

## Chapter 0

### Problems 0.1

1. True;  $-13$  is a negative integer.
2. True, because  $-2$  and  $7$  are integers and  $7 \neq 0$ .
3. False, because  $-3$  is not positive.
4. False, because  $0 = \frac{0}{1}$ .
5. False, it is fairly easy to see (although the details are not relevant for a course using this text) that  $\sqrt{n}$ , for  $n$  an integer, is either an integer (if  $n$  is a perfect square) or irrational (if  $n$  is not a perfect square). The perfect squares are  $1, 4, 9, 16, 25, 36, \dots$  and since  $3$  is not among these,  $\sqrt{3}$  is not rational.
6. False, since a rational number cannot have denominator of zero. In fact,  $\frac{7}{0}$  is not a number at all because we cannot divide by  $0$ .
7. False, because  $\sqrt{25} = 5$ , which is a positive integer.
8. True;  $\sqrt{2}$  is an irrational real number.
9. False; we cannot divide by  $0$ .
10. False, we have  $3 < \pi < 4$  so that  $\pi$  is at best rational and not an integer. It can be shown that  $\pi$  is irrational (but the details are not relevant for our purposes).
11. True
12. False, since the integer  $0$  is neither positive nor negative.

### Problems 0.2

1. False, because  $0$  does not have a reciprocal.
2. True, because  $\frac{7}{3} \cdot \frac{3}{7} = \frac{21}{21} = 1$ .
3. False; the negative of  $7$  is  $-7$  because  $7 + (-7) = 0$ .

4. True;  $1(x \cdot y) = (1 \cdot x)(1 \cdot y)$
5. False;  $-x + y = y + (-x) = y - x$ .
6. True;  $(x + 2)(4) = (x)(4) + (2)(4) = 4x + 8$ .
7. True;  $\frac{x+2}{2} = \frac{x}{2} + \frac{2}{2} = \frac{x}{2} + 1$ .
8. True, because  $a\left(\frac{b}{c}\right) = \frac{ab}{c}$ .
9. False;  $2(x \cdot y) = 2xy$  while  $(2x) \cdot (2y) = (2 \cdot 2) \cdot (x \cdot y) = 4xy$ .
10. True; by the associative and commutative properties,  $x(4y) = (x \cdot 4)y = (4 \cdot x)y = 4xy$ .
11. distributive
12. commutative
13. associative
14. definition of division and commutative property
15. commutative and distributive
16. associative
17. definition of subtraction
18. commutative
19. distributive and commutative
20. distributive
21.  $2x(y - 7) = (2x)y - (2x)7 = 2xy - (7)(2x) = 2xy - (7 \cdot 2)x = 2xy - 14x$
22.  $(a - b) + c = [a + (-b)] + c = a + (-b + c) = a + [c + (-b)] = a + (c - b)$
23.  $(x + y)(2) = 2(x + y) = 2x + 2y$
24.  $a[b + (c + d)] = a[(b + c) + d] = a[d + (b + c)] = a[(d + b) + c]$
25.  $x[(2y + 1) + 3] = x[2y + (1 + 3)] = x[2y + 4] = x(2y) + x(4) = (x \cdot 2)y + 4x = (2x)y + 4x = 2xy + 4x$

26.  $(1 + a)(b + c) = 1(b + c) + a(b + c)$   
 $= 1(b) + 1(c) + a(b) + a(c) = b + c + ab + ac$
27.  $x(y - z + w) = x[(y - z) + w] = x(y - z) + x(w)$   
 $= x[y + (-z)] + xw = x(y) + x(-z) + xw$   
 $= xy - xz + xw$
28.  $-2 + (-4) = -6$
29.  $-a + b = b - a$
30.  $6 + (-4) = 2$
31.  $7 - 2 = 5$
32.  $7 - (-4) = 7 + 4 = 11$
33.  $-5 - (-13) = -5 + 13 = 8$
34.  $-(-a) + (-b) = a - b$
35.  $(-2)(9) = -(2 \cdot 9) = -18$
36.  $7(-9) = -(7 \cdot 9) = -63$
37.  $(-2)(-12) = 2(12) = 24$
38.  $19(-1) = (-1)19 = -(1 \cdot 19) = -19$
39.  $\frac{-1}{-\frac{1}{a}} = -1\left(-\frac{a}{1}\right) = a$
40.  $-(-6 + x) = -(-6) - x = 6 - x$
41.  $-7(x) = -(7x) = -7x$
42.  $-12(x - y) = (-12)x - (-12)(y) = -12x + 12y$   
(or  $12y - 12x$ )
43.  $-[-6 + (-y)] = -(-6) - (-y) = 6 + y$
44.  $-3 \div (3a) = \frac{-3}{3a} = -\frac{1 \cdot 3}{3 \cdot a} = -\frac{1}{a}$
45.  $-9 \div (-27) = \frac{-9}{-27} = \frac{9}{27} = \frac{9 \cdot 1}{9 \cdot 3} = \frac{1}{3}$
46.  $(-a) \div (-b) = \frac{-a}{-b} = \frac{a}{b}$
47.  $2(-6 + 2) = 2(-4) = -8$
48.  $3[-2(3) + 6(2)] = 3[-6 + 12] = 3[6] = 18$
49.  $(-a)(-b)(-1) = ab(-1) = -ab$
50.  $(-12)(-12) = (12)(12) = 144$
51.  $X(1) = X$
52.  $3(x - 4) = 3(x) - 3(4) = 3x - 12$
53.  $4(5 + x) = 4(5) + 4(x) = 20 + 4x$
54.  $-(x - y) = -x + y = y - x$
55.  $0(-x) = 0$
56.  $8\left(\frac{1}{11}\right) = \frac{8 \cdot 1}{11} = \frac{8}{11}$
57.  $\frac{5}{1} = 5$
58.  $\frac{14x}{21y} = \frac{2 \cdot 7 \cdot x}{3 \cdot 7 \cdot y} = \frac{2x}{3y}$
59.  $\frac{2x}{-2} = \frac{2 \cdot x}{-1 \cdot 2} = -x$
60.  $\frac{2}{3} \cdot \frac{1}{x} = \frac{2 \cdot 1}{3 \cdot x} = \frac{2}{3x}$
61.  $\frac{a}{c}(3b) = \frac{a(3b)}{c} = \frac{3ab}{c}$
62.  $(5a)\left(\frac{7}{5a}\right) = 7$
63.  $\frac{-aby}{-ax} = \frac{-a \cdot by}{-a \cdot x} = \frac{by}{x}$
64.  $\frac{a}{b} \cdot \frac{1}{c} = \frac{a \cdot 1}{b \cdot c} = \frac{a}{bc}$
65.  $\frac{2}{x} \cdot \frac{5}{y} = \frac{2 \cdot 5}{x \cdot y} = \frac{10}{xy}$
66.  $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$
67.  $\frac{5}{12} + \frac{3}{4} = \frac{5}{12} + \frac{9}{12} = \frac{5+9}{12} = \frac{14}{12} = \frac{2 \cdot 7}{2 \cdot 6} = \frac{7}{6}$
68.  $\frac{3}{10} - \frac{7}{15} = \frac{9}{30} - \frac{14}{30} = \frac{9-14}{30} = \frac{-5}{30} = -\frac{5 \cdot 1}{5 \cdot 6} = -\frac{1}{6}$

69.  $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

70.  $\frac{X}{\sqrt{5}} - \frac{Y}{\sqrt{5}} = \frac{X-Y}{\sqrt{5}}$

71.  $\frac{3}{2} - \frac{1}{4} + \frac{1}{6} = \frac{18}{12} - \frac{3}{12} + \frac{2}{12} = \frac{18-3+2}{12} = \frac{17}{12}$

72.  $\frac{2}{5} - \frac{3}{8} = \frac{16}{40} - \frac{15}{40} = \frac{16-15}{40} = \frac{1}{40}$

73.  $\frac{6}{\frac{x}{y}} = 6 \div \frac{x}{y} = 6 \cdot \frac{y}{x} = \frac{6y}{x}$

74.  $\frac{\frac{l}{w}}{m} = \frac{l}{w} \div m = \frac{l}{w} \cdot \frac{1}{m} = \frac{l}{wm}$

75.  $\frac{\frac{-x}{y^2}}{\frac{z}{xy}} = -\frac{x}{y^2} \div \frac{z}{xy} = -\frac{x}{y^2} \cdot \frac{xy}{z} = -\frac{x^2}{yz}$

76.  $\frac{7}{0}$  is not defined (we cannot divide by 0).

77.  $\frac{0}{7} = 0$

78.  $\frac{0}{0}$  is not defined (we cannot divide by 0).**Problems 0.3**

1.  $(2^3)(2^2) = 2^{3+2} = 2^5 (= 32)$

2.  $x^6 x^9 = x^{6+9} = x^{15}$

3.  $a^5 a^2 = a^{5+2} = a^7$

4.  $z^3 z z^2 = z^{3+1+2} = z^6$

5.  $\frac{x^3 x^5}{y^9 y^5} = \frac{x^{3+5}}{y^{9+5}} = \frac{x^8}{y^{14}}$

6.  $(x^{12})^4 = x^{12 \cdot 4} = x^{48}$

7.  $\frac{(a^3)^7}{(b^4)^5} = \frac{a^{3 \cdot 7}}{b^{4 \cdot 5}} = \frac{a^{21}}{b^{20}}$

8.  $\left(\frac{w}{w^3}\right)^7 = \left(\frac{w}{w \cdot w^2}\right)^7 = \left(\frac{1}{w^2}\right)^7 = \frac{1^7}{(w^2)^7} = \frac{1}{w^{14}}$

9.  $(2x^2 y^3)^3 = 2^3 (x^2)^3 (y^3)^3 = 8x^{2 \cdot 3} y^{3 \cdot 3} = 8x^6 y^9$

10.  $\left(\frac{w^2 s^3}{y^2}\right)^2 = \frac{(w^2 s^3)^2}{(y^2)^2} = \frac{(w^2)^2 (s^3)^2}{y^{2 \cdot 2}} = \frac{w^{2 \cdot 2} s^{3 \cdot 2}}{y^4} = \frac{w^4 s^6}{y^4}$

11.  $\frac{x^9}{x^5} = x^{9-5} = x^4$

12.  $\left(\frac{2a^4}{7b^5}\right)^6 = \frac{(2a^4)^6}{(7b^5)^6} = \frac{2^6 (a^4)^6}{7^6 (b^5)^6} = \frac{64a^{4 \cdot 6}}{117,649b^{5 \cdot 6}} = \frac{64a^{24}}{117,649b^{30}}$

13.  $\frac{(x^2)^5}{(x^3)^2 x^4} = \frac{x^{2 \cdot 5}}{x^{3 \cdot 2} x^4} = \frac{x^{10}}{x^6 x^4} = \frac{x^{10}}{x^{6+4}} = \frac{x^{10}}{x^{10}} = 1$

14.  $\frac{(x^2)^3 (x^3)^2}{(x^3)^4} = \frac{x^{2 \cdot 3} x^{3 \cdot 2}}{x^{3 \cdot 4}} = \frac{x^6 x^6}{x^{12}} = \frac{x^{12}}{x^{12}} = x^{12-12} = x^0 = 1$

15.  $\sqrt{25} = 5$

16.  $\sqrt[4]{81} = 3$

17.  $\sqrt[7]{-128} = -2$

18.  $\sqrt[3]{0.027} = \sqrt[3]{(0.3)^3} = 0.3$

19.  $\sqrt[4]{\frac{1}{16}} = \frac{\sqrt[4]{1}}{\sqrt[4]{16}} = \frac{1}{2}$

$$20. \sqrt[3]{-\frac{8}{27}} = \sqrt[3]{\frac{-8}{27}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{27}} = \frac{-2}{3} = -\frac{2}{3}$$

$$21. (49)^{1/2} = \sqrt{49} = 7$$

$$22. (64)^{1/3} = \sqrt[3]{64} = 4$$

$$23. 27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$$

$$24. (9)^{-5/2} = \frac{1}{(9)^{5/2}} = \frac{1}{(\sqrt{9})^5} = \frac{1}{3^5} = \frac{1}{243}$$

$$25. (32)^{-2/5} = \frac{1}{(32)^{2/5}} = \frac{1}{(\sqrt[5]{32})^2} = \frac{1}{(2)^2} = \frac{1}{4}$$

$$26. (0.09)^{-1/2} = \frac{1}{(0.09)^{1/2}} = \frac{1}{\sqrt{0.09}} = \frac{1}{0.3} \\ = \frac{1}{\frac{3}{10}} = \frac{10}{3}$$

$$27. \left(\frac{1}{32}\right)^{4/5} = \left(\sqrt[5]{\frac{1}{32}}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$28. \left(-\frac{27}{64}\right)^{4/3} = \left(\sqrt[3]{-\frac{27}{64}}\right)^4 = \left(-\frac{3}{4}\right)^4 = \frac{3^4}{4^4} = \frac{81}{256}$$

$$29. \sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$$

$$30. \sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \sqrt[3]{2} = 3\sqrt[3]{2}$$

$$31. \sqrt[3]{2x^3} = \sqrt[3]{2} \sqrt[3]{x^3} = x \sqrt[3]{2}$$

$$32. \sqrt{4x} = \sqrt{4} \sqrt{x} = 2\sqrt{x}$$

$$33. \sqrt{25y^6} = \sqrt{25} \cdot \sqrt{y^6} = 5y^3$$

$$34. \sqrt[4]{\frac{x}{16}} = \frac{\sqrt[4]{x}}{\sqrt[4]{16}} = \frac{\sqrt[4]{x}}{2}$$

$$35. 2\sqrt{8} - 5\sqrt{27} + \sqrt[3]{128} = 2\sqrt{4 \cdot 2} - 5\sqrt{9 \cdot 3} + \sqrt[3]{64 \cdot 2} \\ = 2 \cdot 2\sqrt{2} - 5 \cdot 3\sqrt{3} + 4\sqrt[3]{2} \\ = 4\sqrt{2} - 15\sqrt{3} + 4\sqrt[3]{2}$$

$$36. \sqrt{\frac{3}{13}} = \sqrt{\frac{3}{13} \cdot \frac{13}{13}} = \sqrt{\frac{39}{13^2}} = \frac{\sqrt{39}}{\sqrt{13^2}} = \frac{\sqrt{39}}{13}$$

$$37. (9z^4)^{1/2} = \sqrt{9z^4} = \sqrt{3^2(z^2)^2} = \sqrt{3^2} \sqrt{(z^2)^2} \\ = 3z^2$$

$$38. (27x^6)^{4/3} = \left(\sqrt[3]{27x^6}\right)^4 = (3x^2)^4 = 81x^8$$

$$39. \left(\frac{27t^3}{8}\right)^{2/3} = \left(\left[\frac{3t}{2}\right]^3\right)^{2/3} = \left[\frac{3t}{2}\right]^2 = \frac{9t^2}{4}$$

$$40. \left(\frac{256}{x^{12}}\right)^{-3/4} = \left(\left[\frac{4}{x^3}\right]^4\right)^{-3/4} = \left[\frac{4}{x^3}\right]^{-3} = \frac{4^{-3}}{(x^3)^{-3}} \\ = \frac{4^{-3}}{x^{-9}} = \frac{x^9}{4^3} = \frac{x^9}{64}$$

$$41. \frac{a^5 b^{-3}}{c^2} = a^5 \cdot b^{-3} \cdot \frac{1}{c^2} = a^5 \cdot \frac{1}{b^3} \cdot \frac{1}{c^2} = \frac{a^5}{b^3 c^2}$$

$$42. \sqrt[5]{x^2 y^3 z^{-10}} = x^{2/5} y^{3/5} z^{-10/5} = \frac{x^{2/5} y^{3/5}}{z^2}$$

$$43. 2a^{-1} b^{-3} = 2 \cdot \frac{1}{a} \cdot \frac{1}{b^3} = \frac{2}{ab^3}$$

$$44. x + y^{-1} = x + \frac{1}{y}$$

$$45. (3t)^{-2} = \frac{1}{(3t)^2} = \frac{1}{9t^2}$$

$$46. (3-z)^{-4} = \frac{1}{(3-z)^4}$$

$$47. \sqrt[5]{5x^2} = (5x^2)^{1/5} = 5^{1/5} (x^2)^{1/5} = 5^{1/5} x^{2/5}$$

$$48. (X^2 Y^{-2})^{-2} = \left(\frac{X^2}{Y^2}\right)^{-2} = \left(\frac{Y^2}{X^2}\right)^2 = \frac{Y^4}{X^4}$$

$$49. \sqrt{x} - \sqrt{y} = x^{1/2} - y^{1/2}$$

$$50. \frac{u^{-2}v^{-6}w^3}{vw^{-5}} = \frac{w^{3-(-5)}}{u^2v^{1-(-6)}} = \frac{w^8}{u^2v^7}$$

$$51. \begin{aligned} x^2\sqrt[4]{xy^{-2}z^3} &= x^2(xy^{-2}z^3)^{1/4} = x^2x^{1/4}y^{-2/4}z^{3/4} \\ &= \frac{x^{9/4}z^{3/4}}{y^{1/2}} \end{aligned}$$

$$52. \begin{aligned} \sqrt[4]{a^{-3}b^{-2}}a^5b^{-4} &= (a^{-3}b^{-2})^{1/4}a^5b^{-4} \\ &= a^{-3/4}b^{-1/2}a^5b^{-4} \\ &= a^{17/4}b^{-9/2} \\ &= \frac{a^{17/4}}{b^{9/2}} \end{aligned}$$

$$53. (a+b-c)^{2/3} = \sqrt[3]{(a+b-c)^2}$$

$$54. (ab^2c^3)^{3/4} = \sqrt[4]{(ab^2c^3)^3} = \sqrt[4]{a^3b^6c^9}$$

$$55. x^{-4/5} = \frac{1}{x^{4/5}} = \frac{1}{\sqrt[5]{x^4}}$$

$$56. 2x^{1/2} - (2y)^{1/2} = 2\sqrt{x} - \sqrt{2y}$$

$$57. \begin{aligned} 3w^{-3/5} - (3w)^{-3/5} &= \frac{3}{w^{3/5}} - \frac{1}{(3w)^{3/5}} \\ &= \frac{3}{\sqrt[5]{w^3}} - \frac{1}{\sqrt[5]{(3w)^3}} = \frac{3}{\sqrt[5]{w^3}} - \frac{1}{\sqrt[5]{27w^3}} \end{aligned}$$

$$58. \begin{aligned} ((x^{-5})^{1/3})^{1/4} &= (x^{-5/3})^{1/4} \\ &= x^{-5/12} \\ &= \frac{1}{x^{5/12}} \\ &= \frac{1}{\sqrt[12]{x^5}} \end{aligned}$$

$$59. \frac{6}{\sqrt{5}} = \frac{6}{5^{1/2}} = \frac{6 \cdot 5^{1/2}}{5^{1/2} \cdot 5^{1/2}} = \frac{6\sqrt{5}}{5}$$

$$60. \frac{3}{\sqrt[4]{8}} = \frac{3}{8^{1/4}} = \frac{3 \cdot 2^{1/4}}{8^{1/4} \cdot 2^{1/4}} = \frac{3\sqrt[4]{2}}{\sqrt[4]{16}} = \frac{3\sqrt[4]{2}}{2}$$

$$61. \begin{aligned} \frac{4}{\sqrt{2x}} &= \frac{4}{(2x)^{1/2}} = \frac{4(2x)^{1/2}}{(2x)^{1/2}(2x)^{1/2}} = \frac{4\sqrt{2x}}{2x} \\ &= \frac{2\sqrt{2x}}{x} \end{aligned}$$

$$62. \begin{aligned} \frac{y}{\sqrt{2y}} &= \frac{y}{(2y)^{1/2}} = \frac{y(2y)^{1/2}}{(2y)^{1/2}(2y)^{1/2}} = \frac{y\sqrt{2y}}{2y} \\ &= \frac{\sqrt{2y}}{2} \end{aligned}$$

$$63. \begin{aligned} \frac{1}{\sqrt[3]{2a}} &= \frac{1}{(2a)^{1/3}} = \frac{1(2a)^{2/3}}{(2a)^{1/3}(2a)^{2/3}} = \frac{(2a)^{2/3}}{2a} \\ &= \frac{\sqrt[3]{4a^2}}{2a} \end{aligned}$$

$$64. \frac{2}{\sqrt[3]{y^2}} = \frac{2}{3y^{2/3}} = \frac{2 \cdot y^{1/3}}{3y^{2/3} \cdot y^{1/3}} = \frac{2y^{1/3}}{3y} = \frac{2\sqrt[3]{y}}{3y}$$

$$65. \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

$$66. \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

$$67. \begin{aligned} \frac{\sqrt[5]{2}}{\sqrt[4]{a^2b}} &= \frac{\sqrt[5]{2}}{a^{2/4}b^{1/4}} = \frac{\sqrt[5]{2} \cdot a^{1/2}b^{3/4}}{a^{1/2}b^{1/4} \cdot a^{1/2}b^{3/4}} \\ &= \frac{2^{1/5}a^{1/2}b^{3/4}}{2^{4/20}a^{10/20}b^{15/20}} \\ &= \frac{ab}{(2^4a^{10}b^{15})^{1/20}} = \frac{ab}{\sqrt[20]{16a^{10}b^{15}}} \end{aligned}$$

$$68. \frac{\sqrt{3}}{\sqrt[3]{2}} = \frac{\sqrt{3}}{2^{1/3}} = \frac{\sqrt{3} \cdot 2^{2/3}}{2^{1/3} \cdot 2^{2/3}} = \frac{\sqrt{3} \cdot \sqrt[3]{2^2}}{2} = \frac{\sqrt{3} \cdot \sqrt[3]{4}}{2}$$

$$69. 2x^2y^{-3}x^4 = 2x^6y^{-3} = \frac{2x^6}{y^3}$$

$$70. \frac{3}{u^{5/2}v^{1/2}} = \frac{3 \cdot u^{1/2}v^{1/2}}{u^{5/2}v^{1/2} \cdot u^{1/2}v^{1/2}} = \frac{3u^{1/2}v^{1/2}}{u^3v}$$

$$71. \frac{\sqrt{243}}{\sqrt{3}} = \sqrt{\frac{243}{3}} = \sqrt{81} = 9$$

$$72. \{(3a^3)^2\}^{-5} = \{[3^2 a^6]^{-5}\}^{-2} \\ = \{3^{-10} a^{-30}\}^{-2} \\ = 3^{20} a^{60}$$

$$73. \frac{3^0}{(3^{-3} x^{1/3} y^{-3})^2} = \frac{1}{3^{-6} x^{2/3} y^{-6}} = \frac{3^6 y^6}{x^{2/3}} \\ = \frac{3^6 y^6 \cdot x^{1/3}}{x^{2/3} \cdot x^{1/3}} = \frac{(3y)^6 x^{1/3}}{x}$$

$$74. \frac{\sqrt{s^5}}{\sqrt[3]{s^2}} = \frac{s^{5/2}}{s^{2/3}} = \frac{s^{15/6}}{s^{4/6}} = s^{11/6}$$

$$75. \sqrt[3]{x^2 y z^3} \sqrt[3]{x y^2} = \sqrt[3]{(x^2 y z^3)(x y^2)} = \sqrt[3]{x^3 y^3 z^3} \\ = xyz$$

$$76. (\sqrt[4]{3})^8 = (3^{1/4})^8 = 3^{8/4} = 3^2 = 9$$

$$77. 3^2 (32)^{-2/5} = 3^2 (2^5)^{-2/5} \\ = 3^2 (2^{-2}) \\ = 3^2 \cdot \frac{1}{2^2} \\ = \frac{9}{4}$$

$$78. (\sqrt[5]{a^2 b})^{3/5} = (a^{2/5} b^{1/5})^{3/5} = a^{6/25} b^{3/25} \\ = (a^6 b^3)^{1/25}$$

$$79. (2x^{-1} y^2)^2 = 2^2 x^{-2} y^4 = \frac{4y^4}{x^2}$$

$$80. \frac{3}{\sqrt[3]{y^4} \sqrt{x}} = \frac{3}{y^{1/3} x^{1/4}} = \frac{3 \cdot y^{2/3} x^{3/4}}{y^{1/3} x^{1/4} \cdot y^{2/3} x^{3/4}} \\ = \frac{3x^{3/4} y^{2/3}}{xy}$$

$$81. \sqrt{x} \sqrt{x^2 y^3} \sqrt{xy^2} = x^{1/2} (x^2 y^3)^{1/2} (xy^2)^{1/2} \\ = x^{1/2} (xy^{3/2})(x^{1/2} y) = x^2 y^{5/2}$$

$$82. \sqrt{75k^4} = (75k^4)^{1/2} = [(25k^4)(3)]^{1/2} \\ = [(5k^2)^2 3]^{1/2} = 5k^2 3^{1/2}$$

$$83. \frac{(a^2 b^{-3} c^4)^5}{(a^{-1} c^{-2})^{-3}} = \frac{a^{10} b^{-15} c^{20}}{a^3 c^6} = \frac{a^7 c^{14}}{b^{15}}$$

$$84. \sqrt[3]{7(49)} = \sqrt[3]{7 \cdot 7^2} = \sqrt[3]{7^3} = 7$$

$$85. \frac{(x^2)^3}{x^4} \div \left[ \frac{x^3}{(x^3)^2} \right]^2 = \frac{x^6}{x^4} \div \frac{(x^3)^2}{(x^6)^2} \\ = x^2 \div \frac{x^6}{x^{12}} = x^2 \div x^{6-12} = x^2 \div x^{-6} \\ = x^2 \div \frac{1}{x^6} = x^2 \cdot x^6 = x^8$$

$$86. \sqrt{(-6)(-6)} = \sqrt{36} = 6$$

Note that  $\sqrt{(-6)^2} \neq -6$  since  $-6 < 0$ .

$$87. -\frac{8s^{-2}}{2s^3} = -\frac{4}{s^3 s^2} = -\frac{4}{s^5}$$

$$88. (x^3 y^{-4} \sqrt{z})^5 = (x^3 y^{-4} z^{1/2})^5 \\ = x^{15} y^{-20} z^{5/2} \\ = \frac{x^{15} z^{5/2}}{y^{20}}$$

$$89. (3x^3 y^2 \div 2y^2 z^{-3})^4 = \left( \frac{3x^3 y^2}{2y^2 z^{-3}} \right)^4 \\ = \left( \frac{3x^3 z^3}{2} \right)^4 \\ = \frac{(3x^3 z^3)^4}{(2)^4} \\ = \frac{3^4 x^{12} z^{12}}{2^4} \\ = \frac{81x^{12} z^{12}}{16}$$

$$90. \frac{1}{\left( \frac{\sqrt{2x^{-2}}}{\sqrt{16x^3}} \right)^2} = \frac{1}{\left( \frac{2^{1/2}}{16^{1/2}} \right)^2 (x^{-2})^2} = \frac{1}{\frac{2x^{-4}}{16x^6}} = \frac{1}{8x^{10}} = 8x^{10}$$

## Problems 0.4

1.  $8x - 4y + 2 + 3x + 2y - 5 = 11x - 2y - 3$
2.  $4a^2 - 2ab + 3 + 5c - 3ab + 7$   
 $= 4a^2 - 5ab + 10 + 5c$
3.  $8t^2 - 6s^2 + 4s^2 - 2t^2 + 6 = 6t^2 - 2s^2 + 6$
4.  $\sqrt{x} + 2\sqrt{x} + \sqrt{x} + 3\sqrt{x} = 7\sqrt{x}$
5.  $\sqrt{a} + 2\sqrt{3b} - \sqrt{c} + 3\sqrt{3b}$   
 $= \sqrt{a} + 5\sqrt{3b} - \sqrt{c}$
6.  $3a + 7b - 9 - 5a - 9b - 21 = -2a - 2b - 30$
7.  $7x^2 + 5xy + \sqrt{2} - 2z + 2xy - \sqrt{2}$   
 $= 7x^2 + 7xy - 2z$
8.  $\sqrt{x} + 2\sqrt{x} - \sqrt{x} - 3\sqrt{x} = -\sqrt{x}$
9.  $\sqrt{x} + \sqrt{2y} - \sqrt{x} - \sqrt{3z} = \sqrt{2y} - \sqrt{3z}$
10.  $8z - 4w - 3w + 6z = 14z - 7w$
11.  $9x + 9y - 21 - 24x + 6y - 6 = -15x + 15y - 27$
12.  $4s - 5t - 2s - 5t + s + 9 = 3s - 10t + 9$
13.  $5x^2 - 5y^2 + xy - 3x^2 - 8xy - 28y^2$   
 $= 2x^2 - 33y^2 - 7xy$
14.  $2 - [3 + 4s - 12] = 2 - [4s - 9] = 2 - 4s + 9$   
 $= 11 - 4s$
15.  $2\{3[3x^2 + 6 - 2x^2 + 10]\} = 2\{3[x^2 + 16]\}$   
 $= 2\{3x^2 + 48\} = 6x^2 + 96$
16.  $4\{3t + 15 - t[1 - t - 1]\} = 4\{3t + 15 - t[-t]\}$   
 $= 4\{3t + 15 + t^2\} = 4t^2 + 12t + 60$
17.  $-2(6u^3 + 6u^2 - 2(u^2 - 5 + 2u))$   
 $= -2(6u^3 + 6u^2 - 2u^2 + 10 - 4u)$   
 $= -2(6u^3 + 4u^2 - 4u + 10)$   
 $= -12u^3 - 8u^2 + 8u - 20$
18.  $-[-6a - 6b + 6 + 10a + 15b - a[2b + 10]]$   
 $= -\{4a + 9b + 6 - 2ab - 10a\}$   
 $= -\{-6a + 9b + 6 - 2ab\}$   
 $= 6a - 9b - 6 + 2ab$
19.  $x^2 + (4+5)x + 4(5) = x^2 + 9x + 20$
20.  $u^2 + (5+2)u + 2(5) = u^2 + 7u + 10$
21.  $w^2 + (-5+2)x + 2(-5) = w^2 - 3w - 10$
22.  $x^2 + (-4+7)x - 28 = x^2 + 3x - 28$
23.  $(2x)(5x) + [(2)(2) + (3)(5)]x + 3(2)$   
 $= 10x^2 + 19x + 6$
24.  $(t)(2t) + [(1)(7) + (-5)(2)]t + (-5)(7)$   
 $= 2t^2 - 3t - 35$
25.  $X^2 + 2(X)(2Y) + (2Y)^2 = X^2 + 4XY + 4Y^2$
26.  $(2x)^2 - 2(2x)(1) + 1^2 = 4x^2 - 4x + 1$
27.  $7^2 - 2(7)(X) + X^2 = 49 - 14X + X^2$
28.  $(1 \cdot 2)(\sqrt{x})^2 + [(1)(5) + (-1)(2)]\sqrt{x} + (-1)(5)$   
 $= 2x + 3\sqrt{x} - 5$
29.  $(\sqrt{3x})^2 + 2(\sqrt{3x})(5) + (5)^2$   
 $= 3x + 10\sqrt{3x} + 25$
30.  $(\sqrt{y})^2 - 3^2 = y - 9$
31.  $(2s)^2 - 1^2 = 4s^2 - 1$
32.  $(a^2)^2 - (2b)^2 = a^4 - 4b^2$
33.  $x^2(x+4) - 3(x+4)$   
 $= x^3 + 4x^2 - 3x - 12$
34.  $x(x^2 + x + 3) + 1(x^2 + x + 3)$   
 $= x^3 + x^2 + 3x + x^2 + x + 3$   
 $= x^3 + 2x^2 + 4x + 3$

$$\begin{aligned}
 35. \quad & x^2(3x^2 + 2x - 1) - 4(3x^2 + 2x - 1) \\
 &= 3x^4 + 2x^3 - x^2 - 12x^2 - 8x + 4 \\
 &= 3x^4 + 2x^3 - 13x^2 - 8x + 4
 \end{aligned}$$

$$\begin{aligned}
 36. \quad & 3y(4y^3 + 2y^2 - 3y) - 2(4y^3 + 2y^2 - 3y) \\
 &= 12y^4 + 6y^3 - 9y^2 - 8y^3 - 4y^2 + 6y \\
 &= 12y^4 - 2y^3 - 13y^2 + 6y
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & t\{3(t^2 - 2t - 8) + 5[3t^2 - 2t]\} \\
 &= t\{3t^2 - 6t - 24 + 15t^2 - 10t\} \\
 &= t\{18t^2 - 11t - 24\} \\
 &= 18t^3 - 11t^2 - 24t
 \end{aligned}$$

$$\begin{aligned}
 38. \quad & [(2z)^2 - 1^2](4z^2 + 1) = [4z^2 - 1](4z^2 + 1) \\
 &= (4z^2)^2 - 1^2 = 16z^4 - 1
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & x(3x + 2y - 4) + y(3x + 2y - 4) + 2(3x + 2y - 4) \\
 &= 3x^2 + 2xy - 4x + 3xy + 2y^2 - 4y + 6x + 4y - 8 \\
 &= 3x^2 + 2y^2 + 5xy + 2x - 8
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & [x^2 + (x+1)]^2 \\
 &= (x^2)^2 + 2x^2(x+1) + (x+1)^2 \\
 &= x^4 + 2x^3 + 2x^2 + x^2 + 2x + 1 \\
 &= x^4 + 2x^3 + 3x^2 + 2x + 1
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & (2a)^3 + 3(2a)^2(3) + 3(2a)(3)^2 + (3)^3 \\
 &= 8a^3 + 36a^2 + 54a + 27
 \end{aligned}$$

$$\begin{aligned}
 42. \quad & (2a)^3 - 3(2a)^2(3) + 3(2a)(3)^2 - 3^3 \\
 &= 8a^3 - 36a^2 + 54a - 27
 \end{aligned}$$

$$\begin{aligned}
 43. \quad & (2x)^3 - 3(2x)^2(3) + 3(2x)(3)^2 - 3^3 \\
 &= 8x^3 - 36x^2 + 54x - 27
 \end{aligned}$$

$$\begin{aligned}
 44. \quad & x^3 + 3x^2(2y) + 3x(2y)^2 + (2y)^3 \\
 &= x^3 + 6x^2y + 12xy^2 + 8y^3
 \end{aligned}$$

$$45. \quad \frac{z^2}{z} - \frac{18z}{z} = z - 18$$

$$46. \quad \frac{2x^3}{x} - \frac{7x}{x} + \frac{4}{x} = 2x^2 - 7 + \frac{4}{x}$$

$$47. \quad \frac{6u^5}{3u^2} + \frac{9u^3}{3u^2} - \frac{1}{3u^2} = 2u^3 + 3u - \frac{1}{3u^2}$$

$$\begin{aligned}
 48. \quad & \frac{3y - 4 - 9y - 5}{3y} \\
 &= \frac{-6y - 9}{3y} \\
 &= \frac{-6y}{3y} - \frac{9}{3y} \\
 &= -2 - \frac{3}{y}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad & x + 5 \sqrt{x^2 + 5x - 3} \\
 & \quad \quad \quad \frac{x^2 + 5x}{x^2 + 5x} \\
 & \quad \quad \quad \quad \quad \quad -3 \\
 \text{Answer: } & x + \frac{-3}{x+5}
 \end{aligned}$$

$$\begin{aligned}
 50. \quad & x - 4 \sqrt{x^2 - 5x + 4} \\
 & \quad \quad \quad \frac{x-1}{x^2 - 5x + 4} \\
 & \quad \quad \quad \frac{x^2 - 4x}{x^2 - 5x + 4} \\
 & \quad \quad \quad \quad \quad \quad -x + 4 \\
 & \quad \quad \quad \quad \quad \quad \quad \quad -x + 4 \\
 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 0 \\
 \text{Answer: } & x - 1
 \end{aligned}$$

$$\begin{aligned}
 51. \quad & x + 2 \sqrt{3x^2 - 8x + 17} \\
 & \quad \quad \quad \frac{3x^2 - 8x + 17}{3x^3 - 2x^2 + x - 3} \\
 & \quad \quad \quad \frac{3x^3 + 6x^2}{3x^3 - 2x^2 + x - 3} \\
 & \quad \quad \quad \quad \quad \quad -8x^2 + x \\
 & \quad \quad \quad \quad \quad \quad \quad \quad -8x^2 - 16x \\
 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 17x - 3 \\
 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 17x + 34 \\
 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad -37 \\
 \text{Answer: } & 3x^2 - 8x + 17 + \frac{-37}{x+2}
 \end{aligned}$$



$$52. \quad x+1 \overline{) \begin{array}{r} x^3 - x^2 + 4x - 4 \\ x^4 + 0x^3 + 3x^2 + 0x + 2 \\ \hline -x^3 + 3x^2 \\ -x^3 - x^2 \\ \hline 4x^2 + 0x \\ 4x^2 + 4x \\ \hline -4x + 2 \\ -4x - 4 \\ \hline 6 \end{array}}$$

$$\text{Answer: } x^3 - x^2 + 4x - 4 + \frac{6}{x+1}$$

$$53. \quad x+2 \overline{) \begin{array}{r} x^2 - 2x + 4 \\ x^3 + 0x^2 + 0x + 0 \\ \hline -2x^2 + 0 \\ -2x^2 - 4x \\ \hline 4x + 0 \\ 4x + 8 \\ \hline -8 \end{array}}$$

$$\text{Answer: } x^2 - 2x + 4 - \frac{8}{x+2}$$

$$54. \quad 2x+3 \overline{) \begin{array}{r} 3x - \frac{1}{2} \\ 6x^2 + 8x + 1 \\ \hline 6x^2 + 9x \\ \hline -x + 1 \\ -x - \frac{3}{2} \\ \hline \frac{5}{2} \end{array}}$$

$$\text{Answer: } 3x - \frac{1}{2} + \frac{\frac{5}{2}}{2x+3}$$

$$55. \quad 3x+2 \overline{) \begin{array}{r} x-2 \\ 3x^2 - 4x + 3 \\ \hline 3x^2 + 2x \\ \hline -6x + 3 \\ -6x - 4 \\ \hline 7 \end{array}}$$

$$\text{Answer: } x - 2 + \frac{7}{3x+2}$$

$$56. \quad z^2 - z + 1 \overline{) \begin{array}{r} z+2 \\ z^3 + z^2 + z \\ \hline z^3 - z^2 + z \\ \hline 2z^2 \\ 2z^2 - 2z + 2 \\ \hline 2z - 2 \end{array}}$$

$$\text{Answer: } z + 2 + \frac{2z-2}{z^2 - z + 1}$$

**Problems 0.5**

1.  $5b(x+1)$
2.  $2y(3y-2)$
3.  $5x(2y+z)$
4.  $3x^2y(1-3xy^2)$
5.  $4bc(2a^3 - 3ab^2d + b^3cd^2)$
6.  $5r^2t^2(s+2rs^2t-3)$
7.  $z^2 - 7^2 = (z+7)(z-7)$
8.  $(x+2)(x-3)$
9.  $(p+3)(p+1)$
10.  $(s-4)(s-2)$
11.  $(5y)^2 - 2^2 = (5y+2)(5y-2)$
12.  $(x+6)(x-4)$
13.  $(a+7)(a+5)$
14.  $(2t)^2 - (3s)^2 = (2t+3s)(2t-3s)$
15.  $x^2 + 2(3)(x) + 3^2 = (x+3)^2$
16.  $(t-6)(t-12)$
17.  $5(x^2 + 5x + 6) = 5(x+3)(x+2)$
18.  $3(t^2 + 4t - 5) = 3(t-1)(t+5)$
19.  $3(x^2 - 1^2) = 3(x+1)(x-1)$

20.  $(3y - 4)(3y - 2)$
21.  $(5x + 1)(x + 3)$
22.  $(4x + 3)(x - 1)$
23.  $2s(6s^2 + 5s - 4) = 2s(3s + 4)(2s - 1)$
24.  $(3z)^2 + 2(3z)(5) + 5^2 = (3z + 5)^2$
25.  $u^{3/5}v(u^2 - 4v^2) = u^{3/5}v(u + 2v)(u - 2v)$
26.  $(2x^{3/5})^2 - 1^2 = (2x^{3/5} + 1)(2x^{3/5} - 1)$
27.  $2x(x^2 + x - 6) = 2x(x + 3)(x - 2)$
28.  $(xy)^2 - 2(xy)(2) + 2^2 = (xy - 2)^2$
29.  $[2(2x + 1)]^2 = 2^2(2x + 1)^2$   
 $= 4(2x + 1)^2$
30.  $2x^2[2x(1 - 2x)]^2$   
 $= 2x^2(2x)^2(1 - 2x)^2$   
 $= 2x^2(4x^2)(1 - 2x)^2$   
 $= 8x^4(1 - 2x)^2$
31.  $x(x^2y^2 - 16xy + 64) = x[(xy)^2 - 2(xy)(8) + 8^2]$   
 $= x(xy - 8)^2$
32.  $x(5x + 2) + 2(5x + 2) = (5x + 2)(x + 2)$
33.  $x(x^2 - 4) + 2(4 - x^2)$   
 $= x(x^2 - 4) - 2(x^2 - 4)$   
 $= (x^2 - 4)(x - 2)$   
 $= (x + 2)(x - 2)(x - 2)$   
 $= (x + 2)(x - 2)^2$
34.  $(x + 1)(x - 1) + (x - 2)(x + 1)$   
 $= (x + 1)[(x - 1) + (x - 2)]$   
 $= (x + 1)(2x - 3)$
35.  $y^2(y^2 + 8y + 16) - (y^2 + 8y + 16)$   
 $= (y^2 + 8y + 16)(y^2 - 1)$   
 $= (y + 4)^2(y + 1)(y - 1)$
36.  $tu(t^2 - 3) + w^2(t^2 - 3)$   
 $= (t^2 - 3)(tu + w^2)$   
 $= (t + \sqrt{3})(t - \sqrt{3})(tu + w^2)$
37.  $b^3 + 4^3 = (b + 4)(b^2 - 4(b) + 4^2)$   
 $= (b + 4)(b^2 - 4b + 16)$
38.  $x^3 - 1^3 = (x - 1)[x^2 + 1(x) + 1^2]$   
 $= (x - 1)(x^2 + x + 1)$
39.  $(x^3)^2 - 1^2 = (x^3 + 1)(x^3 - 1)$   
 $= (x + 1)(x^2 - x + 1)(x - 1)(x^2 + x + 1)$
40.  $3^3 + (2x)^3 = (3 + 2x)[3^2 - 3(2x) + (2x)^2]$   
 $= (3 + 2x)(9 - 6x + 4x^2)$
41.  $(x + 4)^2(x - 2)[(x + 4) + (x - 2)]$   
 $= (x + 4)^2(x - 2)(2x + 2)$   
 $= 2(x + 4)^2(x - 2)(x + 1)$
42.  $(a + 5)^2(a + 1)^2[(a + 5) + (a + 1)]$   
 $= (a + 5)^2(a + 1)^2(2a + 6)$   
 $= 2(a + 5)^2(a + 1)^2(a + 3)$
43.  $[P(1 + r)] + [P(1 + r)]r = [P(1 + r)](1 + r)$   
 $= P(1 + r)^2$
44.  $(3X + 5I)[(X - 3I) - (X + 2I)] = (3X + 5I)(-5I)$   
 $= -5I(3X + 5I)$
45.  $(x^2)^2 - 4^2 = (x^2 + 4)(x^2 - 4)$   
 $= (x^2 + 4)(x + 2)(x - 2)$
46.  $(16y^2)^2 - (z^2)^2 = (16y^2 + z^2)(16y^2 - z^2)$   
 $= (16y^2 + z^2)(4y + z)(4y - z)$

$$\begin{aligned} 47. \quad (y^4)^2 - 1^2 &= (y^4 + 1)(y^4 - 1) \\ &= (y^4 + 1)(y^2 + 1)(y^2 - 1) \\ &= (y^4 + 1)(y^2 + 1)(y + 1)(y - 1) \end{aligned}$$

$$\begin{aligned} 48. \quad (t^2)^2 - 2^2 &= (t^2 + 2)(t^2 - 2) \\ &= (t^2 + 2)\left[t^2 - (\sqrt{2})^2\right] \\ &= (t^2 + 2)(t + \sqrt{2})(t - \sqrt{2}) \end{aligned}$$

$$49. \quad (X^2 + 5)(X^2 - 1) = (X^2 + 5)(X + 1)(X - 1)$$

$$50. \quad (x^2 - 9)(x^2 - 1) = (x + 3)(x - 3)(x + 1)(x - 1)$$

$$\begin{aligned} 51. \quad b(a^4 - 8a^2 + 16) &= b(a^2 - 4)^2 \\ &= b[(a + 2)(a - 2)]^2 \\ &= b(a + 2)^2(a - 2)^2 \end{aligned}$$

$$52. \quad 2x(2x^2 - 3x - 2) = 2x(2x + 1)(x - 2)$$

## Problems 0.6

$$1. \quad \frac{a^2 - 9}{a^2 - 3a} = \frac{(a - 3)(a + 3)}{a(a - 3)} = \frac{a + 3}{a}$$

$$2. \quad \frac{x^2 - 3x - 10}{x^2 - 4} = \frac{(x + 2)(x - 5)}{(x + 2)(x - 2)} = \frac{x - 5}{x - 2}$$

$$3. \quad \frac{x^2 - 9x + 20}{x^2 + x - 20} = \frac{(x - 5)(x - 4)}{(x + 5)(x - 4)} = \frac{x - 5}{x + 5}$$

$$\begin{aligned} 4. \quad \frac{3x^2 - 27x + 24}{2x^3 - 16x^2 + 14x} &= \frac{3(x - 8)(x - 1)}{2x(x - 7)(x - 1)} \\ &= \frac{3(x - 8)}{2x(x - 7)} \end{aligned}$$

$$5. \quad \frac{15x^2 + x - 2}{3x^2 + 20x - 7} = \frac{(5x + 2)(3x - 1)}{(3x - 1)(x + 7)} = \frac{5x + 2}{x + 7}$$

$$6. \quad \frac{12x^2 - 19x + 4}{6x^2 - 17x + 12} = \frac{(4x - 1)(3x - 4)}{(2x - 3)(3x - 4)} = \frac{4x - 1}{2x - 3}$$

$$7. \quad \frac{y^2(-1)}{(y - 3)(y + 2)} = -\frac{y^2}{(y - 3)(y + 2)}$$

$$8. \quad \frac{(t + 3)(t - 3)t^2}{t(t + 3)(t - 3)^2} = \frac{t}{t - 3}$$

$$\begin{aligned} 9. \quad \frac{(ax - b)(c - x)}{(x - c)(ax + b)} &= \frac{(ax - b)(-1)(x - c)}{(x - c)(ax + b)} \\ &= \frac{(ax - b)(-1)}{ax + b} \\ &= \frac{b - ax}{ax + b} \end{aligned}$$

$$10. \quad \frac{(a + b)(a - b)(a - b)^2}{2(a - b)(a + b)} = \frac{(a - b)^2}{2}$$

$$\begin{aligned} 11. \quad \frac{2(x - 1)}{(x - 4)(x + 2)} \cdot \frac{(x + 4)(x + 1)}{(x + 1)(x - 1)} \\ &= \frac{2(x - 1)(x + 4)(x + 1)}{(x - 4)(x + 2)(x + 1)(x - 1)} \\ &= \frac{2(x + 4)}{(x - 4)(x + 2)} \end{aligned}$$

$$\begin{aligned} 12. \quad \frac{x(x + 2)}{3(x - 4)(x - 2)} \cdot \frac{(x - 2)^2}{(x - 3)(x + 2)} \\ &= \frac{x(x + 2)(x - 2)^2}{3(x - 4)(x - 2)(x - 3)(x + 2)} \\ &= \frac{x(x - 2)}{3(x - 4)(x - 3)} \end{aligned}$$

$$13. \quad \frac{X^2}{8} \cdot \frac{4}{X} = \frac{4X^2}{8X} = \frac{X}{2}$$

$$14. \quad \frac{3x^2}{7x} \cdot \frac{14}{x} = \frac{3x}{7} \cdot \frac{14}{x} = \frac{3(14)x}{7x} = 6$$

$$15. \quad \frac{15u}{v^3} \cdot \frac{v^4}{3u} = \frac{15uv^4}{3uv^3} = 5v$$

$$16. \quad \frac{c + d}{c} \cdot \frac{2c}{c - d} = \frac{2c(c + d)}{c(c - d)} = \frac{2(c + d)}{c - d}$$

$$17. \quad \frac{4x}{3} \div 2x = \frac{4x}{3} \cdot \frac{1}{2x} = \frac{4x}{6x} = \frac{2}{3}$$

$$18. \quad \frac{4x}{1} \cdot \frac{2x}{3} = \frac{4x(2x)}{3} = \frac{8x^2}{3}$$

$$19. \frac{-9x^3}{1} \cdot \frac{3}{x} = \frac{-27x^3}{x} = -27x^2$$

$$20. \frac{21r^5}{t^2} \cdot \frac{1}{-7} = \frac{21r^5}{-7t^2} = -3r^5$$

$$21. \frac{x-3}{1} \cdot \frac{x-4}{(x-3)(x-4)} = \frac{x-3}{1} \cdot \frac{1}{x-3} = \frac{x-3}{x-3} = 1$$

$$22. \frac{(x+3)^2}{x} \div (x+3) = \frac{(x+3)^2}{x} \cdot \frac{1}{x+3}$$

$$= \frac{(x+3)^2}{x(x+3)} = \frac{x+3}{x}$$

$$23. \frac{10x^3}{(x+1)(x-1)} \cdot \frac{x+1}{5x} = \frac{10x^3(x+1)}{5x(x+1)(x-1)} = \frac{2x^2}{x-1}$$

$$24. \frac{(x-3)(x+2)}{(x+3)(x-3)} \cdot \frac{(x+3)(x-1)}{(x+2)(x-2)}$$

$$= \frac{x+2}{x+3} \cdot \frac{(x+3)(x-1)}{(x+2)(x-2)}$$

$$= \frac{(x+2)(x+3)(x-1)}{(x+3)(x+2)(x-2)}$$

$$= \frac{x-1}{x-2}$$

$$25. \frac{(x+6)(x+2)}{(x+6)(x+3)} \cdot \frac{(x-5)(x+3)}{(x-5)(x+2)} = \frac{x+2}{x+3} \cdot \frac{x+3}{x+2}$$

$$= \frac{(x+2)(x+3)}{(x+3)(x+2)}$$

$$= 1$$

$$26. \frac{(x+3)^2}{4x-3} \cdot \frac{(3+4x)(3-4x)}{7(x+3)}$$

$$= \frac{(x+3)^2(3+4x)(3-4x)}{7(4x-3)(x+3)}$$

$$= \frac{(x+3)(3+4x)(-1)(4x-3)}{7(4x-3)}$$

$$= -\frac{(x+3)(3+4x)}{7}$$

$$27. \frac{(2x+3)(2x-3)}{(x+4)(x-1)} \cdot \frac{(1+x)(1-x)}{2x-3}$$

$$= \frac{(2x+3)(2x-3)(1+x)(1-x)}{(x+4)(x-1)(2x-3)}$$

$$= \frac{(2x+3)(1+x)(-1)(x-1)}{(x+4)(x-1)}$$

$$= -\frac{(2x+3)(1+x)}{x+4}$$

$$28. \frac{y(6x^2+7x-3)}{x(y-1)+5(y-1)} \cdot \frac{x(y-1)+4(y-1)}{x^2y(x+4)}$$

$$= \frac{y(3x-1)(2x+3)(y-1)(x+4)}{(y-1)(x+5)x^2y(x+4)}$$

$$= \frac{(3x-1)(2x+3)}{x^2(x+5)}$$

$$29. \frac{x^2+5x+6}{x+3} = \frac{(x+3)(x+2)}{x+3} = x+2$$

$$30. \frac{-1+x}{x-1} = \frac{x-1}{x-1} = 1$$

$$31. \text{LCD} = 3t$$

$$\frac{2}{t} + \frac{1}{3t} = \frac{6}{3t} + \frac{1}{3t} = \frac{6+1}{3t} = \frac{7}{3t}$$

$$32. \text{LCD} = X^3$$

$$\frac{9}{X^3} - \frac{1}{X^2} = \frac{9}{X^3} - \frac{X}{X^3} = \frac{9-X}{X^3}$$

$$33. \text{LCD} = x^3 - 1$$

$$1 - \frac{x^3}{x^3-1} = \frac{x^3-1}{x^3-1} - \frac{x^3}{x^3-1}$$

$$= \frac{x^3-1-x^3}{x^3-1}$$

$$= \frac{-1}{x^3-1}$$

$$= \frac{1}{1-x^3}$$

$$34. \text{LCD} = s+4$$

$$\frac{4}{s+4} + s = \frac{4}{s+4} + \frac{s(s+4)}{s+4} = \frac{4+s(s+4)}{s+4}$$

$$= \frac{s^2+4s+4}{s+4} = \frac{(s+2)^2}{s+4}$$

35. LCD =  $(3x - 1)(x + 1)$

$$\begin{aligned}\frac{1}{3x-1} + \frac{x}{x+1} &= \frac{x+1}{(3x-1)(x+1)} + \frac{x(3x-1)}{(3x-1)(x+1)} \\ &= \frac{(x+1) + x(3x-1)}{(3x-1)(x+1)} \\ &= \frac{3x^2 + 1}{(x+1)(3x-1)}\end{aligned}$$

36. LCD =  $(x - 1)(x + 1)$

$$\begin{aligned}\frac{x+1}{x-1} - \frac{x-1}{x+1} &= \frac{(x+1)(x+1)}{(x-1)(x+1)} - \frac{(x-1)(x-1)}{(x-1)(x+1)} \\ &= \frac{(x+1)^2 - (x-1)^2}{(x+1)(x-1)} \\ &= \frac{x^2 + 2x + 1 - (x^2 - 2x + 1)}{(x+1)(x-1)} = \frac{4x}{(x+1)(x-1)}\end{aligned}$$

37. LCD =  $(x - 3)(x + 1)(x + 3)$

$$\begin{aligned}\frac{1}{(x-3)(x+1)} + \frac{1}{(x+3)(x-3)} &= \frac{x+3}{(x-3)(x+1)(x+3)} + \frac{x+1}{(x-3)(x+1)(x+3)} \\ &= \frac{(x+3) + (x+1)}{(x-3)(x+1)(x+3)} \\ &= \frac{2x+4}{(x-3)(x+1)(x+3)} \\ &= \frac{2(x+2)}{(x-3)(x+1)(x+3)}\end{aligned}$$

38. LCD =  $(x - 4)(2x + 1)(2x - 1)$

$$\begin{aligned}\frac{4}{(x-4)(2x+1)} - \frac{x}{(x-4)(2x-1)} &= \frac{4(2x-1)}{(x-4)(2x+1)(2x-1)} - \frac{x(2x+1)}{(x-4)(2x+1)(2x-1)} \\ &= \frac{4(2x-1) - x(2x+1)}{(x-4)(2x+1)(2x-1)} \\ &= \frac{-2x^2 + 7x - 4}{(x-4)(2x+1)(2x-1)}\end{aligned}$$

39. LCD =  $(x - 1)(x + 5)$

$$\begin{aligned}\frac{4}{x-1} - 3 + \frac{-3x^2}{-(x-1)(x+5)} &= \frac{4(x+5)}{(x-1)(x+5)} - \frac{3(x-1)(x+5)}{(x-1)(x+5)} + \frac{3x^2}{(x-1)(x+5)} \\ &= \frac{4x+20 - 3(x^2 + 4x - 5) + 3x^2}{(x-1)(x+5)} \\ &= \frac{35 - 8x}{(x-1)(x+5)}\end{aligned}$$

$$\begin{aligned}
 40. \text{ LCD} &= (2x-1)(x+2)(3x-1) \\
 \frac{x+1}{(2x-1)(x+2)} - \frac{x-1}{(3x-1)(x+2)} + \frac{1}{3x-1} &= \frac{(x+1)(3x-1)}{(2x-1)(x+2)(3x-1)} - \frac{(x-1)(2x-1)}{(2x-1)(x+2)(3x-1)} + \frac{(2x-1)(x+2)}{(2x-1)(x+2)(3x-1)} \\
 &= \frac{(x+1)(3x-1) - (x-1)(2x-1) + (2x-1)(x+2)}{(2x-1)(x+2)(3x-1)} \\
 &= \frac{3x^2 + 8x - 4}{(2x-1)(x+2)(3x-1)}
 \end{aligned}$$

$$41. \left(1 + \frac{1}{x}\right)^2 = \left(\frac{x}{x} + \frac{1}{x}\right)^2 = \left(\frac{x+1}{x}\right)^2 = \frac{x^2 + 2x + 1}{x^2}$$

$$42. \left(\frac{1}{x} + \frac{1}{y}\right)^2 = \left(\frac{y}{xy} + \frac{x}{xy}\right)^2 = \left(\frac{y+x}{xy}\right)^2 = \frac{y^2 + 2xy + x^2}{x^2y^2}$$

$$43. \left(\frac{1}{x} - y\right)^{-1} = \left(\frac{1}{x} - \frac{xy}{x}\right)^{-1} = \left(\frac{1-xy}{x}\right)^{-1} = \frac{x}{1-xy}$$

$$\begin{aligned}
 44. \left(a + \frac{1}{b}\right)^2 &= \left(\frac{ab}{b} + \frac{1}{b}\right)^2 = \left(\frac{ab+1}{b}\right)^2 \\
 &= \frac{a^2b^2 + 2ab + 1}{b^2}
 \end{aligned}$$

$$45. \text{ Multiplying numerator and denominator by } x \text{ gives } \frac{5x+2}{3x}.$$

$$46. \text{ Multiplying numerator and denominator by } x \text{ gives } \frac{x+3}{x^2-9} = \frac{x+3}{(x+3)(x-3)} = \frac{1}{x-3}.$$

$$47. \text{ Multiplying numerator and denominator by } 2x(x+2) \text{ gives } \frac{3(2x)(x+2) - 1(x+2)}{x(2x)(x+2) + x(2x)} = \frac{(x+2)[3(2x)-1]}{2x^2[(x+2)+1]} = \frac{(x+2)(6x-1)}{2x^2(x+3)}.$$

$$\begin{aligned}
 48. \text{ Multiplying numerator and denominator by } 3(x+3)(x+2) \text{ gives} \\
 \frac{3(x-1) - 1(3)(x+3)}{3(3)(x+3)(x+2) + (x-7)(x+3)(x+2)} \\
 = \frac{-12}{(x+3)(x+2)[9+(x-7)]} = -\frac{12}{(x+3)(x+2)^2}.
 \end{aligned}$$

$$\begin{aligned}
 49. \text{ LCD} &= \sqrt[3]{x+h} \cdot \sqrt[3]{x} \\
 \frac{3}{\sqrt[3]{x+h}} - \frac{3}{\sqrt[3]{x}} &= \frac{3\sqrt[3]{x}}{\sqrt[3]{x+h}\sqrt[3]{x}} - \frac{3\sqrt[3]{x+h}}{\sqrt[3]{x+h}\sqrt[3]{x}} \\
 &= \frac{3(\sqrt[3]{x} - \sqrt[3]{x+h})}{\sqrt[3]{x+h}\sqrt[3]{x}}
 \end{aligned}$$

$$50. \text{LCD} = \sqrt{3+x} \cdot \sqrt{x}$$

$$\frac{x\sqrt{x}}{\sqrt{3+x}} + \frac{2}{\sqrt{x}} = \frac{x\sqrt{x} \cdot \sqrt{x}}{\sqrt{3+x}\sqrt{x}} + \frac{2\sqrt{3+x}}{\sqrt{3+x}\sqrt{x}}$$

$$= \frac{x^2 + 2\sqrt{3+x}}{\sqrt{3+x}\sqrt{x}}$$

$$51. \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$52. \frac{1}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{1-2} = \frac{1+\sqrt{2}}{-1} = -1-\sqrt{2}$$

$$53. \frac{\sqrt{2}}{\sqrt{3}-\sqrt{6}} \cdot \frac{\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}}$$

$$= \frac{\sqrt{2}(\sqrt{3}+\sqrt{6})}{3-6} = \frac{\sqrt{6}+\sqrt{12}}{-3} = -\frac{\sqrt{6}+2\sqrt{3}}{3}$$

$$54. \frac{5}{\sqrt{6}+\sqrt{7}} \cdot \frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}-\sqrt{7}} = \frac{5(\sqrt{6}-\sqrt{7})}{6-7}$$

$$= \frac{5(\sqrt{6}-\sqrt{7})}{-1} = 5(\sqrt{7}-\sqrt{6})$$

$$55. \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{5}} \cdot \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}} = \frac{2\sqrt{3}(\sqrt{3}-\sqrt{5})}{3-5}$$

$$= \frac{2 \cdot 3 - 2\sqrt{15}}{-2}$$

$$= -3 + \sqrt{15}$$

$$56. \frac{2\sqrt{5}}{\sqrt{3}-\sqrt{7}} \cdot \frac{\sqrt{3}+\sqrt{7}}{\sqrt{3}+\sqrt{7}}$$

$$= \frac{2\sqrt{5}(\sqrt{3}+\sqrt{7})}{3-7}$$

$$= \frac{2(\sqrt{15}+\sqrt{35})}{-4}$$

$$= -\frac{\sqrt{15}+\sqrt{35}}{2}$$

$$57. \frac{3}{t+\sqrt{7}} \cdot \frac{t-\sqrt{7}}{t-\sqrt{7}} = \frac{3t-3\sqrt{7}}{t^2-7}$$

$$58. \frac{(x-3)+4}{\sqrt{x}-1} = \frac{x+1}{\sqrt{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{(x+1)(\sqrt{x}+1)}{x-1}$$

$$59. \frac{5(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} - \frac{4(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$$

$$= \frac{5(2-\sqrt{3})}{4-3} - \frac{4(1+\sqrt{2})}{1-2}$$

$$= \frac{5(2-\sqrt{3})}{1} - \frac{4(1+\sqrt{2})}{-1}$$

$$= 5(2-\sqrt{3}) + 4(1+\sqrt{2}) = 4\sqrt{2} - 5\sqrt{3} + 14$$

$$60. \frac{5x^2}{4\sqrt{x}+12} = \frac{5x^2}{4\sqrt{x}+12} \cdot \frac{4\sqrt{x}-12}{4\sqrt{x}-12}$$

$$= \frac{20x^2\sqrt{x}-60x^2}{16x-144} = \frac{20x^2(\sqrt{x}-3)}{16(x-9)}$$

$$= \frac{5x^2(\sqrt{x}-3)}{4(x-9)}$$

## Problems 0.7

1.  $9x - x^2 = 0$ ; 1, 0

Set  $x = 1$ :

$$9(1) - (1)^2 \stackrel{?}{=} 0$$

$$9 - 1 \stackrel{?}{=} 0$$

$$8 \neq 0$$

Set  $x = 0$ :

$$9(0) - (0)^2 \stackrel{?}{=} 0$$

$$0 - 0 \stackrel{?}{=} 0$$

$$0 = 0$$

Thus, 0 satisfies the equation, but 1 does not.

2.  $12 - 7x = -x^2$ ; 4, 3

Set  $x = 4$ :

$$12 - 7(4) \stackrel{?}{=} -(4)^2$$

$$12 - 28 \stackrel{?}{=} -16$$

$$-16 = -16$$

Set  $x = 3$ :

$$12 - 7(3) \stackrel{?}{=} -(3)^2$$

$$12 - 21 \stackrel{?}{=} -9$$

$$-9 = -9$$

Thus, 4 and 3 satisfy the equation.