Solution Manual for

Fundamentals of Electromagnetics for Electrical and Computer Engineering

Nannapaneni Narayana Rao

Edward C. Jordan Professor Emeritus of Electrical and Computer Engineering

University of Illinois at Urbana-Champaign, USA

Distinguished Amrita Professor of Engineering

Amrita Vishwa Vidyapeetham (Amrita University), India

Pearson Prentice Hall

Upper Saddle River, New Jersey 07458

CONTENTS

CHAPTER 1

1.1. (a) Total distance $= 1 + \frac{1}{2} + \frac{1}{4} + \cdots = \frac{1}{1-\frac{1}{2}} = 2$ m

(b) Distance north $= 1 - \frac{1}{4} + \frac{1}{16} - \cdots = \frac{1}{1+\frac{1}{4}} = 0.8$ m

Distance east $= \frac{1}{2} - \frac{1}{8} + \frac{1}{32} - \cdots = \frac{1}{2}\left(1 - \frac{1}{4} + \frac{1}{16} - \cdots\right) = 0.4$ m

$\therefore$ Final position is (0.8, 0.4)

(c) Straight line distance $= \sqrt{(0.8)^2 + (0.4)^2} = 0.8944$ m

1.2. $\mathbf{A} + \mathbf{B} + \mathbf{C} = 2\mathbf{a}_1 + 3\mathbf{a}_2 + 2\mathbf{a}_3$ — (1)

$2\mathbf{A} + \mathbf{B} - \mathbf{C} = \mathbf{a}_1 + 3\mathbf{a}_2$ — (2)

$\mathbf{A} - 2\mathbf{B} + 3\mathbf{C} = 4\mathbf{a}_1 + 5\mathbf{a}_2 + \mathbf{a}_3$ — (3)

$(1) + (2) \rightarrow 3\mathbf{A} + 2\mathbf{B} = 3\mathbf{a}_1 + 16\mathbf{a}_2 + 2\mathbf{a}_3$ — (4)

$(2) \times 3 + (3) \rightarrow 7\mathbf{A} + \mathbf{B} = 7\mathbf{a}_1 + 14\mathbf{a}_2 + \mathbf{a}_3$ — (5)

$[(5) \times 2 - (4)] \div 11 \rightarrow \mathbf{A} = \mathbf{a}_1 + 2\mathbf{a}_2$ — (6)

$(5) - (6) \times 7 \rightarrow \quad \mathbf{B} = \mathbf{a}_3$ — (7)

$(1) - (6) - (7) \rightarrow \quad \mathbf{C} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$ — (8)

1.3. $(\mathbf{A} + \mathbf{B}) \bullet (\mathbf{A} - \mathbf{B}) = \mathbf{A} \bullet \mathbf{A} - \mathbf{A} \bullet \mathbf{B} + \mathbf{B} \bullet \mathbf{A} - \mathbf{B} \bullet \mathbf{B} = A^2 - B^2$

$(\mathbf{A} + \mathbf{B}) \times (\mathbf{A} - \mathbf{B}) = \mathbf{A} \times \mathbf{A} - \mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} - \mathbf{B} \times \mathbf{B} = 2\mathbf{B} \times \mathbf{A}$

For $\mathbf{A} = 3\mathbf{a}_1 - 5\mathbf{a}_2 + 4\mathbf{a}_3$ and $\mathbf{B} = \mathbf{a}_1 + \mathbf{a}_2 - 2\mathbf{a}_3$,

$\mathbf{A} + \mathbf{B} = 4\mathbf{a}_1 - 4\mathbf{a}_2 + 2\mathbf{a}_3$, $\mathbf{A} - \mathbf{B} = 2\mathbf{a}_1 - 6\mathbf{a}_2 + 6\mathbf{a}_3$,

$A^2 = 9 + 25 + 16 = 50$, and $B^2 = 1 + 1 + 4 = 6$

$(\mathbf{A} + \mathbf{B}) \bullet (\mathbf{A} - \mathbf{B}) = 8 + 24 + 12 = 44 = A^2 - B^2$

$$(\mathbf{A}+\mathbf{B})\times(\mathbf{A}-\mathbf{B}) = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 4 & -4 & 2 \\ 2 & -6 & 6 \end{vmatrix} = -12\mathbf{a}_x - 20\mathbf{a}_y - 16\mathbf{a}_z$$

$$= 2\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 1 & 1 & -2 \\ 3 & -5 & 4 \end{vmatrix} = 2\mathbf{B}\times\mathbf{A}$$

1.4. $\mathbf{B}\times\mathbf{C} = -4\mathbf{a}_x + 2\mathbf{a}_y + 8\mathbf{a}_z$, $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = 8\mathbf{a}_x + 16\mathbf{a}_y$

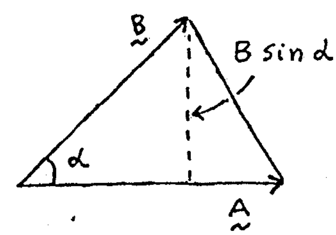
$\mathbf{C}\times\mathbf{A} = -\mathbf{a}_x - 2\mathbf{a}_y + 7\mathbf{a}_z$, $\mathbf{B}\times(\mathbf{C}\times\mathbf{A}) = -12\mathbf{a}_x - 8\mathbf{a}_y - 4\mathbf{a}_z$

$\mathbf{A}\times\mathbf{B} = \mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$, $\mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 4\mathbf{a}_x - 8\mathbf{a}_y + 4\mathbf{a}_z$

$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 0$

In fact, this quantity is zero for any **A**, **B**, and **C**.

1.5. Area $= \frac{1}{2}AB\sin\alpha = \frac{1}{2}|\mathbf{A}\times\mathbf{B}|$



For the points (1, 2, 1), (–3, –4, 5), and (2, –1, –3),

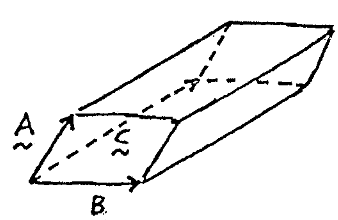
$\mathbf{A} = 4\mathbf{a}_x + 6\mathbf{a}_y - 4\mathbf{a}_z$

$\mathbf{B} = 5\mathbf{a}_x + 3\mathbf{a}_y - 8\mathbf{a}_z$

$\mathbf{A}\times\mathbf{B} = -36\mathbf{a}_x + 12\mathbf{a}_y - 18\mathbf{a}_z$

$\therefore$ Area $= \frac{1}{2}\sqrt{(-36)^2 + (12)^2 + (-18)^2} = 21$ units.

1.6. Area of the base $= |\mathbf{B}\times\mathbf{C}|$



Height of parallelepiped = Projection of **A** onto the normal to the base

$= \mathbf{A}\cdot\dfrac{\mathbf{B}\times\mathbf{C}}{|\mathbf{B}\times\mathbf{C}|}$

$\therefore$ Volume of parallelepiped = Area of base × height = $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C}$

For $\mathbf{A} = 4\mathbf{a}_x$, $\mathbf{B} = 2\mathbf{a}_x + \mathbf{a}_y + 3\mathbf{a}_z$, and $\mathbf{C} = 2\mathbf{a}_y + 6\mathbf{a}_z$, $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = 0$.

Hence, volume of the parallelepiped is zero. The three vectors lie in a plane.

1.7. The vector **A** must be perpendicular to both $(-\mathbf{a}_y + 2\mathbf{a}_z)$ and $(\mathbf{a}_x - 2\mathbf{a}_z)$.

Hence $\mathbf{A} = C(-\mathbf{a}_y + 2\mathbf{a}_z) \times (\mathbf{a}_x - 2\mathbf{a}_z) = C(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ where C is a constant. To find C, we note that $\mathbf{a}_x \times \mathbf{A} = \mathbf{a}_x \times C(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = 2\mathbf{a}_z - \mathbf{a}_y$

$\therefore\ C = 1$ and $\mathbf{A} = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$.

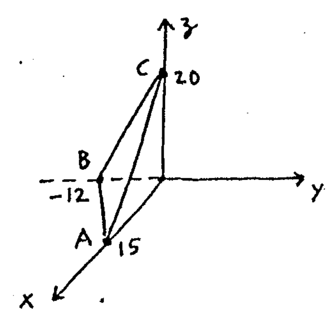
Verification: $\mathbf{a}_y \times \mathbf{A} = \mathbf{a}_y \times (2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = \mathbf{a}_x - 2\mathbf{a}_z$.

1.8. Vector from $A(5, 0, 3)$ to $B(3, 3, 2) = -2\mathbf{a}_x + 3\mathbf{a}_y - \mathbf{a}_z$

Vector from $C(6, 2, 4)$ to $D(3, 3, 6) = -3\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$

Component of **AB** along **CD** = $\mathbf{AB} \bullet \dfrac{\mathbf{CD}}{CD} = \dfrac{6+3-2}{\sqrt{9+1+4}} = 1.8708$

1.9. Writing the equation for the plane as $\dfrac{x}{15} - \dfrac{y}{12} + \dfrac{z}{20} = 1$, we find the intercepts on the x, y, and z-axes to be at 15, –12, and 20, respectively. Thus



$\mathbf{R}_{AB} = -15\mathbf{a}_x - 12\mathbf{a}_y$

$\mathbf{R}_{AC} = -15\mathbf{a}_x + 20\mathbf{a}_z$

$\mathbf{R}_{AC} \times \mathbf{R}_{AB} = 240\mathbf{a}_x - 300\mathbf{a}_y + 180\mathbf{a}_z$

$$\mathbf{a}_n = \frac{\mathbf{R}_{AC} \times \mathbf{R}_{AB}}{|\mathbf{R}_{AC} \times \mathbf{R}_{AB}|} = \frac{4\mathbf{a}_x - 5\mathbf{a}_y + 3\mathbf{a}_z}{5\sqrt{2}}$$

Distance from origin to the plane = $15\mathbf{a}_x \bullet \mathbf{a}_n = 6\sqrt{2}$.

1.10. For $y = 2x$, $z = 4y$, we have $dy = 2\,dx$, $dz = 4\,dy = 8\,dx$.

$\therefore\ d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z = dx\,\mathbf{a}_x + 2\,dx\,\mathbf{a}_y + 8\,dx\,\mathbf{a}_z$

$= (\mathbf{a}_x + 2\mathbf{a}_y + 8\mathbf{a}_z)\,dx$, independent of the point.

1.11. For $x = y = z^2$, we have $dx = dy = 2z\,dz$.

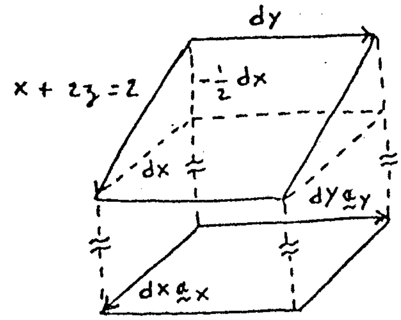
At the point (4, 4, 2), $dx = dy = 4\,dz$

$\therefore\ d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z = 4\,dz\,\mathbf{a}_x + 4\,dz\,\mathbf{a}_y + dz\,\mathbf{a}_z$

$= (4\mathbf{a}_x + 4\mathbf{a}_y + \mathbf{a}_z)\,dz$

1.12. Differential length vector having projection $dy\,\mathbf{a}_y = dy\,\mathbf{a}_y$

Differential length vector having projection $dx\,\mathbf{a}_x$ is



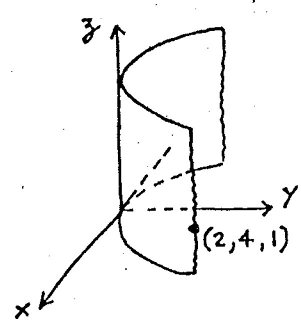
$$dx\,\mathbf{a}_x + dz\,\mathbf{a}_z = dx\,\mathbf{a}_x - \frac{1}{2}\,dx\,\mathbf{a}_z$$

$$= \left(\mathbf{a}_x - \frac{1}{2}\mathbf{a}_z\right) dx,$$

since for $x + 2z = 2$, $dz = -\frac{1}{2}\,dx$, independent of the point.

$$\therefore\ d\mathbf{S} = \left(\mathbf{a}_x - \frac{1}{2}\mathbf{a}_z\right) dx \times dy\,\mathbf{a}_y = \left(\frac{1}{2}\mathbf{a}_x + \mathbf{a}_z\right) dx\,dy\,.$$

1.13. One vector tangential to the surface is $dz\,\mathbf{a}_z$. Another tangential vector is given by



$$d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y$$
$$= dx\,\mathbf{a}_x + 2x\,dx\,\mathbf{a}_y$$
$$= (\mathbf{a}_x + 4\mathbf{a}_y)\,dx$$

$\therefore$ Vector normal to the plane $= (\mathbf{a}_x + 4\mathbf{a}_y)\,dx \times dz\,\mathbf{a}_z$

$$= (4\mathbf{a}_x - \mathbf{a}_y)\,dx\,dz$$

Unit vector normal to the plane $= \dfrac{4\mathbf{a}_x - \mathbf{a}_y}{\sqrt{17}}$.

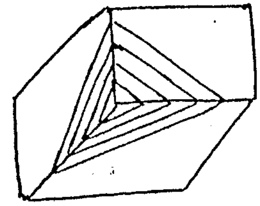
1.14. Denoting $h(x, y)$ to be the height field, we have

$x^2 + y^2 + h^2 = 4,\ x^2 + y^2 \le 4$

or, $h = \sqrt{4 - x^2 - y^2}\ ,\ x^2 + y^2 \le 4.$

1.15. The number field is $x + y + z$.

$\therefore$ Constant magnitude surfaces are the planes $x + y + z =$ constant.

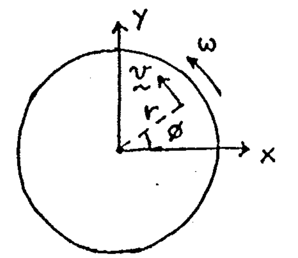


1.16. $\mathbf{d}(x, y, z) = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

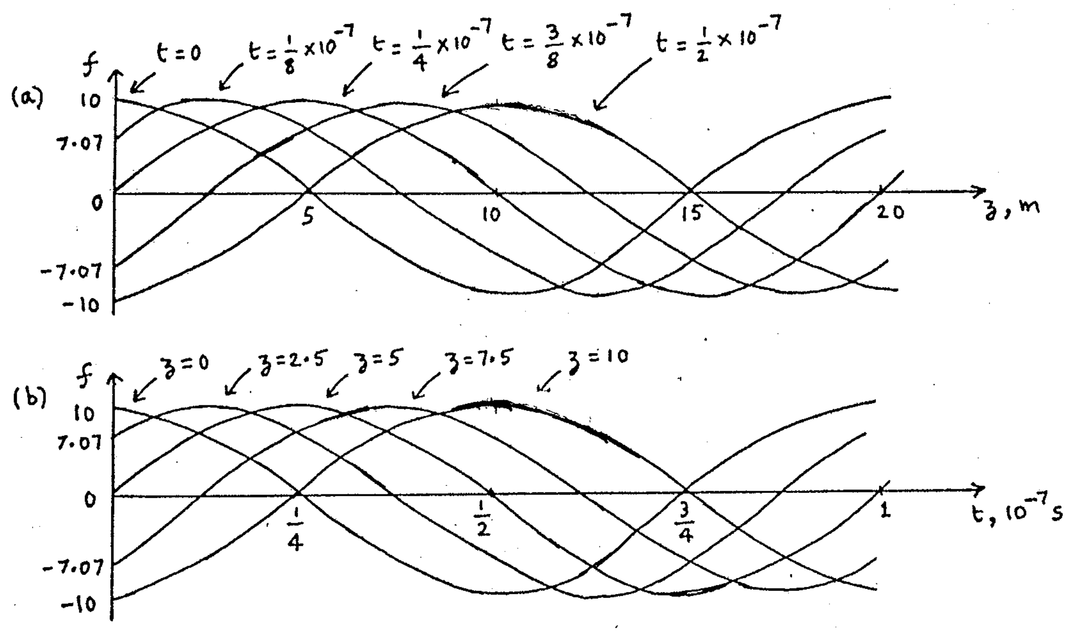
Constant magnitude surfaces are $x^2 + y^2 + z^2$ = constant, and hence are spherical surfaces centered at the corner. Direction lines are radial lines emanating from the corner.

1.17. $\mathbf{v} = -r\omega \sin\phi\, \mathbf{a}_x + r\omega \cos\phi\, \mathbf{a}_y$

$= \omega(-y\mathbf{a}_x + x\mathbf{a}_y)$

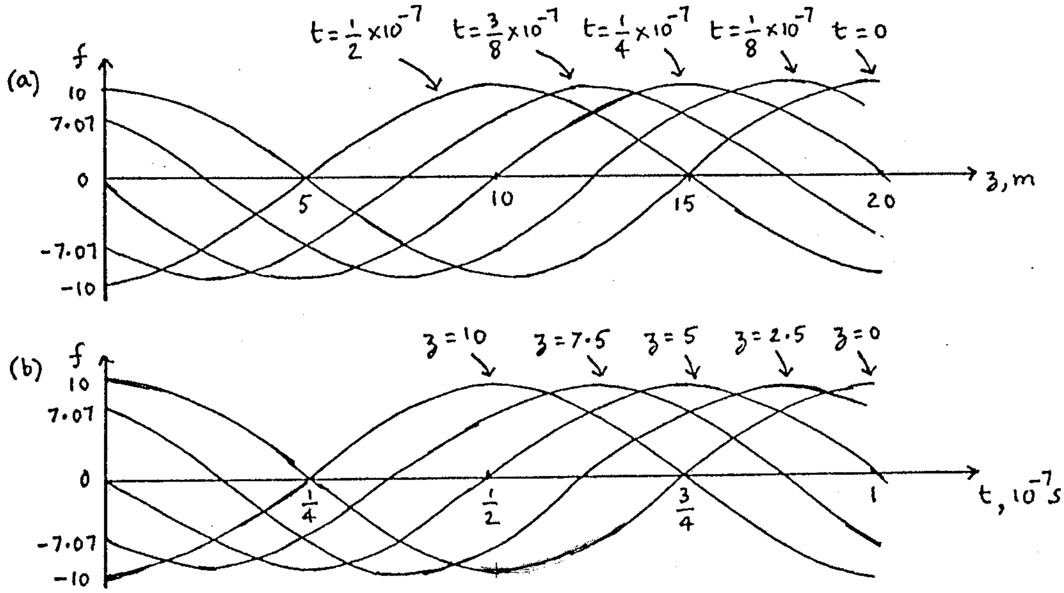


1.18. $f(z, t) = 10\cos(2\pi \times 10^7 t - 0.1\,\pi z)$



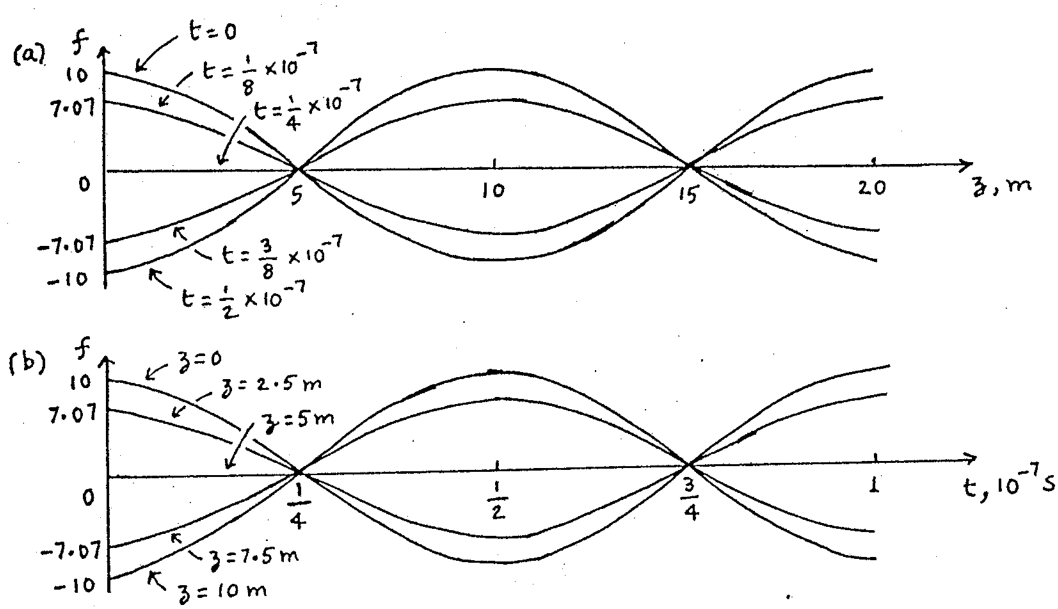
$f(z, t)$ represents a traveling wave progressing with time in the positive z-direction.

1.19. $f(z, t) = 10\cos\,(2\pi \times 10^7 t\ + 0.1\,\pi z)$



$f(z, t)$ represents a traveling wave progressing with time in the negative z-direction.

1.20. $f(z, t) = 10\cos 2\pi \times 10^7 t\ \cos 0.1\,\pi z$



$f(z, t)$ represents a standing wave.

1.21. (a) The two components are in phase; hence, linear polarization.

(b) The two components are perpendicular in direction, differ in phase by 90° and equal in amplitude; hence, circular polarization.

(c) The two components are perpendicular in direction, differ in phase by 90° but unequal in amplitude; hence elliptical polarization.

1.22. $\mathbf{F}_1$ and $\mathbf{F}_2$ differ in phase by 90°.

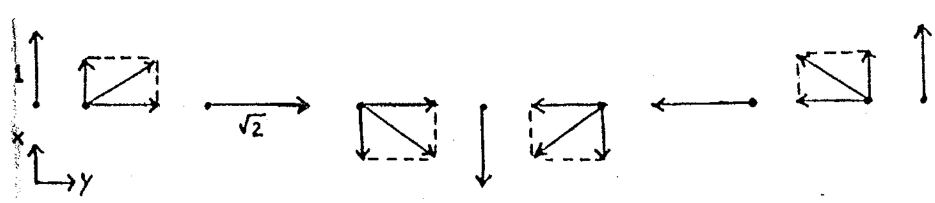
$$|\mathbf{F}_1| = \sqrt{3+1}\ \cos\omega t = 2\cos\omega t;\ |\mathbf{F}_2| = \sqrt{\frac{1}{4}+\frac{3}{4}+3}\ \sin\omega t = 2\sin\omega t\,.$$

$\therefore$ $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal in amplitude.

$$\mathbf{F}_1\cdot\mathbf{F}_2 = -\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2} = 0\,.\ \ \therefore\ \mathbf{F}_1 \text{ is perpendicular to } \mathbf{F}_2.$$

Thus $\mathbf{F}_1 + \mathbf{F}_2$ is circularly polarized.

1.23.

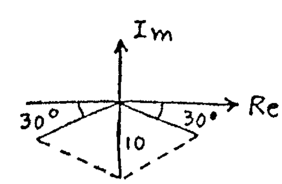


The polarization is elliptical with major axis in the y-direction, minor axis in the x-direction, and eccentricity equal to $\sqrt{2}$.

1.24. $10\cos(\omega t - 30°) + 10\cos(\omega t + 210°)$

$$\begin{array}{ccccc} \downarrow & & \downarrow & & \\ 10\,e^{-j30°} & + & 10\,e^{j210°} & = & 10\,e^{-j90°} \end{array}$$

$\therefore$ The sum is $10\cos(\omega t - 90°) = 10\sin\omega t$.

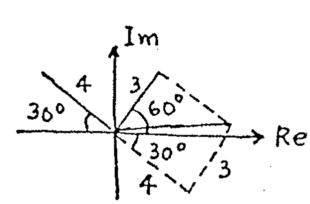


1.25. $3\cos(\omega t + 60°) - 4\cos(\omega t + 150°)$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 3\,e^{j60°} & - & 4\,e^{j150°} \end{array}$$

$= 5\,e^{j(60°-53.13°)} = 5e^{j6.87°}$

$\rightarrow 5\cos(\omega t + 6.87°)$.



1.26. Replacing $\dfrac{di}{dt}$ by $j10^6\overline{I}$, i by $\overline{I}$, and 13 cos $10^6 t$ by $13e^{j0°}$, we have

$$5\times 10^{-6}\times j10^6\overline{I}+12\overline{I}\ =13e^{j0°}$$

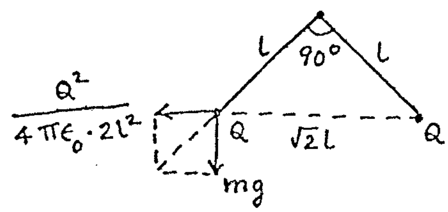
or, $(12+j5)\overline{I}=13,\ \overline{I}=\dfrac{13}{12+j5}=\dfrac{13}{13e^{j22.62°}}=1e^{-j22.62°}$

Thus $i = 1\cos(10^6 t - 22.62°) = 1\cos(10^6 t - 0.126\pi)$

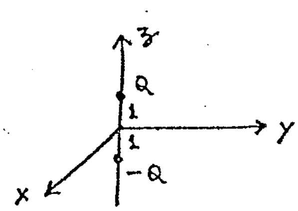
1.27. From the construction shown,

$$\frac{Q^2}{4\pi\varepsilon_0\cdot 2l^2}\bigg/ mg=\tan 45°=1$$

or, $Q=\sqrt{8\pi\,\varepsilon_0 l^2 mg}$



1.28. (a) At the point (0, 0, 100),



$$\mathbf{E}=\frac{Q}{4\pi\varepsilon_0(99)^2}\mathbf{a}_z+\frac{-Q}{4\pi\varepsilon_0(101)^2}\mathbf{a}_z$$

$$=\frac{Q}{4\pi\varepsilon_0}\frac{101^2-99^2}{99^2\times 101^2}\mathbf{a}_z$$

$$=\frac{Q}{4\pi\varepsilon_0}\frac{(100+1)^2-(100-1)^2}{(100-1)^2\times(100+1)^2}\mathbf{a}_z=\frac{Q}{4\pi\varepsilon_0}\frac{400}{(100^2-1)^2}\mathbf{a}_z$$

$$\approx\frac{Q}{4\pi\varepsilon_0}\frac{400}{100^4}\mathbf{a}_z=\frac{Q}{100^3\pi\varepsilon_0}\mathbf{a}_z$$

(b) At the point (100, 0, 0)

$$\mathbf{E}=-\frac{2Q}{4\pi\varepsilon_0(100^2+1^2)^{3/2}}\mathbf{a}_z$$

$$\approx-\frac{Q}{2\pi\varepsilon_0(100^3)}\mathbf{a}_z$$