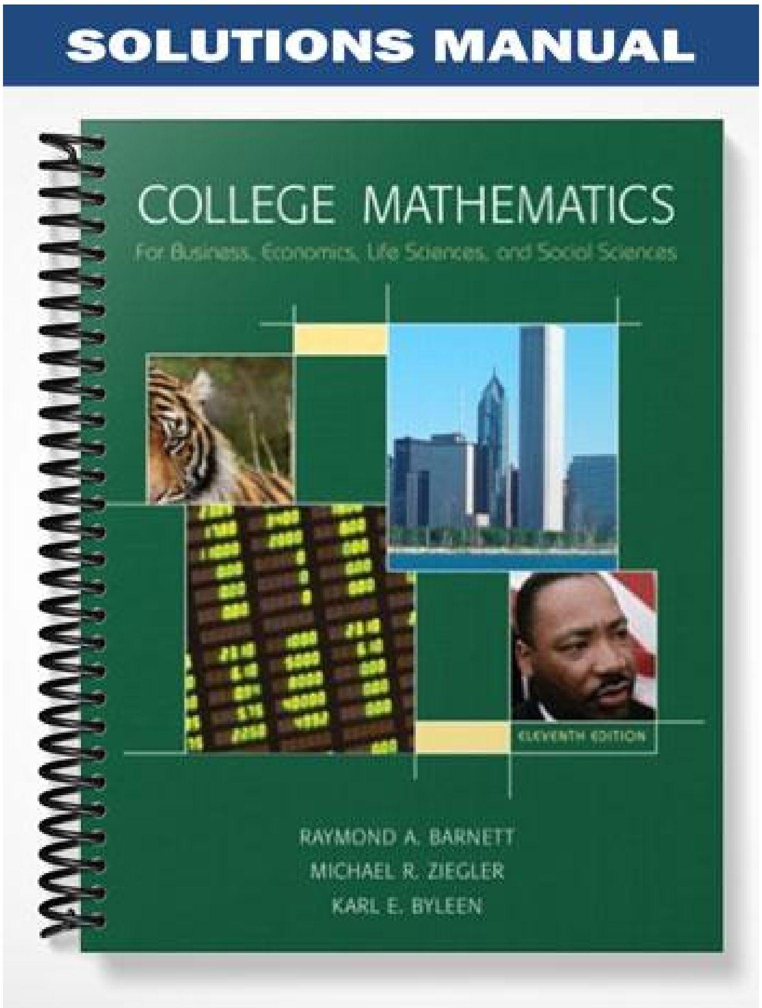
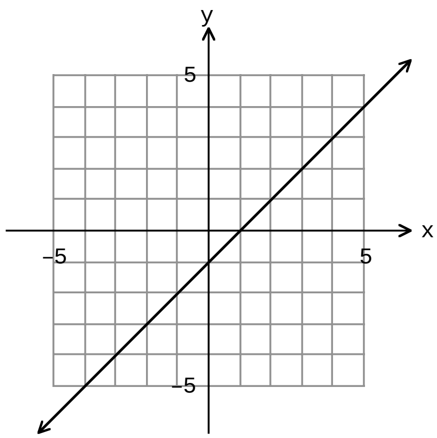

SOLUTIONS MANUAL
COLLEGE MATHEMATICS
For Business, Economics, Life Sciences, and Social Sciences
ELEVENTH EDITION
RAYMOND A. BARNETT
MICHAEL R. ZIEGLER
KARL E. BYLEEN

2 FUNCTIONS AND GRAPHS

EXERCISE 2-1

2.



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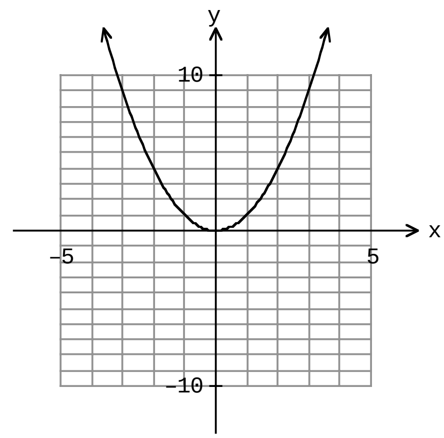

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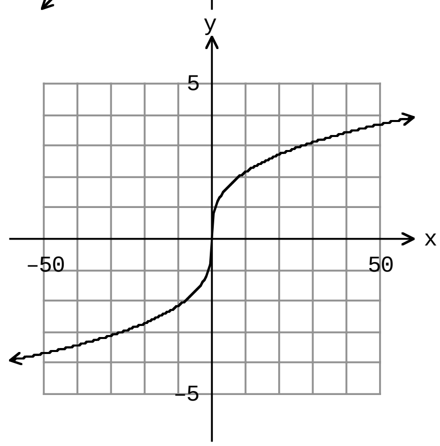

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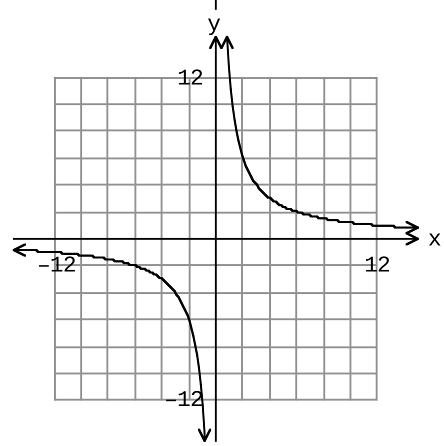


10. The table specifies a function, since for each domain value there corresponds one and only one range value.

12. The table does not specify a function, since more than one range value corresponds to a given domain value.
(Range values 1, 2 correspond to domain value 9.)

14. This is a function.

16. The graph specifies a function; each vertical line in the plane intersects the graph in at most one point.

18. The graph does not specify a function. There are vertical lines which intersect the graph in more than one point. For example, the y-axis intersects the graph in two points.

20. The graph does not specify a function.

22. $y = \pi$ is a constant function.

24. $y = 5x - \frac{1}{2}(4 - x)$

$= 5x - 2 + \frac{1}{2}x$

$= 5.5x - 2$ which is a linear function.

26. $y = 3x + \frac{1}{2}(5 - 6x)$

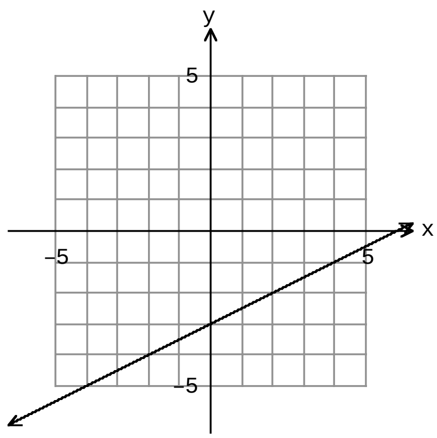
$= 3x + \frac{5}{2} - 3x$

$= \frac{5}{2}$ which is a constant function.

28. $y = \frac{1}{2x + 3}$. It is neither constant nor linear.

30. $y = x^2 - 9$. It is neither constant nor linear.

34.



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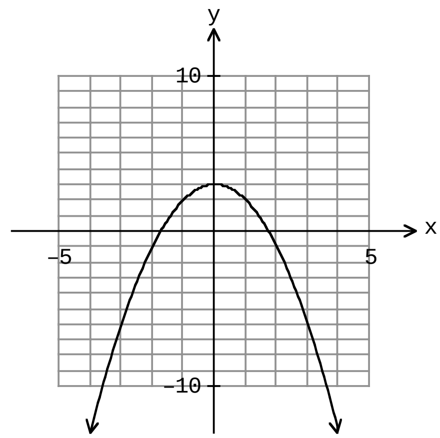

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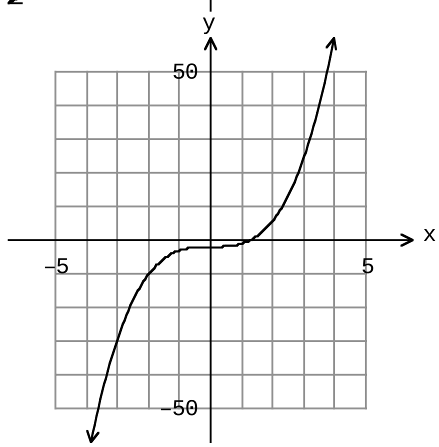

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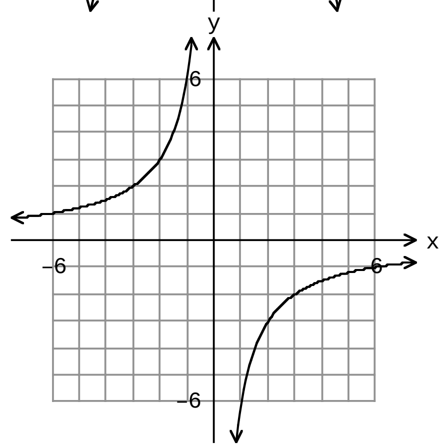
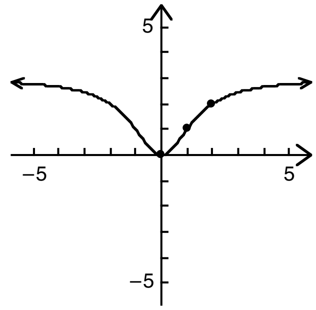


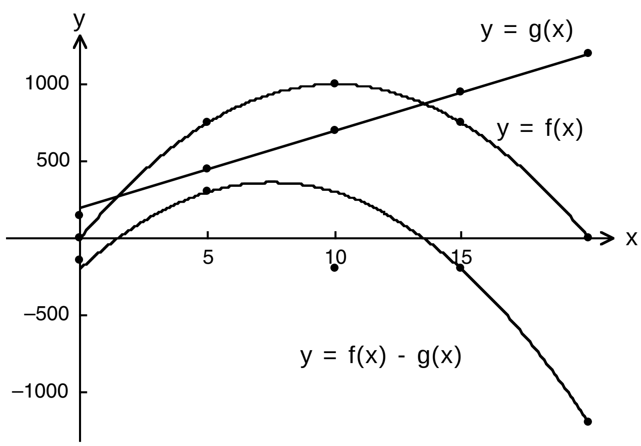
42. $f(x) = \frac{3x^2}{x^2 + 2}$. Since the denominator is bigger than 1, we note that the values of f are between 0 and 3. Furthermore, the function f has the property that $f(-x) = f(x)$. So, adding points $x = 3$, $x = 4$, $x = 5$, we have:

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	2.78	2.67	2.45	2	1	0	1	2	2.45	2.67	2.78

The sketch is: 

44. (A)

x	0	5	10	15	20
$f(x)$	0	750	1,000	750	0
$g(x)$	200	450	700	950	1,200
$f(x) - g(x)$	-200	300	300	-200	-1,200



46. $y = f(4) = 0$

48. $y = f(-2) = 3$

50. $f(x) = 3$, $x < 0$ at $x = -4, -2$

52. $f(x) = 4$ at $x = 5$

54. $f(x) = 2x - 3$
$f(1) = 2(1) - 3 = -1$

56. $g(x) = x^2 + 2x$
$g(1) = (1)^2 + 2(1) = 3$

58. $g(-2) = (-2)^2 + 2(-2) = 0$

60. $f(3) - g(3) = [2(3) - 3] - [(3)^2 + 2(3)] = -12$

62. $g(0) \cdot f(-2) = [(0)^2 - 2(0)] \cdot [2(-2) - 3] = 0$

64. $\frac{g(-3)}{f(2)} = \frac{(-3)^2 + 2(-3)}{2(2) - 3} = \frac{3}{1} = 3$

66. domain: all real numbers or $(-\infty, \infty)$

68. domain: all real numbers except 2

72. f is not defined at the values of x where $x^2 - 9 = 0$, that is, at 3 and -3; f is defined at $x = -2$, $f(-2) = \frac{0}{-5} = 0$.

74. $f(x) = -3x + 4$

76. $F(x) = -8x^3 + 3\sqrt{3}$

78. Function g multiplies the domain element by -2 and adds 7 to the result.

80. Function G multiplies the square root of the domain element by 4 and subtracts the square of the domain element from the result.

82. Given $3y - 7x = 15$. Solving for y, we have:

$$3y = 7x + 15$$
$$y = \frac{7}{3}x + 5$$

Since each input value x determines a unique output value y, the equation specifies a function. The domain is R, the set of real numbers.

84. Given $x - y^2 = 1$. Solving for y, we have:

$$y^2 = x - 1$$
$$y = \pm\sqrt{x-1}$$

This equation does not specify a function, since each value of x, $x > 1$, determines two values of y. For example, corresponding to $x = 5$, we have $y = 2$ and $y = -2$; corresponding to $x = 10$, we have $y = 3$ and $y = -3$.

86. Given $x^2 + y = 10$. Solving for y, we have:

$$y = 10 - x^2$$

This equation specifies a function. The domain is R.

88. Given $xy + y - x = 5$. Solving for y, we have:

$$(x + 1)y = x + 5 \quad \text{or} \quad y = \frac{x + 5}{x + 1}$$

This equation specifies a function. The domain is all real numbers except $x = -1$.

90. Given $x^2 - y^2 = 16$. Solving for y, we have:

$$y^2 = x^2 - 16 \quad \text{or} \quad y = \pm\sqrt{x^2 - 16}$$

Thus, the equation does not specify a function since, for $x = 5$, we have $y = \pm 3$, when $x = 6$, $y = \pm 2\sqrt{5}$, and so on.

92. Given $G(r) = 3 - 5r$. Then:

$$\frac{G(2 + h) - G(2)}{h} = \frac{3 - 5(2 + h) - (3 - 5 \cdot 2)}{h}$$

$$= \frac{-7 - 5h + 7}{h} = \frac{-5h}{h} = -5$$

94. Given $P(x) = 2x^2 - 3x - 7$. Then:

$$\frac{P(3 + h) - P(3)}{h} = \frac{2(3 + h)^2 - 3(3 + h) - 7 - (2\cdot 3^2 - 3\cdot 3 - 7)}{h}$$

$$= \frac{2(9 + 6h + h^2) - 9 - 3h - 7 - (2)}{h}$$

$$= \frac{2h^2 + 9h}{h} = 2h + 9$$

96. $f(x) = x^2 - 1$
$f(-3) = (-3)^2 - 1 = 9 - 1 = 8$

98. $f(x) = x^2 - 1$
$f(3 - 6) = f(-3) = (-3)^2 - 1 = 9 - 1 = 8$

100. $f(x) = x^2 - 1$
$f(3) - f(6) = [(3)^2 - 1] - [(6)^2 - 1] = 9 - 1 - 36 + 1 = -27$

102. $f(x) = x^2 - 1$
$f(f(-2)) = f((-2)^2 - 1) = f(4 - 1) = f(3) = (3)^2 - 1 = 9 - 1 = 8$

104. $f(x) = x^2 - 1$
$f(-3x) = (-3x)^2 - 1 = 9x^2 - 1$

106. $f(x) = x^2 - 1$
$f(1 - x) = (1 - x)^2 - 1 = 1 - 2x + x^2 - 1 = -2x + x^2 = x(x - 2)$

108. (A) $f(x) = -3x + 9$ (B) $f(x + h) = -3x - 3h + 9$
(C) $f(x + h) - f(x) = -3h$ (D) $\frac{f(x + h) - f(x)}{h} = -3$

110. (A) $f(x) = 3x^2 + 5x - 8$ (B) $f(x + h) = 3x^2 + 6xh + 3h^2 + 5x + 5h - 8$
(C) $f(x + h) - f(x) = 6xh + 3h^2 + 5h$
(D) $\frac{f(x + h) - f(x)}{h} = 6x + 3h + 5$

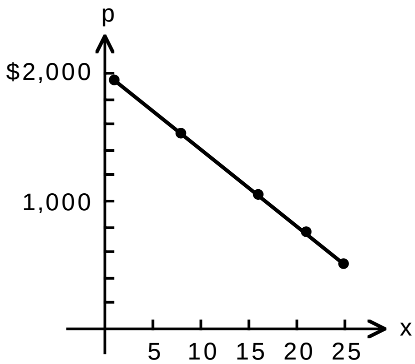
112. (A) $f(x) = x(x + 40) = x^2 + 40x$
(B) $f(x + h) = x^2 + 2xh + h^2 + 40x + 40h$
(C) $f(x + h) - f(x) = 2xh + h^2 + 40h$ (D) $\frac{f(x + h) - f(x)}{h} = 2x + h + 40$

114. Given $A = \bullet w = 81$.
Thus, $w = \frac{81}{l}$. Now $P = 2\bullet + 2w = 2\bullet + 2\left(\frac{81}{l}\right) = 2\bullet + \frac{162}{l}$.
The domain is $\bullet > 0$.

116. Given $P = 2\bullet + 2w = 160$ or $\bullet + w = 80$ and $\bullet = 80 - w$.
Now $A = \bullet w = (80 - w)w$ and $A = 80w - w^2$.

The domain is $0 \le w \le 80$. [Note: $w \le 80$ since $w > 80$ implies ● < 0.]

118.



$p(11) = 1,340$ dollars per computer
$p(18) = 920$ dollars per computer

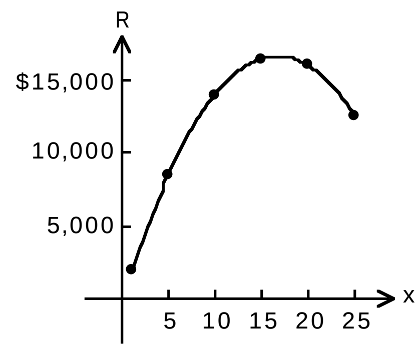
120. (A) $R(x) = xp(x)$
$= x(2,000 - 60x)$ thousands of dollars

Domain: $1 \le x \le 25$

(B) Table 11 Revenue

x(thousands)	$R(x)$(thousands)
1	$1,940
5	8,500
10	14,000
15	16,500
20	16,000
25	12,500

(C)



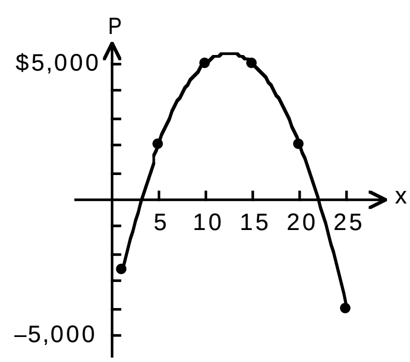
122. (A) $P(x) = R(x) - C(x)$
$= x(2,000 - 60x) - (4,000 + 500x)$ thousand dollars
$= 1,500x - 60x^2 - 4,000$

Domain: $1 \le x \le 25$

(B) Table 13 Profit

x(thousands)	$P(x)$(thousands)
1	-$2,560
5	2,000
10	5,000
15	5,000
20	2,000
25	-4,000

(C)

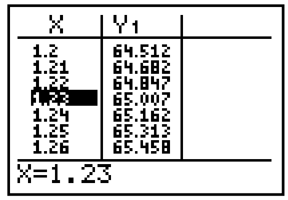


124. (A) 1.2 inches

(B) Evaluate the volume function for $x = 1.21$, 1.22, …, and choose the value of x whose volume is

(C) $x = 1.23$ to two decimal places

closest to 65.



X	Y1
1.2	64.512
1.21	64.682
1.22	64.847
1.23	65.007
1.24	65.162
1.25	65.313
1.26	65.458

X=1.23

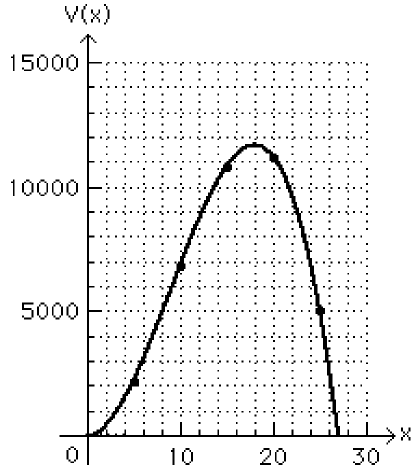
126. (A) $V(x) = x^2(108 - 4x)$

(B) $0 \le x \le 27$

(C) Table 15 Volume

x	$V(x)$
5	2,200
10	6,800
15	10,800
20	11,200
25	5,000

(D)



128. (A) Given $5v - 2s = 1.4$. Solving for v, we have:
$v = 0.4s + 0.28$.
If $s = 0.51$, then $v = 0.4(0.51) + 0.28 = 0.484$ or 48.4%.
(B) Solving the equation for s, we have:
$s = 2.5v - 0.7$.
If $v = 0.51$, then $s = 2.5(0.51) - 0.7 = 0.575$ or 57.5%.

EXERCISE 2-2

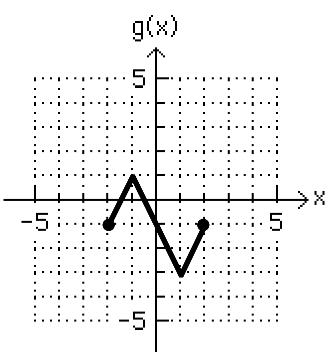
2. $g(x) = -0.3x$
Domain: all real numbers; range: all real numbers

4. $k(x) = 4\sqrt{x}$
Domain: $[0, \infty)$; range: $[0, \infty)$

6. $n(x) = -0.1x^2$
Domain: all real numbers; range: $(-\infty, 0]$

8. $S(x) = 5\sqrt[3]{x}$
Domain: all real numbers; range: all real numbers

10.



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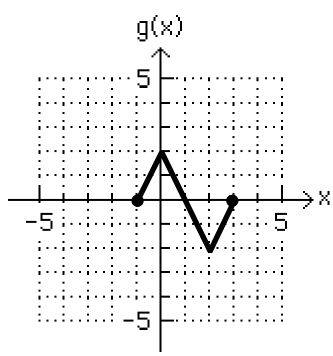

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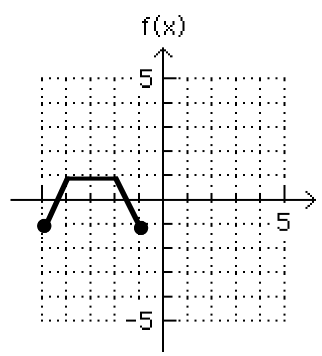

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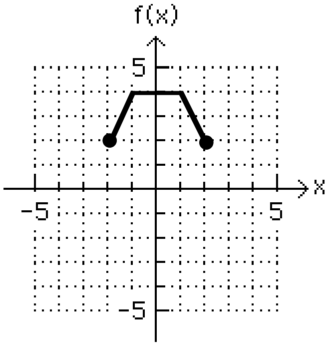

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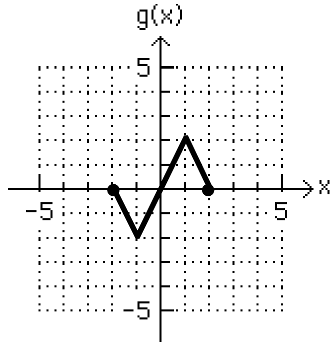

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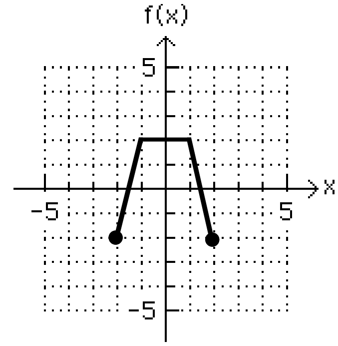
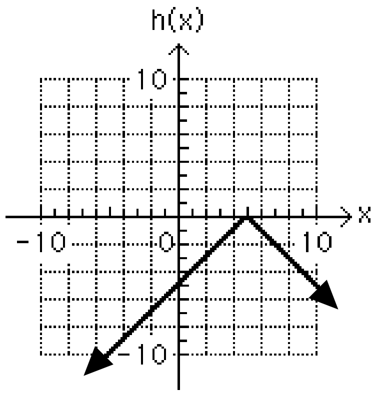
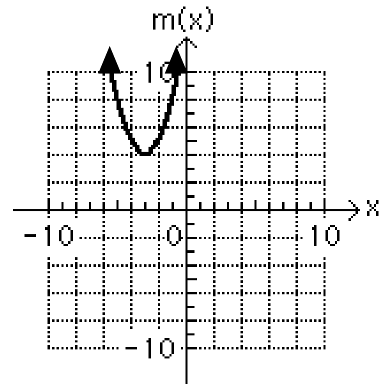


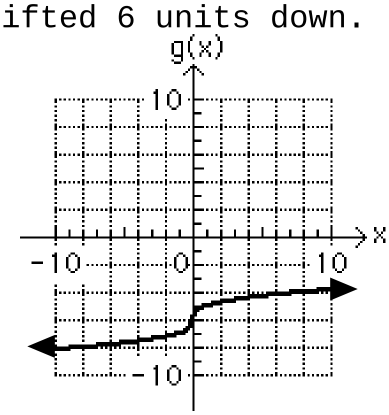
22. The graph of $h(x) = -|x - 5|$ is the graph of $y = |x|$ reflected in the x axis and shifted 5 units to the right.



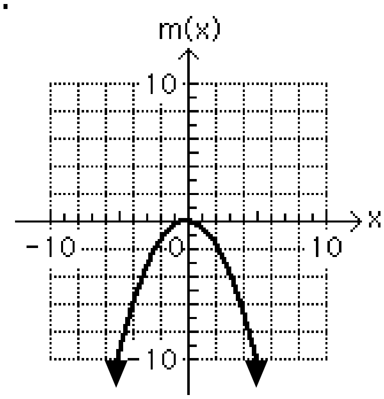
24. The graph of $m(x) = (x + 3)^2 + 4$ is the graph of $y = x^2$ shifted 3 units to the left and 4 units up.



26. The graph of $g(x) = -6 + \sqrt[3]{x}$ is the graph of $y = \sqrt[3]{x}$ shifted 6 units down.



28. The graph of $m(x) = -0.4x^2$ is the same as the graph of $y = x^2$ reflected in the x axis and vertically contracted by a factor of 0.4.



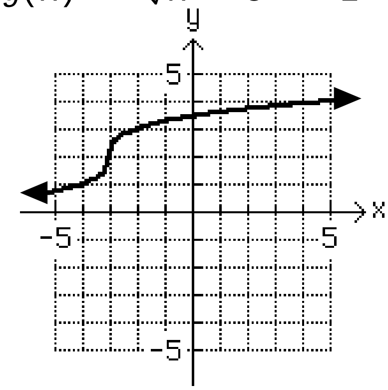
30. The graph of the basic function $y = |x|$ is shifted 3 units to the right and 2 units up. $y = |x - 3| + 2$

32. The graph of the basic function $y = |x|$ is reflected in the x axis, shifted 2 units to the left and 3 units up. Equation: $y = 3 - |x + 2|$

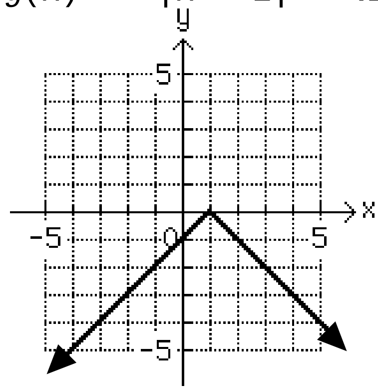
34. The graph of the basic function $\sqrt[3]{x}$ is reflected in the x axis and shifted up 2 units. Equation: $y = 2 - \sqrt[3]{x}$

36. The graph of the basic function $y = x^3$ is reflected in the x axis, shifted to the right 3 units and up 1 unit. Equation: $y = 1 - (x - 3)^3$

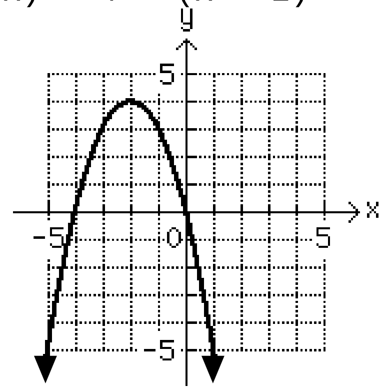
38. $g(x) = \sqrt[3]{x + 3} + 2$



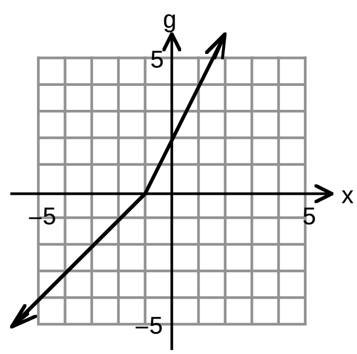
40. $g(x) = -|x - 1|$



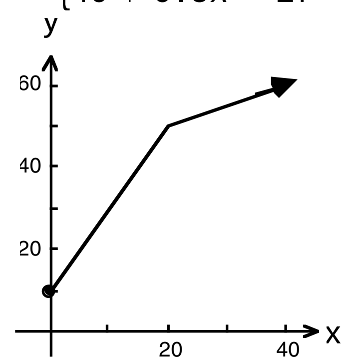
42. $g(x) = 4 - (x + 2)^2$



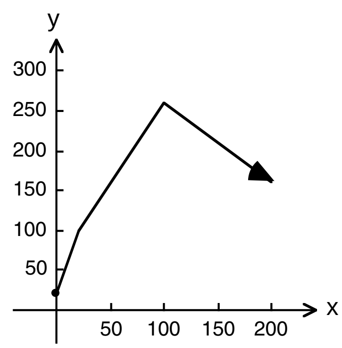
44. $g(x) = \begin{cases} x + 1 & \text{if } x < -1 \\ 2 + 2x & \text{if } x \geq -1 \end{cases}$



46. $h(x) = \begin{cases} 10 + 2x & \text{if } 0 \leq x \leq 20 \\ 40 + 0.5x & \text{if } x > 20 \end{cases}$



48. $h(x) = \begin{cases} 4x + 20 & \text{if } 0 \leq x \leq 20 \\ 2x + 60 & \text{if } 20 < x \leq 100 \\ -x + 360 & \text{if } x > 100 \end{cases}$



50. The graph of the basic function $y = x$ is reflected in the x axis and vertically expanded by a factor of 2. Equation: $y = -2x$

52. The graph of the basic function $y = |x|$ is vertically expanded by a factor of 4. Equation: $y = 4|x|$

54. The graph of the basic function $y = x^3$ is vertically contracted by a factor of 0.25. Equation: $y = 0.25x^3$.

56. Vertical shift, reflection in y axis.
Reversing the order does not change the result. Consider a point

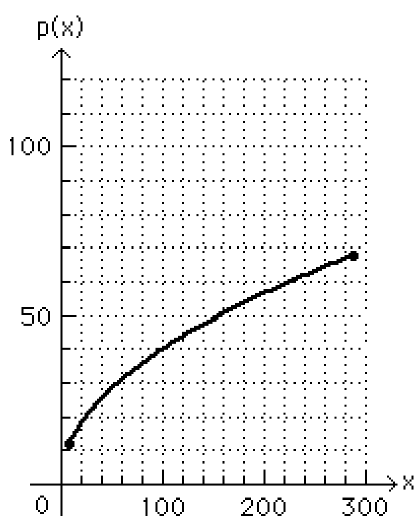
(a, b) in the plane. A vertical shift of k units followed by a reflection in y axis moves (a, b) to $(a, b + k)$ and then to $(-a, b + k)$. In the reverse order, a reflection in y axis followed by a vertical shift of k units moves (a, b) to $(-a, b)$ and then to $(-a, b + k)$. The results are the same.

58. Vertical shift, vertical expansion.
Reversing the order can change the result. For example, let (a, b) be a point in the plane. A vertical shift of k units followed by a vertical expansion of h $(h > 1)$ moves (a, b) to $(a, b + k)$ and then to $(a, bh + kh)$. In the reverse order, a vertical expansion of h followed by a vertical shift of k units moves (a, b) to (a, bh) and then to $(a, bh + k)$; $(a, bh + kh) \neq (a, bh + k)$.

60. Horizontal shift, vertical contraction.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A horizontal shift of k units followed by a vertical contraction of h $(0 < h < 1)$ moves (a, b) to $(a + k, b)$ and then to $(a + k, bh)$. In the reverse order, a vertical contraction of h followed by a horizontal shift of k units moves (a, b) to (a, bh) and then to $(a + k, bh)$. The results are the same.

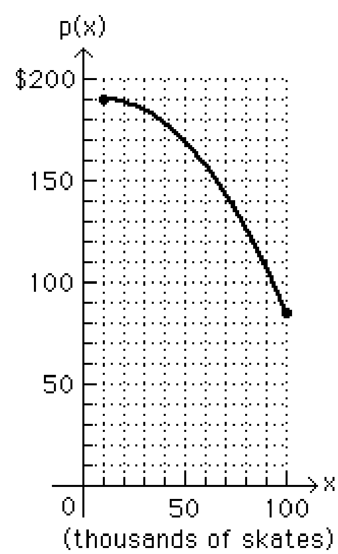
62. (A) The graph of the basic function $y = \sqrt{x}$ is vertically expanded by a factor of 4.

(B)



64. (A) The graph of the basic function $y = x^2$ is reflected in the x axis, vertically contracted by a factor of 0.013, and shifted 10 units to the right and 190 units up.

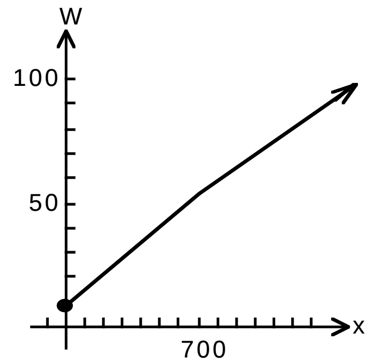
(B)



66. (A) Let x = number of kwh used in a winter month. For $0 \le x \le 700$, the charge is $8.5 + .065x$. At $x = 700$, the charge is \$54. For $x > 700$, the charge is $54 + .053(x - 700) = 16.9 + 0.053x$. Thus,

$$W(x) = \begin{cases} 8.5 + .065x & \text{if} \quad 0 \le x \le 700 \\ 16.9 + 0.053x & \text{if} \quad x > 700 \end{cases}$$

(B)

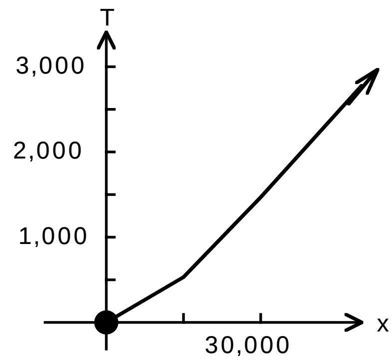


68. (A) Let x = taxable income. If $0 \le x \le 15{,}000$, the tax due is $\$.035x$. At $x = 15{,}000$, the tax due is \$525. For $15{,}000 < x \le 30{,}00$, the tax due is $525 + .0625(x - 15{,}000) = .0625x - 412.5$. For $x > 30{,}000$, the tax due is $1{,}462.5 + .0645(x - 30{,}000) = .0645x - 472.5$.

Thus,

$$T(x) = \begin{cases} 0.035x & \text{if} \quad 0 \le x \le 15{,}000 \\ 0.0625x - 412.5 & \text{if} \quad 15{,}000 < x \le 30{,}000 \\ 0.0645x - 472.5 & \text{if} \quad x > 30{,}000 \end{cases}$$

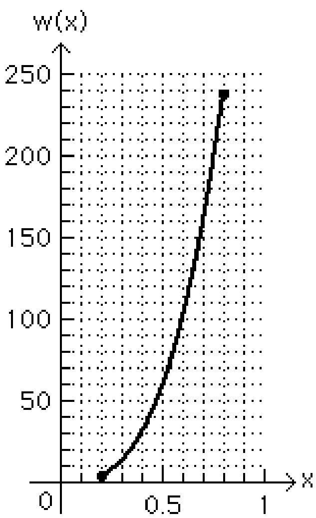
(B)



(C) $T(20{,}000) = \$837.50$
$T(35{,}000) = \$1{,}785$

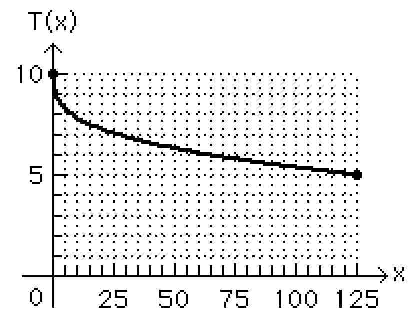
70. (A) The graph of the basic function $y = x^3$ is vertically expanded by a factor of 463.

(B)



72. (A) The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and shifted up 10 units.

(B)



EXERCISE 2-3

2. $x^2 - 2x - 5 = x^2 - 2x + 1 - 6$
$= (x - 1)^2 - 6$

4. $-x^2 + 8x - 9 = -(x^2 - 8x + 9)$
$= -(x^2 - 8x + 16 - 7)$
$= -(x - 4)^2 + 7$

6. The graph of $g(x)$ is the graph of $y = x^2$ shifted right 1 unit and down 6 units.

8. The graph of $n(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 4 units and up 7 units.

10. (A) g (B) m (C) n (D) f

12. (A) x intercepts: -5, -1; y intercept: -5 (B) Vertex: (-3, 4)
(C) Maximum: 4 (D) Range: $y \le 4$ or $(-\infty, 4]$
(E) Increasing interval: $x \le -3$ or $(-\infty, -3]$
(F) Decreasing interval: $x \ge -3$ or $[-3, \infty)$

14. (A) x intercepts: 1, 5; y intercept: 5 (B) Vertex: (3, -4)
(C) Minimum: -4 (D) Range: $y \ge 4$ or $[-4, \infty)$
(E) Increasing interval: $x \ge 3$ or $[3, \infty)$
(F) Decreasing interval: $x \le 3$ or $(-\infty, 3]$

16. $g(x) = -(x + 2)^2 + 3$
(A) x intercepts: $-(x + 2)^2 + 3 = 0$
$(x + 2)^2 = 3$
$x + 2 = \pm\sqrt{3}$
$x = -2 - \sqrt{3}, -2 + \sqrt{3}$
y intercept: -1
(B) Vertex: (-2, 3) (C) Maximum: 3 (D) Range: $y \le 3$ or $(-\infty, 3]$

18. $n(x) = (x - 4)^2 - 3$
(A) x intercepts: $(x - 4)^2 - 3 = 0$
$(x - 4)^2 = 3$
$x - 4 = \pm\sqrt{3}$
$x = 4 - \sqrt{3}, 4 + \sqrt{3}$
y intercept: 13
(B) Vertex: (4, -3) (C) Minimum: -3 (D) Range: $y \ge -3$ or $[-3, \infty)$

20. $y = -(x - 4)^2 + 2$

22. $y = [x - (-3)]^2 + 1$ or $y = (x + 3)^2 + 1$

24. $g(x) = x^2 - 6x + 5 = x^2 - 6x + 9 - 4 = (x - 3)^2 - 4$
(A) x intercepts: $(x - 3)^2 - 4 = 0$

$$(x - 3)^2 = 4$$
$$x - 3 = \pm 2$$
$$x = 1, 5$$

y intercept: 5

(B) Vertex: (3, -4)   (C) Minimum: -4   (D) Range: $y \geq -4$ or $[-4, \infty)$

26. $S(x) = -4x^2 - 8x - 3 = -4\left[x^2 + 2x + \frac{3}{4}\right] = -4\left[x^2 + 2x + 1 - \frac{1}{4}\right]$

$$= -4\left[(x+1)^2 - \frac{1}{4}\right] = -4(x+1)^2 + 1$$

(A) x intercepts: $-4(x + 1)^2 + 1 = 0$

$$4(x+1)^2 = 1$$
$$(x+1)^2 = \frac{1}{4}$$
$$x + 1 = \pm\frac{1}{2}$$
$$x = -\frac{3}{2}, -\frac{1}{2}$$

y intercept: -3

(B) Vertex: (-1, 1) (C) Maximum: 1 (D) Range: $y \le 1$ or $(-\infty, 1]$

28. $V(x) = .5x^2 + 4x + 10 = .5[x^2 + 8x + 20] = .5[x^2 + 8x + 16 + 4]$

$$= .5[(x+4)^2 + 4]$$
$$= .5(x+4)^2 + 2$$

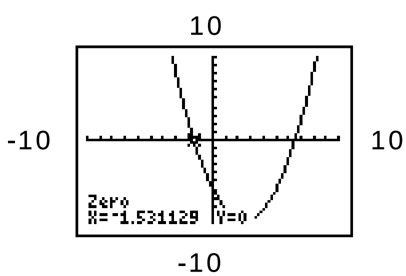
(A) x intercepts: none
y intercept: 10

(B) Vertex: (-4, 2) (C) Minimum: 2 (D) Range: $y \ge 2$ or $[2, \infty)$

30. $g(x) = -0.6x^2 + 3x + 4$

(A) $g(x) = -2$: $-0.6x^2 + 3x + 4 = -2$

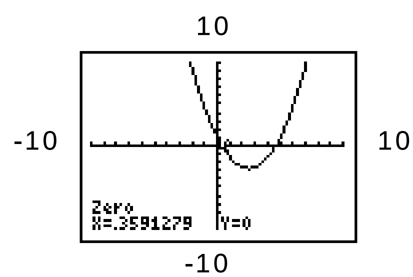
$$0.6x^2 - 3x - 6 = 0$$



$x = -1.53, 6.53$

(B) $g(x) = 5$: $-0.6x^2 + 3x + 4 = 5$

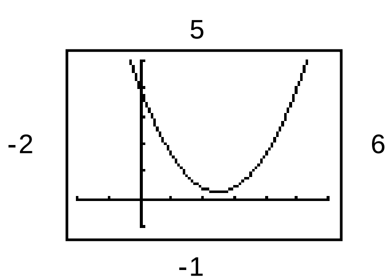
$$-0.6x^2 + 3x - 1 = 0$$
$$0.6x^2 - 3x + 1 = 0$$



$x = 0.36, 4.64$

(C) $g(x) = 8$: $-0.6x^2 + 3x + 4 = 8$

$$-0.6x^2 + 3x - 4 = 0$$
$$0.6x^2 - 3x + 4 = 0$$



No solution

32. Using a graphing utility with $y = 100x - 7x^2 - 10$ and the calculus option with maximum command, we obtain 347.1429 as the maximum value.

34. The graph is entirely above or below the x axis.

36. $m(x) = 0.20x^2 - 1.6x - 1 = 0.20(x^2 - 8x - 5)$
$= 0.20[(x - 4)^2 - 21]$
$= 0.20(x - 4)^2 - 4.2$

(A) x intercepts: $0.20(x - 4)^2 - 4.2 = 0$
$(x - 4)^2 = 21$
$x - 4 = \pm\sqrt{21}$
$x = 4 - \sqrt{21} = -0.6,\ 4 + \sqrt{21} = 8.6;$

y intercept: -1

(B) Vertex: (4, -4.2) (C) Minimum: -4.2
(D) Range: $y \geq -4.2$ or [-4.2, °)

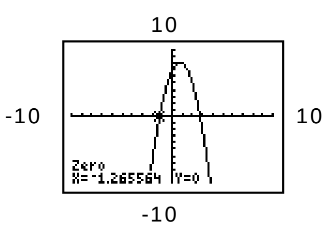
38. $n(x) = -0.15x^2 - 0.90x + 3.3$
$= -0.15(x^2 + 6x - 22)$
$= -0.15[(x + 3)^2 - 31]$
$= -0.15(x + 3)^2 + 4.65$

(A) x intercepts: $-0.15(x + 3)^2 + 4.65 = 0$
$(x + 3)^2 = 31$
$x + 3 = \pm\sqrt{31}$
$x = -3 - \sqrt{31} = -8.6,\ -3 + \sqrt{31}$
$= 2.6;$

y intercept: 3.30

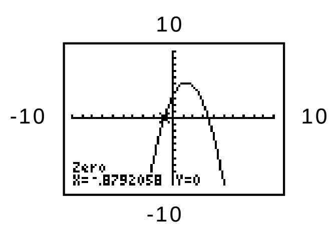
(B) Vertex: (-3, 4.65) (C) Maximum: 3.30
(D) Range: $x \leq 4.65$ or (-°, 4.65]

40.



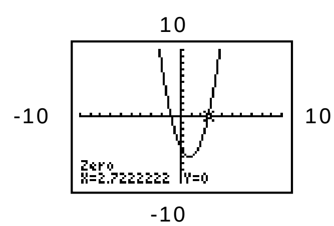
$x = -1.27, 2.77$

42.



$-0.88 \leq x \leq 3.52$

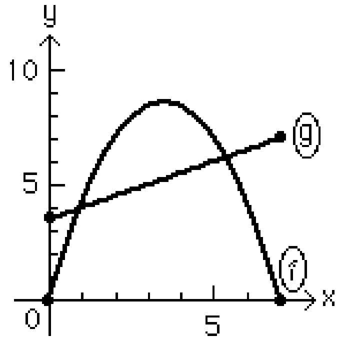
44.



$x < -1$ or $x > 2.72$

46. f is a quadratic function and max $f(x) = f(-3) = -5$
Axis: $x = -3$
Vertex: (-3, -5)
Range: $y \leq -5$ or $(-\infty, -5]$
x intercepts: None

48. (A)



(B) $f(x) = g(x)$: $-0.7x(x - 7) = 0.5x + 3.5$

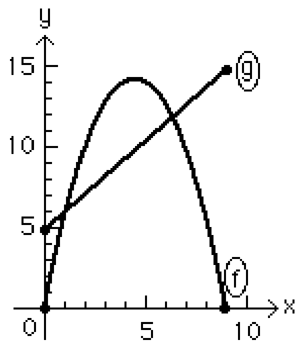
$$-0.7x^2 + 4.4x - 3.5 = 0$$

$$x = \frac{-4.4 \pm \sqrt{(4.4)^2 - (0.7)(3.5)}}{-1.4} = 0.93,\ 5.35$$

(C) $f(x) > g(x)$ for $0.93 < x < 5.35$

(D) $f(x) < g(x)$ for $0 \le x < 0.93$ or $5.35 < x \le 7$

50. (A)



(B) $f(x) = g(x)$: $-0.7x^2 + 6.3x = 1.1x + 4.8$

$$-0.7x^2 - 5.2x - 4.8 = 0$$

$$0.7x^2 + 5.2x + 4.8 = 0$$

$$x = \frac{-5.2 \pm \sqrt{(5.2)^2 - (0.7)(4.8)}}{1.4} = 1.08,\ 6.35$$

(C) $f(x) > g(x)$ for $1.08 < x < 6.35$

(D) $f(x) < g(x)$ for $0 \le x < 1.08$ or $6.35 < x \le 9$

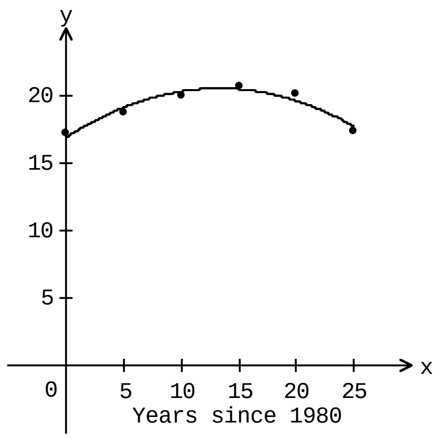
52. $f(x) = x^2$ and $g(x) = -(x - 4)^2$ are two examples. The vertex of the graph is on the x axis.

54. $f(x) = -0.0206x^2 + 0.548x + 16.9$

(A)

x	0	5	10	15	20	25
Market share	17.2	18.8	20.0	20.7	20.2	17.4
$f(x)$	16.9	19.1	20.3	20.5	19.6	17.7

(B)



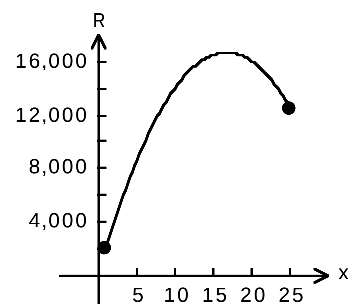
(C) For 2010, $x = 30$ and

$$f(30) = -0.0206(30)^2 + 0.548(30) + 16.9 = 14.8\%$$

For 2015, $x = 35$ and

$$f(35) = -0.0206(35)^2 + 0.548(30) + 16.9 = 10.8\%$$

58. (A)

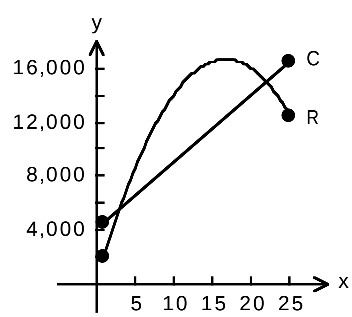


(B)
$$\begin{aligned} R(x) &= 2{,}000x - 60x^2 \\ &= -60\left(x^2 - \frac{100}{3}x\right) \\ &= -60\left[x^2 - \frac{100}{3}x + \frac{2500}{9} - \frac{2500}{9}\right] \\ &= -60\left[\left(x - \frac{50}{3}\right)^2 - \frac{2500}{9}\right] \\ &= -60\left(x - \frac{50}{3}\right) + \frac{50,000}{3} \end{aligned}$$

16.667 thousand computers(16,667 computers);
16,666.667 thousand dollars ($16,666,667)

(C) $p\left(\frac{50}{3}\right) = 2{,}000 - 60\left(\frac{50}{3}\right) = \$1{,}000$

60. (A)



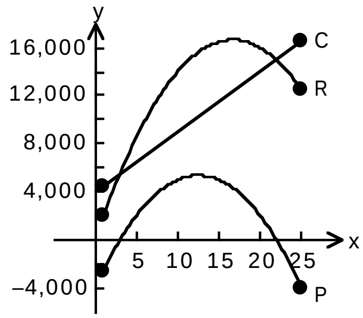
(B)
$$\begin{aligned} R(x) &= C(x) \\ x(2{,}000 - 60x) &= 4{,}000 + 500x \\ 2{,}000x - 60x^2 &= 4{,}000 + 500x \\ 60x^2 - 1{,}500x + 4{,}000 &= 0 \\ 6x^2 - 150x + 400 &= 0 \\ x &= 3.035,\ 21.965 \end{aligned}$$

Break-even at 3.035 thousands (3,035)
and 21.965 thousand (21,965)

(C) Loss: $1 \le x < 3.035$ or $21.965 < x \le 25$;
Profit: $3.035 < x < 21.965$

62. (A) $P(x) = R(x) - C(x)$

$= 1{,}500x - 60x^2 - 4{,}000$



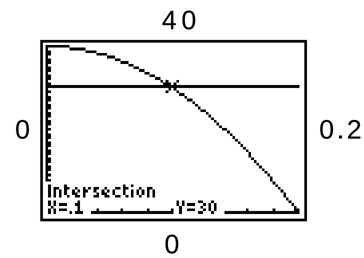
(C) Intercepts and break-even points: 3,035 computers and 21,965 computers

(E) Maximum profit is $5,375,000 when 12,500 computers are produced. This is much smaller than the maximum revenue of $16,666,667.

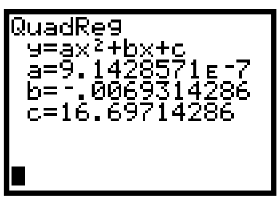
64. (A) Solve: $f(x) = 1{,}000(0.04 - x^2) = 30$

$$40 - 1000x^2 = 30$$
$$1000x^2 = 10$$
$$x^2 = 0.01$$
$$x = 0.10 \text{ cm}$$

(B)



66.



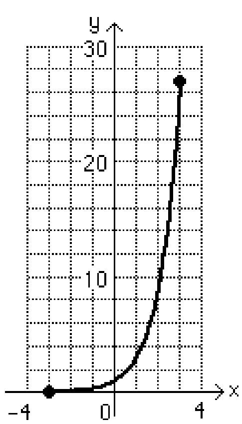
For $x = 2{,}300$, the estimated fuel consumption is $y = a(2{,}300)^2 + b(2{,}300) + c = 5.6$ mpg.

EXERCISE 2-4

2. (A) g (B) f (C) h (D) k

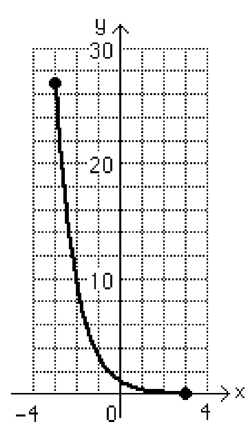
4. $y = 3^x$, $-3 \le x \le 3$

x	y
–3	$\frac{1}{27}$
–1	$\frac{1}{3}$
0	1
1	3
3	27



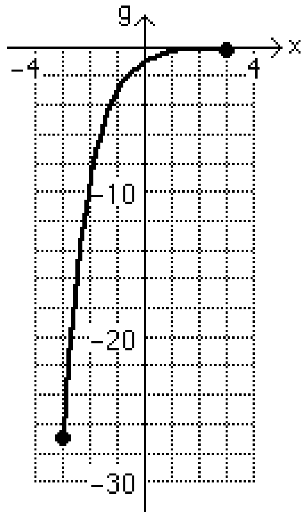
6. $y = \left(\frac{1}{3}\right)^x = 3^{-x}$, $-3 \le x \le 3$

x	y
–3	27
–1	3
0	1
1	$\frac{1}{3}$
3	$\frac{1}{9}$



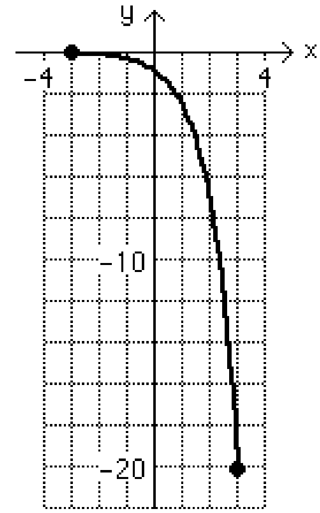
8. $g(x) = -3^{-x}$, $-3 \le x \le 3$

x	y
-3	-27
-1	-3
0	-1
1	$-\frac{1}{2}$
3	$-\frac{1}{27}$



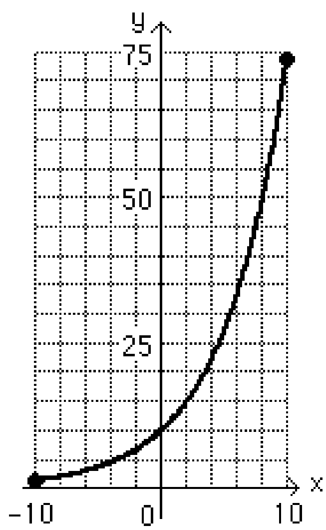
10. $y = -e^{x}$, $-3 \le x \le 3$

x	y
-3	≈ -0.05
-1	≈ -0.4
0	≈ -1
1	≈ -2.7
3	≈ -20



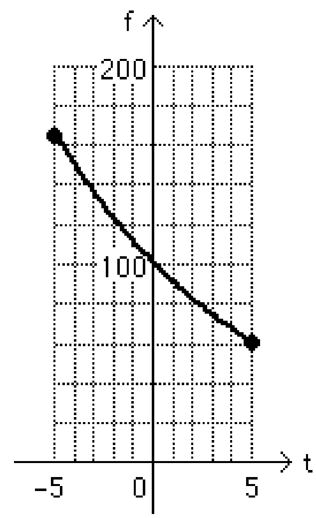
12. $y = 10e^{0.2x}$, $-10 \le x \le 10$

x	y
-10	≈ 1.4
-8	≈ 2.0
-6	≈ 3.0
0	10.0
6	≈ 33.2
8	≈ 49.5
10	≈ 73.9



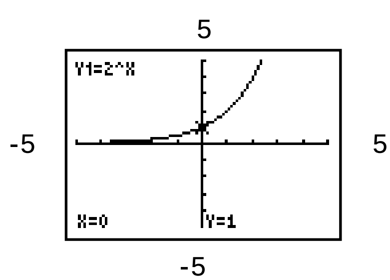
14. $f(t) = 100e^{-0.1t}$, $-5 \le t \le 5$

x	y
-5	≈ 165
-3	≈ 135
-1	≈ 111
0	100
1	≈ 90
3	≈ 74
5	≈ 60

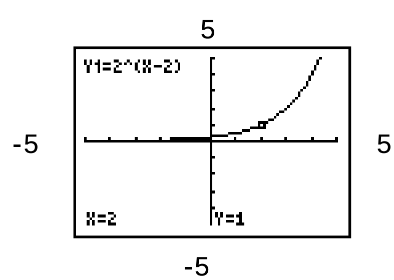


16. $10^{2x + 3}$ **18.** $\dfrac{e^{x}}{e^{1-x}} = e^{x-(1-x)} = e^{2x - 1}$ **20.** $(3e^{-1.4x})^{2} = 9e^{-2.8x}$

22. $g(x) = f(x - 2)$; the graph of g is the graph of f shifted 2 units to the right.

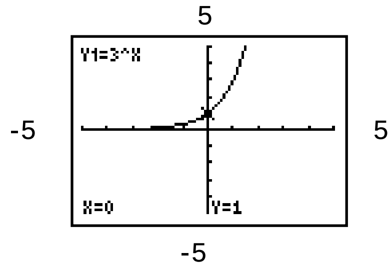


$f(x) = 2^{x}$

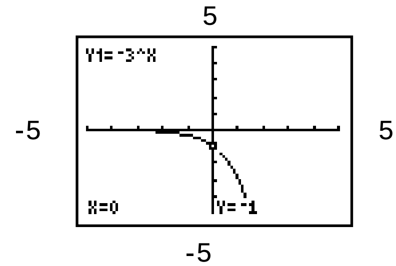


$g(x) = 2^{x-2}$

24. $g(x) = -f(x)$; the graph of g is the graph of f reflected in the x axis.

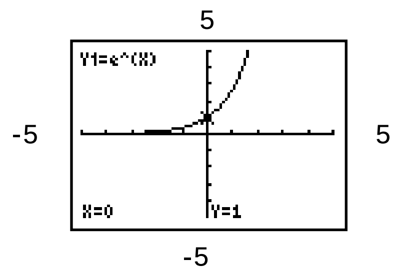


$f(x) = 3^x$

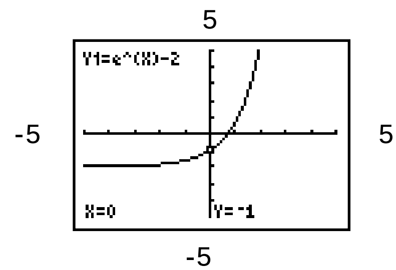


$g(x) = -3^x$

26. $g(x) = f(x) - 2$; the graph of g is the graph of f shifted two units down.

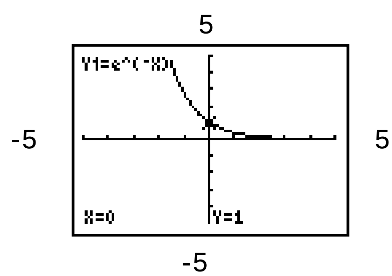


$f(x) = e^x$

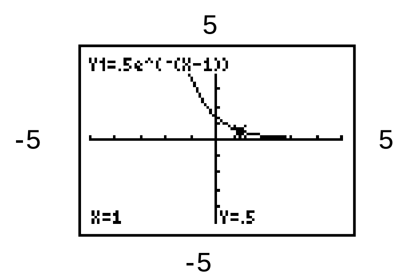


$g(x) = e^x - 2$

28. $g(x) = 0.5f(x - 1)$; the graph of g is the graph of f vertically contracted by a factor of 0.5 and shifted to the right 1 unit.

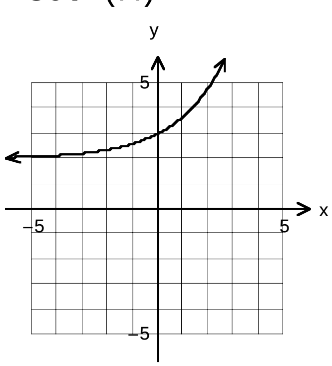


$f(x) = e^{-x}$

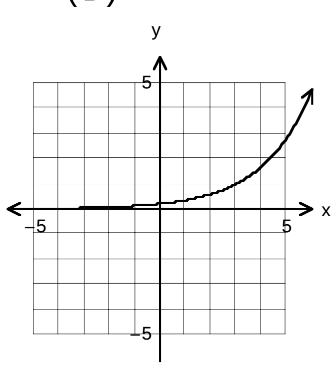


$g(x) = 0.5e^{-(x-1)}$

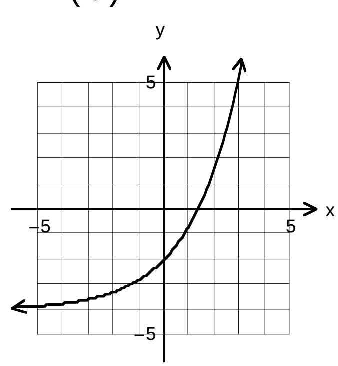
30. (A)



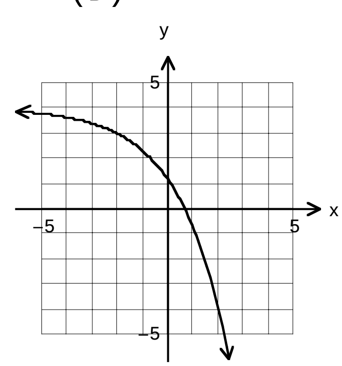
(B)



(C)

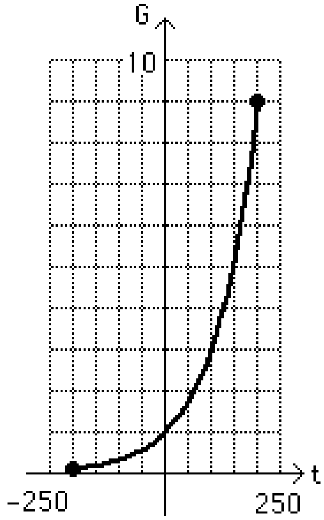


(D)



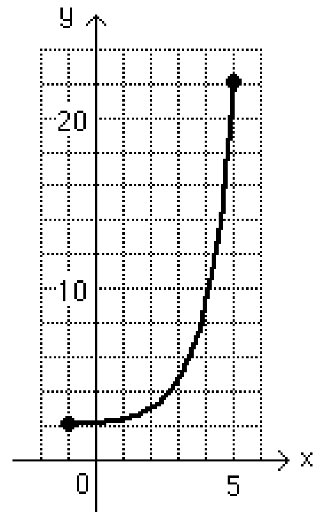
32. $G(t) = 3^{t/100}$, $-200 \le t \le 200$

x	y
-200	$\frac{1}{9}$
-100	$\frac{1}{3}$
0	1
100	3
200	9



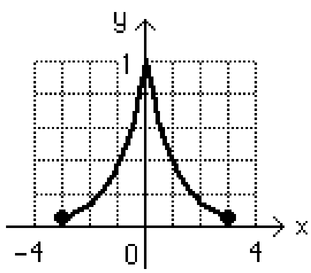
34. $y = 2 + e^{x-2}$, $-1 \le x \le 5$

x	y
-1	≈ 2.0
0	≈ 2.1
1	≈ 2.4
3	≈ 4.7
5	≈ 22.0



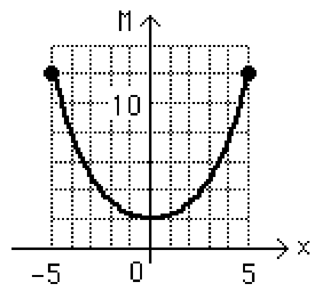
36. $y = e^{-|x|}$, $-3 \le x \le 3$

x	y
-3	≈ 0
-1	≈ 0.4
0	1
1	≈ 0.4
3	≈ 0



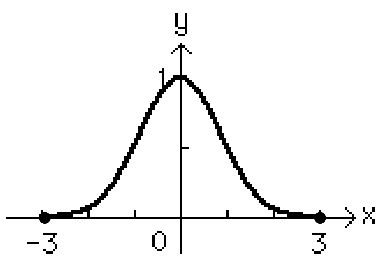
38. $M(x) = e^{x/2} + e^{-x/2}$, $-5 \le x \le 5$

x	y
-5	≈ 12.2
-3	≈ 4.7
-1	≈ 2.3
0	2
1	≈ 2.3
3	≈ 4.7
5	≈ 12.2



40. $y = 2^{-x^2}$, $-3 \le x \le 3$

x	$h(x)$
-3	$\frac{1}{512}$
-1	$\frac{1}{2}$
0	1
1	$\frac{1}{2}$
3	$\frac{1}{512}$



42. $a = 2$, $b = -2$ for example. The exponential function property: For $x \ne 0$, $a^x = b^x$ if and only if $a = b$ assumes $a > 0$, $b > 0$.

44. $5^{3x} = 5^{4x-2}$ implies $3x = 4x - 2$ or $x = 2$

46. $7^{x^2} = 7^{2x+3}$ implies $x^2 = 2x + 3$

$$x^2 - 2x - 3 = 0$$

$(x - 3)(x + 1) = 0$ or $x = -1, 3$

48. $(1 - x)^5 = (2x - 1)^5$ implies $1 - x = 2x - 1$ if $x < 1$ and $x > \frac{1}{2}$.

So $3x = 2$ or $x = \frac{2}{3}$ is a solution since $\frac{1}{2} < x - \frac{2}{3} < 1$.

50. $2xe^{-x} = 0$

$2x = 0$ (since $e^{-x} \neq 0$)

$x = 0$

52. $x^2e^x - 5xe^x = 0$

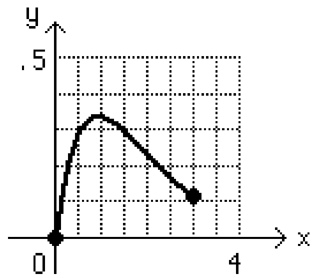
$x(x - 5)e^x = 0$

$x(x - 5) = 0$ (since $e^x \neq 0$)

$x = 0, 5$

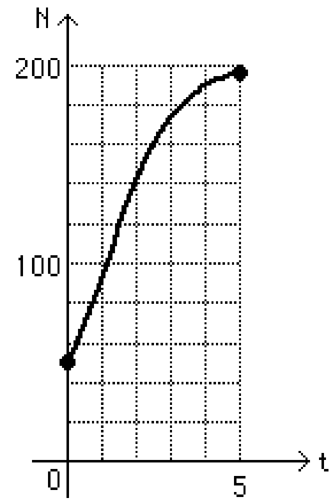
54. $m(x) = x(3^{-x}),\ 0 \le x \le 3$

x	$m(x)$
0	0
1	$\frac{1}{3}$
2	$\frac{2}{9}$
3	$\frac{1}{9}$



56. $N(t) = \dfrac{200}{1 + 3e^{-t}},\ 0 \le t \le 5$

t	$N(t)$
0	50
1	≈ 95
2	≈ 142
3	≈ 174
5	≈ 196



58. $f(x) = 5 - 3^{-x}$

Solve $5 - 3^{-x} = 0$

$x = -1.46$

60. $f(x) = 7 - 2x^2 + 2^{-x}$

Solve $7 - 2x^2 + 2^{-x} = 0$

$x = -6.05, -2.53, 1.91$

62. $A = P\left(1 + \frac{r}{m}\right)^{mt}$, we have:

(A) $P = 4{,}000,\ r = 0.07,\ m = 52,\ t = \frac{1}{2}$

$$A = 4{,}000\left(1 + \frac{0.07}{52}\right)^{(52)(1/2)} = 4{,}142.38$$

Thus, $A = \$4{,}142.38$.

(B) $$A = 4{,}000\left(1 + \frac{0.07}{52}\right)^{(52)(10)} = 8{,}051.22$$

Thus, $A = \$8{,}051.22$.

64. $A = Pe^{rt}$ with $P = 5{,}250$, $r = 0.0745$, we have:

(A) $A = 5{,}250e^{(0.0745)(6.25)} = 8{,}363.30$.

Thus, there will be \$8,363.30 in the account after 6.25 years.

(B) $A = 5{,}250e^{(0.0745)(17)} = 18{,}629.16$

Thus, there will be \$18,629.16 in the account after 17 years.

66. (A) $A = P\left(1 + \frac{r}{m}\right)^{mt}$.

Here $P = \$10,000$, $r = 0.055$, $m = 4$, $t = 5$ years.

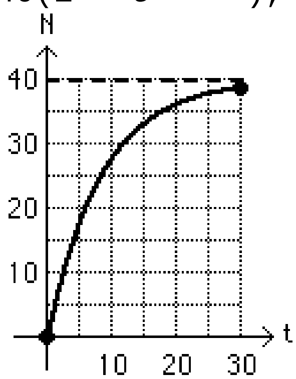
Thus, $A = 10,000\left(1 + \frac{0.055}{4}\right)^{20} = \$13,140.67$.

(B) $A = P\left(1 + \frac{r}{m}\right)^{mt}$, where P and t are as in (A) and $r = 0.0512$, $m = 12$. Thus, $A = 10,000\left(1 + \frac{0.0512}{12}\right)^{60} = \$12,910.49$.

(C) $A = P\left(1 + \frac{r}{m}\right)^{mt}$, where P and t are as before and $r = 0.0486$, $m = 365$. Thus, $A = 10,000\left(1 + \frac{0.0486}{365}\right)^{5(365)} = \$12,750.48$.

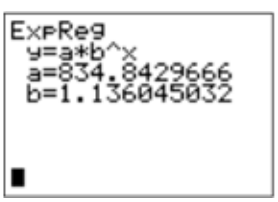
68. Given $N = 40(1 - e^{-0.12t})$, $0 \le t \le 30$

t	N
0	0
10	≈ 28
20	≈ 36
30	≈ 40



Maximum number of boards an average employee can be expected to produce in 1 day is 40.

70. (A)



(B) The model gives an average salary of $2,039,000 in 1997. Inclusion of the data for 1997 gives an average of 20,400,000 in 2015.

72. Given $I = I_0 e^{-0.00942d}$

(A) $I = I_0 e^{-0.00942(50)} = I_0 e^{-0.471} \approx I_0(0.62)$

Thus, about 62% of the surface light will reach a depth of 50 ft.

(B) $I = I_0 e^{-0.00942(100)} = I_0 e^{-0.942} \approx I_0(0.39)$

Thus, about 39% of the surface light will reach a depth of 100 ft.

74. (A) $N = N_0 e^{rt}$, where $N_0 = 25$, $r = 0.01$. Thus

$N = 25e^{0.01t}$

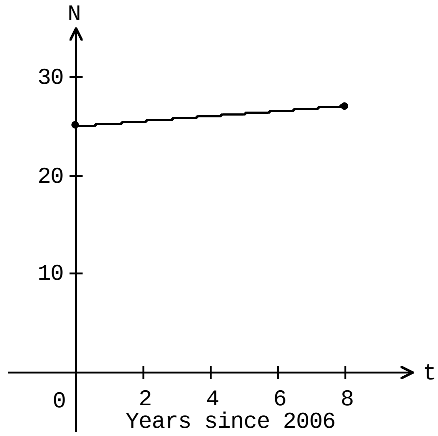
(B) Since 2006 is year 0, for 2002 we need to take $t = -4$. Thus

$N = 25e^{(0.01)(-4)} \approx 24{,}000{,}000$

For 2014, $t = 8$ and

$N = 25e^{0.01(8)} \approx 27{,}000{,}000$

(C)



76. (A) $P = 75e^{0.023t}$

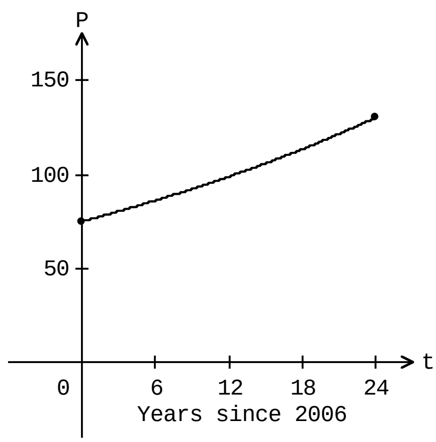
(B) For 2015, $t = 9$ (since 2006 is year 0) we have:

$P = 75e^{0.023(9)} \approx 92$ million.

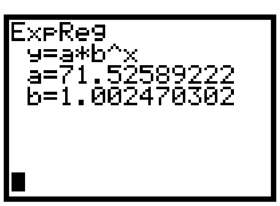
For 2030, $t = 24$ and

$P = 75e^{0.023(24)} \approx 130$ million.

(C)



78.



For $x = 45$, we have

$y = ab^{45} \approx 79.9$ years

2. $32 = 2^5$ **4.** $e^0 = 1$ **6.** $27 = 9^{3/2}$

8. $\log_6 36 = 2$ **10.** $\log_{27} 9 = \frac{2}{3}$ **12.** $\log_b M = x$

14. $\log_e 1 = y$ is equivalent to $e^y = 1$; $y = 0$.

16. $\log_{10} 10 = y$ is equivalent to $10^y = 10$; $y = 1$.

18. $\log_{13} 13 = y$ is equivalent to $13^y = 13$; $y = 1$.

20. $\log_{10} 10^{-5} = -5$ **22.** $\log_3 3^5 = 5$ **24.** $\log_6 36 = \log_6 6^2 = 2$

26. $\log_b FG = \log_b F + \log_b G$ **28.** $\log_b w^{15} = 15 \log_b w$

30. $\text{Log}_3 P = (\text{Log}_R P)(\text{Log}_3 R)$ (change of base formula)
or $\text{Log}_R P = \dfrac{\text{Log}_3 P}{\text{Log}_3 R}$

32. $\log_2 x = 2$
$x = 2^2$
$x = 4$

34. $\log_3 27 = y$
$\log_3 27 = \log_3 3^3 = 3 \log_3 3 = 3$
Thus, $y = 3$

36. $\log_b e^{-2} = -2$
$-2 \log_b e = -2$ or $\log_b e = 1$
Thus, $b = e$

38. $\log_{25} x = \frac{1}{2}$
$x = 25^{1/2}$
$x = 5$

40. $\log_{49}\left(\frac{1}{7}\right) = y$
$\frac{1}{7} = 49^y$
$y = -\frac{1}{2}$

42. $\log_b 4 = \frac{2}{3}$
$4 = b^{2/3}$ Taking square root from both sides, we have
$b^{1/3} = 2$ Cubing both sides yields
$b = 8$

44. False. Take $f(x) = x^3 - x$, then $f(-1) = f(0) = f(1) = 0$.

46. True. Indeed the graph of every function (not necessarily one-to-one) intersects each vertical line exactly once.

48. False. $x = -1$ is in the domain of f, but cannot be in the range of g.

50. True. $y = \log_b x$ implies that $x = b^y$. If $b > 1$, then as y increases so does b^y. Therefore, the inverse of $x = b^y$ which is $y = \log_b x$ must be increasing as well.

52. True. Since g is the inverse of f, then (a, b) is on the graph of f if and only if (b, a) is on the graph of g. Therefore, f is also the inverse of g.

54. $$\begin{aligned}\log_b x &= \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3\\ &= \frac{2}{3}\log_b 3^3 + 2\log_b 2 - \log_b 3\\ &= 2\log_b 3 + 2\log_b 2 - \log_b 3 = \log_b 3 + 2\log_b 2\\ &= \log_b 12\end{aligned}$$

Thus, $x = 12$.

56. $$\begin{aligned}\log_b x &= 3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20\\ &= \log_b 8 + \log_b 5 - \log_b 20\\ &= \log_b 40 - \log_b 20 = \log_b \frac{40}{20} = \log 2\end{aligned}$$

Thus, $x = 2$.

58. $$\begin{aligned}\log_b(x + 2) + \log_b x &= \log_b 24\\ \log_b x(x + 2) &= \log_b 24\end{aligned}$$

$x(x + 2) = 24$ or $x^2 + 2x - 24 = 0$ or $(x - 4)(x + 6) = 4$

Thus, $x = 4$. [Note: $x = -6$ is not a solution since $\log_b(-6)$ is not defined.]

60. $\log_{10}(x + 6) - \log_{10}(x - 3) = 1$

$\log_{10}\frac{x + 6}{x - 3} = 1$ implies that $\frac{x + 6}{x - 3} = 10$ or

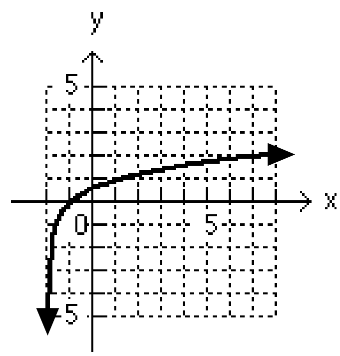
$x + 6 = 10x - 30$ or $9x = 36$ or $x = 4$.

62. $y = \log_3(x + 2)$

$x + 2 = 3^y$

$x = 3^y - 2$

x	y
$-\frac{53}{27}$	-3
$-\frac{17}{9}$	-2
$-\frac{5}{3}$	-1
-1	0
1	1
7	2
25	3



64. The graph of $y = \log_3(x + 2)$ is the graph of $y = \log_3 x$ shifted to the left 2 units.

66. Since logarithmic functions are defined only for positive "inputs", we must have $x - 1 > 0$ or $x > 1$; domain: $(1, \infty)$. The range of $y = \log(x - 1) - 1$ is the set of all real numbers.

68. (A) 1.86096

(B) -1.48095

(C) 10.60304

(D) -5.12836

70. (A) $\log x = 2.0832$

$x = 121.1156$

(B) $\log x = -1.1577$

$x = 0.0696$

(C) $\ln x = 3.1336$

$x = 22.9565$

(D) $\ln x = -4.3281$

$x = 0.0132$

72. $10^x = 153$ (Take common logarithms of both sides)

$\log 10^x = \log 153 = 2.1847$

$x = 2.1847$ ($\log 10^x = x \log 10 = x$; $\log 10 = 1$)

74. $e^x = 0.3059$ (Take natural logarithms of both sides)

$\ln e^x = \ln 0.3059 = -1.1845$

$x = -1.1845$ ($\ln e^x = x \ln e = x$; $\ln e = 1$)

76. $1.075^x = 1.837$ (Take either common or natural logarithms of both sides.) We use natural logarithms.

$\ln 1.075^x = \ln 1.837$

$$x = \frac{\ln 1.837}{\ln 1.075} = 8.4089$$

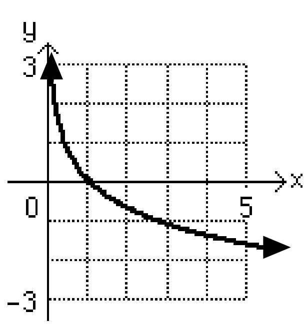
78. $1.02^{4t} = 2$ (Take either common or natural logarithms of both sides.) Here we'll use common logarithms.

$\log 1.02^{4t} = \log 2$

$$t = \frac{\log 2}{4 \log 1.02} = 8.7507$$

80. $y = -\ln x$, $x > 0$

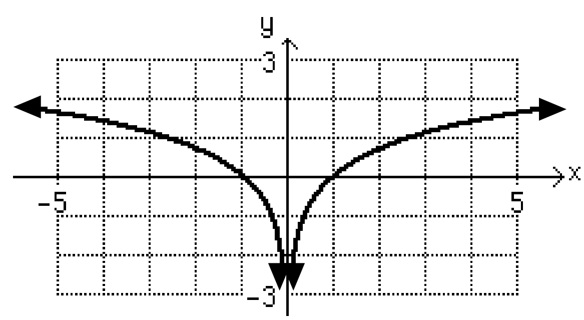
x	y
0.5	≈ 0.69
1	0
2	≈ –0.69
4	≈ –1.39
5	≈ –1.61



decreasing (0, ∞)

82. $y = \ln|x|$

x	y
–5	≈ 1.61
–2	0.69
1	0
2	≈ 0.69
5	≈ 1.61

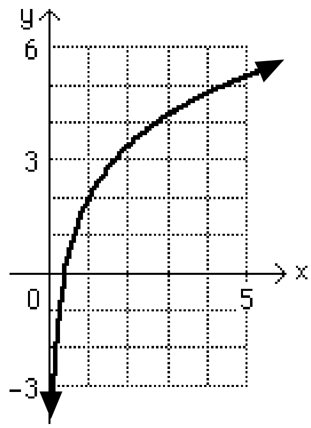


decreasing (-∞, 0)

increasing (0, ∞)

84. $y = 2 \ln x + 2$

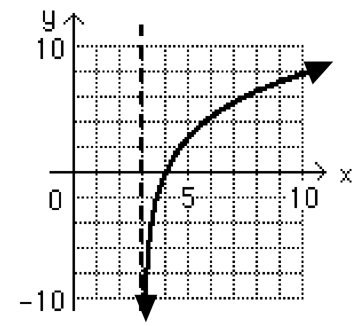
x	y
0.5	$\approx$ 0.62
1	2
2	$\approx$ 3.38
4	$\approx$ 4.78
5	$\approx$ 5.52



increasing $(0, \infty)$

86. $y = 4 \ln(x - 3)$

x	y
4	0
6	$\approx$ 2.77
8	$\approx$ 6.44
10	$\approx$ 7.78
12	$\approx$ 8.79



increasing $(3, \infty)$

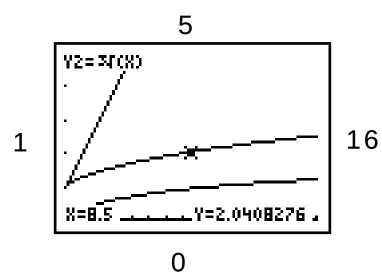
88. It is not possible to find a power of 1 that is an arbitrarily selected real number, because 1 raised to any power is 1.

90. $\log_e x - \log_e 25 = 0.2t$

$\log_e \dfrac{x}{25} = 0.2t$

Therefore, $\dfrac{x}{25} = e^{0.2t}$, and $x = 25e^{0.2t}$

92.



$1 < x \le 16$

A function f is "larger than" a function g on an interval $[a, b]$ if $f(x) > g(x)$ for $a \le x \le b$.
$r(x) > q(x) > p(x)$ for $1 < x \le 16$, that is $x > \sqrt[3]{x} > \log x$.

94. From the compound interest formula $A = P(1 + r)^t$, we have:

$2P = P(1 + 0.0958)^t$ or $(1.0958)^t = 2$

Take the natural log of both sides of this equation:

$\ln(1.0958)^t = \ln 2$ [Note: The common log could have been used instead of the natural log.]

$t \ln(1.0958) = \ln 2$

$$t = \frac{\ln 2}{\ln(1.0958)} \approx \frac{0.69315}{0.09148} \approx 8 \text{ years}$$

96. $A = P\left(1 + \frac{r}{m}\right)^{mt}$.

For P = \$5,000, A = \$7,500, r = 0.08, m = 2, we have:

$$7{,}500 = 5{,}000\left(1 + \frac{0.08}{2}\right)^{2t}$$

$$(1 + 0.04)^{2t} = \frac{7{,}500}{5{,}000} = 1.5$$

$$2t \ln(1.04) = \ln(1.5)$$

$$t = \frac{\ln(1.5)}{2\ln(1.04)} \approx 5.17 \text{ years}$$

If compounded monthly, then m = 12 and

$$7{,}500 = 5{,}000\left(1 + \frac{0.08}{2}\right)^{12t}$$

$$\left(1 + \frac{0.08}{2}\right)^{12t} = 1.5$$

$$12t \ln\left(1 + \frac{0.08}{2}\right) = \ln(1.5)$$

$$t = \frac{\ln(1.5)}{12\ln\left(1 + \frac{0.08}{12}\right)} \approx 5.09 \text{ years}$$

98. 6,145 screwdrivers at \$46.77 each.

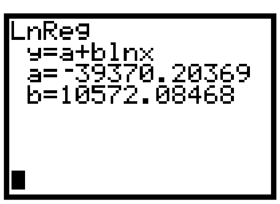
100. (A) $N = 10 \log \frac{I}{I_0} = 10 \log \frac{10^{-13}}{10^{-16}} = 10 \log 10^3 = 30$

(B) $N = 10 \log \frac{3.16 \times 10^{-10}}{10^{-16}} = 10 \log 3.16 \times 10^6 \approx 65$

(C) $N = 10 \log \frac{10^{-8}}{10^{-16}} = 10 \log 10^8 = 80$

(D) $N = 10 \log \frac{10^{-1}}{10^{-16}} = 10 \log 10^{15} = 150$

102.



For 2015, x = 115 and

y = -39,370.20369 + 10,572.08468(115) ≈ 10,794 million bushels.