Chapter 2
Quadratic and Other Nonlinear Functions

Algebra Toolbox

1. **a.** $x^4 \bullet x^3 = x^{4+3} = x^7$

b. $\dfrac{x^{12}}{x^7} = x^{12-7} = x^5$

c. $(4ay)^4 = (4)^4 a^4 y^4 = 256a^4 y^4$

d. $\left(\dfrac{3}{z}\right)^4 = \dfrac{3^4}{z^4} = \dfrac{81}{z^4}$

e. $2^3 \bullet 2^2 = 2^{3+2} = 2^5 = 32$

f. $\left(x^4\right)^2 = x^{4\bullet 2} = x^8$

2. **a.** $y^5 \bullet y = y^5 \bullet y^1 = y^{5+1} = y^6$

b. $\dfrac{w^{10}}{w^4} = w^{10-4} = w^6$

c. $(6bx)^3 = (6)^3 b^3 x^3 = 216b^3 x^3$

d. $\left(\dfrac{5z}{2}\right)^3 = \dfrac{5^3 z^3}{2^3} = \dfrac{125z^3}{8}$

e. $3^2 \bullet 3^3 = 3^{2+3} = 3^5 = 243$

f. $\left(2y^3\right)^4 = 2^4 y^{4\bullet 3} = 16y^{12}$

3. **a.** $\sqrt{16} = \sqrt{4\bullet 4} = 4$

b. $-\sqrt{16} = -\sqrt{4\bullet 4} = -4$

c. $\sqrt{-16}$ has no solution in the real number system, since there is a negative under the even radical.

d. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$

e. $-\sqrt[5]{-32} = -\sqrt[5]{(-2)^5} = -(-2) = 2$

4. **a.** $\sqrt{18} = \sqrt{2\bullet 3^2} = 3\sqrt{2}$

b. $\sqrt{216} = \sqrt{2^2 \bullet 2 \bullet 3^2 \bullet 3} =$
$2\bullet 3\sqrt{2\bullet 3} = 6\sqrt{6}$

c. $\sqrt{(-12)^2 - 4(2)(-3)}$
$= \sqrt{144 + 24}$
$= \sqrt{168}$
$= \sqrt{2^2 \bullet 2\bullet 3\bullet 7}$
$= 2\sqrt{2\bullet 3\bullet 7}$
$= 2\sqrt{42}$

d. $\sqrt[3]{243a^7b^4}$
$= \sqrt[3]{3^5 a^7 b^4}$
$= \sqrt[3]{3^3 \bullet 3^2 \bullet a^6 \bullet a \bullet b^3 \bullet b}$
$= 3a^2b\sqrt[3]{9ab}$

5. **a.** $\sqrt{x^3} = \sqrt[2]{x^3} = x^{\frac{3}{2}}$

b. $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

c. $\sqrt[5]{x^3} = x^{\frac{3}{5}}$

d. $\sqrt[6]{27y^9}$
$= \left(27y^9\right)^{\frac{1}{6}}$
$= \left(3^3 y^9\right)^{\frac{1}{6}}$
$= 3^{\frac{3}{6}} y^{\frac{9}{6}}$
$= 3^{\frac{1}{2}} y^{\frac{3}{2}}$

e. $27\sqrt[6]{y^9} = 27y^{\frac{9}{6}} = 27y^{\frac{3}{2}}$

6. a. $a^{\frac{3}{4}} = \sqrt[4]{a^3}$

b. $-15x^{\frac{5}{8}} = -15\sqrt[8]{x^5}$

c. $(-15x)^{\frac{5}{8}} = \sqrt[8]{(-15x)^5}$

7. The degree of the polynomial is 2, the highest exponent in the expression. The leading coefficient is –3, the coefficient of the highest degree term. The constant term, the term without a variable, is 8. The polynomial contains only one variable, *x*.

8. The degree of the polynomial is 4, the highest exponent in the expression. The leading coefficient is 5, the coefficient of the highest degree term. The constant term, the term without a variable, is –3. The polynomial contains only one variable, *x*.

9. The degree of the polynomial is 7, the greatest sum of exponents in any one term. The leading coefficient is –2, the coefficient of the highest degree term. The constant term, the term without a variable, is –119. The polynomial contains two variables, *x* and *y*.

10. $(4x^2y^3)(-3a^2x^3)$
$= -12a^2x^{2+3}y^3$
$= -12a^2x^5y^3$

11. $2xy^3(2x^2y + 4xz - 3z^2)$
$= (2xy^3)(2x^2y) + (2xy^3)(4xz)$
$\quad -(2xy^3)(3z^2)$
$= 4x^3y^4 + 8x^2y^3z - 6xy^3z^2$

12. $(x-7)(2x+3)$
$x \bullet 2x + x \bullet 3 - 7 \bullet 2x - 7 \bullet 3$
$2x^2 + 3x - 14x - 21$
$2x^2 - 11x - 21$

13. $(k-3)^2$
$(k-3)(k-3)$
$k^2 - 3k - 3k + 9$
$k^2 - 6k + 9$
or
$(k-3)^2$
$(k)^2 - 2(k)(3) + (3)^2$
$k^2 - 6k + 9$

14. $(4x-7y)(4x+7y)$
$16x^2 + 28xy - 28xy - 49y^2$
$16x^2 - 49y^2$
or
$(4x-7y)(4x+7y)$
$(4x)^2 - (7y)^2$
$16x^2 - 49y^2$

15. $3x^2 - 12x = 3x(x-4)$

16. $12x^5 - 24x^3 = 12x^3(x^2 - 2)$

17. Difference of two squares

$9x^2 - 25m^2 = (3x+5m)(3x-5m)$

18. Find two numbers whose product is 15 and whose sum is -8.

$x^2 - 8x + 15 = (x-5)(x-3)$

19. Find two numbers whose product is -35 and whose sum is -2.

$x^2 - 2x - 35 = (x-7)(x+5)$

20. To factor by grouping, first multiply the 2nd degree term by the constant term:

$3x^2 \bullet -2 = -6x^2$.

Then, find two terms whose product is $-6x^2$ and whose sum is $-5x$, the middle term.

$3x^2 - 5x - 2$

$= 3x^2 - 6x + 1x - 2$

$= (3x^2 - 6x) + (1x - 2)$

$= 3x(x-2) + 1(x-2)$

$= (3x+1)(x-2)$

21. To factor by grouping, first multiply the 2nd degree term by the constant term:

$8x^2 \bullet 5 = 40x^2$.

Then, find two terms whose product is $40x^2$ and whose sum is $-22x$, the middle term.

$8x^2 - 22x + 5$

$= 8x^2 - 2x - 20x + 5$

$= (8x^2 - 2x) + (-20x + 5)$

$= 2x(4x-1) + (-5)(4x-1)$

$= (2x-5)(4x-1)$

22. $6n^2 + 18 + 39n$

$= 6n^2 + 39n + 18$

$= 3(2n^2 + 13n + 6)$

To factor by grouping, first multiply the 2nd degree term by the constant term:

$2n^2 \bullet 6 = 12n^2$.

Then, find two terms whose product is $12n^2$ and whose sum is $13n$, the middle term.

$3(2n^2 + 13n + 6)$

$= 3(2n^2 + 1n + 12n + 6)$

$= 3\left[(2n^2 + 1n) + (12n + 6)\right]$

$= 3\left[n(2n+1) + 6(2n+1)\right]$

$= 3(n+6)(2n+1)$

23. $3y^4 + 9y^2 - 12y^2 - 36$
$= 3\left[y^4 + 3y^2 - 4y^2 - 12\right]$
$= 3\left[\left(y^4 + 3y^2\right) + \left(-4y^2 - 12\right)\right]$
$= 3\left[y^2\left(y^2 + 3\right) + (-4)\left(y^2 + 3\right)\right]$
$= 3\left(y^2 - 4\right)\left(y^2 + 3\right)$
$= 3(y-2)(y+2)\left(y^2 + 3\right)$

24. $18p^2 + 12p - 3p - 2$
$= \left(18p^2 + 12p\right) + (-3p - 2)$
$= 6p(3p+2) + (-1)(3p+2)$
$= (6p-1)(3p+2)$

25. $5x - 4xy = 2y$
$5x = 2y + 4xy$
$5x = y(2+4x)$
$\dfrac{5x}{(2+4x)} = \dfrac{y(2+4x)}{(2+4x)}$
$y = \dfrac{5x}{(2+4x)}$

26. $2xy = \dfrac{6}{y} + \dfrac{x}{3}$
The LCM is $3y$.
$3y(2xy) = 3y\left(\dfrac{6}{y} + \dfrac{x}{3}\right)$
$6xy^2 = 18 + xy$
$6xy^2 - xy = 18$
$x\left(6y^2 - y\right) = 18$
$\dfrac{x\left(6y^2 - y\right)}{\left(6y^2 - y\right)} = \dfrac{18}{\left(6y^2 - y\right)}$
$x = \dfrac{18}{\left(6y^2 - y\right)}$

27. **a.** Imaginary. The number has a non-zero real part and pure imaginary part.

b. Pure imaginary. The real part is zero.

c. Real. The imaginary part is zero.

d. Real. $2 - 5i^2 = 2 - 5(-1) = 7$

28. **a.** Imaginary. The number has a non-zero real part and pure imaginary part.

b. Real. The imaginary part is zero.

c. Pure imaginary. The real part is zero.

d. Imaginary. The number has a non-zero real part and pure imaginary part.
$2i^2 - i = 2(-1) - i = -2 - i$

29. $a + bi = 4 + 0i$. Therefore,
$a = 4, b = 0$.

30. $a + 3i = 15 - bi$
Therefore, $a = 15$ and
$-b = 3$ or $b = -3$.

31. $a + bi = 2 + 4i$
Therefore, $a = 2$ and $b = 4$.

32. $(3 + 7i) + (2i - 4)$
$= (3-4) + (7+2)i$
$= -1 + 9i$

33. $(4 - 2i) - (2 - 3i)$
$= 4 - 2i - 2 + 3i$
$= 2 + 1i$
$= 2 + i$

34. $2(3-i)-(7-2i)$
$=6-2i-7+2i$
$=-1$

35. $2(2i+3)-(4i+5)$
$=4i+6-4i-5$
$=1$

36. $(2-i)(2+i)$
$=4+2i-2i-i^2$
$=4-(-1)$
$=5$
or
$(2-i)(2+i)$
$=(2)^2-(i)^2$
$=4-(-1)$
$=5$

37. $(6-3i)(4+5i)$
$=24+30i-12i-15i^2$
$=24+18i-15(-1)$
$=39+18i$

38. $(2i-1)^2$
$=(2i-1)(2i-1)$
$=4i^2-2i-2i+1$
$=4i^2-4i+1$
$=4(-1)+1-4i$
$=-3-4i$
or
$(2i-1)^2$
$(2i)^2-2(2i)(1)+(1)^2$
$=4i^2-4i+1$
$=4(-1)+1-4i$
$=-3-4i$

39. $\frac{4-3i}{3-3i}\bullet\frac{3+3i}{3+3i}$
$=\frac{12+12i-9i-9i^2}{9+9i-9i-9i^2}$
$=\frac{12+3i+9}{9+9}$
$=\frac{21+3i}{18}$
$=\frac{21}{18}+\frac{3}{18}i$
$=\frac{7}{6}+\frac{1}{6}i$

40. $\dfrac{\sqrt{3}-2i}{\sqrt{5}+i}\bullet\dfrac{\sqrt{5}-i}{\sqrt{5}-i}$

$=\dfrac{\sqrt{15}-i\sqrt{3}-2i\sqrt{5}+2i^2}{\sqrt{25}-i\sqrt{5}+i\sqrt{5}-i^2}$

$=\dfrac{\sqrt{15}-i\sqrt{3}-2i\sqrt{5}-2}{5+1}$

$=\dfrac{\sqrt{15}-i\sqrt{3}-2i\sqrt{5}-2}{6}$

$=\dfrac{\left(\sqrt{15}-2\right)-i\left(\sqrt{3}+2\sqrt{5}\right)}{6}$

$=\dfrac{\sqrt{15}-2}{6}-\dfrac{\sqrt{3}+2\sqrt{5}}{6}i$

41. $\sqrt{-36}=\sqrt{36}\bullet\sqrt{-1}=6i$

42. $\sqrt{-98}=\sqrt{98}\bullet\sqrt{-1}=i\sqrt{2\bullet 49}=7i\sqrt{2}$

43. $\left(\sqrt{-4}\right)^2$

$=\left(\sqrt{-4}\right)\left(\sqrt{-4}\right)$

$=\left(i\sqrt{4}\right)\left(i\sqrt{4}\right)$

$=i^2\sqrt{16}$

$=-4$

44. $\sqrt{-6}\sqrt{-15}$

$=\left(i\sqrt{6}\right)\left(i\sqrt{15}\right)$

$=i^2\sqrt{90}$

$=i^2\sqrt{9\bullet 10}$

$=-3\sqrt{10}$

45. $\left(3+\sqrt{-4}\right)+\left(2-\sqrt{9}\right)$

$=\left(3+2\sqrt{-1}\right)+\left(2-3\sqrt{-1}\right)$

$=\left(3+2i\right)+\left(2-3i\right)$

$=5-i$

46. $\left(2+\sqrt{-2}\right)-\left(3-\sqrt{-8}\right)$

$=\left(2+i\sqrt{2}\right)-\left(3-i\sqrt{8}\right)$

$=\left(2+i\sqrt{2}\right)-\left(3-i\sqrt{4\bullet 2}\right)$

$=2+i\sqrt{2}-3+2i\sqrt{2}$

$=-1+3i\sqrt{2}$

47. $\left(1+\sqrt{-4}\right)\left(2-\sqrt{-9}\right)$

$=\left(1+\sqrt{-1\bullet 4}\right)\left(2-\sqrt{-1\bullet 9}\right)$

$=\left(1+2i\right)\left(2-3i\right)$

$=2-3i+4i-6i^2$

$=2+i+6$

$=8+i$

48. $\left(2-3\sqrt{-1}\right)^2$

$=\left(2-3i\right)^2$

$=\left(2-3i\right)\left(2-3i\right)$

$=4-6i-6i+9i^2$

$=4-12i-9$

$=-5-12i$

49. $\dfrac{1+\sqrt{-1}}{\sqrt{-1}}$

$=\dfrac{1+i}{i}$

$=\dfrac{1+i}{i}\bullet\dfrac{-i}{-i}$

$=\dfrac{-i-i^2}{-i^2}$

$=\dfrac{-i+1}{1}$

$=-i+1$

$=1-i$

Section 2.1 Skills Check

1. a. Yes. The equation fits the form $f(x) = ax^2 + bx + c, a \neq 0.$

b. Since $a = 2 > 0$, the graph opens up and is therefore concave up.

c. Since the graph is concave up, the vertex point is a minimum.

2. Not quadratic. The equation does not fit the form $f(x) = ax^2 + bx + c, a \neq 0.$ The highest exponent is 1.

3. Not quadratic. The equation does not fit the form $f(x) = ax^2 + bx + c, a \neq 0.$ The highest exponent is 3.

4. a. Yes. The equation fits the form $f(x) = ax^2 + bx + c, a \neq 0.$

b. Since $a = 1 > 0$, the graph opens up and is therefore concave up.

c. Since the graph is concave up, the vertex point is a minimum.

5. a. Yes. The equation fits the form $f(x) = ax^2 + bx + c, a \neq 0.$

b. Since $a = -5 < 0$, the graph opens down and is therefore concave down.

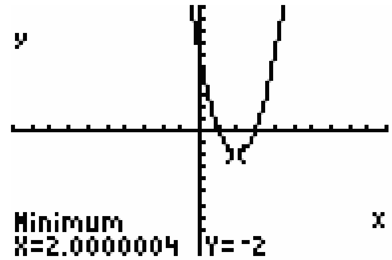
c. Since the graph is concave down, the vertex point is a maximum.

6. a. Yes. The equation fits the form $f(x) = ax^2 + bx + c, a \neq 0.$

b. Since $a = -2 < 0$, the graph opens down and is therefore concave down.

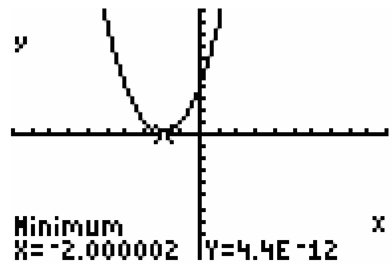
c. Since the graph is concave down, the vertex point is a maximum.

7. a.



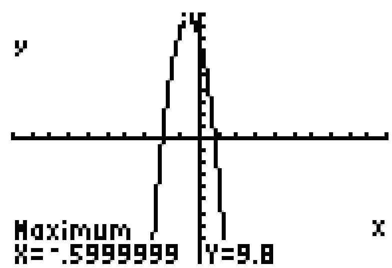
b. Yes.

8. a.



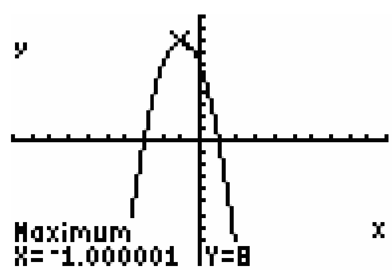
b. Yes.

9. a.



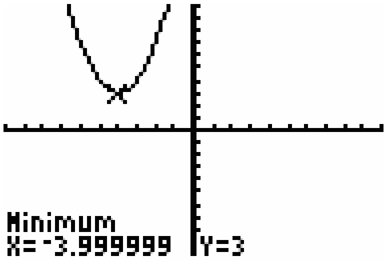
b. Yes.

10. a.



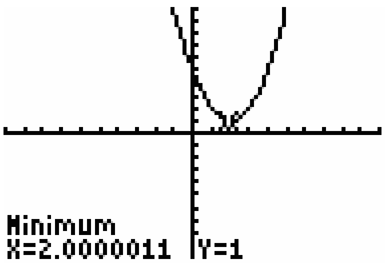
b. Yes.

11. a.



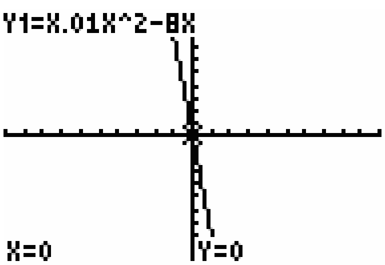
b. Yes.

12. a.



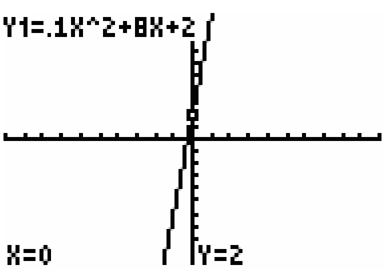
b. Yes.

13. a.



b. No. The complete graph will be a parabola.

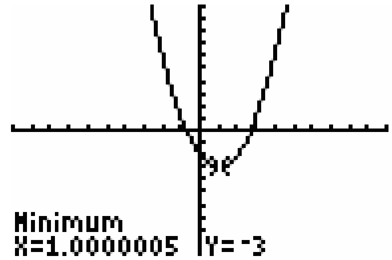
14. a.



b. No. The complete graph will be a parabola.

15. a. $(1,-3)$

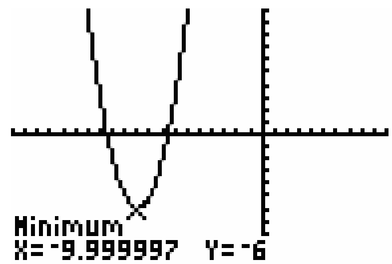
b.



[–10, 10] by [–10, 10]

16. a. $(-10,-6)$

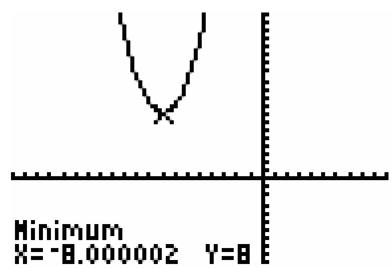
b.



[–20, 10] by [–10, 10]

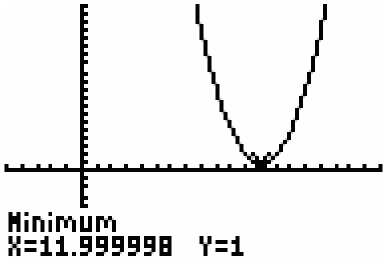
17. a. $(-8,8)$

b.



[–20, 10] by [–10, 20]

18. a. $(12,1)$

b. 

[–5, 20] by [–10, 20]

19. a. $y = 12x - 3x^2$

$y = -3x^2 + 12x$

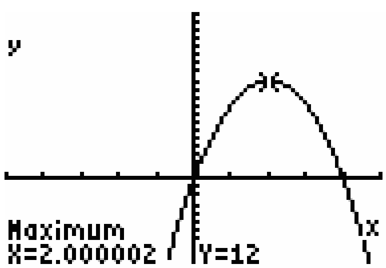
$a = -3, b = 12, c = 0$

$h = \dfrac{-b}{2a} = \dfrac{-12}{2(-3)} = \dfrac{-12}{-6} = 2$

$k = f(h) = f(2) = 12(2) - 3(2)^2 =$

$24 - 3(4) = 24 - 12 = 12$

The vertex is $(2, 12)$.

b. 

[–5, 5] by [–10, 20]

20. a. $y = 3x + 18x^2$

$y = 18x^2 + 3x$

$a = 18, b = 3, c = 0$

$h = \dfrac{-b}{2a} = \dfrac{-3}{2(18)} = \dfrac{-3}{36} = -\dfrac{1}{12}$

$k = f(h)$

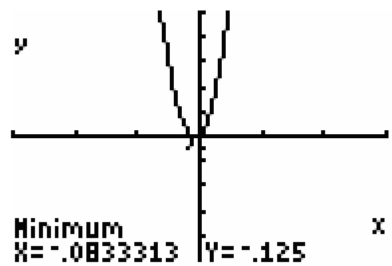
$= f\left(-\dfrac{1}{12}\right)$

$= 3\left(-\dfrac{1}{12}\right) + 18\left(-\dfrac{1}{12}\right)^2$

$= -\dfrac{3}{12} + 18\left(\dfrac{1}{144}\right)$

$= -\dfrac{1}{4} + \dfrac{1}{8} = -\dfrac{2}{8} + \dfrac{1}{8} = -\dfrac{1}{8}$

The vertex is $\left(-\dfrac{1}{12}, -\dfrac{1}{8}\right)$.

b. 

[–3, 3] by [–5, 5]

21. a. $y = 3x^2 + 18x - 3$

$a = 3, b = 18, c = -3$

$h = \dfrac{-b}{2a} = \dfrac{-18}{2(3)} = \dfrac{-18}{6} = -3$

$k = f(h)$

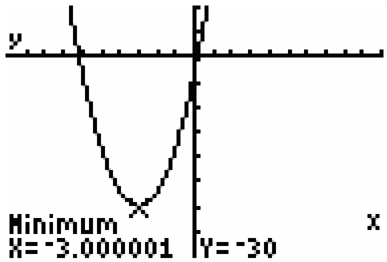
$= f(-3)$

$= 3(-3)^2 + 18(-3) - 3$

$= 3(9) - 54 - 3$

$= 27 - 54 - 3 = -30$

The vertex is $(-3, -30)$.

b. 

[–10, 10] by [–40, 10]

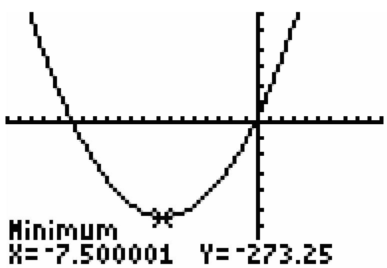
22. a. $y = 5x^2 + 75x + 8$

$a = 5, b = 75, c = 8$

$h = \frac{-b}{2a} = \frac{-75}{2(5)} = \frac{-75}{10} = -7.5$

$$\begin{aligned} k &= f(h) \\ &= f(-7.5) \\ &= 5(-7.5)^2 + 75(-7.5) + 8 \\ &= 5(56.25) - 562.5 + 8 \\ &= -273.25 \end{aligned}$$

The vertex is $(-7.5, -273.25)$.

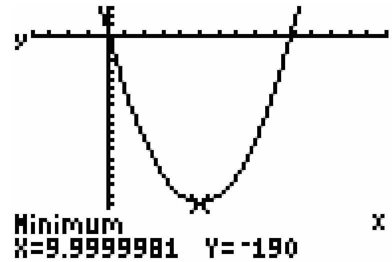
b. 

[–20, 10] by [–400, 300]

23. a. $y = 2x^2 - 40x + 10$

$a = 2, b = -40, c = 10$

$h = \frac{-b}{2a} = \frac{-(-40)}{2(2)} = \frac{40}{4} = 10$

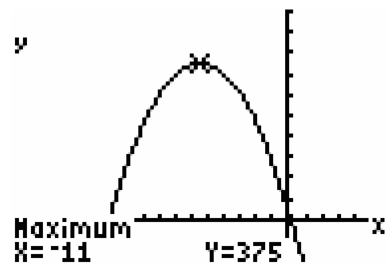
b. 

[–10, 30] by [–250, 30]

24. a. $y = -3x^2 - 66x + 12$

$a = -3, b = -66, c = 12$

$h = \frac{-b}{2a} = \frac{-(-66)}{2(-3)} = \frac{66}{-6} = -11$

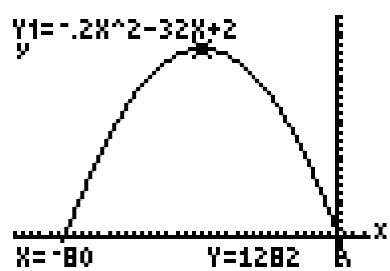
b. 

[–34, 12] by [–50, 500]

25. a. $y = -0.2x^2 - 32x + 2$

$a = -0.2, b = -32, c = 2$

$h = \frac{-b}{2a} = \frac{-(-32)}{2(-0.2)} = \frac{32}{-0.4} = -80$

b. 

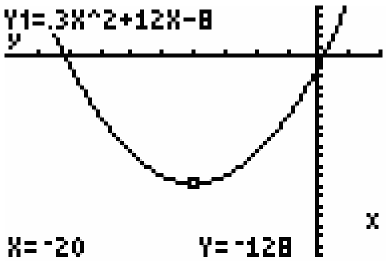
[–190, 30] by [–200, 1500]

26. a. $y = 0.3x^2 + 12x - 8$

$a = 0.3, b = 12, c = -8$

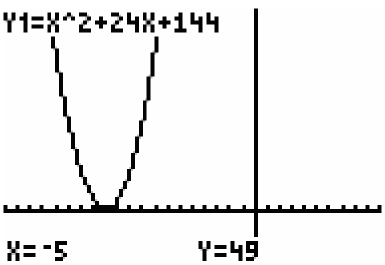
$$h = \frac{-b}{2a} = \frac{-(12)}{2(0.3)} = \frac{-12}{0.6} = -20$$

b.



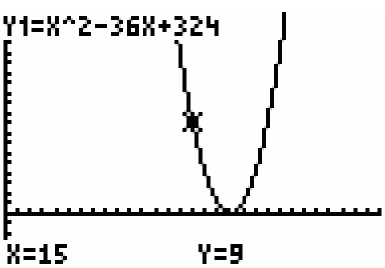
[–50, 10] by [–200, 50]

27.



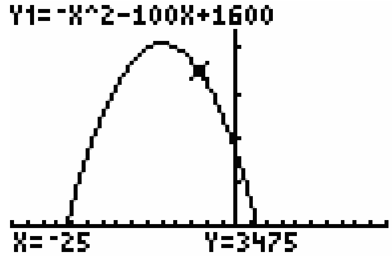
[–20, 10] by [–5, 20]

28.



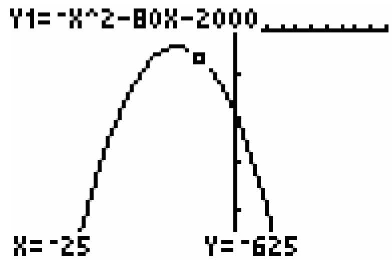
[0, 30] by [–5, 20]

29.



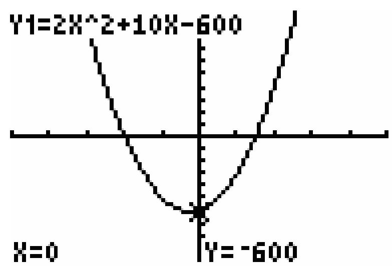
[–150, 100] by [–500, 5000]

30.



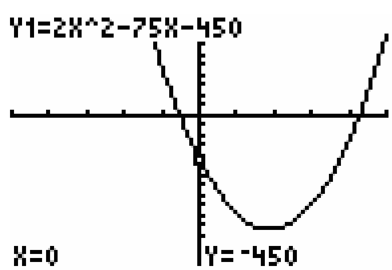
[–150, 100] by [–5000, 500]

31.



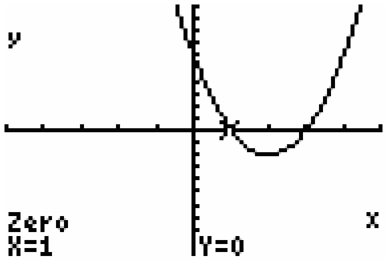
[–50, 50] by [–1000, 1000]

32.

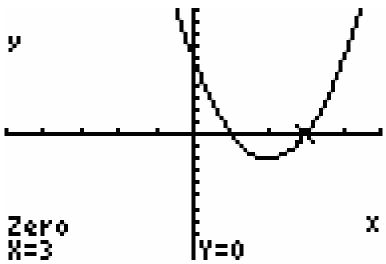


[–50, 50] by [–1500, 1000]

33. $(1,0),(3,0)$

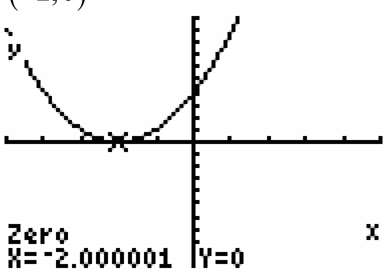


[–5, 5] by [–10, 10]



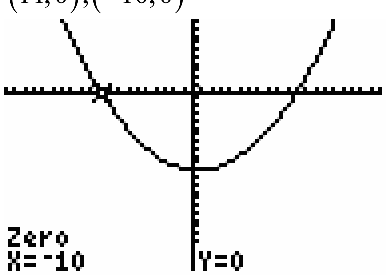
[–5, 5] by [–10, 10]

34. $(-2,0)$

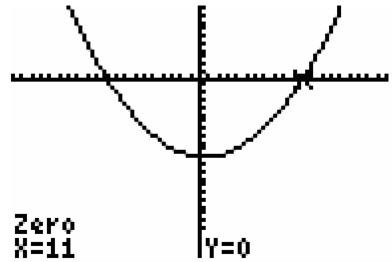


[–5, 5] by [–10, 10]

35. $(11,0),(-10,0)$

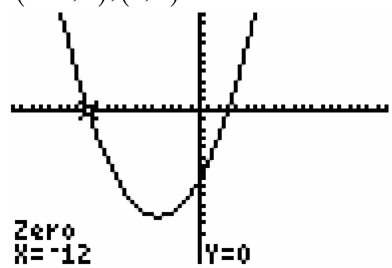


[–20, 20] by [–250, 100]

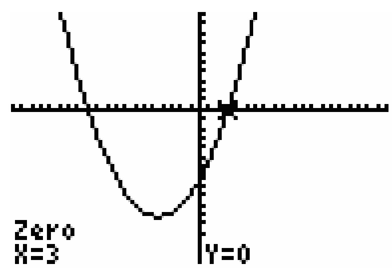


[–20, 20] by [–250, 100]

36. $(-12,0),(3,0)$

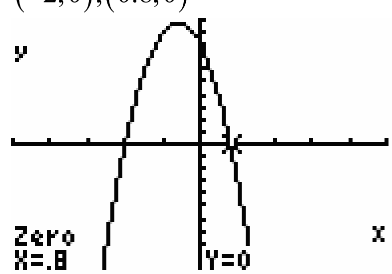


[–20, 20] by [–80, 50]

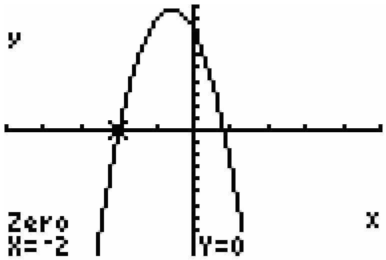


[–20, 20] by [–80, 50]

37. $(-2,0),(0.8,0)$

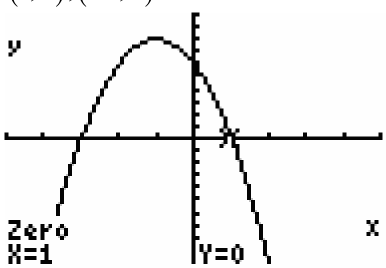


[–5, 5] by [–10, 10]

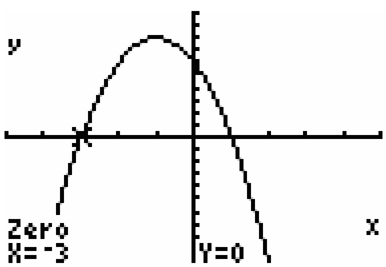


[–5, 5] by [–10, 10]

38. $(1,0),(-3,0)$



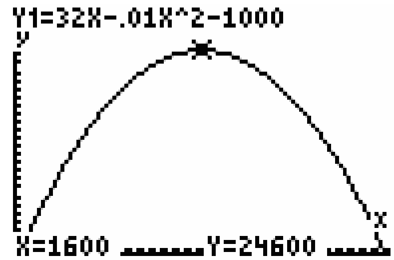
[–5, 5] by [–10, 10]



[–5, 5] by [–10, 10]

Section 2.1 Exercises

39. a.

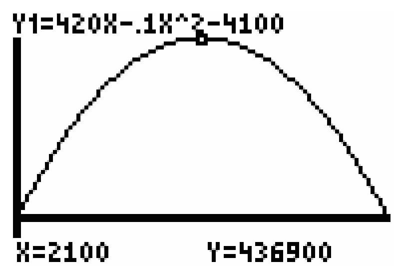


[0, 3200] by [0, 25,000]

b. For x between 1 and 1600, the profit is increasing.

c. For x greater than 1600, the profit decreases.

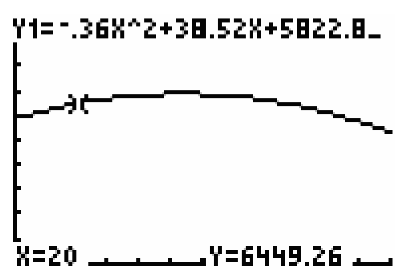
40. a.



[0, 4200] by [0, 500,000]

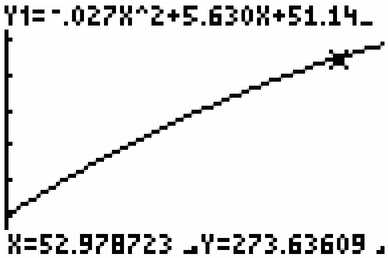
b. The graph is concave down.

41. a.



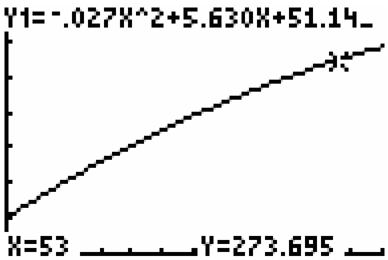
[0, 120] by [0, 10,000]

b. In 2010, x = 2010–1990 =20. Based on the graph of the model in part *a*, the population is 6449.3 million or 6.4493 billion people.

42. a. 

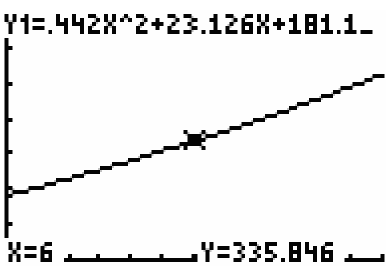
[0, 60] by [0, 350]

b. In 2003, $x = 2003–1950 = 53$.



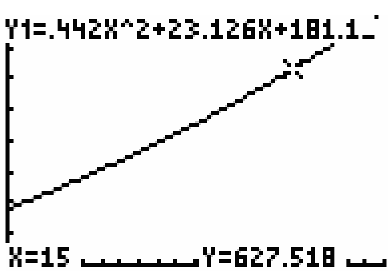
[0, 60] by [0, 350]

The model estimates the number of arrests in 2003 to be 273.695 per 100,000 juveniles.

43. a. 

[0, 12] by [0, 700]

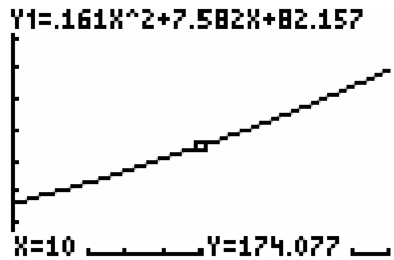
b. In 2002, $x = 2002–1987 = 15$.



[0, 20] by [0, 800]

The projected tourism spending is \$627.52 billion.

c. The calculation part b) is an extrapolation because $x = 15$ is beyond the scope of the original x-values. The model is applicable only in years between 1987 and 1999.

44. a. 

[0, 20] by [0, 400]

b. The function is increasing.

c. The government official must plan on housing more inmates. They need to build more jail space or change the laws so that the fewer people are incarcerated.

45. a. The model is a quadratic function. The graph of the model is a concave down parabola.

b. $h = \dfrac{-b}{2a} = \dfrac{-96}{2(-16)} = \dfrac{-96}{-32} = 3$

$$\begin{aligned} k &= f(h) = f(3) \\ &= 100 + 96(3) - 16(3)^2 \\ &= 100 + 288 - 144 \\ &= 244 \end{aligned}$$

The vertex is $(3, 244)$.

c. Three seconds into the flight of the ball, the ball is 244 feet above the ground. The ball reaches its maximum height of 244 feet in 3 seconds.

46. a. $h = \frac{-b}{2a} = \frac{-39.2}{2(-9.8)} = \frac{-39.2}{-19.6} = 2$

$k = f(h) = f(2)$

$= 30 + 39.2(2) - 9.8(2)^2$

$= 30 + 78.4 - 39.2$

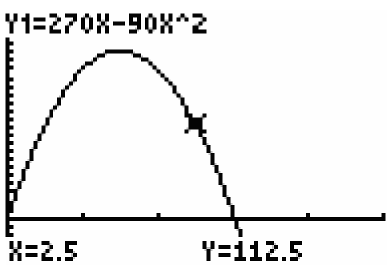
$= 69.2$

The vertex is $(2, 69.2)$.

b. Two seconds into the flight of the ball, the ball is 69.2 meters above the ground. The ball reaches its maximum height of 69.2 meters in 2 seconds.

c. The function is increasing until $t = 2$ seconds. The ball rises for two seconds, at which time it reaches its maximum height. After two seconds the ball falls toward the ground.

47. a.

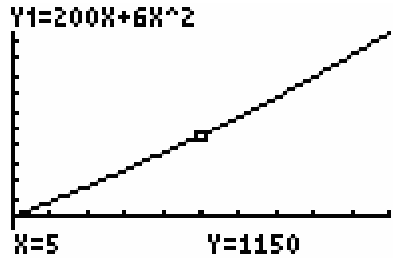


[0, 5] by [–50, 250]

b. $h = \frac{-b}{2a} = \frac{-270}{2(-90)} = \frac{-270}{-180} = 1.5$

1.5 lumens yields the maximum rate of photosynthesis.

48.



[0, 10] by [–500, 3000]

49. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.

a. $h = \frac{-b}{2a} = \frac{-40}{2(-0.01)} = \frac{-40}{-0.02} = 2000$

b. $k = P(h) = P(2000)$

$= 40(2000) - 3000 - 0.01(2000)^2$

$= 80{,}000 - 3000 - 0.01(4{,}000{,}000)$

$= 80{,}000 - 3000 - 40{,}000$

$= 37{,}000$

Producing 2000 units yields a maximum profit of $37,000.

50. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.

a. $h = \frac{-b}{2a} = \frac{-840}{2(-0.4)} = \frac{-840}{-0.8} = 1050$

b. $k = P(h) = P(1050)$

$= 840(1050) - 75.6 - 0.4(1050)^2$

$= 882{,}000 - 75.6 - 441{,}000$

$= 440{,}924.4$

Producing 1050 units yields a maximum profit of $440,924.40.

51. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.

a. $h = \frac{-b}{2a} = \frac{-1500}{2(-0.02)} = \frac{-1500}{-0.04} = 37,500$

b. $k = R(h) = R(37,500)$
$= 1500(37,500) - 0.02(37,500)^2$
$= 56,250,000 - 28,125,000$
$= 28,125,000$

Selling 37,500 units yields a maximum revenue of \$28,125,000.

52. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.

a. $h = \frac{-b}{2a} = \frac{-300}{2(-0.01)} = \frac{-300}{-0.02} = 15,000$

b. $k = R(h) = R(15,000)$
$= 300(15,000) - 0.01(15,000)^2$
$= 4,500,000 - 2,250,000$
$= 2,250,000$

Selling 15,000 units yields a maximum revenue of \$2,250,000.

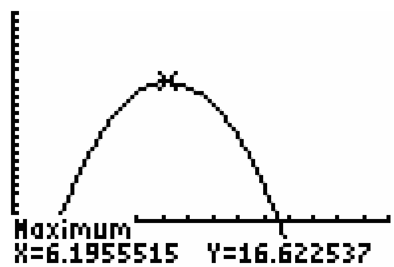
53. a. Yes. $A = x(100 - x) = 100x - x^2$. Note that A fits the form $f(x) = ax^2 + bx + c, a \neq 0$.

b. The maximum area will occur at the vertex of graph of function A.
$h = \frac{-b}{2a} = \frac{-100}{2(-1)} = \frac{-100}{-2} = 50$
$k = A(h) = A(50)$
$= 100(50) - (50)^2$
$= 5000 - 2500$
$= 2500$

The maximum area of the pen is 2500 square feet.

54. $A = (12,500 - x)x$
$A = 12,500x - x^2$
$h = \frac{-b}{2a} = \frac{-12,500}{2(-1)} = \frac{-12,500}{-2} = 6250$
$k = f(h) = A(6250) =$
$12,500(6250) - (6250)^2 =$
$39,062,500$
The maximum area is $39,062,500$ square feet.

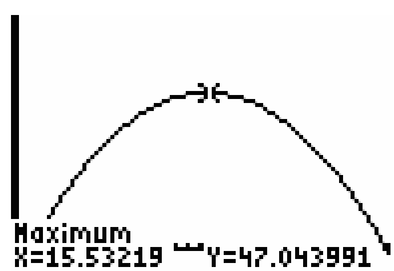
55.



[0, 15] by [–5, 25]

The number of schools with satellite dishes reached a maximum between 1992 and 1997. An x-value of approximately 6.2 corresponds with the year 1996. The maximum number of dishes is approximately 16,623.

56. a.



[0, 30] by [–5, 70]

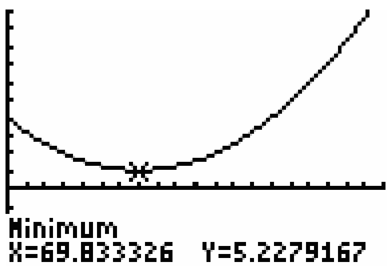
The number of visitors reaches a maximum during the given time frame.

b. Considering the graph in part *a*, the vertex is $(15.53, 47.04)$.

c. When x is 15.53, the year is 1996. In 1996, the number of U.S. visitors reached a maximum of approximately 47.04 million.

57. a. Since $a = 0.003 > 0$, the graph is concave up. The vertex is a minimum.

b. Using a graphing utility yields,



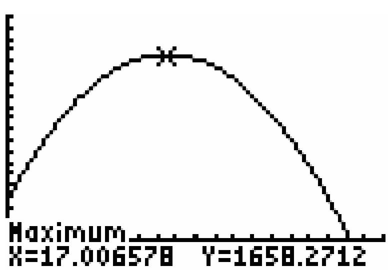
[0, 200] by [–20, 50]

The vertex is approximately $(69.8, 5.2)$.

c. In 1970 (when $x = 69.8$), the U.S. population reached a minimum percentage for foreign born people of 5.2%.

d. Consider part *a* above.

58. a.

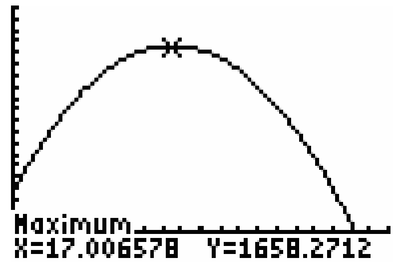


[0, 40] by [–200, 2000]

The maximum occurred when $x = 17$ which corresponds to 1987.

b. The maximum number of abortions is approximately 1658 thousand or 1,650,000.

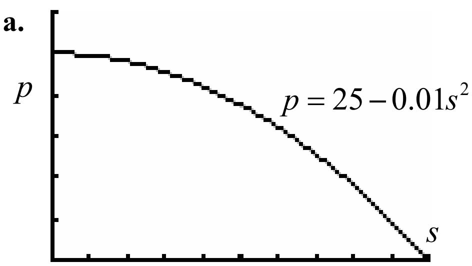
c. The model no longer applies when the number of abortions becomes negative.



[0, 40] by [–200, 2000]

The number of abortions becomes negative in 2007.

59. a.



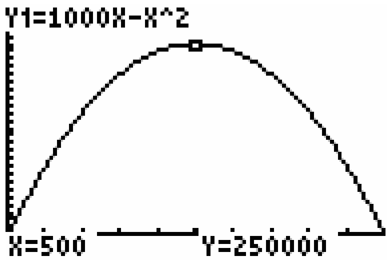
[0, 50] by [0, 30]

b. Decreasing. The graph is falling as s increases.

c. $(0, 25)$.

d. When the wind speed is zero, the amount of particulate pollution is 25 ounces per cubic yard.

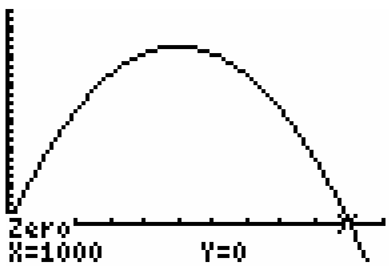
60. a.



[0, 1000] by [0, 300,000]

b. Increasing. The graph is rising as x increases to a value of 500. When x is greater than 500 the graph is decreasing.

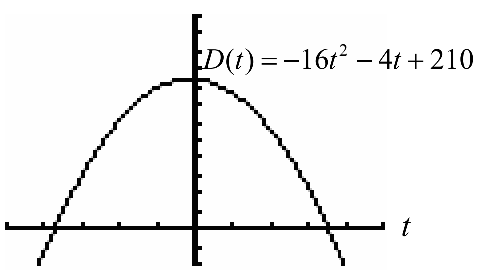
c. $(1000,0)$.



[0, 1100] by [–50,000, 300,000]

d. The sensitivity to the drug drops to zero when the dosage is 1000. This could indicate that the person is overdosed on the drug and no longer able to detect sensitivity to the drug.

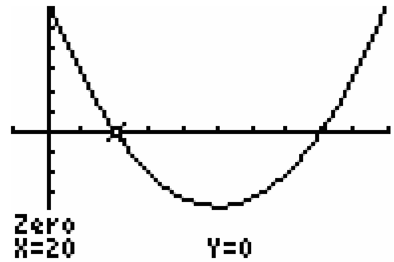
61.



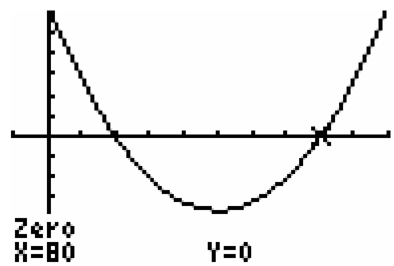
[–5, 5] by [–50, 300]

The t-intercepts are approximately $(-3.75,0)$ and $(3.5,0)$. The $(3.5,0)$ makes sense because time is understood to be positive. In the context of the question, the tennis ball will hit the swimming pool in 3.5 seconds.

62.



[–10, 100] by [–1500, 1500]



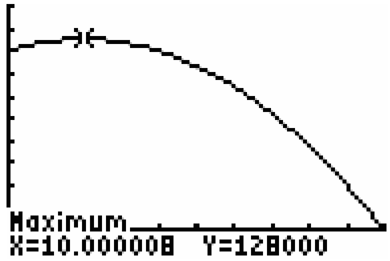
[–10, 100] by [–1500, 1500]

Break-even occurs when the number of units produced and sold is 20 or 80.

63. a.

Rent	Number of Apartments Rented	Total Revenue
\$1200	100	\$120,000
\$1240	98	\$121,520
\$1280	96	\$122,880
\$1320	94	\$124,080

b. Yes. Revenue = Rent multiplied by Number of Apartments Rented. $(1200+40x)$ represents the rent amount, while $(100-2x)$ represents the number of apartments rented.

c. 

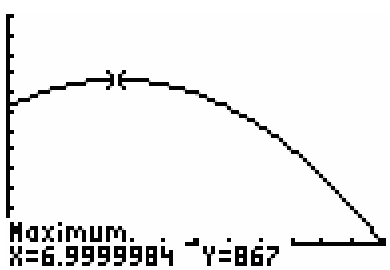
[0, 50] by [–15,000, 150,000]

The maximum occurs when $x = 10$. Therefore the most profitable rent to charge is $1200 + 40(10) = \$1600$.

64. a.

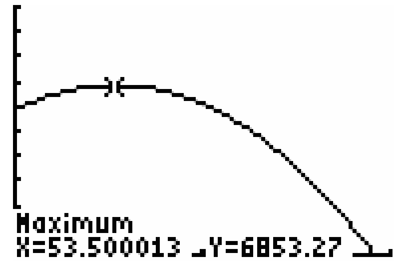
Cost per person	Number of Skaters	Total Revenue
\$11.50	66	\$759
\$11.00	72	\$792
\$10.50	78	\$819

b. Yes. Revenue = Cost multiplied by Number of Skaters. $(12 - 0.5x)$ represents the cost per skater, while $(60 + 6x)$ represents the number of skaters.

c. 

[0, 24] by [–100, 1200]

The maximum occurs when $x = 7$. Therefore the most profitable number of skaters is $60 + 6(7) = 102$.

65. a. 

[0, 200] by [0, 10,000]

The vertex is $(53.5, 6853)$.

b. The model predicts that in 2044 the population of the world will by 6853 million.

c. The model predicts a population increase from 1990 until 2044. After 2004, based on the model, the population of the world will decrease.

Section 2.2 Skills Check

1. $x^2 - 3x - 10 = 0$
$(x-5)(x+2) = 0$
$x - 5 = 0, x + 2 = 0$
$x = 5, x = -2$

2. $x^2 - 9x + 18 = 0$
$(x-6)(x-3) = 0$
$x - 6 = 0, x - 3 = 0$
$x = 6, x = 3$

3. $x^2 - 11x + 24 = 0$
$(x-8)(x-3) = 0$
$x - 8 = 0, x - 3 = 0$
$x = 8, x = 3$

4. $x^2 + 3x - 10 = 0$
$(x+5)(x-2) = 0$
$x + 5 = 0, x - 2 = 0$
$x = -5, x = 2$

5. $2x^2 + 2x - 12 = 0$
$2(x^2 + x - 6) = 0$
$2(x+3)(x-2)$
$x + 3 = 0, x - 2 = 0$
$x = -3, x = 2$

6. $2s^2 + s - 6 = 0$
Note that $2s^2 \bullet -6 = -12s^2$. Look for two terms whose product is $-12s^2$ and whose sum is the middle term, s.
$2s^2 + 4s - 3s - 6 = 0$
$(2s^2 + 4s) + (-3s - 6) = 0$
$2s(s+2) - 3(s+2) = 0$
$(2s-3)(s+2) = 0$
$2s - 3 = 0, s + 2 = 0$
$x = \frac{3}{2}, x = -2$

7. $0 = 2t^2 - 11t + 12$
$2t^2 - 11t + 12 = 0$
Note that $2t^2 \bullet 12 = 24t^2$. Look for two terms whose product is $24t^2$ and whose sum is the middle term, $-11t$.
$2t^2 - 8t - 3t + 12 = 0$
$(2t^2 - 8t) + (-3t + 12) = 0$
$2t(t-4) - 3(t-4) = 0$
$(2t-3)(t-4) = 0$
$2t - 3 = 0, t - 4 = 0$
$x = \frac{3}{2}, x = 4$

8. $6x^2 - 13x + 6 = 0$

Note that $6x^2 \bullet 6 = 36x^2$. Look for two terms whose product is $36x^2$ and whose sum is the middle term, $-13x$.

$6x^2 - 9x - 4x + 6 = 0$

$(6x^2 - 9x) + (-4x + 6) = 0$

$3x(2x - 3) - 2(2x - 3) = 0$

$(3x - 2)(2x - 3) = 0$

$3x - 2 = 0, 2x - 3 = 0$

$x = \frac{2}{3}, x = \frac{3}{2}$

9. $6x^2 + 10x = 4$

$6x^2 + 10x - 4 = 0$

$2(3x^2 + 5x - 2) = 0$

Note that $3x^2 \bullet -2 = -6x^2$. Look for two terms whose product is $-6x^2$ and whose sum is the middle term, $5x$.

$2(3x^2 + 6x - 1x - 2) = 0$

$2[(3x^2 + 6x) + (-1x - 2)] = 0$

$2[3x(x + 2) + (-1)(x + 2)] = 0$

$2(3x - 1)(x + 2) = 0$

$3x - 1 = 0, x + 2 = 0$

$x = \frac{1}{3}, x = -2$

10. $10x^2 + 11x = 6$

$10x^2 + 11x - 6 = 0$

Note that $10x^2 \bullet -6 = -60x^2$. Look for two terms whose product is $-60x^2$ and whose sum is the middle term, $11x$.

$10x^2 + 15x - 4x - 6 = 0$

$(10x^2 + 15x) + (-4x - 6) = 0$

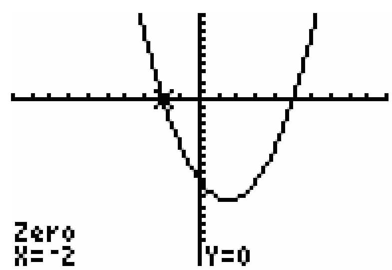
$5x(2x + 3) + (-2)(2x + 3) = 0$

$(2x + 3)(5x - 2) = 0$

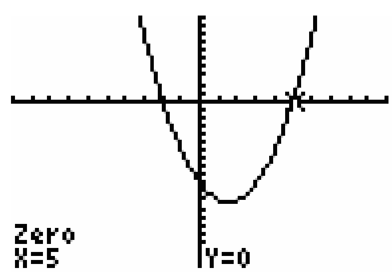
$2x + 3 = 0, 5x - 2 = 0$

$x = -\frac{3}{2}, x = \frac{2}{5}$

11. $y = x^2 - 3x - 10$



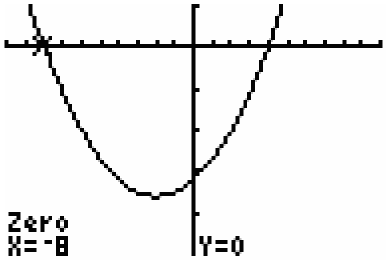
[–10, 10] by [–20, 10]



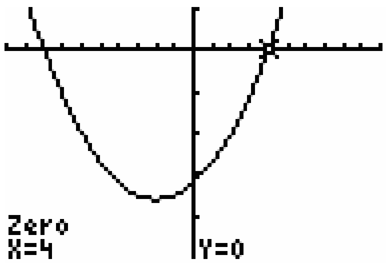
[–10, 10] by [–20, 10]

The x-intercepts are $(-2, 0)$ and $(5, 0)$.

12. $y = x^2 + 4x - 32$



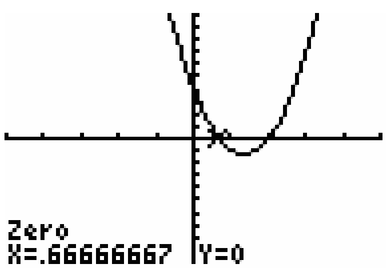
[–10, 10] by [–50, 10]



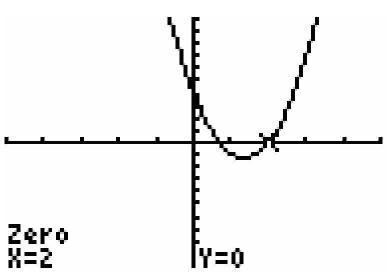
[–10, 10] by [–50, 10]

The x-intercepts are $(-8,0)$ and $(4,0)$.

13. $y = 3x^2 - 8x + 4$



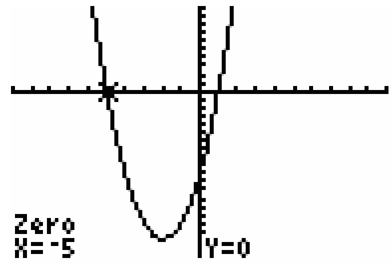
[–5, 5] by [–10, 10]



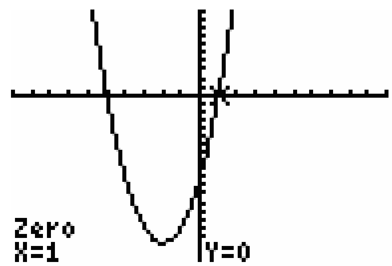
[–5, 5] by [–10, 10]

The x-intercepts are $\left(\frac{2}{3},0\right)$ and $(2,0)$.

14. $y = 2x^2 + 8x - 10$



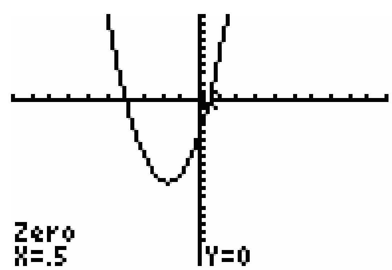
[–10, 10] by [–20, 10]



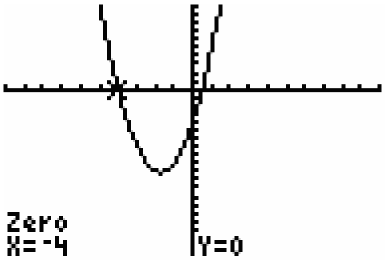
[–10, 10] by [–20, 10]

The x-intercepts are $(1,0)$ and $(-5,0)$.

15. $y = 2x^2 + 7x - 4$



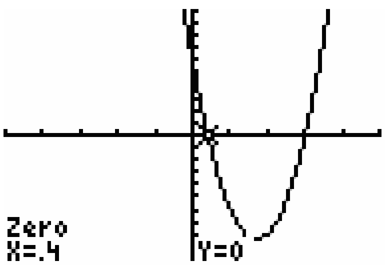
[–10, 10] by [–20, 10]



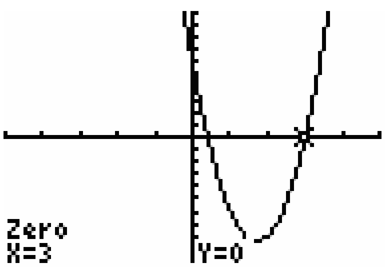
[–10, 10] by [–20, 10]

The x-intercepts are $(-4,0)$ and $(0.5,0)$.

16. $y = 5x^2 - 17x + 6$



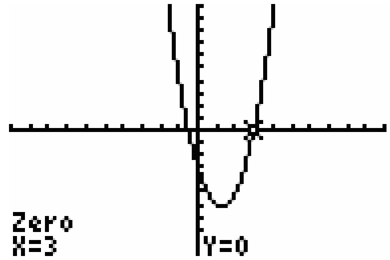
[–5, 5] by [–10, 10]



[–5, 5] by [–10, 10}

The x-intercepts are $\left(\frac{2}{5},0\right)$ and $(3,0)$.

17.



[–10, 10] by [–10, 10]

Since $w = 3$ is an x-intercept, then $w - 3$ is a factor.

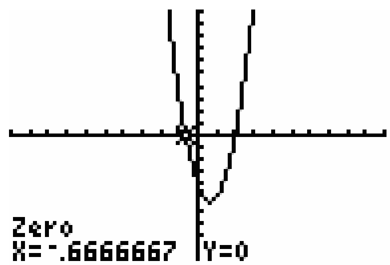
$2w^2 - 5w - 3 = 0$

$(w-3)(2w+1) = 0$

$w - 3 = 0, 2w + 1 = 0$

$w = 3, w = -\frac{1}{2}$

18.



[–10, 10] by [–10, 10]

Since $x = -\frac{2}{3}$ is an x-intercept, then

$x + \frac{2}{3}$ is a factor.

Clearing fractions yields $3x + 2$ as a factor.

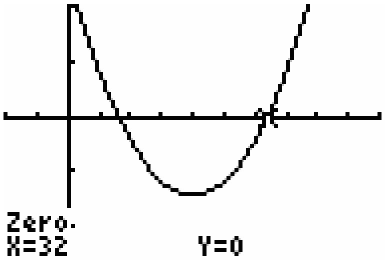
$3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0$

$3x + 2 = 0, x - 2 = 0$

$x = -\frac{2}{3}, x = 2$

19.



[–10, 50] by [–250, 200]

Since $x = 32$ is an x-intercept, then $x - 32$ is a factor.

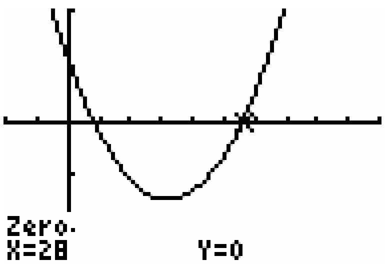
$x^2 - 40x + 256 = 0$

$(x - 32)(x - 8) = 0$

$x - 32 = 0, x - 8 = 0$

$x = 32, x = 8$

20.



[–10, 50] by [–250, 200]

Since $x = 28$ is an x-intercept, then $x - 28$ is a factor.

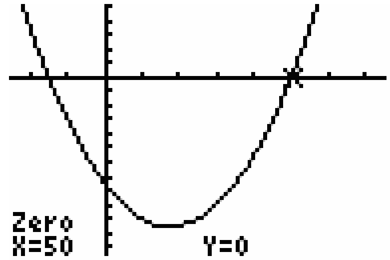
$x^2 - 32x + 112 = 0$

$(x - 28)(x - 4) = 0$

$x - 28 = 0, x - 4 = 0$

$x = 28, x = 4$

21.



[–25, 75] by [–2500, 1000]

Since $x = 50$ is an x-intercept for $2s^2 - 70s - 1500 = 0$, then $s - 50$ is a factor.

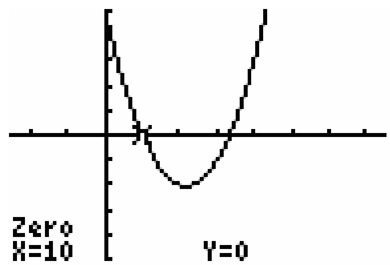
$(s - 50)(2s + 30) = 0$

$2(s - 50)(s + 15) = 0$

$s - 50 = 0, s + 15 = 0$

$s = 50, s = -15$

22.



[–25, 75] by [–1000, 1000]

Since $x = 10$ is an x-intercept for $3s^2 - 130s + 1000 = 0$, then $s - 10$ is a factor.

$(s - 10)(3s - 100) = 0$

$s - 10 = 0, 3s - 100 = 0$

$s = 10, s = \frac{100}{3}$

23. $4x^2 - 9 = 0$

$4x^2 = 9$

$\sqrt{4x^2} = \pm\sqrt{9}$

$2x = \pm 3$

$x = \pm\dfrac{3}{2}$

24. $x^2 - 20 = 0$

$x^2 = 20$

$\sqrt{x^2} = \pm\sqrt{20}$

$x = \pm\sqrt{20} = \pm\sqrt{(4)(5)} = \pm 2\sqrt{5}$

25. $x^2 - 32 = 0$

$x^2 = 32$

$\sqrt{x^2} = \pm\sqrt{32}$

$x = \pm\sqrt{32} = \pm\sqrt{16 \bullet 2} = \pm 4\sqrt{2}$

26. $5x^2 - 25 = 0$

$5x^2 = 25$

$x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

27. $x^2 - 4x - 9 = 0$

$\left(\dfrac{-4}{2}\right)^2 = (-2)^2 = 4$

$x^2 - 4x + 4 - 4 - 9 = 0$

$\left(x^2 - 4x + 4\right) + (-4 - 9) = 0$

$(x-2)^2 - 13 = 0$

$(x-2)^2 = 13$

$\sqrt{(x-2)^2} = \pm\sqrt{13}$

$x - 2 = \pm\sqrt{13}$

$x = 2 \pm \sqrt{13}$

28. $x^2 - 6x + 1 = 0$

$\left(\dfrac{-6}{2}\right)^2 = (-3)^2 = 9$

$x^2 - 6x + 9 - 9 + 1 = 0$

$\left(x^2 - 6x + 9\right) + (-9 + 1) = 0$

$(x-3)^2 - 8 = 0$

$(x-3)^2 = 8$

$\sqrt{(x-3)^2} = \pm\sqrt{8}$

$x - 3 = \pm\sqrt{2^2 \bullet 2}$

$x = 3 \pm 2\sqrt{2}$

29. $x^2 - 3x + 2 = 0$

$\left(\dfrac{-3}{2}\right)^2 = \dfrac{9}{4}$

$x^2 - 3x + \dfrac{9}{4} - \dfrac{9}{4} + 2 = 0$

$\left(x^2 - 3x + \dfrac{9}{4}\right) + \left(-\dfrac{9}{4} + 2\right) = 0$

$\left(x - \dfrac{3}{2}\right)^2 + \left(-\dfrac{9}{4} + \dfrac{8}{4}\right) = 0$

$\left(x - \dfrac{3}{2}\right)^2 - \dfrac{1}{4} = 0$

$\left(x - \dfrac{3}{2}\right)^2 = \dfrac{1}{4}$

$\sqrt{\left(x - \dfrac{3}{2}\right)^2} = \pm\sqrt{\dfrac{1}{4}}$

$x - \dfrac{3}{2} = \pm\dfrac{1}{2}$

$x = \dfrac{3}{2} \pm \dfrac{1}{2}$

$x = \dfrac{3}{2} + \dfrac{1}{2}, x = \dfrac{3}{2} - \dfrac{1}{2}$

$x = 2, x = 1$

30. $2x^2 - 9x + 8 = 0$

$$2\left(x^2 - \frac{9}{2}x\right) + 8 = 0$$

$$\left(-\frac{\left(\frac{9}{2}\right)}{2}\right)^2 = \left(-\frac{9}{4}\right)^2 = \frac{81}{16}$$

$$2\left(x^2 - \frac{9}{4}x + \frac{81}{16} - \frac{81}{16}\right) + 8 = 0$$

$$2\left(x^2 - \frac{9}{4}x + \frac{81}{16}\right) + \left[2\left(-\frac{81}{16}\right) + 8\right] = 0$$

$$2\left(x - \frac{9}{4}\right)^2 + \left(-\frac{162}{16} + \frac{128}{16}\right) = 0$$

$$2\left(x - \frac{9}{4}\right)^2 - \frac{34}{16} = 0$$

$$2\left(x - \frac{9}{4}\right)^2 = \frac{17}{8}$$

$$\sqrt{\left(x - \frac{9}{4}\right)^2} = \pm\sqrt{\frac{17}{16}}$$

$$x - \frac{9}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{9}{4} \pm \frac{\sqrt{17}}{4}$$

$$x = \frac{9 \pm \sqrt{17}}{4}$$

31. $x^2 - 5x + 2 = 0$

$a = 1, b = -5, c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{5 \pm \sqrt{17}}{2}$$

32. $3x^2 - 6x - 12 = 0$

$a = 3, b = -6, c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-12)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 + 144}}{6}$$

$$= \frac{6 \pm \sqrt{180}}{6}$$

$$= \frac{6 \pm \sqrt{2^2 \cdot 3^2 \cdot 5}}{6}$$

$$= \frac{6 \pm (2 \cdot 3)\sqrt{5}}{6}$$

$$= \frac{6 \pm 6\sqrt{5}}{6}$$

$$= \frac{\cancel{6}\left(1 \pm \sqrt{5}\right)}{\cancel{6}}$$

$$= 1 \pm \sqrt{5}$$

33. $5x + 3x^2 = 8$

$3x^2 + 5x - 8 = 0$

$a = 3, b = 5, c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(-8)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 + 96}}{6}$$

$$x = \frac{-5 \pm \sqrt{121}}{6}$$

$$x = \frac{-5 \pm 11}{6}$$

$$x = \frac{-5 + 11}{6}, x = \frac{-5 - 11}{6}$$

$$x = 1, x = -\frac{16}{6} = -\frac{8}{3}$$

34. $3x^2 - 30x - 180 = 0$

$3\left(x^2 - 10x - 60\right) = 0$

$a = 1, b = -10, c = -60$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-60)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 + 240}}{2}$$

$$x = \frac{10 \pm \sqrt{340}}{2}$$

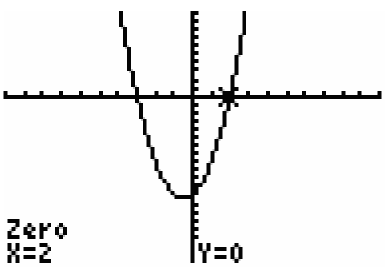
$$x = \frac{10 \pm \sqrt{4 \bullet 85}}{2}$$

$$x = \frac{10 \pm 2\sqrt{85}}{2}$$

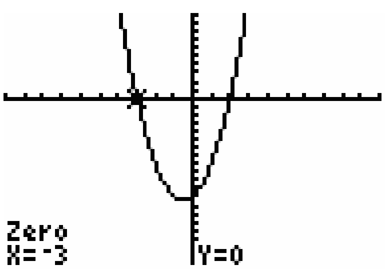
$x = 5 \pm \sqrt{85}$

$x = 5 + \sqrt{85}, x = 5 - \sqrt{85}$

35.



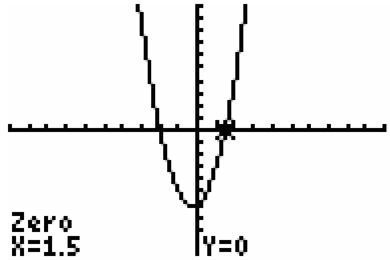
[–10, 10] by [–20, 10]



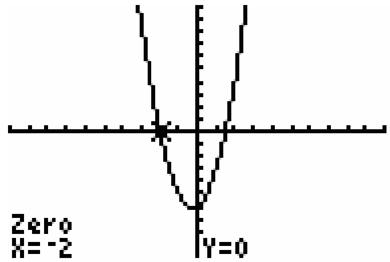
[–10, 10] by [–20, 10]

The solutions are $x = 2$ and $x = -3$.

36.



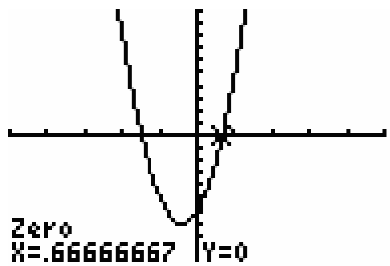
[–10, 10] by [–10, 10]



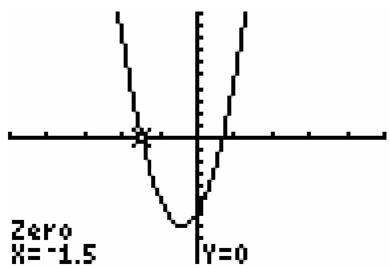
[–10, 10] by [–10, 10]

The solutions are $x = -2$ and $x = 1.5$.

37.



[–5, 5] by [–10, 10]

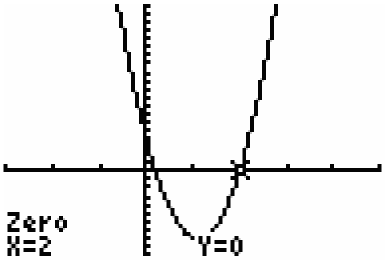


[–5, 5] by [–10, 10]

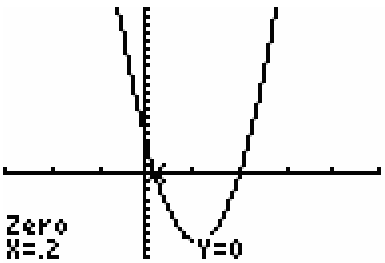
The solutions are $x = \frac{2}{3}$ and $x = -\frac{3}{2}$.

38. $10x^2 = 22x - 4$

$10x^2 - 22x + 4 = 0$



[–3, 5] by [–10, 20]



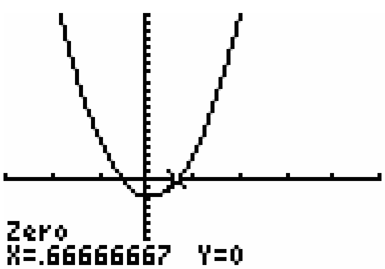
[–3, 5] by [–10, 20]

The solutions are $x = 0.2 = \frac{1}{5}$ and $x = 2$.

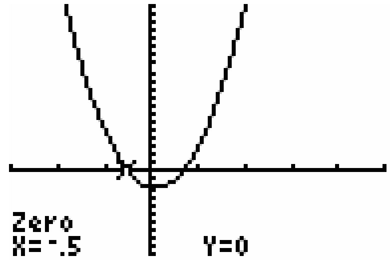
39. $4x + 2 = 6x^2 + 3x$

$0 = 6x^2 + 3x - 4x - 2$

$6x^2 - x - 2 = 0$



[–3, 5] by [–10, 20]



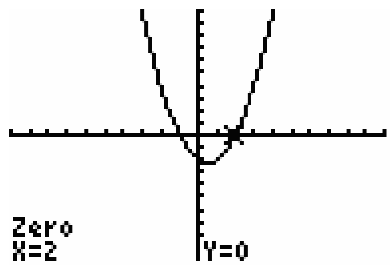
[–3, 5] by [–10, 20]

The solutions are $x = \frac{2}{3}$ and $x = -\frac{1}{2}$.

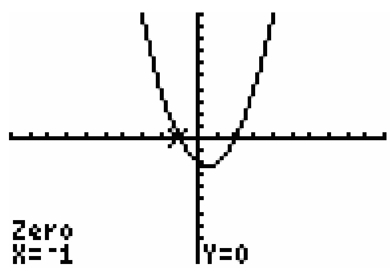
40. $(x-3)(x+2) = -4$

$x^2 + 2x - 3x - 6 = -4$

$x^2 - x - 2 = 0$



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

The solutions are $x = 2$ and $x = -1$.

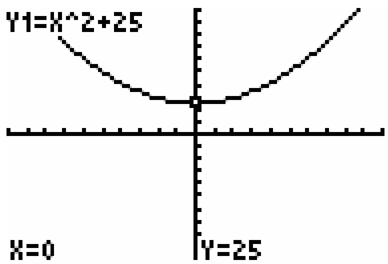
41. $x^2 + 25 = 0$

$x^2 = -25$

$\sqrt{x^2} = \pm\sqrt{-25}$

$x = \pm 5i$

Graphical check



[–10, 10] by [–100, 100]

Note that the graph has no x-intercepts.

42. $2x^2 + 40 = 0$

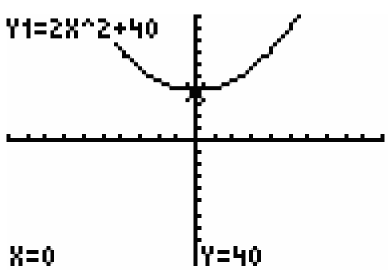
$2x^2 = -40$

$x^2 = 20$

$\sqrt{x^2} = \pm\sqrt{20}$

$x = \pm 2i\sqrt{5}$

Graphical check



[–10, 10] by [–100, 100]

Note that the graph has no x-intercepts.

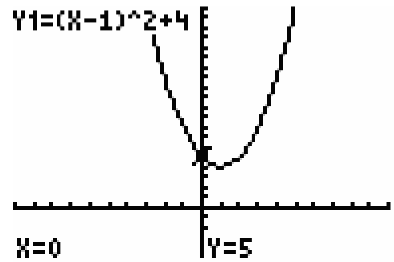
43. $(x-1)^2 = -4$

$\sqrt{(x-1)^2} = \pm\sqrt{-4}$

$x - 1 = \pm 2i$

$x = 1 \pm 2i$

Graphical check



[–10, 10] by [–5, 20]

Note that the graph has no x-intercepts.

44. $(2x+1)^2 + 7 = 0$

$(2x+1)^2 = -7$

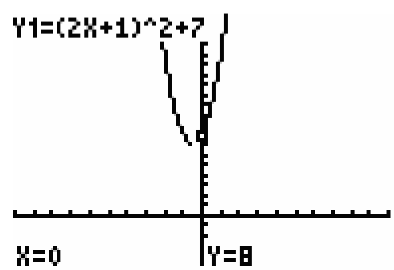
$\sqrt{(2x+1)^2} = \pm\sqrt{-7}$

$2x + 1 = \pm i\sqrt{7}$

$2x = -1 \pm i\sqrt{7}$

$x = \dfrac{-1 \pm i\sqrt{7}}{2}$

Graphical check



[–10, 10] by [–5, 20]

Note that the graph has no x-intercepts.

45. $x^2 + 4x + 8 = 0$

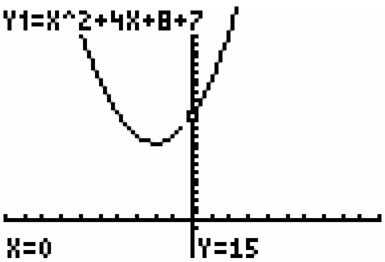
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{-16}}{2}$$

$$x = \frac{-4 \pm 4i}{2} = -2 \pm 2i$$

Graphical check



[–10, 10] by [–5, 30]

Note that the graph has no x-intercepts.

46. $x^2 - 5x + 7 = 0$

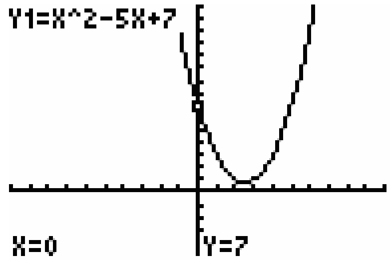
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{-3}}{2}$$

$$x = \frac{5 \pm i\sqrt{3}}{2}$$

Graphical check



[–10, 10] by [–5, 15]

Note that the graph has no x-intercepts.

47. $2x^2 - 8x + 9 = 0$

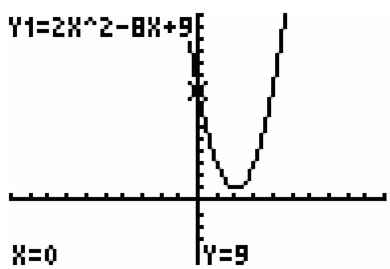
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(9)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{-8}}{4}$$

$$x = \frac{8 \pm 2i\sqrt{2}}{4} = \frac{4 \pm i\sqrt{2}}{2}$$

Graphical check



[–10, 10] by [–5, 15]

Note that the graph has no x-intercepts.

48. $x^2 - x + 1 = 0$

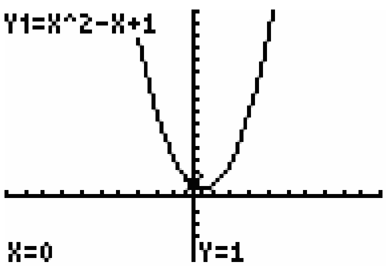
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

Graphical check



[–10, 10] by [–5, 15]

Note that the graph has no x-intercepts.

Section 2.2 Exercises

49. Let $S = 228$ and solve for t.

$228 = 100 + 96t - 16t^2$

$-16t^2 + 96t - 128 = 0$

$-16(t^2 - 6t + 8) = 0$

$-16(t - 4)(t - 2) = 0$

$t - 4 = 0, t - 2 = 0$

$t = 4, t = 2$

The ball is 228-feet high after 2 seconds and 4 seconds.

50. Let $D(t) = 44$ and solve for t.

$44 = -16t^2 - 4t + 200$

$-16t^2 - 4t + 156 = 0$

$-4(4t^2 + t - 39) = 0$

$-4(4t + 13)(t - 3) = 0$

$4t + 13 = 0, t - 3 = 0$

$t = -\frac{13}{4}, t = 3$

Since t represents time and time is not negative the only solution in the given physical context is $t = 3$. The ball is 44-feet high after 3 seconds.

51. Let $P(x) = 0$ and solve for x.

$-12x^2 + 1320x - 21{,}600 = 0$

$-12(x^2 - 110x + 1800) = 0$

$-12(x - 90)(x - 20) = 0$

$x - 90 = 0, x - 20 = 0$

$x = 90, x = 20$

Producing and selling either 20 units or 90 units produces a profit of zero dollars. Therefore 20 units or 90 units represent the break-even point for manufacturing and selling this product.

52. Let $P(x) = 0$ and solve for x.

$-15x^2 + 180x - 405 = 0$

$-15\left(x^2 - 12x + 27\right) = 0$

$-15(x-9)(x-3) = 0$

$x - 9 = 0, x - 3 = 0$

$x = 9, x = 3$

Producing and selling either 3 tons or 9 tons produces a profit of zero dollars. Therefore 3 tons and 9 tons represent the break-even point for manufacturing and selling this product.

53. a. $P(x) = R(x) - C(x)$

$P(x) = 550x - \left(10{,}000 + 30x + x^2\right)$

$P(x) = 550x - 10{,}000 - 30x - x^2$

$P(x) = -x^2 + 520x - 10{,}000$

b. Let $x = 18$.

$P(18) = -(18)^2 + 520(18) - 10{,}000$

$P(18) = -324 + 9360 - 10{,}000$

$p(18) = -964$

When 18 units are produced, there is a loss of \$964.

c. Let $x = 32$.

$P(32) = -(32)^2 + 520(32) - 10{,}000$

$P(32) = -1024 + 16{,}640 - 10{,}000$

$p(32) = 5616$

When 32 units are produced, there is a profit of \$5616.

d. Let $P(x) = 0$ and solve for x.

$0 = -x^2 + 520x - 10{,}000$

$-1\left(x^2 - 520x + 10{,}000\right) = 0$

$-1(x-500)(x-20) = 0$

$x - 500 = 0, x - 20 = 0$

$x = 500, x = 20$

To break even on this product the company needs to manufacture and sell either 20 units or 500 units.

54. a. $P(x) = R(x) - C(x)$

$P(x) = 266x - \left(2000 + 46x + 2x^2\right)$

$P(x) = 266x - 2000 - 46x - 2x^2$

$P(x) = -2x^2 + 220x - 2000$

b. Let $x = 55$

$P(55) = -2(55)^2 + 220(55) - 2000$

$P(55) = -6050 + 12{,}100 - 2000$

$p(55) = 4050$

When 55 units are produced, there is a profit of \$4050 thousand or \$4,050,000.

c. Let $P(x) = 0$ and solve for x.

$0 = -2x^2 + 220x - 2000$

$-2\left(x^2 - 110x + 1000\right) = 0$

$-2(x-100)(x-10) = 0$

$x - 100 = 0, x - 10 = 0$

$x = 100, x = 10$

To break even on this product the company needs to manufacture and sell either 100 units or 10 units.

55. a. Let $p = 0$ and solve for s.

$0 = 25 - 0.01s^2$

$-25 = -0.01s^2$

$\dfrac{-25}{-0.01} = \dfrac{-0.01}{-0.01}s^2$

$2500 = s^2$

$s = \pm\sqrt{2500} = \pm 50$

When s is 50 or –50, $p = 0$. Since a wind speed of –50 does not make physical sense, the only solution is 50. Therefore, when the wind speed is 50 mph the pollution in the air above the power plant is zero.

b. When $p = 0$, the pollution in the air above the power plant is zero.

c. Only the positive solution, $s = 50$, makes sense because wind speed must be positive.

56. a. $0 = 100x - x^2$

$0 = x(100 - x)$

$x = 0, 100 - x = 0$

$x = 0, x = 100$

Dosages of 0 ml and 100 ml give zero sensitivity.

b. When x is zero, there is no amount of drug in a person's system , and therefore no sensitivity to the drug. When x is 100 ml the amount of drug in a person's system is so high that the person may be overdosed on the drug and therefore has no sensitivity to the drug.

57. a. Let $v = 0.02$ and solve for r.

$0.02 = 2(0.01 - r^2)$

$\frac{0.02}{2} = \frac{2(0.01 - r^2)}{2}$

$0.01 = 0.01 - r^2$

$-r^2 = 0$

$r = 0$

A distance of 0 cm produces a velocity of 0.02 cm/sec.

b. Let $v = 0.015$ and solve for r.

$0.015 = 2(0.01 - r^2)$

$\frac{0.015}{2} = \frac{2(0.01 - r^2)}{2}$

$0.0075 = 0.01 - r^2$

$-r^2 = 0.0075 - 0.01$

$-r^2 = -0.0025$

$r^2 = 0.0025$

$r = \pm\sqrt{0.0025} = \pm 0.05$

A distance of 0.05 cm produces a velocity of 0.02 cm/sec. Since r is a distance, only $r = 0.05$ makes sense in the physical context of the question.

c. Let $v = 0$ and solve for r.

$0 = 2(0.01 - r^2)$

$\frac{0}{2} = \frac{2(0.01 - r^2)}{2}$

$0 = 0.01 - r^2$

$r^2 = 0.01$

$r = \pm\sqrt{0.01} = \pm 0.1$

A distance of 0.01 cm produces a velocity of 0 cm/sec. Since r is a distance, only $r = 0.1$ makes sense in the physical context of the question.

58. a. Let $s = 20$

$K^2 = 16(20) + 4$

$K^2 = 324$

$K = \pm\sqrt{324} = \pm 18$

If K is positive, then $K = 18$.

b. Let $s = 60$

$K^2 = 16(60) + 4$

$K^2 = 964$

$K = \pm\sqrt{964} = \pm\sqrt{(4)(241)} = \pm 2\sqrt{241}$

If K is positive, then $K = 2\sqrt{241}$.

c. $\frac{f(b) - f(a)}{b - a}$

$= \frac{f(60) - f(20)}{60 - 20}$

$= \frac{2\sqrt{241} - 18}{40}$

$= 0.3262087348 \approx 0.326$

The average rate of change of the function between 20 and 60 is 0.326 per one unit increase in wind speed.

59. Equilibrium occurs when demand is equal to supply. Solve

$$109.70 - 0.10q = 0.01q^2 + 5.91$$
$$0.01q^2 + 0.10q - 103.79 = 0$$
$$a = 0.01, b = 0.10, c = -103.79$$
$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$q = \frac{-0.10 \pm \sqrt{(0.10)^2 - 4(0.01)(-103.79)}}{2(0.01)}$$
$$q = \frac{-0.10 \pm \sqrt{0.01 + 4.1516}}{0.02}$$
$$q = \frac{-0.10 \pm \sqrt{4.1616}}{0.02}$$
$$q = \frac{-0.10 \pm 2.04}{0.02}$$
$$q = \frac{-0.10 + 2.04}{0.02}, q = \frac{-0.10 - 2.04}{0.02}$$
$$q = 97, q = -107$$

Since q represents the quantity of trees at equilibrium, q must be positive. A q-value of –107 does not make sense in the physical context of the question. Producing 9700 trees, when $q = 97$, creates an equilibrium price. The equilibrium price is given by

$$p = 109.70 - 0.10q$$
$$p = 109.70 - 0.10(97)$$
$$p = 109.70 - 9.70$$
$$p = 100.00 \text{ or } \$100.00 \text{ per tree.}$$

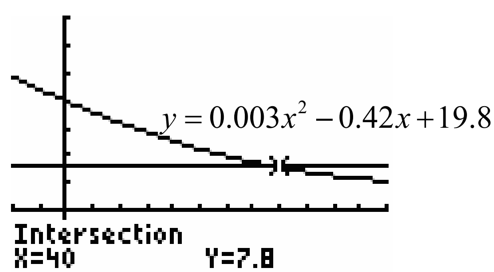
60. Equilibrium occurs when demand is equal to supply. Solve

$$7000 - 2x = 0.01x^2 + 2x + 1000$$
$$0.01x^2 + 4x - 6000 = 0$$
$$a = 0.01, b = 4, c = -6000$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(0.01)(-6000)}}{2(0.01)}$$
$$x = \frac{-4 \pm \sqrt{16 + 240}}{0.02}$$
$$x = \frac{-4 \pm \sqrt{256}}{0.02}$$
$$x = \frac{-4 \pm 16}{0.02}$$
$$x = \frac{-20}{0.02}, x = \frac{12}{0.02}$$
$$x = -1000, x = 600$$

Since x represents the quantity at equilibrium, x must be positive. An x-value of –1000 does not make sense in the physical context of the question. Producing 600 units creates an equilibrium price. The equilibrium price is given by

$$p = 7000 - 2x$$
$$p = 7000 - 2(600)$$
$$p = 7000 - 1200$$
$$p = 5800 \text{ or } \$5800 \text{ per unit.}$$

61. a.



[–10, 60] by [–10, 35]

b. Let $y = 7.8$ and solve for x.

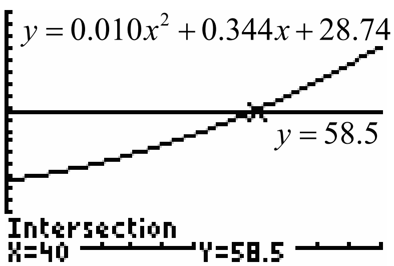
$$7.8 = 0.003x^2 - 0.42x + 19.8$$
$$0.003x^2 - 0.42x + 12 = 0$$
$$0.003\left(x^2 - 140x + 4000\right) = 0$$
$$0.003(x - 40)(x - 100) = 0$$
$$x = 40, x = 100$$

7.8% of the U.S. population is foreign born in 1940 and 2000.

62. a. In 2000 the percentage of people over age 19 who ever smoked and quit is 58.5%

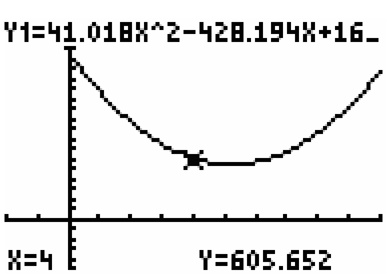
b. Not necessarily. The other solution is before 1960 ($x < 0$) and outside the range of the model. See the graph in part c) below.

c.



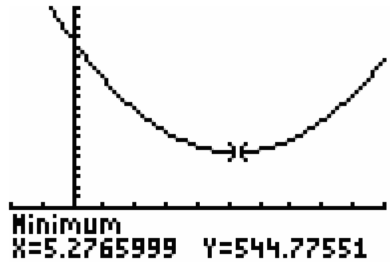
[0, 60] by [0, 100]

63. a. Begin by setting the x-range to [0, 10] in order to capture the years between 1989 and 1997. Then, note that when $x = 0$, $C(0) = 1662.14$. Adjust the y-range accordingly.



[–2, 10] by [–500, 2000]

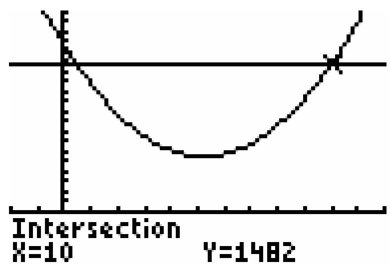
b.



[–2, 10] by [–500, 2000]

The minimum number of complaints occurred in approximately 1989 + 5 = 1994

c.

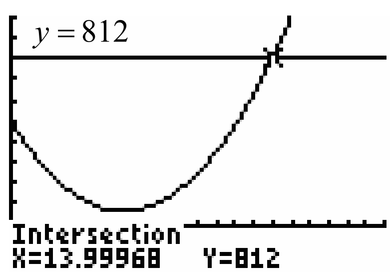


[–2, 12] by [0, 1800]

Baggage complaints are 1482 in 1989 + 10 = 1999.

d. The result in part c) is an extrapolation, since 1999 is beyond the scope of the original data, 1989–1997.

64. a.

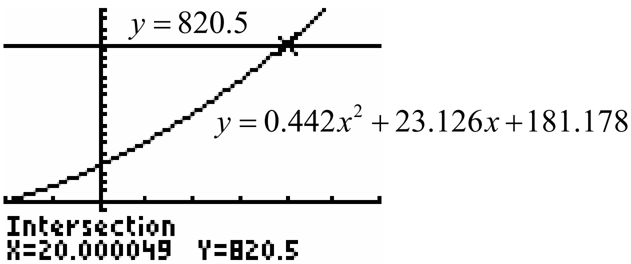


[0, 20] by [–200, 1000]

The model predicts that in 2004 the number of Internet users is 812 million.

b. The number of Internet users is predicted to be greater than the population of the United States.

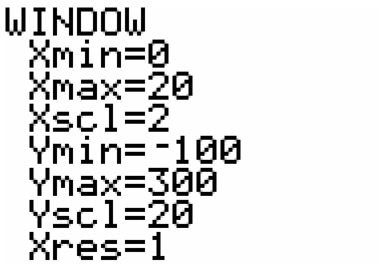
65.



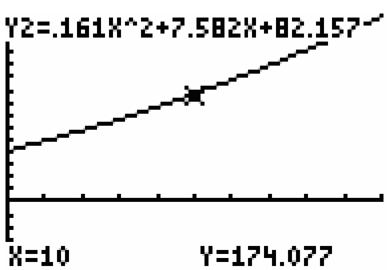
[–10, 30] by [–300, 1000]

In approximately 1987 + 20 = 2007 tourism spending will be $820.5 billion.

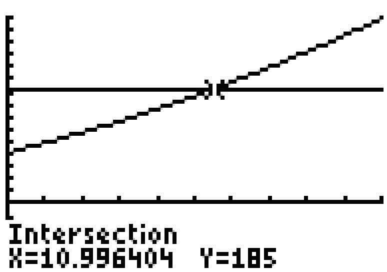
66. a.



Answers may vary. Since $x = 0$ corresponds to 1991, the first year for which the model is valid, $x_{\min}$ should be zero.



b.



The daily number of inmates is 185 in 2002 (1991 + 11).

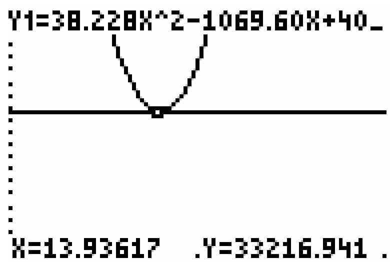
67. a. To determine the change in the number of hospital admissions between 1990 and 1997, calculate $A(1997-1980) - A(1990-1980) = A(17) - A(10)$.

$$A(17) = 38.228(17)^2 - 1069.60(17) + 40,698.547$$
$$= 11,047.892 - 18,183.2 + 40,698.547$$
$$= 33,563.239$$

$$A(10) = 38.228(10)^2 - 1069.60(10) + 40,698.547$$
$$= 3822.8 - 10,696 + 40,698.547$$
$$= 33,825.347$$

$$A(17) - A(10) = 33,563.239 - 33,825.347$$
$$= -262.108 \approx 262$$

There is a decrease of approximately
262 thousand people.

b. 

[10, 20] by [33,100, 33,300]

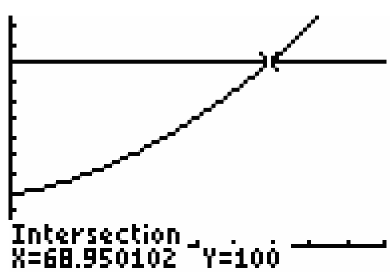
In about 1994 (1980 + 14) hospital admissions were 33,217,000.

c. $A(25) = 38.228(25)^2 - 1069.60(25) + 40,698.547$

$A(25) = 23,892.5 - 26,740 + 40,698.547$

$A(25) = 37,851.047$

Assuming the model continues to be valid beyond 1997, hospital admissions would be about 37,851 thousand or 37,851,000 in 2005.

68. 

[0, 100] by [–10, 125]

In 2029 the percentage of people over age 19 who stop smoking exceeds 100%. Therefore, the model can not be valid after 2028, the final year that the percentage of people over age 19 who stop smoking is less than 100%. Note that the model may become invalid earlier in time due to extrapolation.

69. a. The funds available in 1990 correspond to

$$\begin{aligned} f(10) &= 9.032(10)^2 + 99.970(10) + 3645.90 \\ &= 903.2 + 999.70 + 3645.90 \\ &= 5548.8 \text{ or } \$5548.8 \text{ million} \end{aligned}$$

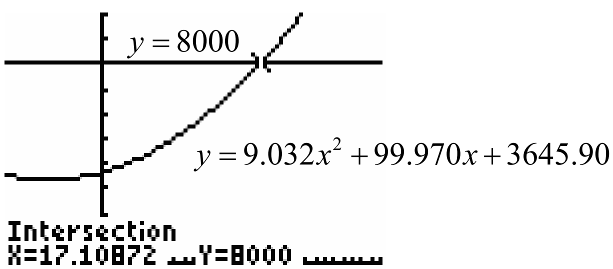
The funds available in 2000 correspond to

$$\begin{aligned} f(20) &= 9.032(20)^2 + 99.970(20) + 3645.90 \\ &= 3612.8 + 1999.4 + 3645.90 \\ &= 9258.10 \text{ or } \$9258.10 \text{ million} \end{aligned}$$

Calculating the change between 1990 and 2000

$$\begin{aligned} &f(20) - f(10) \\ &= 9258.10 - 5548.8 \\ &= 3709.3 \text{ or } \$3709.3 \text{ million} \end{aligned}$$

b.



[–10, 30] by [0, 10,000]

Federal aid exceeds $8000 million in 1998, 17.1 years after 1980.

70. a. In 1997, $t = 1997 - 1980 = 17$.

$$\begin{aligned} B(17) &= 3.268(17)^2 - 45.733(17) + 277.910 \\ &= 3.268(289) - 777.461 + 277.910 \\ &= 944.452 - 777.461 + 277.910 \\ &= 444.901 \approx 445 \end{aligned}$$

In 1997 there were approximately 445 bomb threats in the United States.

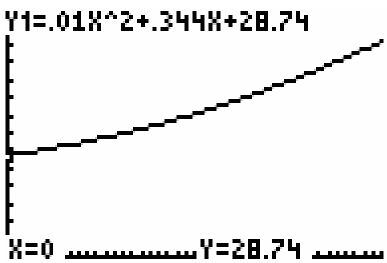
b.

X	Y1	Y2
21	758.71	1177
22	853.5	1177
23	954.82	1177
24	1062.7	1177
25	1177.1	1177
26	1298	1177
27	1425.5	1177

X=25

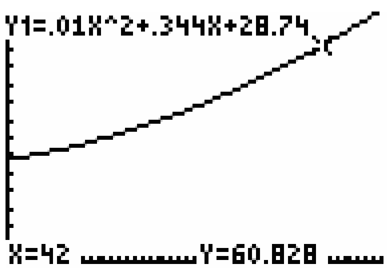
Based on the model, there will be approximately 1177 bomb threats in 2005 (1980 + 25).

71. a.



[0, 42] by [0, 70]

b.



[0, 50] by [0, 70]

In 2002 about 60.8% of the people who smoked after age nineteen will have quit.

c.

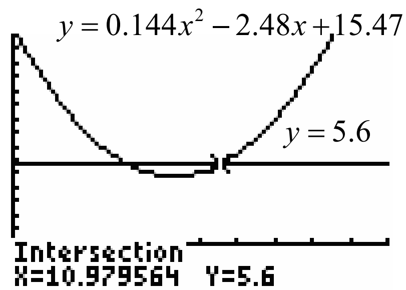
X	Y1	
28	46.212	
29	47.126	
30	48.06	
31	49.014	
32	49.988	
33	50.982	
34	51.996	

X=32

[0, 42] by [0, 70]

In 1992 (when $x = 32$), about 50% of people who smoked after age nineteen will have quit.

72.



[0, 20] by [–3, 15]

When $x = 11$, the unemployment rate is predicted to be 5.6 million. The year is 1991.

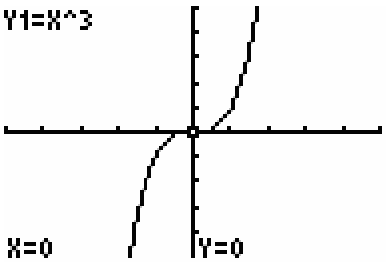
73.

X	Y1	
30	6654.5	
31	6671	
32	6686.9	
33	6702	
34	6716.4	
35	6730.1	
36	6743	

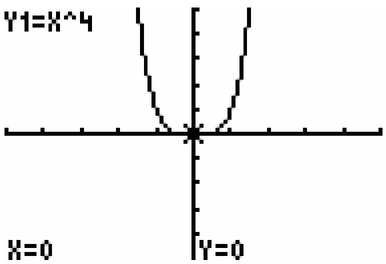
X=33

When $x = 33$, the population is predicted to be 6702 million. The year is 2023.

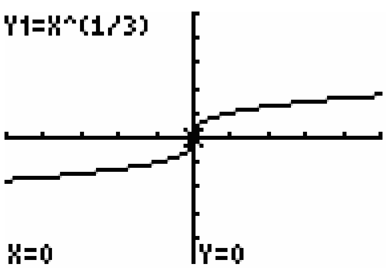
Section 2.3 Skills Check

1. 

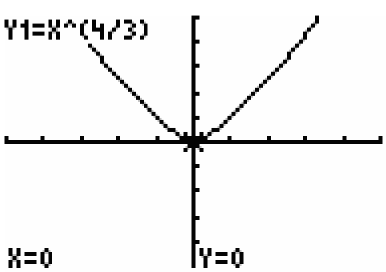
[–5, 5] by [–5, 5]

2. 

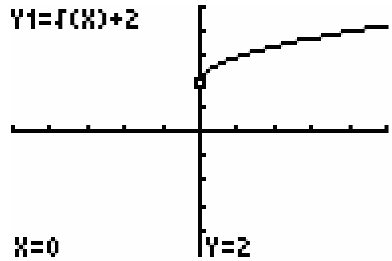
[–5, 5] by [–5, 5]

3. 

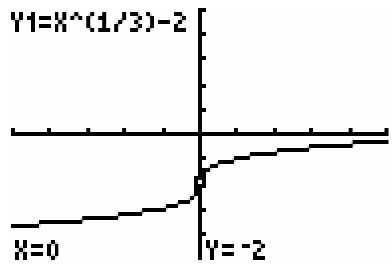
[–5, 5] by [–5, 5]

4. 

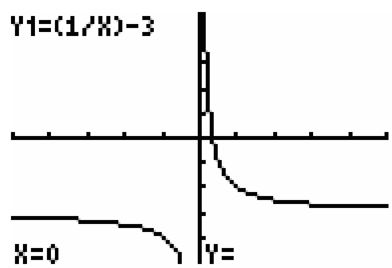
[–5, 5] by [–5, 5]

5. 

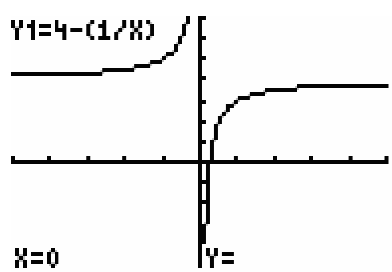
[–5, 5] by [–5, 5]

6. 

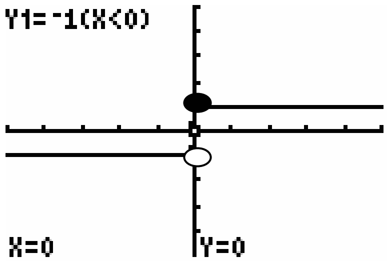
[–5, 5] by [–5, 5]

7. 

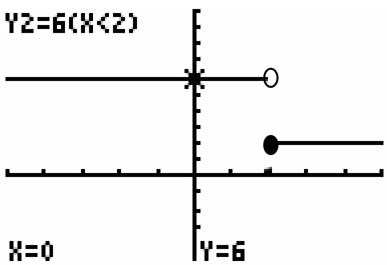
[–5, 5] by [–5, 5]

8. 

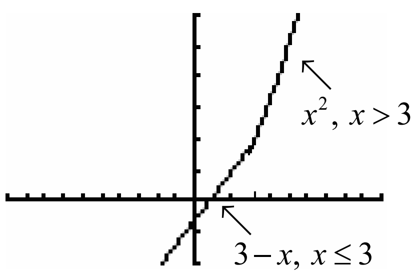
[–5, 5] by [–5, 5]

9. 

[–5, 5] by [–5, 5]

10. 

[–5, 5] by [–5, 10]

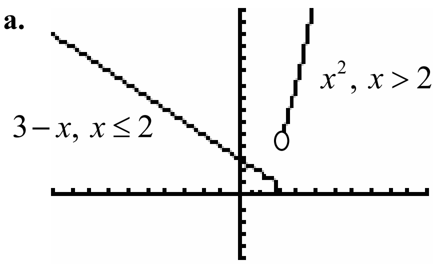
11.a. 

[–10, 10] by [–10, 30]

b. $f(2) = 4(2) - 3 = 8 - 3 = 5$

$f(4) = (4)^2 = 16$

c. Domain: $(-\infty, \infty)$

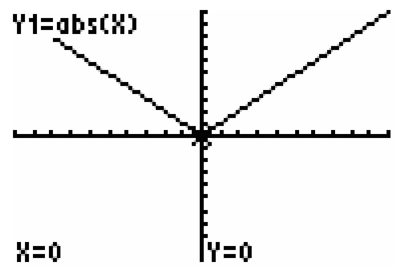
12. a. 

[–10, 10] by [–5, 15]

b. $f(2) = 3 - 2 = 1$

$f(3) = (3)^2 = 9$

c. Domain: $(-\infty, \infty)$

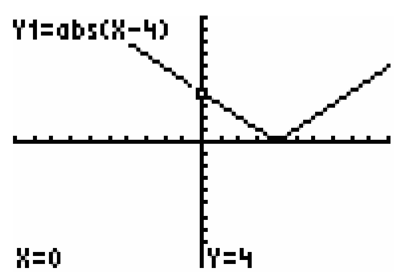
13. a. 

[–10, 10] by [–10, 10]

b. $f(-2) = |-2| = 2$

$f(5) = |5| = 5$

c. Domain: $(-\infty, \infty)$

14. a. 

[–10, 10] by [–10, 10]

b. $f(-2) = |-2 - 4| = |-6| = 6$

$f(5) = |5 - 4| = |1| = 1$

c. Domain: $(-\infty, \infty)$

15. a. $f(-1) = (-1-2)^3 = (-3)^3 = -27$

b. $f(3) = (3-2)^3 = (1)^3 = 1$

16. a. $f(-1) = (-1)^2 + 3 = 1 + 3 = 4$

b. $f(3) = (3)^2 + 3 = 9 + 3 = 12$

17. a. $f(-1) = \sqrt{-1-2} = \sqrt{-3}$
No real number solution

b. $f(3) = \sqrt{3-2} = \sqrt{1} = 1$

18. a. $f(-1) = \sqrt[3]{(-1)^2 - 2} =$
$\sqrt[3]{1-2} = \sqrt[3]{-1} = -1$

b. $f(3) = \sqrt[3]{(3)^2 - 2} = \sqrt[3]{9-2} = \sqrt[3]{7}$

19. a. $f(-1) = \dfrac{1}{-1-2} = \dfrac{1}{-3} = -\dfrac{1}{3}$

b. $f(3) = \dfrac{1}{3-2} = \dfrac{1}{1} = 1$

20. a. $f(-1) = \dfrac{1}{-1+5} = \dfrac{1}{4}$

b. $f(3) = \dfrac{1}{3+5} = \dfrac{1}{8}$

21. a. $$\begin{aligned} f(-1) &= \frac{1}{-1-4} - 9 \\ &= -\frac{1}{5} - 9 \\ &= -9.2 \end{aligned}$$

b. $$\begin{aligned} f(3) &= \frac{1}{3-4} - 9 \\ &= -1 - 9 \\ &= -10 \end{aligned}$$

22. a. $$\begin{aligned} f(-1) &= \frac{1}{-1+3} + 6 \\ &= \frac{1}{2} + 6 \\ &= \frac{13}{2} \text{ or } 6.5 \end{aligned}$$

b. $$\begin{aligned} f(3) &= \frac{1}{3+3} + 6 \\ &= \frac{1}{6} + 6 \\ &= \frac{1}{6} + \frac{36}{6} \\ &= \frac{37}{6} \end{aligned}$$

23. a. $f(-1) = 5$, since $x \le 1$.

b. $f(3) = 6$, since $x > 1$.

24. a. $f(-1) = 4$, since $x \ge -1$.

b. $f(3) = 4$, since $x \ge -1$.

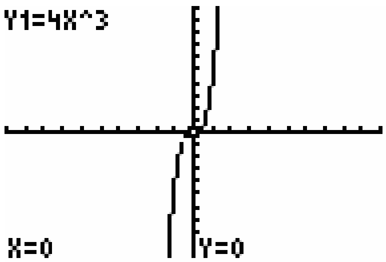
25. a. $f(-1) = (-1)^2 - 1 = 0$, since $x \le 0$.

b. $f(3) = (3)^3 + 2 = 29$, since $x > 0$.

26. a. $f(-1) = 3(-1) + 1 = -2$, since $x < 3$.

b. $f(3) = (3)^2 = 9$, since $x \ge 3$.

27. The function is increasing for all values of x.

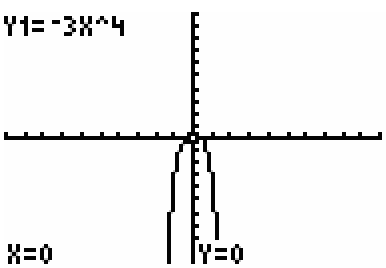


[–10, 10] by [–10, 10]

a. increasing

b. increasing

28. The function is decreasing when $x > 0$ and increasing when $x < 0$.

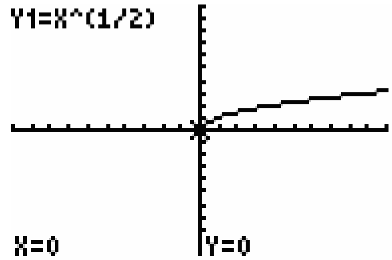


[–10, 10] by [–10, 10]

a. increasing

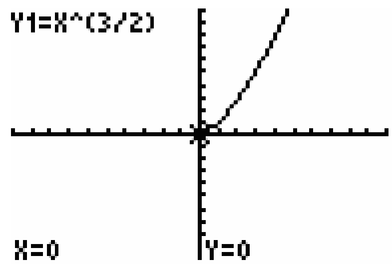
b. decreasing

29. Concave down.



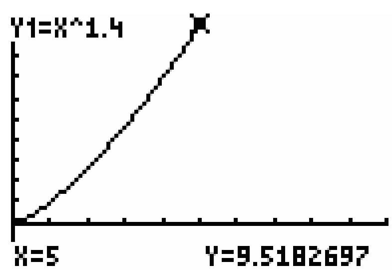
[–10, 10] by [–10, 10]

30. Concave up.



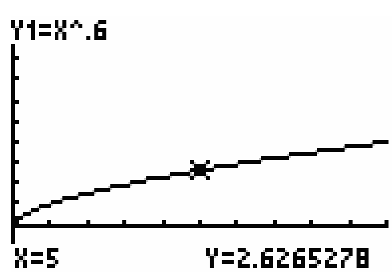
[–10, 10] by [–10, 10]

31. Concave up.

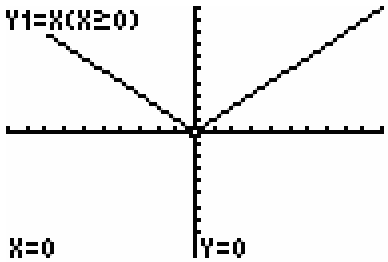


[0, 10] by [–2, 10]

32. Concave down.



[0, 10] by [–2, 10]

33. 

[–10, 10] by [–10, 10]

34. The graphs are the same. The absolute value function is a piecewise function.

Section 2.3 Exercises

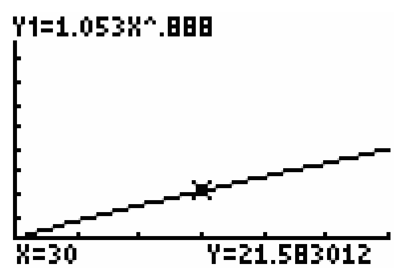
35. a. The given equation is a power function. It fits the form $f(x) = ax^b$, where b is positive.

b. $f(5) = y = 20{,}000(5)^{0.11} = 23{,}873.53$
In 1965 (1960 + 5) the model predicts that there were approximately 23,874 suicides.

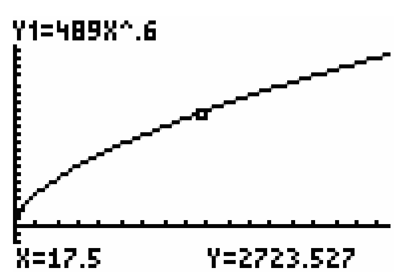
c. In 1982, the x-value is 1982–1960 = 22.
$f(22) = 20{,}000(22)^{0.11} = 28{,}099.36$.
The model predicts approximately 28,099 suicides for 1982.

36. a. $f\left(45\right) = 1.053\left(45\right)^{0.888}$
$= 30.93720559$
In 1995 the percent of all families that are single parent families is approximately 30.94%.

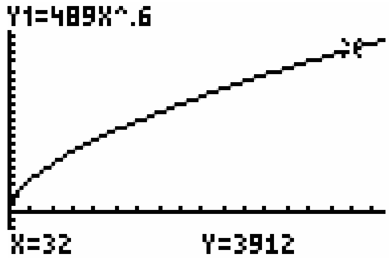
b. Based on the graph of the function below, the model suggests that the percentage is increasing.



[0, 60] by [–10, 100]

37. a. 

[0, 35] by [–1000, 5000]

b. 

[0, 35] by [–1000, 5000]

If the company employs 32 taxi drivers then 3912 miles are driven per day.

c. The function is increasing. As the number of drivers increases, it is reasonable to expect the number of miles to increase.

38. a. Let $x = 25$

$y = 1.053(25)^{0.888}$

$y = 18.35688972 \approx 18.4\%$

b. Let $x = 55$

$y = 1.053(55)^{0.888}$

$y = 36.97178675 \approx 40.0\%$

Since the model is based on data collected between 1960 and 1998, the calculation is an extrapolation. It may not be accurate.

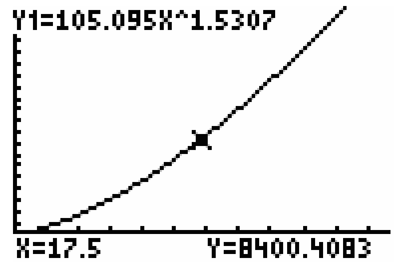
c. The function is concave down since the exponent is in the interval $(0,1)$.

d.

X	Y1
56	37.568
57	38.163
58	38.757
59	39.35
60	39.942
61	40.532
62	41.122

X=60

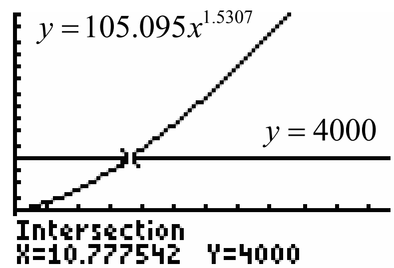
The model predicts the percentage will be 40% in 2010.

39. a. 

[0, 35] by [–2000, 20,000]

The function is increasing.

b. Concave up.

c. 

[0, 35] by [–4000, 15,000]

The model predicts that assets will reach \$4000 billion when x = 10.777542, which corresponds to the year 2001.

40. a. Let $x = 160$

$y = 165.6(160)^{1.345}$

$= 152,616.5572 \approx 152,617$

In 1960 the U.S. population is predicted to by 152, 617 thousand people.

b. The graph of the function is concave up since the exponent is in the interval $(1,\infty)$.

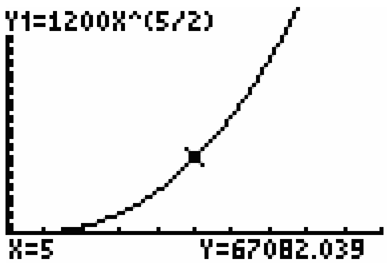
c.

X	Y1
106	87720
107	88834
108	89953
109	91075
110	92201
111	93330
112	94462

X=111

The predicted population is 92.37 million in 1911.

41. a.



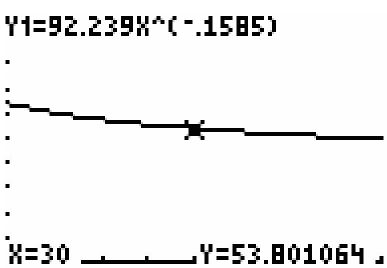
[0, 10] by [0, 200,000]

b. Considering the picture in part *a*, the graph is concave up.

42. a. $B(8) = 6(8+1)^{\frac{3}{2}} = 6(27) = 162$

On May 8th the number of bushels of tomatoes harvested is 162.

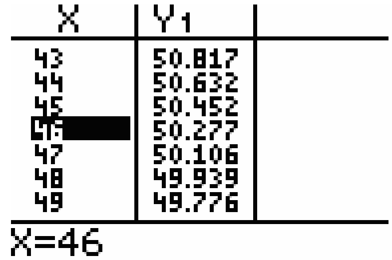
43. a.



[10, 50] by [40, 70]

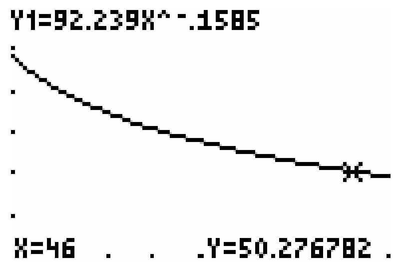
b. Voter turnout is decreasing, since the graph is falling as the number of years increases.

c. Since 1996 corresponds to $x = 46$, the voter turnout in 1996 is predicted to be 50.277%



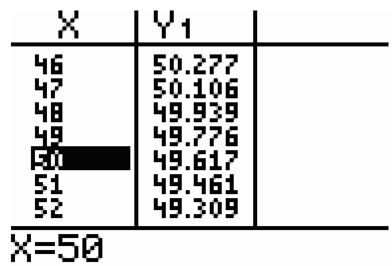
X	Y1
43	50.817
44	50.632
45	50.452
46	50.277
47	50.106
48	49.939
49	49.776

X=46



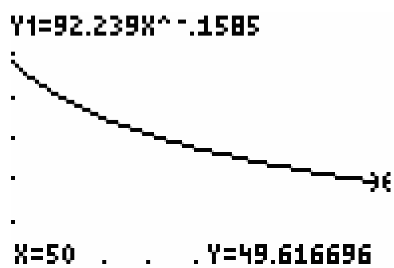
[10, 50] by [40, 70]

d. Since 2000 corresponds to $x = 50$, the voter turnout in 2000 is predicted to be 49.617%



X	Y1
46	50.277
47	50.106
48	49.939
49	49.776
50	49.617
51	49.461
52	49.309

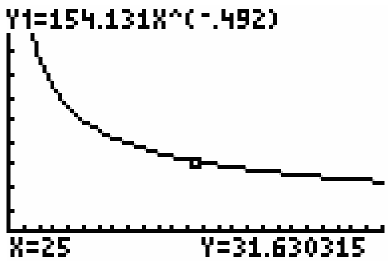
X=50



[10, 50] by [40, 70]

e. The actual voter turnout in 2000 was higher than the value predicted by the model.

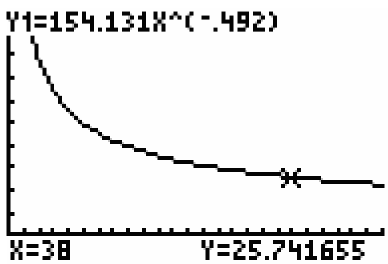
44. a.



[0, 50] by [–10, 100]

b. Trust in the government is decreasing. The graph is going down.

c.



[0, 50] by [–10, 100]

The percentage of people who say they trust the government is 25.7% in 1998.

45. Let $x = 2000$ and calculate $C(x)$.

$$\begin{aligned} C(2000) &= 105 + \frac{50{,}000}{2000} \\ &= 105 + 25 \\ &= 130 \text{ per unit} \end{aligned}$$

46. a. $C(p) = \dfrac{120{,}000}{p} - 1200$

$$\begin{aligned} C(100) &= \frac{120{,}000}{100} - 1200 \\ &= 1200 - 1200 \\ &= 0 \end{aligned}$$

Completely impure water is free!

b. $C(p) = \dfrac{120{,}000}{p} - 1200$

$$\begin{aligned} C(50) &= \frac{120{,}000}{50} - 1200 \\ &= 2400 - 1200 \\ &= 1200 \end{aligned}$$

The cost of drinking water that is 50% impure is \$1200.

47. a. $P(w) = \begin{cases} 0.37 & 0 < w \le 1 \\ 0.60 & 1 < w \le 2 \\ 0.83 & 2 < w \le 3 \\ 1.06 & 3 < w \le 4 \end{cases}$

b. $p(1.2) = 0.60$. The cost of mailing a 1.2 ounce first class letter is \$0.60.

c. Domain: $(0, 4]$

d. $P(2) = 0.60$

$p(2.1) = 0.83$

e. Considering the solution to part *d*, a 2 ounce letter costs \$0.60 to mail first class, while a 2.1 ounce letter costs \$0.83 to mail first class.

48. $P(x) = \begin{cases} 1.14 & 1.5 \le x < 2 \\ 1.16 & 2 \le x < 3 \\ 1.20 & 3 \le x < 4 \\ 1.24 & 4 \le x < 5 \end{cases}$

49. a. $T(x)=\begin{cases} 0.15x & 0 \le x \le 43{,}850 \\ 6577.50+0.28(x-43{,}850) & 43{,}850 < x \le 105{,}950 \end{cases}$

b. $T(42{,}000)=0.15(42{,}000)=6300$

c. $$\begin{aligned} T(55{,}000) &= 6577.50+0.28(55{,}000-43{,}850) \\ &= 6577.50+0.28(11{,}150) \\ &= 9699.50 \end{aligned}$$

The total tax on \$55,000 is \$9699.50.

d. The statement is incorrect.

$$\begin{aligned} T(43{,}850) &= 0.15(43{,}850) \\ &= 6577.50 \end{aligned}$$

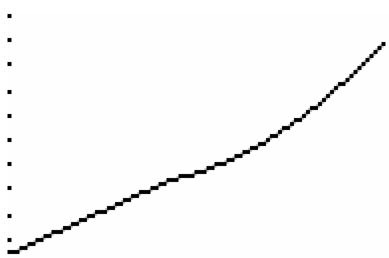
The total tax on \$43,850 is \$6577.50.

$$\begin{aligned} T(43{,}850+1) &= 6577.50+0.28(43{,}851-43{,}850) \\ &= 6577.50+0.28(1) \\ &= 6577.78 \end{aligned}$$

The total tax on \$43,851 is \$6577.78.

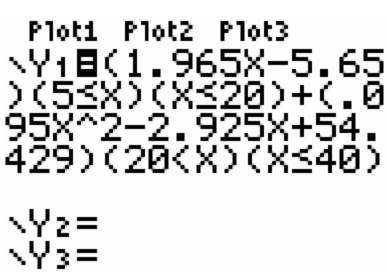
The difference in the tax bills is \$0.28. Only the extra dollar is taxed at the 28% rate.

50. a.



[5, 40] by [0, 100]

The graph above is based on entering the following equation into the TI-83 calculator.



The funding for educational programs increased between 1965 and 2000. The increase followed a linear model until 1985, after which it followed a quadratic model. The overall model is a piecewise function.

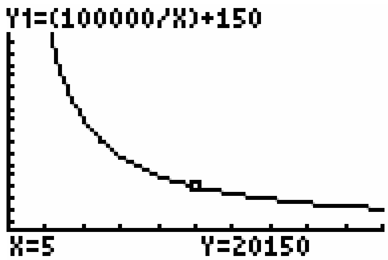
b. $$\begin{aligned} P(1980-1960) &= P(20) \\ &= 1.965(20)-5.65 \\ &= 33.65 \end{aligned}$$

Educational funding in 1980 is \$33.65 million.

c. $$\begin{aligned} &P(1998-1960) \\ &= P(38) \\ &= 0.095(38)^2-2.925(38)+54.429 \\ &= 137.18-111.15+54.429 \\ &= 80.459 \end{aligned}$$

Educational funding in 1998 is \$80.459 million.

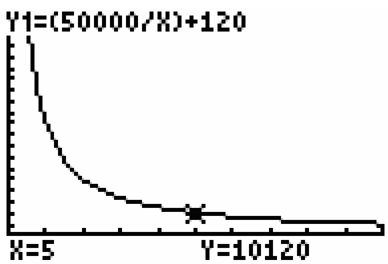
51. a.



[–0, 10] by [–10,000, 101,000]

b. The function, and therefore the average cost, is decreasing.

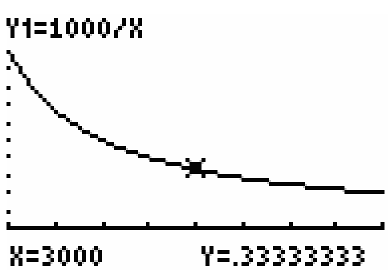
52. a.



[–0, 10] by [–10,000, 101,000]

b. The function, and therefore the average cost, is decreasing.

53. a. $C = \dfrac{1000}{V}$



[1000, 5000] by [–0.2, 1.2]

b. Decrease.

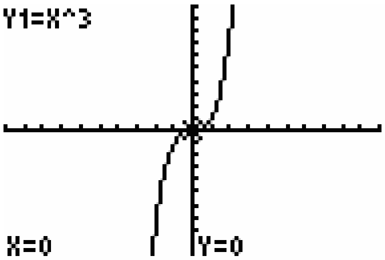
54. a.
$$100C - Cp = 10{,}500$$
$$C(100 - p) = 10{,}500$$
$$C = \frac{10{,}500}{100 - p}$$
$$C(p) = \frac{10{,}500}{100 - p}$$

b. $C(50) = \dfrac{10{,}500}{100 - 50} = \dfrac{10{,}500}{50} = 210$
The daily cost of removing 50% of the pollution is \$210.

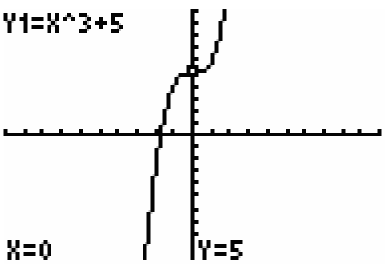
c. $C(99) = \dfrac{10{,}500}{100 - 99} = 10{,}500$
The daily cost of removing 99% of the pollution is \$10,500. The company would resist such a high daily cost.

Section 2.4 Skills Check

1.

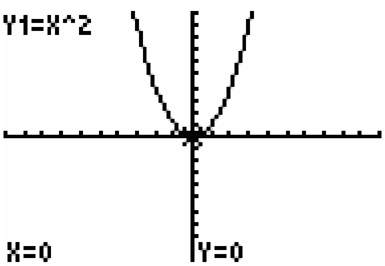


[–10, 10] by [–10, 10]

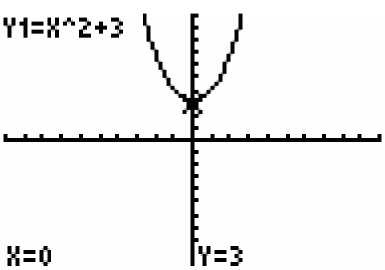


[–10, 10] by [–10, 10]

2.

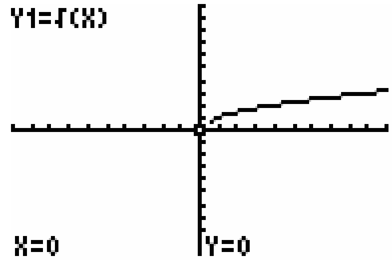


[–10, 10] by [–10, 10]

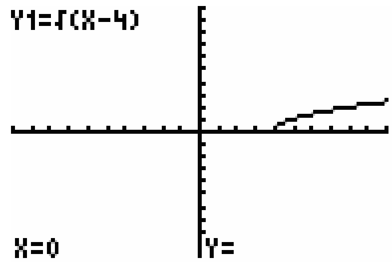


[–10, 10] by [–10, 10]

3.

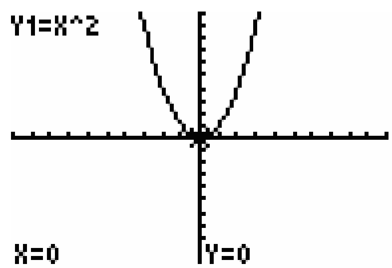


[–10, 10] by [–10, 10]

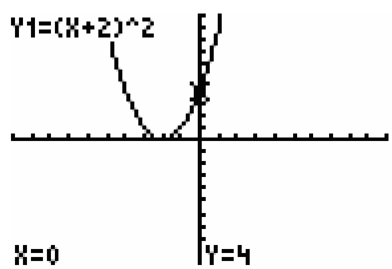


[–10, 10] by [–10, 10]

4.

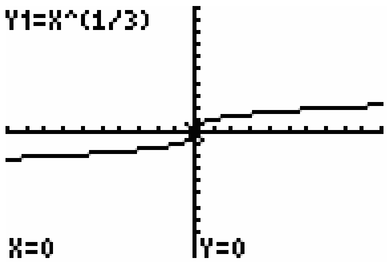


[–10, 10] by [–10, 10]

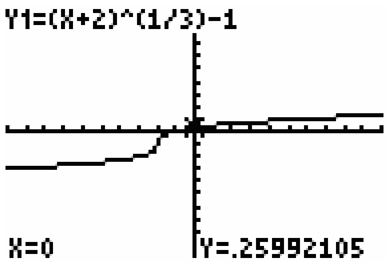


[–10, 10] by [–10, 10]

5.

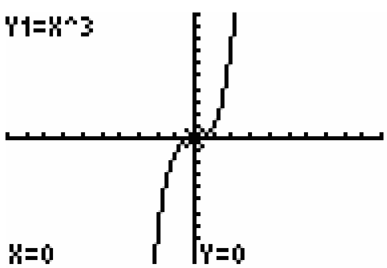


[–10, 10] by [–10, 10]

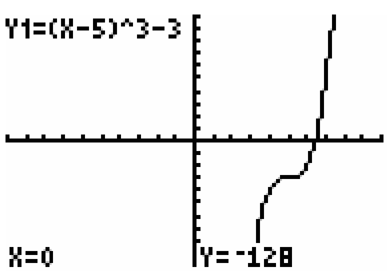


[–10, 10] by [–10, 10]

6.

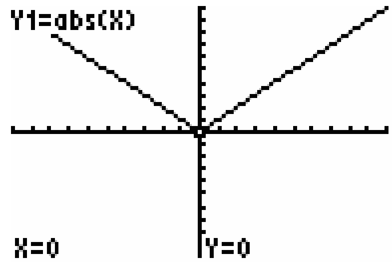


[–10, 10] by [–10, 10]

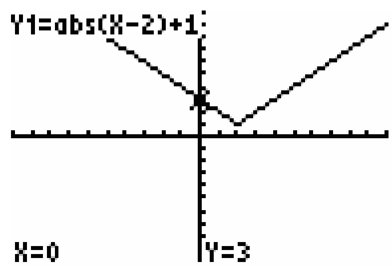


[–10, 10] by [–10, 10]

7.

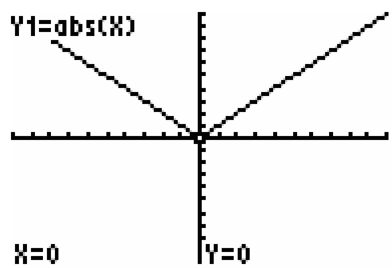


[–10, 10] by [–10, 10]

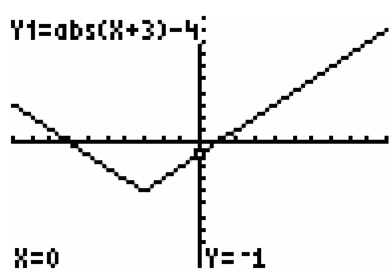


[–10, 10] by [–10, 10]

8.

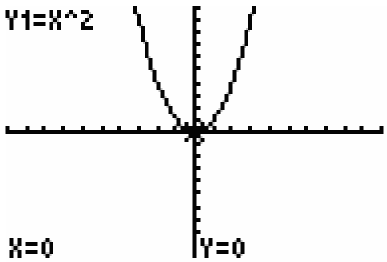


[–10, 10] by [–10, 10]

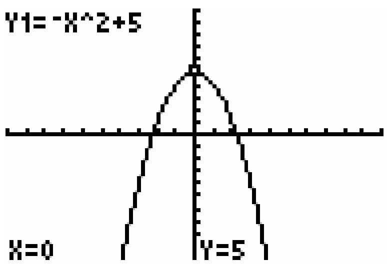


[–10, 10] by [–10, 10]

9.

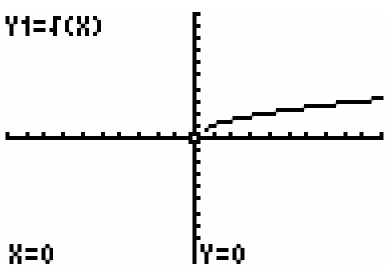


[–10, 10] by [–10, 10]

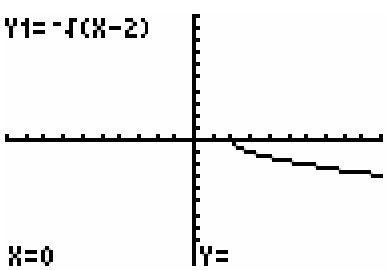


[–10, 10] by [–10, 10]

10.

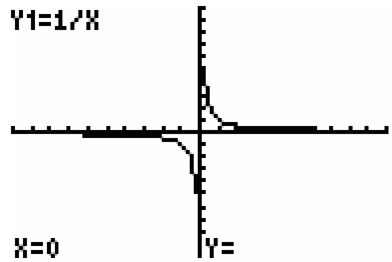


[–10, 10] by [–10, 10]

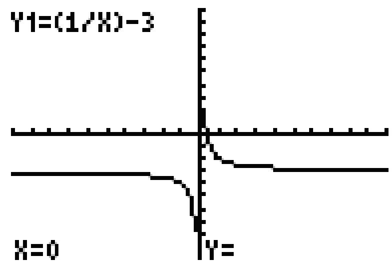


[–10, 10] by [–10, 10]

11.

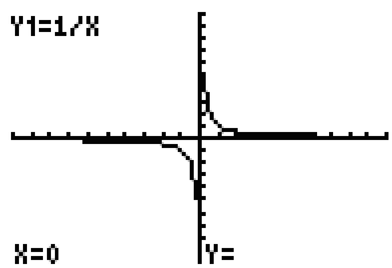


[–10, 10] by [–10, 10]

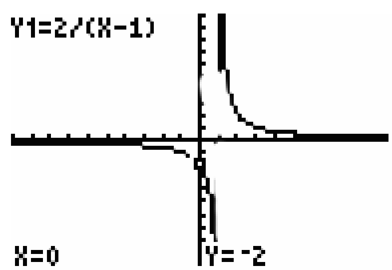


[–10, 10] by [–10, 10]

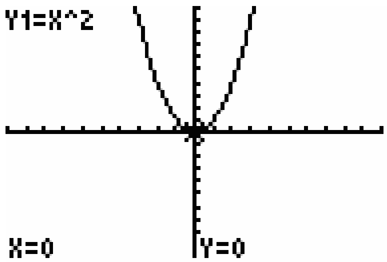
12.



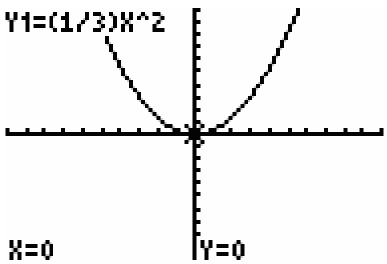
[–10, 10] by [–10, 10]



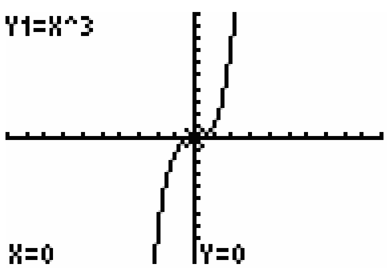
[–10, 10] by [–10, 10]

13. 

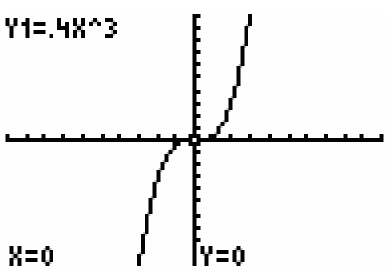
[–10, 10] by [–10, 10]



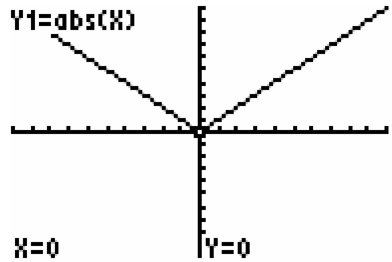
[–10, 10] by [–10, 10]

14. 

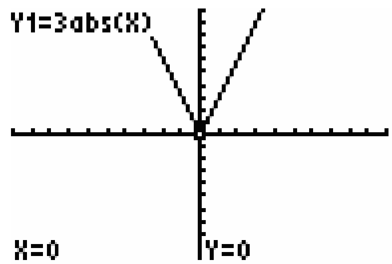
[–10, 10] by [–10, 10]



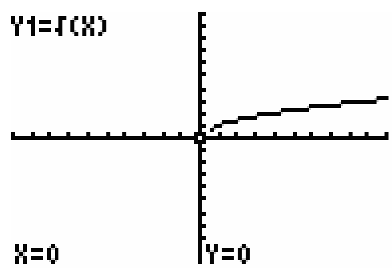
[–10, 10] by [–10, 10]

15. 

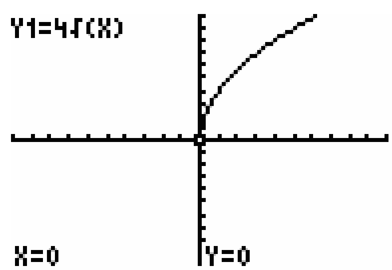
[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

16. 

[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

17. The graph of the function is shifted 2 units right and 3 units up.

18. The graph of the function is shifted 4 units left and 2 units down.

19. $y = (x+4)^{\frac{3}{2}}$

20. $y = (x-4)^{\frac{3}{2}} - 5$

21. $y = 3x^{\frac{3}{2}} + 5$

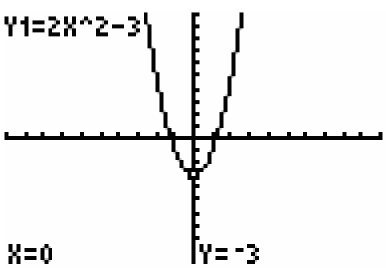
22. $y = \frac{1}{5}(x-6)^{\frac{2}{3}}$

23. x-axis symmetry.

Let $x = -x$

$y = 2(-x)^2 - 3 = 2x^2 - 3$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the x-axis.



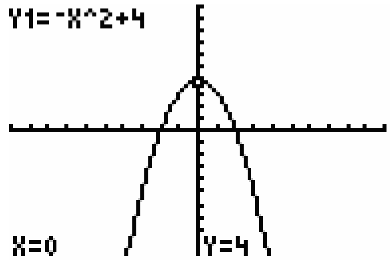
[–10, 10] by [–10, 10]

24. x-axis symmetry.

Let $x = -x$

$y = -(-x)^2 + 4 = -x^2 + 4$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the x-axis.



[–10, 10] by [–10, 10]

25. Origin symmetry.

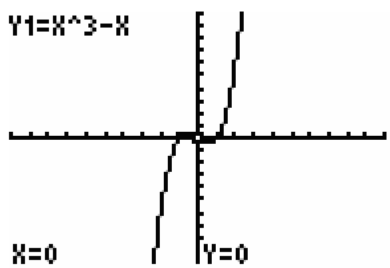
Let $x = -x, y = -y$

$-y = (-x)^3 - (-x)$

$-y = -x^3 + x$

$y = x^3 - x$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.



[–10, 10] by [–10, 10]

26. Origin symmetry.

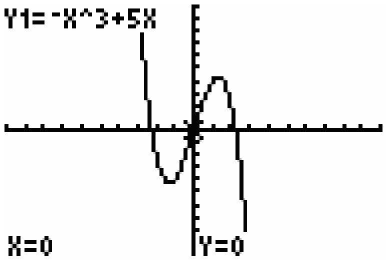
Let $x = -x, y = -y$

$-y = -(-x)^3 + 5(-x)$

$-y = x^3 - 5x$

$y = -x^3 + 5x$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.



[–10, 10] by [–10, 10]

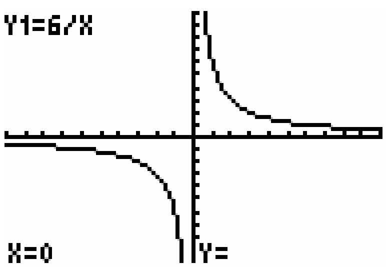
27. Origin symmetry.

Let $x = -x, y = -y$

$$-y = \frac{6}{-x}$$

$$y = \frac{6}{x}$$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.



[–10, 10] by [–10, 10]

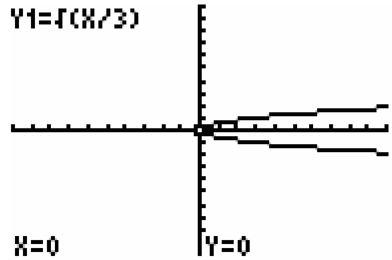
28. y-axis symmetry.

Let $y = -y$

$$x = 3(-y)^2$$

$$x = 3y^2$$

Since the given equation is not a function, it can not be easily graphed using the graphing calculator. The equation must be rewritten as $y = \pm\sqrt{\frac{x}{3}}$



[–10, 10] by [–10, 10]

29. x-axis symmetry.

Let $y = -y$

$$x^2 + (-y)^2 = 25$$

$$x^2 + y^2 = 25$$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the x-axis.

y-axis symmetry.

Let $x = -x$

$$(-x)^2 + y^2 = 25$$

$$x^2 + y^2 = 25$$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the y-axis.

Origin symmetry.

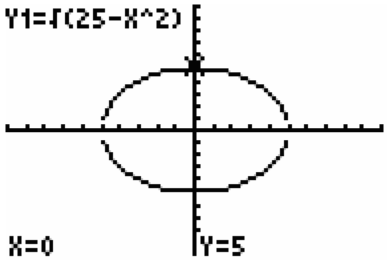
Let $x = -x, y = -y$

$$(-x)^2 + (-y)^2 = 25$$

$$x^2 + y^2 = 25$$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.

Since the given equation is not a function, it can not be easily graphed using the graphing calculator. The equation must be rewritten as $y = \pm\sqrt{25 - x^2}$



[–10, 10] by [–10, 10]

30. y-axis symmetry.

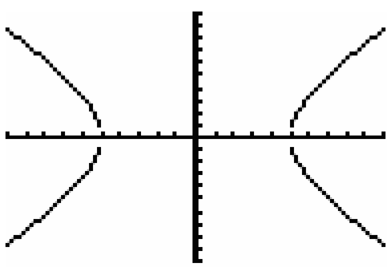
Let $x = -x$

$(-x)^2 - y^2 = 25$

$x^2 - y^2 = 25$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the y-axis.

Since the given equation is not a function, it can not be easily graphed using the graphing calculator. The equation must be rewritten as $y = \pm\sqrt{x^2 - 25}$



[–10, 10] by [–10, 10]

31. $f(-x) = |-x| - 5$
$= |-1 \bullet x| - 5$
$= |-1||x| - 5$
$= |x| - 5$
$= f(x)$

Since $f(-x) = f(x)$, the function is even.

32. $f(-x) = |(-x) - 2|$
$= |-x - 2|$
$= |-1(x+2)|$
$= |x+2|$

Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is neither even nor odd.

33. $f(-x) = \sqrt{(-x)^2 + 3}$
$= \sqrt{x^2 + 3}$

Since $f(-x) = f(x)$, the function is even.

34. $f(-x) = \frac{1}{2}(-x)^3 - (-x)$
$= -\frac{1}{2}x^3 + x$
$= -\left(\frac{1}{2}x^3 - x\right)$

Since $f(-x) = -f(x)$, the function is odd.

35. $f(-x) = \frac{5}{-x}$
$= -\frac{5}{x}$
$= -\left(\frac{5}{x}\right)$

Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is neither even nor odd.

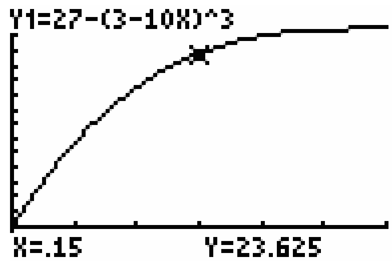
35. $f(-x) = 4(-x) + (-x)^2$
$= -4x + x^2$

Since $f(-x) = -f(x)$, the function is odd.

Section 2.4 Exercises

37. a. $s = t^3 \ (y = x^3)$

b.



[0, 0.3] by [0, 30]

c.
$$\begin{aligned} f(0.3) &= 27-(3-10(0.3))^3 \\ &= 27-(3-3)^3 \\ &= 27-0 \\ &= 27 \end{aligned}$$

After 0.3 seconds the bullet has traveled 27 inches.

38. a. $y = x^2$

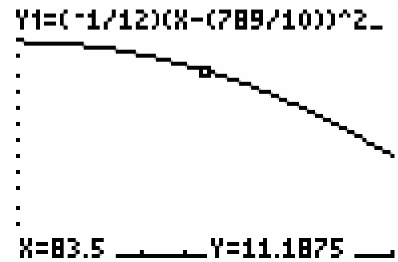
b.
$$\begin{aligned} M(79) &= -\frac{1}{12}\left(79-\frac{789}{10}\right)^2+\frac{15{,}541}{1200} \\ &= -\frac{1}{12}(0.1)^2+\frac{15{,}541}{1200} \\ &= 12.95 \end{aligned}$$

$$\begin{aligned} M(80) &= -\frac{1}{12}\left(80-\frac{789}{10}\right)^2+\frac{15{,}541}{1200} \\ &= -\frac{1}{12}(1.1)^2+\frac{15{,}541}{1200} \\ &= 12.85 \end{aligned}$$

$$\begin{aligned} M(88) &= -\frac{1}{12}\left(88-\frac{789}{10}\right)^2+\frac{15{,}541}{1200} \\ &= -\frac{1}{12}(9.1)^2+\frac{15{,}541}{1200} \\ &= 6.05 \end{aligned}$$

In 1979, 12.95% of people 12 years of age or older used marijuana at least once in the month prior to being surveyed. In 1980, 12.85% of people 12 years of age or older used marijuana at least once in the month prior to being surveyed. In 1988, 6.05% of people 12 years of age or older used marijuana at least once in the month prior to being surveyed.

c.



[79, 88] by [0, 15]

39. a. Since q is in the numerator with an exponent of one, $p = \frac{180+q}{6}$ is a linear function.

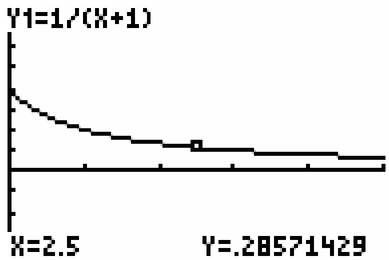
b. Since there is a variable in the denominator, $p = \frac{30{,}000}{q} - 20$ is a shifted reciprocal function.

40. a. Since q is in the numerator and with an exponent of one, $p = 58 + \frac{q}{2}$ is a linear function.

b. Since there is a variable in the denominator, $p = \frac{2555}{q+5}$ is a shifted reciprocal function.

41. a. $y = \frac{1}{x}$**.** Shifted one unit left.

b.



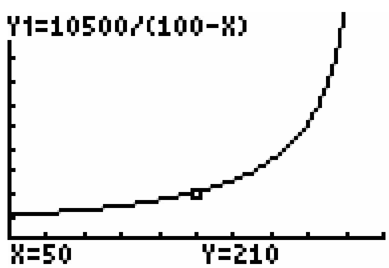
[0, 5] by [–1, 2]

c. The function is decreasing. Therefore, the amount of self-attentiveness of person decreases as the size of the crowd increases.

42. a. $C = -10{,}500\left(\dfrac{1}{p-100}\right)$.

Shifted 100 units right, reflected over the x-axis, and stretched by a factor of 10,500.

b.



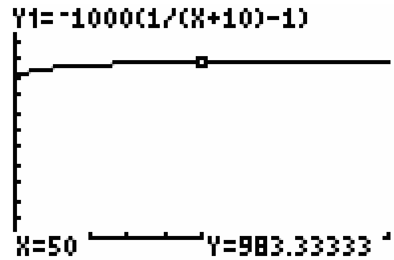
[0, 100] by [–100, 1000]

c.
$$\begin{aligned} C &= \frac{10{,}500}{100-p} \\ &= \frac{10{,}500}{100-80} \\ &= \frac{10{,}500}{20} \\ &= \$525 \end{aligned}$$

The daily cost of removing 80% of the pollution is $525.

43. a. Shifted 10 units left and one unit down. Reflected over the x-axis and stretched by a factor of 1000.

b.



[0, 100] by [–100, 1300]

44. a. Yes. Shifted 110 units right, reflected over the y-axis, and stretched by a factor of 4700.

b. $(-\infty, 110]$

c. See part a) above.

45. Since in the given function x represents the years since 1990, $x+5$ represents the years since 1995. Therefore, the new function would be $f(x) = 105.095(x+5)^{1.5307}$.

46. Since in the given function x represents the years since 1950, $x+10$ represents the years since 1960. Therefore, the new function would be $f(x) = 1.053(x+10)^{0.888}$.

47. Since in the given function x represents the years since 1986, $x+4$ represents the years since 1990. Therefore, the new function would be
$$f(x) = 0.084(x+4)^2 + 1.124(x+4) + 4.028.$$

48. Since in the given function x represents the years since 1970, $x-10$ represents the years since 1960. Therefore, the new function would be
$$f(x) = 0.184(x-10)^2 - 5.437(x-10) + 58.427.$$

49. a. $t = 2005 - 1980 = 25$

$$\begin{aligned} S(25) &= 0.0003488(25)^{4.2077} \\ &\approx 265.882 \end{aligned}$$

The number of subscribers in 2005 is approximately 266 million.

b. Since in the given function *x* represents the years since 1980, $x+5$ represents the years since 1985. Therefore, the new function would be $S(t) = 0.0003488(t+5)^{4.2077}$.

c. $t = 2005 - 1985 = 20$

$$\begin{aligned} S(20) &= 0.0003488(20+5)^{4.2077} \\ &= 0.0003488(25)^{4.2077} \end{aligned}$$

Using the shifted model yields the same result as part a) above.

50. a. $x = 2008 - 1970 = 38$

$$\begin{aligned} P(30) &= 850.36(38)^{0.686} \\ &\approx 10{,}311.797 \end{aligned}$$

The poverty threshold in 2008 is approximately \$10,312.

b. Since in the given function *x* represents the years since 1970, $x+20$ represents the years since 1990. Therefore, the new function would be $P(x) = 850.36(x+20)^{0.686}$.

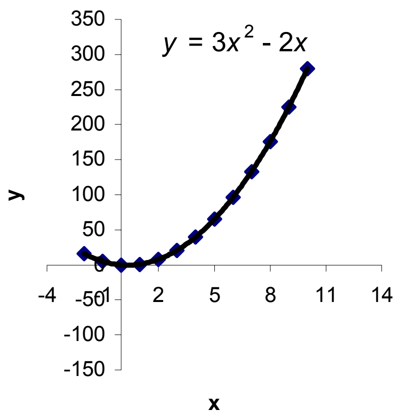
c. $t = 2008 - 1990 = 18$

$$\begin{aligned} P(18) &= 850.36(18+20)^{0.686} \\ &= 850.36(38)^{0.686} \end{aligned}$$

Using the shifted model yields the same result.

Section 2.5 Skills Check

1.



2.

x	$f(x)$	first differences	second differences
0	0		
2	399	399	
4	1601	1202	803
6	3600	1999	797
8	6402	2802	803
10	9998	3596	794

Since the second differences are approximately equal, $f(x)$ is approximately quadratic.

x	$g(x)$	first differences	second differences
0	2.0		
2	0.8	–1.2	
4	1.2	0.4	1.6
6	3.2	2.0	1.6
8	6.8	3.6	1.6
10	12.0	5.2	1.6

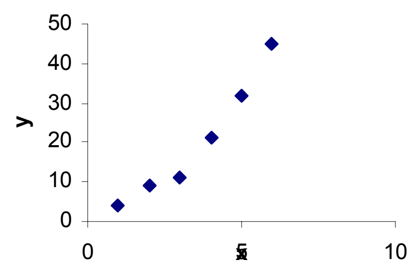
Since the second differences are constant, $g(x)$ is exactly quadratic.

x	$h(x)$	first differences	second differences
0	0		
2	110	110	
4	161	51	–59
6	195	34	–17
8	230	35	1
10	290	60	25

Since the second differences are extremely variable, $h(x)$ is not quadratic.

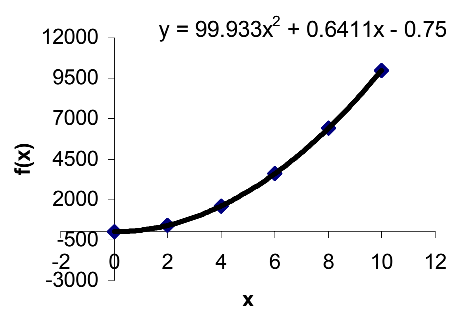
3. The x-values are not equally spaced.

4. a.

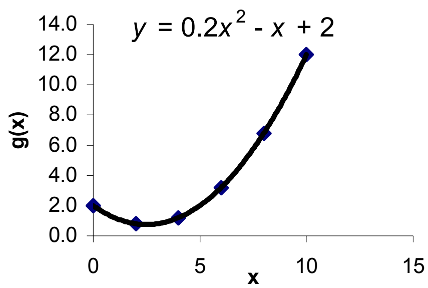


b. Power Model.

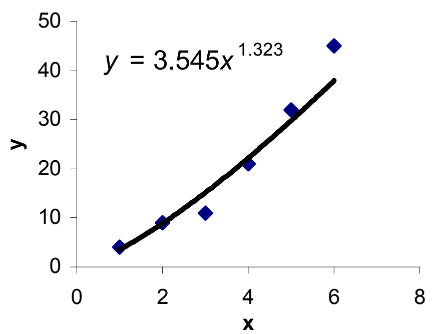
5.



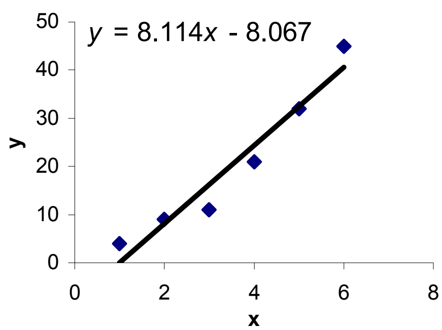
6.



7. a.

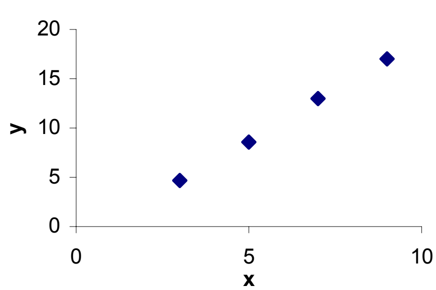


b.



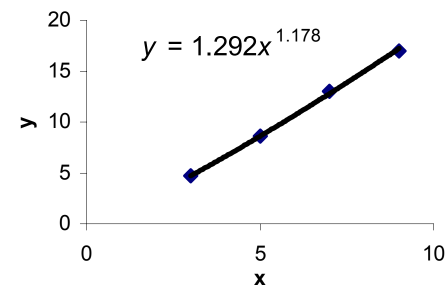
c. The power function is a better fit.

8. a.

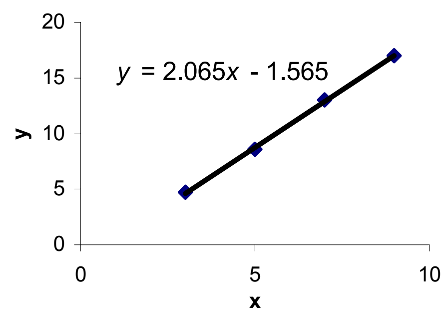


b. The scatter plot suggests that a linear model will fit the data reasonably well.

9. a.

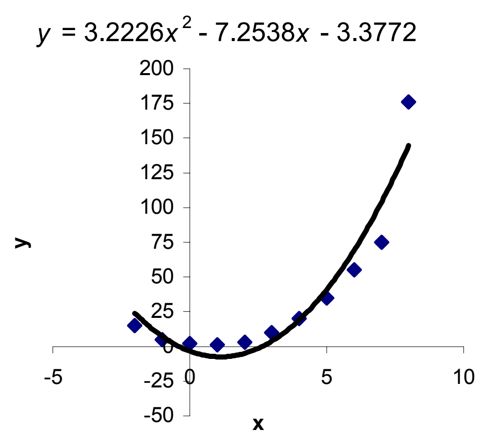


b.

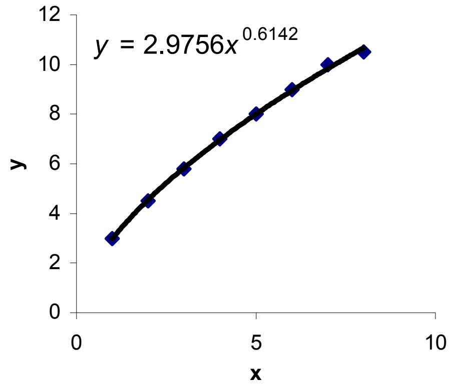


c. Both models appear to be good fits.

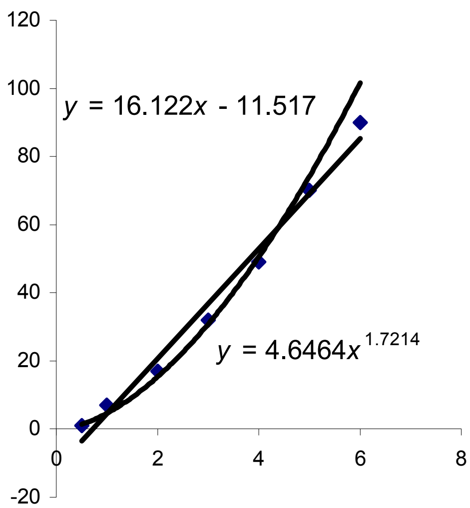
10.



11.



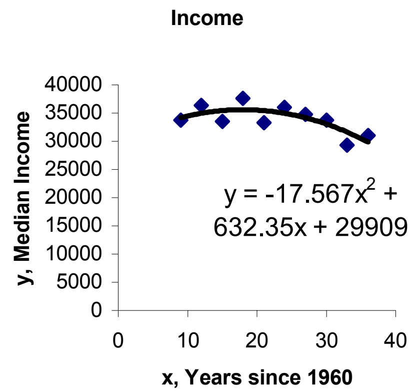
12.



Both models seem to fit the data reasonably well.

Section 2.5 Exercises

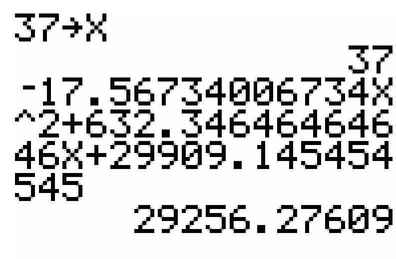
13. a.



b. In 1997, $x = 1997 - 1960 = 37$

$$y = -17.567(37)^2 + 632.35(37) + 29{,}909$$

Using the TI-83 and substituting into the unrounded model



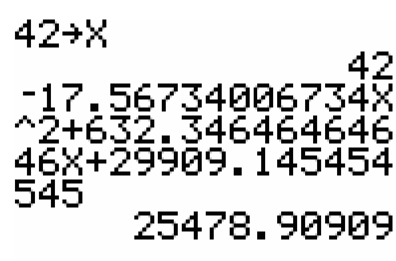
$y \approx 29{,}256$

In 1997 the median annual income is approximately $29,256.

In 2002, $x = 2002 - 1960 = 42$

$$y = -17.567(42)^2 + 632.35(42) + 29{,}909$$

Using the TI-83 and substituting into the unrounded model

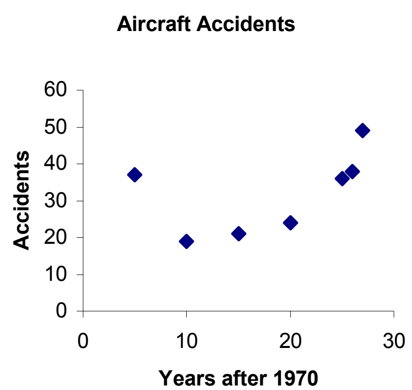


$y \approx 29{,}256$

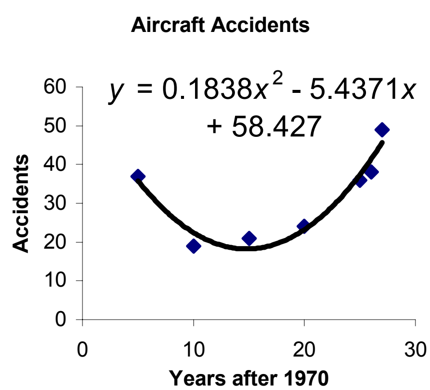
In 2002 the median annual income is approximately \$25,479.

c. No. Since both 1997 and 2002 are beyond the scope of the base data, the extrapolation may not be valid.

14. a.



b.



c. In 1992, $x = 1992 - 1970 = 22$

$y = 0.1838(22)^2 - 5.4371(22) + 58.427$

Using the TI-83 and substituting into the unrounded model

```
22→X
                        22
.18379725631929X
^2+-5.4370555791
962X+58.42667736
7069
           27.76932668
```

$y \approx 28$

The number of aircraft accidents in 1992 is approximately 28.

In 2001, $x = 2001 - 1970 = 31$

$y = 0.1838(31)^2 - 5.4371(31) + 58.427$

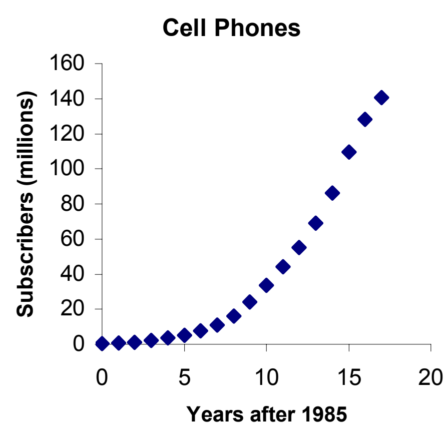
Using the TI-83 and substituting into the unrounded model

```
31→X
                        31
.18379725631929X
^2+-5.4370555791
962X+58.42667736
7069
           66.50711773
```

$y \approx 67$

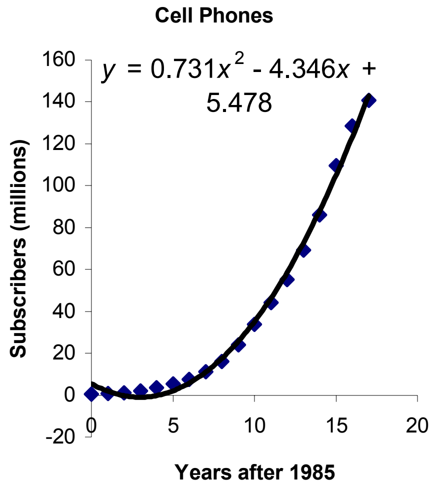
The number of aircraft accidents in 2001 is approximately 67.

15. a.

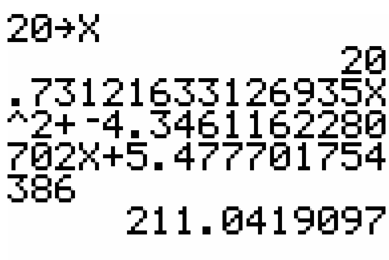


Yes. It appears a quadratic function will fit the data well.

b.



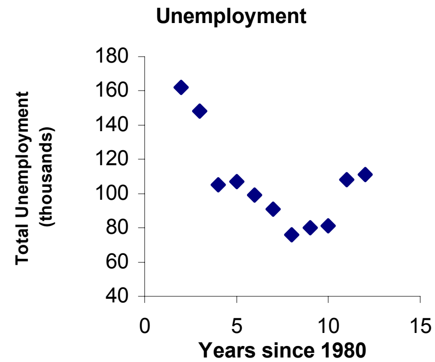
c. In 2005, $x = 2005 - 1985 = 20$
Using the TI-83 and substituting into the unrounded model



$y \approx 211$
The number of cell phone subscribers in 2005 is approximately 211 million.

d. There are about 300 million people in the U.S. Therefore, approximately $\frac{211}{300}$ or $\frac{2}{3}$ have cell phones

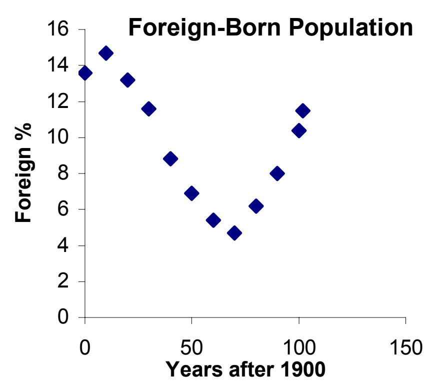
16. a.



b. Yes. A quadratic function will probably fit the data.

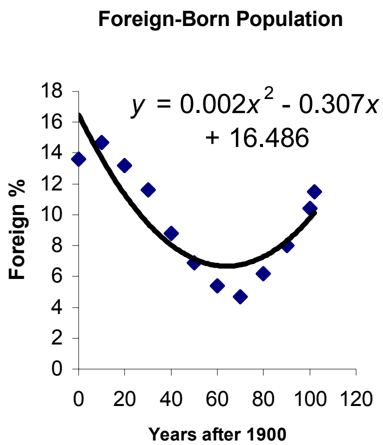
c. Yes. The y-intercept, (0, 225.62) represents the total unemployment in thousands in 1980.

17.a.



b. Yes.

c.

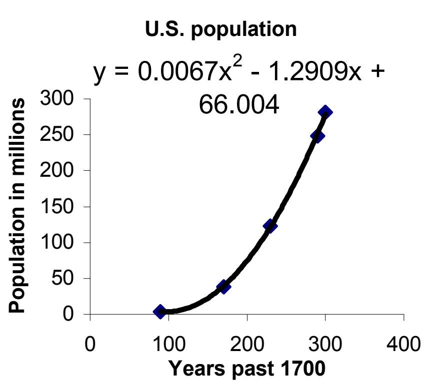


d. In 2005, $x = 2005 - 1900 = 105$
Using the TI-83 and substituting into the unrounded model:

```
105→X
                     105
.00239640183393X
^2+-.30678174110
66X+16.485671659
554
         10.69391906
```

$y \approx 10.69\%$
The percentage of the U.S. population that is foreign born in 2005 is approximately 10.69%.

18. a.



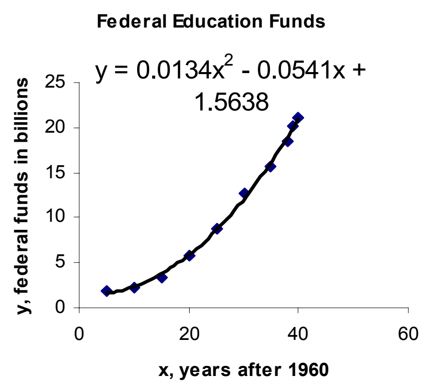
b. Using the TI-83 table feature in conjunction with the unrounded model:

X	Y_1
236	132.67
237	134.53
238	136.41
239	138.3
240	140.2
241	142.12
242	144.05

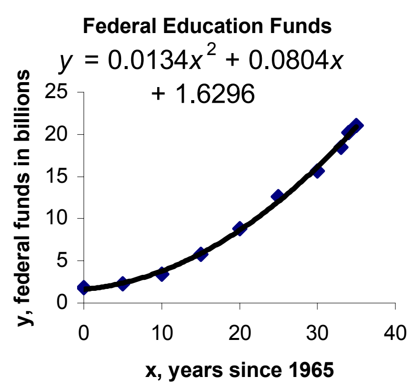
X=240

The population reaches 140.2 million in 1940.

19. a.



b.



c. Using the TI-83 table feature in conjunction with the unrounded model:

X	Y_1
25	8.6162
26	9.2479
27	9.9065
28	10.592
29	11.304
30	12.044
31	12.81

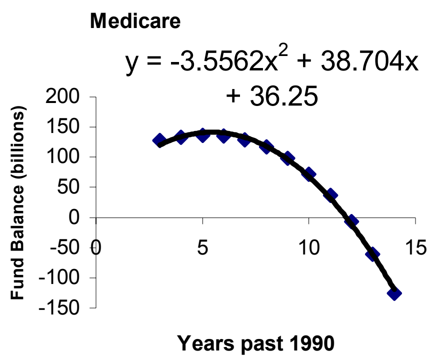
X=28

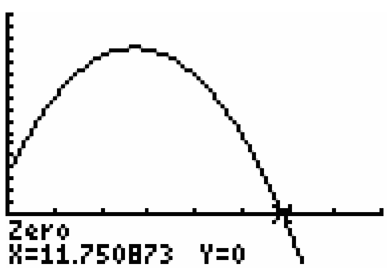
When $x = 28$, federal funding for education exceeds \$10 billion. An x-value of 28 corresponds to year 1988.

d. The results would be the same. The models are equivalent. The function in part b) is shifted five units left in comparison to the function on part a).

20. a. Based on the table, the Medicare trust fund is broke in 2002.

b.



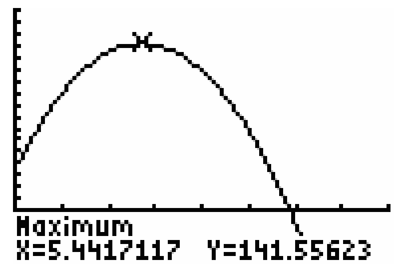


[0, 16] by [0, 170]

The model predicts that the fund will be broke when x is 12. The year is 2002.

c. Assuming the pamphlet was issued in 1994, the data confirm the statement.

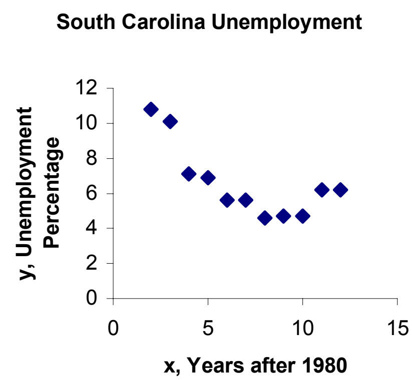
d.



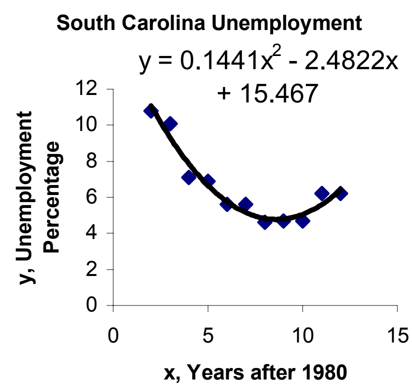
[0, 16] by [–40, 170]

The vertex is approximately $(5.44, 141.56)$. The highest balance of the trust fund is 141.56 billion in 1995.

21. a.

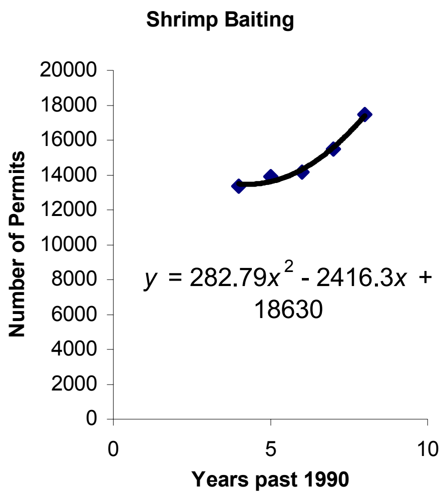


b. Yes. It appears a quadratic model fits the data.



c. Yes. The y-intercept, (0, 15.467) represents the unemployment percentage in 1980.

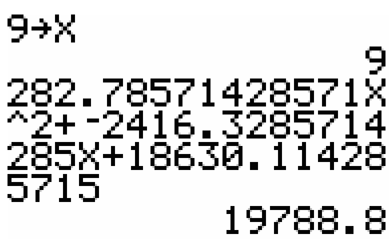
22. a.



b. In 1999, $x = 9$.

$$y = 282.79x^2 - 2416.3x + 18{,}630$$
$$= 282.79(9)^2 - 2416.3(9) + 18{,}630$$

Using the TI-83 and substituting into the unrounded model



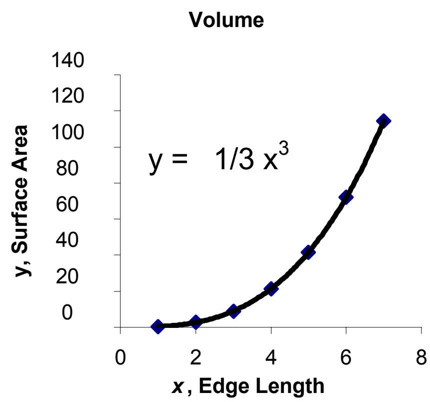
The predicted number of permits issued in 1999 is approximately 19,789.

c. The actual number of permits issued in 1999 was much lower than the number predicted by the model. Extrapolations may not be reliable. When conditions change, models may no longer be valid.

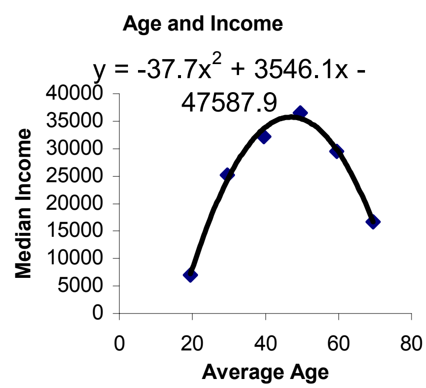
23. a. The second differences are not constant. The situation is not modeled by a quadratic function.

x	y	first differences	second differences
1	1/3		
2	2 2/3	2 1/3	
3	9	6 1/3	4
4	21 1/3	12 1/3	6
5	41 2/3	20 1/3	8
6	72	30 1/3	10

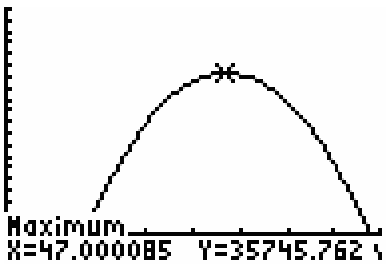
b. Modeling with a power function yields:



24. a.

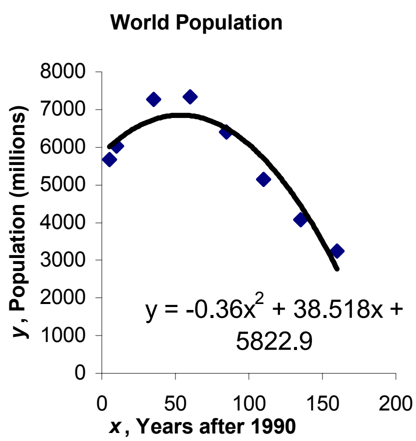


b. The model appears to fit the data points very well.

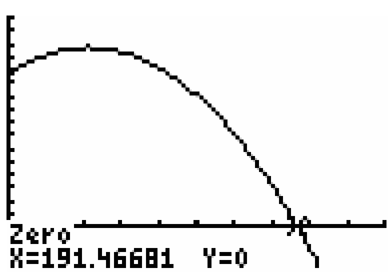
c. 

[0, 80] by [–5000, 50,000]

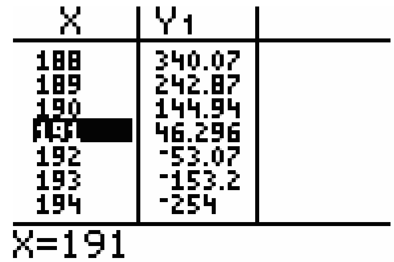
A 47-year old person produces the maximum median income of $35,746.

25. a. 

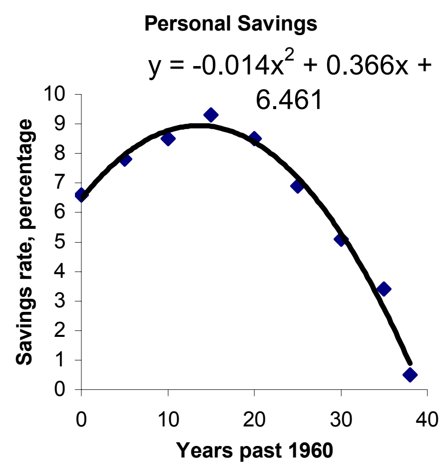
b. The x-intercept is approximately 191. The corresponding year is 2181.

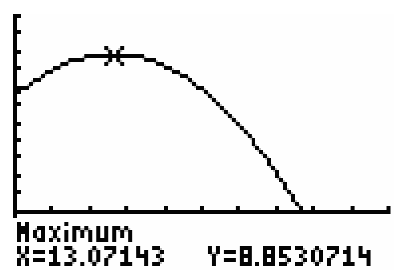


[0, 250] by [–1500, 8000]



c. The model is not valid beyond the year 2181. The model predicts that the population will become negative beginning in 2182!

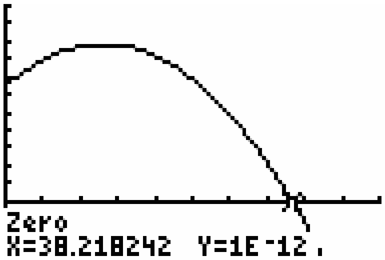
26. a. 

b. 

[0, 50] by [0, 11]

The maximum savings rate of 8.85% occurs during 1973.

c.

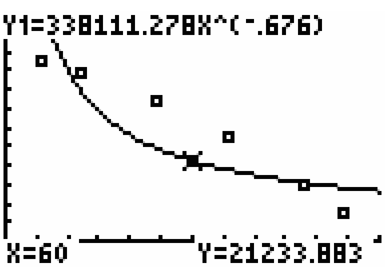


[0, 50] by [–2, 11]

The x-intercept is $(38.2, 0)$.

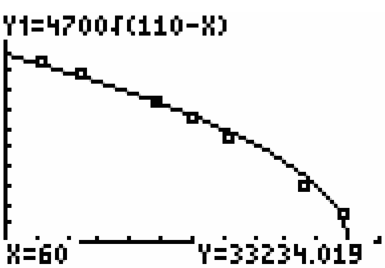
d. The model is no longer valid when the savings rate is negative. For years beyond 1998 $(x > 38)$ the model is not valid.

27. a.



[0, 120] by [–5000, 60,000]

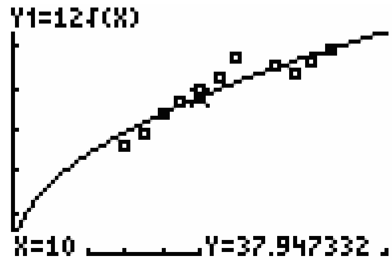
b.



[0, 120] by [–5000, 60,000]

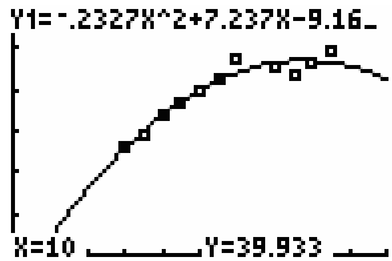
Equation b) fits the data much better.

28. a. (i)



[0, 20] by [0, 60]

(ii)

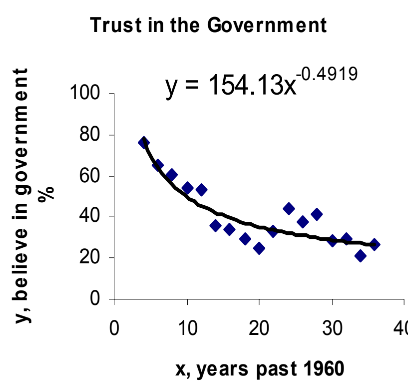


[0, 20] by [0, 60]

The quadratic model appears to be slightly better.

b. Since beyond 1996 the square root model appears to be a better fit, to make a prediction about 2002 apply the square root model

29. a.



b. In the year 2000,

$x = 2000 - 1960 = 40.$

$y = 154.13x^{-0.4919}$

$y = 154.13(40)^{-0.4919}$

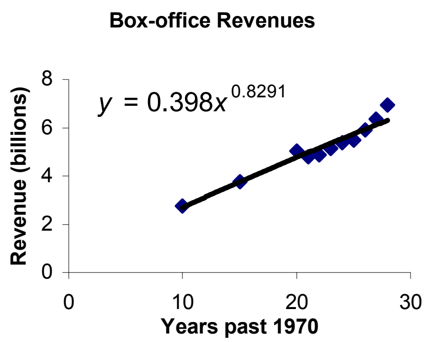
Using the TI-83 and substituting into the unrounded model:

```
40→X
                    40
154.13120343008X
^-.491908526263O
5
     25.10866371
```

$y \approx 25.1\%$

In 2000 approximately 25.1% of people said they trust the government always or most of the time.

30. a.



b. In 2005, $x = 35$.

$y = 0.398(35)^{0.829}$

Using the TI-83 and substituting into the unrounded model:

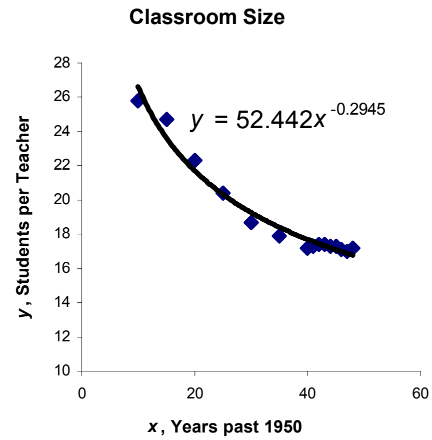
```
35→X
                    35
.39800279252876X
^.82913596218818
      7.58801993
```

$y \approx 7.6$

In 2005 the predicted box-office revenue is $7.6 billion.

c. Since 2005 is outside the range of the original data, the answer in part *b* is an extrapolation. If conditions change in future years, the model may no longer be valid.

31. a.



b. The year 2000 corresponds to an x-value of 50. Therefore,

$y = 52.442x^{-0.2945}$

$y = 52.442(50)^{-0.2945}$

Using the TI-83 and substituting into the unrounded model

```
50→X
                    50
52.44243249087X^
-.29446156335305
     16.57301274
```

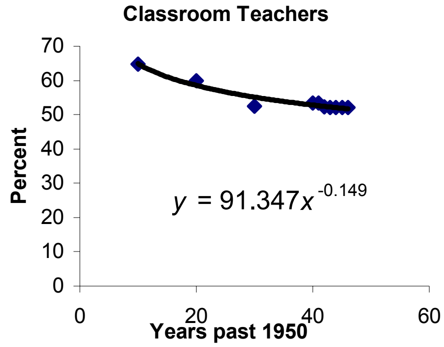
$y \approx 16.6$

The model predicts that in the year 2000, there were 16.6 students per teacher.

c. The function is decreasing.

d. Based on the model, the number of students per teacher will approach zero but will never reach zero.

32. a.



b. In 2008, $x = 58$.

$y = 91.347(58)^{-0.149}$

Using the TI-83 and substituting into the unrounded model

```
58→X
                 58
91.347213822645X
^-.14861294084257
        49.96053558
```

In 2008 the percentage of classroom teachers among full-time school staff is 50.0%.

c. No. The collected data stops in 1996. The calculation in part *b* is an extrapolation.

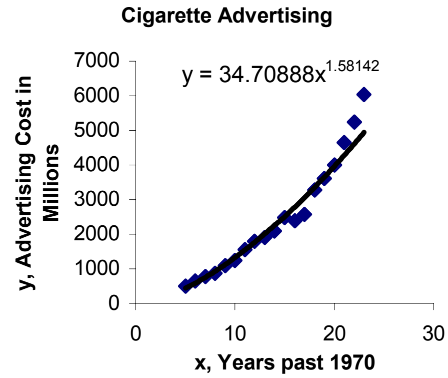
d.

X	Y1
55	50.356
56	50.222
57	50.09
58	49.961
59	49.834
60	49.709
61	49.587

X=58

The percentage of classroom teachers among full-time school staff falls below 50% during 2008.

33. a.



b. The year 2000 corresponds to $x = 30$.

$y = 34.70888x^{1.58142}$

$y = 34.70888(30)^{1.58142}$

Using the TI-83 and substituting into the unrounded model

```
30→X
                 30
34.708877276433X
^1.581419660237 1

        7522.967032
```

$y \approx 7523$

Cigarette advertising in 2000 is predicted to be \$7523 million.

c.

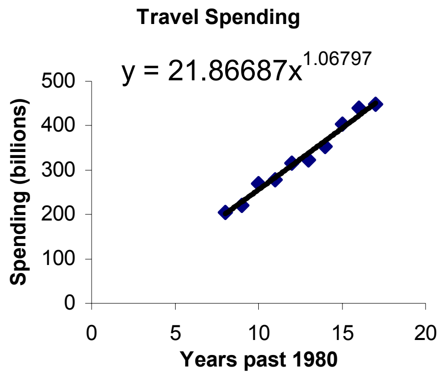
X	Y1
18	3353.9
19	3653.3
20	3962
21	4279.8
22	4606.5
23	4942
24	5286.1

X=20

When $x = 20$, the year is 1990.

d. Cigarette advertising expenditures are modeled by an increasing function.

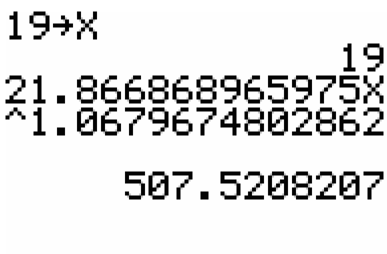
34. a.



b. In 1999, $x = 19$

$y = 21.86687(19)^{1.06797}$

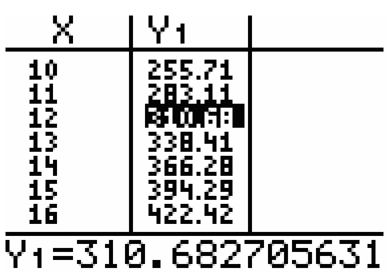
Using the TI-83 and substituting into the unrounded model



$y \approx 507.5$

In 1999 the model predicts travel spending will be $507.5 billion.

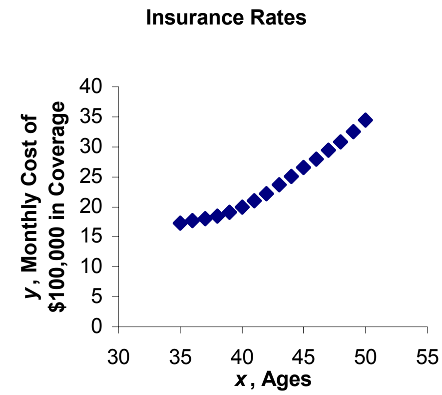
c. Travel spending reaches $300 billion during 1991 (1980 + 11).



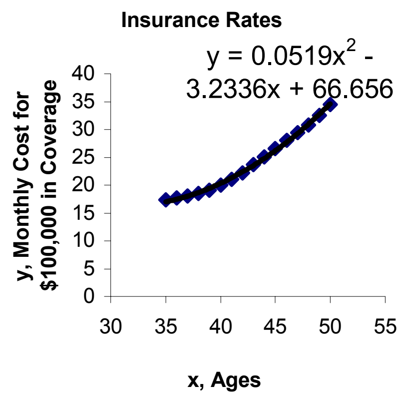
X	Y1	
10	255.71	
11	283.11	
12	310.68	
13	338.41	
14	366.28	
15	394.29	
16	422.42	

Y1=310.682705631

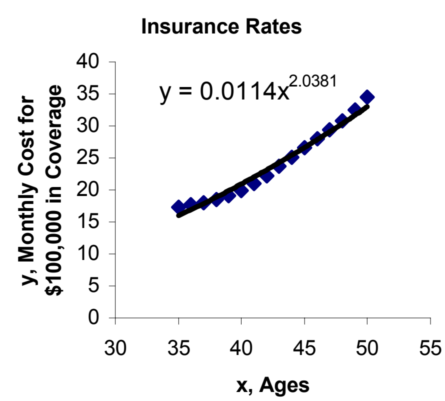
35. a.



b. Yes. It appears that a quadratic function will fit the data.

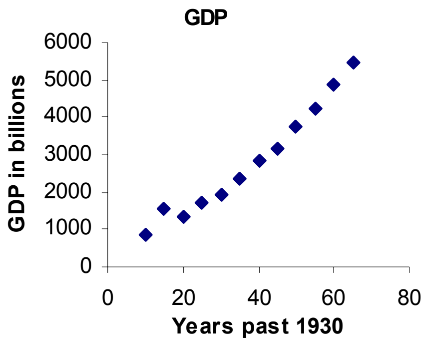


c.

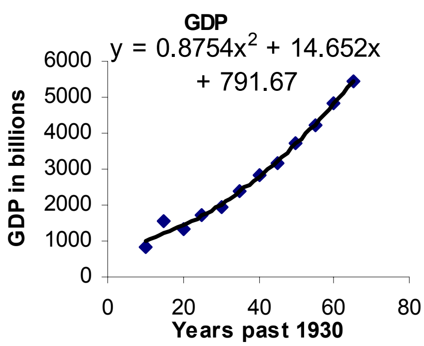


d. It appears that the quadratic model, based on the scatter plots, is the best fit.

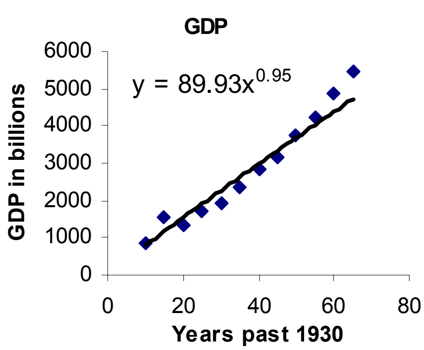
36. a.



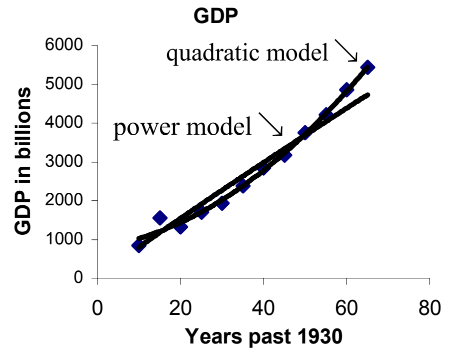
b. Yes. It appears that a quadratic function will fit the data.



c.

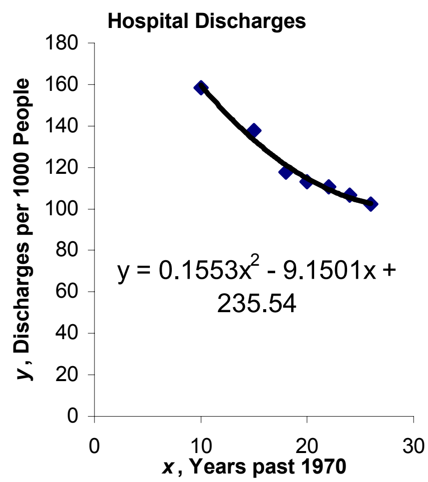


d.

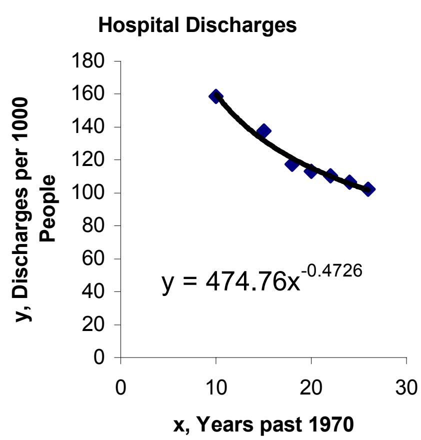


The quadratic model fits the data better.

37. a. Quadratic model



Power model



Based on the graphs, both models fit the data reasonably well. The power model might be slightly better.

b. Since 1988 the discharge rate is deceasing by approximately 2 per thousand people each year. Therefore, 84.3 per thousand people is a reasonable guess for the discharge rate in 2005. The quick estimate for the discharge rate in 2005 is 18 per thousand people less than the discharge rate in 1996.

c. In 2005, $x = 2005–1970 = 35$.
Using the TI-83 and substituting into the unrounded quadratic model

```
35→X
                 35
.1553330963533 3X
^2+-9.1500859401
148X+235.5369897
9592
        105.5670249
```

$y \approx 105.6$ discharges per 1000 people

Using the TI-83 and substituting into the unrounded power model

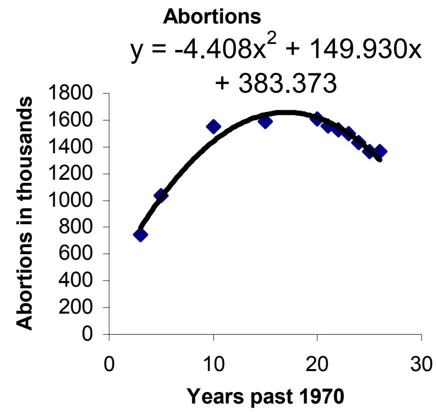
```
35→X
                 35
474.75818816637X
^-.47263475398 19

        88.44886076
```

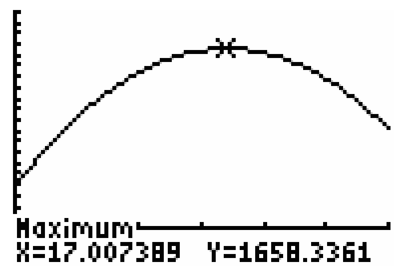
$y \approx 88.4$ discharges per 1000 people

Based on the solution to part *b*, the power function is a better fit to the data. The scatter plots in part *a* also support the conclusion that the power function is a better fit.

38. a.



b.



[0, 30] by [–300, 2000]

In 1997 the number of abortions reaches a maximum of approximately 1660 thousand.

Section 2.6 Skills Check

1. a. $(f+g)(x)$
$= f(x)+g(x)$
$= (3x-5)+(4-x)$
$= 2x-1$

b. $(f-g)(x)$
$= f(x)-g(x)$
$= (3x-5)-(4-x)$
$= 3x-5-4+x$
$= 4x-9$

c. $(f \bullet g)(x)$
$= f(x) \bullet g(x)$
$= (3x-5)(4-x)$
$= -3x^2+17x-20$

d. $\left(\dfrac{f}{g}\right)(x)$
$= \dfrac{f(x)}{g(x)}$
$= \dfrac{3x-5}{4-x}$

e. $g(x) \neq 0$
$4-x=0$
$x=4$
Domain: $(-\infty,4)\cup(4,\infty)$

2. a. $(f+g)(x)$
$= f(x)+g(x)$
$= (2x-3)+(5-x)$
$= x+2$

b. $(f-g)(x)$
$= f(x)-g(x)$
$= (2x-3)-(5-x)$
$= 2x-3-5+x$
$= 3x-8$

c. $(f \bullet g)(x)$
$= f(x) \bullet g(x)$
$= (2x-3)(5-x)$
$= -2x^2+13x-15$

d. $\left(\dfrac{f}{g}\right)(x)$
$= \dfrac{f(x)}{g(x)}$
$= \dfrac{2x-3}{5-x}$

e. $g(x) \neq 0$
$5-x=0$
$x=5$
Domain: $(-\infty,5)\cup(5,\infty)$

3. a. $(f+g)(x)$
$= f(x)+g(x)$
$= (x^2-2x)+(1+x)$
$= x^2-x+1$

b. $(f-g)(x)$
$= f(x)-g(x)$
$= (x^2-2x)-(1+x)$
$= x^2-2x-1-x$
$= x^2-3x-1$

c. $(f \cdot g)(x)$
$= f(x) \cdot g(x)$
$= (x^2 - 2x)(1 + x)$
$= x^3 - x^2 - 2x$

d. $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{x^2 - x}{1 + x}$

e. $g(x) \neq 0$
$1 + x = 0$
$x = -1$
Domain: $(-\infty, -1) \cup (-1, \infty)$

4. a. $(f + g)(x)$
$= f(x) + g(x)$
$= (2x^2 - x) + (2x + 1)$
$= 2x^2 + x + 1$

b. $(f - g)(x)$
$= f(x) - g(x)$
$= (2x^2 - x) - (2x + 1)$
$= 2x^2 - x - 2x - 1$
$= 2x^2 - 3x - 1$

c. $(f \cdot g)(x)$
$= f(x) \cdot g(x)$
$= (2x^2 - x)(2x + 1)$
$= 4x^3 - x$

d. $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{2x^2 - x}{2x + 1}$

e. $g(x) \neq 0$
$2x + 1 = 0$
$x = -\frac{1}{2}$
Domain: $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

5. a. $(f + g)(x)$
$= f(x) + g(x)$
$= \left(\frac{1}{x}\right) + \left(\frac{x+1}{5}\right)$
$LCM: 5x$
$= \frac{5}{5}\left(\frac{1}{x}\right) + \frac{x}{x}\left(\frac{x+1}{5}\right)$
$= \left(\frac{5}{5x}\right) + \left(\frac{x^2 + x}{5x}\right)$
$= \frac{x^2 + x + 5}{5x}$

b. $(f - g)(x)$
$= f(x) - g(x)$
$= \left(\frac{1}{x}\right) - \left(\frac{x+1}{5}\right)$
$LCM: 5x$
$= \frac{5}{5}\left(\frac{1}{x}\right) - \frac{x}{x}\left(\frac{x+1}{5}\right)$
$= \left(\frac{5}{5x}\right) - \left(\frac{x^2 + x}{5x}\right)$
$= \frac{-x^2 - x + 5}{5x}$

c. $(f \bullet g)(x)$
$= f(x) \bullet g(x)$
$= \left(\frac{1}{x}\right)\left(\frac{x+1}{5}\right)$
$= \frac{x+1}{5x}$

d. $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{\frac{1}{x}}{\frac{x+1}{5}}$
$= \frac{1}{x} \bullet \frac{5}{x+1}$
$= \frac{5}{x(x+1)}$

e. $g(x) \neq 0$
$x(x+1) = 0$
$x = 0, -1$
Domain: $(-\infty,-1) \cup (-1,0) \cup (0,\infty)$

6. a. $(f+g)(x)$
$= f(x) + g(x)$
$= \left(\frac{x-2}{3}\right) + \left(\frac{1}{x}\right)$
$LCM : 3x$
$= \frac{x}{x}\left(\frac{x-2}{3}\right) + \frac{3}{3}\left(\frac{1}{x}\right)$
$= \left(\frac{x^2-2x}{3x}\right) + \left(\frac{3}{3x}\right)$
$= \frac{x^2-2x+3}{3x}$

b. $(f-g)(x)$
$= f(x) - g(x)$
$= \left(\frac{x-2}{3}\right) + \left(\frac{1}{x}\right)$
$LCM : 3x$
$= \frac{x}{x}\left(\frac{x-2}{3}\right) - \frac{3}{3}\left(\frac{1}{x}\right)$
$= \left(\frac{x^2-2x}{3x}\right) - \left(\frac{3}{3x}\right)$
$= \frac{x^2-2x-3}{3x}$

c. $(f \bullet g)(x)$
$= f(x) \bullet g(x)$
$= \left(\frac{x-2}{3}\right)\left(\frac{1}{x}\right)$
$= \frac{x-2}{3x}$

d. $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{\frac{x-2}{3}}{\frac{1}{x}}$
$= \frac{x-2}{3} \bullet \frac{x}{1}$
$= \frac{x^2-2x}{3}$

e. $g(x) \neq 0$
$3 = 0$ is impossible.
Note that $g(x)$ is undefined when $x = 0$. Therefore the domain of the composition is not all real numbers.
Domain: $(-\infty,0) \cup (0,\infty)$

7. **a.** $(f+g)(x)$
$= f(x)+g(x)$
$= \left(\sqrt{x}\right)+\left(1-x^2\right)$
$= \sqrt{x}+1-x^2$

b. $(f-g)(x)$
$= f(x)-g(x)$
$= \left(\sqrt{x}\right)-\left(1-x^2\right)$
$= \sqrt{x}-1+x^2$

c. $(f \bullet g)(x)$
$= f(x) \bullet g(x)$
$= \left(\sqrt{x}\right)\left(1-x^2\right)$

d. $\left(\dfrac{f}{g}\right)(x)$
$= \dfrac{f(x)}{g(x)}$
$= \dfrac{\sqrt{x}}{1-x^2}$

e. $g(x) \neq 0$
$1-x^2=0$
$x=-1,1$
Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

8. **a.** $(f+g)(x)$
$= f(x)+g(x)$
$= \left(x^3\right)+\left(\sqrt{x+3}\right)$
$= x^3+\sqrt{x+3}$

b. $(f-g)(x)$
$f(x)-g(x)$
$\left(x^3\right)-\left(\sqrt{x+3}\right)$
$x^3-\sqrt{x+3}$

c. $(f \bullet g)(x)$
$= f(x) \bullet g(x)$
$= \left(x^3\right)\left(\sqrt{x+3}\right)$

d. $\left(\dfrac{f}{g}\right)(x)$
$= \dfrac{f(x)}{g(x)}$
$= \dfrac{x^3}{\sqrt{x+3}}$

e. $g(x) \neq 0$
$x+3=0$
$x=-3$
Additionally, because of the square root in the denominator
$x+3>0$
$x>-3$
Domain: $(-3,\infty)$

9. **a.** $(f+g)(2)$
$= f(2)+g(2)$
$= \left(2^2-5(2)\right)+\left(6-(2)^3\right)$
$= -6-2$
$= -8$

b. $(g-f)(-1)$
$= g(-1)-f(-1)$
$= \left(6-(-1)^3\right)-\left((-1)^2-5(-1)\right)$
$= 7-6$
$= 1$

c. $(f \bullet g)(-2)$
$= f(-2) \bullet g(-2)$
$= \left((-2)^2-5(-2)\right) \bullet \left(6-(-2)^3\right)$
$= (14)(14)$
$= 196$

d. $\left(\frac{g}{f}\right)(3)$
$= \frac{g(3)}{f(3)}$
$= \frac{\left(6-(3)^3\right)}{\left(3^2-5(3)\right)}$
$= \frac{-21}{-6}$
$= 3.5$

d. $\left(\frac{g}{f}\right)(2)$
$= \frac{g(2)}{f(2)}$
$= \frac{\left((2)^3+(2)\right)}{\left(4-(2)^2\right)}$
$= \frac{10}{0}$
Undefined expression

10. a. $(f+g)(1)$
$= f(1)+g(1)$
$= \left(4-(1)^2\right)+\left((1)^3+(1)\right)$
$= 3+2$
$= 5$

b. $(f-g)(-2)$
$= f(-2)-g(-2)$
$= \left(4-(-2)^2\right)-\left((-2)^3+(-2)\right)$
$= 0-(-10)$
$= 10$

c. $(f \bullet g)(-3)$
$= f(-3) \bullet g(-3)$
$= \left(4-(-3)^2\right)\left((-3)^3+(-3)\right)$
$= (-5)(-30)$
$= 150$

11. a. $(f \circ g)(x)$
$= f\left(g(x)\right)$
$= 2(3x-1)-6$
$= 6x-8$

b. $(g \circ f)(x)$
$= g\left(f(x)\right)$
$= 3(2x-6)-1$
$= 6x-19$

12. a. $(f \circ g)(x)$
$= f\left(g(x)\right)$
$= 3(2x-2)-2$
$= 6x-8$

b. $(g \circ f)(x)$
$= g\left(f(x)\right)$
$= 2(3x-2)-2$
$= 6x-6$

13. a. $(f \circ g)(x)$
$= f(g(x))$
$= \left(\frac{1}{x}\right)^2$
$= \frac{1}{x^2}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= \frac{1}{x^2}$

14. a. $(f \circ g)(x)$
$= f(g(x))$
$= \left(\frac{2}{x}\right)^3$
$= \frac{8}{x^3}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= \frac{2}{x^3}$

15. a. $(f \circ g)(x)$
$= f(g(x))$
$= \sqrt{(2x-7)-1}$
$= \sqrt{2x-8}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= 2\left(\sqrt{x-1}\right)-7$
$= 2\sqrt{x-1}-7$

16. a. $(f \circ g)(x)$
$= f(g(x))$
$= \sqrt{3-(x-5)}$
$= \sqrt{8-x}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= \left(\sqrt{3-x}\right)-5$
$= \sqrt{3-x}-5$

17. a. $(f \circ g)(x)$
$= f(g(x))$
$= |(4x)-3|$
$= |4x-3|$

b. $(g \circ f)(x)$
$= g(f(x))$
$= 4(|x-3|)$
$= 4|x-3|$

18. a. $(f \circ g)(x)$
$= f(g(x))$
$= |4-(2x+1)|$
$= |3-2x|$

b. $(g \circ f)(x)$
$= g(f(x))$
$= 2(|4-x|)+1$
$= 2|4-x|+1$

19. a. $(f \circ g)(x)$

$= f(g(x))$

$= \dfrac{3\left(\dfrac{2x-1}{3}\right)+1}{2}$

$= \dfrac{2x-1+1}{2}$

$= \dfrac{2x}{2}$

$= x$

b. $(g \circ f)(x)$

$= g(f(x))$

$= \dfrac{2\left(\dfrac{3x+1}{2}\right)-1}{3}$

$= \dfrac{3x+1-1}{3}$

$= \dfrac{3x}{3}$

$= x$

20. a. $(f \circ g)(x)$

$= f(g(x))$

$= \sqrt[3]{(x^3+1)+1}$

$= \sqrt[3]{x^3+2}$

b. $(g \circ f)(x)$

$= g(f(x))$

$= \left(\sqrt[3]{x+1}\right)^3+1$

$= x+1+1$

$= x+2$

21. a. $f(g(2)) = 2\left(\dfrac{2-5}{3}\right)^2 = 2(-1)^2 = 2$

b. $g(f(-2)) = \dfrac{\left[2(-2)^2\right]-5}{3}$

$= \dfrac{8-5}{3}$

$= \dfrac{3}{3}$

$= 1$

22. a. $f(g(2)) = \left((3(2)-1)-1\right)^2$

$= (5-1)^2$

$= 4^2$

$= 16$

b. $g(f(-2)) = 3\left([-2-1]^2\right)-1$

$= 3\left([-3]^2\right)-1$

$= 3(9)-1$

$= 26$

23. a. $(f+g)(2) = f(2)+g(2) = 1+(-3) = -2$

b. $(fg)(-1) = f(-1) \cdot g(-1) = -2 \cdot 0 = 0$

c. $\left(\dfrac{f}{g}\right)(4) = \dfrac{f(4)}{g(4)} = \dfrac{3}{-1} = -3$

d. $(f \circ g)(1) = f(g(1)) = f(-2) = -3$

e. $(g \circ f)(-2) = g(f(-2)) = g(-3) = 2$

24. a. $(g-f)(-2) = g(-2)-f(-2)$

$= 1-(-3)$

$= 4$

b. $(fg)(3) = f(3) \cdot g(3) = 2 \cdot -2 = -4$

c. $\left(\frac{f}{g}\right)(0)=\frac{f(0)}{g(0)}=\frac{-1}{-1}=1$

d. $(f\circ g)(-2)=f(g(-2))=f(1)=0$

e. $(g\circ f)(2)=g(f(2))=g(1)=-2$

25. a. $$\begin{aligned}P(x)&=R(x)-C(x)\\&=(89x)-(23x+3420)\\&=89x-23x-3420\\&=66x-3420\end{aligned}$$

b. $P(150)=66(150)-3420=6480$
The profit on the production and sale of 150 bicycles is $6480.

26. a. $$\begin{aligned}P(x)&=R(x)-C(x)\\&=(988x)-(189x+5460)\\&=988x-189x-5460\\&=799x-5460\end{aligned}$$

b. $P(80)=799(80)-5460=58,460$
The profit on the production and sale of 80 televisions is $58,460.

27. a. The revenue function is linear, while the cost function is quadratic. Note that *C(x)* fits the form
$f(x)=ax^2+bx+c, a\neq 0.$

b. $$\begin{aligned}P(x)&=R(x)-C(x)\\P(x)&=1050x-(10,000+30x+x^2)\\P(x)&=1050x-10,000-30x-x^2\\P(x)&=-x^2+1020x-10,000\end{aligned}$$

c. Quadratic. Note that *P(x)* fits the form
$f(x)=ax^2+bx+c, a\neq 0.$

28. a. The revenue function is linear, while the cost function is quadratic. Note that *C(x)* fits the form
$f(x)=ax^2+bx+c, a\neq 0.$

b. $$\begin{aligned}&P(x)\\&=R(x)-C(x)\\&=26,600x-(200,000+4600x+2x^2)\\&=26,600x-200,000-4600x-2x^2\\&=-2x^2+22,000x-200,000\end{aligned}$$

c. Quadratic. Note that *P(x)* fits the form
$f(x)=ax^2+bx+c, a\neq 0.$

29. a. $$\begin{aligned}P(x)&=R(x)-C(x)\\P(x)&=550x-(10,000+30x+x^2)\\P(x)&=550x-10,000-30x-x^2\\P(x)&=-x^2+520x-10,000\end{aligned}$$

b. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.
$h=\frac{-b}{2a}=\frac{-520}{2(-1)}=\frac{-520}{-2}=260$

c. $$\begin{aligned}k&=P(h)\\&=P(260)\\&=-(260)^2+520(260)-10,000\\&=-67,600-135,200-10,000\\&=57,600\end{aligned}$$

Producing 260 units yields a maximum profit of $57,600.

30. a. $$\begin{aligned}P(x)&=R(x)-C(x)\\P(x)&=6600x-(2000+4800x+2x^2)\\P(x)&=6600x-2000-4800x-2x^2\\P(x)&=-2x^2+1800x-2000\end{aligned}$$

b. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.

$$h = \frac{-b}{2a} = \frac{-1800}{2(-2)} = \frac{-1800}{-4} = 450$$

c. $$\begin{aligned} k &= P(h) \\ &= P(450) \\ &= -2(450)^2 + 1800(450) - 2000 \\ &= -405{,}000 + 810{,}000 - 2000 \\ &= 403{,}000 \end{aligned}$$

Producing 450 units yields a maximum profit of $403,000.

31. a. $\overline{C}(x)$ fits the form $\left(\frac{f}{g}\right)(x)$ where $f(x) = C(x)$ and $g(x) = x$. Note that $\overline{C}(x) = \frac{C(x)}{x}$.

b. Let $x = 3000$ and calculate $\overline{C}(x)$.

$$\begin{aligned} \overline{C}(3000) &= \frac{50{,}000 + 105(3000)}{3000} \\ &= \frac{365{,}000}{3000} \\ &= 121.\overline{6} \approx \$121.67 \text{ per unit} \end{aligned}$$

32. a. $C(p)$ fits the form $(f - g)(p)$ where $f(p) = \frac{120{,}000}{p}$ and $g(p) = 1200$.

b. $C(p) = \frac{120{,}000}{p} - 1200$

$$\begin{aligned} C(100) &= \frac{120{,}000}{100} - 1200 \\ &= 1200 - 1200 = 0 \end{aligned}$$

Completely impure water is free!

33. a. $\overline{C}(x) = \frac{C(x)}{x} = \frac{3000 + 72x}{x}$

b. $$\begin{aligned} \overline{C}(100) &= \frac{C(100)}{100} \\ &= \frac{3000 + 72(100)}{100} \\ &= \frac{3000 + 7200}{100} \\ &= \frac{10{,}200}{100} \\ &= 102 \text{ or } \$102 \text{ per printer} \end{aligned}$$

34. a. $\overline{C}(x) = \frac{C(x)}{x} = \frac{2.15x + 2350}{x}$

b. $$\begin{aligned} \overline{C}(100) &= \frac{2.15(100) + 2350}{100} \\ &= \frac{215 + 2350}{100} \\ &= \frac{2565}{100} = 25.65 \end{aligned}$$

The average cost for the production of 100 components is $25.65 per component.

35. a. Let $T(p)$ represent the total number of tickets for a home football game.

$$\begin{aligned} &T(p) \\ &= S(p) + N(p) \\ &= (62p + 8500) + (0.5p^2 + 16p + 4400) \\ &= 0.5p^2 + 78p + 12{,}900 \end{aligned}$$

b. Since p represents the winning percentage for the football team, $0 \le p \le 100$. Therefore the domain of the function is $[0, 100]$.

c. $$\begin{aligned} T(90) &= 0.5(90)^2 + 78(90) + 12{,}900 \\ &= 4050 + 7020 + 12{,}900 \\ &= 23{,}970 \end{aligned}$$

The stadium holds 23,970 people.

36. $(T \bullet P)(c) = T(c) \bullet P(c)$

The function represents the number of t-shirts sold multiplied by the price per shirt. The result is the revenue for selling *c* shirts.

37. a. $B(8) = 6(8+1)^{\frac{3}{2}} = 6(27) = 162$

On May 8th the number of bushels of tomatoes harvested is 162.

b. $P(8) = 8.5 - 0.12(8) = 7.54$

On May 8th the price per bushel of tomatoes is \$7.54.

c. $(B \bullet P)(x)$ represents the worth of the tomatoes on the x^{th} day of May.

$$\begin{aligned}(B \bullet P)(8) &= B(8) \bullet P(8)\\ &= 162 \bullet 7.54\\ &= 1221.48\end{aligned}$$

On May 8th the worth is \$1221.48.

d.
$$\begin{aligned}W(x) &= (B \bullet P)(x)\\ &= B(x) \bullet P(x)\\ &= \left[6(x+1)^{\frac{3}{2}}\right] \bullet (8.5 - 0.12x)\\ &= 6(x+1)^{\frac{3}{2}}(8.5 - 0.12x)\end{aligned}$$

38. $C(x) = 3000 + 3.30x^2$

39. a.
$$\begin{aligned}P(x) &= R(x) - C(x)\\ &= (592x) - (32{,}000 + 432x)\\ &= 592x - 32{,}000 - 432x\\ &= 160x - 32{,}000\end{aligned}$$

b. $P(600) = 160(600) - 32{,}000 = 64{,}000$

The profit for producing and selling 600 satellite systems is \$64,000.

c. Since the function is linear, the rate of change is constant. For every one unit increase in production, the profit increases by \$160.

40. a.
$$\begin{aligned}P(x) &= R(x) - C(x)\\ &= (295x) - (87{,}500 + 87x)\\ &= 295x - 87{,}500 - 87x\\ &= 208x - 87{,}500\end{aligned}$$

b. $P(700) = 208(700) - 87{,}500 = 58{,}100$

The profit for producing and selling 700 computers is \$58,100.

c. $(0, -87{,}500)$. It represents the fixed costs for manufacturing and selling the computers. If no computers are manufactured and sold, the company will lose \$87,500 per month.

41. a. Let $P(t)$ represent the total U.S. population under age 5 in millions.

Then, $P(t) = B(t) + G(t)$

$$\begin{aligned}P(t) = {}&\left(0.0076t^2 - 0.1752t + 10.705\right) +\\ &\left(0.0064t^2 - 0.1448t + 10.12\right)\end{aligned}$$

$P(t) = 0.014t^2 - 0.32t + 20.825$

b.
$$\begin{aligned}&P(2003 - 1990)\\ &= P(13)\\ &= 0.014(13)^2 - 0.32(13) + 20.825\\ &= 2.366 - 4.16 + 20.825\\ &= 19.031\end{aligned}$$

The model predicts that in 2003 there are 19.031 million children under age 5.

42. a. No. Adding the percentages is not valid since the percentages are based on different populations of people. Adding the number of males and number of females completing college and then dividing by the total is a legitimate approach.

b. Based on results using the table feature of the TI-83 calculator, the percentage in 1990 is 26.821%, and the percentage in 1999 is 32.050 %.

X	Y1
20	26.821
23	28.132
26	29.875
29	32.05
32	34.657
35	37.696
38	41.167

X=20

c. $1990 \Rightarrow \dfrac{28.0+26.2}{2} = 27.1$

$1999 \Rightarrow \dfrac{31.2+33.0}{2} = 32.1$

The percentages are relatively close but not the same.

43. For a)–d), consider the output from the function.

a. Meat in a Styrofoam container

b. Ground meat.

c. Meat ground and then ground again.

d. Ground meat placed in a Styrofoam container.

e. Meat placed in a Styrofoam container and then ground.

f. Only part d), unless the reader enjoys ground Styrofoam!

44. a. A sock is placed on the left foot.

b. A sock is placed on the left foot, followed by a second sock on the left foot. The result is two socks on the left foot.

c. $(g \circ f)(\text{right foot})$

$= g\left(f(\text{right foot})\right)$

$= g(\text{sock on right foot})$

$=$ sock on right foot removed

The net result is that a sock is placed on the right foot and then removed. Ultimately there is no sock on the right foot.

45. Let $B(x)$ convert a Japanese shoe size, x, into a British shoe size, $B(x)$.

Japanese $\rightarrow$ U.S. $\rightarrow$ British

$$\begin{aligned} B(x) &= (p \circ s)(x) \\ &= p\left(s(x)\right) \\ &= (x-17)-1.5 \\ &= x-18.5 \end{aligned}$$

46. Let $C(x)$ convert a British shoe size, x, into a Continental shoe size, $C(x)$.

British $\rightarrow$ U.S. $\rightarrow$ Continental

$$\begin{aligned} C(x) &= (t \circ d)(x) \\ &= t\left(d(x)\right) \\ &= (x+0.5)+34.5 \\ &= x+35 \end{aligned}$$

47. Chilean pesos $\to$ Austrian schillings $\to$ Russian rubles

Let $R(x) = 1.987376x,$ where x is schillings and $R(x)$ is rubles.
Let $S(x) = 0.025202x,$ where x is pesos and $S(x)$ is schillings.
Let $V(x) = (R \circ S)(x),$ where x is pesos and $V(x)$ is rubles.

$$\begin{aligned} V(x) &= R(S(x)) \\ &= 1.987376(0.025202x) \\ &= 0.05008585x \end{aligned}$$

$$\begin{aligned} V(1000) &= 0.05008585(1000) \\ &= 50.08585 \approx 50.09 \end{aligned}$$

48. French francs $\to$ British pounds $\to$ U.S. dollars

Let $B(x) = 0.096115x,$ where x is francs and $B(x)$ is pounds.
Let $D(x) = 0.1.495701x,$ where x is pounds and $D(x)$ is dollars.
Let $V(x) = (D \circ B)(x),$ where x is francs and $V(x)$ is dollars.

$$\begin{aligned} V(x) &= D(B(x)) \\ &= 1.495701(0.096115x) \\ &= 0.1437593016x \end{aligned}$$

$$\begin{aligned} V(100) &= 0.1437593016(100) \\ &= 14.37593016 \approx 14.37 \end{aligned}$$

49. The function is $100 \bullet \left(\frac{f}{g}\right)(x)$. Note that

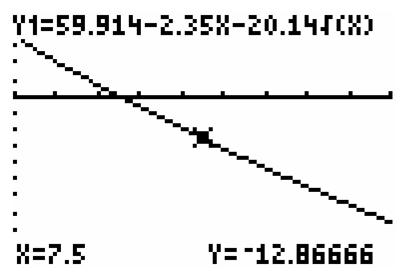
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} =$ the ratio of AOL customers to total Internet users.

Multiplying by 100 creates a percentage.

50. number of homes with computers
$=$ (percentage of homes with computers) $\times$ (number of homes)
In symbols, $(f \bullet g)(x) = f(x) \bullet g(x)$

51. The function is $(f + g)(x) = f(x) + g(x)$.

52. a. If $f_1 = 59.914 - 2.35s$ and $f_2 = 20.14\sqrt{s}$, then $C = f_1 - f_2$.
Answers may vary.

b. 

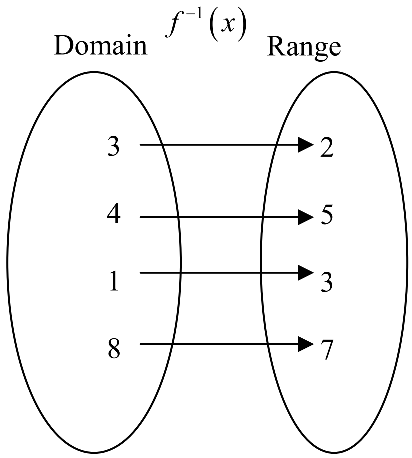
[3, 12] by [–50, 25]

c. Based on the graph in part *b*, the function is deceasing. As *s* increase, *C* decreases.

53. The normal price is $0.50x$ where x represents retail price. Since the sale price is 20% off the normal price, the sale price is
$0.50x - (0.20)(0.50x) = 0.50x - 0.10x = 0.40x.$
Therefore the books are on sale for 40% of retail price.

Section 2.7 Skills Check

1. Yes, the function is one-to-one and has an inverse.



2. No, the function is not one-to-one. Inputs of 4 and 18 correspond with an output of 7.

3. No, the function is not on-to-one. Inputs of 5 and 6 correspond with an output of 2.

4. Yes, the function is one-to-one and has an inverse.
$\{(8,2),(9,3),(10,4),(11,5)\}$

5. **a.** $f(g(x)) = (f \circ g)(x) = 3\left(\dfrac{x}{3}\right) = x$

$g(f(x)) = (g \circ f)(x) = \dfrac{(3x)}{3} = x$

b. Yes, since $(f \circ g)(x) = (g \circ f)(x) = x$.

6. **a.**
$$\begin{aligned} f(g(x)) &= (f \circ g)(x) \\ &= 4\left(\frac{x+1}{4}\right) - 1 \\ &= x + 1 - 1 \\ &= x \end{aligned}$$

b.
$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= \frac{(4x-1)+1}{4} \\ &= \frac{4x}{4} = x \end{aligned}$$

Note that $(f \circ g)(x) = (g \circ f)(x) = x$.
Therefore, yes, *f* and *g* are inverse functions

7. Is $(f \circ g)(x) = (g \circ f)(x) = x$?
$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= \left(\sqrt[3]{x-1}\right)^3 + 1 \\ &= x - 1 + 1 \\ &= x \end{aligned}$$
$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= \sqrt[3]{x^3 + 1 - 1} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$
Yes, *f* and *g* are inverse functions.

8. Is $(f \circ g)(x) = (g \circ f)(x) = x$?
$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= \left(\sqrt[3]{x+2} - 2\right)^3 \end{aligned}$$
$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= \sqrt[3]{(x-2)^3 + 2} \end{aligned}$$
No, *f* and *g* are not inverse functions.

9. See the completed table below.

x	*f(x)*
–1	–7
0	–4
1	–1
2	2
3	5

x	$f^{-1}(x)$
–7	–1
–4	0
–1	1
2	2
5	3

Note that values for x in the second table are the values for $f(x)$ in the first table.

10. See the completed table below.

x	$f(x)$
–2	–17
–1	–3
0	–1
1	1
2	15

x	$f^{-1}(x)$
–17	–2
–3	–1
–1	0
1	1
15	2

Note that values for x in the second table are the values for $f(x)$ in the first table.

11. a.
$$f(x) = 3x - 4$$
$$y = 3x - 4$$
$$x = 3y - 4$$
$$3y = x + 4$$
$$y = \frac{x+4}{3}$$
$$f^{-1}(x) = \frac{x+4}{3}$$

b. Yes. Substituting the x-values from the table into $f^{-1}(x)$ generates the $f^{-1}(x)$ outputs found in the table.

12. a.
$$f(x) = 2x^3 - 1$$
$$y = 2x^3 - 1$$
$$x = 2y^3 - 1$$
$$x + 1 = 2y^3$$
$$\frac{x+1}{2} = y^3$$
$$y = \sqrt[3]{\frac{x+1}{2}}$$
$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}$$

b. Yes. Substituting the x-values from the table into $f^{-1}(x)$ generates the $f^{-1}(x)$ outputs found in the table.

13. Since x and y are interchanged to create the inverse function, if (a, b) is on point on $f(x)$ then (b, a) is a point on $f^{-1}(x)$.

14. $h^{-1}(-2) = 3 \Leftrightarrow h(3) = -2$, if h and h^{-1} are inverse functions.

15.
$$f(x) = \frac{1}{x}$$
$$y = \frac{1}{x}$$
$$x = \frac{1}{y}$$
$$xy = 1$$
$$y = \frac{1}{x}$$
$$f^{-1}(x) = \frac{1}{x}$$
Note that in this example the function and its inverse are the same function!

16.
$$g(x) = 4x + 1$$
$$y = 4x + 1$$
$$x = 4y + 1$$
$$4y = x - 1$$
$$y = \frac{x-1}{4}$$
$$g^{-1}(x) = \frac{x-1}{4}$$

17. $f(x) = 4x^2$

$$y = 4x^2$$
$$x = 4y^2$$
$$y^2 = \frac{x}{4}$$
$$y = \pm\sqrt{\frac{x}{4}}$$

Since $x \geq 0$, the positive solution represents the inverse function.

$$f^{-1}(x) = \sqrt{\frac{x}{4}} = \frac{\sqrt{x}}{2}$$

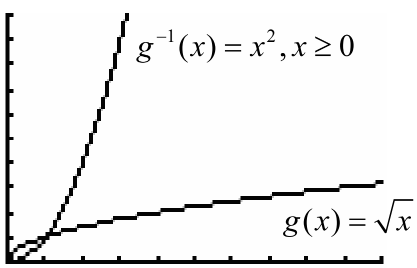
18. $g(x) = x^2 - 3$

$$y = x^2 - 3$$
$$x = y^2 - 3$$
$$y^2 = x + 3$$
$$y = \pm\sqrt{x+3}$$

Since $x \geq 0$, the positive solution represents the inverse function.

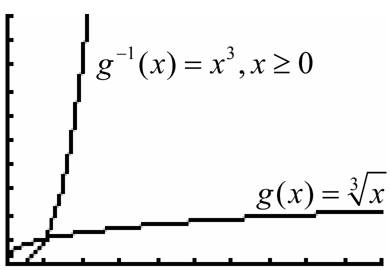
$$g^{-1}(x) = \sqrt{x+3}$$

19.



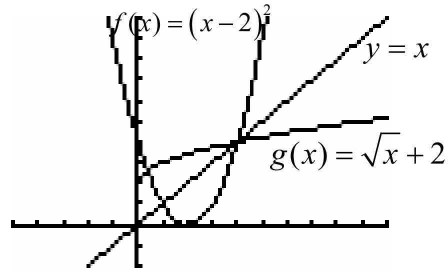
[0,10] by [0, 10]

20.



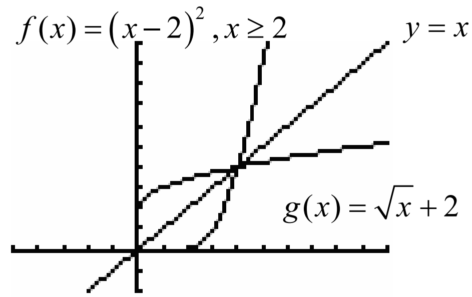
[0,10] by [0, 10]

21.



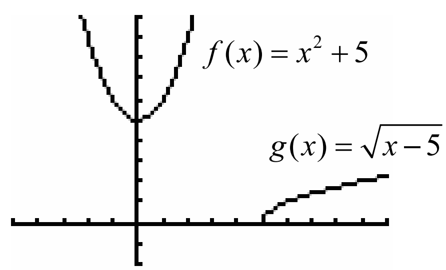
[–5, 10] by [–2, 10]

Restricting the domain of *f* to $[2, \infty)$ and the domain of *g* to $[0, \infty)$ forces *f* and *g* to become inverse functions.



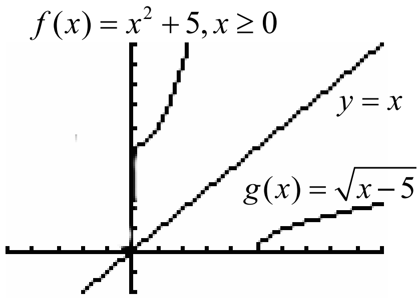
[–5, 10] by [–2, 10]

22.



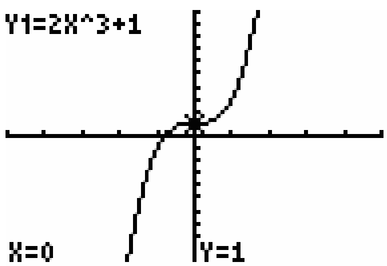
[–5, 10] by [–2, 10]

Restricting the domain of *f* to $[0, \infty)$ and the domain of *g* to $[5, \infty)$ forces *f* and *g* to become inverse functions.



[–5, 10] by [–2, 10]

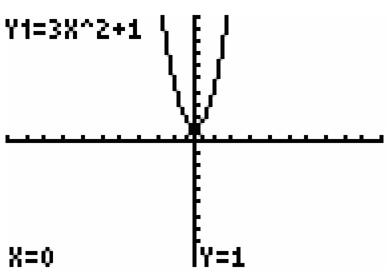
23.



[–5, 5] by [–10, 10]

The function passes the horizontal line test, is one-to-one, and has an inverse function.

24.



[–10, 10] by [–10, 10]

The function does not pass the horizontal line test, is not one-to-one, and does not have an inverse function.

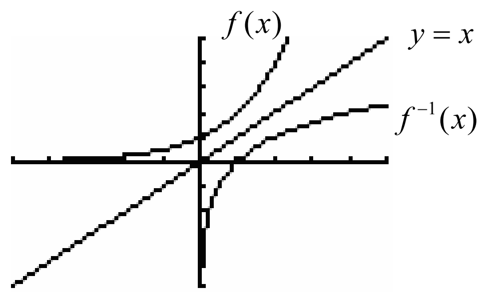
25. Yes. Every x matches with exactly one y, and every y matches with exactly one x.

26. Yes. Every x matches with exactly one y, and every y matches with exactly one x.

27. The graph fails the horizontal line test. It is not a one-to-one function.

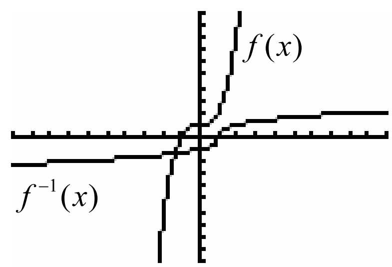
28. The graph fails the horizontal line test. It is not a one-to-one function.

29.



[–5, 5] by [–5, 5]

30.



[–10, 10] by [–10, 10]

Section 2.7 Exercises

31. a.
$$\begin{aligned} d(x) &= x + 0.5 \\ y &= x + 0.5 \\ x &= y + 0.5 \\ y &= x - 0.5 \\ d^{-1}(x) &= x - 0.5 \end{aligned}$$

b. $d^{-1}(8.5) = 8.5 - 0.5 = 8$

The British shoe size is 8.

32. a.
$$\begin{aligned} t(x) &= x + 34.5 \\ y &= x + 34.5 \\ x &= y + 34.5 \\ y &= x - 34.5 \\ t^{-1}(x) &= x - 34.5 \end{aligned}$$

b. $t^{-1}(43) = 43 - 34.5 = 8.5$

The U.S. shoe size is 8.5 or $8\frac{1}{2}$.

33.
$$\begin{aligned} f(t) &= 75.451 - 0.707t \\ y &= 75.451 - 0.707t \\ t &= 75.451 - 0.707y \\ t - 75.451 &= -0.707y \\ y &= \frac{t - 75.451}{-0.707} \\ y &= \frac{75.451 - t}{0.707} \\ f^{-1}(t) &= \frac{75.451 - t}{0.707} \\ f^{-1}(65) &= \frac{75.451 - 65}{0.707} \\ &= \frac{10.451}{0.707} \\ &= 14.78218 \approx 15 \end{aligned}$$

The percentage dropped below 65% in 1990.

34. a.
$$\begin{aligned} S(x) &= x + 0.6x \\ y &= x + 0.6x \\ x &= y + 0.6y \\ x &= 1.6y \\ y &= \frac{x}{1.6} \\ S^{-1}(x) &= \frac{x}{1.6} \end{aligned}$$

b. Given the future value of an investment, the function $S^{-1}(x)$ calculates the amount originally invested.

c. $S^{-1}(24{,}000) = \dfrac{24{,}000}{1.6} = \$15{,}000$

35. a.
$$\begin{aligned} f(x) &= 225.304x + 493.432 \\ y &= 225.304x + 493.432 \\ x &= 225.304y + 493.432 \\ x - 493.432 &= 225.304y \\ y &= \frac{x - 493.432}{225.304} \\ f^{-1}(x) &= \frac{x - 493.432}{225.304} \end{aligned}$$

The inverse function will calculate the number of years beyond 1990 in which Ritalin consumption in grams per 100,000 people reaches a given level.

b. $f^{-1}(1170) = \dfrac{1170 - 493.432}{225.304} = 3.00$

Ritalin consumption equals 1170 grams in 1993.

36. $A(x) = 82.35 + 29.3x$

$$y = 82.35 + 29.3x$$
$$x = 82.35 + 29.3y$$
$$29.3y = x - 82.35$$
$$y = \frac{x - 82.35}{29.3}$$
$$A^{-1}(x) = \frac{x - 82.35}{29.3}$$
$$A^{-1}(97) = \frac{97 - 82.35}{29.3} = 0.5 = 50\%$$

A temperature of 97° corresponds to 50% relative humidity.

37. a. $f(x) = 4\sqrt{4x+1}$

To determine the domain,

solve $4x + 1 \geq 0$.

$4x \geq -1$

$x \geq -\frac{1}{4}$

Domain: $\left[-\frac{1}{4}, \infty\right)$

Consequently, the range is $[0, \infty)$.

b. $f(x) = 4\sqrt{4x+1}$

$$y = 4\sqrt{4x+1}$$
$$x = 4\sqrt{4y+1}$$
$$\frac{x}{4} = \sqrt{4y+1}$$
$$\left(\frac{x}{4}\right)^2 = \left(\sqrt{4y+1}\right)^2$$
$$\frac{x^2}{16} = 4y + 1$$
$$4y = \frac{x^2}{16} - 1$$
$$y = \frac{\frac{x^2 - 16}{16}}{4}$$
$$y = \frac{x^2 - 16}{64}$$
$$f^{-1}(x) = \frac{x^2 - 16}{64}$$

c. The domain of the inverse function is the range of the original function, and the range of the inverse function is the domain of the original function. Therefore,

Domain: $[0, \infty)$

Range: $\left[-\frac{1}{4}, \infty\right)$

d. Both the domain and range would be $[0, \infty)$.

38. a. $W(x) = 0.002x^3$

$$y = 0.002x^3$$
$$x = 0.002y^3$$
$$y^3 = \frac{x}{0.002}$$
$$y = \sqrt[3]{\frac{x}{0.002}} = \sqrt[3]{500x}$$
$$W^{-1}(x) = \sqrt[3]{500x}$$

b. Given the weight, the inverse function calculates the length.

c. $W^{-1}(2) = \sqrt[3]{500(2)} = \sqrt[3]{1000} = 10$
The length of the fish is 10 inches.

d. Both the domain and the range are $(0, \infty)$. The weight and the length must be greater than zero.

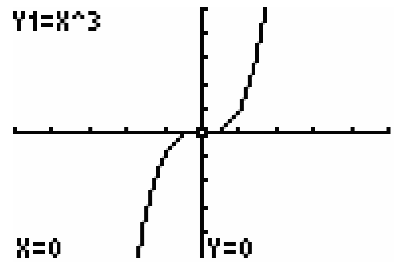
39.
$$\begin{aligned} C(x) &= x+3 \\ y &= x+3 \\ x &= y+3 \\ x-3 &= y \\ y &= x-3 \\ C^{-1}(x) &= x-3 \end{aligned}$$
The decoded numerical sequence is {20 8 5 27 18 5 1 12 27 20 8 9 14 7}, which translates into "The_real_thing."

40.
$$\begin{aligned} C(x) &= 3x+2 \\ y &= 3x+2 \\ x &= 3y+2 \\ 3y &= x-2 \\ y &= \frac{x-2}{3} \\ C^{-1}(x) &= \frac{x-2}{3} \end{aligned}$$
The decoded numerical sequence is {13 1 11 5 27 13 25 27 4 1 25}, which translates into "Make_my_day."

41. Yes. Each person has a unique social security number. Since no two people have the same number, the function is one-to-one.

42. No. The given function is not one-to-one. More than one check could correspond to the same dollar amount.

43. a. Yes. The function is one-to-one. It passes the horizontal line test.



[–5, 5] by [–5, 5]

b.
$$\begin{aligned} f(x) &= x^3 \\ y &= x^3 \\ x &= y^3 \\ \sqrt[3]{x} &= \sqrt[3]{y^3} \\ y &= \sqrt[3]{x} \\ f^{-1}(x) &= \sqrt[3]{x} \end{aligned}$$

c. Both domain and the range are $[0, \infty)$ based on the physical context of the question.

d. The inverse function is used to convert from the volume of the cube into the side length of the cube.

44. a. Yes. The graph of the equation passes the horizontal line test.

b. $f(x)=\frac{4}{3}\pi x^3$

$y=\frac{4}{3}\pi x^3$

$x=\frac{4}{3}\pi y^3$

$3(x)=3\left(\frac{4}{3}\pi y^3\right)$

$3x=4\pi y^3$

$y^3=\frac{3x}{4\pi}$

$y=\sqrt[3]{\frac{3x}{4\pi}}$

$f^{-1}(x)=\sqrt[3]{\frac{3x}{4\pi}}$

c. Both the domain and range are $(0,\infty)$. If the domain is less than or equal to zero, then there is no sphere.

d. Given a volume, the inverse function can be used to calculate the radius of the sphere.

e. $f^{-1}(65{,}450)=\sqrt[3]{\frac{3(65{,}450)}{4\pi}}$

≈ 25

If the volume is 65,450 cubic inches, the radius is approximately 25 inches.

45. a. $f(x)=0.66832x$

$y=0.66832x$

$x=0.66832y$

$y=\frac{x}{0.66832}$

$f^{-1}(x)=\frac{x}{0.66832}$

The inverse function converts U.S. dollars into Canadian dollars.

b. If you convert \$500 from U.S. to Canadian dollars and then convert the money back to U.S. dollars, you will still have \$500 U.S currency. (Note: This assumes there are no transaction fees for the conversion and that the exchange rate has not changed.)

46. a. The model makes sense as long as the length of the side of the cube is not negative. Therefore the domain is $[0,\infty)$.
The function is not one-to-one in general. However, the function is one-to-one over its restricted domain of $[0,\infty)$.

b. $y=6x^2 \quad x\ge 0$

$x=6y^2$

$y^2=\frac{x}{6}$

$y=\pm\sqrt{\frac{x}{6}}$

Based on the original restricted domain, the inverse function is

$y=\sqrt{\frac{x}{6}}$.

c. Given the surface area, the inverse function can be used to calculate the length of the edge of a cube.

47. a. No. The function is not one-to-one. Note that $I(-1)=I(1)=300{,}000$.

b. In the given physical context since x represents distance, the domain is $(0,\infty)$.

c. Yes. Based on the restricted domain $(0,\infty)$, the function is one-to-one.

d. $I(x)=\dfrac{300,000}{x^2}, x>0$

$y=\dfrac{300,000}{x^2}$

$x=\dfrac{300,000}{y^2}$

$xy^2=300,000$

$y^2=\dfrac{300,000}{x}$

$y=\pm\sqrt{\dfrac{300,000}{x}}$

Based on the physical context,

$I^{-1}(x)=\sqrt{\dfrac{300,000}{x}}$

$I^{-1}(75,000)=\sqrt{\dfrac{300,000}{75,000}}$

$=\sqrt{4}$

$=2$

When the distance is 2 feet, the intensity of light is 75,000 candlepower.

48. a. Considering the graph of the function, it is not one-to-one in general.

b. Since the number of units can not be negative, the domain is $[0,\infty)$.

c. Based on the restricted domain arising from the physical context of the problem, the function is one-to-one.

d. $p(x)=\dfrac{1}{4}x^2+20,\ x\ge 0$

$y=\dfrac{1}{4}x^2+20$

$x=\dfrac{1}{4}y^2+20$

$4x=y^2+80$

$y^2=4x-80$

$y=\pm\sqrt{4x-80}$

Based on the restricted domain,

$p^{-1}(x)=\sqrt{4x-80}=2\sqrt{x-20}$

$p^{-1}(101)=2\sqrt{101-20}$

$=2\sqrt{81}$

$=2(9)$

$=18$

The manufacturer will supply 18,000 units if the price is \$101 per unit.

49. a. $f(x)=1.7655x$

$y=1.7655x$

$x=1.7655y$

$y=\dfrac{x}{1.7655}$

$f^{-1}(x)=\dfrac{x}{1.7655}$

The inverse function converts U.S. dollars into British pounds.

b. If you convert \$1000 from U.S. to British currency and then convert the money back to U.S. dollars, you will still have \$1000 U.S currency. (Note: This assumes there are no transaction fees for the conversion and that the exchange rate has not changed.)

50. a. $P(w)=\begin{cases} 37 & 0<w\le 1 \\ 60 & 1<w\le 2 \\ 83 & 2<w\le 3 \end{cases}$

b. No. The function is not one-to-one. The postage is the same for a variety of weights.

51. a. The ball reaches the ground when its height equals zero. Solve $f(x)=0$.

$256+96x-16x^2=0$

$-16\left(x^2-6x-16\right)=0$

$-16(x-8)(x+2)=0$

$x-8=0, x+2=0$

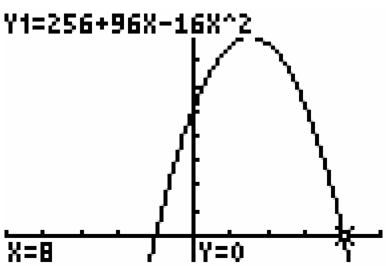
$x=8, x=-2$

Since x represents time, $x \geq 0$.

$x=8$

The ball remains in the air for 8 seconds.

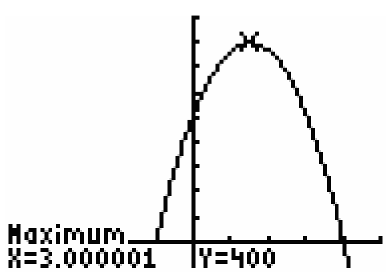
b.



[–10, 10] by [–50, 450]

The function is not one-to-one on the interval $[0,8]$. The graph does not pass the horizontal line test.

c. The function is one-to-one on the interval $[3,\infty)$.



[–10, 10] by [–50, 450]

d. $f(x)=256+96x-16x^2$

$y=-16\left(x^2-6x-16\right)$

$x=-16\left(y^2-6y-16\right)$

Completing the square yields,

$x=-16\left[\left(y^2-6y+9\right)+(-9-16)\right]$

$x=-16\left[(y-3)^2-25\right]$

$x=-16(y-3)^2+400$

$x-400=-16(y-3)^2$

$(y-3)^2=\dfrac{x-400}{-16}$

$\sqrt{(y-3)^2}=\pm\sqrt{\dfrac{x-400}{-16}}$

$y-3=\pm\sqrt{\dfrac{x-400}{-16}}$

$y=3\pm\sqrt{\dfrac{x-400}{-16}}$

For the interval $[0,3]$,

$f^{-1}(x)=3-\sqrt{\dfrac{400-x}{16}}$

The inverse function calculates the time the ball is in air between 0 and 3 seconds, given the height of the ball.

Section 2.8 Skills Check

1. $\sqrt{2x^2-1}-x=0$

$\sqrt{2x^2-1}=x$

$\left(\sqrt{2x^2-1}\right)^2=(x)^2$

$2x^2-1=x^2$

$x^2-1=0$

$(x+1)(x-1)=0$

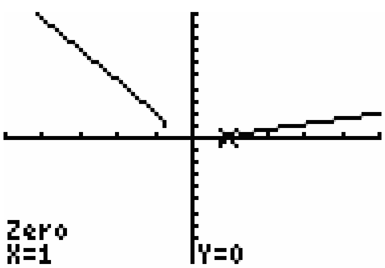
$x=1, x=-1$

-1 does not check

$\sqrt{2(-1)^2-1}-(-1)=$

$\sqrt{2(1)-1}+1=\sqrt{1}+1=2\neq 0$

Applying the intersection of graphs method to check graphically:



[–5, 5] by [–10, 10]

The only solution that checks is $x=1$.

2. $\sqrt{3x^2+4}-2x=0$

$\sqrt{3x^2+4}=2x$

$\left(\sqrt{3x^2+4}\right)^2=(2x)^2$

$3x^2+4=4x^2$

$x^2-4=0$

$x=2, x=-2$

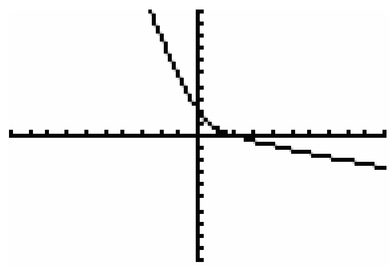
–2 does not check

$\sqrt{3(-2)^2+4}-2(-2)$

$\sqrt{16}+4$

$8\neq 0$

Applying the intersection of graphs method to check graphically:



[–10, 10] by [–10, 10]

There is only one solution. $x=2$.

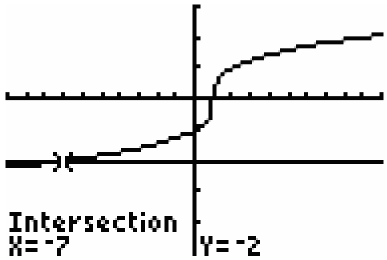
3. $\sqrt[3]{x-1}=-2$

$\left(\sqrt[3]{x-1}\right)^3=(-2)^3$

$x-1=-8$

$x=-7$

Applying the intersection of graphs method to check graphically:



[–10, 10] by [–5, 3]

The solution is $x = -7$.

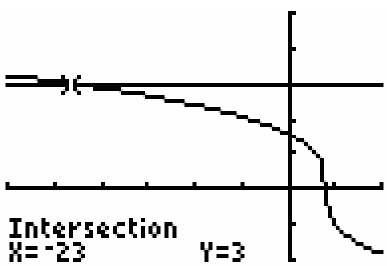
4. $\sqrt[3]{4-x} = 3$

$\left(\sqrt[3]{4-x}\right)^3 = (3)^3$

$4 - x = 27$

$x = -23$

Applying the intersection of graphs method to check graphically:



[–30, 10] by [–2, 5]

The solution is $x = -23$.

5. $\sqrt{3x-2} + 2 = x$

$\sqrt{3x-2} = x - 2$

$\left(\sqrt{3x-2}\right)^2 = (x-2)^2$

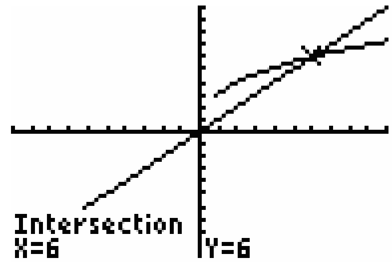
$3x - 2 = x^2 - 4x + 4$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$x = 6, x = 1$

Applying the intersection of graphs method to check graphically:



[–10, 10] by [–10, 10]

There is only one solution. The x-value of 1 does not check. The solution is $x = 6$.

6. $\sqrt{x-2} + 2 = x$

$\sqrt{x-2} = x - 2$

$\left(\sqrt{x-2}\right)^2 = (x-2)^2$

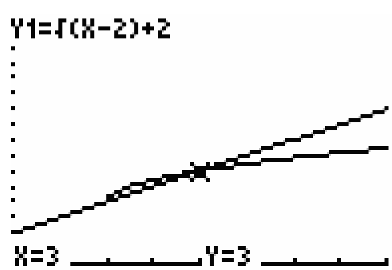
$x - 2 = x^2 - 4x + 4$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3, x = 2$

Applying the intersection of graphs method to check graphically:



[1, 5] by [0, 8]

[1, 5] by [0, 8]

Both solutions check.

7. $\sqrt[3]{4x+5}=\sqrt[3]{x^2-7}$

$\left(\sqrt[3]{4x+5}\right)^3=\left(\sqrt[3]{x^2-7}\right)^3$

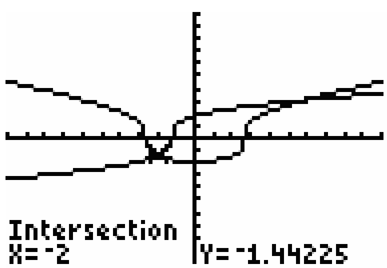
$4x+5=x^2-7$

$x^2-4x-12=0$

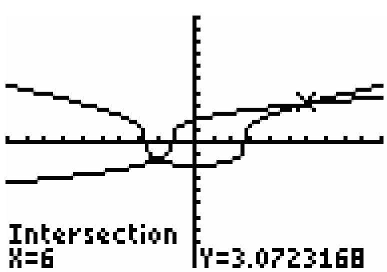
$(x-6)(x+2)=0$

$x=6, x=-2$

Applying the intersection of graphs method to check graphically:



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

Both solutions check.

8. $\sqrt{5x-6}=\sqrt{x^2-2x}$

$\left(\sqrt{5x-6}\right)^2=\left(\sqrt{x^2-2x}\right)^2$

$5x-6=x^2-2x$

$x^2-7x+6=0$

$(x-6)(x-1)=0$

$x=6, x=1$

Checking by substitution

$x=6$

$\sqrt{5x-6}\stackrel{?}{=}\sqrt{x^2-2x}$

$\sqrt{5(6)-6}\stackrel{?}{=}\sqrt{(6)^2-2(6)}$

$\sqrt{24}=\sqrt{24}$

$x=1$

$\sqrt{5x-6}\stackrel{?}{=}\sqrt{x^2-2x}$

$\sqrt{5(1)-6}\stackrel{?}{=}\sqrt{(1)^2-2(1)}$

$\sqrt{-1}=\sqrt{-1}\leftarrow$ Undefined expressions

Only $x = 6$ checks.

9. $\sqrt{x}-1=\sqrt{x-5}$

$\left(\sqrt{x}-1\right)^2=\left(\sqrt{x-5}\right)^2$

$x-2\sqrt{x}+1=x-5$

$-2\sqrt{x}=-6$

$\sqrt{x}=3$

$\left(\sqrt{x}\right)^2=(3)^2$

$x=9$

Checking by substitution

$x=9$

$\sqrt{x}-1\stackrel{?}{=}\sqrt{x-5}$

$\sqrt{9}-1\stackrel{?}{=}\sqrt{9-5}$

$2=2$

The solution is $x=9$.

10. $\sqrt{x}-10=-\sqrt{x-20}$

$\left(\sqrt{x}-10\right)^2=\left(-\sqrt{x-20}\right)^2$

$x-20\sqrt{x}+100=x-20$

$-20\sqrt{x}=-120$

$\sqrt{x}=6$

$\left(\sqrt{x}\right)^2=(6)^2$

$x=36$

Checking by substitution

$x=36$

$\sqrt{x}-10\stackrel{?}{=}-\sqrt{x-20}$

$\sqrt{36}-10\stackrel{?}{=}-\sqrt{36-20}$

$-4=-4$

The solution is $x=36$.

11. $(x+4)^{\frac{2}{3}}=9$

$\sqrt[3]{(x+4)^2}=9$

$\left[\sqrt[3]{(x+4)^2}\right]^3=[9]^3$

$(x+4)^2=729$

$\sqrt{(x+4)^2}=\pm\sqrt{729}$

$x+4=\pm 27$

$x=-4\pm 27$

$x=23, x=-31$

Checking by substitution

$x=23$

$(23+4)^{\frac{2}{3}}=9$

$(27)^{\frac{2}{3}}=9$

$9=9$

$x=-31$

$(-31+4)^{\frac{2}{3}}=9$

$(-27)^{\frac{2}{3}}=9$

$(-3)^2=9$

$9=9$

The solutions are $x=23, x=-31$.

12. $(x-5)^{\frac{3}{2}}=64$

$\sqrt[2]{(x-5)^3}=64$

$\left[\sqrt[2]{(x-5)^3}\right]^2=[64]^2$

$(x-5)^3=4096$

$\sqrt[3]{(x-5)^3}=\sqrt[3]{4096}$

$x-5=16$

$x=21$

Checking by substitution

$x=21$

$(21-5)^{\frac{3}{2}}=64$

$(16)^{\frac{3}{2}}=64$

$(4)^3=64$

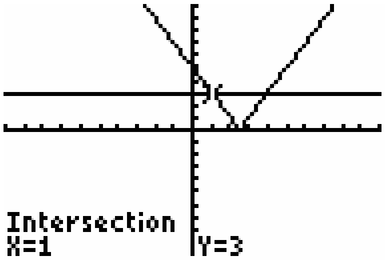
$64=64$

The solution is $x=21$.

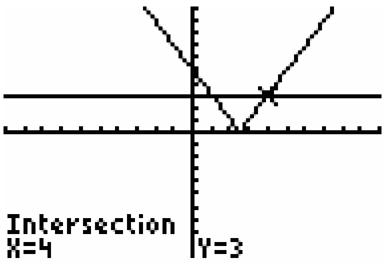
13. $|2x-5|=3$

$$\begin{array}{l|l} 2x-5=3 & 2x-5=-3 \\ 2x=8 & 2x=2 \\ x=4 & x=1 \end{array}$$

Using the intersections of graphs method to check the solutions graphically yields,



[–10, 10] by [–10, 10]

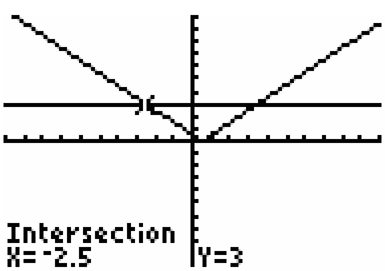


[–10, 10] by [–10, 10]

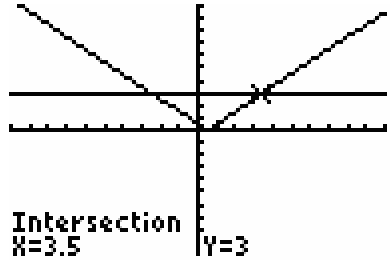
14. $\left|x-\frac{1}{2}\right|=3$

$$\begin{array}{l|l} x-\frac{1}{2}=3 & x-\frac{1}{2}=-3 \\ x=3+\frac{1}{2} & x=-3+\frac{1}{2} \\ x=\frac{7}{2} & x=-\frac{5}{2} \end{array}$$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]



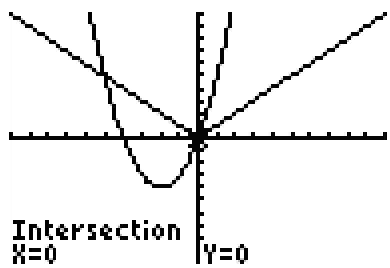
[–10, 10] by [–10, 10]

15. $|x|=x^2+4x$

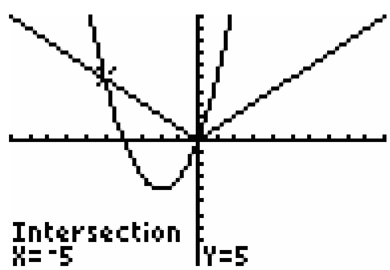
$$\begin{array}{l|l} & x=-\left(x^2+4x\right) \\ x=x^2+4x & x=-x^2-4x \\ x^2+3x=0 & x^2+5x=0 \\ x\left(x+3\right)=0 & x\left(x+5\right)=0 \\ x=0, x=-3 & x=0, x=-5 \end{array}$$

Note that -3 does not check. The solutions are $x=0, x=-5$.

Using the intersections of graphs method to check the solutions graphically yields,



[–10, 10] by [–10, 10]

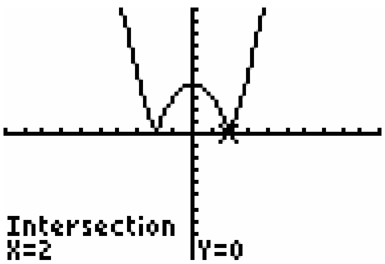


[–10, 10] by [–10, 10]

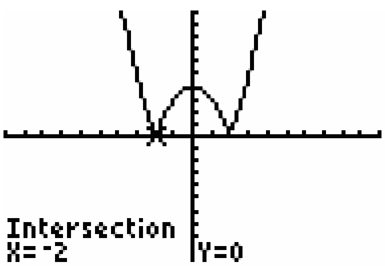
16. $$\left|x^2-4\right|=0$$

$x^2-4=0$	$x^2-4=-0$
$(x+2)(x-2)=0$	$x^2-4=-0$
$x=2, x=-2$	Same solutions

Using the intersections of graphs method to check the solutions graphically yields,



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

17. $$\left|3x-1\right|=4x$$

$3x-1=4x$	$3x-1=-4x$
$-x=1$	$7x=1$
$x=-1$	$x=\dfrac{1}{7}$

Note that only –1 does not check.

$x=-1$

$\left|3(-1)-1\right|=4(-1)$

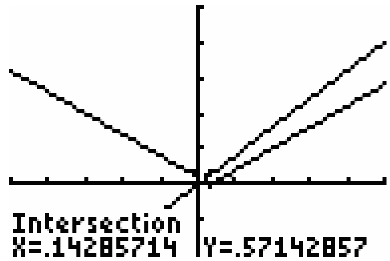
$\left|-3-1\right|=-4$

$\left|-4\right|=-4$

$4\neq-4$

The only solution is $x=\dfrac{1}{7}$.

Using the intersections of graphs method to check the solution graphically yields



[–5, 5] by [–10, 25]

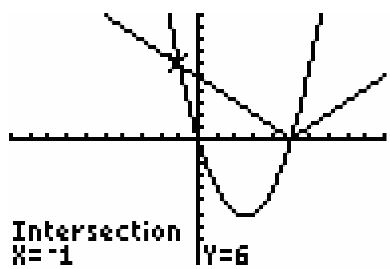
18. $$\left|x-5\right|=x^2-5x$$

$x-5=x^2-5x$	$x-5=-\left(x^2-5x\right)$
$x^2-6x+5=0$	$x-5=-x^2+5x$
$(x-5)(x-1)=0$	$x^2-4x-5=0$
$x=5, x=1$	$(x-5)(x+1)=0$
	$x=5, x=-1$

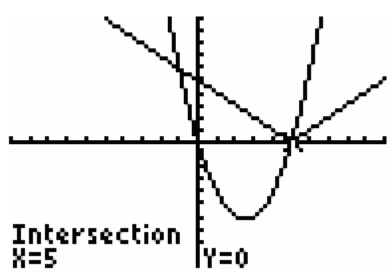
Note that 1 does not check.

The solutions are $x=5, x=-1$.

Using the intersections of graphs method to check the solutions graphically yields,



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

19. $x^2 + 4x < 0$

$x(x+4) < 0$

$x(x+4) = 0$

$x = 0, x = -4$

sign of x $\leftarrow ---_{-4} ---_{0} +++ \rightarrow$

sign of $(x+4)$ $\leftarrow ---_{-4} +++_{0} +++ \rightarrow$

sign of $x(x+4)$ $\leftarrow +++_{-4} ---_{0} +++ \rightarrow$

Considering the inequality symbol in the original question, the solution is $(-4,0)$.

20. $x^2 - 25x < 0$

$x(x-25) < 0$

$x(x-25) = 0$

$x = 0, x = 25$

sign of x $\leftarrow ---_{0} +++_{25} +++ \rightarrow$

sign of $(x-25)$ $\leftarrow ---_{0} ---_{25} +++ \rightarrow$

sign of $x(x-25)$ $\leftarrow +++_{0} ---_{25} +++ \rightarrow$

Considering the inequality symbol in the original question, the solution is $(0,25)$.

21. $9 - x^2 \ge 0$

$-1(x^2 - 9) \ge 0$

$\frac{-1(x^2-9)}{-1} \le \frac{0}{-1}$

$\boxed{x^2 - 9 \le 0}$ $\leftarrow$ Solve the simplified question

$(x+3)(x-3) \le 0$

$(x+3)(x-3) = 0$

$x = -3, x = 3$

sign of $(x+3)$ $\leftarrow ---_{-3} +++_{3} +++ \rightarrow$

sign of $(x-3)$ $\leftarrow ---_{-3} ---_{3} +++ \rightarrow$

sign of $(x+3)(x-3)$ $\leftarrow +++_{-3} ---_{3} +++ \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $[-3,3]$.

22. $x > x^2$

$0 > x^2 - x$

$x^2 - x < 0$

$x(x-1) < 0$

$x(x-1) = 0$

$x = 0, x = 1$

sign of x $\leftarrow ---_{0} +++_{1} +++ \rightarrow$

sign of $(x-1)$ $\leftarrow ---_{0} ---_{1} +++ \rightarrow$

sign of $x(x-1)$ $\leftarrow +++_{0} ---_{1} +++ \rightarrow$

Considering the inequality symbol in the original question, the solution is $(0,1)$.

23. $-x^2 + 9x - 20 > 0$

$-1(x^2 - 9x + 20) > 0$

$\frac{-1(x^2 - 9x + 20)}{-1} < \frac{0}{-1}$

$\boxed{x^2 - 9x + 20 < 0}$ $\leftarrow$ Solve the simplified question

$(x-5)(x-4) < 0$

$(x-5)(x-4) = 0$

$x = 5, x = 4$

sign of $(x-5)$ $\leftarrow ---_{4} ---_{5} +++ \rightarrow$

sign of $(x-4)$ $\leftarrow ---_{4} +++_{5} +++ \rightarrow$

sign of $(x-5)(x-4)$ $\leftarrow +++_{4} ---_{5} +++ \rightarrow$

Considering the inequality symbol in the original question, the solution is $(4,5)$.

24. $2x^2 - 8x < 0$

$2x(x-4) < 0$

$2x(x-4) = 0$

$x = 0, x = 4$

sign of x $\leftarrow ---_{0} +++_{4} +++ \rightarrow$

sign of $(x-4)$ $\leftarrow ---_{0} ---_{4} +++ \rightarrow$

sign of $x(x-4)$ $\leftarrow +++_{0} ---_{4} +++ \rightarrow$

Considering the inequality symbol in the original question, the solution is $(0,4)$.

25. $2x^2 - 8x \ge 24$

$2x^2 - 8x - 24 \ge 0$

$2\left(x^2 - 4x - 12\right) \ge 0$

$2(x-6)(x+2) \ge 0$

$2(x-6)(x+2) = 0$

$2 \ne 0, x = 6, x = -2$

sign of $(x-6)$ $\quad \leftarrow - - -_{-2} - - -_{6} + + + \rightarrow$

sign of $(x+2)$ $\quad \leftarrow - - -_{-2} + + +_{6} + + + \rightarrow$

sign of $(x-6)(x+2)$ $\quad \leftarrow + + +_{-2} - - -_{6} + + + \rightarrow$

Considering the inequality symbol in the original question, the solution is $(-\infty, -2] \cup [6, \infty)$.

26. $t^2 + 17t \le 8t - 14$

$\boxed{t^2 + 9t + 14 \le 0} \leftarrow$ Solve the simplified problem

$(t+7)(t+2) \le 0$

$(t+7)(t+2) = 0$

$t = -7, t = -2$

sign of $(t+7)$ $\quad \leftarrow - - -_{-7} + + +_{-2} + + + \rightarrow$

sign of $(t+2)$ $\quad \leftarrow - - -_{-7} - - -_{-2} + + + \rightarrow$

sign of $(t+7)(t+2)$ $\quad \leftarrow + + +_{-7} - - -_{-2} + + + \rightarrow$

Considering the inequality symbol in the original question, the solution is $[-7, -2]$.

27. $x^2 - 6x < 7$

$\boxed{x^2 - 6x - 7 < 0} \leftarrow$ Solve the simplified problem

$(x-7)(x+1) < 0$

$(x-7)(x+1) = 0$

$t = 7, t = -1$

sign of $(x-7)$ $\quad \leftarrow - - -_{-1} - - -_{7} + + + \rightarrow$

sign of $(x+1)$ $\quad \leftarrow - - -_{-1} + + +_{7} + + + \rightarrow$

sign of $(x-7)(x+1)$ $\quad \leftarrow + + +_{-1} - - -_{7} + + + \rightarrow$

Considering the inequality symbol in the original question, the solution is $(-1, 7)$.

28. $4x^2 - 4x + 1 > 0$

$(2x-1)(2x-1) > 0$

$(2x-1)(2x-1) = 0$

$2x - 1 = 0, 2x - 1 = 0$

$x = \dfrac{1}{2}, x = \dfrac{1}{2}$

sign of $(2x-1)$ $\quad \leftarrow - - - -_{\frac{1}{2}} + + + + \rightarrow$

sign of $(2x-1)$ $\quad \leftarrow - - - -_{\frac{1}{2}} + + + + \rightarrow$

sign of $(2x-1)(2x-1)$ $\quad \leftarrow + + + +_{\frac{1}{2}} + + + + \rightarrow$

Based on the inequality symbol and the sign chart, the equation is greater than zero for all real numbers except $\frac{1}{2}$. When $x = \frac{1}{2}$, the equation equals zeros. Therefore, the solution is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

29. $2x^2 - 7x + 2 \ge 0$

$2x^2 - 7x + 2 = 0$

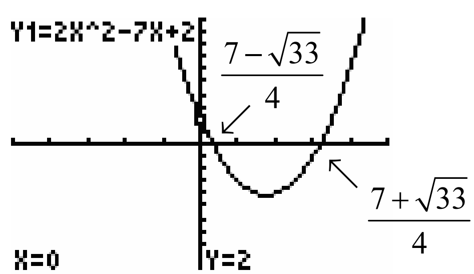
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(2)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{49 - 16}}{4}$$

$$x = \frac{7 \pm \sqrt{33}}{4}$$

Applying the x-intercept method



[–5, 5] by [–10, 10]

Considering the graph, the equation is greater than or equal to zero over the interval $\left(-\infty, \frac{7-\sqrt{33}}{4}\right] \cup \left[\frac{7+\sqrt{33}}{4}, \infty\right)$.

30. $w^2 - 5w + 4 > 0$

$w^2 - 5w + 4 = 0$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)}$$

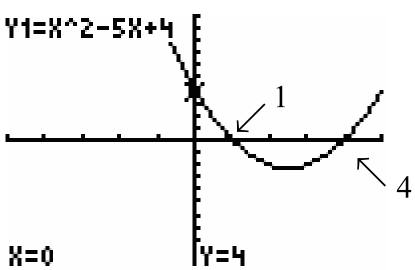
$$w = \frac{5 \pm \sqrt{25-16}}{2}$$

$$w = \frac{5 \pm \sqrt{9}}{2}$$

$$w = \frac{5 \pm 3}{2}$$

$w = 4, w = 1$

Applying the x-intercept method



[–5, 5] by [–10, 10]

Considering the graph, the equation is greater than zero over the interval $(-\infty, 1) \cup (4, \infty)$.

31. $5x^2 \geq 2x + 6$

$\boxed{5x^2 - 2x - 6 \geq 0}$ ← Solve the simplified problem

$5x^2 - 2x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-6)}}{2(5)}$$

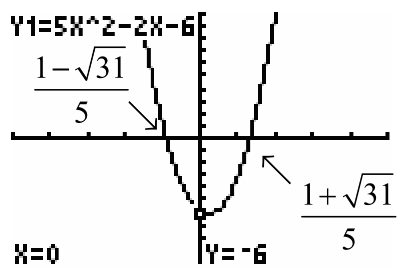
$$x = \frac{2 \pm \sqrt{4+120}}{10}$$

$$x = \frac{2 \pm \sqrt{124}}{10}$$

$$x = \frac{2 \pm 2\sqrt{31}}{10}$$

$$x = \frac{1 \pm \sqrt{31}}{5}$$

Applying the x-intercept method



[–5, 5] by [–10, 10]

Considering the graph, the equation is greater than or equal to zero over the interval $\left(-\infty, \frac{1-\sqrt{31}}{5}\right] \cup \left[\frac{1+\sqrt{31}}{5}, \infty\right)$.

32. $2x^2 \le 5x+6$

$\boxed{2x^2-5x-6\le 0} \leftarrow$ Solve the simplified problem

$2x^2-5x-6=0$

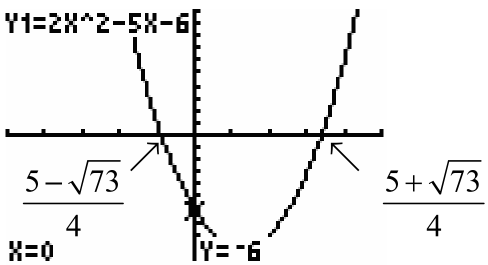
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(-6)}}{2(2)}$$

$$x=\frac{5\pm\sqrt{25+48}}{4}$$

$$x=\frac{5\pm\sqrt{73}}{4}$$

Applying the x-intercept method



[–5, 5] by [–10, 10]

Considering the graph, the equation is less than or equal to zero over the interval $\left[\frac{5-\sqrt{73}}{4},\frac{5+\sqrt{73}}{4}\right]$.

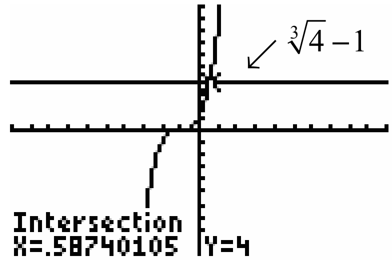
33. $(x+1)^3<4$

$\sqrt[3]{(x+1)^3}<\sqrt[3]{4}$

$x+1<\sqrt[3]{4}$

$x<\sqrt[3]{4}-1$

Applying the intersections of graphs method



[–10, 10] by [–10, 10]

Considering the graph, the solution is $\left(-\infty,\sqrt[3]{4}-1\right)$ or approximately $(-\infty,0.587)$.

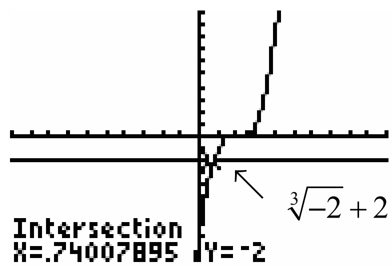
34. $(x-2)^3\ge -2$

$\sqrt[3]{(x-2)^3}\ge\sqrt[3]{-2}$

$x-2\ge\sqrt[3]{-2}$

$x\ge\sqrt[3]{-2}+2$

Applying the intersections of graphs method



[–10, 10] by [–10, 10]

Considering the graph, the solution is $\left[\sqrt[3]{-2}+2,\infty\right)$ or approximately $[0.74,\infty)$.

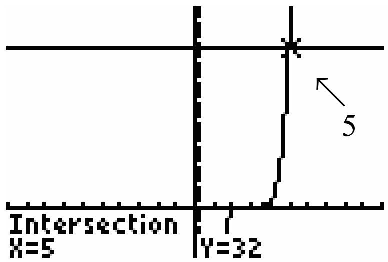
35. $(x-3)^5<32$

$\sqrt[5]{(x-3)^5}<\sqrt[5]{32}$

$x-3<2$

$x<5$

Applying the intersections of graphs method



[–10, 10] by [–10, 40]

Considering the graph, the solution is $(-\infty, 5)$.

36. $(x+5)^4 > 16$

$(x+5)^4 = 16$

$\sqrt[4]{(x+5)^4} = \pm\sqrt[4]{16}$

$x+5 = \pm 2$

$x = -5 \pm 2$

$x = -3, x = -7$

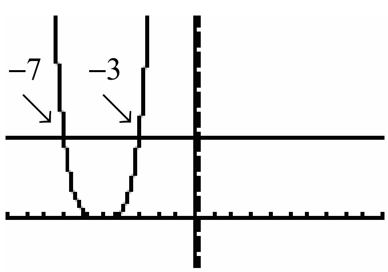
$(x+5)^4 = 16$

Consider $f(x) = (x+5)^4 - 16$

$f(x) > 0$ solves the given inequality

$\leftarrow ++++++_{-7} -----_{-3} ++++++ \rightarrow$

Applying the intersections of graphs method



[–10, 10] by [–10, 40]

Considering the graph, the solution is $(-\infty, -7) \cup (-3, \infty)$.

37. $|2x-1| < 3$

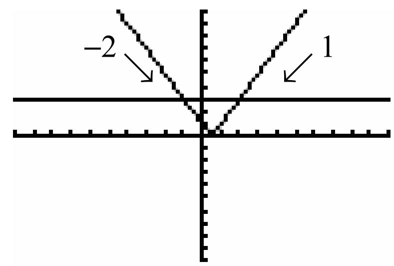
AND

$2x-1<3$	$2x-1>-3$
$2x<4$	$2x>-2$
$x<2$	$x>-1$

$x<2$ and $x>-1$

$(-2,1)$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]

38. $|3x+1| \le 5$

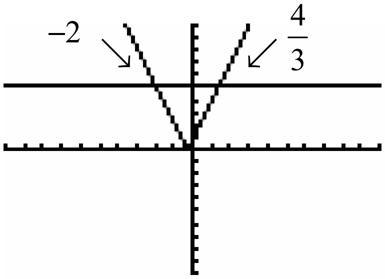
AND

$3x+1 \le 5$	$3x+1 \ge -5$
$3x \le 4$	$3x \ge -6$
$x \le \frac{4}{3}$	$x \ge -2$

$x \le \frac{4}{3}$ and $x \ge -2$

$\left[-2, \frac{4}{3}\right]$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]

39. $|x-6| \geq 2$

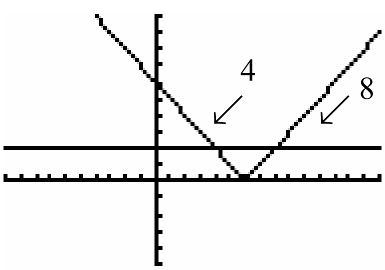
OR

$$\begin{array}{l|l} x-6 \geq 2 & x-6 \leq -2 \\ x \geq 8 & x \leq 4 \end{array}$$

$x \leq 4$ or $x \geq 8$

$(-\infty, 4] \cup [8, \infty)$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 15] by [–5, 10]

40. $|x+8| > 7$

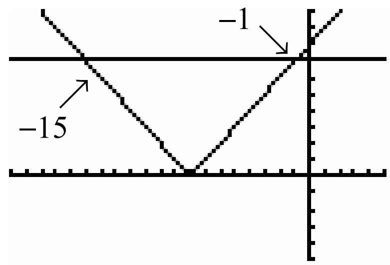
OR

$$\begin{array}{l|l} x+8 > 7 & x+8 < -7 \\ x > -1 & x < -15 \end{array}$$

$x < -15$ or $x > -1$

$(-\infty, -15) \cup (-1, \infty)$

Using the intersections of graphs method to check the solution graphically yields,



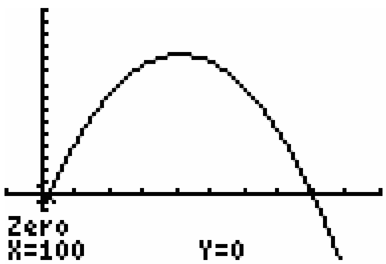
[–20, 5] by [–5, 10]

Section 2.8 Exercises

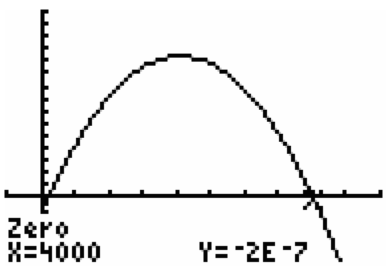
41. $P(x) > 0$

$-0.3x^2 + 1230x - 120{,}000 > 0$

Applying the x-intercept method



[–500, 5000] by [–500,000, 1,500,000]



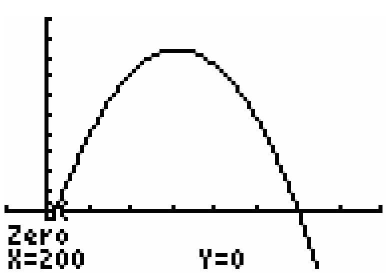
[–500, 5000] by [–500,000, 1,500,000]

Considering the graphs, the function is greater than zero over the interval $(100, 4000)$. Producing and selling between 100 and 4000 units, not inclusive, will result in a profit.

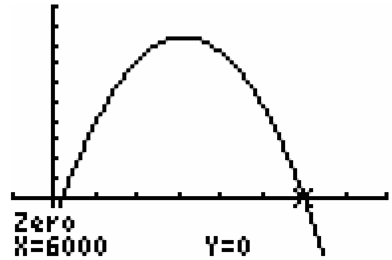
42. $P(x) > 0$

$-0.01x^2 + 62x - 12{,}000 > 0$

Applying the x-intercept method



[–1000, 8000] by [–30,000, 100,000]



[–1000, 8000] by [–30,000, 100,000]

Considering the graphs, the function is greater than zero over the interval $(200, 6000)$. Producing and selling between 200 and 6000 units, not inclusive, will result in a profit.

43. $P(x)$

$= R(x) - C(x)$

$= \left(200x - 0.01x^2\right) - \left(38x + 0.01x^2 + 16{,}000\right)$

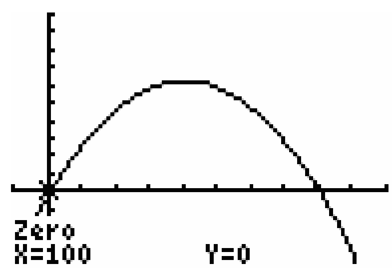
$= 200x - 0.01x^2 - 38x - 0.01x^2 - 16{,}000$

$= -0.02x^2 + 162x - 16{,}000$

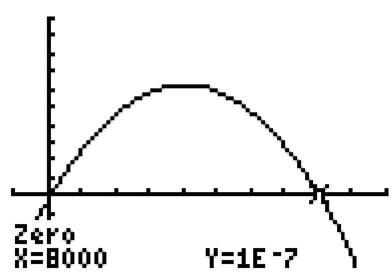
$P(x) > 0$

$-0.02x^2 + 162x - 16{,}000 > 0$

Applying the x-intercept method



[–1000, 10,000] by [–200,000, 500,000]



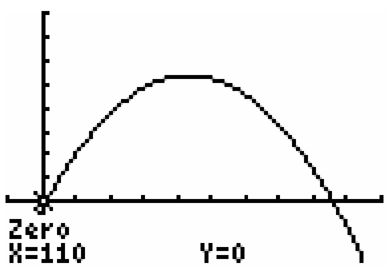
[–1000, 10,000] by [–200,000, 500,000]

Considering the graphs, the function is greater than zero over the interval $(100, 8000)$. Producing and selling between 100 and 8000 units, not inclusive, will result in a profit.

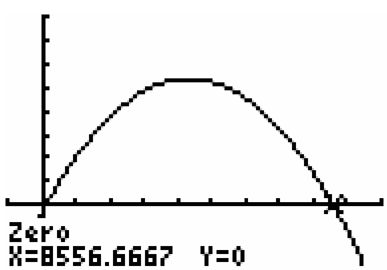
44. $P(x)$

$$= R(x) - C(x)$$
$$= \left(300x - 0.01x^2\right) - \left(40x + 0.02x^2 + 28{,}237\right)$$
$$= 300x - 0.01x^2 - 40x - 0.02x^2 - 28{,}237$$
$$= -0.03x^2 + 260x - 28{,}237$$
$$P(x) > 0$$
$$-0.03x^2 + 260x - 28{,}237 > 0$$

Applying the x-intercept method



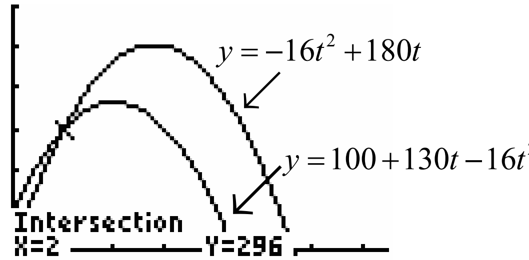
[–1000, 10,000] by [–250,000, 800,000]



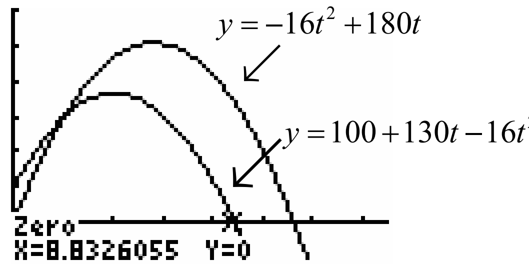
[–1000, 10,000] by [–250,000, 800,000]

Considering the graphs, the function is greater than zero over the interval $(110, 8556.67)$. Producing and selling between 110 and 8557 units, not inclusive, will result in a profit.

45. Applying the intersection of graphs method



[0, 15] by [–10, 600]



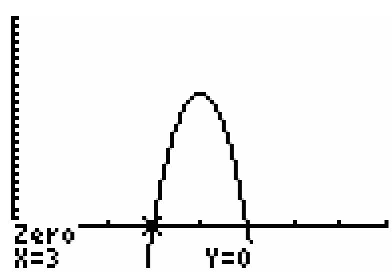
[0, 15] by [–10, 600]

Considering the graphs, the second projectile is above the first projectile over the interval $(2, 8.83)$. Between 2 seconds and 8.83 seconds the height of the second projectile exceeds the height of the first projectile.

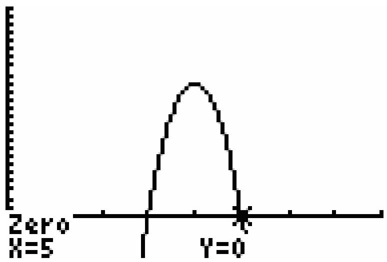
46. $s \ge 240$

$$128t - 16t^2 \ge 240$$
$$-16t^2 + 128t - 240 \ge 0$$

Applying the x-intercept method



[0, 8] by [–5, 25]



[0, 8] by [–5, 25]

Considering the graphs, the function is greater than or equal zero over the interval $[3,5]$. Therefore the height of the rocket is greater than or equal to 240 feet when the time is between 3 and 5 seconds inclusive.

47. If domestic sales are at least 5,940,000 kg, $y \geq 5.94$. Therefore,

$-0.084x^2 + 1.124x + 4.028 \geq 5.94.$

$\boxed{-0.084x^2 + 1.124x - 1.912 < 0}$

Solve the simplified problem above.

$$-0.084x^2 + 1.124x - 1.912 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

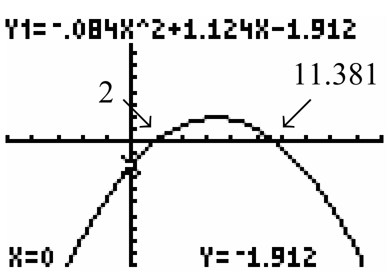
$$x = \frac{-(1.124) \pm \sqrt{(1.124)^2 - 4(-0.084)(-1.912)}}{2(-0.084)}$$

$$x = \frac{-1.124 \pm \sqrt{1.263376 - 0.642432}}{-0.168}$$

$$x = \frac{-1.124 \pm \sqrt{0.620944}}{-0.168}$$

$$x = 2, x = 11.381$$

Applying the x-intercept method



[–10, 20] by [–10, 10]

Considering the graph, the equation is greater than or equal to zero over the interval $[2, 11.381]$. Therefore, tobacco sales total at least 5.94 million kg between 1988 and 1990 inclusive.

48. If the population is above 6,000,000, then $y > 6$. Therefore,

$-0.00036x^2 + 0.0385x + 5.823 < 6.$

$\boxed{-0.00036x^2 + 0.0385x - 0.177 < 0}$

Solve the simplified problem above.

$-0.00036x^2 + 0.0385x - 0.177 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.0385) \pm \sqrt{(0.0385)^2 - 4(-0.00036)(-0.177)}}{2(-0.00036)}$$

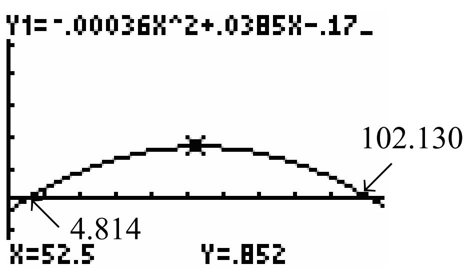
$$x = \frac{-0.0385 \pm \sqrt{0.00148225 - 0.00025488}}{-0.00072}$$

$$x = \frac{-0.0385 \pm \sqrt{0.00122737}}{-0.00072}$$

$$x = \frac{-0.0385 \pm 0.0350338408}{-0.00072}$$

$x = 4.814, x = 102.130$

Applying the x-intercept method



[0, 105] by [–1, 3]

Considering the graph, the equation is greater than zero over the interval $(4.814, 102.130)$. Therefore, world population is above 6 billions between 1995 and 2092 inclusive. In terms of question asked, between 1995 and 2000 inclusive, world population is greater than 6 billion.

49. If the number of airplane crashes is below 3000, then $y < 3$. Therefore,

$0.0057x^2 - 0.197x + 3.613 < 3.$

$\boxed{0.0057x^2 - 0.197x + 0.613 < 0}$

Solve the simplified problem above.

$0.0057x^2 - 0.197x + 0.613 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

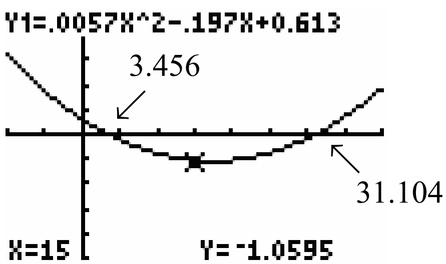
$$x = \frac{-(-0.197) \pm \sqrt{(-0.197)^2 - 4(.0057)(0.613)}}{2(0.0057)}$$

$$x = \frac{0.197 \pm \sqrt{0.038809 - 0.0139764}}{0.0114}$$

$$x = \frac{0.197 \pm \sqrt{0.0248326}}{0.0114}$$

$x = 31.104, x = 3.456$

Applying the x-intercept method



[–10, 40] by [–5, 5]

Considering the graph, the equation is less than zero over the interval $(3.456, 31.104)$. Therefore, the number of plane crashes is less than 3000 between 1984 and 2011 inclusive.

50. If the GDP is above 4 trillion, then $y > 4000$. Therefore,

$0.875x^2 + 23.406x + 886.82 > 4000.$

$\boxed{0.875x^2 + 23.406x - 3113.18 < 0}$

Solve the simplified problem above.

$0.875x^2 + 23.406x - 3113.18 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(23.406) \pm \sqrt{(23.406)^2 - 4(0.875)(-3113.18)}}{2(0.875)}$$

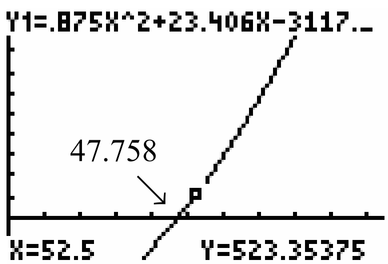
$$x = \frac{-23.406 \pm \sqrt{547.840836 + 10,896.13}}{1.75}$$

$$x = \frac{-23.406 \pm \sqrt{11,443.97084}}{1.75}$$

$$x = \frac{-23.406 \pm 106.9764967}{1.75}$$

$x = 47.758, x = -74.504$

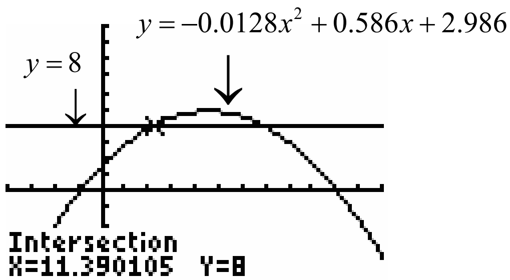
Applying the x-intercept method



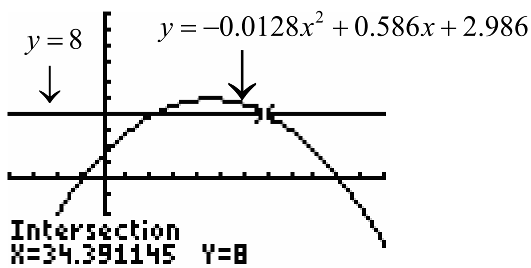
[0, 105] by [–1000, 5000]

Considering the graph, the equation is greater than zero over the interval $(47.792, \infty)$. The equation is also greater than zero for negative x-values which are outside the scope of the question. Therefore the GDP is greater than 4 trillion beginning in 1983. The GDP remains above 4 trillion into the foreseeable future.

51. Applying the intersection of graphs method



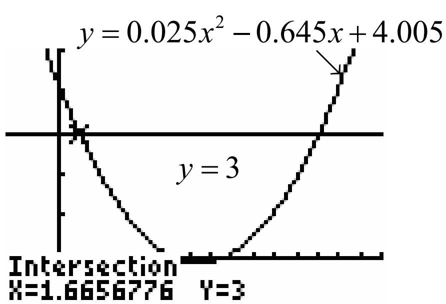
[–20, 60] by [–10, 20]



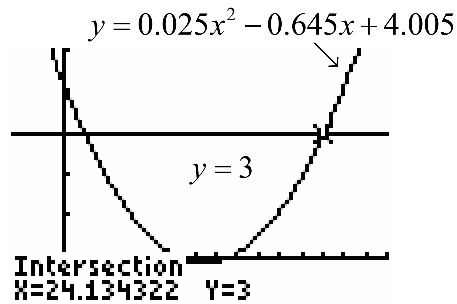
[–20, 60] by [–10, 20]

Considering the graphs, the homicide rate is above 8 per 100,000 people over the interval $(11.39, 34.39)$. Between 1972 and 1994 inclusive the homicide rate is above 8 per 100,000 people.

52. Applying the intersection of graphs method



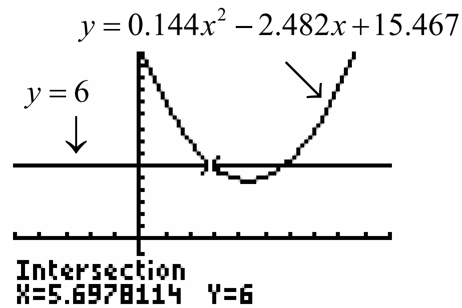
[–5, 30] by [–1, 5]



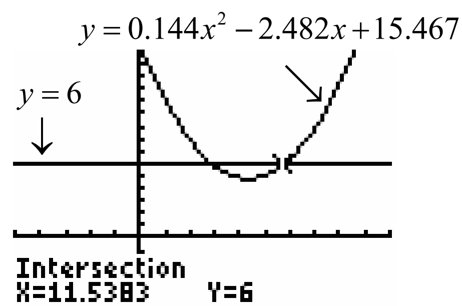
[–5, 30] by [–1, 5]

Considering the graphs, the percentage change in hotel room supply is less than 3% over the interval $(1.67, 24.13)$. Between 2000 and 2022 inclusive percentage change in hotel room supply is less than 3%.

53. Applying the intersection of graphs method



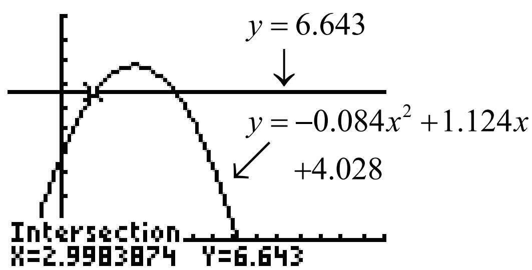
[–10, 20] by [–5, 15]



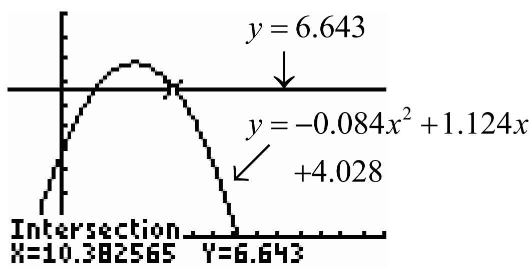
[–10, 20] by [–5, 15]

Considering the graphs, the unemployment rate is below 6% over the interval $(5.70, 11.54)$. Between 1986 and 1991 inclusive the unemployment rate is below 6%.

54. Applying the intersection of graphs method



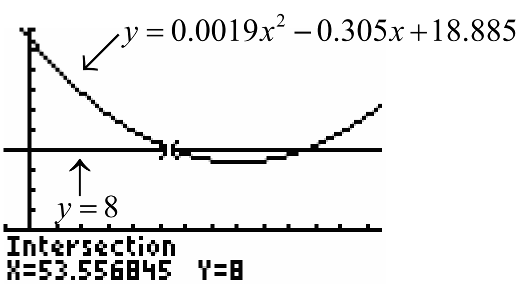
[–5, 30] by [–1, 10]



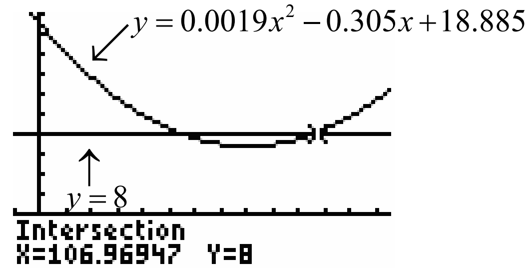
[–5, 30] by [–1, 10]

Considering the graphs, tobacco sales are above 6.643 million kilograms over the interval $(2.998, 10.383)$. Between 1989 and 1996 inclusive tobacco sales are above 6.643 million kilograms.

55. Applying the intersection of graphs method



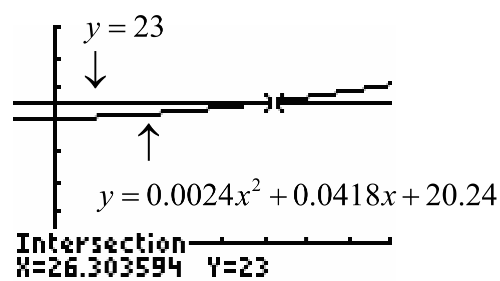
[–10, 135] by [–5, 20]



[–10, 135] by [–5, 20]

Considering the graphs, the percentage of the U.S. population that is foreign born is below 8% over the interval $(53.557, 106.969)$. Between 1954 and 2006 inclusive the percentage of the U.S. population that is foreign born is below 8%.

56. Applying the intersection of graphs method



[–5, 40] by [–5, 35]

Considering the graph, the median age of women at the time of first marriage is at least 32 on the interval $[26.304, \infty)$. For years 1987 and later, the median age of women at the time of first marriage is at least 32.

57. **a.** Let x equal a person's height in inches. Then, $|x - 68| > 8$ represents heights that will be uncomfortable.

b. $|x-68|>8$

OR

$$\begin{array}{l|l} x-68>8 & x-68<-8 \\ x>76 & x<60 \end{array}$$

$x<60$ or $x>76$

$(-\infty,60)\cup(76,\infty)$

In the context of the problem, people with heights below 60 inches or above 76 inches will be uncomfortable

58. a. Let x equal voltage. Then, $|x-220|\le 10$ represents voltages that allow the oven to work normally.

b. $|x-220|\le 10$

AND

$$\begin{array}{l|l} x-220\le 10 & x-220\ge -10 \\ x\le 230 & x\ge 210 \end{array}$$

$x\le 230$ and $x\ge 210$

$[210,230]$

In the context of the problem, voltages between 210 volts and 230 volts inclusive will allow the oven to work normally

Chapter 2 Skills Check

1. $f(x) = 3x^2 - 6x - 24$

$h = \frac{-b}{2a} = \frac{-(-6)}{2(3)} = \frac{6}{6} = 1$

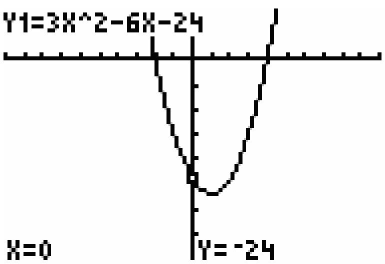
$k = f(h) = f(1)$

$f(1) = 3(1)^2 - 6(1) - 24$

$f(1) = 3 - 6 - 24 = -27$

The vertex is $(1, -27)$.

2.



[–10, 10] by [–40, 10]

3. Algebraically:

Let $f(x) = 0$

$3x^2 - 6x - 24 = 0$

$3(x^2 - 2x - 8) = 0$

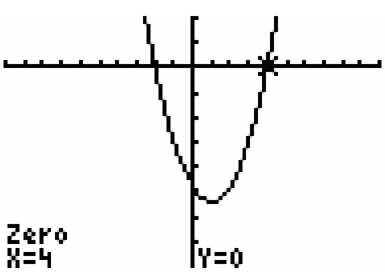
$3(x - 4)(x + 2) = 0$

$x - 4 = 0, x + 2 = 0$

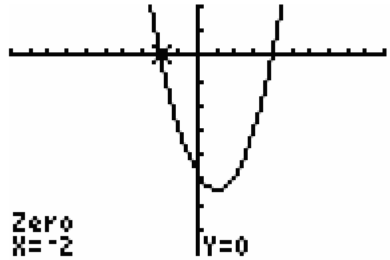
$x = 4, x = -2$

The x-intercepts are $(4, 0)$ and $(-2, 0)$.

Graphically:



[–10, 10] by [–40, 10]



[–10, 10] by [–40, 10]

Again, the x-intercepts are $(4, 0)$ and $(-2, 0)$.

4. Solving $f(x) = 0$ produces the x-intercepts. See problem 3 above. The x-intercepts are $(4, 0)$ and $(-2, 0)$.

5. $x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

$x = 4, x = 1$

6. $6x^2 + x - 2 = 0$

Note that $6x^2 \bullet -2 = -12x^2$. Look for two terms whose product is $-12x^2$ and whose sum is the middle term, x.

$6x^2 + 4x - 3x - 2 = 0$

$(6x^2 + 4x) + (-3x - 2) = 0$

$2x(3x + 2) - 1(3x + 2) = 0$

$(2x - 1)(3x + 2) = 0$

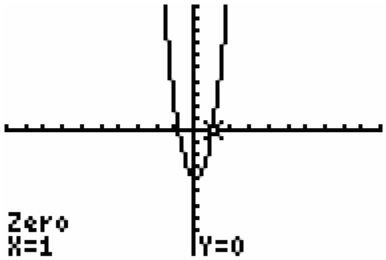
$2x - 1 = 0, 3x + 2 = 0$

$x = \frac{1}{2}, x = \frac{-2}{3}$

7. $5x^2 - x - 4 = 0$

$(x - 1)(5x + 4) = 0$

$x = 1, x = -\frac{4}{5}$

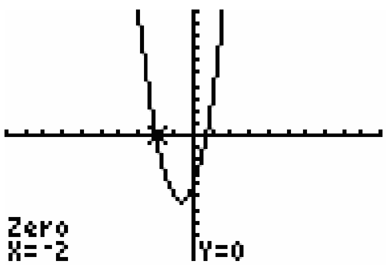


[–10, 10] by [–10, 10]

8. $3x^2 + 4x - 4 = 0$

$(x+2)(3x-2) = 0$

$x = -2, x = \dfrac{2}{3}$



[–10, 10] by [–10, 10]

9. $x^2 - 4x + 3 = 0$

$a = 1, b = -4, c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16-12}}{2}$$

$$x = \frac{4 \pm \sqrt{4}}{2}$$

$$x = \frac{4 \pm 2}{2}$$

$x = 3, x = 1$

10. $4x^2 + 4x - 3 = 0$

$a = 4, b = 4, c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16+48}}{8}$$

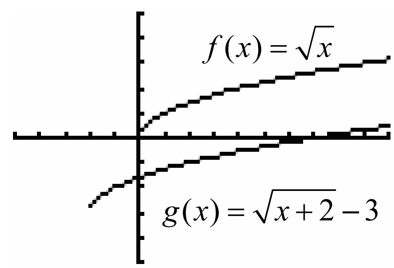
$$x = \frac{-4 \pm \sqrt{64}}{8}$$

$$x = \frac{-4 \pm 8}{8}$$

$$x = \frac{-4+8}{8}, x = \frac{-4-8}{8}$$

$$x = \frac{4}{8} = \frac{1}{2}, x = -\frac{12}{8} = -\frac{3}{2}$$

11. a.



[–5, 10] by [–5, 5]

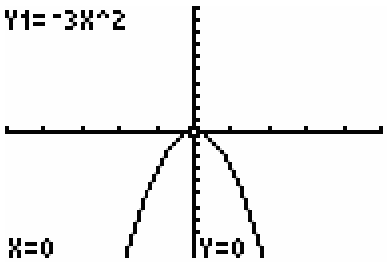
b. The function *g(x)* is shifted two units left and three units down in comparison to *f(x)*.

12. $g(x) = \sqrt{x+2} - 3$

$x + 2 \geq 0$

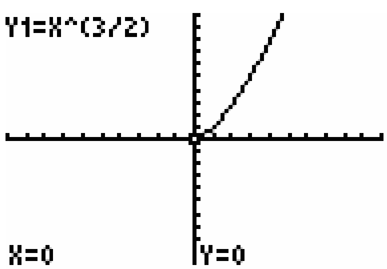
$x \geq -2$

Domain: $[-2, \infty)$.

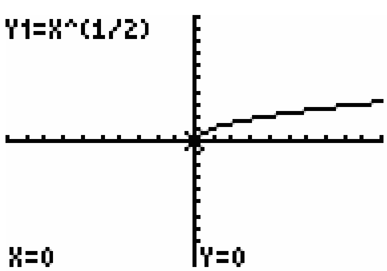
13. 

[–5, 5] by [–10, 10]

The function is increasing when $x < 0$ and decreasing when $x > 0$. In interval notation, the function is increasing on the interval $(-\infty, 0)$ and decreasing on the interval $(0, \infty)$.

14. a. 

[–10, 10] by [–10, 10]
The graph is concave up.

b. 

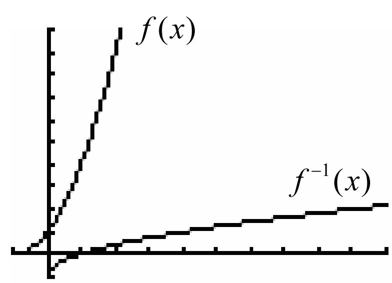
[–10, 10] by [–10, 10]

The graph is concave down.

15.
$$\begin{aligned} f(x) &= 3x - 2 \\ y &= 3x - 2 \\ x &= 3y - 2 \\ 3y &= x + 2 \\ y &= \frac{x+2}{3} \\ f^{-1}(x) &= \frac{x+2}{3} \end{aligned}$$

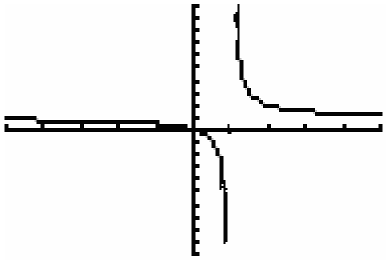
16.
$$\begin{aligned} g(x) &= \sqrt[3]{x-1} \\ y &= \sqrt[3]{x-1} \\ x &= \sqrt[3]{y-1} \\ x^3 &= \left(\sqrt[3]{y-1}\right)^3 \\ x^3 &= y - 1 \\ y &= x^3 + 1 \\ g^{-1}(x) &= x^3 + 1 \end{aligned}$$

17. Note that $f(x)$ is one-to-one on the given interval [–1, 10]. Therefore,
$$\begin{aligned} f(x) &= (x+1)^2 \\ y &= (x+1)^2 \\ x &= (y+1)^2 \\ y + 1 &= \sqrt{x}, \quad x \ge 0, y \ge -1 \\ y &= -1 + \sqrt{x} \\ f^{-1}(x) &= -1 + \sqrt{x} \end{aligned}$$



[–1, 10] by [–1, 10]

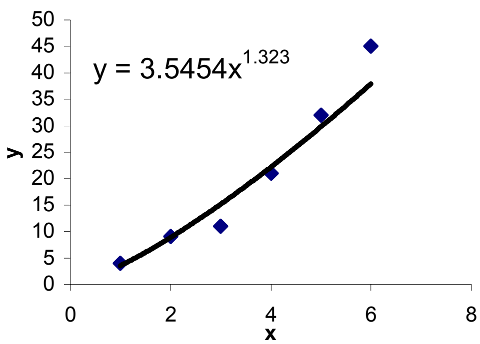
18.



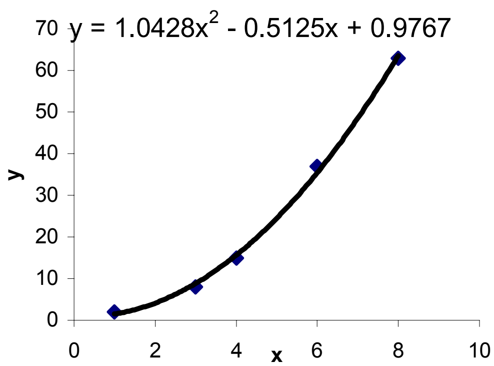
[–5, 5] by [–10, 10]

The function passed the horizontal line test.
It is one-to-one.

19.



20.

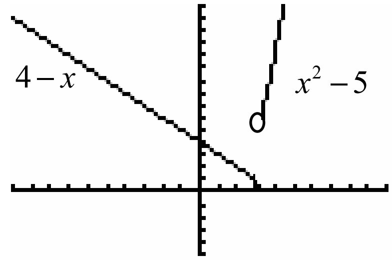


21. f

22. c

23. e

24.



[–10, 10] by [–5, 15]

25. $x^2 - 7x \le 18$

$x^2 - 7x - 18 \le 0$

$x^2 - 7x - 18 = 0$

$(x-9)(x+2) = 0$

$x = 9, x = -2$

sign of $(x-9)$ $\quad \leftarrow - - - _{-2} - - - _{9} + + + \rightarrow$

sign of $(x+2)$ $\quad \leftarrow - - - _{-2} + + + _{9} + + + \rightarrow$

sign of $(x-9)(x+2)$ $\quad \leftarrow + + + _{-2} - - - _{9} + + + \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $\left[-2, 9\right]$.

26. $2x^2 + 5x \ge 3$

$2x^2 + 5x - 3 \ge 0$

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$x = \dfrac{1}{2}, x = -3$

sign of $(2x-1)$ $\quad \leftarrow - - - _{-3} - - - _{\frac{1}{2}} + + + \rightarrow$

sign of $(x+3)$ $\quad \leftarrow - - - _{-3} + + + _{\frac{1}{2}} + + + \rightarrow$

sign of $(2x-1)(x+2)$ $\quad \leftarrow + + + _{-3} - - - _{\frac{1}{2}} + + + \rightarrow$

Considering the inequality symbol in the simplified question, the solution is

$(-\infty, -3] \cup \left[\dfrac{1}{2}, \infty\right)$.

27. $z^2 - 4z + 6 = 0$

$a = 1, b = -4, c = 6$

$z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(6)}}{2(1)}$

$z = \dfrac{4 \pm \sqrt{16 - 24}}{2}$

$z = \dfrac{4 \pm \sqrt{-8}}{2}$

$z = \dfrac{4 \pm 2i\sqrt{2}}{2}$

$z = 2 \pm i\sqrt{2}$

28. $w^2 - 4w + 5 = 0$

$a = 1, b = -4, c = 5$

$w = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$

$w = \dfrac{4 \pm \sqrt{16 - 20}}{2}$

$w = \dfrac{4 \pm \sqrt{-4}}{2}$

$w = \dfrac{4 \pm 2i}{2}$

$w = 2 \pm i$

29. $4x^2 - 5x + 3 = 0$

$a = 4, b = -5, c = 3$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(3)}}{2(4)}$

$x = \dfrac{5 \pm \sqrt{25 - 48}}{8}$

$x = \dfrac{5 \pm \sqrt{-23}}{8}$

$x = \dfrac{5 \pm i\sqrt{23}}{8}$

30. $4x^2 + 2x + 1 = 0$

$a = 4, b = 2, c = 1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(2) \pm \sqrt{(2)^2 - 4(4)(1)}}{2(4)}$

$x = \dfrac{-2 \pm \sqrt{4 - 16}}{8}$

$x = \dfrac{-2 \pm \sqrt{-12}}{8}$

$x = \dfrac{-2 \pm \sqrt{-4 \cdot 3}}{8}$

$x = \dfrac{-2 \pm 2i\sqrt{3}}{8}$

$x = \dfrac{-1 \pm i\sqrt{3}}{4}$

31. $(x - 4)^3 < 4096$

$\sqrt[3]{(x-4)^3} < \sqrt[3]{4096}$

$x - 4 < 16$

$x < 20$

The solution is $(-\infty, 20)$.

32. $(x+2)^2 \ge 512$

$\sqrt{(x+2)^2} = \sqrt{512}$

$x+2 = \pm\sqrt{256 \bullet 2}$

$x+2 = \pm 16\sqrt{2}$

$x = -2 \pm 16\sqrt{2}$

Considering $(x+2)^2 - 512 \ge 0$

$\leftarrow +++++ \; {}_{-2-16\sqrt{2}} \; ---- \; {}_{2+16\sqrt{2}} \; ++++ \rightarrow$

The solution is

$\left(-\infty, -2-16\sqrt{2}\right] \cup \left[-2+16\sqrt{2}, \infty\right)$.

33. $\sqrt{4x^2+1} = 2x+2$

$\left(\sqrt{4x^2+1}\right)^2 = (2x+2)^2$

$4x^2+1 = 4x^2+8x+4$

$8x = -3$

$x = -\dfrac{3}{8}$

34. $\sqrt{3x^2-8} + x = 0$

$\left(\sqrt{3x^2-8}\right)^2 = (-x)^2$

$3x^2 - 8 = x^2$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

Substituting to check

$x = 2$

$\sqrt{3(2)^2-8} + 2 \stackrel{?}{=} 0$

$4 \ne 0$

$x = -2$

$\sqrt{3(-2)^2-8} + (-2) \stackrel{?}{=} 0$

$0 = 0$

The only solution is $x = -2$.

35. $|3x-6| = 24$

$3x-6=24$	$3x-6=-24$
$3x=30$	$3x=-18$
$x=10$	$x=-6$

$x = 10, x = -6$

36. $|2x+3| = 13$

$2x+3=13$	$2x+3=-13$
$2x=10$	$2x=-16$
$x=5$	$x=-8$

$x = 5, x = -8$

37. $|2x-4| \le 8$

AND

$2x-4\le 8$	$2x-4\ge -8$
$2x\le 12$	$2x\ge -4$
$x\le 6$	$x\ge -2$

$x \ge -2$ and $x \le 6$

$[-2, 6]$

38. $|4x-3| \ge 15$

OR

$4x-3\ge 15$	$4x-3\le -15$
$4x\ge 18$	$4x\le -12$
$x\ge \dfrac{9}{2}$	$x\le -3$

$x \ge \dfrac{9}{2}$ or $x \le -3$

$(-\infty, -3] \cup \left[\dfrac{9}{2}, \infty\right)$

Chapter 2 Review Exercises

39. a. The maximum profit occurs at the vertex.

$P(x) = -0.01x^2 + 62x - 12,000$

$h = \frac{-b}{2a} = \frac{-62}{2(-0.01)} = \frac{-62}{-0.02} = 3100$

$P(3100) = -0.01(3100)^2 + 62(3100) - 12,000$

$P(3100) = -96,100 + 192,200 - 12,000$

$p(3100) = 84,100$

The vertex is $(3100, 84,100)$.

Producing and selling 3100 units produces the maximum profit of $84,100.

b. See part *a*. The maximum profit is $84,100.

40. $P(x)$

$= R(x) - C(x)$

$= (200x - 0.01x^2) - (38x + 0.01x^2 + 16,000)$

$= 200x - 0.01x^2 - 38x - 0.01x^2 - 16,000$

$= -0.02x^2 + 162x - 16,000$

Let $P(x) = 0$

$-0.02x^2 + 162x - 16,000 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(162) \pm \sqrt{(162)^2 - 4(-0.02)(-16,000)}}{2(-0.02)}$

$x = \frac{-162 \pm \sqrt{26,244 - 1280}}{-0.04}$

$x = \frac{-162 \pm \sqrt{24,964}}{-0.04}$

$x = \frac{-162 \pm 158}{-0.04}$

$x = \frac{-162 + 158}{-0.04}, x = \frac{-162 - 158}{-0.04}$

$x = 100, x = 8000$

The company will break even producing and selling 100 units or 8000 units.

41. a. $h = \frac{-b}{2a} = \frac{-64}{2(-16)} = 2$. The ball reaches its maximum height in 2 seconds.

b. $S(2) = 192 + 64(2) - 16(2)^2$
$= 256$

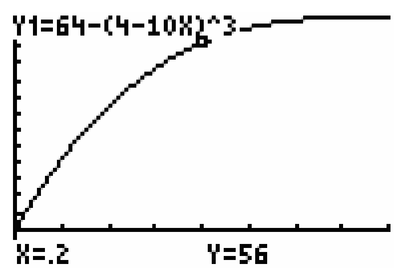
The ball reaches a maximum height of 256 feet.

42. a. $h = \frac{-b}{2a} = \frac{-29.4}{2(-9.8)} = 1.5$. The ball reaches its maximum height in 1.5 seconds.

b. $S(1.5) = 60 + 29.4(1.5) - 9.8(1.5)^2$
$= 82.05$

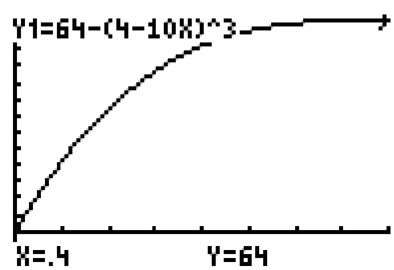
The ball reaches a maximum height of 82.05 meters.

43 a.



[0, 0.4] by [–10, 65]

b.



[0, 0.4] by [–10, 65]

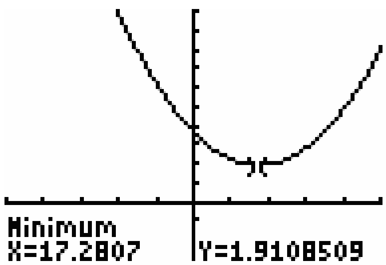
The bullet travels 64 inches in 0.4 seconds.

44 a. Note that the function is quadratic written in standard form, $y = a(x-h)^2 + k$. The vertex is $(h,k) = (4,380)$. Therefore the maximum height occurs 4 seconds into the flight of the rocket.

b. Referring to part *a*, the maximum height of the rocket is 380 feet.

c. In comparison to $y = t^2$ the graph is shifted 4 units right, 380 up, stretched by a factor of 16, and reflected across the *x*-axis.

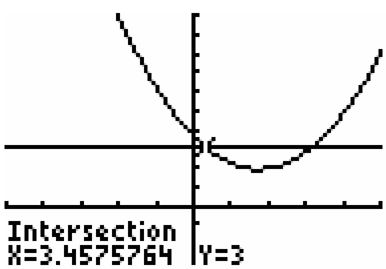
45. a.



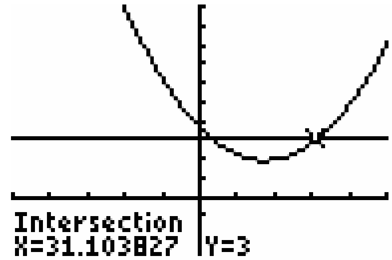
[–50, 50] by [–3, 10]

The model predicts that the minimum number of airplane crashes occurs in 1997.

b.



[–50, 50] by [–3, 10]



[–50, 50] by [–3, 10]

The model predicts that between 1984 and 2011 inclusive the number of plane crashes will be below 3000.

46. a. Using the table feature of the TI-83 calculator along with the given function

X	Y_1
5	6.657
6	5.759
7	5.149
8	4.827
9	4.793
10	5.047
11	5.589

X=9

Unemployment reaches a minimum in approximately 1989.

b. In 1989 the unemployment rate is 4.79%.

c. Using the table feature of the TI-83 calculator along with the given function

X	Y_1
6	5.759
7	5.149
8	4.827
9	4.793
10	5.047
11	5.589
12	6.419

X=10

Based on the table the unemployment rate is 5.047% in 1990.

47. $3600 - 150x + x^2 = 0$

$x^2 - 150x + 3600 = 0$

$(x-30)(x-120) = 0$

$x = 30, x = 120$

Break-even occurs when 30 or 120 units are produced and sold.

48. The ball is on the ground when $s = 0$.

$400 - 16t^2 = 0$

$-16t^2 = -400$

$t^2 = 25$

$t = \pm 5$

In the physical context of the question, $t \geq 0$. The ball reaches the ground in 5 seconds.

49. $-0.3x^2 + 1230x - 120{,}000 = 324{,}000$

$-0.3x^2 + 1230x - 444{,}000 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(1230) \pm \sqrt{(1230)^2 - 4(-0.3)(-444{,}000)}}{2(-0.3)}$$

$$= \frac{-1230 \pm \sqrt{1{,}512{,}900 - 532{,}800}}{-0.6}$$

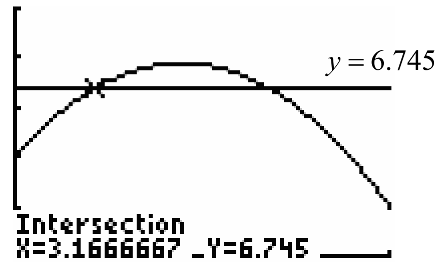
$$= \frac{-1230 \pm \sqrt{980{,}100}}{-0.6}$$

$$= \frac{-1230 \pm 990}{-0.6}$$

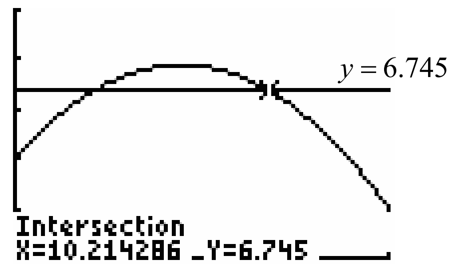
$x = 400, x = 3700$

A profit of \$324,000 occurs when 400 or 3700 units are produced and sold.

50. Applying the intersection of graphs method with the given function and $y = 6.745$



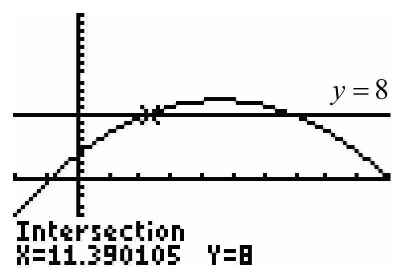
[0, 15] by [0, 10]



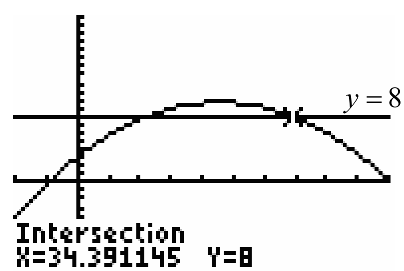
[0, 15] by [0, 10]

Tobacco sales equal 6.745 million in 1989 and 1996.

51. Applying the intersection of graphs method with the given function and $y = 8$



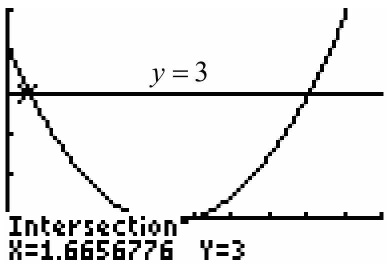
[–10, 50] by [–10, 20]



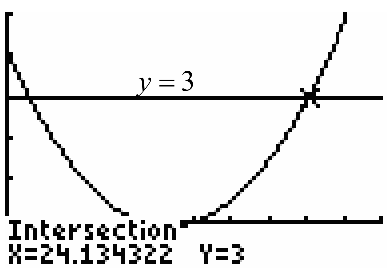
[–10, 50] by [–10, 20]

Between 1972 and 1994 inclusive the homicide rate is above 8 per 100,000 people.

52. Applying the intersection of graphs method with the given function and $y = 3$



[0, 30] by [–1, 5]



[0, 30] by [–1, 5]

Between 2000 and 2022 inclusive the percentage change in hotel room supply is below 3%. In the context of the question, between 2000 and 2010 inclusive, the percentage change in hotel room supply is below 3%.

53. a. Let $x = 1998 - 1980 = 18$

$y = 161{,}488.4931(18)^{-0.4506}$

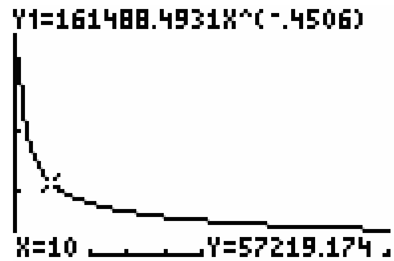
$y = 161{,}488.4931(0.2718780893)$

$y = 43{,}905.18295$

$y \approx 43{,}905$

The model predicts 43,905 injuries in 1998.

b.

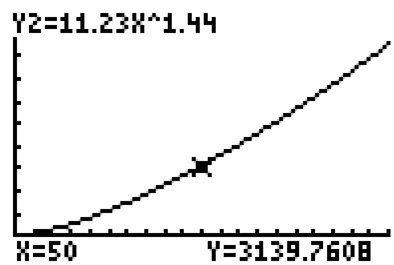


[0, 100] by [–10, 200,000]

In 1990 there are 57,219 predicted injuries.

54. a. Yes. The exponent is greater than one.

b.



[0, 100] by [–1000, 10,000]

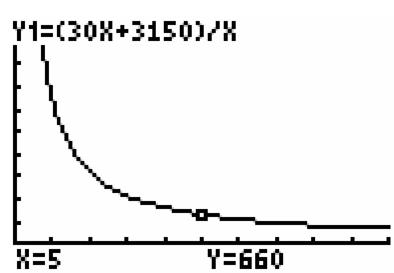
c. The graph is concave up.

d. $S = 11.23(95)^{1.44} \approx 7912.28$

In 1995 the per capita tax is $7912.28.

56. a. $\overline{C}(x) = \dfrac{30x + 3150}{x}$

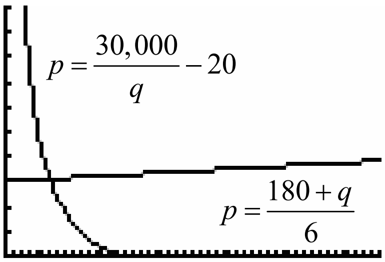
b.



[0, 20] by [–500, 5000]

57. a. Since there is a variable in the denominator $p = \dfrac{30,000}{q} - 20$ is a shifted reciprocal function.

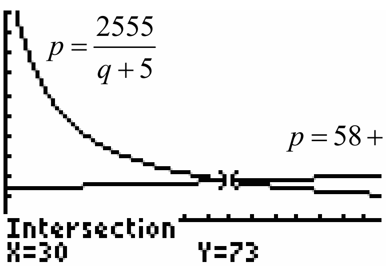
b.



[0, 50] by [0, 100]

58. a. Since there is a variable in the denominator of $p = \dfrac{2555}{q+5}$, the demand function is a shifted reciprocal function.

b.



[0, 50] by [0, 400]

59. a. $P(90) = 2.320(90)^2 - 389(90) + 21,762$

$= 18,792 - 35,010 + 21,762$

$= 5544$

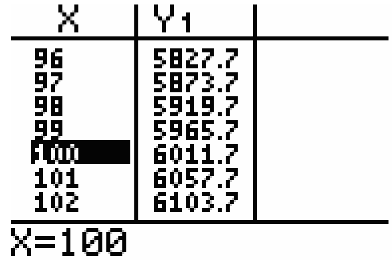
The Indiana population in 1990 is 5544 thousand people or 5,544,000 people.

b. $P(x) = 46.0x + 1411.667$

$6000 = 46.0x + 1411.667$

$46.0x = 4588.333$

$x = \dfrac{4588.333}{46.0} \approx 99.75$



X	Y1
96	5827.7
97	5873.7
98	5919.7
99	5965.7
100	6011.7
101	6057.7
102	6103.7

X=100

In 2000, the population exceeds 6 million. Since the model describes population data between 1980 and 1997, the calculation is an extrapolation. The solution is valid only if the model remains valid in 2000.

60. a. For years between 1979 and 1988 inclusive, the function is quadratic.

b. $M(79) = -\dfrac{1}{12}\left(79 - \dfrac{789}{10}\right)^2 + \dfrac{15,541}{1200}$

$= -\dfrac{1}{12}(0.1)^2 + \dfrac{15,541}{1200}$

$= 12.95$

$M(80) = -\dfrac{1}{12}\left(80 - \dfrac{789}{10}\right)^2 + \dfrac{15,541}{1200}$

$= -\dfrac{1}{12}(1.1)^2 + \dfrac{15,541}{1200}$

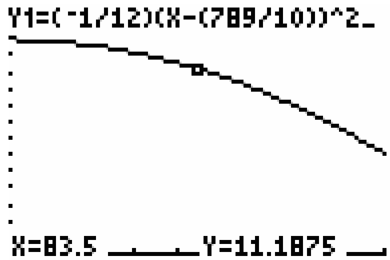
$= 12.85$

$M(88) = -\dfrac{1}{12}\left(88 - \dfrac{789}{10}\right)^2 + \dfrac{15,541}{1200}$

$= -\dfrac{1}{12}(9.1)^2 + \dfrac{15,541}{1200}$

$= 6.05$

In 1979, 12.95% of people 12 years of age or older used marijuana at least once in the month prior to being surveyed. In 1980, 12.85% of people 12 years of age or older used marijuana at least once in the month prior to being surveyed. In 1988, 6.05% of people 12 years of age or older used marijuana at least once in the month prior to being surveyed.

c. 

[79, 88] by [0, 15]

d. The domain is $\{79, 80, 81, 82, ..., 87, 88\}$.

e. The maximum occurs in 1979. Marijuana use decreased until 1988.

61. a. Trade Balance = Exports - Imports

Let $T(x)$ = Trade Balance

$$T(x) = E(x) - I(x)$$
$$= \left(-191.73x^2 + 39{,}882.69x + 377{,}849.85\right) - \left(61{,}487.24x + 435{,}606.84\right)$$
$$= -191.73x^2 - 21{,}604.55x - 57{,}756.99$$

b. Let $x = 10$

$$T(10) = -191.73(10)^2 - 21{,}604.55(10) - 57{,}756.99$$
$$= -19{,}173 - 216{,}045.5 - 57{,}756.99$$
$$= -292{,}975.49$$

The trade deficit in 2000 is $292,975.49 million.

62. a. $R\left(E(t)\right) = 0.165\left(0.017t^2 + 2.164t + 8.061\right) - 0.226$

$$= 0.002805t^2 + 0.35706t + 1.330065 - 0.226$$
$$= 0.002805t^2 + 0.35706t + 1.104065$$

The function calculates the revenue for Southwest Airlines given the number of years past 1990.

b. $R\left(E(t)\right) = 0.002805t^2 + 0.35706t + 1.104065$

$R\left(E(3)\right) = 0.002805(3)^2 + 0.35706(3) + 1.104065$

$R\left(E(3)\right) = 2.2008995 \approx 2.2$

In 1993 Southwest Airlines had revenue of $2.2 billion.

c. $E(7) = 0.017(7)^2 + 2.164(7) + 8.061$

$$= 24.042$$

In 1997 Southwest Airlines has 24,042 employees.

d. $R(E(t)) = 0.002805t^2 + 0.35706t + 1.104065$

$R(E(7)) = 0.002805(7)^2 + 0.35706(7) + 1.104065$

$R(E(7)) = 3.74093 \approx 3.7$

In 1997 Southwest Airlines had revenue of \$3.7 billion.

63. a.

$$\begin{aligned} f(x) &= 0.554x - 2.886 \\ y &= 0.554x - 2.886 \\ x &= 0.554y - 2.886 \\ 0.554y &= x + 2.886 \\ y &= \frac{x + 2.886}{0.554} \\ f^{-1}(x) &= \frac{x + 2.886}{0.554} \end{aligned}$$

b. The inverse function calculates the mean length of the original prison sentence given the mean time spent in prison.

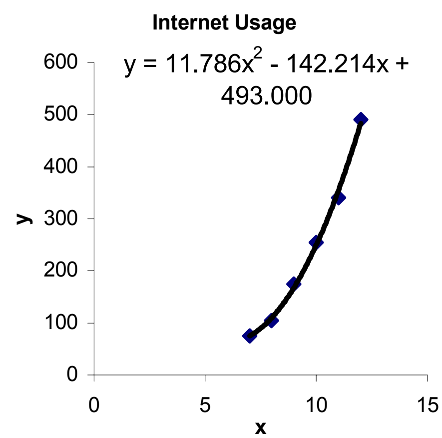
64. a.

$$\begin{aligned} C(x) &= -0.093x + 9.929 \\ y &= -0.093x + 9.929 \\ x &= -0.093y + 9.929 \\ -0.093y &= x - 9.929 \\ y &= \frac{x - 9.929}{-0.093} \\ C^{-1}(x) &= \frac{9.929 - x}{0.093} \end{aligned}$$

b. Given the number of cows used for milk production, the inverse function will calculate the number of years past 1990.

c. $C^{-1}(C(8)) = 8$

65. a.



b. See part a) above.

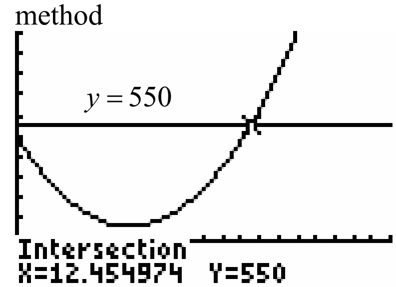
c. Yes. The function seems to fit the data very well.

d. $y = 11.786x^2 - 142.214x + 493$

Let $y = 550$

$550 = 11.786x^2 - 142.214x + 493$

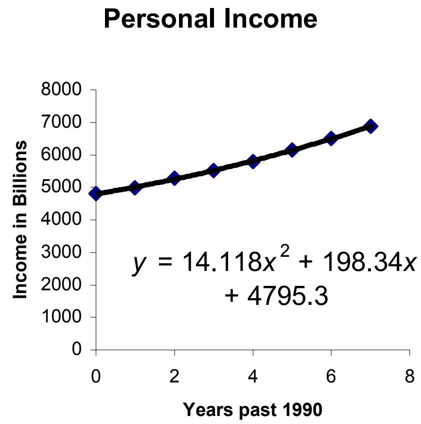
Using the unrounded model and applying the intersection of graphs method



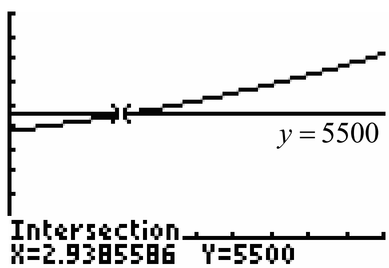
[0, 20] by [–200, 1000]

The model predicts that Internet users will reach 550 billion in 2002. Internet users will exceed 550 billion in 2003.

66. a.



b. Using the unrounded model and applying the intersections of graphs method

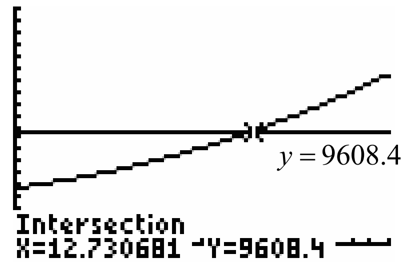


[0, 10] by [–1000, 10,000]

Personal income reaches $5500 billion in 1993.

c. The personal income level in 1990 is $4804.2 billion. Twice that amount is $9608.4 billion.

Using the unrounded model and applying the intersections of graphs method

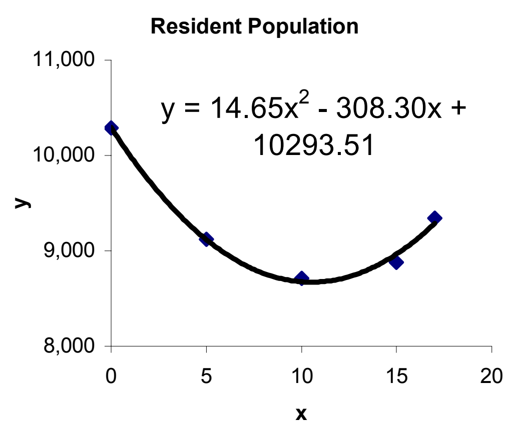


[0, 20] by [–1000, 20,000]
Personal income doubles during 2003.

67.

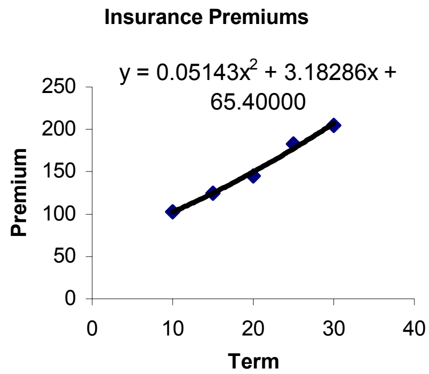
Year	Age 15-19 Population (thousands)	Male Population (%)	15-19 Male Population (thousands)
1980	21,168	48.6	10,287.648
1985	18,727	48.7	9120.049
1990	17,890	48.7	8712.430
1995	18,152	48.9	8876.328
1997	19,068	49.0	9343.320

Years past 1980	15-19 Male Population (thousands)
0	21,168*0.486 = 10,287.648
5	18,727*0.487 = 9120.049
10	17,890*0.487 = 8712.430
15	18152*.0489 = 8876.328
17	19,068*0.490 = 9343.320

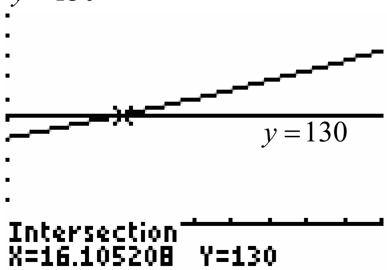


In the quadratic model, x represents the years past 1980 while y represents the 15-19 male population in thousands.

68. a.



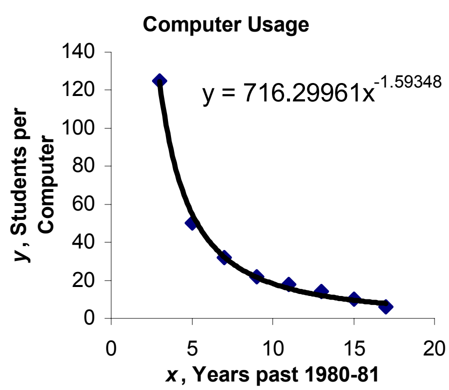
b. Applying the intersection of graphs method using unrounded model and $y = 130$



[10, 30] by [–50, 250]

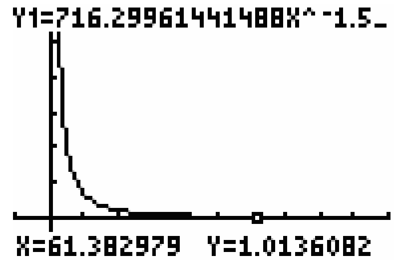
A premium of \$130 would purchase a term of 16 years.

69. a.



The model seems to fit the data well.

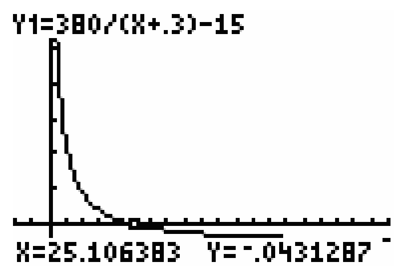
b. Yes. The function is decreasing and will eventually reach a value of one, representing one student per computer. Considering the graph below, this occurs when $x \approx 61.4$ or in school year 2042-2043.



[–10, 100] by [–25, 150]

c. $C(t) = \dfrac{1}{t}$

The basic function is shifted 0.3 units left and 15 units down and is stretched vertically by a factor of 380.



[–10, 100] by [-10, 100]

d. The power function is the better fit for the situation. It remains positive for all school years.

e. Power function (unrounded model)

$y = 716.29961x^{-1.59348}$

$y = 716.3(18)^{-1.5935} \approx 7.1588$

Rational Function

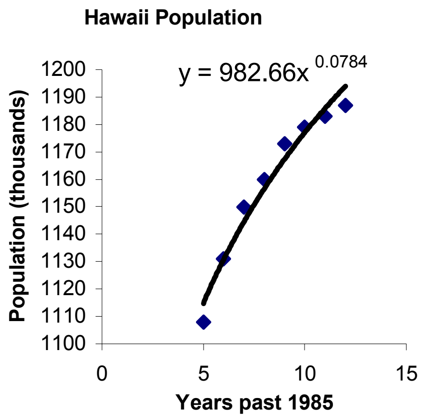
$$C(t) = \frac{380}{t+0.3} - 15$$

$$C(18) = \frac{380}{18+0.3} - 15$$

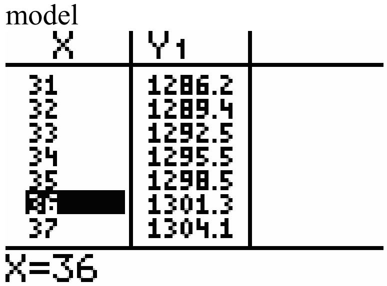
$$= \frac{380}{18.3} - 15 = 5.77$$

$C(t)$ is the better fit for $1998-99$.

70. a.



b. Using the table feature of the TI-83 calculator along with the unrounded model

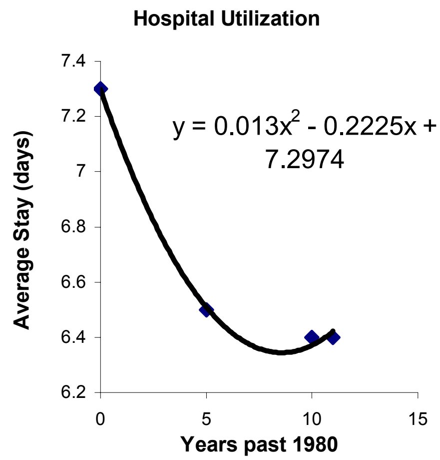


X	Y1
31	1286.2
32	1289.4
33	1292.5
34	1295.5
35	1298.5
36	1301.3
37	1304.1

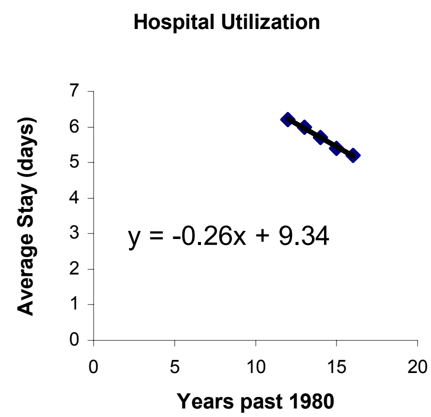
X=36

In 2021 the Hawaiian population first exceeds 1.3 million people.

71. a. Between 1980 and 1991 inclusive, $y = 0.013x^2 - 0.2225x + 7.2974.$



b. Beyond 1991, $y = -0.26x + 9.34.$



c. Combining the solutions to part a) and b) to create a piecewise model:

$$f(x) = \begin{cases} 0.013x^2 - 0.2225x + 7.2974 & 0 \le x \le 11 \\ -0.26x + 9.34 & 12 \le x \le 16 \end{cases}$$

d. i. $f(7) = 0.013(7)^2 - 0.2225(7) + 7.2974$

Using the TI-83 and substituting into the unrounded quadratic model:

```
7→X
                 7
.0129934924078X^
2+-.222537960954
45X+7.2973969631
236
     6.376312364
```

In 1987 the average hospital stay is predicted to be 6.4 days.

ii. $f(x)=6.1$

The year is greater than 1991.

$-0.26x+9.34=6.1$

$-0.26x=-3.24$

$x=12.46154$

The average hospital stay is 6.1 in 1992.

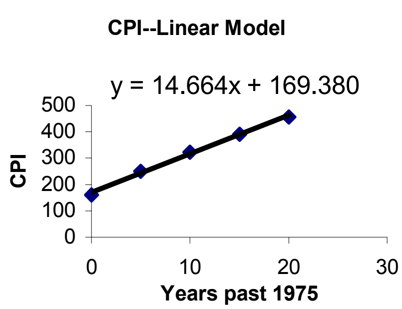
iii. $f(11)=0.013(11)^2-0.2225(11)+7.2974$

Using the TI-83 and substituting into the unrounded quadratic model:

```
11→X
                11
.0129934924078X^
2+-.222537960954
45X+7.2973969631
236
     6.421691974
```

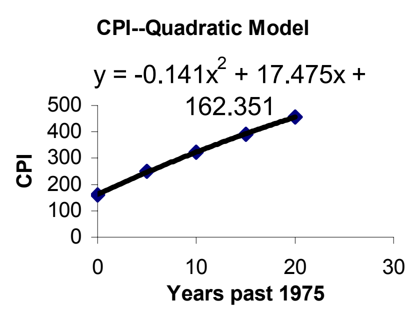
In 1997 the average hospital stay is 6.4 days.

72. a.



The model fits the data well.

b.

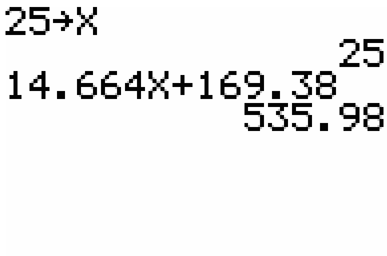


The model fits the data well. The quadratic model seems a little better than the linear model.

c. Using the linear model,

$y = 14.664(25) + 169.380$

Using the TI-83 and substituting into the unrounded linear model:

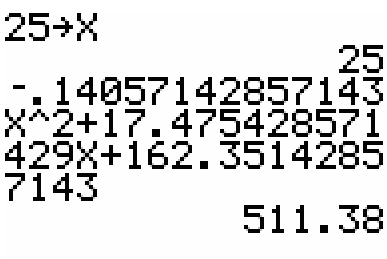


$y = 535.98$

Using the quadratic model,

$y = -0.141(25)^2 + 17.475(25) + 162.351$

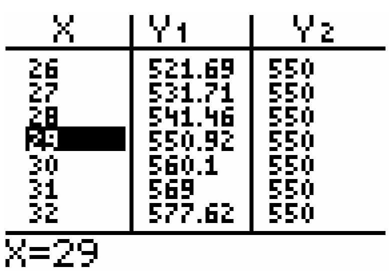
Using the TI-83 and substituting into the unrounded quadratic model:



$y = 511.38$

The quadratic model does a better job of predicting the CPI value for 2000, estimated to be 511.5.

d.



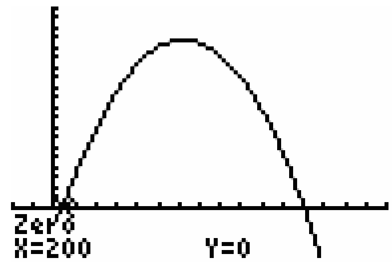
The CPI reaches 550 in 2004.

73. $P(x) = -0.01x^2 + 62x - 12,000$

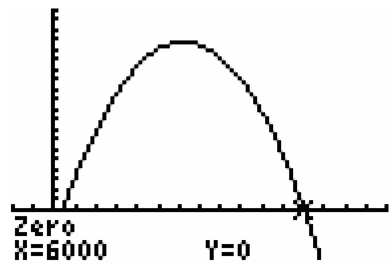
$P(x) > 0$

$-0.01x^2 + 62x - 12,000 > 0$

Applying the x-intercept method



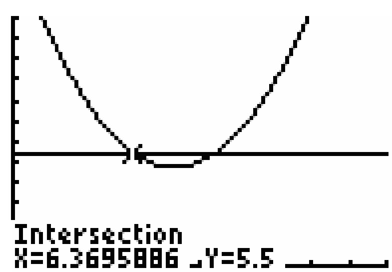
[–1000, 8000] by [–25,000, 100,000]



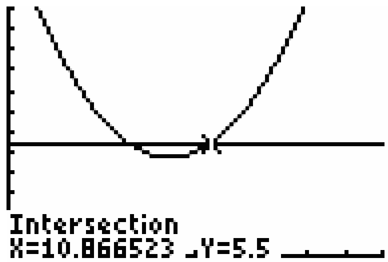
[–1000, 8000] by [–25,000, 100,000]

Manufacturing and producing between 200 and 6000 units will result in a profit.

74. Applying the intersection of graphs method



[0, 20] by [0, 12]

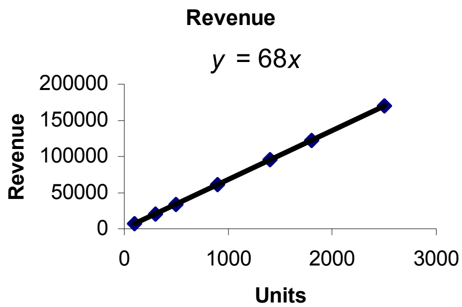


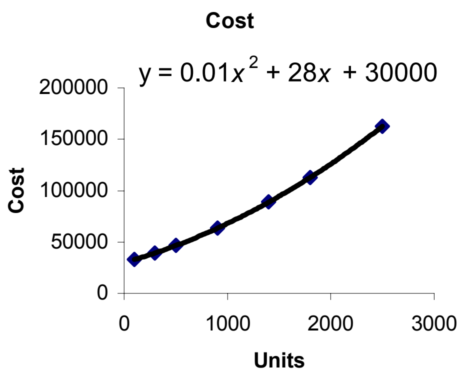
[0, 20] by [0, 12]

In the intervals $(-\infty, 6.37)$ and $(10.87, \infty)$, the unemployment rate is above 5.5%. That implies that prior to 1987 and after 1990 exclusive the unemployment rate is greater than 5.5%.

Group Activity/Extended Application I

1.





2. a. $P(x) = R(x) - C(x)$

$$= 68x - \left(0.01x^2 + 28x + 30{,}000\right)$$

$$= -0.01x^2 + 40x - 30{,}000$$

b.

0	–30,000
100	–26,100
600	–9600
1600	8400
2000	10,000
2500	7500

3. $R(x) = C(x)$

$$68x = 0.01x^2 + 28x + 30{,}000$$

$$0.01x^2 - 40x + 30{,}000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(0.01)(30{,}000)}}{2(0.01)}$$

$$x = \frac{40 \pm \sqrt{1600 - 1200}}{0.02}$$

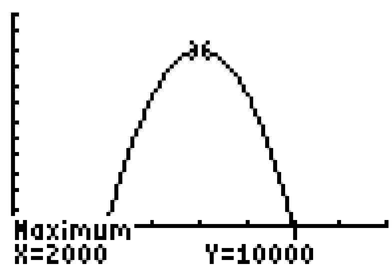
$$x = \frac{40 \pm \sqrt{400}}{0.02}$$

$$x = \frac{40 \pm 20}{0.02}$$

$$x = 3000, x = 1000$$

Producing and selling either 3000 or 1000 units forces the profit to equal the cost.

4.



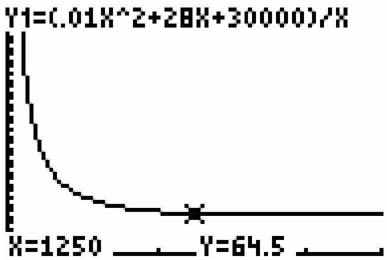
[0, 4000] by [–2000, 12,000]

Producing and selling 2000 units results in a maximum profit of $10,000.

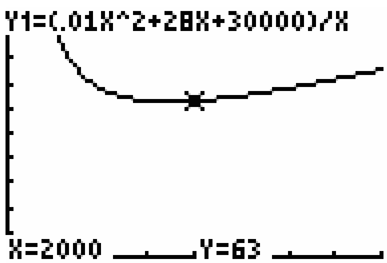
5. a. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{0.01x^2 + 28x + 30{,}000}{x}$

b.

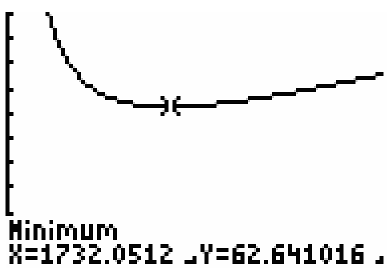
1	30,028.01
100	329.00
300	131.00
1400	63.43
2000	63.00
2500	65.00

c. 

[0, 2500] by [0, 400]

6. 

[0, 4000] by [0, 100]



[0, 4000] by [0, 100]

The minimum average cost is \$62.64 which occurs when 1732 units are produced and sold.

7. The values are different. However, they are relatively close together. The number of units that maximizes profit is most important. While keeping costs low is important, the key to a successful business is generating profit.