) = 0$

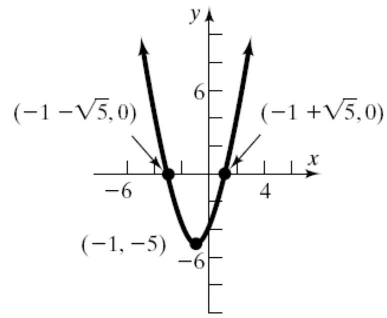
$x^2 + 2x - 4 = 0$

$a = 1, b = 2, c = -4$

$b^2 - 4ac = 2^2 - 4(1)(-4) = 4 + 16 = 20$

$$x = \frac{-2 \pm \sqrt{20}}{2(1)} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$$

The zeros are $-1-\sqrt{5}$ and $-1+\sqrt{5}$.



62. $h(x) = 0$

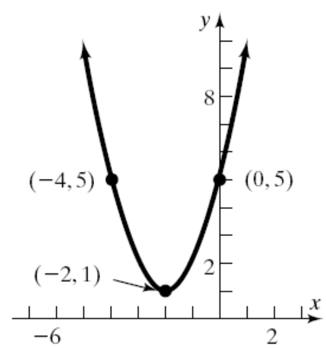
$x^2 + 4x + 5 = 0$

$a = 1, b = 4, c = 5$

$b^2 - 4ac = 4^2 - 4(1)(5) = 16 - 20 = -4$

$$x = \frac{-4 \pm \sqrt{-4}}{2(1)} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

The zeros are $-2-i$ and $-2+i$.



63. $p(x) = 0$

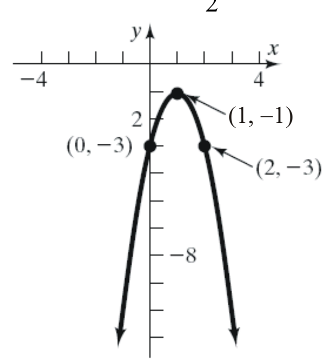
$-2x^2 + 4x - 3 = 0$

$a = -2, b = 4, c = -3$

$b^2 - 4ac = 4^2 - 4(-2)(-3) = 16 - 24 = -8$

$$x = \frac{-4 \pm \sqrt{-8}}{2(-2)} = \frac{-4 \pm 2\sqrt{2}\, i}{-4} = 1 \pm \frac{\sqrt{2}}{2} i$$

The zeros are $1 - \frac{\sqrt{2}}{2} i$ and $1 + \frac{\sqrt{2}}{2} i$.



64. $F(x)=0$

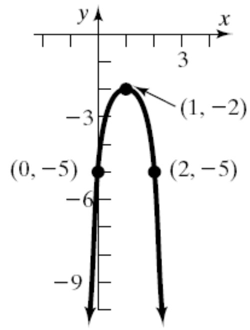
$-3x^2+6x-5=0$

$a=-3, b=6, c=-5$

$b^2-4ac=6^2-4(-3)(-5)=36-60=-24$

$x=\dfrac{-6\pm\sqrt{-24}}{2(-3)}=\dfrac{-6\pm2\sqrt{6}\,i}{-6}=1\pm\dfrac{\sqrt{6}}{3}i$

The zeros are $1-\dfrac{\sqrt{6}}{3}i$ and $1+\dfrac{\sqrt{6}}{3}i$.



65. $f(x)=0$

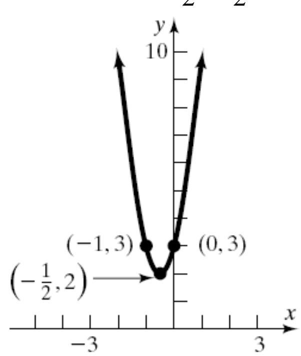
$4x^2+4x+3=0$

$a=4, b=4, c=3$

$b^2-4ac=4^2-4(4)(3)=16-48=-32$

$x=\dfrac{-4\pm\sqrt{-32}}{2(4)}=\dfrac{-4\pm4\sqrt{2}\,i}{8}=-\dfrac{1}{2}\pm\dfrac{\sqrt{2}}{2}i$

The zeros are $-\dfrac{1}{2}-\dfrac{\sqrt{2}}{2}i$ and $-\dfrac{1}{2}+\dfrac{\sqrt{2}}{2}i$.



66. $G(x)=0$

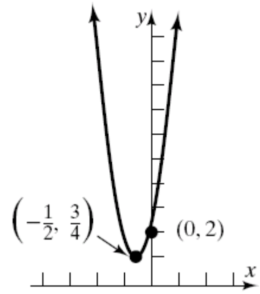
$5x^2+5x+2=0$

$a=5, b=5, c=2$

$b^2-4ac=5^2-4(5)(2)=25-40=-15$

$x=\dfrac{-5\pm\sqrt{-15}}{2(5)}=\dfrac{-5\pm\sqrt{15}\,i}{10}=-\dfrac{1}{2}\pm\dfrac{\sqrt{15}}{10}i$

The zeros are $-\dfrac{1}{2}-\dfrac{\sqrt{15}}{10}i$ and $-\dfrac{1}{2}+\dfrac{\sqrt{15}}{10}i$.



67. $|2x+3|=7$

$2x+3=7$ or $2x+3=-7$

$2x=4$ or $2x=-10$

$x=2$ or $x=-5$

The solution set is $\{-5, 2\}$.

68. $|3x-1|=5$

$3x-1=5$ or $3x-1=-5$

$3x=6$ or $3x=-4$

$x=2$ or $x=-\dfrac{4}{3}$

The solution set is $\left\{-\dfrac{4}{3}, 2\right\}$.

69. $|2-3x|+2=9$

$|2-3x|=7$

$2-3x=7$ or $2-3x=-7$

$-3x=5$ or $-3x=-9$

$x=-\dfrac{5}{3}$ or $x=3$

The solution set is $\left\{-\dfrac{5}{3}, 3\right\}$.

70. $|1-2x|+1=4$

$|1-2x|=3$

$1-2x=3$ or $1-2x=-3$

$-2x=2$ or $-2x=-4$

$x=-1$ or $x=2$

The solution set is $\{-1, 2\}$.

71. $\left|3x+4\right|<\frac{1}{2}$

$$-\frac{1}{2}<3x+4<\frac{1}{2}$$
$$-\frac{9}{2}<\quad 3x \quad<-\frac{7}{2}$$
$$-\frac{3}{2}<\quad x \quad<-\frac{7}{6}$$

$\left\{x\,\middle|\,-\frac{3}{2}<x<-\frac{7}{6}\right\}$ or $\left(-\frac{3}{2},-\frac{7}{6}\right)$

72. $\left|1-2x\right|<\frac{1}{3}$

$$-\frac{1}{3}<1-2x<\frac{1}{3}$$
$$-\frac{4}{3}<-2x<-\frac{2}{3}$$
$$\frac{2}{3}>\quad x \quad>\frac{1}{3}$$

$\left\{x\,\middle|\,\frac{1}{3}<x<\frac{2}{3}\right\}$ or $\left(\frac{1}{3},\frac{2}{3}\right)$

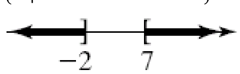
73. $\left|2x-5\right|\geq 9$

$2x-5\leq -9$ or $2x-5\geq 9$

$2x\leq -4$ or $2x\geq 14$

$x\leq -2$ or $x\geq 7$

$\left\{x\,|\,x\leq -2 \text{ or } x\geq 7\right\}$ or $(-\infty,-2]\cup[7,\infty)$



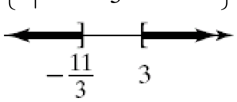
74. $\left|3x+1\right|\geq 10$

$3x+1\leq -10$ or $3x+1\geq 10$

$3x\leq -11$ or $3x\geq 9$

$x\leq -\frac{11}{3}$ or $x\geq 3$

$\left\{x\,\middle|\,x\leq -\frac{11}{3} \text{ or } x\geq 3\right\}$ or $\left(-\infty,-\frac{11}{3}\right]\cup[3,\infty)$



75. $2+\left|2-3x\right|\leq 4$

$$\left|2-3x\right|\leq 2$$
$$-2\leq 2-3x\leq 2$$
$$-4\leq -3x\leq 0$$
$$\frac{4}{3}\geq x\geq 0$$

$\left\{x\,\middle|\,0\leq x\leq\frac{4}{3}\right\}$ or $\left[0,\ \frac{4}{3}\right]$

76. $\frac{1}{2}+\left|\frac{2x-1}{3}\right|\leq 1$

$$\left|\frac{2x-1}{3}\right|\leq\frac{1}{2}$$
$$-\frac{1}{2}\leq\frac{2x-1}{3}\leq\frac{1}{2}$$
$$-\frac{3}{2}\leq 2x-1\leq\frac{3}{2}$$
$$-\frac{1}{2}\leq 2x\leq\frac{5}{2}$$
$$-\frac{1}{4}\leq x\leq\frac{5}{4}$$

$\left\{x\,\middle|\,-\frac{1}{4}\leq x\leq\frac{5}{4}\right\}$ or $\left[-\frac{1}{4},\frac{5}{4}\right]$

77. $1-\left|2-3x\right|<-4$

$$-\left|2-3x\right|<-5$$
$$\left|2-3x\right|>5$$

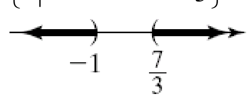
$2-3x<-5$ or $2-3x>5$

$7<3x$ or $-3>3x$

$\frac{7}{3}<x$ or $-1>x$

$x<-1$ or $x>\frac{7}{3}$

$\left\{x\,\middle|\,x<-1 \text{ or } x>\frac{7}{3}\right\}$ or $(-\infty,-1)\cup\left(\frac{7}{3},\infty\right)$



78. $1-\left|\frac{2x-1}{3}\right|<-2$

$-\left|\frac{2x-1}{3}\right|<-3$

$\left|\frac{2x-1}{3}\right|>3$

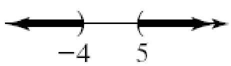
$\frac{2x-1}{3}<-3$ or $\frac{2x-1}{3}>3$

$2x-1<-9$ or $2x-1>9$

$2x<-8$ or $2x>10$

$x<-4$ or $x>5$

$\{x \mid x<-4 \text{ or } x>5\}$ or $(-\infty,-4)\cup(5,\infty)$



79. a. Company A: $C(x)=0.06x+7.00$

Company B: $C(x)=0.08x$

b. $0.06x+7.00=0.08x$

$7.00=0.02x$

$350=x$

The bill from Company A will equal the bill from Company B if 350 minutes are used.

c. $0.08x<0.06x+7.00$

$0.02x<7.00$

$x<350$

The bill from Company B will be less than the bill from Company A if fewer than 350 minutes are used. That is, $0\le x<350$.

80. a. $S(x)=0.01x+15{,}000$

b. $S(1{,}000{,}000)=0.01(1{,}000{,}000)+15{,}000$

$=10{,}000+15{,}000=25{,}000$

In 2005, Bill's salary was \$25,000.

c. $0.01x+15{,}000=100{,}000$

$0.01x=85{,}000$

$x=8{,}500{,}000$

Bill's sales would have to be \$8,500,000 in order to earn \$100,000.

d. $0.01x+15{,}000>150{,}000$

$0.01x>135{,}000$

$x>13{,}500{,}000$

Bill's sales would have to be more than \$13,500,000 in order for his salary to exceed \$150,000.

81. Let $p=$ the monthly payment in dollars, and B = the amount borrowed in dollars. Consider the ordered pair (B,p). We can use the points $(0,0)$ and $(130000,854)$. Now compute the slope:

$$\text{slope}=\frac{\Delta y}{\Delta x}=\frac{854-0}{130000-0}=\frac{854}{130000}\approx 0.00657$$

Therefore we have the linear function $p(B)=0.00657B+0=0.00657B$

If $B=165000$, then $p=(0.00657)(165000)\approx \$1083.92$.

82. Let R = the revenue in dollars, and g = the number of gallons of gasoline sold. Consider the ordered pair (g,R). We can use the points $(0,0)$ and $(13.5,32.13)$. Now compute the slope:

$$\text{slope}=\frac{\Delta y}{\Delta x}=\frac{32.13-0}{13.5-0}=\frac{32.13}{13.5}=2.38$$

Therefore we have the linear function $R(g)=2.38g+0=2.38g$.

If $g=11.2$, then $R=(2.38)(11.2)\approx \$26.66$.

83. Let x represent the number of passengers over 20. Then $20+x$ represents the total number of passengers. $15-0.1x$ represents the fare for each passenger. Solving the equation for total cost (\$482.40), we have:

$(20+x)(15-0.1x)=482.40$

$300+13x-0.1x^2=482.40$

$-0.1x^2+13x-182.40=0$

$x^2-130x+1824=0$

$(x-114)(x-16)=0$

$x=114$ or $x=16$

Since the capacity of the bus is 44, we discard the answer 114. The total number of passengers is 20 + 16 = 36, and the ticket price per passenger is 15 – 0.1(16) = \$13.40.
So 36 senior citizens went on the trip. Each person paid \$13.40.

84. Let $w = 4$. Then,

$$\begin{aligned} l^2 &= 4(l+4) \\ l^2 &= 4l+16 \\ l^2 - 4l - 16 &= 0 \end{aligned}$$

$$l = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-16)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{80}}{2} = \frac{4 \pm 4\sqrt{5}}{2} = 2 \pm 2\sqrt{5}$$

$$\approx 6.47 \text{ or } -2.47$$

Disregard the negative answer since length must be positive. Thus, the length of the plasterboard should be cut down to approximately 6.47 feet.

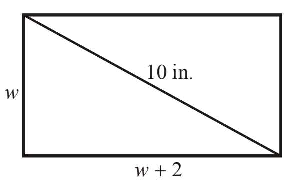
85. length of $\text{leg}_1 = x$, length of $\text{leg}_2 = 17 - x$

By the Pythagorean Theorem, we have:

$$\begin{aligned} x^2 + (17-x)^2 &= (13)^2 \\ x^2 + x^2 - 34x + 289 &= 169 \\ 2x^2 - 34x + 120 &= 0 \\ x^2 - 17x + 60 &= 0 \\ (x-12)(x-5) &= 0 \\ x = 12 \quad \text{or} \quad x &= 5 \end{aligned}$$

Thus, the legs are 5 cm and 12 cm long.

86. Let w = the width. Then $w + 2$ = the length.



By the Pythagorean Theorem we have:

$$\begin{aligned} w^2 + (w+2)^2 &= (10)^2 \\ w^2 + w^2 + 4w + 4 &= 100 \\ 2w^2 + 4w - 96 &= 0 \\ w^2 + 2w - 48 &= 0 \\ (w+8)(w-6) &= 0 \\ w = -8 \quad \text{or} \quad w &= 6 \end{aligned}$$

Disregard the negative answer because the width of a rectangle must be positive. Thus, the width is 6 inches, and the length is 8 inches

87. $C(x) = 4.9x^2 - 617.4x + 19{,}600$;
$a = 4.9, b = -617.4, c = 19{,}600$. Since $a = 4.9 > 0$, the graph opens up, so the vertex is a minimum point.

a. The minimum marginal cost occurs at
$$x = -\frac{b}{2a} = -\frac{-617.40}{2(4.9)} = \frac{617.40}{9.8} = 63.$$
Thus, 63 golf clubs should be manufactured in order to minimize the marginal cost.

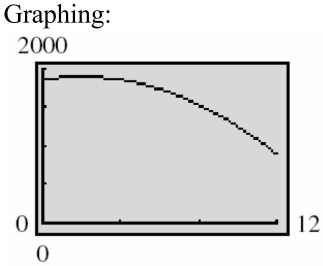
b. The minimum marginal cost is
$$\begin{aligned} C\left(-\frac{b}{2a}\right) &= C(63) \\ &= 4.9(63)^2 - (617.40)(63) + 19600 \\ &= \$151.90 \end{aligned}$$

88. $V(t) = -10.0t^2 + 39.2t + 1862.6$;
$a = -10.0, b = 39.2, c = 1862.6$. Since $a = -10.0 < 0$, the graph opens down, so the vertex is a maximum point.

a. The maximum occurs at
$$t = -\frac{b}{2a} = -\frac{39.2}{2(-10.0)} = -\frac{39.2}{-20} = 1.96.$$
Thus, the most violent crimes were committed during 1992.

b. The maximum number of violent crimes committed was
$$\begin{aligned} V(1.96) &= -10.0(1.96)^2 + 39.2(1.96) + 1862.6 \\ &\approx 1901 \end{aligned}$$
Thus, approximately 1,901 thousand violent crimes were committed in 1992. That is, about 1,901,000 crimes.

c. Graphing:



Violent crimes were decreasing from 1994 to 1998.

89. Since there are 200 feet of border, we know that $2x+2y=200$. The area is to be maximized, so $A=x\cdot y$. Solving the perimeter formula for y:

$$\begin{aligned}2x+2y&=200\\2y&=200-2x\\y&=100-x\end{aligned}$$

The area function is:

$$A(x)=x(100-x)=-x^2+100x$$

The maximum value occurs at the vertex:

$$x=-\frac{b}{2a}=-\frac{100}{2(-1)}=-\frac{100}{-2}=50$$

The pond should be 50 feet by 50 feet for maximum area.

90. Let x represent the length and y represent the width of the rectangle.

$$\begin{aligned}2x+2y&=20\\2y&=20-2x\\y&=10-x\\A=x\cdot y&=16\end{aligned}$$

$$x(10-x)=16$$

Solving the area equation:

$$10x-x^2=16$$

$$x^2-10x+16=0$$

$$(x-8)(x-2)=0$$

$$x=8 \text{ or } x=2$$

The length and width of the rectangle are 8 feet by 2 feet.

91. The area function is:

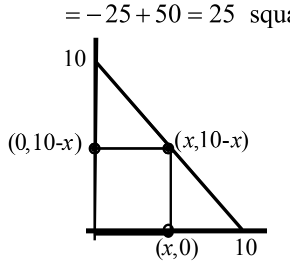
$$A(x)=x(10-x)=-x^2+10x$$

The maximum value occurs at the vertex:

$$x=-\frac{b}{2a}=-\frac{10}{2(-1)}=-\frac{10}{-2}=5$$

The maximum area is:

$$\begin{aligned}A(5)&=-(5)^2+10(5)\\&=-25+50=25 \text{ square units}\end{aligned}$$



92. Locate the origin at the point directly under the highest point of the arch. Then the equation is in the form: $y=-ax^2+k$, where $a>0$. Since the maximum height is 10 feet, when $x=0$, $y=k=10$. Since the point (10, 0) is on the parabola, we can find the constant:

$$0=-a(10)^2+10$$

$$a=\frac{10}{10^2}=\frac{1}{10}=0.10$$

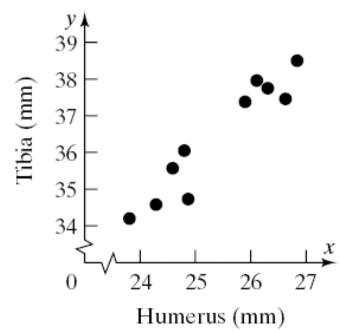
The equation of the parabola is:

$$y=-\frac{1}{10}x^2+10$$

At $x=8$:

$$y=-\frac{1}{10}(8)^2+10=-6.4+10=3.6 \text{ feet}$$

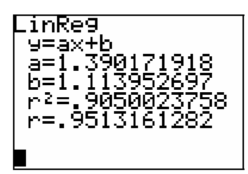
93. **a.**



b. Yes, the two variables appear to have a linear relationship.

c. Using the LINear REGression program, the line of best fit is:

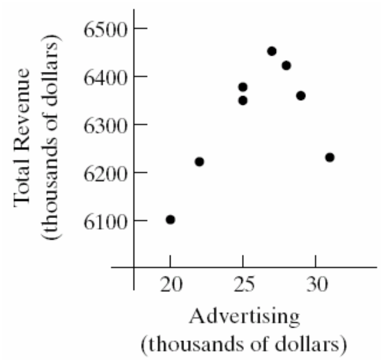
$$y=1.390171918x+1.113952697$$



d. $y=1.390171918(26.5)+1.113952697$

≈ 38.0 mm

94. a.



The data appear to be quadratic with $a < 0$.

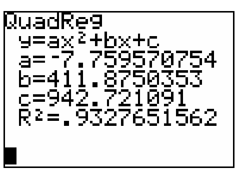
b. The maximum revenue occurs at

$$A = \frac{-b}{2a} = \frac{-(411.88)}{2(-7.76)}$$
$$= \frac{-411.88}{-15.52} \approx \$26.5 \text{ thousand}$$

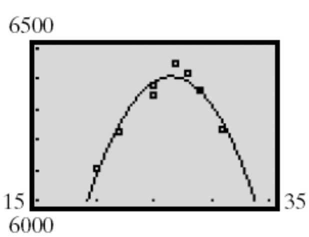
c. The maximum revenue is

$$R\left(\frac{-b}{2a}\right) = R(26.53866)$$
$$= -7.76(26.5)^2 + (411.88)(26.5) + 942.72$$
$$\approx \$6408 \text{ thousand}$$

d. Using the QUADratic REGression program, the quadratic function of best fit is: $y = -7.76x^2 + 411.88x + 942.72$.



e.



Chapter 2 Test

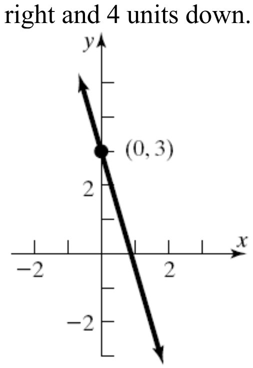
1. $f(x) = -4x + 3$

a.
$$f(x) = 0$$
$$-4x + 3 = 0$$
$$-4x = -3$$
$$x = \frac{3}{4}$$

The zero of f is $\frac{3}{4}$.

b. The slope is negative, so the graph is decreasing.

c. Plot the point $(0, 3)$. Use the slope to find an additional point by moving 1 unit to the right and 4 units down.



2.
$$f(x) = 0$$
$$3x^2 - 2x - 8 = 0$$
$$(3x + 4)(x - 2) = 0$$
$$3x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -\frac{4}{3} \qquad x = 2$$

The zeros of f are $-\frac{4}{3}$ and 2.

3.
$$G(x) = 0$$
$$-2x^2 + 4x + 1 = 0$$
$$a = -2, b = 4, c = 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(-2)(1)}}{2(-2)}$$
$$= \frac{-4 \pm \sqrt{24}}{-4} = \frac{-4 \pm 2\sqrt{6}}{-4} = \frac{2 \pm \sqrt{6}}{2}$$

The zeros of G are $\frac{2 - \sqrt{6}}{2}$ and $\frac{2 + \sqrt{6}}{2}$.

4. $f(x) = g(x)$

$x^2 + 3x = 5x + 3$

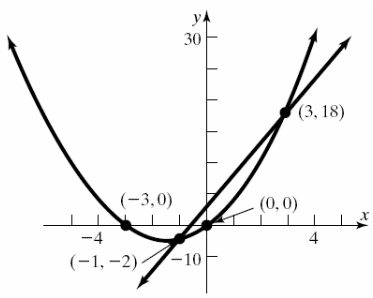
$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x+1=0 \quad \text{or} \quad x-3=0$

$x=-1 \qquad\qquad x=3$

The solution set is $\{-1, 3\}$.



5. $f(x) = 0$

$(x-1)^2 + 5(x-1) + 4 = 0$

Let $u = x-1 \rightarrow u^2 = (x-1)^2$

$u^2 + 5u + 4 = 0$

$(u+4)(u+1) = 0$

$u+4=0 \quad \text{or} \quad u+1=0$

$u=-4 \qquad\qquad u=-1$

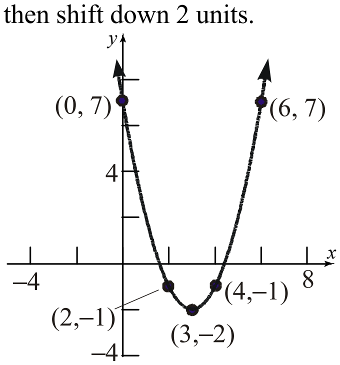
$x-1=-4 \qquad x-1=-1$

$x=-3 \qquad\qquad x=0$

The zeros of G are -3 and 0.

6. $f(x) = (x-3)^2 - 2$

Using the graph of $y = x^2$, shift right 3 units, then shift down 2 units.



7. a. $f(x) = 3x^2 - 12x + 4$

$a = 3, b = -12, c = 4.$ Since $a = 3 > 0,$ the graph opens up.

b. The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{-12}{2(3)} = -\frac{-12}{6} = 2.$$

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f(2) = 3(2)^2 - 12(2) + 4$$
$$= 12 - 24 + 4 = -8$$

Thus, the vertex is $(2, -8)$.

c. The axis of symmetry is the line $x = 2$.

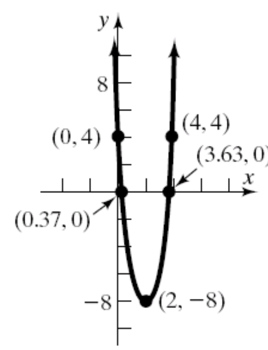
d. The discriminant is:

$b^2 - 4ac = (-12)^2 - 4(3)(4) = 96 > 0$, so the graph has two x-intercepts. The x-intercepts are found by solving: $3x^2 - 12x + 4 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{96}}{2(3)}$$
$$= \frac{12 \pm 4\sqrt{6}}{6} = \frac{6 \pm 2\sqrt{6}}{3}$$

The x-intercepts are $\frac{6 - 2\sqrt{6}}{3} \approx 0.37$ and $\frac{6 \pm 2\sqrt{6}}{3} \approx 3.63$. The y-intercept is $f(0) = 3(0)^2 - 12(0) + 4 = 4$.

e.



8. $f(x) = -2x^2 + 12x + 3$

$a = -2, b = 12, c = 3.$ Since $a = -2 < 0,$ the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = -\frac{12}{-4} = 3.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(3) = -2(3)^2 + 12(3) + 3$$
$$= -18 + 36 + 3 = 21$$

9. $x^2 - 10x + 24 \ge 0$

$f(x) = x^2 - 10x + 24$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x = 4,\ x = 6$ are the zeros of *f*.

Interval	$(-\infty, 4)$	$(4, 6)$	$(6, \infty)$
Test Number	0	5	7
Value of *f*	24	–1	3
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x \le 4 \text{ or } x \ge 6\}$ or, using interval notation, $(-\infty, 4] \cup [6, \infty)$.

10.
$$f(x) = 0$$
$$2x^2 + 4x + 5 = 0$$
$$a = 2, b = 4, c = 5$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(2)(5)}}{2(2)}$$
$$= \frac{-4 \pm \sqrt{-24}}{4} = \frac{-4 \pm 2\sqrt{6}\,i}{4} = -1 \pm \frac{\sqrt{6}}{2} i$$

The complex zeros of *f* are $-1 - \frac{\sqrt{6}}{2} i$ and $-1 + \frac{\sqrt{6}}{2} i$.

11. $|3x+1| = 8$

$3x + 1 = 8$ or $3x + 1 = -8$

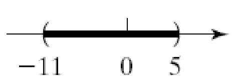
$3x = 7$ or $3x = -9$

$x = \frac{7}{3}$ or $x = -3$

The solution set is $\left\{-3, \frac{7}{3}\right\}$.

12. $\left|\frac{x+3}{4}\right| < 2$
$$-2 < \frac{x+3}{4} < 2$$
$$-8 < x + 3 < 8$$
$$-11 < x < 5$$

$\{x \mid -11 < x < 5\}$ or $(-11, 5)$



13. $|2x+3| - 4 \ge 3$

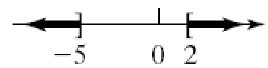
$|2x+3| \ge 7$

$2x + 3 \le -7$ or $2x + 3 \ge 7$

$2x \le -10$ or $2x \ge 4$

$x \le -5$ or $x \ge 2$

$\{x \mid x \le -5 \text{ or } x \ge 2\}$ or $(-\infty, -5] \cup [2, \infty)$



14. **a.** $C(m) = 0.15m + 129.50$

b. $C(860) = 0.15(860) + 129.50$
$$= 129 + 129.50 = 258.50$$
If 860 miles are driven, the rental cost is \$258.50.

c.
$$C(m) = 213.80$$
$$0.15m + 129.50 = 213.80$$
$$0.15m = 84.30$$
$$m = 562$$
The rental cost is \$213.80 if 562 miles were driven.

15. $r(x) = -0.115x^2 + 1.183x + 5.623$;

$a = -0.115, b = 1.183, c = 5.623$

Since $a = -0.115 < 0$, the graph opens down, so the vertex is a maximum point.

a. The maximum interest rate occurs at
$$x = -\frac{b}{2a} = -\frac{-1.183}{2(-0.115)} = \frac{-1.183}{-0.23} \approx 5.14.$$
The maximum interest rate was about
$$r(5.14) = -0.115(5.14)^2 + 1.183(5.14) + 5.623$$
$$\approx 8.67$$
Thus, the interest rate was highest in 1997, and the highest rate at this time was about 8.67%.

b. The year 2010 corresponds to $x = 18$.
$$r(18) = -0.115(18)^2 + 1.183(18) + 5.623$$
$$\approx -10.34$$
The model estimates the rate in 2010 to be -10.34%. This rate does not make sense since an interest rate cannot be negative.

Chapter 2 Projects

Project 1

Answers will vary depending on the stock selected and the time period.

Project 2 (IRC)

a.

x	1	2	3	4	5
$y=-2x+5$	3	1	-1	-3	-5

b. $\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{1-3}{1}=-2$

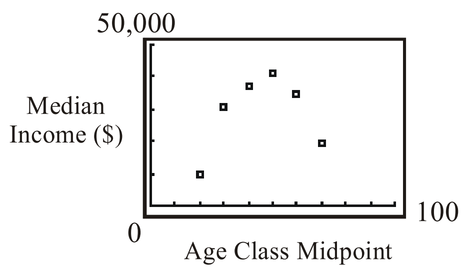
$\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-1-1}{1}=-2$

$\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-3-(-1)}{1}=-2$

$\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-5-(-3)}{1}=-2$

All of the values of $\dfrac{\Delta y}{\Delta x}$ are the same.

c.



d. $\dfrac{\Delta I}{\Delta x}=\dfrac{30633-9548}{10}=2108.50$

$\dfrac{\Delta I}{\Delta x}=\dfrac{37088-30633}{10}=645.50$

$\dfrac{\Delta I}{\Delta x}=\dfrac{41072-37088}{10}=398.40$

$\dfrac{\Delta I}{\Delta x}=\dfrac{34414-41072}{10}=-665.80$

$\dfrac{\Delta I}{\Delta x}=\dfrac{19167-34414}{10}=-1524.70$

These $\dfrac{\Delta I}{\Delta x}$ values are not all equal. The data are not linearly related.

e.

x	-2	-1	0	1	2	3	4
y	23	9	3	5	15	33	59
$\dfrac{\Delta y}{\Delta x}$		-14	-6	2	10	18	26

As x increases, $\dfrac{\Delta y}{\Delta x}$ increases. This makes sense because the parabola is increasing (going up) steeply as x increases.

f.

x	-2	-1	0	1	2	3	4
y	23	9	3	5	15	33	59
$\dfrac{\Delta^2 y}{\Delta x^2}$			8	8	8	8	8

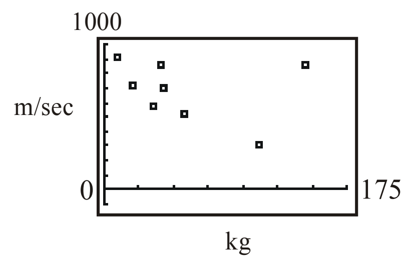
The second differences are all the same.

g. The paragraph should mention at least two observations:

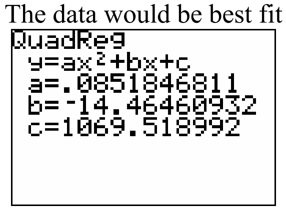
1. The first differences for a linear function are all the same.
2. The second differences for a quadratic function are the same.

Project 3 (IRC)

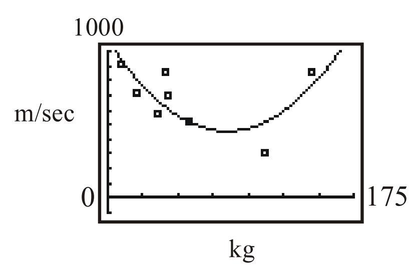
a.



b. The data would be best fit by a quadratic function.



$y=0.085x^2-14.46x+1069.52$



These results seem reasonable since the function fits the data well.

c. $s_0 = 0\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2}v_0t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t$

$s_0 = 200\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2}v_0t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t + 200$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t + 200$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t + 200$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t + 200$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t + 200$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t + 200$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t + 200$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t + 200$

$s_0 = 30\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2}v_0t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t + 30$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t + 30$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t + 30$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t + 30$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t + 30$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t + 30$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t + 30$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t + 30$

Notice that the gun is what makes the difference, not how high it is mounted necessarily. The only way to change the true maximum height that the projectile can go is to change the angle at which it fires.

Project 4 (IRC)

a. – i. Answers will vary , depending on where the CBL is located above the bouncing ball.

j. The ratio of the heights between bounces will be the same.

Chapter 2 Cumulative Review

1. $P = (-1,3); Q = (4,-2)$

Distance between P and Q:

$$d(P,Q) = \sqrt{(4-(-1))^2 + (-2-3)^2}$$
$$= \sqrt{(5)^2 + (5)^2}$$
$$= \sqrt{25+25}$$
$$= \sqrt{50} = 5\sqrt{2}$$

Midpoint between P and Q:

$$\left(\frac{-1+4}{2}, \frac{3-2}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right) = (1.5,\ 0.5)$$

2. $y = x^3 - 3x + 1$

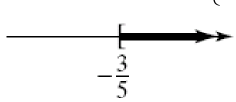
a. $(-2,-1)$: $-1 = (-2)^3 - 3(-2) + 1$
$$-1 = -8 + 6 + 1$$
$$-1 = -1$$
Yes, $(-2,-1)$ is on the graph.

b. $(2,3)$: $3 = (2)^3 - 3(2) + 1$
$$3 = 8 - 6 + 1$$
$$3 = 3$$
Yes, $(2,3)$ is on the graph.

c. $(3,1)$: $1 = (3)^3 - 3(3) + 1$
$$1 = -27 - 9 + 1$$
$$1 \neq -35$$
No, $(3,1)$ is not on the graph.

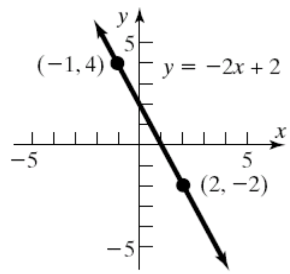
3. $5x + 3 \geq 0$
$$5x \geq -3$$
$$x \geq -\frac{3}{5}$$

The solution set is $\left\{x \,\middle|\, x \geq -\frac{3}{5}\right\}$ or $\left[-\frac{3}{5}, +\infty\right)$.



4. (–1,4) and (2,–2) are points on the line.

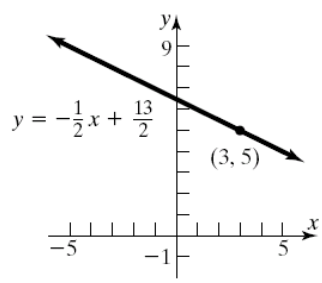
$$\text{Slope} = \frac{-2-4}{2-(-1)} = \frac{-6}{3} = -2$$
$$y - y_1 = m(x - x_1)$$
$$y - 4 = -2(x - (-1))$$
$$y - 4 = -2(x + 1)$$
$$y - 4 = -2x - 2$$
$$y = -2x + 2$$



5. Perpendicular to $y = 2x + 1$;
Containing (3,5)

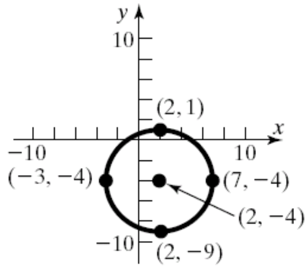
Slope of perpendicular = $-\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - 5 = -\frac{1}{2}(x - 3)$$
$$y - 5 = -\frac{1}{2}x + \frac{3}{2}$$
$$y = -\frac{1}{2}x + \frac{13}{2}$$



6. $x^2 + y^2 - 4x + 8y - 5 = 0$
$$x^2 - 4x + y^2 + 8y = 5$$
$$(x^2 - 4x + 4) + (y^2 + 8y + 16) = 5 + 4 + 16$$
$$(x-2)^2 + (y+4)^2 = 25$$
$$(x-2)^2 + (y+4)^2 = 5^2$$

Center: (2,–4) Radius = 5



7. Yes, this is a function since each x-value is paired with exactly one y-value.

8. $f(x) = x^2 - 4x + 1$

a. $f(2) = 2^2 - 4(2) + 1 = 4 - 8 + 1 = -3$

b. $f(x) + f(2) = x^2 - 4x + 1 + (-3)$
$= x^2 - 4x - 2$

c. $f(-x) = (-x)^2 - 4(-x) + 1 = x^2 + 4x + 1$

d. $-f(x) = -(x^2 - 4x + 1) = -x^2 + 4x - 1$

e. $f(x+2) = (x+2)^2 - 4(x+2) + 1$
$= x^2 + 4x + 4 - 4x - 8 + 1$
$= x^2 - 3$

f. $\dfrac{f(x+h) - f(x)}{h}$

$$= \frac{(x+h)^2 - 4(x+h) + 1 - (x^2 - 4x + 1)}{h}$$
$$= \frac{x^2 + 2xh + h^2 - 4x - 4h + 1 - x^2 + 4x - 1}{h}$$
$$= \frac{2xh + h^2 - 4h}{h}$$
$$= \frac{h(2x + h - 4)}{h} = 2x + h - 4$$

9. $h(z) = \dfrac{3z-1}{6z-7}$

The denominator cannot be zero:
$6z - 7 \neq 0$
$6z \neq 7$
$z \neq \dfrac{7}{6}$

Domain: $\left\{ z \,\middle|\, z \neq \dfrac{7}{6} \right\}$

10. Yes, the graph represents a function since it passes the Vertical Line Test.

11. $f(x) = \dfrac{x}{x+4}$

a. $f(1) = \dfrac{1}{1+4} = \dfrac{1}{5} \neq \dfrac{1}{4}$, so $\left(1, \dfrac{1}{4}\right)$ is not on the graph of f.

b. $f(-2) = \dfrac{-2}{-2+4} = \dfrac{-2}{2} = -1$, so $(-2, -1)$ is a point on the graph of f.

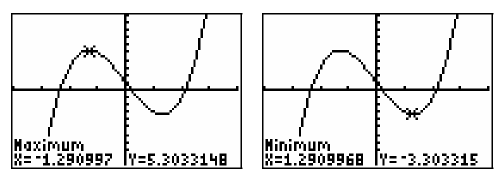
c. Solve for x:
$2 = \dfrac{x}{x+4}$
$2x + 8 = x$
$x = -8$
So, $(-8, 2)$ is a point on the graph of f.

12. $f(x) = \dfrac{x^2}{2x+1}$

$$f(-x) = \frac{(-x)^2}{2(-x)+1} = \frac{x^2}{-2x+1} \neq f(x) \text{ or } -f(x)$$

Therefore, f is neither even nor odd.

13. $f(x) = x^3 - 5x + 4$ on the interval $(-4, 4)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 5x + 4$.



Local maximum is 5.30 and occurs at $x \approx -1.29$;
Local minimum is –3.30 and occurs at $x \approx 1.29$;
f is increasing on $(-4, -1.29)$ or $(1.29, 4)$;
f is decreasing on $(-1.29, 1.29)$.

14. $f(x) = 3x + 5$; $\quad g(x) = 2x + 1$

a. $f(x) = g(x)$
$3x + 5 = 2x + 1$
$3x + 5 = 2x + 1$
$x = -4$

b. $f(x) > g(x)$
$3x + 5 > 2x + 1$
$3x + 5 > 2x + 1$
$x > -4$
The solution set is $\{x \mid x > -4\}$ or $(-4, \infty)$.

15. a. Domain: $\{x \mid -4 \le x \le 4\}$ or $[-4, 4]$
Range: $\{y \mid -1 \le y \le 3\}$ or $[-1, 3]$

b. Intercepts: $(-1,0)$, $(0,-1)$, $(1,0)$
x-intercepts: $-1, 1$
y-intercept: -1

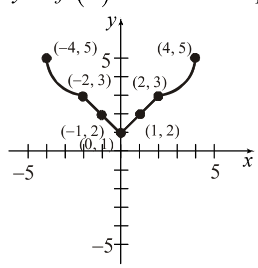
c. The graph is symmetric with respect to the y-axis.

d. When $x = 2$, the function takes on a value of 1. Therefore, $f(2) = 1$.

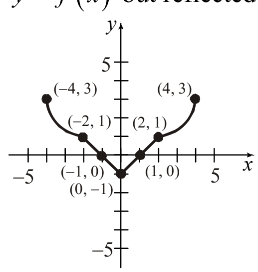
e. The function takes on the value 3 at $x = -4$ and $x = 4$.

f. $f(x) < 0$ means that the graph lies below the x-axis. This happens for x values between -1 and 1. Thus, the solution set is $\{x \mid -1 < x < 1\}$ or $(-1, 1)$.

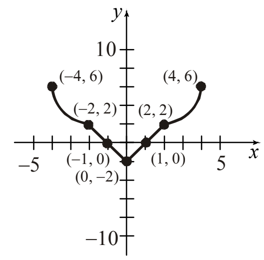
g. The graph of $y = f(x) + 2$ is the graph of $y = f(x)$ but shifted up 2 units.



h. The graph of $y = f(-x)$ is the graph of $y = f(x)$ but reflected about the y-axis.



i. The graph of $y = 2f(x)$ is the graph of $y = f(x)$ but stretched vertically by a factor of 2. That is, the coordinate of each point is multiplied by 2.



j. Since the graph is symmetric about the y-axis, the function is even.

k. The function is increasing on the open interval $(0, 4)$.